

Partnering Matters:

Examining mathematical learning and responsiveness in first-grade peer interactions

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Abstract

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The substantial body of research on cooperative learning suggests its positive effects on student learning and achievement; thus, a growing number of educators and researchers are utilizing cooperative learning in many subject areas. Particularly in mathematics, the role of communication, a key component of cooperative learning, has been recognized as an important aspect of conveying mathematical ideas into more refined and effective forms. This study aims to understand young children's mathematical learning through peer interactions with the goal of determining the effects of certain pairings and responsiveness on cooperative learning. To achieve this aim, I examined peer interactions among three first-grade students and the effect of the processes and quality of those interactions on each students' mathematical learning. The findings from this study suggest that students' ideas and responsiveness move around the classroom over time during the process of peer interactions and eventually lead to an increase in learning opportunities. This result provides a key to understand the potential application and subsequent benefits of cooperative learning settings.

Introduction

Young children learn a lot when they work together. Exchanging their big and small ideas, children gain knowledge and experience from others. In the subject of mathematics, the role of communication has been recognized as an important aspect of conveying mathematical ideas into more refined and effective forms (Chapin, O'Connor, O'Connor, & Anderson, 2009; Kramarski & Mevarech, 2003; Lampert, Rittenhouse, & Crumbaugh, 1996). The Common Core State Standards For Mathematical Practice requires students to be able to “construct viable arguments and critique the reasoning of others” (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, pp. 6-7). In cooperative learning settings, both peer-directed and small group, students in the elementary grades practice how to share their ideas and justify the ideas of others through communication, and respond to each other’s arguments using concrete referents such as objects, drawings, diagrams, and actions. As an observer of this process, teachers and other adults in the classroom can see evidence of the learning process by watching and listening to them closely.

Cooperative learning has been extensively researched and tested in various classroom environments over decades. Researchers and teachers have worked together to develop this instructional arrangement so that it is more effective and practical. As a small contribution, this study examines the development of young children’s mathematical thinking with the goal of determining the effect of certain pairings and responsiveness during peer interactions in order to make recommendations for implementing cooperative learning.

Research Questions

To study young children’s mathematical learning through peer interactions with the goal of determining the effects of certain pairings and responsiveness on cooperative learning, I used

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a qualitative case study approach (Stake, 1995). This study took place within the context of a mathematical activity called “Counting Collections.” Counting Collections is a collaborative activity that promotes higher quality thinking and interaction; it is described in more detail below.

Within peer interactions during the Counting Collections activity, three first-graders’ mathematical ideas and strategies were investigated. The guiding research questions were as follows:

- 1) *How does cooperative learning affect how young children develop their mathematical thinking?*
- 2) *What is the relationship between the partnering and children’s responsiveness?*
- 3) *How does the responsiveness of each partner in the process of peer interaction contribute to young children’s mathematical learning?*

Literature Review

The use of peer learning in educational strategy has been highlighted over the past decades. Peer learning involves gaining skills or knowledge through help and support from fellow learners rather than a formal teacher (Topping, 2005). Learning from and helping other learners at similar ages or status enhances the learning process for all involved. Peer learning can be categorized into two established types: peer tutoring and cooperative learning. This study focuses on cooperative learning and how it can promote the development of a student’s mathematical thinking. In cooperative learning, students in a small group work with given information to achieve a shared goal and are anticipated to have equal participation and responsive interactions. “Cooperative learning is more than “working together” — it has been described as “structuring positive interdependence” (Slavin, 1990)” in order to achieve a specific shared goal (as cited in Topping, 2005, p. 632). The specialization of goals, task, roles and

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rewards is often planned and organized to apply cooperative learning in a way that fosters the interactive learning process. This study examines the mathematical learning and development of first-graders performing a math activity called “Counting Collections” in mathematics classroom. During Counting Collections, students learn through cooperative learning as they share a specific goal — that is, counting objects with a partner.

Many empirical studies put forward the positive effects of cooperative learning on student learning and achievement. Cooperative learning increases students’ opportunity to interact with other students and their ideas. Students have more of a chance to share their ideas and listen to others within the small groups rather than the whole group discussion. Working collectively in the small group to complete a common task, active academic and social engagement allows students to communicate with each other as a way of monitoring and clarifying their ideas, and finally reaching consensus. According to Vygotsky (1978), “a participant might never have truly grasped a concept until having to explain it to another, embodying and crystallizing thought into language. Listening, explaining, questioning summarizing, speculating and hypothesizing are all valuable skills which should be transferable” (as cited in Topping, 2005, p. 637). In this sense, students validate their conceptions and ideas and correct misconceptions and incompleteness of thought through a process of talking to and questioning their peer learners (Forman & Cazden, 1985; Whitebread et al, 2007). Eventually, they will be involved in re-evaluating their ideas and beliefs, seeking new knowledge to complete their understanding, expanding or establishing new ideas, connecting new information to their previous learning, and potentially reaching agreement (Bargh & Schul, 1980; Brown, Campione, Webber, & McGilly, 1992; Chi, 2000). As cooperative learning encourages heavy use of communication skills from the students, they are more likely to “truly” learn a concept or

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lesson, given that providing elaborated explanations of one's own ideas and engaging with others' idea at a high level is positively correlated with learning outcomes (Roscoe & Chi, 2008; Webb et al., 2008, 2009, 2014). Also, during cooperative learning, students have more opportunity to interact with materials which encourages students to deepen their understanding as they relate their learning to real life objects and situations (Webb, 1991).

A majority of mathematics education studies on cooperative learning focus on how cooperative learning can promote student's mathematical achievements (Davidson, 1985; Parker, 1987; Slavin, 1985). In contrast, Yackel, Cobb, & Wood (1991) investigated extensively the process involved in cooperative learning rather than merely focusing on its effects on academic outputs or achievements. Thus, their research lies within what Slavin (1985) described as the "second-generation" research on cooperative learning, which is concerned less with documenting basic effects and more with assessing student's mathematical activities during the process of applying and practicing cooperative learning in math classroom. As a further point, the study concludes that small group interactions can provide learning opportunities and "the use of cognitively based educational activities designed to be problematic for children at a variety of different conceptual levels are the crucial features of a cooperative learning environment in the absence of extrinsic rewards" (Yackel, Cobb, & Wood, 1991, p. 406-407). In parallel with the approach of Yackel, Cobb, & Wood (1991) towards cooperative learning in math education, this study examines the development of young children's mathematical thinking with the goal of determining the effects of certain pairings and responsiveness during peer interactions in order to make recommendations on how best to implement cooperative learning. Thus, this study aims to understand the process involved in cooperative learning and more specifically the development of the responsive and simultaneous interaction among students in the context of math activities.

Conceptual Framework

The conceptual framework for this study brings together ideas about young children’s mathematical development with research on cooperative learning. First, I draw on ideas about how cooperative learning supports children’s learning I focus in two forms of help in peer interaction — “help giving” and “help receiving” (Webb, 1991). I apply these ideas about cooperative learning to research on young children’s mathematical learning trajectory, particularly in the domain of counting in early mathematics.

The Developmental Perspectives on Cooperative Learning

In examining how cooperative learning operates in children’s mathematics learning, this study focuses on the developmental perspective among the four perspectives of cooperative learning introduced by Slavin (1996) (see Figure 1).

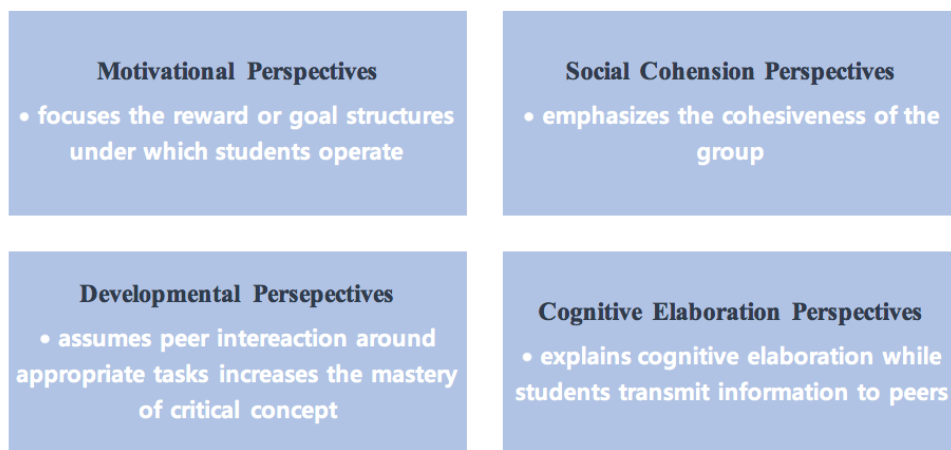


Figure 1. Four major theoretical perspectives to achieve the effects of CL, adapted from Slavin (1996).

The central idea of the developmental perspective is “that interaction among children around appropriate tasks increases their mastery of critical concepts” (Slavin, 1996, p. 48). Based on Piagetian and Vygotskian theories, the developmental perspectives hold that “the task-focused interaction among students enhances learning by creating cognitive conflicts which they must resolve and by exposing students to higher quality thinking that is within their proximal zone of

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development” (Slavin, 1986, p. 1). Collaborative activity among children promotes growth because children of similar ages are likely to be operating within one another’s proximal zones of development, modeling the behaviors and thoughts of more advanced peers. In other words, students will learn from each other within their proximal zone of development by reconciling cognitive conflicts, recognizing inadequate reasoning, and gaining higher quality thinking. In this notion, the effectiveness of cooperative learning on student’s learning and achievement can be primarily due to the use of tasks in collaborative activity, and tasks in collaborative activity is a critical element in cooperative learning that provides opportunities for students to develop higher quality thinking and interaction. In the development perspectives, the math activity called “Counting Collection”, which is discussed in the subsequent section, is an example of the task in the context of cooperative learning. Given the opportunities to count collections of objects with a partner, students communicate, interact, and build their own ideas to resolve their cognitive conflicts. Therefore, in this study, Counting Collections is an important task and opportunity to challenge students’ thinking and to build up responsive and simultaneous interactions.

“Help giving” and “Help receiving”

In peer learning, two kinds of behavior revolve around help: help giving and help receiving. Many studies have shown that when help-giver and help-receiver of similar ages work collaboratively on task-related conversations, peer interaction can help both develop their thinking and increase their mastery of critical concepts (Bargh & Schul, 1980; Bell, Grossen, & Perret-Clermont, 1985; Webb, 1991). Preparing to teach someone or giving explanations encourages the help-giver to learn the material in new ways (Bargh & Schul, 1980). Also, giving help effectively promotes cognitive reconstruction and achievement if the help includes “content-related explanations” of how to solve a problem or how to obtain the correct answer

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(Webb, 1991, p. 372). However, if the help-giver provides information other than content-related explanations, it may not be beneficial for their own learning and achievement (Peterson & Swing, 1985; Webb, 1991). For the help-receivers, receiving explanations may help them not only obtain new information and strategies for solving problems but also correct misunderstandings. If students are of similar ages, they can rephrase difficult vocabulary and expressions using friendly, simplified language that their classmates can easily understand during the peer interaction (Noddings, 1985).

Responsiveness vs. non-responsiveness. The learning involved in receiving explanation is complicated because “receiving explanation is not often adequate to help the student learn how to solve the mathematical problems” (Webb, 1991, p. 375). Thus, to make the received help beneficial for a receiver’s learning, it is essential that the kind of help a student receives be aligned with the student’s request for help. To understand the effectiveness of receiving help, Webb (1991) discusses two types of feedback: responsive and nonresponsive feedback. When the feedback meets the need of the student, it is responsive feedback, while if it is not aligned with the student’s request, it is non-responsive. Webb (1991) describes examples of responsive and nonresponsive feedback as follows:

Sequences of interaction that are considered responsive feedback include making an error and being corrected and receiving an explanation, asking for and receiving an explanation, and asking a procedural question and receiving an answer. Kinds of nonresponsive feedback contain making an error and not being corrected, making an error and being told the correct answer without any explanation of how to obtain it, asking for an explanation and receiving only the correct answer, asking for an explanation and receiving no reply, and asking a procedural question and receiving no reply. (p. 375)

Webb’s examples provide a useful way of assessing the quality of peer interaction, especially in terms of how responsiveness contributes to students’ mathematical learning. This study will examine students’ responsiveness not only to partners’ needs but also to their ideas

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and thinking processes. In addition, rather than just assuming one student only provides help and the other only receives help, this study posits that an individual student can both give and receive help, and that both giving and receiving help can develop thinking and provide learning opportunities. The study will also investigate how young children give and receive help not only through verbal interactions but also through physical actions as they count objects with their partners.

Young Children's Mathematical Thinking and Development

Extensive research on children's thinking (Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993; Fennema et al, 1996; Fuson, 1992; Gutstein and Romberg, 1995) suggests that young children's conceptions of operations are quite different from adults. Their conceptions provide a basis for learning basic mathematical concepts and skills with understanding. Based on their informal and intuitive mathematical knowledge, young children can construct concepts of place value and multi-digit computational procedures (Carpenter, Fennema, & Franke, 1996). Young children devise their own strategies that exhibit the relationship or process involved in the given task or problem rather than using specific knowledge of number facts, mathematical procedures, or algorithms (Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993). Children use their own developing knowledge of number facts to count and/or invent their own strategies. Without direct instruction of a particular strategy or solution that goes with a particular type of problems or tasks, children are capable to naturally come up with (construct) various strategies and solutions to a variety of problems and tasks. But not all children individually construct these strategies by themselves. Carpenter, Fennema, Franke, Levi, & Empson (2014) highlighted that "children adapt strategies as they engage with other children using different strategies and discuss mathematical ideas with their teachers or other children" (p. 4). This perspective on

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children's thinking will help me to understand how the first-graders in the study develop their mathematical thinking differently using their own developing and intuitive knowledge, through the process of peer interactions.

The importance of counting in early mathematics. Counting is an important skill in early mathematical concepts. The counting strategies employed by young children are not only an effective way to solve addition and subtraction problems. They also support the development of understanding numbers by providing the foundation for understanding place value, how numbers are structured, and how they are related to each other (Carpenter, Fennema, Franke, Levi, & Empson, 2014). As children explore different ways to count flexibly, accurately, and efficiently, their procedural fluency improves and along with it, the knowledge of grouping, composing, decomposing, and place value, among other concepts (Mathematics Learning Study Committee, 2001). In this sense, counting provides young children the opportunity to develop their conceptual understanding of numbers. This deep understanding of numbers allows children to use them as they progress to more complex mathematics such as operations. Children's counting skills are one marker of their understanding of number concepts, allowing teachers to evaluate their students' progress.

Method

Research Context

“Counting Collections”

Counting Collections is a simple but rewarding hands-on math activity (Hintz & Latimer, in press; Schwerdtfeger & Chan, 2007). Children are given collections or groups of objects that are familiar to their lives (e.g., pompoms, blocks, shells, coins, and bottle caps), and figure out how to count with a partner or partners. As they count objects, they come up with various ways

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of grouping, sorting, etc. Then, they make a recording of their counting. Counting Collections helps children to develop ideas of mathematical numbers and operations, but also provides rich opportunities to practice oral counting, develop efficient counting strategies, group objects in strategic ways, record numbers, and represent their thinking (Hintz & Latimer, in press; Schwerdtfeger & Chan, 2007). Counting Collections provides multiple entry points for children's learning that is engaging and productive for children of all ages. Counting Collections is a mathematical activity that highly demands collaboration with a partner not only through verbal interactions but also through physical interactions. Working with a partner, students learn how to work together as they learn from and build on each other's ideas, and make decisions together on a strategy to use, how to count, whether to count together, how to keep track of which items they have been counting and which have not, etc. Counting Collections is a cooperative task that provides opportunity for students to develop higher quality thinking and interaction; however, the interactions vary depending on who they are with and how they respond to each other's idea. Therefore, this study will examine the relationship of the development of young children's mathematical thinking to the quality of peer interactions during Counting Collections activity.

Setting and Participants

The study was conducted in a first-grade classroom at Broadview Thomson K-8 in the Seattle Public School District. Three participants who were selected in the study are named Tiffany, Ben, and Jin. Ben and Jin were selected through a convenience sampling, while Tiffany was purposefully selected through the classroom teacher's recommendation. The classroom teacher personally asked Tiffany's parents for their consent to the research. Subsequently, the teacher sent the other students home with research consent forms, and Ben and Jin turned in their

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forms with their parents' signatures the next day. Therefore, I started the interview process with these three students on following day, December 1, 2017. These three students represent a range of performance levels in the mathematics classroom. Tiffany is an African-American female student who underperforms in terms of standard test scores. However, the teacher knows that Tiffany always has her own inventive approaches and thinking when trying to solve math problems and math activities that she does not fully understand; thus, the teacher recommended that I include her as a research participant so that her mathematical strategies and thinking could be carefully observed and documented. Ben is a Caucasian male student who has a positive attitude during math and performs in the 50th percentile in terms of test scores in math class. He tends to have a high level of engagement in mathematics activities and materials given in class. Finally, Jin is an Asian-American female student who generally receives very high grades in math tests. Because the data was collected from the first day of December, 2017, only three months after they started elementary school, the students' performance level in this study might not fully reflect their capability and understanding of mathematics.

As the Counting Collection activity is designed to encourage peer interaction among participants, the classroom teacher was limited to an "observer" role giving only basic guidance and instruction to initiate activity and creating an atmosphere that participants can freely engage themselves without teacher's interruption. For example, in the first couple of weeks, the teacher gave instructions to students on how Counting Collection works and how they can record their results based upon given Counting Collection strategies.

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Data Collection

To understand these three first-graders' development of mathematical thinking through peer interaction, four types of data were collected during Counting Collections sessions. The Counting Collections activity was launched on November 10, 2017 in the first-grade mathematics classroom I observed in. Since then, students have been working on Counting Collections every Friday to develop their understanding of numbers. The data covers the children's peer interaction and their counting strategies over the 20-week period from December 1, 2017 to April 26, 2018. During the 20-week period, I observed the three students engaged in Counting Collections 13 times for Ben, 14 times for Tiffany, and 15 times for Jin. Due to seasonal vacations and various school events, the activity was skipped for several weeks and was occasionally conducted on a day other than Friday. However, most of the four types of data were collected on Fridays.

Observation and field note. The first form of data was observational data to keep track of the three students' counting and recording strategies and their interaction with partners. These notes were recorded every time students worked on the Counting Collections activity in the classroom. The three students were observed while they interacted with their partners, and their strategies and interaction were documented in the field notes. A table was created for field notes to document each student's work during the observation (Appendix A). Each table includes the following: data, name of partner, type of collection, strategies, the number each student counts to and counts by, and accuracy of counting and recording. The same version of the table was saved in Google Docs (see Appendix B) for each student, and I typed the information from my rough paper field notes into the online version on the same day or the day after I took the observational notes. The three final tables were created for each student.

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Video recordings of student interaction. The interactions of the three students with each other (Tiffany–Payton, Payton–Ben, Ben–Tiffany) were video recorded with permission from their parents and/or guardians. Initially, I had planned to video record several formal interview questions asking about students’ thinking and strategies. However, concerned with asking too many questions and consequently interrupting the students’ interactions and focus on counting, I decided to remain an “observer” and just ask simple “why” questions that helped me understand their strategies and interactions better. Mostly, I started their interactions with this question: “What number are we counting by today?” Sometimes, when students got distracted or lost focus, I asked them to focus again on their work. Other than this, I did not provide any strategies or help them keep track of sequences, which could distract their own flow of learning and interactions. Every strategy students came up with was developed from their own ideas, either individually or collaboratively. Initially, I had planned to record videos once a month, but due to vacations and other occasions, it was difficult to record videos that often. In total, 14 videos were recorded and used for documentation. During the video recordings, the documents (see Appendix B) for recording each student’s strategies and interactions were filled in as well.

Recording sheets. Each student’s recording sheets were collected as artifacts (see Appendix C). Recording sheets are great resources to learn about students’ understanding of representing their counting strategies including numbers they count to and count by, and the way they grouped their collections on a piece of paper.

Photographs of Students’ Counting Strategies. In addition to the recording sheets, photographs of students’ counting strategies were taken using the camera in my computer and my phone. It can be very challenging to describe students’ counting strategies in writing. Having photographic evidence made it easier for me to remember what students did and how they

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counted and can also help those who read the paper clearly see and understand students' strategies. The two types of artifact (photographs of students' counting strategies and recording sheets) were also collected with permission from their parents and/or guardians and saved in individuals' documents in Google Docs (see Appendix B).

Data Analysis

My analysis involved an iterative process of reading literature, examining documents that I created to document each student's strategies and interactions during observation and video recording sessions, viewing videos, adding details to the tables, and creating data spreadsheets for each student's strategies and interactions with partners. Phases of this analysis were done individually. To employ triangulation (Merriam, 2009), I used multiple methods of data collection — observational data, videos of students' interactions and work during Counting Collection sessions, and two types of artifacts (students' recording sheets and photographs of their counting strategies).

Phase 1: A review of the literature. Several constructs emerged in the literature review as important to consider in this analysis: *help giving* and *help receiving*, *responsiveness and non-responsiveness*, and *accuracy, flexibility, and efficiency* of student work as part of their procedural fluency.

Phase 2: Analysis of observational data. This phase of the analysis centered on the identification of particular types of help that students provided and received from each other. Each student's interactions and strategies were recorded in his or her document (see Appendix B). Upon analysis, some patterns emerged in the type of help students provided and received. I used different colors to code different types of help in every interaction on each document, and eventually came up with four main types of help: keeping on track in a number sequence, being

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accurate in counting and recording, overcoming challenging moments, and attempting new strategies. Table 1 below shows descriptions and examples for each type of help.

Table 1. Types of Help Given in Peer-to-Peer Interaction.

Types of help	Descriptions	Examples
Keeping on track in a number sequence	Help to keep on track in a certain number sequence.	Tiffany made a mistake when counting by 5s: 5, 10, 20, 25. Ben helped her to keep on track in the correct sequence: 5, 10, 15, 20, 25... and so on.
Being accurate in counting and recording	Help peers to be accurate in their counting and recording by providing strategies or suggesting mathematical tools.	To make sure that the items to be counted were identified and distinguished from items not to be counted, as well as from those that had already been counted, Jin provided a strategy to pile up cups to accurately count collections by distinguishing them.
Overcoming challenging moments	If the partner is stuck on something, provide him/her help to overcome the hurdle.	When she looked at how Ben used a number chart to add groups of different numbers, Tiffany stopped her work and wondered how Ben's strategy worked. Hearing that, Ben provided responsive feedback and showed her how he counted using a number chart.
Attempting new strategies	Encourage new strategies by providing a new idea or mathematical tool that the other student had never used before.	Tiffany suggested counting collections by unfamiliar numbers or grouping them by colors or shapes when working with both Ben and Jin.

With these four types of help, I created a data spreadsheet for each student to identify how frequently each student provided help to the others (see Appendix D).

Phase 3: Primary analysis of video data. For the third phase of analysis, I viewed the 14 videos of students working together and carefully watched their interactions. Then I filled in any missing information (specific words or interactions used) in each student's individual document in Google Docs (see Appendix B). As I observed the students' interactions, I focused

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carefully on the frequency of the four types of help they provided and if these instances of help were given by one student or mutually exchanged with each other. With this step in my analysis, I could determine whether instances of giving and receiving help were mutual or one-sided. I then filled in the data spreadsheet (see Appendix D) for each student with all the information I viewed in the videos and documented.

Phase 4: Analysis of the individual's document. In the fourth phase of analysis, I created another data spreadsheet (see Appendix E) with the information of students' strategies from my analysis of each individual student's document. To understand each child's development of counting and recording strategies over time, I created a data spreadsheet for each child. The data spreadsheets show the children's counting strategy, numbers they count to and by, recording strategies, and accuracy of their counting and recording. It also shows whether the students tried new strategies and whether they were successful or not. All these data spreadsheets are organized chronologically. This analysis facilitated my understanding of the individual student's development of mathematical strategy and thinking over time.

Findings

As I analyzed my data, there are three themes that emerged. First, young children develop their mathematical thinking as they interact with other students and their ideas. Second, the quality of students' responsiveness to each other differs depending on the partner. Last, the peer-to-peer responsiveness influences young children's mathematical learning. In what follows, I provide descriptions of each theme, with the specific examples of individual student's development, student work, and their interactions.

Cooperative Learning and The Development of Young Children's Thinking

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As Carpenter, Fennema, Franke, Levi, & Empson (2014) have suggested would happen, these three students came up with various strategies as they work with their partners (see Appendix E), which allowed them to construct their own learning process. For example, Tiffany started to get curious about counting by threes, sevens, and eights which are not familiar numbers for her to count by, and grappled with different ideas to successfully count by those numbers. At different points in time, the students invented their own methods of sorting collections into different colors and shapes in order to count them. While they demonstrated different processes of learning, all three students began to develop “procedural fluency” in counting and recording over time.

Increasing procedural fluency over time. Procedural fluency refers to the skill to carry out procedures accurately, efficiently, and flexibly. Having sufficient procedural fluency is important for student to deepen their understanding of mathematical ideas (Kilpatrick, Swafford, & Findell, 2001). The following examples of how the individual students developed their procedural fluency over time will refer to the analytic tables of the students’ counting and recording strategies (see Appendix E).

Accuracy. During the individual interview on January 26, 2017, Jin came up with a way to count large collections accurately. Prior to this day, Jin and her partners had been given small collections — the biggest collection had 76 cubes. However, on this day, she got 104 small beads. First she sorted the beads by colors, but suddenly she realized it was challenging to remember each number of colors and add them up together, so she decided to make groups of ten in plastic containers. As she made groups of ten and counted them, she piled a container having a group of ten upon the one she already counted (see Figure 2). She developed this strategy to differentiate between the groups she had already counted and those not counted, so that she could avoid errors

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in counting. Although Jin was consistently accurate in her counting and recordings from the beginning, this strategy contributed to her ability to maintain accuracy on later occasions when counting bigger collections.

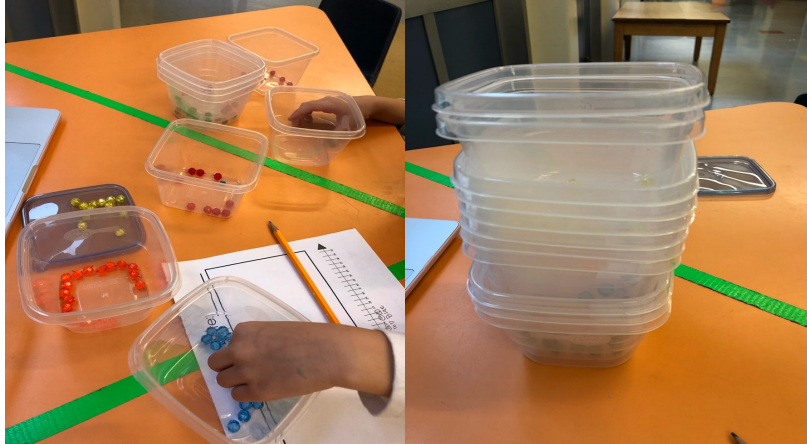


Figure 2. Jin's new strategy of cup stacking.

After this day, every time she worked with other partners, they also used this way of keeping track of the counted and non-counted groups (see Figure 3). When she used this strategy with Tiffany and Ben on different days, this helped the children to accurately count the collections. They continued to use this strategy in their later work (see Figure 4). As students' collections became larger over time, this strategy helped Ben and Tiffany to maintain accuracy in their counting. Even though Jin developed the strategy herself, because she shared this strategy with her peers during their interactions, they all benefited, making use of this technique to improve counting accuracy.



Figure 3 (left). Ben and Jin using Jin's strategy together on March 16th, 2018.
Figure 4 (right). Ben and Tiffany using Jin's strategy together on April 16th, 2018.

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Efficiency. As shown in the analytic tables of the three students' counting and recording strategies (see Appendix E), the numbers that students counted by increased over time. In the first few weeks after they started Counting Collections, they counted collections by ones, twos, and fives. However, as time went by, the numbers increased to counting by tens, and Jin counted her collections by 20s and 50s, as well.

On January 12th, 2018, after Ben and Tiffany counted square blocks by tens as their first strategy and got 69, they discussed what number they should count by for their second strategy. The following conversation between Tiffany and Ben (see Excerpt 1) shows their understanding of efficiency during this time.

Excerpt 1. Ben and Tiffany determining what number they are going to count by.

Tiffany: "8, 20, 40, 60..."

Ben: "That's tens"

Tiffany: "Oh! 80, 81, 82, 83.."

Ben: "That's ones. I don't want to count 69 by ones. I like to count by 5, 10, 15, 20, 25, 30 or 2, 4, 6, 8, 10"

Tiffany: "8, 18, 28,..."

Ben: "Let's count by 20s. I know how. 20, 40, 60"

Tiffany: "I know how to count by 8. 8, 18, 28, 38, 48, 58, 68, 78, 88, 98, 108,... 108..181...? I will just count by 8 and you can just help me set the 8s on."

Ben: "Okay. But I don't know how to count by 8"

Ben's statement "I don't want to count 69 by one" clearly shows his understanding of efficient counting procedures at this point. With sufficient understanding of efficiency, Ben suggested they count by familiar and larger numbers, like 5s and 20s. However, Tiffany did not seem to understand efficient counting procedures yet. On the first day of data collection (December 1, 2017), Tiffany was working with Jin. When Jin suggested counting by tens, Tiffany said, "But that's too much. We are going to run out fast (if we count by tens)." This quote implies Tiffany's own understanding of counting (or lack of understanding of efficiency) at this time — that they should not run out fast (running out fast is an efficient way of counting)

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— which is not a very efficient counting strategy. Later, on January 12, 2018, she suggested that Ben count by eights, which is an unfamiliar number to count by. It is also clearly difficult for Tiffany to determine the sequence of counting by eights. She actually counts by tens starting at eight, and perhaps that's what she means by counting by eights. Dwelling on this idea of counting by unfamiliar numbers, Tiffany made several attempts to count by threes, sevens, and eights with her partners, using number charts and number lines. She finally realized that it is very challenging to count by those unfamiliar numbers. During her last interaction with Ben on April 16, 2018, Tiffany first suggested they count by twos. But Ben suggested they count by fives instead. Tiffany immediately agreed by saying, "Okay, let's count by fives so we can count faster" and started making groups of fives. This situation demonstrates Tiffany's developing understanding of efficient counting procedures, which is significantly more advanced than her thoughts five months earlier.

Not only counting procedures, but students' recording procedures became much more efficient. The shift in Tiffany's recording strategies over times (see Figure 5) clearly illustrates her development of procedural fluency in recording. On the first day of data collection, December 1, 2017, once she had counted 34 pattern blocks by twos with Jin, she drew actual shapes of different pattern blocks by twos on her recording sheets (see Figure 5). Jin said, "Tiffany, you don't have to draw shapes." Tiffany replied, "I just like doing shapes." Two weeks after that, she was still drawing actual shapes of objects that she counted. But she was trying to apply what she was seeing other students doing. On January 12, 2018, she saw that Ben wrote down six 10s and 69 without any shapes or circles on his recording sheet. Looking at Ben's recording, she tried to put the number 10 in some of the square boxes that she drew, but she still did not understand it (see Figure 5). Starting from this day, she was trying to incorporate a new

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method she saw others doing. After a week, Tiffany started to draw groups of 2s, 3s, 3s and 10s with circles rather than drawing actual shapes of objects. On January 19, she drew seven circles with 10s inside and one circle with 4 inside to represent that she had counted 74 pompoms, and this was the first time she was accurate on her recording. By April 16, her recording had become much neater. As she interacted with other students and looked at their recordings for almost two months, it seems she recognized that putting numbers in circles is more efficient and accurate than drawing actual shapes. During the peer interactions over time, Tiffany learned the efficient and accurate way to record collections.

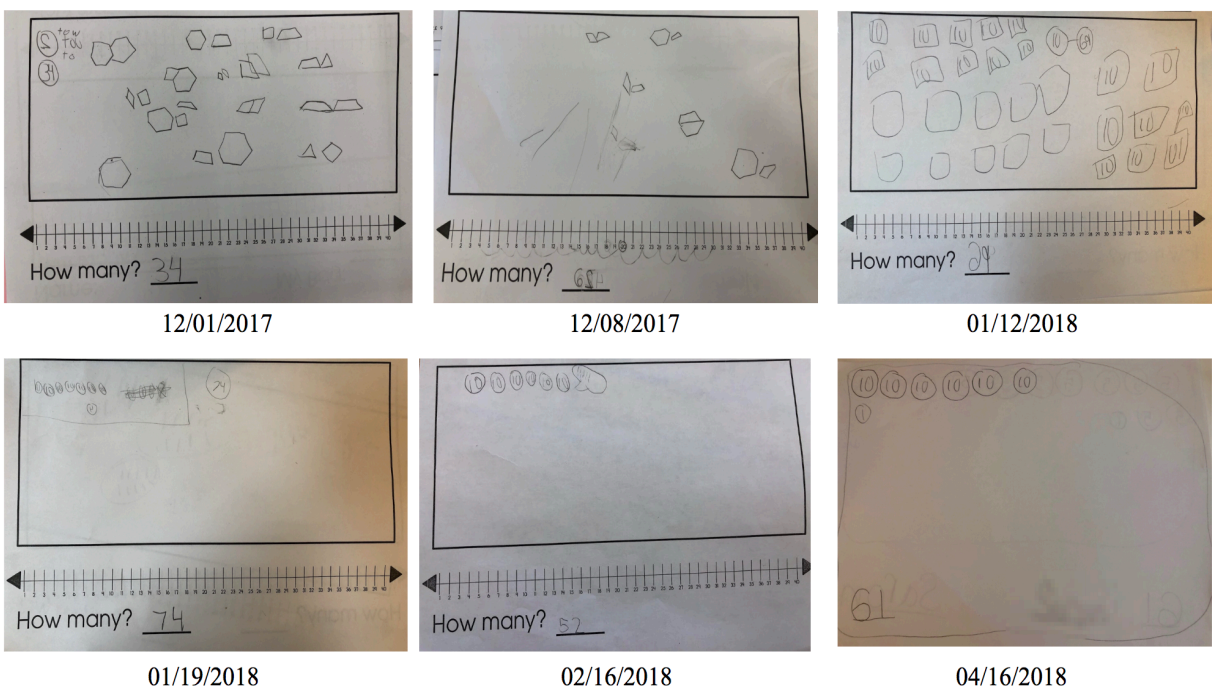


Figure 5. The development of Tiffany's recording strategies over time.

Flexibility. Ben's statement, "Okay, but I don't know how to count by 8s" from excerpt 1 indicates that he had no idea how to count by 8s or by other unfamiliar numbers, as he suggested counting by 2s, 5s, and 20s, which were familiar to him. Although he avoided counting by eights on that day, after being exposed to the idea of counting in unfamiliar numbers

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by Tiffany, Ben worked on counting in unfamiliar numbers with his partners. On February 16, 2018, he accurately counted shaped blocks by sevens using the number line on the bottom of the recording sheet. After that, he realized that using mathematical tools like a number line and number chart is very helpful when counting in unfamiliar numbers.

A month later, on March 16, Ben and Jin counted pompoms in fours using the number chart. Jin was not sure about how to count by fours, but he suggested using the number chart to keep track of the sequence. First, they put four pompoms in each plastic cup. Since he was not familiar with the sequence of counting by fours, Ben started to use the number chart to keep track of the sequence. He colored his own number chart by fours – 4, 8, 12, 16... (see Figure 6) so that they could keep track of the counting sequence as they counted each plastic cup. They counted each cup as referencing the sequence that they had created in their number chart, and eventually that day they were very successful in counting pompoms by fours.

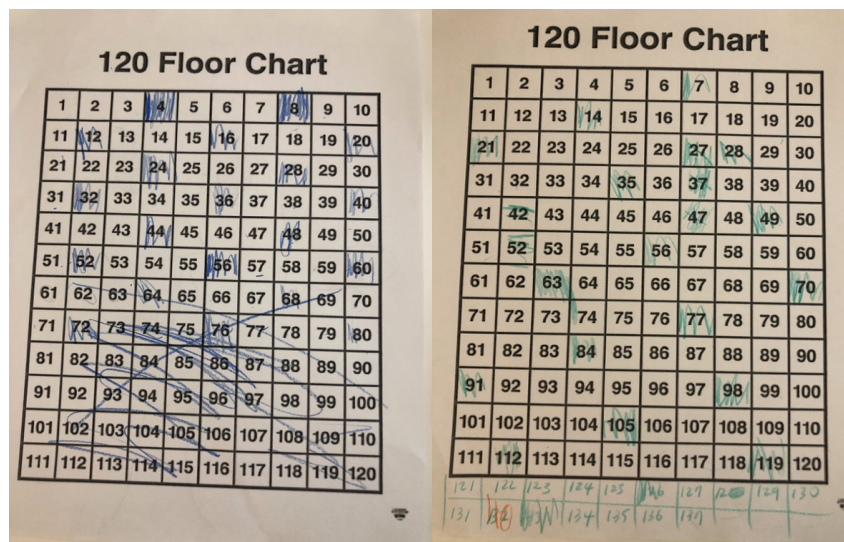


Figure 6. Ben using number charts to count collections by fours and sevens.

On following week, Ben used the same strategy to count by sevens (see Figure 6). This shows Ben's development of procedural fluency, in that he flexibly applied counting procedures for unfamiliar numbers by using a number chart. He knew that the mathematical materials made

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it easier for him to count by unfamiliar numbers. Thus, even though the sequences of fours and sevens were not in his head, he was now able to use appropriate mathematical tools in order to count by these numbers.

Such examples of individual students' development and interaction demonstrate that they develop an understanding of modifying and adapting procedures through peer interaction. Even though the examples are divided into three different parts — *accuracy*, *efficiency*, *flexibility* — they are connected to each other. This means that examples in the accuracy section also demonstrate the students' development of efficiency and flexibility. These examples can be grouped under the term *procedural fluency*. Another incident exemplifies this procedural fluency. On April 16, 2018, Ben and Tiffany counted pompoms by fives by first creating groups of five. For their second strategy, they counted by tens by making groups of ten as they combined two groups of fives that they had already made. This strategy shows their clear understanding of efficiency and flexibility, which helped them reach an accurate number of pompoms (which was also the same number that they had reached counting by fives at first). As this and the other examples have shown, all three students increased their procedural fluency over time as they interacted with each other.

By improving procedural fluency, students develop their knowledge of the structure of numbers as they group objects using different numbers and use those numbers to count by, and this provides them with opportunities to develop a conceptual understanding of numbers and the place value system. In this sense, peer interactions during Counting Collections positively affects individual students in developing their mathematical thinking with respect to specific mathematical concepts around numbers.

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Partnering and Responsiveness

The quality of students' responsiveness to one another differs depending on the partner. The following examples of student interactions shows that partners may or may not be responsive to their peers' help and ideas.

On January 12, while Tiffany was working with Ben, she came up with an interesting idea: "3 plus 2 times 2 is 10." As their first strategy, Tiffany and Ben decided to count square tiles by 10s. First, Tiffany made a group of 10 by arranging square tiles in a certain way, while Ben randomly collected 10 square tiles. Once Tiffany had looked at what Ben was doing, she said, "3 plus 2 times 2 is 10," and she explained why she had created a group of 10 with two rows of five square tiles (see Figure 7).

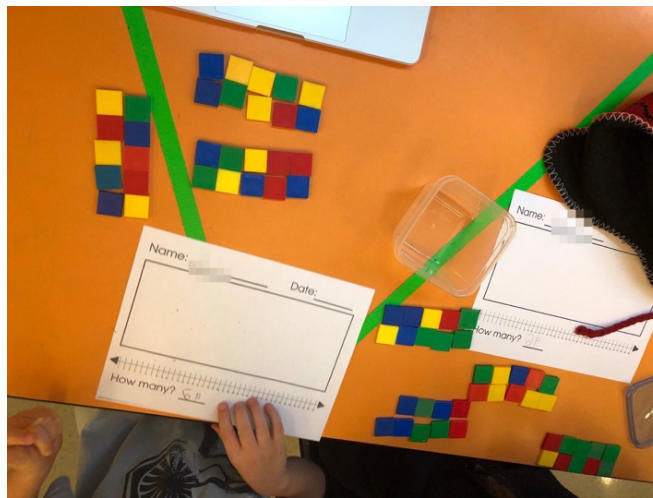


Figure 7. Tiffany's strategy of organizing groups of ten, "3 plus 2 times 2 is 10".

Listening and looking at how Tiffany was organizing groups of 10, Ben started to make groups of 10, just like Tiffany. Once both students had made groups of 10 together, they successfully reached the correct number, which was 69. This was the first time that Tiffany had achieved an accurate number in her counting, and now she could agree and fully understood why the correct number was 69. On the first day of data collections, December 1, even though Tiffany had an accurate answer, she could not agree that there were 34 because the counting then was led

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and mostly done by Jin at the time. However, on this day, with the way that she created to clearly represent groups of 10, she seemed very clear and confident in her answer and about her strategy.

The following week, Tiffany worked with Jin. She continued her idea of “3 plus 2 times 2 is 10.” Tiffany and Jin set about counting pompoms by tens, and Tiffany continued to make two rows of fives to make groups of ten. However, unlike Ben, Jin did not pick up Tiffany’s strategy, and she collected ten pompoms at a time and placed them into different places (on paper, a laptop, the plastic box, the lid of the box, etc.) (see Figure 8). Clearly, Tiffany and Jin used different methods of organizing groups of 10. Tiffany became distracted by the fact that they were not using the same strategy to arrange groups of ten, but she did not change her strategy to adopt Jin’s methods. Instead, she made the groups of tens using her own way until they organized all the pompoms.



Figure 8. Jin collecting ten pompoms into different places while Tiffany making two rows of five square tiles.

These two cases show how two students responded differently to Tiffany’s idea. The idea of “3 plus 2 times 2 is 10” that derived from Tiffany’s intuitive knowledge contains the crucial mathematical idea of addition (“plus”) and multiplication (“times”). Although they had just started to learn addition in math class, the first-graders had not been exposed to the idea of

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multiplication. Tiffany's strategy to organize groups of ten using the idea of "3 plus 2 times 2 is 10" creates a clear representation of groups of ten, which helped to decrease the risk of miscounting when the collections were grouped in a disorderly manner, as Ben and Jin did. This strategy is also very important for Tiffany to advance her learning. Noticing that this way of arranging was helpful, Ben immediately adopted Tiffany's idea and started to build the groups together with her. Thus, together they developed Tiffany's idea to reach an accurate number. In contrast, Jin did not pick up Tiffany's idea, so it was not utilized as a useful strategy in their procedures. In this sense, we can say Ben was responsive to Tiffany's thinking and that they employed mutual exchanges as they arranged groups together, while Jin was not responsive to Tiffany's idea during their interaction.

Another example of Jin working with Tiffany occurred on February 16, when Tiffany suggested Jin that they count heart beads by 8s. They put eight hearts in each cup. Once they had put all heart beads into cups, there were six cups of eight beads and one cup of four (see Figure 9), and Tiffany and Jin started counting: "Eight (pointing out cup 1), "18" (cup 2), "28" (cup 3), and so on up to "58" (cup 6), and then, individually, "59, 60, 61, 62." Suddenly, Jin realized that they had counted by 10s from eight.

"This is not counting by eights," she said, and she started to make groups of ten by herself. Jin knew that $8+2=10$, so she put two more heart beads in each cup and made groups of ten — demonstrating her knowledge of composing and decomposing. While Jin was making groups of 10, Tiffany was still wondering what had gone wrong with their sequence of eight and did not help out Jin to make groups of ten. Jin got five groups of tens with two lefts, which gave her an accurate number, 52.

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Figure 9. Six cups of eight heart beads and one cup of four.

Tiffany, meanwhile, was saying, “I wanted to count by eights.” Even though both Jin and Tiffany wrote down 52 on their recording sheet and were successful in both counting and recording, it is not at all clear that both students learned and developed their mathematical thinking. Since Tiffany did not actually count by eights, nor did she pay attention and become involved in how Jin was doing, it is hard to ascertain whether she learned anything from this interaction. Jin was just giving help to move forward from their challenging moment, but receiving this assistance from Jin was not helpful for Tiffany and her development. This is an example of nonresponsive feedback, which is described by Webb (1991) as “making an error and being told the correct answer without any explanations of how to obtain it” (p. 375). The two students were not being responsive to each other’s thinking, which made their interaction one of Jin giving (to Tiffany) in a way that was not accepted and thus was not helpful (to the receiver).

In contrast, the following interaction between Jin and Ben on March 16 shows their mutual responsiveness to each other’s idea and strategies. A month after the interaction with Tiffany, Jin was partnered with Ben, and he suggested counting by fours. Jin’s first reaction to Ben’s suggestion was not positive. The last time working with Tiffany was actually the first time she had attempted to count objects by an unfamiliar number, and it had not been very successful.

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Following this experience, therefore, Jin was not confident about counting by fours. Nevertheless, she tried again, with Ben. As mentioned above, first, they put four pompoms in each cup, and Ben used the number chart to keep track of the sequence. Looking at how Ben was doing, Jin started to color her own number chart by fours as well: 4, 8, 12, 16... (see Figure 10). Then, as Jin understood what they were doing on the number chart, she suddenly stopped coloring the sequence and she counted the number of cups (while Ben continued coloring the sequence). Jin had realized that they had 16 cups, and she was checking how much of the sequence she had colored on her number chart. She then continued to color 16 spots, so that it matched with the number of cups (see Figure 10). Ben, who colored the sequence by fours until 80, helped her to keep track of the numbers in the number chart. Through this collaboration, they were able to successfully arrive at the correct number, 64.



Figure 10. Jin's (left) and Ben's (right) number charts keeping track of the sequence.

Even though Jin's interactions with Tiffany and Ben both started with the suggestion of counting by unfamiliar numbers, they ended up having different interactions and exchanges. Ben and Jin gave and received help to and from each other as they worked together. They were both very responsive to each other's ideas and strategies, and they made further moves and provided

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further help. This was not the case with Jin and Tiffany, however. Thus, we can say Jin and Ben engaged in mutual exchanges of knowledge and interaction, while Jin and Tiffany did not.

Ideally, both partners should support and benefit from each other; they should develop ideas together through mutual exchanges of knowledge throughout their interaction. The examples above, however, clearly demonstrate that some partnerships work better than others, that some partners may be responsive and other partners not. This suggests the importance of considering who is likely to be responsive to another's ideas and thinking, and who is not.

Responsiveness and Young Children's Mathematical Learning

Peer-to-peer responsiveness effectively promotes young children's mathematical learning. Jin and Ben's interaction described above supports this idea. Before interacting with Ben, Jin struggled to count by unfamiliar numbers. However, after her responsive interaction with Ben and his ideas, she not only improved her mathematical skill in counting by unfamiliar numbers but also developed a deeper mathematical understanding of the numbers. She learned how numbers can be formed and structured in different ways. As first-graders, Jin and the other two students had a strong understanding of how numbers are grouped by ones, twos, fives, and tens. However, it was challenging for them to visualize and understand how numbers are grouped by threes, fours, sixes, sevens, and eights without being taught a certain way or sequence. By using a number chart as a mathematical tool to visualize the groups and sequences of fours and to connect numbers in the chart to actual pompoms, Jin could visualize how numbers are formed by different equal size groups. Doing hands-on mathematical exploration with her partner gave her a deeper understanding than she would have had from isolated facts and methods. Consequently, she developed an important conceptual understanding of numbers: how they can be formed differently and yet are related to each other. Ben's help of providing a number chart and

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demonstrating how to use it was perfectly aligned to Jin’s needs of understanding counting by unfamiliar numbers. Being responsive to each other’s thinking and strategies, Jin and Ben were able to successfully explore the new methods or strategies and arrive at an accurate number, clearly exemplifying how this form of responsive peer interaction provides students mathematical learning opportunities.

The following example demonstrates how active peer-to-peer responsiveness effectively contributes to students’ mathematical learning. On March 2, 2018, Ben and Tiffany were given different shapes and colors of buttons. First, Tiffany suggested counting the collection according to shape. Ben agreed, and they started sorting buttons together by shape into the plastic cups (see Figure 11). Eventually, they came up with 11 groups of shapes, each with a different number of buttons. Tiffany picked up a cup containing circular buttons, counted the number in that cup, and announced there were 17. I asked, “How are you going to keep track that this has 17?” and Ben responded, “We write 17, and then we write [meaning ‘draw’] a circle.” Then they both wrote 17 and drew a circle to remember that there were 17 circular buttons (see Figure 12). Next, they counted 6 hexagonal buttons and so on until they finished counting all 11 groups of button shapes.



Figure 11. Ben and Tiffany sorting buttons by shapes.

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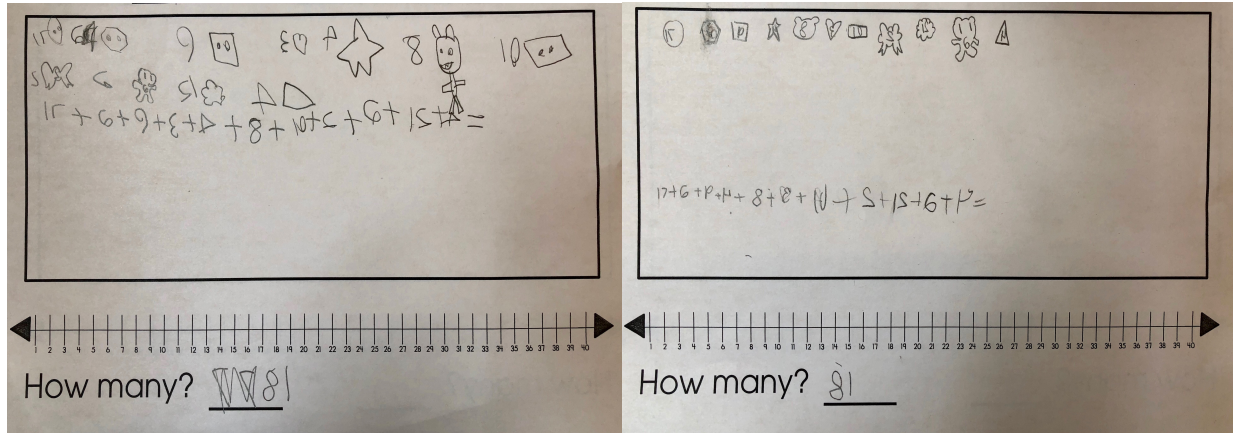


Figure 12. Ben's (left) and Tiffany's (right) recording sheets.

As shown in Figure 12 above, they drew actual shapes and they put the number of buttons for each shape beside (Ben) or inside (Tiffany) their drawings. Ben and Tiffany fully collaborated on all aspects of this counting and recording work. At times, they counted a group of buttons together, and sometimes while Ben counted one group (heart-shaped buttons), Tiffany counted another (bear-shaped buttons), and they exchanged this information. Tiffany also helped Ben to draw some shapes like teddy bears on his recording sheet. They both contributed to this counting and recording work together. This suggests that they were very responsive to each other's needs and ideas. With such great collaborative work on counting and recording, they figured out numbers for each of the 11 groups and recorded the information very accurately in their recording sheets.

Once Ben counted and recorded the last group that had 4 triangular buttons, he started writing down the following equation immediately: $17+6+9+3+4+8+10+2+6+12+4$. By following what Ben was doing, Tiffany also wrote down the equation on her recording sheet (see Figure 12). While Tiffany was working on writing down the equation, Ben grabbed a number chart and started adding numbers. First, he colored 17, and he counted 6 more from 17 and colored 23; then he counted 9 more from 23 and colored 32 (see Figure 13); he continued this method until he reached 81, which was the correct total number of buttons. Figuring out a sum of 11 addends

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using a number chart demonstrates his robust understanding of addition and ability to utilize mathematical tools appropriately. Tiffany, who had no idea what Ben was doing with the number chart, was going to count every single button; however, Ben explained what he was doing, and she eventually understood and followed with his strategy and helped him to move forward until they reached 81.

120 Floor Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120

Figure 13. Ben using a number chart to add different numbers.

In this interaction, Ben and Tiffany both contributed to the process of work as well as to their interaction in meaningful ways. Starting with Tiffany's suggestions, they first tried categorizing, sorting, and grouping collections by attributes, created 11 groups, counted the number of buttons in each group, made an equation with that group, and successfully added those groups of different sizes together, creating a cumulative sum by using an appropriate mathematical tool in order to solve the equation accurately. In order to complete each of these steps accurately, a significant mathematical understanding of numbers is required. The children also developed their conceptual understanding of numbers by doing this. Even though Tiffany could not understand Ben's strategy using the number chart to add numbers at the beginning, Ben provided her with responsive feedback so that she could make sense of how different sizes of

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groups could be added together to finally reach the total 81. Working together but independently from the teacher, Ben and Tiffany created an interesting but much harder problem than what they had covered in class, and they contributed to this very meaningfully together. Each step was derived from one of their suggestions, ideas, or actions, and both students were very responsive to that and worked collaboratively as they built on each other's ideas and suggestions. Opportunities for mathematical learning arose when the children attempted to reach an agreement as they worked together. They were able to accomplish such a complex math problem together because they were listening to and understanding each other's ideas and needs so carefully. This example clearly demonstrates how responsiveness in the process of peer interaction contributes to young children's mathematical learning.

Discussion and Conclusion

In this study, I sought to understand how young children develop their mathematical thinking processes, with the goal of determining the effect of certain pairings and responsiveness during peer interactions in order to make recommendations for implementing cooperative learning.

To achieve this aim, I examined peer interactions among three students in first-grade and the effects of the processes and quality of these interactions on each student's mathematical learning. More specifically, I examined the significance of the relationship between mathematical learning and peer-to-peer responsiveness, which fills a gap in the literature. The process and quality of an interaction differ depending on peer-to-peer responsiveness. As shown in the different cases of peer interaction in this study, the three students would reach an agreement when they became responsive to each other's thinking, strategies, and needs. When they reached an agreement by communicating, clarifying, accepting, and building on each

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other's ideas, they actively engaged in opportunities for mathematical learning (see Figure 14). These learning opportunities allowed the students to acquire new information and connect it to their previous understanding.



Figure 14. Model of peer-to-peer interaction.

Furthermore, my data also show that students' ideas travel around the classroom, changes, and evolve over time during the process of peer-to-peer interaction. Students' ideas should not be considered stationary but rather active and flexible, changing their intangible form and shape very rapidly in different learning environments. This observation also implies that responsiveness should not be a one-time characteristic of a students' effectiveness in partnering with other students. In other words, determining responsiveness based on a one-time interaction could bring a significant bias in understanding complex interactions among students. Thus, this study suggests how ideas and responsiveness move around the classroom over time and eventually lead to an increase in learning opportunities.

While investigating the differing quality of peer interactions, I observed that the students' ideas traveled around the classroom as they interacted with different partners. For example, Tiffany's curiosity and her idea of counting using different numbers traveled around to the other partnerships in this study over time, emerging in many peer interactions; her idea was successfully developed during these interactions (see Figure 15). Tiffany first suggested counting by eights when she was partnering with Ben on January 12. After this, Ben worked on counting

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using unfamiliar numbers with his other partners. On February 16, he accurately counted blocks by sevens using a number chart. On the same day, Tiffany posited the idea of counting by eights to Jin. Even though the interaction between Jin and Tiffany was initially not responsive — they did not have mutual exchanges or successfully count by eights — a month later I observed Jin worked with Ben to complete such a count. Ben, who had continued working on this idea, helped Jin to count by fours and sevens and successfully learned to count using unfamiliar numbers.

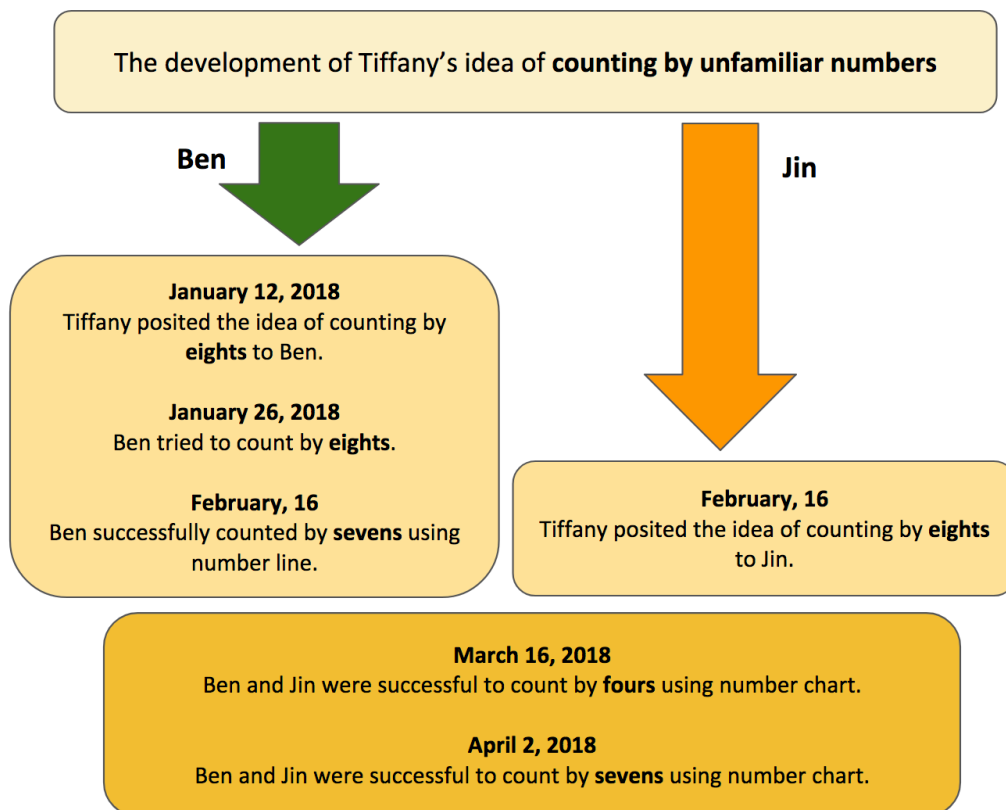


Figure 15. Tiffany's idea traveling around over time.

Starting with Tiffany's curiosity and her suggestion of counting by an unfamiliar number, Ben then figured out a way to keep track of the sequence of unfamiliar numbers. Eventually, he and his partners were able to accurately count objects by unfamiliar numbers using appropriate mathematical tools. He contributed to Tiffany's idea and helped circulate it to the others.

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Regardless of whether they had mutual and responsive interactions with certain peers, Tiffany's idea was shared with different partners in various interactions and thus generated significant learning opportunities (to learn how numbers can be formed and structured in different ways) for each student.

Similarly, Jin's strategy of cup stacking, which evolved over time in various peer interactions, helped the students to maintain accuracy in their counting, especially when their collections of objects became larger. Even though Tiffany and Jin were not responsive to each other while working together, they each developed their own ways of thinking and derived collaborative strategies while working with Ben and other students in the classroom. This is one of the positive effects of cooperation on student learning.

As shown in above example of Tiffany, Ben, and Jin, cooperative learning and peer-to-peer interaction can lead to various positive effects on students' mathematical learning through the social, cognitive, and academic interaction among students. The concept of student's idea circulating and evolving over time in classroom grants us the simple notion that every student's ideas are valuable and could work as a stepping stone to another's idea, which can build gradually over time in various ways. This simple notion may be a key to understand the potential application and resulting benefits of cooperative learning settings. It could also serve as a guide to the important fact that it is not simply possible to label the student and his/her idea. The notion becomes even more impossible after the idea has changed and evolved over time because we cannot fully understand how the ideas add up to and affect each other in a complementary manner. Thus, even a single idea contributes to students' learning in such a way that we cannot easily measure its importance. As a more practical example, the Counting Collection activity in this study was performed in a mixed or heterogeneous ability grouping so that students could

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have the opportunity to communicate with students of different mathematical ability. By ignoring the student's academic label, mixed ability grouping could be equally or more beneficial in cooperative learning compared to its counterpart, homogeneous ability grouping. In mixed ability grouping, the positive effects was possibly due to complementary development of a student's idea over time, not whom this idea is originated from. Comparing the benefits between heterogeneous grouping and homogeneous grouping is somewhat beyond the scope of this study. However, this study tries to show that pre-labeling students based on their academic ability and designing math activities by relying on simple statistical values might lead us to overlook many important facts since we are not able to measure the importance of each student's idea, and the idea traveling around classroom is what drives student's learning, possibly leading to the positive effects of cooperative learning.

Implications for Research and Practice

This study highlights that every student's ideas are valuable, regardless of their various ability levels, because they can build gradually over time in various ways. For examples, Tiffany, who is in the low ability group, has provided important ideas and strategies that circulated around the classroom during the process of peer interactions. Tiffany's ideas generated significant learning opportunities for the other students. Ben, who is in the middle-ability group, made contributions by developing Tiffany's and Jin's initial ideas in several ways, such as in figuring out ways to use appropriate mathematical tools and by being responsive to their ideas and strategies. In addition, Ben and Tiffany solved a complex math problem together, as they both contributed to process of adding 11 groups of different sizes. Hence, Ben and Tiffany have demonstrated remarkable abilities, which prevents us from labeling them as low and middle ability students. This study serves as evidence that students' performance levels in their

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classroom does not represent their thinking. It is problematic that these students cannot demonstrate the value of their ideas on their mathematics tests. One of the limitations of this study is that the three students' performance on their mathematics tests were not carefully investigated; their abilities were ascertained based on data given by the teacher. One possible focus for future research would be to examine what actually occurs when young children take tests (e.g. what factors might affect the performance of children on tests) and why their test performance differs from the demonstrated value of their ideas. Although it is true that the three students in this study received different scores on their mathematics tests, this study suggests that teachers should not simply rely on students' test results, as a specific test cannot adequately measure the value of each student's ideas.

This study is situated within mathematics education but its implications are not limited to the mathematical work of young students. Rather, the implications of this study are applicable for educators and researchers using cooperative learning in a broad range of content areas at all levels of schooling. The findings of this study illustrate that students' ideas circulate around the classroom and develop over time, and eventually lead to an increase in learning opportunities. In other words, the findings contribute to an understanding of the potential application and subsequent benefits of cooperative learning settings — not only in mathematics but also in other subject areas. This study recommends the need for future research on a deeper analysis of students' level of engagement with each other's mathematical ideas during peer interactions and how such engagement develops over time, as the ideas and responsiveness circulate. By filling this gap, educators and researchers could better design and implement cooperative learning settings that encourage every student to engage with each other's idea effectively.

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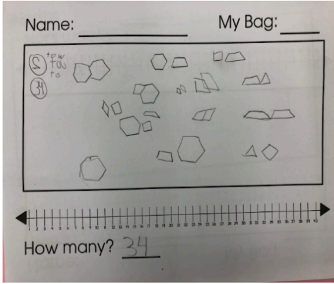
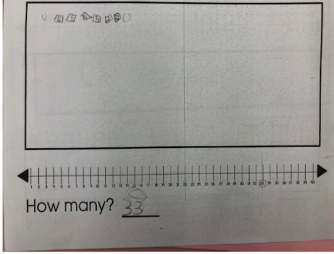
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Appendix A: A table for field notes to document each student's strategies and interactions

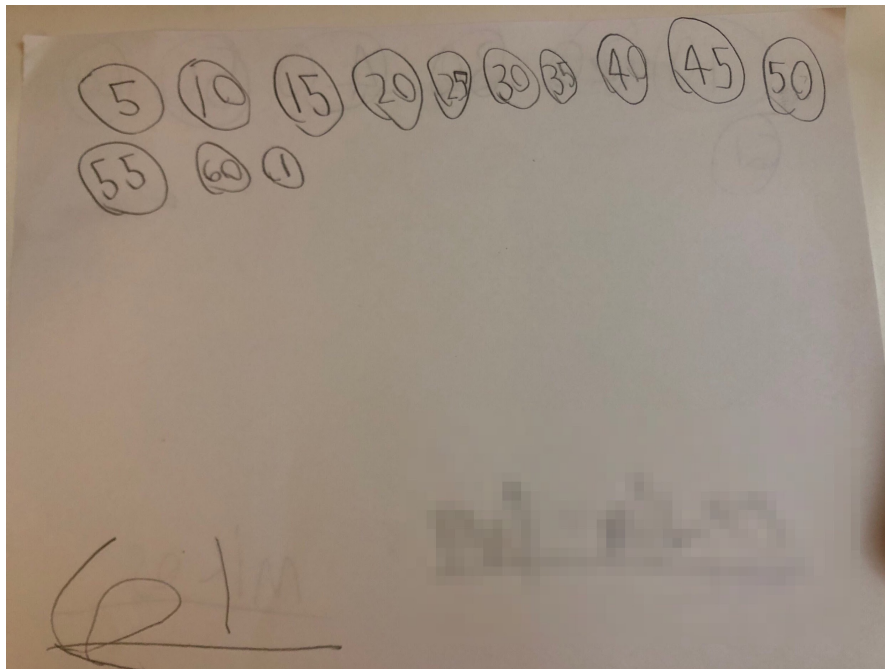
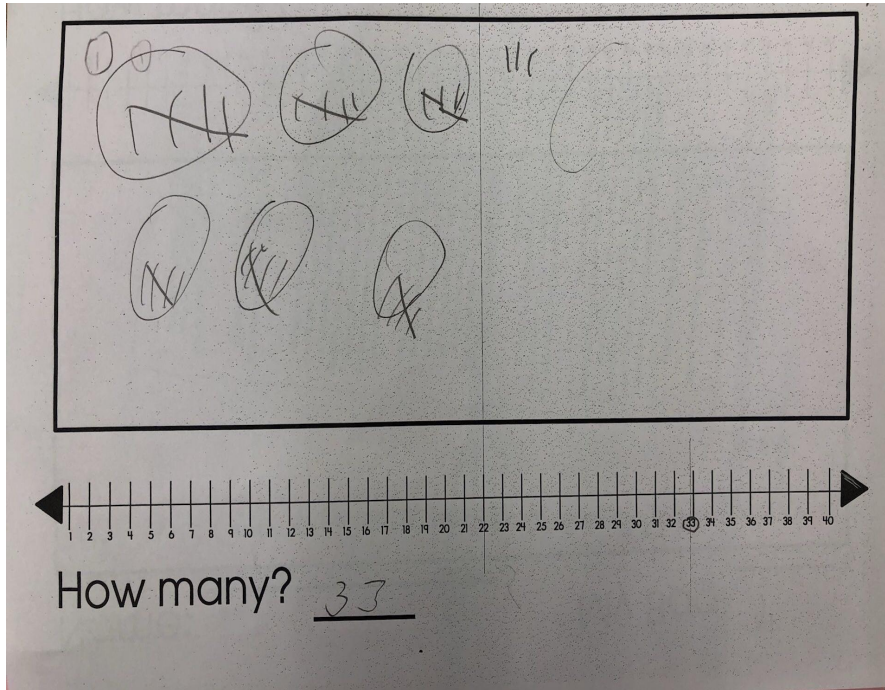
Date	Partner	Collection	Strategies	Count to	Count by	Accuracy - counting	Recording strategy	Accuracy - recording

Appendix B: An example of one day of an individual's document in Google Docs

Date	Partner	Collection	Strategies	Count to	Count by	Accuracy -- Counting	Recordings	Accuracy -- Recording	Other Comments
12/01/2017 (V)	Jin	Pattern blocks	<p>First, they counted blocks by 2s. Jin helped Tiffany to keep track on numbers (18 → 20). Jin who counted last block got 33 but they both forgot the last number so they counted by 1s again to make sure. Then they got 34.</p> <p>Second, Jin and Tiffany decided to count by 10s but Tiffany said, "But that's too much. We are going to run out fast (if we count by 10s)." So they started to count by 5s. They were confused with counting sequence 5 → 10 → 20 → 25. They got 32. However, Jin counted by 1s again to make sure and got 33 which is an accurate number.</p> <p>However, Tiffany could not agree that they have 33 because Jin counted them by ones so Tiffany suggested to count by 5s again so they counted by 5s again with better strategy -- making block towers each has 5 (see below).</p> 	34	2s and 5s	<p>YES</p> <p>Got 34 when counted by 2s (counted again by 1s to make sure)</p> <p>Got 33 when counted by 5s</p> <p>When they counted by 5 again with better strategy, they got 33 again which is an accurate amount.</p>	 	<p>NO -- Tiffany drew shapes to represent her counting strategy. Since she counted by 2s, she drew shapes into pairs but ended up drawing 12 pairs and 2 ones. While she was recording, she kept asking Jin, "how many pairs did you have?"</p> <p>"INTERESTING" Jin: "Tiffany, you don't have to draw shapes"</p> <p>Tiffany: "I just like doing shapes"</p> <p>NO -- Getting some help from Jin's recording, Tiffany drew 6 groups of fives.</p> <p>: Tiffany had hard time representing her counting strategies into paper.</p>	

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Appendix C: Examples of recording sheets



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Appendix D: An example of the data spreadsheet of how frequently each student received help from his/her partners

Getting help in...	Partner	Mutual exchanges of knowledge	Keeping on track in a number sequence	Being accurate in counting and recording	Overcoming challenging moments	Attempting new strategies
12/01/2017	Jin	No -- receiving				
12/08/2017	Student J	No				
12/14/2017	by herself					
01/12/2018	Ben	Yes				
01/19/2018	Jin	No				recording
01/26/2018	by herself					
02/09/2018	Ben	Yes		Recording		
02/09/2018	by herself					
02/16/2018	Jin	No -- receiving		Counting + Recording		
03/02/2018	Ben	Yes				
03/16/2018	Student A	No				
04/02/2018	Jin	No -- mostly receiving				
04/16/2018	Ben	Yes				
04/23/2018	by herself					

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Appendix E: The development of individual student's counting and recording strategies over time

The development of Tiffany's counting and recording strategies

	Partner	Strategies	Count to	Count by	Recording	Accuracy -- Counting	Accuracy -- Recording	new strategies successful	new strategies not successful
12/01/2017	Jin		34	2s and 5s	Drawing actual shapes of pattern blocks and groups of 5s and 2s				
12/08/2017	Student J		62	2s	Drawing actual shapes of pattern blocks				
12/14/2017	by herself	First sorting pompoms by different colors but it was hard to remember and add numbers of colors so counted by 1s	49	1s	Drawing circles one by one (shape of pompoms)				counting & recording
01/12/2018	Ben	"3 plus 2 time 2 is 10"	69	10s and 8s	Started to draw groups of 10			"3 plus 2 times 2 is 10" + recording	counting by 8s + Recording
01/19/2018	Jin	"3 plus 2 time 2 is 10"	74	10s	Groups of 10s and ones			recording	
01/26/2018	by herself		101	10s	Groups of 10s and ones				
02/09/2018	Ben		47	2s	Drawing hearts with groups of two and ones				recording
02/09/2018	by herself	making lines of three hearts under the number line to count by 3s	25	3s and 2s	Groups of 3s		only one strategy -- count by 2s		counting by 3s
02/16/2018	Jin		52	8s and 10s	Groups of 10s and ones		revision		
03/02/2018	Ben	Sorting by shapes and using number chart to keep track on & add numbers of each shapes	81	shapes	drawing shapes of each bottons				
03/16/2018	Student A		62	5s	Groups of 5s and ones				
04/02/2018	Jin	Counting by 3s using number chart	109	3s	Groups of 3s				counting by 3s using number chart
04/16/2018	Ben	making groups of 10s with groups of 5s they already made -- efficiency	61	5s and 10s	Groups of 5s and 10s			making groups of 10s with groups of 5s they already made -- efficiency	
04/23/2018	by herself	Counting by 5s using number chart to keep track on sequence	128	5s	Groups of 5s and ones				

The development of Ben's counting and recording strategies

	Partner	Strategies	Count to	Count by	Recording	Accuracy -- Counting	Accuracy -- Recording	new strategies successful	new strategies not successful
12/01/2017	Student A		35	1s	Drawing shapes/circles one by one				
12/08/2017	Student J		32	10s	Drawing uni-fix cubes 10/10/10/2 Just like how he counted				
12/14/2017	by himself		43	1s and 10s	Groups of 5 marks / groups of 10 marks			recording	
01/08/2018	Jin		76	1s and 5s	one marks / groups of 5 marks			recording	
01/12/2018	Tiffany	"3 plus 2 times 2 is 10"	69	10s and 8s	Writing 10 to represent a group of 10. Six tens and 69.			counting & recording	counting by 8s
01/19/2018	Student A		60	1s	Drawing circles and marks one by one				
01/26/2018	by himself		151	10s and 8s	Writing 10s and one 1 to show 151			recording	
02/09/2018	Tiffany		47	2s	2,4,6,8,10,12,14...46 and 47			recording	
02/16/2018	Student H		28	5s and 7s	Writing 5s and 7s			counting by 7s	
03/02/2018	Tiffany	Sorting by shapes and using number chart to keep track on & add numbers of each shapes	81	shapes	drawing shapes of each bottons				
03/16/2018	Jin	counting by 4s using number chart	64	4s	groups of 4s			counting by 4s using number chart	
04/02/2018	Jin	Counting by 7s using number chart	137	7s	Goups of 10s			counting by 7s using number chart	
04/16/2018	Tiffany	making groups of 10s with groups of 5s they already made -- efficiency	61	5s and 10s	Groups of 5s and 10s				

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The development of Jin's counting and recording strategies

	Partner	Strategies	Count to	Count by	Recording	Accuracy -- Counting	Accuracy -- Recording	new strategies successful	new strategies not successful
12/01/2017	Tiffany		34	2s and 5s	circles with groups of 2s and 5s				
12/08/2017	by herself		54	1s and 2s	circles one by one and circles with groups of 2s <new strategy: checking out to make sure actual objects and her drawings should have same amount.			recording	
12/14/2017	Student M		54	1s and 2s	same from last week				
01/08/2018	Ben		76	1s and 5s	circles one by one and circles with groups of 5s				
01/12/2018	Student J		76	20s	three circles with groups of 20 and one with 16			counting by 20s	
01/19/2018	Tiffany		74	10s	seven circles with groups of 10 and 4 extra <started to write down EXTRA>			recording -- "Extra"	
01/26/2018	by herself	First sorting by colors and put each colors into groups of 10s	104	10s	ten groups of 10 and 4 extras			counting	
02/09/2018	Student N	making groups of 50s with groups of 10s -- combining five groups of tens to make a group of 50s	310	10s and 50s	Groups of 10s and groups of 50s			counting by 50s	
02/16/2018	Tiffany	First tried to count by 8s and made groups of 8s -- but realized it's hard so made groups of 10s with groups of 8s <putting two each>	52	8s and 10s	groups of 10s and two extras			counting by 8s to 10s	
03/02/2018	Student A		180	10s					
03/16/2018	Ben	counting by 4s using number chart	64	4s	groups of 4s			counting by 4s using number chart	
04/02/2018	Tiffany	Counting by 3s using number chart	109	3s	Groups of 3s				Counting by 3s using number chart
04/02/2018	Ben	Counting by 7s using number chart	137	7s	Goups of 10s			counting by 7s using number chart	
04/16/2018	Student A	"3 plus 2 times 2 is 10" & making groups of 20s with groups of 5s they already created	81	5s and 20s	Groups of 5s and 20s			making groups of 20s with 5s	
04/23/2018	by herself	Sorting by colors & 10s	309	10s	groups of 10s			sorting by colors and 10s	