Fixed and Random Effects Models and Multistage Estimation Procedures for Statistical Population Reconstructions

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Abstract

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Age-at-harvest data are routinely collected as part of game-management programs. These data represent a wealth of information regarding demographic processes and trends in wildlife abundance. Use of wildlife age-at-harvest data has blossomed only relatively recently in the literature despite its frequent collection by game management agencies. Statistical models exist for such data, but are limited in their facility, owing to restrictive assumptions regarding constancy of demographic processes, unsuitability of models for the type of data collected, or computing difficulty in fitting models. Current models cannot accommodate the presence of process error (natural variation in demographic processes), or separate this error from sampling error (measurement error that is present whenever the full population cannot be sampled).

I develop and examine a set of statistical models for demographic processes using primarily age-at-harvest data that can be used to estimate survival probability, harvest vulnerability, and recruitment, as well as process error associated with these entities. I conduct thorough simulation studies of these models, and assess them with respect to their ability to accurately and precisely reconstruct abundance. Studies are conducted for fully-aged big game harvest, pooled age-class big-game harvest, and small-game harvest. Results indicate that a mixed-effects model which incorporates random effects in the processes of natural mortality and harvest probability, as well as a likelihood conditional on total cohort capture along with a Horvitz-Thompson abundance estimator outperform other models, and are recommended for use.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

GLOSSARY

AGE-AT-HARVEST DATA: In its simplest form, age-at-harvest data constitute a table of harvest counts for an animal, where the rows are typically the year in which the count was taken, and the columns indicate the age-class of the animal harvested.

DEMOGRAPHIC PROCESSES: A general term for processes that affect wildlife populations, and hence affect our ability to assess their status. Survival, harvest, and reproduction (productivity, or fecundity) are demographic processes.

MRB: Median relative bias (typically computed across simulations)

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Chapter 1

INTRODUCTION

Age-at-harvest data and accompanying hunter/harvest efficacy data are routinely collected for a variety of animal populations, typically as a means to monitor population trends. For populations subject to harvest, management agencies often collect such data directly from hunters shortly after harvest, or through surveys following the harvest season of the population in question. Fishery management agencies also collect such data as a means to assess the needs of a fishery with respect to performance and conservation.

The ability to draw meaningful conclusions regarding trends in animal abundance and/or population structure, efficacy of harvest methods, and subsequent implications of management strategies rests with the ability to extract information from such data sources. Harvestable populations are managed in order to balance the dual requirements of economic and ecologic considerations. Any reasonable management plan will allow the maintenance of a sustainable ecosystem along with the ability for future harvest to be conducted. The accurate and precise estimation of population parameters plays a vital role in protecting these interests, and hence is of key concern for those individuals and entities involved in these pursuits.

Any procedure which seeks to accurately and precisely estimate population parameters from age-at-harvest data must seek to provide the “best” estimate possible. Here, “best” refers to both the utility of resulting estimates and population descriptions, as well as the various statistical considerations regarding unbiasedness, robustness, and variance. Ideally, such a method would not rely on subjective input to the algorithm, lest it be accused of favoring one management concern over another. However, taking into account all possible information, such as prior and/or concurrent studies, is desired for efficient parameter estimation.

A complicating factor in the analysis of such data is the inherent natural variation that is present in every ecosystem. There exist two primary sources of variation in any population reconstruction which relies on age-at-harvest data: variation due to sampling error, and natural (environmental) stochasticity, the latter being a function of our lack of complete understanding regarding the intricate details of an ecosystem's operation. My research seeks to investigate methods of incorporating both sampling error and natural variation into population reconstruction models, determine the relative importance of the two error sources, and in so doing, develop more realistic and accurate assessment techniques. With management concerns in mind, I propose that it is useful to limit the number of assumptions involved in analysis as much as possible, such that any inference is as robust as possible. Out of similar concerns for wildlife management practice, attention will be given to the issue of obtaining adequate model fit such that reasonable projections, at least in the short-term, can be made. I will focus on the management of terrestrial wildlife populations and the unique circumstances and characteristics surrounding analysis of these populations.

A known issue with population reconstruction models is the level of parameterization relative to the available data (Gove, 1997). An overabundance of parameters not only creates an impractical and complicated model, but causes difficulty in model-fitting with numerical methods. Therefore, it is of interest to quantify the level of auxiliary data necessary to provide a "reasonable" degree of accuracy and precision in various types of models. For this purpose, one may need to pair the age-at-harvest data with various auxiliary data sources such as radiotelemetry and/or mark-recovery data to facilitate and enhance the estimation and analysis procedures. Reduced-parameter models will also be examined for their statistical population reconstruction potential.

1.1 Literature Review

Perhaps the first attempt at a reasonable analysis of age-harvest data is described by Quinn and Deriso (1999), who note that a close relative of age-at-harvest analysis (length-frequency analysis, or LFA) has been used since at least 1892 (Quinn and Deriso, 1999, pg. 295). From these humble beginnings, a large variety of techniques which seek to extract information on population dynamics have been developed. In the following, I focus on more modern developments, in effort to place into context the original work in this dissertation.

Quinn and Deriso (1999) describe efforts toward conducting an LFA based on mixture distributions for fish populations, wherein maximum likelihood is used to fit a mixture of probability density functions (pdfs) to length-frequency datasets such that a smooth frequency-by-length curve can be constructed, and population distribution among length classes (a surrogate for age classes) can be inferred. A number of shortcomings of this analysis method are described, including the need to choose a pdf or set of pdfs, and the choice of the number and width of age groups is arbitrary and often difficult. Additional assumptions can be imposed to make the analysis more tractable, but these are subject to their own limitations (Quinn and Deriso, 1999, pg. 297). Another limitation of this method is that it has no mechanism with which to account for interannual variability in a cohesive way, and thus estimation of error is limited to sampling error.

Dupont (1983) proposed a method where each individual is assumed to be susceptible to the competing risks of natural mortality (rate λ) and harvest mortality (rate μ), where a risk can be assigned the value of 0 at any given time. The hazards, as functions of time, are then modeled as

$$\begin{aligned} \lambda_i(t) &= \lambda l_i(t) e^{\mathbf{z}_1(t,i)'\boldsymbol{\beta}} \\ \mu_i(t) &= \lambda m_i(t) e^{\mathbf{z}_2(t,i)'\boldsymbol{\beta}} \end{aligned} \tag{1.1}$$

where i indexes an animal cohort and t is the index for time. Here, l_i and m_i are known functions (that can differ between animal cohorts) that describe the propensity for harvest (l_i) or natural death (m_i) in the time interval (t_j, t_{j+1}), and $\mathbf{z}_1(t,i)'\boldsymbol{\beta}$ and $\mathbf{z}_2(t,i)'\boldsymbol{\beta}$ are linear covariates and associated parameters.

The functions $\lambda_i(t)$ and $\mu_i(t)$ are used to describe the probability of mortality, p_{ij}. Using these p_{ij}, a product-multinomial likelihood can be constructed wherein each cohort is presumed to be multinomially distributed with initial cohort abundance as an additional parameter. Note that initial abundance N_{ij} of each cohort represents both initial population age structure (for the first year of data recorded) and amount of yearly recruitment for cohorts not initially present.

Laake (1992) provides a thorough review of Dupont's model, and also provides extensions to accommodate multiple harvesting methods (such as both rifle and archery hunting) and incomplete classification of animal age (such as grouping older animals into an age $A+$ category). In the work of both Dupont (1983) and Laake (1992), the hazard and natural mortality functions are implemented as step functions, typically with constant natural mortality rate (although it need not be constant).

Laake (1992) also provides a simulation study of such models and examines percent relative bias, coefficient of variation, and confidence interval coverage under various scenarios. Unsurprisingly, results indicate that with relatively higher sample size (either through high average capture probability, high initial cohort size, or high number of harvest years), the maximum likelihood estimates (MLEs) of model parameters generally grew increasingly precise.

Gove et al. (2002) proposed optimizing a joint maximum likelihood function containing components for multinomial age-at-harvest data, an auxiliary data source such as radiotelemetry data, and possibly a reporting likelihood that supplies information on the rates of reporting compliance. This final component may be omitted if reporting compliance is expected to be near 100%.

For this model, the age-at-harvest likelihood for a single cohort can be expressed as

$$\begin{aligned} &L_A(N_{11}, \vec{p}, \vec{R}, s \mid \vec{x}_A) = \\ &\binom{N_{11}}{x_{11}, x_{22}, x_{33}} (p_1R_1)^{x_{11}}((1-p_1)sp_2R_2)^{x_{22}}((1-p_1)s^2(1-p_2)p_3R_3)^{x_{33}} \times \\ &\qquad (1-(p_1R_1+(1-p_1)sp_2R_2+(1-p_1)s^2(1-p_2)p_3R_3))^{N_{11}-\sum_{i=1}^{3} x_{ii}}, \end{aligned} \tag{1.2}$$

where the x_{ij} constitute the harvest data for year i in age class j, p_i describes the probability of harvest in year i, R_i is the probability a harvested animal was reported in year i, and s is the probability of survival from year $i-1$ to i. Note that I've only expressed the likelihood for one cohort and three years of data, and survival is assumed constant for each year. The joint likelihood for all age-at-harvest data is simply the product of the likelihoods for each cohort present in the population

$$L(\vec{N}, \vec{p}, \vec{R}, \vec{s} \mid \boldsymbol{X}) = \prod_{k=1}^{A+Y-1} L_k(\vec{N}, \vec{p}, \vec{R}, \vec{s} \mid \vec{x}_k) \tag{1.3}$$

As multinomial age-at-harvest data do not provide enough minimum sufficient statistics to estimate the parameters involved (Gove et al., 2002), auxiliary data must be provided to facilitate estimation. Gove et al. (2002) used six years of radiotelemetry data for their study of an elk population in northern Idaho, where the random variables corresponding to animals that are harvested (u_i) and those that die from other causes (v_i) are incorporated into the multinomial likelihood

$$L_{auxiliary}(\vec{p}, s \mid \vec{n}, \vec{u}, \vec{v}) = \prod_{i=1}^{6} \begin{pmatrix} n_i \\ u_i, v_i \end{pmatrix} p_i^{u_i}((1-p_i)(1-s))^{v_i}((1-p_i)s)^{n_i-u_i-v_i}. \tag{1.4}$$

Once parameter estimates are obtained via numerical optimization techniques, the population can be "reconstructed" such that estimates of abundance in each age class at each time point can be obtained. The initial abundance values and annual recruitment are directly estimated as parameters, and subsequent abundance values for each cohort can be obtained by computing

$$\widehat{N}_{ij} = \widehat{N}_{i-1,j-1}(1-\widehat{p}_{i-1})\widehat{s}_{i-1}. \tag{1.5}$$

A class of models is available to perform population reconstruction in this manner. One can specify models where survival probability or harvest probability is shared between some age classes or years, or models where independent probabilities of harvest and survival can exist. Clearly, the more parameters one chooses to include in the model, the greater are the auxiliary data needs. A likelihood ratio test can be used to compare nested models to one another in order to determine the "best" such model to fit the observed data, and the Akaike information criterion (AIC) (Akaike, 1974) can be used for model selection.

Skalski et al. (2007) extended the model framework of Gove et al. (2002) to include hunter effort data in order to describe and better estimate harvest probability. In this model formulation, the capture probability is expressed as (Seber, 1982)

$$p_{ij} = 1 - e^{-c_j f_i} \tag{1.6}$$

where f_i is a measure of hunting effort exerted on the population, such as hunter-days, and c_j is a vulnerability parameter, and is permitted to either vary by age class or be fixed among particular age classes (e.g. $c_{juvenile}$ and c_{adult}).

In the absence of auxiliary data sources such as mark-harvest, mark-recapture, line-transect methods, or other methods that can provide additional information to aid parameter estimation, an auxiliary data source can be derived from annual total cohort abundance, wherein individual harvest observations among years are assumed to be binomially-distributed from the total abundance among age classes within a year. The auxiliary likelihood is expressed as

$$L_{Auxiliary} = \prod_{i=1}^{Y} \begin{pmatrix} \sum_{j=1}^{A} N_{ij} \\ x_i \end{pmatrix} (1 - e^{-cf_i})^{x_i} (e^{-cf_i})^{\sum_{j=1}^{A} N_{ij} - x_i}. \tag{1.7}$$

Note that expression (1.7) has been simplified to only incorporate a single vulnerability coefficient for purposes of a concise presentation, although additional coefficients are possible, and might account for changes in hunting regulations or age selectivity in hunting practices (Skalski et al., 2007).

Product-multinomial models of the type described above are notably lacking a feature to accommmodate natural stochastic fluctuation in the processes of survival, harvest, and reproduction (often termed "process error"). Such models can describe only sampling error, the result of the inability to sample an entire population, which is manifest in parameter estimate standard error. In contrast, some age- or length-harvest models often used in the management of fisheries (such as Virtual Population Analysis, or VPA) assume that sampling error is nonexistent, and that harvest is known precisely (Quinn and Deriso, 1999, pg. 323). The result of this is that all variation is attributed to natural fluctuations in the population.

More recently, Conn (2007) developed a Bayesian approach to the population reconstruction problem. In this model, abundance in years following the initial year is expressed as a conditional likelihood. That is, the joint probability mass function (pmf) of year 2 abundance is written as

$$\begin{aligned} &L(\boldsymbol{N_{2}.} \mid \boldsymbol{N_1}.,\ \boldsymbol{S},\ \boldsymbol{C}) = \\ &P(N_{22} = n_{22} \mid N_{11} = n_{11}, S_{11} = s_{11}) \cdots P(N_{2,A+1} = n_{2,A+1} \mid N_{1A} = n_{1A},\ S_{1A} = s_{1A}) \times \\ &P(N_{21} = n_{21} \mid \boldsymbol{N_1}.,\ \boldsymbol{C}) \end{aligned} \tag{1.8}$$

where $\boldsymbol{C}$ represents the age-at-harvest data matrix, $\boldsymbol{S}$ represents the matrix of age and year-specific survival probabilities, and $\boldsymbol{N_{2}.}$ represents the vector of age-specific population sizes in year 2 immediately prior to harvest. The remaining cohorts of age-at-harvest data follow the same pattern; the distribution of abundance in each year is conditional on the abundance in the preceding year, and the vulnerability and survival parameters.

Under assumptions of a binomial sampling model and Poisson recruitment, Conn (2007) writes the age-at-harvest likelihood as

$$\begin{aligned} L(\boldsymbol{S},\ \boldsymbol{r},\ \boldsymbol{f},\ \boldsymbol{N},\ \boldsymbol{D} \mid \boldsymbol{C}) &= \prod_{i=1}^{Y}\prod_{j=1}^{A} \text{Binomial}(D_{ij}; N_{ij}, 1 - S_{ij}) \\ &\times \prod_{i=1}^{Y-1} \text{Poisson}(N_{i+1,1}; f_{i1}N_{i1} + f_{i2}N_{i2} + \cdots + f_{iA}N_{iA} + f_{iA}N_{i,A+1}) \\ &\times \prod_{i=1}^{Y}\prod_{j=1}^{A} \text{Binomial}(c_{ij}; D_{ij}, r_{ij}) \end{aligned} \tag{1.9}$$

where S and N_{ij} are as above, f_{ij} are fecundity parameters, and r_{ij} is a year and cohort-specific reporting probability parameter, and

$$D_{ij} = \begin{cases} N_{ij} - N_{i+1,j+1} & j < A \\ N_{i,A-1} + N_{iA} - D_{i,A-1} - N_{i+1,j} & j = A \end{cases} \tag{1.10}$$

Prior distributions are specified for all $N_{1j}, j = 1, \ldots, A$, and for each S_{ij} and r_{ij}. If instead, one wishes to model a set of parameters as a random effect, such as

$$logit(S_i) = \beta_0 + \epsilon_i \tag{1.11}$$

where $\epsilon_i \sim \text{Normal}(0, \sigma^2)$, then one can avoid fitting an overabundance of survival parameters, provided the necessary assumptions for inclusion of such a construct are satisfied. Model-fitting then proceeds using Markov-chain Monte Carlo (MCMC) methods, and posterior distributions for each parameter are obtained, from which inference may proceed. Models with additional or reduced parameters can be compared using deviance information criteria (DIC) or Bayesian information criteria (BIC).

Conn et al. (2009) performed a simulation study of the Bayesian methods described above, wherein the authors examine large-sample performance with respect to criteria such as Bayesian credible interval coverage, percent relative bias, and coefficient of variation. Results indicate that with long Markov chains, Bayesian analysis resulted in estimators with low bias (absolute percent relative bias less than 20% for parameter value combinations considered) and high precision, as illustrated by near-nominal Bayesian credible interval coverage. Additional simulation results indicate robustness of these techniques to aging errors when aging is assumed to be a random effect, and poor estimation of uncertainty when age-at-harvest data and auxiliary mark-recapture data are correlated (share members of the population).

Except for the most recent work by Conn et al. (2009), wildlife population models and data analysis techniques have not been available to examine natural variability in population-level processes in the presence of sampling error. For that reason, most models (Dupont, 1983; Laake, 1992; Gove et al., 2002; Skalski et al., 2007) assume that the only source of error in model fit is error in the sampling (harvest) process. The Bayesian techniques of Conn et al. (2008, 2009); Conn (2007) rely on a series of prior distributions for implementation, and are subject to the usual difficulties in Bayesian analysis and inference based on MCMC-produced posterior distributions, such as the form of prior distributions and convergence of the sampling process, as well as the time required for the MCMC sampling. In addition, highly correlated parameters (such as natural mortality and harvest mortality) lead to difficulty in MCMC techniques which are not well-understood by wildlife management practitioners, and require a great degree of facility with statistical techniques and Bayesian methods for implementation. Thus, there is a need to further develop and assess statistical models for wildlife populations that can accommodate and describe both measurement error and environmental stochasticity in a frequentist framework.

No previous work has assessed the performance of models for age-at-harvest data typically found in the practice of wildlife management (many years of data, many age classes). In addition, robustness of parameter estimation techniques to deviations from model assumptions must be examined.

The inclusion of a stock-recruitment relationship would facilitate the ability for a model to predict future abundance, at least in the short-term. The ability to predict abundance or trend in abundance (along with assessments of accuracy and precision) is desirable for managers of wildlife populations.

It is also of interest to determine the level of auxiliary data required in order to conduct "successful" analysis with respect to accuracy and precision of estimates, as this also has implications for wildlife population managers.

1.2 Brief Model Description, Significance and Impact

Briefly, the goal of this work is to extend the product-multinomial model formulation of Gove et al. (2002) and Skalski et al. (2007) to

1. Provide the ability to use environmental covariates to aid in the description of demographic processes

2. Provide the ability to easily model demographic processes such as harvest and natural mortality as random effects thereby reducing the parameter space and providing for estimates of process variation and realistic modeling scenarios

3. Include a simple stock-recruit relationship that can help wildlife managers estimate the rate of reproduction

4. Use the stock-recruit relationship to assess the ability of these models to predict future abundance.

5. Examine reduced-parameter models with a second-stage abundance estimation technique for its potential as a replacement for existing model formulations

Each of these pieces provides a signficant extension to existing modeling capability for age-at-harvest data of wildlife populations. Successful model extensions of this nature can greatly enhance the ability to use the easily-collected age-at-harvest data to estimate parameters related to animal demographic processes, and to estimate animal abundance and trends in abundance.

The ability to accurately and precisely model game populations with data collected in an ongoing fashion, with a minimal amount of auxiliary pre-planned population assessment studies (such as mark-recapture, mark-recovery, and radiotelemetry data) can increase the wildlife manager's ability to understand population status, factors affecting population fluctuations, magnitude of population fluctuations, and forecasting of future abundance. In addition, wildlife biologists stand to gain demographic information from statistical population reconstruction that may not otherwise be available, and policymakers will gain strength in evidence upon which to base decisions affecting the population under study.

In the following, I consider the simple stock-recruit relationship of multiplicative fecundity based on the abundance of breeding-age females. More sophisticated stock-recruit relationships are available, such as those used in the management of fisheries. However, for wildlife populations, data are often more sparse than in fisheries stock assessment, and the process of reproduction and its relation to the environment may be poorly-understood. The intent of the inclusion of a simple stock-recruit relationship is to assess the ability of this simple component to reduce the number of parameters that need to be estimated (individual absolute annual abundance amounts), and to predict future abundance. It is understood that a variety of factors affect the reproduction rate of individual animals, and mechanisms to accommodate these additional factors will be provided in a regression-type framework, just as the processes of survival and harvest will include facilities for covariate parameter estimation.

Success of the work contained herein will enable wildlife managers and conservationists to model many demographic processes from age-at-harvest data, and providing a means to obtain detailed demographic information regarding population status and viability. These new models and techniques will provide meaningful advances in the ability to understand and manage game populations.

Chapter 2

MODEL PERFORMANCE WITH FULLY-AGED HARVEST DATA: BIG GAME STUDY

2.1 Introduction

Within game management agencies, it is not uncommon for many years of age-at-harvest data to be available. Harvests of some terrestrial animals are often able to be aged quite precisely, and thus many different age classes can be distinguished. For data of this nature, one typically requires many years of harvest data, particularly for those models where each absolute recruitment amount (each N_{i1}) is estimated independently, and for populations where harvest vulnerability and/or natural mortality are thought to differ among age classes, providing for an increase in the dimension of the parameter space.

A series of models, including some with their lineage in the models of Gove et al. (2002) along with newly-developed models will be fitted to a variety of simulated data intended to represent harvest of a large game animal. These models and their shorthand notation, represent assumptions of both constant (fixed) and random (annually-fluctuating) demographic processes, as well as the different methods of accounting for abundance: in conjunction with a stock-recruit relationship, direct estimates, or with a second-stage Horvitz-Thompson abundance estimator (Table 2.2). Following the model formulation and description (next section), a simulation study will be conducted among this field of candidate models to ascertain which of these has the greatest capability to accurately and precisely estimate abundance and process parameters.

2.2 Detailed Model Formulation and Description

The basic form of the models I examine begins with a product-multinomial formulation, wherein observed harvest data are assigned probabilities of occurrence (to be estimated as parameters), based on the age of the animal, and the year in which the animal was harvested (Gove et al., 2002). Consider an age-at-harvest matrix, such as that in Table (2.1) below. Here, each x_{ij}

represents the observed animal harvest in year i and age class j, the N_{1j} are initial abundance parameters to be estimated, and the N_{i1} are recruitment parameters (total annual recruitment), which also require estimation.

Table 2.1: *Example age-at-harvest data: Three years of data and three age classes, with a single diagonal cohort shaded.*

N_{11}	N_{12}	N_{13}	
N_{21}	x_{11}	x_{12}	x_{13}
N_{31}	x_{21}	x_{22}	x_{23}
	x_{31}	x_{32}	x_{33}

For an individual cohort, such as that which was recruited (age class [column] $j = 1$) in year (row) $i = 1$, the probability of observing the harvest vector $\vec{x} = (x_{11}, x_{22}, \ldots, x_{YA})$ for years $i = 1, \ldots, Y = 3$ and age classes $j = 1, \ldots, A = 3$ is written as

$$\begin{aligned} &L_A(N_{11}, c, s \mid \vec{x}_A) = \\ &\binom{N_{11}}{x_{11}, x_{22}, x_{33}} (p_1)^{x_{11}} ((1-p_1)sp_2)^{x_{22}} ((1-p_1)(1-p_2)s^2 p_3)^{x_{33}} \times \\ &(1-(p_1 + (1-p_1)sp_2 + (1-p_1)(1-p_2)s^2 p_3))^{N_{11} - \sum_{i=1}^{3} x_{ii}}, \end{aligned} \tag{2.1}$$

where each p and s (herein termed “process parameters”) are harvest probability and survival parameters, respectively, that require estimation. Note that harvest probability p has been parameterized as

$$p_i = 1 - e^{-cf_i}$$

for harvest effort data $\vec{f}$ as in Skalski et al. (2007).

The assumption of independence of fates of individual animals is necessary for the multinomial formulation. Additionally, I have assumed that there is no immigration to or emigration from the population, and we thus have a closed population. We may consider permanent emigration to be included in the natural mortality probability $1-s$, but as age-harvest data provide no means of assessing immigration, I do not explicitly consider this here. In addition, I have presumed that some precise measurement of effort (such as number of hunter-days observed, or number of tags sold) is available. This may not always be the case, and modifications may be necessary to accommodate an estimate of effort, such as from post-season mail or telephone surveys.

Of course, it is not always reasonable to assume that survival in every year is the same as that in every other year. Simply adding a subscript for year i to each process parameter indicates a separate such parameter for each year. With this modification, we obtain

$$\begin{aligned} &L_A(N_{11}, c, \vec{s} \mid \vec{x}_A) = \\ &\binom{N_{11}}{x_{11}, x_{22}, x_{33}} (p_1)^{x_{11}}((1-p_1)s_1p_2)^{x_{22}}((1-p_1)s_1s_2(1-p_2)p_3)^{x_{33}} \times \\ &(1-(p_1+(1-p_1)s_1p_2+(1-p_1)s_1s_2(1-p_2)p_3))^{N_{11}-\sum_{i=1}^{3} x_{ii}}. \end{aligned} \tag{2.2}$$

Similarly, a subscript for age class j could be added to each process parameter to permit separate harvest and survival probabilities for each age class. As mentioned previously, any combination of reasonable dimensions of $\vec{s}$ and any number of catch coefficients c can be used, provided requirements on amount of available data are met (such as $s_{juvenile}$ and s_{adult}), and parameters are identifiable in the likelihood equation.

In accordance with standard likelihood theory, and under the assumption of independence of fates of all animals, the likelihood function for the entire age-at-harvest matrix (probability of observation of the data) is simply the product of the likelihood of each individual cohort. For $\boldsymbol{X}$, the small example matrix of age-at-harvest data shown in Table (2.1), we have

$$L(N_{1,1}, \ldots, N_{1,A}, N_{2,1}, \ldots, N_{Y,1}, c, \vec{s} \mid \boldsymbol{X}) = \prod_{k=1}^{A+Y-1} L_k(N_k, c, \vec{s} \mid \vec{x}_k) \tag{2.3}$$

where k indexes the cohort number.

I first make two simple and convenient modifications to this standard model. In order to provide for the ability to include environmental covariates thought to influence the demographic processes, I parameterize each s_i and p_i as

$$s_i = \frac{1}{1 + e^{-(\beta)}} \tag{2.4}$$

and

$$p_i = 1 - e^{-(e^c) f_i} \tag{2.5}$$

such that an exponentiated regression formulation can be used, and covariates $\vec{z} = (z_1, \ldots, z_p)$ along with corresponding regression parameters $\vec{\lambda} = (\lambda_1, \ldots, \lambda_p)$ can be incorporated as

$$s_i = \frac{1}{1 + e^{-(\beta + \lambda_1 z_1 + \cdots + \lambda_p z_p)}} \tag{2.6}$$

and

$$p_i = 1 - e^{-(e^{c + \lambda_1 z_1 + \cdots + \lambda_p z_p}) f_i}. \tag{2.7}$$

in order to provide a mechanism with which to separate environmental influences from interannual or inter-age-class variation and sampling error. These transformations also aid the ability to numerically fit nonlinear models of this fashion by bounding the probability parameters within the interval $[0, 1]$, and maintain that if $f_i = 0$ then $p_i = 0$ (no harvest effort results in no harvest probability).

A notable absence from the standard models used in wildlife for age-at-harvest data is a facility for simultaneous estimation and description of the recruitment process. Instead of attempting to describe the combined processes of recruitment and recruit harvest generating observed harvest data, the traditional models have opted to estimate each year's recruitment individually, rather than attempting to obtain some relationship to prior abundance. This is largely due to the fact that traditional wildlife studies have lacked the necessary data regarding the structure and form of a stock-recruit relationship that would enable its examination. Of course with different wildlife populations, there are challenges in defining the recruitment process, and environmental factors may affect the recruitment process to a different degree than in fisheries. What may be reasonable for a large game population (such as relatively constant recruitment, susceptible only to the most difficult of weather conditions) may not be reasonable at all for small game populations, which may be highly susceptible to factors such as spring rainfall and temperature. For example, Gilbert and Raedeke (2004) found that a population of black-tailed deer experienced recruitment rates ranging from 0.56 fawns per breeding-age female to 1.27 over a span of 20 years. The authors found the annual recruitment rates to be related to weather factors such as mean minimum monthly temperature in May, and days of precipitation in June. In contrast Mauser and Jarvis (1994) estimated mallard recruitment rates of 0.31 and 1.26 in successive years.

Despite these difficulties, there is inherent value in creating a method for such assessment. The ability to gain even a rough understanding of the recruitment process and how it may or may not be changing over time can prove an integral piece of a population assessment, and is worthy of consideration here.

In order to include an assessment of the recruitment process, I make the following modifications to the model. I first consider a simple, rudimentary (yet intuitive) recruitment process wherein the number of recruits in year i, N_{i1}, is linearly dependent on the number of breeding-age adults in year $i-1$. That is

$$N_{i1} = r \sum_{j=b}^{A} N_{i-1,j}, \tag{2.8}$$

where r is a recruitment process parameter to be estimated, and b is the youngest breeding age for the animal in question. The parameter r then represents a sort of "average" recruitment rate for breeding-age adults. Of course, multiple r_j's could be used to describe differing recruitment levels for different ages or age classes of adults. Additionally, if the recruitment process is thought to be non-stationary, one could include a subscript for i to capture a recruitment process that is different among years. Of course, one must not include too many such parameters, lest one return to a model formulation equivalent to the traditional model, wherein absolute recruitment is estimated for each individual year (although there may be some advantage to doing so, at least intially, in order to garner some information about the stock-recruit relationship that is not available when estimating annual total recruitment). Care must be taken when formulating this relationship, such that it is representative of current knowledge of the recruitment process of the species in question. For instance, it may be more reasonable for the recruitment relationship to be a function of females only, or a weighted combination of males and females in the population. Such a construct would require additional information on the population, such as sex ratios. If a constant sex ratio is expected, then the relationship already described may be sufficient.

For this simple recruitment process, I also employ a transformation to facilitate inclusion of environmental covariates, and to aid in numerical optimization techniques. I reparameterize

$$r = e^{\gamma+\lambda_1 z_1+\cdots+\lambda_p z_p} \tag{2.9}$$

to constrain r to be nonnegative during estimation, and allow it to fluctuate according to environmental influences appropriate to the animal being studied. Here, γ has replaced r as the parameter which describes the recruitment process (without covariates, $r = e^{\gamma}$). The regression parameters $\vec{\lambda}$ are not necessarily the same as those considered previously for the survival and harvest processes.

Of course, it is widely held that density-dependent effects may obscure the simple relationship between stock and recruitment as in Equations (2.8) and (2.9). Rather than adding additional structure to the model in the form of different stock-recruit curves such as the Beverton-Holt or Ricker curves (Quinn and Deriso, 1999, pg. 86 - 99), we can instead accommodate density dependence through the covariate structure as in Equation (2.9). Covariates relating to the prior abundance (or functions of the prior abundance, such as polynomial terms) may be included to make recruitment rate (in addition to absolute recruitment total) in year i explicitly depend on breeder abundance in year $i-1$. Thus, the recruitment relationship may be parameterized as

$$r_i = e^{\gamma+\lambda_1 \sum_{j=b}^{A} N_{i-1,j}}, \tag{2.10}$$

and year i absolute recruit abundance would be computed as

$$N_{i1} = \left(e^{\gamma+\lambda_1 \sum_{j=b}^{A} N_{i-1,j}}\right) \sum_{j=b}^{A} N_{i-1,j}. \tag{2.11}$$

Note that the interpretation of the recruitment rate parameter γ is altered in this formulation. In practice, model formulation would be guided by biological information and rough knowledge of abundance relative to carrying capacity. Model selection procedures, which will be discussed in a later section, would then be used to determine the most appropriate formulation.

Wildlife demographers and population managers also have interest in the degree to which natural variation affects the demographic processes of harvested populations, separate from those environmental influences which we have accommodated via reparameterization above. Imposing additional model structure, as described below, will permit us to simultaneously reduce the parameter space (remove the need for many annual parameters for s_i, p_i, and r_i), and also make the model more realistic. In addition, the following modifications will permit some assessment of the magnitude of interannual fluctuations in demographic processes. In order to modify the model, I reconsider the reparameterizations used previously, and add a random error term. I reparameterize the survival process as

$$s_i = \frac{1}{1 + e^{-(\beta + \epsilon_i)}} \tag{2.12}$$

or, with covariates, as

$$s_i = \frac{1}{1 + e^{-(\beta + \lambda_1 z_1 + \cdots + \lambda_p z_p + \epsilon_i)}}, \tag{2.13}$$

where

$$\epsilon_i \sim N(0, \sigma^2_\beta). \tag{2.14}$$

Here, I have removed the need to have different s_i parameters for each year by considering the fluctuations to be random and normally distributed, following a logistic transformation. We have added the parameter σ^2_β, which will produce our estimate of the interannual variation in the survival process. Note that for σ^2_β to truly be an estimate of natural variability, we must account for all other known sources of variation. If we fail to do so, these other sources of variation will be "lumped in" with σ^2_β during estimation.

If there is reason to suspect natural fluctuations (not explainable by covariates alone) in the harvest and recruitment processes, similar modifications can be made for these processes as well. We can reparameterize with

$$p_i = 1 - e^{-e^{(c+\tau_i+\lambda_1 z_1+\cdots+\lambda_p z_p)} f_i} \tag{2.15}$$

and

$$r = e^{\gamma+\delta_i+\lambda_1 z_1+\cdots+\lambda_p z_p} \tag{2.16}$$

where

$$\tau_i \sim N(0, \sigma_c^2) \text{ and } \delta_i \sim N(0, \sigma_r^2) \tag{2.17}$$

assume the role of the random effects, with separate parameters corresponding to interannual variation, which require estimation. Note that recruitment is assumed to be log-normally distributed.

Clearly, a great number of formulations are possible, and random effects $\vec{\epsilon}$, $\vec{\tau}$, and $\vec{\delta}$ need not be normally-distributed. One alternative scenario might be to consider a demographic process such as recruitment as relatively constant, except for large negative disturbances (again, which cannot be explained by environmental covariates alone). One might then hypothesize the magnitude of these random, large, negative deviations to have some Gamma (α, β) distribution, or some other distribution allowing for skewness.

In the absence of harvest, one might typically consider factors such as abundance, survival, and recruitment rate to have some dependence on previous values. That is, an autoregressive process wherein deviations from mean annual abundance are correlated with their lagged counterparts from prior years may be employed to account for correlation between years. If, for example, recruitment rate in year i was thought to be dependent on recruitment rate in year $i-1$ through the relationship

$$\delta_{i+1} = \rho\delta_i + \eta_i \tag{2.18}$$

such that we may iteratively compute

$$\delta_i = \sum_{j=0}^{\infty} \rho^j \eta_{i-j} \tag{2.19}$$

where η_i takes the place of δ_i in Equation (2.17) (independent, normally-distributed, with constant variance). It is well known (Kutner et al., 2005, pg. 484-487) that Equation (2.18) induces the autoregressive correlation structure. We may then represent the variance-covariance matrix for the vector $\vec{\delta}$ as

$$\mathbf{\Sigma}_{\vec{\delta}} = \frac{\sigma_\eta^2}{1-\rho^2} \begin{bmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{Y-2} \\ \rho & 1 & \rho & \ddots & \vdots \\ \rho^2 & \rho & \ddots & \rho & \rho^2 \\ \vdots & \ddots & \rho & 1 & \rho \\ \rho^{Y-2} & \cdots & \rho^2 & \rho & 1 \end{bmatrix}_{(Y-1)\times(Y-1)} \tag{2.20}$$

where ρ is an additional parameter requiring estimation.

Such a process may be considered in the context of statistical population reconstruction models as well. Autocorrelation of random effects for survival and reproduction may be obscured by the presence of harvest and, for extrinsically-controlled species, by the presence of severe weather factors. If, however, an autoregressive nature is exhibited in one or more processes, uncertainty estimation would suffer from its omission in the modeling framework.

One may also consider random effects from different processes to be correlated with one another. For instance, reproduction in year i may be dependent on the survival rate from year $i-1$ to i through temporally-related factors not accounted-for in the covariate structure. In this situation, one may hypothesize that

$$\begin{bmatrix} \epsilon_{i-1} \\ \delta_i \end{bmatrix} \sim MVN\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\beta^2 & \sigma_{\beta\gamma} \\ \sigma_{\beta\gamma} & \sigma_\gamma^2 \end{bmatrix} \right) \tag{2.21}$$

where the covariance parameter $\sigma_{\beta\gamma}$ provides an extra parameter to be estimated. This type of dependency between demographic processes may be obfuscated by density dependent effects, and it is unlikely that data quantity and quality will support both a density-dependent covariate structure or stock-recruit formulation in addition to autoregressive behavior of the stochastic components of demographic processes. Care should be exercised when investigating these possibilities.

Within this document, I do not consider autoregressive demographic process behavior or density dependence, as my primary interest lies in establishing the base-case validity of a new set of statistical population reconstruction models as well as the addition of variance components to their structure. Future work may merit investigation of nontrivial correlation structures for random effects, such as the autoregressive structure or correlated random effects from different demographic processes.

A complete mathematical description of the most complex of the potential models described above (without the use of environmental covariates and with independent normally-distributed random effects), for a single cohort present upon initiation of data collection, is given by

$$
\begin{aligned}
&L_A(N_{11}, \beta, c, \vec{\epsilon}, \vec{\tau}, \sigma_\beta, \sigma_c \mid \vec{x}_A) = \\
&\binom{N_{11}}{x_{11}, x_{22}, x_{33}} (p_1)^{x_{11}} \left((1-p_1)\, s_1 p_2\right)^{x_{22}} \left((1-p_1)\, s_1 \left(1-p_2\right) s_2 p_3\right)^{x_{33}} \times \\
&\left(1 - \left(p_1 + (1-p_1)\, s_1 p_2 + (1-p_1)\, s_1 \left(1-p_2\right) s_2 p_3\right)\right)^{N_{11} - \sum_{i=1}^{3} x_{ii}} \times \\
&\left[\prod_{i=1}^{3} \phi_{\sigma_\tau}\left(\tau_i\right)\right] \left[\prod_{i=1}^{2} \phi_{\sigma_\beta}\left(\epsilon_i\right)\right]
\end{aligned}
\tag{2.22}
$$

where

$$
\begin{aligned}
p_1 &= 1 - e^{-e^{(c+\tau_1)} f_1}, \\
p_2 &= 1 - e^{-e^{(c+\tau_2)} f_2}, \\
p_3 &= 1 - e^{-e^{(c+\tau_3)} f_3}, \\
s_1 &= \frac{1}{1 + e^{-(\beta+\epsilon_1)}}, \\
s_2 &= \frac{1}{1 + e^{-(\beta+\epsilon_2)}}, \\
\epsilon_i &\sim N\left(0, \sigma_\beta^2\right), \\
\tau_i &\sim N\left(0, \sigma_c^2\right),
\end{aligned}
\tag{2.23}
$$

and where $\phi_\sigma(x)$ is the normal distribution with location 0 and scale parameter σ.

Beginning with the oldest age class in year 1, one may enumerate the cohorts in chronological order according to age. The likelihood presented in Equation (2.23), then, corresponds to cohort A. The next cohort, $A+1$, (the first to implement the optional stock-recruitment relationship) is expressed without covariates, as

$$\begin{aligned}
&L_{A+1}(N_{12}, \beta, c, \gamma, \vec{\epsilon}, \vec{\tau}, \vec{\delta}, \sigma_\beta, \sigma_c, \sigma_\gamma \mid \vec{x}_{A+1}) = \\
&\begin{pmatrix} \left(e^{\gamma+\delta_1}\right) \sum_{j=b}^{A} N_{1j} \\ x_{21}, x_{32}, x_{43} \end{pmatrix} p_2^{x_{12}} \left(\left(1-p_2\right) s_2 p_3\right)^{x_{23}} \left(\left(1-p_2\right) s_2 s_3 \left(1-p_3\right)\right) p_4^{x_{34}} \times \\
&\left(\left(1-\left(p_2+\left(1-p_2\right) s_2 p_3\right)+\left(1-p_2\right) s_2 s_3 \left(1-p_3\right)\right) p_4\right)^{N_{11}-\sum_{i=1}^{3} x_{i+1,i}} \times \\
&\left[\prod_{i=2}^{4} \phi_{\sigma_\tau}\left(\tau_i\right)\right] \left[\prod_{i=2}^{3} \phi_{\sigma_\beta}\left(\epsilon_i\right)\right] \left[\phi_{\sigma_\gamma}\left(\delta_1\right)\right]
\end{aligned} \tag{2.24}$$

where

$$\begin{aligned}
p_2 &= 1 - e^{-e^{(c+\tau_2)} f_2}, \\
p_3 &= 1 - e^{-e^{(c+\tau_3)} f_3}, \\
p_4 &= 1 - e^{-e^{(c+\tau_4)} f_4}, \\
s_2 &= \frac{1}{1+e^{-(\beta+\epsilon_2)}}, \\
s_3 &= \frac{1}{1+e^{-(\beta+\epsilon_3)}}, \\
\epsilon_i &\sim N\left(0, \sigma_\beta^2\right), \\
\tau_i &\sim N\left(0, \sigma_c^2\right), \text{ and} \\
\delta_i &\sim N\left(0, \sigma_\gamma^2\right).
\end{aligned} \tag{2.25}$$

Subsequent years follow in the same fashion as Equation (2.25), with indices incrementing with additional years of data. As before, the joint likelihood of each cohort is expressed as

$$L(\vec{N}, \beta, c, \gamma, \vec{\epsilon}, \vec{\tau}, \vec{\delta}, \sigma_\beta, \sigma_c, \sigma_\gamma \mid \boldsymbol{X}) = \prod_{k=1}^{A+Y-1} L_k(\vec{N}, \beta, c, \gamma, \vec{\epsilon}, \vec{\tau}, \vec{\delta}, \sigma_\beta, \sigma_c, \sigma_\gamma \mid \vec{x}_k). \tag{2.26}$$

The number of parameters in the model above (with a single β, a single c, a single γ, 3 variance parameters [σ_β, σ_c, and σ_γ], and A initial abundance parameters) is now $A+6$.

In mixed-effects models such as these, one may form a marginal likelihood by integrating over the random effect terms $(\vec{\epsilon}, \vec{\tau}, \vec{\delta})$ to eliminate them, and thus avoid their direct estimation in the likelihood. This marginal likelihood takes the form

$$\begin{aligned} &L(\vec{N}, \beta, c, \gamma, \sigma_\beta, \sigma_c, \sigma_\gamma \mid \boldsymbol{X}) = \\ &\quad \int_{\epsilon_1} \cdots \int_{\epsilon_{Y-1}} \int_{\tau_1} \cdots \int_{\tau_Y} \int_{\delta_1} \cdots \int_{\delta_{Y-1}} L(\vec{N}, \beta, c, \gamma, \vec{\epsilon}, \vec{\tau}, \vec{\delta}, \sigma_\beta, \sigma_c, \sigma_\gamma \mid \boldsymbol{X}) \; d\vec{\epsilon} \; d\vec{\tau} \; d\vec{\delta}, \end{aligned} \tag{2.27}$$

where I have written $\vec{N}$ to represent $N_{1,1}, N_{1,2}, \ldots, N_{1,A}$. Throughout, likelihoods of this form correspond to the first 4 models in Table (2.2). Additionally, the traditonal models of Gove et al. (2002) (which do not have a stock-recruit relationship, and rely on estimating each annual recruit abundance separately) may be modified such that the harvest and survival probabilities have random-effect terms. Therefore, we may modify the likelihood (1.3) via (2.13) and (2.15) and obtain the marginal (integrated) likelihood for $\boldsymbol{X}$ as

$$\begin{aligned} &L(\vec{N}, \beta, c, \sigma_\beta, \sigma_c \mid \boldsymbol{X}) = \\ &\qquad \int_{\epsilon_1} \cdots \int_{\epsilon_{Y-1}} \int_{\tau_1} \cdots \int_{\tau_Y} L(\vec{N}, \beta, c, \vec{\epsilon}, \vec{\tau}, \sigma_\beta, \sigma_c \mid \boldsymbol{X}) \; d\vec{\epsilon} \; d\vec{\tau}. \end{aligned} \tag{2.28}$$

Since these integrals are not available in closed form, numerical integration techniques such as quadrature or importance sampling might be used to approximate the marginal likelihood (Skaug and Fournier, 2006). For many random effect terms, many of which will be correlated (particularly those relating to harvest and natural mortality), this can be numerically unstable and difficult to evaluate numerically.

Instead, I employ the likelihood approximation technique known as the Laplace approximation (Skaug and Fournier, 2006), which is available in the AD Model Builder software (Fournier et al., 2011). This implementation is used to directly estimate parameters in Equation (2.27). The Laplace approximation is described in detail by Skaug and Fournier (2006), who also describe how automatic differentiation is used to compute the Hessian matrix, which is vital for implementation of the Laplace approximation as well as uncertainty estimation from likelihood models such as these.

Despite the reduction in the number of parameters required to fit the model as in Equations (2.27) and (2.28), there still remains a relatively high number of parameters with respect to the amount of available data, and numerical estimation can be difficult. Abundance parameters ($N_{1,1}, \ldots, N_{1,A}$) can be difficult to estimate as they require explicit constraints (cannot be less than total cohort harvest). Precise parameter estimates generally require a large amount of data relative to the number of parameters in the model. By reducing the number of parameters in the model, it is possible that more precise parameter estimates will be available with the same amount of data. It is therefore of interest to determine if these issues may be remedied by considering reduced-parameter models that allow for abundance estimation outside of the likelihood framework.

An alternative model formulation can be studied by considering the distribution of animal captures in a given cohort conditional on having been captured. That is, each "cell" in the multinomially-distributed vector of cohort harvest is parameterized by the probability of surviving to that age (in the given year) and being harvested in that year *conditional* on having been harvested at some time. The likelihood is formed by computing the probability of ever being harvested for an animal in a given cohort, and dividing the marginal probability in each cell by this amount, according to the standard conditional probability rule. In simple terms,

$$P(\text{age } j \text{ animal harvested in year } i \mid \text{animal was ever harvested}) =$$

$$\frac{P(\text{age } j \text{ animal harvested in year } i \textbf{ and } \text{animal was ever harvested})}{P(\text{animal was ever harvested})} =$$

$$\frac{P(\text{age } j \text{ animal harvested in year } i)}{P(\text{animal was ever harvested})} =$$

$$\frac{s^{i-1}(1-p)^{i-1}p}{p + s(1-p)p + s^2(1-p)^2p + \cdots}$$

A conditional likelihood for a simple 3-age-class harvest may therefore be constructed for cohort A as in Equation (2.2) which relies only on parameters involved in harvest and survival probabilities:

$$
\begin{aligned}
&L_A(\beta, c \mid \vec{x}_A) = \\
&\binom{\sum_i x_{ii}}{x_{11}, x_{22}, \ldots, x_{ij}} \left(\frac{p_1}{p_e}\right)^{x_{11}} \left(\frac{(1-p_1)\, s_1 p_2}{p_e}\right)^{x_{22}} \left(\frac{(1-p_1)\,(1-p_2)\, s_1 s_2 p_3}{p_e}\right)^{x_{33}}
\end{aligned} \tag{2.29}
$$

where the probability of ever being harvested, p_e, is computed as

$$
p_e = p_1 + (1-p_1)s_1p_2 + (1-p_1)(1-p_2)s_1s_2p_3.
$$

Note that a recruitment process is not relevant here, as no abundance estimation takes place with this conditional likelihood.

We may also consider harvest and survival processes to contain random deviations from an overall mean, and parameterize them as random effects. Modifications similar to those above in Equation (2.24) yield a random effects likelihood for the conditional formulation of

$$
\begin{aligned}
&L_A(\beta, c, \sigma_\beta, \sigma_\tau, \epsilon_2, \epsilon_3, \tau_2, \tau_3, \tau_4 \mid \vec{x}_A) = \\
&\binom{\sum_i x_{ii}}{x_{11}, x_{22}, \ldots, x_{ij}} \left(\frac{p_1}{p_e}\right)^{x_{11}} \left(\frac{(1-p_1)\, s_1 p_2}{p_e}\right)^{x_{22}} \left(\frac{(1-p_1)\,(1-p_2)\, s_1 s_2 p_3}{p_e}\right)^{x_{33}} \times \\
&\left[\prod_{i=2}^{4} \phi_{\sigma_\tau}(\tau_i)\right] \left[\prod_{i=2}^{3} \phi_{\sigma_\beta}(\epsilon_i)\right]
\end{aligned} \tag{2.30}
$$

where again

$$
\begin{aligned}
s_i &= \frac{1}{1+e^{-(\beta+\epsilon_i)}}, \\
p_i &= 1 - e^{-e^{(c+\tau_i)} f_i}.
\end{aligned} \tag{2.31}
$$

This leads to a combined (all-cohorts) likelihood of

$$
L(\beta, c, \sigma_\beta, \sigma_\tau, \vec{\epsilon}, \vec{\tau} \mid \boldsymbol{X}) = \prod_{k=1}^{A+Y-1} L_k(\beta, c, \sigma_\beta, \sigma_\tau, \vec{\epsilon}, \vec{\tau} \mid \vec{x}_k) \tag{2.32}
$$

where multidimensional integration over the latent variables $\vec{\epsilon}$ and $\vec{\tau}$ yields the marginal likelihood

$$L(\beta, c, \sigma_\beta, \sigma_c \mid \boldsymbol{X}) = \int_{\epsilon_1} \cdots \int_{\epsilon_{Y-1}} \int_{\tau_1} \cdots \int_{\tau_Y} L(\beta, c, \vec{\epsilon}, \vec{\tau}, \sigma_\beta, \sigma_c \mid \boldsymbol{X})\; d\vec{\epsilon}\; d\vec{\tau}. \tag{2.33}$$

which is approximated with the Laplace approximation and optimized directly for estimation of the parameter vector $\vec{\theta} = (\beta, c, \sigma_\beta, \sigma_\tau)^T$. In the simplest mixed-effects case, only 4 parameters require estimation via numerical methods; if survival and harvest probability are considered to be fixed effects, then only 2 parameters require estimation. This reduction in parameter space can be a very attractive feature for populations with a long harvest history, but limited historical auxiliary data sources. The limitation to fitting age-at-harvest data with these models is that direct estimates of abundance are not available within the likelihood framework.

In order to estimate abundance from the conditional likelihood of Equation (2.32), we may employ a Horvitz-Thompson estimator (Horvitz and Thompson, 1952). In general, a Horvitz-Thompson estimator sums the ratio of a number observed divided by the probability of observing that group (the "inclusion" probability), over the number of groups to estimate a quantity such as abundance (assuming the individuals captured are representative of the superpopulation). Strictly speaking, a Horvitz-Thompson estimator (Horvitz and Thompson, 1952) assumes a *known* inclusion probability, and I employ the method with an *estimated* inclusion (harvest) probability. I will therefore refer to this as the Horvitz-Thompson-type estimator. In the context of statistical population reconstruction, annual abundance is estimated as

$$\widehat{N}_i = \sum_{j=1}^{A} \frac{x_{ij}}{\widehat{p}_{ij}} \tag{2.34}$$

which, in the case of a single vulnerability coefficient c, may be expressed as

$$\widehat{N}_i = \frac{\sum_{j=1}^{A} x_{ij}}{\widehat{p}_i}. \tag{2.35}$$

Note that although estimation of survival and harvest probability requires tracking cohorts through time, the abundance estimation procedure does not rely on computing cohort abundances through time, as in Equation (1.5). If different vulnerability coefficients are necessary, as for $c_{juvenile}$ and c_{adult}, the sum in (2.34) can be broken into the appropriate components for summation as

$$\widehat{N}_i = \frac{x_{i,juvenile}}{\widehat{p}_{i,juvenile}} + \sum_{j=2}^{A} \frac{x_{ij}}{\widehat{p}_{i,adult}}. \tag{2.36}$$

The Horvitz-Thompson approach is a versatile and convenient method to estimate quantities such as abundance, although the properties of such estimators in the statistical population reconstruction context will require detailed examination. Such estimators have been successful in other contexts (Laake and Borchers, 2004; Buckland et al., 2010), and warrant investigation here. Horvitz-Thompson estimators may encounter difficulty if estimated probability of capture is too low, such that the ratio $x_{ij}/\widehat{p}_{ij}$ grows rapidly as $\widehat{p}_{ij}$ shrinks towards zero. Thus, if low harvest rates are expected, caution should be exercised in use of this method. Also, if no animals of a particular age class are harvested, the Horvitz-Thompson estimate of abundance is zero, which is unlikely to be a desirable feature when such zero harvest groups occur frequently in the data.

In models where abundance is estimated directly rather than with a Horvitz-Thompson estimator, the number of parameters is still significantly reduced via the inclusion of a stock-recruitment relationship and the modeling of process parameters as random deviations from a single mean rather than a large number of fixed parameters. Nevertheless, the expected high degree of correlation between the natural mortality (given by β) and harvest mortality (given by the relationship between c and f_i) will still make numerical estimation difficult. It is therefore desired to include any auxiliary data that might be helpful in estimating those parameters, or by assisting the estimation of initial abundance parameters N_{i1}. As mentioned previously, for each of the models considered here, some level of auxiliary data is *required* to fit the models due to identifiability issues in the likelihood function; the more parameters one includes in the model, the greater are the auxiliary data requirements According to standard likelihood theory, independent auxiliary data, perhaps arising from a mark-harvest, mark-recapture, or radiotelemetry study, can be used to augment our age-at-harvest likelihood (Gove et al., 2002) as

$$L_{total} = L_{age-harvest} \times L_{auxiliary} \times L_{catch-effort} \tag{2.37}$$

where $L_{auxiliary}$ is a likelihood function for the auxiliary data, which will typically be from the hypergeometric, binomial, or multinomial families of densities, and $L_{age-harvest}$ is given by any of the three methods detailed above [Equations (2.3), (2.26), or (2.32)], and where $L_{catch-effort}$ is the annual catch-effort likelihood of Equation (1.7), as preliminary investigations suggests this may lead to greater stability in the optimization process.

Model Notation

For simplicity, I will refer to those models that incorporate a stock-recruit relationship as "stock-recruit" models; models most similar to those of Gove et al. (2002) which estimate each initial recruit abundance as a separate parameter will be referred to as "absolute-recruit-abundance" models; those models which employ a conditional-likelihood and which rely on the Horvitz-Thompson abundance estimation approach will be called "conditional-likelihood" models.

The notation used to refer to the models may be interpreted as follows: if the model is listed as $N_x s_y c_z$, then the model estimates abundance within the likelihood framework; if a model is listed as $s_y c_z$, a Horvitz-Thompson approach is used to estimate abundance. The subscript SR refers to the stock-recruit relationship; if it not present, the subscript A, indicating "absolute" recruit abundance is used (the models of Gove et al. (2002)). For all entries in the model shorthand (N, s, and c), a subscript F is used to indicate whether the associated parameters are assumed to have only fixed components, with R indicating random processes.

Software

The statistical software package R (R Development Core Team, 2010) provides a convenient means with which to simulate data and summarize simulation output numerically and graphically. A separate software package, Automatic Differentiation Model Builder (ADMB) (Fournier et al., 2011) is used for optimization of the likelihood functions, due to its ability to implement the Laplace approximation (Skaug and Fournier, 2006) for a general likelihood function, which then permits the inclusion of random effects, such as those in the models described above.

Table 2.2: *List of model shorthand notation.*

Model Reference Name	Model Description
$N_{SR,F}s_Fc_F$	Recruitment relationship, all process parameters fixed
$N_{SR,R}s_Rc_R$	Recruitment relationship, γ, β, and c random
$N_As_Fc_F$	Absolute annual recruit abundance, β and c fixed
$N_As_Rc_R$	Absolute annual recruit abundance, β and c random
s_Fc_F	Absolute annual recruit abundance, β and c fixed, with Horvitz-Thompson abundance estimation
s_Rc_R	Absolute annual recruit abundance, β and c random, with Horvitz-Thompson abundance estimation

ADMB permits the user to write a generic likelihood function (which can be highly nonlinear, as in the models considered here) in the C++ language, and implements the Laplace approximation in conjunction with automatic differentiation, a method to precisely calculate numerical derivatives which are required to estimate the Hessian matrix, the inverse of which constitutes the estimated variance-covariance matrix of the maximum likelihood estimates. Optimization of this approximation to the likelihood is performed via quasi-Newton optimization. ADMB also includes tools for empirical-Bayes estimates of the random effects terms ($\vec{\epsilon}$, $\vec{\tau}$, and $\vec{\delta}$), and importance sampling of the likelihood function to aid in the estimation of random effects terms.

2.2.1 Uncertainty Estimation

Asymptotic theory of maximum likelihood estimates permits us to obtain variance estimates of parameters from the inverse of the Hessian matrix (the matrix of 2nd derivatives of the likelihood function, evaluated at the MLEs). ADMB (Fournier et al., 2011) computes this variance-covariance matrix of estimates automatically following the optimization process, and utilizes automatic differentiation in doing so, which is more accurate than the finite-difference methods employed in many other software packages. ADMB (Fournier et al., 2011) also automatically employs a variance approximation technique known as the delta method for approximating variances of *functions* of parameters. The delta method is comprised of a Taylor-series expansion of the function in question about the estimated value of the parameter(s) involved in the function. For mixed-effects models, variance approximations become more complex. Empirical Bayes estimates of the random effect terms ($\vec{\epsilon}$, $\vec{\tau}$, and $\vec{\delta}$) are available from the multiphasic estimation procedure employed in ADMB, and the uncertainty in these estimates is thus dependent on both uncertainty in their estimation as well as uncertainty of the MLEs upon which they depend. Therefore, the portion of the variance-covariance matrix corresponding to the random effect terms is estimated as (Skaug and Fournier, 2011)

$$\widehat{Cov}\left(\vec{\epsilon}\right) = -\left[\frac{\partial^2 \log p\left(\vec{\epsilon} \mid \boldsymbol{X};\ \widehat{\vec{\theta}}\right)}{\partial\vec{\epsilon}\,\partial\vec{\epsilon}'}\right]^{-1} + \left(\frac{\partial\vec{\epsilon}}{\partial\vec{\theta}}\right)\ \widehat{Cov}\left(\widehat{\vec{\theta}}\right)\left(\frac{\partial\vec{\epsilon}}{\partial\vec{\theta}}\right)'$$

where $\widehat{\vec{\theta}}$ represents the MLEs of the fixed parameters, $\widehat{Cov}\left(\widehat{\vec{\theta}}\right)$ represents the inverse-Hessian variance-covariance matrix of the MLEs, $\frac{\partial\vec{\epsilon}}{\partial\vec{\theta}}$ represents the sensitivity matrix of the estimates of $\vec{\epsilon}$ to the MLEs of $\vec{\theta}$, and $\log p(x \mid y)$ is the log-likelihood of x given y.

Once estimates of uncertainty for all model parameters and random effect terms are available, ADMB (Fournier et al., 2011) employs the delta method to estimate the uncertainty in functions of parameters, ($\widehat{N}_{22}$, for example) that are derived from model-based estimates. However, for conditional-likelihood models, abundance estimates are not included in the model structure as parameters (as for absolute-recruit abundance models) or functions of parameters and prior breeding-age abundance (for stock-recruit models), and a component of variation cannot be specified within the software syntax. To see this, note first that for conditional-likelihood models, abundance is estimated following model fit as in Equation (2.35) as

$$\widehat{N}_i = \frac{\sum_{j=1}^{A} x_{ij}}{\widehat{p}_i}, \tag{2.38}$$

which is a function of both parameter estimates and data. It is easily understood that parameter estimates are uncertain, and we have already seen that estimates of uncertainty of parameters are provided via the Hessian matrix. However, if it were possible to observe the same population of animals under the same conditions as a given year of historical data, with the same amount of harvest effort, one would not necessarily expect the exact same harvest numbers to be exhibited in the data as the first iteration, due to chance alone. Thus, there is an extra-likelihood component of variation in the conditional-likelihood models that is not accounted-for via inverse-Hessian uncertainty alone. For this component of variation, we must account for the binomial sampling model assumed for the harvest data. Therefore, in order to compute the variance estimate $\widehat{Var}\left(\widehat{N}_i\right)$, I use the "law of total variance"

$$\widehat{Var}(\widehat{N}_i) = E_{\boldsymbol{X}}\left(Var_{\widehat{p}_{ij}}\left(\widehat{N}_i \mid \boldsymbol{X}\right)\right) + Var_{\boldsymbol{X}}\left(E_{\widehat{p}_{ij}}\left(\widehat{N}_i \mid \boldsymbol{X}\right)\right). \tag{2.39}$$

I refer readers to Appendix B for details of the variance calculations, and note here that for a fixed-effects-only conditional-likelihood model, with an assumption of a binomial sampling model for annual harvest based on the estimated annual abundance and harvest probability,

$$\widehat{Var}\left(\widehat{N}_i\right) \approx \widehat{Var}\left(\widehat{c}\right)\left(\frac{\partial \widehat{p}_i}{\partial c}\right)^2 \frac{1}{\widehat{p}_i^4}\left(\widehat{N}_i\widehat{p}_i\left(1-\widehat{p}_i\right) + \widehat{N}_i^2\widehat{p}_i^2\right) + \frac{\widehat{N}_i\left(1-\widehat{p}_i\right)}{\widehat{p}_i}. \tag{2.40}$$

For a conditional-likelihood model with a random component for harvest vulnerability,

$$\begin{aligned}\widehat{Var}\left(\widehat{N}_i\right) \approx &\left(\left(\frac{\partial \widehat{p}_i}{\partial c}\right)^2 \widehat{Var}\left(\widehat{c}\right) + \left(\frac{\partial \widehat{p}_i}{\partial \tau_i}\right)^2 \widehat{Var}\left(\widehat{\tau}_i\right) + 2\left(\frac{\partial \widehat{p}_i}{\partial c}\right)\left(\frac{\partial \widehat{p}_i}{\partial \tau_i}\right)\widehat{Cov}\left(\widehat{c},\widehat{\tau}_i\right)\right) \times \\ &\frac{1}{\widehat{p}_i^4}\left(\widehat{N}_i\widehat{p}_i\left(1-\widehat{p}_i\right) + \widehat{N}_i^2\widehat{p}_i^2\right) + \frac{\widehat{N}_i\left(1-\widehat{p}_i\right)}{\widehat{p}_i}.\end{aligned} \tag{2.41}$$

Once variance estimates are obtained for each model formulation, we may appeal to asymptotic normality of maximum likelihood estimates, and compute approximate confidence intervals for annual abundance estimates as

$$\left(\widehat{N}_i - z_{1-\frac{\alpha}{2}}\sqrt{\widehat{Var}\left(\widehat{N}_i\right)},\quad \widehat{N}_i + z_{1-\frac{\alpha}{2}}\sqrt{\widehat{Var}\left(\widehat{N}_i\right)}\right) \tag{2.42}$$

where often we will use $\alpha = 0.05$, which leads to the usual $z_{1-\frac{\alpha}{2}} = z_{0.975} \approx 1.9599$. Abundance estimates produced from conditional-likelihood models via the Horvitz-Thompson estimator are not MLEs, so there is no *a priori* reason to expect asymptotic normality of these estimates. Use of the confidence intervals in Equation (2.42) is not justified by asymptotic statistical theory, but are constructed in this manner for investigative purposes.

2.3 Simulation Model

It is of interest to determine the degree of success the models introduced in the previous chapter attain for age-at-harvest datasets such as these with respect to several criteria, to be discussed later. In order to examine the performance of these models, I constructed a simulation model for the birth/survival/harvest processes of hypothetical large game populations. Within this model, I can specify a variety of values for the relevant process parameters (β, c, γ, σ_β, σ_c, and σ_γ), as well as the size of the population being studied (absolute abundance), and the number of years and age classes of available data.

I begin each simulation with a population at a stable age distribution. To project the population to a stable age distribution, I create a Leslie matrix (Leslie, 1945) of the form

$$\boldsymbol{L}_{A\times A} = \begin{bmatrix} 0 & e^\gamma\left(\frac{1}{1+e^{-\beta}}\right) & e^\gamma\left(\frac{1}{1+e^{-\beta}}\right) & \cdots & e^\gamma\left(\frac{1}{1+e^{-\beta}}\right) \\ \left(\frac{1}{1+e^{-\beta}}\right) & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & \left(\frac{1}{1+e^{-\beta}}\right) & 0 & 0 \\ 0 & 0 & 0 & \left(\frac{1}{1+e^{-\beta}}\right) & 0 \end{bmatrix}$$

where I choose reasonable values of γ and β to represent a feasible large game population (Table 2.4). Note that I have assumed age classes 2 through A are capable of reproduction, and do so at a common rate, and that survival for each age class is assumed to have a common mean. Again, these assumptions are flexible, but I am foremost interested in examining model performance in the simple population dynamics case, and will consider robustness to additional age-class process parameters later. Note that no individuals survive to be older than age A.

Each year's age-class distribution is first subjected to the harvest process, represented by the diagonal harvest matrix

$$\boldsymbol{H}_{A\times A} = diag\left(1 - e^{-e^c f_i}\right). \tag{2.43}$$

Again, I choose reasonable values of c to represent a feasible large game population harvest

(Table 2.4). In conjunction with the chosen value of c, for each year i, I select the amount of harvest effort, f_i, from the $Gamma\,(\alpha, \beta)$ distribution to give a desired mean level of harvest. For numerical purposes, I choose the scale of c and f_i such that $e^c f_i$ is contained in the interval (0, 3) with high probability. Note that harvest effort (hunter-days, # of permits sold, etc.) is assumed known precisely, and need not be estimated.

From the survival/reproduction Leslie matrix and the harvest matrix, I obtain the stable age distribution as the scaled (to sum to 1) dominant right eigenvector of the matrix product

$$\vec{\lambda} = eigen\left(\boldsymbol{L}_{A\times A} \times \boldsymbol{H}_{A\times A}\right).$$

This eigenvector, $\vec{\lambda}$ is then multiplied by a desired level of absolute annual abundance (4000 animals, for example) and rounded to obtain an initial age class abundance for each simulation, $\vec{N}_0$.

For each year of simulated data, I first draw τ_i, the annual fluctuation in harvest probability as

$$\tau_i \sim N(0, \sigma_\tau^2)$$

where σ_τ is chosen to represent a feasible magnitude of interannual variation in the harvest/effort relationship. Then, separately for each age class j, I draw the number harvested as

$$x_{ij} \sim Binomial\left(N_{ij}, p_{ij} = 1 - e^{-e^{(c+\tau_i)} f_i}\right).$$

The number remaining following harvest, $\vec{N}_i - \vec{x}_i$ is then subject to the survival process, where the number surviving N_{ij}^s for age classes $1, \ldots, A-1$ is generated as

$$N_{ij}^s \sim Binomial\left(\vec{N}_i - \vec{x}_i, s_{ij} = 1/(1 + exp(-(\beta + \epsilon_i)))\right)$$

separately for each age class, where ϵ_i is drawn from the $N(0, \sigma_\beta^2)$ distribution. Again, σ_β^2 is chosen to represent a reasonable interannual fluctuation in survival probability, and no members of age class A are assumed to survive. The age-class abundance vector N_i^s is then augmented with a simulated recruit abundance to form the next year's pre-harvest abundance. Recruit abundance is drawn as

$$N_{i1} \sim Poisson\left(e^{(\gamma+\delta_i)}\sum_{j=2}^{A-1} N_{ij}^{s}\right),$$

,

where δ_i is drawn from the $N(0, \sigma_\delta^2)$ distribution, and where σ_δ is chosen to represent a feasible level of interannual variation in recruitment rate. Thus, the simulation model incorporates sampling error of the Binomial and Poisson form, as well as interannual error in the demographic processes of harvest, survival, and reproduction.

I iterate the simulation process for $2Y$ years of "burn-in", and then save the next Y years as the simulated dataset. These final Y years of harvest and abundance data are stored in the matrices $\boldsymbol{X}$ and $\boldsymbol{N}$, respectively; the harvest data to be used in the estimation procedure, and the actual abundance to be compared to estimated abundance reconstructed from the estimated model parameters, following estimation. The reader will note that the "true model" (the model from which the simulated data are derived) for all simulations (except where noted otherwise in the **Robustness** and **Model Selection** sections) is the form of the most complex model listed in Table (2.2), $N_{SR,R}s_Rc_R$, which is the model incorporating a stock-recruit relationship with random effects as well as random-effects in the processes of natural mortality and harvest probability. Therefore, *a priori*, we might expect that if model fit to simulated data is successful, we will frequently see "better" model fits come from the $N_{SR,R}s_Rc_R$ model.

For estimation purposes, initial parameter estimates are required for β, c, and $\vec{N}_{1\cdot}$ (the vector of initial age-class abundances) and γ for some models. Depending on the features of the alternative model choices, initial estimates are required for any or all of the following additional parameters: σ_β, σ_c, σ_γ, $\vec{\epsilon}$, $\vec{\tau}$, and/or $\vec{\delta}$. For simulation purposes, initial parameter estimates for $\vec{N}_{1\cdot}$ (or $\vec{N}_{\cdot 1}$) were drawn from the normal distribution centered about the known parameter value with a standard deviation equal to 10% of the known parameter value, independently for each simulation. For process parameters $(\beta, c, \gamma, \sigma_\beta, \sigma_c, \sigma_\gamma)$, initial parameter estimates were drawn from the normal distribution centered around the known parameter value, with a standard deviation equal to 10% of the the absolute value of the known parameter value, independently for each simulation. This is consistent with the expected level of knowledge of abundances and demographic processes in a harvested population that has been studied for a number of years sufficient to provide age-harvest data, as well as the possibility for the practitioner to refit the model with different starting values to find the optimal fit. For variance components $(\sigma_\beta, \sigma_c, \sigma_\gamma)$, initial parameter estimates are $e^{-5} \approx 0.007$. This number was chosen to be close to 0, but not so close as to cause frequent failure of the numerical estimation technique due to a value of exactly 0. All random effect vectors ($\vec{\epsilon}, \vec{\tau}$, and $\vec{\delta}$ start with an initial value of 0 for all entries.

For what will herein be termed "primary simulation" results (where all model assumptions are satisfied by the simulation model), $n = 1000$ simulations are created, and models are fitted to the resulting data, to be compared by their success in producing precise and accurate parameter estimates and abundance reconstructions. Here, success is measured by a variety of quantities: summaries of percent bias of process parameter estimates as well as reconstructed total annual abundance estimates, precision of estimates (summaries of magnitude of estimated standard error), mean-square error, and confidence interval coverage.

Via this simulation model, a multitude of potential input parameter combinations could be examined. In order to examine model performance under conditions most likely to be pertinent to wildlife population managers, I limited the input parameter combinations to those presented in Table (2.3), in order to draw comparisons regarding model fit between them.

As mentioned previously, auxiliary data are required to fit these models. I chose to simulate two sources of auxiliary radiotelemetry data. To aid in the estimation of harvest probability, I assumed that 20 animals per year were radiotagged for short-term radiotelemetry studies that enabled surviving individuals and deaths due to other causes to be distinguished from harvest deaths, obtained via subtraction from the known number tagged. Assuming that tagged animals behave the same as untagged animals, an appropriate model for these data is the binomial auxiliary likelihood

$$L_{radiotelemetry(p)} = \prod_{i \in \vec{V}} Binomial\left(x_i; M_i, p_i = 1 - e^{-e^{(c+\tau_i)} f_i}\right),$$

where x_i is the number of individuals harvested in year i (those not found to be still alive or dead due to non-harvest causes immediately following the harvest period), M_i is the number of individuals radiotagged in year i, p_i is the harvest probability that has been described previously, and $\vec{V}$ is the set of years for which these radiotelemetry studies were conducted. Under the given level of harvest pressure ($\approx 27\%$) and the desired coefficient of variation of estimates resulting from these data sources (Table 2.3), this resulted in 7 years (140 animals) of radiotelemetry data.

In order to aid the estimation of survival probability, s, I assumed long-term radiotelemetry studies were conducted during the non-harvest season that enabled the number of survivors to be distinguished from those succumbing to non-harvest mortality. An appropriate model for these data is the binomial auxiliary likelihood

$$L_{radiotelemetry(s)} = \prod_{i \in \vec{W}} Binomial\left(y_i; B_i, s_i = \frac{1}{1 + e^{-(\beta + \epsilon_i)}}\right),$$

where y_i is the number of individuals surviving the non-harvest season in year i, B_i is the number of individuals radiotagged in year i, s_i is the survival probability that has been described previously, and $\vec{W}$ is the set of years for which these radiotelemetry studies were conducted. Under the given survival probability and desired coefficient of variation of estimates resulting from these data sources (Table 2.3), this resulted in 4 years (80 animals) of radiotelemetry data simulated. Both of these independent auxiliary radiotelemetry data sources were simulated in the middle of the age-harvest data (eg, 4 years of radiotelemetry data occurred in years 10 - 13 of the 25 years of age-harvest data). The joint likelihood was formed as the product of both of these auxiliary likelihood sources with the age-at-harvest likelihood for the chosen model, as in Equation (2.37). This joint likelihood was used for inference in all scenarios, except where otherwise noted in the **Robustness Simulations** section, which examines model performance under different quantities of available radiotelemetry data.

Following the primary simulations where model assumptions are satisfied, environmental stochasticity is relatively low, and a relatively high level of auxiliary information is available, a series of simulations addresses various questions of robustness, including survival probability that is not independent across years of data, reproduction that is not independent across years of data (although still dependent on prior breeding-age abundance), and where less auxiliary data are available. Some of these robustness simulations will assess the ability of the model to fit trends in abundance other than flat (finite rate of population change $\lambda \neq 1$).

Table 2.3: *Parameter combinations used for primary simulations of big game harvest datasets.*

Simulation Parameter				**Potential Values**
Number of age classes				13
Years of data				25
Desired CV of auxiliary mark-recovery data				15%
Desired CV of auxiliary radiotelemetry data				5%
Finite rate of population change (λ)				≈ 1
			Variation Level	
	None	**Low**	**Med**	**High**
σ_β, σ_c, σ_γ	0.0	0.1	0.2	0.3
Level of harvest[1]				$\approx 27\%$
Level of average survival percentage				84% ($\beta \approx 1.67$)
Total Initial Abundance				$\approx 4,000$
Average recruits per breeding-age female				$e^\gamma = 1$ ($\gamma = 0.0$)

[1] *When evaluated at mean effort. Harvest effort drawn from $Gamma(10, 14)$ then divided by by 100 to give mean effort = 1.4.*

Variation in the process parameters of survival probability, harvest percentage, and fecundity was simulated at a variety of of levels, classified into "none","low", "medium", and "high". (Table 2.4).

2.4 Results

2.4.1 Results: Primary Simulations Under Valid Model Assumptions

Models in Table (2.2) were chosen to examine a mixed-effects version of the models under consideration, as well as their fixed-effects-only counterparts. In the course of preliminary simulation work, it became evident that some "intermediate" models were worthwhile to in-

Table 2.4: *Variation induced in natural demographic parameters of interest. (Harvest probability assessed at mean level of effort.)*

Parameter	Level	Natural Range
Annual Survival Probability ≈ 0.84	$\sigma_\beta = 0.1$	$\mu_s \pm 2\sigma_s$: (0.814, 0.867)
	$\sigma_\beta = 0.2$	$\mu_s \pm 2\sigma_s$: (0.781, 0.888)
	$\sigma_\beta = 0.3$	$\mu_s \pm 2\sigma_s$: (0.745, 0.907)
Annual Harvest Probability $= 0.27$	$\sigma_c = 0.1$	$\mu_p \pm 2\sigma_p$: (0.226, 0.317)
	$\sigma_c = 0.2$	$\mu_p \pm 2\sigma_p$: (0.189, 0.372)
	$\sigma_c = 0.3$	$\mu_p \pm 2\sigma_p$: (0.158, 0.434)
Fecundity (young per female)$= 1.0$	$\sigma_\gamma = 0.1$	$\mu_f \pm 2\sigma_f$: (0.819, 1.221)
	$\sigma_\gamma = 0.2$	$\mu_f \pm 2\sigma_f$: (0.670, 1.492)
	$\sigma_\gamma = 0.3$	$\mu_f \pm 2\sigma_f$: (0.549, 1.822)

vestigate. Specifically, the estimation procedure was frequently unsuccessful in estimating a nonzero variance component for natural survival (to be described in detail later); therefore, the models presented in Table (2.2) were augmented with versions of each model structure where all processes were random *except* for survival, to study the effect, if any, of treating survival as a fixed process. The additional models are : $N_{SR,R}s_Fc_F$, $N_{SR,R}s_Fc_R$, $N_As_Fc_R$, and s_Fc_R (Table 2.5).

In addition, initial investigations concerning the use of the auxiliary catch-effort likelihood of Equation (1.7), it was found that stock-recruit models encountered numerical difficulties during optimization that frequently led to optimization failure when the auxiliary likelihood was used; therefore it was excluded from all fits for stock-recruit models. Preliminary investigations also indicated that results for the absolute-recruit abundance models ($N_As_Fc_F$, $N_As_Fc_R$, and $N_As_Rc_R$) as well as the conditional-likelihood/Horvitz-Thompson models (s_Fc_F, s_Fc_R, and s_Rc_R) differed based on the use of the auxiliary catch-effort likelihood. For these six models, results are presented both with the use of the auxiliary catch-effort likelihood of Equation (1.7),

Table 2.5: *List of additional models fitted.*

Model Reference Name	Model Description
$N_{SR,R} s_F c_F$	Recruitment relationship, γ random, β and c fixed
$N_{SR,R} s_F c_R$	Recruitment relationship, γ and c random, β fixed
$N_A s_F c_R$	Absolute annual recruit abundance, c random, β fixed
$s_F c_R$	Absolute annual recruit abundance, c random, β fixed, with Horvitz-Thompson abundance estimation

and without it.

Throughout this section, some primary tables and figures are presented in-line with the text. Some tables and figures are left for Appendix A, in order to maintain continuity of presentation.

Figure (2.1) contains a sample of simulated annual abundance data for these primary simulations, where average annual abundance is relatively constant at 4000 animals, when averaged over the $n = 1000$ simulations. Ten randomly-chosen abundance trajectories are represented by the ten individual lines at each level of simulated variation (none, low, medium, and high). A variety of trends are represented within the individual simulations, but as noted in Table (2.3), input parameter combinations have been chosen to keep simulated abundance relatively constant, on average.

2.4.2 Estimator Accuracy

Process Parameters

When gauging viability of model formulations via simulation, it is important to assess both the accuracy and precision of parameter estimates. In order to assess accuracy, I provide summaries of the bias exhibited by the maximum likelihood estimates obtained from simulated data (Table 2.7). I chose to assess bias with the median relative bias (MRB) as opposed to the more commonly-used mean relative bias due to the inherent difficulty in numerically optimizing an approximation to a marginal likelihood, which can infrequently lead to clearly erroneous values

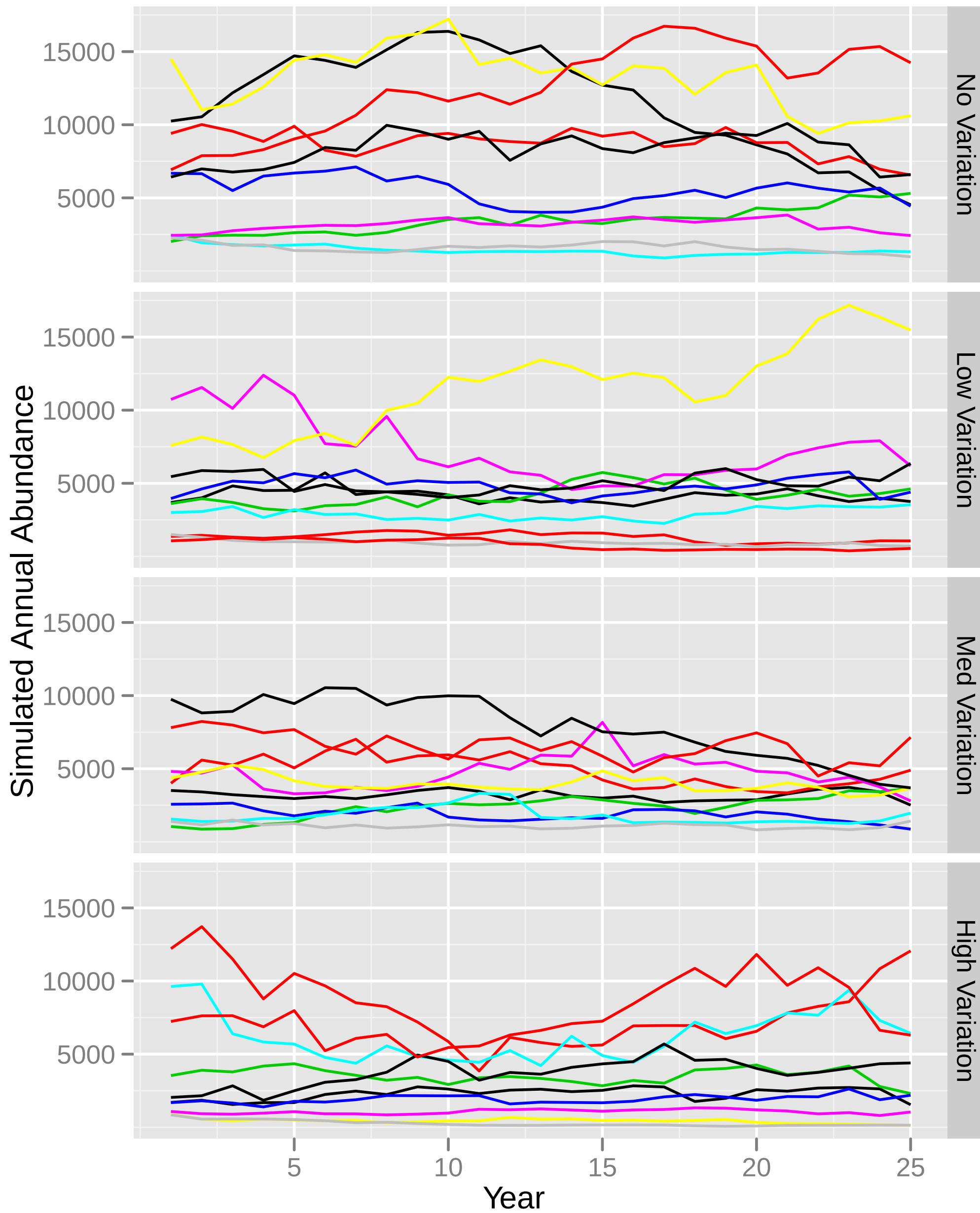


Figure 2.1: *Sample of ten randomly-chosen simulated total annual abundance trajectories.*

(such as $\widehat{s} < 10\%$). In practice, the practitioner will adjust box constraints on parameters as well as optimization intialization values in order to obtain results on a reasonable scale. Over the course of 1000 simulations for each individual simulation scenario, this is not possible, and a few estimates strayed into "unreasonable" territory (e.g. unreasonably large value of the reproductive rate accompanied by unreasonably low value of natural survival probability). To avoid such results from having undue influence on summary measures, I therefore choose the median as a measure of centrality for many simulation results.

With no simulated environmental variation, conditional-likelihood/Horvitz-Thompson models and stock-recruit models show low absolute median relative bias ($< 2\%$) in estimation of parameters for survival probability, harvest vulnerability, and recruitment regardless of the use of the auxiliary catch-effort likelihood component (Tables 2.6 and 2.7). Absolute-recruit abundance models, however, show relatively large bias, particularly for the mixed-effects versions ($N_A s_R c_R$ and $N_A s_F c_R$), while even the fixed-effects version shows 4.7% bias in estimation of survival probability and -9.3% bias in estimation of the harvest vulnerability parameter in the best-case scenario, when the auxiliary catch-effort likelihood is implemented.

Stock-recruit models (which, again, were only able to be fitted when the auxiliary catch-effort likelihood component was excluded) with a random component for survival exhibit moderate positive median relative bias in estimation of the mean survival probability, s (1.5% to 3.9% across all nonzero levels of simulated variation). Absolute-recruit-abundance models with any random effects exhibit relatively large positive median relative bias in estimation of s (between 16% and 17% at all nonzero levels of simulated variation), while the fixed-effects-only member of this model class ($N_A s_F c_F$) has slightly lower positive bias at the low level of simulated variation, with increasingly large median relative bias as the level of simulated variation increases. The conditional-likelihood/Horvitz-Thompson models show nearly-negligible median relative bias for both the fixed-effects and mixed-effects members of the model class, although the magnitude of bias does appear to increase somewhat with an increase in the level of simulated variation when the auxiliary catch-effort likelihood is used (Table 2.6); models show little-to-no-bias whatsoever when the auxiliary catch-effort likelihood component is omitted (Table 2.7). For all model classes, each of these biases in survival probability is balanced by a compensatory bias of the opposite sign in the estimation of the harvest mortality parameter, c, except for some results for model $s_F c_F$, which show low biases of matching sign at some levels of variation. Median relative bias for the exponentiated stock-recruit parameter (e^{γ}) for the relevant model class is relatively low for all models, across all levels of simulated variation (-0.6% to 4.0%).

Table 2.6: *Median relative bias ((estimate minus simulated value)/simulated value) in process parameter estimates, for models utilizing the auxiliary catch-effort likelihood component of Equation (1.7). Results based on n=1000 replicates at s=0.84, p ≈ 0.27, γ=0.0, and total annual abundance ≈ 4000.*

Variation Level	**Model**	$\widehat{s}$	$\widehat{c}$	$\widehat{\sigma}_{\beta}$	$\widehat{\sigma}_{c}$	$\widehat{\sigma}_{\gamma}$
None	$N_A s_F c_F$	4.7	-9.3			
	$s_F c_F$	0.6	-1.2			
	$N_A s_F c_R$	10.5	-18.6			
	$s_F c_R$	0.7	-1.4			
	$N_A s_R c_R$	10.8	-18.7			
	$s_R c_R$	0.7	-1.4			
Low	$N_A s_F c_F$	0.4	-1.2			
	$s_F c_F$	0.4	-1.3			
	$N_A s_F c_R$	16.3	-25.9		1.2	
	$s_F c_R$	0.7	-2.1		-3.6	
	$N_A s_R c_R$	16.2	-25.9	-100.0	1.4	
	$s_R c_R$	0.9	-2.1	-99.9	-6.7	
Medium	$N_A s_F c_F$	-8.8	19.3			
	$s_F c_F$	-0.0	-1.9			
	$N_A s_F c_R$	16.3	-26.4		-6.1	
	$s_F c_R$	0.6	-2.0		-4.2	
	$N_A s_R c_R$	16.4	-26.4	-100.0	-6.0	
	$s_R c_R$	1.1	-2.2	-100.0	-6.5	
High	$N_A s_F c_F$	-14.1	36.1			
	$s_F c_F$	-0.9	-1.6			
	$N_A s_F c_R$	16.4	-26.9		-6.7	
	$s_F c_R$	0.2	-1.4		-5.5	
	$N_A s_R c_R$	16.6	-26.8	-100.0	-6.7	
	$s_R c_R$	1.3	-2.1	-100.0	-7.5	

Table 2.7: *Median relative bias ((estimate minus simulated value)/simulated value) in process parameter estimates, for models omitting the auxiliary catch-effort likelihood component of Equation (1.7). Results based on n=1000 replicates at s=0.84, p $\approx$ 0.27, γ=0.0, and total annual abundance $\approx$ 4000.*

Variation Level	**Model**	$\widehat{s}$	$\widehat{c}$	$e^{\widehat{\gamma}}$	$\widehat{\sigma}_{\beta}$	$\widehat{\sigma}_{c}$	$\widehat{\sigma}_{\gamma}$
None	$N_{SR,F}s_Fc_F$	-1.9	4.0	0.2			
	$N_{SR,R}s_Fc_F$	1.5	-2.8	-0.0			
	$N_{SR,R}s_Fc_R$	1.9	-3.8	0.1			
	$N_{SR,R}s_Rc_R$	1.8	-3.5	0.0			
	$N_As_Fc_F$	7.8	-14.5				
	s_Fc_F	0.0	-0.0				
	$N_As_Fc_R$	9.7	-17.6				
	s_Fc_R	0.0	-0.0				
	$N_As_Rc_R$	9.7	-17.6				
	s_Rc_R	0.1	-0.1				
Low	$N_{SR,F}s_Fc_F$	-14.0	42.8	1.1			
	$N_{SR,R}s_Fc_F$	2.0	-4.5	0.0			21.8
	$N_{SR,R}s_Fc_R$	3.7	-7.7	0.1		-2.7	1.2
	$N_{SR,R}s_Rc_R$	3.6	-7.4	0.1	-100.0	-4.5	1.1
	$N_As_Fc_F$	9.0	-16.9				
	s_Fc_F	-0.4	0.2				
	$N_As_Fc_R$	16.1	-25.7			-3.8	
	s_Fc_R	-0.4	0.7			-3.5	
	$N_As_Rc_R$	16.1	-25.6		-100.0	-3.8	
	s_Rc_R	-0.4	0.7		-99.9	-6.4	
Medium	$N_{SR,F}s_Fc_F$	-21.2	89.6	2.4			
	$N_{SR,R}s_Fc_F$	1.9	-5.1	-0.2			16.8
	$N_{SR,R}s_Fc_R$	3.5	-7.5	-0.1		-5.3	-2.8
	$N_{SR,R}s_Rc_R$	3.9	-8.0	0.1	-100.0	-6.2	-2.8
	$N_As_Fc_F$	11.2	-21.2				
	s_Fc_F	0.0	-1.9				
	$N_As_Fc_R$	16.4	-26.4			-7.6	
	s_Fc_R	-0.3	0.1			-3.1	
	$N_As_Rc_R$	16.6	-26.4		-100.0	-7.5	
	s_Rc_R	0.2	0.1		-99.9	-6.1	
High	$N_{SR,F}s_Fc_F$	-23.3	110.7	4.0			
	$N_{SR,R}s_Fc_F$	0.8	-4.5	-0.6			13.6
	$N_{SR,R}s_Fc_R$	2.1	-4.6	-0.5		-8.7	-5.2
	$N_{SR,R}s_Rc_R$	2.8	-5.4	0.1	-100.0	-9.7	-5.9
	$N_As_Fc_F$	12.8	-22.6				
	s_Fc_F	-1.4	-2.1				
	$N_As_Fc_R$	16.6	-27.0			-7.4	
	s_Fc_R	-0.7	0.8			-3.2	
	$N_As_Rc_R$	16.6	-27.1		-100.0	-7.2	
	s_Rc_R	0.2	0.7		-99.9	-5.9	

Estimation of the magnitude of interannual variation (σ) in process parameters was less successful than for process parameters themselves. Regardless of level of simulated variation, all models that included a random effect to encapsulate variation in survival probability ($N_{SR,R}s_Rc_R$, $N_As_Rc_R$, s_Rc_R) had difficulty estimating a nonzero variance component. In all such models at all levels of simulated variation, 50% or more of estimates were effectively zero, evidenced by the median relative bias of -100%. Interannual variation in harvest vulnerability was estimated more successfully; when the auxiliary catch-effort likelihood is used, at a low level of simulated variation, median relative bias ranged from -6.7% to 1.4%; at a medium level of simulated variation interannual variation median percent bias ranged from -6.5% to -4.2%. At the highest level of simulated variation, median relative bias in estimates of σ_c ranged from -7.5% to -5.5%. When the auxiliary catch-effort likelihood is not used (Table 2.7), results tend to be the same, with low negative bias exhibited for all models, at all levels of simulated variation. No obvious differences between model classes are apparent, except that absolute-recruit abundance models and stock-recruit models tend to exhibit a slightly lower absolute magnitude of bias than the fully-random (both s and c with random componenents) conditional-likelihood/Horvitz-Thompson model. Results regarding magnitude of bias in estimation of variance parameters did not appear to be sensitive to the use of the auxiliary catch-effort likelihood as results for models that omitted this component (Table 2.7) are very similar to the results when it is incorporated (Table 2.6).

Results for interannual variation in the productivity parameter γ indicate that interannual variation is estimated well by those models that incorporate other random processes ($N_{SR,R}s_Fc_R$ and $N_{SR,R}s_Rc_R$), and that significant positive bias is exhibited by the model that assumes reproduction to be the only random process (ranging from 13.6% at the high level of simulated variation to 21.8% at the low level of simulated variation). This positive bias decreases with increasing simulated variation.

In general, it appears that Horvitz-Thompson models, mixed-effects version of stock-recruit models, and in some cases the fixed-effects absolute-recruit-abundance model are capable of accurately estimating process parameters (s, c, and γ); the mixed-effects versions of the absolute-recruit abundance models and the fixed-effects-only stock-recruit model show noticeably poorer behavior. While stock-recruit models required that the auxiliary catch-effort likelihood component be omitted, conditional-likelihood/Horvitz-Thompson models show slightly better behavior (with respect to process parameter estimates) when it is omitted than when it is used, with mixed-effects versions of the absolute-recruit abundance models showing similar behavior regardless of its use.

With respect to relative bias in process parameters, it appears that the conditional-likelihood models are most accurate in estimating the process parameters. The true data-generating model, $N_{SR,R}s_Rc_R$, is neither the best nor the worst model when considering the percent bias metric, although as aforementioned, models $N_As_Rc_R$ and s_Rc_R are in some sense "true" models as well. For absolute recruit abundance models, the fixed-effects only forms appear to be most successful in estimating the mean of process parameters. This is not true for reproductive-parameter models, where $N_{SR,R}s_Fc_F$ appears to be the most successful, or for conditional-likelihood models, where s_Fc_R and s_Rc_R are nearly indistinguishable from one another as the best-of-class in terms of process parameter estimation accuracy. It is worth noting that poor estimates of the primary demographic process parameter do not necessarily indicate a model will provide poor abundance estimates, especially when random effects may be used to compensate for a biased estimate of the mean parameter.

Abundance Reconstruction

The most important goal of statistical population reconstruction relates to the ability of a model and estimation procedure to accurately estimate animal abundance. Previous research has shown that the absolute recruit abundance model formulations perform well in simulation studies that do not include environmental stochasticity with regard to their ability to reconstruct population abundance (Gove, 1997), and that estimates arising from these models can track other indices of population abundance well (Skalski et al., 2007). In this section, I seek to answer similar questions regarding the new model formulations and the inclusion of empirical Bayes estimates of random effects in the estimation of abundance in the presence of simulated environmental stochasticity. Do these provide for a more accurate and precise reconstruction, indicated by reliable absolute abundance estimates, than the previous model formulations? Do such reconstructions track population trends?

Following estimation of parameters, including the annual recruitment or reproductive parameters, stock-recruit or absolute recruit abundance models may reconstruct age-class abundance as

$$\widehat{N}_{22} = \widehat{N}_{11}\widehat{s}_1(1-\widehat{p}_1) = \widehat{N}_{11}\left(\frac{1}{1+e^{-\left(\widehat{\beta}+\widehat{\epsilon}_1\right)}}\right)\left(e^{-e^{(\widehat{c}+\widehat{\tau}_1)}f_1}\right) \tag{2.44}$$

when $\widehat{N}_{11}$ is estimated directly as a parameter (absolute-recruit abundance models), or

$$\widehat{N}_{22} = \left(e^{\widehat{\gamma}+\widehat{\tau}_1}\sum_{j=b}^{A} N_{1j}\right)\widehat{s}_1(1-\widehat{p}_1) = \left(e^{\widehat{\gamma}+\widehat{\tau}_1}\sum_{j=b}^{A} N_{1j}\right)\left(\frac{1}{1+e^{-\left(\widehat{\beta}+\widehat{\epsilon}_1\right)}}\right)\left(e^{-e^{(\widehat{c}+\widehat{\tau}_1)}f_1}\right) \tag{2.45}$$

when the stock-recruit relationship is involved. For conditional-likelihood models, abundance reconstruction proceeds as in Equations (2.34) or (2.35), as appropriate. There are a variety of ways to summarize results of reconstructions based on repeated simulated data. One method involves comparing the median percent difference in annual abundance from the known value for each of the four levels of variation considered here (Figure 2.2, Table A.1). First, for each simulation, total annual abundance is computed for each year of data from which the known (simulated) annual abundance is subtracted. This quantity is then divided by the known annual abundance and multiplied by 100%, and medians are obtained for each year across simulations.

Models exhibiting the lowest consistent bias are the conditional-likelihood models employing the Horvitz-Thompson-type estimator (models s_Fc_F, s_Fc_R, and s_Rc_R), particularly when the auxiliary catch-effort likelihood component is omitted (Figure 2.2). Stock-recruit models show consistent negative bias between about -5% and -10% at all levels of simulated variation. Results for absolute-recruit abundance models indicate consistent large negative bias for mixed-effects versions of the model (-20% to -30%), while the fixed-effects version of this model (the model of Gove et al. (2002)) shows negative bias when the auxiliary catch-effort likelihood is not employed (-15% to -20%) and positive bias for higher-variation simulations when it is employed (up to 70%). With no simulated enviromental stochasticity, this model exhibits negative bias (-10%) and with low simulated environmental stochasticity, this model shows no bias, when the auxiliary catch-effort likelihood is employed. Model $N_{SR,F}s_Fc_F$ is not pictured in Figure (2.2)

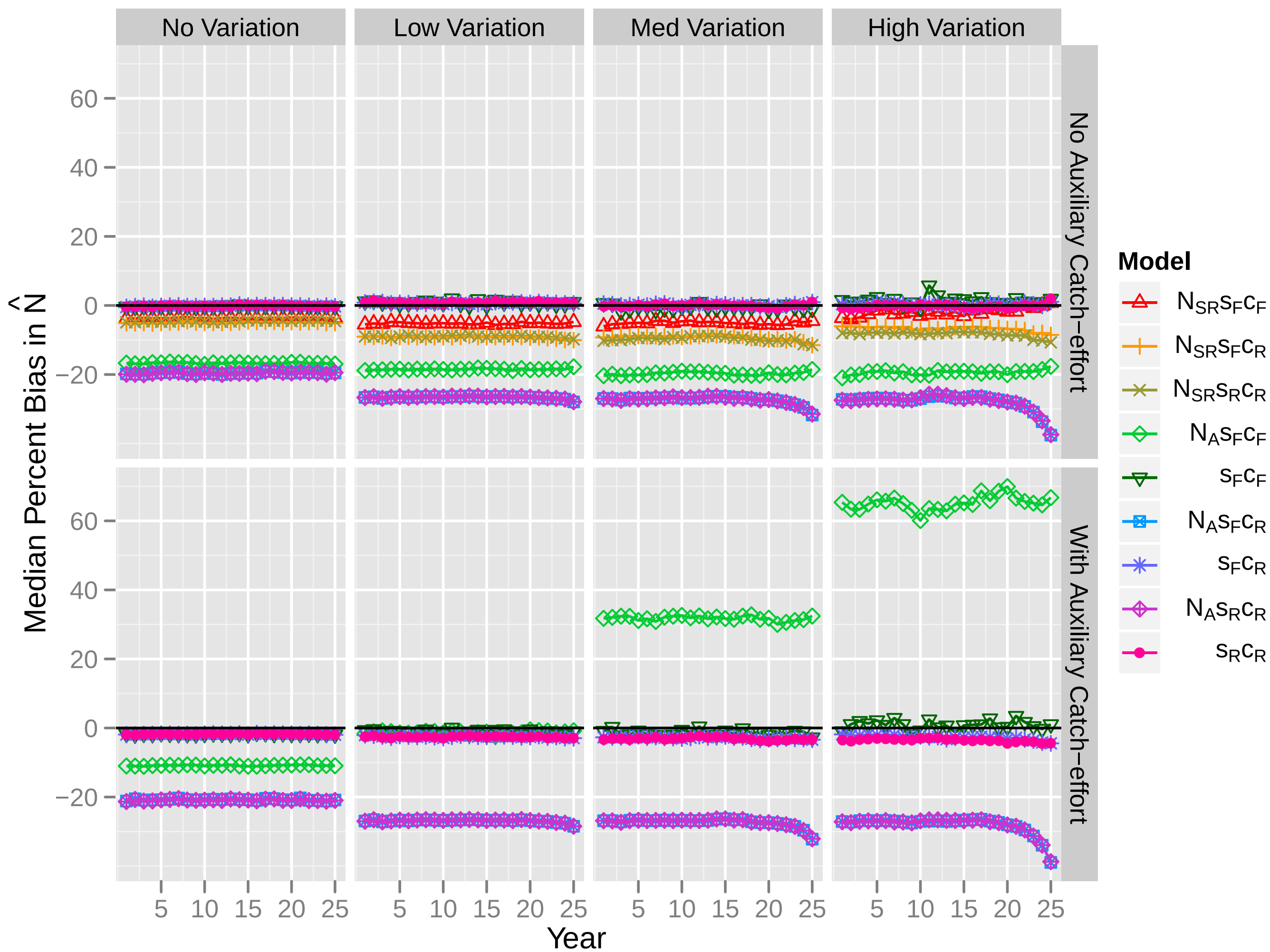


Figure 2.2: *Median relative bias in estimated total annual abundance. Results obtained from 25 years of simulated data with total abundance* $n \approx 4000$ *across years,* $p \approx 27\%$, $s \approx 84\%$.

due to its very high bias that made it difficult to represent differences among other models in the figure. Results can, however, be found in Table (A.1) in Appendix A.

It is worth noting that, by this metric, of the models including a stock-recruit relationship, $N_{SR,R}s_Fc_F$ appears to slightly outperform both $N_{SR,R}s_Fc_R$ and $N_{SR,R}s_Rc_R$. This is interesting because $N_{SR,R}s_Rc_R$ is the model from which data are simulated, and $N_{SR,R}s_Fc_F$ and $N_{SR,R}s_Fc_R$ are submodels of this.

In general, the concordance of results between models which differ only in the specification of survival as fixed or random (for example s_Fc_R with s_Rc_R) can be easily explained by the frequent estimation of survival variation at approximately 0 ($\widehat{\vec{\epsilon}} = \vec{0}$ and $\widehat{\sigma}_\beta = 0$). Essentially, the estimated model with a random effect term has been reduced to its mixed-effects counterpart where survival is assumed fixed, due to the inability of the estimation procedure to detect $\sigma_\beta > 0$ in the presence of variation for harvest vulnerability.

Based on summaries of bias in both process parameters and estimates of annual abundance from simulated data, it appears that the reduced-parameter conditional-likelihood models combined with the Horvitz-Thompson abundance estimation approach tend to estimate abundance as well or better than the heavily-parameterized counterparts, and it appears that including a stock-recruit relationship in a fixed-effects only model ($N_{SR,F}s_Fc_F$, not shown, see Table A.1) induces an unacceptably large magnitude of positive bias in abundance estimates and harvest vulnerability, and the greatest degree of negative bias in estimates of survival probability at all levels of simulated variation.

Alternative Reconstruction Method using the Horvitz-Thompson Estimator

Results detailed above used a cell-based Horvitz-Thompson-type estimator, where each individual year- and age-class abundance was computed by adjusting the observed harvest count by the estimated harvest probability, as $\widehat{N}_{ij} = x_{ij}/\widehat{p}_{ij}$. An alternative to this method involves using the Horvitz-Thompson-type estimator to estimate initial cohort abundance ($\widehat{N}_{1j} = x_{1j}/p_{1j}, j = 1, \ldots, A$ and $\widehat{N}_{i1} = x_{i1}/\widehat{p}_{i1}, i = 2, \ldots, Y$), and then utilizing the harvest probability estimates, the natural survival probability estimates, and the cohort structure to compute age-class abundances as

$$\widehat{N}_{ij} = \widehat{N}_{i-1,j-1}\widehat{s}_{i-1,j-1}(1 - \widehat{p}_{i-1,j-1}).$$

A possible advantage of this cohort-based reconstruction method is that $\widehat{N}_{ij} \leq \widehat{N}_{i-1,j-1}$, whereas this may not necessarily be the case with the cell-based cohort reconstruction method. A disadvantage is that if any initial cohort harvest is zero, not only is its corresponding abundance estimate zero, but all subsequent population abundances in the cohort are estimated to be zero as well. In some case, this may lead to biased estimates of annual abundance.

In order to compare the utility of these two reconstruction approaches to one another, I apply both methods of reconstruction to the model fits obtained for the simulated datasets described in this section, for the models employing the Horvitz-Thompson estimation approach ($s_F c_F$, $s_F c_R$, and $s_R c_R$). A plot of median relative bias for total annual abundance (Figure 2.3) indicates that both models show very similar levels of bias in estimation of total annual abundance, and are only clearly distinguishable from one another for the fixed-effects-only model, wherein the cell-based method tends to estimate slightly lower abundance than the cohort-based method.

As results for mixed-effects models do not indicate a large difference between abundance estimates derived from each of these methods, there is very little reason to recommend one method over another. However, when one moves from comparisons of estimator accuracy to estimator precision (see next section), the cell-based reconstruction method provides a means to more easily employ the delta-method to estimate standard errors for age-class abundances other than the first. More importantly, the cell-based reconstruction method allows computation of the extra-likelihood contribution to the estimate of uncertainty associated with our assumed binomial sampling model. A detailed description follows in the next section.

2.4.3 Estimator Precision

In statistical population reconstruction, assessments of reconstruction quality and process parameter estimate bias are paramount. Assessments of estimator precision are also of great importance. If estimators are known to be unbiased on average but highly uncertain, how can they be employed usefully to inventory populations, or plan management strategies? Therefore

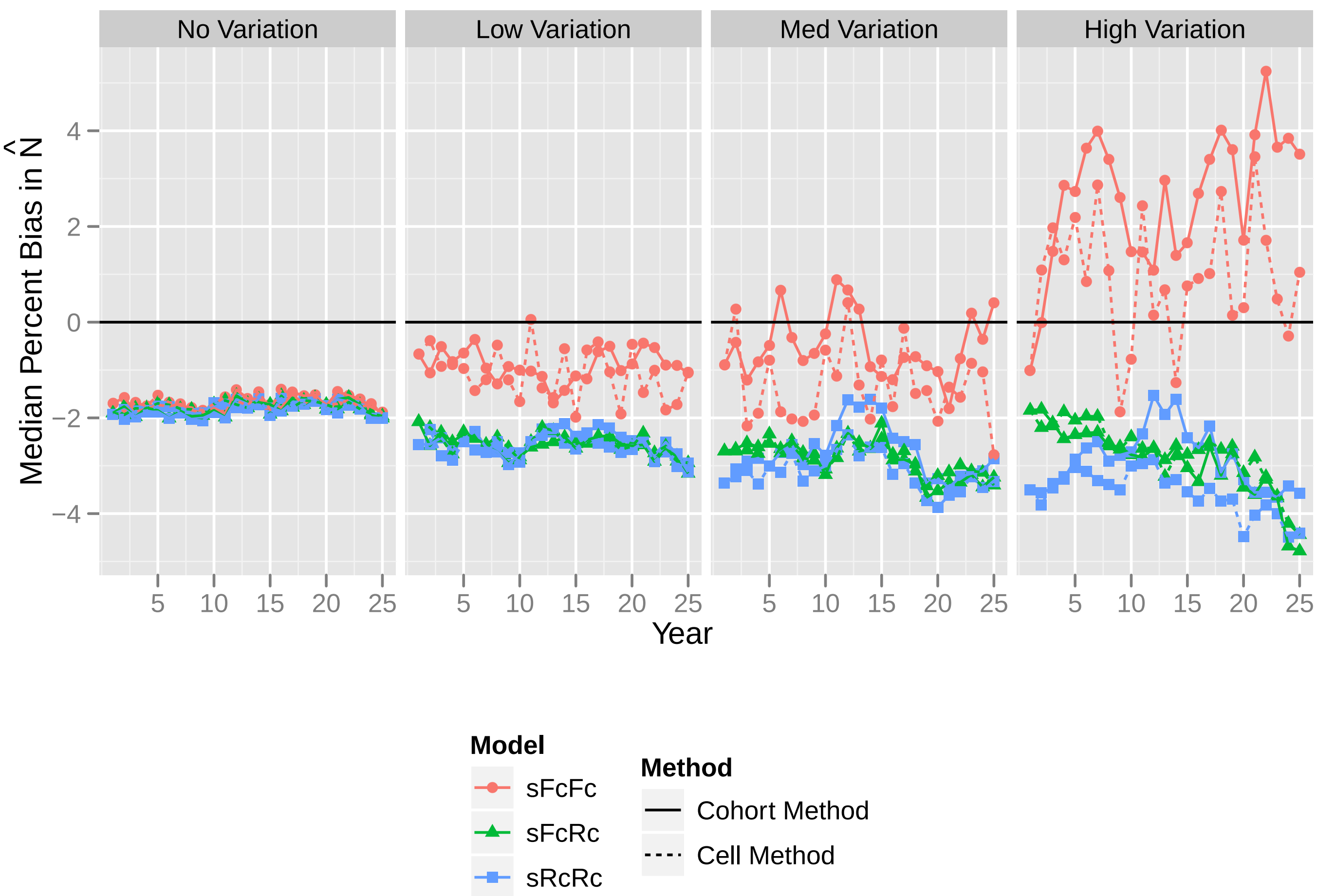


Figure 2.3: *Median relative bias in estimated total annual abundance for models employing the Horvitz-Thompson-type estimator and the cohort-based and cell-based reconstruction methods. Results indicate very similar levels of bias for each model, with only the fixed-effects model at the medium and high level of variation showing any difference between reconstruction method. In this case, the cell method provides for slightly lower abundance estimates than the cell-based method, although the absolute difference in MRB is typically less than 1%. Results obtained from 25 years of simulated data with total abundance* $n \approx 4000$ *across years,* $p \approx 27\%$*,* $s \approx 84\%$*.*

it is of interest to quantify estimator precision among those models found to be “least biased.”If a model produces heavily biased estimates, we are not interested in how precise those estimates are. Among those models presented here, model $N_{SR,F}s_Fc_F$ indicated a high degree of estimation bias, and is therefore excluded from the following precision comparisons.

Abundance Reconstruction

For comparisons of adequacy of uncertainty estimation for annual abundance, it is convenient to compare models by their ability to to provide standard errors that produce confidence intervals with the expected coverage. Models of the form of $N_As_Rc_R$ include absolute recruit abundance estimates as parameters, and subsequent age-class abundances are derived from products of these with harvest and natural survival probabilities as in Equation (2.44). Models of the form of $N_{SR,R}s_Rc_R$ include a parameter for relative recruit abundance, which is multiplied by prior breeding-age abundance to estimate recruit abundance, and the same product of recruit abundance with harvest and natural survival probabilities are used to compute subsequent age-class abundances as in Equation (2.45). These two model structures employ the delta-method to estimate age-class abundances computed via Equations (2.44) and (2.45). For models employing the Horvitz-Thompson estimation approach, the delta-method is employed, and the additional source of uncertainty associated with the assumed binomial sampling model and subsequent non-maximum likelihood estimation of age-class abundance is computed as described in Equations (2.39) through (2.41). Asymptotic 95% confidence intervals are computed as in Equation (2.42).

Bivariate summaries of median relative bias and confidence interval coverage by model and reconstruction year (Figure 2.4) indicate that the mixed-effects conditional-likelihood models with the Horvitz-Thompson abundance estimation approach, s_Fc_R and s_Rc_R, provided very low median relative bias for annual abundance as well as nearest-to-nominal (95%) confidence interval coverage for all levels of variation considered here, and results are slightly better when the auxiliary catch-effort likelihood component is omitted. Median relative bias is very low for all years of reconstruction, and estimated coverage of the asymptotic 95% confidence intervals ranges from 93% to 99% over all levels of simulated variation. Confidence interval coverage for these models is neither extremly subnominal, as for the fixed-effect-only models s_Fc_F and

$N_A s_F c_F$ (even when bias is low, such as at the low level of simulated variation), nor is confidence interval coverage supernominal (indicating confidence intervals that are too wide) as for mixed-effects models including a stock-recruit relationship ($N_{SR,R} s_F c_F$, $N_{SR,R} s_F c_R$, and $N_{SR,R} s_R c_R$), which have a few years of subnominal coverage (corresponding to early years of reconstruction, Table A.2), and a majority of years with supernominal coverage (corresponding to later, more recent years of reconstruction).

The absolute-recruit abundance models $N_A s_F c_F$, $N_A s_F c_R$, and $N_A s_R c_R$ have relatively high absolute median relative bias values (positive bias for the fixed effects model [albeit, not at the low level of simulated variation when the auxiliary catch-effort likelihood is used] and negative bias for the mixed-effects models), and also the poorest confidence interval coverage of those models considered here, below 40% for all models at all levels of simulated variation. For a few specific years of reconstruction, models $N_{SR,R} s_F c_F$, $N_{SR,R} s_F c_R$ or $N_{SR,R} s_R c_R$ might provide somewhat better measures of abundance reconstruction, as they have some points closer to the vertical line at 95%; this would be notable if the years where these stock-recruit models happened to be the most recent few years, which are likely to be the most valuable timepoints for reconstruction for wildlife managers. A close examination of the data used for plotting Figure (2.4), contained in Table (A.2) in Appendix A, indicates this is not the case and the models which most successfully reconstruct population abundance in the latest years of available data are $s_F c_R$ and $s_R c_R$.

Summary

The results discussed in previous sections may be succinctly summarized by implementing a progressive filtering scheme to find the most favorable reconstruction models (Table 2.8). Initially, I reviewed bias in parameter and abundance estimates (columns in Table 2.8 labeled **Bias**). Models [$N_{SR,F} s_F c_F$, $N_{SR,R} s_F c_F$, $N_{SR,R} s_F c_R$, $N_{SR,R} s_R c_R$, $N_A s_F c_R$, $N_A s_R c_R$], [$N_{SR,F} s_F c_F$, $N_{SR,R} s_F c_F$, $N_{SR,R} s_F c_R$, $N_{SR,R} s_R c_R$, $N_A s_F c_F$, $N_A s_F c_R$, $N_A s_R c_R$], and [$N_{SR,F} s_F c_F$, $N_{SR,R} s_F c_R$, $N_{SR,R} s_R c_R$, $N_A s_F c_F$, $N_A s_F c_R$, $N_A s_R c_R$] were eliminated from consideration at low, medium, and high levels of simulation variation, respectively, because their average absolute median relative bias in abundance estimation (median taken across simulations, then averaged across years)

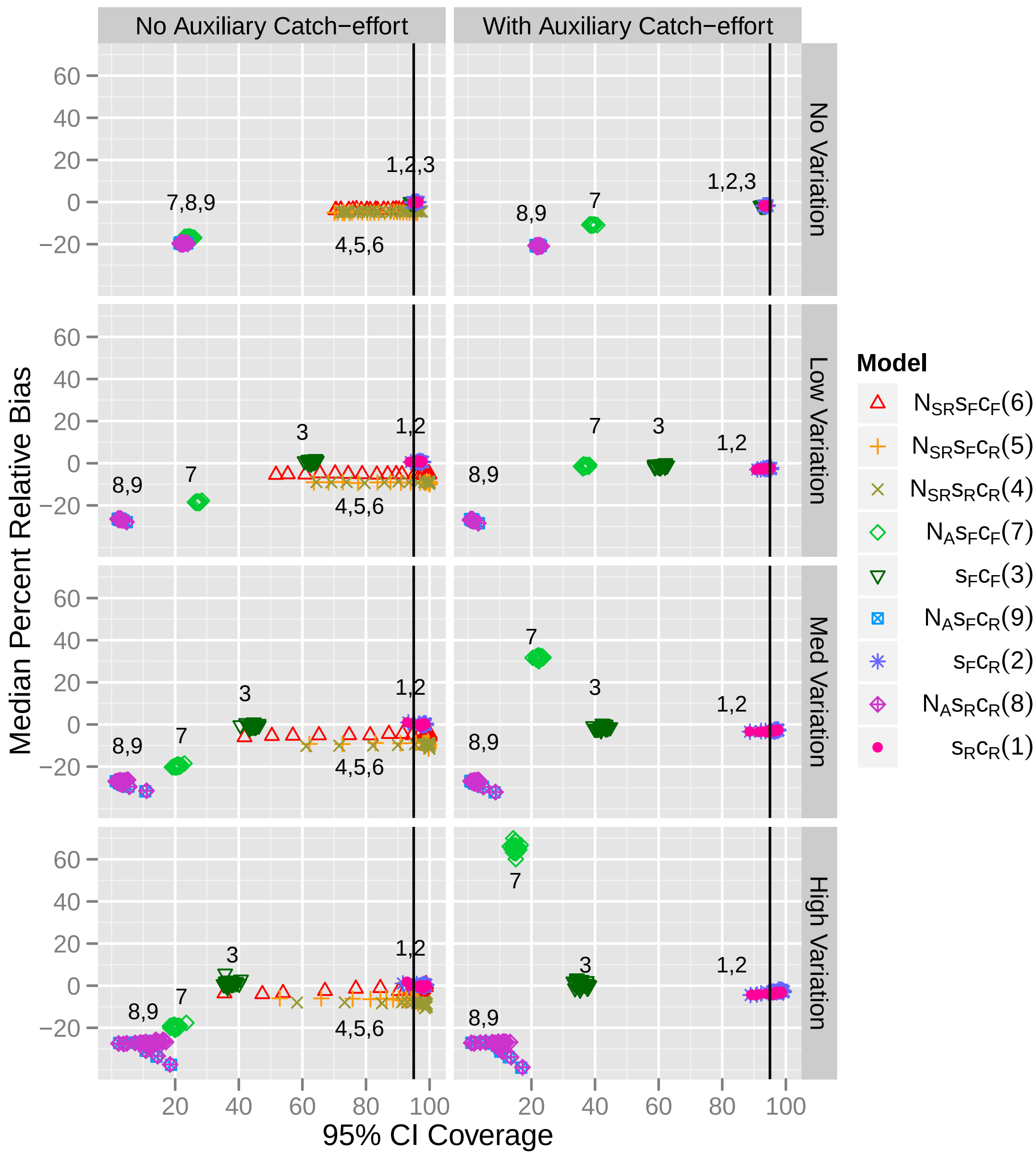


Figure 2.4: *95% CI coverage and median relative bias of total annual abundance. Percent coverage of annual abundance of 95% CIs are on the x-axis. The y-axis contains the median relative bias of the annual abundance estimates. Each model represented by 25 points, one for each year of the 25 years of harvest data. Models that perform best with respect to these two criteria are those with median relative bias near 0% and with 95% coverage CI coverage, models* $s_F c_R$ *and* $s_R c_R$.

is greater than 5%. Once unbiased models have been chosen, the second step relates to the ability to produce estimated variances of the appropriate magnitude, which I have chosen to assess with confidence interval coverage. Here, I compute the average of squared distance between the estimated CI coverage percentage and its expected value, 95, and label it **MSE(CI Coverage)** (Table 2.8). Among models exhibiting low bias, models s_Rc_R and s_Fc_R have nearest-to-nominal 95% asymptotic CI coverage, indicating they produce the most accurate estimates of standard error. While confidence interval coverage is related to bias, some results (e.g. models s_Fc_F and $N_As_Fc_F$) indicate that a model can exhibit low bias in abundance estimation, but produce variance estimates that are incorrect, illustrated by poor confidence interval coverage. In all scenarios, the mixed-effects versions of the conditional-likelihood/Horvitz-Thompson models (s_Fc_R and s_Rc_R) perform the best with respect to these two assessments. These two models also perform best when the auxiliary catch-effort likelihood component is omitted.

The results of this simulation study indicate that mixed-effects conditional-likelihood models combined with a Horvitz-Thompson estimation approach for annual abundance perform best with respect to a variety of criteria. Results also indicate that bias is relatively low for mixed-effects stock-recruit models, but that even though environmental stochasticity (σ_β, σ_c, σ_γ) tends to be underestimated, total variation in annual abundance tends to be overestimated, particularly in the most recent years of reconstruction which are likely to be the most important.

Model Selection

Now that I have chosen a set of models that is successful in reconstructing population abundances and related parameter estimates based on simulation studies (conditional-likelihood models with the Horvitz-Thompson abundance estimator), it is important to understand how these results compare to model selection procedures one might typically employ during a population reconstruction. The issue of model selection with nonlinear mixed models, however, is complex. In many cases, the practitioner may be interested in the estimate of process variation ($\sigma_\beta, \sigma_c, \sigma_\gamma$) directly, and will include random effects for these processes as a matter of course. In other situations these may be regarded as nuisance parameters, and it is of interest to determine whether the true $\sigma > 0$ or if we can reduce the mixed model to a fixed one with

Table 2.8: *Combined summary of model performance. Models with > 5% absolute bias eliminated from consideration. Models performing best with respect to these two criteria (shaded rows) are $s_R c_R$ and $s_F c_R$, at all levels of simulated variation.*

			Bias		MSE(CI Coverage)	
Var.	**Model**	Aux. Likelihood	% Bias[1]	Rank	MSE^2	Rank
None	$s_R c_R$	Without	-0.12	1	0.60	3
	$s_F c_R$	Without	-0.12	2	0.55	2
	$s_F c_F$	Without	-0.14	3	0.22	1
	$s_F c_F$	With	-1.69	4	4.37	6
	$s_F c_R$	With	-1.77	5	1.99	4
	$s_R c_R$	With	-1.81	6	2.03	5
	$N_{SR,R} s_F c_F$	Without	-3.68	7	213.82	9
	$N_{SR,R} s_R c_R$	Without	-4.57	8	144.98	7
	$N_{SR,R} s_F c_R$	Without	-4.89	9	193.56	8
Low	$s_F c_R$	Without	0.70	1	4.32	4
	$s_R c_R$	Without	0.82	2	2.89	2
	$s_F c_F$	Without	0.86	3	1036.46	8
	$s_F c_F$	With	-1.07	4	1175.32	9
	$N_A s_F c_F$	With	-1.27	5	3376.59	10
	$s_F c_R$	With	-2.51	6	1.66	1
	$s_R c_R$	With	-2.63	7	4.03	3
Medium	$s_R c_R$	Without	-0.03	1	7.31	3
	$s_F c_R$	Without	0.15	2	10.06	4
	$s_F c_F$	Without	-0.50	3	2569.65	8
	$s_F c_F$	With	-1.33	4	2774.42	9
	$s_F c_R$	With	-2.85	5	5.46	2
	$s_R c_R$	With	-3.14	6	3.57	1
High	$s_R c_R$	Without	-0.21	1	8.59	2
	$s_F c_R$	Without	0.25	2	9.78	3
	$s_F c_F$	With	0.88	3	3541.05	9
	$s_F c_F$	Without	1.44	4	3334.54	8
	$N_{SR,R} s_F c_F$	Without	-1.90	5	354.85	7
	$s_F c_R$	With	-2.71	6	12.04	4
	$s_R c_R$	With	-3.57	7	7.31	1

[1] *Mean % bias computed first for each year of data (median taken across simulations), then averaged across years.*

[2] *Value computed is* $\frac{1}{25}\sum_{i=1}^{25}(x_i - 95.0)^2$*, where* x_i *is the percent CI coverage estimated for each model.*

a single parameter to describe the process ($\sigma = 0$). There are two general methods for making such comparisons: hypothesis testing and information-theoretic approaches. The former is comprised of statistical tests such as likelihood ratio tests (LRTs) that require a known or approximate distribution under the null hypothesis ($\sigma = 0$) and computation of a test statistic; the latter requires a likelihood value and a penalty term to enforce parsimony in the number of parameters selected into the model, such as AIC, where the penalty term is twice the number of parameters in the model.

In the statistical population reconstruction models considered here, the number of processes considered random is at most three so a stepwise likelihood-ratio test procedure may be justified. (Such stepwise procedures are not generally recommended when considering a "large" amount of parameters being tested for inclusion into the model.) The LRT, a very general and typically powerful hypothesis testing procedure for likelihood-based models, requires that the null value of a hypothesis test not lie on the boundary of its parameter space. In standard situations, twice the absolute difference in log-likelihood values between the model with the parameter of interest and the model without it is approximately χ^2 distributed with degrees of freedom equal to the difference in number of parameters being tested. A test for $\sigma > 0$ from a random effects model does not allow standard LRT procedures to be used, however, because σ is necessarily greater than or equal to zero so that the boundary assumption (the null value for testing must be on the interior of its parameter space) is not satisfied. Self and Liang (1987) show how LRTs can be conducted in such boundary-value situations with a mixture of χ^2 distributions. Based on work by Shapiro (1988), Molenberghs and Verbeke (2007) indicate that for simultaneously testing k parameters $\sigma_i = 0$ vs. $\sigma_i > 0, i = 1, \ldots, k$, the distribution of the likelihood ratio test statistic is a mixture of χ^2 distributions,

$$\sum_{i=0}^{k} 2^{-k} \begin{pmatrix} k \\ i \end{pmatrix} \chi_i^2. \tag{2.46}$$

A special case of this is important for statistical population reconstruction with nonlinear mixed models; when testing k vs. $k+1$ correlated random effects variances (for instance, testing model $N_{SR,R}s_Fc_R$ vs. $N_{SR,R}s_Rc_R$ is a test of 2 vs. 3 random effects terms), the null distribution is $0.5(\chi_k^2 + \chi_{k+1}^2)$ (Molenberghs and Verbeke, 2007). Hypothesis tests are conducted by computing the relevant p-values of the likelihood ratio test statistic for both the χ^2_k and χ^2_{k+1} distributions, and averaging them. For the case of a single random effect, this reduces to halving the p-value from a standard χ^2_1 LRT (Littell et al., 2006; Bolker et al., 2009).

For statistical population reconstruction models where the number of random effects is no greater than three, it may not be of interest to conduct simultaneous testing of multiple random effects for equivalence to zero, and I would instead recommend stepwise comparisons based on comparing models with $k+1$ and k random effects, as this provides more information on *which* random effects may or may not be important for the model. Joint simultaneous testing does not provide any information on *which* processes may be considered constant, as opposed to random.

Two perspectives exist for likelihood-based information theoretic model selection procedures such as Akaike's information criterion (Akaike, 1974). In the context of linear mixed-effects models, Vaida and Blanchard (2005) describe the philosophical differences between use of a marginal AIC (mAIC) and conditional AIC (cAIC). In the context of our statistical population reconstruction models, the mAIC is given by either Equation (2.27), Equation (2.28), or Equation (2.33) evaluated at the MLEs for the fixed effects. The conditional likelihood (conditional on the estimated values of the random effect terms, $\vec{\epsilon}, \vec{\tau}$, and $\vec{\delta}$) is given by evaluating the likelihood in Equation (2.26) evaluated at the MLEs of fixed effects and at the empirical Bayes estimates of the random effect terms. In addition to the difference in likelihood values, the parsimony penalty terms for AIC differ between the two AIC versions. Standard AIC is computed as

$$AIC = 2k - 2ln(L), \tag{2.47}$$

where k is the number of parameters in the model, and serves as a penalty on the likelihood term to enforce parsimony in the number of parameters selected into the model, and $ln(L)$ is the log-likelihood. The mAIC depends only on the fixed effects (as the random effects terms are "integrated out"), and thus mAIC may also be computed the same as in (2.47), where k also includes the variance component fixed parameters ($\sigma_\beta, \sigma_\tau$, and/or σ_γ). The cAIC depends on empirical Bayes estimation of the random effect terms. These terms do not quite have parameter status, as they are technically random variables whose specific realizations are being estimated, however they do contribute directly to the conditional likelihood. Therefore, the parsimony penalty should be somewhere between 1 (for the fixed σ component corresponding to the random effect) and $m+1$, where m is the dimension of the vector of the random effect, such as $\epsilon_i, i = 1, \ldots, m$. There are a number of different methods that have been suggested for computation of the parsimony penalty for cAIC arising from *linear* mixed models; Bolker et al. (2009) provides a brief summary of three common methods including 1) simply using the minimum number of degrees of freedom, corresponding to the number of fixed effects plus the number of σ terms, 2) the Satterthwaite approximation, and 3) the Kenward-Roger approximation. For nonlinear mixed models, no rigorous theory has been developed to accommodate 1) the number of degrees of freedom to use for a vector of random effects, or 2) how to modify AIC to accommodate the "boundary value" problem with random effects models. In general, the use of mAIC for model selection is appropriate for global population inference and prediction that is not specific to the groups or clusters; cAIC is appropriate when the specific levels of the random effects that define groups or clusters (or years) are of interest for inference and prediction.

The choice of model structure (absolute recruit abundance, stock-recruitment or conditional likelihood models) may also be considered from a model selection point of view, where versions of AIC may be used to select among competing structures. However, neither likelihood ratio tests nor information-theoretic approaches may be used to compare models based on different probability distributions, such as the conditional-likelihood models and unconditional absolute-recruit abundance and/or stock-recruit likelihood models. These difficulties notwithstanding, much of the decision regarding choice of model structure will rest on the amount of available data and features of the data at hand, as well as comparisons of estimator accuracy and precision as presented in the previous section. For instance, if many zero-harvests among age classes are present in the data (particularly among juveniles), or if absolute harvest percentage among all age classes is quite low, the Horvitz-Thompson abundance estimation approach of the conditional-likelihood models may provide for very unstable abundance estimates that can be incorrect (too large if harvest probability is very low), and one may choose not to use this method. If recruitment rate is a primary interest of wildlife managers or biologists, one may choose to examine a stock-recruit relationship model.

Because of the inability to compare between model structures with likelihood-based techniques such as hypothesis tests and AIC (and indeed, making these comparisons is the primary goal of the simulation study in previous sections), I limit my investigation of model selection procedures to comparisons within model structures. Model selection with both mAIC and cAIC within model structures is examined for the zero, low, medium, and high levels of simulated variation discussed previously (Sim. 0, Sim. 1, Sim. 2, and Sim. 3), as well as a population whose mean natural survival rate is non-constant and increasing which leads to an increasing population abundance (Sim. 4), a population whose mean natural survival rate is non-constant and decreasing, which leads to a decreasing population abundance (Sim. 5), and a population whose overall trend in abundance is relatively flat (when averaged across simulations), but years of alternatingly increased and decreased recruitment rate are simulated, leading to a periodically fluctuating population abundance (Sim. 6) (Tables 2.9 and 2.10). These latter 3 simulations will be discussed in greater depth in the Robustness section, to follow. For each simulation scenario, the difference in mAIC/cAIC between the lowest-mAIC/cAIC model and each other model (ΔAIC) is computed, then averaged across simulations. Results indicate that regardless of the characteristics of the population simulated here, the mAIC model selection criterion prefers models with the maximum number of random processes, by a large degree above any other model ($s_R c_R$ for conditional-likelihood models, $N_A s_R c_R$ for absolute-recruit abundance models, and $N_{SR,R} s_R c_R$ for stock-recruit models). Among those models and model structures considered here, it appears that the mAIC prefers models with more random components, without exception. Qualitatively, this result does not appear to differ among those models which optionally incorporate the auxiliary catch-effort likelihood component, although the mAIC difference between members of the class of absolute-recruit abundance models that incorporate this component appears to be larger than for absolute-recruit abundance models without this component.

Results from the cAIC comparison (Table 2.10) indicate that models with random processes for harvest vulnerability and reproduction (where relevant), but a fixed survival component are often favored over models with similar components except including a random effect for survival, although sometimes these models appear to be roughly equivalent in terms of their predictive ability by this metric. This is because the cAIC criteria is sensitive to the actual estimated values of the random effect terms, and as previously discussed, many models fail to estimate a nonzero component for survival random effects. The number of parameters used to compute cAIC was the same as that which was used to compute mAIC: number of fixed parameters + number of variance components used to model random effects. As aforementioned, this is an underestimate of the "effective" number of parameters used to fit the model, as it does not account for the number of random effects terms; no concrete methods yet exist to estimate this number for models such as these. A standard "rule of thumb" for AIC differences is given by considering a ΔmAIC > 10 to provide little support for a model, $4 <$ ΔmAIC < 7 to provide moderate support, and $0 <$ ΔmAIC < 2 to provide substantial support (Burnham and Anderson, 2002, pg. 70).

Based on these results, it appears that in standard situations (auxiliary data are available, abundance is not rapidly changing, processes of harvest, survival, and reproduction are stationary) and some nonstandard situations (non-stationary demographic processes, rapidly changing abundance), the mAIC criterion does not consider the "second-place" models of each model class ($s_F c_R$, $N_A s_F c_R$, and $N_{SR,R} s_F c_R$) to even be worth consideration, although in previous sections we have seen that model fits from the fully-random ("best") models and these second-place models typically provide for very similar estimates. The cAIC criterion may be helpful in discerning whether random effects components are helpful in explaining variation in the data, although as previously discussed, this may be examined more rigorously with likelihood ratio tests which attempt to accommodate the boundary value problem of estimating σ values. Based on the simulation results already presented and this new mAIC and cAIC information, it does not appear that mAIC will provide any significant information with respect to model selection. While cAIC may be informative, it is perhaps no more informative than stepwise LRTs for random effects. While previous simulation results appeared to favor models $s_R c_R$ and $s_F c_R$, the degree of separation between model fit success from simulation studies already presented is not as large as is indicated by the average ΔmAIC values (Table 2.9). It should be noted, however, that the mAIC does indeed select the "correct" model within each model structure, as the simulation model was always closest to $N_{SR,R} s_R c_R$, and models $N_A s_R c_R$ and $s_R c_R$ may also be considered "correct" models in some sense. It is also important to note, once again, that AIC comparisons suffer from the same boundary value problems as likelihood ratio tests, when the comparison is being made between models where the submodel has a restriction that lies on the boundary of its parameter space ($\sigma = 0$).

I turn instead to examine likelihood ratio tests (Tables 2.11 and 2.12), and their ability to select the "correct" or "best" model. For each of the 1000 simulations previously considered, for each of the 7 simulation scenarios, likelihood ratio tests were conducted to compare models with random effects to their simplified (nested) counterparts where a random effect in the larger model is assumed to be fixed in the smaller model. Tests were conducted using the procedure outlined above following Equation (2.46), and both the marginal likelihood (Table 2.11) and conditional likelihood (Table 2.12) were used.

Likelihood ratio test results using the marginal likelihood indicate a very high frequency of rejection of simpler models in favor of models with more random effects, for all model structures (Table 2.11). Again, this is interesting in light of the fact that simulation studies in previous sections showed little difference in process parameter and abundance estimates between models with a random effect for survival probability, and models where survival probability is assumed fixed and constant among years. Since each model comparison is between nested models, it is expected that the mAIC criterion and LRT will be in strict concordance (which may be complicated by boundary effects on mAIC); both are shown here to illustrate both the frequency of model selection preference (Table 2.11), as well as the unequivocal nature of results given by the ΔmAIC differences (Table 2.9). Even when no variation is simulated (Sim. 0), marginal likelihood comparisons prefer models with random effects to those without.

Likelihood ratio tests using the conditional likelihood (Table 2.12) indicate a propensity to reject models that omit a random component for harvest vulnerability, but show preference for models that omit a random effect for natural survival, which is in agreement with the frequent estimation of $\widehat{\sigma}_\beta = 0$ (Table 2.7). Where the comparison between model results with and without the auxiliary catch-effort likelihood can be made (absolute-recruit-abundance models and conditional-likelihood/Horvitz-Thompson models), it appears that the results without incorporating this component are either similar or slightly improved, in terms of the ability to detect the true model (eg, for the comparison of $N_A s_R c_R$ vs. $N_A s_F c_R$, 17.1% of LRTs indicated $\sigma_s > 0$ when the auxiliary catch-effort likelihood component was omitted, while 0.0% of LRTs indicated $\sigma_s > 0$ when this component was included, for Sim. 3). Conditional LRT results for stock-recruit models are similar to other model classes in that they select a random effect for harvest probability frequently, but they do not select a random effect for natural survival frequently, although they often select random effects for survival slightly more frequently than other model classes.

Based on these results, it appears that while this testing procedure using the conditional likelihood does not select the model from which the data were generated (a model with a random component for natural survival probability), the use of the conditional likelihood as opposed to the marginal likelihood does a better job of enforcing the parsimony principle, by suggesting removal of a random effect that is estimated to be 0. It should be noted here that the choice of degrees of freedom = 1 for the conditional likelihood LRT may not be appropriate because the difference in the effective number of parameters between models with and without a given random effect may be greater than 1 (Bolker et al., 2009), though no rigorous results for models of this nature have yet been developed. When no variation is simulated (Sim. 0), comparisons between models with a random effect for harvest vulnerability and those with only fixed effects indicate that the conditional likelihood prefers models with the random effect for absolute-recruit abundance models (94.9% of simulations rejected $\sigma_c = 0$ with the auxiliary likelihood, 48.3% of simulations rejected $\sigma_c = 0$ without it), and without a random effect for conditional-likelihood/Horvitz-Thompson models (only 28.5% of simulations rejected $\sigma_c = 0$ with the auxiliary catch-effort likelihood, while 27.4% of simulations rejected σ_0 without it).

In the context of generalized *linear* mixed models, which bear many similarities to the models I consider herein, Zuur et al. (2009, pg. 90 - 92, 130 - 139) recommend first parameterizing the model with a high number of fixed effects that may or not be included in the final model (the model selection set of explanatory environmental variables), then using either of the model selection procedures discussed above (LRTs using mixtures of χ^2 distributions or mAIC/cAIC, which are known to have their own boundary-value problems for constrained parameters) to find the optimal random effects structure. Following this, mAIC is used to select the subset of fixed effects into the model to develop the best model. As these procedures are reasonable from a statistical standpoint and may be conducted with relative ease within the models considered here, and because there is a lack of good alternatives, I recommend a similar protocol be used for model selection for statistical population reconstruction models. As with any likelihood model,

Table 2.9: *Difference in mAIC from lowest-mAIC model and other models (ΔmAIC) within groups defined by model structure. Results indicate high preference for fully-random models, even when no variation is simulated (Sim. 0). "Aux. Like." = Auxiliary likelihood component used (With) or not used (Without) in joint likelihood model.*

		Mean ΔmAIC						
Aux. Like.	**Model**	Sim. 0	Sim. 1	Sim. 2	Sim. 3	Sim. 4	Sim. 5	Sim. 6
Without	$N_A s_F c_F$	87.9	267.0	714.6	1357.3	190.3	198.0	264.4
Without	$N_A s_F c_R$	42.1	49.6	49.7	43.5	42.4	42.7	42.1
Without	$N_A s_R c_R$	0.0	0.0	0.1	0.9	0.0	0.0	0.0
Without	$s_F c_F$	86.5	248.4	697.3	1336.4	184.2	192.8	255.3
Without	$s_F c_R$	42.2	42.4	42.7	44.4	42.4	42.7	42.6
Without	$s_R c_R$	0.0	0.0	0.1	0.2	0.0	0.0	0.0
Without	$N_{SR,R} s_F c_F$	97.7	311.0	752.4	1490.2	354.6	1140.7	359.4
Without	$N_{SR,R} s_F c_R$	62.4	55.5	48.8	160.1	48.5	52.1	62.9
Without	$N_{SR,R} s_R c_R$	1.6	0.4	5.4	53.8	2.1	6.7	6.5
With	$N_A s_F c_F$	98.6	503.6	1501.1	2598.0	649.9	674.1	529.9
With	$N_A s_F c_R$	42.3	44.7	42.5	43.1	42.2	42.1	42.0
With	$N_A s_R c_R$	0.0	0.0	0.0	27.4	0.0	0.1	0.0
With	$s_F c_F$	86.5	251.1	734.4	1277.0	324.3	339.2	263.2
With	$s_F c_R$	42.2	42.4	42.6	42.6	42.4	42.4	42.4
With	$s_R c_R$	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Sim. 0 = No variation, Sim. 1 = Low variation, Sim. 2 = Medium variation, Sim. 3 = High variation, Sim. 4 = s_i (hence N_i) increasing during study, Sim. 5 = s_i (hence N_i) decreasing during study, Sim. 6 = period 4 time series "jump" in recruitment rate.

Table 2.10: *Difference in cAIC from lowest-cAIC model and other models (ΔcAIC) within groups defined by model structure. Results indicate preference for models with random components for harvest probability and recruitment, but no random component for natural survival probability. "Aux. Like." = Auxiliary likelihood component used (With) or not used (Without) in joint likelihood model.*

		Mean ΔcAIC						
Aux. Like.	**Model**	Sim. 0	Sim. 1	Sim. 2	Sim. 3	Sim. 4	Sim. 5	Sim. 6
Without	$N_A s_F c_F$	7.4	231.4	703.2	1359.4	145.2	155.4	229.3
Without	$N_A s_F c_R$	1.8	8.8	10.3	6.6	1.4	2.3	0.5
Without	$N_A s_R c_R$	3.6	2.6	2.3	3.6	2.4	2.5	2.3
Without	$s_F c_F$	1.6	206.4	679.0	1328.8	134.0	143.8	214.5
Without	$s_F c_R$	1.4	0.2	0.6	2.9	0.2	0.5	0.3
Without	$s_R c_R$	3.1	1.7	1.6	1.6	1.7	1.7	1.7
Without	$N_{SR,R} s_F c_F$	17.2	268.0	734.4	1496.4	318.1	1103.9	323.1
Without	$N_{SR,R} s_F c_R$	22.2	14.3	7.0	287.7	7.6	10.6	22.3
Without	$N_{SR,R} s_R c_R$	6.0	3.6	4.5	43.4	5.1	9.4	9.1
With	$N_A s_F c_F$	33.8	487.6	1510.1	2616.4	639.0	665.5	515.2
With	$N_A s_F c_R$	0.7	3.4	1.5	0.0	0.5	0.4	0.4
With	$N_A s_R c_R$	2.2	2.2	2.3	1357.3	2.3	2.4	2.2
With	$s_F c_F$	1.7	210.0	718.3	1271.7	288.7	305.1	223.4
With	$s_F c_R$	1.4	0.2	0.5	0.9	0.1	0.2	0.2
With	$s_R c_R$	3.1	1.7	1.6	1.6	1.7	1.8	1.8

Sim. 0 = No variation, Sim. 1 = Low variation, Sim. 2 = Medium variation, Sim. 3 = High variation, Sim. 4 = s_i (hence N_i) increasing during study, Sim. 5 = s_i (hence N_i) decreasing during study, Sim. 6 = period 4 time series "jump" in recruitment rate.

one may also choose to employ multimodel inference techniques (Burnham and Anderson, 2004) on a group of the best models, provided one is working within a single model structure, so that Akaike weights may be used.

For wild populations subject to harvest, it is unlikely that interannual variation is *not* present in the processes of survival, harvest, and reproduction, regardless of the ability of the estimation method to detect all three sources simultaneously ($\widehat{\sigma}_\beta, \widehat{\sigma}_c, \widehat{\sigma}_\gamma > 0$). For this reason, I now focus my attention not on selecting whether random effects should or should not be included in a statistical population reconstruction model (although a method for doing so with LRTs based on χ^2 mixture distributions has already been described above), but instead focus on selecting the number of fixed parameters necessary for the process parameter transformation functions; in particular, whether the data justify the use of separate harvest vulnerability coefficients for different age classes (such as $c_{juvenile}$ and c_{adult}), or if a single harvest vulnerability coefficient is appropriate. In this case, a model structure is assumed to already have been chosen (a model of the form of $N_{SR,R}s_Rc_R$, $N_As_Rc_R$, s_Rc_R, etc.).

In order to examine the ability for these models to discriminate between a single or multiple harvest vulnerabilities, I added the ability to simulate separate vulnerability parameters for the juveniles (age class 0) and adults (age class 1+) for the same population discussed previously in this chapter, wherein abundance was assumed to be constant at roughly 4000 individuals each year. I then proceeded to simulate 1000 datasets with a low level of process variation and 1000 with a medium level of variation (fitting 1000 models with multiple c's under high variation proved numerically unstable), with parameters listed in Table (2.13). These parameters were chosen to give an adequate difference between $c_{juvenile}$ and c_{adult}, as well as to provide for a relatively flat average population growth rate. It was assumed that tagging juveniles for harvest would be difficult or impossible, so simulated auxiliary mark-harvest and radiotelemetry data applies to adult members of the population only. The only source of information on juvenile harvest vulnerability is in the age-at-harvest data.

In order to examine model selection of the fixed components in mixed-effects models in the context of harvest vulnerability coefficients in statistical population reconstruction models, I first simulate data when only one vulnerability coefficient is used for both juvenile and adult members of the population, and fit the data with the single-c models and the dual-c models and compute the percent of simulations where each model formulation is favored, to examine the Type I error rate. Power is examined by simulating datasets where $c_{juvenile} \neq c_{adult}$, and fitting with both models that assume $c_{juvenile} \neq c_{adult}$ and models that assume $c_{juvenile} = c_{adult}$, and conducting a likelihood ratio test between these two nested models. For these simulations, simulation input parameters are given in Table (2.13).

Type I error rate and power results may be summarized by computing the percent of trials (simulations) where the likelihood ratio test selected the less-complex model when it was the simulation model (in the case of Type I error rate) or the more-complex model when it was the simulation model (in the case of power) (Table 2.14). Likelihood ratio tests with both the conditional likelihood and marginal likelihoods were examined. Fixed-effects comparisons are omitted from the marginal likelihood rows because for fixed-effects-only models, the marginal and conditional likelihoods are identical, and so results match the conditional likelihood results.

Results regarding Type I error rate differ markedly between the model structures being tested, and with the use of the auxiliary catch-effort likelihood (Table 2.14). The critical value used was $\chi^2_{\{\alpha=0.95, df=1\}} = 3.841$, which should provide approximately 5% of simulations where the null hypothesis ($c_{juvenile} = c_{adult}$) is rejected, when in fact simulations were conducted when $c_{juvenile} = c_{adult}$. Both the absolute recruit abundance models and conditional-likelihood models provide Type I error rates much lower than 5% when the auxiliary catch-effort likelihood is used, indicating greater ability to select the less complex model when it is appropriate to do so, for both the fixed and mixed-effects versions of each model structure, and for both the marginal and conditional likelihoods. When the auxiliary catch-effort likelihood is not employed, these two model structures show type I error that is greater than expected for both the marginal and conditional likelihoods. Both fixed and random versions of the stock-recruit models indicate higher-than-expected Type I error rate with either the fixed or marginal likelihood, indicating a propensity for the LRT to incorrectly select the more complex model when it is not the true model. Each of these results are consistent at both levels of simulated variation, although error rates generally increase with the degree of variation. In general, it appears that Type I error rates are somewhat higher with the conditional likelihood than the marginal likelihood, and *without* the auxiliary catch-effort likelihood as opposed to *with* it.

Results regarding power of the likelihood ratio test indicates low power and ability to choose the more complex model at either level of simulated variability for the absolute-recruit abundance and conditional-likelihood model structures, but high power for the stock-recruit models when the auxiliary likelihood is used for both the marginal and conditional likelihoods. Conversely, when the auxiliary catch-effort likelihood is omitted, power appears to be very high for either the marginal or conditional likelihoods. This is dependent on both the relatively high sample size (in terms of number of animals per year, number of years, and amount of auxiliary data) and the magnitude of the difference in simulated harvest vulnerability between juveniles and adults. Lower simulated magnitudes should lead to lower power, although differences in juvenile and adult harvest vulnerability significantly smaller than that which was simulated is unlikely to be meaningful from a wildlife game management perspective.

From the comparison presented, it appears that the absolute-recruit abundance models and conditional-likelihood/Horvitz-Thompson models select a more parsimonious model, even when it is not warranted if one includes the auxiliary catch-effort likelihood. The results also indicate these models are more powerful at detecting when multiple harvest coefficients may be necessary when this extra likelihood component is omitted. The stock-recruit model (only fitted by incorporating the additional likelihood component) indicates relatively high power, but very high Type I error rate, again indicating a propensity to select a more complex model. Increasing stochasticity tends to lead to an increasing Type I error rate as well as a decreased Type II error rate (increased power) for all models considered here. In all cases, the mixed-effects model formulations show equivalent results (as in the case of conditional-likelihood/Horvitz-Thompson models) or outperform their fixed-effects-only counterparts (for other model structures). In general, the practitioner is likely to prefer a higher powered model to detect when age classes need to be considered "different" that also has a higher-than-nominal Type I error rate, instead of an underpowered model with a below-nominal Type I error rate, the latter providing a preference toward lumping age classes together when it may not be warranted. In this case, one would choose to omit the auxiliary likelihood component, as this tends to lead to higher power (along with slightly elevated Type I error rate). The omission of the auxiliary likelihood component has also been associated with lower bias in parameter estimates for the preferred model (the conditional-likelihood/Horvitz-Thompson model) based on these simulation studies.

With respect to environmental covariates that one may choose to include in the process parameter transformations, knowledge of the environment and population will be required to determine which environmental covariates might belong to which process. That is, it may be true that early springtime temperature may affect natural mortality and reproduction, but the effect is more likely to be observed in the rate of absolute amount of reproduction for a small game animal. In general, it is wise to include relevant covariates in only one process, as correlation between parameter estimates may provide numerical difficulties in the parameter estimation process, and to avoid overfitting. Likelihood-ratio tests can again be used to compare nested models in a stepwise fashion to select the most appropriate covariates, or AIC may be used to select among a large class of covariates, or for multimodel inference.

2.4.4 Robustness Simulations

Most previous simulation study results have come from simulations where the average abundance trend is flat; that is, the population is neither growing nor declining when averaged over simulation runs. In order to determine if the results seen in previous sections are in some way dependent on this characteristic of simulated data, this section serves to highlight some simulation results from populations that have been simulated to be non-stationary or periodically fluctuating.

In this section, I highlight three general scenarios. In the "increasing s" scenario, I choose an annual survival probability that slowly increases (on average) over the course of the harvest data. I also examine a "decreasing s" scenario, where natural survival probability gradually declines over the course of the harvest data. The intention of these first two robustness simulations is to determine what effect a changing parameter value may have on resulting parameter estimates, and to see if the random effects for process parameters can accommodate these changes better than their fixed-effects counterparts. Finally, I examine a scenario where overall population abundance is relatively flat (on average), but there is a periodic pulse of births during years 4, 8, 12, 16, 20, and 24, and a complementary crash in recruitment rate during years 2, 6, 10, 14, 18, and 22. The intent of this latter simulation study is to determine if abundance or other process parameter estimates from various model formulations are sensitive to fluctuating behavior in the stock-recruit relationship. Figure 2.5 contains an example of the first ten years of simulated annual abundance from each such scenario.

For simulations where s was gradually increased, the population was simulated to number approximately 4000 individuals in year 1, and survival in year i was computed as

$$s_i = \frac{1}{1 + e^{-(\beta + ib + \epsilon_i)}} \tag{2.48}$$

where ϵ_i is drawn from the $N(0, \sigma = 0.1)$ distribution (as in the low variation simulations previously presented), and b is a constant chosen, along with β, so that the population roughly doubles from year 1 to year 25. For simulations where s was gradually decreasing, the absolute abundance was first simulated to number 8000 individuals, and Equation (2.48) was used with an appropriate choice for the constant b to provide for a population that was approximately halved after 25 years (again, on average).

For simulations where the population experienced a heightened stock-recruitment during years where $modulus$(year, 4) = 0 and a sharp drop in recruitment rate when $modulus$(year, 4) = 2, the simulation drew γ_i from the $N(\gamma, 0.1)$ during odd-numbered years or the $N(\gamma \pm g, 0.1)$ distribution every second year, where $g > 0$ is the constant used, along with the choice of γ, to alter the stock-recruit relationship every 2 years, which are chosen such that the population averages roughly 4000 individuals over years 1 through 25. The remaining simulation parameters required to be set at the start of simulations were set according to the "low variation" parameter values in Table (2.3).

2.4.5 Robustness Results

Abundance reconstruction results for the three simulation studies described above are displayed in Figure (2.6). The least-biased estimates of total annual abundance come from the fixed-effects-only conditional likelihood model, $s_F c_F$. The fixed-effects absolute-recruit abundance model ($N_A s_F c_F$) also shows a low degree of bias for the increasing-s and periodic recruitment simulations. Of the remaining models, the mixed-effects models that implement the Horvitz-Thompson abundance estimator have the lowest absolute median relative bias ($< 5\%$ throughout). The mixed-effects versions of the stock-recruit model show bias of approximately -10%, while the mixed-effects versions of the absolute-recruit abundance models show signifi-

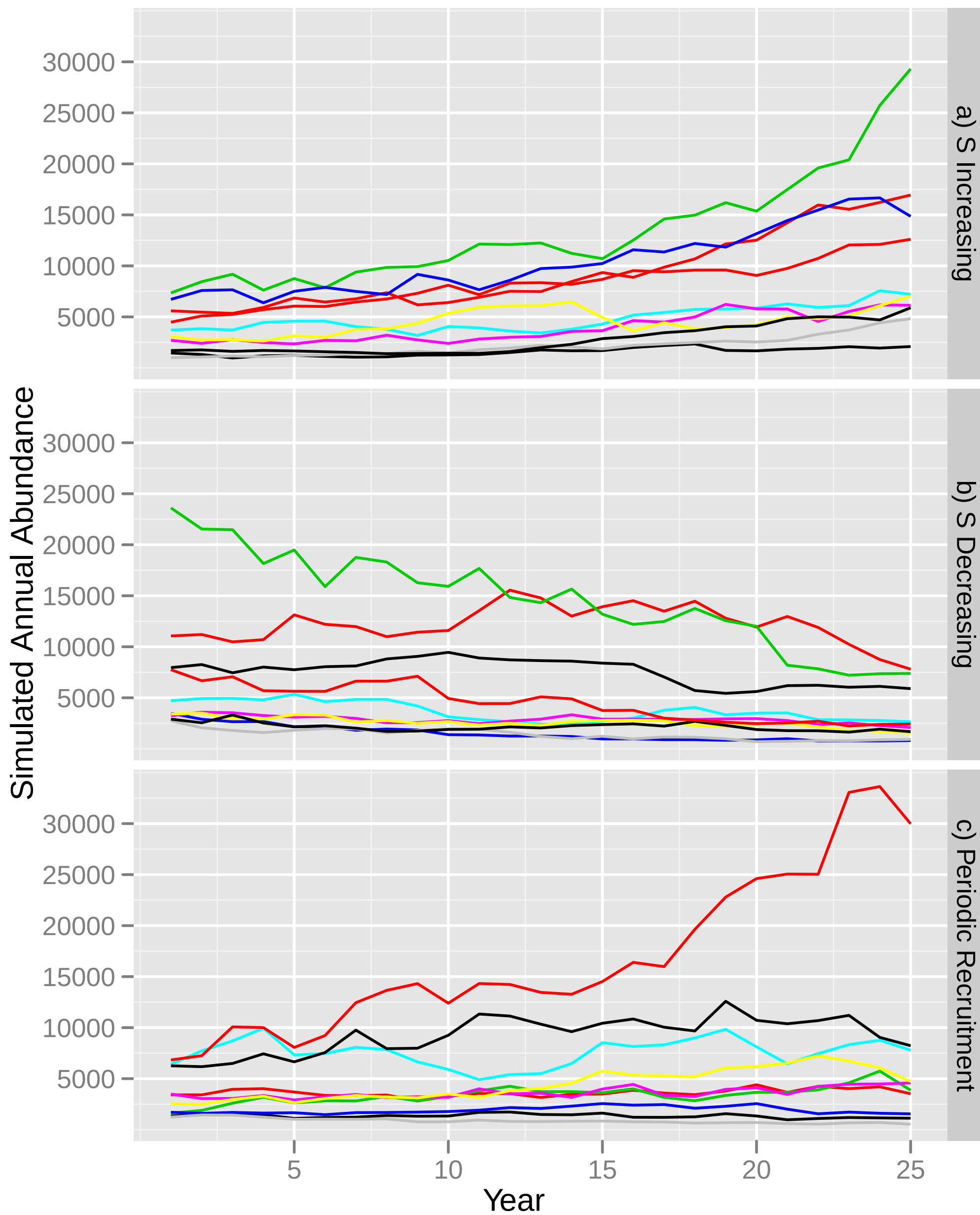


Figure 2.5: *Sample of simulated total annual abundance for robustness simulations where a) s is increasing over time, b) s is decreasing over time, and c) there are periodic fluctuations in recruitment rate.*

cant negative bias, at approximately -25 to -30% for each of the robustness simulations. Percent bias appears to be on the same absolute scale as the primary simulations presented previously (Section 2.4.1), where abundance is simulated to be neither growing nor declining, at low levels of process variation.

A bivariate summary of median relative bias and asymptotic normal-based 95% confidence interval coverage for total annual abundance for each model for these simulations (Figure 2.7) indicates qualitative results similar to those for the primary simulations: the conditional-likelihood models with the second-stage Horvitz-Thompson abundance estimator have the nearest-to-nominal confidence interval coverage and lowest absolute median relative bias, all fixed-effects models underestimate standard errors, mixed-effects absolute-recruit abundance models have poor coverage due (at least partially) to high negative bias, and models with a random stock-recruit relationship overestimate standard errors for most of the later years of abundance reconstruction.

Results for robustness simulations indicate that the relative ranking of models with respect to the statistical criteria considered here, is not sensitive to the deviations from standard model assumptions that I have simulated.

2.4.6 Additional Robustness Results: Identifiability

As mentioned previously, there appears to be some difficulty involved with simultaneous estimation of environmental stochasticity in harvest probability (σ_c) and natural survival (σ_β) when these sources of environmental stochasticity both operate on an annual basis. It is hypothesized that this may be due to an issue of identifiability. That is, when multiple random effects operate on the same levels of a "grouping variable" (here, they are grouped by year), are they simultaneously identifiable (estimable)? Previous simulations indicated that even in circumstances when both σ_β and σ_c were simulated to be greater than zero, model fits frequently provided estimates of $\widehat{\sigma}_\beta = 0$ while $\widehat{\sigma}_c > 0$. In order to determine if the estimation of $\widehat{\sigma}_c > 0$ and $\widehat{\sigma}_\beta = 0$ represents a tradeoff, wherein the estimated total interannual variation can be put in only one of these two "buckets", I chose to simulate from model $N_{SR,R}s_Rc_F$, which is the model where both reproduction and natural survival have random interannual components, but

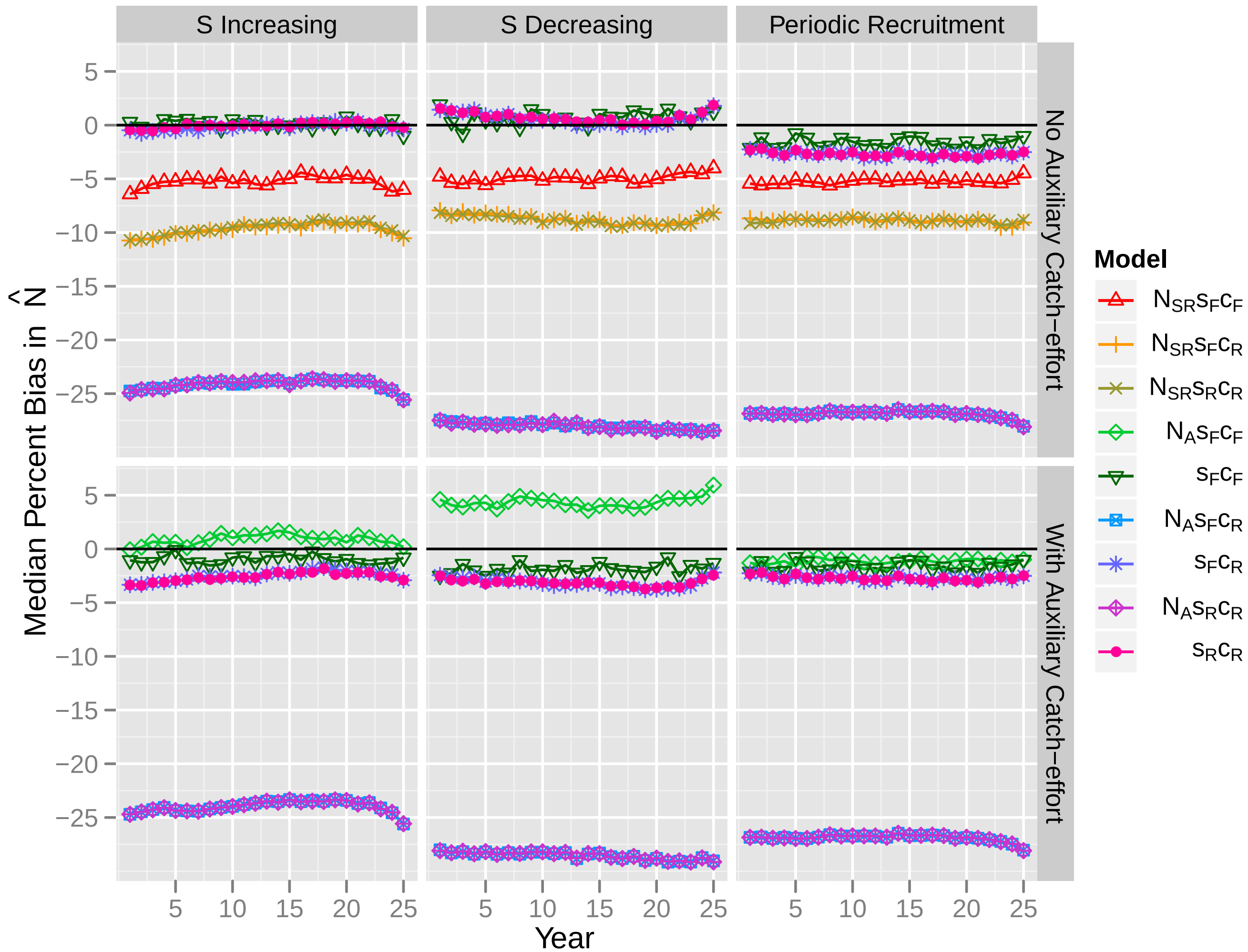


Figure 2.6: *Median relative bias in estimated total annual abundance, robustness simulation studies. Results indicate that conditional-likelihood models with the Horvitz-Thompson estimator abundance estimator have low bias, as does the fixed-effects-only absolute-recruit abundance model of Gove (1997).*

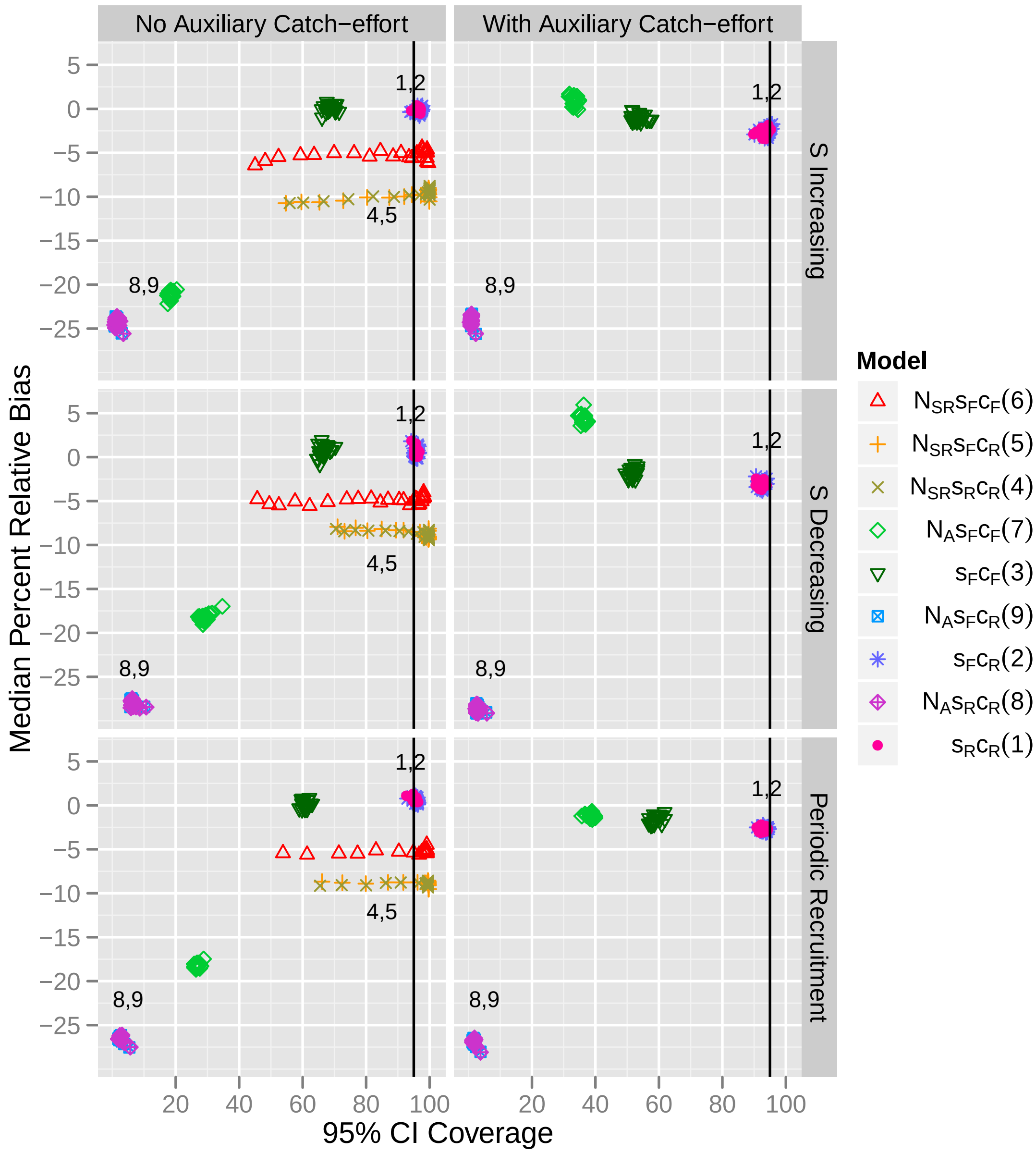


Figure 2.7: *95% confidence interval coverage and median relative bias of estimated total annual abundance for robustness simulations. Mixed-effects conditional-likelihood models s_Rc_R and s_Fc_R have low absolute median relative bias and nearest-nominal (95%) CI coverage. Fixed effects models s_Fc_F and $N_As_Fc_F$ have low absolute median relative bias but also low (subnominal) CI coverage, indicating poor variability estimation.*

harvest probability is described by only the fixed parameter, c. I then fit the set of models that matches this simulation model ($N_{SR,R}s_Rc_F$, $N_As_Rc_F$, and s_Rc_F) in order to determine if instead of $\widehat{\sigma}_c > 0$ and $\widehat{\sigma}_\beta = 0$, results would yield $\widehat{\sigma}_c = 0$ and $\widehat{\sigma}_\beta > 0$. One thousand simulations of the each of the primary simulation input parameters (Table 2.3) was conducted for each of these three models.

Figure (2.8) describes the distribution of estimates of σ_β arising from each model. Results indicate a consistent propensity to severely overestimate variation in natural survival. When compared with Figures (A.4) and (A.5) in Appendix A, it is obvious that the hypothesis that the variation is simply put into one of two "buckets" (σ_c or σ_β) is not true. While models that include a random effect for both natural survival and harvest vulnerability do show some large positively-biased outliers, the estimation of both σ_c and σ_β is generally more reasonable when a random effect for harvest vulnerability is included, and the random effect for natural survival probability is optional.

In order to examine what effect fitting models $N_{SR,R}s_Rc_F$, $N_As_Rc_F$, and s_Rc_F to the simulation model $N_{SR,R}s_Rc_F$ might have on estimates of annual abundance relative to models $N_{SR,R}s_Fc_R$, $N_As_Fc_R$, and s_Fc_R, Figure (2.9) may be compared with Figure (2.2). From this comparison, it is evident that models $N_As_Rc_F$ and s_Rc_F (when simulations come from $N_{SR,R}s_Rc_F$) have greater bias than models $N_As_Fc_R$ and s_Fc_R (when simulations come from $N_{SR,R}s_Rc_F$), which increases as simulated variation increases. Model $N_{SR,R}s_Rc_F$ shows similar bias as model $N_{SR,R}s_Fc_R$ for lower values of simulated variation, but has significantly greater bias than $N_{SR,R}s_Fc_R$ for medium and high values of simulated variation. It is especially surprising to note that models s_Fc_F, s_Fc_R, and s_Rc_R all show similar degrees of bias in all situations, but that model s_Rc_F shows significantly greater bias.

These results appear to indicate that when interannual enviornmental stochasticity operates on the population through harvest effort and vulnerability and/or natural survival, that models that fail to incorporate a random effect term for harvest vulnerability but *do* incorporate a random effect term for natural survival variation may produce severely biased results. For this reason and for reasons presented previously, it is recommended to include a random effect for harvest vulnerability, and optionally include an additional random effect for natural survival (subject to model selection procedures, of course).

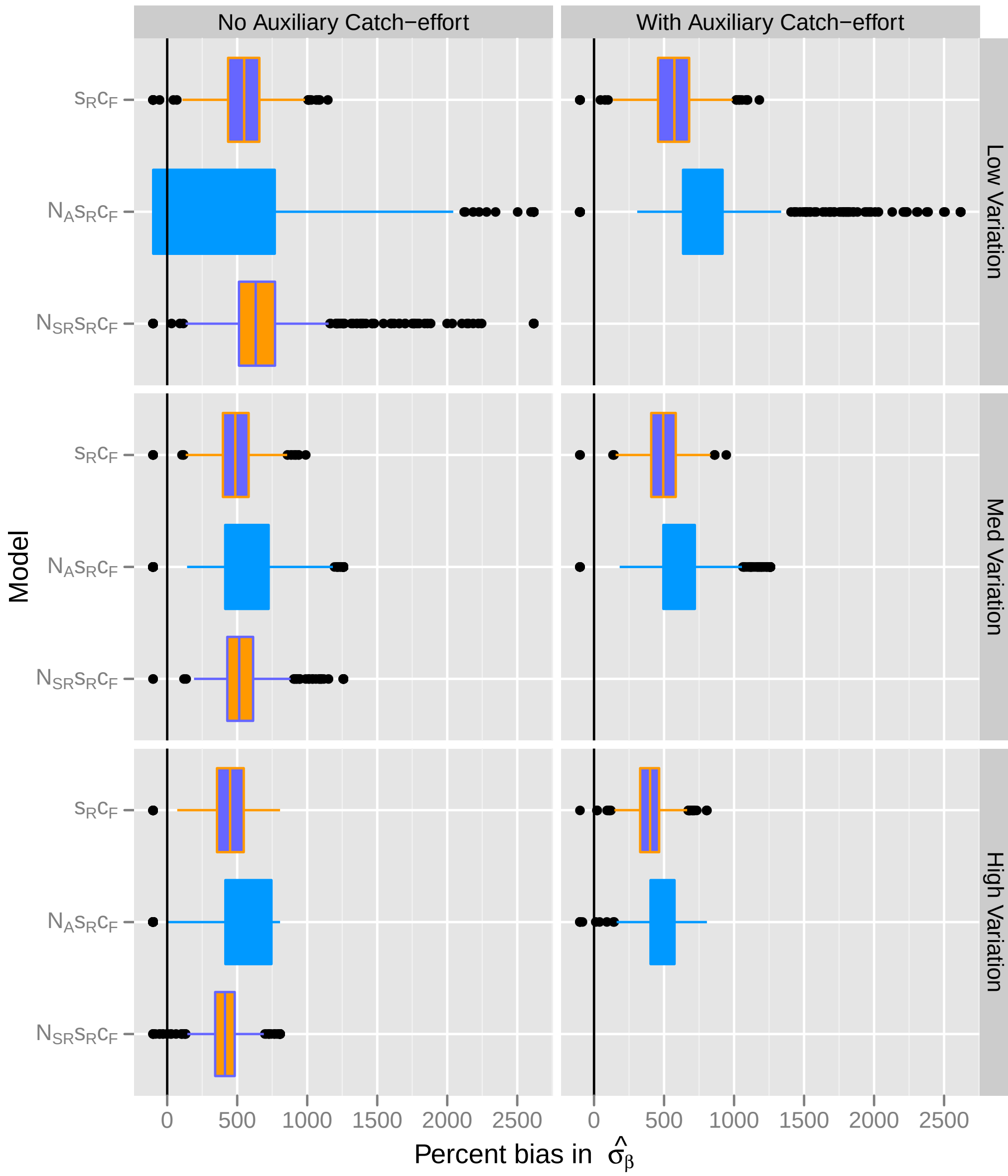


Figure 2.8: *Distribution of estimates of $\widehat{\sigma}_\beta$ from models with no simulated random effect for harvest vulnerability (simulation model matches $N_{SR,R}s_Rc_F$, where $\sigma_\beta > 0$ but $\sigma_c = 0$). Models fit ($N_{SR,R}s_Rc_F$, $N_As_Rc_F$, and s_Rc_F) therefore "match" the simulation model, and should produce reasonable estimates of σ_β, even with the σ_c component omitted from the model. Results obtained from 25 years of simulated data with total abundance $n \approx 4000$ across years, $p \approx .27$, $s \approx .84$. Results indicate extreme positive bias in estimation of σ_β when the σ_c component is omitted, even when σ_c is simulated to be zero, but there is other interannual variation simulated in the reproduction and natural survival processes, indicating a random effect for harvest probability should be included in all models to avoid this estimation bias, at the expense of frequent estimation of $\widehat{\sigma}_\beta = 0$.*

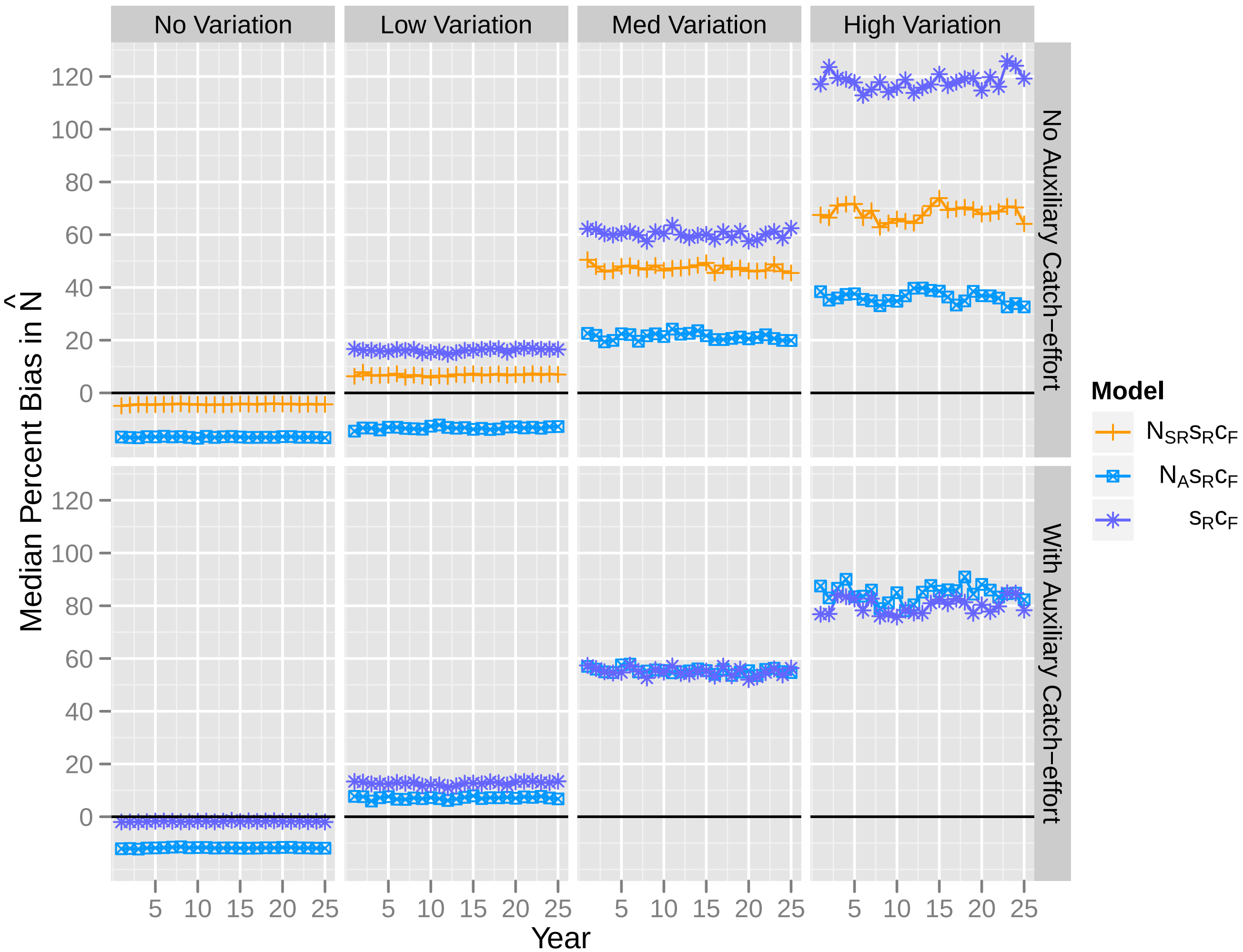


Figure 2.9: *Median relative bias in estimated total annual abundance for models with a random effect for survival and no random effect for harvest probability. Results obtained from 25 years of simulated data with total abundance $n \approx 4000$ across years, $p \approx 27\%$, $s \approx 84\%$. Results indicate increasing positive bias with level of simulated variation.*

2.4.7 Additional Robustness Results: Amount of Auxiliary Data

Primary simulation results and all previous robustness simulation results included 7 years of binomial mark-harvest data intended to aid in the estimation of the harvest vulnerability coefficient, c, as well as 4 years of binomial radiotelemetry data intended to aid in the estimation of natural survival probability, s. This may represent a greater amount of auxiliary data than is available in practice. In order to determine if the results seen in previous simulations are in some way dependent on this amount of auxiliary data available, we may simulate varying degrees of auxiliary data, and determine what effect, if any, this has on the results.

As previous results indicate that mixed-effects versions of the conditional-likelihood/Horvitz-Thompson model ($s_F c_R$ and $s_R c_R$) provide abundance estimates with low bias and confidence intervals of the appropriate width, it is of interest to determine if model performance changes with the level of auxiliary data provided. For this purpose, simulations were conducted with the levels of auxiliary data described in Table (2.15). Each non-empty cell in the table lists the number of years (NY) of auxiliary data used in the simulation. All simulations were conducted with the remaining input parameters as described in Table (2.3). For all simulations, 20 simulated animals are tagged each year (except, of course, when no auxiliary data of one type or the other is simulated).

Results at the low, medium, and high level of simulated variation for each of the scenarios described in Table 2.15 (Figures 2.10, 2.11, and 2.12, respectively) all indicate that regardless of the amount of simulated auxiliary data, annual abundance estimates arising from mixed-effects versions of the conditional-likelihood/Horvitz-Thompson model show low median bias, and estimated confidence interval coverage of asymptotic 95% confidence intervals very near 95%. Some small patterns are apparent in the results; slight subnominal coverage is exhibited when no auxiliary data are available for estimation of harvest vulnerability. In addition, slight positive bias is exhibited when no auxiliary data are available for estimation of survival probability. These deficiencies, however, are minor, especially when compared to the poor behavior of some other models for age-at-harvest data when a greater amount of auxilary data is available, as discussed in previous sections of this chapter.

Based on these simulations, which included some scenarios where only a single year of radio telemetry data was available, the mixed-effects versions of the conditional-likelihood/Horvitz-Thompson indicate the ability to accurately and precisely estimate total annual abundance. The model appears to be robust to varying levels of auxiliary data, when model assumptions are satisfied (independence of animals, stationary, symmetrically fluctuating demographic processes, no immigration or emigration, etc.).

Table 2.15: *Description of simulations to examine amount of auxiliary data. Each non-empty cell specifies the level of auxiliary data for a set of simulations that otherwise follow the simulation study design presented in Table (2.3). Each simulation conducted at the low, medium, and high level of simulated variation in process parameters. NY(s), NY(p) = number of years of auxiliary data available for estimation of s and p, respectively. For each scenario where auxiliary data are simulated, 20 simulated animals are tagged each year.*

		Auxiliary Data for s		
		Zero	Low	High
Aux.	Zero		$NY(s) = 1$	$NY(s) = 4$
Data	Low	$NY(p) = 3$	$NY(s) = 1$, $NY(p) = 3$	
for c	High	$NY(p) = 7$		$NY(s) = 4$, $NY(p) = 7$

2.5 Data Analysis: Michigan Elk Herd

2.5.1 Data Description

In order to illustrate the performance of the recommended model as well as the new stock-recruit model formulation and the addition of random effects to the new models and the classic absolute-abundance models, in this section I fit the relevant models to a dataset regarding elk harvest in the State of Michigan, USA.

The available data (Tables 2.16 and 2.17) are comprised of age-at-harvest data from the elk (*Cervus elaphus*) population in the northern part of the lower peninsula of Michigan, spanning the years 1991 - 2008. The harvest, encompassing both genders and all age classes, was conducted primarily during fall (August and September) and winter (December) hunts, with three

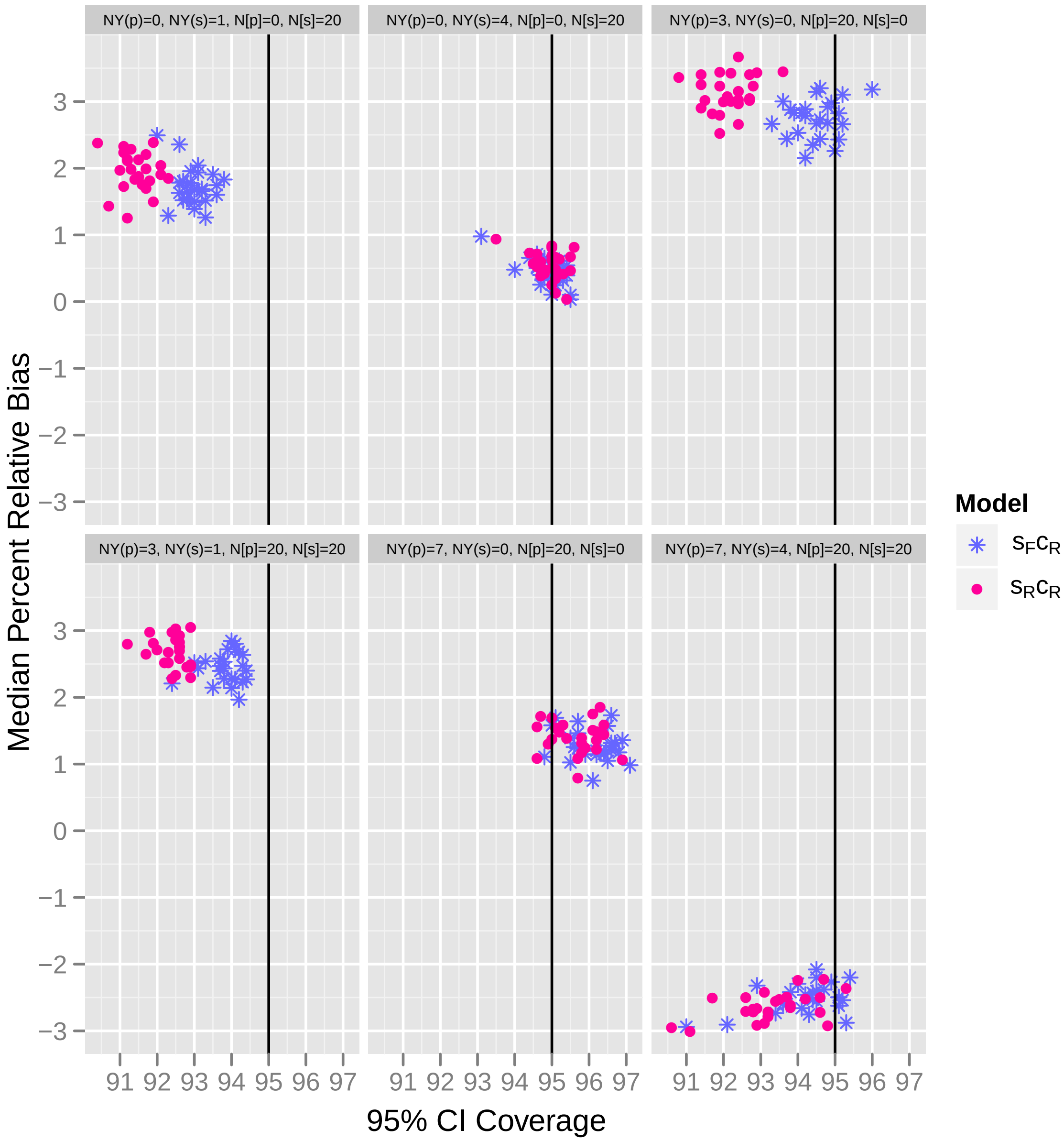


Figure 2.10: *Median relative bias in estimated total annual abundance and estimated asymptotic 95% confidence interval coverage for mixed-effects conditional-likelihood/Horvitz-Thompson models $s_F c_R$ and $s_R c_R$ at varying levels of auxiliary data, at the low level of simulated variation. NY(p), NY(s) = number of years of annual binomial auxiliary data to estimate harvest probability p and survival probability s, respectively. N[p] and N[s] indicate number of animals available* ***per year*** *for auxiliary binomial data to aid in estimation of p and s, respectively. Results obtained from 25 years of simulated data with total abundance $n \approx 4000$ across years, $p \approx 27\%$, $s \approx 84\%$. Results indicate low bias and near-nominal CI coverage for all scenarios.*

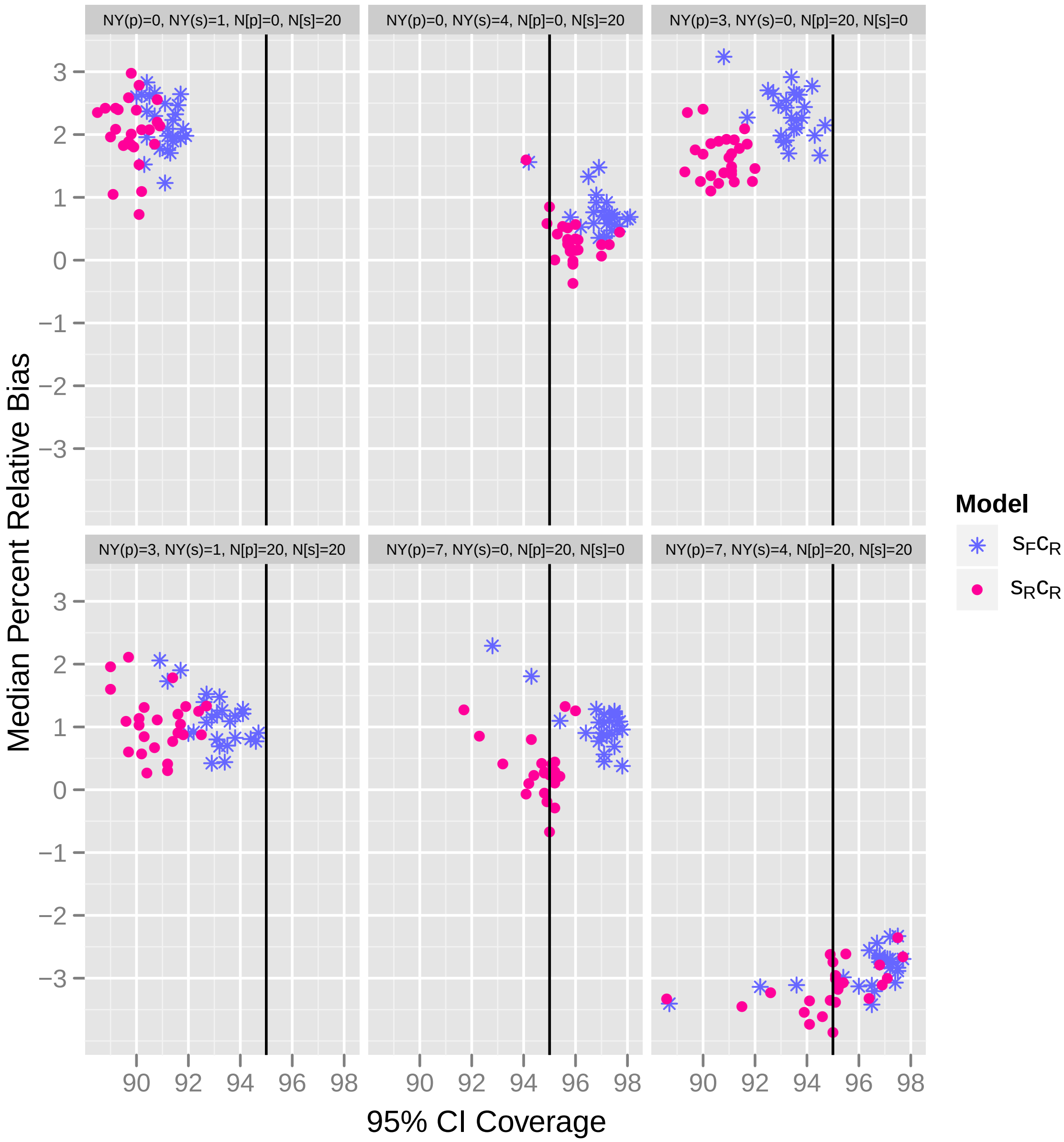


Figure 2.11: *Median relative bias in estimated total annual abundance and estimated asymptotic 95% confidence interval coverage for mixed-effects conditional-likelihood/Horvitz-Thompson models $s_F c_R$ and $s_R c_R$ at varying levels of auxiliary data, at the medium level of simulated variation. NY(p), NY(s) = number of years of annual binomial auxiliary data to estimate harvest probability p and survival probability s, respectively. N[p] and N[s] indicate number of animals available* ***per year*** *for auxiliary binomial data to aid in estimation of p and s, respectively. Results obtained from 25 years of simulated data with total abundance $n \approx 4000$ across years, $p \approx 27\%$, $s \approx 84\%$. Results indicate low bias and near-nominal CI coverage for all scenarios.*

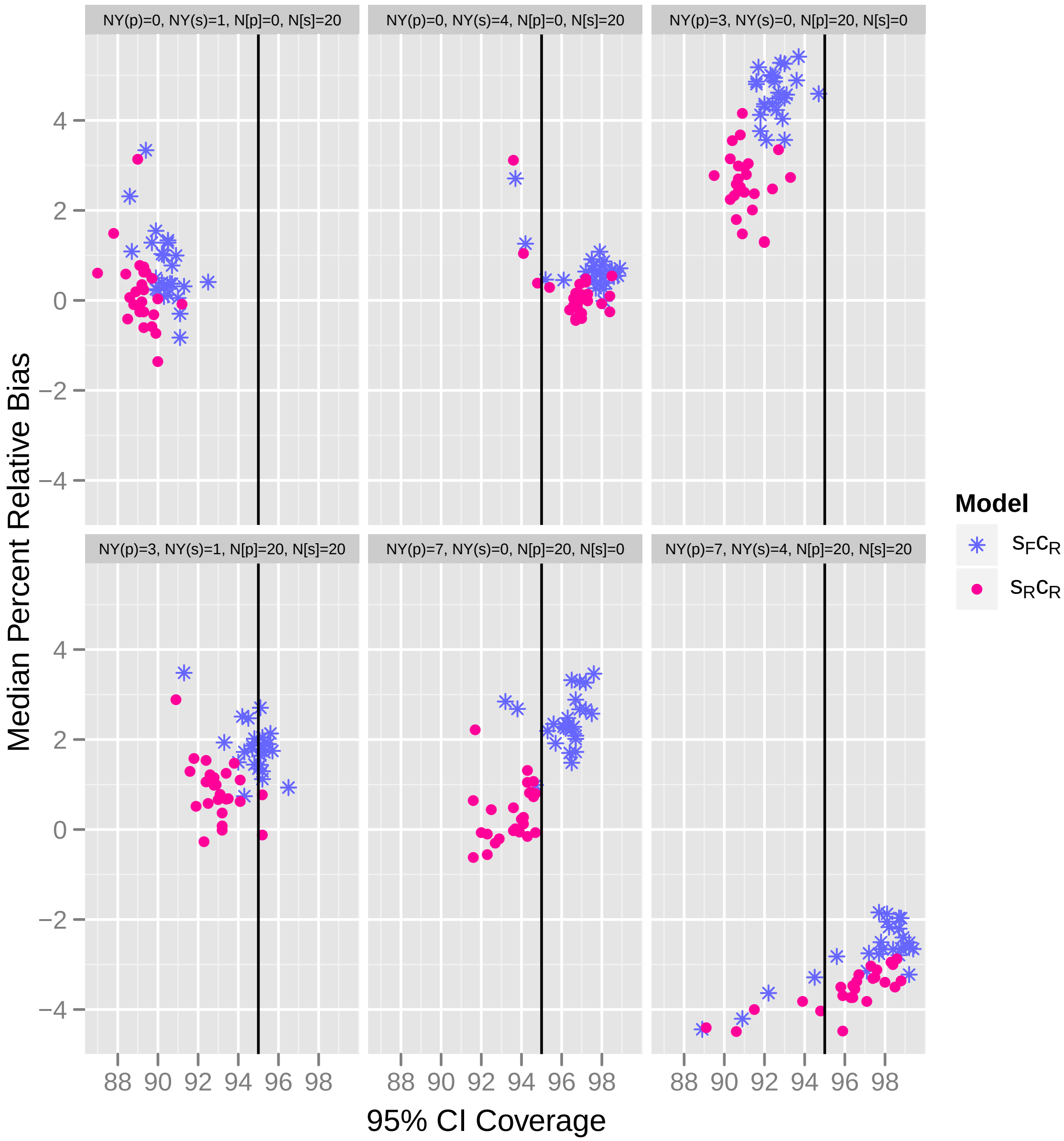


Figure 2.12: *Median relative bias in estimated total annual abundance and estimated asymptotic 95% confidence interval coverage for mixed-effects conditional-likelihood/Horvitz-Thompson models* $s_F c_R$ *and* $s_R c_R$ *at varying levels of auxiliary data, at the high level of simulated variation. NY(p), NY(s) = number of years of annual binomial auxiliary data to estimate harvest probability p and survival probability s, respectively. N[p] and N[s] indicate number of animals available* ***per year*** *for auxiliary binomial data to aid in estimation of p and s, respectively. Results obtained from 25 years of simulated data with total abundance* $n \approx 4000$ *across years,* $p \approx 27\%$, $s \approx 84\%$. *Results indicate low bias and near-nominal CI coverage for all scenarios.*

years of permitted hunting in January (2001, 2007, and 2009) in varying locations. The fall and winter hunts occurred during approximately the same time each year, for approximately the same duration (8 - 13 days for fall hunt, 5 - 7 days for winter hunt), for the same hunting areas. The January hunting periods varied in their spatial location, their duration, and their timing within the month.

Harvested data include a field age, a laboratory-obtained age based on dental annuli (Hamlin et al., 2000; Rosatte et al., 2007), harvest data, gender, and location of the harvest (county, township, range, section). Ages ranged from age 0 (yearling) to 22 years. The rate of aging was high, with 98% of animals having a field age, and 91% of animals having a laboratory-determined age.

As the models considered here are not spatially-explicit, it was not possible to account for the shifting location of the small number of January hunting periods. In order to simplify the analysis and reduce the number of parameters required to fit the separate hunting seasons, I assumed that the combined hunting seasons for a year (where a January hunt in year $i+1$ is included in the winter hunt of year i) occurred in a single hunting period with negligible between-season mortality. Accounting for natural survival between fall and winter hunting periods may be feasible, but the negative correlation between estimates of hunting and natural mortality that are already expected from an analysis such as this would likely be amplified with the inclusion of additional parameters to model the short natural survival process between seasons. Additionally, survival rates for elk tend to be very high, so the impact of this assumption should be low.

Grouping all hunting periods into a single hunting period each year also necessitates the assumption that harvest vulnerability does not change when the area of the open hunting region changes. That is, the animals vulnerable to a January hunt (residing in an open-hunting location) are equally-vulnerable to harvest as animals in the same year from a fall or winter harvest. For fall and winter harvest, the territory open to hunting changed very little. A map of the counties throughout which the hunt was conducted for the period of 1991 to 2008 with an overlayed contour plot representing the density of harvests during this period indicates that the hunt was confined to a relatively small geographic location (Figure 2.13).

A measure of hunting effort is available as the number of permits issued and the length of

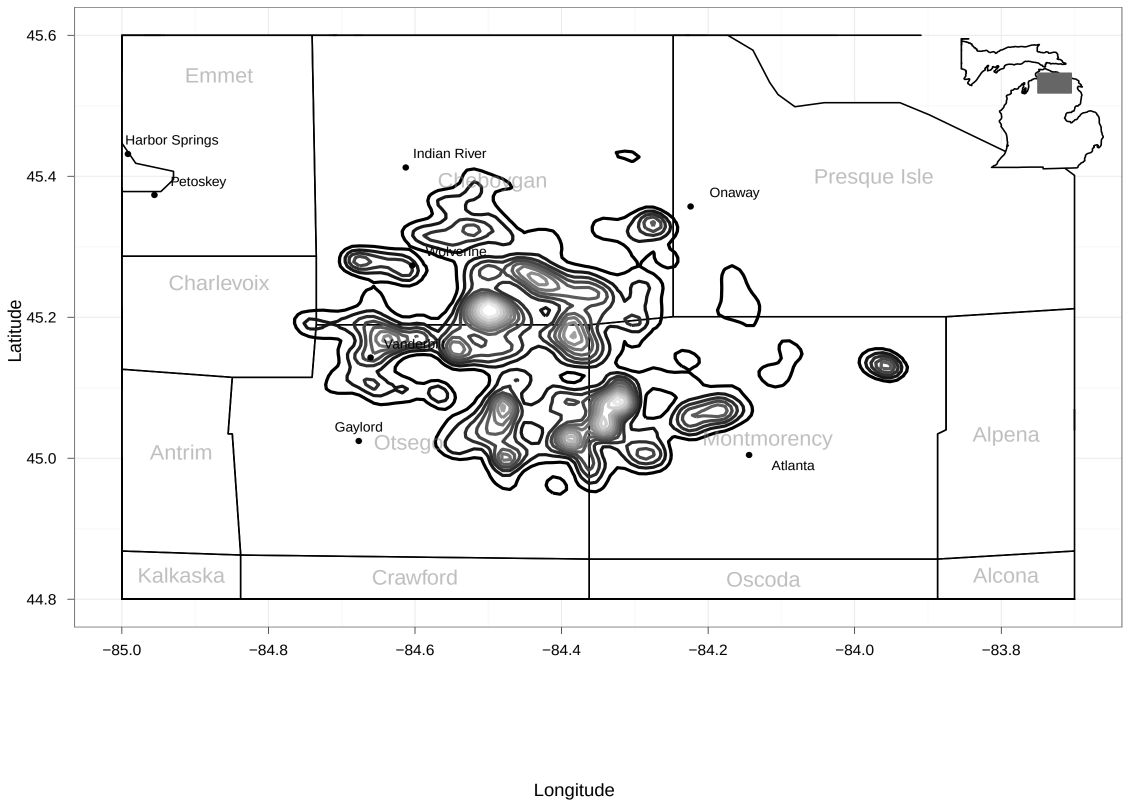


Figure 2.13: *County map with overlayed density contours of elk harvest for the period 1991 to 2008. Higher contours represent greater density of harvest in these locations. Harvest is contained within a relatively small geographic location, and thus it is reasonable to consider this a single, isolated population.*

each hunting season. For each model considered here, hunting effort in year i (the maximum possible number of hunter-days) is computed as

$$f_i = \frac{(\text{number permits issued}) \times (\text{total hunting days in year } i)}{1000}.$$

The scale of hunting effort is chosen to aid in the numerical estimation process.

To aid in the assessment of hunter efficacy, two environmental covariates were examined: minimum temperature during the winter (December) hunt (centered by the mean across years), and average snowfall per day during the winter hunt. These variables were chosen in advance of the modeling process as they were considered likely to have some impact on hunter efficacy. Each covariate was included as a continuous linear predictor following the transformation described in the next subsection for harvest probability. The historical data (Table 2.18) were obtained from the National Oceanic and Atmospheric Administration's National Climatic Data Center website (for 1992 - 2008) and the Michigan State Climatologist's Office website (for 1991) for the Gaylord, Michigan weather station, which was considered to be representative of the weather conditions experienced in the northern part of the lower peninsula during the winter hunting period.

2.5.2 Models

Each of the three model structures (absolute recruit abundance, stock-recruit, and conditional-likelihood/Horvitz-Thompson) were fit to the elk data. It was expected that demographic parameters (especially harvest vulnerability) may differ between genders, so the harvest for each gender was modeled separately, with the joint likelihood computed as the product of the likelihood for the male harvest and the female harvest.

The laboratory-determined age of each harvested animal was used, whenever available. The field age was substituted for the laboratory age whenever necessary and available. Because a low percent ($\approx 2\%$) of ages were not recorded for either aging method, an assumption and model extension were necessary. I assumed that lack of aging was independent of age, such that each age group was equally likely to contain an unaged animal in a given year. An aging probability likelihood was constructed to augment the joint harvest likelihood as

$$L_{i,Aging} = \binom{\sum_{j=1}^{18} x_{ij}}{\sum_{j=1}^{18} x_{ij}^{aged}} a_i^{\sum_{j=1}^{18} x_{ij}^{aged}} (1 - a_i)^{\sum_{j=1}^{18} \left(x_{ij} - x_{ij}^{aged}\right)}, \tag{2.49}$$

where a_i is the probability of an animal being aged in year i, $\sum_{j=1}^{18} n_i^a$ is the total number of aged harvest members in the population in year i, and N_i^h is the total number harvested in year i. The probability a_i (whose MLE is simply $\widehat{a}_i = \frac{\sum_{j=1}^{18} x_{ij}^{aged}}{\sum_{j=1}^{18} x_{ij}}$) is included in the cell probability for the harvest count for each age class of year i, which allows the estimate of parameter uncertainty to include a component for aging uncertainty.

In addition, auxiliary data are available for years 2006 through 2008 in the form of aerial survey data which makes use of a sightability correction model (Walsh, 2007). These data (Table 2.19) were included as an auxiliary likelihood by assuming that each annual abundance estimate as derived by the chosen model during estimation was normally-distributed with a mean equal to the estimate for the auxiliary data, and a standard deviation equal to the estimated standard error from the auxiliary data. That is, if we denote a model-based estimate of total abundance in years 2006 - 2008 as $\widehat{N}_i^{model}$, the auxiliary estimate as $\widehat{N}_i^{aux}$, and the standard error of the auxiliary abundance estimate as $\widehat{\sigma}_{i^{aux}}$ then the auxiliary likelihood of the form

$$L_{auxiliary} = \prod_{i=2006}^{2008} \frac{1}{\widehat{\sigma}_{i^{aux}}\sqrt{2\pi}} e^{\frac{-\left(\widehat{N}_i^{model} - \widehat{N}_i^{aux}\right)^2}{2\widehat{\sigma}_{i^{aux}}^2}} \tag{2.50}$$

was multiplied by the age-at-harvest and aging probability likelihoods to form a complete joint likelihood. The use of this auxiliary likelihood requires computing estimates of abundance for years 2006 through 2008 during each evaluation of the likelihood function during the optimization process. In this respect, the abundance estimation is not strictly a "second-stage" process, as the likelihood optimization process involves these Horvitz-Thompson-based estimators as well. During the optimization process, I compute

$$\widehat{N}_i^{model} = \sum_{j=1}^{A} \widehat{N}_{ij} = \sum_{j=1}^{A} \frac{x_{ij}}{\widehat{p}_{ij}}.$$

For the absolute recruit abundance models based on Gove et al. (2002) and augmented with random effects, the complete likelihood for the at-at-harvest data of cohort A (the cohort on the main diagonal of the age-at-harvest table, beginning with N_{11}) may be expressed as

$$
\begin{aligned}
&L_A = L_A^f \times L_A^m \times L_{i,Aging}^f \times L_{i,Aging}^m = \\
&\binom{N_{11}^f}{x_{11}^{f,aged}, x_{22}^{f,aged}, \ldots, x_{18,18}^{f,aged}} \left(p_{11}^f a_1^f\right)^{x_{11}^{f,aged}} \left((1 - p_{11}^f a_1^f) s_{11}^f p_{22}^f a_2^f\right)^{x_{22}^{f,aged}} \cdots \times \\
&\left(1 - \left(p_{11}^f a_1^f + (1 - p_{11}^f a_1^f) s_{11}^f p_{22}^f a_2^f + \cdots + \sum_{i=3}^{18} \left(\prod_{j=1}^{i} q_{jj}^f s_{jj}^f\right) p_{ii}^f a_i^f\right)\right)^{N_{11}^f - \sum_{i=1}^{18} x_{ii}^{f,aged}} \times \\
&\binom{N_{11}^m}{x_{11}^{m,aged}, x_{22}^{m,aged}, \ldots, x_{18,18}^{m,aged}} (p_{11}^m a_1^m)^{x_{11}^{m,aged}} ((1 - p_{11}^m a_1^m) s_{11}^m p_{22}^m a_2^m)^{x_{22}^{m,aged}} \cdots \times \\
&\left(1 - \left(p_{11}^m a_1^m + (1 - p_{11}^m a_1^m) s_{11}^m p_{22}^m a_2^m + \cdots + \sum_{i=3}^{18} \left(\prod_{j=1}^{i} q_{jj}^m s_{jj}^m\right) p_{ii}^m a_i^m\right)\right)^{N_{11}^m - \sum_{i=1}^{18} x_{ii}^{m,aged}} \times \\
&\left[\prod_{i=1}^{18} \phi_{\sigma_{f,\tau}}(\tau_i^f)\right] \left[\prod_{i=1}^{18} \phi_{\sigma_{f,\beta}}(\epsilon_i^f)\right] \left[\prod_{i=1}^{18} \phi_{\sigma_{m,\tau}}(\tau_i^m)\right] \left[\prod_{i=1}^{18} \phi_{\sigma_{m,\beta}}(\epsilon_i^m)\right] \times \\
&L_{i,Aging}^f \; L_{i,Aging}^m
\end{aligned} \tag{2.51}
$$

where

$$
\begin{aligned}
p_{ij}^f &= 1 - e^{-e^{\left(\left(c_{ij}^f + \tau_i^f\right) + \lambda_1 y_i + \lambda_2 z_i\right) f_i}}, \\
p_{ij}^m &= 1 - e^{-e^{\left(\left(c_{ij}^m + \tau_i^m\right) + \lambda_1 y_i + \lambda_2 z_i\right) f_i}}, \\
s_{ij}^f &= \frac{1}{1 + e^{-\left(\beta + \epsilon_i^f\right)}}, \\
s_{ij}^m &= \frac{1}{1 + e^{-\left(\beta + \epsilon_i^m\right)}}, \\
q_{ij} &= 1 - p_{ij} a_i,
\end{aligned} \tag{2.52}
$$

and where $\phi_{\sigma_x}(x)$ refers to the normal distribution density centered at 0 with a constant variance of σ_x^2, in accordance with the assumption of normality of random effect terms, and where λ_1 and λ_2 are regression coefficients that correspond to the covariates values of centered minimum temperature (y_i) and average snowfall per day (z_i), respectively, during the winter hunting period.

For stock-recruit models, the aging and auxiliary likelihoods are identical, as are the first A cohorts of the age-at-harvest likelihood. For cohorts beginning in or after 1992 (the second year of available data), the replacement is made

$$N_{i1}^{f} = N_{i1}^{m} = \frac{1}{2} e^{\gamma+\delta_i} \left(\sum_{j=2}^{A} N_{i-1,j}^{f} \right). \tag{2.53}$$

That is, recruit abundance in year i has a multiplicative relationship to female abundance in year $i-1$, for females aged 1 year and older (excluding yearlings), and the sex ratio of the recruit class is 1:1. The product-multinomial likelihood proceeds in a manner analogous to Equation (2.51) with the transformations of Equation (2.52). The age-at-harvest likelihood must also be augmented with the product of densities for the assumed distribution of the $\vec{\delta}_i$, which I assume to be distributed as $\delta_i \sim N(0, \sigma_{\delta}^2)$, as in the previous section regarding simulation results (Section 2.2).

For the conditional-likelihood/Horvitz-Thompson models, the age-at-harvest likelihood is constructed as described in Equation (2.30), with the transformations of Equation (2.52). The aging probabilities a_i are also included in the cell probabilities in a manner similar to that of Equation (2.51).

In each case, the complete joint likelihood of age-harvest data, aging probability, and auxiliary abundance estimates may then be written as

$$L_{complete} = \left[\prod_{i=1}^{A+Y-1} L_i^f \; L_i^m \right] \left[\prod_{i=1}^{Y} L_{i,Aging}^{f} \; L_{i,Aging}^{m} \right] L_{auxiliary}. \tag{2.54}$$

The marginal (integrated) likelihood for each model was optimized with ADMB (Fournier et al., 2011) .

2.5.3 Results

Model selection, performed first with the conditional-likelihood model, was implemented following the general recommendations for generalized linear mixed models described in Zuur et al. (2009, pg. 90 - 92, 130 - 139), wherein a model incorporating all feasible random and fixed effects was examined first for reduction of random effects terms, and then fixed effects were selected once a random-effects structure was chosen. Model selection for random effects terms was performed by using likelihood ratio tests as described in the previous section, wherein a mixture of χ^2 p-values was calculated based on the appropriate degrees of freedom. For fixed effects, likelihood ratio tests were used for nested models and both marginal AIC and conditional AIC were examined for nonnested models.

In the initial step, σ_β was estimated to be near 0 and a likelihood ratio test using the maximum conditional log-likelihood value of the form described by Self and Liang (1987) indicated the parameter (and hence the random effect for survival) was not necessary, and thus a random effect for survival was not included in the model. Likelihood ratio tests indicated the model was improved with separate harvest vulnerability random effects for males and females ($\sigma_\tau^f/\vec{\tau}^f$ and $\sigma_\tau^m/\vec{\tau}^m$), and with a random effect for the reproductive parameter ($\sigma_\gamma/\vec{\delta}$). With respect to random effects, the model selected is therefore coded as $s_F c_R$.

Repeated stepwise and joint testing of the number of harvest vulnerability coefficients indicated the optimal model had separate coefficients for gender, and separate coefficients for age classes 0, 1, 2, 3, and 4+ (10 total harvest vulnerability coefficients). This was determined by alternately increasing and decreasing the number of coefficients by testing with LRTs using both the marginal (integrated) and conditional (non-integrated) likelihoods. Results for both mAIC and cAIC were qualitatively similar. Neither environmental covariate was selected into any of the three models, based on AIC comparisons.

A single survival probability s was used for all animals, as no auxiliary data were available to examine the possible separation of juvenile and adult natural survival. Model-based estimates of natural survival did not differ between genders (LRT $p = 1.0$), which may be biologically indicated, or may be a result of the very high estimate for the parameter (Table 2.20), which is near its upper bound.

Model selection for the absolute recruit abundance model ($N_A s_R c_R$) and the stock-recruit model ($N_{SR,R} s_R c_R$) proved difficult due to numerical instability of the likelihood optimization for some parameter combinations. Penalty functions to restrict the magnitude of random effects terms during estimation enhanced stability of optimization, but led to likelihood values that could not be compared to one another due to their different scales. For these reasons, the form of the model for the $s_R c_R$ structure was chosen as a "beginning" model for model selection purposes for models $N_A s_R c_R$ and $N_{SR,R} s_R c_R$. Likelihood ratio tests were then used to selectively drop or add fixed effects terms related to harvest vulnerability, survival, and environmental covariates.

Results for model selection for model $N_A s_R c_R$ indicate that the same process parameters used for $s_F c_R$ should be used for models $N_{SR,R} s_F c_R$ and $N_A s_F c_R$, but that model $N_{SR,R} s_F c_R$ benefited from including an additional pair of harvest vulnerability coefficients, c^f_{5+} and c^m_{5+}. The final set of models is therefore denoted $s_F c_R$, $N_A s_F c_R$, and $N_{SR,R} s_F c_R$. A summary of parameter estimates illustrates very similar parameter estimates for process parameters (Table 2.20).

Total annual abundance estimates are similar for each model structure (Figure 2.14 and Appendix A, Tables A.17 - A.25). The stock-recruit model generally estimates greater abundance in the early part of the reconstruction than the other two models. Models $N_{SR,R} s_F c_R$ and $s_F c_R$ show close agreement with the final two years of the sightability-corrected auxiliary abundance estimates that were used in the estimation procedure (years 2007 - 2008) (Walsh, 2007). In addition, reconstruction results coincide well with estimates based on a combined ground and air survey with a subjective sightability correction factor produced by the Michigan Department of Natural Resources conducted in years 1992-1994, 1996, 1997, and 1999-2001, except for the 1999 estimate (years 1992 - 2001 not used as auxiliary data), which were not used as data for the model, but rather were withheld as a confirmatory dataset. The lower abundance estimates of the absolute-recruit abundance model appears to be in concordance with the negative bias (or, at least, tendency toward lower estimates than the other two model structures) exhibited in the simulation study presented earlier in this chapter.

Uncertainty of annual abundance estimates is illustrated by plots of pointwise confidence intervals about the estimated annual abundance (Figure 2.15). While all models show similar abundance estimates, they differ markedly in their estimates of uncertainty. Previous simulations results regarding much simpler simulated datasets (Figure 2.4) indicated that the absolute-recruit abundance model tended to underestimate uncertainty, which is reflected in the leftmost panel of Figure (2.15), while the conditional-likelihood/Horvitz-Thompson model showed nearest-nominal confidence interval coverage. Previous simulation results (Figure 2.4) also indicated that the model incorporating a stock-recruit relationship tended to overestimate uncertainty in the later years of the analysis, and underestimate it in the early years of the

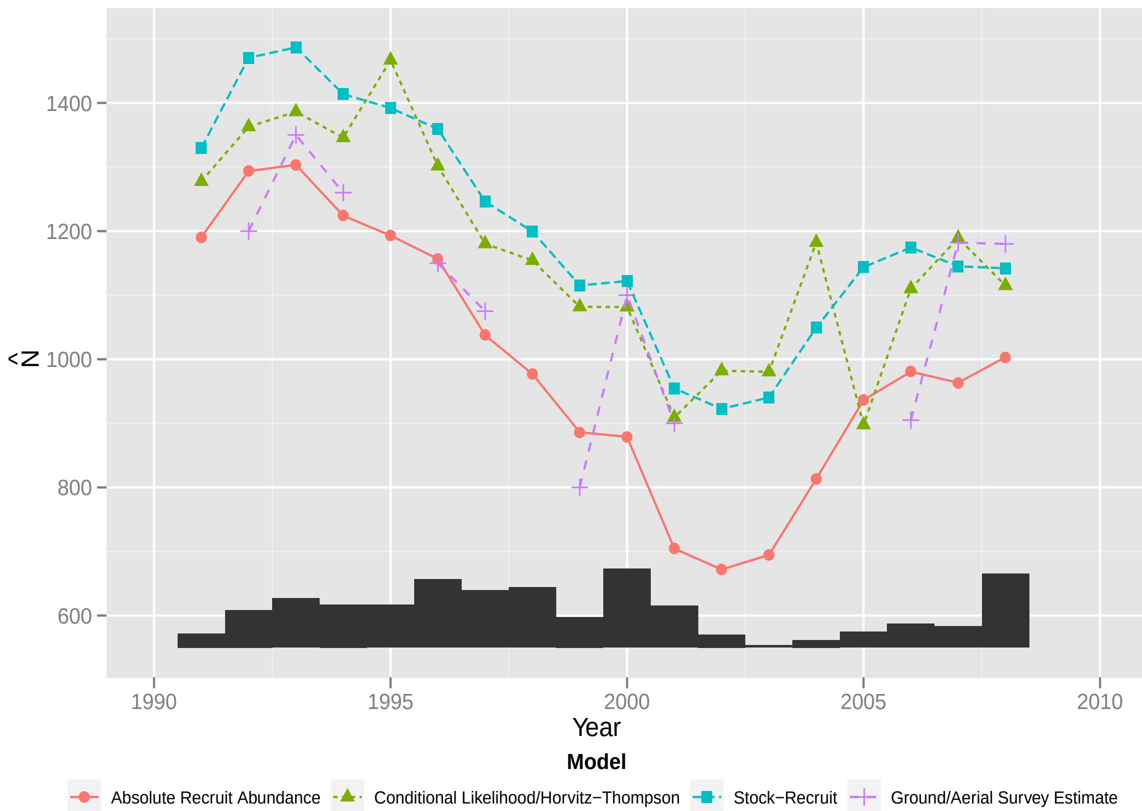


Figure 2.14: *Annual abundance estimates for the Michigan elk population, based on each of three model structures. The Absolute Recruit Abundance model is based on the models of Gove et al. (2002), but has been extended to incorporate covariates and random effects. The Conditional-likelihood/Horvitz-Thompson model is based on a conditional likelihood formulation described in Chapter 2, includes random effects in the harvest process, and implements a second-stage Horvitz-Thompson approach to abundance estimation. The Stock-Recruit model implements a simple multiplicative stock-recruit relationship between recruit abundance and prior female breeding-age abundance. Aerial surveys from 2006 - 2008 incorporated a model-based sightability correction, and were used as auxiliary data. Combined ground and air surveys were conducted in years 1992-1994, 1996, 1997, and 1999-2001 with a subjective sightability correction factor and were not used as auxiliary data, but are plotted for reference purposes. Linearly rescaled harvest effort is presented as vertical bars along the x-axis.*

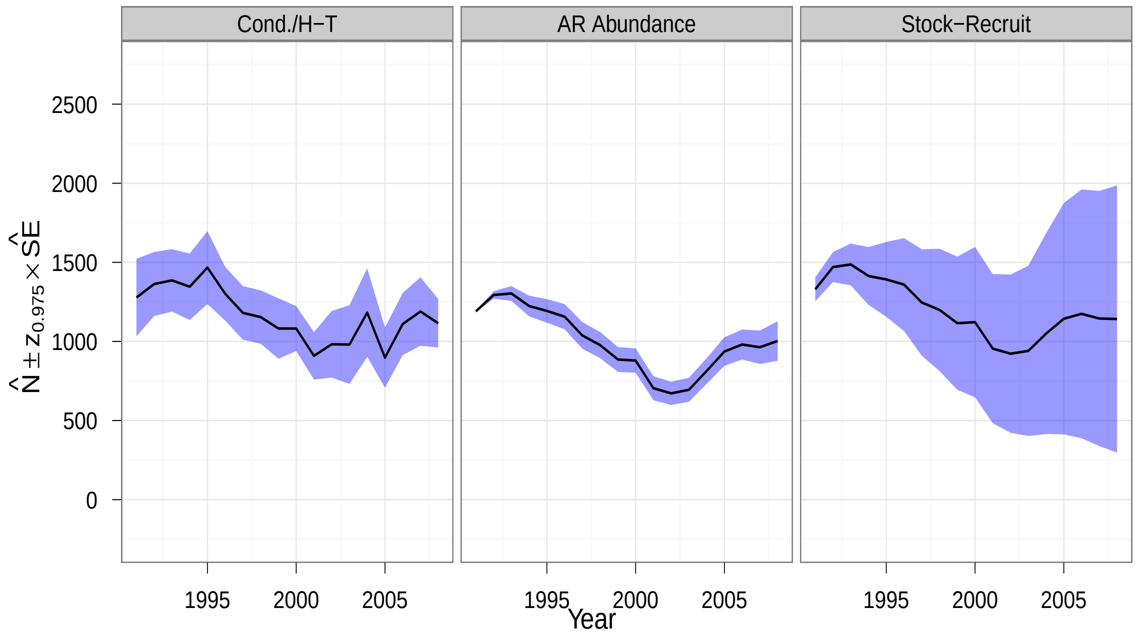


Figure 2.15: *Annual abundance estimates for the Michigan elk population, based on each of three model structures (Cond./H-T = conditional likelihood with Horvitz-Thompson abundance estimation, AR Abundance = absolute-recruit abundance model, Stock-Recruit = stock-recruitment model) with accompanying pointwise 95% confidence intervals.*

analysis, which may also be the case here.

Annual recruitment rate (Figure 2.16), computed as estimated young per breeding-age ($j > 0.5$) female, appears to be relatively stable across all years of reconstruction, although the $s_F c_R$ model shows somewhat greater volatility, perhaps owing to the Horvitz-Thompson abundance estimation approach, and the $N_A s_F c_R$ model shows a sharp decline in recruitment rate for years 2005-2007. The relatively greater stability of the stock-recruitment for model $N_{SR,R} s_F c_R$ is induced by the model assumptions, where on the log scale, recruitment rate is assumed to oscillate symmetrically about a mean value.

Model-based estimates of harvest probability (shown only for model $s_F c_R$, Figure 2.17) indicate generally increasing harvest rates for the first few age classes, which may be a result of hunter selectivity toward larger animals. The model selection procedure indicated that no additional separate harvest vulnerability coefficients were warranted beyond the age 4+ category for this model, so all estimated harvest probabilities are identical for those ages > 4 within a

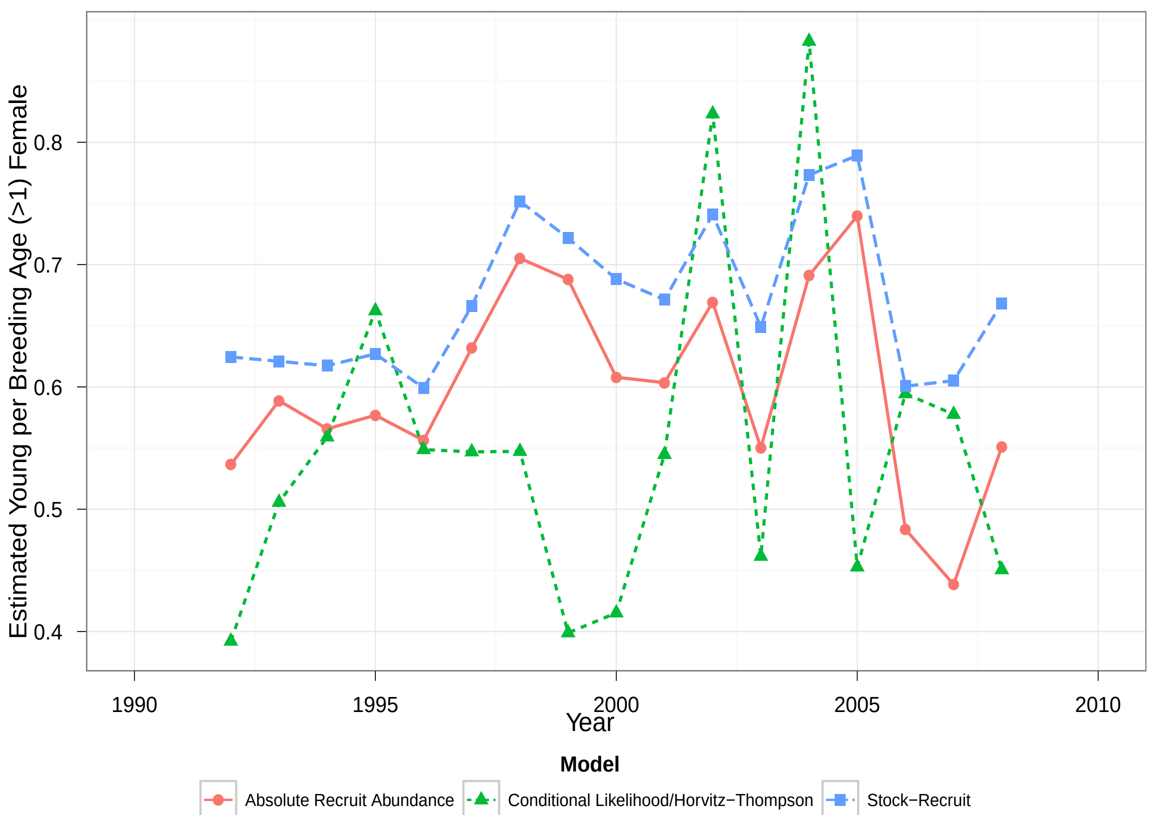


Figure 2.16: *Annual recruitment rate estimates for the Michigan elk population, computed as the estimated number of young per breeding-age (>0.5) female.*

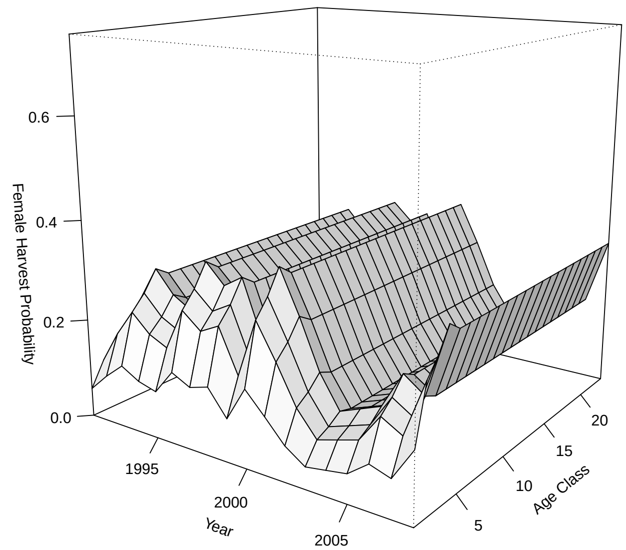
year. Harvest probabilities for adult males tended to be higher than for adult females, while juvenile harvest probabilities were greater for females than for males (Figure 2.17). A similar trend was also exhibited for models $N_A s_F c_R$ and $N_{SR,R} s_F c_R$.

2.5.4 Discussion

Annual abundance estimates indicate a population that declined from approximately 1200-1400 animals in 1992 to approximately 700 - 1000 animals in 2002, which recovered close to its early-1990s level by year 2008. This coincides with a decrease in harvest pressure from years 2001 - 2007, as measured by the maximum number of hunter permit-days applied (Table 2.18). This information, combined with no obvious trend observed in annual recruitment rate, and very low estimated natural mortality, indicates that harvest mortality is the dominant demographic process resulting in changes in Michigan elk abundance, and increased harvest rates lead directly to lower animal abundance, with no observed compensation by the recruitment process. Overall, the elk population appears to have been stable from the 1991 to 2008 endpoint, with a slight drop and accompanying recovery in total abundance, following a period of relatively high hunting pressure.

It is encouraging that such different model structures estimate similar total abundance of elk, and similar trends in abundance, indicating that the population reconstruction models considered here are generally consistent with one another. It is also encouraging to note that model-based estimates of abundance closely match the abundance estimates based on subjective sightability-corrected aerial surveys, which were not incorporated as auxiliary data (Figure 2.14); if different analyses based on different datasets coincide with one another, one tends to have greater confidence in results obtained from them.

Although some simplifying assumptions have been made, the current analysis involves the estimation of many parameters (49 for model $s_R c_R$, 129 for model $N_A s_R c_R$, and 99 for model $N_{SR,R} s_R c_R$), and an approximation to the high-dimensional integrated likelihood. An even more complex analysis may take into account the separate time periods during which harvests were conducted, as well as the spatial location. This was not attempted here, as the purpose of the analysis is to demonstrate the use of models of the form presented in this chapter. In addition,



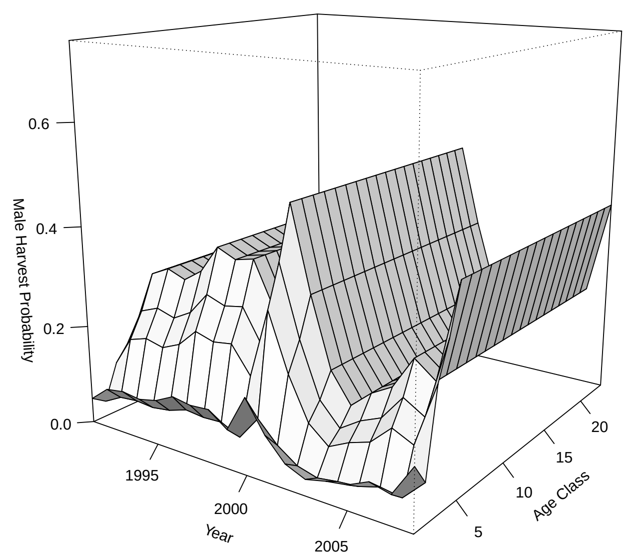


Figure 2.17: *Harvest probability for females (upper plot) and males (lower plot), produced by model $s_F c_R$. Estimated harvest rates were higher for adult males than adult females, but lower for juvenile males than juvenile females. Models $N_A s_F c_R$ and $N_{SR,R} s_F c_R$ exhibited similar trends.*

the sparseness of the resulting dataset would likely involve alternative simplifying assumptions, and would enhance the numerical estimation difficulties already encountered in the analysis presented. Model fit may be slightly improved if auxiliary data were available to inform natural survival rates in order to differentiate them between age classes, and to differentiate them from 100%, although adult elk survival is expected to be high, based on studies of similar populations (Lubow and Smith, 2004).

A primary benefit of the stock-recruit model rests with its ability to facilitate prediction through its relationship with prior breeding-age abundance. Prediction of the size of the recruit class may be made by computing

$$\hat{N}_{1,i+1}^{m+f} = e^{\hat{\gamma}} \sum_{j=2}^{A} \hat{N}_{ij}^{f} \hat{q}_{ij} \hat{s}$$

and under a known level of harvest pressure applied in year i, one may predict future abundance of older age classes via the usual cohort method given in Equation (2.44), separately for each gender. Using model $N_{SR,R}s_Fc_R$, predictions of total abundance were made for years 2009 - 2011. For 2009, the observed harvest effort for 2008 was used to make predictions, and for 2010 and 2011, the mean harvest effort over the course of the reconstruction period (2.12 thousand hunter-days) was used to make predictions. Reconstruction results augmented with these predictions (Figure 2.18) indicate a declining population following the final year of reconstruction, under the assumptions and harvest level just described. Under the estimated population structure and population dynamics from the reconstruction with model $N_{SR,R}s_Fc_R$, and a constant level of harvest effort (maximum of $\approx 2,120$ hunter-days) the population would not be sustainable, and would continue to decline due to over-harvesting. Of course, these predictions do not take into account the fluctuations in hunter efficacy and reproduction that the fitted model indicates are present, as there are no means for predicting these random quantities (τ_i and δ_i for $i = 2009, 2010$), although the model structure I have assumed provides a mean of 0.

The models developed and described in this chapter showed promise for the ability to estimate animal abundance as well as demographic parameters from harvested populations, and such models were fitted with apparent success to the Michigan elk population, producing reasonable estimates that also match independent estimates derived from aerial surveys. Models of

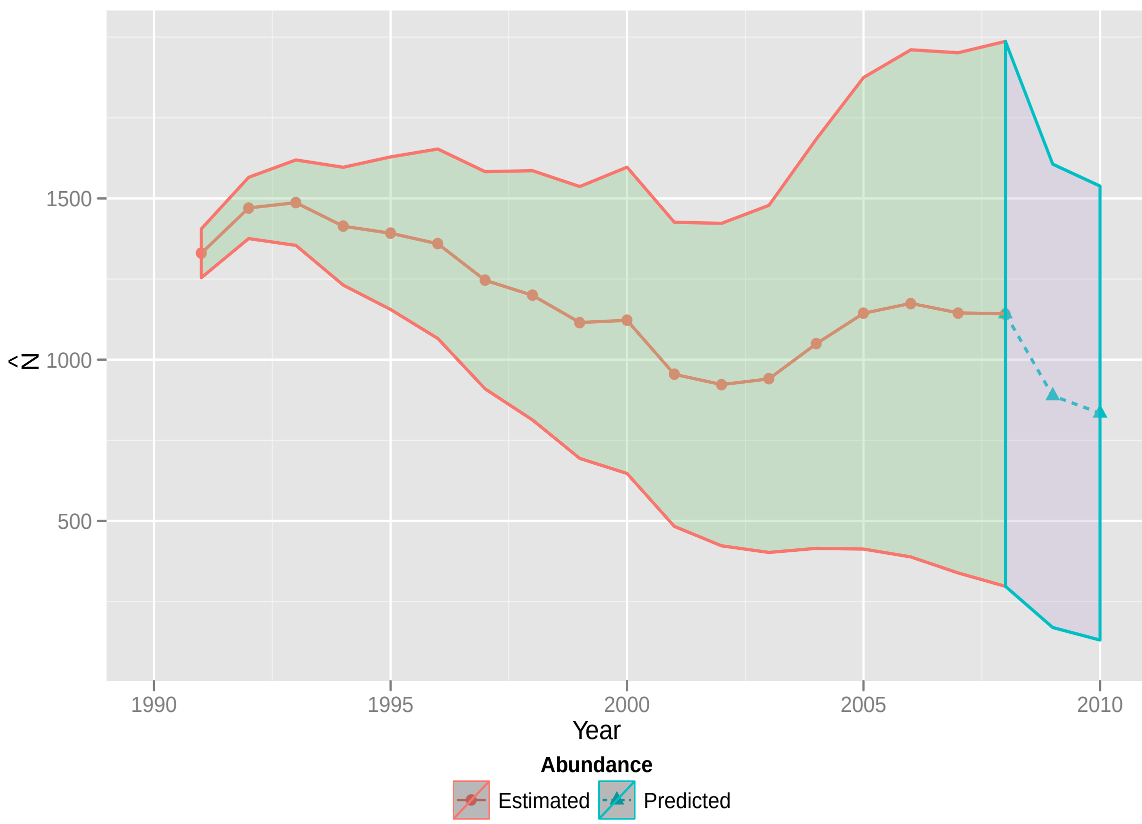


Figure 2.18: *Estimated total annual abundance of Michigan elk population (1991 - 2008) along with predicted abundance (2009 - 2011), using mean historical effort (≈ 2,120 maximum possible hunter-days) from model $N_{SR,RSFCR}$. Results indicate a declining population under estimated population status and dynamics, and this level of harvest effort.*

this nature may find applicability in harvest of animals that is fully (or nearly-fully) aged, as the basic model formulation can be easily modified to accommodate many different characteristics of animal populations and harvest regimes.

Simulation results presented previously in this chapter indicated that each of the models considered here showed low bias in estimates of total population abundance, but that models of the form of $s_F c_R$ produced appropriate confidence intervals with respect to confidence interval coverage. For this reason, I recommend inference be based on the conditional-likelihood models with a Horvitz-Thompson estimator for this elk population. However, as described above, predictions can most sensibly be derived from models of the form $N_{SR,R} s_F c_R$, so there is value in fitting these models as well.

2.6 Discussion

In this chapter, I have presented two new types of models for statistical population reconstruction that utilize aged harvest data; one family of models greatly reduces the number of parameters that must be estimated in exchange for a second-stage Horvitz-Thompson abundance estimator, and another family of models includes a flexible stock-recruit component that can simultaneously reduce the dimension of the parameter space while providing some insight regarding the reproductive process of the animal under study. In addition, I have added the ability to separate interannual variation from sampling variation in demographic process parameters involved in the modeling framework.

Based on results regarding estimator bias (Tables 2.7, A.1, and A.3, as well as Figures A.1 - A.6), mixed-effects models that employ a likelihood function that is conditional on harvest, and implement a Horvitz-Thompson estimator for total annual abundance, $s_R c_R$ and $s_F c_R$ are best among those considered here. Among competing models, these models are typically least-biased for annual abundance as well as for the parameters related to natural survival probability and harvest probability. For these models, annual abundance was very nearly unbiased. Among models including a multiplicative stock-recruit component, model $N_{SR,R} s_F c_F$ which includes a random effect for recruitment and fixed effects for natural survival and harvest probability, showed lowest absolute median relative bias. The other two stock-recruit models considered here, $N_{SR,R} s_R c_R$ and $N_{SR,R} s_F c_R$ showed greater degrees of negative bias. Among the classic models which estimate each annual recruitment as a separate parameter, mixed-effects models indicated negative bias, while the fixed-effects model shows varying levels of bias when the auxiliary catch-effort likelihood is employed, and consistent negative bias when it is not. In general, it appears as though results for conditional-likelihood models employing the Horvitz-Thompson estimator are less-biased when the auxiliary catch-effort likelihood component is omitted (except for the low level of variation for model $N_A s_F c_F$).

For the classic absolute-recruit abundance models of Gove et al. (2002) and Skalski et al. (2007), some of which have been modified to include random effects, there was a tendency to overestimate survival probability while underestimating harvest vulnerability. The conditional-likelihood models showed very low bias in the estimation of survival probability and harvest vulnerability. All stock-recruit models estimated the stock-recruitment parameter with little bias, with most models overestimating survival probability and underestimating harvest vulnerability; the exception was the fixed-effects model $N_{SR,F} s_F c_F$, which typically had the greatest degree of estimation bias for all parameters considered here (Table 2.7).

No model estimated variation in natural survival probability (σ_β) with a great deal of success, although most models with such a component estimated variation in harvest vulnerability (σ_c) and reproductive capacity (σ_γ) with some success. This may be due to numerical difficulty in estimating multiple random effects simultaneously when they each act annually (difficulty in parameter identifiability). That is, it may be difficult for the optimization process to discern if the number of animals in cohort X that survive from year i to $i+1$ is due to a greater-than-zero random effect on survival ($\epsilon_i > 0$) or a lower-than-zero random effect on harvest probability ($\tau_i < 0$). The situation becomes significantly more complex when stock-recruitment random effects are considered. Thus, the expected negative correlation between natural survival and harvest probability is likely causing numerical difficulty in the optimization of the likelihood approximation, which is manifest in the representation of all the interannual variation in σ_c and/or σ_γ. Despite this fact, however, there appears to be little penalty (in terms of bias) to including a random effect for survival, as an estimated $\widehat{\sigma}_\beta = 0$ reduces model $s_R c_R$ to $s_F c_R$ (for example). This explains the tight concordance of simulation results between models that differ only in the status of survival as fixed or random.

Simulation results regarding uncertainty estimation stemming from these models appear to favor the conditional-likelihood models once again. Figure (2.4) illustrates that asymptotic 95% confidence intervals for total annual abundance have closest to 95% coverage for conditional-likelihood models, as opposed to stock-recruit models and absolute-recruit abundance models. All fixed-effect-only models considered here ($N_A s_F c_F$ and $s_F c_F$) underestimate variability in annual abundance, which is indicated by far less-than-nominal confidence interval coverage. In contrast, models that include a random component in the stock-recruit relationship tend to overestimate uncertainty in total annual abundance, particularly in the later years. The reason for this is a propogation of the error in estimating the primary reproductive parameter, γ, as well as the random effect terms δ_i: the uncertainty for each subsequent age class from a single cohort is amplified during the delta-method variance approximation following optimization, and this results in estimated standard errors that are too large, and subsequently in confidence intervals that are too wide. Confidence interval coverage for absolute-recruit-abundance models with random components ($N_A s_R c_R$ and $N_A s_F c_R$) show less-than-nominal coverage, and this is probably mostly related to the large degree of negative bias exhibited by annual abundance estimates.

In addition to simulations where model assumptions are satisfied ("large" samples, symmetric random [stationary] process parameters), I examined a series of robustness simulations where demographic processes were nonstationary. Results from these simulations mimic those from the primary simulation results; the mixed-effects conditional likelihood model has low bias in abundance etimate and reliable estimates of standard error. These results indicate that the model formulation (mixed-effects, conditional-likelihood, product-multinomial) and estimation procedure (multiphasic quasi-Newton-Raphson optimization of Laplace approximation to marginal likelihood) is robust to deviations from model assumptions of this type.

Theoretical results regarding model selection for nonlinear mixed-effects models of this nature are not well-developed. Simulation results presented above operate under the assumption that each process assumed to be random accounts for one degree of freedom when considering information-theoretic or hypothesis-testing model selection. For selection of random effects into the model, results (Tables 2.9 through 2.12) indicate that the marginal likelihood tends to select random effects into the model, when they are perhaps not justified (as when σ_β is estimated to be zero). The conditional likelihood tends to select models without a random effect that is estimated to be zero. It therefore appears that the conditional likelihood may be used to select a parsimonious random effects structure with respect to the number of random effects.

In simulations intended to address selection of fixed effects into the model (one or two harvest vulnerability coefficients), results of likelihood ratio tests using both the marginal and conditional likelihoods (Table 2.14) indicate that power to detect the presence of separate harvest vulnerability coefficients when this is the true model is very high for all models when the auxiliary likelihood of Equation (1.7) is not employed. Results from likelihood ratio tests using both the marginal and conditional likelihoods show power greater than 97% for all models. Type I error, however, differs between models; models employing the Horvitz-Thompson estimator achieve the lowest Type I error rate, at about 6%, whereas absolute-recruit abundance models have Type I error at 15% to 17% (depending on level of simulate variability). Stock-recruit models, which were only able to be fitted when the auxiliary likelihood of Equation (1.7) was incorporated also show high power, but show high Type I error rate for the conditional likelihood (13% to 15% for the mixed-effects model) as well as the marginal likelihood (10% to 12%). These likelihood ratio test results are dependent on the difference in parameter values simulated, as well as the quantity of simulated auxiliary data. Further simulations of this nature may yield more insight regarding the reason for the difference in results between model structures. The results presented here do indicate that likelihood ratio testing procedures with either the conditional or marginal likelihoods have some capability for ascertaining the appropriate model parameterization when sampling error and interannual stochasticity in demographic processes are both present.

Based on the combined assessments presented in this chapter for fully-aged big game harvest data, the most successful population reconstruction models are those models that use a reduced-parameter conditional likelihood formulation with random effects for survival and harvest processes and a second-stage Horvitz-Thompson estimator for annual abundance. Therefore, it is recommended that this class of models be primarily considered for statistical population reconstruction endeavors.

2.7 Summary of Recommendations

Based on simulation results and discussion considered above, I make the following recommendations for statistical population reconstruction arising from fully-aged harvest data:

1. Use random effects to incorporate interannual variation in survival and harvest processes.

2. Use reduced-parameter models utilizing a conditional likelihood function along with a second-stage Horvitz-Thompson abundance estimator, or that which includes a stock-recruit relationship.

3. Use the conditional likelihood with appropriate likelihood ratio tests for bounded parameters to select the number of random processes.

4. Once the number of random processes has been selected from a model with richly-parameterized fixed-effects terms, use likelihood ratio tests with the conditional likelihood (which is sensitive to the actual values of the random effect estimates) to select the appropriate number of age-class vulnerability parameters or survival parameters.

Table 2.11: *Percent of times null hypothesis of adequacy of smaller model ($H_0 : \sigma_x = 0$) rejected out of 1000 simulations, nested model comparisons, using the marginal likelihood. "Aux. Like." = Auxiliary likelihood component used (With) or not used (Without) in joint likelihood model,* $\alpha = 0.05$.

			Percent of Null Hypotheses ($\sigma = 0$) Rejected						
Aux. Like.	**Larger Model**	**Reduced Model**	Sim. 0	Sim. 1	Sim. 2	Sim. 3	Sim. 4	Sim. 5	Sim. 6
Without	$N_{SR,R}s_Fc_R$	$N_{SR,R}s_Fc_F$	94.7	99.2	99.8	97.3	99.3	99.5	98.9
Without	$N_{SR,R}s_Rc_R$	$N_{SR,R}s_Fc_R$	96.9	98.9	96.9	88.9	99.5	98.6	99.2
Without	$N_As_Fc_R$	$N_As_Fc_F$	99.9	100.0	100.0	100.0	99.9	99.9	100.0
Without	$N_As_Rc_R$	$N_As_Fc_R$	99.9	99.8	99.6	98.3	100.0	100.0	100.0
Without	s_Fc_R	s_Fc_F	100.0	100.0	100.0	100.0	100.0	100.0	99.9
Without	s_Rc_R	s_Fc_R	100.0	100.0	99.8	99.9	100.0	100.0	100.0
With	$N_As_Fc_R$	$N_As_Fc_F$	100.0	100.0	100.0	100.0	100.0	100.0	100.0
With	$N_As_Rc_R$	$N_As_Fc_R$	100.0	99.9	99.7	99.3	100.0	99.8	99.9
With	s_Fc_R	s_Fc_F	100.0	100.0	100.0	99.9	100.0	100.0	100.0
With	s_Rc_R	s_Fc_R	100.0	100.0	100.0	99.5	100.0	100.0	100.0

Sim. 0 = No variation, Sim. 1 = Low variation, Sim. 2 = Medium variation, Sim. 3 = High variation, Sim. 4 = s_i (hence N_i) increasing during study, Sim. 5 = s_i (hence N_i) decreasing during study, Sim. 6 = period 4 time series "jump" in recruitment rate.

Table 2.12: *Percent of times null hypothesis of adequacy of smaller model ($H_0 : \sigma_x = 0$) rejected out of 1000 simulations, nested model comparisons, using the conditional likelihood. "Aux. Like." = Auxiliary likelihood component used (With) or not used (Without) in joint likelihood model,* $\alpha = 0.05$.

			Percent of Null Hypotheses ($\sigma = 0$) Rejected						
Aux. Like.	**Larger Model**	**Reduced Model**	Sim. 0	Sim. 1	Sim. 2	Sim. 3	Sim. 4	Sim. 5	Sim. 6
Without	$N_{SR,R}s_Fc_R$	$N_{SR,R}s_Fc_F$	44.5	98.9	99.9	98.2	98.9	99.5	98.4
Without	$N_{SR,R}s_Rc_R$	$N_{SR,R}s_Fc_R$	13.2	8.4	10.6	24.5	9.8	6.3	8.3
Without	$N_As_Fc_R$	$N_As_Fc_F$	48.3	99.5	100.0	99.9	98.7	99.0	99.8
Without	$N_As_Rc_R$	$N_As_Fc_R$	2.3	4.1	12.0	17.1	7.3	7.4	3.0
Without	s_Fc_R	s_Fc_F	27.4	99.8	100.0	100.0	98.9	99.5	99.7
Without	s_Rc_R	s_Fc_R	1.5	2.4	5.6	14.2	1.8	2.3	2.4
With	$N_As_Fc_R$	$N_As_Fc_F$	94.9	100.0	100.0	100.0	100.0	100.0	100.0
With	$N_As_Rc_R$	$N_As_Fc_R$	3.1	3.6	5.0	0.0	4.0	2.0	2.0
With	s_Fc_R	s_Fc_F	28.5	99.9	100.0	99.9	100.0	99.9	99.7
With	s_Rc_R	s_Fc_R	1.6	2.3	4.8	9.3	1.1	1.5	1.9

Sim. 0 = No variation, Sim. 1 = Low variation, Sim. 2 = Medium variation, Sim. 3 = High variation, Sim. 4 = s_i (hence N_i) increasing during study, Sim. 5 = s_i (hence N_i) decreasing during study, Sim. 6 = period 4 time series "jump" in recruitment rate.

Table 2.13: *Parameter combinations used for harvest vulnerability model selection simulation study.*

Simulation Parameter		**Potential Values**
Number of age classes		13
Years of data		25
Desired CV of auxiliary mark-harvest data		15%
Desired CV of auxiliary radiotelemetry data		5%
Dominant eigenvalue of Leslie matrix		≈ 1
	Var. Level	
	Low	**Med**
σ_β, σ_c, σ_γ	0.1	0.2
When harvest vulnerability simulated to be different:		
Level of juvenile harvest[1]		$\approx 23.8\%$ ($c_{juvenile} = -1.64$)
Level of adult harvest[1]		$\approx 28.0\%$ ($c_{adult} = -1.45$)
When harvest vulnerability simulated to be the same:		
Level of juvenile harvest[1] = adult harvest[1]		$\approx 26.8\%$ ($c = -1.5$)
Level of average juvenile survival percentage		84% ($\beta \approx 1.67$)
Level of average adult survival percentage		84% ($\beta \approx 1.67$)
Total Initial Abundance		$\approx 4,000$
Average recruits per breeding-age female		≈ 1.00 ($\gamma = 0.0$)

[1] *When evaluated at mean effort. Harvest effort drawn from $Gamma(10, 14)$ then divided by by 100 to give mean effort = 1.4.*

Table 2.14: *LRT model selection simulation results for single-c versus $c_{juvenile}$ and c_{adult}: Power and Type I error rate.*

	With Auxiliary Likelihood				Without Auxiliary Likelihood			
Variation:	**Low**		**Medium**		**Low**		**Medium**	
Model Comparison	Power[1]	Type I Error[2]	Power[1]	Type I Error[2]	Power[1]	Type I Error[2]	Power[1]	Type I Error[2]
Conditional likelihood								
1fFE1sFE1cFE vs. 1fFE1sFE2cFE	94.8	25.2	95.2	30.9	NA	NA	NA	NA
1fRE1sRE1cRE vs. 1fRE1sRE2cRE	99.3	12.8	99.2	15.2	NA	NA	NA	NA
1sFE1cFE vs. 1sFE2cFE	17.0	0.3	4.9	0.5	99.1	14.5	98.3	21.2
1sRE1cRE vs. 1sRE2cRE	29.4	0.3	23.9	0.4	99.4	15.2	97.4	17.1
1sFE1cFEc vs. 1sFE2cFEc	34.3	0.0	29.4	0.0	99.8	7.1	99.3	12.9
1sRE1cREc vs. 1sRE2cREc	34.0	0.0	28.4	0.0	99.8	6.1	99.2	6.0
Marginal likelihood								
1fRE1sRE1cRE vs. 1fRE1sRE2cRE	99.8	10.2	99.2	11.6	NA	NA	NA	NA
1sRE1cRE vs. 1sRE2cRE	28.3	0.0	23.2	0.5	99.4	15.5	97.7	16.7
1sRE1cREc vs. 1sRE2cREc	33.9	0.2	28.1	0.0	99.8	6.3	99.1	5.2

[1] *Percent of simulations with $c_{juvenile} \neq c_{adult}$ where LRT selects more complex model.*

[2] *Percent of simulations with $c_{juvenile} = c_{adult}$ where LRT selects less complex model.*

NA = Not Available: Model not run due to numerical instability

Table 2.16: *Michigan elk harvest data, females.*

Michigan Elk Harvest Data, Females																								
Age:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Unaged
1991	8	17	10	16	11	6	3	4	2	1	0	1	3	3	1	0	0	0	0	0	0	0	0	0
1992	10	21	30	23	17	13	15	3	7	2	1	2	0	2	2	1	1	0	0	0	0	1	0	1
1993	17	34	17	31	20	18	15	8	12	7	6	5	4	2	2	2	0	0	0	1	2	0	0	0
1994	10	17	26	21	9	16	8	11	9	9	8	0	0	0	0	2	1	1	0	0	1	0	0	0
1995	16	16	21	21	14	13	5	3	3	4	3	4	0	0	0	1	1	0	0	0	0	0	0	2
1996	20	42	31	20	19	17	13	9	9	1	1	3	3	2	2	1	0	0	0	0	0	0	0	0
1997	13	27	18	22	15	14	10	9	5	4	2	1	0	1	2	1	2	0	0	0	0	1	0	1
1998	16	19	32	19	13	12	10	3	5	8	2	3	3	3	0	1	0	1	1	2	0	0	0	0
1999	7	20	12	9	7	11	7	6	4	3	1	0	1	2	0	0	0	0	0	0	0	0	0	0
2000	16	33	23	28	12	5	7	10	4	8	3	2	2	3	0	1	0	2	0	0	0	0	1	0
2001	15	27	18	16	10	4	6	4	6	5	0	2	1	0	0	1	0	0	0	0	0	0	0	0
2002	11	5	15	9	12	8	1	3	4	4	1	1	2	1	1	2	0	1	0	0	0	0	0	0
2003	6	7	11	7	8	3	3	0	1	3	1	0	1	0	0	0	0	0	0	0	0	0	0	0
2004	11	6	11	11	5	2	3	2	2	1	0	0	1	1	1	0	1	0	0	0	0	0	0	0
2005	7	9	9	9	8	4	5	6	2	1	1	0	0	1	1	0	1	0	0	0	0	0	0	0
2006	11	27	18	13	12	6	6	5	4	1	0	3	0	2	3	1	1	0	0	0	0	0	0	0
2007	11	13	13	16	5	11	9	9	5	4	2	0	0	3	0	0	1	0	0	0	0	1	0	1
2008	18	21	25	31	19	19	10	7	4	1	6	5	3	1	1	0	1	1	3	0	0	0	0	2

Table 2.17: *Michigan elk harvest data, males.*

Michigan Elk Harvest Data, Males																							
Age:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Unaged
1991	5	2	6	13	13	9	3	4	4	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0
1992	8	4	15	16	12	11	18	5	6	4	1	1	0	0	0	0	0	0	0	0	0	0	0	0
1993	12	4	8	20	18	12	7	4	8	5	2	2	1	0	0	1	0	0	0	0	0	0	0	0
1994	15	6	10	21	14	16	9	2	1	2	1	1	1	1	0	1	0	0	0	0	0	0	0	0
1995	12	11	27	13	20	10	14	6	7	3	1	2	0	1	0	0	0	0	0	0	0	0	0	2
1996	10	6	25	27	22	13	9	11	2	4	3	1	1	0	0	0	0	0	0	0	0	0	0	0
1997	12	6	16	19	20	15	12	6	6	1	5	2	0	1	0	2	0	0	0	0	0	0	0	0
1998	9	9	25	13	13	12	10	6	6	6	3	2	2	1	0	0	0	0	0	0	0	0	0	0
1999	4	6	18	11	18	12	11	2	2	5	4	1	0	0	0	0	0	0	0	0	0	0	0	1
2000	7	13	28	28	25	17	9	13	8	2	5	1	2	0	0	0	0	0	0	0	0	0	0	0
2001	3	4	7	15	15	9	8	0	5	2	4	1	2	0	0	0	0	0	0	0	0	0	0	0
2002	5	1	6	8	5	2	7	2	3	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0
2003	1	1	7	6	5	2	4	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
2004	6	3	6	2	8	6	8	2	2	0	1	0	1	1	0	1	0	0	0	0	0	0	0	0
2005	2	0	2	11	5	6	12	3	0	1	2	0	1	1	0	0	0	0	0	0	0	0	0	0
2006	7	1	11	6	10	8	6	8	4	3	0	3	1	1	2	0	0	0	0	0	0	0	0	0
2007	6	1	13	13	7	5	7	8	3	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
2008	9	1	12	27	25	18	7	10	6	3	3	2	3	3	0	0	0	0	0	0	0	0	0	2

Table 2.18: *Harvest effort (thousands of hunter permit-days) and historical weather data obtained from the National Oceanic and Atmospheric Administration's National Climatic Data Center website (http://www7.ncdc.noaa.gov/IPS/coop/coop.html) (for 1992 - 2008) and the Michigan State Climatologist's Office website (http://climate.geo.msu.edu/stations/3096/) (for 1991) for the Gaylord, Michigan weather station, which was considered to be representative of the weather conditions experienced in the northern part of the lower peninsula during the winter hunting period.*

Year	**Harvest Effort**	**Centered Min.Temp. (°F)**	**Mean Daily Snowfall (in.)**
1991	1.24	7.6	1.1
1992	2.06	1.5	0.2
1993	2.55	1.5	0.3
1994	2.32	-6.5	2.1
1995	2.31	-16.5	1.0
1996	3.25	13.5	0.5
1997	2.84	5.5	0.0
1998	2.96	11.5	0.3
1999	1.86	14.5	0.7
2000	3.64	-13.5	2.9
2001	2.27	10.5	0.3
2002	1.20	-3.5	0.3
2003	0.80	2.5	1.8
2004	1.00	-2.5	1.4
2005	1.31	-6.5	0.4
2006	1.60	-4.5	1.3
2007	1.50	-12.5	1.1
2008	3.46	-2.5	1.2

Table 2.19: *Michigan elk herd auxiliary abundance estimates resulting from aerial survey with sightability correction model (Walsh, 2007).*

Year	**Estimated Abundance**	**Standard Error**
2006	905	125
2007	1182	167
2008	1180	122

Table 2.20: *Parameter estimates for final models for Michigan elk population reconstruction and accompanying 95% confidence intervals, constructed using asymptotic normality of maximum likelihood estimates.*

	Model					
	$s_F c_R$		$N_A s_F c_R$		$N_{SR,R} s_F c_R$	
	Female	Male	Female	Male	Female	Male
$\widehat{s}$	0.98 (0.94, 1.0)	0.98 (0.94, 1.0)	1.0 (1.0, 1.0)	1.0 (1.0, 1.0)	0.97 (0.95, 0.98)	0.97 (0.95, 0.98)
$\widehat{c}_0$	-3.01 (-3.20, -2.81)	-3.23 (-3.47, -2.99)	-2.90 (-3.07, -2.73)	-3.12 (-3.33, -2.91)	-2.91 (-3.09, -2.74)	-3.49 (-3.68, -3.30)
$\widehat{c}_1$	-2.37 (-2.54, -2.19)	-3.69 (-3.97, -3.42)	-2.25 (-2.39, -2.11)	-3.59 (-3.84, -3.34)	-2.27 (-2.42, -2.12)	-3.92 (-4.15, -3.69)
$\widehat{c}_2$	-2.23 (-2.40, -2.05)	-2.47 (-2.66, -2.29)	-2.10 (-2.24, -1.96)	-2.37 (-2.53, -2.21)	-2.11 (-2.26, -1.96)	-2.67 (-2.82, -2.53)
$\widehat{c}_3$	-2.05 (-2.23, -1.88)	-2.18 (-2.35, -2.00)	-1.91 (-2.06, -1.77)	-2.07 (-2.22, -1.91)	-1.91 (-2.07, -1.76)	-2.35 (-2.49, -2.20)
$\widehat{c}_{4/4+}$	-2.12 (-2.28, -1.97)	-1.85 (-2.00, -1.71)	-1.79 (-1.91, -1.67)	-1.64 (-1.77, -1.51)	-2.03 (-2.21, -1.86)	-2.11 (-2.26, -1.97)
$\widehat{c}_{5+}$	NA	NA	NA	NA	-1.88 (-2.02, -1.74)	-2.07 (-2.19, -1.94)
$\widehat{p}_0^*$	0.10 (0.08, 0.12)	0.08 (0.06, 0.10)	0.11 (0.09, 0.13)	0.09 (0.07, 0.11)	0.11 (0.09, 0.13)	0.06 (0.05, 0.07)
$\widehat{p}_1^*$	0.18 (0.15, 0.21)	0.05 (0.04, 0.07)	0.20 (0.17, 0.22)	0.06 (0.04, 0.07)	0.20 (0.17, 0.22)	0.04 (0.03, 0.05)
$\widehat{p}_2^*$	0.20 (0.17, 0.24)	0.16 (0.14, 0.19)	0.23 (0.20, 0.26)	0.18 (0.15, 0.21)	0.23 (0.20, 0.26)	0.14 (0.12, 0.16)
$\widehat{p}_3^*$	0.24 (0.20, 0.28)	0.21 (0.18, 0.25)	0.27 (0.24, 0.30)	0.24 (0.20, 0.27)	0.27 (0.23, 0.30)	0.18 (0.16, 0.21)
$\widehat{p}_{4/4+}^*$	0.22 (0.19, 0.26)	0.28 (0.25, 0.32)	0.30 (0.27, 0.33)	0.34 (0.30, 0.37)	0.24 (0.21, 0.28)	0.23 (0.20, 0.26)
$\widehat{p}_{5+}^*$	NA	NA	NA	NA	0.28 (0.24, 0.31)	0.24 (0.21, 0.26)
$e^{\widehat{\gamma}} = \widehat{r}$	NA	NA	NA	NA	0.67 (0.62, 0.72)	NA
σ_c	0.12 (0.05, 0.19)	0.11 (0.04, 0.18)	0.14 (0.06, 0.22)	0.13 (0.05, 0.22)	0.21 (0.12, 0.29)	0.12 (0.04, 0.21)
σ_γ	NA	NA	NA	NA	0.10 (0.05, 0.16)	NA
mAIC	2145.30		2289.90		2699.30	
cAIC	2183.44		2318.02		2712.74	

* *Estimated harvest probability computed at mean effort (2.12 thousand hunter-permit-days)*

Chapter 3

BIG GAME, POOLED AGE CLASSES

3.1 Introduction and Models

For large game, aging of harvest data can often be performed readily. In many cases, however, such fine resolution of animal age is not available and animals older than a certain age, A, are lumped into a common age class, "$A+$". The loss of aging information for older age classes means a loss of cohort information that can be used with the models developed in Chapter 2.

There has been some previous work for modeling pooled age-class data of this nature. Laake (1992), using a model formulation for age-harvest data originally proposed by Dupont (1983), proposed optimizing a likelihood via the expectation-maximization (EM) algorithm (Dempster et al., 1977), wherein components of the age $A+$ category in a particular year that belong to different cohorts are included by weighting the expected catch for each cohort relative to the total expected catch of all contributing cohorts. These weights must be estimated and, of course, rely on the current parameter estimates. Therefore, the expected catch (being treated as observed data in the likelihood function) changes during the optimization phase. Optimization of the likelihood function proceeds as usual, using these expected values of catch produced by the weighting, until convergence is achieved.

An alternative solution involves writing a likelihood that is conditional on having been observed (harvested), and using the data and parameters to estimate the probability of having been harvested in a given year as the ratio of harvest in that year to total harvest of the cohort and previous cohorts contributing to the $A+$ category. To see this more clearly, consider again a sample age-at-harvest matrix given in Table (3.1), and consider the case where animal age can only be determined as young-of-the-year (age $\frac{1}{2}$ year at a fall harvest), yearlings (age 1 $\frac{1}{2}$ years at a fall harvest) and adult (age 2+ at a fall harvest). The final category for each year of data is then a composite of the prior year's surviving juveniles plus the prior year's surviving adults, as illustrated in Table (3.2). The difficulty lies in determining the contribution of prior years to the final highlighted cell, $x_{33}^{+} = x_{33} + x_{34} + \cdots + x_{3,A}$, where the "+" in x_{33}^{+} indicates it is a composite of all older age classes.

Table 3.1: *Example age-at-harvest data.*

N_{11}	N_{12}	N_{13}	N_{14}	N_{15}	$\cdots$	
N_{21}	x_{11}	x_{12}	x_{13}	x_{14}	$\cdots$	$x_{1,A}$
N_{31}	x_{21}	x_{22}	x_{23}	x_{24}	$\cdots$	$x_{2,A}$
N_{41}	x_{31}	x_{32}	x_{33}	x_{34}	$\cdots$	$x_{3,A}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$
N_{Y1}	x_{Y1}	x_{Y2}	x_{Y3}	x_{Y4}	$\cdots$	$x_{Y,A}$

Table 3.2: *Example of combined age-at-harvest data.*

N_{11}	N_{12}	N_{13}	
N_{21}	x_{11}	x_{12}	$x_{13} + x_{14} + \cdots + x_{1,A}$
N_{31}	x_{21}	x_{22}	$x_{23} + x_{24} + \cdots + x_{2,A}$
N_{41}	x_{31}	x_{32}	$x_{33} + x_{34} + \cdots + x_{3,A}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
N_{Y1}	x_{Y1}	x_{Y2}	$x_{Y3} + x_{Y4} + \cdots + x_{Y,A}$

Rather than weighting observed catch by the expected proportion of catch belonging to a given cohort, we can instead use the expected value of harvest from each cohort to compute the probability of harvest conditional on being harvested using the current parameter estimates. Consider the shaded main-diagonal cohort of Tables (3.1) and (3.2). If we express the multinomial likelihood of the observed harvest as conditional on harvest, we may write

$$L(\vec{N}_{11}, \vec{p}, s \mid \boldsymbol{X}) =$$
$$\binom{x_{11} + x_{22} + x_{33}^{+}}{x_{11}, x_{22}, x_{33}^{+}} p(1,3|any)^{x_{11}} p(2,3|any)^{x_{22}} p(3,3|any)^{x_{33}^{+}} \tag{3.1}$$
,

where the notation $p(i,j|any)$ indicates the probability of a member of cohort j being harvested in year i given that they were ever harvested. This requires the same assumption we have already employed of independence of fates of each animal within a given year.

A reasonable estimate for $p(i,j|any)$ is the percent of total harvest observed in year i of cohort j. For example, the estimate of $p(1,3|any)$ (cohort $j = 3$ begins with initial abundance N_{11}) is

$$\widehat{p}(1,3|any) = \frac{x_{11}}{x_{11} + x_{22} + \sum_{j=3}^{A} x_{3j}}. \tag{3.2}$$

Standard maximum likelihood theory for multinomial models tells us this is, in fact, the maximum likelihood estimate for the probability of harvest in the x_{11} cell of the age-at-harvest table.

In order to rewrite the cohort likelihood in Equation (3.1) in terms of the parameters to be estimated, we replace the observed harvest values in $p(i,j|any)$ with their expected value, computed as a function of the parameter values. That is, instead of estimating $\widehat{p}(1,3|any)$ as in Equation (3.2), we instead use

$$\widehat{p}(1,3|any) = \frac{E(x_{11})}{E(x_{11}) + E(x_{22}) + E\left(\sum_{j=3}^{A} x_{3j}\right)}. \tag{3.3}$$

In this case, under an assumption that each harvest count is marginally binomially distributed (and harvest and survival probabilities differ only by year and not by age class), we may compute each expected value as

$$
\begin{aligned}
E(x_{11}) &= N_{11}p_1 \\
E(x_{22}) &= N_{11}q_1 s_1 p_2 \\
E(x_{33}^{+}) &= (N_{11} + N_{12} + N_{13})\, q_1 s_1 q_2 s_2 p_3
\end{aligned} \tag{3.4}
$$

where $q_i = 1 - p_i$. As usual, if auxiliary data requirements are met, the probabilities may be made dependent on age class as well as year.

The likelihood for the next cohort may be computed as

$$
L(\vec{N}_{21}, \vec{p}, s \mid \boldsymbol{X}) =
\binom{x_{21} + x_{32} + x_{43}^{+}}{x_{21}, x_{32}, x_{43}^{+}} p(1,4|any)^{x_{21}} p(2,4|any)^{x_{32}} p(3,4|any)^{x_{43}^{+}}, \tag{3.5}
$$

where

$$
\widehat{p}(1,4|any) = \frac{x_{21}}{x_{21} + x_{32} + \sum_{j=3}^{A} x_{4j}} \tag{3.6}
$$

which is estimated by

$$
\widehat{p}(1,4|any) = \frac{E(x_{21})}{E(x_{21}) + E(x_{32}) + E(\sum_{j=3}^{A} x_{4j})} \tag{3.7}
$$

where each expected value is computed as

$$
\begin{aligned}
E(x_{21}) &= N_{21}p_1 \\
E(x_{32}) &= N_{21}q_1 s_1 p_2 \\
E(x_{43}^{+}) &= \big(N_{21}q_1 s_1 q_2 + N_{32}q_1 s_1 q_2 s_2 + E(N_{33}^{+}) q_1 s_1 q_2 s_2\big)\, p_3
\end{aligned} \tag{3.8}
$$

and where in the last line

$$
E(N_{33}^{+}) = N_{11}q_1 s_1 q_2 + N_{12}q_1 s_1 q_2 s_2 + N_{13}q_1 s_1 q_2 s_2 \tag{3.9}
$$

(from Equation (3.4) above). Accordingly, I compute

$$\widehat{p}(2,4|any) = \frac{E(x_{32})}{E(x_{21}) + E(x_{32}) + E(\sum_{j=3}^{A} x_{4j})} \tag{3.10}$$

and

$$\widehat{p}(3,4|any) = \frac{E(\sum_{j=3}^{A} x_{4j})}{E(x_{21}) + E(x_{32}) + E(\sum_{j=3}^{A} x_{4j})}. \tag{3.11}$$

Each expected harvest count is computed in this manner, and the joint likelihood of all cohorts is created as the product of individual cohort likelihoods, as in Equations (2.3) and (2.26). The likelihood contributions for cohorts 1 (the first observation of age class $A+$) and A+Y-1 (the last observation of age class 1) cannot be included in the likelihood in the manner described above, so they are included as binomial components, such as

$$L\left(N_{13}, c \mid x_{13}\right) = \begin{pmatrix} N_{13} \\ x_{13} \end{pmatrix} p_1^{x_{13}} q_1^{N_{13}-x_{13}}.$$

Unconditional multinomial (binomial) likelihood components are included for cohorts $A+Y-1$ and $A+Y-2$ as well.

As in the previous chapter, we may consider adopting the viewpoint that temporal effects on demographic process parameters are random in nature (and, in the work presented here, mutually independent), and thus include these random effects in the model formulation. To do so, we once again create the transformations

$$\begin{aligned} s_i &= \frac{1}{1+e^{-(\beta+\epsilon_i)}} \\ p_i &= 1 - e^{-e^{(c+\tau_i)} f_i} \\ r_i &= e^{\gamma+\delta_i} \end{aligned} \tag{3.12}$$

and hypothesize that

$$\begin{aligned} \epsilon_i &\sim N\left(0, \sigma_\beta^2\right), \\ \tau_i &\sim N\left(0, \sigma_c^2\right), \text{ and} \\ \gamma_i &\sim N\left(0, \sigma_\gamma^2\right) \end{aligned} \tag{3.13}$$

and substitute the members of (3.12) into the appropriate locations in (3.4), and augment the joint likelihood with the normal-distribution penalties, as in (2.22). Once again the marginal likelihood, integrated over the random effects terms, is approximated by the Laplace approximation within ADMB (Fournier et al., 2011) and optimized.

Also as in the previous chapter, we may consider altering this model formulation to include a stock-recruit relationship, wherein years following the first year are dependent on the breeding-age abundance of the prior year. One simply makes the replacement

$$N_{i1} = \left(e^{\gamma+\delta_i}\right) \sum_{j=2}^{A} N_{i-1,j}$$

and proceeds to compute expected harvest counts as in (3.4).

Similarly, the Horvitz-Thompson estimation approach as described in Chapter 2 may be employed here to consider reduced-parameter models. In this case, one first computes the estimate of initial recruit abundance using the Horvitz-Thompson estimator

$$\widehat{N}_{i1} = \frac{x_{i1}}{\widehat{p}_{i1}}$$

where the current parameter values are used to compute $\widehat{p}_{i1}$. Once again, one simply proceeds to compute the likelihood by first computing the expected harvest counts as in (3.4).

As before, the auxiliary catch-effort likelihood described in Equation (1.7) of Chapter 1 (Skalski et al., 2007) is examined for use in the optimization process. The joint likelihood described above is augmented with this auxiliary likelihood as

$$L_{combined} = L_{age-harvest} \times L_{catch-effort}. \tag{3.14}$$

for all models, including stock-recruit models (stock-recruit models in the fully-aged analysis above did not incorporate this auxiliary likelihood due to optimization instability). As with fully-aged harvest data, the use of the auxiliary catch-effort likelihood involves estimation of $\widehat{N}_i = \sum_{j=1}^{A} \widehat{N}_{ij}$ with the Horvitz-Thompson-type estimator in each iteration of the likelihood optimization. This method of first computing an estimated value of the variable $\left(\widehat{N}_i\right)$ and using it as data in the auxiliary likelihood takes the form of an Expectation-Maximization (EM) algorithm. Note that $L_{combined}$ is not truly a joint likelihood of the pooled age-at-harvest likelihood and the auxiliary likelihood, as we have not accounted for the dependence between these two components. In this respect, the likelihood may be considered to be a pseudo-likelihood.

Upon completion of likelihood optimization, standard errors are estimated from the inverse-Hessian matrix (for parameters) and delta-method (for functions of parameters) as described in the previous chapter (Section 2.4.1) for both process parameters and abundance estimates.

For absolute-recruit abundance models and stock-recruit models annual abundance is estimated as described in the previous chapter (Section 2.4), except that the age $A+$ category (the oldest age class) is also carried over into the following year as

$$\widehat{N}_{i,A+} = \left(\widehat{N}_{i-1,2} + \widehat{N}_{i-1,A+}\right)\widehat{q}_{i-1}\widehat{s}_{i-1}$$

The delta method is employed within the ADMB (Fournier et al., 2011) software to estimate standard errors of estimated abundances.

For models employing the Horvitz-Thompson abundance estimator, abundance is estimated as described in the previous chapter (Section 2.2), and standard errors are computed based on a combination of inverse-Hessian standard error estimates for parameters, the delta method for functions of parameters, and the extra-likelihood component described in the previous chapter (Section 2.4) and Appendix B.

In this chapter, I will use the codes $s_R c_R(p)$, $s_F c_R(p)$, and $s_F c_F(p)$ to refer to the models where abundance is estimated with a Horvitz-Thompson estimator, $\widehat{N}_i = \frac{x_{i1}+x_{i2}+x_{3+}}{\widehat{p}_i}$. I will use the codes $N_A s_R c_R(p)$, $N_A s_F c_R(p)$, and $N_A s_F c_F(p)$ to refer to the models where recruit abundance is estimated separately for each year as a parameter. I will use the codes $N_{SR,R} s_R c_R(p)$, $N_{SR,R} s_F c_R(p)$, $N_{SR,R} s_F c_F(p)$ to refer to models where recruit abundance is multiplicatively dependent on prior breeding age abundance. As in the previous chapter, s_R indicates survival is considered randomly-distributed across years, while s_F indicates it is assumed fixed and constant. Similarly, p_R indicates the harvest probability contains a random effect, and p_F indicates it depends on level of effort only, and contains no annual random deviation. The code $N_{SR,F}$ indicates that a stock-recruit function is used in the likelihood model, and the recruitment rate is assumed fixed, while $N_{SR,R}$ indicates the assumption that the recruitment rate differs randomly among years. The code N_A indicates each recruit abundance is estimated as an individual parameter, while the absence of any N from the code list indicates the use of a Horvitz-Thompson estimator for abundance. The code (p) indicates the model formulation described above, where the uppermost age classes are pooled into a single category.

3.2 Simulation Model

The simulation model I employ to create age-harvest and accompanying effort data for this chapter is exactly the same as that which I employ in the previous chapter. Animals are assumed to survive until age A, however harvest data are combined into three age classes: 0.5, 1.5, and 2.5+ (assuming a spring birth pulse and fall harvest). Here, I assume that the first two age classes can be aged exactly (and are not themselves a combination of subclasses), and that the uppermost age class is comprised of the sum of all animals aged 2 and older.

I use the same input parameter combinations as in the previous chapter (Table 2.3), in order to facilitate comparison of model fit success between the two methods. Therefore, results are directly comparable to those in the previous chapter.

3.3 Results

With respect to random-effects structure, the models examined for this pooled age-class simulation study include the same models as in the previous chapter (Tables 2.2 and 2.5). In order to examine the role of the auxiliary catch-effort likelihood of Equation (1.7), all models were fitted both including this component and excluding it, with one exception: for the highest level of simulated variation, the fixed-effects-only stock-recruit model $N_{SR,F}s_Fc_F(p)$ failed to fit with high frequency, so it was removed from consideration.

3.3.1 Estimator Accuracy

Process Parameters

Accuracy of process parameter estimates is assessed by examining median relative bias (Tables 3.3 and 3.4). For the base-case scenario, when no environmental stochasticity is simulated, all models show very low bias in estimation of process parameters, regardless of the use of the auxiliary catch-effort likelihood. Results for nonzero environmental variation indicate that mixed effects versions of the conditional-likelihood/Horvitz-Thompson models and absolute-recruit abundance models show relatively low bias in the estimation of parameters for natural survival and harvest vulnerability; for absolute-recruit abundance models, bias generally appears to be lower (closer to zero) when the auxiliary catch-effort likelihood is not used (Table 3.4). Fixed-effects versions of these same models tend to show significant negative bias in the estimation of survival probability, and significant positive bias in the estimation of harvest vulnerability when the auxiliary catch-effort likelihood is used. When it is not used, these fixed-effects models show negligible bias (never greater in absolute magnitude than 3.5%). Among the 4 models performing best ($N_A s_F c_R(p)$, $s_F c_R(p)$, $N_A s_R c_R(p)$, and $s_R c_R(p)$), models $s_F c_R(p)$ and $s_R c_R(p)$ tend to have the lowest overall magnitude of bias in estimation of demographic process parameters, which is most apparent in the estimation of harvest vulnerability.

Stock-recruit models show negative bias in estimation of survival probability, with accompanying positive bias in estimation of harvest vulnerability, although the magnitude is relatively low for the best-performing models $N_{SR,R} s_F c_R(p)$ and $N_{SR,R} s_R c_R(p)$. As with the fully-aged simulation study of the previous chapter, the fixed-effects-only stock-recruit model shows a great deal of bias in estimation of process parameters. With all stock-recruit models, magnitude of bias increases with the level of simulated variation, while with absolute-recruit models and conditional-likelihood models, the magnitude of median relative bias stays roughly the same, or increases slowly. In general, process parameters tend to be estimated with less bias when the auxiliary catch-effort likelihood of (1.7) is not used in the likelihood function.

All stock-recruit models slightly overestimate the stock-recruitment parameter e^{γ} (with the exception of $N_{SR,R} s_R c_R(p)$ at the highest level of simulated variation) when simulated variation is nonzero, with relative bias increasing with level of simulated variation. Estimates are closest to the true value for the mixed effects models $N_{SR,R} s_F c_R(p)$ and $N_{SR,R} s_R c_R(p)$ at each level of simulated variation.

Interannual variation in process parameters is typically underestimated for each model, except for σ_c for stock-recruit models at the lowest level of simulated variation, and σ_{γ} for model $N_{SR,R} s_F c_R(p)$ at the lowest nonzero level of simulated variation when the auxiliary catch-effort likelihood component is included. Variation in harvest vulnerability (σ_c) is estimated most successfully, and the best of these are from the models that employ a Horvitz-Thompson estimator and the stock-recruit models $N_{SR,R}s_Fc_R(p)$ and $N_{SR,R}s_Rc_R(p)$, with bias ranging from -10.6% to 4.1% across all levels of simulated variation, regardless of use of the auxiliary catch-effort likelihood. Estimates of interannual variation in the stock-recruit relationship are underestimated by models $N_{SR,R}s_Fc_R(p)$ and $N_{SR,R}s_Rc_R(p)$, and severely overestimated by model $N_{SR,R}s_Fc_F(p)$ at all levels of variation, although models $N_{SR,R}s_Fc_R$ and $N_{SR,R}s_Rc_R$ exhibit negative bias of only -12% at the highest level of simulated variation when the auxiliary catch-effort likelihood component is not incorporated. As with the full age class models of the previous chapter, variation in survival probability is poorly estimated by all models, although these pooled age-class models do show some improvement in terms of median relative bias. The model that best estimates σ_β is model $s_Rc_R(p)$ either with or without the auxiliary catch-effort likelihood.

See Appendix A for boxplots of process parameter estimates.

Abundance Reconstruction

As in the previous chapter, quality of model-based abundance reconstruction is assessed by examining the pointwise median relative bias of total annual abundance estimates (Figure 3.1). Note that this plot does not include the absolute-recruit abundance models ($N_As_Fc_F(p)$, $N_As_Rc_R(p)$, and $N_As_Fc_R(p)$) for the simulations where the auxiliary catch-effort likelihood of Equation (1.7) *is* used, due to their significant positive bias, which made presenting the more successful models difficult within the plot (see Appendix A, Table (A.5) for the results in tabular form). Results indicate that lowest bias comes from models $s_Fc_R(p)$ and $s_Rc_R(p)$ without auxiliary catch-effort (though results are very similar when it is used) and models $N_{SR,R}s_Fc_R(p)$, $N_{SR,R}s_Rc_R(p)$, $s_Fc_R(p)$ and $s_Rc_R(p)$ when the auxiliary catch effort is used.

When the auxiliary catch-effort likelihood is not used, at all levels of simulated variation, mixed-effects stock-recruit and mixed- and fixed-effects conditional-likelihood/Horvitz-Thompson models show low median relative bias, with only model $N_{SR,R}s_Fc_F$ exceeding 8% relative bias for a single year of reconstruction (Figure 3.1). When the auxiliary catch effort is used, mixed effects versions of the stock-recruit model and conditional-likelihood/Horvitz-

Table 3.3: *Median relative bias ((estimate minus simulated mean)/simulated mean) in process parameter estimates for analysis of pooled age-class data,* ***with*** *the auxiliary catch-effort likelihood component. Results based on n=1000 replicates at s=0.84, c=-1.5, γ=0.0, and total annual abundance $\approx$ 4000.*

Variation Level	Model	$\widehat{s}$	$\widehat{c}$	$e^{\widehat{\gamma}}$	$\widehat{\sigma}_\beta$	$\widehat{\sigma}_c$	$\widehat{\sigma}_\gamma$
None	$N_{SR,F}s_Fc_F(p)$	-0.4	0.9	-0.0			
	$N_{SR,R}s_Fc_F(p)$	-0.2	0.2	-0.0			
	$N_{SR,R}s_Fc_R(p)$	0.0	-0.1	-0.0			
	$N_{SR,R}s_Rc_R(p)$	0.0	-0.1	-0.0			
	$s_Fc_F(p)$	-0.1	0.3				
	$N_As_Fc_F(p)$	0.6	-1.4				
	$s_Fc_R(p)$	0.3	-0.6				
	$N_As_Fc_R(p)$	0.6	-1.5				
	$s_Rc_R(p)$	0.4	-0.7				
	$N_As_Rc_R(p)$	0.6	-1.5				
Low	$N_{SR,F}s_Fc_F(p)$	-4.9	14.7	0.8			
	$N_{SR,R}s_Fc_F(p)$	-2.3	4.1	1.0			150.4
	$N_{SR,R}s_Fc_R(p)$	-0.0	0.1	0.4		4.1	-35.6
	$N_{SR,R}s_Rc_R(p)$	0.0	0.1	0.4	-83.0	2.9	-35.5
	$s_Fc_F(p)$	-3.8	9.7				
	$N_As_Fc_F(p)$	-1.3	1.5				
	$s_Fc_R(p)$	0.2	-0.6			3.4	
	$N_As_Fc_R(p)$	1.4	-3.4			-11.3	
	$s_Rc_R(p)$	0.5	-0.7		-3.8	-0.5	
	$N_As_Rc_R(p)$	1.4	-3.3		-84.9	-12.8	
Medium	$N_{SR,F}s_Fc_F(p)$	-11.9	42.6	1.8			
	$N_{SR,R}s_Fc_F(p)$	-6.8	12.4	3.2			143.3
	$N_{SR,R}s_Fc_R(p)$	-0.4	0.4	0.2		-1.0	-18.2
	$N_{SR,R}s_Rc_R(p)$	-0.2	0.4	0.2	-80.3	-2.1	-17.9
	$s_Fc_F(p)$	-10.3	31.3				
	$N_As_Fc_F(p)$	-5.2	7.3				
	$s_Fc_R(p)$	-0.3	0.1			-3.1	
	$N_As_Fc_R(p)$	1.3	-4.0			-8.6	
	$s_Rc_R(p)$	0.5	-0.6		-23.9	-3.6	
	$N_As_Rc_R(p)$	1.3	-4.0		-90.3	-9.2	
High	$N_{SR,R}s_Fc_F(p)$	-10.7	17.3	3.9			131.9
	$N_{SR,R}s_Fc_R(p)$	-1.3	1.9	-0.4		-4.5	-15.6
	$N_{SR,R}s_Rc_R(p)$	-0.8	1.8	-0.3	-62.7	-6.0	-16.1
	$s_Fc_F(p)$	-14.0	44.1				
	$N_As_Fc_F(p)$	-8.6	9.4				
	$s_Fc_R(p)$	-1.1	1.1			-5.0	
	$N_As_Fc_R(p)$	0.6	-4.6			-12.6	
	$s_Rc_R(p)$	0.2	0.2		-27.0	-6.3	
	$N_As_Rc_R(p)$	0.7	-4.6		-91.0	-13.3	

Table 3.4: *Median relative bias ((estimate minus simulated mean)/simulated mean) in process parameter estimates for analysis of pooled age-class data,* ***without*** *the auxiliary catch-effort likelihood component. Results based on n=1000 replicates at s=0.84, c=-1.5, γ=0.0, and total annual abundance $\approx$ 4000.*

Variation Level	Model	$\widehat{s}$	$\widehat{c}$	$e^{\widehat{\gamma}}$	$\widehat{\sigma}_{\beta}$	$\widehat{\sigma}_{c}$	$\widehat{\sigma}_{\gamma}$
None	$N_{SR,F}s_Fc_F(p)$	0.0	-0.1	-0.1			
	$N_{SR,R}s_Fc_F(p)$	0.0	-0.1	-0.1			
	$N_{SR,R}s_Fc_R(p)$	0.0	-0.1	-0.1			
	$N_{SR,R}s_Rc_R(p)$	0.0	-0.1	-0.1			
	$s_Fc_F(p)$	-0.0	-0.0				
	$N_As_Fc_F(p)$	0.3	-0.1				
	$s_Fc_R(p)$	-0.0	-0.1				
	$N_As_Fc_R(p)$	0.3	-0.1				
	$s_Rc_R(p)$	0.0	-0.1				
	$N_As_Rc_R(p)$	0.3	-0.1				
Low	$N_{SR,F}s_Fc_F(p)$	-0.9	2.1	0.5			
	$N_{SR,R}s_Fc_F(p)$	-0.6	0.2	0.4			108.4
	$N_{SR,R}s_Fc_R(p)$	-0.4	0.7	0.3		-7.1	-24.2
	$N_{SR,R}s_Rc_R(p)$	-0.2	0.6	0.3	-60.1	-8.8	-24.8
	$s_Fc_F(p)$	-0.7	1.4				
	$N_As_Fc_F(p)$	0.0	-0.4				
	$s_Fc_R(p)$	-0.3	0.4			-5.1	
	$N_As_Fc_R(p)$	-0.2	0.4			-44.1	
	$s_Rc_R(p)$	-0.1	0.4		-23.9	-8.4	
	$N_As_Rc_R(p)$	-0.1	0.3		-94.2	-44.2	
Medium	$N_{SR,F}s_Fc_F(p)$	-1.9	4.2	1.2			
	$N_{SR,R}s_Fc_F(p)$	-1.7	-0.8	0.9			125.7
	$N_{SR,R}s_Fc_R(p)$	-0.8	0.9	0.4		-5.3	-16.6
	$N_{SR,R}s_Rc_R(p)$	-0.3	0.7	0.4	-57.8	-6.3	-15.4
	$s_Fc_F(p)$	-1.6	2.8				
	$N_As_Fc_F(p)$	-0.9	-1.5				
	$s_Fc_R(p)$	-0.7	1.0			-5.4	
	$N_As_Fc_R(p)$	-0.7	0.7			-24.0	
	$s_Rc_R(p)$	-0.1	0.6		-25.2	-7.4	
	$N_As_Rc_R(p)$	-0.6	0.7		-93.5	-24.2	
High	$N_{SR,R}s_Fc_F(p)$	-3.2	-2.4	0.5			127.7
	$N_{SR,R}s_Fc_R(p)$	-1.5	1.2	0.2		-9.9	-12.3
	$N_{SR,R}s_Rc_R(p)$	-0.8	1.3	0.3	-48.2	-11.4	-11.7
	$s_Fc_F(p)$	-2.8	3.4				
	$N_As_Fc_F(p)$	-2.4	-3.1				
	$s_Fc_R(p)$	-1.4	1.2			-8.5	
	$N_As_Fc_R(p)$	-1.5	0.7			-28.3	
	$s_Rc_R(p)$	-0.5	1.4		-23.5	-10.6	
	$N_As_Rc_R(p)$	-1.3	0.6		-86.7	-28.6	

Thompson models once again show very low bias, although the fixed-effects model $s_F c_F(p)$ exhibits significant positive bias increasing with the level of simulated variation. The absolute-recruit abundance models show significantly less bias when the auxiliary catch-effort likelihood is employed (comparing Figure 3.1 with Table A.5), although the bias is still greater in magnitude (negative for mixed-effects, positive for fixed-effects) than the best mixed-effects stock-recruit and conditional-likelihood/Horvitz-Thompson models.

3.3.2 Estimator Precision

Abundance Reconstruction

In order to compare estimates of sampling variation for annual abundance estimates, it is again convenient to compare the ability of each model and accompanying standard error estimates to produce confidence intervals of nominal width. A plot of the median relative bias versus confidence interval width for each model, for each year of abundance reconstruction (Figure 3.2), provides a bivariate summary that simultaneously examines the variance estimates and bias of the abundance estimates. Asymptotic 95% confidence intervals, produced by the methods outlined in Section 2.4 Equation (2.42), have nearest-to-nominal (95%) coverage for models employing the Horvitz-Thompson abundance estimator (Figure 3.2), regardless of the use of the auxiliary catch-effort likelihood. Fixed-effects models ($s_F c_F(p)$ and $N_A s_F c_F(p)$) exhibit CI coverage less than 70% across all nonzero levels of simulated variation, while stock-recruit models exhibit near-100% coverage for many years of reconstruction, indicating confidence intervals that are too wide. Second-best are the mixed-effects models $N_A s_F c_R(p)$ and $N_A s_R c_R(p)$ (as in the full age-class simulations in the previous chapter), which tend to have slightly sub-nominal coverage on average.

Not surprisingly, models that tend to be most-successful in terms of near-95% CI coverage, also tend to have the lowest median relative bias ($s_R c_R(p)$ and $s_F c_R(p)$). Models $N_A s_R c_R(p)$ and $N_A s_F c_R(p)$ also have low median relative bias, while the stock-recruit model $N_{SR,R} s_R c_R(p)$ also has low median relative bias on average, although the latter model has 100% CI coverage for most years of reconstruction when nonzero environmental stochasticity is simulated.

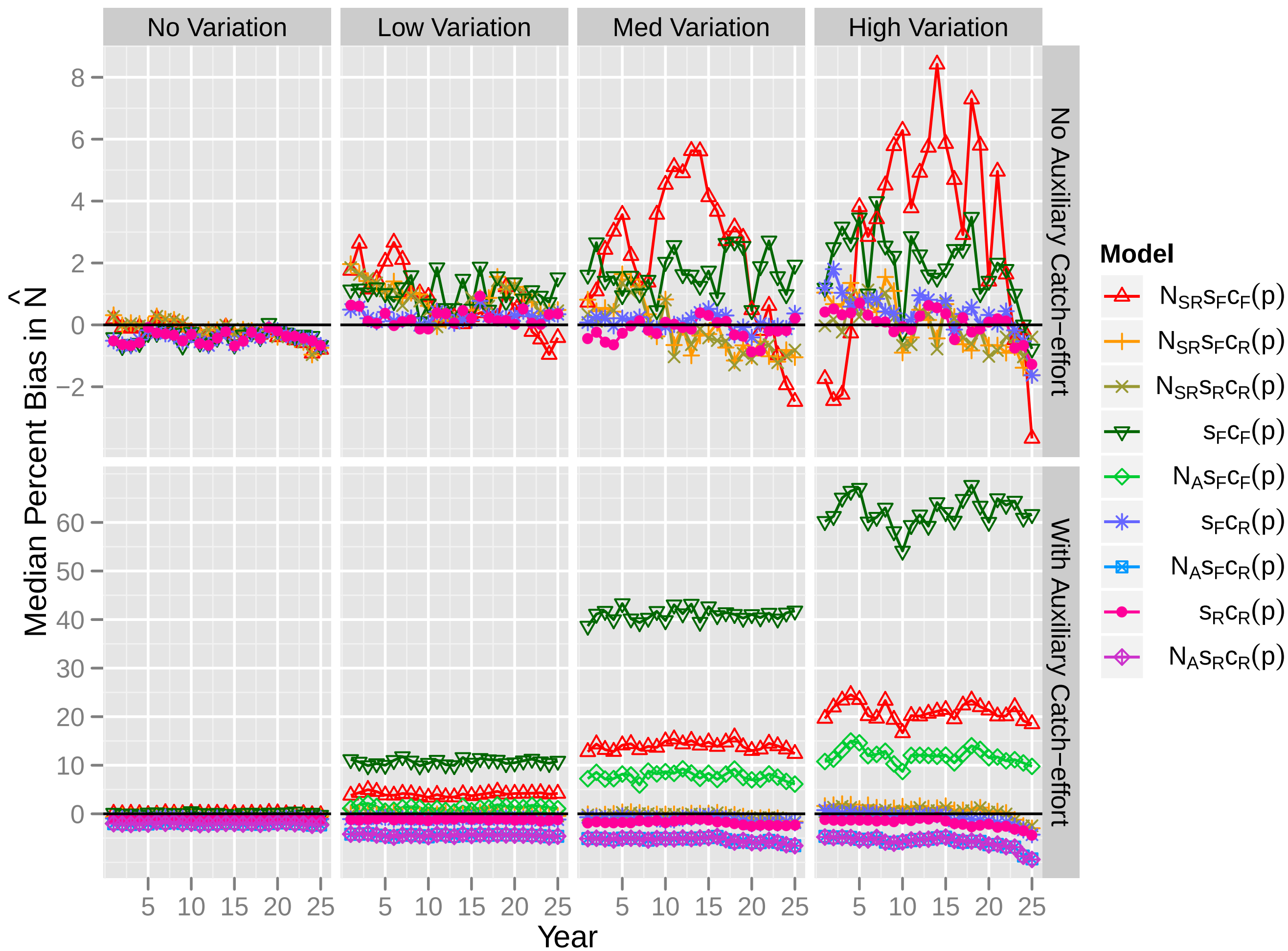


Figure 3.1: *Median relative bias in estimated total annual abundance for pooled age-class data. Models* $N_{As_Fc_F}(p)$, $N_{As_Rc_R}(p)$ *and* $N_{As_Fc_R}(p)$ *not shown for row without auxiliary catch-effort likelihood due to large positive bias (see Appendix B). Results indicate that lowest bias comes from models* $s_Fc_R(p)$ *and* $s_Rc_R(p)$ *without auxiliary catch-effort (though results are very similar when it is used) and models* $N_{SR,R}s_Fc_R(p)$, $N_{SR,R}s_Rc_R(p)$, $s_Fc_R(p)$ *and* $s_Rc_R(p)$ *when the auxiliary catch effort is used.*

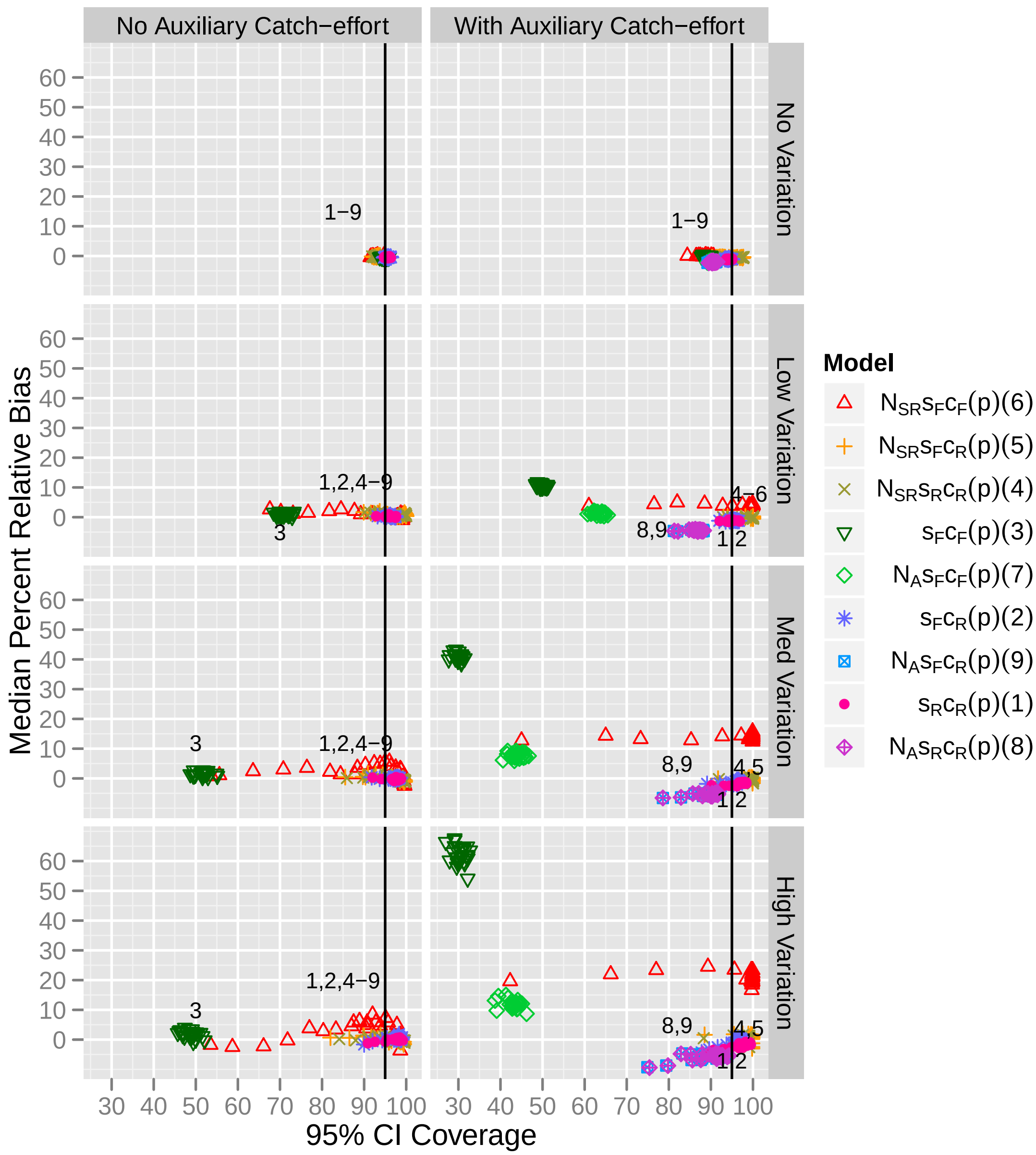


Figure 3.2: *95% CI coverage and median relative bias of total annual abundance, pooled age class models. Percent coverage of annual abundance of 95% CIs are on the x-axis. The y-axis contains the median relative bias of the annual abundance estimates. Each model represented by 25 points, one for each year of data. Models that perform best with respect to these two criteria are those with median relative bias near 0% and with 95% coverage CI coverage, models* s_Fc_R *and* s_Rc_R*.*

Results Summary

As in the simulation study of fully-aged harvest data (Table 2.8), we may implement a progressive filtering scheme to select the models that perform best with respect to the criteria considered here (Table 3.5). First, relative bias in total annual abundance is examined by computing median % bias first for each year of data (averaged across simulations), then averaged across years. Models exhibiting $> 5\%$ absolute bias are eliminated from consideration at this step. The second step examines the mean squared-error (MSE) of confidence interval coverage from its expected value, 95%. In this way, models that systematically overestimate or underestimate standard error will receive a higher MSE, and hence lower rank (in terms of preference). Results indicate that for all levels of simulated variation, the mixed-effects models incorporating a second-stage Horvitz-Thompson abundance estimator have simultaneously among the lowest bias, and nearest-nominal confidence interval coverage; the stock-recruit model $N_{SR,R}s_Fc_R(p)$ also indicates relatively low bias, although its confidence interval coverage tends to be ranked relatively low (Figure 3.2).

Although some differences do exist between results from the fully-aged simulation study and this pooled age-class simulation study (such as less negative bias for absolute-recruit abundance models for pooled data, large positive bias for model $s_Fc_F(p)$ for pooled age-class data, and positive bias for model $N_As_Fc_F(p)$ for pooled age-class data) the primary conclusions mirror those of the full age-class models of the previous chapter (Section 2.4). The reduced-parameter Horvitz-Thompson models tend to be less-biased with more accurate estimates of standard error, and have confidence interval coverage of the appropriate frequency. Overall, these models tend to be most successful in reconstructing population abundance and estimating process parameters and vital rates. The absolute recruit-abundance models exhibit good statistical properties as well, while the stock-recruit models exhibit low bias in some cases, but inappropriate variance estimates in most cases.

As in the case of fully-aged harvest data, the results of this simulation study indicate that mixed-effects models combined with a Horvitz-Thompson estimation approach for annual abundance perform best with respect to a variety of criteria. In addition, the stock-recruit models show good properties with respect to bias, although uncertainty estimation appears to be im-

Table 3.5: *Combined summary of model performance for pooled age-class models. Models with > 5% absolute bias eliminated from comparison. Models performing best with respect to these two criteria (shaded rows) are* s_Rc_R*,* s_Fc_R*, and* $N_{SR,R}s_Fc_R(p)$*, at all levels of simulated variation.*

			Bias		MSE(CI Coverage)	
Var.	**Model**	**Aux. Likelihoo**	% Bias[1]	Rank	MSE^2	Rank
None	$N_{SR,R}s_Fc_F(p)$	With	0.01	1	43.32	16
	$s_Fc_F(p)$	With	0.06	2	37.24	15
	$N_{SR,F}s_Fc_F(p)$	Without	-0.15	3	6.77	11
	$N_{SR,R}s_Fc_R(p)$	Without	-0.17	4	3.37	6
	$N_{SR,R}s_Rc_R(p)$	Without	-0.18	5	3.37	7
	$N_{SR,R}s_Fc_F(p)$	Without	-0.25	6	5.11	8
	$s_Fc_F(p)$	Without	-0.39	7	0.30	1
	$s_Fc_R(p)$	Without	-0.42	8	0.50	4
	$s_Rc_R(p)$	Without	-0.43	9	0.44	2
	$N_{SR,R}s_Rc_R(p)$	With	-0.46	10	5.51	9
	$N_{SR,R}s_Fc_R(p)$	With	-0.46	11	6.24	10
	$N_{SR,F}s_Fc_F(p)$	With	0.69	12	715.50	20
	$s_Fc_R(p)$	With	-1.02	13	0.75	5
	$s_Rc_R(p)$	With	-1.12	14	0.49	3
	$N_As_Fc_F(p)$	With	-1.95	15	23.68	14
	$N_As_Fc_R(p)$	With	-2.02	16	22.21	13
	$N_As_Rc_R(p)$	With	-2.04	17	20.51	12
Low	$N_{SR,R}s_Rc_R(p)$	With	-0.14	1	19.71	8
	$N_{SR,R}s_Fc_R(p)$	With	-0.14	2	19.58	7
	$s_Rc_R(p)$	Without	0.24	3	2.30	3
	$s_Fc_R(p)$	Without	0.27	4	2.61	4
	$N_{SR,R}s_Fc_F(p)$	Without	0.78	5	109.23	14
	$N_{SR,R}s_Rc_R(p)$	Without	0.88	6	13.86	6
	$N_{SR,R}s_Fc_R(p)$	Without	0.88	7	13.60	5
	$s_Fc_F(p)$	Without	1.04	8	573.70	16
	$s_Fc_R(p)$	With	-1.09	9	0.96	1
	$s_Rc_R(p)$	With	-1.18	10	1.03	2
	$N_As_Fc_F(p)$	With	1.24	11	993.96	17
	$N_{SR,F}s_Fc_F(p)$	Without	3.46	12	1126.83	18
	$N_{SR,R}s_Fc_F(p)$	With	4.06	13	85.37	13
	$N_As_Fc_R(p)$	With	-4.42	14	71.85	9
	$N_As_Rc_R(p)$	With	-4.42	15	72.75	10
Medium	$s_Fc_R(p)$	Without	0.08	1	7.50	3
	$N_{SR,R}s_Fc_R(p)$	Without	-0.12	2	18.07	5
	$s_Rc_R(p)$	Without	-0.18	3	7.77	4
	$N_{SR,R}s_Rc_R(p)$	Without	-0.22	4	18.46	6
	$N_{SR,R}s_Fc_R(p)$	With	-0.29	5	21.15	7
	$N_{SR,R}s_Rc_R(p)$	With	-0.37	6	22.56	8
	$s_Fc_R(p)$	With	-1.16	7	3.34	1
	$s_Fc_F(p)$	Without	1.67	8	1890.18	16
	$s_Rc_R(p)$	With	-1.84	9	5.49	2
	$N_{SR,R}s_Fc_F(p)$	Without	2.29	10	229.51	14
High	$N_{SR,R}s_Fc_R(p)$	Without	0.03	1	20.48	7
	$s_Rc_R(p)$	Without	0.05	2	9.44	2
	$N_{SR,R}s_Rc_R(p)$	Without	-0.06	3	18.66	6
	$N_{SR,R}s_Rc_R(p)$	With	0.41	4	16.61	5
	$s_Fc_R(p)$	Without	0.41	5	10.20	3
	$N_{SR,R}s_Fc_R(p)$	With	0.79	6	21.47	8
	$s_Fc_R(p)$	With	-0.81	7	7.01	1
	$s_Rc_R(p)$	With	-1.88	8	11.24	4
	$s_Fc_F(p)$	Without	1.89	9	2146.51	16
	$N_{SR,R}s_Fc_F(p)$	Without	2.92	10	215.41	14

[1] *Median % bias computed first for each year of data (median taken across simulations), then averaged across years.*
[2] *Value computed is* $\frac{1}{25}\sum_{i=1}^{25}(x_i - 95.0)^2$*, where* x_i *is the percent CI coverage estimated for each model.*

precise.

Interestingly, results appear to be better for pooled age class models than models for fully-aged data in some cases. This situation would probably be different if demographic rates for older age classes (> 3 years) differed from younger age classes or from one another, because this information would be "pooled over" in the pooled models, which amounts to a model violation. This is a topic that will be discussed more fully in subsequent chapters.

Model Selection

Again, I turn to the issue of model selection. The discussion of methods presented in Chapter 2 is again relevant here, with respect to the differences between marginal and conditional likelihood values and accompanying likelihood ratio tests and information criteria (mAIC, cAIC). Rather than repeat this information here, the reader is encouraged to review the *Model Selection* section of Chapter 2 before progressing to the results, listed below.

As in the fully-aged simulation study, I present here mean ΔmAIC values (Table 3.6) and mean ΔcAIC values (Table 3.7) to determine the "distance" between models with the same structure, but with different numbers of random effects. Once again, ΔmAIC results indicate a preference towards models with all possible random effects, and ΔcAIC results indicate a preference towards models with random components for harvest vulnerability and recruitment (where relevant), but no random effect for survival probability, which are consistent with the findings of the previous chapter. Given that many estimates of σ_β were zero across model structures and simulations (Table 3.4), this indicates that the cAIC criterion tends to select the more parsimonious model when a random effect variance is estimated to be zero, while the mAIC criteria does tend to select the true simulation model, when variation is nonzero. However, even when environmental stochasticity is simulated to be 0 (Sim. 0), mAIC still prefers models with all possible random effects (the incorrect model) by roughly the same magnitude (difference in mAIC) as scenarios where environmental stochasticity is nonzero, indicating that mAIC always selects random effects into the model, which is not a desirable feature.

As with the fully-aged simulation study of Chapter 2, results show that the mAIC criterion (Table 3.6) indicates a large "distance" between the models of the same class, but with differing number of random effects. According to the cAIC criterion (Table 3.7), the “distance” between models with and without a random effect for survival probability is very low, and that these models tend to be functionally equivalent.

Nested model comparisons based on likelihood ratio tests using the marginal likelihood (Table 3.8) and the conditional likelihood (Table 3.9) support the ΔAIC results, indicating a preference towards use of models with more random effects (frequent rejection of the $\sigma = 0$ null hypothesis) for the marginal likelihood, and preference to select models without a random effect for survival (failing to reject the hypothesis that $\sigma_\beta = 0$) but with random effects for harvest vulnerability for the conditional likelihood. The absolute-recruit abundance model appears to be most successful in selecting a model without random effects when this is the true simulation model (Table 3.9, line 3, column 1). For those models which perform best (models employing the Horvitz-Thompson estimator) by the criteria considered previously (bias, confidence interval coverage), the most parsimonious model structure in terms of random effects may be selected by conducting likelihood ratio tests using the conditional likelihood, which is sensitive to the estimated values of the random effect terms. It is recommended to avoid the marginal likelihood as a means of model selection for these classes of models due to their near-constant preference for inclusion of random effects when they are not justified (as for the simulations with no environmental stochasticity, Sim. 0).

3.3.3 Robustness Simulations

As with the fully-aged simulation study, I use this section to examine model performance under deviation from model assumptions of process parameter stationarity. I have simulated a population where survival probability s is gradually increasing over years of reconstruction (the “increasing s” scenario), a population where survival probability is gradually decreasing (the “decreasing s” scenario), and a population where a periodically-fluctuating recruitment rate leads to a periodically-fluctuating annual abundance that is relatively constant, on average (the “periodic recruitment” scenario). These simulations were conducted in precisely the same manner as the fully-aged simulation study, presented in Chapter 2.

		Mean ΔmAIC						
Aux. Like.	**Model**	Sim. 0	Sim. 1	Sim. 2	Sim. 3	Sim. 4	Sim. 5	Sim. 6
Without	$N_{SR,F}s_Fc_F(p)$	128.5	269.9	657.1	NA	320.3	309.3	335.7
Without	$N_{SR,R}s_Fc_F(p)$	86.2	148.9	300.4	362.2	174.8	167.3	147.6
Without	$N_{SR,R}s_Fc_R(p)$	42.1	42.4	43.9	44.7	42.3	42.2	42.3
Without	$N_{SR,R}s_Rc_R(p)$	0.0	0.0	0.1	0.4	0.0	0.0	0.0
Without	$N_As_Fc_F(p)$	86.1	120.1	256.8	315.6	135.7	134.7	123.9
Without	$N_As_Fc_R(p)$	42.1	42.1	42.1	42.1	42.1	42.1	42.1
Without	$N_As_Rc_R(p)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Without	$s_Fc_F(p)$	86.7	205.2	524.2	650.7	237.6	242.3	213.3
Without	$s_Fc_R(p)$	42.2	42.4	43.3	44.5	42.4	42.4	42.4
Without	$s_Rc_R(p)$	0.0	0.0	0.0	0.1	0.0	0.0	0.0
With	$N_{SR,F}s_Fc_F(p)$	142.6	717.9	2040.9	NA	869.6	1153.6	892.5
With	$N_{SR,R}s_Fc_F(p)$	95.5	302.9	745.6	909.1	344.8	349.1	300.6
With	$N_{SR,R}s_Fc_R(p)$	42.3	42.2	51.2	52.3	45.0	293.3	42.4
With	$N_{SR,R}s_Rc_R(p)$	0.0	0.0	7.1	9.9	7.5	262.3	0.0
With	$N_As_Fc_F(p)$	88.0	253.6	706.8	892.8	323.3	306.0	273.5
With	$N_As_Fc_R(p)$	42.5	42.2	42.5	42.2	42.3	42.2	42.3
With	$N_As_Rc_R(p)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0
With	$s_Fc_F(p)$	93.9	391.2	1071.4	1318.7	466.8	476.3	409.8
With	$s_Fc_R(p)$	42.7	42.7	46.7	48.7	42.8	42.8	42.5
With	$s_Rc_R(p)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Sim. 0 = No variation, Sim. 1 = Low variation, Sim. 2 = Medium variation, Sim. 3 = High variation, Sim. 4 = s_i (hence N_i) increasing during study, Sim. 5 = s_i (hence N_i) decreasing during study, Sim. 6 = period 4 time series "jump" in recruitment rate.

Table 3.6: *Difference in mAIC from lowest-mAIC model and other models (ΔmAIC) within groups defined by model structure for pooled-age simulation study. Results indicate high preference for fully-random models, regardless of use of auxiliary catch-effort likelihood component. "Aux. Like." = Auxiliary likelihood component used (With) or not used (Without) in joint likelihood model.*

		Mean ΔcAIC						
Aux. Like.	**Model**	Sim. 0	Sim. 1	Sim. 2	Sim. 3	Sim. 4	Sim. 5	Sim. 6
Without	$N_{SR,F}s_Fc_F(p)$	1.5	189.5	608.0	NA	1091.8	234.9	268.5
Without	$N_{SR,R}s_Fc_F(p)$	2.3	73.9	234.1	297.3	100.7	94.4	75.6
Without	$N_{SR,R}s_Fc_R(p)$	3.3	0.4	2.6	3.8	0.4	0.3	0.5
Without	$N_{SR,R}s_Rc_R(p)$	5.3	1.9	1.9	2.4	1.9	2.0	2.0
Without	$N_As_Fc_F(p)$	0.1	61.3	225.8	285.9	82.7	82.4	66.5
Without	$N_As_Fc_R(p)$	2.0	0.5	0.2	0.3	0.4	0.3	0.4
Without	$N_As_Rc_R(p)$	4.0	2.5	2.1	2.0	2.4	2.3	2.4
Without	$s_Fc_F(p)$	1.9	156.7	497.5	625.7	193.2	198.8	166.0
Without	$s_Fc_R(p)$	1.3	0.3	1.7	3.2	0.2	0.3	0.3
Without	$s_Rc_R(p)$	3.0	1.7	1.6	1.7	1.8	1.8	1.8
With	$N_{SR,F}s_Fc_F(p)$	33.3	673.5	2035.7	NA	834.2	1117.4	866.0
With	$N_{SR,R}s_Fc_F(p)$	17.3	231.7	690.3	857.8	277.1	280.9	237.2
With	$N_{SR,R}s_Fc_R(p)$	0.2	0.1	9.4	11.0	2.8	251.5	0.5
With	$N_{SR,R}s_Rc_R(p)$	2.2	2.0	9.2	12.0	9.4	264.4	1.9
With	$N_As_Fc_F(p)$	3.5	225.2	704.4	891.5	300.9	283.8	246.7
With	$N_As_Fc_R(p)$	2.3	0.2	0.6	0.3	0.3	0.1	0.3
With	$N_As_Rc_R(p)$	3.6	2.0	2.1	2.0	2.0	2.0	2.0
With	$s_Fc_F(p)$	19.6	361.9	1064.3	1313.3	441.9	451.9	381.6
With	$s_Fc_R(p)$	0.4	0.7	6.1	7.9	0.6	0.8	0.5
With	$s_Rc_R(p)$	1.7	1.8	1.5	1.5	1.8	1.8	1.8

Sim. 0 = No variation, Sim. 1 = Low variation, Sim. 2 = Medium variation, Sim. 3 = High variation, Sim. 4 = s_i (hence N_i) increasing during study, Sim. 5 = s_i (hence N_i) decreasing during study, Sim. 6 = period 4 time series "jump" in recruitment rate.

Table 3.7: *Difference in cAIC from lowest-cAIC model and other models (ΔcAIC) within groups defined by model structure for pooled-age simulation study. Results indicate preference for models with random components for harvest probability and recruitment, but no random component for natural survival probability. "Aux. Like." = Auxiliary likelihood component used (With) or not used (Without) in joint likelihood model.*

			Percent of Null Hypotheses ($\sigma = 0$) Rejected						
Aux. Likelihood	**Larger Model**	**Reduced Model**	Sim. 0	Sim. 1	Sim. 2	Sim. 3	Sim. 4	Sim. 5	Sim. 6
Without	$N_{SR,R}s_Fc_R(p)$	$N_{SR,R}s_Fc_F(p)$	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Without	$N_{SR,R}s_Rc_R(p)$	$N_{SR,R}s_Fc_R(p)$	100.0	100.0	99.8	99.4	100.0	100.0	100.0
Without	$s_Fc_R(p)$	$s_Fc_F(p)$	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Without	$N_As_Fc_R(p)$	$N_As_Fc_F(p)$	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Without	$s_Rc_R(p)$	$s_Fc_R(p)$	100.0	100.0	100.0	99.8	100.0	100.0	100.0
Without	$N_As_Rc_R(p)$	$N_As_Fc_R(p)$	100.0	100.0	100.0	100.0	100.0	100.0	100.0
With	$N_{SR,R}s_Fc_R(p)$	$N_{SR,R}s_Fc_F(p)$	100.0	100.0	99.7	99.9	99.9	99.9	100.0
With	$N_{SR,R}s_Rc_R(p)$	$N_{SR,R}s_Fc_R(p)$	100.0	100.0	100.0	99.5	99.9	99.9	100.0
With	$s_Fc_R(p)$	$s_Fc_F(p)$	100.0	100.0	100.0	100.0	100.0	100.0	100.0
With	$N_As_Fc_R(p)$	$N_As_Fc_F(p)$	100.0	100.0	100.0	100.0	100.0	100.0	100.0
With	$s_Rc_R(p)$	$s_Fc_R(p)$	100.0	100.0	99.9	99.9	100.0	100.0	100.0
With	$N_As_Rc_R(p)$	$N_As_Fc_R(p)$	100.0	100.0	100.0	100.0	100.0	100.0	100.0

Sim. 0 = No variation, Sim. 1 = Low variation, Sim. 2 = Medium variation, Sim. 3 = High variation, Sim. 4 = s_i (hence N_i) increasing during study, Sim. 5 = s_i (hence N_i) decreasing during study, Sim. 6 = period 4 time series "jump" in recruitment rate.

Table 3.8: *Percent of times null hypothesis of adequacy of smaller model ($H_0 : \sigma_x = 0$) rejected out of 1000 simulations, nested model comparisons, using the marginal likelihood.*

			Percent of Null Hypotheses ($\sigma = 0$) Rejected						
Aux. Likelihood	**Larger Model**	**Reduced Model**	Sim. 0	Sim. 1	Sim. 2	Sim. 3	Sim. 4	Sim. 5	Sim. 6
Without	$N_{SR,R}s_Fc_R(p)$	$N_{SR,R}s_Fc_F(p)$	7.9	96.8	99.6	99.1	97.7	97.8	97.8
Without	$N_{SR,R}s_Rc_R(p)$	$N_{SR,R}s_Fc_R(p)$	0.2	1.7	5.0	6.9	1.4	1.7	2.1
Without	$s_Fc_R(p)$	$s_Fc_F(p)$	31.0	99.4	99.9	99.9	99.3	99.8	99.6
Without	$N_As_Fc_R(p)$	$N_As_Fc_F(p)$	0.3	76.3	95.1	94.1	81.7	84.7	77.4
Without	$s_Rc_R(p)$	$s_Fc_R(p)$	1.4	2.9	7.1	10.5	1.6	2.1	2.4
Without	$N_As_Rc_R(p)$	$N_As_Fc_R(p)$	0.0	0.2	1.0	2.5	0.3	0.4	0.2
With	$N_{SR,R}s_Fc_R(p)$	$N_{SR,R}s_Fc_F(p)$	89.3	100.0	99.7	99.8	99.9	99.9	100.0
With	$N_{SR,R}s_Rc_R(p)$	$N_{SR,R}s_Fc_R(p)$	0.1	0.4	2.1	5.7	0.6	1.2	2.3
With	$s_Fc_R(p)$	$s_Fc_F(p)$	91.7	100.0	100.0	100.0	99.9	100.0	100.0
With	$N_As_Fc_R(p)$	$N_As_Fc_F(p)$	11.4	97.7	99.7	99.7	98.3	99.1	98.5
With	$s_Rc_R(p)$	$s_Fc_R(p)$	2.7	4.0	8.7	12.2	2.7	3.3	2.6
With	$N_As_Rc_R(p)$	$N_As_Fc_R(p)$	3.7	1.6	0.8	1.3	1.4	0.8	1.6

Sim. 0 = No variation, Sim. 1 = Low variation, Sim. 2 = Medium variation, Sim. 3 = High variation, Sim. 4 = s_i (hence N_i) increasing during study, Sim. 5 = s_i (hence N_i) decreasing during study, Sim. 6 = period 4 time series "jump" in recruitment rate.

Table 3.9: *Percent of times null hypothesis of adequacy of smaller model ($H_0 : \sigma_x = 0$) rejected out of 1000 simulations, nested model comparisons, using the conditional likelihood.*

3.3.4 Robustness Results

As with the fully-aged data, I examine a series of results intended to investigate robustness to deviations from model assumptions. The simulation models (and indeed, the simulated datasets) for this section match those used for the Robustness Results section of Chapter 2.

Median relative bias for each of the three simulated populations (Figure 3.3) indicates relatively low average absolute relative bias ($< 5\%$) for most models conditional on choosing whether or not the auxiliary catch-effort likelihood is incorporated. The fixed-effects absolute-recruit abundance model $N_A s_F c_F(p)$ shows low bias in each scenario (when the auxiliary likelihood is used), as do the mixed-effects versions of the Horvitz-Thompson model and stock-recruit model (whether or not the auxiliary likelihood is used). As with previous results, the mixed-effects versions of the absolute recruit abundance models of the form $N_A s_R c_R(p)$ show consistent negative bias around -5%, when the auxiliary catch-effort likelihood is used. When the auxiliary catch-effort likelihood component is not used (not pictured, see Table A.7), these models exhibit some large positive bias. For the simulations where recruitment rate was assumed to fluctuate periodically, the stock-recruit models appear to show fluctuating behavior in annual abundance estimates.

In contrast with the robustness results for the fully-aged simulation study (Figure 2.6), pooled age-class models appear to show less bias overall. These results, combined with those from the previous section where deviations from model assumptions were not simulated, appear to indicate there may be some advantage to pooling age classes for harvest data of long-lived animals, and modeling them in the manner described above with respect to accuracy of total annual abundance estimates, assuming that all pooled age classes have the same demographic rates (the scenario that was examined in these simulations).

To examine precision of total annual abundance estimates, I produced a scatterplot of median relative bias vs. confidence interval coverage for each model, for each year of data (Figure 3.4). Once again, results mirror those of the fully-aged simulation study, as well as the results regarding pooled age classes when model assumptions are satisfied; mixed-effects models employing a Horvitz-Thompson estimator have nearest-to-nominal (95%) confidence interval coverage and low bias, while stock-recruit models have relatively low bias, but supernominal

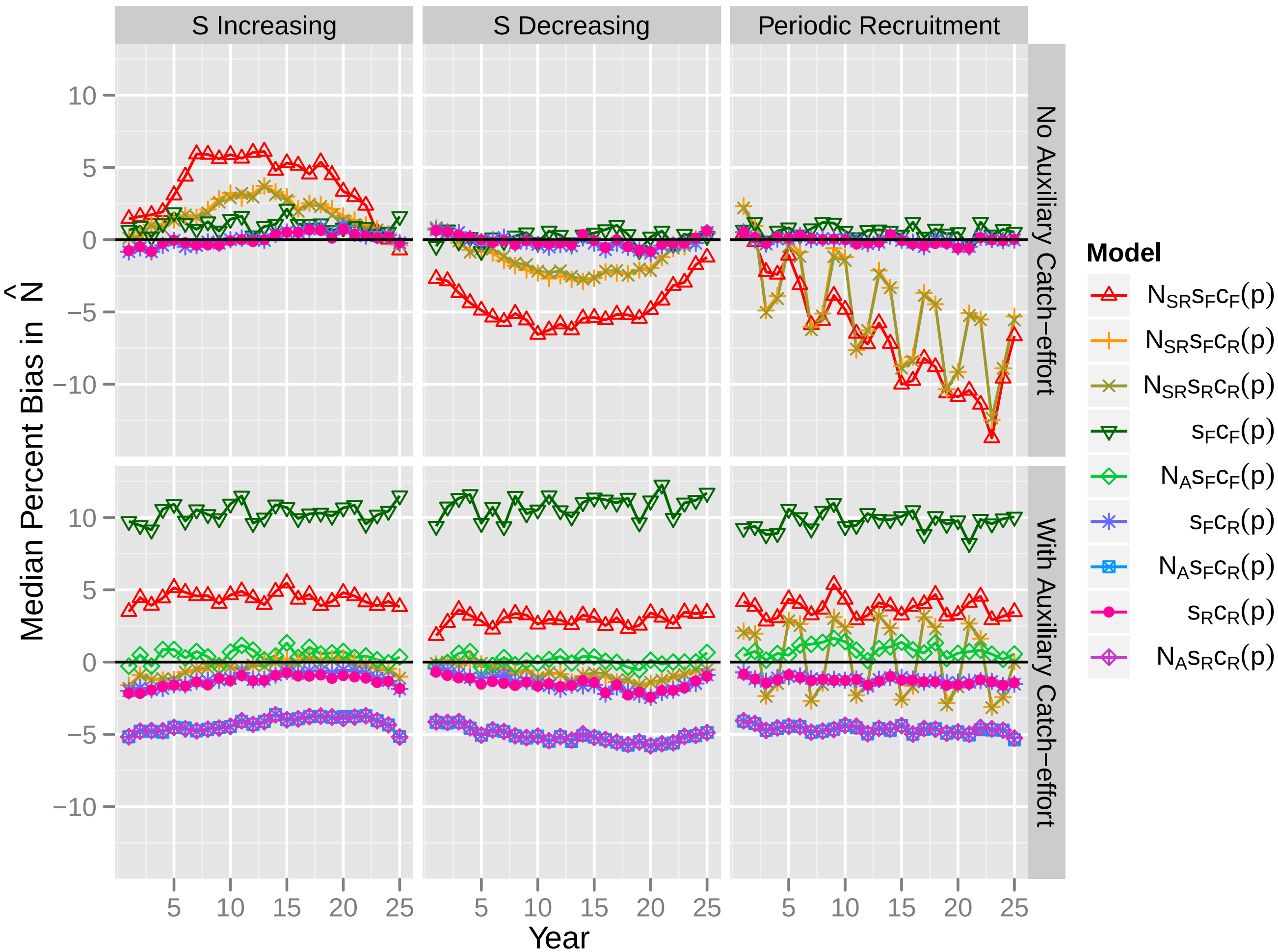


Figure 3.3: *Median relative bias in estimated total annual abundance, robustness simulation studies, pooled age-class models. Results indicate relatively low absolute relative bias for all models, in all scenarios, conditional on the proper use of the auxiliary catch-effort likelihood (except in the case of mixed-effects versions of the conditional-likelihood/Horvitz-Thompson models, which show low bias regardless of its use).*

confidence interval coverage (indicating confidence intervals that are too wide) and fixed-effects versions of each model have low confidence interval coverage (indicating confidence intervals that are too narrow), and a relatively high magnitude of bias in the case of $s_F c_F(p)$. These results indicate that with respect to bias and estimates of parameter estimate uncertainty, the models employing the Horvitz-Thompson estimator have the best performance, while (under a simulation model including random annual deviations for process parameters) fixed effects models consistently underestimate uncertainty, which is manifest in overly-narrow confidence intervals.

3.4 Comparison of Precision of Abundance Estimates with Fully-aged Harvest Data

As mentioned previously, the loss of age information of harvest data is expected to represent a loss of information for the models. This loss of information is expected to have consequences for estimator precision. Generally speaking, precision of model-based parameter estimates from statistical models suffers when information is removed from a dataset, such as when a sample size is decreased. In order to examine what amount of precision is lost, if any, by neglecting the age-class information of adults, I compare the median estimated coefficient of variation of annual abundance estimates ($\widehat{CV}(\hat{N}_i)$) from the pooled age-class models with that from the fully-aged datasets of the previous chapter. While the simulation models for each of these simulation studies was precisely the same, model fit failures were exhibited by some members of each simulation group. Both fully-aged and pooled-age simulations were run until 1000 successful model fits were obtained (no convergence difficulties), independent of one another. The results in this section are comprised of simulations where a simulated dataset produced a successful model fit for both the pooled and fully-aged models. Of the 1000 maximum possible simulations successful in both pooled and full, the zero, low, medium, and high levels of variation allowed for 982, 976, 873, and 562 fits to matching datasets. In order to reduce the number of comparisons made, I limit the models to those which have performed best in Chapters 2 and 3, the models employing the Horvitz-Thompson-type estimators for abundance ($s_R c_R$, $s_R c_R(p)$, $s_F c_R$, $s_F c_R(p)$).

The comparison is summarized by plotting the median $\widehat{CV}(\hat{N}_i)$ for each of the 25 annual

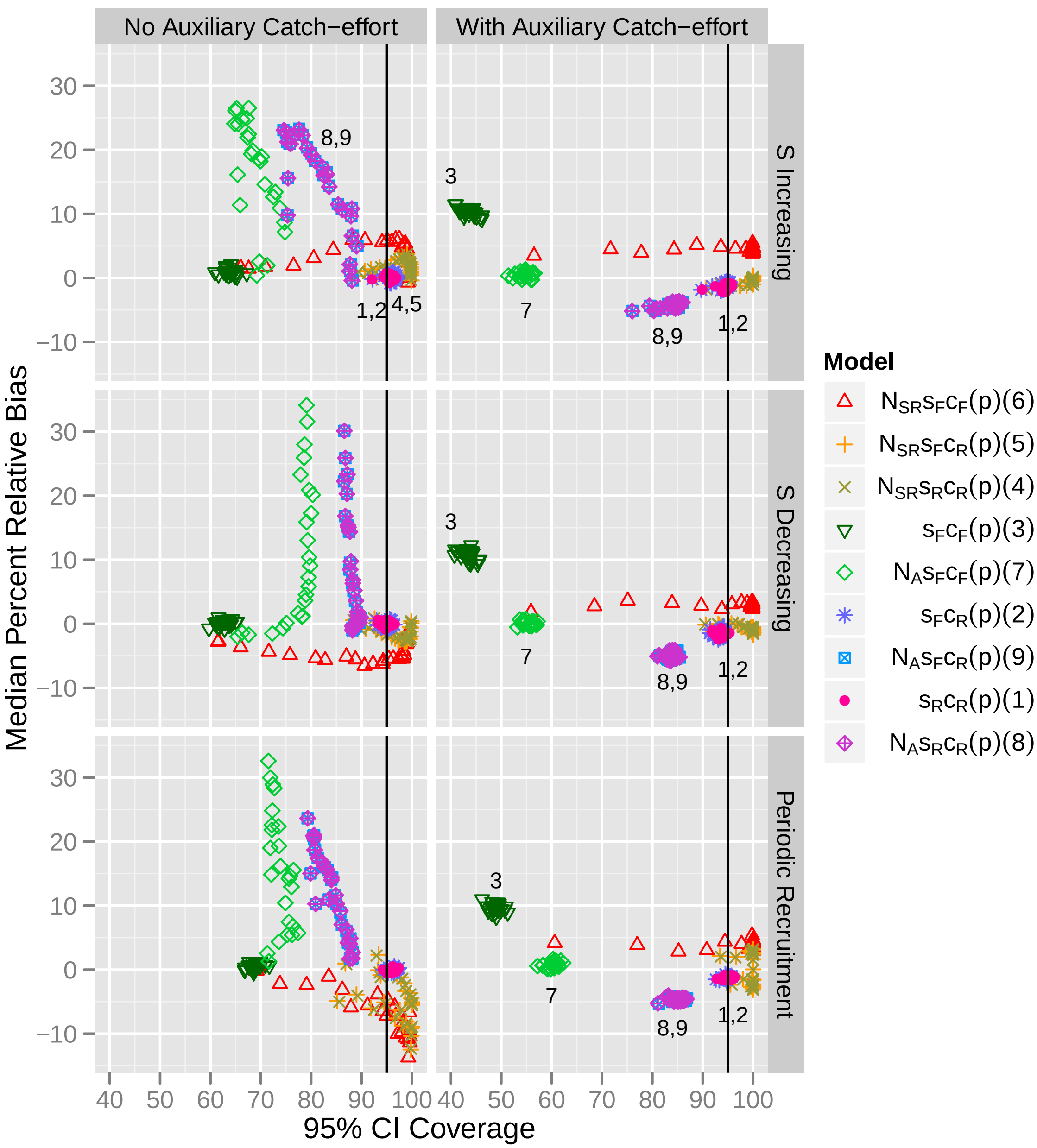


Figure 3.4: *95% confidence interval coverage and median relative bias of estimated total annual abundance for robustness simulations, pooled age-class models. Mixed-effects conditional-likelihood models* $s_R c_R$ *and* $s_F c_R$ *have low bias and nearest-nominal coverage. Fixed effects models* $s_F c_F$ *and* $N_A s_F c_F$ *have lowest (subnominal) CI coverage, while stock-recruit model exhibit* $\approx$ *100% coverage for most years, indicating confidence intervals that are too wide.*

abundance estimates arising from the fully-aged harvest data of Chapter 2 against the median $\widehat{CV}(\widehat{N}_i)$ of each of the 25 annual abundance estimates arising from the pooled age-class models of the current chapter (Figure 3.5). Points that fall on the 45° line indicate that the standard error of estimated abundance relative to the estimated abundance itself is not different between the pooled age-class model and the fully-aged models of Chapter 2; for both models $s_F c_R$ and $s_R c_R$, many points fall on or near this line (Figure 3.5). There appears to be a slight tendency toward lower coefficients of variation for model $s_F c_R$, but the difference between the two models is minimal, which is consistent with previous results concerning bias and confidence interval coverage.

Note that the simulations producing the estimated abundances and standard errors used for plotting Figure (3.5) assumed that harvest vulnerability and survival were identical for all age classes within a given year. For both models, this information loss appears to be minimal for the simulated scenarios. Results would be expected to be different if age classes 3 through 13 required different parameters to model demographic processes. In this case, important information contained in the cohort structure across time would be aggregated over, potentially providing misleading results.

3.5 Discussion

In this chapter, I have presented and discussed statistical population reconstruction models that do not rely on a fully-aged harvest; rather, if animals are accurately aged to three age classes (such as yearling/juvenile/adult), one may apply the methods developed in this chapter.

These models have proven to be both accurate and precise in estimating total annual abundance as well as demographic process parameters regarding harvest probability, survival, and reproductive capacity. In fact, the quality of population reconstruction from the simulation study results from the pooled models developed in this chapter rivals that of the fully-aged simulation study presented in Chapter 2, indicating that there may be little loss of information for populations of the nature simulated here. This would very likely not be the case if demographic parameters differed from one another within adult age classes.

As in the fully-aged simulation study, results regarding estimator bias (Figures 3.1 and 3.3) indicate that the mixed-effects conditional-likelihood models implementing a Horvitz-Thompson

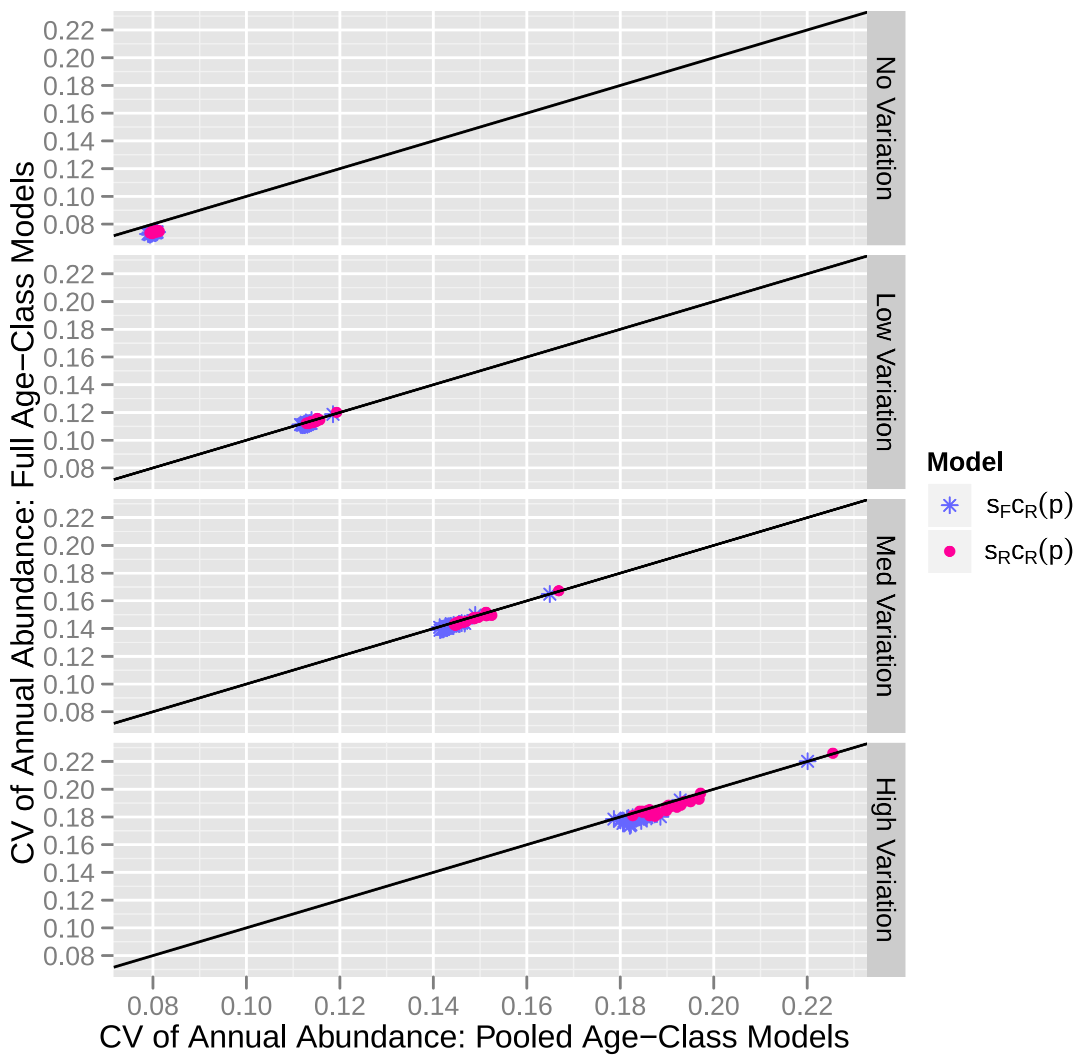


Figure 3.5: *Median estimated coefficient of variation of annual abundance estimates for full age-class models of Chapter 2 (y-axis) plotted against median estimated coefficient of variation of annual abundance estimates for pooled age-class models of Chapter 3 (x-axis) for mixed-effects models employing the Horvitz-Thompson abundance estimator. Each point corresponds to the median of* $\widehat{SE}(\widehat{N}_i)/\widehat{N}_i$ *for each of 25 years of simulated data. The auxiliary catch-effort likelihood of Equation (1.7) was not employed. Points on the 45-degree line indicate no loss of precision by using pooled age-class model. Results indicate low difference in CV for both models, indicating no loss of precision in these simulations, which consider all age classes to have the same harvest vulnerability and survival probability within a given year.*

abundance estimator (models of the form $s_R c_R(p)$) produce accurate estimates of total annual abundance. These models also show nearest-to-nominal confidence interval coverage, indicating appropriate uncertainty estimation (Figures 3.2 and 3.4). Models that estimate initial abundance in year 1 as parameters, and incorporate a stock-recruit function to estimate subsequent recruit abundance (models of the form $N_{SR,R} s_R c_R(p)$) also show low bias (Figures 3.1 and 3.3), but super-nominal CI coverage for most years (Figures 3.2 and 3.4), indicating an over-estimate of standard error, owing to the propogation of estimation error relating to the use of the stock-recruit parameter and the delta-method to estimate standard errors. The absolute-recruit abundance models (models of the form $N_A s_R c_R(p)$) show the greatest magnitude of bias (Figures 3.1 and 3.3 and Tables A.5 and A.7), leading to confidence intervals with subnominal coverage (Figures 3.2 and 3.4). Fixed effects models from the Horvitz-Thompson and stock-recruit models show relatively large bias when the auxiliary catch-effort is used, and confidence interval coverage that is less than or equal to 60%. When the auxiliary catch-effort is not used, model $s_F c_F(p)$ shows low bias, but still subnominal confidence interval coverage ($< 75\%$).

As in the fully-aged simulation study, models showed difficulty in estimating $\sigma_\beta > 0$, although there is some improvement over the fully-aged models of Chapter 2 (Tables 2.7 and 3.4). Akaike information criteria and LRTs using the marginal likelihood tended to select the true model (that which included a random effect for survival probability, as well as for other demographic processes), but AIC and LRTs using the conditional likelihood indicated a preference towards models that removed the random effect for survival, which is consistent with the estimate of survival random effect variation estimated to be zero. For purposes of model selection, one may use the conditional likelihood to select a random effects structure that is most parsimonious.

Among the class of best-performing models (mixed-effects models employing the Horvitz-Thompson estimator), there does not appear to be a good reason to use the auxiliary catch-effort likelihood of Equation (1.7), so I recommended it be omitted. For stock-recruit models that include a random-effects component in the harvest probability transformation and absolute-recruit abundance models, it appears that the use of this extra component in the joint likelihood provides for less-biased and more stable parameter estimates.

Combined results from simulations where model assumptions are satisfied, and when some deviations from model assumptions are considered indicate that the models employing a Horvitz-Thompson abundance estimator are the most successful in reconstructing annual abundance from pooled age-class harvest data.

3.6 Data Analysis: Michigan Elk Herd

As with the fully-aged dataset, the simulation studies indicate that the preferred model is the mixed-effects model that utilizes the Horvitz-Thompson abundance estimator. In order to illustrate the use of this model for pooled age-class datasets, I collapse the Michigan elk data from the previous chapter into 3 age classes for each gender: yearling, juvenile, and adult. Given the results from the previous chapter, where 10 vulnerability coefficients were used (1 for each age class for each gender, up to age 4, and then a single coefficient for each gender for age 4+), we expect to see some deviation from the previous analysis because age classes which are believed to have different harvest vulnerabilities (according to the fully-aged analysis in Section 2.8) are being pooled together. Nevertheless, it is interesting to determine the magnitude of difference exhibited between the two analyses.

3.6.1 Model Details

The aging probability likelihood of Equation (2.49) and the auxiliary abundance likelihood of Equation (2.50) (using auxiliary abundance estimates described in Table 2.19) are used here, to accompany the joint age-at-harvest likelihood, formed as the product of the male and female likelihoods. Each gender's age-at-harvest likelihood is formulated as in Equation (3.1) through Equation (3.11), with one significant modification considered; rather than incorporating an individual vulnerability coefficient for each gender and age class present in the data (which can cause problems with parameter identifiability in the likelihood), we may instead consider that there exists an overall mean vulnerability, and that differential vulnerability between age classes are simply deviations from this mean vulnerability. That is, we may hypothesize that

$$\begin{aligned}
&p_{ij}^{f} = 1 - e^{-e^{\left(c_f + \tau_i^f + \nu_j\right)} f_i} \text{ and} \\
&p_{ij}^{m} = 1 - e^{-e^{\left(c_m + \tau_i^m + \nu_{j+3}\right)} f_i} \text{ for } i = 1, \ldots, Y, j = 1, 2, 3 \\
&\quad \text{and where} \\
&\nu_j \sim Normal\left(0, \sigma_\nu^2\right) \text{ and} \\
&\tau_i \sim Normal\left(0, \sigma_\tau^2\right).
\end{aligned} \tag{3.15}$$

This permits the use of a single fixed effect for harvest vulnerability for each gender, along with the estimation of a variance component (σ_ν) corresponding to the variance of the 6 random effects, $\vec{\nu}$. Note that each ν_j is assumed to be independent of each other in this case, although covariance structures (such as constant covariance between ν_j within genders) may be considered.

The joint likelihood of Equation (2.54) (with L_i^f and L_i^m replaced by the appropriate pooled components) was then optimized with ADMB software (Fournier et al., 2011) to produce parameter estimates and the population reconstruction.

3.6.2 Results

Model selection was performed as for the fully-aged dataset, described in Section 2.8.3, wherein a model incorporating all feasible random and fixed effects first for reduction of random effects terms, and then fixed effects were selected once a random-effects structure was chosen. Model selection for random effects terms was performed by using likelihood ratio tests wherein a test comprised of a mixture of χ^2 p-values was conducted using the conditional likelihood. For fixed effects, likelihood ratio tests were used for nested models and both marginal AIC and conditional AIC were examined for nonnested models.

The final model chosen for the pooled elk data is coded $s_F c_R(p)$, and includes 2 fixed harvest vulnerability parameters (one for each gender, c_f and c_m) as well as interannual random effects for harvest vulnerability ($\vec{\tau}^f/\sigma_c^f$ and $\vec{\tau}^m/\sigma_c^m$). In addition, 6 inter-age random effects ($\vec{\nu}/\sigma_\nu$) were included, one for each combination of gender and age class represented in the pooled data. No random effects for natural mortality were justified by conditional likelihood LRT, and only a single fixed survival parameter was required. No environmental covariates were selected into the model.

Parameter estimate results (Table 3.10) closely mirror those of the fully-aged dataset analysis (Table 2.20). Harvest probability appears to be higher for females than for males, except perhaps for the combined older age class.

Table 3.10: *Parameter estimates for final models for Michigan elk population reconstruction and accompanying 95% confidence intervals, constructed using asymptotic normality of maximum likelihood estimates using pooled age-class harvest data and auxiliary abundance estimates.*

	Female	Male
$\widehat{s}$	1.0 (1.0, 1.0)	1.0 (1.0, 1.0)
$\widehat{c}$	-2.35 (-3.10, -1.61)	-2.85 (-3.62, -2.08)
$\widehat{p}_0^*$	0.11 (0.0, 0.29)	0.09 (0.0, 0.23)
$\widehat{p}_1^*$	0.19 (0.0, 0.45)	0.05 (0.0, 0.15)
$\widehat{p}_2^*$	0.27 (0.0, 0.63)	0.29 (0.0, 0.75)
σ_c	0.20 (0.8, 0.32)	0.06 (0.03, 0.08)
σ_ν	0.64 (0.61, 0.66)	
mAIC	437.79	
cAIC	475.60	

* *Estimated harvest probability computed at mean effort (2.12 thousand hunter-permit-days)*

Total annual abundance estimates reflect the same trend as the model-based results from the fully-aged dataset (Figure 3.6), although abundance estimates are generally somewhat lower for the pooled age-class model. Both pooled and fully-aged model-based estimates of total abundance appear to track the abundance estimates derived from aerial surveys with a subjective sightability correction, which were not used as auxiliary data in the model fit (years 1992-1994, 1996, 1997, 1999-2001).

A penalty of loss of precision associated with pooling older age classes is evident in a comparison of confidence interval width arising from annual abundance estimates (shaded areas, Figure 3.6); pointwise 95% confidence interval widths for estimates arising from the pooled

Figure 3.6: *Annual abundance estimates for the Michigan elk population, comparison of estimates from pooled age-class model fit to fully-aged model fit of Chapter 2. Both models are based on a conditional-likelihood model formulation that utilizes a second-stage Horvitz-Thompson approach to abundance estimation. Both models illustrate a similar trend, although the pooled age-class model estimates lower abundance than the model using the fully-aged dataset, which appears to be consistent with simulation results indicating some negative bias for the pooled model for various levels of simulated variation (Figure 3.1) with little bias or slight positive bias for the model for fully-aged data (Figure 2.2). Alternatively, difference in results between fully-aged and pooled-age analyses may arise from pooling over age classes with differential harvest vulnerabilities. Aerial surveys from 2006 - 2008 incorporated a model-based sightability correction, and were used as auxiliary data. Combined ground and air surveys were conducted in years 1992-1994, 1996, 1997, and 1999-2001 with a subjective sightability correction factor and were not used as auxiliary data, but are plotted for reference purposes. Shaded areas indicate 95% normal-based pointwise confidence intervals (dark = results from fully-aged models of Chapter 2, light = results from pooled age-class models of Chapter 3).*

model are significantly wider than those arising from the fully-aged analysis of Chapter 2.

As with the fully-aged harvest model considered in Chapter 2, results from the pooled age-class analysis indicate a population that declines gradually from the early 1990's to the early 2000's, corresponding to an increase in permits issued during this period (Table 2.18). A gradual increase in estimated total annual abundance accompanies a period of relatively low harvest effort, in the mid-2000's. Estimated annual recruitment rate from the chosen model ($s_R c_R(p)$), measured as the total estimated members of age class 0 in year i divided by the estimated number of breeding-age females in year $i-1$, is very similar for both the pooled and fully-aged models (Figure 3.7), although confidence intervals (95% pointwise intervals computed by estimating the standard error of the ratio via the delta-method) arising from the fully-aged analysis are significantly narrower than those arising from the pooled age-class model.

3.6.3 Discussion

Due to budgetary, personnel, or time restrictions, it may not be possible for a wildlife management agency to accurately age all harvested animals. In such cases, the statistical population reconstruction models for long-lived animals described in this chapter provide a means to assess population status and trends when only 3 age classes can be determined; young-of-the-year, juvenile, and adult. Simulation studies conducted with a reasonable operating model with satisfactory model assumptions, as well as deviations from standard model assumptions indicate that abundance and demographic parameter estimates may be recovered successfully with models designed to accommodate the reduced cohort information typically contained in a fully-aged harvest dataset. Harvest probability may be related to effort or harvest quotas imposed on the population, as well as measured and unmeasured environmental factors through the use of covariates and random effects.

As the elk data analysis section shows, much information may be lost when adult age classes cannot be differentiated; analysis of the fully-aged dataset in Chapter 2 indicated that harvest vulnerability parameters differed among even adult age classes, which were combined into a single adult age class for the pooled analysis of this chapter. However estimates of population demographics, including abundance and recruitment, as well as trends and fluctuations in

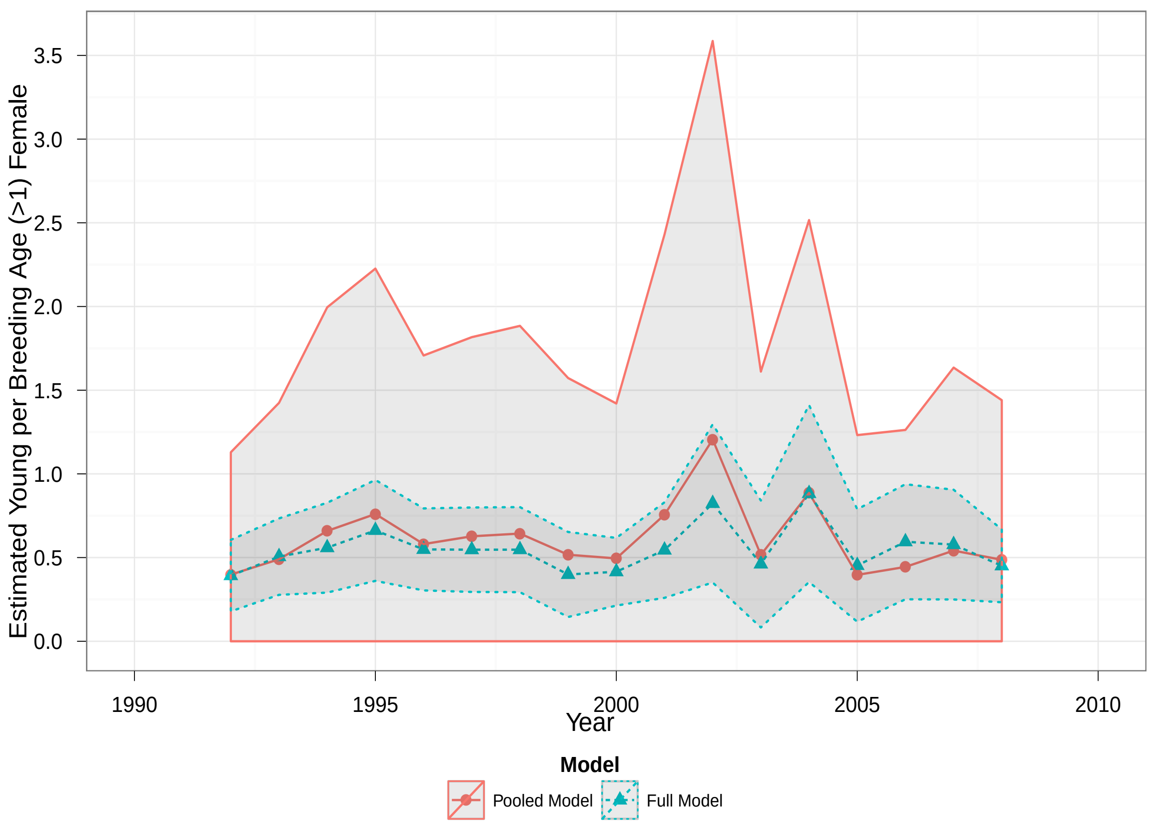


Figure 3.7: *Annual recruitment rate estimates for the Michigan elk population, computed as the estimated number of young per breeding-age (> 0.5) female. Results shown are for the pooled model (solid line) and fully-aged model (dashed line). Results indicate similar trends and estimates in recruitment rate, but significantly wider 95% pointwise confidence intervals from the pooled age-class analysis.*

abundance may be estimated as successfully as with a fully-aged harvest dataset.

The comparison of the three types of models (absolute-recruit abundance, stock-recruit, and conditional-likelihood/Horvitz-Thompson) for pooled age class data mirrors the results of the fully-aged harvest data: the mixed-effects model that utilizes a Horvitz-Thompson-style estimator outperforms the other models. Bias in total abundance estimation is comparable in magnitude among most models, although some show significant bias (Figures 2.2, 2.6, 3.1 and 3.3). The most obvious result separating models from one another in terms of reconstruction quality is that accuracy of estimation of variability in abundance estimates is greatly improved by the use of the model with the Horvitz-Thompson abundance estimator, as evidenced by simulation studies of confidence interval coverage, which show nearest-to-nominal coverage of 95% asymptotically-normal confidence intervals for this model when compared to other models.

For harvested large-game animals, it appears that the best model for statistical population reconstruction is the model of the form $s_R c_R$ or its pooled-age-class counterpart $s_R c_R(p)$. This result is consistent when harvest may be fully aged (Chapter 2) or only partially aged (Chapter 3), although analysis of the Michigan elk data suggests precision of estimates is greatly improved by the additional information contained in the fully-aged dataset (Figures 3.6 and 3.7).

Chapter 4

MODEL PERFORMANCE IN SMALL SAMPLES: SMALL GAME STUDY

4.1 Introduction

Intuition suggests that large datasets which provide a great deal of data should provide greater quality of model fit than smaller datasets, where information may be lacking. It is not unreasonable to hypothesize that when a great deal of age structure information is available, such as for the large game populations studied in Chapters 2 and 3 of this dissertation, and when parameters are shared between age classes, more precise and accurate estimates can be produced than in scenarios where such data is lacking. It is therefore interesting to study how much information is lost when the cohort structure is nearly completely absent.

In this chapter, I seek to understand how the models and estimation procedures proposed herein can be applied to small-game datasets, where age structure is typically very limited; this includes small mammals such as rabbits, squirrels, and most game-bird species. For these populations, there may only be 2 age classes available for animals such as these (i.e., young-of-the-year or adults), which is likely to have an impact on model fit.

The previous chapter (Chapter 3) involved model development and assessment for large game animals when the number of age classes contained in the pooled group was large, and natural mortality was relatively low. The same model structure and assumptions may also be considered for small game animals, where animals may only be placed into two age categories, juvenile and adult. In this case, the number of age classes included in the "pooled" group may only be 1 or 2, and certainly less than 5. Information regarding age structure is therefore significantly reduced. In addition, small game animals are more susceptible to extrinsic factors in their reproduction and survival. Some extrinsic factors may be incorporated via covariates in the relevant demographic process parameter transformations presented in previous chapters, but some factors affecting the population will remain unmeasured, making assessments more difficult.

In the following, I assess the ability of the age-harvest models proposed above to estimate demographic process parameters and abundance in situations where age-at-harvest data are limited to young-of-the-year or adults, and when environmental stochasticity is "large."

4.2 Models

The age-at-harvest model including random effects for harvest and demographic process parameters described in Chapter 3 for pooled data are not significantly altered for use in small game parameter and abundance estimation, except that the number of age classes has been reduced to 2. Therefore, the likelihood in Equation (3.1) is re-expressed (for only two recorded age classes, for the cohort on the main diagonal of the harvest matrix) as

$$L(\vec{N}_{11}, \vec{p}, s \mid \boldsymbol{X}) = \binom{x_{11} + x_{22}^{+}}{x_{11}, x_{22}^{+}} p(1,2|any)^{x_{11}} p(2,2|any)^{x_{22}^{+}} \tag{4.1}$$

,

where

$$p(1,2|any) = \frac{x_{11}}{x_{11} + x_{22}}. \tag{4.2}$$

is estimated by

$$\widehat{p}(1,2|any) = \frac{E(x_{11})}{E(x_{11}) + E(x_{22})} \tag{4.3}$$

where the expected harvest counts are computed as

$$\begin{aligned} E(x_{11}) &= N_{11} p_1 \\ E(x_{22}) &= (N_{11} q_1 s_1 + N_{12} q_1 s_1)\, p_2 = (N_{11} + N_{12})\, q_1 s_1 p_2 \end{aligned} \tag{4.4}$$

Similar modifications to other cohort likelihoods are made, and the joint likelihood is formed as the product of individual cohort likelihoods along with the product of available auxiliary likelihoods.

Table 4.1: *List of model shorthand notation.*

Model Reference Name	Model Description
$N_A s_F c_F$	Absolute annual recruit abundance, β and c fixed
$N_A s_F c_R$	Absolute annual recruit abundance, c random, β fixed
$N_A s_R c_R$	Absolute annual recruit abundance, β and c random
$s_F c_F$	Absolute annual recruit abundance, β and c fixed, with Horvitz-Thompson abundance estimation
$s_F c_R$	Absolute annual recruit abundance, c random, β fixed, with Horvitz-Thompson abundance estimation
$s_R c_R$	Absolute annual recruit abundance, β and c random, with Horvitz-Thompson abundance estimation

Table (4.1) contains the list of shorthand notation used to refer to the models in this chapter, as well as an explanation of what each model entails, in terms of random effects and abundance estimation technique.

For each model, upon completion of likelihood optimization, standard errors are estimated from the inverse-Hessian matrix (for parameters) and delta-method (for functions of parameters) as described in Chapter 2 (Section 2.2.1) for both process parameters and abundance estimates.

For absolute-recruit abundance models ($N_A s_R c_R(p)$, etc.) annual abundance is estimated as described in Chapter 2 (Section 2.4), except that the age $A+$ category (the oldest age class) is also carried over into the following year as

$$\widehat{N}_{i,2+} = \left(\widehat{N}_{i-1,1} + \widehat{N}_{i-1,2+}\right)\widehat{q}_{i-1}\widehat{s}_{i-1}$$

The delta method is employed within the ADMB (Fournier et al., 2011) software to estimate standard errors of estimated abundances.

For models employing the Horvitz-Thompson abundance estimator ($s_R c_R(p)$, etc.), abundance is estimated as described in Chapter 2 (Section 2.2), and standard errors are computed based on a combination of inverse-Hessian standard error estimates for parameters, the delta method for functions of parameters, and the extra-likelihood component described in Chapter 2 (Section 2.2.1) and Appendix B.

4.3 Simulation Model

The simulation model for this small game study is constructed in the same manner as the large game population (see Chapter 2), but different combinations of input parameter values are used to simulate abundance and harvest data, which are more likely to be reasonable for small game populations. Table (4.2) lists the parameter value input combinations which were used for this simulation study, while Table (4.3) details the magnitude of interannual fluctuations simulated.

The amount of simulated auxiliary data available constituted another dimension for this simulation study. As small game animals are often easier to tag, it was of interest to examine model performance under conditions when a large amount of auxiliary data were available. For this reason, simulations were conducted with 6 years of 30 animals each of radiotelemetry data that may be used to aid in estimation of harvest probability. It is also of interest to determine how sensitive the simulation study results are to the amount of available data. For this reason, simulations were also conducted with only a single year of radiotelemetry data comprised of 30 tagged animals, providing a coefficient of variation of approximately 25% for the harvest probability, p. In both cases, the auxiliary data occurs in the middle of the 25 years of simulated age-at-harvest data (year 12 for the single-year case, years 9 through 14 when 6 years are available). In both cases, the number of animals detected as alive following the harvest season and the known number tagged yields (by subtraction) a binomial sample from the number tagged with probability of harvest equal to the probability of harvest of untagged animals (see, for example, Equation 2.3).

4.4 Results

Mixed-effects stock-recruit models proved numerically unstable during optimization. This, combined with the questionable usefulness of such models in real data situations (which hypothesize

Table 4.2: *Parameter combinations used for simulation of small game harvest datasets.*

Simulation Parameter			Potential Values
Number of age classes			2
Years of data			25
Years(# per year) of radiotelemetry data to help estimate harvest vulnerability			6(30), 1(30)
Finite rate of population change (λ)			≈ 1
		Variation Level	
	Low	**Med**	**High**
σ_β, σ_c, σ_γ	0.1	0.2	0.3
Level of harvest[1]			$\approx 40\%$
Level of average survival percentage			50% ($\beta = 0.0$)
Total Initial Abundance			$\approx 40,000$
Average recruits per breeding-age female			≈ 7.4 ($\gamma = 2.0$)

[1] *When evaluated at mean effort. Harvest effort drawn from $Gamma(10, 10)$ then divided by by 100 to give mean effort = 1.0.*

Table 4.3: *Variation induced in natural demographic parameters of interest for small game simulation study. (Harvest probability assessed at mean level of effort.)*

Parameter	Level	Natural Range
Annual Survival Probability = 0.50	$\sigma_\beta = 0.1$	$\mu_s \pm 2\sigma_s$: (0.450, 0.550)
	$\sigma_\beta = 0.2$	$\mu_s \pm 2\sigma_s$: (0.401, 0.599)
	$\sigma_\beta = 0.3$	$\mu_s \pm 2\sigma_s$: (0.354, 0.646)
Annual Harvest Probability = 0.40	$\sigma_c = 0.1$	$\mu_p \pm 2\sigma_p$: (0.338, 0.460)
	$\sigma_c = 0.2$	$\mu_p \pm 2\sigma_p$: (0.287, 0.529)
	$\sigma_c = 0.3$	$\mu_p \pm 2\sigma_p$: (0.242, 0.601)
Fecundity (young per female)= $e^{2.0}$	$\sigma_\gamma = 0.1$	$\mu_f \pm 2\sigma_f$: (6.050, 9.025)
	$\sigma_\gamma = 0.2$	$\mu_f \pm 2\sigma_f$: (4.953, 11.023)
	$\sigma_\gamma = 0.3$	$\mu_f \pm 2\sigma_f$: (4.055, 13.464)

a mean recruitment rate as a simple multiplicative function of prior breeding-age abundance), lead to their exclusion in all results that follow. Therefore, the models fitted to the simulated small-game data are the absolute-recruit abundance models $N_A s_F c_F(p)$, $N_A s_F c_R(p)$, and $N_A s_R c_R(p)$, as well as the models employing the Horvitz-Thompson estimator, $s_F c_F(p)$, $s_F c_R(p)$, and $s_R c_R(p)$. All models were fitted both including the auxiliary catch-effort likelihood component of Equation (1.7) and without this component, with one exception: when the auxiliary catch-effort likelihood component was omitted, the fixed-effects-only absolute-recruit abundance model $N_A s_F c_F(p)$ became unstable during numerical optimization, and it was therefore excluded from consideration.

4.4.1 Estimator Accuracy

Process Parameters

Once again, I examine median relative bias of process parameters, in order to assess the accuracy of their estimation in the small-game scenario (Tables 4.5 and 4.4). Results for both levels of simulated auxiliary data indicate that the models with lowest bias in the estimation of survival probability, s and harvest vulnerability, c are the mixed-effects models that employ the Horvitz-Thompson abundance estimation approach, without the use of the auxiliary catch-effort likelihood of Equation (1.7). At nonzero levels of simulated variation, bias in estimation of both s and c from models that assume a random effect for harvest probability ($s_F c_R(p)$ and $s_R c_R(p)$) ranges from -2.3% to 1.9% when a high level of simulated auxiliary data is available (Table 4.4), and from -5.5% to 16.1% when a low level of simulated auxiliary data is available (Table 4.5). Both of these ranges are largest for the highest level of simulated variation, and are significantly lower when lower environmental stochasticity is simulated. When a low amount of auxiliary data is available, and a high level of variation is simulated, the fixed-effects model that employs the Horvitz-Thompson abundance estimator ($s_F c_F(p)$) has lower estimated bias in estimation of s and c than any other model. In general, the model that assumes natural survival probability is fixed but harvest probability contains a random component ($s_F c_R(p)$) has lower bias than the model which assumes both processes contain random interannual components ($s_R c_R(p)$).

Mixed-effects models that estimate each initial recruit abundance as a parameter (models $N_A s_F c_R(p)$ and $N_A s_R c_R(p)$) indicate large negative bias in the estimation of s when the auxiliary catch-effort likelihood is not employed, but large negative bias in estimation of c when it is employed, for both levels of simulated auxiliary data, and all levels of simulated variation. The fixed-effects version of this model ($N_A s_F c_F(p)$) often provides estimates of s and c with lower estimated bias than its mixed-effect counterparts.

When no environmental variation is simulated, all models show very low bias in the estimation of s and c, however the models employing the Horvitz-Thompson estimators still outperform the absolute-recruit abundance models.

Variability in harvest vulnerability (σ_c) is generally underestimated by all models except for harvest vulnerability for model $s_F c_R(p)$, which slightly overestimates this quantity when a high level of simulated auxiliary data is available (Table 4.4). Models $s_F c_R(p)$ and $s_R c_R(p)$ produce relatively successful estimates of variation in the harvest probability parameter, ranging from -18.7% to 6.0% across levels of simulated variation for the high level of simulated auxiliary data, and from -8.4% to 2.9% for the low level of simulated auxliary data, regardless of use of the catch-effort auxiliary likelihood. All models indicate that $> 50\%$ of estimates of σ_β are effectively zero (bias of approximately -100.0%), with one exception: model $s_R c_R(p)$ indicates positive bias for the medium and high levels of simulated variation when a large level of simulated auxiliary data is available for estimation of c. This positive bias increases with the level of simulated variation.

Based only on these results, it appears as though the mixed-effects model that employs the Horvitz-Thompson abundance estimation approach and an interannual random component for harvest vulnerability but a fixed component for natural survival (model $s_F c_R(p)$) provides the least-biased estimates of the primary process parameters. Model $s_R c_R(p)$ also performs well in many situations, but the positive bias associated with the estimation of σ_β may be quite high. Of course, based on the results of Table (4.4), all alternative models have estimated negative bias of $\approx$ -100.0%. Bias for these results is lowest when the auxiliary likelihood component of Equation (1.7) is omitted.

Abundance Reconstruction

The most important indicator of model performance, median relative bias in abundance reconstruction (Figures 4.1 and 4.2), indicates that annual abundance estimates with the lowest bias are those coming from models employing the Horvitz-Thompson abundance estimator, without the use of the auxiliary catch-effort likelihood of Equation (1.7). When a large amount of auxiliary data are available, both the fixed- and mixed-effects versions of this model have bias between -3% and 2% at all levels of simulated variation (Figure 4.1). When simulated auxiliary data is at a much lower level, the fixed-effects version of this model again has negligible bias (between -3% and 2%), while the mixed-effects versions show similar bias except at the highest level of simulated variation, where bias is approximately 4% to 9%. When the auxiliary catch-

Table 4.4: *Median relative bias (((estimate minus simulated value)/simulated value) in process parameter estimates for analysis of small game data with 6 years of radiotelemetry data (30 tagged animals each) available for harvest probability. Results based on n=1000 replicates at s=0.5,* $c \approx -0.685$*,* γ*=2.0, and total annual abundance* $\approx$ *40,000.*

Aux. Likelihood	Variation Level	Model	$\widehat{s}$	$\widehat{c}$	$\widehat{\sigma}_{\beta}$	$\widehat{\sigma}_{c}$
Without	None	$s_F c_F(p)$	0.1	0.0		
		$s_F c_R(p)$	0.1	-0.0		
		$N_A s_F c_R(p)$	0.4	2.4		
		$s_R c_R(p)$	0.1	-0.0		
		$N_A s_R c_R(p)$	0.4	2.4		
	Low	$s_F c_F(p)$	0.4	-2.0		
		$s_F c_R(p)$	-0.1	-0.3		4.1
		$N_A s_F c_R(p)$	-5.8	1.8		-100.0
		$s_R c_R(p)$	-0.4	1.2	-99.8	-6.5
		$N_A s_R c_R(p)$	-5.4	1.8	-100.0	-100.0
	Medium	$s_F c_F(p)$	-1.0	-1.0		
		$s_F c_R(p)$	-0.4	-0.3		3.9
		$N_A s_F c_R(p)$	-20.8	-0.6		-100.0
		$s_R c_R(p)$	0.3	-0.0	20.3	-11.4
		$N_A s_R c_R(p)$	-20.8	-0.5	-100.0	-100.0
	High	$s_F c_F(p)$	-3.1	-1.7		
		$s_F c_R(p)$	-1.6	0.1		0.2
		$N_A s_F c_R(p)$	-33.5	-0.9		-100.0
		$s_R c_R(p)$	1.9	-2.3	73.9	-18.7
		$N_A s_R c_R(p)$	-33.5	-0.8	-100.0	-100.0
With	None	$s_F c_F(p)$	0.5	-1.5		
		$N_A s_F c_F(p)$	0.6	-1.5		
		$s_F c_R(p)$	0.3	-0.9		
		$N_A s_F c_R(p)$	0.6	-1.5		
		$s_R c_R(p)$	0.3	-0.9		
		$N_A s_R c_R(p)$	0.6	-1.5		
	Low	$s_F c_F(p)$	-6.1	17.0		
		$N_A s_F c_F(p)$	-0.2	-2.0		
		$s_F c_R(p)$	1.9	-6.1		6.0
		$N_A s_F c_R(p)$	6.0	-16.8		-4.8
		$s_R c_R(p)$	1.6	-5.2	-99.8	-5.4
		$N_A s_R c_R(p)$	5.8	-16.1	-100.0	-5.4
	Medium	$s_F c_F(p)$	-19.3	66.1		
		$N_A s_F c_F(p)$	-3.9	1.7		
		$s_F c_R(p)$	2.2	-7.2		4.8
		$N_A s_F c_R(p)$	8.3	-23.1		-3.5
		$s_R c_R(p)$	3.3	-8.1	34.3	-11.0
		$N_A s_R c_R(p)$	7.2	-19.8	-100.0	-7.6
	High	$s_F c_F(p)$	-26.0	94.7		
		$N_A s_F c_F(p)$	-10.3	5.5		
		$s_F c_R(p)$	0.8	-7.1		2.0
		$N_A s_F c_R(p)$	7.4	-24.2		-9.3
		$s_R c_R(p)$	6.1	-12.4	96.7	-17.3
		$N_A s_R c_R(p)$	6.6	-20.3	-100.0	-16.3

Table 4.5: *Median relative bias (((estimate minus simulated value)/simulated value) in process parameter estimates for analysis of small game data with 1 year of radiotelemetry data (30 tagged animals) available for harvest probability. Results based on n=1000 replicates at s=0.5,* $c \approx -0.685$*,* γ*=2.0, and total annual abundance* $\approx$ *40,000.*

Aux. Likelihood	Variation Level	Model	$\hat{s}$	$\hat{c}$	$\hat{\sigma}_\beta$	$\hat{\sigma}_c$
Without	None	$s_F c_F(p)$	0.1	-0.5		
		$s_F c_R(p)$	0.1	-0.5		
		$N_A s_F c_R(p)$	2.8	-7.9		
		$s_R c_R(p)$	0.1	-0.5		
		$N_A s_R c_R(p)$	3.3	-7.9		
	Low	$s_F c_F(p)$	0.4	-2.0		
		$s_F c_R(p)$	1.1	-3.2		-2.0
		$N_A s_F c_R(p)$	-2.4	-7.8		-100.0
		$s_R c_R(p)$	0.0	-0.8	-99.9	-8.4
		$N_A s_R c_R(p)$	-1.6	-7.8	-100.0	-100.0
	Medium	$s_F c_F(p)$	0.1	-2.4		
		$s_F c_R(p)$	0.2	-1.1		2.9
		$N_A s_F c_R(p)$	-18.9	-3.7		-100.0
		$s_R c_R(p)$	-0.6	1.8	-99.8	-4.5
		$N_A s_R c_R(p)$	-18.8	-4.6	-100.0	-100.0
	High	$s_F c_F(p)$	-1.8	-4.9		
		$s_F c_R(p)$	-5.3	12.4		-0.7
		$N_A s_F c_R(p)$	-34.1	6.5		-100.0
		$s_R c_R(p)$	-5.5	16.1	-99.9	-8.1
		$N_A s_R c_R(p)$	-34.1	6.2	-100.0	-100.0
With	None	$s_F c_F(p)$	0.6	-1.7		
		$N_A s_F c_F(p)$	0.6	-2.0		
		$s_F c_R(p)$	0.4	-1.2		
		$N_A s_F c_R(p)$	0.6	-2.0		
		$s_R c_R(p)$	0.5	-1.3		
		$N_A s_R c_R(p)$	0.6	-2.0		
	Low	$s_F c_F(p)$	-6.5	18.0		
		$N_A s_F c_F(p)$	0.5	-3.0		
		$s_F c_R(p)$	6.0	-16.0		-0.6
		$N_A s_F c_R(p)$	17.4	-41.0		-9.3
		$s_R c_R(p)$	5.8	-15.7	-99.9	-8.0
		$N_A s_R c_R(p)$	16.7	-39.7	-100.0	-10.4
	Medium	$s_F c_F(p)$	-22.8	88.8		
		$N_A s_F c_F(p)$	-3.6	1.0		
		$s_F c_R(p)$	12.1	-31.0		1.5
		$N_A s_F c_R(p)$	44.7	-82.4		-8.7
		$s_R c_R(p)$	11.8	-28.9	-99.9	-4.3
		$N_A s_R c_R(p)$	40.0	-77.3	-100.0	-13.6
	High	$s_F c_F(p)$	-32.7	163.6		
		$N_A s_F c_F(p)$	-10.4	6.4		
		$s_F c_R(p)$	9.3	-27.0		-1.3
		$N_A s_F c_R(p)$	56.1	-95.7		-17.4
		$s_R c_R(p)$	9.9	-25.6	-99.9	-8.4
		$N_A s_R c_R(p)$	44.4	-85.1	-100.0	-24.0

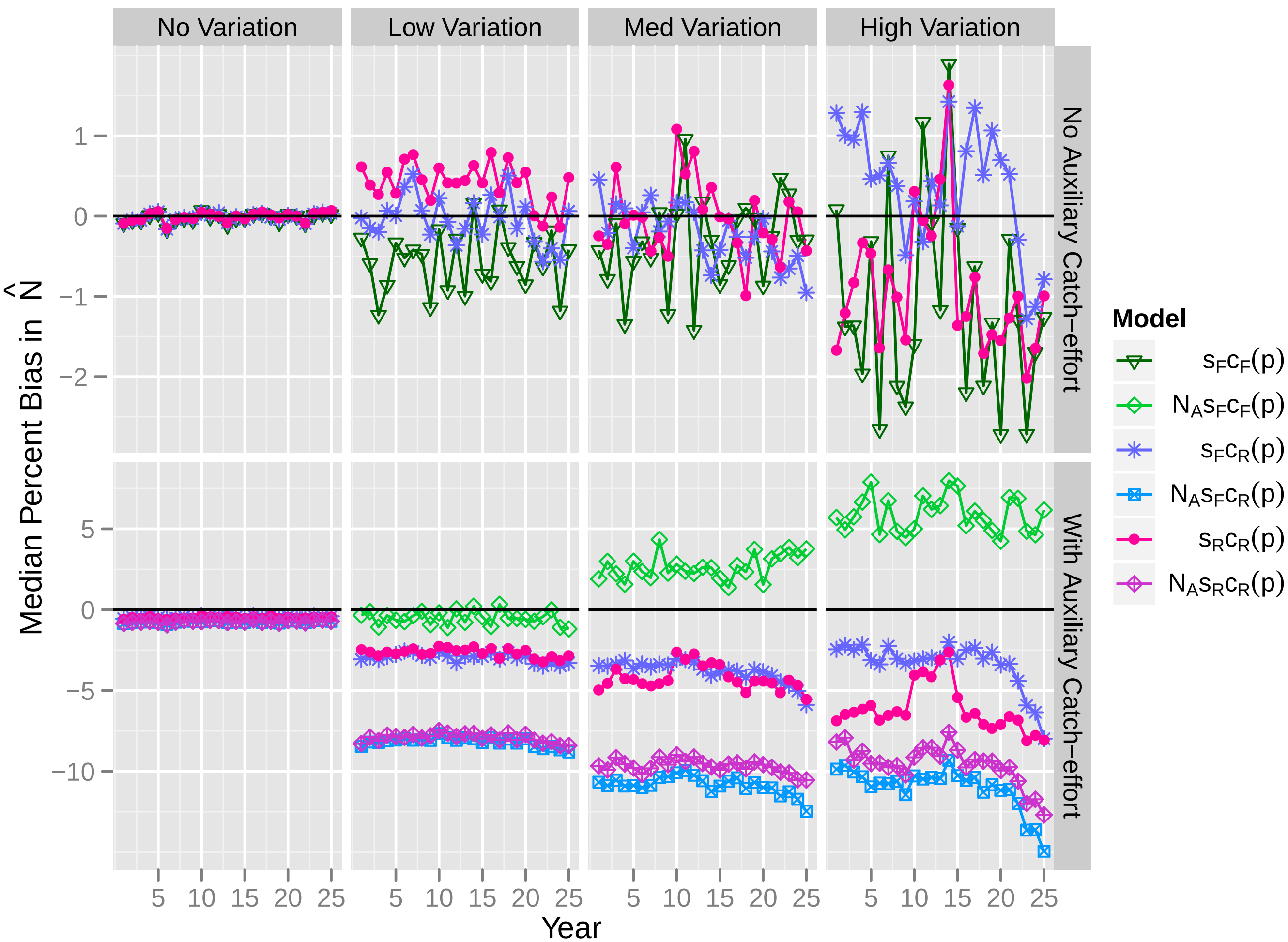


Figure 4.1: *Median relative bias in estimated total annual abundance for simulated small game data, when a large amount of auxiliary data is available for estimation of c. Results indicate lowest bias for models employing the Horvitz-Thompson abundance estimator.*

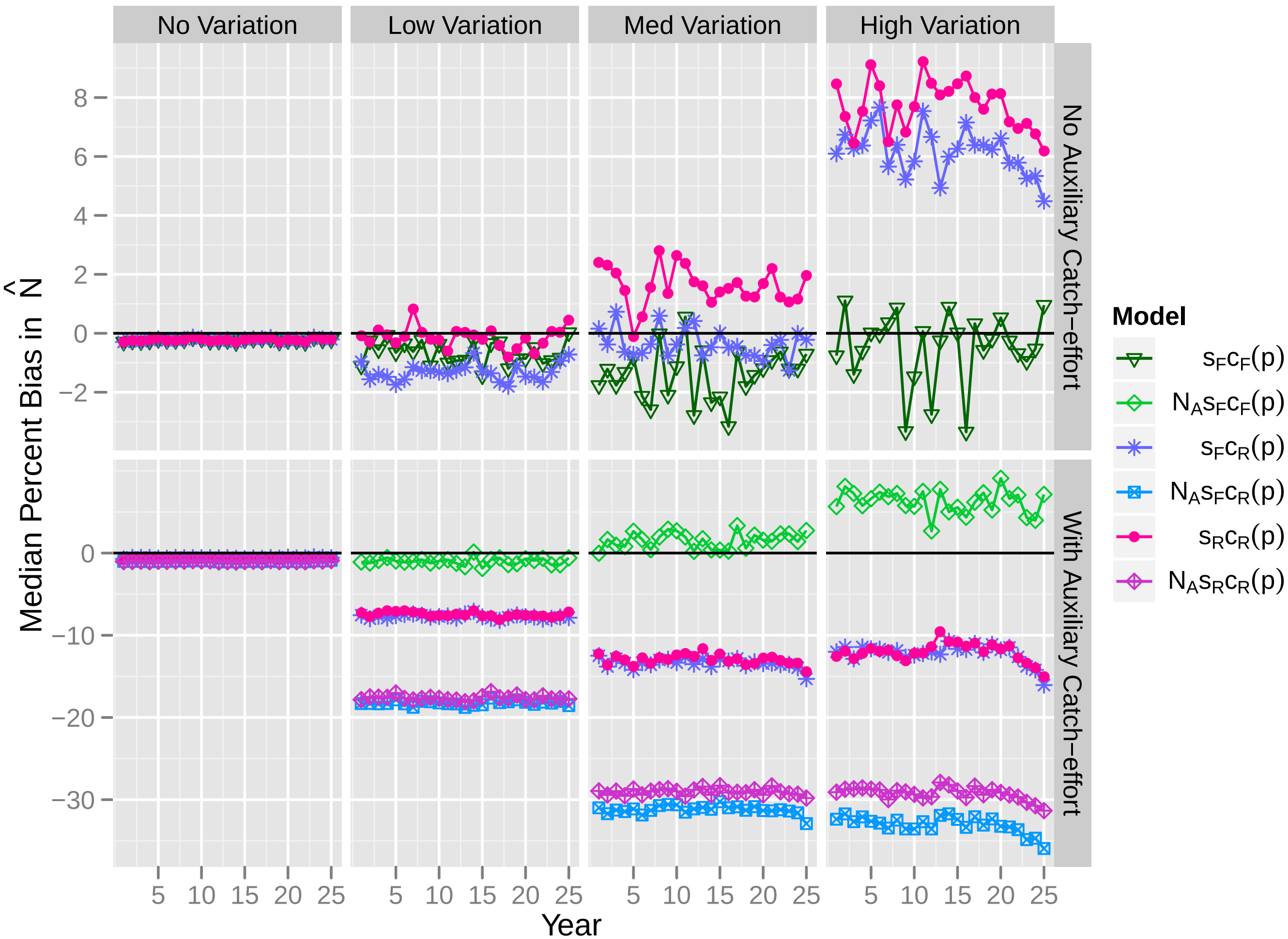


Figure 4.2: *Median relative bias in estimated total annual abundance for simulated small game data, when a low amount of auxiliary data is available for estimation of c. Results indicate lowest bias for models employing the Horvitz-Thompson abundance estimator, or in some cases, the absolute-recruit abundance model of Gove et al. (2002).*

effort likelihood of Equation (1.7) is employed, bias for these models is negative and somewhat large (between -2% and -15% across all simulation scenarios). Note that model $s_F c_F(p)$, *with* the auxiliary likelihood of Equation (1.7), has been removed from Figures (4.1) and (4.2) because the high degree of bias exhibited by this model made visualization of more successful models difficult in the figures. Similarly, absolute-recruit abundance models $N_A s_F c_R(p)$ and $N_A s_R c_R(p)$, without the auxiliary catch-effort likelihood of Equation (1.7), have been removed from Figures (4.1) and (4.2). Results for these models can be found in Tables A.9 and A.10.

Absolute-recruit abundance models indicate relatively low bias for the fixed-effects-only version (with some positive bias at the highest level of simulated variation, between 5% and 10% depending on the level of simulated auxiliary data), but high negative bias for the mixed-effects versions. Models $N_A s_F c_R(p)$ and $N_A s_R c_R(p)$ show bias between 7% and 15% when a large amount of auxiliary data is available to estimate c, and between 18% and 35% when a low level of simulated auxiliary data is available (at nonzero levels of simulated environmental stochasticity).

When no environmental stochasticity is simulated, all models show negligible bias in estimation of total annual abundance, in all simulated scenarios.

4.4.2 Estimator Precision

Abundance Reconstruction

In order to compare estimates of total variation for annual abundance estimates, I compare the ability of each model and accompanying standard error estimates to produce confidence intervals with nominal coverage. A plot of the median relative bias versus confidence interval coverage for each model, for each year of abundance reconstruction (Figures 4.3 and 4.4), provides a bivariate summary that simultaneously examines the confidence interval coverage and bias of the abundance estimates.

Asymptotic 95% confidence intervals, produced by the methods outlined in Section 2.4 Equation (2.42) is nearest-nominal (often by a large degree) for the mixed-effects models employing the Horvitz-Thompson estimator. When a high level of simulated auxiliary data for estimation of c is available (Figure 4.3), results indicate confidence interval coverage for the least-biased models (models employing the Horvitz-Thompson abundance estimation approach, with the auxiliary catch-effort likelihood of Equation (1.7)) hovers between 85% and 95% across all levels of simulated variation, with generally higher coverage at lower levels of simulated variation. When a low level of simulated auxiliary data is available for estimation of c (Figure 4.4), confidence interval coverage for total annual abundance suffers, with coverage ranging between 80% and 90% for these models.

Absolute-recruit abundance models consistently show poor confidence interval coverage, ranging from 20% to 75% depending on the level of simulated auxiliary data and the level of simulated enviornmental stochasticity. For the mixed-effects models, this low confidence interval coverage is at least partially due to the large negative bias exhibited in annual abundance estimates. The fixed-effects model, $N_A s_F c_F(p)$, has low bias in many scenarios, but has extremely low confidence interval coverage (20% to 35% across all simulated scenarios), indicating underestimation of total variablity in annual abundance.

Summary

In concordance with previous chapters, I combine the results of abundance estimator accuracy and precision to determine a model or class of models that performs best overall, with respect to these criteria (Tables 4.6 and 4.7). Simulation results indicate that the models that employ a Horvitz-Thompson estimator for abundance ($s_R c_R(p)$,. etc.) tend to outperform models that estimate initial cohort abundance as a parameter ($N_A s_R c_R(p)$, etc.) in terms of the combined criteria of bias and confidence interval coverage, at both the low and high levels of simulated auxiliary data for estimation of c. Generally, results show both lower absolute bias and better confidence interval coverage (lower MSE(CI Coverage)) for this class of models.

For the low level of simulated auxiliary data and at the highest level of simulated variation, bias is comparable for the mixed-effects versions of both model structures, but the confidence interval coverage is significantly better for models $s_F c_R(p)$ and $s_R c_R(p)$. Also for the low level of simulated auxiliary data, at at the high level of simulated variation, the fixed-effects-only model indicates equivalent or lower estimated bias, but the fixed-effects model with lowest estimated bias tends to have the worst or nearly the worst confidence interval coverage. In this

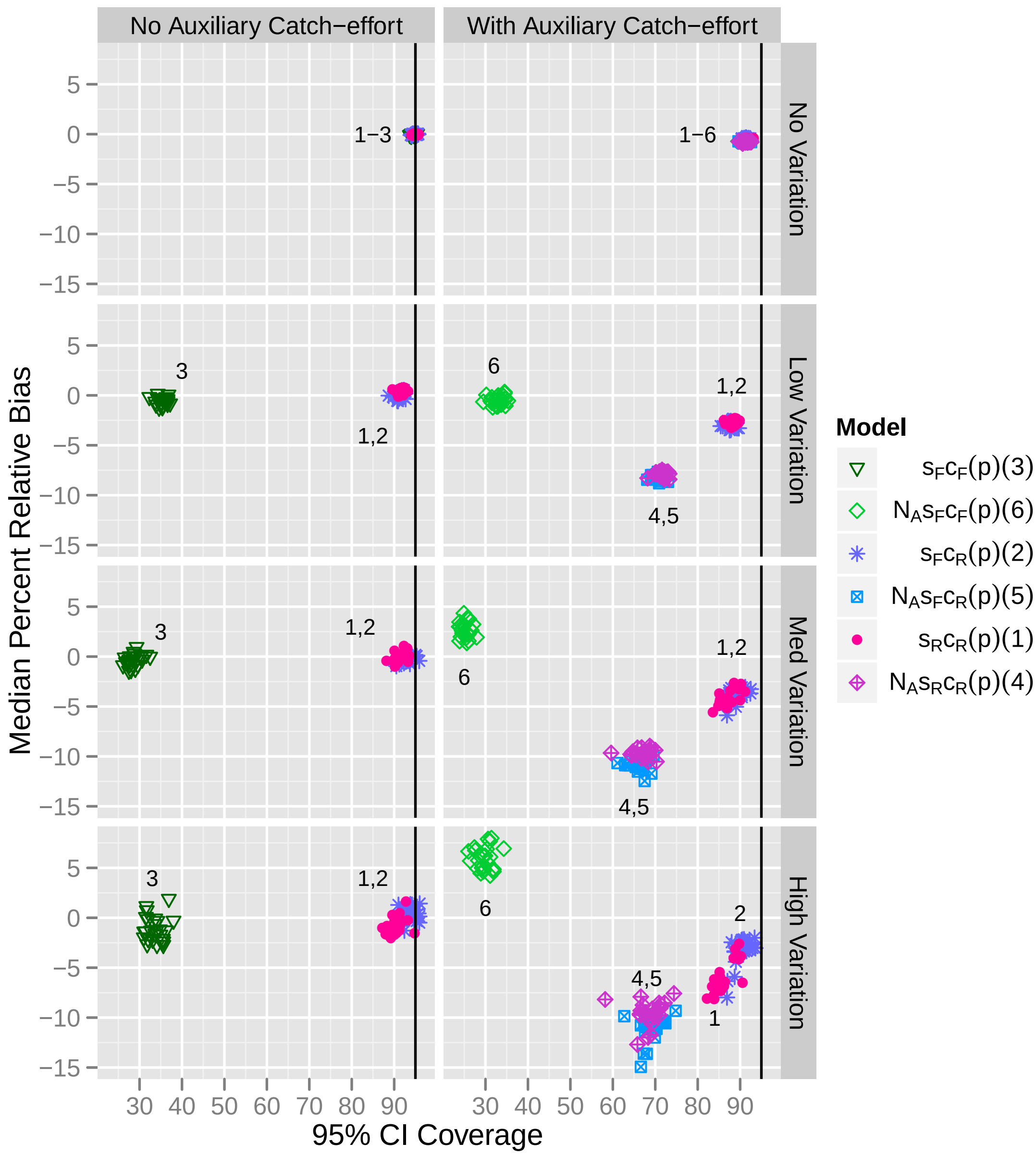


Figure 4.3: *95% CI coverage and median relative bias of total annual abundance, for small game models when a high amount of auxiliary data is available for estimation of c. Percent coverage of annual abundance of 95% CIs are on the x-axis. The y-axis contains the median relative bias of the annual abundance estimates. Each model represented by 25 points, one for each year of data. Models that perform best with respect to these two criteria are those with 95% coverage and low absolute median relative bias, models $s_F c_R(p)$ and $s_R c_R(p)$.*

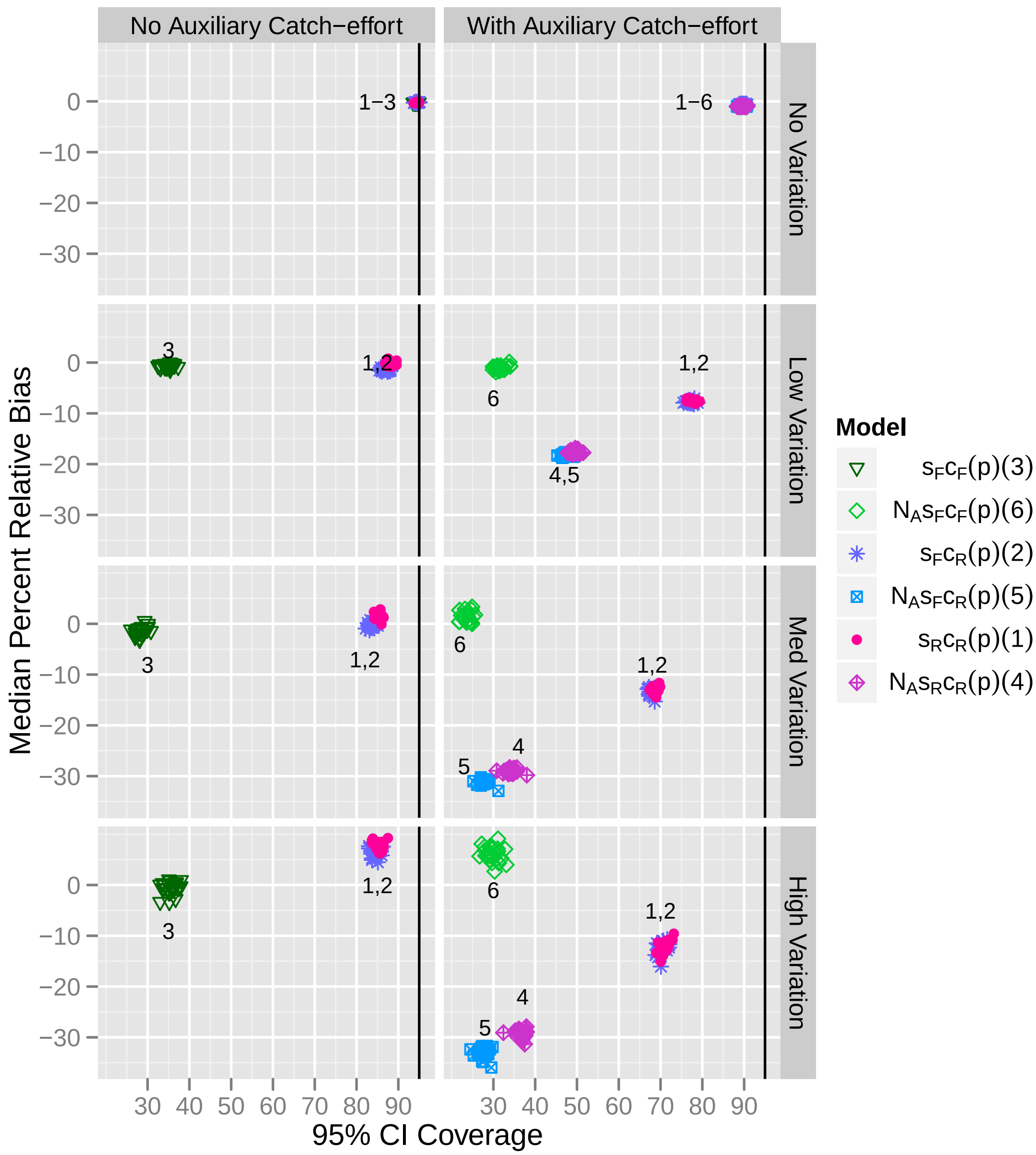


Figure 4.4: *95% CI coverage and median relative bias of total annual abundance, for small game models when a low amount of auxiliary data is available for estimation of c. Percent coverage of annual abundance of 95% CIs are on the x-axis. The y-axis contains the median relative bias of the annual abundance estimates. Each model represented by 25 points, one for each year of data. Models that perform best with respect to these two criteria are those with 95% coverage and low absolute median relative bias, models $s_F c_R(p)$ and $s_R c_R(p)$.*

case, the mixed-effects models that employ a Horvitz-Thompson abundance estimator strike a compromise between these two criteria (bias and CI coverage) for model assessment.

Results in this section are consistent with results from previous chapters. Simulation studies indicate that abundance is best estimated with a Horvitz-Thompson estimator which explicitly takes into account the binomial sampling variation associated with the harvest process at each age class and time point. The extension of each model to incorporate random effects provides nearer-to-nominal confidence interval coverage under a realistic simulation model that includes environmental stochasticity in the demographic processes of survival, reproduction, and harvest.

4.4.3 Robustness Simulations

As in previous chapters, I take this opportunity to examine model performance when deviations from model assumptions are simulated. Also as in previous chapters, I examine three scenarios in order to determine if abundance estimates are robust to these simulated deviations: the "increasing s" scenario, where mean annual survival tends to increase over the course of the age-at-harvest data, the "decreasing s" scenario, where mean annual survival tends to decrease over the course of the age-at-harvest data, and the "periodic recruitment" scenario, where recruitment tends to exhibit a sharp jump every fourth year, alternating with years including a sharp drop and years of "average" recruitment (see section 2.5 for more details). Each of 1000 robustness simulations per scenario was conducted at the low level of simulated variation (Table 4.2). These scenarios are also simulated with the "high" and "low" levels of auxiliary data for the estimation of c, as in previous sections of this chapter.

4.4.4 Robustness Results

In order to examine the effect of the simulated deviations from model assumptions, I once again examine the median relative bias in annual abundance estimates for the "increasing-s", "decreasing-s" and periodic-recruitment models described in Chapters 2 and 3 (Figures 4.5 and 4.6). Results indicate that the fixed-effects model with a parameter for each initial cohort abundance ($N_A s_F c_F(p)$) has nearly 0 median bias for each scenario (when the auxiliary catch-effort likelihood is employed), while the mixed-effects models that employ a Horvitz-Thompson

Table 4.6: *Combined summary of model performance for small game models when a high amount of auxiliary data for estimating c was simulated; models exhibiting > 10% absolute relative bias eliminated from consideration. Models performing best with respect to these two criteria (shaded rows) are s_Rc_R and s_Fc_R, at all levels of simulated variation. "Aux. Like." = Auxiliary catch-effort likelihood used (With) or not used (Without).*

			Bias		MSE(CI Coverage)	
Var.	**Aux. Like.**	**Model**	% Bias[1]	Rank	MSE^2	Rank
None	Without	$s_Rc_R(p)$	-0.01	1	0.27	1
	Without	$s_Fc_R(p)$	-0.02	2	0.28	2
	Without	$s_Fc_F(p)$	-0.02	3	0.35	3
	With	$s_Fc_R(p)$	-0.45	4	14.25	5
	With	$s_Rc_R(p)$	-0.50	5	9.44	4
	With	$N_As_Fc_F(p)$	-0.74	6	15.04	7
	With	$N_As_Fc_R(p)$	-0.74	7	15.04	8
	With	$N_As_Rc_R(p)$	-0.74	8	14.26	6
Low	Without	$s_Fc_R(p)$	-0.04	1	14.25	4
	Without	$s_Rc_R(p)$	0.41	2	11.78	3
	With	$N_As_Fc_F(p)$	-0.51	3	3852.53	10
	Without	$s_Fc_F(p)$	-0.57	4	3555.38	9
	With	$s_Rc_R(p)$	-2.67	5	43.52	5
	With	$s_Fc_R(p)$	-2.96	6	52.68	6
	With	$N_As_Rc_R(p)$	-7.92	7	556.45	7
	With	$N_As_Fc_R(p)$	-8.17	8	597.34	8
Medium	Without	$s_Rc_R(p)$	-0.04	1	15.42	2
	Without	$s_Fc_R(p)$	-0.22	2	5.93	1
	Without	$s_Fc_F(p)$	-0.33	3	4402.79	9
	With	$N_As_Fc_F(p)$	2.65	4	4850.26	10
	With	$s_Fc_R(p)$	-3.81	5	35.50	3
	With	$s_Rc_R(p)$	-4.22	6	66.13	4
High	Without	$s_Fc_R(p)$	0.37	1	4.91	1
	Without	$s_Rc_R(p)$	-0.89	2	26.37	3
	Without	$s_Fc_F(p)$	-1.10	3	3770.68	9
	With	$s_Fc_R(p)$	-3.34	4	25.34	2
	With	$N_As_Fc_F(p)$	5.89	5	4256.04	10
	With	$s_Rc_R(p)$	-6.13	6	87.27	4
	With	$N_As_Rc_R(p)$	-9.55	7	700.12	6

[1] *Median % bias computed first for each year of data (median taken across simulations), then averaged across years.*

[2] *Value computed is $\frac{1}{25}\sum_{i=1}^{25}(x_i - 95.0)^2$, where x_i is the percent CI coverage estimated for each model.*

Table 4.7: *Combined summary of model performance for small game models when a low amount of auxiliary data for estimating c was simulated; models exhibiting > 10% absolute relative bias eliminated from consideration. Models performing best with respect to these two criteria (shaded rows) are s_Rc_R and s_Fc_R, at all levels of simulated variation. "Aux. Like." = Auxiliary catch-effort likelihood used (With) or not used (Without).*

			Bias		**MSE(CI Coverage)**	
Var.	**Aux. Like.**	**Model**	% Bias[1]	Rank	MSE^2	Rank
None	Without	$s_Fc_R(p)$	-0.22	1	0.35	2
	Without	$s_Rc_R(p)$	-0.22	2	0.24	1
	Without	$s_Fc_F(p)$	-0.23	3	0.65	3
	With	$s_Fc_R(p)$	-0.58	4	29.75	5
	With	$s_Rc_R(p)$	-0.67	5	23.06	4
	With	$s_Fc_F(p)$	-0.85	6	312.94	11
	With	$N_As_Fc_R(p)$	-0.99	7	31.06	8
	With	$N_As_Fc_F(p)$	-0.99	8	31.06	7
	With	$N_As_Rc_R(p)$	-1.00	9	30.78	6
	Without	$N_As_Rc_R(p)$	3.51	10	51.29	10
Low	Without	$s_Rc_R(p)$	-0.14	1	48.83	3
	Without	$s_Fc_F(p)$	-0.66	2	3620.61	9
	With	$N_As_Fc_F(p)$	-1.01	3	4015.96	10
	Without	$s_Fc_R(p)$	-1.32	4	66.65	4
	With	$s_Rc_R(p)$	-7.48	5	309.58	5
	With	$s_Fc_R(p)$	-7.73	6	321.71	6
	With	$s_Fc_F(p)$	9.91	7	5159.71	11
Medium	Without	$s_Fc_R(p)$	-0.34	1	129.53	2
	Without	$s_Fc_F(p)$	-1.40	2	4435.94	8
	With	$N_As_Fc_F(p)$	1.54	3	5092.27	10
	Without	$s_Rc_R(p)$	1.61	4	95.48	1
High	Without	$s_Fc_F(p)$	-0.48	1	3535.14	8
	Without	$s_Fc_R(p)$	6.17	2	111.49	2
	With	$N_As_Fc_F(p)$	6.25	3	4254.02	9
	Without	$s_Rc_R(p)$	7.73	4	95.75	1
	Without	$N_As_Rc_R(p)$	-7.81	5	2749.34	5
	Without	$N_As_Fc_R(p)$	-7.85	6	2817.33	6

[1] *Median % bias computed first for each year of data (median taken across simulations), then averaged across years.*

[2] *Value computed is* $\frac{1}{25}\sum_{i=1}^{25}(x_i - 95.0)^2$, *where* x_i *is the percent CI coverage estimated for each model.*

abundance estimator are second-best; for the high level of simulated auxiliary data (Figure 4.5), bias is very low for the mixed-effects versions of the Horvitz-Thompson models (between -1.5% and 1.0% in all scenarios), when the catch-effort likelihood of Equation (1.7) is omitted. When a low level of auxiliary data is simulated (Figure 4.6), bias for these models is approximately -7.5% for each of the three scenarios when the auxiliary catch-effort likelihood is used, but a bias between -2% and 2% when it is omitted. Mixed-effects versions of the absolute-recruit abundance model exhibit negative bias of nearly -20%, while the fixed-effects-only version of the model employing the Horvitz-Thompson estimator exhibits median relative bias of 10% to 12% when the auxiliary catch-effort likelihood component is used, but bias between -2% and 1% when it is omitted. As in previous simulation results in this chapter, absolute-recruit abundance models $N_A s_F c_R(p)$ and $N_A s_R c_R(p)$, without the auxiliary catch-effort likelihood of Equation (1.7), have been removed from Figures (4.1) and (4.2) due to their high bias.

Uncertainty estimation of annual abundance estimates is again described by estimated confidence interval coverage, plotted against the median relative bias of annual abundance for each year of data (Figures 4.7 and 4.8). Not surprisingly, the results indicate that the fixed-effects models have significantly subnominal confidence interval coverage, around 20% - 30%, regardless of the level of simulated auxiliary data. While still exhibiting subnominal confidence interval coverage, the best-performing models are once again the models that employ a Horvitz-Thompson abundance estimator, $s_F c_R(p)$ and $s_R c_R(p)$, which exhibit confidence interval coverage roughly between 75% and 85% when the auxiliary catch-effort likelihood component is included, and coverage between 80% and 85% when it is omitted, across levels of simulated auxiliary data. Absolute-recruit abundance models have estimated confidence interval coverage between 40% and 50%, at least partially due to their large estimated negative bias.

4.5 Discussion

The analysis of age-at-harvest data for small game animals presents a number of challenges in addition to those presented by large game data. Adult age classes are typically not able to be separated, so only two age classes may be available. In addition to a lack of age specificity, auxiliary data sources may be in abundance, or may be limited; small game animals are not often collared for radiotelemetry studies, nor are independent survey data frequently available.

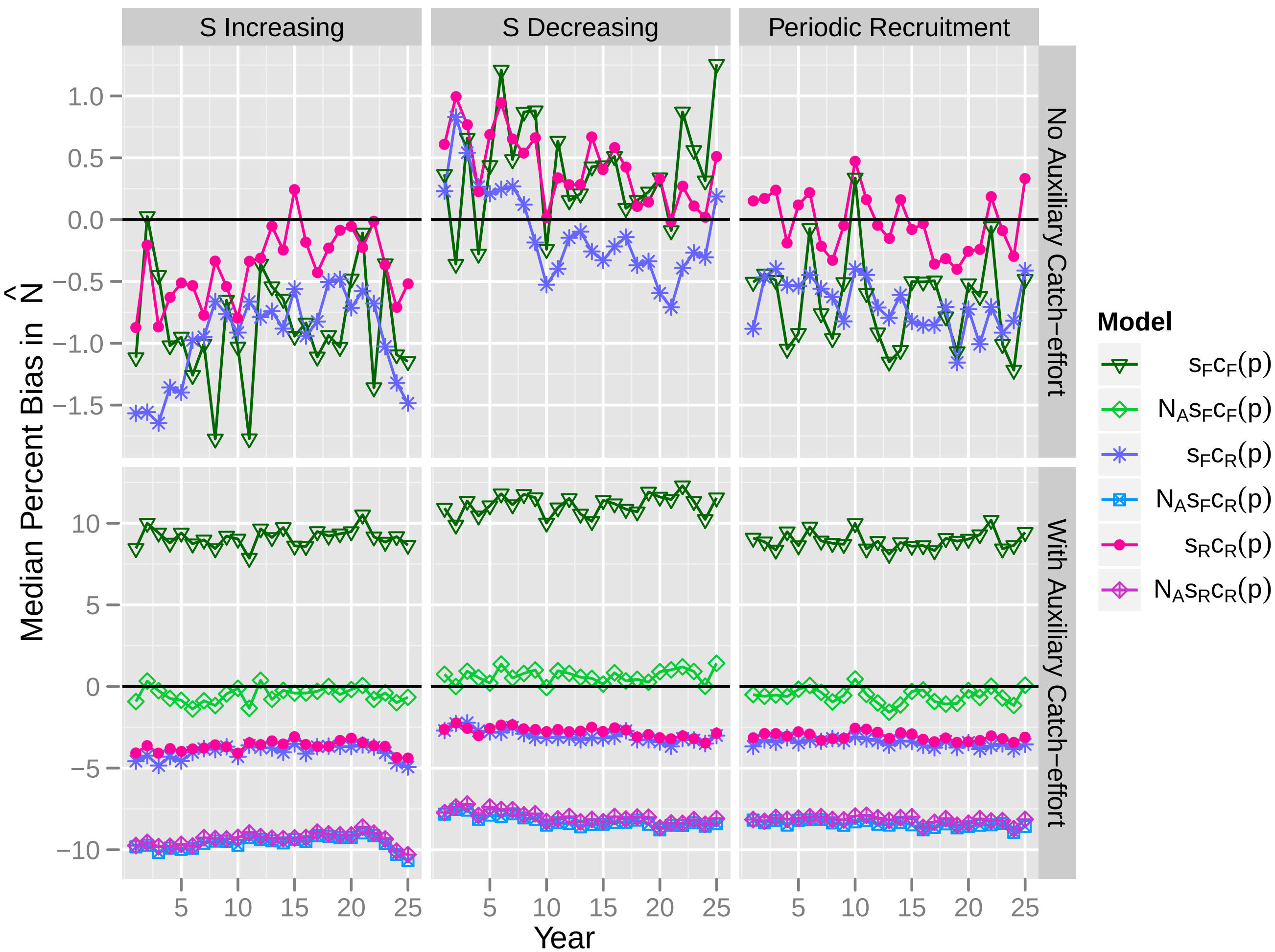


Figure 4.5: *Median percent bias in estimated total annual abundance, robustness simulation studies, for small game models when a high amount of auxiliary data is available for estimation of c. Results indicate consistent bias in all three scenarios, with the fixed-effects-only absolute-recruit abundance model ($N_A s_F c_F(p)$, with the inclusion of the auxiliary catch-effort likelihood component), and the fixed- and mixed-effects Horvitz-Thompson models (when the auxiliary catch-effort likelihood component is omitted) performing the best.*

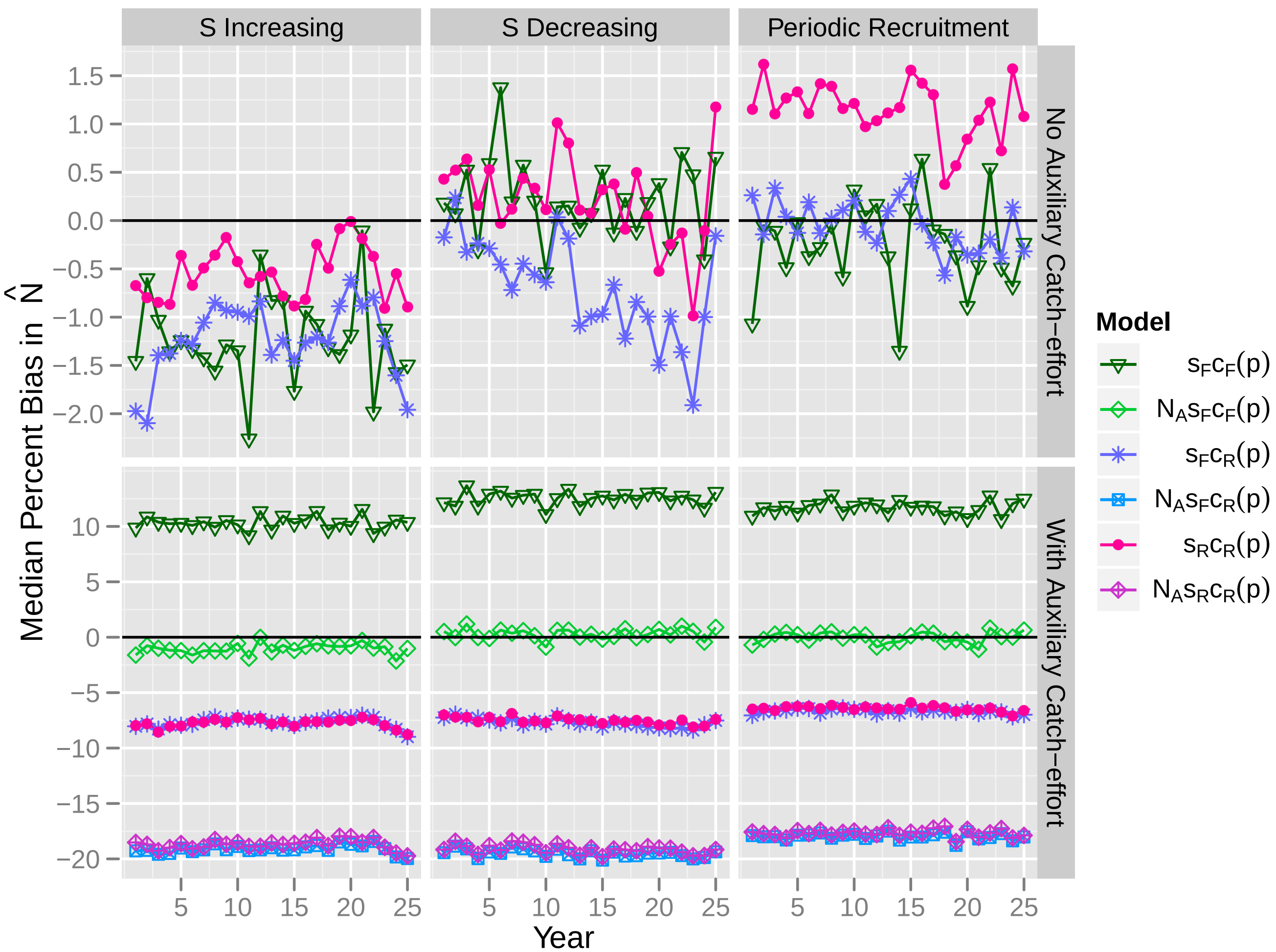


Figure 4.6: *Median percent bias in estimated total annual abundance, robustness simulation studies, for small game models when a low amount of auxiliary data is available for estimation of c. Results indicate consistent bias in all three scenarios, with the fixed-effects-only absolute-recruit abundance model ($N_A s_F c_F(p)$, with the inclusion of the auxiliary catch-effort likelihood component), and the fixed- and mixed-effects Horvitz-Thompson models (when the auxiliary catch-effort likelihood component is omitted) performing the best.*

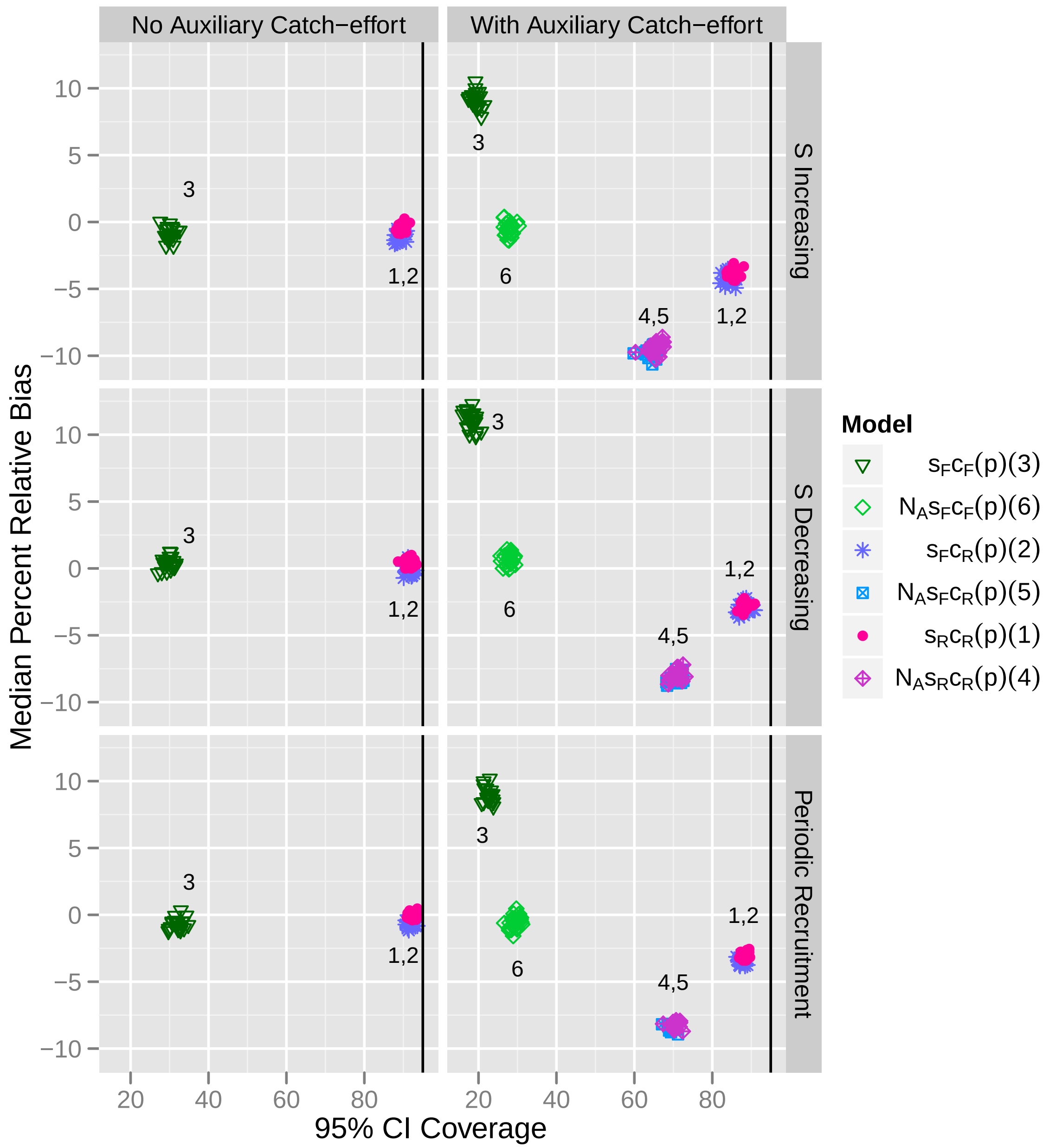


Figure 4.7: *95% confidence interval coverage and median relative bias of estimated total annual abundance for robustness simulations, small game models, when a high amount of auxiliary data is simulated for estimation of c. Mixed-effects conditional-likelihood models s_Rc_R and s_Fc_R have lowest median relative bias and nearest-nominal coverage. Fixed effects models s_Fc_F and $N_As_Fc_F$ have low bias, but also and lowest (subnominal) CI coverage.*

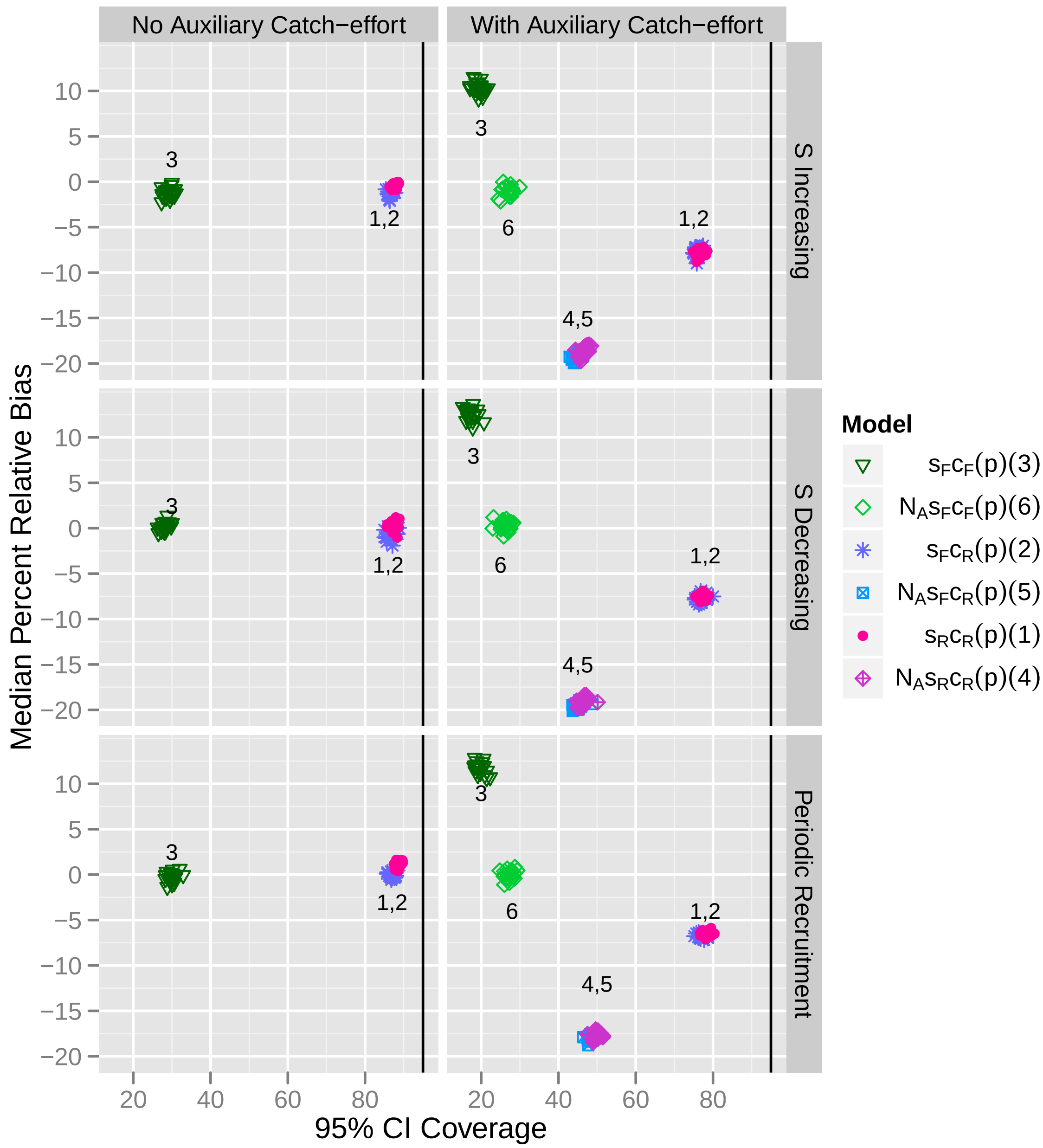


Figure 4.8: *95% confidence interval coverage and median relative bias of estimated total annual abundance for robustness simulations, small game models, when a low amount of auxiliary data is simulated for estimation of c. Mixed-effects conditional-likelihood models $s_R c_R$ and $s_F c_R$ have lowest median relative bias and nearest-nominal coverage. Fixed effects models $s_F c_F$ and $N_A s_F c_F$ have low bias, but also and lowest (subnominal) CI coverage.*

The simulations conducted here assumed a) only one year or b) six years of radiotelemetry data were available to augment the 25 years of age-at-harvest data to aid in the estimation of harvest probability, which span the breadth of potential scenarios for small game studies. In addition to low cohort information, small game animals tend to be impacted by extrinsic factors related to habitat status and quality, as well as environmental factors such as temperature and rainfall. Many such factors may be incorporated as covariates in the analysis (although this simulation study did not examine it), but many factors will remain unmeasured or measured with error. In addition, the functional relationship between the extrinsic factors affecting a particular demographic process (such as survival, harvest, or reproduction) is typically unknown.

Despite the low cohort information, the simulation studies presented above indicate that the extension to mixed-effects statistical population reconstruction models described in this chapter are able to provide reasonably successful estimates of animal abundance, with respect to accuracy and precision of maximum likelihood estimates, and second-stage Horvitz-Thompson-type abundance estimates. Estimation accuracy was generally improved with the greater amount of auxiliary data simulated. The models that incorporate a Horvitz-Thompson-type estimator ($s_R c_R(p)$, etc.) rather than estimating each recruit abundance individually ($N_A s_R c_R(p)$, etc.) tend to show greater success in accurately and precisely estimating animal abundance when paired with the use of random effects to aid the description of interannual variability in demographic processes. Clearly, specific cases will warrant unique considerations, such as gender-specific reconstructions, different types and amounts of available data, and availability of measured extrinsic factors that are known or hypothesized to influence population dynamics.

It appears that the auxiliary catch-effort likelihood of Equation (1.7) induces some negative bias in the total annual abundance estimates for the Horvitz-Thompson models $s_F c_F(p)$, $s_F c_R(p)$, and $s_R c_R(p)$, and it is therefore recommended to exclude it. This auxiliary likelihood component was found to be necessary to provide reliable fits to absolute-recruit abundance models $N_A s_F c_F(p)$, $N_A s_F c_R(p)$, and $N_A s_R c_R(p)$.

Maximum likelihood estimates of process parameters (survival, harvest probability) as well as second-stage Horvitz-Thompson abundance estimators have proven to be capable of providing successful process parameter and abundance estimates, and have also proven to be robust to the types of deviations from standard model assumptions described in this chapter. Therefore, these new models are recommended to be used in statistical population reconstruction of harvested small game populations.

4.6 Data Analysis: Missouri Turkey

4.6.1 Introduction

In order to demonstrate the use of the recommended model (that which employs a Horvitz-Thompson estimator for abundance), I examine an age-at-harvest dataset of male wild turkey (*Meleagri gallopavo*) collected from the East Ozarks turkey productivity region in the State of Missouri, USA (Table 4.8). Two harvests are conducted in this area annually, in spring and in fall. The spring harvest is constituted primarily of males, while both males and females comprise the fall harvest. In addition, both the spring and fall hunts allow for unpermitted landowner harvest (Table 4.9). Gender and age class (juvenile or adult) are determined for each harvested member of the population at hunter check stations, or more recently, by hunters themselves, which are then entered in an online database. In the fall, juvenile males and females may be difficult to distinguish from one another, and therefore the count of juvenile males harvested in the fall contains observation error around the true harvest of juvenile males. The period of data used for statistical population reconstruction extends from 1996 to 2010, with 5 years of auxiliary radio telemetry data (1996 to 2000, Table 4.10). In addition, an estimated poult-to-hen ratio is available for each year of reconstruction, as well as an index of abundance arising from fall archery hunter counts (Table 4.8). Hunter effort information for the spring permitted hunt (Table 4.8) is presented as the number of hunter trips/10,000, estimated from a post-season survey.

4.6.2 Model Details

A joint likelihood model of the multiple data sources was formed. The primary likelihood component for the age-at-harvest data was formed as described in Equations (4.1) - (4.4), which follow from a reduction in number of available age classes of the model presented in Equations (3.1) through (3.11). Separate harvest vulnerability coefficients (c_J for juveniles, c_A for adults) were used to model separate harvest probabilities for juveniles and adults.

Table 4.8: *Spring harvest count of male wild turkey, East Ozarks turkey productivity region, Missouri, USA. Effort = survey estimated hunter trips per 10,000. Effort SE = standard error of estimated hunter effort. Poult/Hen = Landowner estimated poult per hen count. State Index = abundance index based on statewide fall archer counts. East Ozarks Index = abundance index based on East Ozarks fall archer counts.*

Year	**Juvenile Harvest**	**Adult Harvest**	**Effort**	**Effort SE**	**Poult/Hen**	**State Index**	**East Ozarks Index**
1995					1.4		
1996	626	2703	6.9298	0.519	2.1	477	364
1997	596	1764	6.5852	0.442	1.8	541	188
1998	847	2855	6.5854	0.384	3.1	730	541
1999	1833	1716	7.1273	0.401	2.1	644	263
2000	767	4237	6.2601	0.340	2.8	656	365
2001	1580	2781	7.6457	0.448	2.6	637	565
2002	1388	3399	7.4921	0.462	1.3	774	359
2003	884	4287	6.935	0.409	1.7	646	384
2004	1046	3289	8.1414	0.984	2.0	620	521
2005	887	2877	8.2299	0.527	1.5	480	224
2006	829	3033	6.7334	0.431	2.2	543	294
2007	904	2077	8.0017	0.483	1.2	460	342
2008	501	3011	6.0511	0.381	1.3	377	207
2009	791	2429	6.7649	0.394	1.6	418	326
2010	840	2233	6.2808	0.344	1.1	342	161

The auxiliary radiotelemetry harvest data were assumed to be binomially-distributed from the number at risk, with probability of harvest equal to probability of harvest of unmarked animals. Therefore, the auxiliary likelihood component was included as

Table 4.9: *Landowner harvest of East Ozarks Wild Turkey. Fall juvenile harvest is estimated, as it is difficult to distinguish juvenile males from females in the fall.*

	Spring		**Fall**	
Year	**Juvenile**	**Adult**	**Juvenile**	**Adult**
1996	53	279	220	167
1997	60	174	104	81
1998	102	255	363	89
1999	185	197	130	134
2000	101	406	289	107
2001	174	324	286	223
2002	191	398	175	196
2003	119	470	173	153
2004	118	382	249	226
2005	244	559	181	140
2006	233	602	283	169
2007	238	522	312	357
2008	196	722	135	93
2009	191	528	234	168
2010	223	558	146	103

$$\begin{aligned} L_{Radiotelemetry} = & \prod_{i=1996}^{2000} \begin{pmatrix} R_i^J \\ r_i^J \end{pmatrix} (p_i^J)^{r_i^J} (1 - p_i^J)^{R_i^J - r_i^J} \\ & \times \prod_{i=1996}^{2000} \begin{pmatrix} R_i^A \\ r_i^a \end{pmatrix} (p_i^A)^{r_i^A} (1 - p_i^A)^{R_i^A - r_i^A}. \end{aligned} \tag{4.5}$$

where R_i^J and R_i^A represent the known number of tagged juvenile and adult male turkeys, respectively, at risk in year i, r_i^J and r_i^A represent the known harvest count of the number of adult juveniles and male, respectively, at risk, and p_i^J and p_i^A represent the harvest probabilities

Table 4.10: *Radiotelemetry data for East Ozarks spring turkey harvest.*

Year	**Juvenile Males at Risk**	**Juvenile Males Harvested**	**Adult Males at Risk**	**Adult Males Harvested**
1996	23	0	32	7
1997	12	2	46	13
1998	4	2	16	8
1999	30	3	9	5
2000	8	2	33	17

of juveniles and adults, respectively, in year i.

For both the age-at-harvest and auxiliary radiotelemetry data, harvest probability was parameterized as

$$\begin{aligned} p_i^J &= 1 - e^{-e^{(c_J + \tau_i)} f_i} \quad \text{and} \\ p_i^A &= 1 - e^{-e^{(c_A + \tau_i)} f_i} \end{aligned} \tag{4.6}$$

where it was assumed

$$\tau_i \sim Normal\left(0, \sigma_\tau^2\right). \tag{4.7}$$

As no auxiliary information were available to support examination of separate natural survival probabilities for juveniles and adults, a single survival parameter, β, was used for both age classes. As there are two survival periods (the period between spring and fall harvest of year i [22 weeks, summer], and the period between fall harvest in year i and spring harvest of year $i+1$ [24 weeks, winter]), it was assumed that survival rate, β, was constant across time within a year, but that interannual variation is present across years. Survival probability was therefore parameterized as

$$
\begin{aligned}
s_i^S &= e^{-\left(e^{(\beta+\epsilon_i)}\times 22\right)} \\
s_i^W &= e^{-\left(e^{(\beta+\epsilon_i)}\times 24\right)}
\end{aligned}
\tag{4.8}
$$

to maintain bounding between (0,1), as well as to make survival probability dependent on the length of the interval the turkey must survive, in weeks (S = summer, W = winter). It was assumed that

$$\epsilon_i \sim Normal\left(0, \sigma_\epsilon^2\right). \tag{4.9}$$

Modifications to the age-at-harvest likelihood were necessary to account for the known number of spring and fall removals, which where considered known removals for this analysis, because 1) no measure of effort is available for these harvests, and 2) this would depend heavily on the interaction between hunter effort and area hunted, which is unlikely to be constant across years. It was assumed that spring landowner removals occurred prior to the permitted spring harvest season. Each turkey is assumed to have the life history detailed in Figure (4.9).

Therefore, in order for juveniles (age $>$ 6 months) in year i to be available for harvest as adults (age $>$ 1 year) in year $i+1$, they must not be removed in the spring landowner harvest and survive the spring permitted harvest. After the spring permitted harvest (when turkey are greater than age 1 year) they are classified as "adults", where they then must survive until the fall harvest, survive the fall harvest, and survive until just prior to the following year's spring landowner removal period. As adults may survive more than 1 year, there is a possibility for older adults to stay in the adult age class, rather than being removed from the population. Therefore, the number of adults available for harvest in year $i+1$ is

$$N_{i+1}^A = \left(\left(N_i^J - SR_i^J\right)\left(1-p_i^J\right)s_i^S + \left(N_i^A - SR_i^A\right)\left(1-p_i^A\right)s_i^S - FR_i^A\right)s_i^W \tag{4.10}$$

where

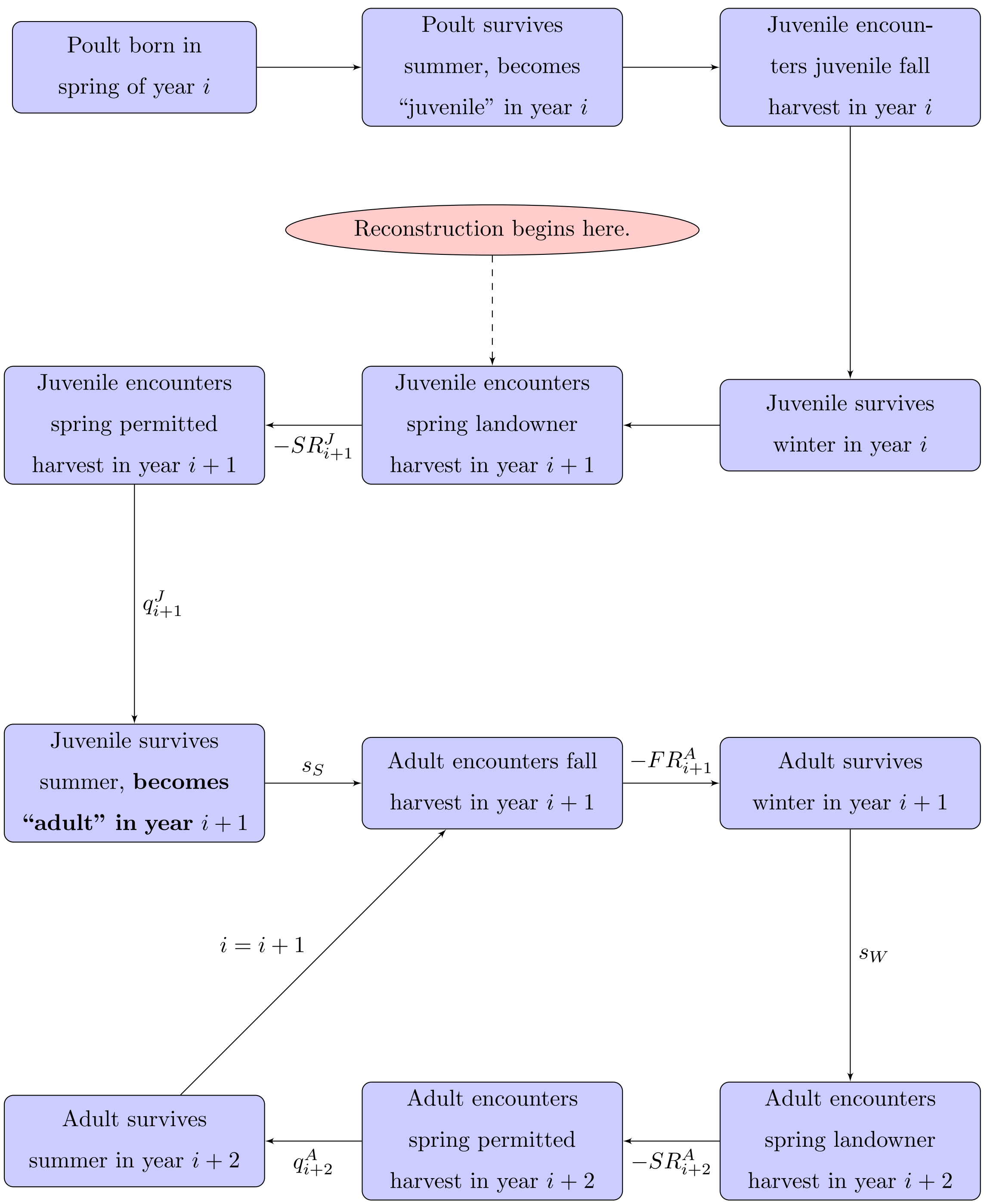


Figure 4.9: *Turkey life history flowchart. Death is not explicit, but can occur at many possible nodes (natural mortality in summer or winter, harvest in spring or fall).*

$$\begin{aligned}
N_i^A &= \text{ adult abundance in year } i \\
N_i^J &= \text{ juvenile abundance in year } i \\
SR_i^J &= \text{ juvenile spring removals in year } i \\
SR_i^A &= \text{ adult spring removals in year } i \\
FR_i^A &= \text{ adult fall removals in year } i.
\end{aligned} \tag{4.11}$$

Note that the number of juveniles removed in fall does not enter the equation, because fall juvenile removals are confounded with the number of male poults produced, and the number of these that survive the summer. Since an estimate of this survival probability (and data that might be used to estimate this survival probability) is not available, an estimate of poult abundance is not available. Abundance of juveniles in year i, then, is the abundance immediately prior to the spring juvenile removal, when the turkey are roughly 1 year old and have already undergone a fall harvest at age $\approx$ 6 months.

Annual abundance is estimated as male abundance prior to the spring harvest season. Therefore, in the age-at-harvest likelihood component, annual abundance is estimated as

$$\widehat{N}_i = \left(\frac{h_i^J}{p_i^J} + SR_i^J\right) + \left(\frac{h_i^A}{p_i^A} + SR_i^A\right) \tag{4.12}$$

such that the Horvitz-Thompson estimator is used to scale up the harvest counts (h_i^J and h_i^A for juveniles and adults, respectively) via the probability of harvest, and the spring landowner removals are added to this to obtain the total estimated number of turkey in the spring of year i.

Fall harvest counts of juvenile males are not known precisely, as it can often be difficult to distinguish a male juvenile from a female. The fall count of juvenile male harvest therefore represents an *estimate* of the true harvest count. There is additional variability resulting from the use of these estimates in our likelihood function, however because fall harvest is considered a known removal in this model (because there is no measure of effort to associate with the harvest count), the estimated count cannot be written as a function of model parameters. The consequence of this is that variability in annual abundance will be slightly underestimated, because the variability associated with the fall juvenile male removal is not accounted for.

The joint likelihood is written as:

$$L_{joint} = L_{Age-at-Harvest} \times L_{Radiotelemetry} \times \prod_{i=1}^{Y} \phi_{\sigma_\tau}(\tau_i) \times \prod_{i=1}^{Y-1} \phi_{\sigma_\epsilon}(\epsilon_i) \tag{4.13}$$

where $\phi_{\sigma_x}(x_i)$ represents the normal density of x_i with mean 0 and variance σ_x^2, which constitutes the likelihood component owing to the random effects terms $\vec{\tau}$ and $\vec{\epsilon}$.

As before, the marginal likelihood

$$L_{joint,\ marginal} = \int_{\epsilon_1}\int_{\tau_1}\cdots\int_{\epsilon_{Y-1}}\int_{\tau_{Y-1}}\int_{\tau_Y} L_{joint}\ \partial\epsilon_1\ \partial\tau_1\ \cdots\partial\epsilon_{Y-1}\ \partial\tau_{Y-1}\ \partial\tau_Y \tag{4.14}$$

is approximated with the Laplace approximation, which is then optimized numerically with ADMB (Fournier et al., 2011) software.

Model selection results using methods outlined in Chapter 2 indicated the best model is that which uses two harvest vulnerability coefficients ($c_{juvenile}$ and c_{adult}), separate random effects terms for each age class ($\vec{\tau}^{juvenile}$ and $\vec{\tau}^{adult}$), and no random effects for natural survival ($\sigma_\epsilon = 0$).

Spring abundance estimates of juvenile and adult male turkey indicate the population numbered approximately 13,513 individuals (95% CI: [8012, 19,014]) in the mid 1990's, increased to approximately 21,373 individuals (95% CI: [12,917, 29,828]) in 2002, and slowly declined to approximately 14,542 individuals (95% CI: [8,534, 20,551]) in 2010 (Figure 4.10). The decline in estimated abundance corresponds to an increase in estimated hunter effort between 2001 and 2007 (Figure 4.10) from 76,457 hunter-trips to 80,017 hunter-trips. Fall abundance appears to track the rescaled fall archer indices well, until approximately 2005, when estimated abundance appears to indicate a population that is declining more slowly than the archer index would suggest. The scaling factor for the archer indices is obtained by regressing total annual abundance on each index value without an intercept term.

An estimated poult-to-hen ratio provides another index available for comparison with estimates arising from the model fit. To compare this index with a data estimate, I plot the

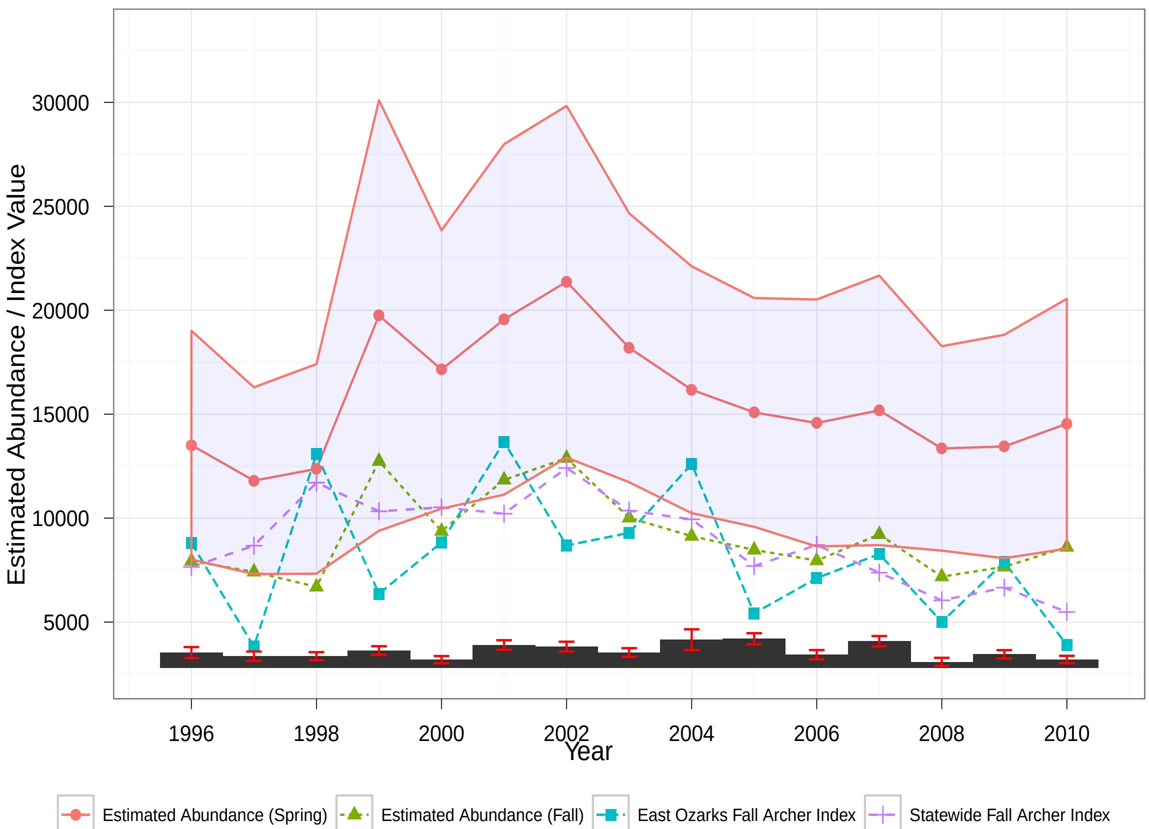


Figure 4.10: *Estimated total annual abundance of juvenile and adult male turkey, East Ozarks turkey productivity region, for both spring and fall (with a confidence interval band for the spring abundance). Shown also are two indices of abundance (East Ozarks fall archer count index and statewide fall archer count index), which have been rescaled to be shown on the same axis as estimated fall abundance. Linearly rescaled hunter effort is presented as vertical bars along the x-axis, with accompanying error bars indicating estimated effort ± SE.*

estimated spring male turkey abundance in year i alongside a rescaled version of the estimated poult-to-hen ratio from late spring/early summer of year $i-1$ (Figure 4.11). The scaling factor is obtained by regressing juvenile abundance on the poult-to-hen ratio without an intercept term. This is because the poult-to-hen ratio is obtained when turkey are approximately 3 - 4 months old, whereas the earliest spring abundance available from the model fit is juveniles that are roughly 1 year old, and therefore there is a lag of approximately 1 year. Perfect concordance of the two estimates is not expected, because 1) the poult-to-hen ratio includes both males and females, and 2) it is possible (likely) that interannual fluctuations are observed in the percent of male turkey surviving from age 3 - 4 months to age 12 months. In addition, it is possible that males and females survive at different rates, which would provide another source of error in the attempt to match the poult-to-hen ratio with the juvenile male abundance. Despite these limitations, results indicate a high degree of concordance between the two estimates Figure (4.11). The sample correlation between these two estimates is quite high, at $\rho = 0.81$.

Parameter estimates (Table 4.11) indicate that male turkey survival is approximately 60% over the course of one year, and that adult male turkey are harvested at a rate of approximately 41%, while juvenile male turkey are harvested at approximately 12% at the mean level of hunter effort, 70,500 hunter trips. Interannual variation in the relationship between hunter effort and harvest vulnerability appears to be higher for adults than for juveniles, although this may be related to the lower estimated value.

Overall, the male turkey abundance in the East Ozarks appears to be at sustainable levels; the population is estimated to be slightly larger in 2010 than it was at the beginning of the reconstruction, in 1996. Total annual abundance, however, is estimated to have increased in the late 1990's, and then declined over the early 2000's as increased hunting pressure was exerted. Natural survival (for which only a single parameter could be fitted for both age classes) as well as harvest rates (estimated separately for each age class) appear to be within expectations for similar populations from a recent study (Diefenbach and Vreeland, 2010). Confidence intervals around total annual abundance as well as parameters defining the demographic processes are wide. Possible reasons for wide confidence intervals include influence from extrinsic factors that manifests itself in the uncertainty estimation for harvest probability and survival parameters and random effects, low availability of cohort information, and the models themselves as well as

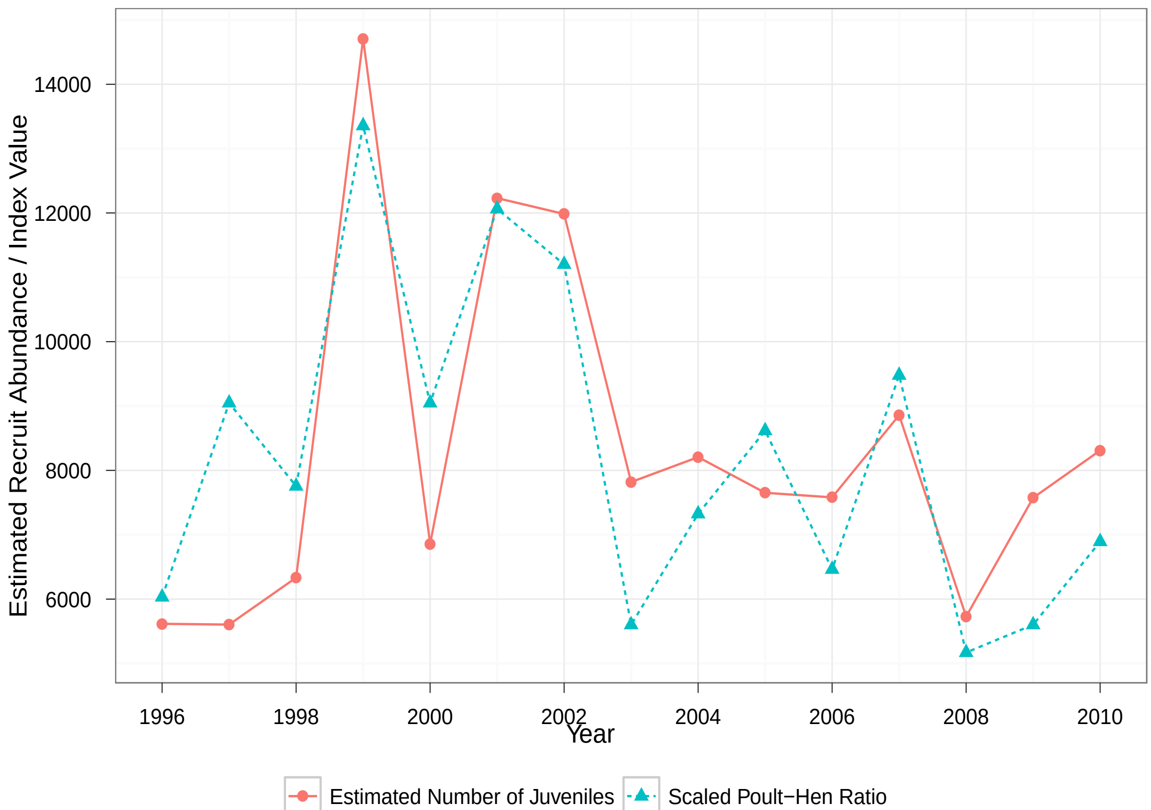


Figure 4.11: *Estimated annual spring abundance of juvenile male turkey and scaled poult-to-hen ratio estimated from landowner counts, East Ozarks turkey productivity region. Points plotted are estimated male juvenile abundance in year i (age 1 year), and estimated poult-to-hen ratio in summer of year $i-1$ (age 3 - 4 months), which includes both males and females. Perfect concordance is not expected due to the possibility of differential survival between males and females in the first year of life, as well as the fall harvest of juveniles at age $\approx$ 6 months.*

	Juvenile	Adult
$\widehat{s}$	0.60 (0.31, 0.89)	
$\widehat{c}$	-4.05 (-4.64, -3.46)	-2.61 (-2.98, -2.23)
$\widehat{p}^{*}$	0.12 (0.05, 0.18)	0.41 (0.29, 0.52)
σ_c	0.18 (0.01, 0.36)	0.25 (0.12, 0.39)
mAIC	330.87	
cAIC	332.31	

** Estimated harvest probability computed at mean effort (70,500 hunter trips)*

Table 4.11: *Parameter estimates for final model for Missouri turkey population reconstruction and accompanying 95% confidence intervals, constructed using asymptotic normality of maximum likelihood estimates. Model fit was* $s_F c_R(p)$.

the estimation technique employed here. A more extensive dataset may permit the same model to be augmented with data regarding female members of the population, which may permit examination of a stock-recruit relationship, which could incorporate extrinsic factors related to recruitment of poults, such as spring rainfall. In addition, radiotelemetry studies of survival would provide an auxiliary data source with which to investigate estimating separate natural survival parameters for juveniles and adults.

Chapter 5

MANAGEMENT IMPLICATIONS AND RECOMMENDATIONS

Existing models for statistical population reconstruction (Gove et al., 2002; Broms et al., 2010; Skalski et al., 2007) assumed harvest vulnerability and survival probability were fixed across time and age class. Inter-age and interannual differences could only be detected if specific data sources were available to estimate these parameters independently. Because such datasets are rare, models were limited to parameterizations that assumed these important demographic parameters were fixed and constant.

The work contained in this dissertation has compared these previous models to some new models that provide statistical population reconstructions of harvested wildlife populations. These new models and model extensions have provided a way to model age- and time-dependent effects on vital rates, and these have proven in simulation studies to provide improved statistical population reconstructions, by a variety of criteria. These improved reconstructions also contain the ability to estimate natural variability (owing to environmental influences that were not or cannot be measured) in processes affecting population growth and sustainability, separate from sampling error inherent in the harvest process.

In addition to the model extension which accounts for environmental stochasticity, a new model class was developed based on the multinomial cohort framework which uses a likelihood that is conditional on total cohort harvest. This model, coded $s_R c_R$, hypothesizes a binomial sampling scheme for each age class in each year that leads to a second-stage Horvitz-Thompson-type estimator for abundance. Simulation studies have indicated this model produces annual abundance estimates with lower bias than other models. In addition, uncertainty estimates of annual abundance admit confidence intervals that produce appropriate coverage; that is, estimated coverage of 95% confidence intervals is very near 95%. For this model, abundance estimates ($\widehat{N}$) are obtained by adjusting the observed harvest count (x) by the estimated probability of harvest ($\widehat{p}$), as $\widehat{N} = x/\widehat{p}$.

An additional model extension explicitly incorporates a relationship between estimated breeding-age animal abundance in year i, and the new members of the population in year $i+1$. The use of this relationship in statistical population reconstruction models allows the researcher to investigate hypotheses regarding the relationship between other population dynamics (such as abundance) and the production of new individuals, as well as the relationship of reproduction to environmental influences.

These developments in the modeling of harvested wildlife populations represent improvements over previous methods in the case when harvest is fully-aged (such as the Michigan elk example of Chapter 2), as well as for the case when long-lived animals are aged to only 3 age classes (juvenile, subadult, adult, as in Chapter 3). The simulations in Chapter 4 as well as the Missouri turkey example also indicate that the model developments are capable of improving the estimates arising from a small game dataset with only two recorded age classes (juvenile and adult). In each of these scenarios, simulation studies have indicated that the use of random effects to model interannual environmental stochasticity as well as the Horvitz-Thompson estimation approach is a robust and powerful framework with which to model and subsequently manage harvested wildlife populations. It is therefore recommended that this be the preferred model for modeling age-at-harvest data of harvested wildlife populations.

5.1 Recommendations

5.1.1 Use of Random Effects

A primary goal of the model developments in this work has been to provide for more "realistic" models for populations that experience a wide variety of influences on their demographic rates. Of course, all models are simplified views of reality, but one of the primary developments has been to explicitly admit, through the use of random effects, that there are sources of variability that cannot be measured. Simulation studies have indicated that when random interannual deviations from an overall mean in process parameters do exist, models that incorporate random effects are able to accommodate this additional source of variation. When no random deviations in process parameters are simulated, however, the use of random effects does not detract from the ability of the model and estimation procedure to provide accurate and precise estimates of abundance and demographic rates. Therefore, it is recommended to use random effects in the modeling of demographic processes for harvested wildlife populations, and to use model selection procedures to remove those random effects that are estimated to be zero. When paired with the Horvitz-Thompson estimation approach random effects models (herein coded $s_R c_R$ and $s_F c_R$) have proven optimal in many respects, in simulation studies.

In concordance with the recommendation to use random effects, I also recommend *against* using the fixed-effects model that estimates each initial cohort abundance as a separate parameter (the model $N_A s_F c_F$ of Gove et al. (2002); Skalski et al. (2007)). In simulation studies, this model has shown that it can be biased when environmental stochasticity is not incorporated. Even when no environmental stochasticity was simulated and the model produced annual abundance estimates with low bias, estimated coverage of asymptotic 95% confidence intervals was still well below the expected value of 95%, indicating poor uncertainty estimation.

As only a single software package, ADMB (Fournier et al., 2011), is available for fitting nonlinear mixed-effects models of this nature, results contained herein must be interpreted in the context of the methods employed in this software. That is, the likelihood is first approximated with the Laplace approximation, and this approximation to the likelihood is optimized in separate phases for fixed-effects and random-effects, yielding empirical Bayes estimators of the random effects. I recommend use of ADMB for fitting the recommended models, as it is the only software package available for models of this nature.

5.1.2 Reconstruction of Age Class Abundance with the Horvitz-Thompson Estimator

When using the Horvitz-Thompson-type estimator to reconstruct age-class abundance, one can choose to use the estimated harvest probability for each cell of the age-at-harvest matrix, or use the Horvitz-Thomspon estimator for the initial cohort abundances and use the estimated harvest survival $(1-p)$ and natural survival (s) probabilities to estimate the number of animals surviving from age j in year i to age $j+1$ in year $i+1$. Simulation studies based on fully-aged big game data (Chapter 2) indicate that both methods appear to have a similar level of bias (very low). However, the cell-based reconstruction method permits simple expressions to compute the extra-likelihood variability associated with the binomial sampling model upon which abundance estimates are based. A method for computing this estimate would be significantly more complex for the cohort-based reconstruction method. Therefore, it is recommended to compute the Horvitz-Thompson estimate of abundance in each cell as $\widehat{N}_{ij} = x_{ij}/\widehat{p}_{ij}$, rather than utilizing the cohort structure to produce abundance estimates as $\widehat{N}_{ij} = \widehat{N}_{i-1,j-1}\widehat{s}_{i-1}(1 - \widehat{p}_{i-1,j-1})$.

5.1.3 Model Selection

Model selection for nonlinear mixed-effects models of this nature has not been fully developed. No rigorous theory has been developed regarding the use of likelihood ratio tests or information-theoretic approaches such as AIC for the case where comparisons involve the inclusion or exclusion of random effect terms. Despite uncertainties regarding the theory of such model selection procedures, simulations have indicated that likelihood ratio tests using the conditional likelihood (as described in Chapter 2) provide a means to exclude random effects which are estimated to be zero. In contrast, likelihood ratio tests using the marginal likelihood select random effects into the model even when none are justified (no environmental variation is present, as in some simulation study scenarios). Therefore, it is recommended to use the conditional likelihood for model selection with regard to the random effects structure.

When selecting fixed effects to incorporate into the model, both the marginal and conditional likelihoods showed high power in selecting the more complex model when it was the true simulation model (Table 2.14). However, this was for the single simulated difference and fully-aged large game simulation presented in Chapter 2, when the auxiliary catch-effort likelihood was omitted (except for the stock-recruit model, which required this likelihood component). Type I error rate was larger than expected for both the marginal and conditional likelihoods for each model structure, but this is an acceptable tradeoff for high power when one seeks to understand what differences, if any, exist between fixed-effects structures (such as age-class-specific harvest vulnerability coefficients) in these models. Since no obvious differences exist between the use of the marginal and conditional likelihood for this simulation study, the conditional likelihood may be used for model selection here, as it was found to be useful in the context of selecting random effects terms into the model. Further simulation studies of all model selection scenarios is warranted to understand the performance of marginal and conditional likelihood-based likelihood ratio tests.

5.1.4 Auxiliary Catch-Effort Likelihood

The auxiliary catch-effort likelihood of Equation (1.7) was found to be useful in providing stable model fits for the stock-recruit model, but a consistent level of bias was exhibited by these models. If it is desired to model the relationship between breeding-age stock and recruitment of new individuals, one may choose to use the stock-recruit model ($N_{SR,R}s_Rc_R$) and may need to use this additional likelihood component, with the knowledge that parameter estimates may be biased.

For other model structures, including the conditional-likelihood/Horvitz-Thompson modeling approach recommended here, this additional likelihood component either did not change the results (in the case of the mixed-effects absolute-recruit abundance models), induced an inconsistent level of bias (for the fixed-effects absolute-recruit abundance models), or unnecessarily induced a low level of bias in results (for the conditional-likelihood/Horvitz-Thompson models). It is therefore recommended that this additional likelihood component be omitted for all models, except perhaps for stock-recruit models, if necessary to achieve stability in the optimization process.

5.1.5 Modeling of the Stock-Recruit Relationship

In most simulation study scenarios presented in this work, use of the stock-recruit model with random-effects terms lead to a consistent level of negative bias, even when the statistical reconstruction model perfectly matched the simulation model (for example, Figure 2.2). Despite this, each of these models (in particular, the mixed-effects versions of these models: $N_{SR,R}s_Rc_R$, $N_{SR,R}s_Fc_R$, and $N_{SR,R}s_Fc_R$) showed low bias in the estimation of the stock-recruit parameter (Table 2.7). Therefore, if it is desired to model the stock-recruit relationship specifically, one may use the stock-recruit model, but expect that abundance estimates derived from this model may be biased. Of course, more sophisticated relationships between breeding-age stock and number of recruits may be considered, but use of these would require further simulations study.

5.2 Areas for Future Research

Despite the strides already made in the modeling of these populations, a number of further developments are immediately indicated. For instance, all simulation work and examples contained herein hypothesize that the random effects on harvest probability are independent, the random effects on survival are independent, and that these two sources of random variation are independent from one another. There may be scenarios where, for instance, increased probability of harvest due to random factors other than effort lead to a small population over the winter months, which may lead to increased survival probability for the remaining members of the population in food- or habitat-limited situations. Modeling random effects as, say, multivariate normal with an appropriately-chosen covariance structure would provide a way to capture this dependence through the covariance parameter.

In addition, the use of environmental covariates was not examined rigorously, and this may provide an alternative way to account for some of the variation in demographic processes rather than simply lumping all such sources into a random effect term. This would provide an additional way to relate demographic processes to environmental controls in order to understand the *source* of the variation and how it may be influenced.

As mentioned previously, an area for theoretical research that would benefit the modeling of harvested wildlife populations includes the development of theory for model selection of nonlinear mixed-effects models. Such a development would allow more rigorous hypothesis testing in the frequentist framework, which would enable practitioners to better understand what factors may influence the estimates and predictions arising from the model.

Appendix A
SUPPLEMENTARY FIGURES AND TABLES

A.1 Figures

Note that for some boxplots, ranges have been set such that a few outliers may not be shown, in order to show better resolution for the majority of data.

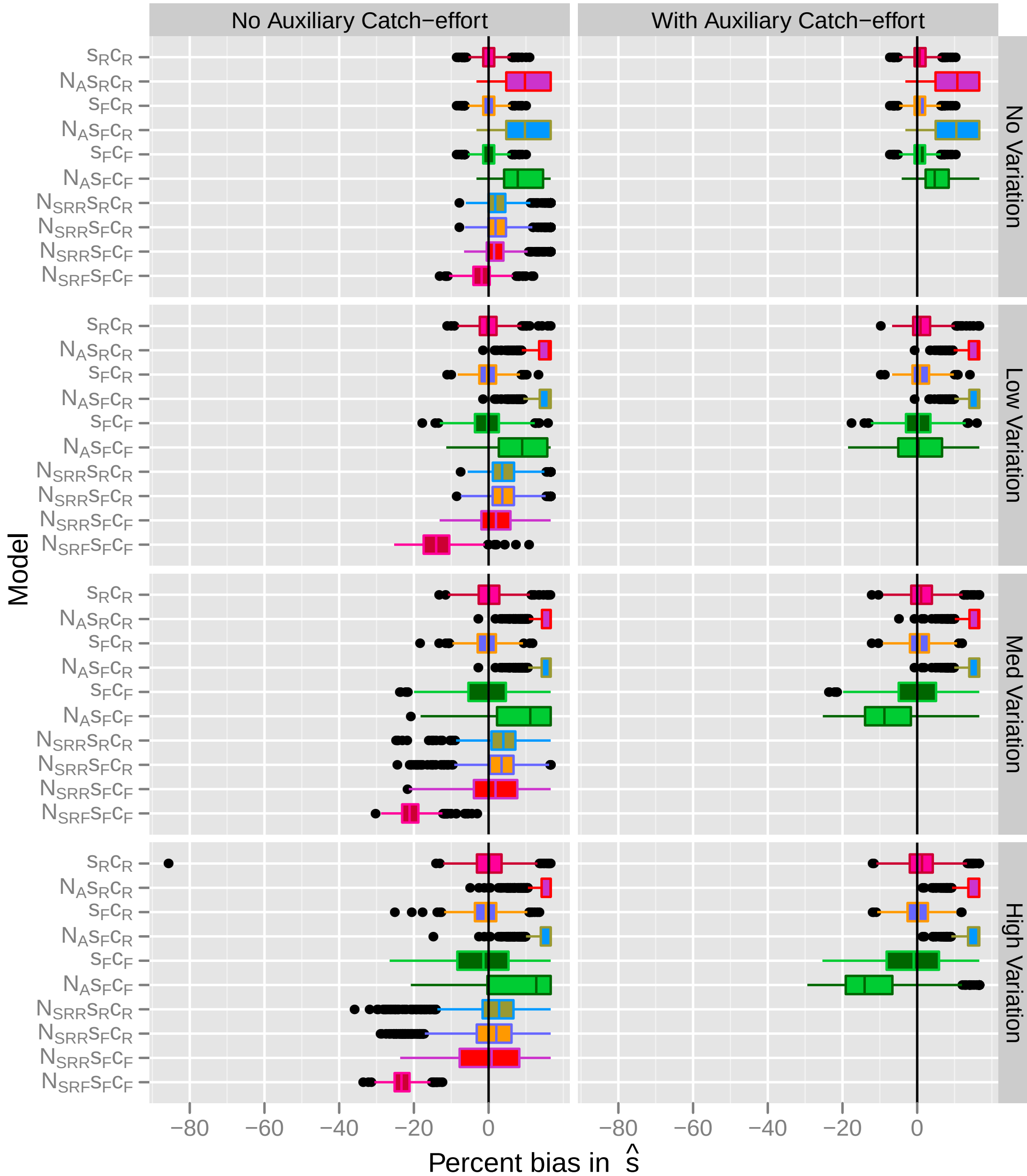


Figure A.1: *Percent bias in estimated survival probability, s, for big game models. Results indicate that conditional-likelihood/Horvitz-Thompson models show low bias, which is not heavily affected by level of simulated variation. Fixed-effects models* $N_{A}s_{F}c_{F}$ *and* $N_{SR,F}s_{F}c_{F}$ *show relatively large bias. Results based on 25 years of data for 13 age classes, and 1000 simulations.*

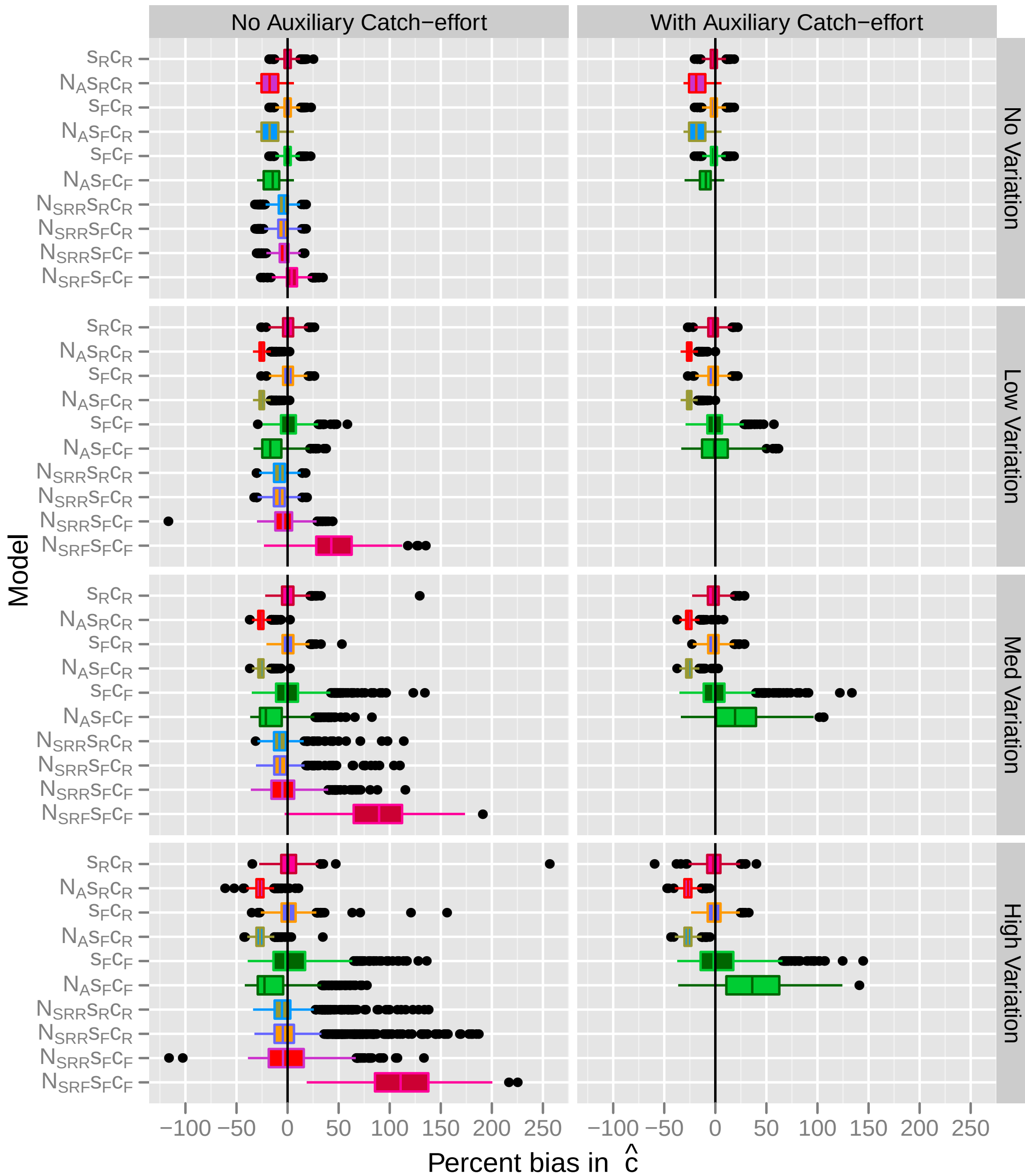


Figure A.2: *Percent bias in estimated harvest vulnerability, c, for big game models. Results indicate that conditional-likelihood/Horvitz-Thompson models show low bias, which is not heavily affected by level of simulated variation. Fixed-effects models* $N_A s_F c_F$ *and* $N_{SR,F} s_F c_F$ *show relatively large bias. Many large outlying observations are exhibited by stock-recruit models. Results based on 25 years of data for 13 age classes, and 1000 simulations.*

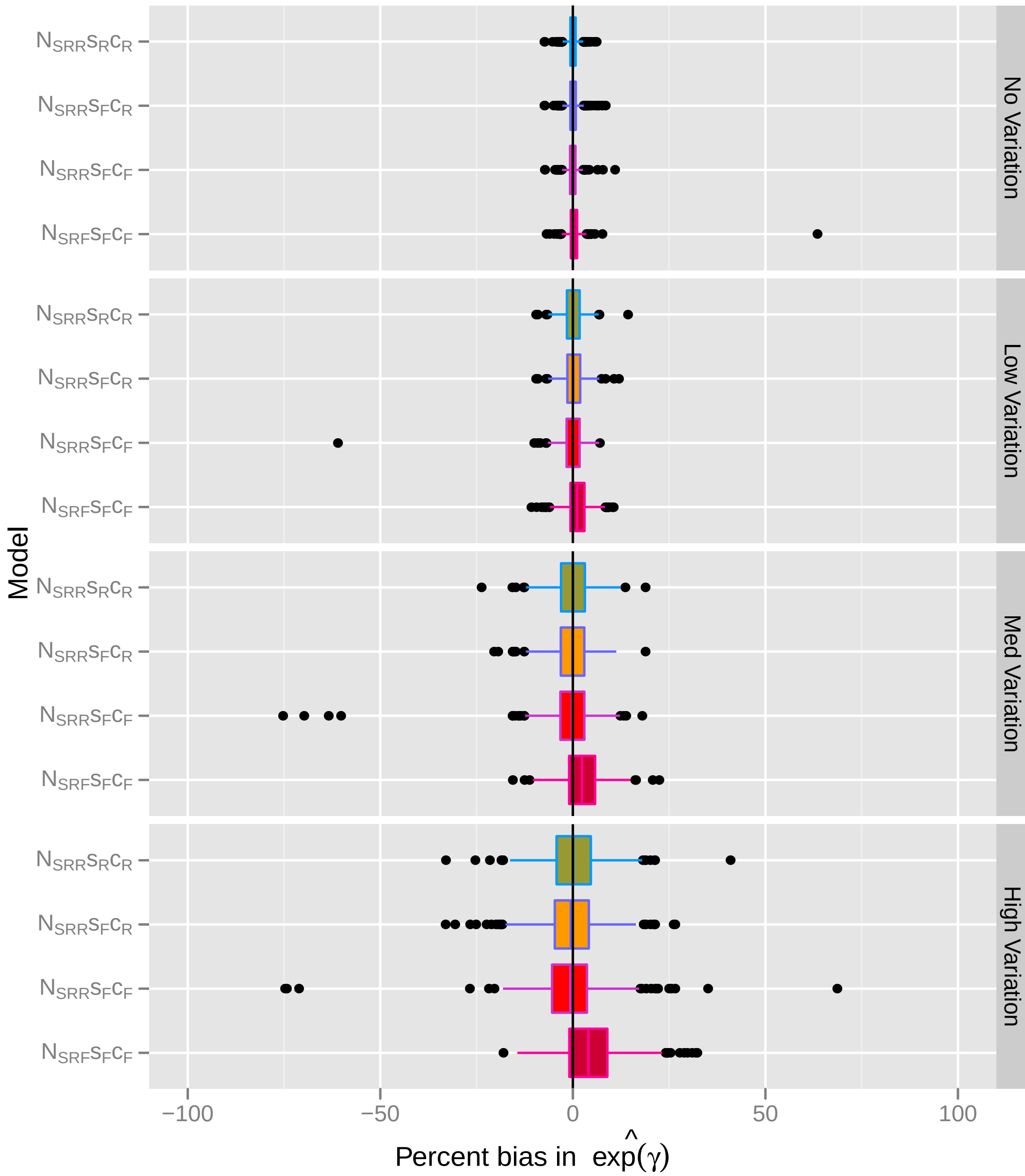


Figure A.3: *Percent bias in estimated productivity, e^{γ}, for big game models. All models show low mean bias, except for the fixed-effects model $N_{SR,F}s_Fc_F$ at the highest level of simulated variation. Results based on 25 years of data for 13 age classes, and 1000 simulations.*

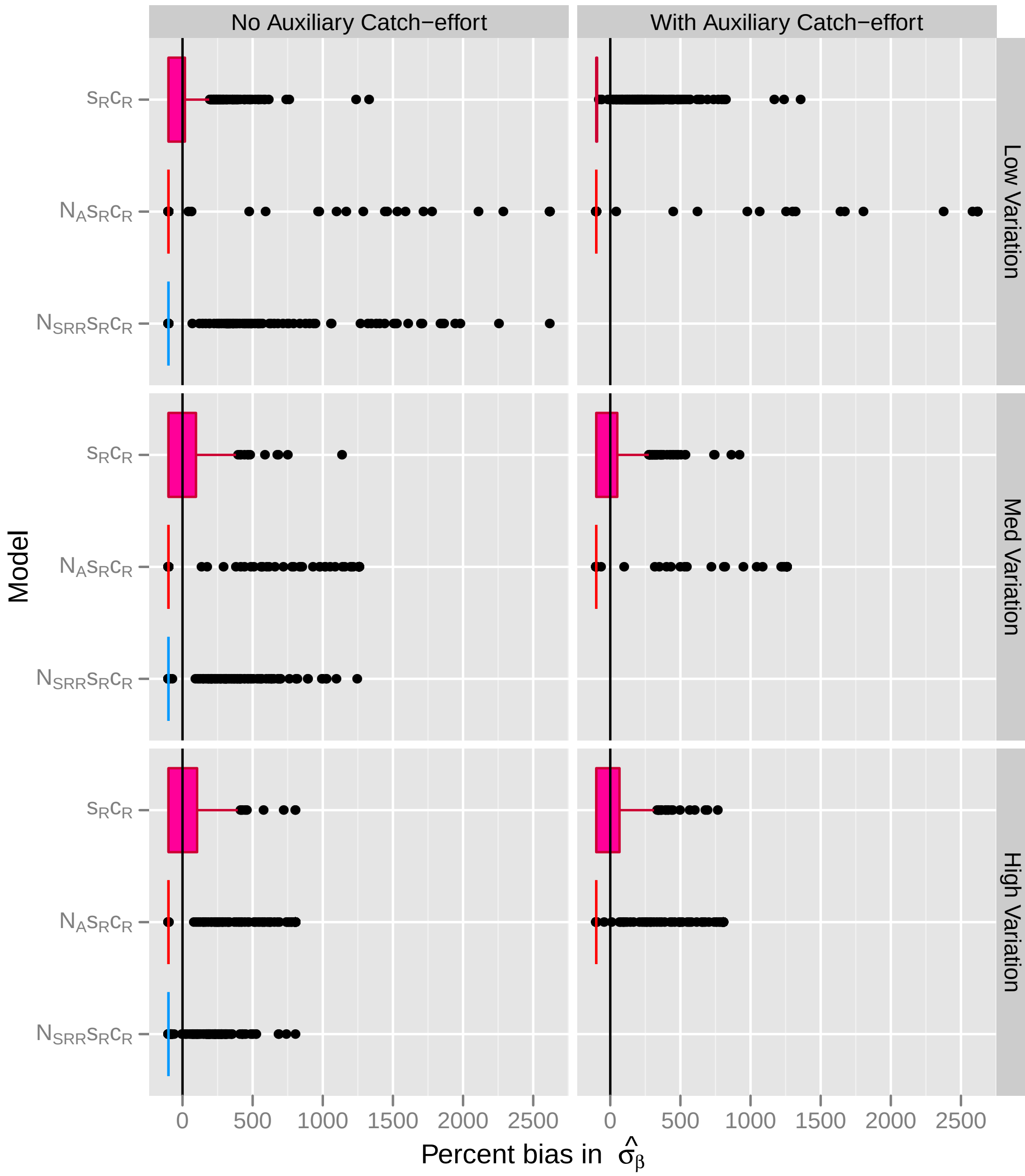


Figure A.4: *Percent bias in estimated interannual variation in survival probability, σ_β, for big game models. Results indicate poor ability to estimate environmental stochasticity in survival probability. Model $s_R c_R$ performs better than $N_A s_R c_R$ or $N_{SR,R} s_R c_R$. Results based on 25 years of data for 13 age classes, and 1000 simulations.*

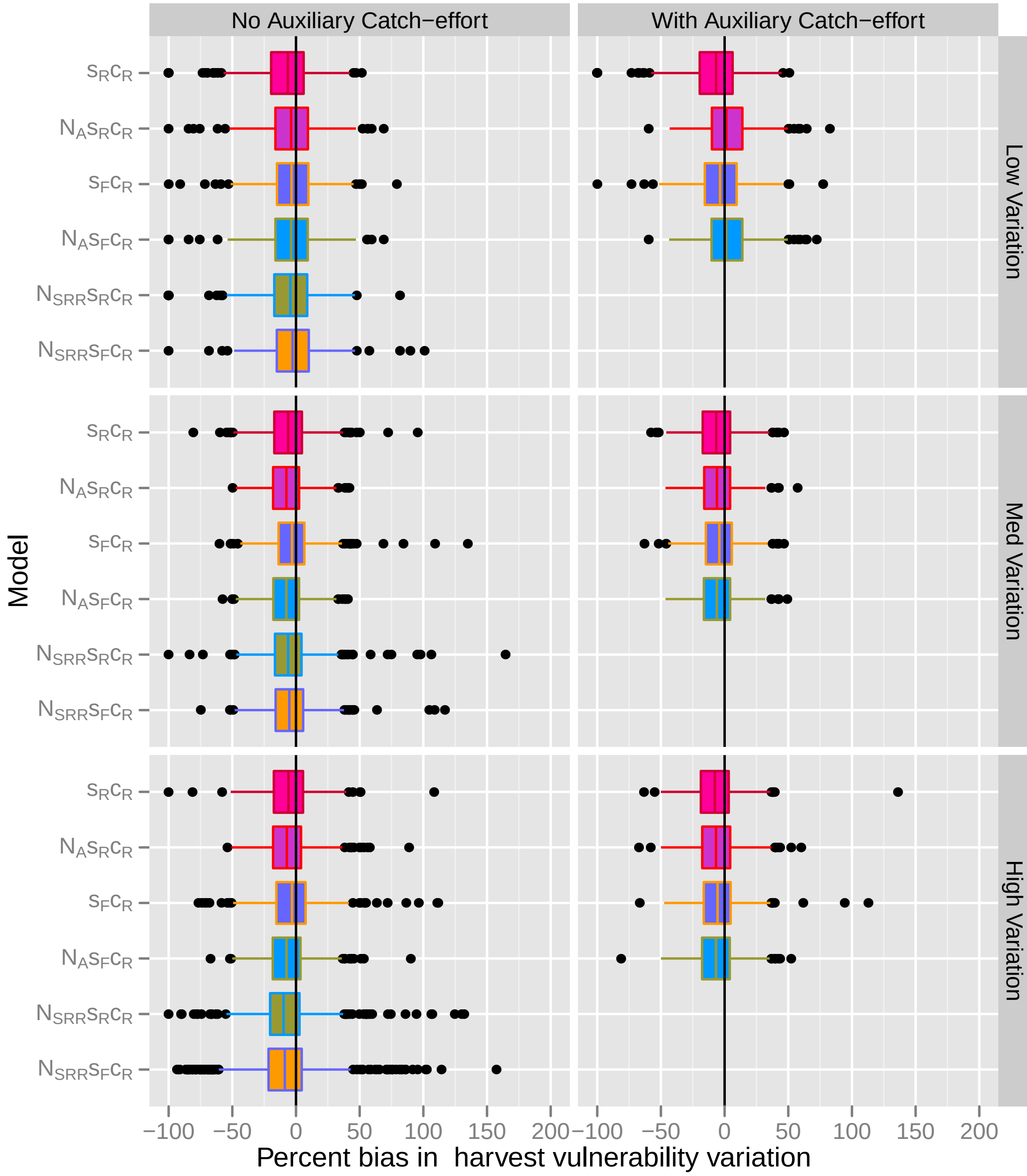


Figure A.5: *Percent bias in estimated interannual variation in harvest vulnerability, σ_c, for big game models. Models tend to show negative bias in estimation fo σ_c. Stock-recruit models show greater variation in estimates, as evidenced by greater frequency of outlying observations in boxplots. Results based on 25 years of data for 13 age classes, and 1000 simulations.*

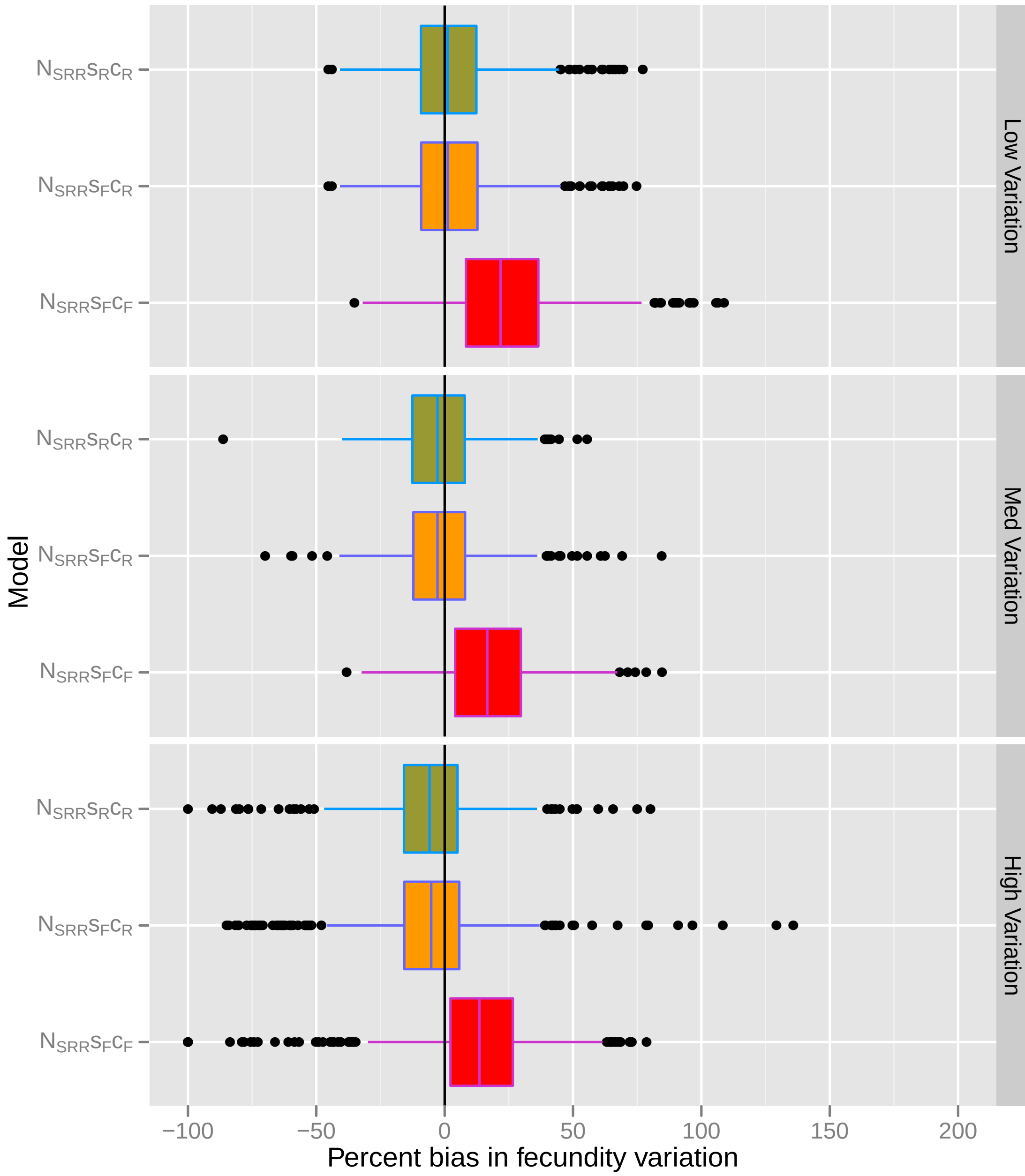


Figure A.6: *Percent bias in estimated variation in productivity, σ_γ, for big game models. Models that incorporate random effects other than productivity (models $N_{SR,R}s_Rc_R$ and $N_{SR,R}s_Fc_R$) indicate low median bias in estimating environmental variation in productivity, while model $N_{SR,R}s_Fc_F$ appears to consistently overestimate environmental variation in productivity. Results based on 25 years of data for 13 age classes, and 1000 simulations.*

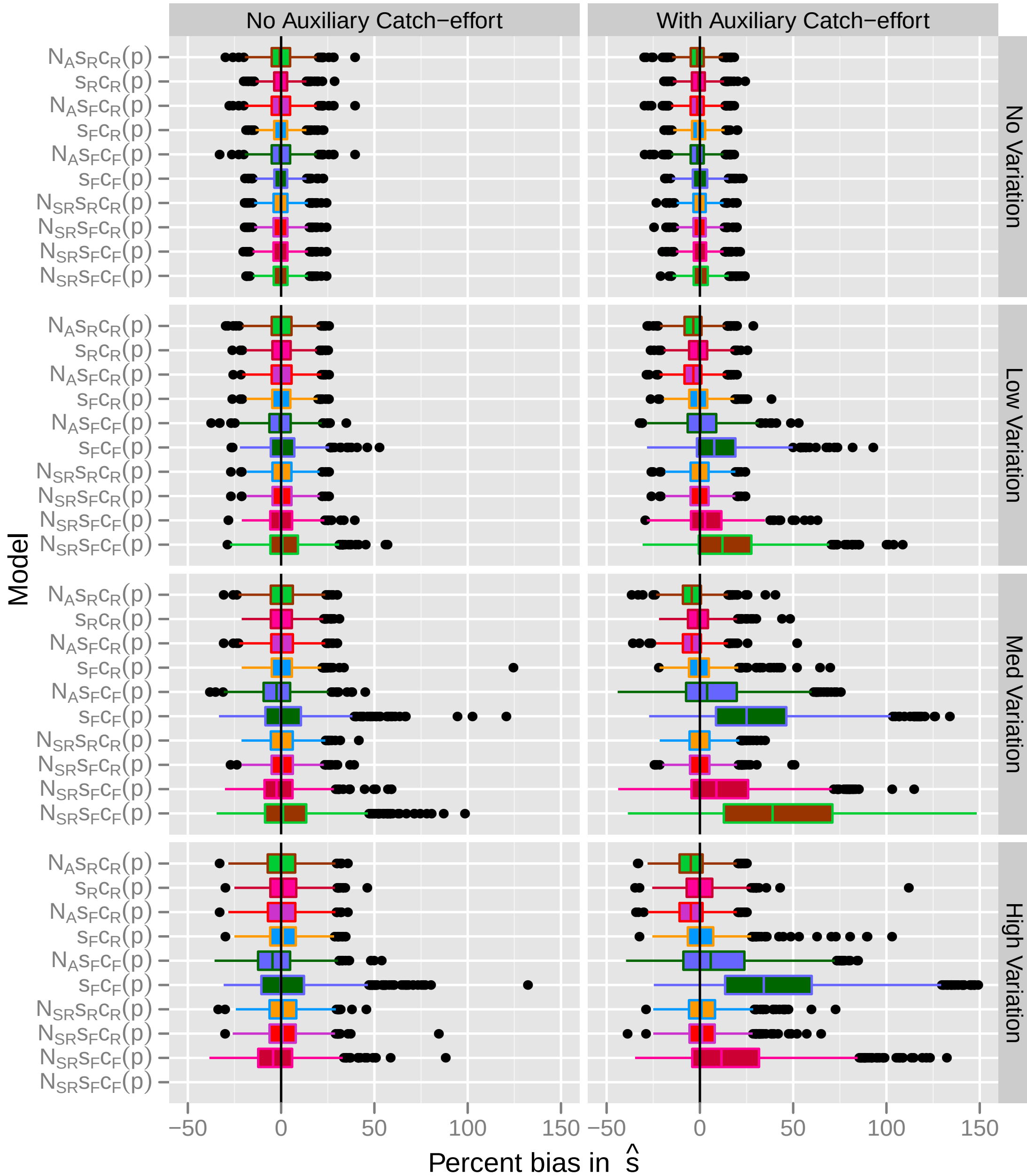


Figure A.7: *Percent bias in estimated survival probability, s, for pooled age class models. Results indicate low median bias in estimation when the auxiliary catch-effort likelihood component of Equation (1.7) is omitted, for all models. Results based on 25 years of data for 13 age classes (pooled into 1, 2, and 3+), and 1000 simulations. Model $N_{SR,F}s_Fc_F$ not fitted at the high level of simulated variation.*

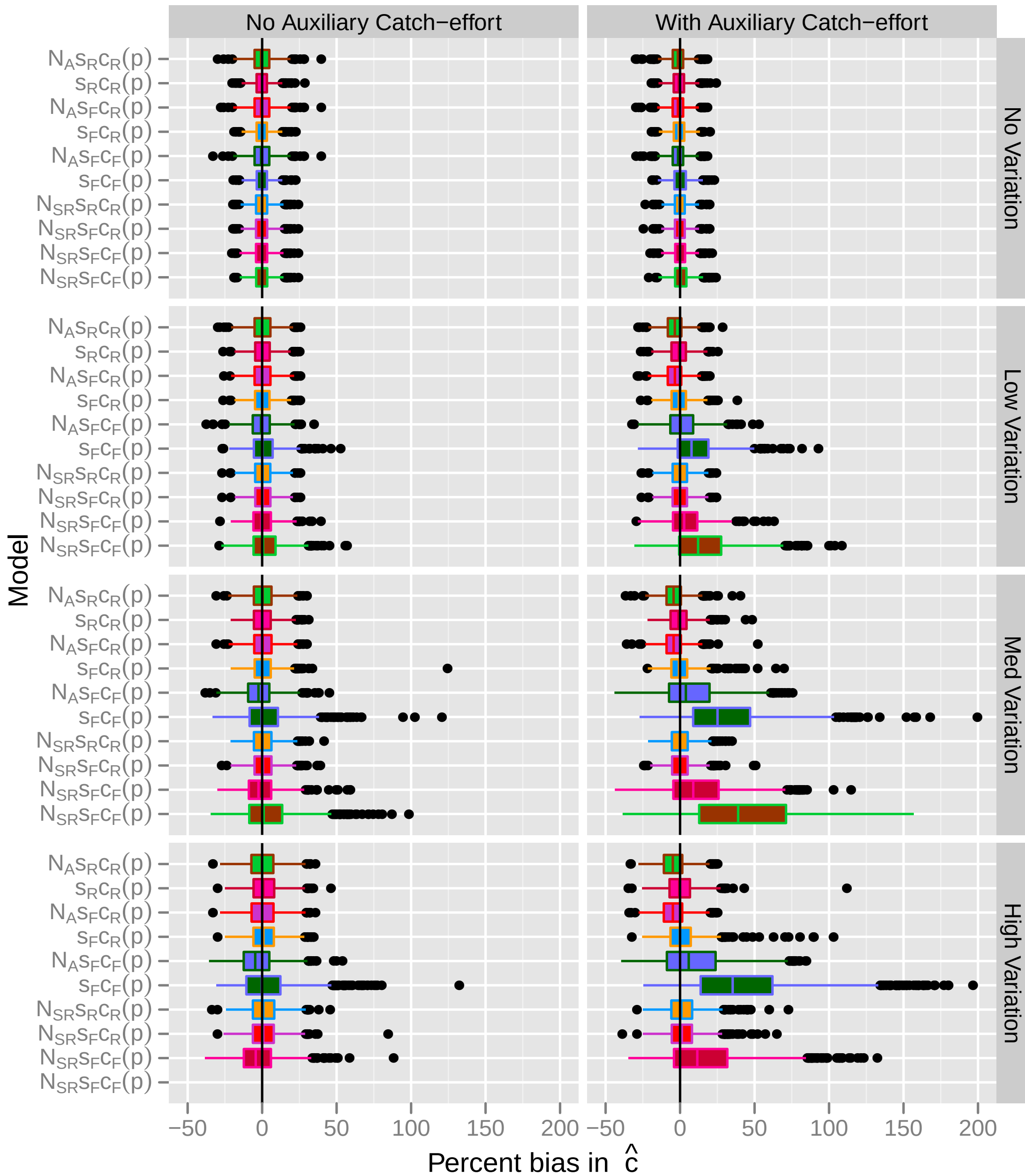


Figure A.8: *Percent bias in estimated harvest vulnerability, c, for pooled age class models. Results indicate low median bias in estimation when the auxiliary catch-effort likelihood component of Equation (1.7) is omitted, for all models. Results based on 25 years of data for 13 age classes (pooled into 1, 2, and 3+), and 1000 simulations. Model* $N_{SR,F} s_F c_F$ *not fitted at the high level of simulated variation.*

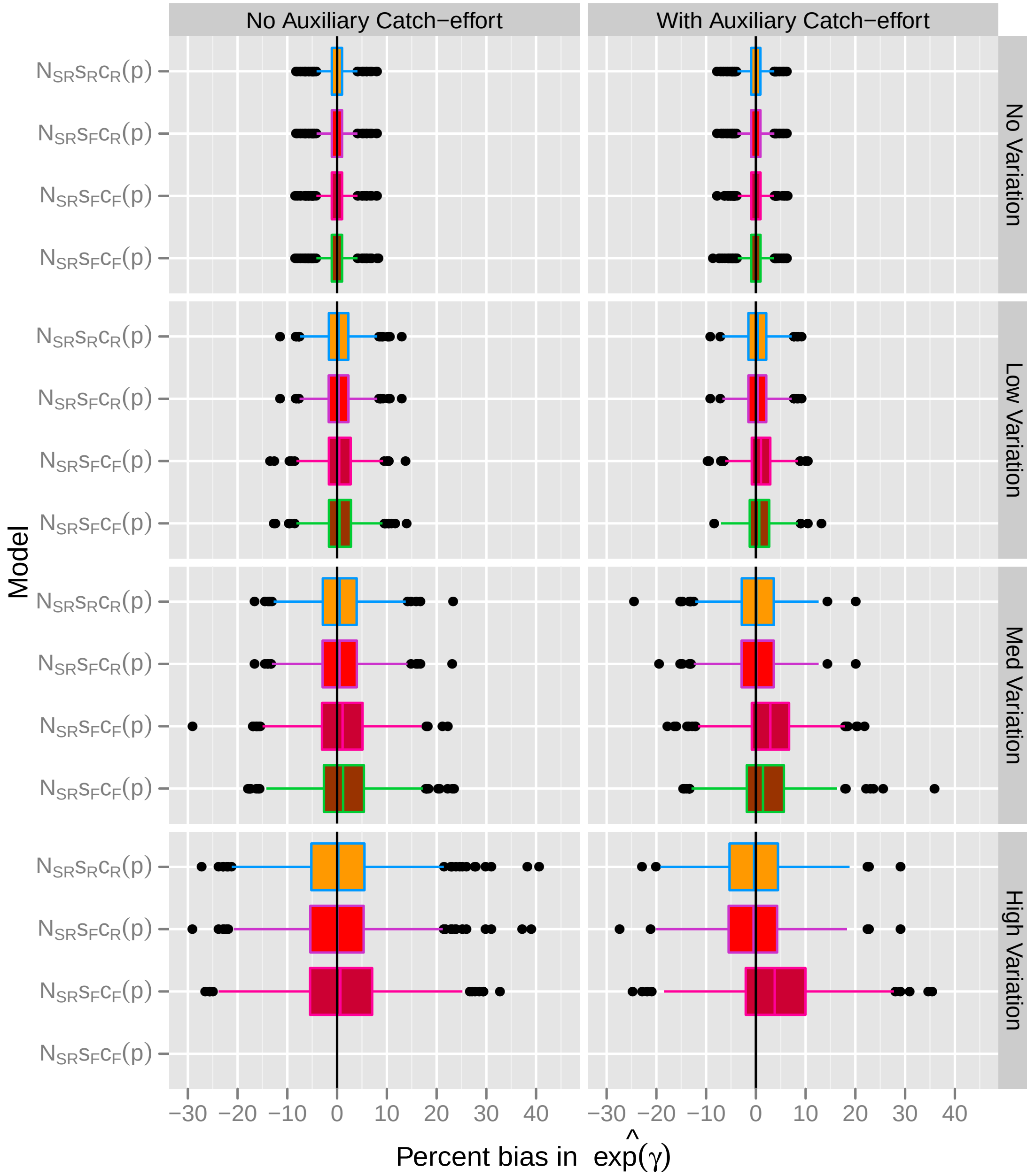


Figure A.9: *Percent bias in estimated productivity, e^{γ}, for pooled age class models. Results indicate low bias for all models, at all levels of simulated variation. Results based on 25 years of data for 13 age classes (pooled into 1, 2, and 3+), and 1000 simulations. Model $N_{SR,F}s_Fc_F$ not fitted at the high level of simulated variation.*

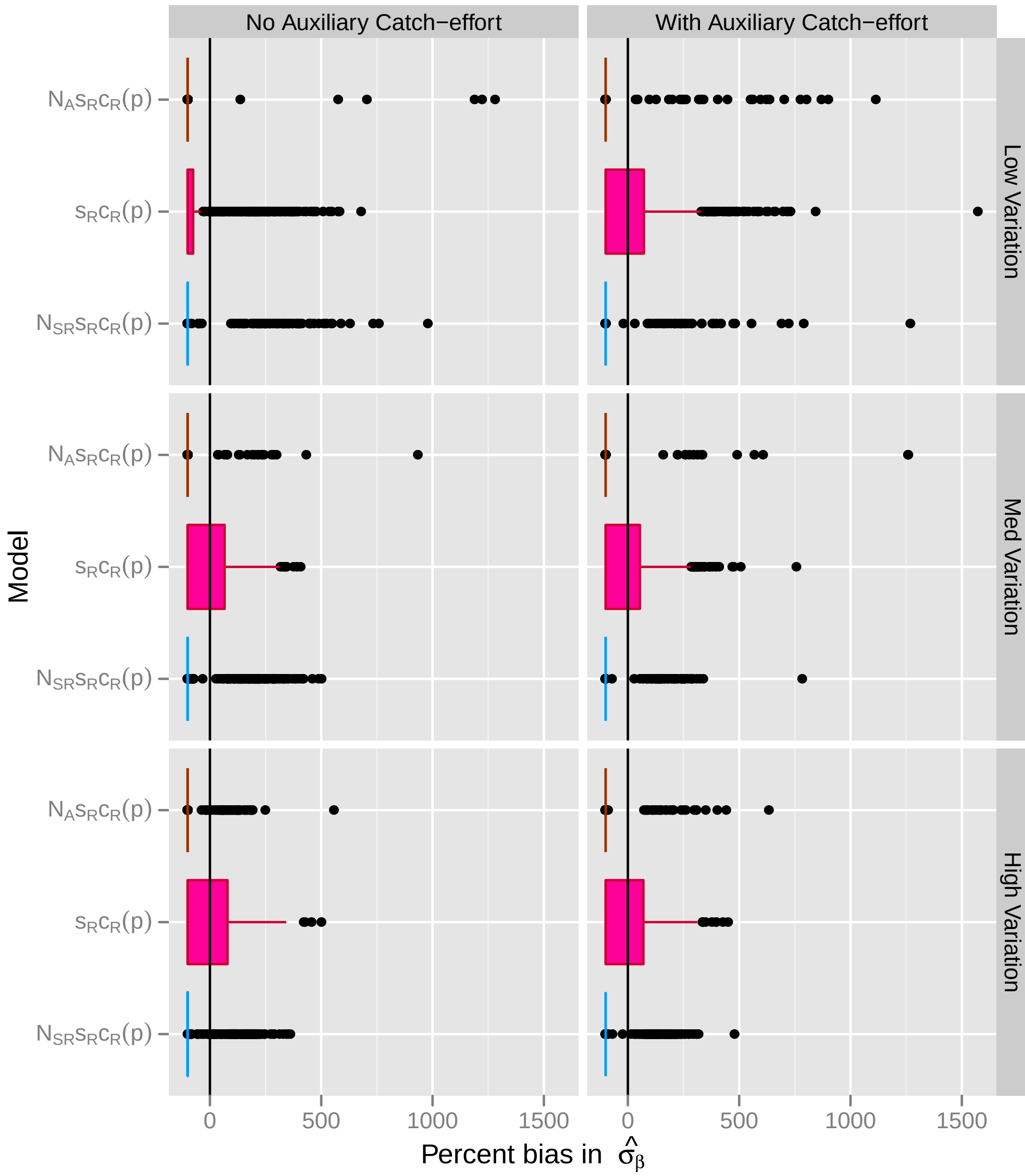


Figure A.10: *Percent bias in estimated interannual variation in survival probability, σ_β, for pooled age class models. Results indicate poor ability to estimate environmental stochasticity in survival probability, with many results estimated near 0, and frequent estimates with large positive bias. Model $s_R c_R(p)$ outperforms models $N_A s_R c_R(p)$ and $N_{SR,R} s_R c_R(p)$. Results based on 25 years of data for 13 age classes (pooled into 1, 2, and 3+), and 1000 simulations.*

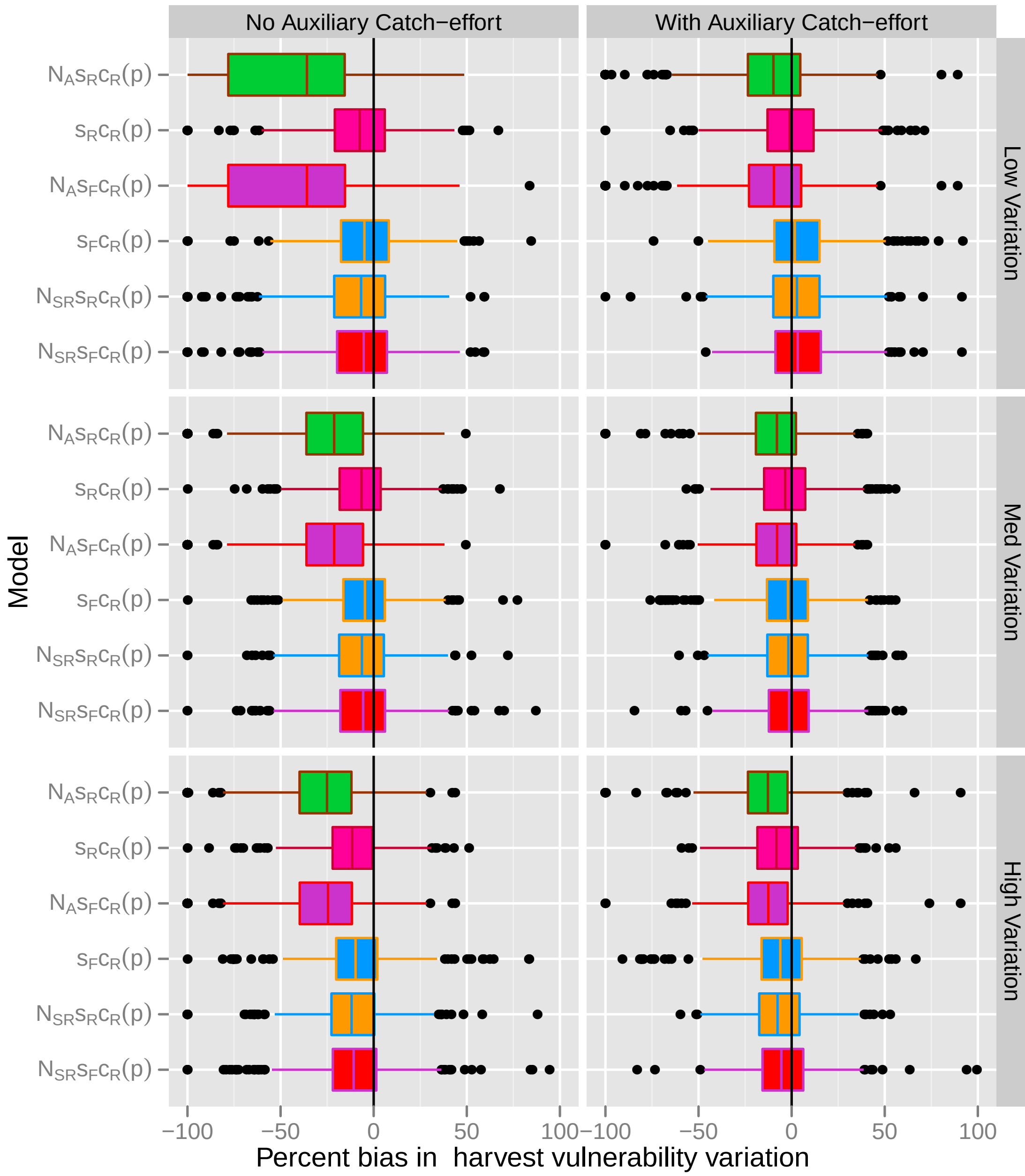


Figure A.11: *Percent bias in estimated interannual variation in harvest vulnerability, σ_c, for pooled age class models. Results indicate that environmental variation in harvest probability is slightly underestimated by most models; the underestimation is greater for absolute-recruit abundance models $N_A s_F c_R(p)$ and $N_A s_R c_R(p)$. Stock-recruit and Horvitz-Thompson models perform the best. Results based on 25 years of data for 13 age classes (pooled into 1, 2, and 3+), and 1000 simulations.*

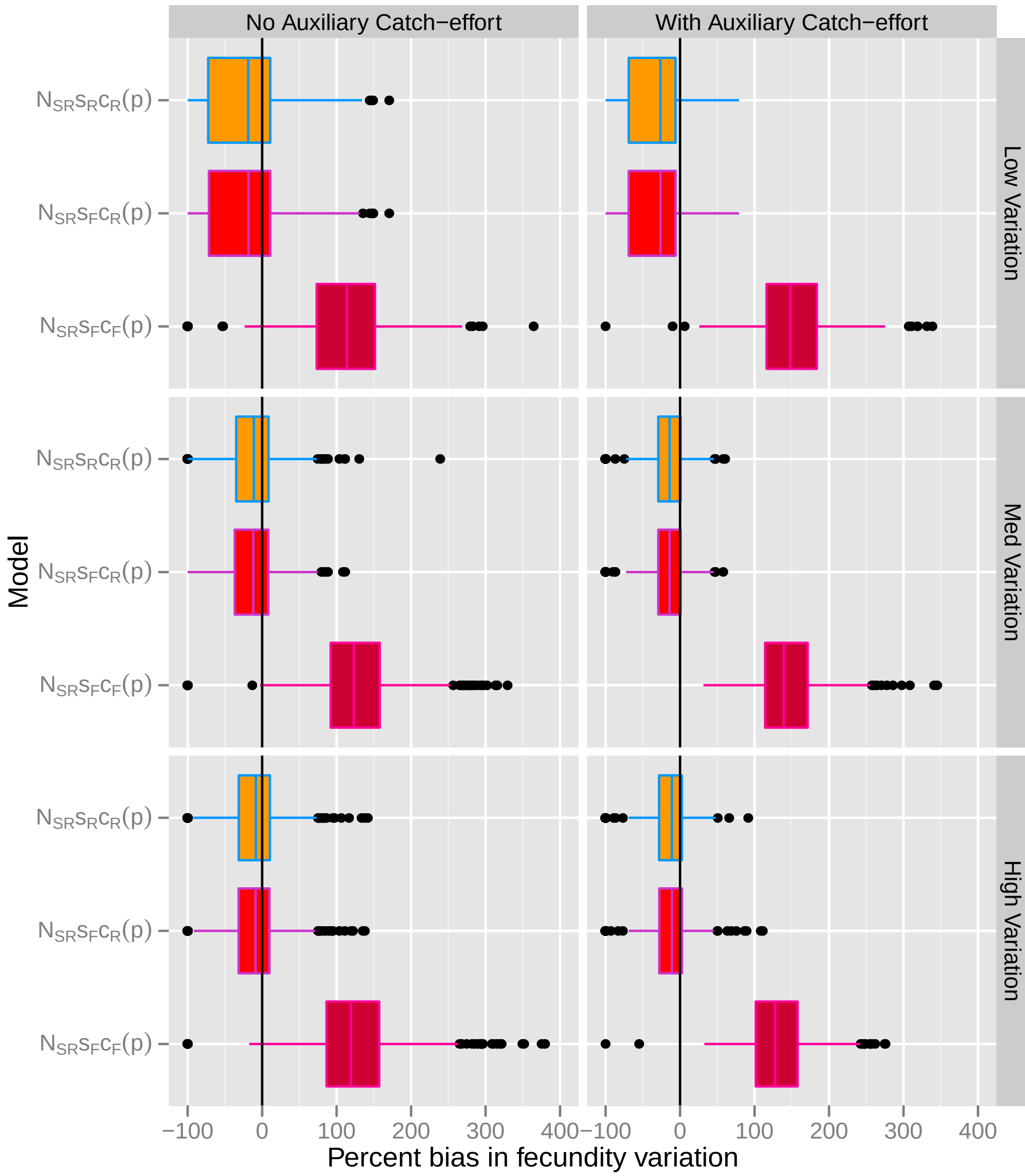


Figure A.12: *Percent bias in estimated variation in productivity, σ_γ, for pooled age class models. Results indicate that models incorporating another random effect for harvest probability and/or survival probability slightlyunder-estimate environmental variation in reproduction, while the model that incorporates a single random effect for reproduction ($N_{SR,R}s_Fc_F(p)$) shows a large degree of positive bias. Results based on 25 years of data for 13 age classes (pooled into 1, 2, and 3+), and 1000 simulations.*

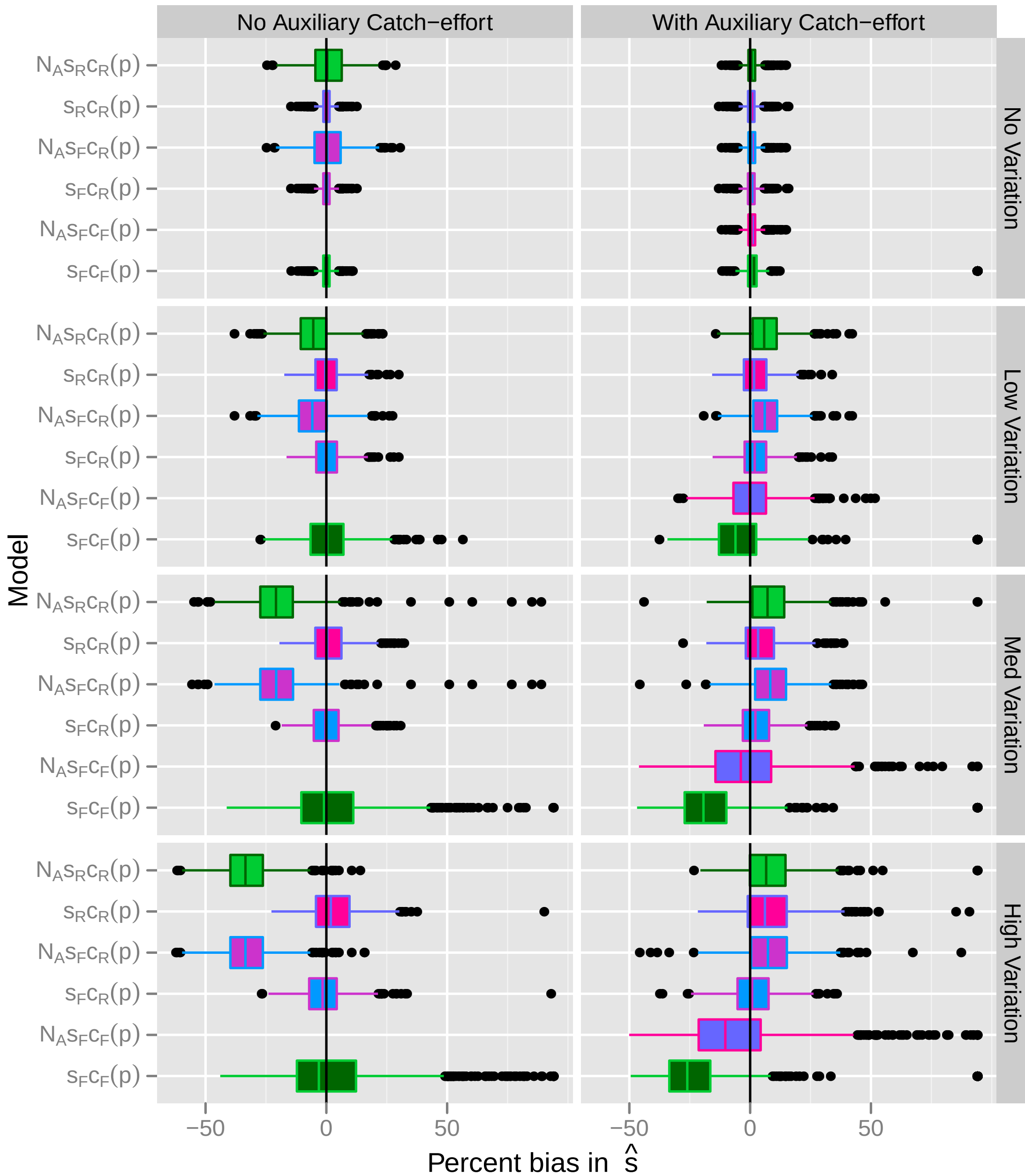


Figure A.13: *Percent bias in estimated survival probability, s, for small game models when a high amount of auxiliary data has been simulated for estimation of c. Results indicate low bias for models employing the Horvitz-Thompson estimator when the auxiliary catch-effort likelihood of Equation (1.7) is omitted. At medium and high levels of simulated variation, absolute-recruit abundance models ($N_A s_F c_F(p)$, $N_A s_F c_R(p)$, and $N_A s_R c_R(p)$) show negative bias when the auxiliary catch-effort likelihood is omitted and positive bias when it is included. Results based on 25 years of data for 2 age classes (young-of-the-year and adult aged 1+), and 1000 simulations. Model $N_A s_F c_F(p)$ was only fitted by using the auxiliary catch-effort likelihood due to numerical stability issues.*

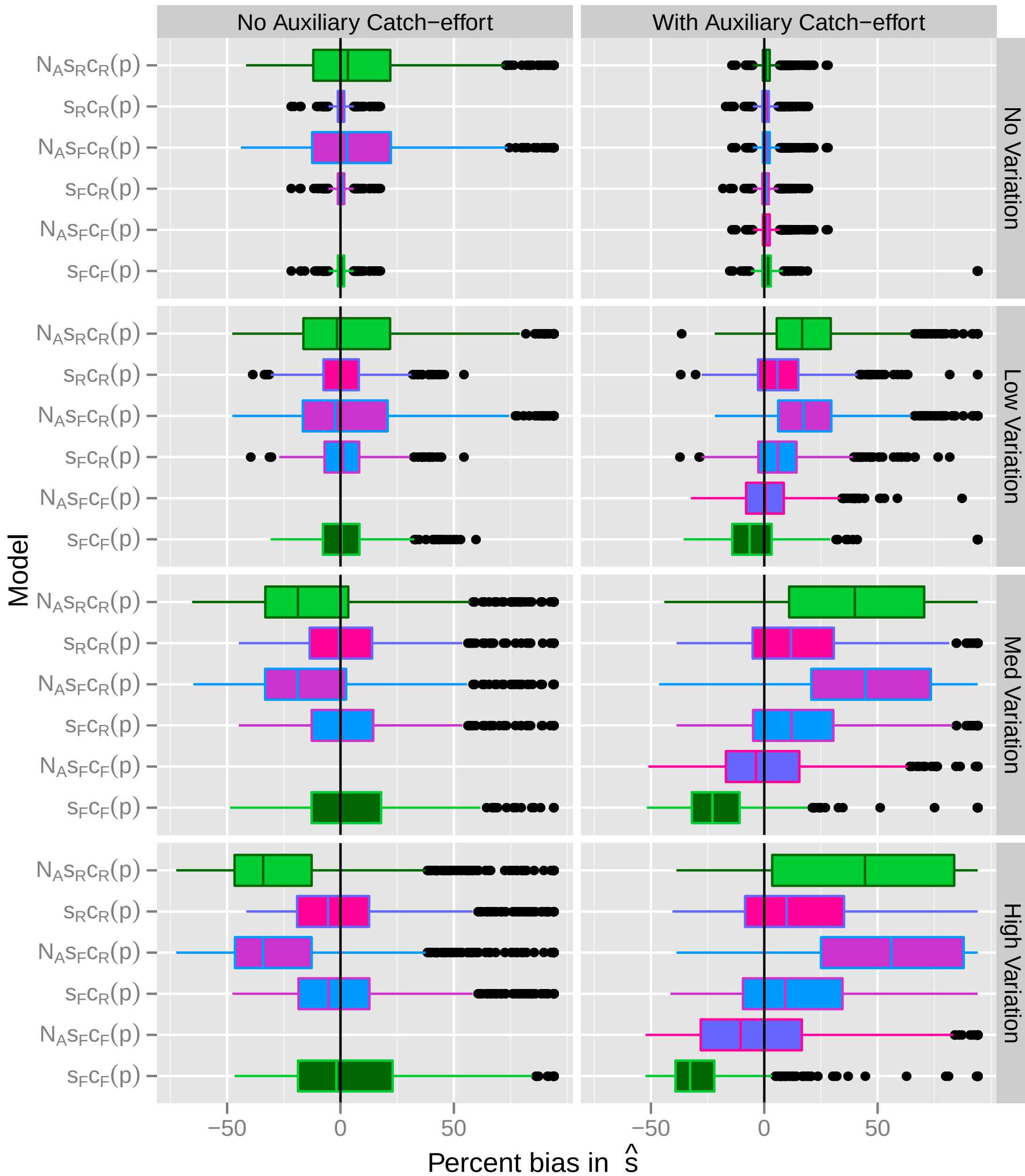


Figure A.14: *Percent bias in estimated survival probability, s, for small game models when a low amount of auxiliary data has been simulated for estimation of c. Results indicate low bias for models employing the Horvitz-Thompson estimator when the auxiliary catch-effort likelihood of Equation (1.7) is omitted. At medium and high levels of simulated variation, absolute-recruit abundance models ($N_As_Fc_F(p)$, $N_As_Fc_R(p)$, and $N_As_Rc_R(p)$ show negative bias when the auxiliary catch-effort likelihood is omitted and positive bias when it is included. Results based on 25 years of data for 2 age classes (young-of-the-year and adult aged 1+), and 1000 simulations. Model $N_As_Fc_F(p)$ was only fitted by using the auxiliary catch-effort likelihood due to numerical stability issues.*

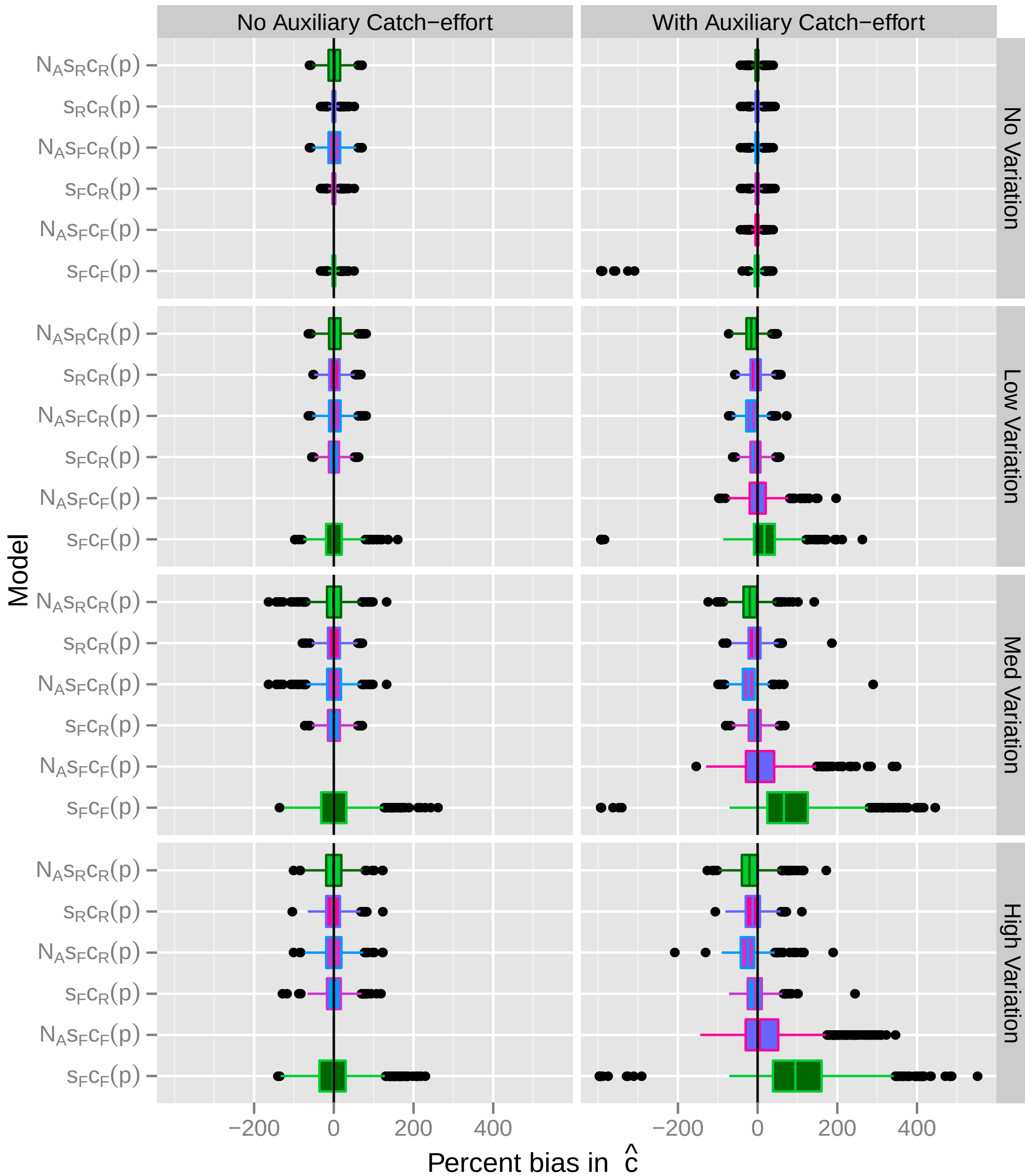


Figure A.15: *Percent bias in estimated harvest vulnerability, c, for small game models when a high amount of auxiliary data has been simulated for estimation of c. Results indicate low bias in estimation when the auxiliary catch-effort likelihood of Equation (1.7) is omitted, for all models. Variation in bias, however, is lowest for models employing the Horvitz-Thompson estimator. Some bias is exhibited for all models at nonzero levels of simulated variation when the auxilairy catch-effort likelihood is employed. Results based on 25 years of data for 2 age classes (young-of-the-year and adult aged 1+), and 1000 simulations. Model $N_As_Fc_F(p)$ was only fitted by using the auxiliary catch-effort likelihood due to numerical stability issues.*

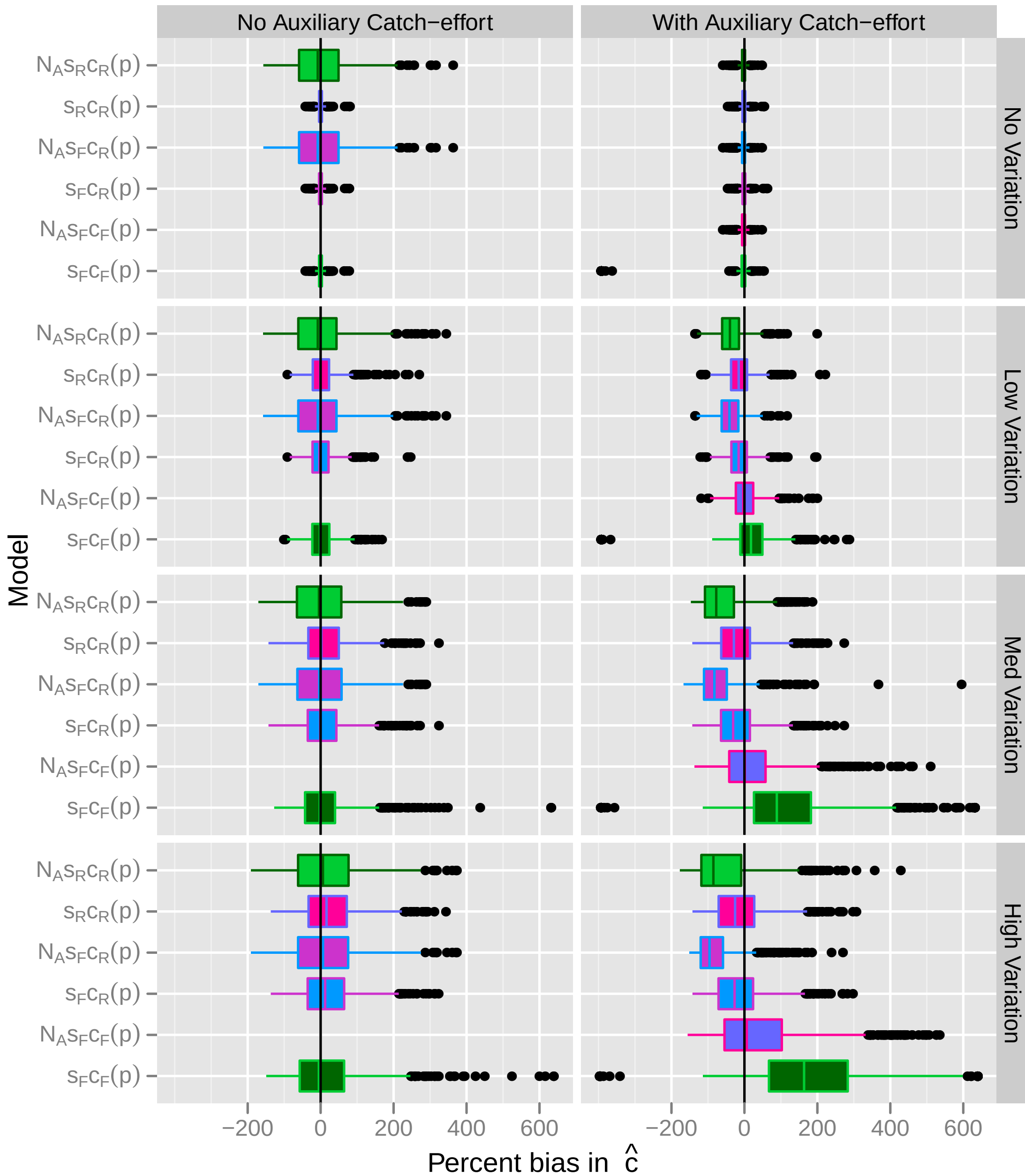


Figure A.16: *Percent bias in estimated harvest vulnerability, c, for small game models when a low amount of auxiliary data has been simulated for estimation of c. Results indicate low bias in estimation when the auxiliary catch-effort likelihood of Equation (1.7) is omitted, for all models. Variation in bias, however, is lowest for models employing the Horvitz-Thompson estimator. Some bias is exhibited for all models at nonzero levels of simulated variation when the auxilairy catch-effort likelihood is employed. Results based on 25 years of data for 2 age classes (young-of-the-year and adult aged 1+), and 1000 simulations. Model* $N_A s_F c_F(p)$ *was only fitted by using the auxiliary catch-effort likelihood due to numerical stability issues.*

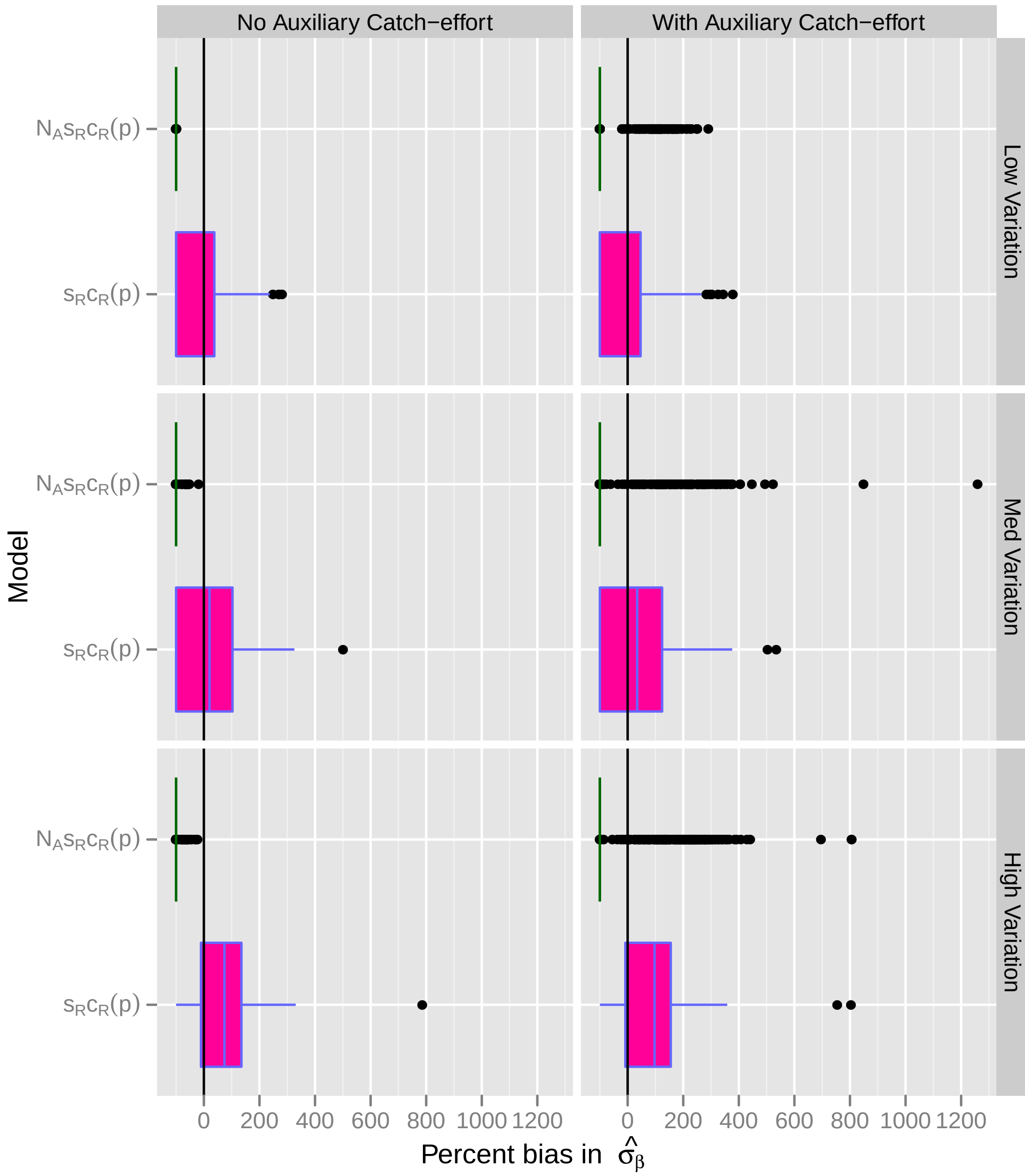


Figure A.17: *Percent bias in estimated interannual variation in survival probability, σ_β, for small game models when a high amount of auxiliary data has been simulated for estimation of c. Results indicate poor ability to estimate environmental variation in survival probability, although model $s_R c_R(p)$ slightly outperforms $N_A s_R c_R(p)$. For both models, many estimates are near zero for low values of simulated variability. For higher values of simulated variability, model $N_A s_R c_R(p)$ still frequently estimates $\sigma_\beta \approx 0$, while model $s_R c_R(p)$ shows a propensity to overestimate survival variation. Both models show a some positively-biased outliers. Results based on 25 years of data for 2 age classes (young-of-the-year and adult aged 1+), and 1000 simulations.*

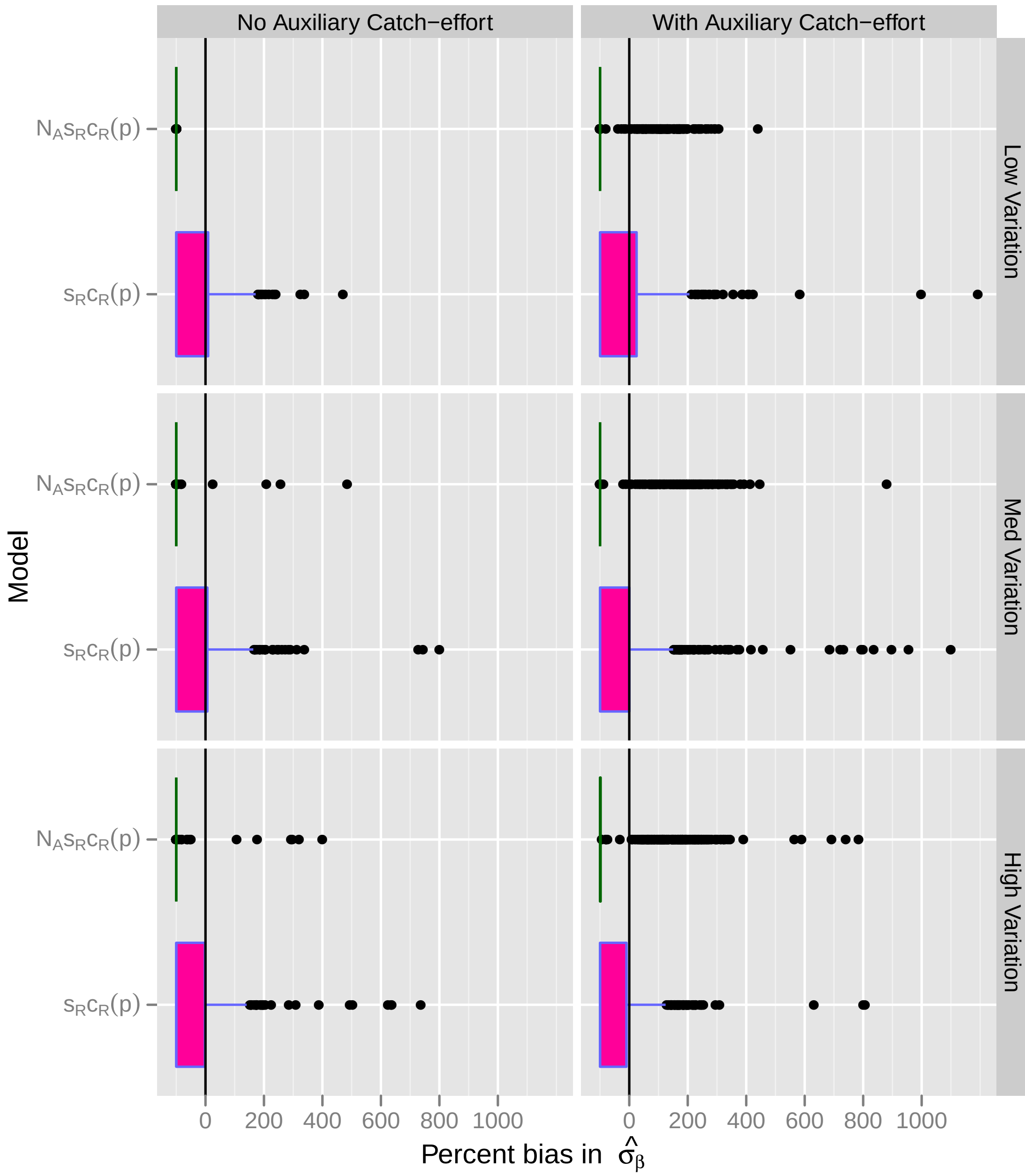


Figure A.18: *Percent bias in estimated interannual variation in survival probability, σ_β, for small game models when a low amount of auxiliary data has been simulated for estimation of c. Results indicate poor ability to estimate environmental variation in survival probability, although model $s_R c_R(p)$ slightly outperforms $N_A s_R c_R(p)$. For both models, many estimates zero, with both models showing a propensity for positively-biased outliers. Results based on 25 years of data for 2 age classes (young-of-the-year and adult aged 1+), and 1000 simulations.*

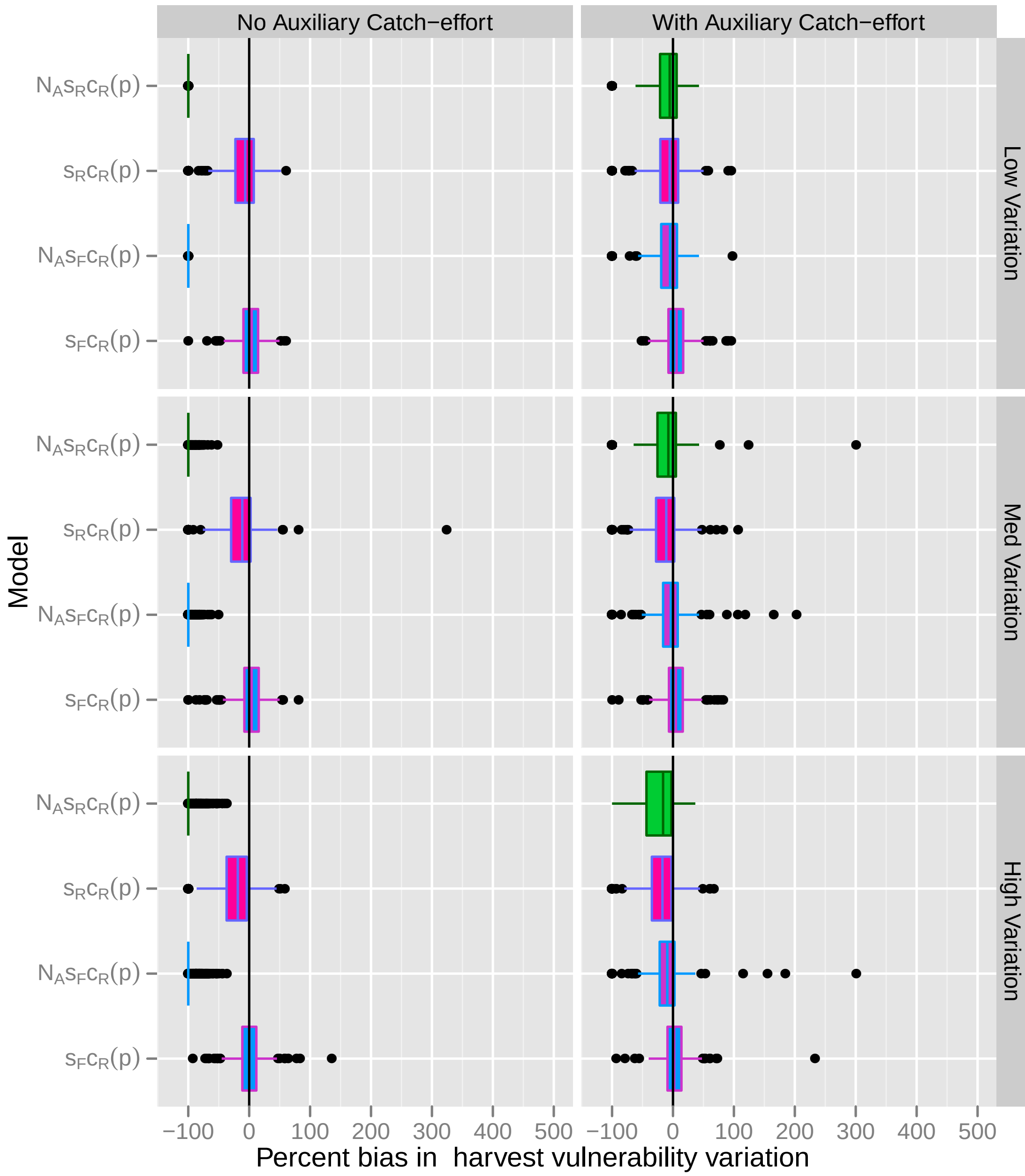


Figure A.19: *Percent bias in estimated interannual variation in harvest vulnerability, σ_c, for small game models when a high amount of auxiliary data has been simulated for estimation of c. When the auxiliary catch-effort likelihood of Equation (1.7) is omitted, results indicate underestimation of environmental variation for harvest vulnerability for absolute-recruit abundance models $N_A s_R c_R(p)$ and $N_A s_F c_R(p)$, while Horvitz-Thompson models $s_R c_R(p)$ and $s_F c_R(p)$, show low estimation bias. When the auxiliary catch-effort likelihood component is included, both sets of models show only slight underestimation of environmental variation in harvest probability. Results based on 25 years of data for 2 age classes (young-of-the-year and adult aged 1+), and 1000 simulations.*

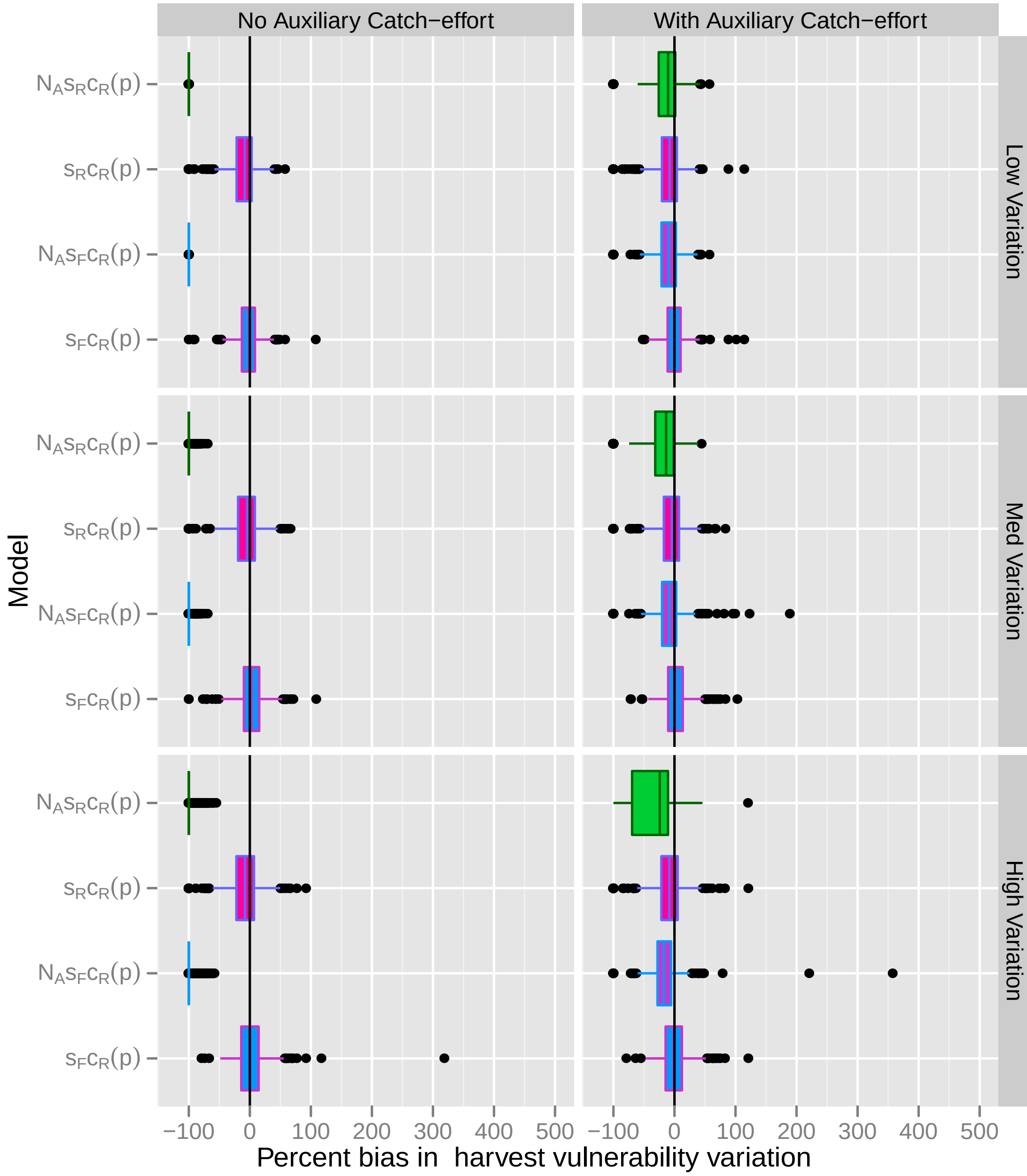


Figure A.20: *Percent bias in estimated interannual variation in harvest vulnerability, σ_c, for small game models when a low amount of auxiliary data has been simulated for estimation of c. When the auxiliary catch-effort likelihood of Equation (1.7) is omitted, results indicate underestimation of environmental variation for harvest vulnerability for absolute-recruit abundance models $N_A s_R c_R(p)$ and $N_A s_F c_R(p)$, while Horvitz-Thompson models $s_R c_R(p)$ and $s_F c_R(p)$, show low estimation bias. When the auxiliary catch-effort likelihood component is included, both sets of models show only slight underestimation of environmental variation in harvest probability. Results based on 25 years of data for 2 age classes (young-of-the-year and adult aged 1+), and 1000 simulations.*

A.2 Tables

Table (A.1) contains the data used for plotting Figure (2.2), the relative bias in estimation of annual abundance for each model, for each year of data.

Table A.1: *Median relative bias in total annual abundance estimates. Results indicate negligible bias for conditional-likelihood/Horvitz-Thompson models, consistent low negative bias for stock-recruit models, and large negative bias for absolute-recruit abundance models, with one exception: model $N_{A}s_{F}c_{F}$ shows low negative bias with no simulated variation, but increasing positive bias as simulated variation increases. Results based on $n = 1000$ replicates, with $s = 0.84$, $c = -1.5$, $\gamma = 0.0$, and total annual abundance ≈ 4000. "Aux. Like." = Auxiliary catch-effort likelihood used (With) or not used (Without).*

Table A.1
Annual Abundance Percent Bias

Aux. Like.	Model	Year	No Variation	Low Variation	Medium Variation	High Variation
Without	$N_{SR,F}s_{F}c_{F}$	1	5.44%	80.29%	256.03%	394.61%
Without	$N_{SR,F}s_{F}c_{F}$	2	5.2%	80.31%	254.25%	395.25%
Without	$N_{SR,F}s_{F}c_{F}$	3	5.26%	81.65%	249.6%	389.5%
Without	$N_{SR,F}s_{F}c_{F}$	4	5.17%	80%	249.72%	393.7%
Without	$N_{SR,F}s_{F}c_{F}$	5	5.21%	78.35%	251.61%	384.06%
Without	$N_{SR,F}s_{F}c_{F}$	6	5.25%	80.25%	246.18%	395.16%
Without	$N_{SR,F}s_{F}c_{F}$	7	5.03%	79.22%	253.44%	388.61%
Without	$N_{SR,F}s_{F}c_{F}$	8	5.16%	81.65%	253.74%	397.32%
Without	$N_{SR,F}s_{F}c_{F}$	9	5.32%	82.24%	249.1%	396.49%
Without	$N_{SR,F}s_{F}c_{F}$	10	5.43%	80.53%	254.59%	397.76%
Without	$N_{SR,F}s_{F}c_{F}$	11	5.54%	80.24%	257%	395.37%
Without	$N_{SR,F}s_{F}c_{F}$	12	5.22%	80.3%	257.61%	397.44%
Without	$N_{SR,F}s_{F}c_{F}$	13	5.19%	80.84%	263.46%	394.38%
Without	$N_{SR,F}s_{F}c_{F}$	14	5.34%	79.21%	255.08%	390.44%
Without	$N_{SR,F}s_{F}c_{F}$	15	5.46%	80.6%	252.22%	392.69%
Without	$N_{SR,F}s_{F}c_{F}$	16	5.27%	78.31%	252.86%	385.73%
Without	$N_{SR,F}s_{F}c_{F}$	17	5.31%	78.82%	253.9%	390.28%
Without	$N_{SR,F}s_{F}c_{F}$	18	5.1%	76.94%	252.87%	397.52%
Without	$N_{SR,F}s_{F}c_{F}$	19	5.36%	79.51%	248.57%	388.54%
Without	$N_{SR,F}s_{F}c_{F}$	20	5.2%	78.1%	251.3%	392.94%
Without	$N_{SR,F}s_{F}c_{F}$	21	5.38%	79.07%	250.74%	387.34%
Without	$N_{SR,F}s_{F}c_{F}$	22	4.94%	79.36%	251.32%	400.5%
Without	$N_{SR,F}s_{F}c_{F}$	23	5.15%	79.42%	253.64%	416.78%
Without	$N_{SR,F}s_{F}c_{F}$	24	5.27%	80.69%	252.37%	402.78%
Without	$N_{SR,F}s_{F}c_{F}$	25	5.38%	81.86%	253.32%	418.68%
Without	$N_{SR,R}s_{F}c_{F}$	1	-3.84%	-5.48%	-6.05%	-3.62%
Without	$N_{SR,R}s_{F}c_{F}$	2	-3.81%	-5.21%	-5.43%	-3.99%
Without	$N_{SR,R}s_{F}c_{F}$	3	-3.63%	-5.33%	-5.3%	-3.48%
Without	$N_{SR,R}s_{F}c_{F}$	4	-3.77%	-4.76%	-5.11%	-2.48%
Without	$N_{SR,R}s_{F}c_{F}$	5	-3.75%	-4.85%	-5.03%	-1.42%
Without	$N_{SR,R}s_{F}c_{F}$	6	-3.76%	-5.04%	-5.14%	-1.1%
Without	$N_{SR,R}s_{F}c_{F}$	7	-3.56%	-5.16%	-4.46%	-2.52%
Without	$N_{SR,R}s_{F}c_{F}$	8	-3.38%	-5.35%	-4.45%	-2.31%
Without	$N_{SR,R}s_{F}c_{F}$	9	-3.74%	-5.17%	-5.02%	-1.85%
Without	$N_{SR,R}s_{F}c_{F}$	10	-3.66%	-5.14%	-4.58%	-2.98%
Without	$N_{SR,R}s_{F}c_{F}$	11	-3.94%	-5.25%	-4.41%	-2.58%
Without	$N_{SR,R}s_{F}c_{F}$	12	-3.73%	-5.18%	-4.85%	-1.8%
Without	$N_{SR,R}s_{F}c_{F}$	13	-3.61%	-5.49%	-4.77%	-2.57%

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Table A.1 – continued from previous page

Annual Abundance Percent Bias						
Aux. Like.	**Model**	**Year**	**No Variation**	**Low Variation**	**Medium Variation**	**High Variation**
Without	$N_{SR,R}s_Fc_F$	14	-3.76%	-5.42%	-4.79%	-1.84%
Without	$N_{SR,R}s_Fc_F$	15	-3.67%	-4.96%	-5.11%	-3.05%
Without	$N_{SR,R}s_Fc_F$	16	-3.66%	-5.63%	-5.12%	-1.41%
Without	$N_{SR,R}s_Fc_F$	17	-3.71%	-5.37%	-5.39%	-2.38%
Without	$N_{SR,R}s_Fc_F$	18	-3.62%	-5.38%	-5.07%	-1.03%
Without	$N_{SR,R}s_Fc_F$	19	-3.68%	-4.75%	-5.59%	-1.32%
Without	$N_{SR,R}s_Fc_F$	20	-3.5%	-5.12%	-5.42%	-1.76%
Without	$N_{SR,R}s_Fc_F$	21	-3.6%	-5%	-5.64%	-1.75%
Without	$N_{SR,R}s_Fc_F$	22	-3.56%	-5.19%	-5.54%	-0.15%
Without	$N_{SR,R}s_Fc_F$	23	-3.54%	-5.32%	-4.5%	-0.7%
Without	$N_{SR,R}s_Fc_F$	24	-3.78%	-5.22%	-4.89%	-0.04%
Without	$N_{SR,R}s_Fc_F$	25	-3.67%	-4.77%	-4.46%	0.74%
Without	$N_{SR,R}s_Fc_R$	1	-5.06%	-9.03%	-9.24%	-6.02%
Without	$N_{SR,R}s_Fc_R$	2	-5.14%	-9.06%	-9.3%	-6.04%
Without	$N_{SR,R}s_Fc_R$	3	-5.02%	-8.92%	-8.79%	-6.2%
Without	$N_{SR,R}s_Fc_R$	4	-5.17%	-9.4%	-8.97%	-6.41%
Without	$N_{SR,R}s_Fc_R$	5	-5.04%	-9.12%	-8.79%	-6.23%
Without	$N_{SR,R}s_Fc_R$	6	-4.91%	-9%	-8.92%	-6.17%
Without	$N_{SR,R}s_Fc_R$	7	-4.9%	-9.18%	-8.87%	-6.55%
Without	$N_{SR,R}s_Fc_R$	8	-4.51%	-9.19%	-8.98%	-6.17%
Without	$N_{SR,R}s_Fc_R$	9	-4.89%	-9.18%	-8.91%	-6.92%
Without	$N_{SR,R}s_Fc_R$	10	-5%	-9.17%	-9.15%	-7.09%
Without	$N_{SR,R}s_Fc_R$	11	-5.04%	-9.15%	-8.93%	-6.63%
Without	$N_{SR,R}s_Fc_R$	12	-5.03%	-9.06%	-8.83%	-6.91%
Without	$N_{SR,R}s_Fc_R$	13	-4.93%	-8.79%	-8.57%	-6.62%
Without	$N_{SR,R}s_Fc_R$	14	-4.82%	-9.13%	-8.62%	-6.25%
Without	$N_{SR,R}s_Fc_R$	15	-4.61%	-9.16%	-8.47%	-6.45%
Without	$N_{SR,R}s_Fc_R$	16	-4.74%	-9.1%	-8.87%	-6.17%
Without	$N_{SR,R}s_Fc_R$	17	-4.68%	-9.11%	-8.75%	-6.26%
Without	$N_{SR,R}s_Fc_R$	18	-4.69%	-9.18%	-9.41%	-6.62%
Without	$N_{SR,R}s_Fc_R$	19	-4.86%	-9.08%	-9.5%	-6.56%
Without	$N_{SR,R}s_Fc_R$	20	-4.78%	-9.29%	-9.86%	-6.81%
Without	$N_{SR,R}s_Fc_R$	21	-4.86%	-9.21%	-9.76%	-6.83%
Without	$N_{SR,R}s_Fc_R$	22	-4.78%	-9.46%	-10.15%	-7.16%
Without	$N_{SR,R}s_Fc_R$	23	-4.74%	-9.59%	-9.81%	-8.18%
Without	$N_{SR,R}s_Fc_R$	24	-5.04%	-10%	-10.68%	-7.96%
Without	$N_{SR,R}s_Fc_R$	25	-5.1%	-10.04%	-11.5%	-8.53%
Without	$N_{SR,R}s_Rc_R$	1	-4.88%	-8.97%	-10.26%	-8.03%
Without	$N_{SR,R}s_Rc_R$	2	-4.89%	-9.05%	-10.06%	-7.93%
Without	$N_{SR,R}s_Rc_R$	3	-4.6%	-8.92%	-10.04%	-8.34%
Without	$N_{SR,R}s_Rc_R$	4	-4.62%	-9.43%	-9.8%	-7.85%
Without	$N_{SR,R}s_Rc_R$	5	-4.75%	-9.03%	-9.42%	-7.93%
Without	$N_{SR,R}s_Rc_R$	6	-4.66%	-8.66%	-9.38%	-7.82%
Without	$N_{SR,R}s_Rc_R$	7	-4.56%	-9.01%	-9.5%	-8.14%
Without	$N_{SR,R}s_Rc_R$	8	-4.45%	-9.18%	-9.77%	-7.75%
Without	$N_{SR,R}s_Rc_R$	9	-4.64%	-8.95%	-9.22%	-7.98%
Without	$N_{SR,R}s_Rc_R$	10	-4.64%	-8.98%	-9.42%	-8.31%
Without	$N_{SR,R}s_Rc_R$	11	-4.88%	-8.71%	-8.95%	-8.27%
Without	$N_{SR,R}s_Rc_R$	12	-4.79%	-8.85%	-8.9%	-7.95%
Without	$N_{SR,R}s_Rc_R$	13	-4.66%	-8.52%	-8.82%	-8.07%
Without	$N_{SR,R}s_Rc_R$	14	-4.6%	-8.99%	-8.79%	-7.61%
Without	$N_{SR,R}s_Rc_R$	15	-4.28%	-9.13%	-9.04%	-7.78%
Without	$N_{SR,R}s_Rc_R$	16	-4.38%	-8.96%	-9.4%	-7.73%
Without	$N_{SR,R}s_Rc_R$	17	-4.38%	-8.9%	-9.3%	-7.77%
Without	$N_{SR,R}s_Rc_R$	18	-4.46%	-9.01%	-9.89%	-8.24%
Without	$N_{SR,R}s_Rc_R$	19	-4.37%	-8.78%	-9.94%	-8.38%
Without	$N_{SR,R}s_Rc_R$	20	-4.54%	-9.06%	-10.33%	-8.68%
Without	$N_{SR,R}s_Rc_R$	21	-4.34%	-9.11%	-10.07%	-8.58%
Without	$N_{SR,R}s_Rc_R$	22	-4.48%	-9.1%	-10.33%	-8.75%

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Table A.1 – continued from previous page

Annual Abundance Percent Bias						
Aux. Like.	**Model**	**Year**	**No Variation**	**Low Variation**	**Medium Variation**	**High Variation**
Without	$N_{SR,R}s_Rc_R$	23	-4.34%	-9.33%	-9.87%	-9.95%
Without	$N_{SR,R}s_Rc_R$	24	-4.4%	-9.49%	-11.23%	-10.18%
Without	$N_{SR,R}s_Rc_R$	25	-4.54%	-9.8%	-11.5%	-10.7%
Without	$N_As_Fc_F$	1	-16.7%	-18.84%	-20.35%	-20.97%
Without	$N_As_Fc_F$	2	-16.89%	-18.7%	-19.91%	-20.13%
Without	$N_As_Fc_F$	3	-17%	-18.61%	-20.36%	-20.01%
Without	$N_As_Fc_F$	4	-16.55%	-18.49%	-20.13%	-19.19%
Without	$N_As_Fc_F$	5	-16.63%	-18.45%	-20.13%	-19.13%
Without	$N_As_Fc_F$	6	-16.36%	-18.71%	-20.03%	-18.88%
Without	$N_As_Fc_F$	7	-16.56%	-18.43%	-19.51%	-19.59%
Without	$N_As_Fc_F$	8	-16.5%	-18.65%	-19.59%	-19.18%
Without	$N_As_Fc_F$	9	-16.8%	-18.36%	-19.35%	-20.12%
Without	$N_As_Fc_F$	10	-17.07%	-18.56%	-19.02%	-20.08%
Without	$N_As_Fc_F$	11	-16.6%	-18.7%	-19.22%	-20.21%
Without	$N_As_Fc_F$	12	-16.86%	-18.46%	-19.15%	-18.83%
Without	$N_As_Fc_F$	13	-16.61%	-18.47%	-19.38%	-19.23%
Without	$N_As_Fc_F$	14	-16.52%	-18.1%	-19.56%	-19.1%
Without	$N_As_Fc_F$	15	-16.69%	-18.23%	-19.63%	-19.04%
Without	$N_As_Fc_F$	16	-16.79%	-18.33%	-20.2%	-19.19%
Without	$N_As_Fc_F$	17	-16.85%	-18.49%	-20.1%	-19.68%
Without	$N_As_Fc_F$	18	-16.79%	-18.87%	-20.25%	-19.2%
Without	$N_As_Fc_F$	19	-16.82%	-18.27%	-20.26%	-19.22%
Without	$N_As_Fc_F$	20	-16.47%	-18.67%	-19.49%	-20.05%
Without	$N_As_Fc_F$	21	-16.45%	-18.52%	-20.01%	-19.14%
Without	$N_As_Fc_F$	22	-16.81%	-18.55%	-20.13%	-19.11%
Without	$N_As_Fc_F$	23	-16.75%	-18.34%	-19.6%	-19.14%
Without	$N_As_Fc_F$	24	-16.81%	-18.54%	-19.42%	-18.55%
Without	$N_As_Fc_F$	25	-16.95%	-17.76%	-18.54%	-17.65%
Without	s_Fc_F	1	-0.4%	1.15%	0.64%	1.54%
Without	s_Fc_F	2	-0.29%	1.44%	0.51%	1.01%
Without	s_Fc_F	3	-0.12%	0.86%	-1.72%	1.06%
Without	s_Fc_F	4	-0.32%	0.94%	-1.12%	1.64%
Without	s_Fc_F	5	-0.16%	0.39%	-0.39%	2.39%
Without	s_Fc_F	6	-0.15%	0.6%	-0.23%	0.65%
Without	s_Fc_F	7	-0.09%	0.97%	-1.35%	1.86%
Without	s_Fc_F	8	-0.24%	1.37%	-0.91%	0.14%
Without	s_Fc_F	9	-0.38%	1.06%	-1.01%	0.83%
Without	s_Fc_F	10	-0.19%	0.61%	-0.35%	-0.74%
Without	s_Fc_F	11	-0.2%	2.01%	-0.35%	5.74%
Without	s_Fc_F	12	0.01%	0.47%	0.97%	2.9%
Without	s_Fc_F	13	-0.08%	-0.11%	-0.52%	1.02%
Without	s_Fc_F	14	0.27%	1.78%	-0.9%	1.96%
Without	s_Fc_F	15	-0.14%	-0.04%	-0.35%	1.77%
Without	s_Fc_F	16	0.05%	1.61%	-0.36%	1.15%
Without	s_Fc_F	17	-0.04%	0.84%	-0.5%	2.36%
Without	s_Fc_F	18	-0.02%	1.33%	-0.48%	0.93%
Without	s_Fc_F	19	0.06%	0.53%	0.28%	0.46%
Without	s_Fc_F	20	-0.09%	0.86%	-1.21%	0.69%
Without	s_Fc_F	21	-0.06%	0.38%	-1.01%	2.1%
Without	s_Fc_F	22	-0.23%	0.9%	0.21%	1.09%
Without	s_Fc_F	23	-0.29%	0.21%	-0.5%	0.59%
Without	s_Fc_F	24	-0.09%	0.33%	-0.95%	0.92%
Without	s_Fc_F	25	-0.24%	0.97%	-1.01%	1.89%
Without	$N_As_Fc_R$	1	-19.81%	-26.64%	-26.92%	-27.21%
Without	$N_As_Fc_R$	2	-19.89%	-26.44%	-26.96%	-27.33%
Without	$N_As_Fc_R$	3	-19.92%	-26.93%	-27.42%	-27.1%
Without	$N_As_Fc_R$	4	-19.59%	-26.59%	-26.83%	-26.92%
Without	$N_As_Fc_R$	5	-19.43%	-26.48%	-26.96%	-26.82%
Without	$N_As_Fc_R$	6	-19.42%	-26.67%	-26.87%	-26.88%

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Table A.1 – continued from previous page

Annual Abundance Percent Bias						
Aux. Like.	**Model**	**Year**	**No Variation**	**Low Variation**	**Medium Variation**	**High Variation**
Without	$N_A s_F c_R$	7	-19.22%	-26.53%	-26.79%	-27.04%
Without	$N_A s_F c_R$	8	-19.74%	-26.41%	-26.72%	-27.43%
Without	$N_A s_F c_R$	9	-19.85%	-26.41%	-26.57%	-27.45%
Without	$N_A s_F c_R$	10	-19.61%	-26.63%	-26.85%	-26.98%
Without	$N_A s_F c_R$	11	-19.74%	-26.4%	-26.87%	-26.41%
Without	$N_A s_F c_R$	12	-20%	-26.41%	-26.74%	-26.2%
Without	$N_A s_F c_R$	13	-19.7%	-26.22%	-26.47%	-26.33%
Without	$N_A s_F c_R$	14	-19.7%	-26.33%	-26.31%	-26.82%
Without	$N_A s_F c_R$	15	-19.58%	-26.49%	-26.33%	-26.76%
Without	$N_A s_F c_R$	16	-19.63%	-26.36%	-26.59%	-26.37%
Without	$N_A s_F c_R$	17	-19.06%	-26.38%	-26.63%	-26.41%
Without	$N_A s_F c_R$	18	-19.16%	-26.55%	-26.92%	-26.98%
Without	$N_A s_F c_R$	19	-19.42%	-26.43%	-27.34%	-27.34%
Without	$N_A s_F c_R$	20	-19.59%	-26.56%	-27.25%	-27.83%
Without	$N_A s_F c_R$	21	-19.3%	-26.75%	-27.58%	-28.27%
Without	$N_A s_F c_R$	22	-19.44%	-26.89%	-28.05%	-29.23%
Without	$N_A s_F c_R$	23	-19.4%	-26.95%	-28.72%	-30.88%
Without	$N_A s_F c_R$	24	-19.7%	-27.13%	-29.53%	-33.67%
Without	$N_A s_F c_R$	25	-19.53%	-27.94%	-31.74%	-37.56%
Without	$s_F c_R$	1	-0.46%	0.84%	0.53%	0.36%
Without	$s_F c_R$	2	-0.24%	1.03%	0.33%	-0.2%
Without	$s_F c_R$	3	-0.1%	0.99%	0%	0.37%
Without	$s_F c_R$	4	-0.42%	0.57%	0.07%	0.09%
Without	$s_F c_R$	5	-0.16%	0.75%	0.22%	0.83%
Without	$s_F c_R$	6	-0.06%	0.63%	-0.05%	0.78%
Without	$s_F c_R$	7	-0.02%	0.6%	0.44%	0.46%
Without	$s_F c_R$	8	-0.21%	0.52%	0.56%	0.46%
Without	$s_F c_R$	9	-0.25%	0.66%	-0.11%	-0.01%
Without	$s_F c_R$	10	-0.18%	0.41%	-0.32%	0.27%
Without	$s_F c_R$	11	-0.23%	0.77%	0.16%	0.41%
Without	$s_F c_R$	12	-0.09%	0.72%	0.41%	0.26%
Without	$s_F c_R$	13	-0.11%	0.5%	0.19%	-0.08%
Without	$s_F c_R$	14	0.24%	0.44%	0.37%	0.23%
Without	$s_F c_R$	15	-0.08%	0.32%	0.33%	-0.53%
Without	$s_F c_R$	16	0.08%	1.01%	0.02%	-0.37%
Without	$s_F c_R$	17	0.03%	0.7%	0.04%	-0.31%
Without	$s_F c_R$	18	0.09%	0.91%	0.05%	0.58%
Without	$s_F c_R$	19	0.06%	0.88%	-0.34%	0.49%
Without	$s_F c_R$	20	-0.06%	0.74%	-0.2%	-0.07%
Without	$s_F c_R$	21	-0.03%	0.96%	-0.4%	0.21%
Without	$s_F c_R$	22	-0.15%	0.77%	0.02%	0.74%
Without	$s_F c_R$	23	-0.33%	0.68%	0.36%	0.33%
Without	$s_F c_R$	24	-0.07%	0.56%	0.01%	-0.08%
Without	$s_F c_R$	25	-0.26%	0.55%	0.96%	1.06%
Without	$N_A s_R c_R$	1	-19.93%	-26.67%	-26.97%	-27.46%
Without	$N_A s_R c_R$	2	-19.94%	-26.41%	-27.09%	-27.65%
Without	$N_A s_R c_R$	3	-20.07%	-26.91%	-27.51%	-27.38%
Without	$N_A s_R c_R$	4	-19.67%	-26.61%	-27.02%	-27.19%
Without	$N_A s_R c_R$	5	-19.49%	-26.39%	-27.14%	-27.09%
Without	$N_A s_R c_R$	6	-19.38%	-26.64%	-27%	-27.06%
Without	$N_A s_R c_R$	7	-19.36%	-26.51%	-26.8%	-27.16%
Without	$N_A s_R c_R$	8	-19.76%	-26.38%	-26.73%	-27.44%
Without	$N_A s_R c_R$	9	-20%	-26.37%	-26.46%	-27.33%
Without	$N_A s_R c_R$	10	-19.5%	-26.52%	-26.7%	-26.7%
Without	$N_A s_R c_R$	11	-19.62%	-26.24%	-26.68%	-25.72%
Without	$N_A s_R c_R$	12	-20%	-26.37%	-26.61%	-25.73%
Without	$N_A s_R c_R$	13	-19.65%	-26.19%	-26.3%	-26.1%
Without	$N_A s_R c_R$	14	-19.67%	-26.26%	-26.31%	-26.72%
Without	$N_A s_R c_R$	15	-19.6%	-26.49%	-26.61%	-26.99%

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Table A.1 – continued from previous page

Annual Abundance Percent Bias						
Aux. Like.	**Model**	**Year**	**No Variation**	**Low Variation**	**Medium Variation**	**High Variation**
Without	$N_A s_R c_R$	16	-19.77%	-26.38%	-26.92%	-26.63%
Without	$N_A s_R c_R$	17	-19.06%	-26.33%	-26.75%	-26.65%
Without	$N_A s_R c_R$	18	-19.16%	-26.51%	-27.06%	-27.14%
Without	$N_A s_R c_R$	19	-19.42%	-26.36%	-27.47%	-27.55%
Without	$N_A s_R c_R$	20	-19.56%	-26.54%	-27.26%	-27.98%
Without	$N_A s_R c_R$	21	-19.34%	-26.68%	-27.66%	-28.29%
Without	$N_A s_R c_R$	22	-19.44%	-26.78%	-28.1%	-29.17%
Without	$N_A s_R c_R$	23	-19.46%	-26.92%	-28.67%	-30.88%
Without	$N_A s_R c_R$	24	-19.9%	-27.12%	-29.56%	-33.42%
Without	$N_A s_R c_R$	25	-19.46%	-27.9%	-31.45%	-37.41%
Without	$s_R c_R$	1	-0.45%	0.85%	-0.25%	-0.72%
Without	$s_R c_R$	2	-0.33%	1.39%	-0.04%	-1.06%
Without	$s_R c_R$	3	-0.1%	0.98%	-0.35%	-0.76%
Without	$s_R c_R$	4	-0.4%	0.59%	-0.23%	-0.54%
Without	$s_R c_R$	5	-0.1%	0.93%	0.06%	0.04%
Without	$s_R c_R$	6	0%	0.56%	-0.23%	-0.02%
Without	$s_R c_R$	7	-0.03%	0.63%	0.25%	0.07%
Without	$s_R c_R$	8	-0.21%	0.54%	0.47%	-0.11%
Without	$s_R c_R$	9	-0.22%	0.66%	-0.12%	-0.4%
Without	$s_R c_R$	10	-0.12%	0.49%	-0.02%	0.17%
Without	$s_R c_R$	11	-0.2%	1.02%	0.21%	-0.08%
Without	$s_R c_R$	12	-0.09%	0.85%	0.58%	0.14%
Without	$s_R c_R$	13	-0.16%	0.62%	0.08%	0.04%
Without	$s_R c_R$	14	0.26%	0.83%	0.35%	0%
Without	$s_R c_R$	15	0%	0.65%	0.2%	-0.54%
Without	$s_R c_R$	16	0.1%	1.24%	-0.11%	-0.81%
Without	$s_R c_R$	17	0.07%	0.8%	-0.05%	-0.49%
Without	$s_R c_R$	18	-0.01%	1.04%	-0.15%	-0.5%
Without	$s_R c_R$	19	0.13%	1.07%	-0.56%	-0.3%
Without	$s_R c_R$	20	-0.06%	0.69%	-0.65%	-0.89%
Without	$s_R c_R$	21	-0.12%	1.2%	-0.99%	-0.3%
Without	$s_R c_R$	22	-0.17%	0.71%	-0.29%	0%
Without	$s_R c_R$	23	-0.33%	0.8%	0.18%	-0.16%
Without	$s_R c_R$	24	-0.14%	0.85%	0.04%	-0.06%
Without	$s_R c_R$	25	-0.29%	0.61%	0.97%	1.96%
With	$N_A s_F c_F$	1	-11.01%	-1.67%	31.76%	65.36%
With	$N_A s_F c_F$	2	-11.04%	-1.11%	32.03%	63.38%
With	$N_A s_F c_F$	3	-11.11%	-0.81%	32.38%	63.32%
With	$N_A s_F c_F$	4	-10.88%	-1.04%	32.35%	64.86%
With	$N_A s_F c_F$	5	-10.85%	-1.43%	31.13%	66.09%
With	$N_A s_F c_F$	6	-10.77%	-1.48%	31.65%	65.69%
With	$N_A s_F c_F$	7	-10.85%	-1.4%	30.85%	66.61%
With	$N_A s_F c_F$	8	-10.65%	-0.93%	32.1%	65.03%
With	$N_A s_F c_F$	9	-10.74%	-1.01%	32.38%	63.04%
With	$N_A s_F c_F$	10	-11.03%	-1.43%	32.6%	60.09%
With	$N_A s_F c_F$	11	-10.81%	-0.95%	31.9%	63.58%
With	$N_A s_F c_F$	12	-10.83%	-1.16%	32.46%	63.31%
With	$N_A s_F c_F$	13	-10.64%	-1.9%	31.61%	62.88%
With	$N_A s_F c_F$	14	-11.02%	-1.32%	32.15%	64.83%
With	$N_A s_F c_F$	15	-11.09%	-1.17%	31.7%	65.21%
With	$N_A s_F c_F$	16	-11.07%	-2.38%	31.49%	64.74%
With	$N_A s_F c_F$	17	-10.95%	-2.02%	32.41%	68.69%
With	$N_A s_F c_F$	18	-10.83%	-1.62%	32.79%	65.81%
With	$N_A s_F c_F$	19	-10.76%	-1.4%	31.48%	68.6%
With	$N_A s_F c_F$	20	-10.7%	-0.53%	31.81%	69.93%
With	$N_A s_F c_F$	21	-10.66%	-0.76%	30.02%	66.59%
With	$N_A s_F c_F$	22	-10.64%	-0.84%	30.6%	65.62%
With	$N_A s_F c_F$	23	-10.95%	-1.43%	31.13%	65.12%
With	$N_A s_F c_F$	24	-10.85%	-1.13%	31.4%	64.61%

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Table A.1 – continued from previous page

Annual Abundance Percent Bias						
Aux. Like.	**Model**	**Year**	**No Variation**	**Low Variation**	**Medium Variation**	**High Variation**
With	$N_A s_F c_F$	25	-10.95%	-0.79%	32.43%	66.74%
With	$s_F c_F$	1	-1.7%	-0.67%	-0.89%	-1.01%
With	$s_F c_F$	2	-1.87%	-0.39%	0.27%	1.09%
With	$s_F c_F$	3	-1.68%	-0.92%	-2.17%	1.97%
With	$s_F c_F$	4	-1.75%	-0.89%	-1.9%	1.3%
With	$s_F c_F$	5	-1.53%	-0.97%	-0.8%	2.19%
With	$s_F c_F$	6	-1.68%	-1.43%	-1.88%	0.85%
With	$s_F c_F$	7	-1.74%	-1.2%	-2.02%	2.87%
With	$s_F c_F$	8	-1.79%	-0.48%	-2.07%	1.07%
With	$s_F c_F$	9	-1.84%	-1.21%	-1.94%	-1.87%
With	$s_F c_F$	10	-1.71%	-1.66%	-0.59%	-0.77%
With	$s_F c_F$	11	-1.56%	0.06%	-1.13%	2.43%
With	$s_F c_F$	12	-1.64%	-1.38%	0.41%	0.15%
With	$s_F c_F$	13	-1.75%	-1.58%	-1.32%	0.68%
With	$s_F c_F$	14	-1.46%	-0.55%	-2.03%	-1.26%
With	$s_F c_F$	15	-1.83%	-1.99%	-0.79%	0.76%
With	$s_F c_F$	16	-1.4%	-0.58%	-1.77%	0.91%
With	$s_F c_F$	17	-1.52%	-0.41%	-0.13%	1.01%
With	$s_F c_F$	18	-1.71%	-1.04%	-1.49%	2.73%
With	$s_F c_F$	19	-1.56%	-1.92%	-1.43%	0.14%
With	$s_F c_F$	20	-1.72%	-0.46%	-2.07%	0.3%
With	$s_F c_F$	21	-1.71%	-1.47%	-1.36%	3.46%
With	$s_F c_F$	22	-1.63%	-1.01%	-1.57%	1.71%
With	$s_F c_F$	23	-1.77%	-1.83%	-0.86%	0.48%
With	$s_F c_F$	24	-1.71%	-1.72%	-1.04%	-0.29%
With	$s_F c_F$	25	-1.91%	-1.05%	-2.77%	1.04%
With	$N_A s_F c_R$	1	-21.15%	-26.94%	-26.86%	-27.09%
With	$N_A s_F c_R$	2	-20.51%	-26.54%	-26.84%	-27.24%
With	$N_A s_F c_R$	3	-20.89%	-27.17%	-27.17%	-26.86%
With	$N_A s_F c_R$	4	-20.81%	-26.92%	-26.73%	-26.83%
With	$N_A s_F c_R$	5	-20.77%	-26.69%	-26.66%	-26.87%
With	$N_A s_F c_R$	6	-20.65%	-26.89%	-26.82%	-26.77%
With	$N_A s_F c_R$	7	-20.29%	-26.67%	-26.82%	-26.96%
With	$N_A s_F c_R$	8	-20.74%	-26.6%	-26.66%	-27.19%
With	$N_A s_F c_R$	9	-20.91%	-26.72%	-26.83%	-27.51%
With	$N_A s_F c_R$	10	-20.76%	-26.84%	-26.72%	-27.05%
With	$N_A s_F c_R$	11	-20.67%	-26.72%	-26.83%	-26.97%
With	$N_A s_F c_R$	12	-20.71%	-26.64%	-26.84%	-26.91%
With	$N_A s_F c_R$	13	-20.53%	-26.54%	-26.77%	-27.06%
With	$N_A s_F c_R$	14	-20.73%	-26.59%	-26.42%	-26.88%
With	$N_A s_F c_R$	15	-20.77%	-26.76%	-26.29%	-26.69%
With	$N_A s_F c_R$	16	-20.99%	-26.73%	-26.6%	-26.56%
With	$N_A s_F c_R$	17	-20.37%	-26.71%	-26.54%	-26.42%
With	$N_A s_F c_R$	18	-20.46%	-26.82%	-27.16%	-26.97%
With	$N_A s_F c_R$	19	-20.79%	-26.57%	-27.41%	-27.36%
With	$N_A s_F c_R$	20	-20.88%	-26.82%	-27.42%	-28.03%
With	$N_A s_F c_R$	21	-20.26%	-27.01%	-27.62%	-28.5%
With	$N_A s_F c_R$	22	-20.98%	-27.05%	-27.98%	-29.52%
With	$N_A s_F c_R$	23	-20.97%	-27.4%	-28.5%	-31.28%
With	$N_A s_F c_R$	24	-21.15%	-27.58%	-29.61%	-33.96%
With	$N_A s_F c_R$	25	-20.84%	-28.46%	-32.2%	-38.94%
With	$s_F c_R$	1	-1.9%	-2.08%	-2.69%	-1.84%
With	$s_F c_R$	2	-1.94%	-2.2%	-2.69%	-2.2%
With	$s_F c_R$	3	-1.8%	-2.42%	-2.67%	-2.17%
With	$s_F c_R$	4	-1.81%	-2.75%	-2.74%	-1.87%
With	$s_F c_R$	5	-1.7%	-2.29%	-2.34%	-2.05%
With	$s_F c_R$	6	-1.73%	-2.43%	-2.73%	-1.96%
With	$s_F c_R$	7	-1.81%	-2.55%	-2.56%	-1.97%
With	$s_F c_R$	8	-1.82%	-2.4%	-2.84%	-2.51%

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Table A.1 – continued from previous page

Annual Abundance Percent Bias						
Aux. Like.	**Model**	**Year**	**No Variation**	**Low Variation**	**Medium Variation**	**High Variation**
With	$s_F c_R$	9	-1.95%	-2.62%	-2.89%	-2.65%
With	$s_F c_R$	10	-1.71%	-2.88%	-3.07%	-2.4%
With	$s_F c_R$	11	-1.63%	-2.5%	-2.83%	-2.64%
With	$s_F c_R$	12	-1.72%	-2.2%	-2.33%	-2.62%
With	$s_F c_R$	13	-1.79%	-2.27%	-2.7%	-3.22%
With	$s_F c_R$	14	-1.61%	-2.5%	-2.64%	-2.79%
With	$s_F c_R$	15	-1.92%	-2.54%	-2.44%	-2.76%
With	$s_F c_R$	16	-1.54%	-2.53%	-2.76%	-2.66%
With	$s_F c_R$	17	-1.78%	-2.38%	-2.69%	-2.5%
With	$s_F c_R$	18	-1.63%	-2.46%	-3.11%	-2.66%
With	$s_F c_R$	19	-1.66%	-2.66%	-3.42%	-2.75%
With	$s_F c_R$	20	-1.72%	-2.6%	-3.21%	-3.14%
With	$s_F c_R$	21	-1.81%	-2.32%	-3.13%	-2.82%
With	$s_F c_R$	22	-1.67%	-2.73%	-2.99%	-3.29%
With	$s_F c_R$	23	-1.81%	-2.59%	-3.11%	-3.63%
With	$s_F c_R$	24	-1.91%	-2.91%	-3.14%	-4.21%
With	$s_F c_R$	25	-1.96%	-2.94%	-3.41%	-4.44%
With	$N_A s_R c_R$	1	-21.31%	-27%	-26.86%	-27.16%
With	$N_A s_R c_R$	2	-20.7%	-26.57%	-26.91%	-27.42%
With	$N_A s_R c_R$	3	-21.18%	-27.17%	-27.33%	-27.02%
With	$N_A s_R c_R$	4	-21.14%	-26.91%	-26.82%	-26.92%
With	$N_A s_R c_R$	5	-20.82%	-26.69%	-26.74%	-27.02%
With	$N_A s_R c_R$	6	-20.69%	-26.87%	-26.81%	-26.82%
With	$N_A s_R c_R$	7	-20.47%	-26.66%	-26.79%	-27.23%
With	$N_A s_R c_R$	8	-20.88%	-26.56%	-26.75%	-27.22%
With	$N_A s_R c_R$	9	-20.96%	-26.66%	-26.82%	-27.48%
With	$N_A s_R c_R$	10	-20.92%	-26.77%	-26.7%	-26.88%
With	$N_A s_R c_R$	11	-20.79%	-26.6%	-26.78%	-26.59%
With	$N_A s_R c_R$	12	-21.01%	-26.59%	-26.8%	-26.6%
With	$N_A s_R c_R$	13	-20.55%	-26.51%	-26.72%	-26.75%
With	$N_A s_R c_R$	14	-20.78%	-26.62%	-26.33%	-26.79%
With	$N_A s_R c_R$	15	-20.88%	-26.8%	-26.33%	-26.87%
With	$N_A s_R c_R$	16	-21.11%	-26.69%	-26.68%	-26.69%
With	$N_A s_R c_R$	17	-20.64%	-26.66%	-26.55%	-26.6%
With	$N_A s_R c_R$	18	-20.54%	-26.83%	-27.23%	-27.15%
With	$N_A s_R c_R$	19	-20.97%	-26.52%	-27.41%	-27.4%
With	$N_A s_R c_R$	20	-21%	-26.8%	-27.41%	-28.09%
With	$N_A s_R c_R$	21	-20.55%	-26.95%	-27.67%	-28.47%
With	$N_A s_R c_R$	22	-21.03%	-27.05%	-28.02%	-29.51%
With	$N_A s_R c_R$	23	-21.03%	-27.36%	-28.5%	-31.31%
With	$N_A s_R c_R$	24	-21.2%	-27.56%	-29.7%	-33.91%
With	$N_A s_R c_R$	25	-20.97%	-28.46%	-32.09%	-38.72%
With	$s_R c_R$	1	-1.93%	-2.56%	-3.36%	-3.5%
With	$s_R c_R$	2	-2.03%	-2.25%	-3.07%	-3.82%
With	$s_R c_R$	3	-1.86%	-2.79%	-3.09%	-3.37%
With	$s_R c_R$	4	-1.85%	-2.89%	-3.39%	-3.23%
With	$s_R c_R$	5	-1.76%	-2.5%	-3.01%	-3.03%
With	$s_R c_R$	6	-1.77%	-2.67%	-3.14%	-3.11%
With	$s_R c_R$	7	-1.88%	-2.72%	-2.74%	-3.31%
With	$s_R c_R$	8	-1.9%	-2.49%	-3.32%	-3.4%
With	$s_R c_R$	9	-1.9%	-2.72%	-3.11%	-3.51%
With	$s_R c_R$	10	-1.68%	-2.93%	-3.01%	-3%
With	$s_R c_R$	11	-1.72%	-2.5%	-2.66%	-2.95%
With	$s_R c_R$	12	-1.79%	-2.37%	-2.35%	-2.87%
With	$s_R c_R$	13	-1.79%	-2.23%	-2.79%	-3.37%
With	$s_R c_R$	14	-1.6%	-2.52%	-2.61%	-3.29%
With	$s_R c_R$	15	-1.94%	-2.65%	-2.62%	-3.55%
With	$s_R c_R$	16	-1.59%	-2.42%	-3.18%	-3.74%
With	$s_R c_R$	17	-1.75%	-2.53%	-2.96%	-3.47%

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Table A.1 – continued from previous page

Annual Abundance Percent Bias						
Aux. Like.	**Model**	**Year**	**No Variation**	**Low Variation**	**Medium Variation**	**High Variation**
With	$s_R c_R$	18	-1.71%	-2.61%	-3.36%	-3.74%
With	$s_R c_R$	19	-1.66%	-2.72%	-3.73%	-3.7%
With	$s_R c_R$	20	-1.74%	-2.66%	-3.87%	-4.48%
With	$s_R c_R$	21	-1.9%	-2.51%	-3.62%	-4.04%
With	$s_R c_R$	22	-1.74%	-2.91%	-3.54%	-3.82%
With	$s_R c_R$	23	-1.82%	-2.71%	-3.23%	-4.01%
With	$s_R c_R$	24	-2.01%	-3.01%	-3.45%	-4.49%
With	$s_R c_R$	25	-2%	-2.95%	-3.33%	-4.4%

Table (A.2) contains the data used for plotting Figure (2.4), the median relative bias (MRB) and estimated asymptotic 95% confidence interval coverage for each model, for each year of data, for each level of simulated variation.

Table A.2: *95% confidence interval coverage and median relative bias in total annual abundance estimates. Results indicate nearest-nominal confidence interval coverage for conditional-likelihood/Horvitz-Thompson models with subnominal coverage for absolute-recruit abundance models. Stock-recruit models show low coverage for early years, supernominal coverage for later years of population reconstruction. Results based on* $n = 1000$ *replicates, with* $s = 0.84$, $c = -1.5$, $\gamma = 0.0$, *and total annual abundance* ≈ 4000. *"Aux. Like." = Auxiliary catch-effort likelihood used (With) or not used (Without).*

Table A.2 - Annual Abundance 95% CI Coverage and Median Relative Bias

			No Variation		Low Variation		Medium Variation		High Variation	
Aux. Like.	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
Without	$N_{SR,R}s_Fc_F$	1	70.40	-3.84	51.70	-5.48	41.80	-6.05	35.50	-3.62
Without	$N_{SR,R}s_Fc_F$	2	70.60	-3.81	55.40	-5.21	50.40	-5.43	47.40	-3.99
Without	$N_{SR,R}s_Fc_F$	3	72.10	-3.63	60.90	-5.33	57.00	-5.30	53.90	-3.48
Without	$N_{SR,R}s_Fc_F$	4	72.00	-3.77	65.50	-4.76	65.20	-5.11	67.10	-2.48
Without	$N_{SR,R}s_Fc_F$	5	74.60	-3.75	70.30	-4.85	74.70	-5.03	76.80	-1.42
Without	$N_{SR,R}s_Fc_F$	6	74.60	-3.76	74.40	-5.04	81.30	-5.14	84.50	-1.10
Without	$N_{SR,R}s_Fc_F$	7	75.90	-3.56	78.80	-5.16	87.20	-4.46	90.30	-2.52
Without	$N_{SR,R}s_Fc_F$	8	77.00	-3.38	83.40	-5.35	91.50	-4.45	92.30	-2.31
Without	$N_{SR,R}s_Fc_F$	9	78.50	-3.74	86.80	-5.17	94.60	-5.02	94.70	-1.85
Without	$N_{SR,R}s_Fc_F$	10	80.30	-3.66	89.50	-5.14	96.90	-4.58	96.20	-2.98
Without	$N_{SR,R}s_Fc_F$	11	80.30	-3.94	91.30	-5.25	98.50	-4.41	96.80	-2.58
Without	$N_{SR,R}s_Fc_F$	12	81.30	-3.73	95.30	-5.18	98.90	-4.85	97.20	-1.80
Without	$N_{SR,R}s_Fc_F$	13	83.00	-3.61	96.20	-5.49	99.50	-4.77	97.50	-2.57
Without	$N_{SR,R}s_Fc_F$	14	83.60	-3.76	97.10	-5.42	99.40	-4.79	97.60	-1.84
Without	$N_{SR,R}s_Fc_F$	15	85.60	-3.67	98.10	-4.96	99.40	-5.11	97.70	-3.05
Without	$N_{SR,R}s_Fc_F$	16	85.50	-3.66	99.00	-5.63	99.70	-5.12	97.30	-1.41
Without	$N_{SR,R}s_Fc_F$	17	86.90	-3.71	99.20	-5.37	99.80	-5.39	97.40	-2.38
Without	$N_{SR,R}s_Fc_F$	18	88.50	-3.62	99.70	-5.38	99.90	-5.07	97.20	-1.03
Without	$N_{SR,R}s_Fc_F$	19	89.30	-3.68	99.70	-4.75	99.90	-5.59	97.70	-1.32
Without	$N_{SR,R}s_Fc_F$	20	89.50	-3.50	99.70	-5.12	99.90	-5.42	97.90	-1.76
Without	$N_{SR,R}s_Fc_F$	21	90.40	-3.60	99.70	-5.00	99.90	-5.64	98.00	-1.75
Without	$N_{SR,R}s_Fc_F$	22	91.60	-3.56	99.90	-5.19	100.00	-5.54	98.60	-0.15
Without	$N_{SR,R}s_Fc_F$	23	91.80	-3.54	99.80	-5.32	100.00	-4.50	98.80	-0.70
Without	$N_{SR,R}s_Fc_F$	24	92.00	-3.78	99.90	-5.22	100.00	-4.89	98.70	-0.04
Without	$N_{SR,R}s_Fc_F$	25	93.20	-3.67	99.80	-4.77	100.00	-4.46	98.90	0.74
Without	$N_{SR,R}s_Fc_R$	1	70.10	-5.06	63.60	-9.03	62.20	-9.24	52.90	-6.02
Without	$N_{SR,R}s_Fc_R$	2	71.10	-5.14	68.20	-9.06	72.60	-9.30	65.90	-6.04
Without	$N_{SR,R}s_Fc_R$	3	72.60	-5.02	72.70	-8.92	83.00	-8.79	75.80	-6.20
Without	$N_{SR,R}s_Fc_R$	4	73.20	-5.17	77.50	-9.40	90.80	-8.97	81.40	-6.41
Without	$N_{SR,R}s_Fc_R$	5	74.80	-5.04	83.60	-9.12	95.70	-8.79	84.50	-6.23
Without	$N_{SR,R}s_Fc_R$	6	74.90	-4.91	87.70	-9.00	97.90	-8.92	86.40	-6.17
Without	$N_{SR,R}s_Fc_R$	7	75.60	-4.90	91.00	-9.18	98.40	-8.87	88.50	-6.55
Without	$N_{SR,R}s_Fc_R$	8	77.50	-4.51	94.00	-9.19	98.70	-8.98	90.30	-6.17
Without	$N_{SR,R}s_Fc_R$	9	78.80	-4.89	97.00	-9.18	99.00	-8.91	91.30	-6.92
Without	$N_{SR,R}s_Fc_R$	10	80.40	-5.00	98.40	-9.17	99.50	-9.15	92.40	-7.09
Without	$N_{SR,R}s_Fc_R$	11	81.40	-5.04	99.00	-9.15	99.50	-8.93	92.50	-6.63
Without	$N_{SR,R}s_Fc_R$	12	82.60	-5.03	99.70	-9.06	99.50	-8.83	92.60	-6.91
Without	$N_{SR,R}s_Fc_R$	13	84.00	-4.93	99.90	-8.79	99.60	-8.57	93.00	-6.62
Without	$N_{SR,R}s_Fc_R$	14	85.90	-4.82	99.90	-9.13	99.60	-8.62	94.20	-6.25

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Table A.2 – continued from previous page

			No Variation		Low Variation		Medium Variation		High Variation	
Aux. Like.	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
Without	$N_{SR,R}s_Fc_R$	15	87.60	-4.61	100.00	-9.16	99.60	-8.47	94.50	-6.45
Without	$N_{SR,R}s_Fc_R$	16	88.80	-4.74	100.00	-9.10	99.60	-8.87	94.50	-6.17
Without	$N_{SR,R}s_Fc_R$	17	89.80	-4.68	100.00	-9.11	99.60	-8.75	94.90	-6.26
Without	$N_{SR,R}s_Fc_R$	18	90.70	-4.69	100.00	-9.18	99.60	-9.41	95.10	-6.62
Without	$N_{SR,R}s_Fc_R$	19	91.70	-4.86	100.00	-9.08	99.60	-9.50	95.60	-6.56
Without	$N_{SR,R}s_Fc_R$	20	92.80	-4.78	100.00	-9.29	99.60	-9.86	96.00	-6.81
Without	$N_{SR,R}s_Fc_R$	21	93.70	-4.86	100.00	-9.21	99.60	-9.76	96.10	-6.83
Without	$N_{SR,R}s_Fc_R$	22	94.70	-4.78	100.00	-9.46	99.70	-10.15	96.10	-7.16
Without	$N_{SR,R}s_Fc_R$	23	94.90	-4.74	100.00	-9.59	99.70	-9.81	96.60	-8.18
Without	$N_{SR,R}s_Fc_R$	24	95.70	-5.04	100.00	-10.00	99.70	-10.68	96.50	-7.96
Without	$N_{SR,R}s_Fc_R$	25	96.10	-5.10	100.00	-10.04	99.70	-11.50	96.30	-8.53
Without	$N_{SR,R}s_Rc_R$	1	72.10	-4.88	64.40	-8.97	61.20	-10.26	58.30	-8.03
Without	$N_{SR,R}s_Rc_R$	2	73.00	-4.89	69.10	-9.05	71.60	-10.06	73.30	-7.93
Without	$N_{SR,R}s_Rc_R$	3	73.80	-4.60	73.90	-8.92	82.20	-10.04	85.00	-8.34
Without	$N_{SR,R}s_Rc_R$	4	74.40	-4.62	79.60	-9.43	90.00	-9.80	91.30	-7.85
Without	$N_{SR,R}s_Rc_R$	5	76.70	-4.75	85.80	-9.03	95.40	-9.42	93.10	-7.93
Without	$N_{SR,R}s_Rc_R$	6	77.80	-4.66	90.20	-8.66	97.80	-9.38	94.80	-7.82
Without	$N_{SR,R}s_Rc_R$	7	78.90	-4.56	93.60	-9.01	98.40	-9.50	96.00	-8.14
Without	$N_{SR,R}s_Rc_R$	8	80.60	-4.45	96.40	-9.18	99.00	-9.77	96.30	-7.75
Without	$N_{SR,R}s_Rc_R$	9	81.80	-4.64	98.30	-8.95	99.20	-9.22	96.40	-7.98
Without	$N_{SR,R}s_Rc_R$	10	83.00	-4.64	99.30	-8.98	99.60	-9.42	97.20	-8.31
Without	$N_{SR,R}s_Rc_R$	11	84.10	-4.88	99.60	-8.71	99.50	-8.95	97.40	-8.27
Without	$N_{SR,R}s_Rc_R$	12	86.40	-4.79	99.90	-8.85	99.70	-8.90	97.40	-7.95
Without	$N_{SR,R}s_Rc_R$	13	88.00	-4.66	99.90	-8.52	99.90	-8.82	98.10	-8.07
Without	$N_{SR,R}s_Rc_R$	14	89.30	-4.60	99.90	-8.99	99.80	-8.79	98.20	-7.61
Without	$N_{SR,R}s_Rc_R$	15	91.00	-4.28	99.80	-9.13	99.90	-9.04	98.40	-7.78
Without	$N_{SR,R}s_Rc_R$	16	92.00	-4.38	99.90	-8.96	99.90	-9.40	98.50	-7.73
Without	$N_{SR,R}s_Rc_R$	17	92.80	-4.38	99.90	-8.90	99.90	-9.30	98.60	-7.77
Without	$N_{SR,R}s_Rc_R$	18	93.80	-4.46	99.90	-9.01	99.90	-9.89	98.70	-8.24
Without	$N_{SR,R}s_Rc_R$	19	94.80	-4.37	99.80	-8.78	99.90	-9.94	98.90	-8.38
Without	$N_{SR,R}s_Rc_R$	20	95.70	-4.54	99.90	-9.06	99.90	-10.33	99.10	-8.68
Without	$N_{SR,R}s_Rc_R$	21	96.20	-4.34	99.90	-9.11	99.90	-10.07	99.00	-8.58
Without	$N_{SR,R}s_Rc_R$	22	97.10	-4.48	99.90	-9.10	99.90	-10.33	99.10	-8.75
Without	$N_{SR,R}s_Rc_R$	23	97.00	-4.34	99.90	-9.33	99.90	-9.87	99.10	-9.95
Without	$N_{SR,R}s_Rc_R$	24	97.00	-4.40	99.90	-9.49	99.90	-11.23	99.00	-10.18
Without	$N_{SR,R}s_Rc_R$	25	97.60	-4.54	99.90	-9.80	99.90	-11.50	98.50	-10.70
Without	$N_As_Fc_F$	1	23.10	-16.70	26.70	-18.84	20.00	-20.35	20.00	-20.97
Without	$N_As_Fc_F$	2	23.40	-16.89	26.60	-18.70	21.00	-19.91	20.70	-20.13
Without	$N_As_Fc_F$	3	23.70	-17.00	26.90	-18.61	20.40	-20.36	18.50	-20.01
Without	$N_As_Fc_F$	4	23.10	-16.55	26.80	-18.49	20.80	-20.13	19.20	-19.19
Without	$N_As_Fc_F$	5	22.90	-16.63	27.90	-18.45	20.20	-20.13	19.60	-19.13
Without	$N_As_Fc_F$	6	24.30	-16.36	27.80	-18.71	21.50	-20.03	19.10	-18.88
Without	$N_As_Fc_F$	7	24.90	-16.56	26.50	-18.43	21.00	-19.51	19.00	-19.59
Without	$N_As_Fc_F$	8	24.50	-16.50	27.20	-18.65	21.70	-19.59	19.90	-19.18
Without	$N_As_Fc_F$	9	24.10	-16.80	27.10	-18.36	21.00	-19.35	21.00	-20.12
Without	$N_As_Fc_F$	10	23.20	-17.07	26.50	-18.56	20.90	-19.02	20.80	-20.08
Without	$N_As_Fc_F$	11	24.00	-16.60	26.60	-18.70	20.60	-19.22	19.70	-20.21
Without	$N_As_Fc_F$	12	23.50	-16.86	27.80	-18.46	20.60	-19.15	20.00	-18.83
Without	$N_As_Fc_F$	13	23.40	-16.61	27.00	-18.47	21.50	-19.38	19.70	-19.23
Without	$N_As_Fc_F$	14	23.60	-16.52	26.80	-18.10	20.30	-19.56	20.70	-19.10
Without	$N_As_Fc_F$	15	23.70	-16.69	26.90	-18.23	19.90	-19.63	21.00	-19.04
Without	$N_As_Fc_F$	16	23.40	-16.79	27.50	-18.33	19.50	-20.20	21.50	-19.19
Without	$N_As_Fc_F$	17	23.20	-16.85	27.00	-18.49	19.50	-20.10	19.60	-19.68
Without	$N_As_Fc_F$	18	23.40	-16.79	27.50	-18.87	20.20	-20.25	19.10	-19.20
Without	$N_As_Fc_F$	19	23.70	-16.82	27.40	-18.27	19.00	-20.26	19.90	-19.22
Without	$N_As_Fc_F$	20	24.80	-16.47	27.40	-18.67	20.30	-19.49	20.20	-20.05
Without	$N_As_Fc_F$	21	24.00	-16.45	26.20	-18.52	19.90	-20.01	18.40	-19.14
Without	$N_As_Fc_F$	22	24.40	-16.81	27.50	-18.55	20.90	-20.13	20.00	-19.11
Without	$N_As_Fc_F$	23	24.60	-16.75	27.40	-18.34	21.40	-19.60	19.40	-19.14

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Table A.2 – continued from previous page

			No Variation		Low Variation		Medium Variation		High Variation	
Aux. Like.	Model	Year	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB
Without	$N_A s_F c_F$	24	25.30	-16.81	27.90	-18.54	22.00	-19.42	19.70	-18.55
Without	$N_A s_F c_F$	25	26.00	-16.95	28.40	-17.76	22.90	-18.54	23.50	-17.65
Without	$s_F c_F$	1	95.10	-0.40	64.20	1.15	42.30	0.64	38.50	1.54
Without	$s_F c_F$	2	95.10	-0.29	62.60	1.44	44.20	0.51	35.80	1.01
Without	$s_F c_F$	3	95.10	-0.12	62.70	0.86	43.50	-1.72	35.90	1.06
Without	$s_F c_F$	4	95.20	-0.32	63.00	0.94	44.30	-1.12	36.00	1.64
Without	$s_F c_F$	5	94.90	-0.16	62.50	0.39	40.50	-0.39	37.00	2.39
Without	$s_F c_F$	6	94.90	-0.15	62.30	0.60	43.70	-0.23	37.80	0.65
Without	$s_F c_F$	7	94.50	-0.09	63.90	0.97	43.10	-1.35	37.40	1.86
Without	$s_F c_F$	8	95.60	-0.24	64.40	1.37	43.60	-0.91	36.70	0.14
Without	$s_F c_F$	9	95.40	-0.38	64.50	1.06	44.90	-1.01	36.10	0.83
Without	$s_F c_F$	10	94.70	-0.19	63.60	0.61	44.40	-0.35	36.50	-0.74
Without	$s_F c_F$	11	94.50	-0.20	64.10	2.01	44.10	-0.35	35.70	5.74
Without	$s_F c_F$	12	94.40	0.01	63.70	0.47	44.70	0.97	40.70	2.90
Without	$s_F c_F$	13	94.40	-0.08	62.80	-0.11	46.30	-0.52	37.50	1.02
Without	$s_F c_F$	14	95.00	0.27	62.80	1.78	43.70	-0.90	35.90	1.96
Without	$s_F c_F$	15	95.40	-0.14	62.60	-0.04	46.00	-0.35	39.10	1.77
Without	$s_F c_F$	16	95.20	0.05	61.90	1.61	44.90	-0.36	39.30	1.15
Without	$s_F c_F$	17	95.80	-0.04	61.60	0.84	45.70	-0.50	36.60	2.36
Without	$s_F c_F$	18	94.90	-0.02	63.50	1.33	43.70	-0.48	38.40	0.93
Without	$s_F c_F$	19	94.00	0.06	61.60	0.53	46.20	0.28	35.80	0.46
Without	$s_F c_F$	20	95.00	-0.09	62.10	0.86	45.30	-1.21	35.20	0.69
Without	$s_F c_F$	21	94.40	-0.06	61.90	0.38	44.10	-1.01	37.60	2.10
Without	$s_F c_F$	22	94.50	-0.23	62.00	0.90	45.30	0.21	37.40	1.09
Without	$s_F c_F$	23	94.10	-0.29	62.90	0.21	45.70	-0.50	36.80	0.59
Without	$s_F c_F$	24	95.40	-0.09	62.60	0.33	43.70	-0.95	40.10	0.92
Without	$s_F c_F$	25	94.60	-0.24	60.70	0.97	44.20	-1.01	38.00	1.89
Without	$N_A s_F c_R$	1	21.70	-19.81	2.10	-26.64	1.30	-26.92	2.40	-27.21
Without	$N_A s_F c_R$	2	21.90	-19.89	1.90	-26.44	1.50	-26.96	4.00	-27.33
Without	$N_A s_F c_R$	3	22.20	-19.92	2.40	-26.93	2.10	-27.42	5.00	-27.10
Without	$N_A s_F c_R$	4	21.30	-19.59	2.30	-26.59	2.50	-26.83	7.60	-26.92
Without	$N_A s_F c_R$	5	22.20	-19.43	2.10	-26.48	2.70	-26.96	8.80	-26.82
Without	$N_A s_F c_R$	6	22.80	-19.42	2.40	-26.67	1.90	-26.87	9.20	-26.88
Without	$N_A s_F c_R$	7	23.50	-19.22	2.20	-26.53	2.20	-26.79	9.80	-27.04
Without	$N_A s_F c_R$	8	22.50	-19.74	2.40	-26.41	2.40	-26.72	10.00	-27.43
Without	$N_A s_F c_R$	9	22.50	-19.85	2.50	-26.41	2.70	-26.57	9.50	-27.45
Without	$N_A s_F c_R$	10	21.90	-19.61	2.10	-26.63	2.30	-26.85	9.70	-26.98
Without	$N_A s_F c_R$	11	22.80	-19.74	2.40	-26.40	3.00	-26.87	10.70	-26.41
Without	$N_A s_F c_R$	12	22.40	-20.00	2.50	-26.41	2.90	-26.74	12.60	-26.20
Without	$N_A s_F c_R$	13	21.80	-19.70	2.30	-26.22	3.00	-26.47	12.40	-26.33
Without	$N_A s_F c_R$	14	22.10	-19.70	2.20	-26.33	3.20	-26.31	13.70	-26.82
Without	$N_A s_F c_R$	15	22.20	-19.58	2.60	-26.49	3.00	-26.33	13.20	-26.76
Without	$N_A s_F c_R$	16	21.60	-19.63	2.80	-26.36	3.20	-26.59	12.60	-26.37
Without	$N_A s_F c_R$	17	21.60	-19.06	2.90	-26.38	3.40	-26.63	12.70	-26.41
Without	$N_A s_F c_R$	18	22.10	-19.16	2.80	-26.55	3.00	-26.92	11.70	-26.98
Without	$N_A s_F c_R$	19	22.50	-19.42	2.40	-26.43	3.10	-27.34	12.00	-27.34
Without	$N_A s_F c_R$	20	22.80	-19.59	2.50	-26.56	2.60	-27.25	11.00	-27.83
Without	$N_A s_F c_R$	21	22.50	-19.30	2.40	-26.75	2.40	-27.58	10.40	-28.27
Without	$N_A s_F c_R$	22	22.60	-19.44	3.00	-26.89	2.50	-28.05	11.40	-29.23
Without	$N_A s_F c_R$	23	23.20	-19.40	3.00	-26.95	3.40	-28.72	10.70	-30.88
Without	$N_A s_F c_R$	24	23.70	-19.70	3.70	-27.13	5.30	-29.53	14.10	-33.67
Without	$N_A s_F c_R$	25	23.80	-19.53	4.80	-27.94	10.60	-31.74	18.70	-37.56
Without	$s_F c_R$	1	95.40	-0.46	96.80	0.84	98.20	0.53	98.00	0.36
Without	$s_F c_R$	2	96.00	-0.24	97.00	1.03	98.40	0.33	98.50	-0.20
Without	$s_F c_R$	3	95.80	-0.10	96.50	0.99	98.30	0.00	98.20	0.37
Without	$s_F c_R$	4	95.50	-0.42	96.70	0.57	98.60	0.07	98.40	0.09
Without	$s_F c_R$	5	95.30	-0.16	96.60	0.75	98.70	0.22	98.30	0.83
Without	$s_F c_R$	6	95.60	-0.06	96.80	0.63	98.70	-0.05	98.60	0.78
Without	$s_F c_R$	7	95.20	-0.02	97.20	0.60	98.40	0.44	99.00	0.46

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Table A.2 – continued from previous page

			No Variation		Low Variation		Medium Variation		High Variation	
Aux. Like.	Model	Year	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB
Without	$s_F c_R$	8	96.10	-0.21	97.60	0.52	98.70	0.56	98.40	0.46
Without	$s_F c_R$	9	96.20	-0.25	97.80	0.66	98.70	-0.11	98.50	-0.01
Without	$s_F c_R$	10	95.30	-0.18	97.70	0.41	98.80	-0.32	98.70	0.27
Without	$s_F c_R$	11	94.90	-0.23	97.40	0.77	98.60	0.16	98.80	0.41
Without	$s_F c_R$	12	95.70	-0.09	98.00	0.72	98.50	0.41	98.90	0.26
Without	$s_F c_R$	13	95.50	-0.11	97.90	0.50	98.90	0.19	98.70	-0.08
Without	$s_F c_R$	14	95.80	0.24	97.00	0.44	98.50	0.37	97.70	0.23
Without	$s_F c_R$	15	96.10	-0.08	96.90	0.32	98.30	0.33	98.10	-0.53
Without	$s_F c_R$	16	96.10	0.08	97.00	1.01	97.90	0.02	98.50	-0.37
Without	$s_F c_R$	17	96.60	0.03	97.10	0.70	98.00	0.04	98.30	-0.31
Without	$s_F c_R$	18	95.70	0.09	97.30	0.91	97.90	0.05	97.90	0.58
Without	$s_F c_R$	19	94.60	0.06	97.30	0.88	98.10	-0.34	97.80	0.49
Without	$s_F c_R$	20	95.70	-0.06	97.00	0.74	97.90	-0.20	97.00	-0.07
Without	$s_F c_R$	21	95.10	-0.03	96.60	0.96	97.80	-0.40	97.20	0.21
Without	$s_F c_R$	22	95.60	-0.15	96.70	0.77	97.10	0.02	96.00	0.74
Without	$s_F c_R$	23	94.50	-0.33	96.50	0.68	96.60	0.36	95.90	0.33
Without	$s_F c_R$	24	95.60	-0.07	95.90	0.56	95.20	0.01	93.50	-0.08
Without	$s_F c_R$	25	95.30	-0.26	94.10	0.55	93.30	0.96	91.60	1.06
Without	$N_A s_R c_R$	1	21.50	-19.93	2.00	-26.67	1.30	-26.97	2.20	-27.46
Without	$N_A s_R c_R$	2	22.00	-19.94	1.90	-26.41	1.60	-27.09	3.80	-27.65
Without	$N_A s_R c_R$	3	22.20	-20.07	2.30	-26.91	2.10	-27.51	4.80	-27.38
Without	$N_A s_R c_R$	4	21.40	-19.67	2.20	-26.61	2.30	-27.02	7.40	-27.19
Without	$N_A s_R c_R$	5	22.20	-19.49	2.00	-26.39	2.50	-27.14	9.00	-27.09
Without	$N_A s_R c_R$	6	22.60	-19.38	2.30	-26.64	1.80	-27.00	9.00	-27.06
Without	$N_A s_R c_R$	7	23.50	-19.36	2.10	-26.51	2.10	-26.80	9.80	-27.16
Without	$N_A s_R c_R$	8	22.70	-19.76	2.30	-26.38	2.30	-26.73	10.40	-27.44
Without	$N_A s_R c_R$	9	22.70	-20.00	2.40	-26.37	2.70	-26.46	9.80	-27.33
Without	$N_A s_R c_R$	10	22.00	-19.50	2.00	-26.52	2.50	-26.70	10.70	-26.70
Without	$N_A s_R c_R$	11	22.60	-19.62	2.30	-26.24	3.80	-26.68	13.90	-25.72
Without	$N_A s_R c_R$	12	22.50	-20.00	2.60	-26.37	4.40	-26.61	16.20	-25.73
Without	$N_A s_R c_R$	13	22.20	-19.65	2.50	-26.19	5.20	-26.30	16.60	-26.10
Without	$N_A s_R c_R$	14	22.10	-19.67	2.60	-26.26	4.90	-26.31	17.10	-26.72
Without	$N_A s_R c_R$	15	22.70	-19.60	3.10	-26.49	4.50	-26.61	15.90	-26.99
Without	$N_A s_R c_R$	16	21.70	-19.77	3.00	-26.38	3.90	-26.92	14.70	-26.63
Without	$N_A s_R c_R$	17	21.80	-19.06	2.80	-26.33	3.50	-26.75	13.60	-26.65
Without	$N_A s_R c_R$	18	22.30	-19.16	2.80	-26.51	2.80	-27.06	12.20	-27.14
Without	$N_A s_R c_R$	19	22.60	-19.42	2.30	-26.36	2.90	-27.47	12.90	-27.55
Without	$N_A s_R c_R$	20	23.00	-19.56	2.40	-26.54	2.60	-27.26	11.20	-27.98
Without	$N_A s_R c_R$	21	22.50	-19.34	2.30	-26.68	2.40	-27.66	10.80	-28.29
Without	$N_A s_R c_R$	22	22.80	-19.44	3.00	-26.78	2.50	-28.10	11.50	-29.17
Without	$N_A s_R c_R$	23	23.40	-19.46	2.90	-26.92	3.30	-28.67	10.80	-30.88
Without	$N_A s_R c_R$	24	23.90	-19.90	3.50	-27.12	5.60	-29.56	14.50	-33.42
Without	$N_A s_R c_R$	25	23.90	-19.46	4.70	-27.90	11.00	-31.45	18.40	-37.41
Without	$s_R c_R$	1	95.70	-0.45	96.00	0.85	97.10	-0.25	97.40	-0.72
Without	$s_R c_R$	2	96.10	-0.33	96.20	1.39	98.10	-0.04	98.00	-1.06
Without	$s_R c_R$	3	95.80	-0.10	96.10	0.98	97.70	-0.35	97.80	-0.76
Without	$s_R c_R$	4	95.60	-0.40	95.90	0.59	97.80	-0.23	97.80	-0.54
Without	$s_R c_R$	5	95.50	-0.10	96.30	0.93	98.00	0.06	98.00	0.04
Without	$s_R c_R$	6	95.70	0.00	96.00	0.56	98.20	-0.23	98.80	-0.02
Without	$s_R c_R$	7	95.20	-0.03	96.60	0.63	97.90	0.25	98.90	0.07
Without	$s_R c_R$	8	96.10	-0.21	97.50	0.54	98.20	0.47	98.30	-0.11
Without	$s_R c_R$	9	96.00	-0.22	97.50	0.66	98.30	-0.12	98.90	-0.40
Without	$s_R c_R$	10	95.40	-0.12	97.40	0.49	98.50	-0.02	99.20	0.17
Without	$s_R c_R$	11	94.90	-0.20	97.30	1.02	98.50	0.21	99.20	-0.08
Without	$s_R c_R$	12	95.90	-0.09	97.80	0.85	98.50	0.58	98.80	0.14
Without	$s_R c_R$	13	95.70	-0.16	97.50	0.62	98.70	0.08	98.50	0.04
Without	$s_R c_R$	14	95.80	0.26	96.30	0.83	97.80	0.35	97.50	0.00
Without	$s_R c_R$	15	96.20	0.00	96.40	0.65	97.50	0.20	97.10	-0.54
Without	$s_R c_R$	16	96.20	0.10	96.40	1.24	97.20	-0.11	97.70	-0.81

Continued on next page

Table A.2 – continued from previous page

			No Variation		Low Variation		Medium Variation		High Variation	
Aux. Like.	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
Without	$s_R c_R$	17	96.50	0.07	96.50	0.80	97.10	-0.05	97.80	-0.49
Without	$s_R c_R$	18	95.60	-0.01	96.80	1.04	97.20	-0.15	97.30	-0.50
Without	$s_R c_R$	19	94.80	0.13	97.00	1.07	97.30	-0.56	97.40	-0.30
Without	$s_R c_R$	20	95.80	-0.06	96.80	0.69	97.10	-0.65	97.00	-0.89
Without	$s_R c_R$	21	95.20	-0.12	96.10	1.20	97.40	-0.99	97.30	-0.30
Without	$s_R c_R$	22	95.60	-0.17	96.40	0.71	97.00	-0.29	96.80	-0.00
Without	$s_R c_R$	23	94.70	-0.33	96.10	0.80	96.60	0.18	96.60	-0.16
Without	$s_R c_R$	24	95.60	-0.14	95.70	0.85	94.70	0.04	93.90	-0.06
Without	$s_R c_R$	25	95.50	-0.29	93.70	0.61	93.00	0.97	92.90	1.96
With	$N_A s_F c_F$	1	38.60	-11.01	38.00	-1.67	23.50	31.76	15.70	65.36
With	$N_A s_F c_F$	2	38.90	-11.04	37.20	-1.11	23.40	32.03	14.60	63.38
With	$N_A s_F c_F$	3	38.90	-11.11	37.20	-0.81	23.30	32.38	14.20	63.32
With	$N_A s_F c_F$	4	38.30	-10.88	37.30	-1.04	21.90	32.35	13.80	64.86
With	$N_A s_F c_F$	5	38.60	-10.85	36.40	-1.43	21.50	31.13	13.20	66.09
With	$N_A s_F c_F$	6	39.10	-10.77	35.60	-1.48	20.40	31.65	14.10	65.69
With	$N_A s_F c_F$	7	39.40	-10.85	36.40	-1.40	21.60	30.85	14.20	66.61
With	$N_A s_F c_F$	8	39.90	-10.65	37.90	-0.93	21.60	32.10	13.90	65.03
With	$N_A s_F c_F$	9	38.20	-10.74	38.00	-1.01	21.60	32.38	14.70	63.04
With	$N_A s_F c_F$	10	38.70	-11.03	37.60	-1.43	22.50	32.60	15.10	60.09
With	$N_A s_F c_F$	11	38.70	-10.81	37.90	-0.95	22.00	31.90	14.30	63.58
With	$N_A s_F c_F$	12	39.50	-10.83	37.20	-1.16	22.00	32.46	15.50	63.31
With	$N_A s_F c_F$	13	38.50	-10.64	36.90	-1.90	22.60	31.61	15.10	62.88
With	$N_A s_F c_F$	14	39.60	-11.02	37.30	-1.32	22.00	32.15	14.10	64.83
With	$N_A s_F c_F$	15	39.00	-11.09	36.40	-1.17	21.50	31.70	14.80	65.21
With	$N_A s_F c_F$	16	38.60	-11.07	36.20	-2.38	22.40	31.49	14.60	64.74
With	$N_A s_F c_F$	17	38.80	-10.95	36.40	-2.02	22.10	32.41	14.90	68.69
With	$N_A s_F c_F$	18	39.00	-10.83	36.60	-1.62	22.20	32.79	15.40	65.81
With	$N_A s_F c_F$	19	38.50	-10.76	36.20	-1.40	23.60	31.48	15.10	68.60
With	$N_A s_F c_F$	20	39.10	-10.70	36.40	-0.53	23.80	31.81	14.30	69.93
With	$N_A s_F c_F$	21	39.10	-10.66	36.30	-0.76	22.30	30.02	15.10	66.59
With	$N_A s_F c_F$	22	38.70	-10.64	36.40	-0.84	22.20	30.60	16.10	65.62
With	$N_A s_F c_F$	23	39.20	-10.95	36.40	-1.43	21.40	31.13	15.70	65.12
With	$N_A s_F c_F$	24	39.00	-10.85	36.00	-1.13	22.80	31.40	16.20	64.61
With	$N_A s_F c_F$	25	40.70	-10.95	38.20	-0.79	22.40	32.43	16.60	66.74
With	$s_F c_F$	1	92.60	-1.70	61.20	-0.67	43.30	-0.89	37.60	-1.01
With	$s_F c_F$	2	92.40	-1.87	61.80	-0.39	42.60	0.27	33.50	1.09
With	$s_F c_F$	3	92.50	-1.68	60.40	-0.92	40.20	-2.17	33.20	1.97
With	$s_F c_F$	4	93.50	-1.75	60.40	-0.89	42.00	-1.90	35.60	1.30
With	$s_F c_F$	5	93.30	-1.53	60.10	-0.97	39.40	-0.80	37.40	2.19
With	$s_F c_F$	6	92.50	-1.68	60.10	-1.43	41.20	-1.88	34.90	0.85
With	$s_F c_F$	7	92.80	-1.74	62.10	-1.20	39.90	-2.02	34.30	2.87
With	$s_F c_F$	8	93.30	-1.79	61.80	-0.48	40.80	-2.07	34.60	1.07
With	$s_F c_F$	9	92.60	-1.84	60.30	-1.21	43.40	-1.94	35.30	-1.87
With	$s_F c_F$	10	93.30	-1.71	62.00	-1.66	43.30	-0.59	37.80	-0.77
With	$s_F c_F$	11	93.00	-1.56	62.10	0.06	42.70	-1.13	35.90	2.43
With	$s_F c_F$	12	93.00	-1.64	61.00	-1.38	42.30	0.41	38.20	0.15
With	$s_F c_F$	13	92.00	-1.75	59.10	-1.58	44.80	-1.32	37.70	0.68
With	$s_F c_F$	14	92.70	-1.46	60.90	-0.55	41.00	-2.03	33.70	-1.26
With	$s_F c_F$	15	92.50	-1.83	60.50	-1.99	43.30	-0.79	36.60	0.76
With	$s_F c_F$	16	93.50	-1.40	58.80	-0.58	44.00	-1.77	37.40	0.91
With	$s_F c_F$	17	93.50	-1.52	60.50	-0.41	43.30	-0.13	33.70	1.01
With	$s_F c_F$	18	93.40	-1.71	62.50	-1.04	41.70	-1.49	35.70	2.73
With	$s_F c_F$	19	93.40	-1.56	58.80	-1.92	43.60	-1.43	34.00	0.14
With	$s_F c_F$	20	93.20	-1.72	61.80	-0.46	41.80	-2.07	34.80	0.30
With	$s_F c_F$	21	93.30	-1.71	60.20	-1.47	43.10	-1.36	34.30	3.46
With	$s_F c_F$	22	92.50	-1.63	62.70	-1.01	43.80	-1.57	34.80	1.71
With	$s_F c_F$	23	92.30	-1.77	60.60	-1.83	43.00	-0.86	36.70	0.48
With	$s_F c_F$	24	93.60	-1.71	60.00	-1.72	42.20	-1.04	35.40	-0.29
With	$s_F c_F$	25	93.30	-1.91	58.70	-1.05	41.90	-2.77	34.70	1.04

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Table A.2 – continued from previous page

			No Variation		Low Variation		Medium Variation		High Variation	
Aux. Like.	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
With	$N_A s_F c_R$	1	21.50	-21.15	0.90	-26.94	0.80	-26.86	1.10	-27.09
With	$N_A s_F c_R$	2	21.30	-20.51	1.00	-26.54	0.80	-26.84	2.10	-27.24
With	$N_A s_F c_R$	3	21.70	-20.89	1.40	-27.17	1.20	-27.17	3.70	-26.86
With	$N_A s_F c_R$	4	21.50	-20.81	1.60	-26.92	1.70	-26.73	5.70	-26.83
With	$N_A s_F c_R$	5	21.50	-20.77	1.50	-26.69	1.70	-26.66	7.70	-26.87
With	$N_A s_F c_R$	6	21.30	-20.65	1.40	-26.89	1.50	-26.82	7.40	-26.77
With	$N_A s_F c_R$	7	22.20	-20.29	1.30	-26.67	1.60	-26.82	8.40	-26.96
With	$N_A s_F c_R$	8	22.00	-20.74	1.50	-26.60	2.10	-26.66	8.60	-27.19
With	$N_A s_F c_R$	9	22.00	-20.91	1.60	-26.72	1.70	-26.83	7.60	-27.51
With	$N_A s_F c_R$	10	21.90	-20.76	1.30	-26.84	1.70	-26.72	6.90	-27.05
With	$N_A s_F c_R$	11	22.20	-20.67	1.30	-26.72	2.10	-26.83	7.50	-26.97
With	$N_A s_F c_R$	12	22.10	-20.71	1.20	-26.64	2.00	-26.84	8.90	-26.91
With	$N_A s_F c_R$	13	21.30	-20.53	0.70	-26.54	2.20	-26.77	9.60	-27.06
With	$N_A s_F c_R$	14	21.50	-20.73	0.90	-26.59	2.20	-26.42	10.70	-26.88
With	$N_A s_F c_R$	15	21.80	-20.77	1.00	-26.76	2.50	-26.29	9.80	-26.69
With	$N_A s_F c_R$	16	22.20	-20.99	1.00	-26.73	2.30	-26.60	10.10	-26.56
With	$N_A s_F c_R$	17	21.30	-20.37	1.60	-26.71	2.10	-26.54	10.20	-26.42
With	$N_A s_F c_R$	18	22.60	-20.46	1.50	-26.82	2.00	-27.16	10.10	-26.97
With	$N_A s_F c_R$	19	22.10	-20.79	1.70	-26.57	1.80	-27.41	9.50	-27.36
With	$N_A s_F c_R$	20	22.50	-20.88	1.60	-26.82	2.00	-27.42	9.80	-28.03
With	$N_A s_F c_R$	21	22.50	-20.26	1.30	-27.01	2.00	-27.62	9.10	-28.50
With	$N_A s_F c_R$	22	22.30	-20.98	1.60	-27.05	1.90	-27.98	9.90	-29.52
With	$N_A s_F c_R$	23	22.40	-20.97	1.70	-27.40	2.90	-28.50	10.20	-31.28
With	$N_A s_F c_R$	24	22.00	-21.15	2.10	-27.58	4.80	-29.61	12.90	-33.96
With	$N_A s_F c_R$	25	23.00	-20.84	3.50	-28.46	8.40	-32.20	16.80	-38.94
With	$s_F c_R$	1	93.70	-1.90	94.50	-2.08	97.10	-2.69	97.70	-1.84
With	$s_F c_R$	2	93.10	-1.94	94.50	-2.20	97.70	-2.69	98.70	-2.20
With	$s_F c_R$	3	92.80	-1.80	93.80	-2.42	96.80	-2.67	98.20	-2.17
With	$s_F c_R$	4	93.90	-1.81	94.30	-2.75	96.80	-2.74	98.10	-1.87
With	$s_F c_R$	5	93.90	-1.70	94.00	-2.29	97.20	-2.34	98.10	-2.05
With	$s_F c_R$	6	93.30	-1.73	94.40	-2.43	97.30	-2.73	98.70	-1.96
With	$s_F c_R$	7	93.70	-1.81	94.50	-2.55	96.40	-2.56	98.80	-1.97
With	$s_F c_R$	8	93.90	-1.82	94.50	-2.40	97.20	-2.84	99.20	-2.51
With	$s_F c_R$	9	93.50	-1.95	95.10	-2.62	97.50	-2.89	99.40	-2.65
With	$s_F c_R$	10	94.00	-1.71	95.30	-2.88	97.40	-3.07	98.90	-2.40
With	$s_F c_R$	11	93.90	-1.63	95.10	-2.50	97.50	-2.83	98.80	-2.64
With	$s_F c_R$	12	93.80	-1.72	95.40	-2.20	97.50	-2.33	99.20	-2.62
With	$s_F c_R$	13	93.00	-1.79	94.90	-2.27	97.20	-2.70	99.20	-3.22
With	$s_F c_R$	14	92.90	-1.61	95.10	-2.50	96.80	-2.64	98.70	-2.79
With	$s_F c_R$	15	93.90	-1.92	95.20	-2.54	96.70	-2.44	97.70	-2.76
With	$s_F c_R$	16	94.30	-1.54	94.40	-2.53	97.20	-2.76	98.40	-2.66
With	$s_F c_R$	17	94.00	-1.78	94.70	-2.38	96.80	-2.69	97.80	-2.50
With	$s_F c_R$	18	93.50	-1.63	94.20	-2.46	96.50	-3.11	97.90	-2.66
With	$s_F c_R$	19	93.40	-1.66	94.10	-2.66	96.50	-3.42	97.20	-2.75
With	$s_F c_R$	20	93.90	-1.72	93.70	-2.60	96.60	-3.21	97.10	-3.14
With	$s_F c_R$	21	93.90	-1.81	92.90	-2.32	96.00	-3.13	95.60	-2.82
With	$s_F c_R$	22	93.50	-1.67	93.40	-2.73	95.40	-2.99	94.50	-3.29
With	$s_F c_R$	23	93.40	-1.81	93.60	-2.59	93.60	-3.11	92.20	-3.63
With	$s_F c_R$	24	94.20	-1.91	92.10	-2.91	92.20	-3.14	90.90	-4.21
With	$s_F c_R$	25	93.70	-1.96	91.00	-2.94	88.70	-3.41	88.90	-4.44
With	$N_A s_R c_R$	1	21.70	-21.31	0.70	-27.00	0.90	-26.86	1.10	-27.16
With	$N_A s_R c_R$	2	21.60	-20.70	0.90	-26.57	0.90	-26.91	2.00	-27.42
With	$N_A s_R c_R$	3	21.70	-21.18	1.10	-27.17	1.30	-27.33	3.90	-27.02
With	$N_A s_R c_R$	4	21.60	-21.14	1.30	-26.91	1.60	-26.82	5.60	-26.92
With	$N_A s_R c_R$	5	21.70	-20.82	1.10	-26.69	1.70	-26.74	7.80	-27.02
With	$N_A s_R c_R$	6	21.50	-20.69	1.20	-26.87	1.60	-26.81	7.70	-26.82
With	$N_A s_R c_R$	7	22.30	-20.47	1.10	-26.66	1.60	-26.79	8.60	-27.23
With	$N_A s_R c_R$	8	22.20	-20.88	1.20	-26.56	2.20	-26.75	8.80	-27.22
With	$N_A s_R c_R$	9	22.10	-20.96	1.30	-26.66	1.90	-26.82	8.00	-27.48

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Table A.2 – continued from previous page

			No Variation		Low Variation		Medium Variation		High Variation	
Aux. Like.	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
With	$N_A s_R c_R$	10	22.00	-20.92	1.20	-26.77	2.00	-26.70	7.70	-26.88
With	$N_A s_R c_R$	11	22.30	-20.79	1.10	-26.60	2.80	-26.78	9.60	-26.59
With	$N_A s_R c_R$	12	22.10	-21.01	1.30	-26.59	3.00	-26.80	11.00	-26.60
With	$N_A s_R c_R$	13	21.50	-20.55	0.90	-26.51	3.60	-26.72	12.00	-26.75
With	$N_A s_R c_R$	14	21.70	-20.78	1.30	-26.62	3.20	-26.33	13.30	-26.79
With	$N_A s_R c_R$	15	22.20	-20.88	1.10	-26.80	3.30	-26.33	12.20	-26.87
With	$N_A s_R c_R$	16	22.40	-21.11	0.80	-26.69	2.70	-26.68	11.70	-26.69
With	$N_A s_R c_R$	17	21.30	-20.64	1.40	-26.66	2.40	-26.55	11.20	-26.60
With	$N_A s_R c_R$	18	22.60	-20.54	1.20	-26.83	2.20	-27.23	10.90	-27.15
With	$N_A s_R c_R$	19	22.40	-20.97	1.50	-26.52	2.10	-27.41	10.30	-27.40
With	$N_A s_R c_R$	20	22.80	-21.00	1.40	-26.80	2.10	-27.41	10.40	-28.09
With	$N_A s_R c_R$	21	22.80	-20.55	1.00	-26.95	2.20	-27.67	9.50	-28.47
With	$N_A s_R c_R$	22	22.40	-21.03	1.30	-27.05	2.00	-28.02	10.40	-29.51
With	$N_A s_R c_R$	23	22.20	-21.03	1.40	-27.36	3.00	-28.50	10.80	-31.31
With	$N_A s_R c_R$	24	22.10	-21.20	1.90	-27.56	4.90	-29.70	13.60	-33.91
With	$N_A s_R c_R$	25	23.30	-20.97	3.30	-28.46	8.70	-32.09	17.20	-38.72
With	$s_R c_R$	1	93.80	-1.93	93.40	-2.56	94.90	-3.36	95.80	-3.50
With	$s_R c_R$	2	93.20	-2.03	94.00	-2.25	95.40	-3.07	97.10	-3.82
With	$s_R c_R$	3	93.20	-1.86	93.20	-2.79	95.30	-3.09	96.60	-3.37
With	$s_R c_R$	4	94.00	-1.85	93.10	-2.89	95.10	-3.39	96.70	-3.23
With	$s_R c_R$	5	94.00	-1.76	92.60	-2.50	95.10	-3.01	97.30	-3.03
With	$s_R c_R$	6	93.20	-1.77	92.80	-2.67	95.20	-3.14	97.60	-3.11
With	$s_R c_R$	7	93.30	-1.88	92.80	-2.72	95.00	-2.74	97.40	-3.31
With	$s_R c_R$	8	93.70	-1.90	93.70	-2.49	96.40	-3.32	98.00	-3.40
With	$s_R c_R$	9	93.60	-1.90	94.60	-2.72	96.90	-3.11	98.50	-3.51
With	$s_R c_R$	10	94.10	-1.68	94.80	-2.93	97.10	-3.01	98.40	-3.00
With	$s_R c_R$	11	94.00	-1.72	94.60	-2.50	97.70	-2.66	98.30	-2.95
With	$s_R c_R$	12	93.70	-1.79	95.30	-2.37	97.50	-2.35	98.60	-2.87
With	$s_R c_R$	13	93.10	-1.79	94.70	-2.23	96.80	-2.79	98.80	-3.37
With	$s_R c_R$	14	93.00	-1.60	94.20	-2.52	95.50	-2.61	97.50	-3.29
With	$s_R c_R$	15	93.70	-1.94	93.80	-2.65	94.90	-2.62	96.50	-3.55
With	$s_R c_R$	16	94.20	-1.59	93.10	-2.42	95.20	-3.18	96.30	-3.74
With	$s_R c_R$	17	93.90	-1.75	93.50	-2.53	95.10	-2.96	96.40	-3.47
With	$s_R c_R$	18	93.60	-1.71	93.80	-2.61	94.10	-3.36	96.40	-3.74
With	$s_R c_R$	19	93.40	-1.66	93.20	-2.72	94.10	-3.73	95.90	-3.70
With	$s_R c_R$	20	93.80	-1.74	92.90	-2.66	95.00	-3.87	95.90	-4.48
With	$s_R c_R$	21	93.70	-1.90	91.70	-2.51	94.60	-3.62	94.80	-4.04
With	$s_R c_R$	22	93.30	-1.74	92.90	-2.91	93.90	-3.54	93.90	-3.82
With	$s_R c_R$	23	93.20	-1.82	92.60	-2.71	92.60	-3.23	91.50	-4.01
With	$s_R c_R$	24	94.10	-2.01	91.10	-3.01	91.50	-3.45	90.60	-4.49
With	$s_R c_R$	25	93.70	-2.00	90.60	-2.95	88.60	-3.33	89.10	-4.40

Table (A.3) contains the data used for plotting Figure (2.6), the median relative bias (MRB) in estimation of annual abundance for each model, for each year of data from robustness simulations.

Table A.3: *Median relative bias in total annual abundance estimates from robustness simulations. Results indicate negligible bias for conditional-likelihood/Horvitz-Thompson models, low consistent negative bias for stock-recruit models, and large negative bias for mixed-effects versions of the absolute-recruit abundance models. The fixed-effects absolute-recruit abundance model* $N_{As_Fc_F}$ *shows low bias. Results based on* $n = 1000$ *replicates. "Aux. Like." = Auxiliary catch-effort likelihood used (With) or not used (Without).*

Table A.3
Annual Abundance Percent Bias

Aux. Like.	Model	Year	S Increasing	S Decreasing	Periodic Recruitment
Without	N_{SR,Rs_Fc_F}	1	-6.43%	-4.78%	-5.44%
Without	N_{SR,Rs_Fc_F}	2	-5.92%	-5.36%	-5.6%
Without	N_{SR,Rs_Fc_F}	3	-5.47%	-5.48%	-5.48%
Without	N_{SR,Rs_Fc_F}	4	-5.28%	-5.04%	-5.49%
Without	N_{SR,Rs_Fc_F}	5	-5.24%	-5.57%	-5.12%
Without	N_{SR,Rs_Fc_F}	6	-5.04%	-5.11%	-5.26%
Without	N_{SR,Rs_Fc_F}	7	-5.05%	-4.81%	-5.36%
Without	N_{SR,Rs_Fc_F}	8	-5.43%	-4.75%	-5.61%
Without	N_{SR,Rs_Fc_F}	9	-4.79%	-4.71%	-5.35%
Without	N_{SR,Rs_Fc_F}	10	-5.41%	-5.16%	-5.16%
Without	N_{SR,Rs_Fc_F}	11	-5.02%	-4.85%	-5.05%
Without	N_{SR,Rs_Fc_F}	12	-5.5%	-4.87%	-5.03%
Without	N_{SR,Rs_Fc_F}	13	-5.6%	-4.91%	-5.3%
Without	N_{SR,Rs_Fc_F}	14	-5.06%	-5.47%	-5.15%
Without	N_{SR,Rs_Fc_F}	15	-5.01%	-4.96%	-5.12%
Without	N_{SR,Rs_Fc_F}	16	-4.42%	-4.73%	-5.02%
Without	N_{SR,Rs_Fc_F}	17	-4.65%	-4.81%	-5.44%
Without	N_{SR,Rs_Fc_F}	18	-4.92%	-5.43%	-5.05%
Without	N_{SR,Rs_Fc_F}	19	-4.94%	-5.33%	-5.39%
Without	N_{SR,Rs_Fc_F}	20	-4.62%	-5.02%	-5.13%
Without	N_{SR,Rs_Fc_F}	21	-4.98%	-4.71%	-5.28%
Without	N_{SR,Rs_Fc_F}	22	-4.96%	-4.47%	-5.34%
Without	N_{SR,Rs_Fc_F}	23	-5.57%	-4.35%	-5.41%
Without	N_{SR,Rs_Fc_F}	24	-6.17%	-4.56%	-5.08%
Without	N_{SR,Rs_Fc_F}	25	-6.01%	-4.01%	-4.48%
Without	N_{SR,Rs_Fc_R}	1	-10.74%	-7.93%	-8.68%
Without	N_{SR,Rs_Fc_R}	2	-10.59%	-8.44%	-8.8%
Without	N_{SR,Rs_Fc_R}	3	-10.63%	-8.05%	-8.91%
Without	N_{SR,Rs_Fc_R}	4	-10.44%	-8.38%	-8.76%
Without	N_{SR,Rs_Fc_R}	5	-10.08%	-8.18%	-8.76%
Without	N_{SR,Rs_Fc_R}	6	-10.1%	-8.31%	-8.76%
Without	N_{SR,Rs_Fc_R}	7	-9.98%	-8.3%	-8.82%
Without	N_{SR,Rs_Fc_R}	8	-9.75%	-8.5%	-8.83%
Without	N_{SR,Rs_Fc_R}	9	-9.84%	-8.52%	-8.78%
Without	N_{SR,Rs_Fc_R}	10	-9.73%	-8.96%	-8.56%
Without	N_{SR,Rs_Fc_R}	11	-9.26%	-8.81%	-8.8%
Without	N_{SR,Rs_Fc_R}	12	-9.49%	-8.67%	-9%
Without	N_{SR,Rs_Fc_R}	13	-9.48%	-9.14%	-8.9%
Without	N_{SR,Rs_Fc_R}	14	-9.32%	-8.81%	-8.7%
Without	N_{SR,Rs_Fc_R}	15	-9.25%	-8.8%	-8.86%
Without	N_{SR,Rs_Fc_R}	16	-9.59%	-9.33%	-9.09%
Without	N_{SR,Rs_Fc_R}	17	-9.17%	-9.34%	-8.92%

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Table A.3 – continued from previous page

Annual Abundance Percent Bias					
Aux. Like.	**Model**	**Year**	**S Increasing**	**S Decreasing**	**Periodic Recruitment**
Without	$N_{SR,R}s_Fc_R$	18	-9%	-9.08%	-8.7%
Without	$N_{SR,R}s_Fc_R$	19	-9.3%	-9.12%	-8.96%
Without	$N_{SR,R}s_Fc_R$	20	-9.16%	-9.36%	-9.05%
Without	$N_{SR,R}s_Fc_R$	21	-9.23%	-9.25%	-8.76%
Without	$N_{SR,R}s_Fc_R$	22	-9.18%	-9.03%	-8.98%
Without	$N_{SR,R}s_Fc_R$	23	-9.74%	-9.16%	-9.53%
Without	$N_{SR,R}s_Fc_R$	24	-10.05%	-8.38%	-9.53%
Without	$N_{SR,R}s_Fc_R$	25	-10.53%	-8.14%	-9.07%
Without	$N_{SR,R}s_Rc_R$	1	-10.72%	-8.16%	-9.14%
Without	$N_{SR,R}s_Rc_R$	2	-10.67%	-8.44%	-9.07%
Without	$N_{SR,R}s_Rc_R$	3	-10.51%	-8.31%	-9.1%
Without	$N_{SR,R}s_Rc_R$	4	-10.29%	-8.37%	-8.82%
Without	$N_{SR,R}s_Rc_R$	5	-9.97%	-8.36%	-8.78%
Without	$N_{SR,R}s_Rc_R$	6	-10.02%	-8.41%	-8.84%
Without	$N_{SR,R}s_Rc_R$	7	-9.83%	-8.48%	-8.92%
Without	$N_{SR,R}s_Rc_R$	8	-9.83%	-8.72%	-8.86%
Without	$N_{SR,R}s_Rc_R$	9	-9.71%	-8.54%	-8.78%
Without	$N_{SR,R}s_Rc_R$	10	-9.5%	-9.06%	-8.58%
Without	$N_{SR,R}s_Rc_R$	11	-9.3%	-8.77%	-8.62%
Without	$N_{SR,R}s_Rc_R$	12	-9.34%	-8.74%	-9%
Without	$N_{SR,R}s_Rc_R$	13	-9.24%	-9.26%	-8.77%
Without	$N_{SR,R}s_Rc_R$	14	-9.11%	-8.9%	-8.62%
Without	$N_{SR,R}s_Rc_R$	15	-9.22%	-9.02%	-8.87%
Without	$N_{SR,R}s_Rc_R$	16	-9.31%	-9.39%	-9.12%
Without	$N_{SR,R}s_Rc_R$	17	-8.93%	-9.45%	-8.96%
Without	$N_{SR,R}s_Rc_R$	18	-8.78%	-9.1%	-8.78%
Without	$N_{SR,R}s_Rc_R$	19	-9.09%	-9.17%	-8.94%
Without	$N_{SR,R}s_Rc_R$	20	-9.09%	-9.38%	-8.97%
Without	$N_{SR,R}s_Rc_R$	21	-9.09%	-9.25%	-8.8%
Without	$N_{SR,R}s_Rc_R$	22	-8.94%	-9.28%	-8.89%
Without	$N_{SR,R}s_Rc_R$	23	-9.56%	-9.14%	-9.33%
Without	$N_{SR,R}s_Rc_R$	24	-9.79%	-8.47%	-9.21%
Without	$N_{SR,R}s_Rc_R$	25	-10.32%	-8.28%	-8.81%
Without	s_Fc_F	1	0.29%	1.93%	-2.13%
Without	s_Fc_F	2	-0.16%	0.29%	-1.15%
Without	s_Fc_F	3	-0.33%	-0.81%	-2.14%
Without	s_Fc_F	4	0.55%	1.17%	-2.06%
Without	s_Fc_F	5	0.4%	0.45%	-0.77%
Without	s_Fc_F	6	0.57%	0.19%	-1.18%
Without	s_Fc_F	7	0.22%	0.68%	-2.03%
Without	s_Fc_F	8	0.37%	-0.25%	-1.92%
Without	s_Fc_F	9	-0.38%	1.47%	-1.19%
Without	s_Fc_F	10	0.53%	1.03%	-1.54%
Without	s_Fc_F	11	0.18%	0.46%	-1.87%
Without	s_Fc_F	12	0.47%	0.69%	-1.79%
Without	s_Fc_F	13	-0.12%	0.22%	-2.14%
Without	s_Fc_F	14	-0.05%	-0.19%	-1.21%
Without	s_Fc_F	15	-0.04%	1.04%	-1.05%
Without	s_Fc_F	16	0.17%	0.77%	-1.11%
Without	s_Fc_F	17	-0.29%	0.72%	-1.89%
Without	s_Fc_F	18	0.26%	1.36%	-1.65%
Without	s_Fc_F	19	-0.12%	1.11%	-2.21%
Without	s_Fc_F	20	0.79%	0.56%	-1.52%
Without	s_Fc_F	21	0.11%	1.52%	-2.25%
Without	s_Fc_F	22	-0.24%	0.82%	-1.31%
Without	s_Fc_F	23	-0.24%	0.37%	-1.68%
Without	s_Fc_F	24	0.54%	1.16%	-1.45%
Without	s_Fc_F	25	-0.99%	1.24%	-1.03%
Without	$N_As_Fc_R$	1	-24.76%	-27.46%	-26.85%

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Table A.3 – continued from previous page

Annual Abundance Percent Bias					
Aux. Like.	**Model**	**Year**	**S Increasing**	**S Decreasing**	**Periodic Recruitment**
Without	$N_A s_F c_R$	2	-24.65%	-27.59%	-26.81%
Without	$N_A s_F c_R$	3	-24.48%	-27.64%	-26.96%
Without	$N_A s_F c_R$	4	-24.51%	-27.8%	-26.84%
Without	$N_A s_F c_R$	5	-24.24%	-27.73%	-26.93%
Without	$N_A s_F c_R$	6	-24.16%	-27.89%	-26.96%
Without	$N_A s_F c_R$	7	-23.98%	-27.71%	-26.83%
Without	$N_A s_F c_R$	8	-24.05%	-27.85%	-26.6%
Without	$N_A s_F c_R$	9	-23.87%	-27.6%	-26.72%
Without	$N_A s_F c_R$	10	-24.12%	-27.9%	-26.73%
Without	$N_A s_F c_R$	11	-24.11%	-27.74%	-26.71%
Without	$N_A s_F c_R$	12	-23.92%	-28.01%	-26.73%
Without	$N_A s_F c_R$	13	-23.81%	-27.78%	-26.84%
Without	$N_A s_F c_R$	14	-23.77%	-28.14%	-26.47%
Without	$N_A s_F c_R$	15	-24.04%	-28%	-26.7%
Without	$N_A s_F c_R$	16	-23.78%	-28.19%	-26.68%
Without	$N_A s_F c_R$	17	-23.62%	-28.2%	-26.63%
Without	$N_A s_F c_R$	18	-23.72%	-28.09%	-26.68%
Without	$N_A s_F c_R$	19	-23.79%	-28.12%	-26.92%
Without	$N_A s_F c_R$	20	-23.78%	-28.43%	-26.87%
Without	$N_A s_F c_R$	21	-23.82%	-28.25%	-26.93%
Without	$N_A s_F c_R$	22	-23.84%	-28.37%	-27.07%
Without	$N_A s_F c_R$	23	-24.48%	-28.33%	-27.23%
Without	$N_A s_F c_R$	24	-24.71%	-28.54%	-27.49%
Without	$N_A s_F c_R$	25	-25.54%	-28.41%	-28.04%
Without	$s_F c_R$	1	-0.48%	1.43%	-2.26%
Without	$s_F c_R$	2	-0.77%	1.28%	-2.25%
Without	$s_F c_R$	3	-0.59%	1.22%	-2.58%
Without	$s_F c_R$	4	-0.48%	1.42%	-2.78%
Without	$s_F c_R$	5	-0.55%	0.91%	-2.47%
Without	$s_F c_R$	6	-0.27%	0.76%	-2.68%
Without	$s_F c_R$	7	-0.52%	1.02%	-2.76%
Without	$s_F c_R$	8	-0.1%	0.5%	-2.5%
Without	$s_F c_R$	9	-0.29%	0.43%	-2.66%
Without	$s_F c_R$	10	-0.3%	0.45%	-2.53%
Without	$s_F c_R$	11	0%	0.51%	-3.04%
Without	$s_F c_R$	12	-0.05%	0.38%	-2.95%
Without	$s_F c_R$	13	0.04%	-0.04%	-3.03%
Without	$s_F c_R$	14	0.1%	0.05%	-2.43%
Without	$s_F c_R$	15	-0.25%	0.06%	-2.73%
Without	$s_F c_R$	16	0.13%	0.23%	-2.75%
Without	$s_F c_R$	17	0.23%	-0.15%	-3.07%
Without	$s_F c_R$	18	0.18%	0.12%	-2.68%
Without	$s_F c_R$	19	0.35%	-0.15%	-2.76%
Without	$s_F c_R$	20	0.19%	0.06%	-2.79%
Without	$s_F c_R$	21	0.34%	0.07%	-2.94%
Without	$s_F c_R$	22	-0.03%	0.71%	-2.69%
Without	$s_F c_R$	23	0.06%	0.47%	-2.6%
Without	$s_F c_R$	24	-0.28%	0.99%	-2.88%
Without	$s_F c_R$	25	-0.36%	1.8%	-2.51%
Without	$N_A s_R c_R$	1	-24.94%	-27.48%	-26.85%
Without	$N_A s_R c_R$	2	-24.62%	-27.76%	-26.84%
Without	$N_A s_R c_R$	3	-24.56%	-27.64%	-26.94%
Without	$N_A s_R c_R$	4	-24.56%	-27.85%	-26.91%
Without	$N_A s_R c_R$	5	-24.22%	-27.83%	-26.96%
Without	$N_A s_R c_R$	6	-24.18%	-27.96%	-26.96%
Without	$N_A s_R c_R$	7	-23.96%	-27.89%	-26.83%
Without	$N_A s_R c_R$	8	-24%	-27.92%	-26.61%
Without	$N_A s_R c_R$	9	-23.87%	-27.73%	-26.73%
Without	$N_A s_R c_R$	10	-23.96%	-27.89%	-26.72%

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Table A.3 – continued from previous page

Annual Abundance Percent Bias					
Aux. Like.	**Model**	**Year**	**S Increasing**	**S Decreasing**	**Periodic Recruitment**
Without	$N_A s_R c_R$	11	-23.94%	-27.56%	-26.72%
Without	$N_A s_R c_R$	12	-23.8%	-27.89%	-26.72%
Without	$N_A s_R c_R$	13	-23.79%	-27.71%	-26.84%
Without	$N_A s_R c_R$	14	-23.78%	-28.16%	-26.47%
Without	$N_A s_R c_R$	15	-24.16%	-28.07%	-26.67%
Without	$N_A s_R c_R$	16	-23.82%	-28.35%	-26.66%
Without	$N_A s_R c_R$	17	-23.62%	-28.2%	-26.64%
Without	$N_A s_R c_R$	18	-23.69%	-28.23%	-26.69%
Without	$N_A s_R c_R$	19	-23.86%	-28.16%	-26.94%
Without	$N_A s_R c_R$	20	-23.78%	-28.52%	-26.84%
Without	$N_A s_R c_R$	21	-23.78%	-28.23%	-26.94%
Without	$N_A s_R c_R$	22	-23.84%	-28.37%	-27.06%
Without	$N_A s_R c_R$	23	-24.35%	-28.43%	-27.25%
Without	$N_A s_R c_R$	24	-24.67%	-28.57%	-27.46%
Without	$N_A s_R c_R$	25	-25.58%	-28.44%	-28.09%
Without	$s_R c_R$	1	-0.45%	1.54%	-2.32%
Without	$s_R c_R$	2	-0.5%	1.37%	-2.21%
Without	$s_R c_R$	3	-0.55%	1.14%	-2.59%
Without	$s_R c_R$	4	-0.24%	1.28%	-2.82%
Without	$s_R c_R$	5	-0.35%	0.75%	-2.33%
Without	$s_R c_R$	6	0.1%	0.84%	-2.68%
Without	$s_R c_R$	7	-0.1%	1.01%	-2.82%
Without	$s_R c_R$	8	-0.02%	0.62%	-2.6%
Without	$s_R c_R$	9	-0.1%	0.76%	-2.78%
Without	$s_R c_R$	10	-0.05%	0.57%	-2.53%
Without	$s_R c_R$	11	0.07%	0.62%	-2.89%
Without	$s_R c_R$	12	-0.12%	0.54%	-2.86%
Without	$s_R c_R$	13	-0.1%	0.31%	-2.96%
Without	$s_R c_R$	14	0.14%	0.28%	-2.52%
Without	$s_R c_R$	15	-0.17%	0.44%	-2.81%
Without	$s_R c_R$	16	0.21%	0.52%	-2.88%
Without	$s_R c_R$	17	0.25%	0.03%	-3.05%
Without	$s_R c_R$	18	0.18%	0.22%	-2.71%
Without	$s_R c_R$	19	0.11%	0.07%	-2.98%
Without	$s_R c_R$	20	0.22%	0.38%	-2.92%
Without	$s_R c_R$	21	0.38%	0.29%	-3.11%
Without	$s_R c_R$	22	0.16%	0.9%	-2.78%
Without	$s_R c_R$	23	0.3%	0.52%	-2.64%
Without	$s_R c_R$	24	-0.14%	1.2%	-2.8%
Without	$s_R c_R$	25	-0.22%	1.86%	-2.49%
With	$N_A s_F c_F$	1	-0.08%	4.6%	-1.3%
With	$N_A s_F c_F$	2	0.16%	4.07%	-1.51%
With	$N_A s_F c_F$	3	0.67%	3.91%	-1.38%
With	$N_A s_F c_F$	4	0.58%	4.26%	-1.14%
With	$N_A s_F c_F$	5	0.62%	4.3%	-1.6%
With	$N_A s_F c_F$	6	0.14%	3.72%	-0.7%
With	$N_A s_F c_F$	7	0.58%	4.42%	-0.78%
With	$N_A s_F c_F$	8	0.88%	4.89%	-1.03%
With	$N_A s_F c_F$	9	1.44%	4.72%	-0.97%
With	$N_A s_F c_F$	10	1.04%	4.54%	-1.11%
With	$N_A s_F c_F$	11	1.27%	4.47%	-1.23%
With	$N_A s_F c_F$	12	1.26%	4.12%	-1.43%
With	$N_A s_F c_F$	13	1.41%	4.13%	-1.32%
With	$N_A s_F c_F$	14	1.69%	3.57%	-1.18%
With	$N_A s_F c_F$	15	1.54%	4.01%	-1.13%
With	$N_A s_F c_F$	16	1.14%	4.06%	-0.91%
With	$N_A s_F c_F$	17	0.99%	4.01%	-1.17%
With	$N_A s_F c_F$	18	0.91%	3.77%	-1.33%
With	$N_A s_F c_F$	19	1.04%	3.88%	-1.09%

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Table A.3 – continued from previous page

Annual Abundance Percent Bias					
Aux. Like.	**Model**	**Year**	**S Increasing**	**S Decreasing**	**Periodic Recruitment**
With	$N_A s_F c_F$	20	0.61%	4.34%	-0.94%
With	$N_A s_F c_F$	21	1.26%	4.72%	-0.94%
With	$N_A s_F c_F$	22	1.05%	4.7%	-1.32%
With	$N_A s_F c_F$	23	0.68%	4.73%	-1.05%
With	$N_A s_F c_F$	24	0.6%	4.87%	-1.22%
With	$N_A s_F c_F$	25	0.2%	5.94%	-0.99%
With	$s_F c_F$	1	-1.06%	-2.5%	-2.13%
With	$s_F c_F$	2	-1.24%	-2.27%	-1.15%
With	$s_F c_F$	3	-1.23%	-1.41%	-2.14%
With	$s_F c_F$	4	-0.68%	-2.01%	-2.06%
With	$s_F c_F$	5	-0.11%	-2.52%	-0.77%
With	$s_F c_F$	6	-1.33%	-1.87%	-1.18%
With	$s_F c_F$	7	-1.25%	-2.22%	-2.03%
With	$s_F c_F$	8	-1.54%	-1.05%	-1.92%
With	$s_F c_F$	9	-1.43%	-2.06%	-1.19%
With	$s_F c_F$	10	-0.8%	-1.94%	-1.54%
With	$s_F c_F$	11	-0.69%	-2.04%	-1.87%
With	$s_F c_F$	12	-1.28%	-1.51%	-1.79%
With	$s_F c_F$	13	-0.66%	-2.22%	-2.14%
With	$s_F c_F$	14	-0.69%	-1.99%	-1.21%
With	$s_F c_F$	15	-0.46%	-1.23%	-1.05%
With	$s_F c_F$	16	-1%	-1.8%	-1.11%
With	$s_F c_F$	17	-0.26%	-1.97%	-1.89%
With	$s_F c_F$	18	-0.87%	-2.05%	-1.65%
With	$s_F c_F$	19	-1.16%	-2.15%	-2.21%
With	$s_F c_F$	20	-0.94%	-1.67%	-1.52%
With	$s_F c_F$	21	-1.32%	-0.8%	-2.25%
With	$s_F c_F$	22	-1.46%	-2.55%	-1.31%
With	$s_F c_F$	23	-1.36%	-1.5%	-1.68%
With	$s_F c_F$	24	-1.29%	-1.84%	-1.45%
With	$s_F c_F$	25	-0.81%	-1.31%	-1.03%
With	$N_A s_F c_R$	1	-24.7%	-28.01%	-26.85%
With	$N_A s_F c_R$	2	-24.47%	-28.3%	-26.81%
With	$N_A s_F c_R$	3	-24.26%	-28.19%	-26.96%
With	$N_A s_F c_R$	4	-24.06%	-28.42%	-26.84%
With	$N_A s_F c_R$	5	-24.35%	-28.2%	-26.93%
With	$N_A s_F c_R$	6	-24.39%	-28.43%	-26.96%
With	$N_A s_F c_R$	7	-24.42%	-28.27%	-26.83%
With	$N_A s_F c_R$	8	-24.2%	-28.42%	-26.6%
With	$N_A s_F c_R$	9	-24.07%	-28.23%	-26.72%
With	$N_A s_F c_R$	10	-23.96%	-28.24%	-26.73%
With	$N_A s_F c_R$	11	-23.8%	-28.37%	-26.71%
With	$N_A s_F c_R$	12	-23.67%	-28.26%	-26.73%
With	$N_A s_F c_R$	13	-23.48%	-28.83%	-26.84%
With	$N_A s_F c_R$	14	-23.52%	-28.41%	-26.47%
With	$N_A s_F c_R$	15	-23.33%	-28.31%	-26.7%
With	$N_A s_F c_R$	16	-23.53%	-28.65%	-26.68%
With	$N_A s_F c_R$	17	-23.4%	-28.82%	-26.63%
With	$N_A s_F c_R$	18	-23.5%	-28.61%	-26.68%
With	$N_A s_F c_R$	19	-23.33%	-29.01%	-26.92%
With	$N_A s_F c_R$	20	-23.41%	-28.8%	-26.87%
With	$N_A s_F c_R$	21	-23.7%	-29.15%	-26.93%
With	$N_A s_F c_R$	22	-23.6%	-29.07%	-27.07%
With	$N_A s_F c_R$	23	-24.11%	-29.15%	-27.23%
With	$N_A s_F c_R$	24	-24.56%	-28.74%	-27.49%
With	$N_A s_F c_R$	25	-25.6%	-29.04%	-28.04%
With	$s_F c_R$	1	-3.34%	-2.45%	-2.26%
With	$s_F c_R$	2	-3.31%	-2.62%	-2.25%
With	$s_F c_R$	3	-3.11%	-2.73%	-2.58%

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Table A.3 – continued from previous page

Annual Abundance Percent Bias					
Aux. Like.	**Model**	**Year**	**S Increasing**	**S Decreasing**	**Periodic Recruitment**
With	$s_F c_R$	4	-3.07%	-2.6%	-2.78%
With	$s_F c_R$	5	-2.96%	-3.04%	-2.47%
With	$s_F c_R$	6	-2.84%	-2.78%	-2.68%
With	$s_F c_R$	7	-2.51%	-2.94%	-2.76%
With	$s_F c_R$	8	-2.57%	-2.99%	-2.5%
With	$s_F c_R$	9	-2.58%	-3.05%	-2.66%
With	$s_F c_R$	10	-2.52%	-3.23%	-2.53%
With	$s_F c_R$	11	-2.59%	-3.41%	-3.04%
With	$s_F c_R$	12	-2.68%	-3.38%	-2.95%
With	$s_F c_R$	13	-2.32%	-3.31%	-3.03%
With	$s_F c_R$	14	-2.18%	-3.22%	-2.43%
With	$s_F c_R$	15	-2.26%	-3.2%	-2.73%
With	$s_F c_R$	16	-2.14%	-3.62%	-2.75%
With	$s_F c_R$	17	-1.84%	-3.53%	-3.07%
With	$s_F c_R$	18	-1.74%	-3.6%	-2.68%
With	$s_F c_R$	19	-1.94%	-3.81%	-2.76%
With	$s_F c_R$	20	-2.05%	-3.76%	-2.79%
With	$s_F c_R$	21	-2.15%	-3.67%	-2.94%
With	$s_F c_R$	22	-2.16%	-3.65%	-2.69%
With	$s_F c_R$	23	-2.38%	-3.4%	-2.6%
With	$s_F c_R$	24	-2.38%	-2.74%	-2.88%
With	$s_F c_R$	25	-2.91%	-2.19%	-2.51%
With	$N_A s_R c_R$	1	-24.7%	-28.09%	-26.85%
With	$N_A s_R c_R$	2	-24.49%	-28.28%	-26.84%
With	$N_A s_R c_R$	3	-24.28%	-28.15%	-26.94%
With	$N_A s_R c_R$	4	-24.09%	-28.37%	-26.91%
With	$N_A s_R c_R$	5	-24.35%	-28.18%	-26.96%
With	$N_A s_R c_R$	6	-24.39%	-28.44%	-26.96%
With	$N_A s_R c_R$	7	-24.42%	-28.29%	-26.83%
With	$N_A s_R c_R$	8	-24.21%	-28.35%	-26.61%
With	$N_A s_R c_R$	9	-24.07%	-28.2%	-26.73%
With	$N_A s_R c_R$	10	-23.98%	-28.2%	-26.72%
With	$N_A s_R c_R$	11	-23.81%	-28.37%	-26.72%
With	$N_A s_R c_R$	12	-23.68%	-28.25%	-26.72%
With	$N_A s_R c_R$	13	-23.49%	-28.79%	-26.84%
With	$N_A s_R c_R$	14	-23.59%	-28.36%	-26.47%
With	$N_A s_R c_R$	15	-23.36%	-28.37%	-26.67%
With	$N_A s_R c_R$	16	-23.54%	-28.71%	-26.66%
With	$N_A s_R c_R$	17	-23.49%	-28.83%	-26.64%
With	$N_A s_R c_R$	18	-23.51%	-28.63%	-26.69%
With	$N_A s_R c_R$	19	-23.36%	-29%	-26.94%
With	$N_A s_R c_R$	20	-23.41%	-28.79%	-26.84%
With	$N_A s_R c_R$	21	-23.75%	-29.09%	-26.94%
With	$N_A s_R c_R$	22	-23.64%	-29.03%	-27.06%
With	$N_A s_R c_R$	23	-24.2%	-29.15%	-27.25%
With	$N_A s_R c_R$	24	-24.52%	-28.77%	-27.46%
With	$N_A s_R c_R$	25	-25.58%	-29.13%	-28.09%
With	$s_R c_R$	1	-3.37%	-2.52%	-2.32%
With	$s_R c_R$	2	-3.41%	-2.9%	-2.21%
With	$s_R c_R$	3	-3.15%	-3%	-2.59%
With	$s_R c_R$	4	-3.1%	-2.83%	-2.82%
With	$s_R c_R$	5	-2.96%	-3.25%	-2.33%
With	$s_R c_R$	6	-2.88%	-3.06%	-2.68%
With	$s_R c_R$	7	-2.67%	-3.08%	-2.82%
With	$s_R c_R$	8	-2.85%	-2.95%	-2.6%
With	$s_R c_R$	9	-2.75%	-3%	-2.78%
With	$s_R c_R$	10	-2.6%	-3.13%	-2.53%
With	$s_R c_R$	11	-2.67%	-3.19%	-2.89%
With	$s_R c_R$	12	-2.67%	-3.25%	-2.86%

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Table A.3 – continued from previous page

Annual Abundance Percent Bias					
Aux. Like.	**Model**	**Year**	**S Increasing**	**S Decreasing**	**Periodic Recruitment**
With	$s_R c_R$	13	-2.35%	-3.2%	-2.96%
With	$s_R c_R$	14	-2.17%	-3.12%	-2.52%
With	$s_R c_R$	15	-2.35%	-3.14%	-2.81%
With	$s_R c_R$	16	-2.15%	-3.47%	-2.88%
With	$s_R c_R$	17	-2.17%	-3.39%	-3.05%
With	$s_R c_R$	18	-1.85%	-3.53%	-2.71%
With	$s_R c_R$	19	-2.4%	-3.74%	-2.98%
With	$s_R c_R$	20	-2.29%	-3.61%	-2.92%
With	$s_R c_R$	21	-2.21%	-3.49%	-3.11%
With	$s_R c_R$	22	-2.17%	-3.58%	-2.78%
With	$s_R c_R$	23	-2.57%	-3.22%	-2.64%
With	$s_R c_R$	24	-2.62%	-2.81%	-2.8%
With	$s_R c_R$	25	-2.91%	-2.45%	-2.49%

Table (A.4) contains the data used for plotting Figure (2.7), the median relative bias (MRB) and estimated asymptotic 95% confidence interval coverage for each model, for each year of data, for each level of simulated variation from robustness simulations.

Table A.4: *95% confidence interval coverage and median relative bias in total annual abundance estimates from robustness simulations. Results indicate nearest nominal coverage for conditional-likelihood/Horvitz-Thompson models, with low CI coverage for absolute-recruit abundance models, and CI coverage that ranges from too low to too high for stock-recruit models. Results based on* $n = 1000$ *replicates. "Aux. Like." = Auxiliary catch-effort likelihood used (With) or not used (Without).*

Table A.4 - Annual Abundance 95% CI Coverage and Median Relative Bias								
			S Increasing		**S Decreasing**		**Periodic Recruitment**	
Aux. Like.	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
Without	$N_{SR,R}s_Fc_F$	1	45.00	-6.43	45.70	-4.78	53.80	-5.44
Without	$N_{SR,R}s_Fc_F$	2	48.20	-5.92	49.50	-5.36	61.40	-5.60
Without	$N_{SR,R}s_Fc_F$	3	52.40	-5.47	52.50	-5.48	71.40	-5.48
Without	$N_{SR,R}s_Fc_F$	4	59.30	-5.28	57.60	-5.04	77.30	-5.49
Without	$N_{SR,R}s_Fc_F$	5	63.60	-5.24	62.20	-5.57	83.10	-5.12
Without	$N_{SR,R}s_Fc_F$	6	69.90	-5.04	67.90	-5.11	90.30	-5.26
Without	$N_{SR,R}s_Fc_F$	7	76.20	-5.05	73.90	-4.81	94.90	-5.36
Without	$N_{SR,R}s_Fc_F$	8	81.10	-5.43	77.50	-4.75	96.70	-5.61
Without	$N_{SR,R}s_Fc_F$	9	84.50	-4.79	81.60	-4.71	97.50	-5.35
Without	$N_{SR,R}s_Fc_F$	10	88.50	-5.41	84.50	-5.16	98.10	-5.16
Without	$N_{SR,R}s_Fc_F$	11	91.00	-5.02	86.90	-4.85	98.30	-5.05
Without	$N_{SR,R}s_Fc_F$	12	93.50	-5.50	90.40	-4.87	98.60	-5.03
Without	$N_{SR,R}s_Fc_F$	13	94.40	-5.60	91.80	-4.91	98.90	-5.30
Without	$N_{SR,R}s_Fc_F$	14	96.00	-5.06	93.80	-5.47	98.90	-5.15
Without	$N_{SR,R}s_Fc_F$	15	96.80	-5.01	95.00	-4.96	99.10	-5.12
Without	$N_{SR,R}s_Fc_F$	16	97.60	-4.42	95.40	-4.73	99.10	-5.02
Without	$N_{SR,R}s_Fc_F$	17	97.90	-4.65	95.90	-4.81	99.10	-5.44
Without	$N_{SR,R}s_Fc_F$	18	98.30	-4.92	96.50	-5.43	99.10	-5.05
Without	$N_{SR,R}s_Fc_F$	19	98.80	-4.94	96.80	-5.33	99.10	-5.39
Without	$N_{SR,R}s_Fc_F$	20	99.20	-4.62	97.50	-5.02	99.10	-5.13
Without	$N_{SR,R}s_Fc_F$	21	99.00	-4.98	97.90	-4.71	99.10	-5.28
Without	$N_{SR,R}s_Fc_F$	22	99.20	-4.96	98.20	-4.47	99.10	-5.34
Without	$N_{SR,R}s_Fc_F$	23	99.10	-5.57	98.00	-4.35	99.10	-5.41
Without	$N_{SR,R}s_Fc_F$	24	99.60	-6.17	98.30	-4.56	99.10	-5.08
Without	$N_{SR,R}s_Fc_F$	25	99.20	-6.01	98.10	-4.01	99.10	-4.48
Without	$N_{SR,R}s_Fc_R$	1	54.70	-10.74	71.00	-7.93	66.10	-8.68
Without	$N_{SR,R}s_Fc_R$	2	59.60	-10.59	73.20	-8.44	72.50	-8.80
Without	$N_{SR,R}s_Fc_R$	3	65.30	-10.63	76.70	-8.05	79.90	-8.91
Without	$N_{SR,R}s_Fc_R$	4	72.80	-10.44	80.40	-8.38	86.90	-8.76
Without	$N_{SR,R}s_Fc_R$	5	80.30	-10.08	84.90	-8.18	91.70	-8.76
Without	$N_{SR,R}s_Fc_R$	6	87.30	-10.10	89.40	-8.31	96.20	-8.76
Without	$N_{SR,R}s_Fc_R$	7	92.00	-9.98	91.80	-8.30	98.30	-8.82
Without	$N_{SR,R}s_Fc_R$	8	94.40	-9.75	94.80	-8.50	98.70	-8.83
Without	$N_{SR,R}s_Fc_R$	9	97.20	-9.84	96.90	-8.52	99.30	-8.78
Without	$N_{SR,R}s_Fc_R$	10	98.30	-9.73	97.60	-8.96	99.50	-8.56
Without	$N_{SR,R}s_Fc_R$	11	99.20	-9.26	98.30	-8.81	99.60	-8.80
Without	$N_{SR,R}s_Fc_R$	12	99.50	-9.49	98.60	-8.67	99.60	-9.00
Without	$N_{SR,R}s_Fc_R$	13	99.60	-9.48	98.90	-9.14	99.60	-8.90
Without	$N_{SR,R}s_Fc_R$	14	99.70	-9.32	99.40	-8.81	99.60	-8.70
Without	$N_{SR,R}s_Fc_R$	15	99.70	-9.25	99.50	-8.80	99.60	-8.86
Without	$N_{SR,R}s_Fc_R$	16	99.70	-9.59	99.60	-9.33	99.60	-9.09

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Table A.4 – continued from previous page

			S Increasing		S Decreasing		Periodic Recruitment	
Aux. Like.	Model	Year	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB
Without	$N_{SR,R}s_Fc_R$	17	99.70	-9.17	99.70	-9.34	99.60	-8.92
Without	$N_{SR,R}s_Fc_R$	18	99.70	-9.00	99.70	-9.08	99.60	-8.70
Without	$N_{SR,R}s_Fc_R$	19	99.70	-9.30	99.60	-9.12	99.60	-8.96
Without	$N_{SR,R}s_Fc_R$	20	99.70	-9.16	99.70	-9.36	99.60	-9.05
Without	$N_{SR,R}s_Fc_R$	21	99.80	-9.23	99.70	-9.25	99.60	-8.76
Without	$N_{SR,R}s_Fc_R$	22	99.70	-9.18	99.70	-9.03	99.60	-8.98
Without	$N_{SR,R}s_Fc_R$	23	99.80	-9.74	99.70	-9.16	99.70	-9.53
Without	$N_{SR,R}s_Fc_R$	24	99.90	-10.05	99.70	-8.38	99.70	-9.53
Without	$N_{SR,R}s_Fc_R$	25	99.90	-10.53	99.70	-8.14	99.70	-9.07
Without	$N_{SR,R}s_Rc_R$	1	55.90	-10.72	70.50	-8.16	65.50	-9.14
Without	$N_{SR,R}s_Rc_R$	2	60.20	-10.67	73.10	-8.44	72.30	-9.07
Without	$N_{SR,R}s_Rc_R$	3	66.60	-10.51	76.60	-8.31	80.00	-9.10
Without	$N_{SR,R}s_Rc_R$	4	74.40	-10.29	80.70	-8.37	86.20	-8.82
Without	$N_{SR,R}s_Rc_R$	5	82.20	-9.97	86.10	-8.36	90.90	-8.78
Without	$N_{SR,R}s_Rc_R$	6	88.80	-10.02	90.50	-8.41	96.60	-8.84
Without	$N_{SR,R}s_Rc_R$	7	93.60	-9.83	93.40	-8.48	98.70	-8.92
Without	$N_{SR,R}s_Rc_R$	8	96.90	-9.83	96.00	-8.72	98.80	-8.86
Without	$N_{SR,R}s_Rc_R$	9	98.80	-9.71	98.00	-8.54	99.30	-8.78
Without	$N_{SR,R}s_Rc_R$	10	99.40	-9.50	98.40	-9.06	99.50	-8.58
Without	$N_{SR,R}s_Rc_R$	11	99.90	-9.30	99.30	-8.77	99.50	-8.62
Without	$N_{SR,R}s_Rc_R$	12	100.00	-9.34	99.60	-8.74	99.50	-9.00
Without	$N_{SR,R}s_Rc_R$	13	100.00	-9.24	99.70	-9.26	99.50	-8.77
Without	$N_{SR,R}s_Rc_R$	14	100.00	-9.11	99.70	-8.90	99.50	-8.62
Without	$N_{SR,R}s_Rc_R$	15	100.00	-9.22	99.70	-9.02	99.50	-8.87
Without	$N_{SR,R}s_Rc_R$	16	100.00	-9.31	99.70	-9.39	99.50	-9.12
Without	$N_{SR,R}s_Rc_R$	17	100.00	-8.93	99.80	-9.45	99.50	-8.96
Without	$N_{SR,R}s_Rc_R$	18	100.00	-8.78	99.80	-9.10	99.50	-8.78
Without	$N_{SR,R}s_Rc_R$	19	100.00	-9.09	99.80	-9.17	99.50	-8.94
Without	$N_{SR,R}s_Rc_R$	20	100.00	-9.09	99.70	-9.38	99.50	-8.97
Without	$N_{SR,R}s_Rc_R$	21	100.00	-9.09	99.80	-9.25	99.50	-8.80
Without	$N_{SR,R}s_Rc_R$	22	100.00	-8.94	99.70	-9.28	99.50	-8.89
Without	$N_{SR,R}s_Rc_R$	23	100.00	-9.56	99.80	-9.14	99.50	-9.33
Without	$N_{SR,R}s_Rc_R$	24	100.00	-9.79	99.80	-8.47	99.50	-9.21
Without	$N_{SR,R}s_Rc_R$	25	100.00	-10.32	99.80	-8.28	99.50	-8.81
Without	$N_As_Fc_F$	1	17.50	-22.18	30.00	-18.48	26.40	-18.65
Without	$N_As_Fc_F$	2	18.40	-21.84	30.40	-17.81	26.30	-18.33
Without	$N_As_Fc_F$	3	18.20	-21.66	29.20	-18.16	27.60	-18.54
Without	$N_As_Fc_F$	4	18.20	-21.21	29.00	-18.72	26.70	-18.45
Without	$N_As_Fc_F$	5	18.00	-21.36	27.50	-18.29	26.20	-18.51
Without	$N_As_Fc_F$	6	18.70	-21.05	28.90	-18.54	27.30	-17.95
Without	$N_As_Fc_F$	7	18.50	-20.64	28.60	-19.04	26.60	-18.51
Without	$N_As_Fc_F$	8	17.50	-21.01	28.00	-18.55	26.60	-18.32
Without	$N_As_Fc_F$	9	18.50	-20.74	27.50	-18.14	26.70	-17.93
Without	$N_As_Fc_F$	10	18.20	-20.84	27.60	-18.47	26.60	-18.00
Without	$N_As_Fc_F$	11	18.30	-20.96	28.50	-18.07	25.90	-18.47
Without	$N_As_Fc_F$	12	19.20	-21.32	28.90	-18.10	26.90	-18.28
Without	$N_As_Fc_F$	13	18.70	-21.17	28.20	-18.42	27.20	-18.20
Without	$N_As_Fc_F$	14	18.10	-21.18	27.70	-18.50	25.90	-18.34
Without	$N_As_Fc_F$	15	17.40	-21.23	27.10	-18.15	26.60	-18.07
Without	$N_As_Fc_F$	16	18.10	-21.23	28.50	-18.34	28.00	-18.29
Without	$N_As_Fc_F$	17	18.50	-21.40	29.30	-18.13	27.60	-18.01
Without	$N_As_Fc_F$	18	17.90	-21.13	29.90	-18.26	27.30	-18.17
Without	$N_As_Fc_F$	19	18.10	-21.04	28.90	-18.14	26.30	-18.20
Without	$N_As_Fc_F$	20	18.00	-20.75	29.50	-17.96	26.90	-18.04
Without	$N_As_Fc_F$	21	18.20	-20.84	29.50	-18.49	26.10	-18.08
Without	$N_As_Fc_F$	22	18.40	-20.62	29.20	-18.56	27.60	-18.06
Without	$N_As_Fc_F$	23	18.60	-20.81	30.90	-17.84	25.80	-18.05
Without	$N_As_Fc_F$	24	19.20	-20.75	31.50	-17.75	27.10	-17.94
Without	$N_As_Fc_F$	25	20.30	-20.56	34.70	-16.98	28.80	-17.48

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Table A.4 – continued from previous page

			S Increasing		S Decreasing		Periodic Recruitment	
Aux. Like.	Model	Year	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB
Without	$s_F c_F$	1	66.60	0.29	66.00	1.93	61.40	-0.43
Without	$s_F c_F$	2	70.20	-0.16	67.00	0.29	60.30	0.57
Without	$s_F c_F$	3	71.50	-0.33	65.40	-0.81	59.90	0.22
Without	$s_F c_F$	4	69.20	0.55	65.50	1.17	58.90	-0.34
Without	$s_F c_F$	5	68.00	0.40	66.20	0.45	61.70	0.59
Without	$s_F c_F$	6	70.20	0.57	66.50	0.19	60.30	0.62
Without	$s_F c_F$	7	70.30	0.22	65.20	0.68	60.20	-0.24
Without	$s_F c_F$	8	68.30	0.37	64.90	-0.25	61.50	0.02
Without	$s_F c_F$	9	67.40	-0.38	66.10	1.47	61.50	0.58
Without	$s_F c_F$	10	70.70	0.53	67.90	1.03	62.60	0.21
Without	$s_F c_F$	11	69.10	0.18	65.90	0.46	59.80	0.19
Without	$s_F c_F$	12	68.40	0.47	66.80	0.69	60.60	-0.07
Without	$s_F c_F$	13	68.40	-0.12	65.40	0.22	61.80	-0.04
Without	$s_F c_F$	14	66.00	-0.05	64.50	-0.19	63.00	0.17
Without	$s_F c_F$	15	68.60	-0.04	67.10	1.04	59.60	0.68
Without	$s_F c_F$	16	68.60	0.17	67.20	0.77	62.10	0.83
Without	$s_F c_F$	17	70.30	-0.29	68.60	0.72	60.30	-0.22
Without	$s_F c_F$	18	68.70	0.26	67.80	1.36	60.50	0.15
Without	$s_F c_F$	19	67.80	-0.12	67.40	1.11	60.70	-0.43
Without	$s_F c_F$	20	67.60	0.79	67.40	0.56	61.10	-0.11
Without	$s_F c_F$	21	68.00	0.11	64.90	1.52	59.80	-0.46
Without	$s_F c_F$	22	70.50	-0.24	69.10	0.82	59.90	0.38
Without	$s_F c_F$	23	68.00	-0.24	66.90	0.37	61.10	-0.08
Without	$s_F c_F$	24	68.30	0.54	70.30	1.16	60.30	-0.02
Without	$s_F c_F$	25	66.10	-0.99	68.20	1.24	59.90	0.77
Without	$N_A s_F c_R$	1	0.80	-24.76	6.10	-27.46	1.90	-26.56
Without	$N_A s_F c_R$	2	0.70	-24.65	5.70	-27.59	2.00	-26.51
Without	$N_A s_F c_R$	3	0.70	-24.48	6.20	-27.64	2.30	-26.65
Without	$N_A s_F c_R$	4	1.00	-24.51	6.50	-27.80	2.20	-26.71
Without	$N_A s_F c_R$	5	0.90	-24.24	6.50	-27.73	2.30	-26.67
Without	$N_A s_F c_R$	6	1.00	-24.16	6.40	-27.89	2.40	-26.65
Without	$N_A s_F c_R$	7	1.10	-23.98	6.60	-27.71	2.30	-26.71
Without	$N_A s_F c_R$	8	1.00	-24.05	6.30	-27.85	2.50	-26.32
Without	$N_A s_F c_R$	9	1.20	-23.87	5.80	-27.60	2.70	-26.39
Without	$N_A s_F c_R$	10	1.00	-24.12	5.90	-27.90	2.80	-26.45
Without	$N_A s_F c_R$	11	1.20	-24.11	6.10	-27.74	2.40	-26.39
Without	$N_A s_F c_R$	12	1.20	-23.92	6.10	-28.01	2.40	-26.26
Without	$N_A s_F c_R$	13	1.20	-23.81	6.10	-27.78	2.40	-26.40
Without	$N_A s_F c_R$	14	1.50	-23.77	6.50	-28.14	2.80	-26.15
Without	$N_A s_F c_R$	15	1.60	-24.04	6.20	-28.00	2.40	-26.28
Without	$N_A s_F c_R$	16	1.60	-23.78	6.50	-28.19	2.60	-26.32
Without	$N_A s_F c_R$	17	1.10	-23.62	6.90	-28.20	2.60	-26.30
Without	$N_A s_F c_R$	18	1.10	-23.72	5.90	-28.09	2.40	-26.35
Without	$N_A s_F c_R$	19	1.20	-23.79	6.10	-28.12	2.30	-26.53
Without	$N_A s_F c_R$	20	1.50	-23.78	5.70	-28.43	2.40	-26.53
Without	$N_A s_F c_R$	21	1.10	-23.82	5.80	-28.25	2.30	-26.48
Without	$N_A s_F c_R$	22	1.60	-23.84	6.40	-28.37	2.30	-26.55
Without	$N_A s_F c_R$	23	1.70	-24.48	7.30	-28.33	3.00	-26.85
Without	$N_A s_F c_R$	24	2.00	-24.71	7.90	-28.54	3.80	-27.16
Without	$N_A s_F c_R$	25	3.00	-25.54	10.10	-28.41	5.40	-27.54
Without	$s_F c_R$	1	96.40	-0.48	96.30	1.43	95.80	0.94
Without	$s_F c_R$	2	96.80	-0.77	96.20	1.28	95.60	1.08
Without	$s_F c_R$	3	97.00	-0.59	96.50	1.22	95.80	0.57
Without	$s_F c_R$	4	97.00	-0.48	96.20	1.42	95.40	0.18
Without	$s_F c_R$	5	96.40	-0.55	96.00	0.91	95.80	0.68
Without	$s_F c_R$	6	97.10	-0.27	96.20	0.76	96.40	0.28
Without	$s_F c_R$	7	97.10	-0.52	96.00	1.02	96.30	0.35
Without	$s_F c_R$	8	97.70	-0.10	95.80	0.50	96.20	0.89
Without	$s_F c_R$	9	97.50	-0.29	96.50	0.43	96.20	0.56

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Table A.4 – continued from previous page

			S Increasing		S Decreasing		Periodic Recruitment	
Aux. Like.	Model	Year	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB
Without	s_Fc_R	10	97.40	-0.30	96.50	0.45	95.90	0.66
Without	s_Fc_R	11	96.80	-0.00	96.60	0.51	96.30	0.10
Without	s_Fc_R	12	97.30	-0.05	95.90	0.38	96.30	0.31
Without	s_Fc_R	13	96.60	0.04	95.50	-0.04	96.10	0.66
Without	s_Fc_R	14	97.20	0.10	96.10	0.05	96.00	0.77
Without	s_Fc_R	15	96.40	-0.25	95.80	0.06	95.80	0.49
Without	s_Fc_R	16	97.20	0.13	95.30	0.23	95.70	0.39
Without	s_Fc_R	17	97.30	0.23	95.90	-0.15	95.80	0.22
Without	s_Fc_R	18	97.20	0.18	95.50	0.12	96.20	0.52
Without	s_Fc_R	19	97.70	0.35	96.20	-0.15	95.80	0.56
Without	s_Fc_R	20	97.40	0.19	95.20	0.06	95.80	0.41
Without	s_Fc_R	21	96.20	0.34	95.20	0.07	95.80	0.32
Without	s_Fc_R	22	96.70	-0.03	95.50	0.71	95.70	0.57
Without	s_Fc_R	23	96.50	0.06	95.10	0.47	94.90	0.70
Without	s_Fc_R	24	94.50	-0.28	95.20	0.99	94.70	0.61
Without	s_Fc_R	25	93.90	-0.36	94.20	1.80	93.00	0.77
Without	$N_As_Rc_R$	1	1.00	-24.94	6.30	-27.48	1.90	-26.59
Without	$N_As_Rc_R$	2	0.80	-24.62	6.30	-27.76	2.00	-26.55
Without	$N_As_Rc_R$	3	0.80	-24.56	6.50	-27.64	2.30	-26.63
Without	$N_As_Rc_R$	4	1.00	-24.56	6.60	-27.85	2.20	-26.65
Without	$N_As_Rc_R$	5	0.90	-24.22	6.60	-27.83	2.30	-26.64
Without	$N_As_Rc_R$	6	1.10	-24.18	6.60	-27.96	2.50	-26.59
Without	$N_As_Rc_R$	7	1.30	-23.96	6.60	-27.89	2.40	-26.69
Without	$N_As_Rc_R$	8	1.10	-24.00	6.70	-27.92	2.60	-26.25
Without	$N_As_Rc_R$	9	1.40	-23.87	5.90	-27.73	2.60	-26.37
Without	$N_As_Rc_R$	10	1.10	-23.96	6.10	-27.89	2.70	-26.40
Without	$N_As_Rc_R$	11	1.40	-23.94	6.20	-27.56	2.30	-26.37
Without	$N_As_Rc_R$	12	1.60	-23.80	6.60	-27.89	2.40	-26.19
Without	$N_As_Rc_R$	13	1.50	-23.79	6.70	-27.71	2.50	-26.33
Without	$N_As_Rc_R$	14	1.90	-23.78	7.30	-28.16	3.10	-26.13
Without	$N_As_Rc_R$	15	2.50	-24.16	6.90	-28.07	2.70	-26.28
Without	$N_As_Rc_R$	16	2.10	-23.82	6.80	-28.35	3.00	-26.28
Without	$N_As_Rc_R$	17	1.50	-23.62	6.90	-28.20	2.70	-26.29
Without	$N_As_Rc_R$	18	1.30	-23.69	6.10	-28.23	2.50	-26.31
Without	$N_As_Rc_R$	19	1.30	-23.86	6.20	-28.16	2.30	-26.50
Without	$N_As_Rc_R$	20	1.50	-23.78	5.90	-28.52	2.40	-26.47
Without	$N_As_Rc_R$	21	1.10	-23.78	6.00	-28.23	2.30	-26.42
Without	$N_As_Rc_R$	22	1.50	-23.84	6.60	-28.37	2.40	-26.55
Without	$N_As_Rc_R$	23	1.80	-24.35	7.50	-28.43	3.10	-26.82
Without	$N_As_Rc_R$	24	2.20	-24.67	8.70	-28.57	3.90	-27.02
Without	$N_As_Rc_R$	25	3.50	-25.58	10.70	-28.44	5.70	-27.51
Without	s_Rc_R	1	95.80	-0.45	95.70	1.54	94.90	1.11
Without	s_Rc_R	2	96.70	-0.50	95.60	1.37	94.80	1.28
Without	s_Rc_R	3	97.10	-0.55	95.80	1.14	94.90	0.74
Without	s_Rc_R	4	96.80	-0.24	95.60	1.28	95.10	0.46
Without	s_Rc_R	5	96.00	-0.35	96.00	0.75	95.60	0.74
Without	s_Rc_R	6	96.90	0.10	95.90	0.84	95.90	0.68
Without	s_Rc_R	7	96.70	-0.10	95.80	1.01	96.00	0.36
Without	s_Rc_R	8	97.10	-0.02	95.80	0.62	95.90	0.89
Without	s_Rc_R	9	97.20	-0.10	96.40	0.76	96.00	0.66
Without	s_Rc_R	10	96.80	-0.05	96.60	0.57	95.60	0.70
Without	s_Rc_R	11	96.70	0.07	96.80	0.62	96.30	0.41
Without	s_Rc_R	12	97.20	-0.12	96.10	0.54	96.40	0.39
Without	s_Rc_R	13	96.30	-0.10	95.80	0.31	95.70	0.54
Without	s_Rc_R	14	96.50	0.14	96.30	0.28	95.40	0.85
Without	s_Rc_R	15	95.70	-0.17	95.80	0.44	94.90	0.57
Without	s_Rc_R	16	96.00	0.21	95.20	0.52	94.80	0.68
Without	s_Rc_R	17	96.70	0.25	95.20	0.03	95.10	0.44
Without	s_Rc_R	18	96.70	0.18	95.30	0.22	95.90	0.65

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Table A.4 – continued from previous page

			S Increasing		S Decreasing		Periodic Recruitment	
Aux. Like.	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
Without	s_Rc_R	19	97.10	0.11	96.30	0.07	95.40	0.56
Without	s_Rc_R	20	96.60	0.22	95.10	0.38	94.80	0.50
Without	s_Rc_R	21	95.90	0.38	95.40	0.29	95.00	0.36
Without	s_Rc_R	22	96.40	0.16	95.10	0.90	95.20	0.72
Without	s_Rc_R	23	96.30	0.30	95.40	0.52	94.50	0.70
Without	s_Rc_R	24	94.60	-0.14	95.30	1.20	94.50	0.84
Without	s_Rc_R	25	94.20	-0.22	94.30	1.86	92.70	1.10
With	$N_As_Fc_F$	1	34.50	-0.08	36.40	4.60	38.10	-1.30
With	$N_As_Fc_F$	2	33.10	0.16	36.80	4.07	38.30	-1.51
With	$N_As_Fc_F$	3	33.90	0.67	35.90	3.91	38.70	-1.38
With	$N_As_Fc_F$	4	34.00	0.58	36.20	4.26	38.40	-1.14
With	$N_As_Fc_F$	5	33.00	0.62	36.70	4.30	39.00	-1.60
With	$N_As_Fc_F$	6	33.90	0.14	36.90	3.72	38.90	-0.70
With	$N_As_Fc_F$	7	32.90	0.58	36.20	4.42	38.70	-0.78
With	$N_As_Fc_F$	8	34.70	0.88	35.50	4.89	38.60	-1.03
With	$N_As_Fc_F$	9	34.20	1.44	36.70	4.72	39.10	-0.97
With	$N_As_Fc_F$	10	33.50	1.04	36.60	4.54	39.60	-1.11
With	$N_As_Fc_F$	11	32.30	1.27	36.30	4.47	39.80	-1.23
With	$N_As_Fc_F$	12	32.50	1.26	35.90	4.12	39.90	-1.43
With	$N_As_Fc_F$	13	31.70	1.41	36.20	4.13	39.90	-1.32
With	$N_As_Fc_F$	14	31.80	1.69	35.40	3.57	38.50	-1.18
With	$N_As_Fc_F$	15	33.20	1.54	36.90	4.01	38.90	-1.13
With	$N_As_Fc_F$	16	33.80	1.14	37.60	4.06	38.20	-0.91
With	$N_As_Fc_F$	17	34.80	0.99	37.20	4.01	37.20	-1.17
With	$N_As_Fc_F$	18	34.50	0.91	36.30	3.77	38.10	-1.33
With	$N_As_Fc_F$	19	33.90	1.04	36.10	3.88	38.20	-1.09
With	$N_As_Fc_F$	20	33.80	0.61	36.50	4.34	39.20	-0.94
With	$N_As_Fc_F$	21	33.20	1.26	34.60	4.72	38.50	-0.94
With	$N_As_Fc_F$	22	33.90	1.05	35.00	4.70	38.40	-1.32
With	$N_As_Fc_F$	23	33.70	0.68	35.50	4.73	36.80	-1.05
With	$N_As_Fc_F$	24	33.40	0.60	35.50	4.87	35.70	-1.22
With	$N_As_Fc_F$	25	32.80	0.20	36.30	5.94	36.70	-0.99
With	s_Fc_F	1	52.70	-1.06	51.70	-2.50	60.90	-2.13
With	s_Fc_F	2	56.40	-1.24	51.30	-2.27	59.30	-1.15
With	s_Fc_F	3	57.70	-1.23	50.70	-1.41	57.50	-2.14
With	s_Fc_F	4	54.50	-0.68	51.00	-2.01	56.80	-2.06
With	s_Fc_F	5	51.50	-0.11	50.40	-2.52	61.80	-0.77
With	s_Fc_F	6	53.80	-1.33	49.40	-1.87	59.30	-1.18
With	s_Fc_F	7	57.10	-1.25	50.40	-2.22	58.10	-2.03
With	s_Fc_F	8	54.30	-1.54	53.30	-1.05	58.30	-1.92
With	s_Fc_F	9	53.10	-1.43	51.20	-2.06	59.60	-1.19
With	s_Fc_F	10	52.00	-0.80	51.70	-1.94	61.90	-1.54
With	s_Fc_F	11	54.10	-0.69	52.00	-2.04	57.90	-1.87
With	s_Fc_F	12	56.10	-1.28	52.50	-1.51	58.60	-1.79
With	s_Fc_F	13	55.60	-0.66	51.20	-2.22	58.50	-2.14
With	s_Fc_F	14	54.60	-0.69	51.90	-1.99	60.20	-1.21
With	s_Fc_F	15	53.80	-0.46	51.30	-1.23	60.10	-1.05
With	s_Fc_F	16	54.90	-1.00	53.10	-1.80	60.70	-1.11
With	s_Fc_F	17	51.70	-0.26	52.10	-1.97	58.20	-1.89
With	s_Fc_F	18	52.70	-0.87	51.70	-2.05	58.50	-1.65
With	s_Fc_F	19	51.60	-1.16	50.30	-2.15	57.80	-2.21
With	s_Fc_F	20	53.90	-0.94	51.50	-1.67	59.00	-1.52
With	s_Fc_F	21	52.40	-1.32	52.40	-0.80	57.50	-2.25
With	s_Fc_F	22	51.70	-1.46	52.60	-2.55	60.00	-1.31
With	s_Fc_F	23	53.10	-1.36	53.20	-1.50	60.20	-1.68
With	s_Fc_F	24	51.30	-1.29	51.90	-1.84	56.90	-1.45
With	s_Fc_F	25	51.30	-0.81	53.20	-1.31	58.40	-1.03
With	$N_As_Fc_R$	1	0.80	-24.70	2.50	-28.01	1.40	-26.85
With	$N_As_Fc_R$	2	0.80	-24.47	2.60	-28.30	1.60	-26.81

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Table A.4 – continued from previous page

			S Increasing		S Decreasing		Periodic Recruitment	
Aux. Like.	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
With	$N_A s_F c_R$	3	0.60	-24.26	2.90	-28.19	1.30	-26.96
With	$N_A s_F c_R$	4	0.80	-24.06	3.00	-28.42	1.40	-26.84
With	$N_A s_F c_R$	5	0.90	-24.35	2.90	-28.20	1.30	-26.93
With	$N_A s_F c_R$	6	0.90	-24.39	3.00	-28.43	1.30	-26.96
With	$N_A s_F c_R$	7	0.90	-24.42	2.90	-28.27	1.40	-26.83
With	$N_A s_F c_R$	8	1.20	-24.20	3.10	-28.42	1.50	-26.60
With	$N_A s_F c_R$	9	1.20	-24.07	2.70	-28.23	1.30	-26.72
With	$N_A s_F c_R$	10	0.90	-23.96	2.60	-28.24	1.30	-26.73
With	$N_A s_F c_R$	11	0.80	-23.80	2.70	-28.37	1.50	-26.71
With	$N_A s_F c_R$	12	0.90	-23.67	2.70	-28.26	1.50	-26.73
With	$N_A s_F c_R$	13	1.00	-23.48	2.50	-28.83	1.40	-26.84
With	$N_A s_F c_R$	14	1.00	-23.52	2.60	-28.41	1.60	-26.47
With	$N_A s_F c_R$	15	1.00	-23.33	2.50	-28.31	1.40	-26.70
With	$N_A s_F c_R$	16	1.00	-23.53	2.60	-28.65	1.90	-26.68
With	$N_A s_F c_R$	17	0.90	-23.40	2.40	-28.82	1.90	-26.63
With	$N_A s_F c_R$	18	1.00	-23.50	2.20	-28.61	1.80	-26.68
With	$N_A s_F c_R$	19	1.10	-23.33	2.60	-29.01	1.70	-26.92
With	$N_A s_F c_R$	20	0.90	-23.41	2.70	-28.80	1.80	-26.87
With	$N_A s_F c_R$	21	1.00	-23.70	2.50	-29.15	1.70	-26.93
With	$N_A s_F c_R$	22	1.00	-23.60	3.10	-29.07	1.50	-27.07
With	$N_A s_F c_R$	23	1.00	-24.11	3.00	-29.15	1.90	-27.23
With	$N_A s_F c_R$	24	1.20	-24.56	3.70	-28.74	2.40	-27.49
With	$N_A s_F c_R$	25	2.30	-25.60	5.60	-29.04	3.80	-28.04
With	$s_F c_R$	1	93.30	-3.34	94.00	-2.45	92.90	-2.26
With	$s_F c_R$	2	94.30	-3.31	93.40	-2.62	92.80	-2.25
With	$s_F c_R$	3	93.80	-3.11	92.90	-2.73	93.40	-2.58
With	$s_F c_R$	4	94.20	-3.07	93.50	-2.60	93.60	-2.78
With	$s_F c_R$	5	94.50	-2.96	93.90	-3.04	94.40	-2.47
With	$s_F c_R$	6	93.70	-2.84	93.10	-2.78	94.60	-2.68
With	$s_F c_R$	7	94.10	-2.51	93.30	-2.94	94.20	-2.76
With	$s_F c_R$	8	94.20	-2.57	93.20	-2.99	94.10	-2.50
With	$s_F c_R$	9	94.50	-2.58	93.60	-3.05	94.10	-2.66
With	$s_F c_R$	10	94.80	-2.52	92.50	-3.23	93.40	-2.53
With	$s_F c_R$	11	94.50	-2.59	93.30	-3.41	94.00	-3.04
With	$s_F c_R$	12	94.60	-2.68	93.10	-3.38	92.50	-2.95
With	$s_F c_R$	13	95.30	-2.32	93.10	-3.31	94.10	-3.03
With	$s_F c_R$	14	94.70	-2.18	92.60	-3.22	93.10	-2.43
With	$s_F c_R$	15	95.50	-2.26	93.00	-3.20	92.80	-2.73
With	$s_F c_R$	16	94.10	-2.14	92.90	-3.62	93.20	-2.75
With	$s_F c_R$	17	94.80	-1.84	92.50	-3.53	93.70	-3.07
With	$s_F c_R$	18	95.70	-1.74	92.30	-3.60	93.10	-2.68
With	$s_F c_R$	19	95.20	-1.94	92.60	-3.81	92.50	-2.76
With	$s_F c_R$	20	94.80	-2.05	92.60	-3.76	93.20	-2.79
With	$s_F c_R$	21	94.00	-2.15	92.50	-3.67	92.90	-2.94
With	$s_F c_R$	22	93.60	-2.16	91.90	-3.65	92.90	-2.69
With	$s_F c_R$	23	94.00	-2.38	90.80	-3.40	92.90	-2.60
With	$s_F c_R$	24	91.50	-2.38	91.20	-2.74	91.90	-2.88
With	$s_F c_R$	25	90.10	-2.91	90.60	-2.19	90.90	-2.51
With	$N_A s_R c_R$	1	0.70	-24.70	2.60	-28.09	1.40	-26.85
With	$N_A s_R c_R$	2	0.70	-24.49	2.60	-28.28	1.60	-26.84
With	$N_A s_R c_R$	3	0.50	-24.28	2.90	-28.15	1.30	-26.94
With	$N_A s_R c_R$	4	0.80	-24.09	3.00	-28.37	1.40	-26.91
With	$N_A s_R c_R$	5	0.80	-24.35	2.80	-28.18	1.30	-26.96
With	$N_A s_R c_R$	6	0.80	-24.39	2.80	-28.44	1.30	-26.96
With	$N_A s_R c_R$	7	0.80	-24.42	2.80	-28.29	1.30	-26.83
With	$N_A s_R c_R$	8	1.00	-24.21	3.10	-28.35	1.50	-26.61
With	$N_A s_R c_R$	9	1.10	-24.07	2.60	-28.20	1.30	-26.73
With	$N_A s_R c_R$	10	0.80	-23.98	2.50	-28.20	1.30	-26.72
With	$N_A s_R c_R$	11	0.80	-23.81	2.60	-28.37	1.50	-26.72

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Table A.4 – continued from previous page

			S Increasing		S Decreasing		Periodic Recruitment	
Aux. Like.	Model	Year	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB
With	$N_A s_R c_R$	12	0.90	-23.68	2.70	-28.25	1.80	-26.72
With	$N_A s_R c_R$	13	1.10	-23.49	2.50	-28.79	1.80	-26.84
With	$N_A s_R c_R$	14	1.00	-23.59	2.60	-28.36	1.90	-26.47
With	$N_A s_R c_R$	15	0.90	-23.36	2.50	-28.37	1.60	-26.67
With	$N_A s_R c_R$	16	0.90	-23.54	2.60	-28.71	2.10	-26.66
With	$N_A s_R c_R$	17	0.80	-23.49	2.40	-28.83	2.00	-26.64
With	$N_A s_R c_R$	18	0.90	-23.51	2.30	-28.63	1.80	-26.69
With	$N_A s_R c_R$	19	1.00	-23.36	2.60	-29.00	1.70	-26.94
With	$N_A s_R c_R$	20	0.80	-23.41	2.70	-28.79	1.80	-26.84
With	$N_A s_R c_R$	21	0.90	-23.75	2.40	-29.09	1.70	-26.94
With	$N_A s_R c_R$	22	0.90	-23.64	3.00	-29.03	1.50	-27.06
With	$N_A s_R c_R$	23	0.80	-24.20	3.00	-29.15	1.90	-27.25
With	$N_A s_R c_R$	24	1.10	-24.52	3.70	-28.77	2.50	-27.46
With	$N_A s_R c_R$	25	2.30	-25.58	5.80	-29.13	3.80	-28.09
With	$s_R c_R$	1	92.60	-3.37	93.10	-2.52	91.90	-2.32
With	$s_R c_R$	2	93.60	-3.41	92.70	-2.90	92.20	-2.21
With	$s_R c_R$	3	92.90	-3.15	92.00	-3.00	92.30	-2.59
With	$s_R c_R$	4	93.00	-3.10	92.80	-2.83	92.50	-2.82
With	$s_R c_R$	5	94.20	-2.96	93.40	-3.25	93.70	-2.33
With	$s_R c_R$	6	93.70	-2.88	92.60	-3.06	94.40	-2.68
With	$s_R c_R$	7	93.90	-2.67	93.00	-3.08	93.80	-2.82
With	$s_R c_R$	8	93.40	-2.85	92.70	-2.95	93.80	-2.60
With	$s_R c_R$	9	93.60	-2.75	93.70	-3.00	93.70	-2.78
With	$s_R c_R$	10	94.00	-2.60	92.80	-3.13	93.40	-2.53
With	$s_R c_R$	11	94.60	-2.67	93.50	-3.19	94.20	-2.89
With	$s_R c_R$	12	94.40	-2.67	94.00	-3.25	93.00	-2.86
With	$s_R c_R$	13	95.30	-2.35	93.20	-3.20	94.00	-2.96
With	$s_R c_R$	14	93.60	-2.17	93.10	-3.12	92.30	-2.52
With	$s_R c_R$	15	95.00	-2.35	92.70	-3.14	92.10	-2.81
With	$s_R c_R$	16	93.40	-2.15	92.60	-3.47	92.20	-2.88
With	$s_R c_R$	17	93.60	-2.17	92.30	-3.39	93.00	-3.05
With	$s_R c_R$	18	94.50	-1.85	91.70	-3.53	92.40	-2.71
With	$s_R c_R$	19	94.10	-2.40	92.40	-3.74	91.50	-2.98
With	$s_R c_R$	20	93.80	-2.29	91.60	-3.61	92.50	-2.92
With	$s_R c_R$	21	93.20	-2.21	92.10	-3.49	92.00	-3.11
With	$s_R c_R$	22	92.30	-2.17	91.60	-3.58	91.90	-2.78
With	$s_R c_R$	23	92.80	-2.57	90.50	-3.22	92.20	-2.64
With	$s_R c_R$	24	90.70	-2.62	91.30	-2.81	91.70	-2.80
With	$s_R c_R$	25	89.80	-2.91	90.60	-2.45	90.80	-2.49

Table (A.5) contains the data used for plotting Figure (3.1), the relative bias in estimation of annual abundance for each model, for each year of data for pooled age-class models.

Table A.5: *Median relative bias in total annual abundance estimates for pooled age-class models. Results indicate low median bias for all models when the auxiliary catch-effort likelihood component of Equation (1.7) is omitted, but positive bias for models* $N_A s_F c_F(p)$, $N_{SR,R} s_F c_F(p)$ *and* $s_F c_F(p)$ *when it is included. Results based on* $n = 1000$ *replicates, with* $s = 0.84$, $c = -1.5$, $\gamma = 0.0$, *and total annual abundance* ≈ 4000. *"Aux. Like." = Auxiliary catch-effort likelihood component used (With) or not used (Without).*

Table A.5
Annual Abundance Percent Bias

Aux. Like.	Model	Year	No Variation	Low Variation	Medium Variation	High Variation
Without	$N_{SR,R} s_F c_F(p)$	1	0.19%	1.76%	0.73%	-1.74%
Without	$N_{SR,R} s_F c_F(p)$	2	-0.08%	2.63%	1.09%	-2.45%
Without	$N_{SR,R} s_F c_F(p)$	3	-0.11%	1.15%	2.44%	-2.24%
Without	$N_{SR,R} s_F c_F(p)$	4	-0.07%	1.48%	3.02%	-0.27%
Without	$N_{SR,R} s_F c_F(p)$	5	-0.22%	2.05%	3.56%	3.82%
Without	$N_{SR,R} s_F c_F(p)$	6	0.2%	2.66%	2.24%	2.85%
Without	$N_{SR,R} s_F c_F(p)$	7	-0.01%	2.11%	1.39%	3.42%
Without	$N_{SR,R} s_F c_F(p)$	8	0.05%	1%	1.38%	4.51%
Without	$N_{SR,R} s_F c_F(p)$	9	-0.08%	1.02%	3.57%	5.79%
Without	$N_{SR,R} s_F c_F(p)$	10	-0.22%	0.91%	4.54%	6.28%
Without	$N_{SR,R} s_F c_F(p)$	11	-0.34%	0.41%	5.11%	3.78%
Without	$N_{SR,R} s_F c_F(p)$	12	-0.33%	0.21%	4.91%	4.93%
Without	$N_{SR,R} s_F c_F(p)$	13	-0.25%	0.25%	5.63%	5.73%
Without	$N_{SR,R} s_F c_F(p)$	14	-0.07%	0.03%	5.62%	8.42%
Without	$N_{SR,R} s_F c_F(p)$	15	-0.36%	0.26%	4.14%	5.86%
Without	$N_{SR,R} s_F c_F(p)$	16	-0.23%	0.39%	3.66%	4.69%
Without	$N_{SR,R} s_F c_F(p)$	17	-0.26%	0.43%	2.72%	2.91%
Without	$N_{SR,R} s_F c_F(p)$	18	-0.3%	0.24%	3.16%	7.3%
Without	$N_{SR,R} s_F c_F(p)$	19	-0.17%	1.23%	2.82%	5.8%
Without	$N_{SR,R} s_F c_F(p)$	20	-0.39%	0.43%	0.49%	1.41%
Without	$N_{SR,R} s_F c_F(p)$	21	-0.33%	0.89%	-0.18%	4.96%
Without	$N_{SR,R} s_F c_F(p)$	22	-0.49%	-0.22%	0.62%	1.64%
Without	$N_{SR,R} s_F c_F(p)$	23	-0.58%	-0.46%	-0.97%	-0.48%
Without	$N_{SR,R} s_F c_F(p)$	24	-0.92%	-0.96%	-1.94%	-0.16%
Without	$N_{SR,R} s_F c_F(p)$	25	-0.79%	-0.41%	-2.48%	-3.67%
Without	$N_{SR,R} s_F c_R(p)$	1	0.31%	1.96%	0.81%	1.04%
Without	$N_{SR,R} s_F c_R(p)$	2	-0.01%	1.66%	0.44%	0.67%
Without	$N_{SR,R} s_F c_R(p)$	3	0.06%	1.42%	0.55%	0.59%
Without	$N_{SR,R} s_F c_R(p)$	4	0.03%	1.35%	0.43%	1.35%
Without	$N_{SR,R} s_F c_R(p)$	5	-0.01%	1.08%	1.64%	0.89%
Without	$N_{SR,R} s_F c_R(p)$	6	0.28%	1.4%	1.47%	0.72%
Without	$N_{SR,R} s_F c_R(p)$	7	0.18%	0.86%	1.24%	0.41%
Without	$N_{SR,R} s_F c_R(p)$	8	0.16%	1.03%	0.34%	1.55%
Without	$N_{SR,R} s_F c_R(p)$	9	0.04%	0.98%	-0.37%	1.1%
Without	$N_{SR,R} s_F c_R(p)$	10	-0.2%	0.62%	0.82%	-0.9%
Without	$N_{SR,R} s_F c_R(p)$	11	-0.32%	-0.05%	-0.65%	-0.41%
Without	$N_{SR,R} s_F c_R(p)$	12	-0.16%	0.15%	-0.07%	0.49%
Without	$N_{SR,R} s_F c_R(p)$	13	-0.17%	0.13%	-0.99%	0.16%
Without	$N_{SR,R} s_F c_R(p)$	14	-0.06%	0.34%	-0.35%	-0.44%
Without	$N_{SR,R} s_F c_R(p)$	15	-0.3%	0.48%	-0.3%	0.67%
Without	$N_{SR,R} s_F c_R(p)$	16	-0.15%	0.87%	0.06%	0.26%
Without	$N_{SR,R} s_F c_R(p)$	17	-0.19%	0.87%	-0.73%	-0.63%
Without	$N_{SR,R} s_F c_R(p)$	18	-0.3%	1.55%	-1.15%	-0.83%
Without	$N_{SR,R} s_F c_R(p)$	19	-0.14%	1.15%	-0.66%	-0.04%

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Table A.5 – continued from previous page

Annual Abundance Percent Bias						
Aux. Like.	**Model**	**Year**	**No Variation**	**Low Variation**	**Medium Variation**	**High Variation**
Without	$N_{SR,R}s_Fc_R(p)$	20	-0.39%	1.22%	-0.99%	-0.66%
Without	$N_{SR,R}s_Fc_R(p)$	21	-0.29%	0.99%	-0.54%	-0.67%
Without	$N_{SR,R}s_Fc_R(p)$	22	-0.46%	0.73%	-1.01%	-0.9%
Without	$N_{SR,R}s_Fc_R(p)$	23	-0.55%	0.38%	-1.19%	-0.72%
Without	$N_{SR,R}s_Fc_R(p)$	24	-0.96%	0.42%	-0.81%	-1.38%
Without	$N_{SR,R}s_Fc_R(p)$	25	-0.74%	0.48%	-1.05%	-1.63%
Without	$N_{SR,R}s_Rc_R(p)$	1	0.27%	1.85%	0.35%	-0.03%
Without	$N_{SR,R}s_Rc_R(p)$	2	-0.01%	1.63%	0.12%	0.18%
Without	$N_{SR,R}s_Rc_R(p)$	3	0.04%	1.5%	0.29%	-0.23%
Without	$N_{SR,R}s_Rc_R(p)$	4	-0.03%	1.36%	0.47%	0.96%
Without	$N_{SR,R}s_Rc_R(p)$	5	-0.05%	1.1%	1.35%	0.26%
Without	$N_{SR,R}s_Rc_R(p)$	6	0.28%	1.12%	1.05%	1.13%
Without	$N_{SR,R}s_Rc_R(p)$	7	0.18%	0.62%	1.14%	0.72%
Without	$N_{SR,R}s_Rc_R(p)$	8	0.16%	0.95%	0.15%	1.03%
Without	$N_{SR,R}s_Rc_R(p)$	9	0.06%	0.77%	-0.26%	0.23%
Without	$N_{SR,R}s_Rc_R(p)$	10	-0.2%	0.45%	0.8%	-0.66%
Without	$N_{SR,R}s_Rc_R(p)$	11	-0.32%	-0.06%	-1.03%	-0.64%
Without	$N_{SR,R}s_Rc_R(p)$	12	-0.16%	0.43%	-0.07%	0.41%
Without	$N_{SR,R}s_Rc_R(p)$	13	-0.19%	0.34%	-0.64%	0.34%
Without	$N_{SR,R}s_Rc_R(p)$	14	-0.01%	0.56%	-0.13%	-0.78%
Without	$N_{SR,R}s_Rc_R(p)$	15	-0.3%	0.88%	-0.42%	0.7%
Without	$N_{SR,R}s_Rc_R(p)$	16	-0.13%	0.75%	-0.51%	0.21%
Without	$N_{SR,R}s_Rc_R(p)$	17	-0.16%	0.61%	-0.52%	-0.42%
Without	$N_{SR,R}s_Rc_R(p)$	18	-0.29%	1.43%	-1.3%	-0.61%
Without	$N_{SR,R}s_Rc_R(p)$	19	-0.13%	1.2%	-0.92%	-0.19%
Without	$N_{SR,R}s_Rc_R(p)$	20	-0.39%	1.26%	-1.11%	-1.02%
Without	$N_{SR,R}s_Rc_R(p)$	21	-0.29%	1.06%	-0.65%	-0.83%
Without	$N_{SR,R}s_Rc_R(p)$	22	-0.46%	0.83%	-0.64%	-0.39%
Without	$N_{SR,R}s_Rc_R(p)$	23	-0.55%	0.39%	-1.22%	-0.39%
Without	$N_{SR,R}s_Rc_R(p)$	24	-0.96%	0.48%	-1%	-1.03%
Without	$N_{SR,R}s_Rc_R(p)$	25	-0.76%	0.46%	-0.81%	-0.39%
Without	$s_Fc_F(p)$	1	-0.41%	1.13%	1.61%	1.19%
Without	$s_Fc_F(p)$	2	-0.69%	1.16%	2.66%	2.5%
Without	$s_Fc_F(p)$	3	-0.63%	1.04%	1.42%	3.16%
Without	$s_Fc_F(p)$	4	-0.59%	1.2%	1.56%	2.65%
Without	$s_Fc_F(p)$	5	-0.27%	1.01%	0.95%	3.46%
Without	$s_Fc_F(p)$	6	-0.26%	0.76%	1.58%	1%
Without	$s_Fc_F(p)$	7	-0.18%	1.21%	1%	3.99%
Without	$s_Fc_F(p)$	8	-0.27%	1.59%	1.44%	2.55%
Without	$s_Fc_F(p)$	9	-0.68%	0.1%	0.46%	2.22%
Without	$s_Fc_F(p)$	10	-0.24%	0.68%	2.03%	-0.24%
Without	$s_Fc_F(p)$	11	-0.58%	1.85%	2.57%	2.85%
Without	$s_Fc_F(p)$	12	-0.62%	0.53%	1.63%	2.27%
Without	$s_Fc_F(p)$	13	-0.42%	0.53%	1.61%	1.62%
Without	$s_Fc_F(p)$	14	-0.18%	1.48%	1.25%	1.51%
Without	$s_Fc_F(p)$	15	-0.65%	0.51%	1.74%	1.82%
Without	$s_Fc_F(p)$	16	-0.33%	1.87%	0.88%	2.44%
Without	$s_Fc_F(p)$	17	-0.25%	0.48%	2.64%	2.44%
Without	$s_Fc_F(p)$	18	-0.4%	1.56%	2.68%	3.48%
Without	$s_Fc_F(p)$	19	0.05%	0.74%	2.54%	1.02%
Without	$s_Fc_F(p)$	20	-0.17%	1.36%	0.47%	1.42%
Without	$s_Fc_F(p)$	21	-0.26%	0.84%	1.89%	2%
Without	$s_Fc_F(p)$	22	-0.38%	1.11%	2.71%	1.8%
Without	$s_Fc_F(p)$	23	-0.33%	0.93%	1.56%	1%
Without	$s_Fc_F(p)$	24	-0.37%	0.73%	0.98%	0%
Without	$s_Fc_F(p)$	25	-0.66%	1.52%	1.93%	-0.78%
Without	$N_As_Fc_F(p)$	1	0.57%	1.79%	0.91%	-2.16%
Without	$N_As_Fc_F(p)$	2	0.87%	2.81%	2.99%	3.17%
Without	$N_As_Fc_F(p)$	3	1.06%	2.75%	6.7%	8.54%

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Table A.5 – continued from previous page

Annual Abundance Percent Bias						
Aux. Like.	**Model**	**Year**	**No Variation**	**Low Variation**	**Medium Variation**	**High Variation**
Without	$N_A s_F c_F(p)$	4	2.22%	4.14%	13.77%	13.11%
Without	$N_A s_F c_F(p)$	5	4.07%	5.79%	18.25%	24.53%
Without	$N_A s_F c_F(p)$	6	4.48%	9.2%	18.21%	31.45%
Without	$N_A s_F c_F(p)$	7	5.8%	10.58%	21.18%	45.58%
Without	$N_A s_F c_F(p)$	8	6.08%	10.62%	25.47%	54.38%
Without	$N_A s_F c_F(p)$	9	6.42%	11.87%	32.33%	63.39%
Without	$N_A s_F c_F(p)$	10	7.29%	12.4%	36.67%	74.05%
Without	$N_A s_F c_F(p)$	11	6.56%	14.2%	42.32%	90.59%
Without	$N_A s_F c_F(p)$	12	7.86%	16%	49.16%	101.64%
Without	$N_A s_F c_F(p)$	13	7.51%	18.14%	51.49%	110.25%
Without	$N_A s_F c_F(p)$	14	7.62%	18.86%	55.09%	138.55%
Without	$N_A s_F c_F(p)$	15	9.19%	19.96%	59.24%	165.65%
Without	$N_A s_F c_F(p)$	16	9.72%	20.8%	71.69%	177.56%
Without	$N_A s_F c_F(p)$	17	10.15%	22.98%	69.43%	195.49%
Without	$N_A s_F c_F(p)$	18	11.01%	24.71%	83%	231.21%
Without	$N_A s_F c_F(p)$	19	12.45%	26.37%	91.37%	271.54%
Without	$N_A s_F c_F(p)$	20	13.58%	28.11%	95.42%	287.74%
Without	$N_A s_F c_F(p)$	21	14.89%	31.7%	100.88%	368.94%
Without	$N_A s_F c_F(p)$	22	13.3%	32.92%	106.04%	423.39%
Without	$N_A s_F c_F(p)$	23	15.06%	31.75%	110.28%	502.51%
Without	$N_A s_F c_F(p)$	24	10%	20.09%	67.35%	307.13%
Without	$N_A s_F c_F(p)$	25	6.64%	13.05%	42.85%	179.11%
Without	$s_F c_R(p)$	1	-0.51%	0.49%	0.07%	1.19%
Without	$s_F c_R(p)$	2	-0.65%	0.58%	0.25%	1.8%
Without	$s_F c_R(p)$	3	-0.67%	0.21%	0.21%	1.04%
Without	$s_F c_R(p)$	4	-0.48%	0.1%	-0.03%	0.59%
Without	$s_F c_R(p)$	5	-0.17%	0.41%	0.21%	0.73%
Without	$s_F c_R(p)$	6	-0.25%	0.12%	0.14%	0.81%
Without	$s_F c_R(p)$	7	-0.28%	0.17%	0.25%	0.84%
Without	$s_F c_R(p)$	8	-0.35%	0.22%	-0.04%	0.43%
Without	$s_F c_R(p)$	9	-0.54%	-0.01%	-0.13%	0.37%
Without	$s_F c_R(p)$	10	-0.32%	0.05%	-0.06%	-0.03%
Without	$s_F c_R(p)$	11	-0.55%	0.43%	0.02%	0.17%
Without	$s_F c_R(p)$	12	-0.66%	0.32%	0.06%	0.96%
Without	$s_F c_R(p)$	13	-0.42%	0.06%	0.21%	0.81%
Without	$s_F c_R(p)$	14	-0.22%	0.18%	0.4%	0.57%
Without	$s_F c_R(p)$	15	-0.64%	0.15%	0.52%	0.78%
Without	$s_F c_R(p)$	16	-0.52%	0.77%	0.32%	-0.15%
Without	$s_F c_R(p)$	17	-0.26%	0.28%	0.3%	0.35%
Without	$s_F c_R(p)$	18	-0.41%	0.27%	-0.32%	0.56%
Without	$s_F c_R(p)$	19	-0.14%	0.19%	-0.07%	0%
Without	$s_F c_R(p)$	20	-0.19%	0.3%	-0.4%	0.32%
Without	$s_F c_R(p)$	21	-0.31%	0.45%	0.04%	0.03%
Without	$s_F c_R(p)$	22	-0.39%	0.28%	-0.05%	0.44%
Without	$s_F c_R(p)$	23	-0.45%	0.13%	-0.04%	-0.18%
Without	$s_F c_R(p)$	24	-0.45%	0.3%	-0.13%	-0.53%
Without	$s_F c_R(p)$	25	-0.67%	0.34%	0.37%	-1.63%
Without	$N_A s_F c_R(p)$	1	0.57%	2%	0.92%	2.18%
Without	$N_A s_F c_R(p)$	2	0.87%	2.98%	2.43%	3.14%
Without	$N_A s_F c_R(p)$	3	1.06%	3.94%	3.64%	6.15%
Without	$N_A s_F c_R(p)$	4	2.22%	5.72%	6.21%	11.71%
Without	$N_A s_F c_R(p)$	5	4.07%	6.46%	7.7%	13.85%
Without	$N_A s_F c_R(p)$	6	4.48%	9.28%	7.61%	18.39%
Without	$N_A s_F c_R(p)$	7	5.8%	9.96%	9.45%	23.67%
Without	$N_A s_F c_R(p)$	8	6.08%	10.52%	11.87%	27.14%
Without	$N_A s_F c_R(p)$	9	6.42%	12.28%	15.8%	30.68%
Without	$N_A s_F c_R(p)$	10	7.29%	12.57%	17.37%	38.29%
Without	$N_A s_F c_R(p)$	11	6.56%	13.66%	19.63%	44.34%
Without	$N_A s_F c_R(p)$	12	7.86%	14.84%	21.57%	51.55%

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Table A.5 – continued from previous page

Annual Abundance Percent Bias						
Aux. Like.	**Model**	**Year**	**No Variation**	**Low Variation**	**Medium Variation**	**High Variation**
Without	$N_A s_F c_R(p)$	13	7.51%	15.42%	22.6%	55.53%
Without	$N_A s_F c_R(p)$	14	7.62%	17.48%	27.01%	67.95%
Without	$N_A s_F c_R(p)$	15	9.19%	18.38%	30.28%	82.16%
Without	$N_A s_F c_R(p)$	16	9.72%	18.99%	33.42%	85.63%
Without	$N_A s_F c_R(p)$	17	10.15%	20.97%	35.23%	88.06%
Without	$N_A s_F c_R(p)$	18	11.01%	22.06%	39.06%	99.69%
Without	$N_A s_F c_R(p)$	19	12.45%	22.83%	43.03%	117.05%
Without	$N_A s_F c_R(p)$	20	13.58%	24%	45.18%	130.87%
Without	$N_A s_F c_R(p)$	21	14.89%	27.53%	45.04%	143.83%
Without	$N_A s_F c_R(p)$	22	13.3%	29.48%	45.42%	158.03%
Without	$N_A s_F c_R(p)$	23	15.06%	30.75%	56.36%	192.49%
Without	$N_A s_F c_R(p)$	24	10%	19.36%	36.41%	123.65%
Without	$N_A s_F c_R(p)$	25	6.64%	12.38%	26.83%	80.54%
Without	$s_R c_R(p)$	1	-0.52%	0.63%	-0.45%	0.41%
Without	$s_R c_R(p)$	2	-0.64%	0.6%	-0.24%	0.51%
Without	$s_R c_R(p)$	3	-0.67%	0.12%	-0.56%	0.33%
Without	$s_R c_R(p)$	4	-0.58%	0.05%	-0.65%	0.38%
Without	$s_R c_R(p)$	5	-0.09%	0.37%	-0.27%	0.7%
Without	$s_R c_R(p)$	6	-0.26%	-0.03%	-0.04%	0.31%
Without	$s_R c_R(p)$	7	-0.29%	0.1%	0.13%	0.11%
Without	$s_R c_R(p)$	8	-0.33%	0.16%	-0.18%	0.09%
Without	$s_R c_R(p)$	9	-0.52%	-0.13%	-0.28%	-0.22%
Without	$s_R c_R(p)$	10	-0.32%	-0.13%	0.08%	-0.06%
Without	$s_R c_R(p)$	11	-0.6%	0.38%	0.02%	-0.16%
Without	$s_R c_R(p)$	12	-0.66%	0.36%	-0.09%	0.28%
Without	$s_R c_R(p)$	13	-0.43%	0.06%	-0.14%	0.62%
Without	$s_R c_R(p)$	14	-0.22%	0.46%	0.39%	0.55%
Without	$s_R c_R(p)$	15	-0.68%	0.21%	0.31%	0.36%
Without	$s_R c_R(p)$	16	-0.53%	0.92%	0.07%	-0.48%
Without	$s_R c_R(p)$	17	-0.24%	0.22%	0.12%	0.23%
Without	$s_R c_R(p)$	18	-0.44%	0.13%	-0.32%	-0.23%
Without	$s_R c_R(p)$	19	-0.09%	0.14%	-0.37%	-0.14%
Without	$s_R c_R(p)$	20	-0.2%	0.02%	-0.87%	0.09%
Without	$s_R c_R(p)$	21	-0.37%	0.51%	-0.84%	0.19%
Without	$s_R c_R(p)$	22	-0.37%	0.05%	-0.2%	0.13%
Without	$s_R c_R(p)$	23	-0.43%	0.01%	-0.2%	-0.75%
Without	$s_R c_R(p)$	24	-0.51%	0.34%	-0.19%	-0.67%
Without	$s_R c_R(p)$	25	-0.67%	0.36%	0.21%	-1.28%
Without	$N_A s_R c_R(p)$	1	0.57%	2%	0.81%	1.91%
Without	$N_A s_R c_R(p)$	2	0.87%	2.98%	2.43%	2.81%
Without	$N_A s_R c_R(p)$	3	1.06%	3.94%	3.64%	6.34%
Without	$N_A s_R c_R(p)$	4	2.22%	5.72%	6.21%	12.26%
Without	$N_A s_R c_R(p)$	5	4.07%	6.46%	7.7%	13.79%
Without	$N_A s_R c_R(p)$	6	4.48%	9.44%	7.61%	18.57%
Without	$N_A s_R c_R(p)$	7	5.8%	10.02%	9.18%	24.05%
Without	$N_A s_R c_R(p)$	8	6.08%	10.83%	11.56%	27.57%
Without	$N_A s_R c_R(p)$	9	6.42%	12.35%	15.73%	30.7%
Without	$N_A s_R c_R(p)$	10	7.29%	12.57%	17.09%	38.29%
Without	$N_A s_R c_R(p)$	11	6.56%	13.66%	19.44%	44.6%
Without	$N_A s_R c_R(p)$	12	7.86%	14.84%	20.04%	51.78%
Without	$N_A s_R c_R(p)$	13	7.51%	15.42%	22.38%	56.18%
Without	$N_A s_R c_R(p)$	14	7.62%	17.48%	26.57%	67.95%
Without	$N_A s_R c_R(p)$	15	9.19%	18.38%	30.18%	81.62%
Without	$N_A s_R c_R(p)$	16	9.72%	18.99%	33.42%	88.63%
Without	$N_A s_R c_R(p)$	17	10.15%	20.97%	34.87%	88.2%
Without	$N_A s_R c_R(p)$	18	11.01%	22.06%	38.81%	100.94%
Without	$N_A s_R c_R(p)$	19	12.45%	22.83%	42.11%	119.73%
Without	$N_A s_R c_R(p)$	20	13.58%	24%	45.06%	131.97%
Without	$N_A s_R c_R(p)$	21	14.89%	27.53%	44.74%	144.72%

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Table A.5 – continued from previous page

Annual Abundance Percent Bias						
Aux. Like.	Model	Year	No Variation	Low Variation	Medium Variation	High Variation
Without	$N_A s_R c_R(p)$	22	13.3%	29.48%	45.42%	160.62%
Without	$N_A s_R c_R(p)$	23	15.06%	30.75%	56.36%	196.68%
Without	$N_A s_R c_R(p)$	24	10%	19.36%	36.41%	124.24%
Without	$N_A s_R c_R(p)$	25	6.64%	12.38%	26.83%	81.9%
With	$N_{SR,R} s_F c_F(p)$	1	0.07%	3.8%	12.77%	19.63%
With	$N_{SR,R} s_F c_F(p)$	2	-0.03%	4.36%	14.36%	21.96%
With	$N_{SR,R} s_F c_F(p)$	3	0.06%	4.97%	13.22%	23.39%
With	$N_{SR,R} s_F c_F(p)$	4	0.01%	4.57%	12.82%	24.51%
With	$N_{SR,R} s_F c_F(p)$	5	-0.19%	3.85%	14.15%	23.5%
With	$N_{SR,R} s_F c_F(p)$	6	-0.1%	3.82%	14.42%	20.18%
With	$N_{SR,R} s_F c_F(p)$	7	0.19%	4.18%	13.18%	19.66%
With	$N_{SR,R} s_F c_F(p)$	8	0.08%	4.09%	13.88%	23.33%
With	$N_{SR,R} s_F c_F(p)$	9	-0.03%	3.76%	13.67%	19.41%
With	$N_{SR,R} s_F c_F(p)$	10	0.19%	3.41%	14.97%	16.67%
With	$N_{SR,R} s_F c_F(p)$	11	0.05%	3.96%	15.38%	20.21%
With	$N_{SR,R} s_F c_F(p)$	12	0.05%	3.47%	14.37%	20.12%
With	$N_{SR,R} s_F c_F(p)$	13	0.08%	3.45%	15.09%	20.67%
With	$N_{SR,R} s_F c_F(p)$	14	0%	4.09%	14.12%	21.11%
With	$N_{SR,R} s_F c_F(p)$	15	0.03%	3.71%	14.77%	21.45%
With	$N_{SR,R} s_F c_F(p)$	16	-0.06%	3.98%	13.85%	19.53%
With	$N_{SR,R} s_F c_F(p)$	17	0.06%	4.25%	14.74%	22.37%
With	$N_{SR,R} s_F c_F(p)$	18	-0.01%	4.63%	15.78%	23.39%
With	$N_{SR,R} s_F c_F(p)$	19	0.24%	4.05%	13.78%	22.03%
With	$N_{SR,R} s_F c_F(p)$	20	0.14%	4.18%	13.02%	21.33%
With	$N_{SR,R} s_F c_F(p)$	21	-0.04%	4.2%	13.31%	20.12%
With	$N_{SR,R} s_F c_F(p)$	22	-0.12%	4.24%	14.5%	20.14%
With	$N_{SR,R} s_F c_F(p)$	23	0%	4.33%	13.99%	22.03%
With	$N_{SR,R} s_F c_F(p)$	24	-0.25%	4.05%	13.32%	19.15%
With	$N_{SR,R} s_F c_F(p)$	25	-0.24%	4.19%	12.39%	18.52%
With	$N_{SR,R} s_F c_R(p)$	1	-0.67%	0.07%	-0.07%	1.61%
With	$N_{SR,R} s_F c_R(p)$	2	-0.39%	0.03%	-0.49%	1.8%
With	$N_{SR,R} s_F c_R(p)$	3	-0.4%	0.13%	-0.13%	2%
With	$N_{SR,R} s_F c_R(p)$	4	-0.41%	-0.2%	0.01%	1.9%
With	$N_{SR,R} s_F c_R(p)$	5	-0.46%	-0.05%	0.33%	1.01%
With	$N_{SR,R} s_F c_R(p)$	6	-0.3%	0.24%	0.38%	1.77%
With	$N_{SR,R} s_F c_R(p)$	7	-0.29%	-0.47%	0.21%	1.27%
With	$N_{SR,R} s_F c_R(p)$	8	-0.24%	-0.23%	-0.09%	1.22%
With	$N_{SR,R} s_F c_R(p)$	9	-0.35%	-0.16%	-0.12%	1.15%
With	$N_{SR,R} s_F c_R(p)$	10	-0.4%	-0.26%	-0.1%	1.49%
With	$N_{SR,R} s_F c_R(p)$	11	-0.43%	-0.06%	-0.08%	0.91%
With	$N_{SR,R} s_F c_R(p)$	12	-0.5%	0.07%	-0.25%	1.61%
With	$N_{SR,R} s_F c_R(p)$	13	-0.58%	-0.27%	0.08%	1.1%
With	$N_{SR,R} s_F c_R(p)$	14	-0.55%	-0.15%	0.06%	1.19%
With	$N_{SR,R} s_F c_R(p)$	15	-0.57%	-0.25%	0.09%	1.58%
With	$N_{SR,R} s_F c_R(p)$	16	-0.49%	0.3%	0.24%	0.73%
With	$N_{SR,R} s_F c_R(p)$	17	-0.41%	0.13%	-0.21%	0.72%
With	$N_{SR,R} s_F c_R(p)$	18	-0.46%	-0.22%	-0.2%	0.9%
With	$N_{SR,R} s_F c_R(p)$	19	-0.56%	-0.26%	-0.55%	1.27%
With	$N_{SR,R} s_F c_R(p)$	20	-0.45%	-0.15%	-0.94%	0.47%
With	$N_{SR,R} s_F c_R(p)$	21	-0.48%	-0.13%	-0.87%	0.48%
With	$N_{SR,R} s_F c_R(p)$	22	-0.7%	-0.44%	-0.73%	0.2%
With	$N_{SR,R} s_F c_R(p)$	23	-0.36%	-0.3%	-0.89%	-1.23%
With	$N_{SR,R} s_F c_R(p)$	24	-0.61%	-0.53%	-1.44%	-2.49%
With	$N_{SR,R} s_F c_R(p)$	25	-0.52%	-0.42%	-1.62%	-2.98%
With	$N_{SR,R} s_R c_R(p)$	1	-0.74%	0.01%	-0.24%	0.46%
With	$N_{SR,R} s_R c_R(p)$	2	-0.36%	0.14%	-0.64%	1.34%
With	$N_{SR,R} s_R c_R(p)$	3	-0.42%	0.02%	-0.26%	1.54%
With	$N_{SR,R} s_R c_R(p)$	4	-0.38%	-0.22%	-0.21%	1.29%
With	$N_{SR,R} s_R c_R(p)$	5	-0.41%	-0.09%	0.27%	0.41%

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Table A.5 – continued from previous page

Annual Abundance Percent Bias						
Aux. Like.	Model	Year	No Variation	Low Variation	Medium Variation	High Variation
With	$N_{SR,R}s_Rc_R(p)$	6	-0.25%	0.32%	0.19%	0.8%
With	$N_{SR,R}s_Rc_R(p)$	7	-0.26%	-0.42%	0.02%	0.51%
With	$N_{SR,R}s_Rc_R(p)$	8	-0.24%	-0.27%	0.02%	0.78%
With	$N_{SR,R}s_Rc_R(p)$	9	-0.32%	-0.2%	-0.16%	0.61%
With	$N_{SR,R}s_Rc_R(p)$	10	-0.46%	-0.23%	-0.14%	0.5%
With	$N_{SR,R}s_Rc_R(p)$	11	-0.54%	-0.06%	-0.08%	0.79%
With	$N_{SR,R}s_Rc_R(p)$	12	-0.55%	0.1%	-0.22%	1.29%
With	$N_{SR,R}s_Rc_R(p)$	13	-0.61%	-0.26%	-0.04%	0.66%
With	$N_{SR,R}s_Rc_R(p)$	14	-0.54%	-0.12%	-0.12%	0.63%
With	$N_{SR,R}s_Rc_R(p)$	15	-0.47%	-0.2%	-0.09%	1.25%
With	$N_{SR,R}s_Rc_R(p)$	16	-0.45%	0.33%	0.2%	0.31%
With	$N_{SR,R}s_Rc_R(p)$	17	-0.36%	0.13%	-0.23%	0.64%
With	$N_{SR,R}s_Rc_R(p)$	18	-0.4%	-0.23%	-0.32%	0.63%
With	$N_{SR,R}s_Rc_R(p)$	19	-0.46%	-0.26%	-0.61%	1.27%
With	$N_{SR,R}s_Rc_R(p)$	20	-0.42%	-0.22%	-1.02%	0.3%
With	$N_{SR,R}s_Rc_R(p)$	21	-0.47%	-0.15%	-0.8%	0.39%
With	$N_{SR,R}s_Rc_R(p)$	22	-0.68%	-0.41%	-0.77%	-0.04%
With	$N_{SR,R}s_Rc_R(p)$	23	-0.35%	-0.28%	-0.82%	-1.24%
With	$N_{SR,R}s_Rc_R(p)$	24	-0.66%	-0.53%	-1.64%	-2.49%
With	$N_{SR,R}s_Rc_R(p)$	25	-0.6%	-0.42%	-1.64%	-2.47%
With	$s_Fc_F(p)$	1	0.14%	11.18%	38.66%	60.22%
With	$s_Fc_F(p)$	2	-0.09%	10.66%	41.13%	61.26%
With	$s_Fc_F(p)$	3	0%	9.92%	41.72%	65.05%
With	$s_Fc_F(p)$	4	-0.28%	10.33%	39.98%	66.44%
With	$s_Fc_F(p)$	5	0.18%	10.01%	43.29%	67.05%
With	$s_Fc_F(p)$	6	0.23%	11.01%	40.15%	60.11%
With	$s_Fc_F(p)$	7	0.1%	11.78%	39.38%	61.07%
With	$s_Fc_F(p)$	8	0.09%	10.89%	40.3%	62.98%
With	$s_Fc_F(p)$	9	0.05%	9.89%	41.69%	58.12%
With	$s_Fc_F(p)$	10	0.46%	10.41%	39.8%	54.11%
With	$s_Fc_F(p)$	11	0.1%	11.04%	43.04%	59.4%
With	$s_Fc_F(p)$	12	0.01%	10.11%	41.21%	61.56%
With	$s_Fc_F(p)$	13	0.12%	9.96%	43.18%	59.23%
With	$s_Fc_F(p)$	14	-0.08%	11.6%	39.46%	64.1%
With	$s_Fc_F(p)$	15	-0.31%	10.45%	42.65%	62.07%
With	$s_Fc_F(p)$	16	0.1%	11.41%	40.83%	60.29%
With	$s_Fc_F(p)$	17	0.16%	11.14%	41.44%	64.77%
With	$s_Fc_F(p)$	18	-0.13%	11.06%	41.07%	67.66%
With	$s_Fc_F(p)$	19	0.18%	10.37%	40.29%	63.37%
With	$s_Fc_F(p)$	20	0.11%	10.51%	41.05%	60.05%
With	$s_Fc_F(p)$	21	0.2%	10.92%	40.38%	64.91%
With	$s_Fc_F(p)$	22	0.39%	11.27%	41.33%	63.56%
With	$s_Fc_F(p)$	23	-0.19%	10.72%	40.14%	64.39%
With	$s_Fc_F(p)$	24	0.14%	10.4%	41.37%	60.9%
With	$s_Fc_F(p)$	25	-0.28%	10.85%	41.79%	61.65%
With	$N_As_Fc_F(p)$	1	-1.98%	1.13%	7.23%	10.77%
With	$N_As_Fc_F(p)$	2	-2.01%	1.85%	8.55%	11.26%
With	$N_As_Fc_F(p)$	3	-2.09%	2.14%	7.15%	13.12%
With	$N_As_Fc_F(p)$	4	-1.89%	1.5%	7.24%	14.99%
With	$N_As_Fc_F(p)$	5	-2.09%	0.55%	8.19%	14.61%
With	$N_As_Fc_F(p)$	6	-1.54%	0.74%	8%	11.95%
With	$N_As_Fc_F(p)$	7	-1.86%	1.28%	5.93%	12.24%
With	$N_As_Fc_F(p)$	8	-1.61%	1.45%	8.8%	12.88%
With	$N_As_Fc_F(p)$	9	-1.95%	1.25%	8.22%	10.28%
With	$N_As_Fc_F(p)$	10	-1.86%	0.54%	8.68%	8.7%
With	$N_As_Fc_F(p)$	11	-2.15%	1.11%	8.33%	12.03%
With	$N_As_Fc_F(p)$	12	-2.07%	0.7%	9.26%	12.06%
With	$N_As_Fc_F(p)$	13	-2.03%	0.37%	8.41%	12.01%
With	$N_As_Fc_F(p)$	14	-1.78%	1.59%	7.3%	11.84%

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Table A.5 – continued from previous page

Annual Abundance Percent Bias						
Aux. Like.	**Model**	**Year**	**No Variation**	**Low Variation**	**Medium Variation**	**High Variation**
With	$N_A s_F c_F(p)$	15	-1.92%	0.64%	8.35%	12.09%
With	$N_A s_F c_F(p)$	16	-2.08%	1.19%	7.06%	10.48%
With	$N_A s_F c_F(p)$	17	-1.9%	1.23%	8.19%	12.31%
With	$N_A s_F c_F(p)$	18	-2.1%	1.86%	9.21%	14.01%
With	$N_A s_F c_F(p)$	19	-1.9%	1.54%	7.46%	13.26%
With	$N_A s_F c_F(p)$	20	-1.82%	1.39%	6.99%	11.55%
With	$N_A s_F c_F(p)$	21	-1.77%	1.28%	7.08%	11.7%
With	$N_A s_F c_F(p)$	22	-1.99%	1.75%	8.21%	10.9%
With	$N_A s_F c_F(p)$	23	-1.97%	1.51%	7.55%	11.14%
With	$N_A s_F c_F(p)$	24	-2.16%	1.3%	6.67%	10.47%
With	$N_A s_F c_F(p)$	25	-2.19%	1.05%	6.11%	9.76%
With	$s_F c_R(p)$	1	-0.96%	-1.09%	-0.71%	0.75%
With	$s_F c_R(p)$	2	-1.02%	-1.03%	-0.79%	0.48%
With	$s_F c_R(p)$	3	-0.95%	-0.96%	-0.99%	0.12%
With	$s_F c_R(p)$	4	-0.97%	-1.04%	-1.08%	0.36%
With	$s_F c_R(p)$	5	-0.92%	-0.72%	-0.46%	-0.05%
With	$s_F c_R(p)$	6	-1.03%	-1.11%	-1.16%	0.25%
With	$s_F c_R(p)$	7	-0.69%	-1.09%	-0.76%	0.04%
With	$s_F c_R(p)$	8	-0.97%	-1.1%	-0.93%	-0.04%
With	$s_F c_R(p)$	9	-0.95%	-1.21%	-1.02%	-0.45%
With	$s_F c_R(p)$	10	-1.04%	-1.23%	-1.52%	-0.42%
With	$s_F c_R(p)$	11	-1.18%	-1.14%	-1.24%	-0.58%
With	$s_F c_R(p)$	12	-1.15%	-1.12%	-0.91%	-0.05%
With	$s_F c_R(p)$	13	-0.9%	-1.2%	-0.78%	-0.34%
With	$s_F c_R(p)$	14	-1.08%	-0.96%	-0.62%	-0.19%
With	$s_F c_R(p)$	15	-1.09%	-0.93%	-0.5%	-0.28%
With	$s_F c_R(p)$	16	-1.21%	-1.05%	-0.74%	-1.44%
With	$s_F c_R(p)$	17	-1.09%	-1.15%	-1.03%	-0.92%
With	$s_F c_R(p)$	18	-1.04%	-1%	-1.19%	-1.11%
With	$s_F c_R(p)$	19	-1.14%	-0.86%	-1.39%	-1.25%
With	$s_F c_R(p)$	20	-0.97%	-1.15%	-2.05%	-1.4%
With	$s_F c_R(p)$	21	-0.94%	-1.12%	-1.9%	-1.83%
With	$s_F c_R(p)$	22	-0.88%	-1.05%	-1.81%	-1.74%
With	$s_F c_R(p)$	23	-1.23%	-1.34%	-1.88%	-2.56%
With	$s_F c_R(p)$	24	-1.1%	-1.41%	-1.66%	-3.3%
With	$s_F c_R(p)$	25	-1.07%	-1.19%	-1.83%	-4.3%
With	$N_A s_F c_R(p)$	1	-2.05%	-4.2%	-5.09%	-4.64%
With	$N_A s_F c_R(p)$	2	-2.06%	-4.2%	-5.01%	-4.95%
With	$N_A s_F c_R(p)$	3	-2.2%	-4.09%	-5.14%	-4.74%
With	$N_A s_F c_R(p)$	4	-1.97%	-4.38%	-5.36%	-4.83%
With	$N_A s_F c_R(p)$	5	-2.1%	-4.51%	-4.99%	-5.25%
With	$N_A s_F c_R(p)$	6	-1.6%	-4.82%	-5.06%	-5.22%
With	$N_A s_F c_R(p)$	7	-1.96%	-4.43%	-5.06%	-5%
With	$N_A s_F c_R(p)$	8	-1.74%	-4.27%	-5.43%	-5.84%
With	$N_A s_F c_R(p)$	9	-1.99%	-4.5%	-5.04%	-6.05%
With	$N_A s_F c_R(p)$	10	-1.95%	-4.75%	-5.13%	-5.78%
With	$N_A s_F c_R(p)$	11	-2.17%	-4.29%	-5.17%	-5.53%
With	$N_A s_F c_R(p)$	12	-2.12%	-4.38%	-5.02%	-5.29%
With	$N_A s_F c_R(p)$	13	-2%	-4.66%	-5.21%	-5.23%
With	$N_A s_F c_R(p)$	14	-1.94%	-4.26%	-4.97%	-4.94%
With	$N_A s_F c_R(p)$	15	-1.96%	-4.49%	-4.9%	-4.79%
With	$N_A s_F c_R(p)$	16	-2.11%	-4.25%	-4.65%	-5.5%
With	$N_A s_F c_R(p)$	17	-1.96%	-4.41%	-5.29%	-5.88%
With	$N_A s_F c_R(p)$	18	-2.18%	-4.27%	-5.88%	-5.66%
With	$N_A s_F c_R(p)$	19	-2.02%	-4.36%	-5.49%	-5.68%
With	$N_A s_F c_R(p)$	20	-1.88%	-4.36%	-5.91%	-6.38%
With	$N_A s_F c_R(p)$	21	-1.86%	-4.39%	-5.94%	-6.3%
With	$N_A s_F c_R(p)$	22	-2.03%	-4.46%	-5.47%	-6.81%
With	$N_A s_F c_R(p)$	23	-2.09%	-4.49%	-5.89%	-6.85%

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Table A.5 – continued from previous page

Annual Abundance Percent Bias						
Aux. Like.	**Model**	**Year**	**No Variation**	**Low Variation**	**Medium Variation**	**High Variation**
With	$N_A s_F c_R(p)$	24	-2.36%	-4.73%	-6.37%	-8.7%
With	$N_A s_F c_R(p)$	25	-2.29%	-4.63%	-6.58%	-9.28%
With	$s_R c_R(p)$	1	-1.14%	-1.18%	-1.9%	-1.19%
With	$s_R c_R(p)$	2	-1.29%	-1.33%	-1.67%	-1.34%
With	$s_R c_R(p)$	3	-1.18%	-1.16%	-1.81%	-1.45%
With	$s_R c_R(p)$	4	-1.25%	-1.05%	-1.88%	-1.29%
With	$s_R c_R(p)$	5	-0.97%	-0.74%	-1.79%	-1.32%
With	$s_R c_R(p)$	6	-1.18%	-1.27%	-1.85%	-1.33%
With	$s_R c_R(p)$	7	-0.91%	-1.1%	-1.41%	-1.49%
With	$s_R c_R(p)$	8	-0.97%	-1.22%	-1.66%	-1.4%
With	$s_R c_R(p)$	9	-1.16%	-1.23%	-1.42%	-1.64%
With	$s_R c_R(p)$	10	-1.14%	-1.43%	-1.84%	-1.08%
With	$s_R c_R(p)$	11	-1.39%	-1.14%	-1.54%	-1.39%
With	$s_R c_R(p)$	12	-1.19%	-1.12%	-1.26%	-0.94%
With	$s_R c_R(p)$	13	-0.92%	-1.12%	-1.24%	-1.08%
With	$s_R c_R(p)$	14	-1.12%	-0.85%	-1.13%	-0.72%
With	$s_R c_R(p)$	15	-1.09%	-1.16%	-1.28%	-1.49%
With	$s_R c_R(p)$	16	-1.16%	-1.03%	-1.66%	-2.03%
With	$s_R c_R(p)$	17	-1.19%	-1.32%	-1.73%	-2.18%
With	$s_R c_R(p)$	18	-1.07%	-1.08%	-2.03%	-2.68%
With	$s_R c_R(p)$	19	-1.16%	-1.11%	-2.3%	-2.37%
With	$s_R c_R(p)$	20	-1.01%	-1.34%	-2.58%	-2.14%
With	$s_R c_R(p)$	21	-0.98%	-1.24%	-2.37%	-2.76%
With	$s_R c_R(p)$	22	-0.84%	-1.2%	-2.36%	-2.57%
With	$s_R c_R(p)$	23	-1.15%	-1.54%	-2.45%	-3.19%
With	$s_R c_R(p)$	24	-1.25%	-1.38%	-2.43%	-3.51%
With	$s_R c_R(p)$	25	-1.26%	-1.24%	-2.35%	-4.41%
With	$N_A s_R c_R(p)$	1	-2.02%	-4.19%	-5.12%	-4.79%
With	$N_A s_R c_R(p)$	2	-2.12%	-4.2%	-5.03%	-4.9%
With	$N_A s_R c_R(p)$	3	-2.2%	-4.03%	-5.17%	-4.86%
With	$N_A s_R c_R(p)$	4	-1.97%	-4.37%	-5.4%	-4.89%
With	$N_A s_R c_R(p)$	5	-2.13%	-4.5%	-5.02%	-5.37%
With	$N_A s_R c_R(p)$	6	-1.66%	-4.82%	-5.03%	-5.39%
With	$N_A s_R c_R(p)$	7	-1.96%	-4.37%	-5.07%	-4.86%
With	$N_A s_R c_R(p)$	8	-1.66%	-4.45%	-5.42%	-5.84%
With	$N_A s_R c_R(p)$	9	-1.99%	-4.5%	-5.1%	-6.05%
With	$N_A s_R c_R(p)$	10	-1.97%	-4.71%	-5.13%	-5.78%
With	$N_A s_R c_R(p)$	11	-2.24%	-4.24%	-5.17%	-5.39%
With	$N_A s_R c_R(p)$	12	-2.14%	-4.37%	-4.96%	-5.27%
With	$N_A s_R c_R(p)$	13	-2.08%	-4.65%	-5.12%	-5.23%
With	$N_A s_R c_R(p)$	14	-1.97%	-4.21%	-4.97%	-4.91%
With	$N_A s_R c_R(p)$	15	-2%	-4.47%	-4.9%	-4.79%
With	$N_A s_R c_R(p)$	16	-2.12%	-4.25%	-4.66%	-5.49%
With	$N_A s_R c_R(p)$	17	-1.96%	-4.44%	-5.32%	-5.72%
With	$N_A s_R c_R(p)$	18	-2.18%	-4.3%	-5.82%	-5.59%
With	$N_A s_R c_R(p)$	19	-2%	-4.36%	-5.49%	-5.78%
With	$N_A s_R c_R(p)$	20	-1.88%	-4.4%	-5.94%	-6.42%
With	$N_A s_R c_R(p)$	21	-1.86%	-4.4%	-5.97%	-6.22%
With	$N_A s_R c_R(p)$	22	-2.05%	-4.46%	-5.47%	-6.77%
With	$N_A s_R c_R(p)$	23	-2.09%	-4.51%	-5.89%	-6.86%
With	$N_A s_R c_R(p)$	24	-2.36%	-4.78%	-6.37%	-8.74%
With	$N_A s_R c_R(p)$	25	-2.28%	-4.61%	-6.57%	-9.4%

Table (A.6) contains the data used for plotting Figure (3.2), the median relative bias (MRB) and estimated asymptotic 95% confidence interval coverage for each model, for each year of data, for each level of simulated variation for pooled age-class models.

Table A.6: *95% confidence interval coverage and median relative bias in total annual abundance estimates for pooled age-class models. Results indicate near-nominal coverage for models employing the Horvitz-Thompson abundance estimator, with subnominal coverage for absolute-recruit abundance models, and supernominal coverage for later years of stock-recruit models (regardless of the use of the auxiliary catch-effort likelihood of Equation (1.7)). Results based on* $n = 1000$ *replicates, with* $s = 0.84$, $c = -1.5$, $\gamma = 0.0$, *and total annual abundance* $\approx$ *4000. "Aux. Like." = Auxiliary catch-effort likelihood component used (With) or not used (Without).*

Table A.6 - Annual Abundance 95% CI Coverage and Median Relative Bias

			No Variation		Low Variation		Medium Variation		High Variation	
Aux. Like.	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
Without	$N_{SR,R}s_Fc_F(p)$	1	94.70	0.19	70.20	1.76	53.80	0.73	53.50	-1.74
Without	$N_{SR,R}s_Fc_F(p)$	2	93.00	-0.08	67.60	2.63	55.60	1.09	58.70	-2.45
Without	$N_{SR,R}s_Fc_F(p)$	3	93.40	-0.11	73.10	1.15	63.60	2.44	66.10	-2.24
Without	$N_{SR,R}s_Fc_F(p)$	4	93.40	-0.07	76.70	1.48	70.80	3.02	71.80	-0.27
Without	$N_{SR,R}s_Fc_F(p)$	5	93.30	-0.22	81.70	2.05	76.40	3.56	77.00	3.82
Without	$N_{SR,R}s_Fc_F(p)$	6	93.20	0.20	84.50	2.66	81.90	2.24	80.30	2.85
Without	$N_{SR,R}s_Fc_F(p)$	7	93.30	-0.01	87.70	2.11	84.40	1.39	83.30	3.42
Without	$N_{SR,R}s_Fc_F(p)$	8	93.10	0.05	89.20	1.00	87.80	1.38	87.00	4.51
Without	$N_{SR,R}s_Fc_F(p)$	9	92.80	-0.08	91.90	1.02	88.40	3.57	87.50	5.79
Without	$N_{SR,R}s_Fc_F(p)$	10	93.20	-0.22	93.40	0.91	90.30	4.54	88.90	6.28
Without	$N_{SR,R}s_Fc_F(p)$	11	92.90	-0.34	94.60	0.41	92.40	5.11	90.50	3.78
Without	$N_{SR,R}s_Fc_F(p)$	12	92.90	-0.33	95.40	0.21	93.80	4.91	90.80	4.93
Without	$N_{SR,R}s_Fc_F(p)$	13	93.10	-0.25	96.30	0.25	95.00	5.63	90.70	5.73
Without	$N_{SR,R}s_Fc_F(p)$	14	92.10	-0.07	97.70	0.03	96.00	5.62	92.00	8.42
Without	$N_{SR,R}s_Fc_F(p)$	15	92.50	-0.36	97.80	0.26	96.40	4.14	92.80	5.86
Without	$N_{SR,R}s_Fc_F(p)$	16	91.60	-0.23	98.00	0.39	97.50	3.66	93.80	4.69
Without	$N_{SR,R}s_Fc_F(p)$	17	92.60	-0.26	98.60	0.43	97.80	2.72	94.10	2.91
Without	$N_{SR,R}s_Fc_F(p)$	18	92.10	-0.30	98.70	0.24	98.60	3.16	95.10	7.30
Without	$N_{SR,R}s_Fc_F(p)$	19	91.70	-0.17	98.70	1.23	98.80	2.82	95.70	5.80
Without	$N_{SR,R}s_Fc_F(p)$	20	91.50	-0.39	98.90	0.43	99.20	0.49	97.00	1.41
Without	$N_{SR,R}s_Fc_F(p)$	21	92.10	-0.33	99.20	0.89	99.20	-0.18	97.80	4.96
Without	$N_{SR,R}s_Fc_F(p)$	22	92.10	-0.49	99.20	-0.22	99.50	0.62	98.60	1.64
Without	$N_{SR,R}s_Fc_F(p)$	23	93.20	-0.58	98.90	-0.46	99.40	-0.97	98.60	-0.48
Without	$N_{SR,R}s_Fc_F(p)$	24	93.80	-0.92	99.10	-0.96	99.50	-1.94	98.70	-0.16
Without	$N_{SR,R}s_Fc_F(p)$	25	94.50	-0.79	98.90	-0.41	99.60	-2.48	98.60	-3.67
Without	$N_{SR,R}s_Fc_R(p)$	1	95.10	0.31	93.70	1.96	92.80	0.81	90.00	1.04
Without	$N_{SR,R}s_Fc_R(p)$	2	93.20	-0.01	89.90	1.66	85.50	0.44	82.00	0.67
Without	$N_{SR,R}s_Fc_R(p)$	3	93.50	0.06	91.60	1.42	89.70	0.55	86.50	0.59
Without	$N_{SR,R}s_Fc_R(p)$	4	93.70	0.03	92.30	1.35	90.30	0.43	89.80	1.35
Without	$N_{SR,R}s_Fc_R(p)$	5	93.70	-0.01	92.60	1.08	90.90	1.64	91.70	0.89
Without	$N_{SR,R}s_Fc_R(p)$	6	93.40	0.28	93.20	1.40	92.70	1.47	92.30	0.72
Without	$N_{SR,R}s_Fc_R(p)$	7	93.80	0.18	94.70	0.86	94.40	1.24	93.20	0.41
Without	$N_{SR,R}s_Fc_R(p)$	8	93.40	0.16	95.40	1.03	96.70	0.34	93.90	1.55
Without	$N_{SR,R}s_Fc_R(p)$	9	93.10	0.04	95.60	0.98	97.30	-0.37	95.00	1.10
Without	$N_{SR,R}s_Fc_R(p)$	10	93.40	-0.20	95.70	0.62	97.80	0.82	95.90	-0.90
Without	$N_{SR,R}s_Fc_R(p)$	11	93.10	-0.32	96.40	-0.05	98.40	-0.65	96.50	-0.41
Without	$N_{SR,R}s_Fc_R(p)$	12	93.20	-0.16	97.90	0.15	98.20	-0.07	97.30	0.49
Without	$N_{SR,R}s_Fc_R(p)$	13	93.50	-0.17	98.50	0.13	98.50	-0.99	97.90	0.16
Without	$N_{SR,R}s_Fc_R(p)$	14	92.80	-0.06	99.10	0.34	98.90	-0.35	98.00	-0.44

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Table A.6 – continued from previous page

			No Variation		Low Variation		Medium Variation		High Variation	
Aux. Like.	Model	Year	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB
Without	$N_{SR,R}s_Fc_R(p)$	15	92.80	-0.30	99.10	0.48	99.00	-0.30	98.10	0.67
Without	$N_{SR,R}s_Fc_R(p)$	16	91.90	-0.15	99.30	0.87	99.10	0.06	98.30	0.26
Without	$N_{SR,R}s_Fc_R(p)$	17	93.20	-0.19	99.60	0.87	99.30	-0.73	98.40	-0.63
Without	$N_{SR,R}s_Fc_R(p)$	18	92.40	-0.30	99.60	1.55	99.40	-1.15	98.70	-0.83
Without	$N_{SR,R}s_Fc_R(p)$	19	92.60	-0.14	99.70	1.15	99.60	-0.66	98.90	-0.04
Without	$N_{SR,R}s_Fc_R(p)$	20	92.30	-0.39	99.70	1.22	99.60	-0.99	99.00	-0.66
Without	$N_{SR,R}s_Fc_R(p)$	21	92.70	-0.29	99.90	0.99	99.60	-0.54	99.20	-0.67
Without	$N_{SR,R}s_Fc_R(p)$	22	93.00	-0.46	99.90	0.73	99.60	-1.01	99.20	-0.90
Without	$N_{SR,R}s_Fc_R(p)$	23	93.80	-0.55	100.00	0.38	99.60	-1.19	99.40	-0.72
Without	$N_{SR,R}s_Fc_R(p)$	24	94.40	-0.96	100.00	0.42	99.60	-0.81	99.40	-1.38
Without	$N_{SR,R}s_Fc_R(p)$	25	95.30	-0.74	99.90	0.48	99.60	-1.05	99.40	-1.63
Without	$N_{SR,R}s_Rc_R(p)$	1	95.10	0.27	93.70	1.85	92.90	0.35	90.90	-0.03
Without	$N_{SR,R}s_Rc_R(p)$	2	93.20	-0.01	90.10	1.63	85.90	0.12	84.10	0.18
Without	$N_{SR,R}s_Rc_R(p)$	3	93.50	0.04	91.50	1.50	89.80	0.29	88.00	-0.23
Without	$N_{SR,R}s_Rc_R(p)$	4	93.70	-0.03	92.40	1.36	91.10	0.47	91.00	0.96
Without	$N_{SR,R}s_Rc_R(p)$	5	93.70	-0.05	92.80	1.10	91.20	1.35	92.80	0.26
Without	$N_{SR,R}s_Rc_R(p)$	6	93.40	0.28	93.40	1.12	93.20	1.05	93.30	1.13
Without	$N_{SR,R}s_Rc_R(p)$	7	93.80	0.18	94.90	0.62	95.10	1.14	93.90	0.72
Without	$N_{SR,R}s_Rc_R(p)$	8	93.40	0.16	95.60	0.95	97.10	0.15	94.90	1.03
Without	$N_{SR,R}s_Rc_R(p)$	9	93.10	0.06	95.90	0.77	97.70	-0.26	95.80	0.23
Without	$N_{SR,R}s_Rc_R(p)$	10	93.40	-0.20	96.20	0.45	98.10	0.80	96.70	-0.66
Without	$N_{SR,R}s_Rc_R(p)$	11	93.10	-0.32	96.90	-0.06	98.60	-1.03	97.40	-0.64
Without	$N_{SR,R}s_Rc_R(p)$	12	93.20	-0.16	98.10	0.43	98.50	-0.07	98.20	0.41
Without	$N_{SR,R}s_Rc_R(p)$	13	93.50	-0.19	98.80	0.34	98.80	-0.64	98.90	0.34
Without	$N_{SR,R}s_Rc_R(p)$	14	92.80	-0.01	99.20	0.56	99.20	-0.13	98.90	-0.78
Without	$N_{SR,R}s_Rc_R(p)$	15	92.80	-0.30	99.30	0.88	99.40	-0.42	99.00	0.70
Without	$N_{SR,R}s_Rc_R(p)$	16	91.90	-0.13	99.40	0.75	99.30	-0.51	98.90	0.21
Without	$N_{SR,R}s_Rc_R(p)$	17	93.20	-0.16	99.60	0.61	99.60	-0.52	98.80	-0.42
Without	$N_{SR,R}s_Rc_R(p)$	18	92.40	-0.29	99.60	1.43	99.60	-1.30	99.10	-0.61
Without	$N_{SR,R}s_Rc_R(p)$	19	92.60	-0.13	99.70	1.20	99.80	-0.92	99.40	-0.19
Without	$N_{SR,R}s_Rc_R(p)$	20	92.30	-0.39	99.70	1.26	99.70	-1.11	99.50	-1.02
Without	$N_{SR,R}s_Rc_R(p)$	21	92.70	-0.29	99.90	1.06	99.70	-0.65	99.50	-0.83
Without	$N_{SR,R}s_Rc_R(p)$	22	93.00	-0.46	99.90	0.83	99.70	-0.64	99.50	-0.39
Without	$N_{SR,R}s_Rc_R(p)$	23	93.80	-0.55	100.00	0.39	99.70	-1.22	99.70	-0.39
Without	$N_{SR,R}s_Rc_R(p)$	24	94.40	-0.96	100.00	0.48	99.70	-1.00	99.60	-1.03
Without	$N_{SR,R}s_Rc_R(p)$	25	95.30	-0.76	99.90	0.46	99.70	-0.81	99.60	-0.39
Without	$s_Fc_F(p)$	1	95.20	-0.41	73.20	1.13	51.70	1.61	51.60	1.19
Without	$s_Fc_F(p)$	2	95.30	-0.69	69.80	1.16	50.20	2.66	46.80	2.50
Without	$s_Fc_F(p)$	3	95.40	-0.63	70.30	1.04	48.70	1.42	46.30	3.16
Without	$s_Fc_F(p)$	4	95.80	-0.59	71.40	1.20	51.80	1.56	49.20	2.65
Without	$s_Fc_F(p)$	5	94.90	-0.27	71.10	1.01	49.40	0.95	49.10	3.46
Without	$s_Fc_F(p)$	6	94.80	-0.26	72.20	0.76	51.30	1.58	49.00	1.00
Without	$s_Fc_F(p)$	7	94.40	-0.18	71.70	1.21	49.60	1.00	47.40	3.99
Without	$s_Fc_F(p)$	8	95.40	-0.27	72.60	1.59	51.40	1.44	47.00	2.55
Without	$s_Fc_F(p)$	9	94.60	-0.68	72.90	0.10	52.90	0.46	45.70	2.22
Without	$s_Fc_F(p)$	10	94.70	-0.24	72.00	0.68	51.40	2.03	52.10	-0.24
Without	$s_Fc_F(p)$	11	94.20	-0.58	73.30	1.85	51.90	2.57	46.10	2.85
Without	$s_Fc_F(p)$	12	95.00	-0.62	70.20	0.53	52.70	1.63	51.10	2.27
Without	$s_Fc_F(p)$	13	93.70	-0.42	70.90	0.53	54.90	1.61	49.20	1.62
Without	$s_Fc_F(p)$	14	94.00	-0.18	72.00	1.48	49.90	1.25	46.90	1.51
Without	$s_Fc_F(p)$	15	94.90	-0.65	70.40	0.51	52.80	1.74	49.50	1.82
Without	$s_Fc_F(p)$	16	95.40	-0.33	69.90	1.87	55.10	0.88	50.10	2.44
Without	$s_Fc_F(p)$	17	95.40	-0.25	69.20	0.48	51.90	2.64	47.90	2.44
Without	$s_Fc_F(p)$	18	95.90	-0.40	72.60	1.56	49.50	2.68	48.40	3.48
Without	$s_Fc_F(p)$	19	95.30	0.05	69.70	0.74	52.80	2.54	48.60	1.02
Without	$s_Fc_F(p)$	20	94.90	-0.17	71.40	1.36	51.60	0.47	48.80	1.42
Without	$s_Fc_F(p)$	21	94.70	-0.26	70.00	0.84	52.60	1.89	50.40	2.00
Without	$s_Fc_F(p)$	22	94.50	-0.38	71.90	1.11	51.60	2.71	49.60	1.80
Without	$s_Fc_F(p)$	23	94.20	-0.33	70.60	0.93	52.20	1.56	47.30	1.00

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Table A.6 – continued from previous page

			No Variation		Low Variation		Medium Variation		High Variation	
Aux. Like.	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
Without	$s_F c_F(p)$	24	94.80	-0.37	69.30	0.73	49.60	0.98	50.00	-0.00
Without	$s_F c_F(p)$	25	95.40	-0.66	68.50	1.52	51.30	1.93	49.40	-0.78
Without	$s_F c_R(p)$	1	95.70	-0.51	96.10	0.49	97.20	0.07	98.10	1.19
Without	$s_F c_R(p)$	2	95.80	-0.65	95.90	0.58	98.50	0.25	98.30	1.80
Without	$s_F c_R(p)$	3	95.80	-0.67	96.20	0.21	98.30	0.21	97.90	1.04
Without	$s_F c_R(p)$	4	96.40	-0.48	96.30	0.10	98.30	-0.03	97.90	0.59
Without	$s_F c_R(p)$	5	96.30	-0.17	96.00	0.41	97.80	0.21	98.30	0.73
Without	$s_F c_R(p)$	6	94.90	-0.25	96.50	0.12	98.00	0.14	99.00	0.81
Without	$s_F c_R(p)$	7	95.00	-0.28	96.40	0.17	97.90	0.25	98.20	0.84
Without	$s_F c_R(p)$	8	95.90	-0.35	97.40	0.22	98.10	-0.04	98.40	0.43
Without	$s_F c_R(p)$	9	95.60	-0.54	97.50	-0.01	98.20	-0.13	98.80	0.37
Without	$s_F c_R(p)$	10	95.40	-0.32	97.20	0.05	98.30	-0.06	98.90	-0.03
Without	$s_F c_R(p)$	11	94.70	-0.55	97.40	0.43	97.80	0.02	98.60	0.17
Without	$s_F c_R(p)$	12	95.60	-0.66	97.30	0.32	98.30	0.06	98.40	0.96
Without	$s_F c_R(p)$	13	94.90	-0.42	97.40	0.06	98.20	0.21	98.30	0.81
Without	$s_F c_R(p)$	14	95.20	-0.22	96.70	0.18	98.00	0.40	98.20	0.57
Without	$s_F c_R(p)$	15	95.50	-0.64	96.40	0.15	97.80	0.52	98.40	0.78
Without	$s_F c_R(p)$	16	96.10	-0.52	96.20	0.77	97.60	0.32	98.70	-0.15
Without	$s_F c_R(p)$	17	95.80	-0.26	96.30	0.28	97.30	0.30	98.00	0.35
Without	$s_F c_R(p)$	18	96.20	-0.41	96.40	0.27	96.90	-0.32	97.70	0.56
Without	$s_F c_R(p)$	19	95.70	-0.14	96.70	0.19	97.20	-0.07	97.70	0.00
Without	$s_F c_R(p)$	20	95.70	-0.19	96.80	0.30	97.70	-0.40	97.40	0.32
Without	$s_F c_R(p)$	21	95.30	-0.31	95.30	0.45	97.20	0.04	97.10	0.03
Without	$s_F c_R(p)$	22	95.20	-0.39	96.30	0.28	96.30	-0.05	95.40	0.44
Without	$s_F c_R(p)$	23	95.20	-0.45	95.30	0.13	95.80	-0.04	94.30	-0.18
Without	$s_F c_R(p)$	24	95.30	-0.45	94.70	0.30	93.70	-0.13	92.00	-0.53
Without	$s_F c_R(p)$	25	95.70	-0.67	93.20	0.34	91.80	0.37	90.00	-1.63
Without	$s_R c_R(p)$	1	95.70	-0.52	95.80	0.63	96.80	-0.45	97.70	0.41
Without	$s_R c_R(p)$	2	95.70	-0.64	95.60	0.60	97.90	-0.24	97.80	0.51
Without	$s_R c_R(p)$	3	95.70	-0.67	96.00	0.12	97.90	-0.56	97.60	0.33
Without	$s_R c_R(p)$	4	96.40	-0.58	96.20	0.05	98.10	-0.65	97.70	0.38
Without	$s_R c_R(p)$	5	96.10	-0.09	96.10	0.37	98.10	-0.27	98.50	0.70
Without	$s_R c_R(p)$	6	94.90	-0.26	96.00	-0.03	97.80	-0.04	98.90	0.31
Without	$s_R c_R(p)$	7	94.90	-0.29	95.50	0.10	97.90	0.13	98.60	0.11
Without	$s_R c_R(p)$	8	95.80	-0.33	97.30	0.16	98.70	-0.18	98.40	0.09
Without	$s_R c_R(p)$	9	95.50	-0.52	97.50	-0.13	98.60	-0.28	99.20	-0.22
Without	$s_R c_R(p)$	10	95.30	-0.32	97.50	-0.13	98.70	0.08	99.00	-0.06
Without	$s_R c_R(p)$	11	94.80	-0.60	97.60	0.38	98.70	0.02	98.90	-0.16
Without	$s_R c_R(p)$	12	95.70	-0.66	97.50	0.36	98.80	-0.09	98.70	0.28
Without	$s_R c_R(p)$	13	94.70	-0.43	97.20	0.06	98.30	-0.14	98.40	0.62
Without	$s_R c_R(p)$	14	95.30	-0.22	96.20	0.46	97.60	0.39	98.10	0.55
Without	$s_R c_R(p)$	15	95.30	-0.68	95.90	0.21	97.70	0.31	98.00	0.36
Without	$s_R c_R(p)$	16	96.00	-0.53	95.90	0.92	97.40	0.07	98.40	-0.48
Without	$s_R c_R(p)$	17	95.60	-0.24	95.60	0.22	97.40	0.12	97.50	0.23
Without	$s_R c_R(p)$	18	96.20	-0.44	96.40	0.13	97.00	-0.32	97.70	-0.23
Without	$s_R c_R(p)$	19	95.80	-0.09	96.70	0.14	97.20	-0.37	97.40	-0.14
Without	$s_R c_R(p)$	20	95.60	-0.20	96.40	0.02	97.20	-0.87	96.80	0.09
Without	$s_R c_R(p)$	21	95.30	-0.37	95.30	0.51	97.20	-0.84	97.30	0.19
Without	$s_R c_R(p)$	22	95.30	-0.37	96.10	0.05	96.70	-0.20	95.40	0.13
Without	$s_R c_R(p)$	23	95.20	-0.43	95.10	0.01	96.60	-0.20	95.10	-0.75
Without	$s_R c_R(p)$	24	95.20	-0.51	95.00	0.34	94.10	-0.19	92.60	-0.67
Without	$s_R c_R(p)$	25	95.70	-0.67	92.90	0.36	92.00	0.21	90.90	-1.28
With	$N_{SR,R} s_F c_F(p)$	1	84.40	0.07	61.00	3.80	45.00	12.77	42.30	19.63
With	$N_{SR,R} s_F c_F(p)$	2	86.50	-0.03	76.50	4.36	65.00	14.36	66.20	21.96
With	$N_{SR,R} s_F c_F(p)$	3	87.10	0.06	82.00	4.97	73.30	13.22	77.00	23.39
With	$N_{SR,R} s_F c_F(p)$	4	86.60	0.01	88.50	4.57	85.30	12.82	89.30	24.51
With	$N_{SR,R} s_F c_F(p)$	5	88.00	-0.19	92.80	3.85	92.70	14.15	95.60	23.50
With	$N_{SR,R} s_F c_F(p)$	6	87.50	-0.10	95.10	3.82	97.20	14.42	98.40	20.18
With	$N_{SR,R} s_F c_F(p)$	7	88.70	0.19	97.50	4.18	99.00	13.18	99.60	19.66

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Table A.6 – continued from previous page

			No Variation		Low Variation		Medium Variation		High Variation	
Aux. Like.	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
With	$N_{SR,R}s_Fc_F(p)$	8	88.90	0.08	99.10	4.09	99.10	13.88	99.60	23.33
With	$N_{SR,R}s_Fc_F(p)$	9	88.60	-0.03	99.30	3.76	99.60	13.67	99.80	19.41
With	$N_{SR,R}s_Fc_F(p)$	10	88.70	0.19	99.70	3.41	99.70	14.97	99.70	16.67
With	$N_{SR,R}s_Fc_F(p)$	11	89.30	0.05	99.70	3.96	99.90	15.38	99.80	20.21
With	$N_{SR,R}s_Fc_F(p)$	12	87.50	0.05	99.80	3.47	99.90	14.37	99.80	20.12
With	$N_{SR,R}s_Fc_F(p)$	13	88.70	0.08	99.70	3.45	99.90	15.09	99.80	20.67
With	$N_{SR,R}s_Fc_F(p)$	14	88.70	-0.00	100.00	4.09	99.90	14.12	99.90	21.11
With	$N_{SR,R}s_Fc_F(p)$	15	89.40	0.03	99.90	3.71	99.90	14.77	99.90	21.45
With	$N_{SR,R}s_Fc_F(p)$	16	89.00	-0.06	99.90	3.98	99.90	13.85	99.90	19.53
With	$N_{SR,R}s_Fc_F(p)$	17	89.30	0.06	99.90	4.25	99.90	14.74	99.90	22.37
With	$N_{SR,R}s_Fc_F(p)$	18	89.00	-0.01	99.90	4.63	99.90	15.78	99.90	23.39
With	$N_{SR,R}s_Fc_F(p)$	19	88.80	0.24	99.90	4.05	99.90	13.78	99.80	22.03
With	$N_{SR,R}s_Fc_F(p)$	20	90.10	0.14	99.90	4.18	99.90	13.02	99.80	21.33
With	$N_{SR,R}s_Fc_F(p)$	21	89.10	-0.04	99.90	4.20	99.80	13.31	99.80	20.12
With	$N_{SR,R}s_Fc_F(p)$	22	90.30	-0.12	99.90	4.24	99.90	14.50	99.80	20.14
With	$N_{SR,R}s_Fc_F(p)$	23	90.60	0.00	99.90	4.33	99.90	13.99	99.80	22.03
With	$N_{SR,R}s_Fc_F(p)$	24	90.10	-0.25	99.90	4.05	99.90	13.32	99.80	19.15
With	$N_{SR,R}s_Fc_F(p)$	25	89.00	-0.24	99.90	4.19	99.90	12.39	99.80	18.52
With	$N_{SR,R}s_Fc_R(p)$	1	90.20	-0.67	93.00	0.07	91.70	-0.07	88.50	1.61
With	$N_{SR,R}s_Fc_R(p)$	2	90.30	-0.39	94.00	0.03	96.70	-0.49	95.50	1.80
With	$N_{SR,R}s_Fc_R(p)$	3	91.30	-0.40	95.10	0.13	97.90	-0.13	97.50	2.00
With	$N_{SR,R}s_Fc_R(p)$	4	91.70	-0.41	97.10	-0.20	98.90	0.01	98.90	1.90
With	$N_{SR,R}s_Fc_R(p)$	5	92.20	-0.46	98.20	-0.05	99.10	0.33	99.20	1.01
With	$N_{SR,R}s_Fc_R(p)$	6	92.10	-0.30	98.20	0.24	99.60	0.38	99.50	1.77
With	$N_{SR,R}s_Fc_R(p)$	7	92.20	-0.29	99.50	-0.47	99.80	0.21	99.60	1.27
With	$N_{SR,R}s_Fc_R(p)$	8	92.70	-0.24	99.60	-0.23	99.80	-0.09	99.70	1.22
With	$N_{SR,R}s_Fc_R(p)$	9	93.10	-0.35	99.60	-0.16	99.80	-0.12	99.80	1.15
With	$N_{SR,R}s_Fc_R(p)$	10	93.40	-0.40	99.80	-0.26	99.90	-0.10	99.80	1.49
With	$N_{SR,R}s_Fc_R(p)$	11	94.30	-0.43	99.90	-0.06	99.90	-0.08	99.80	0.91
With	$N_{SR,R}s_Fc_R(p)$	12	94.80	-0.50	100.00	0.07	99.90	-0.25	99.80	1.61
With	$N_{SR,R}s_Fc_R(p)$	13	95.30	-0.58	100.00	-0.27	99.90	0.08	99.80	1.10
With	$N_{SR,R}s_Fc_R(p)$	14	95.60	-0.55	100.00	-0.15	99.90	0.06	99.80	1.19
With	$N_{SR,R}s_Fc_R(p)$	15	96.00	-0.57	100.00	-0.25	99.90	0.09	99.80	1.58
With	$N_{SR,R}s_Fc_R(p)$	16	96.00	-0.49	100.00	0.30	99.90	0.24	99.80	0.73
With	$N_{SR,R}s_Fc_R(p)$	17	96.30	-0.41	100.00	0.13	99.90	-0.21	99.80	0.72
With	$N_{SR,R}s_Fc_R(p)$	18	97.20	-0.46	100.00	-0.22	99.90	-0.20	99.80	0.90
With	$N_{SR,R}s_Fc_R(p)$	19	96.90	-0.56	100.00	-0.26	99.90	-0.55	99.80	1.27
With	$N_{SR,R}s_Fc_R(p)$	20	97.20	-0.45	100.00	-0.15	99.90	-0.94	99.80	0.47
With	$N_{SR,R}s_Fc_R(p)$	21	97.40	-0.48	100.00	-0.13	99.90	-0.87	99.80	0.48
With	$N_{SR,R}s_Fc_R(p)$	22	97.50	-0.70	100.00	-0.44	99.90	-0.73	99.80	0.20
With	$N_{SR,R}s_Fc_R(p)$	23	97.70	-0.36	100.00	-0.30	99.90	-0.89	99.80	-1.23
With	$N_{SR,R}s_Fc_R(p)$	24	97.60	-0.61	100.00	-0.53	99.90	-1.44	99.80	-2.49
With	$N_{SR,R}s_Fc_R(p)$	25	97.60	-0.52	100.00	-0.42	99.90	-1.62	99.80	-2.98
With	$N_{SR,R}s_Rc_R(p)$	1	90.50	-0.74	92.90	0.01	91.90	-0.24	88.30	0.46
With	$N_{SR,R}s_Rc_R(p)$	2	90.80	-0.36	94.00	0.14	97.30	-0.64	95.40	1.34
With	$N_{SR,R}s_Rc_R(p)$	3	91.70	-0.42	95.00	0.02	98.50	-0.26	97.60	1.54
With	$N_{SR,R}s_Rc_R(p)$	4	92.10	-0.38	97.10	-0.22	99.30	-0.21	98.80	1.29
With	$N_{SR,R}s_Rc_R(p)$	5	92.80	-0.41	98.30	-0.09	99.40	0.27	98.90	0.41
With	$N_{SR,R}s_Rc_R(p)$	6	92.60	-0.25	98.40	0.32	99.90	0.19	98.90	0.80
With	$N_{SR,R}s_Rc_R(p)$	7	92.80	-0.26	99.50	-0.42	100.00	0.02	99.00	0.51
With	$N_{SR,R}s_Rc_R(p)$	8	93.20	-0.24	99.60	-0.27	100.00	0.02	99.10	0.78
With	$N_{SR,R}s_Rc_R(p)$	9	93.50	-0.32	99.60	-0.20	100.00	-0.16	99.10	0.61
With	$N_{SR,R}s_Rc_R(p)$	10	93.60	-0.46	99.90	-0.23	100.00	-0.14	99.10	0.50
With	$N_{SR,R}s_Rc_R(p)$	11	94.40	-0.54	99.90	-0.06	100.00	-0.08	99.10	0.79
With	$N_{SR,R}s_Rc_R(p)$	12	95.00	-0.55	100.00	0.10	100.00	-0.22	99.10	1.29
With	$N_{SR,R}s_Rc_R(p)$	13	95.50	-0.61	100.00	-0.26	100.00	-0.04	99.10	0.66
With	$N_{SR,R}s_Rc_R(p)$	14	95.80	-0.54	100.00	-0.12	100.00	-0.12	99.10	0.63
With	$N_{SR,R}s_Rc_R(p)$	15	96.10	-0.47	100.00	-0.20	100.00	-0.09	99.10	1.25
With	$N_{SR,R}s_Rc_R(p)$	16	96.20	-0.45	100.00	0.33	100.00	0.20	99.10	0.31

Continued on next page

Table A.6 – continued from previous page

			No Variation		Low Variation		Medium Variation		High Variation	
Aux. Like.	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
With	$N_{SR,R}s_Rc_R(p)$	17	96.40	-0.36	100.00	0.13	100.00	-0.23	99.10	0.64
With	$N_{SR,R}s_Rc_R(p)$	18	97.40	-0.40	100.00	-0.23	100.00	-0.32	99.10	0.63
With	$N_{SR,R}s_Rc_R(p)$	19	97.10	-0.46	100.00	-0.26	100.00	-0.61	99.10	1.27
With	$N_{SR,R}s_Rc_R(p)$	20	97.40	-0.42	100.00	-0.22	100.00	-1.02	99.10	0.30
With	$N_{SR,R}s_Rc_R(p)$	21	97.60	-0.47	100.00	-0.15	100.00	-0.80	99.10	0.39
With	$N_{SR,R}s_Rc_R(p)$	22	97.60	-0.68	100.00	-0.41	100.00	-0.77	99.10	-0.04
With	$N_{SR,R}s_Rc_R(p)$	23	97.80	-0.35	100.00	-0.28	100.00	-0.82	99.10	-1.24
With	$N_{SR,R}s_Rc_R(p)$	24	97.70	-0.66	100.00	-0.53	100.00	-1.64	99.10	-2.49
With	$N_{SR,R}s_Rc_R(p)$	25	97.70	-0.60	100.00	-0.42	100.00	-1.64	99.10	-2.47
With	$s_Fc_F(p)$	1	88.80	0.14	48.40	11.18	30.70	38.66	27.90	60.22
With	$s_Fc_F(p)$	2	89.20	-0.09	48.50	10.66	30.20	41.13	29.40	61.26
With	$s_Fc_F(p)$	3	89.20	-0.00	49.40	9.92	30.70	41.72	29.10	65.05
With	$s_Fc_F(p)$	4	89.10	-0.28	51.30	10.33	27.70	39.98	27.00	66.44
With	$s_Fc_F(p)$	5	88.60	0.18	51.00	10.01	29.40	43.29	29.10	67.05
With	$s_Fc_F(p)$	6	89.10	0.23	48.40	11.01	30.00	40.15	29.80	60.11
With	$s_Fc_F(p)$	7	89.20	0.10	48.80	11.78	30.80	39.38	29.80	61.07
With	$s_Fc_F(p)$	8	89.60	0.09	51.10	10.89	29.30	40.30	28.70	62.98
With	$s_Fc_F(p)$	9	89.30	0.05	49.60	9.89	30.20	41.69	29.60	58.12
With	$s_Fc_F(p)$	10	88.20	0.46	50.30	10.41	29.80	39.80	32.20	54.11
With	$s_Fc_F(p)$	11	89.40	0.10	49.90	11.04	28.80	43.04	29.80	59.40
With	$s_Fc_F(p)$	12	89.00	0.01	50.60	10.11	30.70	41.21	30.50	61.56
With	$s_Fc_F(p)$	13	88.40	0.12	50.00	9.96	29.40	43.18	31.50	59.23
With	$s_Fc_F(p)$	14	89.70	-0.08	49.50	11.60	29.90	39.46	31.20	64.10
With	$s_Fc_F(p)$	15	88.70	-0.31	50.80	10.45	30.10	42.65	31.90	62.07
With	$s_Fc_F(p)$	16	88.60	0.10	49.80	11.41	30.90	40.83	30.50	60.29
With	$s_Fc_F(p)$	17	89.90	0.16	49.30	11.14	27.90	41.44	29.80	64.77
With	$s_Fc_F(p)$	18	90.00	-0.13	49.50	11.06	30.90	41.07	29.10	67.66
With	$s_Fc_F(p)$	19	88.30	0.18	50.50	10.37	29.40	40.29	30.50	63.37
With	$s_Fc_F(p)$	20	89.00	0.11	50.20	10.51	31.00	41.05	31.30	60.05
With	$s_Fc_F(p)$	21	88.30	0.20	49.70	10.92	31.50	40.38	32.10	64.91
With	$s_Fc_F(p)$	22	88.10	0.39	48.90	11.27	29.00	41.33	32.80	63.56
With	$s_Fc_F(p)$	23	87.80	-0.19	49.70	10.72	31.40	40.14	31.20	64.39
With	$s_Fc_F(p)$	24	88.40	0.14	48.70	10.40	30.60	41.37	32.20	60.90
With	$s_Fc_F(p)$	25	89.20	-0.28	50.30	10.85	29.20	41.79	32.20	61.65
With	$N_As_Fc_F(p)$	1	89.80	-1.98	63.40	1.13	45.00	7.23	43.00	10.77
With	$N_As_Fc_F(p)$	2	90.10	-2.01	62.60	1.85	43.40	8.55	43.00	11.26
With	$N_As_Fc_F(p)$	3	90.60	-2.09	62.20	2.14	42.50	7.15	38.70	13.12
With	$N_As_Fc_F(p)$	4	91.20	-1.89	64.80	1.50	45.90	7.24	41.30	14.99
With	$N_As_Fc_F(p)$	5	90.80	-2.09	62.90	0.55	43.90	8.19	39.50	14.61
With	$N_As_Fc_F(p)$	6	89.90	-1.54	63.90	0.74	42.90	8.00	44.10	11.95
With	$N_As_Fc_F(p)$	7	89.60	-1.86	64.00	1.28	43.30	5.93	42.90	12.24
With	$N_As_Fc_F(p)$	8	90.40	-1.61	64.40	1.45	45.20	8.80	42.70	12.88
With	$N_As_Fc_F(p)$	9	91.20	-1.95	64.50	1.25	43.90	8.22	43.90	10.28
With	$N_As_Fc_F(p)$	10	90.60	-1.86	63.70	0.54	45.80	8.68	46.20	8.70
With	$N_As_Fc_F(p)$	11	90.10	-2.15	63.90	1.11	42.80	8.33	44.30	12.03
With	$N_As_Fc_F(p)$	12	90.80	-2.07	65.50	0.70	43.70	9.26	45.10	12.06
With	$N_As_Fc_F(p)$	13	89.80	-2.03	64.60	0.37	44.90	8.41	43.40	12.01
With	$N_As_Fc_F(p)$	14	89.50	-1.78	64.30	1.59	42.90	7.30	42.20	11.84
With	$N_As_Fc_F(p)$	15	89.60	-1.92	64.00	0.64	44.50	8.35	44.50	12.09
With	$N_As_Fc_F(p)$	16	90.10	-2.08	61.50	1.19	43.40	7.06	44.10	10.48
With	$N_As_Fc_F(p)$	17	89.80	-1.90	65.00	1.23	41.60	8.19	43.80	12.31
With	$N_As_Fc_F(p)$	18	90.80	-2.10	64.10	1.86	41.70	9.21	41.80	14.01
With	$N_As_Fc_F(p)$	19	89.80	-1.90	61.80	1.54	45.10	7.46	44.10	13.26
With	$N_As_Fc_F(p)$	20	90.90	-1.82	63.90	1.39	45.50	6.99	43.30	11.55
With	$N_As_Fc_F(p)$	21	90.00	-1.77	62.50	1.28	43.80	7.08	42.90	11.70
With	$N_As_Fc_F(p)$	22	89.60	-1.99	63.10	1.75	44.10	8.21	43.50	10.90
With	$N_As_Fc_F(p)$	23	89.80	-1.97	64.50	1.51	46.70	7.55	43.30	11.14
With	$N_As_Fc_F(p)$	24	90.10	-2.16	61.60	1.30	44.50	6.67	42.80	10.47
With	$N_As_Fc_F(p)$	25	89.20	-2.19	60.70	1.05	40.60	6.11	39.10	9.76

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Table A.6 – continued from previous page

			No Variation		Low Variation		Medium Variation		High Variation	
Aux. Like.	Model	Year	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB
With	$s_Fc_R(p)$	1	94.20	-0.96	95.00	-1.09	96.30	-0.71	96.00	0.75
With	$s_Fc_R(p)$	2	94.70	-1.02	95.10	-1.03	96.50	-0.79	97.00	0.48
With	$s_Fc_R(p)$	3	94.70	-0.95	94.60	-0.96	96.20	-0.99	96.80	0.12
With	$s_Fc_R(p)$	4	94.70	-0.97	95.40	-1.04	97.00	-1.08	96.70	0.36
With	$s_Fc_R(p)$	5	94.70	-0.92	94.80	-0.72	96.20	-0.46	96.50	-0.05
With	$s_Fc_R(p)$	6	93.90	-1.03	95.40	-1.11	96.20	-1.16	97.20	0.25
With	$s_Fc_R(p)$	7	94.20	-0.69	95.30	-1.09	96.30	-0.76	96.90	0.04
With	$s_Fc_R(p)$	8	95.00	-0.97	95.20	-1.10	96.40	-0.93	97.20	-0.04
With	$s_Fc_R(p)$	9	94.40	-0.95	96.70	-1.21	95.90	-1.02	97.80	-0.45
With	$s_Fc_R(p)$	10	94.10	-1.04	96.40	-1.23	96.70	-1.52	97.80	-0.42
With	$s_Fc_R(p)$	11	93.80	-1.18	96.10	-1.14	96.50	-1.24	97.40	-0.58
With	$s_Fc_R(p)$	12	95.10	-1.15	96.10	-1.12	96.80	-0.91	97.60	-0.05
With	$s_Fc_R(p)$	13	93.60	-0.90	96.20	-1.20	96.60	-0.78	97.60	-0.34
With	$s_Fc_R(p)$	14	94.00	-1.08	95.80	-0.96	96.60	-0.62	96.90	-0.19
With	$s_Fc_R(p)$	15	93.90	-1.09	95.50	-0.93	96.60	-0.50	97.00	-0.28
With	$s_Fc_R(p)$	16	94.20	-1.21	95.80	-1.05	96.20	-0.74	97.40	-1.44
With	$s_Fc_R(p)$	17	94.40	-1.09	95.80	-1.15	95.90	-1.03	96.60	-0.92
With	$s_Fc_R(p)$	18	94.30	-1.04	95.20	-1.00	96.30	-1.19	96.50	-1.11
With	$s_Fc_R(p)$	19	94.20	-1.14	95.60	-0.86	95.30	-1.39	96.00	-1.25
With	$s_Fc_R(p)$	20	94.50	-0.97	94.90	-1.15	96.20	-2.05	95.30	-1.40
With	$s_Fc_R(p)$	21	94.60	-0.94	94.40	-1.12	95.40	-1.90	95.30	-1.83
With	$s_Fc_R(p)$	22	93.30	-0.88	94.70	-1.05	94.90	-1.81	93.70	-1.74
With	$s_Fc_R(p)$	23	94.10	-1.23	94.50	-1.34	93.60	-1.88	91.60	-2.56
With	$s_Fc_R(p)$	24	94.50	-1.10	93.50	-1.41	92.10	-1.66	89.60	-3.30
With	$s_Fc_R(p)$	25	93.40	-1.07	92.00	-1.19	89.10	-1.83	87.80	-4.30
With	$N_As_Fc_R(p)$	1	90.30	-2.05	85.30	-4.20	86.10	-5.09	83.20	-4.64
With	$N_As_Fc_R(p)$	2	90.40	-2.06	85.60	-4.20	87.10	-5.01	85.10	-4.95
With	$N_As_Fc_R(p)$	3	90.70	-2.20	86.30	-4.09	88.60	-5.14	88.00	-4.74
With	$N_As_Fc_R(p)$	4	91.30	-1.97	86.80	-4.38	89.20	-5.36	89.70	-4.83
With	$N_As_Fc_R(p)$	5	90.90	-2.10	86.80	-4.51	91.10	-4.99	90.20	-5.25
With	$N_As_Fc_R(p)$	6	90.20	-1.60	87.10	-4.82	89.60	-5.06	92.30	-5.22
With	$N_As_Fc_R(p)$	7	89.70	-1.96	87.20	-4.43	90.40	-5.06	91.60	-5.00
With	$N_As_Fc_R(p)$	8	90.60	-1.74	88.20	-4.27	90.50	-5.43	91.90	-5.84
With	$N_As_Fc_R(p)$	9	91.30	-1.99	88.20	-4.50	91.00	-5.04	92.50	-6.05
With	$N_As_Fc_R(p)$	10	90.80	-1.95	87.90	-4.75	91.50	-5.13	93.80	-5.78
With	$N_As_Fc_R(p)$	11	90.40	-2.17	87.50	-4.29	91.60	-5.17	93.90	-5.53
With	$N_As_Fc_R(p)$	12	91.10	-2.12	88.00	-4.38	90.90	-5.02	92.30	-5.29
With	$N_As_Fc_R(p)$	13	89.80	-2.00	88.30	-4.66	90.90	-5.21	91.10	-5.23
With	$N_As_Fc_R(p)$	14	89.70	-1.94	87.80	-4.26	91.60	-4.97	91.30	-4.94
With	$N_As_Fc_R(p)$	15	89.90	-1.96	87.10	-4.49	91.50	-4.90	91.10	-4.79
With	$N_As_Fc_R(p)$	16	90.10	-2.11	87.30	-4.25	91.50	-4.65	91.60	-5.50
With	$N_As_Fc_R(p)$	17	90.00	-1.96	87.60	-4.41	91.20	-5.29	91.30	-5.88
With	$N_As_Fc_R(p)$	18	90.60	-2.18	86.90	-4.27	90.40	-5.88	90.50	-5.66
With	$N_As_Fc_R(p)$	19	90.00	-2.02	87.20	-4.36	91.50	-5.49	91.30	-5.68
With	$N_As_Fc_R(p)$	20	90.80	-1.88	87.70	-4.36	90.00	-5.91	91.30	-6.38
With	$N_As_Fc_R(p)$	21	90.20	-1.86	85.80	-4.39	90.30	-5.94	87.90	-6.30
With	$N_As_Fc_R(p)$	22	89.80	-2.03	87.40	-4.46	90.40	-5.47	87.60	-6.81
With	$N_As_Fc_R(p)$	23	90.00	-2.09	86.20	-4.49	88.10	-5.89	85.40	-6.85
With	$N_As_Fc_R(p)$	24	90.10	-2.36	82.00	-4.73	82.90	-6.37	79.40	-8.70
With	$N_As_Fc_R(p)$	25	89.20	-2.29	81.20	-4.63	78.60	-6.58	74.90	-9.28
With	$s_Rc_R(p)$	1	94.60	-1.14	94.70	-1.18	96.60	-1.90	96.70	-1.19
With	$s_Rc_R(p)$	2	94.90	-1.29	94.70	-1.33	97.40	-1.67	98.10	-1.34
With	$s_Rc_R(p)$	3	94.60	-1.18	95.00	-1.16	96.80	-1.81	97.80	-1.45
With	$s_Rc_R(p)$	4	94.40	-1.25	95.10	-1.05	97.60	-1.88	98.00	-1.29
With	$s_Rc_R(p)$	5	94.90	-0.97	94.30	-0.74	97.00	-1.79	98.10	-1.32
With	$s_Rc_R(p)$	6	94.10	-1.18	94.70	-1.27	97.10	-1.85	98.70	-1.33
With	$s_Rc_R(p)$	7	94.10	-0.91	94.50	-1.10	97.40	-1.41	98.70	-1.49
With	$s_Rc_R(p)$	8	95.10	-0.97	95.20	-1.22	97.30	-1.66	98.90	-1.40
With	$s_Rc_R(p)$	9	94.70	-1.16	96.60	-1.23	97.50	-1.42	99.50	-1.64

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Table A.6 – continued from previous page

			No Variation		Low Variation		Medium Variation		High Variation	
Aux. Like.	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
With	$s_Rc_R(p)$	10	94.20	-1.14	96.90	-1.43	98.40	-1.84	99.30	-1.08
With	$s_Rc_R(p)$	11	93.90	-1.39	96.70	-1.14	98.40	-1.54	99.20	-1.39
With	$s_Rc_R(p)$	12	95.10	-1.19	96.40	-1.12	98.40	-1.26	99.30	-0.94
With	$s_Rc_R(p)$	13	94.30	-0.92	96.30	-1.12	98.10	-1.24	99.20	-1.08
With	$s_Rc_R(p)$	14	94.30	-1.12	95.30	-0.85	97.50	-1.13	98.30	-0.72
With	$s_Rc_R(p)$	15	93.70	-1.09	94.90	-1.16	96.70	-1.28	98.10	-1.49
With	$s_Rc_R(p)$	16	94.80	-1.16	95.30	-1.03	96.80	-1.66	98.60	-2.03
With	$s_Rc_R(p)$	17	94.50	-1.19	94.80	-1.32	96.80	-1.73	97.60	-2.18
With	$s_Rc_R(p)$	18	94.90	-1.07	94.70	-1.08	96.60	-2.03	97.60	-2.68
With	$s_Rc_R(p)$	19	94.40	-1.16	95.60	-1.11	95.60	-2.30	96.90	-2.37
With	$s_Rc_R(p)$	20	94.80	-1.01	94.60	-1.34	96.20	-2.58	96.20	-2.14
With	$s_Rc_R(p)$	21	94.80	-0.98	94.20	-1.24	96.20	-2.37	96.70	-2.76
With	$s_Rc_R(p)$	22	93.70	-0.84	94.50	-1.20	95.70	-2.36	94.90	-2.57
With	$s_Rc_R(p)$	23	94.40	-1.15	94.30	-1.54	94.40	-2.45	93.10	-3.19
With	$s_Rc_R(p)$	24	94.40	-1.25	93.80	-1.38	93.10	-2.43	90.60	-3.51
With	$s_Rc_R(p)$	25	93.50	-1.26	92.10	-1.24	90.20	-2.35	89.60	-4.41
With	$N_As_Rc_R(p)$	1	90.40	-2.02	84.80	-4.19	85.70	-5.12	82.90	-4.79
With	$N_As_Rc_R(p)$	2	90.60	-2.12	85.60	-4.20	87.10	-5.03	85.20	-4.90
With	$N_As_Rc_R(p)$	3	90.90	-2.20	86.30	-4.03	88.70	-5.17	88.00	-4.86
With	$N_As_Rc_R(p)$	4	91.50	-1.97	86.70	-4.37	89.30	-5.40	89.60	-4.89
With	$N_As_Rc_R(p)$	5	91.20	-2.13	86.60	-4.50	91.00	-5.02	90.40	-5.37
With	$N_As_Rc_R(p)$	6	90.20	-1.66	86.90	-4.82	89.30	-5.03	92.30	-5.39
With	$N_As_Rc_R(p)$	7	89.80	-1.96	87.20	-4.37	90.30	-5.07	91.60	-4.86
With	$N_As_Rc_R(p)$	8	90.60	-1.66	88.30	-4.45	90.40	-5.42	92.00	-5.84
With	$N_As_Rc_R(p)$	9	91.50	-1.99	88.20	-4.50	91.00	-5.10	92.60	-6.05
With	$N_As_Rc_R(p)$	10	91.20	-1.97	87.90	-4.71	91.50	-5.13	93.90	-5.78
With	$N_As_Rc_R(p)$	11	90.70	-2.24	87.50	-4.24	91.70	-5.17	94.30	-5.39
With	$N_As_Rc_R(p)$	12	91.20	-2.14	88.00	-4.37	91.00	-4.96	92.70	-5.27
With	$N_As_Rc_R(p)$	13	90.20	-2.08	88.20	-4.65	90.90	-5.12	91.40	-5.23
With	$N_As_Rc_R(p)$	14	89.90	-1.97	87.40	-4.21	91.60	-4.97	91.80	-4.91
With	$N_As_Rc_R(p)$	15	89.90	-2.00	86.90	-4.47	91.40	-4.90	91.10	-4.79
With	$N_As_Rc_R(p)$	16	90.10	-2.12	87.20	-4.25	91.50	-4.66	91.50	-5.49
With	$N_As_Rc_R(p)$	17	89.90	-1.96	87.60	-4.44	91.20	-5.32	91.10	-5.72
With	$N_As_Rc_R(p)$	18	90.70	-2.18	86.90	-4.30	90.40	-5.82	90.60	-5.59
With	$N_As_Rc_R(p)$	19	90.40	-2.00	87.20	-4.36	91.60	-5.49	91.30	-5.78
With	$N_As_Rc_R(p)$	20	91.20	-1.88	87.70	-4.40	90.00	-5.94	91.40	-6.42
With	$N_As_Rc_R(p)$	21	90.30	-1.86	85.90	-4.40	90.30	-5.97	88.10	-6.22
With	$N_As_Rc_R(p)$	22	90.20	-2.05	87.40	-4.46	90.50	-5.47	87.60	-6.77
With	$N_As_Rc_R(p)$	23	90.20	-2.09	86.00	-4.51	88.00	-5.89	85.60	-6.86
With	$N_As_Rc_R(p)$	24	90.30	-2.36	82.20	-4.78	82.90	-6.37	79.80	-8.74
With	$N_As_Rc_R(p)$	25	89.50	-2.28	81.30	-4.61	78.60	-6.57	75.40	-9.40

Table (A.7) contains the data used for plotting Figure (3.3), the relative bias in estimation of annual abundance for each model, for each year of data from pooled age-class robustness simulations.

Table A.7: *Median relative bias in total annual abundance estimates from pooled age-class robustness simulations. Results indicate negligible bias for mixed-effects models employing the Horvitz-Thompson abundance estimator, positive bias for the fixed-effects model employing the Horvitz-Thompson abundance estimator (when the auxilary catch-effort likelihood of Equation (1.7) is included). Stock-recruit models generally show low bias except for the Period Recruitment scenario, where abundance estimates show periodic behavior with increasing negative bias. Results based on* $n = 1000$ *replicates. "Aux. Like." = Auxiliary catch-effort like component used (With) or not used (Without).*

Table A.7
Annual Abundance Percent Bias

Aux. Like.	Model	Year	S Increasing	S Decreasing	Periodic Recruitment
Without	$N_{SR,R}s_{F}c_{F}(p)$	1	1.44%	-2.69%	0.46%
Without	$N_{SR,R}s_{F}c_{F}(p)$	2	1.61%	-2.85%	-0.15%
Without	$N_{SR,R}s_{F}c_{F}(p)$	3	1.72%	-3.68%	-2.22%
Without	$N_{SR,R}s_{F}c_{F}(p)$	4	1.93%	-4.37%	-2.39%
Without	$N_{SR,R}s_{F}c_{F}(p)$	5	3.09%	-4.88%	-1.09%
Without	$N_{SR,R}s_{F}c_{F}(p)$	6	4.39%	-5.36%	-3.12%
Without	$N_{SR,R}s_{F}c_{F}(p)$	7	5.92%	-5.67%	-5.89%
Without	$N_{SR,R}s_{F}c_{F}(p)$	8	5.9%	-5.11%	-5.59%
Without	$N_{SR,R}s_{F}c_{F}(p)$	9	5.59%	-5.56%	-3.86%
Without	$N_{SR,R}s_{F}c_{F}(p)$	10	5.89%	-6.56%	-4.82%
Without	$N_{SR,R}s_{F}c_{F}(p)$	11	5.63%	-6.25%	-6.48%
Without	$N_{SR,R}s_{F}c_{F}(p)$	12	6.03%	-5.83%	-7.21%
Without	$N_{SR,R}s_{F}c_{F}(p)$	13	6.11%	-6.24%	-5.76%
Without	$N_{SR,R}s_{F}c_{F}(p)$	14	4.79%	-5.43%	-7.18%
Without	$N_{SR,R}s_{F}c_{F}(p)$	15	5.31%	-5.39%	-10%
Without	$N_{SR,R}s_{F}c_{F}(p)$	16	5.14%	-5.55%	-9.74%
Without	$N_{SR,R}s_{F}c_{F}(p)$	17	4.54%	-5.17%	-8.2%
Without	$N_{SR,R}s_{F}c_{F}(p)$	18	5.37%	-5.2%	-8.81%
Without	$N_{SR,R}s_{F}c_{F}(p)$	19	4.48%	-5.44%	-10.61%
Without	$N_{SR,R}s_{F}c_{F}(p)$	20	3.34%	-4.83%	-10.87%
Without	$N_{SR,R}s_{F}c_{F}(p)$	21	2.96%	-4.19%	-10.42%
Without	$N_{SR,R}s_{F}c_{F}(p)$	22	2.37%	-3.17%	-11.39%
Without	$N_{SR,R}s_{F}c_{F}(p)$	23	0.53%	-2.95%	-13.71%

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Table A.7 – continued from previous page

Annual Abundance Percent Bias					
Aux. Like.	**Model**	**Year**	**S Increasing**	**S Decreasing**	**Periodic Recruitment**
Without	$N_{SR,R}s_Fc_F(p)$	24	0.03%	-1.74%	-9.59%
Without	$N_{SR,R}s_Fc_F(p)$	25	-0.72%	-1.21%	-6.65%
Without	$N_{SR,R}s_Fc_R(p)$	1	0.06%	0.81%	2.33%
Without	$N_{SR,R}s_Fc_R(p)$	2	0.31%	0.56%	0.95%
Without	$N_{SR,R}s_Fc_R(p)$	3	1.02%	-0.2%	-4.9%
Without	$N_{SR,R}s_Fc_R(p)$	4	1.08%	-0.7%	-3.88%
Without	$N_{SR,R}s_Fc_R(p)$	5	1.36%	-0.79%	-0.1%
Without	$N_{SR,R}s_Fc_R(p)$	6	1.68%	-0.93%	-0.86%
Without	$N_{SR,R}s_Fc_R(p)$	7	1.55%	-1.44%	-6.04%
Without	$N_{SR,R}s_Fc_R(p)$	8	2.09%	-1.8%	-5.11%
Without	$N_{SR,R}s_Fc_R(p)$	9	2.82%	-2.1%	-0.59%
Without	$N_{SR,R}s_Fc_R(p)$	10	3.24%	-2.31%	-1.21%
Without	$N_{SR,R}s_Fc_R(p)$	11	2.88%	-2.68%	-7.63%
Without	$N_{SR,R}s_Fc_R(p)$	12	3.2%	-2.5%	-6.27%
Without	$N_{SR,R}s_Fc_R(p)$	13	3.72%	-2.74%	-2.13%
Without	$N_{SR,R}s_Fc_R(p)$	14	3.32%	-2.88%	-3.33%
Without	$N_{SR,R}s_Fc_R(p)$	15	2.98%	-2.66%	-8.74%
Without	$N_{SR,R}s_Fc_R(p)$	16	2.13%	-2.23%	-8.12%
Without	$N_{SR,R}s_Fc_R(p)$	17	2.52%	-2.31%	-3.68%
Without	$N_{SR,R}s_Fc_R(p)$	18	2.45%	-2.35%	-4.47%
Without	$N_{SR,R}s_Fc_R(p)$	19	2.13%	-2.09%	-10.34%
Without	$N_{SR,R}s_Fc_R(p)$	20	1.64%	-1.97%	-9.15%
Without	$N_{SR,R}s_Fc_R(p)$	21	1.34%	-1.21%	-5.08%
Without	$N_{SR,R}s_Fc_R(p)$	22	1.02%	-0.62%	-5.51%
Without	$N_{SR,R}s_Fc_R(p)$	23	0.78%	-0.48%	-12.49%
Without	$N_{SR,R}s_Fc_R(p)$	24	0.43%	0.08%	-8.9%
Without	$N_{SR,R}s_Fc_R(p)$	25	-0.39%	0.41%	-5.31%
Without	$N_{SR,R}s_Rc_R(p)$	1	0.1%	0.83%	2.19%
Without	$N_{SR,R}s_Rc_R(p)$	2	0.32%	0.62%	0.84%
Without	$N_{SR,R}s_Rc_R(p)$	3	0.99%	-0.1%	-5.02%
Without	$N_{SR,R}s_Rc_R(p)$	4	0.77%	-0.87%	-4.08%
Without	$N_{SR,R}s_Rc_R(p)$	5	1.34%	-0.64%	-0.45%
Without	$N_{SR,R}s_Rc_R(p)$	6	1.71%	-0.63%	-1.16%
Without	$N_{SR,R}s_Rc_R(p)$	7	1.57%	-1.15%	-6.24%
Without	$N_{SR,R}s_Rc_R(p)$	8	1.86%	-1.62%	-5.28%
Without	$N_{SR,R}s_Rc_R(p)$	9	2.6%	-1.72%	-1.18%

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Table A.7 – continued from previous page

Annual Abundance Percent Bias					
Aux. Like.	**Model**	**Year**	**S Increasing**	**S Decreasing**	**Periodic Recruitment**
Without	$N_{SR,R}s_Rc_R(p)$	10	2.89%	-2.17%	-1.42%
Without	$N_{SR,R}s_Rc_R(p)$	11	3.2%	-2.29%	-7.57%
Without	$N_{SR,R}s_Rc_R(p)$	12	2.94%	-2.15%	-6.27%
Without	$N_{SR,R}s_Rc_R(p)$	13	3.7%	-2.53%	-2.39%
Without	$N_{SR,R}s_Rc_R(p)$	14	3.11%	-2.73%	-3.33%
Without	$N_{SR,R}s_Rc_R(p)$	15	2.74%	-2.63%	-8.88%
Without	$N_{SR,R}s_Rc_R(p)$	16	1.97%	-2.17%	-8.4%
Without	$N_{SR,R}s_Rc_R(p)$	17	2.45%	-2.14%	-3.87%
Without	$N_{SR,R}s_Rc_R(p)$	18	2.29%	-2.45%	-4.47%
Without	$N_{SR,R}s_Rc_R(p)$	19	1.69%	-2.02%	-10.29%
Without	$N_{SR,R}s_Rc_R(p)$	20	1.35%	-2.14%	-9.15%
Without	$N_{SR,R}s_Rc_R(p)$	21	1.05%	-1.27%	-5.24%
Without	$N_{SR,R}s_Rc_R(p)$	22	0.7%	-0.58%	-5.54%
Without	$N_{SR,R}s_Rc_R(p)$	23	0.69%	-0.42%	-12.34%
Without	$N_{SR,R}s_Rc_R(p)$	24	0.38%	0.13%	-8.9%
Without	$N_{SR,R}s_Rc_R(p)$	25	-0.35%	0.38%	-5.55%
Without	$s_Fc_F(p)$	1	0.66%	-0.44%	0.67%
Without	$s_Fc_F(p)$	2	0.92%	0.69%	1.2%
Without	$s_Fc_F(p)$	3	0.2%	-0.14%	-0.09%
Without	$s_Fc_F(p)$	4	1.11%	0.17%	0.6%
Without	$s_Fc_F(p)$	5	1.88%	-0.8%	0.83%
Without	$s_Fc_F(p)$	6	1.14%	0.13%	0.33%
Without	$s_Fc_F(p)$	7	0.72%	-0.11%	0.76%
Without	$s_Fc_F(p)$	8	1.24%	0.26%	1.19%
Without	$s_Fc_F(p)$	9	0.6%	0.48%	1.15%
Without	$s_Fc_F(p)$	10	1.42%	-0.19%	0.59%
Without	$s_Fc_F(p)$	11	1.63%	0.6%	0.15%
Without	$s_Fc_F(p)$	12	0.27%	0.31%	0.64%
Without	$s_Fc_F(p)$	13	0.92%	-0.21%	0.68%
Without	$s_Fc_F(p)$	14	1.06%	0.33%	0.59%
Without	$s_Fc_F(p)$	15	2.12%	0.46%	0.32%
Without	$s_Fc_F(p)$	16	1.06%	0.7%	1.21%
Without	$s_Fc_F(p)$	17	1.08%	1%	0.18%
Without	$s_Fc_F(p)$	18	1.11%	0.36%	0.74%
Without	$s_Fc_F(p)$	19	0.64%	-0.73%	0.4%
Without	$s_Fc_F(p)$	20	0.83%	0.19%	0.5%
Without	$s_Fc_F(p)$	21	0.92%	0.6%	-0.43%

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Table A.7 – continued from previous page

Annual Abundance Percent Bias					
Aux. Like.	**Model**	**Year**	**S Increasing**	**S Decreasing**	**Periodic Recruitment**
Without	$s_F c_F(p)$	22	0.87%	-0.31%	1.21%
Without	$s_F c_F(p)$	23	0.39%	0.38%	0.33%
Without	$s_F c_F(p)$	24	0.53%	0.09%	0.71%
Without	$s_F c_F(p)$	25	1.61%	0.25%	0.52%
Without	$N_A s_F c_F(p)$	1	0.38%	-2.03%	1.28%
Without	$N_A s_F c_F(p)$	2	2.54%	-1.4%	2.54%
Without	$N_A s_F c_F(p)$	3	1.95%	-1.7%	4.35%
Without	$N_A s_F c_F(p)$	4	7.15%	-1.52%	5.46%
Without	$N_A s_F c_F(p)$	5	8.67%	-0.69%	5.43%
Without	$N_A s_F c_F(p)$	6	10.9%	0.11%	5.74%
Without	$N_A s_F c_F(p)$	7	13.43%	1.66%	6.77%
Without	$N_A s_F c_F(p)$	8	12.65%	1.06%	7.45%
Without	$N_A s_F c_F(p)$	9	14.61%	1.21%	10.42%
Without	$N_A s_F c_F(p)$	10	18.94%	3.63%	12.93%
Without	$N_A s_F c_F(p)$	11	18.22%	4.53%	15.54%
Without	$N_A s_F c_F(p)$	12	18.58%	5.82%	14.19%
Without	$N_A s_F c_F(p)$	13	19.87%	7.27%	14.64%
Without	$N_A s_F c_F(p)$	14	19.35%	9.08%	16.17%
Without	$N_A s_F c_F(p)$	15	21.93%	10.39%	19.31%
Without	$N_A s_F c_F(p)$	16	22.44%	13.02%	22.36%
Without	$N_A s_F c_F(p)$	17	26.53%	17.26%	21.82%
Without	$N_A s_F c_F(p)$	18	24.92%	20.15%	22.57%
Without	$N_A s_F c_F(p)$	19	24.72%	20.89%	24.82%
Without	$N_A s_F c_F(p)$	20	24.01%	25.93%	28.9%
Without	$N_A s_F c_F(p)$	21	26.5%	28%	28.33%
Without	$N_A s_F c_F(p)$	22	26.09%	31.56%	29.98%
Without	$N_A s_F c_F(p)$	23	24.06%	34.11%	32.58%
Without	$N_A s_F c_F(p)$	24	16.14%	23.29%	19%
Without	$N_A s_F c_F(p)$	25	11.37%	15.86%	14.85%
Without	$s_F c_R(p)$	1	-0.83%	0.71%	0.52%
Without	$s_F c_R(p)$	2	-0.56%	0.51%	0%
Without	$s_F c_R(p)$	3	-0.83%	0.51%	-0.3%
Without	$s_F c_R(p)$	4	-0.39%	0.07%	0.05%
Without	$s_F c_R(p)$	5	-0.04%	-0.13%	0.06%
Without	$s_F c_R(p)$	6	-0.48%	-0.01%	0.27%
Without	$s_F c_R(p)$	7	-0.41%	0.16%	0.01%

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Table A.7 – continued from previous page

Annual Abundance Percent Bias					
Aux. Like.	**Model**	**Year**	**S Increasing**	**S Decreasing**	**Periodic Recruitment**
Without	$s_F c_R(p)$	8	-0.15%	-0.24%	0.09%
Without	$s_F c_R(p)$	9	-0.24%	-0.12%	0.05%
Without	$s_F c_R(p)$	10	-0.01%	-0.33%	0.04%
Without	$s_F c_R(p)$	11	0.11%	-0.51%	-0.09%
Without	$s_F c_R(p)$	12	0.08%	-0.33%	-0.28%
Without	$s_F c_R(p)$	13	-0.04%	-0.43%	-0.18%
Without	$s_F c_R(p)$	14	0.37%	0.13%	0.25%
Without	$s_F c_R(p)$	15	0.53%	-0.24%	-0.03%
Without	$s_F c_R(p)$	16	0.53%	-0.7%	-0.19%
Without	$s_F c_R(p)$	17	0.72%	-0.25%	-0.48%
Without	$s_F c_R(p)$	18	0.83%	-0.51%	-0.01%
Without	$s_F c_R(p)$	19	0.35%	-0.73%	-0.12%
Without	$s_F c_R(p)$	20	0.96%	-0.93%	-0.49%
Without	$s_F c_R(p)$	21	0.43%	-0.4%	-0.48%
Without	$s_F c_R(p)$	22	0.26%	-0.38%	0.16%
Without	$s_F c_R(p)$	23	0.28%	-0.25%	0.02%
Without	$s_F c_R(p)$	24	0.3%	-0.24%	-0.06%
Without	$s_F c_R(p)$	25	-0.21%	0.51%	-0.02%
Without	$N_A s_F c_R(p)$	1	-0.39%	0.18%	1.76%
Without	$N_A s_F c_R(p)$	2	1.06%	-0.24%	1.71%
Without	$N_A s_F c_R(p)$	3	2.19%	-1.01%	2.68%
Without	$N_A s_F c_R(p)$	4	4.96%	-0.67%	4.2%
Without	$N_A s_F c_R(p)$	5	6.59%	-0.79%	3.98%
Without	$N_A s_F c_R(p)$	6	9.66%	-0.43%	4.86%
Without	$N_A s_F c_R(p)$	7	10.88%	0.7%	6.06%
Without	$N_A s_F c_R(p)$	8	10.74%	1.25%	6.98%
Without	$N_A s_F c_R(p)$	9	11.54%	0.85%	8.91%
Without	$N_A s_F c_R(p)$	10	14.39%	1.77%	10.22%
Without	$N_A s_F c_R(p)$	11	16.54%	3.55%	11.54%
Without	$N_A s_F c_R(p)$	12	16.04%	5.14%	10.96%
Without	$N_A s_F c_R(p)$	13	17.25%	6.33%	13.99%
Without	$N_A s_F c_R(p)$	14	18.27%	6.84%	14.39%
Without	$N_A s_F c_R(p)$	15	19.43%	8.47%	15.56%
Without	$N_A s_F c_R(p)$	16	20.38%	9.54%	16%
Without	$N_A s_F c_R(p)$	17	22.31%	14.36%	15.98%
Without	$N_A s_F c_R(p)$	18	23.29%	15.36%	17.42%
Without	$N_A s_F c_R(p)$	19	22.45%	16.81%	18.68%

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Table A.7 – continued from previous page

Annual Abundance Percent Bias					
Aux. Like.	**Model**	**Year**	**S Increasing**	**S Decreasing**	**Periodic Recruitment**
Without	$N_A s_F c_R(p)$	20	20.92%	22.23%	20.47%
Without	$N_A s_F c_R(p)$	21	21.27%	23.33%	20.99%
Without	$N_A s_F c_R(p)$	22	22.74%	25.87%	20.89%
Without	$N_A s_F c_R(p)$	23	23.08%	30.13%	23.62%
Without	$N_A s_F c_R(p)$	24	15.57%	20.28%	15.02%
Without	$N_A s_F c_R(p)$	25	9.78%	15.05%	10.26%
Without	$s_R c_R(p)$	1	-0.76%	0.62%	0.49%
Without	$s_R c_R(p)$	2	-0.48%	0.55%	0.15%
Without	$s_R c_R(p)$	3	-0.82%	0.32%	-0.24%
Without	$s_R c_R(p)$	4	-0.24%	0.19%	0.16%
Without	$s_R c_R(p)$	5	-0.04%	-0.02%	0.1%
Without	$s_R c_R(p)$	6	-0.21%	-0.18%	0.3%
Without	$s_R c_R(p)$	7	-0.38%	-0.08%	0.09%
Without	$s_R c_R(p)$	8	-0.36%	-0.35%	0%
Without	$s_R c_R(p)$	9	-0.38%	-0.07%	0.04%
Without	$s_R c_R(p)$	10	-0.08%	-0.24%	-0.01%
Without	$s_R c_R(p)$	11	-0.01%	-0.28%	-0.3%
Without	$s_R c_R(p)$	12	-0.13%	-0.23%	-0.24%
Without	$s_R c_R(p)$	13	0%	-0.34%	-0.18%
Without	$s_R c_R(p)$	14	0.35%	0.36%	0.34%
Without	$s_R c_R(p)$	15	0.52%	-0.06%	-0.06%
Without	$s_R c_R(p)$	16	0.5%	-0.55%	-0.3%
Without	$s_R c_R(p)$	17	0.68%	0%	-0.43%
Without	$s_R c_R(p)$	18	0.63%	-0.48%	-0.27%
Without	$s_R c_R(p)$	19	0.11%	-0.72%	-0.26%
Without	$s_R c_R(p)$	20	0.7%	-0.82%	-0.59%
Without	$s_R c_R(p)$	21	0.36%	-0.32%	-0.59%
Without	$s_R c_R(p)$	22	0.26%	-0.23%	0.14%
Without	$s_R c_R(p)$	23	0.12%	-0.24%	-0.02%
Without	$s_R c_R(p)$	24	0.16%	0.11%	-0.06%
Without	$s_R c_R(p)$	25	-0.21%	0.62%	0.01%
Without	$N_A s_R c_R(p)$	1	-0.4%	0.18%	1.76%
Without	$N_A s_R c_R(p)$	2	1.03%	-0.24%	1.71%
Without	$N_A s_R c_R(p)$	3	2.12%	-1.01%	2.68%
Without	$N_A s_R c_R(p)$	4	4.95%	-0.35%	4.2%
Without	$N_A s_R c_R(p)$	5	6.56%	-0.76%	3.98%

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Table A.7 – continued from previous page

Annual Abundance Percent Bias					
Aux. Like.	**Model**	**Year**	**S Increasing**	**S Decreasing**	**Periodic Recruitment**
Without	$N_{A}s_{R}c_{R}(p)$	6	9.64%	-0.38%	4.89%
Without	$N_{A}s_{R}c_{R}(p)$	7	10.82%	0.76%	6.24%
Without	$N_{A}s_{R}c_{R}(p)$	8	10.63%	1.3%	7.03%
Without	$N_{A}s_{R}c_{R}(p)$	9	11.47%	0.98%	9.17%
Without	$N_{A}s_{R}c_{R}(p)$	10	14.22%	1.86%	10.3%
Without	$N_{A}s_{R}c_{R}(p)$	11	16.18%	3.62%	11.62%
Without	$N_{A}s_{R}c_{R}(p)$	12	15.99%	5.22%	10.96%
Without	$N_{A}s_{R}c_{R}(p)$	13	17.25%	6.33%	13.99%
Without	$N_{A}s_{R}c_{R}(p)$	14	18.27%	6.84%	14.39%
Without	$N_{A}s_{R}c_{R}(p)$	15	19.25%	8.47%	15.56%
Without	$N_{A}s_{R}c_{R}(p)$	16	20.24%	9.75%	16%
Without	$N_{A}s_{R}c_{R}(p)$	17	22.25%	14.36%	16.8%
Without	$N_{A}s_{R}c_{R}(p)$	18	23.12%	15.36%	17.42%
Without	$N_{A}s_{R}c_{R}(p)$	19	22.31%	16.81%	18.68%
Without	$N_{A}s_{R}c_{R}(p)$	20	20.91%	22.23%	20.47%
Without	$N_{A}s_{R}c_{R}(p)$	21	21.27%	23.33%	20.99%
Without	$N_{A}s_{R}c_{R}(p)$	22	22.74%	25.87%	20.89%
Without	$N_{A}s_{R}c_{R}(p)$	23	23.08%	30.13%	23.62%
Without	$N_{A}s_{R}c_{R}(p)$	24	15.57%	20.28%	15.02%
Without	$N_{A}s_{R}c_{R}(p)$	25	9.78%	15.05%	10.26%
With	$N_{SR,F}s_{F}c_{F}(p)$	1	13.06%	15.44%	24.25%
With	$N_{SR,F}s_{F}c_{F}(p)$	2	12.77%	15.77%	24.44%
With	$N_{SR,F}s_{F}c_{F}(p)$	3	13.8%	14.92%	12.12%
With	$N_{SR,F}s_{F}c_{F}(p)$	4	13.66%	14.6%	16.72%
With	$N_{SR,F}s_{F}c_{F}(p)$	5	14.34%	14.14%	25.19%
With	$N_{SR,F}s_{F}c_{F}(p)$	6	15.41%	13.97%	21.4%
With	$N_{SR,F}s_{F}c_{F}(p)$	7	15.55%	14.47%	11.15%
With	$N_{SR,F}s_{F}c_{F}(p)$	8	16.4%	14.65%	15.43%
With	$N_{SR,F}s_{F}c_{F}(p)$	9	16.37%	13.92%	25.2%
With	$N_{SR,F}s_{F}c_{F}(p)$	10	16.1%	14.46%	21.27%
With	$N_{SR,F}s_{F}c_{F}(p)$	11	15.67%	14.22%	11.88%
With	$N_{SR,F}s_{F}c_{F}(p)$	12	15.26%	14.32%	15.43%
With	$N_{SR,F}s_{F}c_{F}(p)$	13	15.6%	15.4%	24.43%
With	$N_{SR,F}s_{F}c_{F}(p)$	14	15.14%	14.94%	22.18%
With	$N_{SR,F}s_{F}c_{F}(p)$	15	15.5%	14.6%	11.76%
With	$N_{SR,F}s_{F}c_{F}(p)$	16	16.37%	15.58%	17.12%
With	$N_{SR,F}s_{F}c_{F}(p)$	17	15.81%	13.58%	25.72%

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Table A.7 – continued from previous page

Annual Abundance Percent Bias					
Aux. Like.	**Model**	**Year**	**S Increasing**	**S Decreasing**	**Periodic Recruitment**
With	$N_{SR,F}s_Fc_F(p)$	18	15.5%	13.16%	21.49%
With	$N_{SR,F}s_Fc_F(p)$	19	15.96%	14.58%	11.26%
With	$N_{SR,F}s_Fc_F(p)$	20	15.71%	14.37%	16.01%
With	$N_{SR,F}s_Fc_F(p)$	21	15.4%	14.34%	24.79%
With	$N_{SR,F}s_Fc_F(p)$	22	15.41%	14.46%	20.11%
With	$N_{SR,F}s_Fc_F(p)$	23	15.17%	15.34%	11.02%
With	$N_{SR,F}s_Fc_F(p)$	24	14.99%	16.19%	14.46%
With	$N_{SR,F}s_Fc_F(p)$	25	14.77%	16.82%	24.28%
With	$N_{SR,R}s_Fc_F(p)$	1	3.48%	1.82%	4.18%
With	$N_{SR,R}s_Fc_F(p)$	2	4.46%	2.74%	3.84%
With	$N_{SR,R}s_Fc_F(p)$	3	3.92%	3.62%	2.83%
With	$N_{SR,R}s_Fc_F(p)$	4	4.43%	3.23%	3.06%
With	$N_{SR,R}s_Fc_F(p)$	5	5.14%	2.84%	4.36%
With	$N_{SR,R}s_Fc_F(p)$	6	4.82%	2.27%	4.03%
With	$N_{SR,R}s_Fc_F(p)$	7	4.57%	3.05%	3.26%
With	$N_{SR,R}s_Fc_F(p)$	8	4.59%	3.36%	3.65%
With	$N_{SR,R}s_Fc_F(p)$	9	4.04%	3.25%	5.36%
With	$N_{SR,R}s_Fc_F(p)$	10	4.64%	2.62%	4.34%
With	$N_{SR,R}s_Fc_F(p)$	11	4.9%	2.96%	2.91%
With	$N_{SR,R}s_Fc_F(p)$	12	4.43%	2.9%	3.23%
With	$N_{SR,R}s_Fc_F(p)$	13	3.97%	2.58%	4.1%
With	$N_{SR,R}s_Fc_F(p)$	14	4.88%	3.24%	3.87%
With	$N_{SR,R}s_Fc_F(p)$	15	5.46%	3.09%	3.24%
With	$N_{SR,R}s_Fc_F(p)$	16	4.34%	2.53%	3.84%
With	$N_{SR,R}s_Fc_F(p)$	17	4.67%	3.07%	4.02%
With	$N_{SR,R}s_Fc_F(p)$	18	3.89%	2.31%	4.67%
With	$N_{SR,R}s_Fc_F(p)$	19	4.2%	2.54%	3.17%
With	$N_{SR,R}s_Fc_F(p)$	20	4.8%	3.4%	3.27%
With	$N_{SR,R}s_Fc_F(p)$	21	4.57%	3.1%	4.12%
With	$N_{SR,R}s_Fc_F(p)$	22	4.15%	2.65%	4.55%
With	$N_{SR,R}s_Fc_F(p)$	23	3.89%	3.45%	2.92%
With	$N_{SR,R}s_Fc_F(p)$	24	4.19%	3.35%	3.16%
With	$N_{SR,R}s_Fc_F(p)$	25	3.82%	3.42%	3.47%
With	$N_{SR,R}s_Fc_R(p)$	1	-1.61%	-0.14%	2.14%
With	$N_{SR,R}s_Fc_R(p)$	2	-0.9%	-0.02%	1.97%
With	$N_{SR,R}s_Fc_R(p)$	3	-1.2%	-0.19%	-2.35%

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Table A.7 – continued from previous page

Annual Abundance Percent Bias					
Aux. Like.	**Model**	**Year**	**S Increasing**	**S Decreasing**	**Periodic Recruitment**
With	$N_{SR,Rs_Fc_R}(p)$	4	-1.24%	0.06%	-1.44%
With	$N_{SR,Rs_Fc_R}(p)$	5	-1.22%	-0.13%	2.89%
With	$N_{SR,Rs_Fc_R}(p)$	6	-0.7%	-0.53%	2.64%
With	$N_{SR,Rs_Fc_R}(p)$	7	-0.57%	-0.41%	-2.69%
With	$N_{SR,Rs_Fc_R}(p)$	8	-0.34%	-0.78%	-1.46%
With	$N_{SR,Rs_Fc_R}(p)$	9	-0.25%	-0.59%	3.1%
With	$N_{SR,Rs_Fc_R}(p)$	10	-0.32%	-1.26%	2.41%
With	$N_{SR,Rs_Fc_R}(p)$	11	-0.52%	-0.8%	-2.3%
With	$N_{SR,Rs_Fc_R}(p)$	12	-0.1%	-0.83%	-1.55%
With	$N_{SR,Rs_Fc_R}(p)$	13	0.05%	-1.38%	3.2%
With	$N_{SR,Rs_Fc_R}(p)$	14	0.3%	-0.96%	2.35%
With	$N_{SR,Rs_Fc_R}(p)$	15	0.19%	-0.98%	-2.6%
With	$N_{SR,Rs_Fc_R}(p)$	16	0.11%	-1%	-1.67%
With	$N_{SR,Rs_Fc_R}(p)$	17	0.33%	-1.38%	3.09%
With	$N_{SR,Rs_Fc_R}(p)$	18	0.21%	-1.34%	2.44%
With	$N_{SR,Rs_Fc_R}(p)$	19	0.24%	-1.69%	-2.84%
With	$N_{SR,Rs_Fc_R}(p)$	20	0.32%	-1.47%	-1.55%
With	$N_{SR,Rs_Fc_R}(p)$	21	0.06%	-1.35%	2.66%
With	$N_{SR,Rs_Fc_R}(p)$	22	-0.25%	-1.12%	1.62%
With	$N_{SR,Rs_Fc_R}(p)$	23	-0.39%	-0.85%	-3.13%
With	$N_{SR,Rs_Fc_R}(p)$	24	-0.47%	-0.67%	-2.43%
With	$N_{SR,Rs_Fc_R}(p)$	25	-1.01%	-0.53%	0.04%
With	$N_{SR,Rs_Rc_R}(p)$	1	-1.8%	-0.09%	2.03%
With	$N_{SR,Rs_Rc_R}(p)$	2	-0.9%	0%	1.91%
With	$N_{SR,Rs_Rc_R}(p)$	3	-1.23%	-0.18%	-2.37%
With	$N_{SR,Rs_Rc_R}(p)$	4	-1.13%	0.08%	-1.49%
With	$N_{SR,Rs_Rc_R}(p)$	5	-1.16%	-0.08%	2.85%
With	$N_{SR,Rs_Rc_R}(p)$	6	-0.7%	-0.5%	2.52%
With	$N_{SR,Rs_Rc_R}(p)$	7	-0.49%	-0.44%	-2.64%
With	$N_{SR,Rs_Rc_R}(p)$	8	-0.39%	-0.73%	-1.57%
With	$N_{SR,Rs_Rc_R}(p)$	9	-0.15%	-0.48%	3.01%
With	$N_{SR,Rs_Rc_R}(p)$	10	-0.34%	-1.08%	2.35%
With	$N_{SR,Rs_Rc_R}(p)$	11	-0.54%	-0.71%	-2.33%
With	$N_{SR,Rs_Rc_R}(p)$	12	-0.2%	-0.85%	-1.52%
With	$N_{SR,Rs_Rc_R}(p)$	13	-0.29%	-1.36%	3.2%
With	$N_{SR,Rs_Rc_R}(p)$	14	-0.01%	-0.88%	2.28%
With	$N_{SR,Rs_Rc_R}(p)$	15	-0.24%	-0.75%	-2.57%

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Table A.7 – continued from previous page

Annual Abundance Percent Bias					
Aux. Like.	**Model**	**Year**	**S Increasing**	**S Decreasing**	**Periodic Recruitment**
With	$N_{SR,R}s_Rc_R(p)$	16	-0.12%	-0.79%	-1.72%
With	$N_{SR,R}s_Rc_R(p)$	17	0.16%	-1.24%	3.19%
With	$N_{SR,R}s_Rc_R(p)$	18	-0.09%	-1.21%	2.28%
With	$N_{SR,R}s_Rc_R(p)$	19	0.13%	-1.6%	-2.96%
With	$N_{SR,R}s_Rc_R(p)$	20	0.21%	-1.39%	-1.71%
With	$N_{SR,R}s_Rc_R(p)$	21	-0.29%	-1.25%	2.61%
With	$N_{SR,R}s_Rc_R(p)$	22	-0.27%	-0.98%	1.56%
With	$N_{SR,R}s_Rc_R(p)$	23	-0.48%	-0.85%	-3.16%
With	$N_{SR,R}s_Rc_R(p)$	24	-0.58%	-0.51%	-2.43%
With	$N_{SR,R}s_Rc_R(p)$	25	-1.11%	-0.52%	-0.03%
With	$s_Fc_F(p)$	1	9.73%	9.4%	9.25%
With	$s_Fc_F(p)$	2	9.45%	10.74%	9.38%
With	$s_Fc_F(p)$	3	9.15%	11.33%	8.82%
With	$s_Fc_F(p)$	4	10.57%	11.59%	8.9%
With	$s_Fc_F(p)$	5	10.92%	9.6%	10.58%
With	$s_Fc_F(p)$	6	9.76%	10.71%	10%
With	$s_Fc_F(p)$	7	10.54%	9.37%	9.22%
With	$s_Fc_F(p)$	8	10.21%	11.47%	10.45%
With	$s_Fc_F(p)$	9	9.92%	10.27%	10.99%
With	$s_Fc_F(p)$	10	10.93%	10.53%	9.38%
With	$s_Fc_F(p)$	11	11.49%	11.51%	9.46%
With	$s_Fc_F(p)$	12	9.61%	10.47%	10.28%
With	$s_Fc_F(p)$	13	9.97%	10.05%	9.88%
With	$s_Fc_F(p)$	14	10.88%	11.04%	9.85%
With	$s_Fc_F(p)$	15	10.69%	11.36%	10.07%
With	$s_Fc_F(p)$	16	9.93%	11.22%	10.48%
With	$s_Fc_F(p)$	17	10.26%	11.01%	8.84%
With	$s_Fc_F(p)$	18	10.31%	11.34%	10.07%
With	$s_Fc_F(p)$	19	10.1%	9.64%	9.54%
With	$s_Fc_F(p)$	20	10.67%	11.15%	9.78%
With	$s_Fc_F(p)$	21	10.84%	12.26%	8.22%
With	$s_Fc_F(p)$	22	9.55%	9.95%	9.88%
With	$s_Fc_F(p)$	23	10.18%	11.01%	9.57%
With	$s_Fc_F(p)$	24	10.44%	11.19%	9.92%
With	$s_Fc_F(p)$	25	11.5%	11.69%	10.04%
With	$N_As_Fc_F(p)$	1	-0.29%	-0.31%	0.46%

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Table A.7 – continued from previous page

Annual Abundance Percent Bias					
Aux. Like.	**Model**	**Year**	**S Increasing**	**S Decreasing**	**Periodic Recruitment**
With	$N_{A}s_{F}c_{F}(p)$	2	0.5%	-0.03%	0.74%
With	$N_{A}s_{F}c_{F}(p)$	3	-0.28%	0.61%	0.13%
With	$N_{A}s_{F}c_{F}(p)$	4	0.87%	0.73%	0.59%
With	$N_{A}s_{F}c_{F}(p)$	5	0.87%	-0.33%	0.47%
With	$N_{A}s_{F}c_{F}(p)$	6	0.28%	-0.23%	1.18%
With	$N_{A}s_{F}c_{F}(p)$	7	0.73%	0.31%	1.23%
With	$N_{A}s_{F}c_{F}(p)$	8	0.36%	-0.19%	1.37%
With	$N_{A}s_{F}c_{F}(p)$	9	-0.27%	0.1%	1.64%
With	$N_{A}s_{F}c_{F}(p)$	10	0.7%	-0.08%	1.41%
With	$N_{A}s_{F}c_{F}(p)$	11	1.16%	0.18%	0.82%
With	$N_{A}s_{F}c_{F}(p)$	12	0.83%	0.37%	0.1%
With	$N_{A}s_{F}c_{F}(p)$	13	0.16%	-0.07%	0.93%
With	$N_{A}s_{F}c_{F}(p)$	14	0.42%	0.4%	1.05%
With	$N_{A}s_{F}c_{F}(p)$	15	1.32%	0.35%	1.37%
With	$N_{A}s_{F}c_{F}(p)$	16	0.32%	0.06%	0.85%
With	$N_{A}s_{F}c_{F}(p)$	17	1.03%	-0.02%	0.62%
With	$N_{A}s_{F}c_{F}(p)$	18	0.56%	-0.36%	1.33%
With	$N_{A}s_{F}c_{F}(p)$	19	0.64%	-0.55%	0.24%
With	$N_{A}s_{F}c_{F}(p)$	20	0.73%	0.13%	0.61%
With	$N_{A}s_{F}c_{F}(p)$	21	0.32%	-0.14%	0.72%
With	$N_{A}s_{F}c_{F}(p)$	22	0.42%	-0.01%	0.82%
With	$N_{A}s_{F}c_{F}(p)$	23	0.17%	0%	0.54%
With	$N_{A}s_{F}c_{F}(p)$	24	-0.05%	0.01%	0.18%
With	$N_{A}s_{F}c_{F}(p)$	25	0.35%	0.66%	0.56%
With	$s_{F}c_{R}(p)$	1	-2%	-0.46%	-0.71%
With	$s_{F}c_{R}(p)$	2	-1.79%	-0.59%	-1.17%
With	$s_{F}c_{R}(p)$	3	-1.91%	-0.85%	-1.33%
With	$s_{F}c_{R}(p)$	4	-1.82%	-1.08%	-1.09%
With	$s_{F}c_{R}(p)$	5	-1.57%	-1.1%	-1.01%
With	$s_{F}c_{R}(p)$	6	-1.58%	-1.05%	-0.97%
With	$s_{F}c_{R}(p)$	7	-1.33%	-0.92%	-1.14%
With	$s_{F}c_{R}(p)$	8	-1.29%	-1.36%	-1.21%
With	$s_{F}c_{R}(p)$	9	-1.24%	-1.34%	-1.2%
With	$s_{F}c_{R}(p)$	10	-1.14%	-1.49%	-1.26%
With	$s_{F}c_{R}(p)$	11	-0.75%	-1.64%	-1.19%
With	$s_{F}c_{R}(p)$	12	-1.23%	-1.87%	-1.67%
With	$s_{F}c_{R}(p)$	13	-1.19%	-1.59%	-1.2%

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Table A.7 – continued from previous page

Annual Abundance Percent Bias					
Aux. Like.	**Model**	**Year**	**S Increasing**	**S Decreasing**	**Periodic Recruitment**
With	$s_F c_R(p)$	14	-0.9%	-1.65%	-1.07%
With	$s_F c_R(p)$	15	-0.67%	-1.58%	-1.05%
With	$s_F c_R(p)$	16	-0.77%	-2.21%	-1.19%
With	$s_F c_R(p)$	17	-0.62%	-1.73%	-1.37%
With	$s_F c_R(p)$	18	-0.62%	-2.04%	-1.25%
With	$s_F c_R(p)$	19	-0.75%	-2.23%	-1.42%
With	$s_F c_R(p)$	20	-0.54%	-2.45%	-1.39%
With	$s_F c_R(p)$	21	-0.63%	-2.1%	-1.39%
With	$s_F c_R(p)$	22	-0.76%	-1.94%	-1.28%
With	$s_F c_R(p)$	23	-1.11%	-1.63%	-1.34%
With	$s_F c_R(p)$	24	-1.29%	-1.48%	-1.63%
With	$s_F c_R(p)$	25	-1.86%	-0.89%	-1.52%
With	$N_A s_F c_R(p)$	1	-5.17%	-4.16%	-4.09%
With	$N_A s_F c_R(p)$	2	-4.79%	-4.22%	-4.25%
With	$N_A s_F c_R(p)$	3	-4.79%	-4.15%	-4.73%
With	$N_A s_F c_R(p)$	4	-4.84%	-4.58%	-4.59%
With	$N_A s_F c_R(p)$	5	-4.51%	-5.06%	-4.43%
With	$N_A s_F c_R(p)$	6	-4.55%	-4.72%	-4.46%
With	$N_A s_F c_R(p)$	7	-4.76%	-4.81%	-4.85%
With	$N_A s_F c_R(p)$	8	-4.65%	-5.11%	-4.78%
With	$N_A s_F c_R(p)$	9	-4.53%	-5.26%	-4.64%
With	$N_A s_F c_R(p)$	10	-4.47%	-5.13%	-4.39%
With	$N_A s_F c_R(p)$	11	-4.1%	-5.48%	-4.55%
With	$N_A s_F c_R(p)$	12	-4.31%	-5.17%	-4.97%
With	$N_A s_F c_R(p)$	13	-4.1%	-5.49%	-4.59%
With	$N_A s_F c_R(p)$	14	-3.62%	-5.07%	-4.71%
With	$N_A s_F c_R(p)$	15	-3.99%	-5.2%	-4.4%
With	$N_A s_F c_R(p)$	16	-3.93%	-5.36%	-4.99%
With	$N_A s_F c_R(p)$	17	-3.78%	-5.54%	-4.66%
With	$N_A s_F c_R(p)$	18	-3.75%	-5.75%	-4.61%
With	$N_A s_F c_R(p)$	19	-3.83%	-5.52%	-4.93%
With	$N_A s_F c_R(p)$	20	-3.76%	-5.79%	-4.84%
With	$N_A s_F c_R(p)$	21	-3.75%	-5.67%	-5.05%
With	$N_A s_F c_R(p)$	22	-3.72%	-5.6%	-4.68%
With	$N_A s_F c_R(p)$	23	-4.06%	-5.16%	-4.7%
With	$N_A s_F c_R(p)$	24	-4.35%	-5.1%	-4.71%
With	$N_A s_F c_R(p)$	25	-5.15%	-4.88%	-5.38%

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Table A.7 – continued from previous page

Annual Abundance Percent Bias					
Aux. Like.	**Model**	**Year**	**S Increasing**	**S Decreasing**	**Periodic Recruitment**
With	$s_R c_R(p)$	1	-2.15%	-0.71%	-0.85%
With	$s_R c_R(p)$	2	-2.18%	-0.93%	-1.18%
With	$s_R c_R(p)$	3	-1.93%	-1.1%	-1.46%
With	$s_R c_R(p)$	4	-1.7%	-1.12%	-1.24%
With	$s_R c_R(p)$	5	-1.6%	-1.53%	-0.88%
With	$s_R c_R(p)$	6	-1.66%	-1.37%	-1.07%
With	$s_R c_R(p)$	7	-1.39%	-1.47%	-1.29%
With	$s_R c_R(p)$	8	-1.59%	-1.62%	-1.21%
With	$s_R c_R(p)$	9	-1.12%	-1.4%	-1.28%
With	$s_R c_R(p)$	10	-1.29%	-1.67%	-1.28%
With	$s_R c_R(p)$	11	-0.93%	-1.51%	-1.22%
With	$s_R c_R(p)$	12	-1.3%	-1.71%	-1.6%
With	$s_R c_R(p)$	13	-1.28%	-1.61%	-1.33%
With	$s_R c_R(p)$	14	-0.92%	-1.28%	-0.99%
With	$s_R c_R(p)$	15	-0.75%	-1.42%	-1.26%
With	$s_R c_R(p)$	16	-0.97%	-2.12%	-1.26%
With	$s_R c_R(p)$	17	-0.97%	-1.58%	-1.38%
With	$s_R c_R(p)$	18	-0.89%	-2.29%	-1.37%
With	$s_R c_R(p)$	19	-1.13%	-2.08%	-1.61%
With	$s_R c_R(p)$	20	-0.94%	-2.43%	-1.59%
With	$s_R c_R(p)$	21	-1.03%	-1.97%	-1.52%
With	$s_R c_R(p)$	22	-1.11%	-1.96%	-1.24%
With	$s_R c_R(p)$	23	-1.42%	-1.79%	-1.37%
With	$s_R c_R(p)$	24	-1.33%	-1.32%	-1.59%
With	$s_R c_R(p)$	25	-1.84%	-0.99%	-1.44%
With	$N_A s_R c_R(p)$	1	-5.17%	-4.13%	-4.06%
With	$N_A s_R c_R(p)$	2	-4.79%	-4.18%	-4.25%
With	$N_A s_R c_R(p)$	3	-4.74%	-4.12%	-4.73%
With	$N_A s_R c_R(p)$	4	-4.76%	-4.54%	-4.55%
With	$N_A s_R c_R(p)$	5	-4.52%	-5.06%	-4.45%
With	$N_A s_R c_R(p)$	6	-4.63%	-4.69%	-4.47%
With	$N_A s_R c_R(p)$	7	-4.76%	-4.81%	-4.85%
With	$N_A s_R c_R(p)$	8	-4.65%	-5.1%	-4.78%
With	$N_A s_R c_R(p)$	9	-4.57%	-5.22%	-4.67%
With	$N_A s_R c_R(p)$	10	-4.45%	-5.13%	-4.37%
With	$N_A s_R c_R(p)$	11	-4.06%	-5.45%	-4.44%

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Table A.7 – continued from previous page

Annual Abundance Percent Bias					
Aux. Like.	**Model**	**Year**	**S Increasing**	**S Decreasing**	**Periodic Recruitment**
With	$N_A s_R c_R(p)$	12	-4.31%	-5.16%	-4.95%
With	$N_A s_R c_R(p)$	13	-4.13%	-5.4%	-4.59%
With	$N_A s_R c_R(p)$	14	-3.65%	-4.96%	-4.65%
With	$N_A s_R c_R(p)$	15	-4%	-5.21%	-4.39%
With	$N_A s_R c_R(p)$	16	-3.93%	-5.36%	-4.98%
With	$N_A s_R c_R(p)$	17	-3.77%	-5.54%	-4.6%
With	$N_A s_R c_R(p)$	18	-3.74%	-5.71%	-4.65%
With	$N_A s_R c_R(p)$	19	-3.8%	-5.52%	-4.93%
With	$N_A s_R c_R(p)$	20	-3.9%	-5.79%	-4.84%
With	$N_A s_R c_R(p)$	21	-3.81%	-5.65%	-4.99%
With	$N_A s_R c_R(p)$	22	-3.72%	-5.58%	-4.52%
With	$N_A s_R c_R(p)$	23	-4.06%	-5.14%	-4.62%
With	$N_A s_R c_R(p)$	24	-4.35%	-5.06%	-4.72%
With	$N_A s_R c_R(p)$	25	-5.19%	-4.88%	-5.27%

Table (A.8) contains the data used for plotting Figure (3.4), the median relative bias (MRB) and estimated asymptotic 95% confidence interval coverage for each model, for each year of data, for each level of simulated variation from pooled age-class robustness simulations.

Table A.8: *95% confidence interval coverage and median relative bias in total annual abundance estimates from pooled age-class robustness simulations. Results indicate nearst nominal coverage for models employing the Horvitz-Thompson abundance estimator in all scenarios. Stock-recruit models generally show supernominal coverage, while absolute-recruit abundance models generally show subnominal coverage. Results based on* $n = 1000$ *replicates. "Aux. Like." = Auxiliary catch-effort likelihood component used (With) or not used (Without).*

Table A.8 - Annual Abundance 95% CI Coverage and Median Relative Bias								
			S Increasing		S Decreasing		Periodic Recruitment	
Aux. Like.	Model	Year	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB
Without	$N_{SR,R s_F c_F}(p)$	1	67.60	1.44	61.60	-2.69	69.50	0.46
Without	$N_{SR,R s_F c_F}(p)$	2	66.00	1.61	61.50	-2.85	69.20	-0.15
Without	$N_{SR,R s_F c_F}(p)$	3	70.90	1.72	66.00	-3.68	73.80	-2.22
Without	$N_{SR,R s_F c_F}(p)$	4	76.50	1.93	71.60	-4.37	79.10	-2.39
Without	$N_{SR,R s_F c_F}(p)$	5	80.50	3.09	75.80	-4.88	83.50	-1.09
Without	$N_{SR,R s_F c_F}(p)$	6	84.40	4.39	80.90	-5.36	86.20	-3.12
Without	$N_{SR,R s_F c_F}(p)$	7	88.20	5.92	82.80	-5.67	87.90	-5.89
Without	$N_{SR,R s_F c_F}(p)$	8	90.70	5.90	87.00	-5.11	91.20	-5.59
Without	$N_{SR,R s_F c_F}(p)$	9	94.10	5.59	88.80	-5.56	93.20	-3.86
Without	$N_{SR,R s_F c_F}(p)$	10	95.10	5.89	90.60	-6.56	95.30	-4.82
Without	$N_{SR,R s_F c_F}(p)$	11	96.00	5.63	92.30	-6.25	94.20	-6.48
Without	$N_{SR,R s_F c_F}(p)$	12	96.80	6.03	94.30	-5.83	95.00	-7.21
Without	$N_{SR,R s_F c_F}(p)$	13	97.50	6.11	94.20	-6.24	96.60	-5.76
Without	$N_{SR,R s_F c_F}(p)$	14	98.00	4.79	95.40	-5.43	96.80	-7.18
Without	$N_{SR,R s_F c_F}(p)$	15	98.50	5.31	96.30	-5.39	97.20	-10.00
Without	$N_{SR,R s_F c_F}(p)$	16	98.60	5.14	97.40	-5.55	98.00	-9.74
Without	$N_{SR,R s_F c_F}(p)$	17	99.00	4.54	98.10	-5.17	98.30	-8.20
Without	$N_{SR,R s_F c_F}(p)$	18	98.70	5.37	98.20	-5.20	98.60	-8.81
Without	$N_{SR,R s_F c_F}(p)$	19	99.10	4.48	98.20	-5.44	98.80	-10.61
Without	$N_{SR,R s_F c_F}(p)$	20	99.10	3.34	98.50	-4.83	99.30	-10.87
Without	$N_{SR,R s_F c_F}(p)$	21	99.30	2.96	98.30	-4.19	99.60	-10.42
Without	$N_{SR,R s_F c_F}(p)$	22	99.30	2.37	98.90	-3.17	99.60	-11.39
Without	$N_{SR,R s_F c_F}(p)$	23	99.20	0.53	99.00	-2.95	99.30	-13.71
Without	$N_{SR,R s_F c_F}(p)$	24	99.20	0.03	99.30	-1.74	99.50	-9.59
Without	$N_{SR,R s_F c_F}(p)$	25	99.20	-0.72	99.30	-1.21	99.50	-6.65
Without	$N_{SR,R s_F c_R}(p)$	1	93.00	0.06	92.60	0.81	93.40	2.33
Without	$N_{SR,R s_F c_R}(p)$	2	88.70	0.31	88.40	0.56	86.80	0.95
Without	$N_{SR,R s_F c_R}(p)$	3	88.90	1.02	88.50	-0.20	85.20	-4.90
Without	$N_{SR,R s_F c_R}(p)$	4	90.70	1.08	89.30	-0.70	89.00	-3.88
Without	$N_{SR,R s_F c_R}(p)$	5	91.90	1.36	90.80	-0.79	93.30	-0.10
Without	$N_{SR,R s_F c_R}(p)$	6	93.60	1.68	92.90	-0.93	93.80	-0.86
Without	$N_{SR,R s_F c_R}(p)$	7	94.70	1.55	94.00	-1.44	91.90	-6.04
Without	$N_{SR,R s_F c_R}(p)$	8	95.80	2.09	94.90	-1.80	94.50	-5.11
Without	$N_{SR,R s_F c_R}(p)$	9	96.70	2.82	96.00	-2.10	97.00	-0.59
Without	$N_{SR,R s_F c_R}(p)$	10	97.40	3.24	96.70	-2.31	97.80	-1.21
Without	$N_{SR,R s_F c_R}(p)$	11	98.10	2.88	97.30	-2.68	96.60	-7.63
Without	$N_{SR,R s_F c_R}(p)$	12	98.50	3.20	97.70	-2.50	97.60	-6.27
Without	$N_{SR,R s_F c_R}(p)$	13	99.00	3.72	98.50	-2.74	98.50	-2.13
Without	$N_{SR,R s_F c_R}(p)$	14	99.00	3.32	98.90	-2.88	98.70	-3.33
Without	$N_{SR,R s_F c_R}(p)$	15	99.30	2.98	99.00	-2.66	98.30	-8.74
Without	$N_{SR,R s_F c_R}(p)$	16	99.50	2.13	99.10	-2.23	98.90	-8.12

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Table A.8 – continued from previous page

			S Increasing		S Decreasing		Periodic Recruitment	
Aux. Like.	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
Without	$N_{SR,R}s_Fc_R(p)$	17	99.60	2.52	99.50	-2.31	99.70	-3.68
Without	$N_{SR,R}s_Fc_R(p)$	18	99.70	2.45	99.60	-2.35	99.90	-4.47
Without	$N_{SR,R}s_Fc_R(p)$	19	99.80	2.13	99.50	-2.09	99.90	-10.34
Without	$N_{SR,R}s_Fc_R(p)$	20	99.80	1.64	99.80	-1.97	99.90	-9.15
Without	$N_{SR,R}s_Fc_R(p)$	21	99.80	1.34	99.70	-1.21	100.00	-5.08
Without	$N_{SR,R}s_Fc_R(p)$	22	99.80	1.02	99.80	-0.62	100.00	-5.51
Without	$N_{SR,R}s_Fc_R(p)$	23	99.80	0.78	99.70	-0.48	99.80	-12.49
Without	$N_{SR,R}s_Fc_R(p)$	24	99.80	0.43	99.90	0.08	100.00	-8.90
Without	$N_{SR,R}s_Fc_R(p)$	25	99.90	-0.39	99.90	0.41	100.00	-5.31
Without	$N_{SR,R}s_Rc_R(p)$	1	93.30	0.10	92.50	0.83	93.00	2.19
Without	$N_{SR,R}s_Rc_R(p)$	2	88.70	0.32	88.40	0.62	87.00	0.84
Without	$N_{SR,R}s_Rc_R(p)$	3	89.40	0.99	88.60	-0.10	85.60	-5.02
Without	$N_{SR,R}s_Rc_R(p)$	4	91.30	0.77	89.80	-0.87	89.20	-4.08
Without	$N_{SR,R}s_Rc_R(p)$	5	92.30	1.34	91.50	-0.64	93.60	-0.45
Without	$N_{SR,R}s_Rc_R(p)$	6	93.90	1.71	93.40	-0.63	93.80	-1.16
Without	$N_{SR,R}s_Rc_R(p)$	7	94.80	1.57	94.00	-1.15	92.50	-6.24
Without	$N_{SR,R}s_Rc_R(p)$	8	95.90	1.86	95.20	-1.62	94.70	-5.28
Without	$N_{SR,R}s_Rc_R(p)$	9	96.90	2.60	96.20	-1.72	97.30	-1.18
Without	$N_{SR,R}s_Rc_R(p)$	10	97.50	2.89	97.00	-2.17	98.10	-1.42
Without	$N_{SR,R}s_Rc_R(p)$	11	98.10	3.20	97.90	-2.29	96.80	-7.57
Without	$N_{SR,R}s_Rc_R(p)$	12	98.50	2.94	98.00	-2.15	97.90	-6.27
Without	$N_{SR,R}s_Rc_R(p)$	13	99.00	3.70	98.80	-2.53	98.70	-2.39
Without	$N_{SR,R}s_Rc_R(p)$	14	99.00	3.11	99.10	-2.73	98.90	-3.33
Without	$N_{SR,R}s_Rc_R(p)$	15	99.30	2.74	99.20	-2.63	98.50	-8.88
Without	$N_{SR,R}s_Rc_R(p)$	16	99.60	1.97	99.20	-2.17	98.90	-8.40
Without	$N_{SR,R}s_Rc_R(p)$	17	99.60	2.45	99.60	-2.14	99.70	-3.87
Without	$N_{SR,R}s_Rc_R(p)$	18	99.70	2.29	99.70	-2.45	99.90	-4.47
Without	$N_{SR,R}s_Rc_R(p)$	19	99.80	1.69	99.60	-2.02	99.90	-10.29
Without	$N_{SR,R}s_Rc_R(p)$	20	99.80	1.35	99.80	-2.14	99.90	-9.15
Without	$N_{SR,R}s_Rc_R(p)$	21	99.80	1.05	99.80	-1.27	100.00	-5.24
Without	$N_{SR,R}s_Rc_R(p)$	22	99.80	0.70	99.80	-0.58	100.00	-5.54
Without	$N_{SR,R}s_Rc_R(p)$	23	99.80	0.69	99.80	-0.42	99.80	-12.34
Without	$N_{SR,R}s_Rc_R(p)$	24	99.80	0.38	99.90	0.13	100.00	-8.90
Without	$N_{SR,R}s_Rc_R(p)$	25	99.90	-0.35	99.90	0.38	100.00	-5.55
Without	$s_Fc_F(p)$	1	63.10	0.66	61.70	-0.44	68.80	0.67
Without	$s_Fc_F(p)$	2	65.20	0.92	63.70	0.69	68.90	1.20
Without	$s_Fc_F(p)$	3	65.30	0.20	61.20	-0.14	66.80	-0.09
Without	$s_Fc_F(p)$	4	63.40	1.11	62.10	0.17	68.40	0.60
Without	$s_Fc_F(p)$	5	63.10	1.88	62.80	-0.80	69.10	0.83
Without	$s_Fc_F(p)$	6	64.50	1.14	60.90	0.13	68.90	0.33
Without	$s_Fc_F(p)$	7	67.20	0.72	64.10	-0.11	69.20	0.76
Without	$s_Fc_F(p)$	8	64.90	1.24	65.10	0.26	67.70	1.19
Without	$s_Fc_F(p)$	9	64.10	0.60	63.10	0.48	68.40	1.15
Without	$s_Fc_F(p)$	10	63.20	1.42	62.10	-0.19	71.70	0.59
Without	$s_Fc_F(p)$	11	63.30	1.63	62.50	0.60	68.70	0.15
Without	$s_Fc_F(p)$	12	64.90	0.27	65.30	0.31	67.60	0.64
Without	$s_Fc_F(p)$	13	63.30	0.92	63.70	-0.21	67.90	0.68
Without	$s_Fc_F(p)$	14	64.60	1.06	61.40	0.33	69.90	0.59
Without	$s_Fc_F(p)$	15	64.10	2.12	63.10	0.46	66.90	0.32
Without	$s_Fc_F(p)$	16	63.70	1.06	64.30	0.70	69.50	1.21
Without	$s_Fc_F(p)$	17	63.10	1.08	61.60	1.00	68.40	0.18
Without	$s_Fc_F(p)$	18	64.10	1.11	63.30	0.36	70.90	0.74
Without	$s_Fc_F(p)$	19	63.20	0.64	59.70	-0.73	68.70	0.40
Without	$s_Fc_F(p)$	20	63.90	0.83	63.70	0.19	67.60	0.50
Without	$s_Fc_F(p)$	21	62.70	0.92	63.50	0.60	68.60	-0.43
Without	$s_Fc_F(p)$	22	60.90	0.87	61.60	-0.31	69.90	1.21
Without	$s_Fc_F(p)$	23	63.60	0.39	63.00	0.38	68.70	0.33
Without	$s_Fc_F(p)$	24	61.60	0.53	63.90	0.09	69.40	0.71
Without	$s_Fc_F(p)$	25	63.10	1.61	62.60	0.25	68.60	0.52

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Table A.8 – continued from previous page

			S Increasing		S Decreasing		Periodic Recruitment	
Aux. Like.	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
Without	$N_A s_F c_F(p)$	1	69.20	0.38	65.40	-2.03	71.60	1.28
Without	$N_A s_F c_F(p)$	2	69.70	2.54	66.30	-1.40	71.30	2.54
Without	$N_A s_F c_F(p)$	3	71.30	1.95	67.60	-1.70	73.60	4.35
Without	$N_A s_F c_F(p)$	4	74.80	7.15	72.30	-1.52	75.30	5.46
Without	$N_A s_F c_F(p)$	5	74.70	8.67	74.40	-0.69	76.20	5.43
Without	$N_A s_F c_F(p)$	6	73.80	10.90	75.10	0.11	77.40	5.74
Without	$N_A s_F c_F(p)$	7	72.90	13.43	77.40	1.66	76.40	6.77
Without	$N_A s_F c_F(p)$	8	72.50	12.65	78.20	1.06	75.60	7.45
Without	$N_A s_F c_F(p)$	9	70.80	14.61	78.30	1.21	74.90	10.42
Without	$N_A s_F c_F(p)$	10	70.20	18.94	78.80	3.63	76.10	12.93
Without	$N_A s_F c_F(p)$	11	69.90	18.22	79.00	4.53	76.50	15.54
Without	$N_A s_F c_F(p)$	12	69.60	18.58	79.50	5.82	75.60	14.19
Without	$N_A s_F c_F(p)$	13	68.40	19.87	79.50	7.27	75.80	14.64
Without	$N_A s_F c_F(p)$	14	68.10	19.35	79.80	9.08	73.90	16.17
Without	$N_A s_F c_F(p)$	15	67.40	21.93	79.60	10.39	73.60	19.31
Without	$N_A s_F c_F(p)$	16	67.60	22.44	79.30	13.02	73.50	22.36
Without	$N_A s_F c_F(p)$	17	67.60	26.53	80.00	17.26	72.20	21.82
Without	$N_A s_F c_F(p)$	18	67.20	24.92	80.30	20.15	72.20	22.57
Without	$N_A s_F c_F(p)$	19	66.30	24.72	79.60	20.89	72.30	24.82
Without	$N_A s_F c_F(p)$	20	65.50	24.01	78.60	25.93	72.40	28.90
Without	$N_A s_F c_F(p)$	21	65.20	26.50	78.70	28.00	72.70	28.33
Without	$N_A s_F c_F(p)$	22	65.00	26.09	79.20	31.56	71.90	29.98
Without	$N_A s_F c_F(p)$	23	64.80	24.06	79.10	34.11	71.50	32.58
Without	$N_A s_F c_F(p)$	24	65.40	16.14	77.90	23.29	71.90	19.00
Without	$N_A s_F c_F(p)$	25	65.90	11.37	79.10	15.86	72.10	14.85
Without	$s_F c_R(p)$	1	95.70	-0.83	95.60	0.71	96.50	0.52
Without	$s_F c_R(p)$	2	95.90	-0.56	95.80	0.51	95.10	-0.00
Without	$s_F c_R(p)$	3	95.90	-0.83	96.00	0.51	96.50	-0.30
Without	$s_F c_R(p)$	4	96.00	-0.39	95.40	0.07	96.60	0.05
Without	$s_F c_R(p)$	5	96.90	-0.04	95.20	-0.13	96.50	0.06
Without	$s_F c_R(p)$	6	96.60	-0.48	95.90	-0.01	97.40	0.27
Without	$s_F c_R(p)$	7	96.50	-0.41	95.00	0.16	96.90	0.01
Without	$s_F c_R(p)$	8	96.80	-0.15	95.80	-0.24	97.20	0.09
Without	$s_F c_R(p)$	9	96.90	-0.24	96.50	-0.12	96.70	0.05
Without	$s_F c_R(p)$	10	95.90	-0.01	95.40	-0.33	96.70	0.04
Without	$s_F c_R(p)$	11	96.90	0.11	95.50	-0.51	96.70	-0.09
Without	$s_F c_R(p)$	12	95.90	0.08	95.80	-0.33	96.40	-0.28
Without	$s_F c_R(p)$	13	96.90	-0.04	95.10	-0.43	96.50	-0.18
Without	$s_F c_R(p)$	14	97.00	0.37	95.20	0.13	96.70	0.25
Without	$s_F c_R(p)$	15	96.70	0.53	94.80	-0.24	96.10	-0.03
Without	$s_F c_R(p)$	16	96.30	0.53	94.80	-0.70	96.00	-0.19
Without	$s_F c_R(p)$	17	95.90	0.72	94.60	-0.25	96.40	-0.48
Without	$s_F c_R(p)$	18	96.50	0.83	95.20	-0.51	96.90	-0.01
Without	$s_F c_R(p)$	19	96.40	0.35	95.40	-0.73	96.40	-0.12
Without	$s_F c_R(p)$	20	96.30	0.96	94.90	-0.93	95.30	-0.49
Without	$s_F c_R(p)$	21	96.40	0.43	94.90	-0.40	96.00	-0.48
Without	$s_F c_R(p)$	22	95.20	0.26	94.40	-0.38	96.10	0.16
Without	$s_F c_R(p)$	23	95.60	0.28	93.80	-0.25	96.10	0.02
Without	$s_F c_R(p)$	24	94.50	0.30	93.20	-0.24	94.90	-0.06
Without	$s_F c_R(p)$	25	92.20	-0.21	93.00	0.51	94.40	-0.02
Without	$N_A s_F c_R(p)$	1	88.30	-0.39	89.60	0.18	88.20	1.76
Without	$N_A s_F c_R(p)$	2	87.80	1.06	88.50	-0.24	87.70	1.71
Without	$N_A s_F c_R(p)$	3	87.90	2.19	88.20	-1.01	88.30	2.68
Without	$N_A s_F c_R(p)$	4	89.20	4.96	88.20	-0.67	87.30	4.20
Without	$N_A s_F c_R(p)$	5	88.30	6.59	88.40	-0.79	87.70	3.98
Without	$N_A s_F c_R(p)$	6	88.00	9.66	88.90	-0.43	87.80	4.86
Without	$N_A s_F c_R(p)$	7	88.10	10.88	89.30	0.70	87.00	6.06
Without	$N_A s_F c_R(p)$	8	86.10	10.74	89.10	1.25	86.10	6.98
Without	$N_A s_F c_R(p)$	9	85.30	11.54	89.80	0.85	85.80	8.91

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Table A.8 – continued from previous page

			S Increasing		S Decreasing		Periodic Recruitment	
Aux. Like.	Model	Year	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB
Without	$N_A s_F c_R(p)$	10	83.60	14.39	89.10	1.77	85.00	10.22
Without	$N_A s_F c_R(p)$	11	83.10	16.54	88.70	3.55	84.80	11.54
Without	$N_A s_F c_R(p)$	12	82.40	16.04	88.40	5.14	83.50	10.96
Without	$N_A s_F c_R(p)$	13	82.20	17.25	88.10	6.33	84.00	13.99
Without	$N_A s_F c_R(p)$	14	80.80	18.27	88.10	6.84	84.20	14.39
Without	$N_A s_F c_R(p)$	15	80.00	19.43	87.60	8.47	83.30	15.56
Without	$N_A s_F c_R(p)$	16	79.20	20.38	87.70	9.54	82.70	16.00
Without	$N_A s_F c_R(p)$	17	78.30	22.31	87.50	14.36	82.20	15.98
Without	$N_A s_F c_R(p)$	18	77.60	23.29	87.20	15.36	81.30	17.42
Without	$N_A s_F c_R(p)$	19	76.70	22.45	86.70	16.81	80.80	18.68
Without	$N_A s_F c_R(p)$	20	75.70	20.92	86.50	22.23	80.60	20.47
Without	$N_A s_F c_R(p)$	21	75.20	21.27	87.20	23.33	80.60	20.99
Without	$N_A s_F c_R(p)$	22	75.00	22.74	86.80	25.87	80.40	20.89
Without	$N_A s_F c_R(p)$	23	74.50	23.08	86.60	30.13	79.30	23.62
Without	$N_A s_F c_R(p)$	24	75.40	15.57	87.10	20.28	79.90	15.02
Without	$N_A s_F c_R(p)$	25	75.30	9.78	87.40	15.05	80.90	10.26
Without	$s_R c_R(p)$	1	95.50	-0.76	94.80	0.62	96.20	0.49
Without	$s_R c_R(p)$	2	95.80	-0.48	95.40	0.55	95.30	0.15
Without	$s_R c_R(p)$	3	95.50	-0.82	95.90	0.32	96.50	-0.24
Without	$s_R c_R(p)$	4	95.60	-0.24	95.20	0.19	96.90	0.16
Without	$s_R c_R(p)$	5	96.80	-0.04	94.90	-0.02	96.50	0.10
Without	$s_R c_R(p)$	6	96.60	-0.21	96.00	-0.18	97.40	0.30
Without	$s_R c_R(p)$	7	96.40	-0.38	95.10	-0.08	97.10	0.09
Without	$s_R c_R(p)$	8	96.50	-0.36	95.80	-0.35	97.20	0.00
Without	$s_R c_R(p)$	9	96.50	-0.38	96.60	-0.07	96.90	0.04
Without	$s_R c_R(p)$	10	95.90	-0.08	95.40	-0.24	97.00	-0.01
Without	$s_R c_R(p)$	11	96.90	-0.01	95.60	-0.28	96.70	-0.30
Without	$s_R c_R(p)$	12	96.20	-0.13	95.70	-0.23	96.30	-0.24
Without	$s_R c_R(p)$	13	96.80	-0.00	95.10	-0.34	96.40	-0.18
Without	$s_R c_R(p)$	14	96.50	0.35	95.40	0.36	96.50	0.34
Without	$s_R c_R(p)$	15	96.10	0.52	95.30	-0.06	95.70	-0.06
Without	$s_R c_R(p)$	16	95.80	0.50	94.70	-0.55	95.90	-0.30
Without	$s_R c_R(p)$	17	94.90	0.68	94.50	-0.00	96.20	-0.43
Without	$s_R c_R(p)$	18	95.70	0.63	94.90	-0.48	96.90	-0.27
Without	$s_R c_R(p)$	19	95.40	0.11	95.00	-0.72	96.00	-0.26
Without	$s_R c_R(p)$	20	95.40	0.70	94.70	-0.82	95.10	-0.59
Without	$s_R c_R(p)$	21	96.00	0.36	94.90	-0.32	96.00	-0.59
Without	$s_R c_R(p)$	22	94.80	0.26	94.60	-0.23	96.00	0.14
Without	$s_R c_R(p)$	23	95.00	0.12	93.90	-0.24	95.80	-0.02
Without	$s_R c_R(p)$	24	94.40	0.16	93.20	0.11	94.80	-0.06
Without	$s_R c_R(p)$	25	92.10	-0.21	93.30	0.62	94.30	0.01
Without	$N_A s_R c_R(p)$	1	88.10	-0.40	89.60	0.18	88.20	1.76
Without	$N_A s_R c_R(p)$	2	87.60	1.03	88.50	-0.24	87.70	1.71
Without	$N_A s_R c_R(p)$	3	87.70	2.12	88.20	-1.01	88.30	2.68
Without	$N_A s_R c_R(p)$	4	89.00	4.95	88.20	-0.35	87.30	4.20
Without	$N_A s_R c_R(p)$	5	88.10	6.56	88.40	-0.76	87.70	3.98
Without	$N_A s_R c_R(p)$	6	87.90	9.64	88.90	-0.38	87.80	4.89
Without	$N_A s_R c_R(p)$	7	88.10	10.82	89.40	0.76	87.00	6.24
Without	$N_A s_R c_R(p)$	8	86.20	10.63	89.20	1.30	86.10	7.03
Without	$N_A s_R c_R(p)$	9	85.40	11.47	89.90	0.98	85.80	9.17
Without	$N_A s_R c_R(p)$	10	83.60	14.22	89.20	1.86	85.10	10.30
Without	$N_A s_R c_R(p)$	11	83.10	16.18	88.90	3.62	84.90	11.62
Without	$N_A s_R c_R(p)$	12	82.50	15.99	88.60	5.22	83.50	10.96
Without	$N_A s_R c_R(p)$	13	82.30	17.25	88.30	6.33	84.00	13.99
Without	$N_A s_R c_R(p)$	14	80.80	18.27	88.30	6.84	84.10	14.39
Without	$N_A s_R c_R(p)$	15	80.00	19.25	87.80	8.47	83.20	15.56
Without	$N_A s_R c_R(p)$	16	79.20	20.24	87.90	9.75	82.70	16.00
Without	$N_A s_R c_R(p)$	17	78.30	22.25	87.70	14.36	82.20	16.80
Without	$N_A s_R c_R(p)$	18	77.60	23.12	87.30	15.36	81.30	17.42

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Table A.8 – continued from previous page

			S Increasing		S Decreasing		Periodic Recruitment	
Aux. Like.	Model	Year	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB
Without	$N_A s_R c_R(p)$	19	76.70	22.31	86.80	16.81	80.70	18.68
Without	$N_A s_R c_R(p)$	20	75.90	20.91	86.60	22.23	80.60	20.47
Without	$N_A s_R c_R(p)$	21	75.30	21.27	87.20	23.33	80.60	20.99
Without	$N_A s_R c_R(p)$	22	75.00	22.74	86.80	25.87	80.40	20.89
Without	$N_A s_R c_R(p)$	23	74.60	23.08	86.60	30.13	79.30	23.62
Without	$N_A s_R c_R(p)$	24	75.40	15.57	87.10	20.28	79.90	15.02
Without	$N_A s_R c_R(p)$	25	75.30	9.78	87.40	15.05	80.90	10.26
With	$N_{SR,R} s_F c_F(p)$	1	56.50	3.48	55.90	1.82	60.60	4.18
With	$N_{SR,R} s_F c_F(p)$	2	71.70	4.46	68.50	2.74	77.00	3.84
With	$N_{SR,R} s_F c_F(p)$	3	77.80	3.92	75.10	3.62	85.20	2.83
With	$N_{SR,R} s_F c_F(p)$	4	84.30	4.43	83.90	3.23	90.80	3.06
With	$N_{SR,R} s_F c_F(p)$	5	88.80	5.14	89.70	2.84	94.40	4.36
With	$N_{SR,R} s_F c_F(p)$	6	93.60	4.82	93.80	2.27	97.70	4.03
With	$N_{SR,R} s_F c_F(p)$	7	96.50	4.57	95.80	3.05	99.10	3.26
With	$N_{SR,R} s_F c_F(p)$	8	98.60	4.59	97.70	3.36	99.50	3.65
With	$N_{SR,R} s_F c_F(p)$	9	99.30	4.04	98.80	3.25	99.80	5.36
With	$N_{SR,R} s_F c_F(p)$	10	99.50	4.64	99.50	2.62	100.00	4.34
With	$N_{SR,R} s_F c_F(p)$	11	99.90	4.90	99.60	2.96	99.90	2.91
With	$N_{SR,R} s_F c_F(p)$	12	99.90	4.43	99.80	2.90	99.90	3.23
With	$N_{SR,R} s_F c_F(p)$	13	99.90	3.97	99.80	2.58	100.00	4.10
With	$N_{SR,R} s_F c_F(p)$	14	99.90	4.88	99.70	3.24	100.00	3.87
With	$N_{SR,R} s_F c_F(p)$	15	99.90	5.46	99.80	3.09	100.00	3.24
With	$N_{SR,R} s_F c_F(p)$	16	99.90	4.34	99.80	2.53	100.00	3.84
With	$N_{SR,R} s_F c_F(p)$	17	100.00	4.67	99.90	3.07	100.00	4.02
With	$N_{SR,R} s_F c_F(p)$	18	100.00	3.89	99.90	2.31	100.00	4.67
With	$N_{SR,R} s_F c_F(p)$	19	100.00	4.20	99.90	2.54	100.00	3.17
With	$N_{SR,R} s_F c_F(p)$	20	100.00	4.80	99.80	3.40	100.00	3.27
With	$N_{SR,R} s_F c_F(p)$	21	100.00	4.57	99.90	3.10	100.00	4.12
With	$N_{SR,R} s_F c_F(p)$	22	100.00	4.15	99.90	2.65	100.00	4.55
With	$N_{SR,R} s_F c_F(p)$	23	99.90	3.89	99.80	3.45	100.00	2.92
With	$N_{SR,R} s_F c_F(p)$	24	99.90	4.19	99.90	3.35	100.00	3.16
With	$N_{SR,R} s_F c_F(p)$	25	99.90	3.82	99.90	3.42	100.00	3.47
With	$N_{SR,R} s_F c_R(p)$	1	90.70	-1.61	90.60	-0.14	93.40	2.14
With	$N_{SR,R} s_F c_R(p)$	2	93.40	-0.90	93.10	-0.02	96.60	1.97
With	$N_{SR,R} s_F c_R(p)$	3	95.60	-1.20	93.30	-0.19	95.50	-2.35
With	$N_{SR,R} s_F c_R(p)$	4	97.40	-1.24	95.70	0.06	98.00	-1.44
With	$N_{SR,R} s_F c_R(p)$	5	98.70	-1.22	96.90	-0.13	99.20	2.89
With	$N_{SR,R} s_F c_R(p)$	6	99.50	-0.70	97.70	-0.53	99.60	2.64
With	$N_{SR,R} s_F c_R(p)$	7	99.80	-0.57	97.90	-0.41	99.60	-2.69
With	$N_{SR,R} s_F c_R(p)$	8	99.90	-0.34	98.90	-0.78	99.80	-1.46
With	$N_{SR,R} s_F c_R(p)$	9	99.90	-0.25	99.60	-0.59	99.90	3.10
With	$N_{SR,R} s_F c_R(p)$	10	99.90	-0.32	99.70	-1.26	100.00	2.41
With	$N_{SR,R} s_F c_R(p)$	11	100.00	-0.52	99.80	-0.80	100.00	-2.30
With	$N_{SR,R} s_F c_R(p)$	12	99.90	-0.10	99.90	-0.83	100.00	-1.55
With	$N_{SR,R} s_F c_R(p)$	13	99.90	0.05	100.00	-1.38	100.00	3.20
With	$N_{SR,R} s_F c_R(p)$	14	100.00	0.30	100.00	-0.96	100.00	2.35
With	$N_{SR,R} s_F c_R(p)$	15	99.90	0.19	100.00	-0.98	100.00	-2.60
With	$N_{SR,R} s_F c_R(p)$	16	100.00	0.11	100.00	-1.00	100.00	-1.67
With	$N_{SR,R} s_F c_R(p)$	17	100.00	0.33	100.00	-1.38	100.00	3.09
With	$N_{SR,R} s_F c_R(p)$	18	100.00	0.21	100.00	-1.34	100.00	2.44
With	$N_{SR,R} s_F c_R(p)$	19	100.00	0.24	100.00	-1.69	100.00	-2.84
With	$N_{SR,R} s_F c_R(p)$	20	100.00	0.32	100.00	-1.47	100.00	-1.55
With	$N_{SR,R} s_F c_R(p)$	21	100.00	0.06	100.00	-1.35	100.00	2.66
With	$N_{SR,R} s_F c_R(p)$	22	100.00	-0.25	100.00	-1.12	100.00	1.62
With	$N_{SR,R} s_F c_R(p)$	23	100.00	-0.39	100.00	-0.85	100.00	-3.13
With	$N_{SR,R} s_F c_R(p)$	24	100.00	-0.47	100.00	-0.67	100.00	-2.43
With	$N_{SR,R} s_F c_R(p)$	25	100.00	-1.01	100.00	-0.53	100.00	0.04
With	$N_{SR,R} s_R c_R(p)$	1	90.70	-1.80	90.50	-0.09	93.60	2.03
With	$N_{SR,R} s_R c_R(p)$	2	93.50	-0.90	93.00	-0.00	96.50	1.91

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Table A.8 – continued from previous page

			S Increasing		S Decreasing		Periodic Recruitment	
Aux. Like.	Model	Year	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB
With	$N_{SR,R}s_Rc_R(p)$	3	95.60	-1.23	93.20	-0.18	95.80	-2.37
With	$N_{SR,R}s_Rc_R(p)$	4	97.20	-1.13	96.00	0.08	98.10	-1.49
With	$N_{SR,R}s_Rc_R(p)$	5	98.80	-1.16	97.20	-0.08	99.40	2.85
With	$N_{SR,R}s_Rc_R(p)$	6	99.50	-0.70	98.00	-0.50	99.70	2.52
With	$N_{SR,R}s_Rc_R(p)$	7	99.80	-0.49	98.30	-0.44	99.60	-2.64
With	$N_{SR,R}s_Rc_R(p)$	8	99.90	-0.39	99.20	-0.73	99.80	-1.57
With	$N_{SR,R}s_Rc_R(p)$	9	99.90	-0.15	99.60	-0.48	99.90	3.01
With	$N_{SR,R}s_Rc_R(p)$	10	99.90	-0.34	100.00	-1.08	100.00	2.35
With	$N_{SR,R}s_Rc_R(p)$	11	100.00	-0.54	100.00	-0.71	100.00	-2.33
With	$N_{SR,R}s_Rc_R(p)$	12	99.90	-0.20	100.00	-0.85	100.00	-1.52
With	$N_{SR,R}s_Rc_R(p)$	13	99.90	-0.29	100.00	-1.36	100.00	3.20
With	$N_{SR,R}s_Rc_R(p)$	14	99.90	-0.01	100.00	-0.88	100.00	2.28
With	$N_{SR,R}s_Rc_R(p)$	15	99.90	-0.24	100.00	-0.75	100.00	-2.57
With	$N_{SR,R}s_Rc_R(p)$	16	99.90	-0.12	100.00	-0.79	100.00	-1.72
With	$N_{SR,R}s_Rc_R(p)$	17	100.00	0.16	100.00	-1.24	100.00	3.19
With	$N_{SR,R}s_Rc_R(p)$	18	100.00	-0.09	100.00	-1.21	100.00	2.28
With	$N_{SR,R}s_Rc_R(p)$	19	100.00	0.13	100.00	-1.60	100.00	-2.96
With	$N_{SR,R}s_Rc_R(p)$	20	100.00	0.21	100.00	-1.39	100.00	-1.71
With	$N_{SR,R}s_Rc_R(p)$	21	100.00	-0.29	100.00	-1.25	100.00	2.61
With	$N_{SR,R}s_Rc_R(p)$	22	100.00	-0.27	100.00	-0.98	100.00	1.56
With	$N_{SR,R}s_Rc_R(p)$	23	100.00	-0.48	100.00	-0.85	100.00	-3.16
With	$N_{SR,R}s_Rc_R(p)$	24	100.00	-0.58	100.00	-0.51	100.00	-2.43
With	$N_{SR,R}s_Rc_R(p)$	25	100.00	-1.11	100.00	-0.52	100.00	-0.03
With	$s_Fc_F(p)$	1	44.60	9.73	43.90	9.40	49.90	9.25
With	$s_Fc_F(p)$	2	45.80	9.45	40.70	10.74	49.30	9.38
With	$s_Fc_F(p)$	3	46.10	9.15	41.40	11.33	48.10	8.82
With	$s_Fc_F(p)$	4	44.10	10.57	40.80	11.59	51.30	8.90
With	$s_Fc_F(p)$	5	44.00	10.92	43.50	9.60	48.20	10.58
With	$s_Fc_F(p)$	6	46.20	9.76	43.70	10.71	48.60	10.00
With	$s_Fc_F(p)$	7	43.90	10.54	45.30	9.37	47.40	9.22
With	$s_Fc_F(p)$	8	43.10	10.21	42.00	11.47	49.10	10.45
With	$s_Fc_F(p)$	9	44.70	9.92	43.80	10.27	46.20	10.99
With	$s_Fc_F(p)$	10	43.20	10.93	42.00	10.53	50.70	9.38
With	$s_Fc_F(p)$	11	40.80	11.49	42.80	11.51	49.90	9.46
With	$s_Fc_F(p)$	12	45.10	9.61	43.50	10.47	49.50	10.28
With	$s_Fc_F(p)$	13	43.60	9.97	45.70	10.05	47.20	9.88
With	$s_Fc_F(p)$	14	44.40	10.88	42.50	11.04	50.90	9.85
With	$s_Fc_F(p)$	15	42.00	10.69	44.10	11.36	49.20	10.07
With	$s_Fc_F(p)$	16	44.80	9.93	43.60	11.22	49.30	10.48
With	$s_Fc_F(p)$	17	44.50	10.26	42.90	11.01	48.70	8.84
With	$s_Fc_F(p)$	18	44.70	10.31	43.70	11.34	49.10	10.07
With	$s_Fc_F(p)$	19	42.80	10.10	43.70	9.64	48.30	9.54
With	$s_Fc_F(p)$	20	43.40	10.67	42.40	11.15	49.30	9.78
With	$s_Fc_F(p)$	21	41.40	10.84	44.00	12.26	49.00	8.22
With	$s_Fc_F(p)$	22	42.60	9.55	45.40	9.95	48.70	9.88
With	$s_Fc_F(p)$	23	44.90	10.18	44.30	11.01	49.00	9.57
With	$s_Fc_F(p)$	24	41.70	10.44	44.20	11.19	48.20	9.92
With	$s_Fc_F(p)$	25	41.00	11.50	43.20	11.69	49.50	10.04
With	$N_As_Fc_F(p)$	1	54.10	-0.29	55.50	-0.31	61.40	0.46
With	$N_As_Fc_F(p)$	2	56.00	0.50	54.20	-0.03	58.30	0.74
With	$N_As_Fc_F(p)$	3	56.10	-0.28	54.40	0.61	59.20	0.13
With	$N_As_Fc_F(p)$	4	55.00	0.87	54.90	0.73	60.60	0.59
With	$N_As_Fc_F(p)$	5	56.30	0.87	55.80	-0.33	59.50	0.47
With	$N_As_Fc_F(p)$	6	55.60	0.28	54.30	-0.23	61.10	1.18
With	$N_As_Fc_F(p)$	7	56.60	0.73	54.20	0.31	60.40	1.23
With	$N_As_Fc_F(p)$	8	54.90	0.36	57.00	-0.19	60.60	1.37
With	$N_As_Fc_F(p)$	9	55.90	-0.27	56.60	0.10	60.30	1.64
With	$N_As_Fc_F(p)$	10	54.80	0.70	56.80	-0.08	61.80	1.41
With	$N_As_Fc_F(p)$	11	54.70	1.16	54.80	0.18	60.50	0.82

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Table A.8 – continued from previous page

			S Increasing		S Decreasing		Periodic Recruitment	
Aux. Like.	Model	Year	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB
With	$N_A s_F c_F(p)$	12	54.50	0.83	57.20	0.37	59.80	0.10
With	$N_A s_F c_F(p)$	13	54.70	0.16	55.40	-0.07	59.90	0.93
With	$N_A s_F c_F(p)$	14	55.00	0.42	56.40	0.40	62.30	1.05
With	$N_A s_F c_F(p)$	15	54.80	1.32	56.10	0.35	60.60	1.37
With	$N_A s_F c_F(p)$	16	53.80	0.32	56.80	0.06	60.20	0.85
With	$N_A s_F c_F(p)$	17	53.80	1.03	56.70	-0.02	58.20	0.62
With	$N_A s_F c_F(p)$	18	54.20	0.56	56.00	-0.36	60.90	1.33
With	$N_A s_F c_F(p)$	19	52.70	0.64	53.20	-0.55	60.50	0.24
With	$N_A s_F c_F(p)$	20	56.20	0.73	56.00	0.13	60.30	0.61
With	$N_A s_F c_F(p)$	21	53.70	0.32	55.80	-0.14	60.10	0.72
With	$N_A s_F c_F(p)$	22	54.50	0.42	56.20	-0.01	60.20	0.82
With	$N_A s_F c_F(p)$	23	53.80	0.17	56.00	0.00	61.30	0.54
With	$N_A s_F c_F(p)$	24	52.30	-0.05	54.70	0.01	59.10	0.18
With	$N_A s_F c_F(p)$	25	51.40	0.35	53.70	0.66	57.20	0.56
With	$s_F c_R(p)$	1	93.60	-2.00	94.20	-0.46	94.50	-0.71
With	$s_F c_R(p)$	2	93.80	-1.79	94.00	-0.59	93.70	-1.17
With	$s_F c_R(p)$	3	93.80	-1.91	93.70	-0.85	94.40	-1.33
With	$s_F c_R(p)$	4	94.20	-1.82	93.70	-1.08	94.70	-1.09
With	$s_F c_R(p)$	5	95.00	-1.57	94.40	-1.10	95.20	-1.01
With	$s_F c_R(p)$	6	93.90	-1.58	94.10	-1.05	95.70	-0.97
With	$s_F c_R(p)$	7	94.90	-1.33	94.00	-0.92	95.80	-1.14
With	$s_F c_R(p)$	8	95.00	-1.29	94.50	-1.36	94.90	-1.21
With	$s_F c_R(p)$	9	95.20	-1.24	94.60	-1.34	95.50	-1.20
With	$s_F c_R(p)$	10	94.60	-1.14	93.40	-1.49	95.10	-1.26
With	$s_F c_R(p)$	11	95.30	-0.75	93.80	-1.64	95.20	-1.19
With	$s_F c_R(p)$	12	94.60	-1.23	93.40	-1.87	94.70	-1.67
With	$s_F c_R(p)$	13	95.30	-1.19	93.70	-1.59	95.20	-1.20
With	$s_F c_R(p)$	14	94.90	-0.90	93.10	-1.65	95.20	-1.07
With	$s_F c_R(p)$	15	95.40	-0.67	93.50	-1.58	95.10	-1.05
With	$s_F c_R(p)$	16	94.60	-0.77	93.30	-2.21	94.80	-1.19
With	$s_F c_R(p)$	17	94.70	-0.62	92.70	-1.73	94.10	-1.37
With	$s_F c_R(p)$	18	95.00	-0.62	93.60	-2.04	94.20	-1.25
With	$s_F c_R(p)$	19	94.50	-0.75	93.50	-2.23	94.60	-1.42
With	$s_F c_R(p)$	20	94.80	-0.54	93.10	-2.45	94.40	-1.39
With	$s_F c_R(p)$	21	94.40	-0.63	93.00	-2.10	94.50	-1.39
With	$s_F c_R(p)$	22	93.70	-0.76	92.00	-1.94	94.80	-1.28
With	$s_F c_R(p)$	23	94.00	-1.11	92.00	-1.63	94.60	-1.34
With	$s_F c_R(p)$	24	91.90	-1.29	91.40	-1.48	93.30	-1.63
With	$s_F c_R(p)$	25	89.70	-1.86	91.50	-0.89	92.60	-1.52
With	$N_A s_F c_R(p)$	1	80.60	-5.17	85.00	-4.16	83.80	-4.09
With	$N_A s_F c_R(p)$	2	80.90	-4.79	83.80	-4.22	83.30	-4.25
With	$N_A s_F c_R(p)$	3	81.50	-4.79	84.20	-4.15	84.60	-4.73
With	$N_A s_F c_R(p)$	4	82.70	-4.84	83.90	-4.58	85.70	-4.59
With	$N_A s_F c_R(p)$	5	84.50	-4.51	85.00	-5.06	85.10	-4.43
With	$N_A s_F c_R(p)$	6	84.30	-4.55	84.30	-4.72	86.90	-4.46
With	$N_A s_F c_R(p)$	7	84.60	-4.76	84.60	-4.81	86.70	-4.85
With	$N_A s_F c_R(p)$	8	85.30	-4.65	85.00	-5.11	85.30	-4.78
With	$N_A s_F c_R(p)$	9	84.90	-4.53	85.50	-5.26	86.70	-4.64
With	$N_A s_F c_R(p)$	10	85.10	-4.47	84.60	-5.13	86.20	-4.39
With	$N_A s_F c_R(p)$	11	84.40	-4.10	84.70	-5.48	86.00	-4.55
With	$N_A s_F c_R(p)$	12	84.80	-4.31	85.40	-5.17	85.70	-4.97
With	$N_A s_F c_R(p)$	13	85.00	-4.10	84.20	-5.49	85.20	-4.59
With	$N_A s_F c_R(p)$	14	85.30	-3.62	84.30	-5.07	86.60	-4.71
With	$N_A s_F c_R(p)$	15	84.50	-3.99	82.60	-5.20	86.00	-4.40
With	$N_A s_F c_R(p)$	16	84.90	-3.93	82.80	-5.36	85.10	-4.99
With	$N_A s_F c_R(p)$	17	84.70	-3.78	83.40	-5.54	86.10	-4.66
With	$N_A s_F c_R(p)$	18	85.40	-3.75	83.80	-5.75	85.40	-4.61
With	$N_A s_F c_R(p)$	19	86.00	-3.83	83.00	-5.52	86.10	-4.93
With	$N_A s_F c_R(p)$	20	84.70	-3.76	83.50	-5.79	84.20	-4.84

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Table A.8 – continued from previous page

			S Increasing		S Decreasing		Periodic Recruitment	
Aux. Like.	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
With	$N_A s_F c_R(p)$	21	84.90	-3.75	83.40	-5.67	84.00	-5.05
With	$N_A s_F c_R(p)$	22	84.00	-3.72	83.10	-5.60	84.20	-4.68
With	$N_A s_F c_R(p)$	23	83.10	-4.06	84.50	-5.16	85.70	-4.70
With	$N_A s_F c_R(p)$	24	79.50	-4.35	81.40	-5.10	83.30	-4.71
With	$N_A s_F c_R(p)$	25	76.10	-5.15	81.50	-4.88	81.30	-5.38
With	$s_R c_R(p)$	1	93.90	-2.15	93.70	-0.71	94.40	-0.85
With	$s_R c_R(p)$	2	94.60	-2.18	93.70	-0.93	94.00	-1.18
With	$s_R c_R(p)$	3	93.80	-1.93	93.50	-1.10	94.80	-1.46
With	$s_R c_R(p)$	4	94.30	-1.70	93.50	-1.12	95.10	-1.24
With	$s_R c_R(p)$	5	95.30	-1.60	94.50	-1.53	95.80	-0.88
With	$s_R c_R(p)$	6	94.60	-1.66	94.40	-1.37	96.30	-1.07
With	$s_R c_R(p)$	7	95.50	-1.39	93.90	-1.47	96.20	-1.29
With	$s_R c_R(p)$	8	95.20	-1.59	95.00	-1.62	95.60	-1.21
With	$s_R c_R(p)$	9	95.50	-1.12	95.30	-1.40	96.30	-1.28
With	$s_R c_R(p)$	10	95.20	-1.29	94.20	-1.67	95.80	-1.28
With	$s_R c_R(p)$	11	95.80	-0.93	94.70	-1.51	95.80	-1.22
With	$s_R c_R(p)$	12	95.20	-1.30	94.20	-1.71	95.70	-1.60
With	$s_R c_R(p)$	13	95.80	-1.28	94.50	-1.61	95.50	-1.33
With	$s_R c_R(p)$	14	95.10	-0.92	93.60	-1.28	95.50	-0.99
With	$s_R c_R(p)$	15	95.10	-0.75	93.90	-1.42	94.90	-1.26
With	$s_R c_R(p)$	16	94.50	-0.97	93.70	-2.12	94.90	-1.26
With	$s_R c_R(p)$	17	94.40	-0.97	93.20	-1.58	94.00	-1.38
With	$s_R c_R(p)$	18	94.40	-0.89	93.80	-2.29	94.30	-1.37
With	$s_R c_R(p)$	19	94.70	-1.13	93.50	-2.08	94.60	-1.61
With	$s_R c_R(p)$	20	94.70	-0.94	93.10	-2.43	93.90	-1.59
With	$s_R c_R(p)$	21	94.50	-1.03	93.40	-1.97	94.50	-1.52
With	$s_R c_R(p)$	22	94.10	-1.11	93.00	-1.96	94.90	-1.24
With	$s_R c_R(p)$	23	94.10	-1.42	92.50	-1.79	94.30	-1.37
With	$s_R c_R(p)$	24	92.50	-1.33	91.90	-1.32	93.90	-1.59
With	$s_R c_R(p)$	25	89.90	-1.84	92.00	-0.99	92.80	-1.44
With	$N_A s_R c_R(p)$	1	80.30	-5.17	84.50	-4.13	83.20	-4.06
With	$N_A s_R c_R(p)$	2	80.80	-4.79	83.40	-4.18	82.80	-4.25
With	$N_A s_R c_R(p)$	3	81.60	-4.74	84.00	-4.12	84.30	-4.73
With	$N_A s_R c_R(p)$	4	83.00	-4.76	83.80	-4.54	85.40	-4.55
With	$N_A s_R c_R(p)$	5	84.50	-4.52	84.90	-5.06	84.90	-4.45
With	$N_A s_R c_R(p)$	6	84.20	-4.63	84.10	-4.69	86.70	-4.47
With	$N_A s_R c_R(p)$	7	84.60	-4.76	84.40	-4.81	86.30	-4.85
With	$N_A s_R c_R(p)$	8	85.20	-4.65	84.90	-5.10	85.00	-4.78
With	$N_A s_R c_R(p)$	9	84.90	-4.57	85.40	-5.22	86.70	-4.67
With	$N_A s_R c_R(p)$	10	85.40	-4.45	84.40	-5.13	86.10	-4.37
With	$N_A s_R c_R(p)$	11	84.60	-4.06	84.70	-5.45	86.10	-4.44
With	$N_A s_R c_R(p)$	12	85.10	-4.31	85.20	-5.16	85.60	-4.95
With	$N_A s_R c_R(p)$	13	85.20	-4.13	84.10	-5.40	85.20	-4.59
With	$N_A s_R c_R(p)$	14	85.30	-3.65	84.30	-4.96	86.70	-4.65
With	$N_A s_R c_R(p)$	15	84.40	-4.00	82.50	-5.21	85.90	-4.39
With	$N_A s_R c_R(p)$	16	84.90	-3.93	82.90	-5.36	85.10	-4.98
With	$N_A s_R c_R(p)$	17	84.70	-3.77	83.20	-5.54	86.00	-4.60
With	$N_A s_R c_R(p)$	18	85.30	-3.74	83.70	-5.71	85.20	-4.65
With	$N_A s_R c_R(p)$	19	86.00	-3.80	82.90	-5.52	85.80	-4.93
With	$N_A s_R c_R(p)$	20	84.70	-3.90	83.60	-5.79	84.10	-4.84
With	$N_A s_R c_R(p)$	21	84.90	-3.81	83.30	-5.65	84.30	-4.99
With	$N_A s_R c_R(p)$	22	84.00	-3.72	83.10	-5.58	84.10	-4.52
With	$N_A s_R c_R(p)$	23	83.10	-4.06	84.50	-5.14	85.50	-4.62
With	$N_A s_R c_R(p)$	24	79.40	-4.35	81.00	-5.06	83.30	-4.72
With	$N_A s_R c_R(p)$	25	76.00	-5.19	81.30	-4.88	81.10	-5.27

Table (A.9) contains the data used for plotting Figure (4.1), the relative bias in estimation of annual abundance for each model, for each year of data for small game models when a high level of simulated auxiliary data is available for estimating c. Table (A.10) contains the data used for plotting Figure (4.2), the relative bias in estimation of annual abundance for each model, for each year of data for small game models when a low level of auxiliary data is available for estimating c.

Table A.9: *Median relative bias in total annual abundance estimates for small game models, when a high level of simulated auxiliary data is available for estimation of c. Results indicate low bias for mixed-effects versions of the models employing the Horvitz-Thompson estimator (when the catch-effort likelihood of Equation (1.7) is omitted), except for the highest level of simulated variation, which shows bias between 5% and 9%. The fixed-effects version of the model employing the Horvitz-Thomspon abundance estimator shows low bias when the auxiliary catch-effort likelihood component is omitted, in all scenarios. The mixed-effects absolute-recruit abundance models show negative bias that increases in magnitude with the level of simulated variation, while the fixed-effects version of this model shows negligible bias which increases to low positive bias at higher levels of simulated variation. Results based on* $n = 1000$ *replicates, with* $s \approx 0.50$, $c \approx -0.685$, $\gamma = 2.0$, *and total annual abundance ≈ 40,000. "Aux. Like." = Auxiliary catch-effort likelihood used (With) or not used (Without).*

Table A.9
Annual Abundance Percent Bias

Aux. Like.	Model	Year	No Variation	Low Variation	Medium Variation	High Variation
Without	$s_F c_F(p)$	1	-0.09%	-0.27%	-0.43%	0.08%
Without	$s_F c_F(p)$	2	-0.05%	-0.59%	-0.79%	-1.38%
Without	$s_F c_F(p)$	3	-0.06%	-1.23%	-0.09%	-1.37%
Without	$s_F c_F(p)$	4	0.01%	-0.86%	-1.35%	-1.97%
Without	$s_F c_F(p)$	5	0.04%	-0.33%	-0.56%	-0.32%
Without	$s_F c_F(p)$	6	-0.17%	-0.52%	-0.32%	-2.66%
Without	$s_F c_F(p)$	7	-0.05%	-0.42%	-0.52%	0.75%
Without	$s_F c_F(p)$	8	-0.03%	-0.47%	0.04%	-2.12%
Without	$s_F c_F(p)$	9	-0.05%	-1.14%	-1.22%	-2.38%
Without	$s_F c_F(p)$	10	0.07%	-0.17%	0.03%	-1.6%
Without	$s_F c_F(p)$	11	-0.01%	-0.93%	0.96%	1.17%
Without	$s_F c_F(p)$	12	0.02%	-0.29%	-1.42%	-0.08%
Without	$s_F c_F(p)$	13	-0.11%	-1%	0.18%	-1.17%
Without	$s_F c_F(p)$	14	-0.03%	0.16%	-0.3%	1.9%
Without	$s_F c_F(p)$	15	-0.04%	-0.72%	-0.85%	-0.15%
Without	$s_F c_F(p)$	16	0.03%	-0.81%	-0.61%	-2.2%
Without	$s_F c_F(p)$	17	0.04%	0.08%	-0.04%	-0.63%
Without	$s_F c_F(p)$	18	0%	-0.39%	0.1%	-2.12%
Without	$s_F c_F(p)$	19	-0.08%	-0.62%	-0.02%	-1.33%
Without	$s_F c_F(p)$	20	0.02%	-0.85%	-0.87%	-2.72%
Without	$s_F c_F(p)$	21	0%	-0.33%	-0.25%	-0.29%
Without	$s_F c_F(p)$	22	-0.09%	-0.63%	0.47%	-1.29%
Without	$s_F c_F(p)$	23	0.01%	-0.18%	0.28%	-2.72%
Without	$s_F c_F(p)$	24	0.04%	-1.18%	-0.3%	-1.7%
Without	$s_F c_F(p)$	25	0.02%	-0.42%	-0.3%	-1.26%

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Table A.9 – continued from previous page

Annual Abundance Percent Bias						
Aux. Like.	**Model**	**Year**	**No Variation**	**Low Variation**	**Medium Variation**	**High Variation**
Without	$s_F c_R(p)$	1	-0.09%	-0.02%	0.45%	1.29%
Without	$s_F c_R(p)$	2	-0.05%	-0.15%	-0.21%	1.01%
Without	$s_F c_R(p)$	3	-0.05%	-0.2%	0.15%	0.95%
Without	$s_F c_R(p)$	4	0.03%	0.07%	0.1%	1.3%
Without	$s_F c_R(p)$	5	0.05%	0.01%	-0.4%	0.46%
Without	$s_F c_R(p)$	6	-0.16%	0.36%	0.05%	0.51%
Without	$s_F c_R(p)$	7	-0.05%	0.52%	0.26%	0.66%
Without	$s_F c_R(p)$	8	-0.02%	0.07%	-0.19%	0.38%
Without	$s_F c_R(p)$	9	-0.03%	-0.23%	-0.07%	-0.49%
Without	$s_F c_R(p)$	10	0.05%	0.22%	0.17%	0.18%
Without	$s_F c_R(p)$	11	0.01%	-0.07%	0.17%	-0.32%
Without	$s_F c_R(p)$	12	0.04%	-0.36%	0.05%	0.44%
Without	$s_F c_R(p)$	13	-0.08%	-0.16%	-0.43%	0.13%
Without	$s_F c_R(p)$	14	-0.01%	0.16%	-0.74%	1.43%
Without	$s_F c_R(p)$	15	-0.04%	-0.23%	-0.42%	-0.13%
Without	$s_F c_R(p)$	16	0.02%	0.27%	-0.06%	0.81%
Without	$s_F c_R(p)$	17	0.03%	-0.01%	-0.26%	1.35%
Without	$s_F c_R(p)$	18	-0.01%	0.5%	-0.52%	0.51%
Without	$s_F c_R(p)$	19	-0.04%	-0.16%	-0.26%	1.07%
Without	$s_F c_R(p)$	20	0.01%	0.12%	-0.03%	0.7%
Without	$s_F c_R(p)$	21	0%	-0.31%	-0.44%	0.52%
Without	$s_F c_R(p)$	22	-0.09%	-0.57%	-0.77%	-0.29%
Without	$s_F c_R(p)$	23	0.03%	-0.4%	-0.65%	-1.28%
Without	$s_F c_R(p)$	24	0.05%	-0.55%	-0.49%	-1.13%
Without	$s_F c_R(p)$	25	0.03%	0.06%	-0.95%	-0.79%
Without	$N_A s_F c_R(p)$	1	3.51%	-8.93%	-28.38%	-37.39%
Without	$N_A s_F c_R(p)$	2	8.55%	-10.08%	-55.8%	-66.29%
Without	$N_A s_F c_R(p)$	3	16.17%	-15.54%	-69.72%	-74.97%
Without	$N_A s_F c_R(p)$	4	23.38%	-17.25%	-75.16%	-70.18%
Without	$N_A s_F c_R(p)$	5	30.43%	-19.43%	-79.13%	-50.07%
Without	$N_A s_F c_R(p)$	6	34.65%	-24.99%	-81.37%	-7.7%
Without	$N_A s_F c_R(p)$	7	46.77%	-26%	-81.04%	24.84%
Without	$N_A s_F c_R(p)$	8	54.08%	-25.54%	-81.66%	27.54%
Without	$N_A s_F c_R(p)$	9	68.25%	-29.97%	-82.86%	11.13%
Without	$N_A s_F c_R(p)$	10	80.36%	-34.83%	-79.78%	13.14%
Without	$N_A s_F c_R(p)$	11	85.8%	-38.89%	-78.62%	-1.35%
Without	$N_A s_F c_R(p)$	12	101%	-39.03%	-80.81%	8.24%
Without	$N_A s_F c_R(p)$	13	111.59%	-44.67%	-81.19%	3.98%
Without	$N_A s_F c_R(p)$	14	113.55%	-43.12%	-81.88%	15.36%
Without	$N_A s_F c_R(p)$	15	131.55%	-45.53%	-82.14%	22.95%
Without	$N_A s_F c_R(p)$	16	150.21%	-41.77%	-84.44%	12.6%
Without	$N_A s_F c_R(p)$	17	167.05%	-41.6%	-86.98%	-10.41%
Without	$N_A s_F c_R(p)$	18	186.01%	-42.1%	-89.84%	-19.7%
Without	$N_A s_F c_R(p)$	19	207.06%	-46.33%	-90.08%	-11.34%
Without	$N_A s_F c_R(p)$	20	235.54%	-49.46%	-92%	-40.36%
Without	$N_A s_F c_R(p)$	21	254.13%	-46.93%	-92.94%	-52.58%
Without	$N_A s_F c_R(p)$	22	274.29%	-41.12%	-92.04%	-54.91%
Without	$N_A s_F c_R(p)$	23	284.63%	-33.58%	-89.99%	-51.48%
Without	$N_A s_F c_R(p)$	24	328.78%	-25.79%	-90.32%	-32.18%
Without	$N_A s_F c_R(p)$	25	109.38%	-2.91%	-14.03%	6.65%
Without	$s_R c_R(p)$	1	-0.09%	0.61%	-0.25%	-1.67%
Without	$s_R c_R(p)$	2	-0.05%	0.39%	-0.35%	-1.21%
Without	$s_R c_R(p)$	3	-0.05%	0.27%	0.61%	-0.83%
Without	$s_R c_R(p)$	4	0.03%	0.55%	-0.1%	-0.33%
Without	$s_R c_R(p)$	5	0.05%	0.28%	0.01%	-0.47%
Without	$s_R c_R(p)$	6	-0.15%	0.71%	-0.01%	-1.64%
Without	$s_R c_R(p)$	7	-0.04%	0.77%	-0.44%	-0.67%
Without	$s_R c_R(p)$	8	-0.02%	0.45%	-0.27%	-1.01%
Without	$s_R c_R(p)$	9	-0.03%	0.19%	-0.5%	-1.54%

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Table A.9 – continued from previous page

Annual Abundance Percent Bias						
Aux. Like.	**Model**	**Year**	**No Variation**	**Low Variation**	**Medium Variation**	**High Variation**
Without	$s_R c_R(p)$	10	0.05%	0.6%	1.08%	0.31%
Without	$s_R c_R(p)$	11	0.02%	0.41%	0.52%	-0.06%
Without	$s_R c_R(p)$	12	0%	0.41%	0.8%	-0.25%
Without	$s_R c_R(p)$	13	-0.08%	0.44%	0.08%	0.46%
Without	$s_R c_R(p)$	14	0%	0.63%	0.35%	1.63%
Without	$s_R c_R(p)$	15	-0.04%	0.41%	-0.01%	-1.36%
Without	$s_R c_R(p)$	16	0.02%	0.79%	-0.04%	-1.25%
Without	$s_R c_R(p)$	17	0.05%	0.29%	-0.33%	-0.76%
Without	$s_R c_R(p)$	18	0.01%	0.73%	-0.99%	-1.71%
Without	$s_R c_R(p)$	19	-0.02%	0.41%	0.19%	-1.48%
Without	$s_R c_R(p)$	20	0.02%	0.55%	-0.21%	-1.55%
Without	$s_R c_R(p)$	21	0%	0%	-0.29%	-1.27%
Without	$s_R c_R(p)$	22	-0.09%	-0.12%	-0.64%	-1%
Without	$s_R c_R(p)$	23	0.03%	0.24%	0.18%	-2.02%
Without	$s_R c_R(p)$	24	0.05%	-0.14%	0.05%	-1.65%
Without	$s_R c_R(p)$	25	0.07%	0.48%	-0.43%	-1%
Without	$N_A s_R c_R(p)$	1	4.72%	-6.78%	-28.29%	-37.42%
Without	$N_A s_R c_R(p)$	2	10.6%	-9.21%	-55.85%	-65.95%
Without	$N_A s_R c_R(p)$	3	17.53%	-11.5%	-69.26%	-75.12%
Without	$N_A s_R c_R(p)$	4	27.46%	-11.19%	-75.05%	-70.89%
Without	$N_A s_R c_R(p)$	5	35.54%	-12.31%	-77.91%	-51.4%
Without	$N_A s_R c_R(p)$	6	43.77%	-16.54%	-80.99%	-11.57%
Without	$N_A s_R c_R(p)$	7	57.79%	-19.65%	-80.28%	20.42%
Without	$N_A s_R c_R(p)$	8	66.52%	-20.14%	-81.32%	26.93%
Without	$N_A s_R c_R(p)$	9	76.78%	-23.62%	-82.9%	-0.45%
Without	$N_A s_R c_R(p)$	10	86.97%	-17.78%	-81.22%	11.44%
Without	$N_A s_R c_R(p)$	11	97.33%	-17.83%	-76.52%	-1.99%
Without	$N_A s_R c_R(p)$	12	118.27%	-20.21%	-79.05%	5.74%
Without	$N_A s_R c_R(p)$	13	129.25%	-25.3%	-79.18%	-6.58%
Without	$N_A s_R c_R(p)$	14	146.28%	-24.2%	-81.66%	11.87%
Without	$N_A s_R c_R(p)$	15	167.32%	-25.4%	-82.51%	17.76%
Without	$N_A s_R c_R(p)$	16	185.49%	-22.49%	-84.14%	3.43%
Without	$N_A s_R c_R(p)$	17	206%	-15.31%	-85%	-17.59%
Without	$N_A s_R c_R(p)$	18	232.65%	-11.81%	-88.33%	-16.93%
Without	$N_A s_R c_R(p)$	19	269.32%	-11.69%	-89.63%	-12.07%
Without	$N_A s_R c_R(p)$	20	297.14%	-13.64%	-91.73%	-42.22%
Without	$N_A s_R c_R(p)$	21	319.71%	-13.86%	-92.85%	-55.85%
Without	$N_A s_R c_R(p)$	22	354.43%	-13.71%	-91.85%	-57.75%
Without	$N_A s_R c_R(p)$	23	385.87%	2.13%	-88.43%	-49.26%
Without	$N_A s_R c_R(p)$	24	412.95%	13.51%	-89.58%	-31.98%
Without	$N_A s_R c_R(p)$	25	130.37%	8.72%	-13.32%	7.68%
With	$s_F c_F(p)$	1	-0.83%	10.12%	45.14%	67.06%
With	$s_F c_F(p)$	2	-0.82%	9.39%	45.88%	68.68%
With	$s_F c_F(p)$	3	-0.86%	8.55%	46.87%	67.27%
With	$s_F c_F(p)$	4	-0.79%	9.1%	44.81%	69.63%
With	$s_F c_F(p)$	5	-0.8%	9.44%	45.45%	69.25%
With	$s_F c_F(p)$	6	-0.79%	9.12%	46.4%	67.31%
With	$s_F c_F(p)$	7	-0.83%	10.05%	46.82%	70.34%
With	$s_F c_F(p)$	8	-0.77%	9.49%	47.63%	70.11%
With	$s_F c_F(p)$	9	-0.76%	9.7%	45.31%	64.87%
With	$s_F c_F(p)$	10	-0.72%	10.24%	47.54%	70.49%
With	$s_F c_F(p)$	11	-0.69%	8.33%	46.73%	71.93%
With	$s_F c_F(p)$	12	-0.8%	9.21%	45.94%	71.2%
With	$s_F c_F(p)$	13	-0.84%	8.39%	44.03%	67.78%
With	$s_F c_F(p)$	14	-0.73%	9.46%	48.57%	75.96%
With	$s_F c_F(p)$	15	-0.85%	9.17%	43.72%	69.47%
With	$s_F c_F(p)$	16	-0.78%	9.75%	44.82%	67.94%
With	$s_F c_F(p)$	17	-0.77%	9.63%	47.62%	69.8%
With	$s_F c_F(p)$	18	-0.63%	9.31%	44.95%	66.98%

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Table A.9 – continued from previous page

Annual Abundance Percent Bias						
Aux. Like.	**Model**	**Year**	**No Variation**	**Low Variation**	**Medium Variation**	**High Variation**
With	$s_F c_F(p)$	19	-0.77%	9.54%	46.19%	67.36%
With	$s_F c_F(p)$	20	-0.63%	9%	44.7%	67.37%
With	$s_F c_F(p)$	21	-0.73%	9.38%	46.64%	69.26%
With	$s_F c_F(p)$	22	-0.71%	9.06%	47.57%	69.51%
With	$s_F c_F(p)$	23	-0.71%	9.41%	49.09%	66.94%
With	$s_F c_F(p)$	24	-0.77%	8.42%	48%	73.59%
With	$s_F c_F(p)$	25	-0.65%	9.02%	48.1%	72.18%
With	$N_A s_F c_F(p)$	1	-0.85%	-0.34%	1.9%	5.7%
With	$N_A s_F c_F(p)$	2	-0.78%	-0.12%	2.98%	4.94%
With	$N_A s_F c_F(p)$	3	-0.76%	-1.08%	2.22%	5.74%
With	$N_A s_F c_F(p)$	4	-0.72%	-0.36%	1.57%	6.66%
With	$N_A s_F c_F(p)$	5	-0.79%	-0.65%	2.99%	7.89%
With	$N_A s_F c_F(p)$	6	-0.93%	-0.74%	2.36%	4.64%
With	$N_A s_F c_F(p)$	7	-0.77%	-0.38%	1.98%	6.74%
With	$N_A s_F c_F(p)$	8	-0.67%	-0.1%	4.34%	4.85%
With	$N_A s_F c_F(p)$	9	-0.72%	-0.92%	2.26%	4.46%
With	$N_A s_F c_F(p)$	10	-0.74%	-0.2%	2.82%	5%
With	$N_A s_F c_F(p)$	11	-0.66%	-1.11%	2.37%	7.04%
With	$N_A s_F c_F(p)$	12	-0.67%	0.05%	2.24%	6.2%
With	$N_A s_F c_F(p)$	13	-0.79%	-0.79%	2.62%	6.43%
With	$N_A s_F c_F(p)$	14	-0.73%	0.21%	2.59%	7.97%
With	$N_A s_F c_F(p)$	15	-0.78%	-0.43%	1.93%	7.65%
With	$N_A s_F c_F(p)$	16	-0.63%	-1.05%	1.37%	5.19%
With	$N_A s_F c_F(p)$	17	-0.78%	0.33%	2.73%	6.1%
With	$N_A s_F c_F(p)$	18	-0.71%	-0.54%	2.33%	5.51%
With	$N_A s_F c_F(p)$	19	-0.83%	-0.55%	3.72%	4.9%
With	$N_A s_F c_F(p)$	20	-0.68%	-0.6%	1.55%	4.23%
With	$N_A s_F c_F(p)$	21	-0.7%	-0.72%	3.14%	6.93%
With	$N_A s_F c_F(p)$	22	-0.81%	-0.44%	3.46%	6.88%
With	$N_A s_F c_F(p)$	23	-0.67%	-0.01%	3.85%	4.85%
With	$N_A s_F c_F(p)$	24	-0.64%	-1.09%	3.23%	4.63%
With	$N_A s_F c_F(p)$	25	-0.72%	-1.2%	3.76%	6.16%
With	$s_F c_R(p)$	1	-0.55%	-3.07%	-3.46%	-2.45%
With	$s_F c_R(p)$	2	-0.45%	-2.91%	-3.49%	-2.19%
With	$s_F c_R(p)$	3	-0.43%	-3.08%	-3.38%	-2.48%
With	$s_F c_R(p)$	4	-0.46%	-2.89%	-3.14%	-2.16%
With	$s_F c_R(p)$	5	-0.48%	-2.77%	-3.64%	-3.12%
With	$s_F c_R(p)$	6	-0.54%	-2.56%	-3.37%	-3.38%
With	$s_F c_R(p)$	7	-0.52%	-2.59%	-3.55%	-2.25%
With	$s_F c_R(p)$	8	-0.42%	-2.83%	-3.38%	-3.03%
With	$s_F c_R(p)$	9	-0.5%	-2.95%	-3.43%	-3.33%
With	$s_F c_R(p)$	10	-0.35%	-2.57%	-3.06%	-3.16%
With	$s_F c_R(p)$	11	-0.45%	-2.85%	-3.08%	-3.04%
With	$s_F c_R(p)$	12	-0.47%	-3.29%	-3.26%	-2.97%
With	$s_F c_R(p)$	13	-0.47%	-2.87%	-3.69%	-3.03%
With	$s_F c_R(p)$	14	-0.44%	-2.9%	-4.07%	-2.01%
With	$s_F c_R(p)$	15	-0.55%	-2.9%	-3.84%	-3.05%
With	$s_F c_R(p)$	16	-0.35%	-2.64%	-3.73%	-2.48%
With	$s_F c_R(p)$	17	-0.43%	-3.06%	-3.8%	-2.35%
With	$s_F c_R(p)$	18	-0.4%	-2.65%	-4.18%	-3.03%
With	$s_F c_R(p)$	19	-0.45%	-2.96%	-3.7%	-2.63%
With	$s_F c_R(p)$	20	-0.42%	-2.87%	-3.84%	-3.42%
With	$s_F c_R(p)$	21	-0.5%	-3.33%	-4.07%	-3.35%
With	$s_F c_R(p)$	22	-0.5%	-3.49%	-4.46%	-4.41%
With	$s_F c_R(p)$	23	-0.38%	-3.28%	-4.66%	-5.92%
With	$s_F c_R(p)$	24	-0.4%	-3.46%	-5.03%	-6.35%
With	$s_F c_R(p)$	25	-0.41%	-3.29%	-5.88%	-7.98%
With	$N_A s_F c_R(p)$	1	-0.85%	-8.45%	-10.67%	-9.86%
With	$N_A s_F c_R(p)$	2	-0.78%	-8.18%	-10.88%	-9.64%

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Table A.9 – continued from previous page

Annual Abundance Percent Bias						
Aux. Like.	**Model**	**Year**	**No Variation**	**Low Variation**	**Medium Variation**	**High Variation**
With	$N_A s_F c_R(p)$	3	-0.76%	-8.21%	-10.55%	-10.04%
With	$N_A s_F c_R(p)$	4	-0.72%	-8.09%	-10.91%	-10.33%
With	$N_A s_F c_R(p)$	5	-0.79%	-8.06%	-10.88%	-10.96%
With	$N_A s_F c_R(p)$	6	-0.93%	-7.98%	-11%	-10.72%
With	$N_A s_F c_R(p)$	7	-0.77%	-8.07%	-10.86%	-10.77%
With	$N_A s_F c_R(p)$	8	-0.67%	-8.03%	-10.38%	-10.61%
With	$N_A s_F c_R(p)$	9	-0.72%	-8.08%	-10.32%	-11.44%
With	$N_A s_F c_R(p)$	10	-0.74%	-7.66%	-10.09%	-10.28%
With	$N_A s_F c_R(p)$	11	-0.66%	-7.91%	-9.94%	-10.47%
With	$N_A s_F c_R(p)$	12	-0.67%	-8.08%	-10.23%	-10.38%
With	$N_A s_F c_R(p)$	13	-0.79%	-7.92%	-10.58%	-10.44%
With	$N_A s_F c_R(p)$	14	-0.73%	-7.98%	-11.24%	-9.32%
With	$N_A s_F c_R(p)$	15	-0.78%	-8.21%	-10.91%	-10.27%
With	$N_A s_F c_R(p)$	16	-0.63%	-7.89%	-10.6%	-10.56%
With	$N_A s_F c_R(p)$	17	-0.78%	-8.25%	-10.4%	-10.37%
With	$N_A s_F c_R(p)$	18	-0.71%	-8%	-11.06%	-11.28%
With	$N_A s_F c_R(p)$	19	-0.83%	-8.25%	-10.69%	-10.83%
With	$N_A s_F c_R(p)$	20	-0.68%	-7.99%	-11%	-11.17%
With	$N_A s_F c_R(p)$	21	-0.7%	-8.47%	-11.02%	-11.13%
With	$N_A s_F c_R(p)$	22	-0.81%	-8.6%	-11.52%	-11.99%
With	$N_A s_F c_R(p)$	23	-0.67%	-8.46%	-11.26%	-13.63%
With	$N_A s_F c_R(p)$	24	-0.64%	-8.67%	-11.72%	-13.61%
With	$N_A s_F c_R(p)$	25	-0.72%	-8.8%	-12.46%	-14.94%
With	$s_R c_R(p)$	1	-0.6%	-2.47%	-4.97%	-6.88%
With	$s_R c_R(p)$	2	-0.49%	-2.62%	-4.56%	-6.48%
With	$s_R c_R(p)$	3	-0.55%	-2.85%	-3.69%	-6.35%
With	$s_R c_R(p)$	4	-0.43%	-2.63%	-4.27%	-6.16%
With	$s_R c_R(p)$	5	-0.56%	-2.74%	-4.32%	-5.93%
With	$s_R c_R(p)$	6	-0.61%	-2.58%	-4.58%	-6.84%
With	$s_R c_R(p)$	7	-0.53%	-2.43%	-4.72%	-6.54%
With	$s_R c_R(p)$	8	-0.53%	-2.76%	-4.58%	-6.31%
With	$s_R c_R(p)$	9	-0.53%	-2.71%	-4.39%	-6.52%
With	$s_R c_R(p)$	10	-0.36%	-2.27%	-2.62%	-4.05%
With	$s_R c_R(p)$	11	-0.45%	-2.33%	-3.07%	-3.84%
With	$s_R c_R(p)$	12	-0.49%	-2.54%	-2.73%	-4.16%
With	$s_R c_R(p)$	13	-0.43%	-2.51%	-3.48%	-3.12%
With	$s_R c_R(p)$	14	-0.47%	-2.3%	-3.29%	-2.63%
With	$s_R c_R(p)$	15	-0.54%	-2.72%	-3.39%	-5.44%
With	$s_R c_R(p)$	16	-0.44%	-2.41%	-4.15%	-6.66%
With	$s_R c_R(p)$	17	-0.54%	-3.01%	-4.49%	-6.42%
With	$s_R c_R(p)$	18	-0.4%	-2.4%	-5.13%	-7.11%
With	$s_R c_R(p)$	19	-0.55%	-2.75%	-4.43%	-7.35%
With	$s_R c_R(p)$	20	-0.47%	-2.52%	-4.42%	-7.11%
With	$s_R c_R(p)$	21	-0.52%	-3.06%	-4.54%	-6.61%
With	$s_R c_R(p)$	22	-0.52%	-3.23%	-5.14%	-6.84%
With	$s_R c_R(p)$	23	-0.47%	-2.9%	-4.36%	-8.12%
With	$s_R c_R(p)$	24	-0.47%	-3.15%	-4.67%	-7.78%
With	$s_R c_R(p)$	25	-0.46%	-2.85%	-5.55%	-8.07%
With	$N_A s_R c_R(p)$	1	-0.85%	-8.29%	-9.66%	-8.18%
With	$N_A s_R c_R(p)$	2	-0.78%	-7.88%	-9.91%	-7.91%
With	$N_A s_R c_R(p)$	3	-0.76%	-8.1%	-9.14%	-9.3%
With	$N_A s_R c_R(p)$	4	-0.73%	-7.76%	-9.53%	-8.75%
With	$N_A s_R c_R(p)$	5	-0.79%	-7.85%	-9.79%	-9.52%
With	$N_A s_R c_R(p)$	6	-0.93%	-7.89%	-10.19%	-9.49%
With	$N_A s_R c_R(p)$	7	-0.77%	-7.72%	-9.81%	-9.75%
With	$N_A s_R c_R(p)$	8	-0.67%	-7.94%	-9.12%	-9.65%
With	$N_A s_R c_R(p)$	9	-0.72%	-7.81%	-9.58%	-10.21%
With	$N_A s_R c_R(p)$	10	-0.74%	-7.47%	-8.97%	-9.13%
With	$N_A s_R c_R(p)$	11	-0.66%	-7.64%	-9.38%	-8.54%

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Table A.9 – continued from previous page

Annual Abundance Percent Bias						
Aux. Like.	**Model**	**Year**	**No Variation**	**Low Variation**	**Medium Variation**	**High Variation**
With	$N_A s_R c_R(p)$	12	-0.67%	-7.85%	-9.12%	-8.53%
With	$N_A s_R c_R(p)$	13	-0.79%	-7.68%	-9.52%	-9.06%
With	$N_A s_R c_R(p)$	14	-0.73%	-7.65%	-9.73%	-7.58%
With	$N_A s_R c_R(p)$	15	-0.78%	-7.97%	-9.92%	-8.69%
With	$N_A s_R c_R(p)$	16	-0.63%	-7.74%	-9.55%	-9.76%
With	$N_A s_R c_R(p)$	17	-0.78%	-8.07%	-9.48%	-9.27%
With	$N_A s_R c_R(p)$	18	-0.71%	-7.63%	-9.84%	-9.37%
With	$N_A s_R c_R(p)$	19	-0.83%	-8.05%	-9.42%	-9.37%
With	$N_A s_R c_R(p)$	20	-0.68%	-7.71%	-9.58%	-9.94%
With	$N_A s_R c_R(p)$	21	-0.7%	-8.07%	-9.75%	-9.74%
With	$N_A s_R c_R(p)$	22	-0.82%	-8.26%	-10.04%	-10.6%
With	$N_A s_R c_R(p)$	23	-0.67%	-8.16%	-10.11%	-11.98%
With	$N_A s_R c_R(p)$	24	-0.64%	-8.41%	-10.53%	-11.73%
With	$N_A s_R c_R(p)$	25	-0.71%	-8.4%	-10.54%	-12.69%

Table A.10: *Median relative bias in total annual abundance estimates for small game models, when a low level of simulated auxiliary data is available for estimation of c. Results indicate low bias for mixed-effects versions of the models employing the Horvitz-Thompson estimator (when the catch-effort likelihood of Equation (1.7) is omitted), except for the highest level of simulated variation, which shows bias between 5% and 9%. The fixed-effects version of the model employing the Horvitz-Thomspon abundance estimator shows low bias when the auxiliary catch-effort likelihood component is omitted, in all scenarios. The mixed-effects absolute-recruit abundance models show negative bias that increases in magnitude with the level of simulated variation, while the fixed-effects version of this model shows negligible bias which increases to low positive bias at higher levels of simulated variation. Results based on $n = 1000$ replicates, with $s \approx 0.50$, $c \approx -0.685$, $\gamma = 2.0$, and total annual abundance $\approx$ 40,000. "Aux. Like." = Auxiliary catch-effort likelihood used (With) or not used (Without).*

Table A.10 Annual Abundance Percent Bias						
Aux. Like.	**Model**	**Year**	**No Variation**	**Low Variation**	**Medium Variation**	**High Variation**
Without	$s_F c_F(p)$	1	-0.29%	-1.1%	-1.77%	-0.76%
Without	$s_F c_F(p)$	2	-0.26%	-0.25%	-1.21%	1.1%
Without	$s_F c_F(p)$	3	-0.28%	-0.55%	-1.76%	-1.39%
Without	$s_F c_F(p)$	4	-0.26%	-0.07%	-1.32%	-0.6%
Without	$s_F c_F(p)$	5	-0.2%	-0.66%	-0.82%	0.02%
Without	$s_F c_F(p)$	6	-0.24%	-0.35%	-2.13%	-0.03%
Without	$s_F c_F(p)$	7	-0.23%	-0.62%	-2.59%	0.36%
Without	$s_F c_F(p)$	8	-0.21%	-0.23%	-0.01%	0.86%
Without	$s_F c_F(p)$	9	-0.14%	-1.1%	-2.1%	-3.33%
Without	$s_F c_F(p)$	10	-0.19%	-0.37%	-1.13%	-1.47%
Without	$s_F c_F(p)$	11	-0.26%	-1%	0.56%	0.07%
Without	$s_F c_F(p)$	12	-0.26%	-0.93%	-2.79%	-2.75%
Without	$s_F c_F(p)$	13	-0.23%	-0.91%	-0.59%	-0.25%
Without	$s_F c_F(p)$	14	-0.3%	-0.21%	-2.35%	0.89%
Without	$s_F c_F(p)$	15	-0.21%	-1.43%	-2.15%	0.02%
Without	$s_F c_F(p)$	16	-0.19%	-0.34%	-3.16%	-3.35%

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Table A.10 – continued from previous page

Annual Abundance Percent Bias						
Aux. Like.	**Model**	**Year**	**No Variation**	**Low Variation**	**Medium Variation**	**High Variation**
Without	$s_F c_F(p)$	17	-0.18%	-0.28%	-0.63%	0.33%
Without	$s_F c_F(p)$	18	-0.19%	-1.2%	-1.8%	-0.58%
Without	$s_F c_F(p)$	19	-0.29%	-0.88%	-1.42%	-0.19%
Without	$s_F c_F(p)$	20	-0.23%	-0.86%	-1.2%	0.53%
Without	$s_F c_F(p)$	21	-0.24%	-0.48%	-0.92%	-0.26%
Without	$s_F c_F(p)$	22	-0.3%	-1.02%	-0.63%	-0.69%
Without	$s_F c_F(p)$	23	-0.14%	-0.92%	-1.21%	-0.95%
Without	$s_F c_F(p)$	24	-0.22%	-0.84%	-1.21%	-0.53%
Without	$s_F c_F(p)$	25	-0.21%	0.02%	-0.7%	0.95%
Without	$s_F c_R(p)$	1	-0.29%	-0.97%	0.16%	6.09%
Without	$s_F c_R(p)$	2	-0.25%	-1.56%	-0.38%	6.74%
Without	$s_F c_R(p)$	3	-0.27%	-1.39%	0.73%	6.27%
Without	$s_F c_R(p)$	4	-0.23%	-1.48%	-0.63%	6.37%
Without	$s_F c_R(p)$	5	-0.19%	-1.74%	-0.72%	7.22%
Without	$s_F c_R(p)$	6	-0.25%	-1.57%	-0.67%	7.66%
Without	$s_F c_R(p)$	7	-0.23%	-1.15%	-0.37%	5.66%
Without	$s_F c_R(p)$	8	-0.21%	-1.27%	0.59%	6.39%
Without	$s_F c_R(p)$	9	-0.13%	-1.27%	-0.76%	5.22%
Without	$s_F c_R(p)$	10	-0.18%	-1.32%	-0.38%	5.84%
Without	$s_F c_R(p)$	11	-0.24%	-1.37%	0.18%	7.54%
Without	$s_F c_R(p)$	12	-0.25%	-1.25%	0.42%	6.66%
Without	$s_F c_R(p)$	13	-0.22%	-1.16%	-0.75%	4.93%
Without	$s_F c_R(p)$	14	-0.29%	-0.67%	-0.47%	5.99%
Without	$s_F c_R(p)$	15	-0.21%	-1.3%	0.01%	6.26%
Without	$s_F c_R(p)$	16	-0.19%	-1.35%	-0.48%	7.15%
Without	$s_F c_R(p)$	17	-0.18%	-1.66%	-0.44%	6.38%
Without	$s_F c_R(p)$	18	-0.14%	-1.8%	-0.75%	6.39%
Without	$s_F c_R(p)$	19	-0.28%	-1.1%	-0.75%	6.23%
Without	$s_F c_R(p)$	20	-0.22%	-1.47%	-0.94%	6.61%
Without	$s_F c_R(p)$	21	-0.23%	-1.49%	-0.4%	5.78%
Without	$s_F c_R(p)$	22	-0.28%	-1.65%	-0.22%	5.79%
Without	$s_F c_R(p)$	23	-0.13%	-1.31%	-1.26%	5.25%
Without	$s_F c_R(p)$	24	-0.19%	-0.93%	-0.01%	5.34%
Without	$s_F c_R(p)$	25	-0.2%	-0.72%	-0.22%	4.48%
Without	$N_A s_F c_R(p)$	1	-1.44%	-13.15%	-26.01%	-30.47%
Without	$N_A s_F c_R(p)$	2	-5.1%	-19.01%	-50.83%	-58.89%
Without	$N_A s_F c_R(p)$	3	-13.77%	-26.91%	-63.22%	-72.43%
Without	$N_A s_F c_R(p)$	4	-16%	-30.4%	-71.65%	-69.46%
Without	$N_A s_F c_R(p)$	5	-20.72%	-32.63%	-72.58%	-56.84%
Without	$N_A s_F c_R(p)$	6	-23.97%	-39.31%	-78.14%	-41.36%
Without	$N_A s_F c_R(p)$	7	-29.48%	-40.98%	-78.76%	-2.97%
Without	$N_A s_F c_R(p)$	8	-32.35%	-47.04%	-79.56%	38.56%
Without	$N_A s_F c_R(p)$	9	-35.49%	-50.84%	-80.89%	24.2%
Without	$N_A s_F c_R(p)$	10	-44.15%	-52.81%	-79%	26.75%
Without	$N_A s_F c_R(p)$	11	-48.01%	-57.81%	-80.34%	31.35%
Without	$N_A s_F c_R(p)$	12	-53.02%	-60.42%	-83.31%	8.48%
Without	$N_A s_F c_R(p)$	13	-54.9%	-63.4%	-79.08%	5.01%
Without	$N_A s_F c_R(p)$	14	-54.48%	-65.74%	-78.93%	27.65%
Without	$N_A s_F c_R(p)$	15	-58.37%	-69.7%	-82.22%	27.29%
Without	$N_A s_F c_R(p)$	16	-59.39%	-69.07%	-84.36%	13.67%
Without	$N_A s_F c_R(p)$	17	-62.2%	-66.81%	-85.28%	-9.75%
Without	$N_A s_F c_R(p)$	18	-64.1%	-70.69%	-88.28%	-20.15%
Without	$N_A s_F c_R(p)$	19	-64.95%	-74.08%	-89.04%	-4.38%
Without	$N_A s_F c_R(p)$	20	-67.98%	-74.68%	-91.04%	-25.74%
Without	$N_A s_F c_R(p)$	21	-70.38%	-77.99%	-90.96%	-36.25%
Without	$N_A s_F c_R(p)$	22	-69.49%	-79.74%	-90.36%	-20.54%
Without	$N_A s_F c_R(p)$	23	-69.87%	-79.18%	-85.63%	-27.83%
Without	$N_A s_F c_R(p)$	24	-71.14%	-76.3%	-84.69%	10.91%
Without	$N_A s_F c_R(p)$	25	5.55%	-2.77%	5.89%	66.83%

Continued on next page

Table A.10 – continued from previous page

Annual Abundance Percent Bias						
Aux. Like.	**Model**	**Year**	**No Variation**	**Low Variation**	**Medium Variation**	**High Variation**
Without	$s_R c_R(p)$	1	-0.29%	-0.09%	2.4%	8.46%
Without	$s_R c_R(p)$	2	-0.25%	-0.3%	2.31%	7.35%
Without	$s_R c_R(p)$	3	-0.26%	0.11%	2.04%	6.45%
Without	$s_R c_R(p)$	4	-0.23%	-0.06%	1.45%	7.53%
Without	$s_R c_R(p)$	5	-0.17%	-0.32%	-0.11%	9.11%
Without	$s_R c_R(p)$	6	-0.25%	-0.11%	0.56%	8.4%
Without	$s_R c_R(p)$	7	-0.24%	0.82%	1.55%	6.5%
Without	$s_R c_R(p)$	8	-0.22%	0.03%	2.8%	7.75%
Without	$s_R c_R(p)$	9	-0.13%	-0.2%	1.35%	6.82%
Without	$s_R c_R(p)$	10	-0.19%	-0.23%	2.64%	7.69%
Without	$s_R c_R(p)$	11	-0.26%	-0.61%	2.37%	9.22%
Without	$s_R c_R(p)$	12	-0.24%	0.06%	1.75%	8.48%
Without	$s_R c_R(p)$	13	-0.23%	0.03%	1.61%	8.09%
Without	$s_R c_R(p)$	14	-0.3%	-0.06%	1.05%	8.21%
Without	$s_R c_R(p)$	15	-0.22%	-0.21%	1.4%	8.47%
Without	$s_R c_R(p)$	16	-0.18%	0.08%	1.52%	8.73%
Without	$s_R c_R(p)$	17	-0.18%	-0.41%	1.72%	8%
Without	$s_R c_R(p)$	18	-0.15%	-0.81%	1.26%	7.6%
Without	$s_R c_R(p)$	19	-0.29%	-0.53%	1.23%	8.12%
Without	$s_R c_R(p)$	20	-0.22%	-0.16%	1.69%	8.13%
Without	$s_R c_R(p)$	21	-0.24%	-0.69%	2.2%	7.18%
Without	$s_R c_R(p)$	22	-0.29%	-0.34%	1.23%	6.95%
Without	$s_R c_R(p)$	23	-0.13%	0.06%	1.06%	7.12%
Without	$s_R c_R(p)$	24	-0.2%	0.03%	1.16%	6.77%
Without	$s_R c_R(p)$	25	-0.2%	0.44%	1.96%	6.19%
Without	$N_A s_R c_R(p)$	1	-0.49%	-12.09%	-26.47%	-29.95%
Without	$N_A s_R c_R(p)$	2	3.3%	-15.59%	-50.84%	-58.08%
Without	$N_A s_R c_R(p)$	3	3.35%	-18.89%	-63.52%	-70.97%
Without	$N_A s_R c_R(p)$	4	1.88%	-20.63%	-72.05%	-69.68%
Without	$N_A s_R c_R(p)$	5	2.87%	-25.29%	-74.22%	-54.92%
Without	$N_A s_R c_R(p)$	6	4.47%	-27.15%	-78.28%	-34.1%
Without	$N_A s_R c_R(p)$	7	6.24%	-26.4%	-78.31%	-2.86%
Without	$N_A s_R c_R(p)$	8	7.15%	-29.39%	-78.65%	46.89%
Without	$N_A s_R c_R(p)$	9	7.67%	-32.54%	-78.19%	26.35%
Without	$N_A s_R c_R(p)$	10	6.13%	-31.78%	-74.92%	29.95%
Without	$N_A s_R c_R(p)$	11	5.69%	-39.84%	-76.73%	28.42%
Without	$N_A s_R c_R(p)$	12	3.62%	-38.76%	-79.15%	21.63%
Without	$N_A s_R c_R(p)$	13	1.57%	-41.08%	-76.53%	9.75%
Without	$N_A s_R c_R(p)$	14	-3.34%	-44.06%	-76.55%	28.71%
Without	$N_A s_R c_R(p)$	15	0.48%	-44.4%	-79.37%	30.47%
Without	$N_A s_R c_R(p)$	16	0.71%	-44.91%	-81.99%	7.02%
Without	$N_A s_R c_R(p)$	17	2.12%	-44.33%	-85.25%	-6.87%
Without	$N_A s_R c_R(p)$	18	-1.1%	-46.2%	-88.15%	-20.31%
Without	$N_A s_R c_R(p)$	19	3.82%	-48.97%	-88.26%	-13.91%
Without	$N_A s_R c_R(p)$	20	4.11%	-40.01%	-90.57%	-28.21%
Without	$N_A s_R c_R(p)$	21	3.78%	-37.14%	-90.26%	-42.46%
Without	$N_A s_R c_R(p)$	22	0.64%	-37.52%	-86.32%	-29.81%
Without	$N_A s_R c_R(p)$	23	-0.24%	-30.93%	-80.28%	-32.16%
Without	$N_A s_R c_R(p)$	24	-1.33%	-31.17%	-79.31%	2.59%
Without	$N_A s_R c_R(p)$	25	24.67%	13.18%	9.32%	67.32%
With	$s_F c_F(p)$	1	-0.91%	10.09%	65.62%	159.79%
With	$s_F c_F(p)$	2	-0.86%	9.2%	67.4%	155.16%
With	$s_F c_F(p)$	3	-0.79%	9.29%	68.14%	148.59%
With	$s_F c_F(p)$	4	-0.87%	10.48%	63.69%	151.55%
With	$s_F c_F(p)$	5	-0.84%	9.73%	68.7%	149.31%
With	$s_F c_F(p)$	6	-0.8%	10.27%	66.3%	154.49%
With	$s_F c_F(p)$	7	-0.87%	9.5%	64.3%	158.06%
With	$s_F c_F(p)$	8	-0.8%	10.41%	67.97%	152.05%
With	$s_F c_F(p)$	9	-0.83%	9.15%	65.9%	155.53%

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Table A.10 – continued from previous page

Aux. Like.	Model	Year	No Variation	Low Variation	Medium Variation	High Variation
			Annual Abundance Percent Bias			
With	$s_F c_F(p)$	10	-0.85%	10.12%	68.76%	152.74%
With	$s_F c_F(p)$	11	-0.83%	9.28%	67.63%	163.47%
With	$s_F c_F(p)$	12	-0.81%	9.16%	65.29%	154.54%
With	$s_F c_F(p)$	13	-0.78%	9.21%	62.73%	159.74%
With	$s_F c_F(p)$	14	-0.84%	10.91%	65.17%	158.73%
With	$s_F c_F(p)$	15	-0.94%	9.32%	60.24%	153.88%
With	$s_F c_F(p)$	16	-0.88%	9.51%	63.15%	151.79%
With	$s_F c_F(p)$	17	-0.84%	10.66%	65.79%	162.63%
With	$s_F c_F(p)$	18	-0.79%	9.8%	63.32%	153.28%
With	$s_F c_F(p)$	19	-0.91%	10%	63.89%	161.71%
With	$s_F c_F(p)$	20	-0.86%	10.15%	66.37%	155.73%
With	$s_F c_F(p)$	21	-0.87%	10.76%	65.39%	157.25%
With	$s_F c_F(p)$	22	-0.94%	10.33%	66.88%	148.89%
With	$s_F c_F(p)$	23	-0.82%	10.14%	66.87%	154.1%
With	$s_F c_F(p)$	24	-0.8%	10.02%	69.34%	153.31%
With	$s_F c_F(p)$	25	-0.83%	10.22%	69.7%	151.23%
With	$N_A s_F c_F(p)$	1	-1.02%	-1.09%	-0.03%	5.66%
With	$N_A s_F c_F(p)$	2	-1.01%	-1.22%	1.65%	8.1%
With	$N_A s_F c_F(p)$	3	-0.99%	-0.9%	0.95%	7.25%
With	$N_A s_F c_F(p)$	4	-1.05%	-0.55%	0.8%	5.76%
With	$N_A s_F c_F(p)$	5	-1%	-0.97%	2.66%	6.61%
With	$N_A s_F c_F(p)$	6	-1%	-1.11%	1.66%	7.42%
With	$N_A s_F c_F(p)$	7	-0.96%	-1.03%	0.41%	6.85%
With	$N_A s_F c_F(p)$	8	-0.98%	-0.79%	1.95%	7.25%
With	$N_A s_F c_F(p)$	9	-0.91%	-1.22%	2.9%	5.78%
With	$N_A s_F c_F(p)$	10	-0.94%	-0.98%	2.7%	5.68%
With	$N_A s_F c_F(p)$	11	-0.94%	-0.81%	1.95%	7.5%
With	$N_A s_F c_F(p)$	12	-1.07%	-1.25%	0.21%	2.69%
With	$N_A s_F c_F(p)$	13	-1.05%	-1.67%	1.75%	7.75%
With	$N_A s_F c_F(p)$	14	-1.09%	0.12%	0.38%	5%
With	$N_A s_F c_F(p)$	15	-1.02%	-1.87%	0.37%	5.57%
With	$N_A s_F c_F(p)$	16	-0.98%	-0.79%	0.24%	4.36%
With	$N_A s_F c_F(p)$	17	-1.05%	-0.59%	3.34%	6.19%
With	$N_A s_F c_F(p)$	18	-0.91%	-1.41%	0.58%	7.34%
With	$N_A s_F c_F(p)$	19	-1.04%	-1.3%	2.19%	5.24%
With	$N_A s_F c_F(p)$	20	-0.99%	-0.68%	1.57%	9.1%
With	$N_A s_F c_F(p)$	21	-1.04%	-0.91%	1.43%	6.62%
With	$N_A s_F c_F(p)$	22	-1.05%	-0.65%	2.34%	7.07%
With	$N_A s_F c_F(p)$	23	-0.94%	-1.43%	2.34%	4.31%
With	$N_A s_F c_F(p)$	24	-0.96%	-1.43%	1.39%	3.98%
With	$N_A s_F c_F(p)$	25	-0.89%	-0.6%	2.74%	7.14%
With	$s_F c_R(p)$	1	-0.71%	-7.56%	-12.49%	-12.01%
With	$s_F c_R(p)$	2	-0.53%	-7.99%	-13.81%	-11.44%
With	$s_F c_R(p)$	3	-0.54%	-7.68%	-12.86%	-12.87%
With	$s_F c_R(p)$	4	-0.54%	-7.91%	-13.37%	-11.43%
With	$s_F c_R(p)$	5	-0.62%	-7.62%	-14.2%	-11.65%
With	$s_F c_R(p)$	6	-0.63%	-7.47%	-13.34%	-11.69%
With	$s_F c_R(p)$	7	-0.53%	-7.41%	-13.59%	-12.1%
With	$s_F c_R(p)$	8	-0.57%	-7.57%	-13%	-11.85%
With	$s_F c_R(p)$	9	-0.57%	-7.8%	-12.91%	-12.66%
With	$s_F c_R(p)$	10	-0.55%	-7.72%	-13.33%	-12.42%
With	$s_F c_R(p)$	11	-0.56%	-7.66%	-12.63%	-12.22%
With	$s_F c_R(p)$	12	-0.59%	-7.92%	-13.51%	-12.03%
With	$s_F c_R(p)$	13	-0.57%	-7.41%	-12.86%	-12.32%
With	$s_F c_R(p)$	14	-0.64%	-7.08%	-13.82%	-10.72%
With	$s_F c_R(p)$	15	-0.67%	-7.76%	-12.87%	-11.66%
With	$s_F c_R(p)$	16	-0.62%	-7.93%	-13.14%	-11.74%
With	$s_F c_R(p)$	17	-0.65%	-8.19%	-12.82%	-10.98%
With	$s_F c_R(p)$	18	-0.55%	-7.8%	-13.7%	-12.09%

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Table A.10 – continued from previous page

Aux. Like.	Model	Year	No Variation	Low Variation	Medium Variation	High Variation
			Annual Abundance Percent Bias			
With	$s_F c_R(p)$	19	-0.63%	-7.53%	-13.22%	-11.1%
With	$s_F c_R(p)$	20	-0.5%	-7.73%	-13.46%	-11.78%
With	$s_F c_R(p)$	21	-0.61%	-7.79%	-13.39%	-11.47%
With	$s_F c_R(p)$	22	-0.59%	-8.04%	-13.56%	-12.63%
With	$s_F c_R(p)$	23	-0.45%	-7.95%	-13.54%	-13.79%
With	$s_F c_R(p)$	24	-0.53%	-7.79%	-13.95%	-14.22%
With	$s_F c_R(p)$	25	-0.55%	-7.86%	-15.3%	-16.06%
With	$N_A s_F c_R(p)$	1	-1.02%	-18.3%	-30.99%	-32.37%
With	$N_A s_F c_R(p)$	2	-1.01%	-18.29%	-31.7%	-31.72%
With	$N_A s_F c_R(p)$	3	-0.99%	-18.33%	-31.27%	-32.67%
With	$N_A s_F c_R(p)$	4	-1.05%	-18.3%	-31.44%	-32.09%
With	$N_A s_F c_R(p)$	5	-1%	-17.86%	-31.12%	-32.63%
With	$N_A s_F c_R(p)$	6	-1%	-18.34%	-31.87%	-32.85%
With	$N_A s_F c_R(p)$	7	-0.96%	-18.76%	-31.3%	-33.45%
With	$N_A s_F c_R(p)$	8	-0.98%	-18.04%	-30.71%	-32.48%
With	$N_A s_F c_R(p)$	9	-0.91%	-18.08%	-30.58%	-33.56%
With	$N_A s_F c_R(p)$	10	-0.94%	-18.23%	-30.62%	-33.56%
With	$N_A s_F c_R(p)$	11	-0.94%	-18.33%	-31.52%	-32.68%
With	$N_A s_F c_R(p)$	12	-1.07%	-18.35%	-31.12%	-33.56%
With	$N_A s_F c_R(p)$	13	-1.05%	-18.77%	-30.94%	-31.91%
With	$N_A s_F c_R(p)$	14	-1.09%	-18.52%	-31.17%	-31.71%
With	$N_A s_F c_R(p)$	15	-1.02%	-18.46%	-30.25%	-32.4%
With	$N_A s_F c_R(p)$	16	-0.98%	-17.64%	-30.99%	-33.37%
With	$N_A s_F c_R(p)$	17	-1.05%	-18.19%	-30.86%	-32.08%
With	$N_A s_F c_R(p)$	18	-0.91%	-18.09%	-31.29%	-33.09%
With	$N_A s_F c_R(p)$	19	-1.04%	-17.85%	-30.85%	-32.34%
With	$N_A s_F c_R(p)$	20	-0.99%	-18.14%	-31.32%	-33.24%
With	$N_A s_F c_R(p)$	21	-1.04%	-18.4%	-31.35%	-33.32%
With	$N_A s_F c_R(p)$	22	-1.05%	-18.14%	-31.22%	-33.65%
With	$N_A s_F c_R(p)$	23	-0.94%	-18.25%	-31.38%	-34.86%
With	$N_A s_F c_R(p)$	24	-0.96%	-18.05%	-31.6%	-34.68%
With	$N_A s_F c_R(p)$	25	-0.89%	-18.56%	-32.91%	-35.94%
With	$s_R c_R(p)$	1	-0.79%	-7.29%	-12.26%	-12.57%
With	$s_R c_R(p)$	2	-0.61%	-7.73%	-13.61%	-11.95%
With	$s_R c_R(p)$	3	-0.65%	-7.32%	-12.55%	-12.85%
With	$s_R c_R(p)$	4	-0.69%	-7.03%	-13.01%	-12.21%
With	$s_R c_R(p)$	5	-0.66%	-7.1%	-13.79%	-11.61%
With	$s_R c_R(p)$	6	-0.73%	-7.01%	-12.75%	-11.97%
With	$s_R c_R(p)$	7	-0.57%	-7.18%	-13.42%	-11.83%
With	$s_R c_R(p)$	8	-0.7%	-7.32%	-12.76%	-12.49%
With	$s_R c_R(p)$	9	-0.68%	-7.69%	-12.94%	-13.1%
With	$s_R c_R(p)$	10	-0.62%	-7.6%	-12.39%	-12.2%
With	$s_R c_R(p)$	11	-0.62%	-7.62%	-12.22%	-12.17%
With	$s_R c_R(p)$	12	-0.68%	-7.43%	-12.58%	-11.39%
With	$s_R c_R(p)$	13	-0.73%	-7.56%	-11.64%	-9.58%
With	$s_R c_R(p)$	14	-0.75%	-7.03%	-13.06%	-10.76%
With	$s_R c_R(p)$	15	-0.77%	-7.66%	-12.28%	-10.83%
With	$s_R c_R(p)$	16	-0.68%	-7.65%	-13.19%	-11.33%
With	$s_R c_R(p)$	17	-0.7%	-8.1%	-12.86%	-10.96%
With	$s_R c_R(p)$	18	-0.6%	-7.68%	-13.57%	-12.01%
With	$s_R c_R(p)$	19	-0.76%	-7.5%	-13.44%	-11.14%
With	$s_R c_R(p)$	20	-0.6%	-7.58%	-12.76%	-11.67%
With	$s_R c_R(p)$	21	-0.63%	-7.61%	-12.68%	-11.3%
With	$s_R c_R(p)$	22	-0.68%	-7.68%	-13.07%	-12.76%
With	$s_R c_R(p)$	23	-0.56%	-7.8%	-13.38%	-13.41%
With	$s_R c_R(p)$	24	-0.6%	-7.64%	-13.41%	-13.97%
With	$s_R c_R(p)$	25	-0.58%	-7.18%	-14.44%	-15.04%
With	$N_A s_R c_R(p)$	1	-1.03%	-17.83%	-28.94%	-29.09%
With	$N_A s_R c_R(p)$	2	-1.01%	-17.5%	-29.42%	-28.73%

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Table A.10 – continued from previous page

Annual Abundance Percent Bias						
Aux. Like.	**Model**	**Year**	**No Variation**	**Low Variation**	**Medium Variation**	**High Variation**
With	$N_A s_R c_R(p)$	3	-0.99%	-17.53%	-28.96%	-28.68%
With	$N_A s_R c_R(p)$	4	-1.05%	-17.56%	-29.53%	-28.57%
With	$N_A s_R c_R(p)$	5	-1%	-17.02%	-28.71%	-28.72%
With	$N_A s_R c_R(p)$	6	-1%	-17.69%	-29.41%	-28.81%
With	$N_A s_R c_R(p)$	7	-0.97%	-17.87%	-28.96%	-29.98%
With	$N_A s_R c_R(p)$	8	-0.98%	-17.71%	-28.77%	-28.9%
With	$N_A s_R c_R(p)$	9	-0.91%	-17.55%	-28.67%	-29.06%
With	$N_A s_R c_R(p)$	10	-0.94%	-17.67%	-28.98%	-29.36%
With	$N_A s_R c_R(p)$	11	-0.94%	-17.84%	-29.58%	-29.79%
With	$N_A s_R c_R(p)$	12	-1.07%	-17.88%	-28.8%	-29.65%
With	$N_A s_R c_R(p)$	13	-1.05%	-18.09%	-28.4%	-27.9%
With	$N_A s_R c_R(p)$	14	-1.08%	-17.97%	-29.41%	-28.2%
With	$N_A s_R c_R(p)$	15	-1.03%	-17.45%	-28.29%	-28.94%
With	$N_A s_R c_R(p)$	16	-0.98%	-16.86%	-29.14%	-29.74%
With	$N_A s_R c_R(p)$	17	-1.05%	-17.6%	-29.1%	-28.35%
With	$N_A s_R c_R(p)$	18	-0.91%	-17.64%	-29.15%	-29.4%
With	$N_A s_R c_R(p)$	19	-1.04%	-17.26%	-28.8%	-28.8%
With	$N_A s_R c_R(p)$	20	-0.98%	-17.83%	-29.41%	-29.13%
With	$N_A s_R c_R(p)$	21	-1.04%	-17.86%	-28.33%	-29.42%
With	$N_A s_R c_R(p)$	22	-1.05%	-17.37%	-29.04%	-29.68%
With	$N_A s_R c_R(p)$	23	-0.94%	-17.73%	-29.28%	-30.32%
With	$N_A s_R c_R(p)$	24	-0.96%	-17.69%	-29.33%	-30.74%
With	$N_A s_R c_R(p)$	25	-0.91%	-17.73%	-29.8%	-31.33%

Table (A.11) contains the data used for plotting Figure (4.3), the median relative bias (MRB) and estimated asymptotic 95% confidence interval coverage for each model, for each year of data, for each level of simulated variation for small game models when a high level of simulated auxiliary data is available for estimating c. Table (A.12) contains the data used for plotting Figure (4.4), the median relative bias (MRB) and estimated asymptotic 95% confidence interval coverage for each model, for each year of data, for each level of simulated variation for small game models when a low level of simulated auxiliary data is available for estimating c.

Table A.11: *95% confidence interval coverage and median relative bias in total annual abundance estimates for small game models when a high amount of simulated auxiliary data is available for estimating c. Results indicate nearest nominal coverage for models employing the Horvitz-Thompson abundance estimator, although these remain subnominal (between 80% and 90%) for nonzero levels of simulated variation. All absolute-recruit abundance models show low confidence interval coverage (between 20% and 50%). Results based on* $n = 1000$ *replicates, with* $s \approx 0.50$, $c \approx -0.685$, $\gamma = 2.0$, *and total annual abundance ≈ 40,000. "Aux. Like." = Auxiliary catch-effort likelihood used (With) or not used (Without).*

Table A.11 - Annual Abundance 95% CI Coverage and Median Relative Bias										
			No Variation		Low Variation		Medium Variation		High Variation	
Aux. Like.	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
Without	$s_F c_F(p)$	1	93.70	-0.09	36.10	-0.27	27.50	-0.43	31.50	0.08
Without	$s_F c_F(p)$	2	94.80	-0.05	36.60	-0.59	27.70	-0.79	31.10	-1.38
Without	$s_F c_F(p)$	3	95.00	-0.06	34.60	-1.23	26.50	-0.09	31.40	-1.37
Without	$s_F c_F(p)$	4	94.50	0.01	34.80	-0.86	28.00	-1.35	31.00	-1.97
Without	$s_F c_F(p)$	5	95.40	0.04	34.70	-0.33	26.80	-0.56	34.20	-0.32
Without	$s_F c_F(p)$	6	94.00	-0.17	35.80	-0.52	27.60	-0.32	31.80	-2.66
Without	$s_F c_F(p)$	7	94.00	-0.05	35.50	-0.42	27.20	-0.52	31.70	0.75
Without	$s_F c_F(p)$	8	95.20	-0.03	36.00	-0.47	28.70	0.04	32.00	-2.12
Without	$s_F c_F(p)$	9	94.80	-0.05	35.30	-1.14	29.00	-1.22	35.60	-2.38
Without	$s_F c_F(p)$	10	94.20	0.07	32.30	-0.17	27.70	0.03	33.70	-1.60
Without	$s_F c_F(p)$	11	94.60	-0.01	33.90	-0.93	29.30	0.96	31.60	1.17
Without	$s_F c_F(p)$	12	95.20	0.02	34.30	-0.29	27.50	-1.42	33.70	-0.08
Without	$s_F c_F(p)$	13	94.80	-0.11	35.60	-1.00	31.60	0.18	34.80	-1.17
Without	$s_F c_F(p)$	14	94.60	-0.03	34.30	0.16	29.30	-0.30	36.90	1.90
Without	$s_F c_F(p)$	15	93.90	-0.04	35.20	-0.72	29.60	-0.85	32.00	-0.15
Without	$s_F c_F(p)$	16	94.70	0.03	37.20	-0.81	28.00	-0.61	33.10	-2.20
Without	$s_F c_F(p)$	17	94.90	0.04	36.80	0.08	30.30	-0.04	33.60	-0.63
Without	$s_F c_F(p)$	18	95.30	-0.00	36.60	-0.39	30.40	0.10	35.50	-2.12
Without	$s_F c_F(p)$	19	94.60	-0.08	33.70	-0.62	32.50	-0.02	33.90	-1.33
Without	$s_F c_F(p)$	20	94.20	0.02	36.60	-0.85	26.10	-0.87	34.10	-2.72
Without	$s_F c_F(p)$	21	95.30	-0.00	34.40	-0.33	28.60	-0.25	38.00	-0.29
Without	$s_F c_F(p)$	22	94.90	-0.09	36.10	-0.63	28.60	0.47	32.70	-1.29
Without	$s_F c_F(p)$	23	95.20	0.01	36.30	-0.18	28.80	0.28	35.60	-2.72
Without	$s_F c_F(p)$	24	94.50	0.04	35.40	-1.18	30.60	-0.30	34.90	-1.70
Without	$s_F c_F(p)$	25	94.10	0.02	36.50	-0.42	28.70	-0.30	36.20	-1.26
Without	$s_F c_R(p)$	1	93.90	-0.09	88.70	-0.02	92.50	0.45	91.00	1.29
Without	$s_F c_R(p)$	2	95.00	-0.05	89.70	-0.15	92.30	-0.21	92.90	1.01
Without	$s_F c_R(p)$	3	95.30	-0.05	90.80	-0.20	92.60	0.15	93.60	0.95

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Table A.11 – continued from previous page

			No Variation		Low Variation		Medium Variation		High Variation	
Auxiliary	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
Without	$s_F c_R(p)$	4	94.80	0.03	91.40	0.07	91.90	0.10	94.20	1.30
Without	$s_F c_R(p)$	5	95.60	0.05	91.70	0.01	91.90	-0.40	92.80	0.46
Without	$s_F c_R(p)$	6	94.20	-0.16	91.20	0.36	93.20	0.05	92.20	0.51
Without	$s_F c_R(p)$	7	94.00	-0.05	92.30	0.52	92.90	0.26	92.80	0.66
Without	$s_F c_R(p)$	8	95.60	-0.02	91.20	0.07	93.30	-0.19	93.40	0.38
Without	$s_F c_R(p)$	9	95.30	-0.03	91.20	-0.23	94.40	-0.07	95.90	-0.49
Without	$s_F c_R(p)$	10	94.30	0.05	91.20	0.22	95.00	0.17	93.40	0.18
Without	$s_F c_R(p)$	11	94.80	0.01	91.80	-0.07	94.00	0.17	95.30	-0.32
Without	$s_F c_R(p)$	12	95.40	0.04	92.70	-0.36	95.30	0.05	95.70	0.44
Without	$s_F c_R(p)$	13	95.40	-0.08	92.20	-0.16	95.90	-0.43	95.90	0.13
Without	$s_F c_R(p)$	14	95.40	-0.01	92.60	0.16	93.70	-0.74	96.00	1.43
Without	$s_F c_R(p)$	15	94.20	-0.04	91.00	-0.23	93.10	-0.42	94.70	-0.13
Without	$s_F c_R(p)$	16	95.20	0.02	92.20	0.27	93.80	-0.06	95.40	0.81
Without	$s_F c_R(p)$	17	95.20	0.03	91.30	-0.01	92.00	-0.26	93.80	1.35
Without	$s_F c_R(p)$	18	95.70	-0.01	91.80	0.50	91.70	-0.52	93.10	0.51
Without	$s_F c_R(p)$	19	95.10	-0.04	92.10	-0.16	92.80	-0.26	92.50	1.07
Without	$s_F c_R(p)$	20	94.50	0.01	90.70	0.12	91.90	-0.03	92.00	0.70
Without	$s_F c_R(p)$	21	95.60	-0.00	91.20	-0.31	92.00	-0.44	91.90	0.52
Without	$s_F c_R(p)$	22	95.10	-0.09	90.90	-0.57	91.20	-0.77	91.60	-0.29
Without	$s_F c_R(p)$	23	95.40	0.03	90.90	-0.40	92.30	-0.65	92.40	-1.28
Without	$s_F c_R(p)$	24	94.90	0.05	90.70	-0.55	92.80	-0.49	91.10	-1.13
Without	$s_F c_R(p)$	25	94.50	0.03	91.60	0.06	90.50	-0.95	91.80	-0.79
Without	$N_A s_F c_R(p)$	1	99.80	3.51	97.50	-8.93	49.90	-28.38	30.60	-37.39
Without	$N_A s_F c_R(p)$	2	99.60	8.55	97.70	-10.08	45.40	-55.80	24.80	-66.29
Without	$N_A s_F c_R(p)$	3	99.60	16.17	97.30	-15.54	43.60	-69.72	26.00	-74.97
Without	$N_A s_F c_R(p)$	4	99.50	23.38	97.00	-17.25	42.00	-75.16	26.90	-70.18
Without	$N_A s_F c_R(p)$	5	99.50	30.43	97.10	-19.43	41.90	-79.13	28.90	-50.07
Without	$N_A s_F c_R(p)$	6	99.50	34.65	97.20	-24.99	42.60	-81.37	30.80	-7.70
Without	$N_A s_F c_R(p)$	7	99.50	46.77	97.10	-26.00	43.50	-81.04	34.20	24.84
Without	$N_A s_F c_R(p)$	8	99.50	54.08	96.90	-25.54	45.60	-81.66	35.90	27.54
Without	$N_A s_F c_R(p)$	9	99.50	68.25	96.70	-29.97	45.90	-82.86	38.30	11.13
Without	$N_A s_F c_R(p)$	10	99.50	80.36	96.70	-34.83	49.20	-79.78	39.30	13.14
Without	$N_A s_F c_R(p)$	11	99.50	85.80	96.60	-38.89	52.40	-78.62	41.90	-1.35
Without	$N_A s_F c_R(p)$	12	99.50	101.00	96.80	-39.03	54.40	-80.81	43.00	8.24
Without	$N_A s_F c_R(p)$	13	99.50	111.59	96.60	-44.67	56.00	-81.19	43.10	3.98
Without	$N_A s_F c_R(p)$	14	99.40	113.55	96.40	-43.12	56.30	-81.88	45.50	15.36
Without	$N_A s_F c_R(p)$	15	99.40	131.55	96.40	-45.53	56.10	-82.14	47.00	22.95
Without	$N_A s_F c_R(p)$	16	99.30	150.21	96.70	-41.77	57.10	-84.44	49.00	12.60
Without	$N_A s_F c_R(p)$	17	99.10	167.05	96.80	-41.60	57.40	-86.98	50.00	-10.41
Without	$N_A s_F c_R(p)$	18	99.00	186.01	96.90	-42.10	58.30	-89.84	51.30	-19.70
Without	$N_A s_F c_R(p)$	19	99.10	207.06	97.10	-46.33	59.00	-90.08	52.90	-11.34
Without	$N_A s_F c_R(p)$	20	99.00	235.54	96.60	-49.46	60.20	-92.00	51.90	-40.36
Without	$N_A s_F c_R(p)$	21	98.90	254.13	96.20	-46.93	60.20	-92.94	47.60	-52.58
Without	$N_A s_F c_R(p)$	22	98.60	274.29	96.40	-41.12	59.60	-92.04	46.00	-54.91
Without	$N_A s_F c_R(p)$	23	98.20	284.63	95.90	-33.58	61.60	-89.99	45.90	-51.48
Without	$N_A s_F c_R(p)$	24	97.90	328.78	96.50	-25.79	60.40	-90.32	43.20	-32.18
Without	$N_A s_F c_R(p)$	25	98.50	109.38	96.40	-2.91	66.10	-14.03	52.80	6.65
Without	$s_R c_R(p)$	1	94.10	-0.09	89.60	0.61	90.10	-0.25	88.40	-1.67
Without	$s_R c_R(p)$	2	95.00	-0.05	90.50	0.39	90.50	-0.35	90.10	-1.21
Without	$s_R c_R(p)$	3	95.40	-0.05	91.30	0.27	90.10	0.61	88.30	-0.83
Without	$s_R c_R(p)$	4	94.70	0.03	90.80	0.55	91.10	-0.10	90.10	-0.33
Without	$s_R c_R(p)$	5	95.60	0.05	91.50	0.28	90.80	0.01	90.80	-0.47
Without	$s_R c_R(p)$	6	94.40	-0.15	91.40	0.71	90.90	-0.01	88.70	-1.64
Without	$s_R c_R(p)$	7	94.10	-0.04	92.60	0.77	89.80	-0.44	91.40	-0.67
Without	$s_R c_R(p)$	8	95.70	-0.02	92.60	0.45	91.20	-0.27	89.60	-1.01
Without	$s_R c_R(p)$	9	95.20	-0.03	92.40	0.19	93.30	-0.50	94.80	-1.54
Without	$s_R c_R(p)$	10	94.40	0.05	91.90	0.60	92.30	1.08	89.60	0.31
Without	$s_R c_R(p)$	11	94.80	0.02	93.30	0.41	92.50	0.52	91.40	-0.06
Without	$s_R c_R(p)$	12	95.50	-0.00	93.20	0.41	93.10	0.80	93.20	-0.25

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Table A.11 – continued from previous page

			No Variation		Low Variation		Medium Variation		High Variation	
Auxiliary	Model	Year	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB
Without	$s_R c_R(p)$	13	95.60	-0.08	92.60	0.44	93.60	0.08	91.30	0.46
Without	$s_R c_R(p)$	14	95.50	0.00	92.50	0.63	92.80	0.35	92.80	1.63
Without	$s_R c_R(p)$	15	94.20	-0.04	91.00	0.41	90.70	-0.01	88.30	-1.36
Without	$s_R c_R(p)$	16	95.50	0.02	92.20	0.79	91.60	-0.04	91.20	-1.25
Without	$s_R c_R(p)$	17	95.20	0.05	91.80	0.29	91.30	-0.33	89.60	-0.76
Without	$s_R c_R(p)$	18	95.70	0.01	91.60	0.73	90.20	-0.99	90.10	-1.71
Without	$s_R c_R(p)$	19	95.10	-0.02	91.80	0.41	92.00	0.19	90.60	-1.48
Without	$s_R c_R(p)$	20	94.70	0.02	91.00	0.55	90.90	-0.21	89.30	-1.55
Without	$s_R c_R(p)$	21	95.80	-0.00	92.20	0.00	90.80	-0.29	89.40	-1.27
Without	$s_R c_R(p)$	22	95.10	-0.09	91.00	-0.12	90.90	-0.64	90.50	-1.00
Without	$s_R c_R(p)$	23	95.50	0.03	90.70	0.24	91.20	0.18	89.20	-2.02
Without	$s_R c_R(p)$	24	94.90	0.05	91.00	-0.14	91.60	0.05	88.00	-1.65
Without	$s_R c_R(p)$	25	94.80	0.07	91.50	0.48	88.20	-0.43	87.20	-1.00
Without	$N_A s_R c_R(p)$	1	99.70	4.72	97.50	-6.78	50.20	-28.29	31.20	-37.42
Without	$N_A s_R c_R(p)$	2	99.60	10.60	97.60	-9.21	44.70	-55.85	24.20	-65.95
Without	$N_A s_R c_R(p)$	3	99.60	17.53	97.00	-11.50	43.40	-69.26	25.90	-75.12
Without	$N_A s_R c_R(p)$	4	99.60	27.46	96.60	-11.19	41.30	-75.05	26.80	-70.89
Without	$N_A s_R c_R(p)$	5	99.60	35.54	97.00	-12.31	41.40	-77.91	28.20	-51.40
Without	$N_A s_R c_R(p)$	6	99.60	43.77	96.90	-16.54	42.20	-80.99	30.80	-11.57
Without	$N_A s_R c_R(p)$	7	99.60	57.79	96.90	-19.65	43.70	-80.28	34.40	20.42
Without	$N_A s_R c_R(p)$	8	99.40	66.52	96.60	-20.14	45.40	-81.32	34.70	26.93
Without	$N_A s_R c_R(p)$	9	99.40	76.78	96.50	-23.62	46.20	-82.90	37.10	-0.45
Without	$N_A s_R c_R(p)$	10	99.30	86.97	96.90	-17.78	48.60	-81.22	38.90	11.44
Without	$N_A s_R c_R(p)$	11	99.20	97.33	96.80	-17.83	51.70	-76.52	41.90	-1.99
Without	$N_A s_R c_R(p)$	12	99.20	118.27	96.50	-20.21	54.50	-79.05	42.90	5.74
Without	$N_A s_R c_R(p)$	13	99.20	129.25	96.30	-25.30	56.40	-79.18	43.50	-6.58
Without	$N_A s_R c_R(p)$	14	99.20	146.28	96.30	-24.20	56.50	-81.66	46.40	11.87
Without	$N_A s_R c_R(p)$	15	99.10	167.32	96.70	-25.40	57.40	-82.51	46.90	17.76
Without	$N_A s_R c_R(p)$	16	99.10	185.49	96.90	-22.49	58.90	-84.14	49.00	3.43
Without	$N_A s_R c_R(p)$	17	99.00	206.00	96.70	-15.31	59.00	-85.00	50.80	-17.59
Without	$N_A s_R c_R(p)$	18	99.00	232.65	96.90	-11.81	60.40	-88.33	52.40	-16.93
Without	$N_A s_R c_R(p)$	19	98.90	269.32	96.90	-11.69	61.30	-89.63	54.20	-12.07
Without	$N_A s_R c_R(p)$	20	98.40	297.14	96.60	-13.64	62.70	-91.73	53.30	-42.22
Without	$N_A s_R c_R(p)$	21	98.20	319.71	96.60	-13.86	62.70	-92.85	48.10	-55.85
Without	$N_A s_R c_R(p)$	22	98.10	354.43	96.80	-13.71	63.40	-91.85	46.10	-57.75
Without	$N_A s_R c_R(p)$	23	98.00	385.87	96.60	2.13	64.50	-88.43	46.30	-49.26
Without	$N_A s_R c_R(p)$	24	97.90	412.95	96.50	13.51	63.60	-89.58	44.20	-31.98
Without	$N_A s_R c_R(p)$	25	98.00	130.37	96.60	8.72	68.00	-13.32	53.90	7.68
With	$s_F c_F(p)$	1	76.70	-0.83	21.70	10.12	14.80	45.14	16.70	67.06
With	$s_F c_F(p)$	2	76.60	-0.82	23.00	9.39	15.40	45.88	17.00	68.68
With	$s_F c_F(p)$	3	76.80	-0.86	25.20	8.55	13.00	46.87	17.20	67.27
With	$s_F c_F(p)$	4	78.30	-0.79	23.70	9.10	16.00	44.81	17.40	69.63
With	$s_F c_F(p)$	5	78.00	-0.80	22.00	9.44	14.40	45.45	16.40	69.25
With	$s_F c_F(p)$	6	77.50	-0.79	23.90	9.12	15.80	46.40	17.50	67.31
With	$s_F c_F(p)$	7	77.40	-0.83	22.00	10.05	16.40	46.82	16.10	70.34
With	$s_F c_F(p)$	8	77.40	-0.77	23.00	9.49	15.70	47.63	16.90	70.11
With	$s_F c_F(p)$	9	77.30	-0.76	22.70	9.70	17.30	45.31	18.00	64.87
With	$s_F c_F(p)$	10	77.40	-0.72	22.40	10.24	14.70	47.54	17.50	70.49
With	$s_F c_F(p)$	11	77.20	-0.69	24.10	8.33	16.00	46.73	16.80	71.93
With	$s_F c_F(p)$	12	77.70	-0.80	25.10	9.21	16.60	45.94	18.80	71.20
With	$s_F c_F(p)$	13	78.00	-0.84	25.50	8.39	16.30	44.03	16.50	67.78
With	$s_F c_F(p)$	14	77.20	-0.73	23.10	9.46	17.30	48.57	17.60	75.96
With	$s_F c_F(p)$	15	77.00	-0.85	24.00	9.17	17.60	43.72	18.00	69.47
With	$s_F c_F(p)$	16	77.10	-0.78	23.10	9.75	16.70	44.82	18.40	67.94
With	$s_F c_F(p)$	17	77.80	-0.77	24.10	9.63	17.70	47.62	19.20	69.80
With	$s_F c_F(p)$	18	78.40	-0.63	24.60	9.31	17.60	44.95	20.40	66.98
With	$s_F c_F(p)$	19	77.60	-0.77	22.20	9.54	16.10	46.19	18.70	67.36
With	$s_F c_F(p)$	20	77.10	-0.63	23.60	9.00	18.20	44.70	20.90	67.37
With	$s_F c_F(p)$	21	77.30	-0.73	23.20	9.38	17.80	46.64	19.90	69.26

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Table A.11 – continued from previous page

			No Variation		Low Variation		Medium Variation		High Variation	
Auxiliary	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
With	$s_F c_F(p)$	22	77.50	-0.71	24.00	9.06	18.30	47.57	18.70	69.51
With	$s_F c_F(p)$	23	78.40	-0.71	24.40	9.41	16.00	49.09	19.10	66.94
With	$s_F c_F(p)$	24	78.10	-0.77	24.20	8.42	17.40	48.00	18.20	73.59
With	$s_F c_F(p)$	25	76.20	-0.65	24.30	9.02	17.20	48.10	21.40	72.18
With	$N_A s_F c_F(p)$	1	90.40	-0.85	34.50	-0.34	25.50	1.90	26.40	5.70
With	$N_A s_F c_F(p)$	2	90.20	-0.78	32.80	-0.12	24.00	2.98	28.10	4.94
With	$N_A s_F c_F(p)$	3	91.20	-0.76	32.60	-1.08	24.90	2.22	28.30	5.74
With	$N_A s_F c_F(p)$	4	90.80	-0.72	34.80	-0.36	26.00	1.57	26.00	6.66
With	$N_A s_F c_F(p)$	5	92.10	-0.79	29.50	-0.65	23.80	2.99	30.60	7.89
With	$N_A s_F c_F(p)$	6	90.60	-0.93	31.50	-0.74	24.60	2.36	29.50	4.64
With	$N_A s_F c_F(p)$	7	90.90	-0.77	33.70	-0.38	24.20	1.98	27.60	6.74
With	$N_A s_F c_F(p)$	8	91.50	-0.67	34.30	-0.10	24.90	4.34	29.70	4.85
With	$N_A s_F c_F(p)$	9	91.20	-0.72	33.70	-0.92	25.80	2.26	28.90	4.46
With	$N_A s_F c_F(p)$	10	91.30	-0.74	31.50	-0.20	24.50	2.82	29.20	5.00
With	$N_A s_F c_F(p)$	11	92.40	-0.66	32.70	-1.11	25.50	2.37	27.40	7.04
With	$N_A s_F c_F(p)$	12	91.70	-0.67	30.20	0.05	26.00	2.24	29.90	6.20
With	$N_A s_F c_F(p)$	13	92.60	-0.79	33.20	-0.79	26.70	2.62	28.90	6.43
With	$N_A s_F c_F(p)$	14	91.70	-0.73	34.50	0.21	24.50	2.59	31.40	7.97
With	$N_A s_F c_F(p)$	15	90.40	-0.78	32.20	-0.43	27.90	1.93	31.00	7.65
With	$N_A s_F c_F(p)$	16	90.60	-0.63	34.70	-1.05	25.60	1.37	29.40	5.19
With	$N_A s_F c_F(p)$	17	90.70	-0.78	34.50	0.33	26.40	2.73	31.10	6.10
With	$N_A s_F c_F(p)$	18	91.20	-0.71	35.30	-0.54	25.60	2.33	30.50	5.51
With	$N_A s_F c_F(p)$	19	91.70	-0.83	31.20	-0.55	26.00	3.72	31.80	4.90
With	$N_A s_F c_F(p)$	20	91.60	-0.68	32.30	-0.60	23.90	1.55	31.10	4.23
With	$N_A s_F c_F(p)$	21	91.80	-0.70	33.50	-0.72	25.00	3.14	34.30	6.93
With	$N_A s_F c_F(p)$	22	91.20	-0.81	32.90	-0.44	23.90	3.46	30.30	6.88
With	$N_A s_F c_F(p)$	23	90.80	-0.67	33.00	-0.01	25.70	3.85	31.90	4.85
With	$N_A s_F c_F(p)$	24	91.40	-0.64	32.90	-1.09	27.10	3.23	31.90	4.63
With	$N_A s_F c_F(p)$	25	89.60	-0.72	31.70	-1.20	26.10	3.76	29.50	6.16
With	$s_F c_R(p)$	1	90.50	-0.55	85.40	-3.07	88.60	-3.46	88.00	-2.45
With	$s_F c_R(p)$	2	90.50	-0.45	86.40	-2.91	87.80	-3.49	90.50	-2.19
With	$s_F c_R(p)$	3	91.10	-0.43	85.90	-3.08	87.40	-3.38	90.70	-2.48
With	$s_F c_R(p)$	4	91.30	-0.46	87.60	-2.89	87.90	-3.14	90.90	-2.16
With	$s_F c_R(p)$	5	90.30	-0.48	87.90	-2.77	88.30	-3.64	89.40	-3.12
With	$s_F c_R(p)$	6	91.20	-0.54	87.10	-2.56	89.00	-3.37	89.40	-3.38
With	$s_F c_R(p)$	7	90.90	-0.52	87.40	-2.59	90.60	-3.55	90.30	-2.25
With	$s_F c_R(p)$	8	92.40	-0.42	89.00	-2.83	90.60	-3.38	90.30	-3.03
With	$s_F c_R(p)$	9	91.40	-0.50	87.50	-2.95	91.30	-3.43	91.50	-3.33
With	$s_F c_R(p)$	10	91.40	-0.35	87.80	-2.57	91.20	-3.06	92.10	-3.16
With	$s_F c_R(p)$	11	91.00	-0.45	88.30	-2.85	90.40	-3.08	93.60	-3.04
With	$s_F c_R(p)$	12	92.00	-0.47	89.30	-3.29	92.50	-3.26	93.40	-2.97
With	$s_F c_R(p)$	13	91.70	-0.47	88.80	-2.87	92.40	-3.69	92.90	-3.03
With	$s_F c_R(p)$	14	91.40	-0.44	88.70	-2.90	90.40	-4.07	93.40	-2.01
With	$s_F c_R(p)$	15	91.40	-0.55	87.60	-2.90	90.40	-3.84	91.80	-3.05
With	$s_F c_R(p)$	16	91.30	-0.35	88.40	-2.64	90.30	-3.73	92.20	-2.48
With	$s_F c_R(p)$	17	91.70	-0.43	87.60	-3.06	88.50	-3.80	91.10	-2.35
With	$s_F c_R(p)$	18	92.00	-0.40	86.90	-2.65	88.10	-4.18	89.10	-3.03
With	$s_F c_R(p)$	19	90.80	-0.45	88.50	-2.96	89.60	-3.70	88.90	-2.63
With	$s_F c_R(p)$	20	91.30	-0.42	88.60	-2.87	86.40	-3.84	88.70	-3.42
With	$s_F c_R(p)$	21	91.60	-0.50	87.90	-3.33	89.00	-4.07	88.70	-3.35
With	$s_F c_R(p)$	22	91.50	-0.50	87.50	-3.49	87.70	-4.46	89.00	-4.41
With	$s_F c_R(p)$	23	91.50	-0.38	87.60	-3.28	87.50	-4.66	88.70	-5.92
With	$s_F c_R(p)$	24	90.90	-0.40	87.80	-3.46	89.00	-5.03	87.00	-6.35
With	$s_F c_R(p)$	25	90.40	-0.41	89.70	-3.29	86.90	-5.88	86.90	-7.98
With	$N_A s_F c_R(p)$	1	90.40	-0.85	68.10	-8.45	61.10	-10.67	62.70	-9.86
With	$N_A s_F c_R(p)$	2	90.20	-0.78	69.00	-8.18	62.90	-10.88	68.40	-9.64
With	$N_A s_F c_R(p)$	3	91.20	-0.76	69.80	-8.21	64.10	-10.55	68.10	-10.04
With	$N_A s_F c_R(p)$	4	90.80	-0.72	70.20	-8.09	63.90	-10.91	67.70	-10.33
With	$N_A s_F c_R(p)$	5	92.10	-0.79	70.30	-8.06	63.40	-10.88	68.00	-10.96

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Table A.11 – continued from previous page

			No Variation		Low Variation		Medium Variation		High Variation	
Auxiliary	Model	Year	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB
With	$N_A s_F c_R(p)$	6	90.60	-0.93	69.30	-7.98	65.80	-11.00	68.00	-10.72
With	$N_A s_F c_R(p)$	7	90.90	-0.77	69.10	-8.07	66.90	-10.86	66.60	-10.77
With	$N_A s_F c_R(p)$	8	91.50	-0.67	69.70	-8.03	65.20	-10.38	67.40	-10.61
With	$N_A s_F c_R(p)$	9	91.20	-0.72	71.40	-8.08	67.40	-10.32	67.60	-11.44
With	$N_A s_F c_R(p)$	10	91.30	-0.74	70.60	-7.66	68.60	-10.09	68.70	-10.28
With	$N_A s_F c_R(p)$	11	92.40	-0.66	70.90	-7.91	69.60	-9.94	71.20	-10.47
With	$N_A s_F c_R(p)$	12	91.70	-0.67	72.20	-8.08	68.50	-10.23	72.00	-10.38
With	$N_A s_F c_R(p)$	13	92.60	-0.79	72.50	-7.92	67.50	-10.58	71.60	-10.44
With	$N_A s_F c_R(p)$	14	91.70	-0.73	71.60	-7.98	66.40	-11.24	74.80	-9.32
With	$N_A s_F c_R(p)$	15	90.40	-0.78	70.60	-8.21	66.50	-10.91	71.70	-10.27
With	$N_A s_F c_R(p)$	16	90.60	-0.63	70.30	-7.89	66.40	-10.60	72.40	-10.56
With	$N_A s_F c_R(p)$	17	90.70	-0.78	71.40	-8.25	67.40	-10.40	71.30	-10.37
With	$N_A s_F c_R(p)$	18	91.20	-0.71	70.50	-8.00	65.40	-11.06	69.70	-11.28
With	$N_A s_F c_R(p)$	19	91.70	-0.83	69.90	-8.25	67.80	-10.69	69.90	-10.83
With	$N_A s_F c_R(p)$	20	91.60	-0.68	69.00	-7.99	66.60	-11.00	69.70	-11.17
With	$N_A s_F c_R(p)$	21	91.80	-0.70	71.00	-8.47	66.90	-11.02	70.30	-11.13
With	$N_A s_F c_R(p)$	22	91.20	-0.81	71.40	-8.60	65.90	-11.52	69.90	-11.99
With	$N_A s_F c_R(p)$	23	90.80	-0.67	72.00	-8.46	67.20	-11.26	68.00	-13.63
With	$N_A s_F c_R(p)$	24	91.40	-0.64	73.00	-8.67	69.10	-11.72	67.30	-13.61
With	$N_A s_F c_R(p)$	25	89.60	-0.72	70.90	-8.80	67.50	-12.46	66.60	-14.94
With	$s_R c_R(p)$	1	90.80	-0.60	86.20	-2.47	84.90	-4.97	83.40	-6.88
With	$s_R c_R(p)$	2	91.50	-0.49	87.20	-2.62	85.20	-4.56	84.50	-6.48
With	$s_R c_R(p)$	3	92.30	-0.55	86.50	-2.85	85.10	-3.69	84.50	-6.35
With	$s_R c_R(p)$	4	91.90	-0.43	88.30	-2.63	86.00	-4.27	83.90	-6.16
With	$s_R c_R(p)$	5	91.60	-0.56	88.80	-2.74	85.30	-4.32	85.30	-5.93
With	$s_R c_R(p)$	6	91.70	-0.61	87.60	-2.58	85.90	-4.58	86.20	-6.84
With	$s_R c_R(p)$	7	91.70	-0.53	89.00	-2.43	85.90	-4.72	85.90	-6.54
With	$s_R c_R(p)$	8	92.70	-0.53	89.60	-2.76	88.10	-4.58	85.70	-6.31
With	$s_R c_R(p)$	9	92.10	-0.53	89.30	-2.71	90.00	-4.39	90.60	-6.52
With	$s_R c_R(p)$	10	91.60	-0.36	88.70	-2.27	88.60	-2.62	88.50	-4.05
With	$s_R c_R(p)$	11	91.90	-0.45	89.20	-2.33	89.10	-3.07	90.20	-3.84
With	$s_R c_R(p)$	12	93.10	-0.49	89.90	-2.54	90.20	-2.73	89.70	-4.16
With	$s_R c_R(p)$	13	92.40	-0.43	89.60	-2.51	91.20	-3.48	88.90	-3.12
With	$s_R c_R(p)$	14	92.20	-0.47	88.90	-2.30	89.70	-3.29	89.80	-2.63
With	$s_R c_R(p)$	15	91.70	-0.54	87.70	-2.72	87.70	-3.39	85.20	-5.44
With	$s_R c_R(p)$	16	92.10	-0.44	89.00	-2.41	87.10	-4.15	86.00	-6.66
With	$s_R c_R(p)$	17	92.30	-0.54	88.80	-3.01	86.10	-4.49	86.40	-6.42
With	$s_R c_R(p)$	18	93.20	-0.40	88.00	-2.40	87.00	-5.13	85.70	-7.11
With	$s_R c_R(p)$	19	91.00	-0.55	88.80	-2.75	86.80	-4.43	85.40	-7.35
With	$s_R c_R(p)$	20	92.30	-0.47	89.20	-2.52	86.70	-4.42	84.20	-7.11
With	$s_R c_R(p)$	21	92.90	-0.52	88.70	-3.06	86.70	-4.54	84.30	-6.61
With	$s_R c_R(p)$	22	92.00	-0.52	87.90	-3.23	86.90	-5.14	84.40	-6.84
With	$s_R c_R(p)$	23	92.00	-0.47	87.50	-2.90	86.30	-4.36	83.90	-8.12
With	$s_R c_R(p)$	24	91.90	-0.47	88.30	-3.15	86.80	-4.67	83.80	-7.78
With	$s_R c_R(p)$	25	90.80	-0.46	89.00	-2.85	83.60	-5.55	82.20	-8.07
With	$N_A s_R c_R(p)$	1	90.40	-0.85	68.20	-8.29	59.60	-9.66	58.20	-8.18
With	$N_A s_R c_R(p)$	2	90.40	-0.78	69.90	-7.88	64.40	-9.91	66.60	-7.91
With	$N_A s_R c_R(p)$	3	91.30	-0.76	71.10	-8.10	65.80	-9.14	66.60	-9.30
With	$N_A s_R c_R(p)$	4	91.00	-0.73	70.90	-7.76	64.70	-9.53	67.10	-8.75
With	$N_A s_R c_R(p)$	5	92.30	-0.79	70.60	-7.85	64.20	-9.79	67.60	-9.52
With	$N_A s_R c_R(p)$	6	90.60	-0.93	70.40	-7.89	66.70	-10.19	67.50	-9.49
With	$N_A s_R c_R(p)$	7	91.00	-0.77	70.10	-7.72	66.70	-9.81	66.50	-9.75
With	$N_A s_R c_R(p)$	8	91.50	-0.67	70.80	-7.94	66.80	-9.12	66.40	-9.65
With	$N_A s_R c_R(p)$	9	91.40	-0.72	72.20	-7.81	67.60	-9.58	68.40	-10.21
With	$N_A s_R c_R(p)$	10	91.40	-0.74	71.60	-7.47	68.70	-8.97	69.30	-9.13
With	$N_A s_R c_R(p)$	11	92.50	-0.66	72.10	-7.64	70.00	-9.38	70.90	-8.54
With	$N_A s_R c_R(p)$	12	91.70	-0.67	73.30	-7.85	69.10	-9.12	72.20	-8.53
With	$N_A s_R c_R(p)$	13	92.60	-0.79	72.70	-7.68	68.70	-9.52	71.20	-9.06
With	$N_A s_R c_R(p)$	14	91.80	-0.73	73.00	-7.65	68.40	-9.73	74.40	-7.58

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Table A.11 – continued from previous page

			No Variation		Low Variation		Medium Variation		High Variation	
Auxiliary	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
With	$N_A s_R c_R(p)$	15	90.70	-0.78	71.30	-7.97	67.60	-9.92	71.30	-8.69
With	$N_A s_R c_R(p)$	16	90.70	-0.63	71.20	-7.74	67.80	-9.55	71.00	-9.76
With	$N_A s_R c_R(p)$	17	90.80	-0.78	72.20	-8.07	68.50	-9.48	70.40	-9.27
With	$N_A s_R c_R(p)$	18	91.30	-0.71	71.00	-7.63	65.30	-9.84	70.20	-9.37
With	$N_A s_R c_R(p)$	19	91.80	-0.83	71.10	-8.05	67.50	-9.42	70.30	-9.37
With	$N_A s_R c_R(p)$	20	91.70	-0.68	70.20	-7.71	67.50	-9.58	69.80	-9.94
With	$N_A s_R c_R(p)$	21	91.80	-0.70	71.80	-8.07	68.20	-9.75	69.60	-9.74
With	$N_A s_R c_R(p)$	22	91.30	-0.82	72.70	-8.26	67.20	-10.04	69.20	-10.60
With	$N_A s_R c_R(p)$	23	90.90	-0.67	72.20	-8.16	67.80	-10.11	68.30	-11.98
With	$N_A s_R c_R(p)$	24	91.50	-0.64	73.30	-8.41	70.30	-10.53	68.90	-11.73
With	$N_A s_R c_R(p)$	25	89.70	-0.71	72.10	-8.40	68.30	-10.54	65.80	-12.69

Table A.12: *95% confidence interval coverage and median relative bias in total annual abundance estimates for small game models when a low amount of simulated auxiliary data is available for estimating c. Results indicate nearest nominal coverage for models employing the Horvitz-Thompson abundance estimator, although these remain subnominal (between 80% and 90%) for nonzero levels of simulated variation. All absolute-recruit abundance models show low confidence interval coverage (between 20% and 50%). Results based on* $n = 1000$ *replicates, with* $s \approx 0.50$, $c \approx -0.685$, $\gamma = 2.0$, *and total annual abundance* $\approx$ *40,000. "Aux. Like." = Auxiliary catch-effort likelihood used (With) or not used (Without).*

Table A.12 - Annual Abundance 95% CI Coverage and Median Relative Bias

			No Variation		Low Variation		Medium Variation		High Variation	
Aux. Like.	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
Without	$s_F c_F(p)$	1	93.60	-0.29	35.20	-1.10	28.10	-1.77	33.60	-0.76
Without	$s_F c_F(p)$	2	93.60	-0.26	34.10	-0.25	27.80	-1.21	35.10	1.10
Without	$s_F c_F(p)$	3	94.00	-0.28	36.00	-0.55	27.10	-1.76	35.40	-1.39
Without	$s_F c_F(p)$	4	94.50	-0.26	35.60	-0.07	27.60	-1.32	33.90	-0.60
Without	$s_F c_F(p)$	5	94.50	-0.20	32.50	-0.66	28.30	-0.82	36.10	0.02
Without	$s_F c_F(p)$	6	94.70	-0.24	34.30	-0.35	27.30	-2.13	35.10	-0.03
Without	$s_F c_F(p)$	7	94.10	-0.23	36.40	-0.62	27.00	-2.59	33.60	0.36
Without	$s_F c_F(p)$	8	94.50	-0.21	34.60	-0.23	30.10	-0.01	35.30	0.86
Without	$s_F c_F(p)$	9	93.80	-0.14	33.10	-1.10	27.40	-2.10	33.00	-3.33
Without	$s_F c_F(p)$	10	94.40	-0.19	32.90	-0.37	26.00	-1.13	35.10	-1.47
Without	$s_F c_F(p)$	11	94.20	-0.26	33.00	-1.00	29.30	0.56	33.00	0.07
Without	$s_F c_F(p)$	12	94.10	-0.26	34.10	-0.93	28.10	-2.79	36.70	-2.75
Without	$s_F c_F(p)$	13	94.90	-0.23	34.50	-0.91	30.10	-0.59	35.90	-0.25
Without	$s_F c_F(p)$	14	94.60	-0.30	36.00	-0.21	27.30	-2.35	35.40	0.89
Without	$s_F c_F(p)$	15	94.50	-0.21	35.40	-1.43	28.70	-2.15	34.10	0.02
Without	$s_F c_F(p)$	16	94.10	-0.19	36.00	-0.34	28.10	-3.16	35.20	-3.35
Without	$s_F c_F(p)$	17	94.20	-0.18	36.40	-0.28	28.60	-0.63	36.80	0.33
Without	$s_F c_F(p)$	18	94.00	-0.19	35.30	-1.20	29.10	-1.80	36.00	-0.58
Without	$s_F c_F(p)$	19	94.20	-0.29	34.50	-0.88	30.80	-1.42	35.90	-0.19
Without	$s_F c_F(p)$	20	94.40	-0.23	37.30	-0.86	27.10	-1.20	36.80	0.53
Without	$s_F c_F(p)$	21	94.40	-0.24	34.00	-0.48	27.70	-0.92	37.90	-0.26
Without	$s_F c_F(p)$	22	94.20	-0.30	34.60	-1.02	29.90	-0.63	37.20	-0.69
Without	$s_F c_F(p)$	23	94.60	-0.14	35.00	-0.92	28.90	-1.21	36.20	-0.95
Without	$s_F c_F(p)$	24	94.30	-0.22	34.90	-0.84	29.90	-1.21	37.70	-0.53

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Table A.12 – continued from previous page

			No Variation		Low Variation		Medium Variation		High Variation	
Auxiliary	Model	Year	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB
Without	$s_F c_F(p)$	25	94.00	-0.21	35.30	0.02	29.90	-0.70	38.00	0.95
Without	$s_F c_R(p)$	1	93.70	-0.29	85.80	-0.97	82.70	0.16	84.50	6.09
Without	$s_F c_R(p)$	2	93.90	-0.25	85.50	-1.56	83.90	-0.38	85.50	6.74
Without	$s_F c_R(p)$	3	94.50	-0.27	87.10	-1.39	83.30	0.73	84.80	6.27
Without	$s_F c_R(p)$	4	94.50	-0.23	85.90	-1.48	83.10	-0.63	84.30	6.37
Without	$s_F c_R(p)$	5	94.70	-0.19	86.10	-1.74	83.90	-0.72	83.00	7.22
Without	$s_F c_R(p)$	6	94.90	-0.25	86.00	-1.57	83.40	-0.67	83.00	7.66
Without	$s_F c_R(p)$	7	94.10	-0.23	86.10	-1.15	85.10	-0.37	84.60	5.66
Without	$s_F c_R(p)$	8	94.80	-0.21	88.00	-1.27	84.00	0.59	84.30	6.39
Without	$s_F c_R(p)$	9	94.20	-0.13	87.00	-1.27	84.10	-0.76	84.50	5.22
Without	$s_F c_R(p)$	10	94.70	-0.18	86.60	-1.32	84.20	-0.38	86.00	5.84
Without	$s_F c_R(p)$	11	94.60	-0.24	87.00	-1.37	83.90	0.18	86.30	7.54
Without	$s_F c_R(p)$	12	94.40	-0.25	86.50	-1.25	84.20	0.42	85.80	6.66
Without	$s_F c_R(p)$	13	95.00	-0.22	86.90	-1.16	83.40	-0.75	83.70	4.93
Without	$s_F c_R(p)$	14	94.80	-0.29	87.50	-0.67	82.80	-0.47	84.90	5.99
Without	$s_F c_R(p)$	15	94.90	-0.21	88.20	-1.30	83.50	0.01	85.30	6.26
Without	$s_F c_R(p)$	16	94.30	-0.19	87.60	-1.35	83.70	-0.48	83.20	7.15
Without	$s_F c_R(p)$	17	94.40	-0.18	87.70	-1.66	83.10	-0.44	83.80	6.38
Without	$s_F c_R(p)$	18	94.10	-0.14	87.40	-1.80	83.60	-0.75	83.90	6.39
Without	$s_F c_R(p)$	19	94.60	-0.28	86.50	-1.10	83.90	-0.75	83.80	6.23
Without	$s_F c_R(p)$	20	94.60	-0.22	86.10	-1.47	82.20	-0.94	84.80	6.61
Without	$s_F c_R(p)$	21	94.70	-0.23	86.50	-1.49	83.60	-0.40	84.40	5.78
Without	$s_F c_R(p)$	22	94.40	-0.28	88.10	-1.65	83.20	-0.22	84.20	5.79
Without	$s_F c_R(p)$	23	95.10	-0.13	86.70	-1.31	83.10	-1.26	83.70	5.25
Without	$s_F c_R(p)$	24	94.70	-0.19	86.60	-0.93	84.50	-0.01	84.50	5.34
Without	$s_F c_R(p)$	25	94.30	-0.20	88.50	-0.72	84.50	-0.22	85.10	4.48
Without	$N_A s_F c_R(p)$	1	93.60	-1.44	93.30	-13.15	57.80	-26.01	43.90	-30.47
Without	$N_A s_F c_R(p)$	2	93.80	-5.10	92.70	-19.01	48.20	-50.83	30.80	-58.89
Without	$N_A s_F c_R(p)$	3	93.80	-13.77	92.50	-26.91	43.40	-63.22	26.20	-72.43
Without	$N_A s_F c_R(p)$	4	93.80	-16.00	92.10	-30.40	41.40	-71.65	26.90	-69.46
Without	$N_A s_F c_R(p)$	5	93.30	-20.72	92.30	-32.63	40.70	-72.58	28.00	-56.84
Without	$N_A s_F c_R(p)$	6	92.90	-23.97	92.00	-39.31	40.70	-78.14	29.10	-41.36
Without	$N_A s_F c_R(p)$	7	93.10	-29.48	91.80	-40.98	42.70	-78.76	33.40	-2.97
Without	$N_A s_F c_R(p)$	8	93.10	-32.35	91.90	-47.04	43.90	-79.56	36.60	38.56
Without	$N_A s_F c_R(p)$	9	92.20	-35.49	91.20	-50.84	44.70	-80.89	40.00	24.20
Without	$N_A s_F c_R(p)$	10	92.00	-44.15	91.70	-52.81	45.70	-79.00	41.50	26.75
Without	$N_A s_F c_R(p)$	11	91.50	-48.01	91.20	-57.81	49.00	-80.34	42.30	31.35
Without	$N_A s_F c_R(p)$	12	91.20	-53.02	90.70	-60.42	51.20	-83.31	44.90	8.48
Without	$N_A s_F c_R(p)$	13	90.60	-54.90	91.00	-63.40	53.70	-79.08	44.40	5.01
Without	$N_A s_F c_R(p)$	14	89.90	-54.48	90.60	-65.74	54.40	-78.93	47.10	27.65
Without	$N_A s_F c_R(p)$	15	89.40	-58.37	90.70	-69.70	53.70	-82.22	49.30	27.29
Without	$N_A s_F c_R(p)$	16	88.80	-59.39	90.60	-69.07	55.20	-84.36	51.50	13.67
Without	$N_A s_F c_R(p)$	17	87.90	-62.20	91.00	-66.81	56.30	-85.28	50.70	-9.75
Without	$N_A s_F c_R(p)$	18	87.50	-64.10	89.90	-70.69	56.70	-88.28	52.10	-20.15
Without	$N_A s_F c_R(p)$	19	86.40	-64.95	89.40	-74.08	57.60	-89.04	51.80	-4.38
Without	$N_A s_F c_R(p)$	20	85.80	-67.98	89.50	-74.68	58.90	-91.04	51.60	-25.74
Without	$N_A s_F c_R(p)$	21	84.30	-70.38	89.30	-77.99	58.60	-90.96	48.20	-36.25
Without	$N_A s_F c_R(p)$	22	83.30	-69.49	89.20	-79.74	60.20	-90.36	45.40	-20.54
Without	$N_A s_F c_R(p)$	23	82.90	-69.87	89.90	-79.18	61.40	-85.63	44.80	-27.83
Without	$N_A s_F c_R(p)$	24	81.70	-71.14	89.30	-76.30	58.70	-84.69	45.10	10.91
Without	$N_A s_F c_R(p)$	25	82.80	5.55	89.70	-2.77	68.70	5.89	62.30	66.83
Without	$s_R c_R(p)$	1	93.70	-0.29	86.90	-0.09	84.20	2.40	85.10	8.46
Without	$s_R c_R(p)$	2	94.10	-0.25	86.90	-0.30	85.50	2.31	86.50	7.35
Without	$s_R c_R(p)$	3	94.50	-0.26	87.90	0.11	85.10	2.04	86.10	6.45
Without	$s_R c_R(p)$	4	94.70	-0.23	87.20	-0.06	85.00	1.45	85.00	7.53
Without	$s_R c_R(p)$	5	94.80	-0.17	87.40	-0.32	86.00	-0.11	83.90	9.11
Without	$s_R c_R(p)$	6	94.90	-0.25	87.30	-0.11	85.60	0.56	83.70	8.40
Without	$s_R c_R(p)$	7	94.20	-0.24	87.60	0.82	86.30	1.55	85.30	6.50
Without	$s_R c_R(p)$	8	94.80	-0.22	89.50	0.03	85.70	2.80	85.10	7.75

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Table A.12 – continued from previous page

			No Variation		Low Variation		Medium Variation		High Variation	
Auxiliary	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
Without	$s_R c_R(p)$	9	94.30	-0.13	88.00	-0.20	85.80	1.35	85.10	6.82
Without	$s_R c_R(p)$	10	94.70	-0.19	87.10	-0.23	85.70	2.64	86.50	7.69
Without	$s_R c_R(p)$	11	94.60	-0.26	87.70	-0.61	85.40	2.37	87.50	9.22
Without	$s_R c_R(p)$	12	94.70	-0.24	87.90	0.06	85.80	1.75	86.50	8.48
Without	$s_R c_R(p)$	13	95.00	-0.23	87.50	0.03	84.50	1.61	83.90	8.09
Without	$s_R c_R(p)$	14	94.90	-0.30	88.50	-0.06	84.30	1.05	85.50	8.21
Without	$s_R c_R(p)$	15	94.90	-0.22	89.40	-0.21	84.90	1.40	86.10	8.47
Without	$s_R c_R(p)$	16	94.60	-0.18	89.20	0.08	85.10	1.52	83.70	8.73
Without	$s_R c_R(p)$	17	94.60	-0.18	89.60	-0.41	85.30	1.72	84.30	8.00
Without	$s_R c_R(p)$	18	94.30	-0.15	88.40	-0.81	85.20	1.26	84.70	7.60
Without	$s_R c_R(p)$	19	94.70	-0.29	87.90	-0.53	86.50	1.23	84.90	8.12
Without	$s_R c_R(p)$	20	94.70	-0.22	87.40	-0.16	84.50	1.69	85.80	8.13
Without	$s_R c_R(p)$	21	94.70	-0.24	87.40	-0.69	85.20	2.20	85.00	7.18
Without	$s_R c_R(p)$	22	94.50	-0.29	89.00	-0.34	84.90	1.23	85.30	6.95
Without	$s_R c_R(p)$	23	95.00	-0.13	88.20	0.06	84.30	1.06	84.80	7.12
Without	$s_R c_R(p)$	24	94.90	-0.20	88.10	0.03	84.70	1.16	85.60	6.77
Without	$s_R c_R(p)$	25	94.50	-0.20	89.60	0.44	85.70	1.96	85.60	6.19
Without	$N_A s_R c_R(p)$	1	93.00	-0.49	93.00	-12.09	56.80	-26.47	44.60	-29.95
Without	$N_A s_R c_R(p)$	2	93.30	3.30	92.80	-15.59	47.50	-50.84	31.50	-58.08
Without	$N_A s_R c_R(p)$	3	93.40	3.35	92.40	-18.89	43.80	-63.52	26.90	-70.97
Without	$N_A s_R c_R(p)$	4	93.40	1.88	91.90	-20.63	41.90	-72.05	26.90	-69.68
Without	$N_A s_R c_R(p)$	5	92.90	2.87	91.90	-25.29	40.80	-74.22	28.20	-54.92
Without	$N_A s_R c_R(p)$	6	92.60	4.47	91.40	-27.15	40.60	-78.28	30.50	-34.10
Without	$N_A s_R c_R(p)$	7	92.60	6.24	91.40	-26.40	42.80	-78.31	33.90	-2.86
Without	$N_A s_R c_R(p)$	8	92.50	7.15	91.50	-29.39	43.90	-78.65	36.80	46.89
Without	$N_A s_R c_R(p)$	9	92.20	7.67	90.90	-32.54	44.80	-78.19	40.70	26.35
Without	$N_A s_R c_R(p)$	10	92.30	6.13	91.40	-31.78	44.70	-74.92	41.40	29.95
Without	$N_A s_R c_R(p)$	11	91.60	5.69	90.20	-39.84	48.50	-76.73	43.10	28.42
Without	$N_A s_R c_R(p)$	12	90.50	3.62	89.40	-38.76	50.60	-79.15	45.20	21.63
Without	$N_A s_R c_R(p)$	13	89.70	1.57	89.80	-41.08	52.80	-76.53	45.20	9.75
Without	$N_A s_R c_R(p)$	14	89.00	-3.34	90.10	-44.06	55.00	-76.55	47.80	28.71
Without	$N_A s_R c_R(p)$	15	88.00	0.48	89.70	-44.40	54.30	-79.37	50.10	30.47
Without	$N_A s_R c_R(p)$	16	87.50	0.71	89.90	-44.91	55.80	-81.99	52.00	7.02
Without	$N_A s_R c_R(p)$	17	87.30	2.12	89.90	-44.33	57.50	-85.25	51.00	-6.87
Without	$N_A s_R c_R(p)$	18	86.40	-1.10	89.10	-46.20	57.60	-88.15	52.50	-20.31
Without	$N_A s_R c_R(p)$	19	86.00	3.82	88.50	-48.97	58.20	-88.26	52.50	-13.91
Without	$N_A s_R c_R(p)$	20	85.60	4.11	88.90	-40.01	58.60	-90.57	53.70	-28.21
Without	$N_A s_R c_R(p)$	21	84.00	3.78	88.60	-37.14	58.80	-90.26	50.80	-42.46
Without	$N_A s_R c_R(p)$	22	83.20	0.64	88.60	-37.52	61.10	-86.32	46.90	-29.81
Without	$N_A s_R c_R(p)$	23	82.70	-0.24	89.30	-30.93	61.50	-80.28	45.90	-32.16
Without	$N_A s_R c_R(p)$	24	82.10	-1.33	88.80	-31.17	59.60	-79.31	44.40	2.59
Without	$N_A s_R c_R(p)$	25	82.70	24.67	90.10	13.18	70.30	9.32	62.10	67.32
With	$s_F c_F(p)$	1	76.20	-0.91	23.00	10.09	14.80	65.62	18.60	159.79
With	$s_F c_F(p)$	2	76.60	-0.86	23.00	9.20	13.90	67.40	18.90	155.16
With	$s_F c_F(p)$	3	76.00	-0.79	24.10	9.29	13.20	68.14	20.20	148.59
With	$s_F c_F(p)$	4	78.30	-0.87	21.70	10.48	14.60	63.69	19.10	151.55
With	$s_F c_F(p)$	5	77.60	-0.84	24.20	9.73	14.50	68.70	18.90	149.31
With	$s_F c_F(p)$	6	77.50	-0.80	22.40	10.27	15.40	66.30	19.40	154.49
With	$s_F c_F(p)$	7	77.60	-0.87	23.30	9.50	17.40	64.30	18.50	158.06
With	$s_F c_F(p)$	8	77.30	-0.80	23.40	10.41	14.10	67.97	19.20	152.05
With	$s_F c_F(p)$	9	77.60	-0.83	22.90	9.15	15.60	65.90	20.10	155.53
With	$s_F c_F(p)$	10	76.90	-0.85	22.70	10.12	12.50	68.76	18.70	152.74
With	$s_F c_F(p)$	11	76.90	-0.83	23.50	9.28	14.30	67.63	20.10	163.47
With	$s_F c_F(p)$	12	77.10	-0.81	22.90	9.16	15.50	65.29	19.90	154.54
With	$s_F c_F(p)$	13	76.70	-0.78	22.20	9.21	13.90	62.73	19.30	159.74
With	$s_F c_F(p)$	14	77.30	-0.84	22.50	10.91	14.90	65.17	18.40	158.73
With	$s_F c_F(p)$	15	77.80	-0.94	23.60	9.32	15.20	60.24	20.20	153.88
With	$s_F c_F(p)$	16	77.40	-0.88	23.30	9.51	14.00	63.15	20.20	151.79
With	$s_F c_F(p)$	17	77.90	-0.84	23.40	10.66	15.70	65.79	19.60	162.63

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Table A.12 – continued from previous page

			No Variation		Low Variation		Medium Variation		High Variation	
Auxiliary	Model	Year	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB
With	$s_F c_F(p)$	18	77.40	-0.79	24.10	9.80	15.60	63.32	20.10	153.28
With	$s_F c_F(p)$	19	77.40	-0.91	24.30	10.00	15.20	63.89	20.00	161.71
With	$s_F c_F(p)$	20	78.30	-0.86	22.10	10.15	14.90	66.37	21.00	155.73
With	$s_F c_F(p)$	21	76.80	-0.87	22.70	10.76	15.20	65.39	18.90	157.25
With	$s_F c_F(p)$	22	78.20	-0.94	23.10	10.33	17.20	66.88	20.00	148.89
With	$s_F c_F(p)$	23	78.10	-0.82	23.60	10.14	13.50	66.87	22.00	154.10
With	$s_F c_F(p)$	24	77.30	-0.80	23.70	10.02	16.30	69.34	19.10	153.31
With	$s_F c_F(p)$	25	76.80	-0.83	23.60	10.22	15.60	69.70	23.00	151.23
With	$N_A s_F c_F(p)$	1	88.90	-1.02	32.30	-1.09	24.90	-0.03	26.70	5.66
With	$N_A s_F c_F(p)$	2	88.30	-1.01	30.70	-1.22	23.50	1.65	27.20	8.10
With	$N_A s_F c_F(p)$	3	88.30	-0.99	32.40	-0.90	24.30	0.95	29.70	7.25
With	$N_A s_F c_F(p)$	4	89.10	-1.05	33.10	-0.55	23.00	0.80	28.10	5.76
With	$N_A s_F c_F(p)$	5	89.50	-1.00	30.30	-0.97	21.90	2.66	30.40	6.61
With	$N_A s_F c_F(p)$	6	89.30	-1.00	31.20	-1.11	22.30	1.66	29.60	7.42
With	$N_A s_F c_F(p)$	7	89.40	-0.96	31.10	-1.03	21.90	0.41	28.00	6.85
With	$N_A s_F c_F(p)$	8	89.80	-0.98	31.20	-0.79	23.60	1.95	29.90	7.25
With	$N_A s_F c_F(p)$	9	89.10	-0.91	30.70	-1.22	23.20	2.90	28.60	5.78
With	$N_A s_F c_F(p)$	10	89.80	-0.94	31.50	-0.98	21.90	2.70	29.20	5.68
With	$N_A s_F c_F(p)$	11	89.70	-0.94	29.90	-0.81	23.40	1.95	27.90	7.50
With	$N_A s_F c_F(p)$	12	89.40	-1.07	30.60	-1.25	24.80	0.21	30.30	2.69
With	$N_A s_F c_F(p)$	13	90.40	-1.05	31.40	-1.67	25.60	1.75	29.40	7.75
With	$N_A s_F c_F(p)$	14	88.60	-1.09	33.80	0.12	21.80	0.38	29.20	5.00
With	$N_A s_F c_F(p)$	15	89.50	-1.02	30.60	-1.87	24.50	0.37	30.30	5.57
With	$N_A s_F c_F(p)$	16	88.90	-0.98	34.00	-0.79	23.60	0.24	29.70	4.36
With	$N_A s_F c_F(p)$	17	90.00	-1.05	31.70	-0.59	24.90	3.34	29.20	6.19
With	$N_A s_F c_F(p)$	18	89.80	-0.91	29.90	-1.41	23.80	0.58	29.20	7.34
With	$N_A s_F c_F(p)$	19	90.20	-1.04	31.80	-1.30	23.60	2.19	31.90	5.24
With	$N_A s_F c_F(p)$	20	90.00	-0.99	34.00	-0.68	23.80	1.57	31.10	9.10
With	$N_A s_F c_F(p)$	21	89.20	-1.04	30.30	-0.91	22.90	1.43	31.10	6.62
With	$N_A s_F c_F(p)$	22	90.60	-1.05	33.00	-0.65	24.00	2.34	32.80	7.07
With	$N_A s_F c_F(p)$	23	90.60	-0.94	32.00	-1.43	24.00	2.34	31.30	4.31
With	$N_A s_F c_F(p)$	24	89.70	-0.96	32.60	-1.43	25.20	1.39	33.10	3.98
With	$N_A s_F c_F(p)$	25	88.50	-0.89	30.90	-0.60	24.80	2.74	31.00	7.14
With	$s_F c_R(p)$	1	88.70	-0.71	78.00	-7.56	67.20	-12.49	69.90	-12.01
With	$s_F c_R(p)$	2	89.60	-0.53	77.00	-7.99	68.00	-13.81	71.80	-11.44
With	$s_F c_R(p)$	3	89.10	-0.54	76.00	-7.68	67.50	-12.86	71.50	-12.87
With	$s_F c_R(p)$	4	89.40	-0.54	75.50	-7.91	68.20	-13.37	70.60	-11.43
With	$s_F c_R(p)$	5	89.40	-0.62	76.20	-7.62	67.30	-14.20	69.40	-11.65
With	$s_F c_R(p)$	6	88.80	-0.63	77.10	-7.47	67.60	-13.34	69.70	-11.69
With	$s_F c_R(p)$	7	89.70	-0.53	76.80	-7.41	68.30	-13.59	70.80	-12.10
With	$s_F c_R(p)$	8	89.90	-0.57	76.30	-7.57	68.00	-13.00	71.50	-11.85
With	$s_F c_R(p)$	9	88.90	-0.57	76.30	-7.80	68.90	-12.91	69.90	-12.66
With	$s_F c_R(p)$	10	89.10	-0.55	76.40	-7.72	68.60	-13.33	70.70	-12.42
With	$s_F c_R(p)$	11	89.10	-0.56	76.40	-7.66	68.50	-12.63	70.80	-12.22
With	$s_F c_R(p)$	12	89.50	-0.59	76.50	-7.92	67.50	-13.51	70.80	-12.03
With	$s_F c_R(p)$	13	90.00	-0.57	76.90	-7.41	67.90	-12.86	72.00	-12.32
With	$s_F c_R(p)$	14	89.60	-0.64	78.10	-7.08	67.20	-13.82	71.60	-10.72
With	$s_F c_R(p)$	15	89.90	-0.67	77.10	-7.76	66.90	-12.87	72.10	-11.66
With	$s_F c_R(p)$	16	90.60	-0.62	78.90	-7.93	67.90	-13.14	70.90	-11.74
With	$s_F c_R(p)$	17	90.80	-0.65	78.00	-8.19	67.10	-12.82	70.60	-10.98
With	$s_F c_R(p)$	18	89.60	-0.55	77.90	-7.80	68.50	-13.70	70.60	-12.09
With	$s_F c_R(p)$	19	89.70	-0.63	77.60	-7.53	68.10	-13.22	70.70	-11.10
With	$s_F c_R(p)$	20	89.40	-0.50	77.40	-7.73	68.70	-13.46	69.70	-11.78
With	$s_F c_R(p)$	21	89.20	-0.61	77.50	-7.79	67.40	-13.39	69.10	-11.47
With	$s_F c_R(p)$	22	90.20	-0.59	77.50	-8.04	67.50	-13.56	69.90	-12.63
With	$s_F c_R(p)$	23	90.40	-0.45	77.40	-7.95	67.80	-13.54	68.80	-13.79
With	$s_F c_R(p)$	24	89.80	-0.53	76.90	-7.79	67.80	-13.95	69.40	-14.22
With	$s_F c_R(p)$	25	88.90	-0.55	77.30	-7.86	68.60	-15.30	70.10	-16.06
With	$N_A s_F c_R(p)$	1	88.90	-1.02	45.30	-18.30	25.20	-30.99	24.50	-32.37

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Table A.12 – continued from previous page

			No Variation		Low Variation		Medium Variation		High Variation	
Auxiliary	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
With	$N_A s_F c_R(p)$	2	88.30	-1.01	46.10	-18.29	26.10	-31.70	27.30	-31.72
With	$N_A s_F c_R(p)$	3	88.30	-0.99	46.80	-18.33	27.30	-31.27	28.00	-32.67
With	$N_A s_F c_R(p)$	4	89.10	-1.05	46.40	-18.30	27.60	-31.44	27.00	-32.09
With	$N_A s_F c_R(p)$	5	89.50	-1.00	47.50	-17.86	26.70	-31.12	27.20	-32.63
With	$N_A s_F c_R(p)$	6	89.30	-1.00	48.00	-18.34	27.00	-31.87	26.30	-32.85
With	$N_A s_F c_R(p)$	7	89.40	-0.96	46.60	-18.76	26.80	-31.30	26.70	-33.45
With	$N_A s_F c_R(p)$	8	89.80	-0.98	46.80	-18.04	27.80	-30.71	26.60	-32.48
With	$N_A s_F c_R(p)$	9	89.10	-0.91	47.00	-18.08	27.10	-30.58	28.20	-33.56
With	$N_A s_F c_R(p)$	10	89.80	-0.94	47.00	-18.23	27.90	-30.62	26.40	-33.56
With	$N_A s_F c_R(p)$	11	89.70	-0.94	46.60	-18.33	27.90	-31.52	27.80	-32.68
With	$N_A s_F c_R(p)$	12	89.40	-1.07	48.80	-18.35	27.20	-31.12	25.30	-33.56
With	$N_A s_F c_R(p)$	13	90.40	-1.05	46.50	-18.77	28.60	-30.94	29.80	-31.91
With	$N_A s_F c_R(p)$	14	88.60	-1.09	48.80	-18.52	27.70	-31.17	28.50	-31.71
With	$N_A s_F c_R(p)$	15	89.50	-1.02	46.30	-18.46	27.00	-30.25	28.80	-32.40
With	$N_A s_F c_R(p)$	16	88.90	-0.98	47.20	-17.64	28.40	-30.99	28.40	-33.37
With	$N_A s_F c_R(p)$	17	90.00	-1.05	46.50	-18.19	29.10	-30.86	28.40	-32.08
With	$N_A s_F c_R(p)$	18	89.80	-0.91	46.40	-18.09	27.30	-31.29	28.50	-33.09
With	$N_A s_F c_R(p)$	19	90.20	-1.04	46.80	-17.85	28.20	-30.85	29.20	-32.34
With	$N_A s_F c_R(p)$	20	90.00	-0.99	47.20	-18.14	27.20	-31.32	27.90	-33.24
With	$N_A s_F c_R(p)$	21	89.20	-1.04	47.60	-18.40	28.80	-31.35	28.80	-33.32
With	$N_A s_F c_R(p)$	22	90.60	-1.05	47.20	-18.14	27.40	-31.22	27.90	-33.65
With	$N_A s_F c_R(p)$	23	90.60	-0.94	47.90	-18.25	28.20	-31.38	27.50	-34.86
With	$N_A s_F c_R(p)$	24	89.70	-0.96	49.40	-18.05	27.60	-31.60	27.30	-34.68
With	$N_A s_F c_R(p)$	25	88.50	-0.89	49.20	-18.56	31.20	-32.91	29.50	-35.94
With	$s_R c_R(p)$	1	89.40	-0.79	78.60	-7.29	68.20	-12.26	70.40	-12.57
With	$s_R c_R(p)$	2	89.70	-0.61	76.80	-7.73	68.90	-13.61	71.90	-11.95
With	$s_R c_R(p)$	3	90.10	-0.65	76.60	-7.32	68.70	-12.55	71.60	-12.85
With	$s_R c_R(p)$	4	90.10	-0.69	76.30	-7.03	69.30	-13.01	70.40	-12.21
With	$s_R c_R(p)$	5	90.40	-0.66	77.00	-7.10	68.20	-13.79	69.80	-11.61
With	$s_R c_R(p)$	6	90.00	-0.73	76.80	-7.01	67.80	-12.75	70.20	-11.97
With	$s_R c_R(p)$	7	90.50	-0.57	77.30	-7.18	69.20	-13.42	71.90	-11.83
With	$s_R c_R(p)$	8	90.50	-0.70	78.00	-7.32	68.70	-12.76	71.70	-12.49
With	$s_R c_R(p)$	9	90.00	-0.68	77.90	-7.69	69.20	-12.94	69.60	-13.10
With	$s_R c_R(p)$	10	89.90	-0.62	76.60	-7.60	69.90	-12.39	70.50	-12.20
With	$s_R c_R(p)$	11	89.70	-0.62	76.70	-7.62	69.60	-12.22	70.90	-12.17
With	$s_R c_R(p)$	12	90.10	-0.68	76.60	-7.43	68.60	-12.58	71.60	-11.39
With	$s_R c_R(p)$	13	90.50	-0.73	76.70	-7.56	69.70	-11.64	73.20	-9.58
With	$s_R c_R(p)$	14	90.60	-0.75	77.30	-7.03	68.30	-13.06	72.40	-10.76
With	$s_R c_R(p)$	15	90.50	-0.77	76.70	-7.66	68.10	-12.28	72.90	-10.83
With	$s_R c_R(p)$	16	90.90	-0.68	79.40	-7.65	69.50	-13.19	71.00	-11.33
With	$s_R c_R(p)$	17	91.10	-0.70	78.40	-8.10	67.80	-12.86	71.50	-10.96
With	$s_R c_R(p)$	18	90.30	-0.60	78.30	-7.68	68.70	-13.57	71.50	-12.01
With	$s_R c_R(p)$	19	89.90	-0.76	78.20	-7.50	69.10	-13.44	71.70	-11.14
With	$s_R c_R(p)$	20	89.90	-0.60	78.10	-7.58	69.40	-12.76	70.00	-11.67
With	$s_R c_R(p)$	21	90.00	-0.63	78.10	-7.61	67.70	-12.68	69.40	-11.30
With	$s_R c_R(p)$	22	90.50	-0.68	78.00	-7.68	67.40	-13.07	70.40	-12.76
With	$s_R c_R(p)$	23	90.80	-0.56	77.40	-7.80	68.30	-13.38	69.00	-13.41
With	$s_R c_R(p)$	24	90.40	-0.60	76.30	-7.64	68.10	-13.41	70.60	-13.97
With	$s_R c_R(p)$	25	89.60	-0.58	77.50	-7.18	69.00	-14.44	70.10	-15.04
With	$N_A s_R c_R(p)$	1	88.90	-1.03	47.80	-17.83	30.80	-28.94	32.40	-29.09
With	$N_A s_R c_R(p)$	2	88.40	-1.01	48.40	-17.50	32.30	-29.42	35.20	-28.73
With	$N_A s_R c_R(p)$	3	88.30	-0.99	48.90	-17.53	34.50	-28.96	36.70	-28.68
With	$N_A s_R c_R(p)$	4	89.10	-1.05	48.00	-17.56	34.70	-29.53	36.30	-28.57
With	$N_A s_R c_R(p)$	5	89.60	-1.00	50.20	-17.02	33.40	-28.71	35.30	-28.72
With	$N_A s_R c_R(p)$	6	89.30	-1.00	50.40	-17.69	34.00	-29.41	35.50	-28.81
With	$N_A s_R c_R(p)$	7	89.40	-0.97	49.40	-17.87	32.60	-28.96	36.60	-29.98
With	$N_A s_R c_R(p)$	8	89.80	-0.98	48.90	-17.71	35.30	-28.77	35.70	-28.90
With	$N_A s_R c_R(p)$	9	89.10	-0.91	49.50	-17.55	33.90	-28.67	35.50	-29.06
With	$N_A s_R c_R(p)$	10	89.80	-0.94	48.80	-17.67	34.60	-28.98	35.90	-29.36

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Table A.12 – continued from previous page

			No Variation		Low Variation		Medium Variation		High Variation	
Auxiliary	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
With	$N_A s_R c_R(p)$	11	89.80	-0.94	48.70	-17.84	33.60	-29.58	36.40	-29.79
With	$N_A s_R c_R(p)$	12	89.40	-1.07	50.80	-17.88	33.20	-28.80	35.50	-29.65
With	$N_A s_R c_R(p)$	13	90.40	-1.05	48.90	-18.09	34.90	-28.40	37.90	-27.90
With	$N_A s_R c_R(p)$	14	88.60	-1.08	50.70	-17.97	34.10	-29.41	37.60	-28.20
With	$N_A s_R c_R(p)$	15	89.50	-1.03	48.40	-17.45	33.90	-28.29	38.00	-28.94
With	$N_A s_R c_R(p)$	16	88.90	-0.98	49.60	-16.86	34.20	-29.14	37.60	-29.74
With	$N_A s_R c_R(p)$	17	90.00	-1.05	48.50	-17.60	35.60	-29.10	36.10	-28.35
With	$N_A s_R c_R(p)$	18	89.80	-0.91	48.50	-17.64	33.90	-29.15	36.20	-29.40
With	$N_A s_R c_R(p)$	19	90.30	-1.04	48.50	-17.26	34.80	-28.80	37.70	-28.80
With	$N_A s_R c_R(p)$	20	90.10	-0.98	49.20	-17.83	34.30	-29.41	37.60	-29.13
With	$N_A s_R c_R(p)$	21	89.20	-1.04	49.80	-17.86	35.60	-28.33	37.60	-29.42
With	$N_A s_R c_R(p)$	22	90.60	-1.05	49.60	-17.37	33.40	-29.04	36.50	-29.68
With	$N_A s_R c_R(p)$	23	90.80	-0.94	49.30	-17.73	34.80	-29.28	37.00	-30.32
With	$N_A s_R c_R(p)$	24	89.70	-0.96	50.70	-17.69	34.60	-29.33	36.90	-30.74
With	$N_A s_R c_R(p)$	25	88.50	-0.91	51.50	-17.73	38.00	-29.80	37.50	-31.33

Table (A.13) contains the data used for plotting Figure (4.5), the relative bias in estimation of annual abundance for each model, for each year of data from small game robustness simulations when a high level of simulated auxiliary data is available for estimating c. Table (A.14) contains the data used for plotting Figure (4.6), the relative bias in estimation of annual abundance for each model, for each year of data from small game robustness simulations when a low level of simulated auxiliary data is available for estimating c.

Table A.13: *Median relative bias in total annual abundance estimates from small game robustness simulations when a high amount of simulated auxiliary data is available for estimating c. Results indicate low bias for models employing the Horvitz-Thompson abundance estimator when the auxiliary catch-effort likelihood of Equation (1.7) is omitted. Results for mixed-effects versions of the absolute-recruit abundance models show large negative bias for all scenarios, while the fixed-effects version of this model shows negligible bias in all scenarios. When the auxiliary catch-effort likelihoos is employed, the models employing the Horvitz-Thompson estimator sow negative bias between -5% and -10%. Results based on n = 1000 replicates. "Aux. Like." = Auxiliary catch-effort likelihood used (With) or not used (Without).*

Table A.13
Annual Abundance Percent Bias

Aux. Like.	Model	Year	S Increasing	S Decreasing	Periodic Recruitment
Without	$s_F c_F(p)$	1	-1.12%	0.37%	-0.51%
Without	$s_F c_F(p)$	2	0.03%	-0.36%	-0.44%
Without	$s_F c_F(p)$	3	-0.45%	0.66%	-0.49%
Without	$s_F c_F(p)$	4	-1.02%	-0.28%	-1.05%
Without	$s_F c_F(p)$	5	-0.95%	0.44%	-0.92%
Without	$s_F c_F(p)$	6	-1.26%	1.21%	-0.07%
Without	$s_F c_F(p)$	7	-1.01%	0.49%	-0.76%
Without	$s_F c_F(p)$	8	-1.77%	0.87%	-0.96%
Without	$s_F c_F(p)$	9	-0.65%	0.88%	-0.51%
Without	$s_F c_F(p)$	10	-1.03%	-0.24%	0.34%
Without	$s_F c_F(p)$	11	-1.77%	0.63%	-0.6%
Without	$s_F c_F(p)$	12	-0.36%	0.15%	-0.92%
Without	$s_F c_F(p)$	13	-0.54%	0.21%	-1.15%
Without	$s_F c_F(p)$	14	-0.64%	0.43%	-1.06%
Without	$s_F c_F(p)$	15	-0.94%	0.44%	-0.5%
Without	$s_F c_F(p)$	16	-0.84%	0.51%	-0.5%
Without	$s_F c_F(p)$	17	-1.11%	0.09%	-0.49%
Without	$s_F c_F(p)$	18	-0.94%	0.16%	-0.79%
Without	$s_F c_F(p)$	19	-1.03%	0.22%	-1.07%
Without	$s_F c_F(p)$	20	-0.48%	0.34%	-0.52%
Without	$s_F c_F(p)$	21	-0.11%	-0.09%	-0.62%
Without	$s_F c_F(p)$	22	-1.36%	0.87%	-0.06%
Without	$s_F c_F(p)$	23	-0.36%	0.56%	-1.01%
Without	$s_F c_F(p)$	24	-1.09%	0.31%	-1.22%
Without	$s_F c_F(p)$	25	-1.15%	1.26%	-0.48%
Without	$s_F c_R(p)$	1	-1.57%	0.23%	-0.88%
Without	$s_F c_R(p)$	2	-1.56%	0.83%	-0.47%
Without	$s_F c_R(p)$	3	-1.65%	0.54%	-0.4%
Without	$s_F c_R(p)$	4	-1.36%	0.27%	-0.53%

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Table A.13 – continued from previous page

Annual Abundance Percent Bias					
Aux. Like.	**Model**	**Year**	**S Increasing**	**S Decreasing**	**Periodic Recruitment**
Without	$s_F c_R(p)$	5	-1.4%	0.21%	-0.54%
Without	$s_F c_R(p)$	6	-0.98%	0.25%	-0.45%
Without	$s_F c_R(p)$	7	-0.95%	0.27%	-0.56%
Without	$s_F c_R(p)$	8	-0.66%	0.12%	-0.63%
Without	$s_F c_R(p)$	9	-0.76%	-0.19%	-0.82%
Without	$s_F c_R(p)$	10	-0.91%	-0.53%	-0.4%
Without	$s_F c_R(p)$	11	-0.66%	-0.4%	-0.45%
Without	$s_F c_R(p)$	12	-0.79%	-0.15%	-0.71%
Without	$s_F c_R(p)$	13	-0.74%	-0.1%	-0.79%
Without	$s_F c_R(p)$	14	-0.88%	-0.26%	-0.61%
Without	$s_F c_R(p)$	15	-0.56%	-0.33%	-0.82%
Without	$s_F c_R(p)$	16	-0.94%	-0.22%	-0.85%
Without	$s_F c_R(p)$	17	-0.82%	-0.14%	-0.85%
Without	$s_F c_R(p)$	18	-0.5%	-0.37%	-0.71%
Without	$s_F c_R(p)$	19	-0.48%	-0.34%	-1.16%
Without	$s_F c_R(p)$	20	-0.71%	-0.59%	-0.72%
Without	$s_F c_R(p)$	21	-0.58%	-0.71%	-1.01%
Without	$s_F c_R(p)$	22	-0.68%	-0.39%	-0.71%
Without	$s_F c_R(p)$	23	-1.03%	-0.27%	-0.91%
Without	$s_F c_R(p)$	24	-1.32%	-0.3%	-0.82%
Without	$s_F c_R(p)$	25	-1.48%	0.19%	-0.41%
Without	$s_R c_R(p)$	1	-0.87%	0.61%	0.15%
Without	$s_R c_R(p)$	2	-0.21%	0.99%	0.17%
Without	$s_R c_R(p)$	3	-0.87%	0.77%	0.24%
Without	$s_R c_R(p)$	4	-0.63%	0.23%	-0.19%
Without	$s_R c_R(p)$	5	-0.51%	0.69%	0.12%
Without	$s_R c_R(p)$	6	-0.53%	0.94%	0.22%
Without	$s_R c_R(p)$	7	-0.77%	0.65%	-0.22%
Without	$s_R c_R(p)$	8	-0.34%	0.54%	-0.33%
Without	$s_R c_R(p)$	9	-0.54%	0.66%	-0.05%
Without	$s_R c_R(p)$	10	-0.8%	0.01%	0.47%
Without	$s_R c_R(p)$	11	-0.34%	0.34%	0.16%
Without	$s_R c_R(p)$	12	-0.31%	0.28%	-0.05%
Without	$s_R c_R(p)$	13	-0.05%	0.28%	-0.15%
Without	$s_R c_R(p)$	14	-0.25%	0.67%	0.16%
Without	$s_R c_R(p)$	15	0.24%	0.4%	-0.08%
Without	$s_R c_R(p)$	16	-0.18%	0.58%	-0.03%
Without	$s_R c_R(p)$	17	-0.43%	0.42%	-0.36%
Without	$s_R c_R(p)$	18	-0.23%	0.11%	-0.32%
Without	$s_R c_R(p)$	19	-0.09%	0.14%	-0.4%
Without	$s_R c_R(p)$	20	-0.05%	0.33%	-0.26%
Without	$s_R c_R(p)$	21	-0.22%	-0.02%	-0.24%
Without	$s_R c_R(p)$	22	-0.01%	0.27%	0.18%
Without	$s_R c_R(p)$	23	-0.37%	0.11%	-0.09%
Without	$s_R c_R(p)$	24	-0.71%	0.02%	-0.3%
Without	$s_R c_R(p)$	25	-0.52%	0.51%	0.33%
With	$s_F c_F(p)$	1	8.46%	10.92%	9.09%
With	$s_F c_F(p)$	2	10%	9.89%	8.86%
With	$s_F c_F(p)$	3	9.41%	11.36%	8.36%
With	$s_F c_F(p)$	4	8.79%	10.45%	9.48%
With	$s_F c_F(p)$	5	9.41%	11.07%	8.62%
With	$s_F c_F(p)$	6	8.74%	11.8%	9.77%
With	$s_F c_F(p)$	7	8.98%	11.14%	8.92%
With	$s_F c_F(p)$	8	8.44%	11.77%	8.77%
With	$s_F c_F(p)$	9	9.21%	11.57%	8.71%
With	$s_F c_F(p)$	10	9.04%	10.01%	9.98%
With	$s_F c_F(p)$	11	7.86%	10.95%	8.43%
With	$s_F c_F(p)$	12	9.66%	11.52%	8.89%
With	$s_F c_F(p)$	13	9.13%	10.57%	8.11%

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Table A.13 – continued from previous page

Annual Abundance Percent Bias					
Aux. Like.	Model	Year	S Increasing	S Decreasing	Periodic Recruitment
With	$s_F c_F(p)$	14	9.74%	10.12%	8.82%
With	$s_F c_F(p)$	15	8.61%	11.4%	8.59%
With	$s_F c_F(p)$	16	8.58%	11.19%	8.63%
With	$s_F c_F(p)$	17	9.49%	10.86%	8.32%
With	$s_F c_F(p)$	18	9.22%	10.7%	9.07%
With	$s_F c_F(p)$	19	9.35%	11.91%	8.9%
With	$s_F c_F(p)$	20	9.49%	11.61%	9.03%
With	$s_F c_F(p)$	21	10.51%	11.45%	9.3%
With	$s_F c_F(p)$	22	9.17%	12.3%	10.19%
With	$s_F c_F(p)$	23	8.84%	11.34%	8.43%
With	$s_F c_F(p)$	24	9.18%	10.24%	8.64%
With	$s_F c_F(p)$	25	8.66%	11.56%	9.44%
With	$N_A s_F c_F(p)$	1	-0.92%	0.75%	-0.5%
With	$N_A s_F c_F(p)$	2	0.33%	0%	-0.6%
With	$N_A s_F c_F(p)$	3	-0.26%	0.94%	-0.52%
With	$N_A s_F c_F(p)$	4	-0.73%	0.54%	-0.62%
With	$N_A s_F c_F(p)$	5	-0.84%	0.2%	-0.17%
With	$N_A s_F c_F(p)$	6	-1.38%	1.37%	0.06%
With	$N_A s_F c_F(p)$	7	-0.87%	0.52%	-0.36%
With	$N_A s_F c_F(p)$	8	-1.17%	0.8%	-0.95%
With	$N_A s_F c_F(p)$	9	-0.46%	1.01%	-0.55%
With	$N_A s_F c_F(p)$	10	-0.11%	-0.06%	0.46%
With	$N_A s_F c_F(p)$	11	-1.34%	0.96%	-0.49%
With	$N_A s_F c_F(p)$	12	0.38%	0.8%	-1.01%
With	$N_A s_F c_F(p)$	13	-0.79%	0.57%	-1.58%
With	$N_A s_F c_F(p)$	14	-0.24%	0.5%	-1.13%
With	$N_A s_F c_F(p)$	15	-0.41%	0.15%	-0.3%
With	$N_A s_F c_F(p)$	16	-0.4%	0.84%	-0.21%
With	$N_A s_F c_F(p)$	17	-0.3%	0.34%	-0.92%
With	$N_A s_F c_F(p)$	18	0%	0.44%	-1.08%
With	$N_A s_F c_F(p)$	19	-0.49%	0.27%	-1.04%
With	$N_A s_F c_F(p)$	20	-0.19%	0.91%	-0.24%
With	$N_A s_F c_F(p)$	21	0.06%	1.02%	-0.67%
With	$N_A s_F c_F(p)$	22	-0.8%	1.2%	0.02%
With	$N_A s_F c_F(p)$	23	-0.39%	0.92%	-0.71%
With	$N_A s_F c_F(p)$	24	-0.99%	0.02%	-1.16%
With	$N_A s_F c_F(p)$	25	-0.66%	1.42%	0.08%
With	$s_F c_R(p)$	1	-4.58%	-2.7%	-3.67%
With	$s_F c_R(p)$	2	-4.23%	-2.27%	-3.27%
With	$s_F c_R(p)$	3	-4.84%	-2.22%	-3.42%
With	$s_F c_R(p)$	4	-4.29%	-2.71%	-3.14%
With	$s_F c_R(p)$	5	-4.57%	-2.78%	-3.45%
With	$s_F c_R(p)$	6	-4.04%	-2.81%	-3.25%
With	$s_F c_R(p)$	7	-3.81%	-2.5%	-3.36%
With	$s_F c_R(p)$	8	-3.86%	-2.89%	-3.19%
With	$s_F c_R(p)$	9	-3.69%	-3.13%	-3.33%
With	$s_F c_R(p)$	10	-4.3%	-3.13%	-3.08%
With	$s_F c_R(p)$	11	-3.6%	-3.13%	-3.22%
With	$s_F c_R(p)$	12	-3.74%	-3.11%	-3.33%
With	$s_F c_R(p)$	13	-3.77%	-3.27%	-3.6%
With	$s_F c_R(p)$	14	-4.03%	-3.14%	-3.33%
With	$s_F c_R(p)$	15	-3.52%	-3.17%	-3.37%
With	$s_F c_R(p)$	16	-4.11%	-3.05%	-3.6%
With	$s_F c_R(p)$	17	-3.69%	-2.71%	-3.75%
With	$s_F c_R(p)$	18	-3.65%	-3.27%	-3.36%
With	$s_F c_R(p)$	19	-3.68%	-3.27%	-3.74%
With	$s_F c_R(p)$	20	-3.68%	-3.4%	-3.43%
With	$s_F c_R(p)$	21	-3.58%	-3.67%	-3.83%
With	$s_F c_R(p)$	22	-3.71%	-3.18%	-3.64%

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Table A.13 – continued from previous page

Annual Abundance Percent Bias					
Aux. Like.	**Model**	**Year**	**S Increasing**	**S Decreasing**	**Periodic Recruitment**
With	$s_F c_R(p)$	23	-4.07%	-3.28%	-3.51%
With	$s_F c_R(p)$	24	-4.7%	-3.5%	-3.83%
With	$s_F c_R(p)$	25	-4.93%	-3%	-3.55%
With	$N_A s_F c_R(p)$	1	-9.82%	-7.82%	-8.18%
With	$N_A s_F c_R(p)$	2	-9.72%	-7.53%	-8.34%
With	$N_A s_F c_R(p)$	3	-10.17%	-7.57%	-8.21%
With	$N_A s_F c_R(p)$	4	-9.9%	-8.14%	-8.49%
With	$N_A s_F c_R(p)$	5	-9.98%	-7.88%	-8.2%
With	$N_A s_F c_R(p)$	6	-9.91%	-7.96%	-8.16%
With	$N_A s_F c_R(p)$	7	-9.61%	-7.8%	-8.16%
With	$N_A s_F c_R(p)$	8	-9.47%	-8.05%	-8.35%
With	$N_A s_F c_R(p)$	9	-9.48%	-8.13%	-8.51%
With	$N_A s_F c_R(p)$	10	-9.73%	-8.49%	-8.29%
With	$N_A s_F c_R(p)$	11	-9.25%	-8.34%	-8.21%
With	$N_A s_F c_R(p)$	12	-9.36%	-8.4%	-8.45%
With	$N_A s_F c_R(p)$	13	-9.45%	-8.59%	-8.49%
With	$N_A s_F c_R(p)$	14	-9.58%	-8.46%	-8.32%
With	$N_A s_F c_R(p)$	15	-9.38%	-8.44%	-8.47%
With	$N_A s_F c_R(p)$	16	-9.51%	-8.33%	-8.77%
With	$N_A s_F c_R(p)$	17	-9.13%	-8.32%	-8.64%
With	$N_A s_F c_R(p)$	18	-9.17%	-8.18%	-8.41%
With	$N_A s_F c_R(p)$	19	-9.26%	-8.45%	-8.65%
With	$N_A s_F c_R(p)$	20	-9.25%	-8.77%	-8.58%
With	$N_A s_F c_R(p)$	21	-8.97%	-8.5%	-8.5%
With	$N_A s_F c_R(p)$	22	-9.13%	-8.52%	-8.47%
With	$N_A s_F c_R(p)$	23	-9.62%	-8.34%	-8.4%
With	$N_A s_F c_R(p)$	24	-10.28%	-8.56%	-8.94%
With	$N_A s_F c_R(p)$	25	-10.65%	-8.4%	-8.58%
With	$s_R c_R(p)$	1	-4.07%	-2.63%	-3.15%
With	$s_R c_R(p)$	2	-3.63%	-2.24%	-2.89%
With	$s_R c_R(p)$	3	-4.09%	-2.57%	-2.89%
With	$s_R c_R(p)$	4	-3.8%	-3.02%	-3.05%
With	$s_R c_R(p)$	5	-3.97%	-2.57%	-2.77%
With	$s_R c_R(p)$	6	-3.83%	-2.37%	-2.92%
With	$s_R c_R(p)$	7	-3.78%	-2.35%	-3.29%
With	$s_R c_R(p)$	8	-3.58%	-2.6%	-3.2%
With	$s_R c_R(p)$	9	-3.7%	-2.64%	-3.18%
With	$s_R c_R(p)$	10	-4.09%	-2.77%	-2.56%
With	$s_R c_R(p)$	11	-3.44%	-2.64%	-2.61%
With	$s_R c_R(p)$	12	-3.57%	-2.77%	-2.81%
With	$s_R c_R(p)$	13	-3.33%	-2.74%	-3.18%
With	$s_R c_R(p)$	14	-3.53%	-2.5%	-2.84%
With	$s_R c_R(p)$	15	-3.08%	-2.76%	-2.91%
With	$s_R c_R(p)$	16	-3.56%	-2.54%	-3.24%
With	$s_R c_R(p)$	17	-3.68%	-2.67%	-3.39%
With	$s_R c_R(p)$	18	-3.67%	-3.1%	-3.16%
With	$s_R c_R(p)$	19	-3.31%	-2.95%	-3.43%
With	$s_R c_R(p)$	20	-3.17%	-3.14%	-3.39%
With	$s_R c_R(p)$	21	-3.44%	-3.21%	-3.31%
With	$s_R c_R(p)$	22	-3.63%	-3.03%	-3.03%
With	$s_R c_R(p)$	23	-3.67%	-3.2%	-3.2%
With	$s_R c_R(p)$	24	-4.35%	-3.46%	-3.43%
With	$s_R c_R(p)$	25	-4.39%	-2.87%	-3.11%
With	$N_A s_R c_R(p)$	1	-9.74%	-7.72%	-8.15%
With	$N_A s_R c_R(p)$	2	-9.54%	-7.38%	-8.26%
With	$N_A s_R c_R(p)$	3	-9.81%	-7.2%	-8.02%
With	$N_A s_R c_R(p)$	4	-9.78%	-7.9%	-8.14%
With	$N_A s_R c_R(p)$	5	-9.67%	-7.38%	-8.07%
With	$N_A s_R c_R(p)$	6	-9.77%	-7.54%	-7.98%

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Table A.13 – continued from previous page

Annual Abundance Percent Bias					
Aux. Like.	**Model**	**Year**	**S Increasing**	**S Decreasing**	**Periodic Recruitment**
With	$N_A s_R c_R(p)$	7	-9.26%	-7.54%	-7.97%
With	$N_A s_R c_R(p)$	8	-9.31%	-7.87%	-8.15%
With	$N_A s_R c_R(p)$	9	-9.34%	-7.79%	-8.17%
With	$N_A s_R c_R(p)$	10	-9.24%	-8.27%	-7.94%
With	$N_A s_R c_R(p)$	11	-8.98%	-8.11%	-7.89%
With	$N_A s_R c_R(p)$	12	-9.19%	-7.96%	-8.04%
With	$N_A s_R c_R(p)$	13	-9.3%	-8.29%	-8.2%
With	$N_A s_R c_R(p)$	14	-9.32%	-8.14%	-8.03%
With	$N_A s_R c_R(p)$	15	-9.27%	-8.28%	-7.98%
With	$N_A s_R c_R(p)$	16	-9.25%	-7.96%	-8.63%
With	$N_A s_R c_R(p)$	17	-8.92%	-8.12%	-8.32%
With	$N_A s_R c_R(p)$	18	-9.03%	-8%	-8.09%
With	$N_A s_R c_R(p)$	19	-9.09%	-8.01%	-8.5%
With	$N_A s_R c_R(p)$	20	-9.1%	-8.66%	-8.37%
With	$N_A s_R c_R(p)$	21	-8.61%	-8.36%	-8.09%
With	$N_A s_R c_R(p)$	22	-8.98%	-8.38%	-8.24%
With	$N_A s_R c_R(p)$	23	-9.34%	-8.15%	-8.21%
With	$N_A s_R c_R(p)$	24	-10.08%	-8.44%	-8.71%
With	$N_A s_R c_R(p)$	25	-10.29%	-8.1%	-8.14%

Table A.14: *Median relative bias in total annual abundance estimates from small game robustness simulations when a low amount of simulated auxiliary data is available for estimating c. Results indicate low bias for models employing the Horvitz-Thompson abundance estimator when the auxiliary catch-effort likelihood of Equation (1.7) is omitted. Results for mixed-effects versions of the absolute-recruit abundance models show large negative bias for all scenarios, while the fixed-effects version of this model shows negligible bias in all scenarios. When the auxiliary catch-effort likelihoos is employed, the models employing the Horvitz-Thompson estimator sow negative bias between -5% and -10%. Results based on* n = 1000 *replicates. "Aux. Like." = Auxiliary catch-effort likelihood used (With) or not used (Without).*

Table A.14 Annual Abundance Percent Bias					
Aux. Like.	**Model**	**Year**	**S Increasing**	**S Decreasing**	**Periodic Recruitment**
Without	$s_F c_F(p)$	1	-1.46%	0.18%	-1.07%
Without	$s_F c_F(p)$	2	-0.6%	0.07%	-0.05%
Without	$s_F c_F(p)$	3	-1.03%	0.52%	-0.11%
Without	$s_F c_F(p)$	4	-1.36%	-0.3%	-0.49%
Without	$s_F c_F(p)$	5	-1.24%	0.59%	-0.02%
Without	$s_F c_F(p)$	6	-1.33%	1.38%	-0.37%
Without	$s_F c_F(p)$	7	-1.42%	0.19%	-0.28%
Without	$s_F c_F(p)$	8	-1.56%	0.57%	-0.05%
Without	$s_F c_F(p)$	9	-1.29%	0.2%	-0.58%
Without	$s_F c_F(p)$	10	-1.35%	-0.54%	0.32%
Without	$s_F c_F(p)$	11	-2.26%	0.14%	0.05%
Without	$s_F c_F(p)$	12	-0.36%	0.15%	0.17%
Without	$s_F c_F(p)$	13	-0.83%	-0.08%	-0.37%
Without	$s_F c_F(p)$	14	-0.82%	0.07%	-1.35%

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Table A.14 – continued from previous page

Annual Abundance Percent Bias					
Aux. Like.	**Model**	**Year**	**S Increasing**	**S Decreasing**	**Periodic Recruitment**
Without	$s_F c_F(p)$	15	-1.77%	0.52%	0.12%
Without	$s_F c_F(p)$	16	-0.94%	-0.13%	0.64%
Without	$s_F c_F(p)$	17	-1.07%	0.23%	-0.1%
Without	$s_F c_F(p)$	18	-1.32%	-0.11%	-0.14%
Without	$s_F c_F(p)$	19	-1.39%	0.19%	-0.36%
Without	$s_F c_F(p)$	20	-1.18%	0.38%	-0.89%
Without	$s_F c_F(p)$	21	-0.11%	-0.27%	-0.47%
Without	$s_F c_F(p)$	22	-1.98%	0.71%	0.54%
Without	$s_F c_F(p)$	23	-1.12%	0.48%	-0.49%
Without	$s_F c_F(p)$	24	-1.57%	-0.41%	-0.68%
Without	$s_F c_F(p)$	25	-1.49%	0.66%	-0.23%
Without	$s_F c_R(p)$	1	-1.97%	-0.17%	0.26%
Without	$s_F c_R(p)$	2	-2.1%	0.24%	-0.14%
Without	$s_F c_R(p)$	3	-1.39%	-0.33%	0.34%
Without	$s_F c_R(p)$	4	-1.38%	-0.24%	0.04%
Without	$s_F c_R(p)$	5	-1.24%	-0.29%	-0.13%
Without	$s_F c_R(p)$	6	-1.28%	-0.46%	0.19%
Without	$s_F c_R(p)$	7	-1.06%	-0.72%	-0.13%
Without	$s_F c_R(p)$	8	-0.85%	-0.45%	0.02%
Without	$s_F c_R(p)$	9	-0.93%	-0.56%	0.11%
Without	$s_F c_R(p)$	10	-0.95%	-0.64%	0.21%
Without	$s_F c_R(p)$	11	-0.99%	0.03%	-0.12%
Without	$s_F c_R(p)$	12	-0.84%	-0.19%	-0.23%
Without	$s_F c_R(p)$	13	-1.39%	-1.09%	0.1%
Without	$s_F c_R(p)$	14	-1.24%	-0.99%	0.27%
Without	$s_F c_R(p)$	15	-1.45%	-0.97%	0.43%
Without	$s_F c_R(p)$	16	-1.26%	-0.67%	-0.04%
Without	$s_F c_R(p)$	17	-1.21%	-1.22%	-0.23%
Without	$s_F c_R(p)$	18	-1.26%	-0.84%	-0.57%
Without	$s_F c_R(p)$	19	-0.89%	-0.99%	-0.17%
Without	$s_F c_R(p)$	20	-0.61%	-1.5%	-0.36%
Without	$s_F c_R(p)$	21	-0.88%	-0.99%	-0.34%
Without	$s_F c_R(p)$	22	-0.8%	-1.36%	-0.2%
Without	$s_F c_R(p)$	23	-1.25%	-1.91%	-0.39%
Without	$s_F c_R(p)$	24	-1.6%	-1%	0.14%
Without	$s_F c_R(p)$	25	-1.96%	-0.16%	-0.32%
Without	$s_R c_R(p)$	1	-0.67%	0.43%	1.15%
Without	$s_R c_R(p)$	2	-0.8%	0.52%	1.62%
Without	$s_R c_R(p)$	3	-0.85%	0.64%	1.1%
Without	$s_R c_R(p)$	4	-0.87%	0.16%	1.27%
Without	$s_R c_R(p)$	5	-0.36%	0.53%	1.33%
Without	$s_R c_R(p)$	6	-0.67%	-0.03%	1.11%
Without	$s_R c_R(p)$	7	-0.49%	0.12%	1.42%
Without	$s_R c_R(p)$	8	-0.36%	0.44%	1.39%
Without	$s_R c_R(p)$	9	-0.18%	0.33%	1.16%
Without	$s_R c_R(p)$	10	-0.43%	0.11%	1.21%
Without	$s_R c_R(p)$	11	-0.64%	1.01%	0.97%
Without	$s_R c_R(p)$	12	-0.58%	0.8%	1.03%
Without	$s_R c_R(p)$	13	-0.53%	0.11%	1.11%
Without	$s_R c_R(p)$	14	-0.78%	0.07%	1.17%
Without	$s_R c_R(p)$	15	-0.89%	0.32%	1.56%
Without	$s_R c_R(p)$	16	-0.82%	0.38%	1.42%
Without	$s_R c_R(p)$	17	-0.25%	-0.09%	1.3%
Without	$s_R c_R(p)$	18	-0.49%	0.5%	0.37%
Without	$s_R c_R(p)$	19	-0.09%	0.04%	0.57%
Without	$s_R c_R(p)$	20	-0.01%	-0.53%	0.84%
Without	$s_R c_R(p)$	21	-0.19%	-0.25%	1.04%
Without	$s_R c_R(p)$	22	-0.37%	-0.13%	1.23%
Without	$s_R c_R(p)$	23	-0.91%	-0.99%	0.72%

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Table A.14 – continued from previous page

Annual Abundance Percent Bias					
Aux. Like.	**Model**	**Year**	**S Increasing**	**S Decreasing**	**Periodic Recruitment**
Without	$s_R c_R(p)$	24	-0.55%	-0.1%	1.57%
Without	$s_R c_R(p)$	25	-0.89%	1.18%	1.08%
With	$s_F c_F(p)$	1	9.86%	12.15%	10.93%
With	$s_F c_F(p)$	2	10.86%	11.84%	11.7%
With	$s_F c_F(p)$	3	10.38%	13.67%	11.39%
With	$s_F c_F(p)$	4	10.24%	11.84%	11.82%
With	$s_F c_F(p)$	5	10.29%	12.92%	11.21%
With	$s_F c_F(p)$	6	10.09%	13.18%	11.9%
With	$s_F c_F(p)$	7	10.42%	12.56%	12.03%
With	$s_F c_F(p)$	8	9.93%	12.81%	12.85%
With	$s_F c_F(p)$	9	10.54%	12.91%	11.33%
With	$s_F c_F(p)$	10	10.14%	11.08%	11.83%
With	$s_F c_F(p)$	11	9.17%	12.52%	12.13%
With	$s_F c_F(p)$	12	11.35%	13.36%	11.95%
With	$s_F c_F(p)$	13	9.68%	11.81%	11.23%
With	$s_F c_F(p)$	14	10.94%	12.54%	12.36%
With	$s_F c_F(p)$	15	10.28%	12.75%	11.74%
With	$s_F c_F(p)$	16	10.61%	12.39%	11.84%
With	$s_F c_F(p)$	17	11.36%	12.9%	11.79%
With	$s_F c_F(p)$	18	9.7%	12.37%	10.95%
With	$s_F c_F(p)$	19	10.31%	13.02%	11.32%
With	$s_F c_F(p)$	20	10.02%	13.07%	10.72%
With	$s_F c_F(p)$	21	11.54%	12.31%	11.45%
With	$s_F c_F(p)$	22	9.37%	12.74%	12.76%
With	$s_F c_F(p)$	23	9.95%	12.4%	10.63%
With	$s_F c_F(p)$	24	10.58%	11.66%	12.05%
With	$s_F c_F(p)$	25	10.37%	13.09%	12.47%
With	$N_A s_F c_F(p)$	1	-1.61%	0.51%	-0.7%
With	$N_A s_F c_F(p)$	2	-0.76%	-0.03%	-0.2%
With	$N_A s_F c_F(p)$	3	-0.99%	1.19%	0.28%
With	$N_A s_F c_F(p)$	4	-1.18%	-0.03%	0.44%
With	$N_A s_F c_F(p)$	5	-1.21%	-0.11%	0.23%
With	$N_A s_F c_F(p)$	6	-1.61%	0.64%	-0.27%
With	$N_A s_F c_F(p)$	7	-1.22%	0.36%	0.38%
With	$N_A s_F c_F(p)$	8	-1.24%	0.59%	0.49%
With	$N_A s_F c_F(p)$	9	-1.26%	0.13%	-0.09%
With	$N_A s_F c_F(p)$	10	-0.56%	-0.89%	0.23%
With	$N_A s_F c_F(p)$	11	-1.89%	0.62%	0.21%
With	$N_A s_F c_F(p)$	12	-0.02%	0.64%	-0.88%
With	$N_A s_F c_F(p)$	13	-1.33%	0.01%	-0.49%
With	$N_A s_F c_F(p)$	14	-0.72%	0.27%	-0.4%
With	$N_A s_F c_F(p)$	15	-1.19%	-0.18%	0.13%
With	$N_A s_F c_F(p)$	16	-0.83%	0.07%	0.47%
With	$N_A s_F c_F(p)$	17	-0.59%	0.77%	0.36%
With	$N_A s_F c_F(p)$	18	-0.78%	-0.04%	-0.4%
With	$N_A s_F c_F(p)$	19	-0.83%	0.25%	-0.23%
With	$N_A s_F c_F(p)$	20	-0.78%	0.71%	-0.44%
With	$N_A s_F c_F(p)$	21	-0.29%	0.21%	-1.1%
With	$N_A s_F c_F(p)$	22	-0.98%	1.01%	0.82%
With	$N_A s_F c_F(p)$	23	-0.86%	0.54%	0.05%
With	$N_A s_F c_F(p)$	24	-2.14%	-0.44%	0.01%
With	$N_A s_F c_F(p)$	25	-1.01%	0.89%	0.64%
With	$s_F c_R(p)$	1	-8.04%	-7.25%	-7.05%
With	$s_F c_R(p)$	2	-7.82%	-6.94%	-6.74%
With	$s_F c_R(p)$	3	-8.29%	-7.28%	-6.66%
With	$s_F c_R(p)$	4	-7.88%	-7.28%	-6.57%
With	$s_F c_R(p)$	5	-7.94%	-7.47%	-6.41%
With	$s_F c_R(p)$	6	-7.85%	-7.73%	-6.44%
With	$s_F c_R(p)$	7	-7.44%	-7.34%	-6.86%

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Table A.14 – continued from previous page

Annual Abundance Percent Bias					
Aux. Like.	**Model**	**Year**	**S Increasing**	**S Decreasing**	**Periodic Recruitment**
With	$s_F c_R(p)$	8	-7.19%	-7.91%	-6.48%
With	$s_F c_R(p)$	9	-7.58%	-7.58%	-6.38%
With	$s_F c_R(p)$	10	-7.3%	-7.85%	-6.5%
With	$s_F c_R(p)$	11	-7.4%	-7.09%	-6.51%
With	$s_F c_R(p)$	12	-7.4%	-7.52%	-6.94%
With	$s_F c_R(p)$	13	-7.75%	-7.86%	-6.62%
With	$s_F c_R(p)$	14	-7.64%	-7.66%	-6.86%
With	$s_F c_R(p)$	15	-7.96%	-8.09%	-6.43%
With	$s_F c_R(p)$	16	-7.66%	-7.68%	-6.73%
With	$s_F c_R(p)$	17	-7.54%	-7.83%	-6.55%
With	$s_F c_R(p)$	18	-7.29%	-7.9%	-6.66%
With	$s_F c_R(p)$	19	-7.22%	-8.09%	-6.77%
With	$s_F c_R(p)$	20	-7.24%	-8.18%	-6.51%
With	$s_F c_R(p)$	21	-7.04%	-8.25%	-6.89%
With	$s_F c_R(p)$	22	-7.21%	-8.17%	-6.63%
With	$s_F c_R(p)$	23	-7.92%	-8.39%	-6.73%
With	$s_F c_R(p)$	24	-8.3%	-7.89%	-7.18%
With	$s_F c_R(p)$	25	-8.98%	-7.52%	-6.99%
With	$N_A s_F c_R(p)$	1	-19.27%	-19.44%	-17.9%
With	$N_A s_F c_R(p)$	2	-19.23%	-18.85%	-18%
With	$N_A s_F c_R(p)$	3	-19.58%	-19.09%	-17.99%
With	$N_A s_F c_R(p)$	4	-19.51%	-19.97%	-18.29%
With	$N_A s_F c_R(p)$	5	-19.02%	-19.37%	-17.86%
With	$N_A s_F c_R(p)$	6	-19.34%	-19.5%	-17.85%
With	$N_A s_F c_R(p)$	7	-19.18%	-18.92%	-17.66%
With	$N_A s_F c_R(p)$	8	-18.64%	-19.08%	-18.13%
With	$N_A s_F c_R(p)$	9	-19.17%	-19.28%	-17.86%
With	$N_A s_F c_R(p)$	10	-18.88%	-19.77%	-17.78%
With	$N_A s_F c_R(p)$	11	-19.25%	-19.12%	-18.15%
With	$N_A s_F c_R(p)$	12	-19.18%	-19.62%	-17.94%
With	$N_A s_F c_R(p)$	13	-18.99%	-20%	-17.49%
With	$N_A s_F c_R(p)$	14	-19.21%	-19.27%	-18.3%
With	$N_A s_F c_R(p)$	15	-19.18%	-20.11%	-18.02%
With	$N_A s_F c_R(p)$	16	-18.91%	-19.39%	-18.02%
With	$N_A s_F c_R(p)$	17	-18.79%	-19.74%	-17.82%
With	$N_A s_F c_R(p)$	18	-19.23%	-19.71%	-17.59%
With	$N_A s_F c_R(p)$	19	-18.47%	-19.48%	-18.78%
With	$N_A s_F c_R(p)$	20	-18.68%	-19.47%	-17.55%
With	$N_A s_F c_R(p)$	21	-18.82%	-19.41%	-18.19%
With	$N_A s_F c_R(p)$	22	-18.43%	-19.7%	-18.07%
With	$N_A s_F c_R(p)$	23	-19.07%	-20%	-17.72%
With	$N_A s_F c_R(p)$	24	-19.82%	-19.87%	-18.37%
With	$N_A s_F c_R(p)$	25	-19.97%	-19.36%	-18.02%
With	$s_R c_R(p)$	1	-7.98%	-7.01%	-6.48%
With	$s_R c_R(p)$	2	-7.82%	-7.21%	-6.42%
With	$s_R c_R(p)$	3	-8.57%	-7.23%	-6.61%
With	$s_R c_R(p)$	4	-8.05%	-7.64%	-6.27%
With	$s_R c_R(p)$	5	-8.02%	-7.24%	-6.25%
With	$s_R c_R(p)$	6	-7.65%	-7.62%	-6.26%
With	$s_R c_R(p)$	7	-7.66%	-6.88%	-6.45%
With	$s_R c_R(p)$	8	-7.41%	-7.68%	-6.15%
With	$s_R c_R(p)$	9	-7.66%	-7.56%	-6.35%
With	$s_R c_R(p)$	10	-7.25%	-7.73%	-6.53%
With	$s_R c_R(p)$	11	-7.46%	-7.1%	-6.3%
With	$s_R c_R(p)$	12	-7.33%	-7.37%	-6.38%
With	$s_R c_R(p)$	13	-7.83%	-7.44%	-6.48%
With	$s_R c_R(p)$	14	-7.65%	-7.57%	-6.51%
With	$s_R c_R(p)$	15	-8.03%	-7.79%	-5.9%
With	$s_R c_R(p)$	16	-7.62%	-7.51%	-6.39%

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Table A.14 – continued from previous page

Annual Abundance Percent Bias					
Aux. Like.	**Model**	**Year**	**S Increasing**	**S Decreasing**	**Periodic Recruitment**
With	$s_Rc_R(p)$	17	-7.63%	-7.67%	-6.16%
With	$s_Rc_R(p)$	18	-7.67%	-7.5%	-6.38%
With	$s_Rc_R(p)$	19	-7.49%	-7.65%	-6.7%
With	$s_Rc_R(p)$	20	-7.52%	-7.92%	-6.56%
With	$s_Rc_R(p)$	21	-7.22%	-7.93%	-6.55%
With	$s_Rc_R(p)$	22	-7.44%	-7.48%	-6.4%
With	$s_Rc_R(p)$	23	-7.97%	-8.11%	-6.76%
With	$s_Rc_R(p)$	24	-8.39%	-8.01%	-7.11%
With	$s_Rc_R(p)$	25	-8.79%	-7.41%	-6.62%
With	$N_As_Rc_R(p)$	1	-18.52%	-19.16%	-17.57%
With	$N_As_Rc_R(p)$	2	-18.7%	-18.41%	-17.73%
With	$N_As_Rc_R(p)$	3	-19.26%	-18.85%	-17.8%
With	$N_As_Rc_R(p)$	4	-18.98%	-19.6%	-18.11%
With	$N_As_Rc_R(p)$	5	-18.62%	-18.8%	-17.45%
With	$N_As_Rc_R(p)$	6	-19.09%	-19.22%	-17.72%
With	$N_As_Rc_R(p)$	7	-18.92%	-18.4%	-17.42%
With	$N_As_Rc_R(p)$	8	-18.26%	-18.5%	-17.84%
With	$N_As_Rc_R(p)$	9	-18.69%	-18.72%	-17.61%
With	$N_As_Rc_R(p)$	10	-18.51%	-19.45%	-17.49%
With	$N_As_Rc_R(p)$	11	-18.88%	-18.64%	-17.77%
With	$N_As_Rc_R(p)$	12	-18.87%	-18.99%	-17.8%
With	$N_As_Rc_R(p)$	13	-18.56%	-19.68%	-17.19%
With	$N_As_Rc_R(p)$	14	-18.71%	-19%	-17.88%
With	$N_As_Rc_R(p)$	15	-18.6%	-19.81%	-17.59%
With	$N_As_Rc_R(p)$	16	-18.5%	-19.11%	-17.7%
With	$N_As_Rc_R(p)$	17	-18.08%	-19.18%	-17.23%
With	$N_As_Rc_R(p)$	18	-18.81%	-19.27%	-17.07%
With	$N_As_Rc_R(p)$	19	-17.96%	-18.9%	-18.45%
With	$N_As_Rc_R(p)$	20	-17.99%	-19.02%	-17.34%
With	$N_As_Rc_R(p)$	21	-18.5%	-19.04%	-18.04%
With	$N_As_Rc_R(p)$	22	-18.07%	-19.39%	-17.65%
With	$N_As_Rc_R(p)$	23	-18.95%	-19.73%	-17.26%
With	$N_As_Rc_R(p)$	24	-19.47%	-19.7%	-18.11%
With	$N_As_Rc_R(p)$	25	-19.73%	-19.16%	-17.88%

Table (A.15) contains the data used for plotting Figure (4.7), the median relative bias (MRB) and estimated asymptotic 95% confidence interval coverage for each model, for each year of data, for each level of simulated variation from small game robustness simulations when a high amount of simulated auxiliary data are available for estimating c. Table (A.16) contains the data used for plotting Figure (4.8), the median relative bias (MRB) and estimated asymptotic 95% confidence interval coverage for each model, for each year of data, for each level of simulated variation from small game robustness simulations when a low amount of simulated auxiliary data are available for estimating c.

Table A.15: *95% confidence interval coverage and median relative bias in total annual abundance estimates from small game robustness simulations when a high amount of simulated auxiliary data is available for estimating c. Results indicate nearest nominal coverage for models employing the Horvitz-Thompson abundance estimator in all scenarios, although it remains subnominal (between 85% and 90% when the auxiliary catch-effort likelihood of Equation (1.7) is omitted). Absolute-recruit abundance models show low confidence interval coverage in each scenario (between 20% and 50%). Results based on* $n = 1000$ *replicates. "Aux. Like." = Auxiliary catch-effort likelihood used (With) or not used (Without).*

Table A.15 - Annual Abundance 95% CI Coverage and Median Relative Bias

			S Increasing		S Decreasing		Periodic Recruitment	
Aux. Like.	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
Without	$s_F c_F(p)$	1	30.20	-1.12	31.60	0.37	32.00	-0.51
Without	$s_F c_F(p)$	2	27.60	0.03	27.00	-0.36	31.40	-0.44
Without	$s_F c_F(p)$	3	30.60	-0.45	28.20	0.66	31.50	-0.49
Without	$s_F c_F(p)$	4	29.50	-1.02	28.00	-0.28	29.80	-1.05
Without	$s_F c_F(p)$	5	30.70	-0.95	28.40	0.44	32.80	-0.92
Without	$s_F c_F(p)$	6	30.90	-1.26	30.30	1.21	31.40	-0.07
Without	$s_F c_F(p)$	7	30.50	-1.01	30.00	0.49	34.80	-0.76
Without	$s_F c_F(p)$	8	31.00	-1.77	30.50	0.87	33.00	-0.96
Without	$s_F c_F(p)$	9	31.90	-0.65	30.00	0.88	33.30	-0.51
Without	$s_F c_F(p)$	10	28.80	-1.03	29.30	-0.24	32.90	0.34
Without	$s_F c_F(p)$	11	29.10	-1.77	29.00	0.63	30.70	-0.60
Without	$s_F c_F(p)$	12	29.50	-0.36	31.40	0.15	30.30	-0.92
Without	$s_F c_F(p)$	13	29.30	-0.54	30.40	0.21	32.80	-1.15
Without	$s_F c_F(p)$	14	32.60	-0.64	28.70	0.43	30.00	-1.06
Without	$s_F c_F(p)$	15	30.40	-0.94	29.80	0.44	30.70	-0.50
Without	$s_F c_F(p)$	16	30.10	-0.84	29.60	0.51	33.40	-0.50
Without	$s_F c_F(p)$	17	30.00	-1.11	31.20	0.09	32.20	-0.49
Without	$s_F c_F(p)$	18	30.90	-0.94	28.80	0.16	31.90	-0.79
Without	$s_F c_F(p)$	19	30.20	-1.03	31.50	0.22	32.20	-1.07
Without	$s_F c_F(p)$	20	30.80	-0.48	31.20	0.34	33.20	-0.52
Without	$s_F c_F(p)$	21	30.10	-0.11	30.30	-0.09	31.90	-0.62
Without	$s_F c_F(p)$	22	29.70	-1.36	30.40	0.87	34.30	-0.06
Without	$s_F c_F(p)$	23	30.60	-0.36	31.00	0.56	33.70	-1.01
Without	$s_F c_F(p)$	24	29.40	-1.09	29.30	0.31	29.70	-1.22
Without	$s_F c_F(p)$	25	30.30	-1.15	30.10	1.26	33.50	-0.48
Without	$s_F c_R(p)$	1	88.50	-1.57	90.60	0.23	91.10	-0.88

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Table A.15 – continued from previous page

			S Increasing		S Decreasing		Periodic Recruitment	
Aux. Like.	Model	Year	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB
Without	$s_F c_R(p)$	2	88.20	-1.56	91.20	0.83	91.60	-0.47
Without	$s_F c_R(p)$	3	87.80	-1.65	91.20	0.54	91.00	-0.40
Without	$s_F c_R(p)$	4	87.70	-1.36	90.60	0.27	91.90	-0.53
Without	$s_F c_R(p)$	5	88.50	-1.40	91.00	0.21	91.00	-0.54
Without	$s_F c_R(p)$	6	87.80	-0.98	91.00	0.25	91.10	-0.45
Without	$s_F c_R(p)$	7	88.70	-0.95	92.00	0.27	91.70	-0.56
Without	$s_F c_R(p)$	8	89.60	-0.66	91.40	0.12	92.00	-0.63
Without	$s_F c_R(p)$	9	88.80	-0.76	91.70	-0.19	93.60	-0.82
Without	$s_F c_R(p)$	10	90.60	-0.91	92.30	-0.53	92.60	-0.40
Without	$s_F c_R(p)$	11	90.90	-0.66	92.80	-0.40	93.00	-0.45
Without	$s_F c_R(p)$	12	90.10	-0.79	93.30	-0.15	92.00	-0.71
Without	$s_F c_R(p)$	13	90.30	-0.74	92.10	-0.10	93.00	-0.79
Without	$s_F c_R(p)$	14	88.90	-0.88	91.50	-0.26	92.10	-0.61
Without	$s_F c_R(p)$	15	89.70	-0.56	92.40	-0.33	91.40	-0.82
Without	$s_F c_R(p)$	16	89.30	-0.94	92.30	-0.22	91.30	-0.85
Without	$s_F c_R(p)$	17	89.50	-0.82	92.40	-0.14	93.10	-0.85
Without	$s_F c_R(p)$	18	88.40	-0.50	91.60	-0.37	91.80	-0.71
Without	$s_F c_R(p)$	19	89.50	-0.48	91.90	-0.34	91.40	-1.16
Without	$s_F c_R(p)$	20	89.10	-0.71	92.10	-0.59	90.60	-0.72
Without	$s_F c_R(p)$	21	89.10	-0.58	90.10	-0.71	91.10	-1.01
Without	$s_F c_R(p)$	22	88.50	-0.68	91.60	-0.39	92.00	-0.71
Without	$s_F c_R(p)$	23	89.10	-1.03	90.50	-0.27	91.10	-0.91
Without	$s_F c_R(p)$	24	90.20	-1.32	91.20	-0.30	92.50	-0.82
Without	$s_F c_R(p)$	25	90.70	-1.48	90.00	0.19	90.60	-0.41
Without	$s_R c_R(p)$	1	89.40	-0.87	91.40	0.61	91.50	0.15
Without	$s_R c_R(p)$	2	88.80	-0.21	92.10	0.99	92.40	0.17
Without	$s_R c_R(p)$	3	88.60	-0.87	90.60	0.77	92.00	0.24
Without	$s_R c_R(p)$	4	88.10	-0.63	90.70	0.23	90.90	-0.19
Without	$s_R c_R(p)$	5	88.80	-0.51	91.60	0.69	91.10	0.12
Without	$s_R c_R(p)$	6	88.50	-0.53	91.50	0.94	92.60	0.22
Without	$s_R c_R(p)$	7	89.20	-0.77	91.80	0.65	92.70	-0.22
Without	$s_R c_R(p)$	8	89.70	-0.34	92.10	0.54	92.10	-0.33
Without	$s_R c_R(p)$	9	88.60	-0.54	92.90	0.66	93.80	-0.05
Without	$s_R c_R(p)$	10	90.70	-0.80	92.00	0.01	93.60	0.47
Without	$s_R c_R(p)$	11	90.80	-0.34	93.30	0.34	93.60	0.16
Without	$s_R c_R(p)$	12	90.60	-0.31	93.40	0.28	92.60	-0.05
Without	$s_R c_R(p)$	13	91.70	-0.05	92.50	0.28	93.80	-0.15
Without	$s_R c_R(p)$	14	89.90	-0.25	91.90	0.67	92.60	0.16
Without	$s_R c_R(p)$	15	90.30	0.24	92.40	0.40	91.20	-0.08
Without	$s_R c_R(p)$	16	90.30	-0.18	92.10	0.58	91.70	-0.03
Without	$s_R c_R(p)$	17	90.30	-0.43	92.80	0.42	93.30	-0.36
Without	$s_R c_R(p)$	18	89.30	-0.23	92.60	0.11	92.00	-0.32
Without	$s_R c_R(p)$	19	89.40	-0.09	92.20	0.14	92.30	-0.40
Without	$s_R c_R(p)$	20	90.20	-0.05	91.90	0.33	91.40	-0.26
Without	$s_R c_R(p)$	21	90.10	-0.22	90.50	-0.02	91.30	-0.24
Without	$s_R c_R(p)$	22	90.20	-0.01	91.50	0.27	92.00	0.18
Without	$s_R c_R(p)$	23	90.50	-0.37	91.00	0.11	91.40	-0.09
Without	$s_R c_R(p)$	24	89.70	-0.71	91.20	0.02	91.60	-0.30
Without	$s_R c_R(p)$	25	90.40	-0.52	88.70	0.51	91.60	0.33
With	$s_F c_F(p)$	1	20.80	8.46	18.00	10.92	22.40	9.09
With	$s_F c_F(p)$	2	19.20	10.00	19.30	9.89	23.00	8.86
With	$s_F c_F(p)$	3	19.80	9.41	19.40	11.36	20.80	8.36
With	$s_F c_F(p)$	4	19.20	8.79	17.40	10.45	21.50	9.48
With	$s_F c_F(p)$	5	20.40	9.41	18.30	11.07	23.00	8.62
With	$s_F c_F(p)$	6	21.50	8.74	16.10	11.80	21.30	9.77
With	$s_F c_F(p)$	7	19.20	8.98	19.20	11.14	23.10	8.92
With	$s_F c_F(p)$	8	19.60	8.44	17.00	11.77	22.20	8.77
With	$s_F c_F(p)$	9	17.40	9.21	17.10	11.57	22.50	8.71
With	$s_F c_F(p)$	10	18.20	9.04	17.70	10.01	21.30	9.98

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Table A.15 – continued from previous page

			S Increasing		S Decreasing		Periodic Recruitment	
Aux. Like.	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
With	$s_F c_F(p)$	11	20.70	7.86	19.20	10.95	21.30	8.43
With	$s_F c_F(p)$	12	19.30	9.66	15.90	11.52	22.70	8.89
With	$s_F c_F(p)$	13	19.50	9.13	17.00	10.57	23.80	8.11
With	$s_F c_F(p)$	14	20.10	9.74	19.40	10.12	22.70	8.82
With	$s_F c_F(p)$	15	20.10	8.61	17.80	11.40	22.50	8.59
With	$s_F c_F(p)$	16	19.50	8.58	18.80	11.19	23.60	8.63
With	$s_F c_F(p)$	17	18.30	9.49	18.30	10.86	23.40	8.32
With	$s_F c_F(p)$	18	18.90	9.22	18.30	10.70	22.60	9.07
With	$s_F c_F(p)$	19	17.60	9.35	17.00	11.91	22.60	8.90
With	$s_F c_F(p)$	20	18.70	9.49	18.70	11.61	23.60	9.03
With	$s_F c_F(p)$	21	19.20	10.51	18.30	11.45	23.20	9.30
With	$s_F c_F(p)$	22	19.70	9.17	18.40	12.30	22.90	10.19
With	$s_F c_F(p)$	23	18.60	8.84	18.20	11.34	23.80	8.43
With	$s_F c_F(p)$	24	18.70	9.18	20.70	10.24	22.30	8.64
With	$s_F c_F(p)$	25	19.70	8.66	18.30	11.56	22.00	9.44
With	$N_A s_F c_F(p)$	1	27.00	-0.92	28.00	0.75	29.70	-0.50
With	$N_A s_F c_F(p)$	2	26.50	0.33	26.30	-0.00	28.00	-0.60
With	$N_A s_F c_F(p)$	3	28.40	-0.26	25.70	0.94	30.40	-0.52
With	$N_A s_F c_F(p)$	4	27.40	-0.73	25.80	0.54	26.60	-0.62
With	$N_A s_F c_F(p)$	5	28.80	-0.84	27.30	0.20	30.50	-0.17
With	$N_A s_F c_F(p)$	6	27.80	-1.38	28.30	1.37	30.40	0.06
With	$N_A s_F c_F(p)$	7	28.30	-0.87	28.00	0.52	30.80	-0.36
With	$N_A s_F c_F(p)$	8	28.40	-1.17	29.10	0.80	29.80	-0.95
With	$N_A s_F c_F(p)$	9	27.50	-0.46	27.90	1.01	31.00	-0.55
With	$N_A s_F c_F(p)$	10	27.10	-0.11	27.80	-0.06	29.70	0.46
With	$N_A s_F c_F(p)$	11	27.50	-1.34	26.70	0.96	29.00	-0.49
With	$N_A s_F c_F(p)$	12	26.60	0.38	29.10	0.80	27.80	-1.01
With	$N_A s_F c_F(p)$	13	27.60	-0.79	28.70	0.57	28.90	-1.58
With	$N_A s_F c_F(p)$	14	28.60	-0.24	27.10	0.50	28.40	-1.13
With	$N_A s_F c_F(p)$	15	28.40	-0.41	27.20	0.15	29.50	-0.30
With	$N_A s_F c_F(p)$	16	27.70	-0.40	27.90	0.84	30.90	-0.21
With	$N_A s_F c_F(p)$	17	30.30	-0.30	28.30	0.34	29.00	-0.92
With	$N_A s_F c_F(p)$	18	29.90	0.00	28.40	0.44	29.90	-1.08
With	$N_A s_F c_F(p)$	19	27.30	-0.49	29.40	0.27	28.50	-1.04
With	$N_A s_F c_F(p)$	20	28.00	-0.19	29.20	0.91	28.70	-0.24
With	$N_A s_F c_F(p)$	21	27.90	0.06	28.40	1.02	29.60	-0.67
With	$N_A s_F c_F(p)$	22	28.30	-0.80	28.40	1.20	30.20	0.02
With	$N_A s_F c_F(p)$	23	26.50	-0.39	29.30	0.92	31.20	-0.71
With	$N_A s_F c_F(p)$	24	26.80	-0.99	28.10	0.02	27.90	-1.16
With	$N_A s_F c_F(p)$	25	27.70	-0.66	27.30	1.42	29.00	0.08
With	$s_F c_R(p)$	1	82.10	-4.58	87.30	-2.70	87.20	-3.67
With	$s_F c_R(p)$	2	82.80	-4.23	87.90	-2.27	87.40	-3.27
With	$s_F c_R(p)$	3	83.30	-4.84	88.70	-2.22	86.60	-3.42
With	$s_F c_R(p)$	4	83.50	-4.29	86.80	-2.71	86.20	-3.14
With	$s_F c_R(p)$	5	84.30	-4.57	87.60	-2.78	86.90	-3.45
With	$s_F c_R(p)$	6	83.90	-4.04	88.70	-2.81	88.00	-3.25
With	$s_F c_R(p)$	7	82.30	-3.81	88.60	-2.50	87.90	-3.36
With	$s_F c_R(p)$	8	85.50	-3.86	88.60	-2.89	87.00	-3.19
With	$s_F c_R(p)$	9	85.30	-3.69	89.50	-3.13	87.40	-3.33
With	$s_F c_R(p)$	10	86.20	-4.30	89.10	-3.13	88.10	-3.08
With	$s_F c_R(p)$	11	85.70	-3.60	90.90	-3.13	88.10	-3.22
With	$s_F c_R(p)$	12	85.10	-3.74	90.50	-3.11	88.10	-3.33
With	$s_F c_R(p)$	13	85.00	-3.77	89.10	-3.27	88.20	-3.60
With	$s_F c_R(p)$	14	84.30	-4.03	87.40	-3.14	86.80	-3.33
With	$s_F c_R(p)$	15	84.00	-3.52	87.60	-3.17	87.30	-3.37
With	$s_F c_R(p)$	16	84.70	-4.11	87.60	-3.05	88.10	-3.60
With	$s_F c_R(p)$	17	84.40	-3.69	88.90	-2.71	89.00	-3.75
With	$s_F c_R(p)$	18	83.50	-3.65	88.30	-3.27	87.30	-3.36
With	$s_F c_R(p)$	19	85.20	-3.68	87.20	-3.27	86.90	-3.74

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Table A.15 – continued from previous page

			S Increasing		S Decreasing		Periodic Recruitment	
Aux. Like.	Model	Year	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB
With	$s_F c_R(p)$	20	84.30	-3.68	86.60	-3.40	87.80	-3.43
With	$s_F c_R(p)$	21	85.30	-3.58	86.90	-3.67	87.10	-3.83
With	$s_F c_R(p)$	22	84.90	-3.71	86.80	-3.18	88.50	-3.64
With	$s_F c_R(p)$	23	84.50	-4.07	86.10	-3.28	87.00	-3.51
With	$s_F c_R(p)$	24	85.10	-4.70	88.10	-3.50	88.40	-3.83
With	$s_F c_R(p)$	25	86.00	-4.93	88.30	-3.00	87.50	-3.55
With	$N_A s_F c_R(p)$	1	59.80	-9.82	70.90	-7.82	67.10	-8.18
With	$N_A s_F c_R(p)$	2	62.70	-9.72	70.70	-7.53	68.70	-8.34
With	$N_A s_F c_R(p)$	3	63.60	-10.17	72.40	-7.57	68.50	-8.21
With	$N_A s_F c_R(p)$	4	62.90	-9.90	71.10	-8.14	69.10	-8.49
With	$N_A s_F c_R(p)$	5	64.20	-9.98	70.60	-7.88	70.10	-8.20
With	$N_A s_F c_R(p)$	6	65.10	-9.91	71.00	-7.96	69.50	-8.16
With	$N_A s_F c_R(p)$	7	63.10	-9.61	71.80	-7.80	69.70	-8.16
With	$N_A s_F c_R(p)$	8	64.70	-9.47	71.70	-8.05	70.10	-8.35
With	$N_A s_F c_R(p)$	9	66.30	-9.48	71.10	-8.13	70.10	-8.51
With	$N_A s_F c_R(p)$	10	65.80	-9.73	71.40	-8.49	70.70	-8.29
With	$N_A s_F c_R(p)$	11	66.60	-9.25	71.30	-8.34	69.90	-8.21
With	$N_A s_F c_R(p)$	12	66.10	-9.36	70.50	-8.40	69.30	-8.45
With	$N_A s_F c_R(p)$	13	64.70	-9.45	70.80	-8.59	69.40	-8.49
With	$N_A s_F c_R(p)$	14	64.90	-9.58	69.80	-8.46	69.00	-8.32
With	$N_A s_F c_R(p)$	15	64.20	-9.38	69.00	-8.44	69.60	-8.47
With	$N_A s_F c_R(p)$	16	64.60	-9.51	70.20	-8.33	69.40	-8.77
With	$N_A s_F c_R(p)$	17	65.00	-9.13	70.90	-8.32	69.30	-8.64
With	$N_A s_F c_R(p)$	18	64.80	-9.17	69.30	-8.18	69.40	-8.41
With	$N_A s_F c_R(p)$	19	65.00	-9.26	68.50	-8.45	68.90	-8.65
With	$N_A s_F c_R(p)$	20	64.70	-9.25	68.40	-8.77	70.90	-8.58
With	$N_A s_F c_R(p)$	21	66.30	-8.97	68.20	-8.50	70.70	-8.50
With	$N_A s_F c_R(p)$	22	66.10	-9.13	69.00	-8.52	70.10	-8.47
With	$N_A s_F c_R(p)$	23	65.40	-9.62	69.90	-8.34	69.40	-8.40
With	$N_A s_F c_R(p)$	24	65.60	-10.28	72.00	-8.56	71.20	-8.94
With	$N_A s_F c_R(p)$	25	64.60	-10.65	72.60	-8.40	69.90	-8.58
With	$s_R c_R(p)$	1	83.80	-4.07	87.80	-2.63	87.30	-3.15
With	$s_R c_R(p)$	2	84.00	-3.63	88.20	-2.24	88.00	-2.89
With	$s_R c_R(p)$	3	84.90	-4.09	89.00	-2.57	88.00	-2.89
With	$s_R c_R(p)$	4	84.30	-3.80	87.30	-3.02	87.50	-3.05
With	$s_R c_R(p)$	5	85.10	-3.97	87.30	-2.57	87.30	-2.77
With	$s_R c_R(p)$	6	84.50	-3.83	88.50	-2.37	88.40	-2.92
With	$s_R c_R(p)$	7	83.70	-3.78	87.80	-2.35	89.00	-3.29
With	$s_R c_R(p)$	8	84.80	-3.58	89.10	-2.60	87.60	-3.20
With	$s_R c_R(p)$	9	86.20	-3.70	89.40	-2.64	88.30	-3.18
With	$s_R c_R(p)$	10	87.40	-4.09	89.00	-2.77	89.50	-2.56
With	$s_R c_R(p)$	11	86.20	-3.44	90.90	-2.64	88.90	-2.61
With	$s_R c_R(p)$	12	86.80	-3.57	90.40	-2.77	89.50	-2.81
With	$s_R c_R(p)$	13	88.10	-3.33	89.30	-2.74	89.70	-3.18
With	$s_R c_R(p)$	14	85.80	-3.53	88.00	-2.50	89.10	-2.84
With	$s_R c_R(p)$	15	85.50	-3.08	88.50	-2.76	87.80	-2.91
With	$s_R c_R(p)$	16	84.90	-3.56	87.80	-2.54	88.80	-3.24
With	$s_R c_R(p)$	17	86.00	-3.68	89.40	-2.67	88.90	-3.39
With	$s_R c_R(p)$	18	84.80	-3.67	88.80	-3.10	88.00	-3.16
With	$s_R c_R(p)$	19	85.50	-3.31	88.50	-2.95	87.90	-3.43
With	$s_R c_R(p)$	20	85.30	-3.17	87.60	-3.14	87.80	-3.39
With	$s_R c_R(p)$	21	86.00	-3.44	87.60	-3.21	88.30	-3.31
With	$s_R c_R(p)$	22	86.30	-3.63	87.90	-3.03	88.90	-3.03
With	$s_R c_R(p)$	23	84.80	-3.67	86.40	-3.20	86.90	-3.20
With	$s_R c_R(p)$	24	85.30	-4.35	88.00	-3.46	88.60	-3.43
With	$s_R c_R(p)$	25	85.90	-4.39	87.60	-2.87	87.40	-3.11
With	$N_A s_R c_R(p)$	1	60.30	-9.74	71.30	-7.72	67.40	-8.15
With	$N_A s_R c_R(p)$	2	63.60	-9.54	71.20	-7.38	69.60	-8.26
With	$N_A s_R c_R(p)$	3	64.40	-9.81	72.50	-7.20	69.90	-8.02

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Table A.15 – continued from previous page

			S Increasing		S Decreasing		Periodic Recruitment	
Aux. Like.	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
With	$N_A s_R c_R(p)$	4	64.00	-9.78	71.40	-7.90	69.70	-8.14
With	$N_A s_R c_R(p)$	5	64.70	-9.67	71.00	-7.38	71.70	-8.07
With	$N_A s_R c_R(p)$	6	66.30	-9.77	71.70	-7.54	70.60	-7.98
With	$N_A s_R c_R(p)$	7	64.10	-9.26	71.90	-7.54	70.60	-7.97
With	$N_A s_R c_R(p)$	8	65.50	-9.31	71.80	-7.87	70.90	-8.15
With	$N_A s_R c_R(p)$	9	67.50	-9.34	71.40	-7.79	71.00	-8.17
With	$N_A s_R c_R(p)$	10	66.70	-9.24	72.00	-8.27	71.70	-7.94
With	$N_A s_R c_R(p)$	11	67.50	-8.98	71.60	-8.11	70.70	-7.89
With	$N_A s_R c_R(p)$	12	66.80	-9.19	71.30	-7.96	70.40	-8.04
With	$N_A s_R c_R(p)$	13	65.80	-9.30	71.00	-8.29	70.10	-8.20
With	$N_A s_R c_R(p)$	14	65.90	-9.32	70.70	-8.14	70.00	-8.03
With	$N_A s_R c_R(p)$	15	64.90	-9.27	69.90	-8.28	70.50	-7.98
With	$N_A s_R c_R(p)$	16	65.30	-9.25	71.00	-7.96	70.20	-8.63
With	$N_A s_R c_R(p)$	17	65.60	-8.92	71.30	-8.12	70.60	-8.32
With	$N_A s_R c_R(p)$	18	65.30	-9.03	69.70	-8.00	70.00	-8.09
With	$N_A s_R c_R(p)$	19	65.90	-9.09	68.80	-8.01	70.30	-8.50
With	$N_A s_R c_R(p)$	20	65.80	-9.10	68.70	-8.66	71.60	-8.37
With	$N_A s_R c_R(p)$	21	67.20	-8.61	68.90	-8.36	70.90	-8.09
With	$N_A s_R c_R(p)$	22	66.90	-8.98	69.20	-8.38	71.00	-8.24
With	$N_A s_R c_R(p)$	23	66.70	-9.34	70.20	-8.15	70.50	-8.21
With	$N_A s_R c_R(p)$	24	66.40	-10.08	72.30	-8.44	72.40	-8.71
With	$N_A s_R c_R(p)$	25	65.50	-10.29	73.10	-8.10	71.10	-8.14

Table A.16: *95% confidence interval coverage and median relative bias in total annual abundance estimates from small game robustness simulations when a low amount of simulated auxiliary data is available for estimating c. Results indicate nearest nominal coverage for models employing the Horvitz-Thompson abundance estimator in all scenarios, although it remains subnominal (between 85% and 90% when the auxiliary catch-effort likelihood of Equation (1.7) is omitted). Absolute-recruit abundance models show low confidence interval coverage in each scenario (between 20% and 50%). Results based on* $n = 1000$ *replicates. "Aux. Like." = Auxiliary catch-effort likelihood used (With) or not used (Without).*

Table A.16 - Annual Abundance 95% CI Coverage and Median Relative Bias

			S Increasing		S Decreasing		Periodic Recruitment	
Aux. Like.	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
Without	$s_F c_F(p)$	1	29.50	-1.46	29.80	0.18	30.00	-1.07
Without	$s_F c_F(p)$	2	27.30	-0.60	26.30	0.07	28.40	-0.05
Without	$s_F c_F(p)$	3	30.00	-1.03	27.70	0.52	30.30	-0.11
Without	$s_F c_F(p)$	4	29.60	-1.36	27.30	-0.30	28.30	-0.49
Without	$s_F c_F(p)$	5	29.70	-1.24	27.50	0.59	31.00	-0.02
Without	$s_F c_F(p)$	6	30.00	-1.33	28.80	1.38	30.00	-0.37
Without	$s_F c_F(p)$	7	30.00	-1.42	28.30	0.19	30.90	-0.28
Without	$s_F c_F(p)$	8	30.60	-1.56	30.00	0.57	32.90	-0.05
Without	$s_F c_F(p)$	9	31.00	-1.29	28.90	0.20	29.80	-0.58
Without	$s_F c_F(p)$	10	27.50	-1.35	26.50	-0.54	28.60	0.32
Without	$s_F c_F(p)$	11	27.30	-2.26	27.80	0.14	30.10	0.05
Without	$s_F c_F(p)$	12	29.90	-0.36	28.90	0.15	29.70	0.17
Without	$s_F c_F(p)$	13	29.00	-0.83	28.40	-0.08	30.40	-0.37

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Table A.16 – continued from previous page

			S Increasing		S Decreasing		Periodic Recruitment	
Aux. Like.	Model	Year	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB
Without	$s_F c_F(p)$	14	30.80	-0.82	27.40	0.07	28.80	-1.35
Without	$s_F c_F(p)$	15	28.50	-1.77	28.80	0.52	30.10	0.12
Without	$s_F c_F(p)$	16	28.40	-0.94	28.10	-0.13	32.00	0.64
Without	$s_F c_F(p)$	17	28.50	-1.07	28.30	0.23	29.80	-0.10
Without	$s_F c_F(p)$	18	28.70	-1.32	27.00	-0.11	30.30	-0.14
Without	$s_F c_F(p)$	19	30.30	-1.39	27.60	0.19	30.10	-0.36
Without	$s_F c_F(p)$	20	28.60	-1.18	28.60	0.38	30.60	-0.89
Without	$s_F c_F(p)$	21	30.00	-0.11	28.40	-0.27	29.60	-0.47
Without	$s_F c_F(p)$	22	29.50	-1.98	28.30	0.71	30.20	0.54
Without	$s_F c_F(p)$	23	29.10	-1.12	29.60	0.48	29.50	-0.49
Without	$s_F c_F(p)$	24	27.60	-1.57	28.10	-0.41	29.90	-0.68
Without	$s_F c_F(p)$	25	29.50	-1.49	29.20	0.66	30.40	-0.23
Without	$s_F c_R(p)$	1	86.30	-1.97	85.00	-0.17	85.80	0.26
Without	$s_F c_R(p)$	2	86.40	-2.10	85.90	0.24	86.50	-0.14
Without	$s_F c_R(p)$	3	86.40	-1.39	87.40	-0.33	86.70	0.34
Without	$s_F c_R(p)$	4	86.90	-1.38	86.60	-0.24	87.20	0.04
Without	$s_F c_R(p)$	5	86.30	-1.24	86.40	-0.29	87.30	-0.13
Without	$s_F c_R(p)$	6	85.90	-1.28	87.20	-0.46	87.60	0.19
Without	$s_F c_R(p)$	7	86.00	-1.06	86.70	-0.72	86.70	-0.13
Without	$s_F c_R(p)$	8	87.00	-0.85	86.70	-0.45	87.30	0.02
Without	$s_F c_R(p)$	9	87.20	-0.93	86.60	-0.56	87.10	0.11
Without	$s_F c_R(p)$	10	87.40	-0.95	88.40	-0.64	87.90	0.21
Without	$s_F c_R(p)$	11	86.90	-0.99	88.70	0.03	88.00	-0.12
Without	$s_F c_R(p)$	12	85.40	-0.84	86.10	-0.19	86.60	-0.23
Without	$s_F c_R(p)$	13	86.60	-1.39	85.50	-1.09	85.70	0.10
Without	$s_F c_R(p)$	14	87.20	-1.24	85.10	-0.99	87.30	0.27
Without	$s_F c_R(p)$	15	86.50	-1.45	85.60	-0.97	88.30	0.43
Without	$s_F c_R(p)$	16	86.80	-1.26	86.10	-0.67	86.90	-0.04
Without	$s_F c_R(p)$	17	87.80	-1.21	86.90	-1.22	87.90	-0.23
Without	$s_F c_R(p)$	18	86.90	-1.26	86.20	-0.84	86.80	-0.57
Without	$s_F c_R(p)$	19	86.70	-0.89	85.90	-0.99	86.30	-0.17
Without	$s_F c_R(p)$	20	87.50	-0.61	85.60	-1.50	87.60	-0.36
Without	$s_F c_R(p)$	21	87.50	-0.88	86.50	-0.99	86.70	-0.34
Without	$s_F c_R(p)$	22	86.10	-0.80	86.90	-1.36	86.40	-0.20
Without	$s_F c_R(p)$	23	87.50	-1.25	87.10	-1.91	87.00	-0.39
Without	$s_F c_R(p)$	24	86.40	-1.60	87.20	-1.00	87.50	0.14
Without	$s_F c_R(p)$	25	86.30	-1.96	87.80	-0.16	87.70	-0.32
Without	$s_R c_R(p)$	1	87.30	-0.67	85.80	0.43	87.50	1.15
Without	$s_R c_R(p)$	2	88.10	-0.80	86.50	0.52	88.10	1.62
Without	$s_R c_R(p)$	3	87.50	-0.85	88.20	0.64	88.80	1.10
Without	$s_R c_R(p)$	4	88.00	-0.87	87.10	0.16	88.70	1.27
Without	$s_R c_R(p)$	5	87.20	-0.36	87.40	0.53	88.80	1.33
Without	$s_R c_R(p)$	6	87.50	-0.67	87.90	-0.03	88.70	1.11
Without	$s_R c_R(p)$	7	87.60	-0.49	87.50	0.12	88.40	1.42
Without	$s_R c_R(p)$	8	88.50	-0.36	87.40	0.44	88.90	1.39
Without	$s_R c_R(p)$	9	87.30	-0.18	88.00	0.33	88.60	1.16
Without	$s_R c_R(p)$	10	87.90	-0.43	88.70	0.11	88.80	1.21
Without	$s_R c_R(p)$	11	87.10	-0.64	88.90	1.01	89.00	0.97
Without	$s_R c_R(p)$	12	86.40	-0.58	86.90	0.80	88.30	1.03
Without	$s_R c_R(p)$	13	87.20	-0.53	85.70	0.11	88.20	1.11
Without	$s_R c_R(p)$	14	87.70	-0.78	86.30	0.07	89.50	1.17
Without	$s_R c_R(p)$	15	87.60	-0.89	87.50	0.32	89.70	1.56
Without	$s_R c_R(p)$	16	88.20	-0.82	87.70	0.38	88.40	1.42
Without	$s_R c_R(p)$	17	88.60	-0.25	87.90	-0.09	89.80	1.30
Without	$s_R c_R(p)$	18	87.60	-0.49	87.40	0.50	88.60	0.37
Without	$s_R c_R(p)$	19	88.20	-0.09	87.30	0.04	87.80	0.57
Without	$s_R c_R(p)$	20	88.60	-0.01	87.20	-0.53	89.00	0.84
Without	$s_R c_R(p)$	21	88.80	-0.19	87.50	-0.25	88.20	1.04
Without	$s_R c_R(p)$	22	87.80	-0.37	87.10	-0.13	88.10	1.23

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Table A.16 – continued from previous page

			S Increasing		S Decreasing		Periodic Recruitment	
Aux. Like.	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
Without	$s_R c_R(p)$	23	88.00	-0.91	88.40	-0.99	88.60	0.72
Without	$s_R c_R(p)$	24	87.00	-0.55	87.50	-0.10	88.90	1.57
Without	$s_R c_R(p)$	25	86.90	-0.89	88.00	1.18	88.20	1.08
With	$s_F c_F(p)$	1	20.40	9.86	16.60	12.15	20.90	10.93
With	$s_F c_F(p)$	2	18.70	10.86	17.80	11.84	20.10	11.70
With	$s_F c_F(p)$	3	20.10	10.38	17.90	13.67	18.50	11.39
With	$s_F c_F(p)$	4	18.40	10.24	16.90	11.84	19.50	11.82
With	$s_F c_F(p)$	5	21.70	10.29	17.30	12.92	19.70	11.21
With	$s_F c_F(p)$	6	20.90	10.09	16.60	13.18	19.50	11.90
With	$s_F c_F(p)$	7	19.70	10.42	19.30	12.56	18.40	12.03
With	$s_F c_F(p)$	8	19.70	9.93	17.10	12.81	18.30	12.85
With	$s_F c_F(p)$	9	17.10	10.54	16.40	12.91	19.40	11.33
With	$s_F c_F(p)$	10	19.10	10.14	17.80	11.08	19.40	11.83
With	$s_F c_F(p)$	11	19.30	9.17	18.30	12.52	19.00	12.13
With	$s_F c_F(p)$	12	19.90	11.35	15.20	13.36	20.70	11.95
With	$s_F c_F(p)$	13	20.00	9.68	16.10	11.81	19.80	11.23
With	$s_F c_F(p)$	14	19.20	10.94	18.30	12.54	20.30	12.36
With	$s_F c_F(p)$	15	20.60	10.28	17.20	12.75	19.30	11.74
With	$s_F c_F(p)$	16	19.90	10.61	18.20	12.39	18.20	11.84
With	$s_F c_F(p)$	17	18.50	11.36	17.40	12.90	19.40	11.79
With	$s_F c_F(p)$	18	18.60	9.70	18.20	12.37	19.10	10.95
With	$s_F c_F(p)$	19	17.10	10.31	16.00	13.02	19.50	11.32
With	$s_F c_F(p)$	20	19.30	10.02	19.00	13.07	22.30	10.72
With	$s_F c_F(p)$	21	18.00	11.54	17.90	12.31	21.50	11.45
With	$s_F c_F(p)$	22	20.40	9.37	17.40	12.74	20.60	12.76
With	$s_F c_F(p)$	23	19.10	9.95	16.70	12.40	21.40	10.63
With	$s_F c_F(p)$	24	18.20	10.58	20.70	11.66	19.60	12.05
With	$s_F c_F(p)$	25	20.00	10.37	17.60	13.09	18.80	12.47
With	$N_A s_F c_F(p)$	1	27.70	-1.61	26.70	0.51	26.90	-0.70
With	$N_A s_F c_F(p)$	2	25.60	-0.76	26.00	-0.03	25.90	-0.20
With	$N_A s_F c_F(p)$	3	28.20	-0.99	23.20	1.19	28.00	0.28
With	$N_A s_F c_F(p)$	4	27.10	-1.18	23.00	-0.03	24.80	0.44
With	$N_A s_F c_F(p)$	5	27.60	-1.21	25.10	-0.11	27.30	0.23
With	$N_A s_F c_F(p)$	6	27.30	-1.61	25.20	0.64	28.20	-0.27
With	$N_A s_F c_F(p)$	7	27.80	-1.22	25.20	0.36	28.10	0.38
With	$N_A s_F c_F(p)$	8	26.70	-1.24	28.30	0.59	27.80	0.49
With	$N_A s_F c_F(p)$	9	26.90	-1.26	26.50	0.13	27.50	-0.09
With	$N_A s_F c_F(p)$	10	26.30	-0.56	25.80	-0.89	27.70	0.23
With	$N_A s_F c_F(p)$	11	24.50	-1.89	26.20	0.62	27.10	0.21
With	$N_A s_F c_F(p)$	12	25.70	-0.02	27.70	0.64	27.30	-0.88
With	$N_A s_F c_F(p)$	13	27.00	-1.33	27.10	0.01	27.30	-0.49
With	$N_A s_F c_F(p)$	14	27.90	-0.72	25.30	0.27	26.70	-0.40
With	$N_A s_F c_F(p)$	15	26.70	-1.19	26.80	-0.18	28.60	0.13
With	$N_A s_F c_F(p)$	16	27.60	-0.83	25.20	0.07	29.30	0.47
With	$N_A s_F c_F(p)$	17	29.90	-0.59	27.10	0.77	26.00	0.36
With	$N_A s_F c_F(p)$	18	27.90	-0.78	27.50	-0.04	28.60	-0.40
With	$N_A s_F c_F(p)$	19	27.20	-0.83	26.90	0.25	26.90	-0.23
With	$N_A s_F c_F(p)$	20	27.60	-0.78	26.10	0.71	27.30	-0.44
With	$N_A s_F c_F(p)$	21	27.60	-0.29	26.70	0.21	26.00	-1.10
With	$N_A s_F c_F(p)$	22	27.60	-0.98	26.50	1.01	28.70	0.82
With	$N_A s_F c_F(p)$	23	25.30	-0.86	27.20	0.54	27.30	0.05
With	$N_A s_F c_F(p)$	24	25.00	-2.14	27.00	-0.44	26.00	0.01
With	$N_A s_F c_F(p)$	25	27.30	-1.01	25.60	0.89	26.70	0.64
With	$s_F c_R(p)$	1	75.60	-8.04	76.60	-7.25	76.80	-7.05
With	$s_F c_R(p)$	2	74.90	-7.82	76.80	-6.94	76.50	-6.74
With	$s_F c_R(p)$	3	75.40	-8.29	77.30	-7.28	77.00	-6.66
With	$s_F c_R(p)$	4	76.10	-7.88	77.40	-7.28	78.80	-6.57
With	$s_F c_R(p)$	5	76.70	-7.94	76.50	-7.47	77.40	-6.41
With	$s_F c_R(p)$	6	75.80	-7.85	77.30	-7.73	77.60	-6.44

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Table A.16 – continued from previous page

			S Increasing		S Decreasing		Periodic Recruitment	
Aux. Like.	Model	Year	CI Cov.	MRB	CI Cov.	MRB	CI Cov.	MRB
With	$s_F c_R(p)$	7	75.30	-7.44	77.20	-7.34	76.60	-6.86
With	$s_F c_R(p)$	8	75.60	-7.19	77.10	-7.91	75.70	-6.48
With	$s_F c_R(p)$	9	75.50	-7.58	76.00	-7.58	76.30	-6.38
With	$s_F c_R(p)$	10	76.90	-7.30	78.30	-7.85	78.10	-6.50
With	$s_F c_R(p)$	11	76.90	-7.40	78.30	-7.09	77.60	-6.51
With	$s_F c_R(p)$	12	75.40	-7.40	77.10	-7.52	76.70	-6.94
With	$s_F c_R(p)$	13	76.30	-7.75	75.30	-7.86	76.50	-6.62
With	$s_F c_R(p)$	14	77.80	-7.64	75.40	-7.66	78.80	-6.86
With	$s_F c_R(p)$	15	77.10	-7.96	75.90	-8.09	78.90	-6.43
With	$s_F c_R(p)$	16	76.80	-7.66	77.10	-7.68	77.80	-6.73
With	$s_F c_R(p)$	17	76.00	-7.54	77.30	-7.83	78.30	-6.55
With	$s_F c_R(p)$	18	75.90	-7.29	78.40	-7.90	77.40	-6.66
With	$s_F c_R(p)$	19	77.10	-7.22	75.90	-8.09	75.20	-6.77
With	$s_F c_R(p)$	20	76.50	-7.24	76.70	-8.18	76.50	-6.51
With	$s_F c_R(p)$	21	77.40	-7.04	77.10	-8.25	76.10	-6.89
With	$s_F c_R(p)$	22	75.40	-7.21	76.60	-8.17	78.00	-6.63
With	$s_F c_R(p)$	23	75.00	-7.92	76.40	-8.39	77.60	-6.73
With	$s_F c_R(p)$	24	76.30	-8.30	77.00	-7.89	77.70	-7.18
With	$s_F c_R(p)$	25	75.80	-8.98	80.00	-7.52	78.70	-6.99
With	$N_A s_F c_R(p)$	1	42.90	-19.27	44.00	-19.44	46.40	-17.90
With	$N_A s_F c_R(p)$	2	43.80	-19.23	46.70	-18.85	47.50	-18.00
With	$N_A s_F c_R(p)$	3	43.40	-19.58	45.80	-19.09	48.70	-17.99
With	$N_A s_F c_R(p)$	4	44.20	-19.51	45.20	-19.97	47.60	-18.29
With	$N_A s_F c_R(p)$	5	45.20	-19.02	45.10	-19.37	48.70	-17.86
With	$N_A s_F c_R(p)$	6	45.00	-19.34	44.60	-19.50	50.30	-17.85
With	$N_A s_F c_R(p)$	7	44.50	-19.18	45.40	-18.92	48.20	-17.66
With	$N_A s_F c_R(p)$	8	44.90	-18.64	45.60	-19.08	48.40	-18.13
With	$N_A s_F c_R(p)$	9	46.40	-19.17	45.70	-19.28	49.20	-17.86
With	$N_A s_F c_R(p)$	10	45.50	-18.88	45.30	-19.77	48.90	-17.78
With	$N_A s_F c_R(p)$	11	45.10	-19.25	46.10	-19.12	48.80	-18.15
With	$N_A s_F c_R(p)$	12	45.40	-19.18	45.90	-19.62	48.20	-17.94
With	$N_A s_F c_R(p)$	13	45.90	-18.99	44.10	-20.00	48.60	-17.49
With	$N_A s_F c_R(p)$	14	45.80	-19.21	44.60	-19.27	48.30	-18.30
With	$N_A s_F c_R(p)$	15	45.30	-19.18	43.70	-20.11	48.30	-18.02
With	$N_A s_F c_R(p)$	16	45.40	-18.91	44.40	-19.39	48.30	-18.02
With	$N_A s_F c_R(p)$	17	45.70	-18.79	44.20	-19.74	48.50	-17.82
With	$N_A s_F c_R(p)$	18	46.30	-19.23	45.40	-19.71	48.20	-17.59
With	$N_A s_F c_R(p)$	19	46.60	-18.47	45.30	-19.48	47.70	-18.78
With	$N_A s_F c_R(p)$	20	45.90	-18.68	43.60	-19.47	48.90	-17.55
With	$N_A s_F c_R(p)$	21	46.00	-18.82	44.90	-19.41	48.50	-18.19
With	$N_A s_F c_R(p)$	22	46.90	-18.43	44.10	-19.70	49.40	-18.07
With	$N_A s_F c_R(p)$	23	45.80	-19.07	44.90	-20.00	48.10	-17.72
With	$N_A s_F c_R(p)$	24	44.60	-19.82	45.20	-19.87	48.20	-18.37
With	$N_A s_F c_R(p)$	25	44.00	-19.97	48.80	-19.36	49.70	-18.02
With	$s_R c_R(p)$	1	76.30	-7.98	77.10	-7.01	77.40	-6.48
With	$s_R c_R(p)$	2	76.30	-7.82	76.70	-7.21	77.20	-6.42
With	$s_R c_R(p)$	3	76.50	-8.57	77.60	-7.23	78.50	-6.61
With	$s_R c_R(p)$	4	76.40	-8.05	77.20	-7.64	78.60	-6.27
With	$s_R c_R(p)$	5	77.20	-8.02	77.50	-7.24	79.00	-6.25
With	$s_R c_R(p)$	6	75.50	-7.65	78.10	-7.62	78.20	-6.26
With	$s_R c_R(p)$	7	74.90	-7.66	77.50	-6.88	76.70	-6.45
With	$s_R c_R(p)$	8	76.10	-7.41	78.40	-7.68	77.20	-6.15
With	$s_R c_R(p)$	9	75.80	-7.66	76.90	-7.56	76.90	-6.35
With	$s_R c_R(p)$	10	76.70	-7.25	77.70	-7.73	78.70	-6.53
With	$s_R c_R(p)$	11	76.40	-7.46	77.90	-7.10	79.50	-6.30
With	$s_R c_R(p)$	12	76.10	-7.33	77.60	-7.37	77.70	-6.38
With	$s_R c_R(p)$	13	77.20	-7.83	75.60	-7.44	77.50	-6.48
With	$s_R c_R(p)$	14	78.50	-7.65	75.50	-7.57	80.40	-6.51
With	$s_R c_R(p)$	15	78.20	-8.03	76.80	-7.79	79.50	-5.90

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Table A.16 – continued from previous page

			S Increasing		S Decreasing		Periodic Recruitment	
Aux. Like.	**Model**	**Year**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**	**CI Cov.**	**MRB**
With	$s_R c_R(p)$	16	78.00	-7.62	78.30	-7.51	79.00	-6.39
With	$s_R c_R(p)$	17	77.20	-7.63	78.40	-7.67	78.60	-6.16
With	$s_R c_R(p)$	18	77.00	-7.67	78.50	-7.50	78.20	-6.38
With	$s_R c_R(p)$	19	76.90	-7.49	76.30	-7.65	76.80	-6.70
With	$s_R c_R(p)$	20	77.30	-7.52	77.40	-7.92	78.40	-6.56
With	$s_R c_R(p)$	21	77.60	-7.22	78.40	-7.93	76.70	-6.55
With	$s_R c_R(p)$	22	76.40	-7.44	76.80	-7.48	79.40	-6.40
With	$s_R c_R(p)$	23	75.40	-7.97	76.70	-8.11	79.50	-6.76
With	$s_R c_R(p)$	24	76.70	-8.39	78.10	-8.01	78.20	-7.11
With	$s_R c_R(p)$	25	75.80	-8.79	78.90	-7.41	79.30	-6.62
With	$N_A s_R c_R(p)$	1	44.40	-18.52	45.20	-19.16	47.50	-17.57
With	$N_A s_R c_R(p)$	2	45.30	-18.70	47.20	-18.41	48.90	-17.73
With	$N_A s_R c_R(p)$	3	45.10	-19.26	46.80	-18.85	50.60	-17.80
With	$N_A s_R c_R(p)$	4	46.10	-18.98	46.40	-19.60	48.90	-18.11
With	$N_A s_R c_R(p)$	5	47.00	-18.62	46.40	-18.80	50.40	-17.45
With	$N_A s_R c_R(p)$	6	46.50	-19.09	45.90	-19.22	51.40	-17.72
With	$N_A s_R c_R(p)$	7	46.00	-18.92	46.70	-18.40	49.60	-17.42
With	$N_A s_R c_R(p)$	8	46.50	-18.26	46.80	-18.50	50.00	-17.84
With	$N_A s_R c_R(p)$	9	47.80	-18.69	47.30	-18.72	51.00	-17.61
With	$N_A s_R c_R(p)$	10	46.80	-18.51	46.40	-19.45	50.90	-17.49
With	$N_A s_R c_R(p)$	11	46.40	-18.88	47.60	-18.64	50.20	-17.77
With	$N_A s_R c_R(p)$	12	46.50	-18.87	47.40	-18.99	50.10	-17.80
With	$N_A s_R c_R(p)$	13	47.40	-18.56	45.50	-19.68	50.20	-17.19
With	$N_A s_R c_R(p)$	14	47.40	-18.71	46.00	-19.00	49.90	-17.88
With	$N_A s_R c_R(p)$	15	46.10	-18.60	44.90	-19.81	50.20	-17.59
With	$N_A s_R c_R(p)$	16	46.40	-18.50	45.60	-19.11	49.70	-17.70
With	$N_A s_R c_R(p)$	17	47.00	-18.08	45.60	-19.18	50.10	-17.23
With	$N_A s_R c_R(p)$	18	47.40	-18.81	46.40	-19.27	49.60	-17.07
With	$N_A s_R c_R(p)$	19	47.90	-17.96	46.40	-18.90	49.00	-18.45
With	$N_A s_R c_R(p)$	20	47.40	-17.99	44.70	-19.02	50.10	-17.34
With	$N_A s_R c_R(p)$	21	47.70	-18.50	46.30	-19.04	50.20	-18.04
With	$N_A s_R c_R(p)$	22	48.40	-18.07	45.70	-19.39	50.60	-17.65
With	$N_A s_R c_R(p)$	23	47.00	-18.95	46.10	-19.73	49.50	-17.26
With	$N_A s_R c_R(p)$	24	45.70	-19.47	46.30	-19.70	49.60	-18.11
With	$N_A s_R c_R(p)$	25	45.80	-19.73	50.10	-19.16	51.50	-17.88

Table (A.17) contains the estimated abundance of female elk in Michigan, based on the Conditional-likelihood/Horvitz-Thompson model. Table (A.18) contains the estimated abundance of male elk in Michigan, based on the Conditional-likelihood/Horvitz-Thompson model. Table (A.19) contains the estimated abundance of all elk in Michigan, based on the Conditional-likelihood/Horvitz-Thompson model.

Table (A.20) contains the estimated abundance of female elk in Michigan, based on the absolute recruit abundance model. Table (A.21) contains the estimated abundance of male elk in Michigan, based on the absolute recruit abundance model. Table (A.21) contains the estimated abundance of all elk in Michigan, based on the absolute recruit abundance model.

Table (A.23) contains the estimated abundance of female elk in Michigan, based on the stock-recruit model. Table (A.24) contains the estimated abundance of male elk in Michigan, based on the stock-recruit model. Table (A.24) contains the estimated abundance of all elk in Michigan, based on the stock-recruit model.

Table A.17: *Estimated age-class abundance of female elk, Michigan population, based on the conditional-likelihood/Horvitz-Thompson model.*

Year	Age Class																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Total
1991	139.9	160.9	82.9	112.7	82.9	45.2	22.6	30.1	15.1	7.5	0	7.5	22.6	22.6	7.5	0	0	0	0	0	0	0	0	760
1992	107.3	124	156.1	102.4	80.7	61.7	71.2	14.2	33.2	9.5	4.7	9.5	0	9.5	9.5	4.7	4.7	0	0	0	0	4.7	0	807.6
1993	135.9	151.8	67.2	105.5	72.4	65.1	54.3	29	43.4	25.3	21.7	18.1	14.5	7.2	7.2	7.2	0	0	0	3.6	7.2	0	0	836.6
1994	95.9	90.1	121.6	84.3	38.5	68.4	34.2	47	38.5	38.5	34.2	0	0	0	0	8.5	4.3	4.3	0	0	4.3	0	0	712.6
1995	172.7	95.1	110	94.2	67	62.2	23.9	14.3	14.3	19.1	14.3	19.1	0	0	0	4.8	4.8	0	0	0	0	0	0	715.8
1996	139.4	164.9	108	60.3	60.8	54.4	41.6	28.8	28.8	3.2	3.2	9.6	9.6	6.4	6.4	3.2	0	0	0	0	0	0	0	728.6
1997	105	121.8	71.9	75.7	54.9	51.2	36.6	32.9	18.3	14.6	7.3	3.7	0	3.7	7.3	3.7	7.3	0	0	0	0	3.7	0	619.6
1998	117.4	78.3	116.8	59.9	43.5	40.2	33.5	10	16.7	26.8	6.7	10	10	10	0	3.3	0	3.3	3.3	6.7	0	0	0	596.4
1999	81.4	127.5	67.3	43.2	35.8	56.3	35.8	30.7	20.5	15.3	5.1	0	5.1	10.2	0	0	0	0	0	0	0	0	0	534.2
2000	103.7	121.1	75	79.2	36	15	21	30	12	24	9	6	6	9	0	3	0	6	0	0	0	0	3	559
2001	132.5	132.4	78	59.6	39.6	15.9	23.8	15.9	23.8	19.8	0	7.9	4	0	0	4	0	0	0	0	0	0	0	557.2
2002	157.1	38.9	102.3	52.3	74.5	49.7	6.2	18.6	24.8	24.8	6.2	6.2	12.4	6.2	6.2	12.4	0	6.2	0	0	0	0	0	605
2003	138.9	87.1	119.7	64.5	78.9	29.6	29.6	0	9.9	29.6	9.9	0	9.9	0	0	0	0	0	0	0	0	0	0	607.6
2004	220.6	64.9	104.1	88.3	42.9	17.2	25.8	17.2	17.2	8.6	0	0	8.6	8.6	8.6	0	8.6	0	0	0	0	0	0	641.2
2005	123.6	85.9	75.3	63.9	60.8	30.4	38	45.6	15.2	7.6	7.6	0	0	7.6	7.6	0	7.6	0	0	0	0	0	0	576.7
2006	125.8	169.4	99.4	61.4	60.4	30.2	30.2	25.2	20.1	5	0	15.1	0	10.1	15.1	5	5	0	0	0	0	0	0	677.4
2007	149.2	96.2	84.5	88.7	29.6	65.1	53.3	53.3	29.6	23.7	11.8	0	0	17.8	0	0	5.9	0	0	0	0	5.9	0	714.6
2008	132	86.6	91.5	98.1	63.8	63.8	33.6	23.5	13.4	3.4	20.1	16.8	10.1	3.4	3.4	0	3.4	3.4	10.1	0	0	0	0	680.4

Table A.18: *Estimated age-class abundance of male elk, Michigan population, based on the conditional-likelihood/Horvitz-Thompson model.*

Year	**Age Class**																						
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Total
1991	101	63.8	58.7	96.1	71.4	49.5	16.5	22	22	5.5	5.5	0	5.5	0	0	0	0	0	0	0	0	0	0	517.5
1992	100.5	78.8	92.8	75.7	42.9	39.3	64.3	17.9	21.4	14.3	3.6	3.6	0	0	0	0	0	0	0	0	0	0	0	555.1
1993	140.2	73.3	46.2	88.5	60.4	40.2	23.5	13.4	26.8	16.8	6.7	6.7	3.4	0	0	3.4	0	0	0	0	0	0	0	549.5
1994	185.5	116.4	61	97.9	49.3	56.4	31.7	7	3.5	7	3.5	3.5	3.5	3.5	0	3.5	0	0	0	0	0	0	0	633.2
1995	136.4	195.8	152.2	56.2	65.7	32.9	46	19.7	23	9.9	3.3	6.6	0	3.3	0	0	0	0	0	0	0	0	0	751
1996	92.5	86.6	115.9	96.7	60.4	35.7	24.7	30.2	5.5	11	8.2	2.7	2.7	0	0	0	0	0	0	0	0	0	0	572.8
1997	117.5	91.8	78.2	71.6	57.6	43.2	34.6	17.3	17.3	2.9	14.4	5.8	0	2.9	0	5.8	0	0	0	0	0	0	0	560.9
1998	85.6	133.7	118.9	47.7	36.5	33.7	28.1	16.9	16.9	16.9	8.4	5.6	5.6	2.8	0	0	0	0	0	0	0	0	0	557.3
1999	52.1	122.7	115.4	53.9	66.6	44.4	40.7	7.4	7.4	18.5	14.8	3.7	0	0	0	0	0	0	0	0	0	0	0	547.6
2000	46.4	133.4	95.5	75	52.5	35.7	18.9	27.3	16.8	4.2	10.5	2.1	4.2	0	0	0	0	0	0	0	0	0	0	522.5
2001	33.3	69.5	38.5	63.3	48.1	28.9	25.6	0	16	6.4	12.8	3.2	6.4	0	0	0	0	0	0	0	0	0	0	352
2002	104.1	32.9	60.5	60.9	28.3	11.3	39.6	11.3	17	5.7	0	0	0	0	5.7	0	0	0	0	0	0	0	0	377.3
2003	31.6	50	106	68.2	41.8	16.7	33.5	8.4	0	0	8.4	8.4	0	0	0	0	0	0	0	0	0	0	0	373
2004	144.9	114.5	69.8	17.5	51.9	38.9	51.9	13	13	0	6.5	0	6.5	6.5	0	6.5	0	0	0	0	0	0	0	541.4
2005	41.7	0	20.2	83.8	28.3	33.9	67.9	17	0	5.7	11.3	0	5.7	5.7	0	0	0	0	0	0	0	0	0	321.2
2006	104.3	23.5	80.2	33.3	41.6	33.3	25	33.3	16.6	12.5	0	12.5	4.2	4.2	8.3	0	0	0	0	0	0	0	0	432.8
2007	104.6	27.5	110.2	83.6	33.6	24	33.6	38.4	14.4	0	0	0	4.8	0	0	0	0	0	0	0	0	0	0	474.7
2008	76	13.1	51.2	89.5	64	46.1	17.9	25.6	15.4	7.7	7.7	5.1	7.7	7.7	0	0	0	0	0	0	0	0	0	434.7

Table A.19: *Estimated age-class abundance of all elk, Michigan population, based on the conditional-likelihood/Horvitz-Thompson model.*

Year	Age Class																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Total
1991	241	224.7	141.6	208.8	154.3	94.6	39.1	52.1	37	13	5.5	7.5	28.1	22.6	7.5	0	0	0	0	0	0	0	0	1277
1992	207.7	202.9	248.9	178.1	123.6	101	135.5	32.1	54.7	23.8	8.3	13.1	0	9.5	9.5	4.7	4.7	0	0	0	0	4.7	0	1363
1993	276.1	225.1	113.4	194.1	132.7	105.4	77.8	42.4	70.3	42.1	28.4	24.8	17.8	7.2	7.2	10.6	0	0	0	3.6	7.2	0	0	1386
1994	281.4	206.6	182.6	182.2	87.8	124.8	65.9	54.1	42	45.5	37.7	3.5	3.5	3.5	0	12.1	4.3	4.3	0	0	4.3	0	0	1346
1995	309.1	290.9	262.2	150.4	132.7	95	69.9	34.1	37.4	29	17.6	25.7	0	3.3	0	4.8	4.8	0	0	0	0	0	0	1467
1996	231.9	251.6	223.9	157	121.2	90.1	66.3	59	34.3	14.2	11.4	12.3	12.3	6.4	6.4	3.2	0	0	0	0	0	0	0	1302
1997	222.5	213.6	150.1	147.3	112.5	94.4	71.1	50.2	35.6	17.5	21.7	9.4	0	6.5	7.3	9.4	7.3	0	0	0	0	3.7	0	1180
1998	203	211.9	235.8	107.6	80.1	73.9	61.6	26.9	33.6	43.7	15.1	15.7	15.7	12.9	0	3.3	0	3.3	3.3	6.7	0	0	0	1154
1999	133.5	250.3	182.8	97	102.4	100.6	76.5	38.1	27.9	33.8	19.9	3.7	5.1	10.2	0	0	0	0	0	0	0	0	0	1082
2000	150.1	254.6	170.5	154.2	88.5	50.7	39.9	57.3	28.8	28.2	19.5	8.1	10.2	9	0	3	0	6	0	0	0	0	3	1082
2001	165.8	202	116.5	122.9	87.7	44.7	49.4	15.9	39.8	26.2	12.8	11.1	10.4	0	0	4	0	0	0	0	0	0	0	909.2
2002	261.2	71.7	162.8	113.2	102.7	61	45.8	29.9	41.8	30.5	6.2	6.2	12.4	6.2	11.9	12.4	0	6.2	0	0	0	0	0	982.1
2003	170.5	137.1	225.6	132.7	120.8	46.3	63.1	8.4	9.9	29.6	18.2	8.4	9.9	0	0	0	0	0	0	0	0	0	0	980.5
2004	365.4	179.4	173.9	105.8	94.8	56.1	77.7	30.2	30.2	8.6	6.5	0	15.1	15.1	8.6	6.5	8.6	0	0	0	0	0	0	1182
2005	165.3	85.9	95.5	147.7	89.1	64.3	105.9	62.6	15.2	13.3	18.9	0	5.7	13.3	7.6	0	7.6	0	0	0	0	0	0	897.9
2006	230.1	192.8	179.6	94.6	102	63.5	55.2	58.5	36.8	17.5	0	27.6	4.2	14.2	23.4	5	5	0	0	0	0	0	0	1110
2007	253.8	123.7	194.8	172.2	63.2	89.1	86.8	91.6	44	23.7	11.8	0	4.8	17.8	0	0	5.9	0	0	0	0	5.9	0	1189
2008	208	99.8	142.7	187.5	127.8	109.9	51.5	49.1	28.8	11	27.8	21.9	17.8	11	3.4	0	3.4	3.4	10.1	0	0	0	0	1115

Table A.20: *Estimated age-class abundance of female elk, Michigan population, based on the absolute recruit abundance model.*

Year	Age Class																						
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Total
1991	129.8	141.7	101.6	80.5	64.5	50.4	37.5	33.5	16.4	0	9.4	4.3	11.5	0	2.4	0	0	1.7	0	0	1.1	0	0	686.3
1992	156	121.9	125.8	88.4	68.1	53.3	41.7	31	27.7	13.6	0	7.8	3.6	9.5	0	2	0	0	1.4	0	0	0.9	0	752.7
1993	132.2	140.6	100	99.9	66.9	49.7	39	30.5	22.6	20.2	9.9	0	5.7	2.6	7	0	1.5	0	0	1	0	0	0.6	729.9
1994	125.5	114.6	107.1	72.8	68	43.4	32.2	25.3	19.8	14.7	13.1	6.4	0	3.7	1.7	4.5	0	1	0	0	0.7	0	0	654.5
1995	124	111.5	91.5	82.3	53	47.6	30.4	22.6	17.7	13.8	10.3	9.2	4.5	0	2.6	1.2	3.2	0	0.7	0	0	0.5	0	626.6
1996	112.6	111.6	91.3	72.4	62.2	38.6	34.7	22.1	16.4	12.9	10.1	7.5	6.7	3.3	0	1.9	0.9	2.3	0	0.5	0	0	0.3	608.3
1997	107.7	95.4	81.4	63.2	46.4	37.6	23.4	21	13.4	9.9	7.8	6.1	4.5	4	2	0	1.1	0.5	1.4	0	0.3	0	0	527.1
1998	118.1	93.2	72.4	59	42.9	30	24.3	15.1	13.5	8.6	6.4	5	3.9	2.9	2.6	1.3	0	0.7	0.3	0.9	0	0.2	0	501.3
1999	99.7	100.3	68.2	50.3	38	26.1	18.3	14.8	9.2	8.2	5.3	3.9	3.1	2.4	1.8	1.6	0.8	0	0.4	0.2	0.5	0	0.1	453.2
2000	103.5	90.1	82.6	54.4	38.3	28	19.2	13.4	10.9	6.8	6.1	3.9	2.9	2.3	1.8	1.3	1.2	0.6	0	0.3	0.2	0.4	0	468.2
2001	85	85.1	62	53.5	32.2	21.2	15.5	10.6	7.4	6	3.7	3.4	2.1	1.6	1.2	1	0.7	0.6	0.3	0	0.2	0.1	0.2	393.6
2002	89.6	73.7	64.8	45.2	36.5	20.9	13.8	10	6.9	4.8	3.9	2.4	2.2	1.4	1	0.8	0.6	0.5	0.4	0.2	0	0.1	0.1	379.8
2003	78.4	81.9	62.1	53.1	35.5	27.8	15.9	10.5	7.7	5.3	3.7	3	1.9	1.7	1.1	0.8	0.6	0.5	0.4	0.3	0.2	0	0.1	392.5
2004	107.6	74.3	74	55.2	46.1	30.2	23.7	13.6	8.9	6.5	4.5	3.1	2.5	1.6	1.4	0.9	0.7	0.5	0.4	0.3	0.3	0.1	0	456.4
2005	133.3	101.3	66.3	64.8	47	38.4	25.2	19.7	11.3	7.4	5.4	3.7	2.6	2.1	1.3	1.2	0.8	0.6	0.4	0.3	0.3	0.2	0.1	533.7
2006	106.3	125	89.5	57.4	54.4	38.6	31.6	20.7	16.2	9.3	6.1	4.5	3.1	2.1	1.7	1.1	1	0.6	0.5	0.4	0.3	0.2	0.2	570.8
2007	102.2	96.7	104.4	72.6	44.5	40.9	29	23.7	15.6	12.2	7	4.6	3.4	2.3	1.6	1.3	0.8	0.7	0.5	0.3	0.3	0.2	0.2	565
2008	131.8	93.7	81.9	86.1	57.5	34.3	31.5	22.3	18.2	12	9.4	5.4	3.5	2.6	1.8	1.2	1	0.6	0.6	0.4	0.3	0.2	0.2	596.5

Table A.21: *Estimated age-class abundance of male elk, Michigan population, based on the absolute recruit abundance model.*

Year	Age Class																						
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Total
1991	97.5	92.4	100.4	61.3	50.3	41.3	20.3	19.3	10.3	7.3	3.2	0	0	0.2	0	0	0	0	0	0	0	0	0	503.8
1992	98.6	92.2	89.3	89.3	52.3	39.4	32.4	15.9	15.1	8.1	5.7	2.5	0	0	0.2	0	0	0	0	0	0	0	0	541
1993	136.9	90.2	87.2	73.9	69.1	35.4	26.7	21.9	10.8	10.2	5.5	3.9	1.7	0	0	0.1	0	0	0	0	0	0	0	573.5
1994	109.1	124.7	85	71.5	56.5	45.9	23.5	17.7	14.5	7.1	6.8	3.6	2.6	1.1	0	0	0.1	0	0	0	0	0	0	569.7
1995	101.4	99.8	117.9	70.5	55.5	38.3	31.1	15.9	12	9.9	4.8	4.6	2.5	1.7	0.8	0	0	0.1	0	0	0	0	0	566.8
1996	101	91.7	93.7	95.3	52.8	35.7	24.7	20	10.2	7.7	6.3	3.1	3	1.6	1.1	0.5	0	0	0	0	0	0	0	548.4
1997	97.1	89.5	85	72.6	67.5	31.2	21.1	14.6	11.8	6.1	4.6	3.7	1.8	1.8	0.9	0.7	0.3	0	0	0	0	0	0	510.3
1998	86.5	86.5	83.3	66.6	52.2	40.8	18.8	12.7	8.8	7.1	3.7	2.8	2.3	1.1	1.1	0.6	0.4	0.2	0	0	0	0	0	475.5
1999	74.4	76.6	80.3	64.5	47.2	30.8	24	11.1	7.5	5.2	4.2	2.2	1.6	1.3	0.7	0.6	0.3	0.2	0.1	0	0	0	0	432.8
2000	63.2	67.9	72.4	66.1	49.7	31.6	20.6	16.1	7.4	5	3.5	2.8	1.4	1.1	0.9	0.4	0.4	0.2	0.2	0.1	0	0	0	411
2001	50	52.2	60.3	48.4	38.3	21.6	13.7	9	7	3.2	2.2	1.5	1.2	0.6	0.5	0.4	0.2	0.2	0.1	0.1	0	0	0	310.7
2002	55.4	44.7	48.7	47.5	35	23.4	13.2	8.4	5.5	4.3	2	1.3	0.9	0.7	0.4	0.3	0.2	0.1	0.1	0.1	0	0	0	292.2
2003	49.3	52.3	43.1	43.1	40.2	27.2	18.1	10.2	6.5	4.2	3.3	1.5	1	0.7	0.6	0.3	0.2	0.2	0.1	0.1	0	0	0	302.2
2004	82.2	47.5	51.1	39.8	38.7	34.2	23.1	15.4	8.7	5.5	3.6	2.8	1.3	0.9	0.6	0.5	0.3	0.2	0.2	0.1	0.1	0	0	356.8
2005	88	78.2	46.1	46	34.5	31.2	27.5	18.6	12.4	7	4.4	2.9	2.3	1	0.7	0.5	0.4	0.2	0.2	0.1	0.1	0.1	0	402.4
2006	57.9	83.3	75.5	41	39.2	27.1	24.5	21.6	14.6	9.7	5.5	3.5	2.3	1.8	0.8	0.6	0.4	0.3	0.2	0.1	0.1	0	0	410
2007	58.3	53.7	79.5	64.4	33	28.2	19.5	17.6	15.5	10.5	7	3.9	2.5	1.6	1.3	0.6	0.4	0.3	0.2	0.1	0.1	0.1	0	398.3
2008	72.6	54.5	51.5	68.9	53.1	24.6	21	14.5	13.1	11.6	7.8	5.2	2.9	1.9	1.2	1	0.4	0.3	0.2	0.2	0.1	0.1	0.1	406.8

Table A.22: *Estimated age-class abundance of all elk, Michigan population, based on the absolute recruit abundance model.*

Year	Age Class																						
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Total
1991	227.3	234.2	202	141.9	114.8	91.7	57.8	52.8	26.7	7.3	12.7	4.3	11.5	0.2	2.4	0	0	1.7	0	0	1.1	0	0	1190
1992	254.7	214.1	215	177.6	120.4	92.8	74.1	46.9	42.8	21.7	5.7	10.3	3.6	9.5	0.2	2	0	0	1.4	0	0	0.9	0	1294
1993	269.1	230.8	187.2	173.8	136	85.1	65.6	52.4	33.4	30.5	15.4	3.9	7.4	2.6	7	0.1	1.5	0	0	1	0	0	0.6	1303
1994	234.6	239.2	192.1	144.3	124.6	89.3	55.7	43	34.3	21.8	19.9	10.1	2.6	4.8	1.7	4.5	0.1	1	0	0	0.7	0	0	1224
1995	225.4	211.3	209.4	152.8	108.5	85.9	61.4	38.5	29.7	23.7	15.1	13.8	7	1.7	3.4	1.2	3.2	0.1	0.7	0	0	0.5	0	1193
1996	213.6	203.3	185	167.7	115	74.3	59.3	42.1	26.7	20.6	16.4	10.6	9.7	4.9	1.1	2.4	0.9	2.3	0	0.5	0	0	0.3	1157
1997	204.9	184.9	166.4	135.8	114	68.8	44.5	35.6	25.2	16	12.4	9.8	6.4	5.8	2.9	0.7	1.4	0.5	1.4	0	0.3	0	0	1038
1998	204.6	179.7	155.7	125.6	95.1	70.7	43.1	27.8	22.3	15.8	10.1	7.8	6.2	4	3.7	1.8	0.4	0.9	0.3	0.9	0	0.2	0	976.7
1999	174.1	176.9	148.4	114.8	85.2	56.9	42.3	25.9	16.7	13.4	9.5	6.1	4.7	3.7	2.4	2.2	1.1	0.2	0.6	0.2	0.5	0	0.1	885.9
2000	166.7	158	155	120.5	88	59.6	39.8	29.5	18.3	11.8	9.5	6.7	4.3	3.3	2.7	1.7	1.6	0.8	0.2	0.4	0.2	0.4	0	879
2001	135	137.4	122.3	101.8	70.5	42.8	29.2	19.6	14.4	9.3	5.9	4.9	3.4	2.2	1.7	1.4	0.9	0.8	0.4	0.1	0.2	0.1	0.2	704.5
2002	145	118.3	113.5	92.6	71.5	44.3	26.9	18.4	12.4	9.1	5.9	3.8	3.1	2.1	1.4	1.1	0.9	0.6	0.5	0.3	0	0.1	0.1	671.9
2003	127.7	134.2	105.2	96.2	75.7	55	34.1	20.7	14.2	9.5	7	4.5	2.9	2.4	1.6	1.1	0.8	0.7	0.4	0.4	0.2	0	0.1	694.6
2004	189.8	121.8	125.1	95	84.8	64.4	46.8	29	17.6	12	8.1	5.9	3.8	2.5	2	1.4	0.9	0.7	0.6	0.4	0.3	0.2	0	813.1
2005	221.4	179.5	112.3	110.7	81.5	69.6	52.7	38.3	23.7	14.4	9.9	6.6	4.9	3.2	2	1.7	1.1	0.8	0.6	0.5	0.3	0.3	0.1	936.1
2006	164.2	208.2	165.1	98.3	93.6	65.7	56	42.3	30.8	19	11.6	8	5.3	3.9	2.6	1.6	1.4	0.9	0.6	0.5	0.4	0.3	0.2	980.5
2007	160.6	150.4	183.8	137	77.6	69.1	48.5	41.3	31.1	22.7	14	8.5	5.9	3.9	2.9	1.9	1.2	1	0.7	0.5	0.4	0.3	0.2	963.5
2008	204.4	148.3	133.4	155	110.6	58.8	52.5	36.8	31.3	23.5	17.2	10.6	6.5	4.5	3	2.2	1.4	0.9	0.8	0.5	0.4	0.3	0.2	1003

Table A.23: *Estimated age-class abundance of female elk, Michigan population, based on the stock-recruit model.*

Year	Age Class																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Total
1991	146.3	157.6	112.5	88.9	71.2	55.6	41.4	37	18.3	10.5	0	0	12.9	9.6	0	1.7	0	0	2.2	0	1.2	0	0	766.9
1992	162.4	133.4	136.2	95.3	73.2	59.7	45.5	33.9	30.3	15	8.6	0	0	10.5	7.9	0	1.4	0	0	1.8	0	1	0	816.1
1993	154.4	142.4	107	105.7	70.5	55.8	43.8	33.4	24.8	22.2	11	6.3	0	0	7.7	5.8	0	1	0	0	1.3	0	0.7	793.8
1994	137	130.4	106.4	76.4	70.7	49.2	36.9	28.9	22	16.4	14.7	7.2	4.2	0	0	5.1	3.8	0	0.7	0	0	0.9	0	710.9
1995	133.6	118.9	102.6	80.7	55	52.6	35	26.3	20.6	15.7	11.7	10.5	5.2	3	0	0	3.6	2.7	0	0.5	0	0	0.6	678.8
1996	125.5	117.7	96.2	80.4	60.5	42.4	39.1	26	19.5	15.3	11.7	8.7	7.8	3.8	2.2	0	0	2.7	2	0	0.4	0	0	661.9
1997	120.5	104.8	86	66.9	52.1	41	27.1	25	16.6	12.5	9.8	7.5	5.5	5	2.4	1.4	0	0	1.7	1.3	0	0.2	0	587.3
1998	125	102.8	79.8	62.8	46	37.2	27.8	18.4	17	11.3	8.5	6.6	5.1	3.8	3.4	1.7	1	0	0	1.2	0.9	0	0.2	560.5
1999	107.2	104.7	75.7	56	41.1	31.4	24	18	11.9	10.9	7.3	5.5	4.3	3.3	2.4	2.2	1.1	0.6	0	0	0.8	0.6	0	509
2000	107.5	94.7	85.3	59.8	42.3	31.9	23.6	18	13.5	8.9	8.2	5.5	4.1	3.2	2.5	1.8	1.6	0.8	0.5	0	0	0.6	0.4	514.7
2001	87.7	87.7	66.3	56.4	36.4	27.2	19.2	14.1	10.8	8.1	5.3	4.9	3.3	2.5	1.9	1.5	1.1	1	0.5	0.3	0	0	0.3	436.5
2002	90.8	74.3	65.9	47.6	38	25.6	18.1	12.8	9.4	7.2	5.4	3.6	3.3	2.2	1.6	1.3	1	0.7	0.7	0.3	0.2	0	0	410
2003	80.2	80.1	60.3	51.9	35.9	29.5	19.1	13.5	9.5	7	5.4	4	2.7	2.4	1.6	1.2	1	0.7	0.5	0.5	0.2	0.1	0	407.3
2004	107.2	73.5	69.8	51.6	43.3	30.4	24.4	15.8	11.2	7.9	5.8	4.5	3.3	2.2	2	1.4	1	0.8	0.6	0.5	0.4	0.2	0.1	457.9
2005	114.9	97.4	63.1	58.7	42.1	36	24.7	19.8	12.8	9.1	6.4	4.7	3.6	2.7	1.8	1.6	1.1	0.8	0.6	0.5	0.4	0.3	0.2	503.3
2006	95	103.5	82.3	52	46.8	34.3	28.5	19.5	15.7	10.2	7.2	5.1	3.7	2.9	2.1	1.4	1.3	0.9	0.7	0.5	0.4	0.3	0.3	514.6
2007	93.8	82.2	81	62	37.2	34.6	24.2	20.1	13.8	11.1	7.2	5.1	3.6	2.6	2	1.5	1	0.9	0.6	0.5	0.4	0.3	0.2	485.9
2008	98.2	81.7	65.1	62	45.2	27.9	24.9	17.5	14.5	10	8	5.2	3.7	2.6	1.9	1.5	1.1	0.7	0.7	0.4	0.3	0.3	0.2	473.6

Table A.24: *Estimated age-class abundance of male elk, Michigan population, based on the stock-recruit model.*

Year	Age Class																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Total
1991	114.7	106.4	113.1	68.5	56	46	22.8	21.7	11.7	0	0	0	1.6	0.5	0	0	0	0	0	0	0	0	0	563
1992	162.4	106.2	100	99	57.6	45.4	37	18.3	17.5	9.4	0	0	0	1.3	0.4	0	0	0	0	0	0	0	0	654.5
1993	154.4	146.3	98.1	82.3	76.5	42	32.7	26.6	13.2	12.6	6.8	0	0	0	0.9	0.3	0	0	0	0	0	0	0	692.7
1994	137	138.4	134.7	79.9	62.7	54.7	29.7	23.1	18.8	9.3	8.9	4.8	0	0	0	0.7	0.2	0	0	0	0	0	0	702.9
1995	133.6	123.6	128	111.3	62	46	39.7	21.5	16.7	13.6	6.8	6.4	3.5	0	0	0	0.5	0.1	0	0	0	0	0	713.3
1996	125.5	119.4	113.6	103.4	83.9	43.8	32.1	27.7	15	11.7	9.5	4.7	4.5	2.4	0	0	0	0.3	0.1	0	0	0	0	697.6
1997	120.5	110.6	108.7	88.9	74.6	56	28.8	21.1	18.2	9.8	7.7	6.2	3.1	2.9	1.6	0	0	0	0.2	0.1	0	0	0	659
1998	125	106.7	101	86.1	65.2	50.8	37.6	19.3	14.1	12.2	6.6	5.1	4.2	2.1	2	1.1	0	0	0	0.1	0	0	0	639.2
1999	107.2	110.6	97.5	79.9	63	44.3	34.1	25.2	12.9	9.5	8.2	4.4	3.4	2.8	1.4	1.3	0.7	0	0	0	0.1	0	0	606.5
2000	107.5	97	102.5	81.2	62.8	46.9	32.6	25.1	18.5	9.5	7	6	3.3	2.5	2.1	1	1	0.5	0	0	0	0.1	0	607.1
2001	87.7	91.5	86.4	74.3	52.6	36.6	26.8	18.6	14.3	10.6	5.4	4	3.4	1.9	1.4	1.2	0.6	0.6	0.3	0	0	0	0	518.2
2002	90.8	79.1	84.6	71.3	57.6	38.5	26.5	19.4	13.5	10.3	7.7	3.9	2.9	2.5	1.3	1	0.9	0.4	0.4	0.2	0	0	0	512.8
2003	80.2	84.7	74.7	75.4	61.6	48.3	32.1	22.1	16.1	11.2	8.6	6.4	3.3	2.4	2.1	1.1	0.9	0.7	0.4	0.3	0.2	0	0	532.8
2004	107.2	75.8	80.7	68.5	67.7	54.2	42.4	28.1	19.4	14.1	9.8	7.6	5.6	2.9	2.1	1.8	1	0.8	0.6	0.3	0.3	0.2	0	591.1
2005	114.9	100.5	71.9	72.7	60.1	57.8	46.1	36	23.9	16.4	12	8.4	6.4	4.7	2.4	1.8	1.5	0.8	0.6	0.5	0.3	0.2	0.1	640
2006	95	107.4	95.1	64.3	63.1	50.6	48.5	38.6	30.2	20	13.8	10.1	7	5.4	4	2	1.5	1.3	0.7	0.5	0.4	0.2	0.2	659.9
2007	93.8	87.8	100.9	83.1	54	51	40.6	38.9	31	24.2	16.1	11.1	8.1	5.6	4.3	3.2	1.6	1.2	1	0.6	0.4	0.4	0.2	659.1
2008	98.2	87.2	82.7	89.1	70.8	44.5	41.8	33.3	31.8	25.4	19.8	13.2	9.1	6.6	4.6	3.5	2.6	1.3	1	0.8	0.5	0.4	0.3	668.5

Table A.25: *Estimated age-class abundance of all elk, Michigan population, based on the stock-recruit model.*

Year	Age Class																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Total
1991	261	264	225.6	157.4	127.1	101.6	64.1	58.7	30	10.5	0	0	14.5	10.1	0	1.7	0	0	2.2	0	1.2	0	0	1330
1992	324.8	239.5	236.2	194.2	130.8	105.1	82.5	52.2	47.7	24.4	8.6	0	0	11.8	8.2	0	1.4	0	0	1.8	0	1	0	1470
1993	308.8	288.7	205.1	188	147.1	97.9	76.5	60	38	34.8	17.8	6.3	0	0	8.7	6	0	1	0	0	1.3	0	0.7	1487
1994	274.1	268.8	241.1	156.3	133.4	103.9	66.5	52	40.9	25.7	23.6	12	4.2	0	0	5.8	4	0	0.7	0	0	0.9	0	1414
1995	267.3	242.5	230.5	192	117	98.6	74.7	47.8	37.3	29.3	18.4	16.9	8.6	3	0	0	4.1	2.9	0	0.5	0	0	0.6	1392
1996	251	237.1	209.8	183.8	144.4	86.2	71.1	53.7	34.5	27	21.2	13.4	12.2	6.3	2.2	0	0	3	2.1	0	0.4	0	0	1359
1997	241	215.3	194.7	155.8	126.7	97	55.9	46	34.8	22.3	17.4	13.7	8.6	7.9	4	1.4	0	0	1.9	1.4	0	0.2	0	1246
1998	249.9	209.5	180.8	148.9	111.2	88	65.4	37.7	31.1	23.5	15.1	11.8	9.3	5.8	5.3	2.7	1	0	0	1.3	0.9	0	0.2	1199
1999	214.3	215.3	173.1	135.9	104.1	75.8	58.1	43.2	24.8	20.4	15.5	9.9	7.7	6.1	3.8	3.5	1.8	0.6	0	0	0.9	0.6	0	1115
2000	215	191.7	187.8	141	105.2	78.9	56.2	43.1	32	18.4	15.2	11.5	7.4	5.7	4.5	2.8	2.6	1.3	0.5	0	0	0.6	0.4	1122
2001	175.3	179.2	152.7	130.6	89	63.8	45.9	32.7	25.1	18.7	10.8	8.9	6.7	4.3	3.4	2.6	1.7	1.5	0.8	0.3	0	0	0.4	954.4
2002	181.5	153.3	150.5	119	95.7	64.1	44.6	32.1	22.9	17.5	13	7.5	6.2	4.7	3	2.3	1.8	1.2	1.1	0.5	0.2	0	0	922.7
2003	160.5	164.8	135.1	127.3	97.5	77.8	51.2	35.6	25.7	18.2	14	10.4	5.9	4.8	3.7	2.3	1.8	1.4	0.9	0.8	0.4	0.1	0	940.2
2004	214.5	149.3	150.5	120.2	111	84.7	66.8	44	30.6	22	15.7	12	8.9	5.1	4.1	3.2	2	1.6	1.2	0.8	0.7	0.4	0.1	1049
2005	229.8	198	135	131.4	102.2	93.8	70.7	55.8	36.7	25.5	18.4	13.1	10	7.5	4.2	3.4	2.6	1.7	1.3	1	0.6	0.6	0.3	1144
2006	189.9	210.9	177.4	116.3	109.9	85	77	58.2	45.9	30.2	21	15.1	10.8	8.2	6.1	3.5	2.8	2.2	1.4	1.1	0.8	0.5	0.5	1175
2007	187.6	170	181.9	145.1	91.1	85.6	64.9	59	44.8	35.3	23.3	16.1	11.7	8.3	6.3	4.7	2.6	2.1	1.6	1	0.8	0.6	0.4	1145
2008	196.4	168.9	147.9	151.1	116	72.4	66.7	50.7	46.4	35.3	27.8	18.3	12.7	9.2	6.5	5	3.7	2.1	1.6	1.3	0.8	0.6	0.5	1142

Appendix B

VARIANCE CALCULATIONS FOR ABUNDANCE ESTIMATES BASED ON THE HORVITZ-THOMPSON ESTIMATOR

For conditional-likelihood models, abundance is estimated as in Equation (2.35) as

$$\widehat{N}_i = \frac{\sum_{j=1}^{A} x_{ij}}{\widehat{p}_i}, \tag{B.1}$$

which is a function of both parameter estimates and data. In order to compute the variance estimate $\widehat{Var}\left(\widehat{N}_i\right)$, we use the "law of total variance"

$$\widehat{Var}(\widehat{N}_i) = E_{\boldsymbol{X}}\left(Var_{\widehat{p}_i}\left(\widehat{N}_i \mid \boldsymbol{X}\right)\right) + Var_{\boldsymbol{X}}\left(E_{\widehat{p}_i}\left(\widehat{N}_i \mid \boldsymbol{X}\right)\right). \tag{B.2}$$

For the first component, we estimate

$$Var_{\widehat{p}_i}\left(\widehat{N}_i \mid \boldsymbol{X}\right) = Var\left(\frac{\sum_{j=1}^{A} x_{ij}}{\widehat{p}_i} \mid \boldsymbol{X}\right) = \left(\sum_{j=1}^{A} x_{ij}\right)^2 Var\left(\frac{1}{\widehat{p}_i}\right) \approx$$
$$\left(\sum_{j=1}^{A} x_{ij}\right)^2 \left(\frac{1}{\widehat{p}_i}\right)^4 Var\left(\widehat{p}_i\right)$$

and we may estimate $Var\left(\widehat{p}_i\right)$ via the delta method.

$$Var\left(\widehat{p}_i\right) \approx \left(\frac{\partial \widehat{p}_i}{\partial c}\right)^2 \widehat{Var}\left(\widehat{c}\right)$$

where the derivative of $\widehat{p}_i$ will depend on the specific transformation used to relate c to p_i.

This leaves us with

$$
\begin{aligned}
&Var_{\widehat{p}_i}\left(\widehat{N}_i \mid \boldsymbol{X}\right) \approx \left(\sum_{j=1}^{A} x_{ij}\right)^2 \left(\frac{1}{\widehat{p}_i}\right)^4 \widehat{Var}\left(\widehat{p}_i\right) = \\
&\quad \left(\sum_{j=1}^{A} x_{ij}\right)^2 \left(\frac{1}{\widehat{p}_i}\right)^4 \left(\frac{\partial \widehat{p}_i}{\partial c}\right)^2 \widehat{Var}\left(\widehat{c}\right) \Rightarrow \\
&E_{\boldsymbol{X}}\left(Var_{\widehat{p}_i}\left(\widehat{N}_i \mid \boldsymbol{X}\right)\right) = \\
&\quad \left(\frac{1}{\widehat{p}_i}\right)^4 \left(\frac{\partial \widehat{p}_i}{\partial c}\right)^2 \widehat{Var}\left(\widehat{c}\right) E_{\boldsymbol{X}}\left(\left(\sum_{j=1}^{A} x_{ij}\right)^2\right) = \\
&\quad \left(\frac{1}{\widehat{p}_i}\right)^4 \left(\frac{\partial \widehat{p}_i}{\partial c}\right)^2 \widehat{Var}\left(\widehat{c}\right) \left(Var\left(\sum_{j=1}^{A} x_{ij}\right) + \left(E_{\boldsymbol{X}}\left(\sum_{j=1}^{A} x_{ij}\right)\right)^2\right)
\end{aligned}
$$

Here, we may assume that total annual harvest is binomially-distributed from total annual abundance. That is,

$$
\sum_{j=1}^{A} x_{ij} \sim Binomial\left(\widehat{N}_i, \widehat{p}_i\right). \tag{B.3}
$$

Then

$$
\begin{aligned}
&\left(\frac{1}{\widehat{p}_i}\right)^4 \left(\frac{\partial \widehat{p}_i}{\partial c}\right)^2 \widehat{Var}\left(\widehat{c}\right) \left(Var\left(\sum_{j=1}^{A} x_{ij}\right) + \left(E_{\boldsymbol{X}}\left(\sum_{j=1}^{A} x_{ij}\right)\right)^2\right) = \\
&\left(\frac{1}{\widehat{p}_i}\right)^4 \left(\frac{\partial \widehat{p}_i}{\partial c}\right)^2 \widehat{Var}\left(\widehat{c}\right) \left(\widehat{N}_i\widehat{p}_i\left(1-\widehat{p}_i\right) + \left(\widehat{N}_i\widehat{p}_i\right)^2\right).
\end{aligned}
$$

From the second component of Equation (B.2)

$$
\begin{aligned}
&Var_{\boldsymbol{X}}\left(E_{\widehat{p}_i}\left(\widehat{N}_i \mid \boldsymbol{X}\right)\right) \approx Var_{\boldsymbol{X}}\left(\widehat{N}_i\right) = Var_{\boldsymbol{X}}\left(\frac{\sum_{j=1}^{A} x_{ij}}{\widehat{p}_i}\right) \\
&\quad \left(\frac{1}{\widehat{p}_i^2}\right) Var_{\boldsymbol{X}}\left(\sum_{j=1}^{A} x_{ij}\right) = \left(\frac{1}{\widehat{p}_i^2}\right)\left(\widehat{N}_i\widehat{p}_i\left(1-\widehat{p}_i\right)\right) = \frac{\widehat{N}_i\left(1-\widehat{p}_i\right)}{\widehat{p}_i}
\end{aligned} \tag{B.4}
$$

Combining all of the above portions we obtain

$$\widehat{Var}_{FE}\left(\widehat{N}_i\right) \approx \widehat{Var}\left(\widehat{c}\right)\left(\frac{\partial \widehat{p}_i}{\partial c}\right)^2 \frac{1}{\widehat{p}_i^4}\left(\widehat{N}_i\widehat{p}_i\left(1-\widehat{p}_i\right)+\widehat{N}_i^2\widehat{p}_i^2\right)+\frac{\widehat{N}_i\left(1-\widehat{p}_i\right)}{\widehat{p}_i}.$$

If harvest probability is modeled with a random effect (τ_i), we simply make the replacement

$$\widehat{Var}\left(\widehat{p}_i\right)=\left(\frac{\partial \widehat{p}_i}{\partial c}\right)^2 \widehat{Var}\left(\widehat{c}\right)+\left(\frac{\partial \widehat{p}_i}{\partial \tau_i}\right)^2 \widehat{Var}\left(\widehat{\tau}_i\right)+2\left(\frac{\partial \widehat{p}_i}{\partial c}\right)\left(\frac{\partial \widehat{p}_i}{\partial \tau_i}\right)\widehat{Cov}\left(\widehat{c},\widehat{\tau}_i\right)$$

where each of $\widehat{Var}\left(\widehat{c}\right)$, $\widehat{Var}\left(\widehat{\tau}_i\right)$, and $\widehat{Cov}\left(\widehat{c},\widehat{\tau}_i\right)$ may be obtained from the inverse-Hessian.

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