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Sheh Lit Chang

An investigation of student difficulties in qualitative and quantitative physics problems: Examples from linear kinematics, rolling motion, fluid mechanics, and vector superposition

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Abstract

An investigation of student difficulties in qualitative and quantitative physics problems: Examples from linear kinematics, rolling motion, fluid mechanics, and vector superposition

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This dissertation consists of two primary studies on student understanding of physics. These include a broad investigation of student understanding of the motion of different kinds of objects (particle, rigid body, and fluid) and results from two preliminary efforts of incorporate the use of technology into the learning and teaching of physics and physics education research. The first section of this dissertation presents results from several detailed studies into the existence and/or persistence of student understanding of particle, rigid body, and fluid motion. These studies, for the most part, began as investigations to examine research results that had previously been reported, but they led to the identification of additional student difficulties. In one case, the findings guided the design of instructional materials to promote student understanding of rolling motion. The second section of the dissertation consists of two preliminary studies: the first from a sequence of

online instructional interventions designed to improve student understanding of vector superposition. The second study is on the use of natural language processing techniques to examine student reasoning in physics education research. The population across all these studies range from students in high school to those in algebra- and calculus-based introductory physics courses.

TABLE OF CONTENTS

List of Figures xiii

List of Tables xxi

Chapter 1. Introduction 1

1.1 Motivation for research 3

1.2 Context of investigation 4

1.3 Iteration of research and curriculum development 9

1.4 Organization of the dissertation 11

Chapter 2. Research Methodology & Statistical Analysis 13

2.1 Data collection 13

2.1.1 Pretests 13

2.1.2 Posttests 15

2.1.3 In-class observations 16

2.2 Experimental design 16

2.2.1 Cross-sectional design 16

2.2.2 Design of pretest-posttest studies 17

2.2.3 Longitudinal design 18

2.3 Data analysis methods 19

2.3.1 Quantitative methods 19

2.3.2 Qualitative analysis 23

2.4 Possible sources of threats to validity 24

2.4.1 Missing data 24
2.4.2 Lack of randomization 26
2.4.3 Variations in student performance 27
2.5 Summary 28
Part I Investigation of Student Understanding of Particle Motion 29
Chapter 3. Background & Prior Research 31
3.1 Review of kinematical quantities relevant to this investigation 31
3.2 Prior research 33
3.2.1 Research on student conceptual understanding of kinematics 33
3.2.2 Research on student understanding of rates 35
3.2.3 Research on student problem-solving 36
3.2.4 Commentary 38
3.3 Motivation and context of research 38
Chapter 4. Preliminary Investigation of Student Understanding of Speed and Related Tasks 41
4.1 Student responses on the speed calculation task 42
4.1.1 Results and discussions from the written format 43
4.1.2 Results from the multiple-choice format 48
4.2 Student responses on kinematics graph task 53
4.2.1 Response from the written pretest 54
4.2.2 Student responses on the time-interval ranking task 60
4.3 Summary of the preliminary investigation 63
Chapter 5. In-depth Investigation of Student Understanding of Speeds and Relevant Ideas 65

5.1 Investigating student understanding of speed through isomorphic problems 66
5.1.1 The isomorphic problems – description and administration 66
5.1.2 Results and discussions 70
5.1.3 Commentary 74
5.2 Additional investigation involving spring-block & cart-on-ramp systems 75
5.2.1 Modifications and results for the SB context 75
5.2.2 Results from the CR context 78
5.2.3 Implications of this study 85
5.3 Students' ability to perform consistency check in online pretests 87
5.3.1 Methodology 88
5.3.2 Discussion of preliminary results 90
5.3.3 Results and discussions from further investigation 96
5.3.4 Commentary 100
5.4 Modification of curriculum to address speed misconception 102
5.4.1 Description of the curriculum 102
5.4.2 Assessment of curriculum 104
5.4.3 Additional discussions 107
5.5 Summary and conclusion 108
Conclusion for Part I 111
Part II Investigation of Student Understanding OF Rolling Motion 115
Chapter 6. Background & Prior Research 117
6.1 Overview of relevant physics underlying rotational and rolling motion 117

6.1.1 Rotational motion of a point object 118
6.1.2 Rotation of a rigid body about a fixed axis 118
6.1.3 Rolling motion 121
6.1.4 Commentary on the traditional approach to instruction on rolling without slipping.... 123
6.2 Prior research on student understanding of rotational and rolling motion 124
6.2.1 Student difficulties with rotational dynamics and rolling motion 124
6.2.2 Curriculum development and instructional strategies for rolling motion 129
6.3 Prior research on student understanding of Galilean relativity 129
6.4 Commentary on prior research 132
6.5 Motivation and Context of Investigation 133
Chapter 7. Investigation oF Student Difficulties with Rolling Motion I – Replication studies & Results from past examinations 135
7.1 Rolling wheel question in RRMCS 136
7.1.1 Description of the Rolling Motion Task (RMT) 136
7.1.2 Correct response 137
7.1.3 Administration of the RMT 138
7.1.4 Results for the RMT-Direction task 139
7.1.5 Results for the RMT-Magnitude task 141
7.1.6 Discussion 144
7.2 Student explanations on the RMT on pretests 146
7.2.1 Student reasoning in the RMT-Direction task 147
7.2.2 Student reasoning in the RMT-Magnitude task 149

7.2.3 Discussion 153
7.3 Student understanding of friction in rolling motion 154
7.3.1 Friction on a rolling object (horizontal) 154
7.3.2 Friction on a rolling object (inclined plane) 157
7.3.3 Friction on a slipping object (horizontal) 159
7.3.4 Commentary 160
7.4 Summary 161
Chapter 8. Investigation of Student Difficulties with Rolling Motion II – Further investigations Related to Rolling Motion Tasks 163
8.1 Investigation 1 – Tendency of students to use the tangent to the shape of an object to determine the velocity of a point on the object 165
8.1.1 Study design and task administration 165
8.1.2 Results and discussion 167
8.1.3 Commentary 173
8.2 Investigation 2 – The modified RMT-Direction task 174
8.2.1 Student responses to the modified RMT-Direction (displacement) task on pretest 174
8.2.2 Student responses to the modified RMT-Direction (trajectory) task on exam 177
8.2.3 Commentary 180
8.3 Investigation 3 – RMT involving different rigid bodies 181
8.3.1 RMT-Direction task involving different rigid bodies 181
8.3.2 RMT-Magnitude task involving different rigid bodies 185
8.3.3 Commentary 188
8.4 Investigation 4 – Correlation between relative motion and RMT 189

8.4.1 Student performance in typical relative motion problems 190
8.4.2 Student performance in relative motion tasks analogous to RMT-Direction task .. 199
8.4.3 Correlation with the RMT-Direction Task 203
8.4.4 Commentary 204
8.5 Summary 205
Chapter 9. Investigation of Student Difficulties with Rolling Motion III – Others Studies 207
9.1 Student responses on the cylinder-on-board (CoB) task 208
9.1.1 Description and administration of the CoB task 208
9.1.2 Results 210
9.1.3 Student explanations 211
9.1.4 Discussions 214
9.2 Student responses on the board-on-cylinder (BoC) task 215
9.2.1 Description and administration of the BoC task 215
9.2.2 Results of the BoC task on pretests 217
9.2.3 Student explanations 218
9.2.4 Discussions 221
9.3 Further investigations involving the board-on-cylinder (BoC) task 223
9.3.1 Results for the modified board-on-cylinder (mBoC) task during lectures 223
9.3.2 Student responses for the mBoC task on pretests 225
9.3.3 Results from additional investigations 228
9.3.4 Commentary 230
9.4 Summary 231

Chapter 10. Addressing Student Difficulties with Rolling Motion 233

10.1 Student difficulties with rolling motion and learning objectives of the tutorial curriculum 234

10.1.1 Difficulty in applying relative motion concepts in rolling motion 234

10.1.2 Difficulty in thinking of the dynamics (forces) involved in rolling motion 235

10.1.3 Difficulty in applying the no-slip condition in situations involving rolling without slipping 236

10.1.4 Learning objectives for the tutorial curriculum 236

10.2 The preliminary version of the tutorial curriculum 237

10.2.1 Description of the tutorial sequence 238

10.2.2 Description of the homework sequence 242

10.3 Assessment of preliminary version of the tutorial 243

10.3.1 Learning objective 1 – students can apply relative motion concepts to rolling motion or rigid body motion 244

10.3.2 Learning objective 2 – students can define the no-slip condition and show how it applies to different situations 247

10.3.3 Learning objective 3 – students can recognize the role of friction in rolling without slipping and rolling with slipping situations 253

10.3.4 Learning objective 4 – students can perform better in questions that involve relative motion ideas 255

10.3.5 Commentary of assessment of preliminary version of the tutorial 257

10.4 The modified version of the tutorial curriculum 258

10.4.1 Description of the modified tutorial 259

10.4.2 Description of the modified homework 264
10.5 Assessment of the modified tutorial 271
10.5.1 Learning objective 1 – students can apply relative motion concepts to rolling motion or rigid body motion 272
10.5.2 Learning objective 2 – students can define the no-slip condition and show how it applies to different situations 275
10.5.3 Learning objective 3 – students can recognize the role of friction in rolling without slipping and rolling with slipping situations 277
10.5.4 Commentary 281
10.6 Summary 282
Conclusion for Part II 285
Part III Investigation of Student Understanding in Fluid Motion 289
Chapter 11. Background & Prior Research 291
11.1 Physics of fluid mechanics 291
11.1.1 Fluid statics 292
11.1.2 Fluid dynamics 294
11.2 Prior research on student understanding in fluid mechanics 296
11.2.1 Misinterpretation of Bernoulli's equation 297
11.2.2 Research on student difficulties in fluid dynamics 297
11.3 Motivation and context of the investigation 300
Chapter 12. Investigation of Student Difficulties in Fluid Dynamics I – Replication Studies and Observations from Exams 303

12.1 Student responses to selected FMCI questions on pretests 304
12.1.1 The selected FMCI questions in our investigation 304
12.1.2 Student responses on the HN2W question 306
12.1.3 Student responses on the VDN2W question 309
12.1.4 Student responses on the VUW2N question 313
12.1.5 Discussions 315
12.1.6 Commentary 317
12.2 Student responses to fluid dynamics questions on exams 318
12.2.1 Exam question involving continuity equation only 318
12.2.2 Exam questions involving both the continuity and Bernoulli's equations 320
12.2.3 Student responses to the FMCI questions on exams 322
12.3 Summary 324
Chapter 13. Investigation of Student Difficulties in Fluid Dynamics II – Additional Studies ... 325
13.1 Student responses to the modified FMCI questions 327
13.1.1 Student responses to the modified HN2W question 327
13.1.2 Student responses to the modified VDN2W question 330
13.1.3 Student responses to the modified VUW2N question 332
13.1.4 Student responses to vertical pipe flow of uniform cross-section context 336
13.1.5 Student responses on modified fluid questions (mathematical) 339
13.1.6 Commentary 346
13.2 Student responses to additional fluid mechanics tasks 348
13.2.1 Continuity equation in a traffic context 348
13.2.2 One-channel to two-channel context 354

13.3 Summary of student difficulties in fluid dynamics in our investigation 358
13.3.1 Gravity-related reasoning 358
13.3.2 Pressure-related reasoning 358
13.3.3 Proportional reasoning 360
13.4 Instructional implications 360
Conclusion for Part III 363
Part IV Other Investigations 367
Chapter 14. Improving Student Ability to apply Vector Superposition in Algebra-based Physics through Online Instruction 369
14.1 Literature review, motivation, and research questions 370
14.1.1 Literature review of student difficulties in vector superposition 370
14.1.2 Literature review on instructions to improve vector superposition skill 372
14.1.3 Motivations and research questions 373
14.1.4 Commentary and organization of the chapter 374
14.2 Overview of research methodology 375
14.2.1 The vector superposition tasks used in the investigation 375
14.2.2 Student populations and data collection 378
14.2.3 Procedures for research questions (RQs) 1, 2, and 3 379
14.2.4 Procedure for the research question (RQ) 4 380
14.2.5 Statistical method for analyzing longitudinal data 385
14.3 Results and discussions 388

14.3.1 Research question 1 – How does the performance in vector superposition in the calculus-based students compare to that in the algebra-based students? 388
14.3.2 Research question 2 – What is the progression of the student performance in vector superposition in an algebra-based physics course? 393
14.3.3 Research question 3 – How does the prior knowledge in vectors affect the progression of the student performance in an algebra-based physics course? 394
14.3.4 Research question 4 – The effectiveness of interventions through online pretests 394
14.3.5 Additional results – intervention B in the Wi'20 quarter 406
14.3.6 Discussions 410
14.4 Instructional implications and limitations of our study 414
14.4.1 Instructional implications of intervention B 414
14.4.2 Limitations to our study 416
14.5 Summary, future work, and conclusions 417
14.5.1 Summary of findings 417
14.5.2 Future work 420
14.5.3 Conclusion 421
Chapter 15. The Use of Computational Text Analysis in Physics Education Research 423
15.1 Background and motivation 423
15.2 Literature review of NLP work related to physics education 425
15.3 Methodology 428
15.3.1 Data processing 429
15.3.2 Exploratory data analysis 431

15.3.3 Quantitative analysis in NLP 432
15.4 Using NLP to determine similarity of student explanations 433
15.4.1 Summary of results of the investigation 434
15.4.2 Methodology 437
15.4.3 Results and discussions 439
15.5 Summary and future work 443
Conclusion for Part IV 445
Chapter 16. Conclusion and Future Work 449
16.1 Summary of specific difficulties 450
16.2 Implications for instruction and the development of curriculum 453
16.3 Assessment of student learning 454
16.4 Questions for further research 455
Bibliography 457
Appendix A. Notes on the Statistics used in the dissertation 465
Appendix B. Pretests, Posttests, and Curriculum for Part I 477
Appendix C. Pretests, Posttests, Curriculum, and Additional Notes for Part II 537
Appendix D. Pretests and Posttests for Part III 639
Appendix E. Pretests, Posttests, and Additional Notes for Part IV 653

LIST OF FIGURES

Figure 1.1. The iterative process of curriculum development in the UWPEG. 9
Figure 4.1. The speed calculation task – cart-on-ramp (CR) context. 42
Figure 4.2. The CR context in the multiple-choice format. 48
Figure 4.3. The kinematics graph task. 53
Figure 4.4. The correct position-time and speed-time graphs for the kinematics graph task. 55
Figure 4.5. The time ranking task on pretest and midterm. 61
Figure 5.1. Student responses across all the speed isomorphic tasks. The contexts for the isomorphic tasks are described in Table 5.12. 71
Figure 5.2. Students' responses across all the speed isomorphic tasks in greater detail. . 73
Figure 5.3. The modifications on the SB context – (a) block ends at point S, (b) block starts with speed v. 76
Figure 5.4. Options for the modified SB context on pretests..... 76
Figure 5.5. The modified CR context on a pretest. 79
Figure 5.6. Results involving the original and modified CR contexts on pretests – (a) version 1 and (b) version 2. 81
Figure 5.7. Schematic diagram of the study to investigate students' ability to perform a consistency check on online pretests. 88
Figure 5.8. The speed-time graph task (elicit stage). 89
Figure 5.9. Example of the confrontation and resolution stage in the pretest. 90
Figure 5.10. Modification of the confrontation and resolution stage by prompting students to consider the area under graphs in determining the consistency of answers..... 96
Figure 5.11. Modification of the confrontation and resolution stage in pretest by highlighting the constancy and non-constancy of speeds at points Q, R, and S in both graphs. 97
Figure 5.12. Homework curriculum (Vel-HW) to address students' misconceptions on average and instantaneous velocity. 103
Figure 5.13. HB context on written posttest. 105

Figure 6.1. Rotation of rigid body about a fixed axis passing through the origin along the *z*-direction. 119
Figure 6.2. The trajectory of a point on the rim of a cylinder (black dot) and the center-of-mass of the cylinder when it rolls without slipping. 121
Figure 6.3. Applying relative motion ideas to illustrate rolling motion. 122
Figure 7.1. Rolling Motion Task in the Rotational and Rolling Motion Conceptual Survey (RRMCS) [12]. 137
Figure 7.2. Friction on rolling objects in RRMCS. The correct answer is B. 155
Figure 7.3. Friction on rolling cylinder (horizontal). The correct answer is D. 156
Figure 7.4. Sphere rolling down the ramp. The correct answer is D. 157
Figure 7.5. Wheel rolling up the incline. The correct answer is C. 158
Figure 7.6. Slipping ball. The correct answer is A. 160
Figure 8.1. The schematics of the study design to find out the prevalence of students using the given curve to determine the direction of velocity. 166
Figure 8.2. The results of the study in both the Sp'18 and Sp'19 quarters. In the Sp'18 quarter, students attempted Task 2 **before** they had lecture instruction. Students attempted Task 2 **after** they had lecture instruction in the Sp'19 quarter. The arrows refer to the answers chosen by the students. The percentages represent the sample proportion. 169
Figure 8.3. The modified RMT-Direction (displacement) task. Students are first asked to think about the direction of the displacement before the direction of velocity on the pretest. 175
Figure 8.4. Modified RMT-Direction (trajectory) on the final exam. 178
Figure 8.5. Rolling Motion Task (Direction) with different rigid bodies: (a) hoop, (b) rod, (c) L-shaped rod, and (d) 'plus'-shaped object. 182
Figure 8.6. Rolling Motion Task (Magnitude) with different rigid bodies: (a) hoop and (b) plus-shaped object. 186
Figure 8.7. The moving-ball-on-moving-boat problem (ball-boat context). 191
Figure 8.8. The moving-boat-in-moving-water problem (boat-water context). 193
Figure 8.9. The moving-ball-on-moving-block problem – (a) circular track and (b) straight track. 200

Figure 9.1. Cylinder-on-board task on a pretest. 209
Figure 9.2. The BoC task (displacement) on a pretest. 216
Figure 9.3. The mBoC task on a pretest. It is also used as a clicker question during the lecture. 224
Figure 9.4. The sequence of administration of the BoC and mBoC tasks in pretests and in-class clicker questions in the Au'18, Sp'19, and Au'19 quarters. 228
Figure 10.1. Tutorial exercise in which students review Galilean transformation of velocity in a one-dimensional context. 239
Figure 10.2. Tutorial exercise in which students analyze the direction of velocities of points A, B, and C in the center-of-mass frame for a rotating hoop moving through the air. 239
Figure 10.3. Tutorial exercise in which students apply Galilean relativity to determine the possible magnitudes and directions of the velocities of points A, B, and C in the ground frame. 240
Figure 10.4. Tutorial exercise in which students predict and observe how the motion of the cylinder changes with time for three different initial conditions: $v\mathrm{CM} = R\omega$, $v\mathrm{CM} > R\omega$, and $v\mathrm{CM} < R\omega$. 241
Figure 10.5. Homework exercise in which students determine the answers to the RMT-Direction and Magnitude tasks. 242
Figure 10.6. Homework exercise in which students determine the relationship between the velocity/acceleration of the board and the velocity/acceleration of the CM of the cylinder in the board-on-cylinder task. 243
Figure 10.7. Board-on-cylinder task in the final exam. 247
Figure 10.8. The ball-on-moving-belt task on the final exam. 251
Figure 10.9. The friction on rolling cylinder (FoC) problem. The correct answer is D. 253
Figure 10.10. Relative motion in 2D problem (fish-in-river) in the final exam. 256
Figure 10.11. Vector triangle for the fish-in-river problem in Figure 10.10. 256
Figure 10.12. Section I of the modified tutorial (parts A, B, and C) – students predict the direction of vP and the path taken by point *P*, and then sketch the cycloid using a gear rack set. 260

Figure 10.13. The cycloid trajectory is sketched using the 3D-printed gear rack set. Note that this trajectory is not a perfect cycloid. 261
Figure 10.14. Parts D and E of section I of the modified tutorial – student interact with two discussion statements and determine the direction of vP after the cylinder rolls for some time. 262
Figure 10.15. Parts A and B of section II of the modified tutorial – students determine the velocity addition equation based on the exercise in section I. 263
Figure 10.16. The last two questions of the last section of the modified tutorial – students revisit the case in which the cylinder rolls without slipping on the horizontal surface. 264
Figure 10.17. The first question of the modified homework – students revisit the case in which the cylinder rolls without slipping on the horizontal surface, the cylinder rolls to the left. 265
Figure 10.18. Question 3 of the modified homework – a guided exercise for students to determine the direction of frictional force acting on the cylinder as it rolls down an incline. 266
Figure 10.19. Question 4 of the modified homework – a guided exercise for students to determine the direction of frictional force acting on the cylinder as it rolls up an incline. 267
Figure 10.20. Question 5 of the modified homework – students revisit the observations from the class demonstration and determine the directions of center-of-mass acceleration and angular acceleration. 268
Figure 10.21. Question 6 of the modified homework – students practice applying what they learned from the previous questions on the cylinder-on-board context. 269
Figure 10.22. Question 7 of the modified homework – students apply the no-slip condition to the block-spool experiment from the *Dynamics of rigid bodies* tutorial. 271
Figure 10.23. The BoC task on the pretests and final exams in Au'19 quarters. 275
Figure 10.24. The friction on the rolling cylinder (FoC) problem..... 278
Figure 10.25. The original and modified RRMCS question on the effects of friction on rolling objects. The changes in the modified RRMCS question is italicized. 280

Figure 11.1. A sample of liquid with density ρ and the forces acting on it. The arrows represent the forces exerted on the fluid sample, but the lengths do not represent the magnitudes. 292
Figure 11.2. Fluid flow in a pipe segment that is filled with fluid completely. The shaded part shows the parcels of fluid that are flowing in the time interval Δt past points 1 and 2.295
Figure 11.3. Fluid flow in a horizontal pipe segment, from a region of a wide cross-section to a region with a narrow cross-section (from Martin J. *et al.* [20])..... 299
Figure 12.1. The three FMCI questions used in our investigation [20]. (a) HN2W question: Fluid flow in a horizontal pipe segment, from a region of narrow cross-section to a region of wide cross-section. (b) VDN2W question: Downward flow in a vertical pipe segment, from a region of narrow cross-section to a region of wide cross-section. (c) VUW2N question: Upward flow in a vertical pipe segment, from a region of wide cross-section to a region of narrow cross-section. 305
Figure 12.2. The HN2W question in FMCI. Fluid flow in a horizontal pipe segment, from a region of narrow cross-section to a region of wide cross-section [20]. 306
Figure 12.3. The VDN2W question in FMCI. Downward fluid flow in a vertical pipe segment, from a region of narrow cross-section to a region with wide cross-section [20].... 310
Figure 12.4. The VUW2N question. Upward fluid flow in a vertical pipe segment, from a region of wide cross-section to a region with narrow cross-section [20]..... 313
Figure 12.5. The greatest-speed task for fluid flow in a pipe system. The correct answer is E. 319
Figure 12.6. Fluid flow in a horizontal pipe system. Case 1 is from a region of wide cross-section to a region of narrow cross-section. Case 2 is from narrow cross-section to wide cross-section. 321
Figure 12.7. Results on the FMCI questions (HN2W, VDN2W, and VUW2N) in PHYS115 and PHYS 123 final exams. The error bars are twice the standard error. 322
Figure 13.1. The modified HN2W question. 328
Figure 13.2. The VDU question. Fluid flows downward along a vertical pipe with uniform cross-section. 336

Figure 13.3. The VDU-with-context question. Water flowing down from a tank along a vertical pipe with a uniform cross-section. The pressure comparison task is modified to contain options (7 bullet points) that are expressed mathematically. 340
Figure 13.4. The VUW2N-with-context question. Water flowing upward along a vertical pipe from a wide channel to a narrow channel. 344
Figure 13.5. Continuity equation in a traffic context (Versions A and B)..... 349
Figure 13.6. The 1C2C task. Water flowing from one channel to two-channel context in midterm 2 of the Sp'17 quarter. 354
Figure 13.7. Water flowing from one channel to two-channel context in the final exam of Sp'17. 357
Figure 14.1. The vector superposition tasks used in our investigation. 2v-NC stands for two-vectors-no-context. 2v4v-NC stands for two-vectors-four-vectors-no-context. The F, T, and E in the next three refer to force, tension, and electric field contexts, respectively. 376
Figure 14.2. The tip-to-tail method to compare the magnitude of the resultant (in red) of the vectors in the 2v4v-NC task. 377
Figure 14.3. An example of the implementation of limiting case intervention in improving student vector superposition ability on pretests. Students see the helper question on page 1 and the actual question on page 2 of the pretest. 382
Figure 14.4. The outline of the sequence of questions in intervention B. 383
Figure 14.5. The sequence of questions when students answered the actual task incorrectly. Students are informed that their answer is incorrect, then they are demonstrated the tip-to-tail vector addition method. Students are also asked to comment on the relationship between the magnitude of the resultant of two vectors and the angle between the two vectors. 384
Figure 14.6. The retest in which students redo the actual task the second time. 384
Figure 14.7. The page in which students see if they did the retest incorrectly. Students are informed that their answer is incorrect, then they are given the correct answer and retest for the third time. 385

Figure 14.8. The results of each vector superposition task from the calculus-based physics students. Error bars indicate ± 2 SE. 389
Figure 14.9. Results of each vector superposition task from PHYS 114 in the Au'18 and Wi'19 quarters. N = 391 / 234 refers to there are 391 students and 234 students who learned/did not learned vectors before registering for PHYS 114. 'All respondents' refers to the results of all students who responded to that task. Error bars indicate ± 2 SE. 391
Figure 14.10. Results of each vector superposition task from the PHYS 115 students in the Wi'19 quarter. Error bars indicate ± 2 SE. 392
Figure 14.11. Results of intervention A from PHYS 114 class in the Wi'19 quarter. ... 395
Figure 14.12. The results of intervention B for PHYS 114 in the Au'19 quarter. Vector Y/N refers to students who have/have not learned about vectors before attending PHYS 114. N = 290 / 171 refers to there are 290 students and 171 students who learned/did not learned vectors before registering for PHYS 114. 'All respondents' refers to the results of the students who responded to that task. Error bars indicate ± 2 SE. 396
Figure 14.13. The results of intervention B for PHYS 115 in the Au'19 quarter. Error bars indicate ± 2 SE. 397
Figure 14.14. The student performance of vector superposition in the first four pretests of students in the no-intervention, intervention-A, and intervention-B groups. Error bars indicate ± 2 SE. 399
Figure 14.15. Results among students with missing data imputed as "correct" for pretests 1 and 2, and "incorrect" for pretests 3 and 4. Error bars indicate ± 2 SE. 403
Figure 14.16. Results among students with missing data in the no-intervention group imputed as "correct," and missing data in the intervention group imputed as "incorrect." Error bars indicate ± 2 SE. 405
Figure 14.17. The results of intervention B for PHYS 114 in the Wi'20 quarter. Vector Y/N refers to students who have/have not learned about vectors before attending PHYS 114. N = 140 / 94 refers to there are 140 students and 94 students who learned/did not learned vectors before registering for PHYS 114. 'All respondents' refers to the results of the students who responded to that task. Error bars indicate ± 2 SE. 407

Figure 14.18. The 2v-E and 2v4v-E tasks in PHYS 115 midterm 2 of the Wi'20 quarter. Q_1 and Q_2 are positive charges of magnitude Q_o. 408
Figure 14.19. The results of intervention B for PHYS 115 in the Wi'20 quarter. Error bars indicate ± 2 SE. 409
Figure 14.20. The original and modified 2v-E tasks. The electric field vectors at point *P* due to each individual charge were drawn on the modified 2v-E task. 413
Figure 15.1. An example of (a) a word cloud and (b) a network of co-occurrence of words. On the word cloud, the phrase "tangent to" is the most common. On the network, the phrases "tangent to" and "circular motion" tend to appear together in student explanations.432
Figure 15.2. The schematics of the study design to find out the prevalence of students using the given curve to determine the direction of velocity. 435
Figure 15.3. The results of investigating student tendency to use the tangent to the shape of the given object to determine the velocity of a point. The results from the Sp'18 and Sp'19 quarters are combined. 436
Figure 15.4. The top six commonly used two-word phrases in the explanations for selected answers in Task 1A, Task 1B, and Task 2. 440

LIST OF TABLES

Table 1.1. Summary of the course structure and mode of instruction in Physics 11X and 12X courses..... 5
Table 3.2. Operational definitions of linear kinematical quantities..... 32
Table 4.3. HS and introductory students' responses for the CR task on the written pretest and written exam. The correct answer and the corresponding percentages are in bold. 44
Table 4.4. Percentage of score distribution for the CR task (written format) for high school and introductory physics students..... 47
Table 4.5. Student responses for the CR task (multiple-choice version) on the pretest and final exam..... 49
Table 4.6. Selected student explanations for the CR task on pretests..... 52
Table 4.7. Categorizations of students' position-time graphs. Students used A–D as labels because those were given on the written pretest, with D as the starting point. The labels were changed to 1–4 to prevent confusion with the answer options A–E on a multiple-choice exam. 56
Table 4.8. Categorizations of students' speed-time graphs. Students used A-D as labels because those were given on the written pretest, with D as the starting point. We changed the labels to 1 – 4 to prevent confusion with the answer options A-E on a multiple-choice exam. 57
Table 4.9. Students' time interval ranking on the kinematic graphs. 58
Table 4.10. Correlation table between students' speed-time graphs and speed calculations. 59
Table 4.11. Students' responses for the time ranking task on pretest and midterm. 62
Table 5.12. Isomorphic problems on average speed and instantaneous speed. 68
Table 5.13. Approaches used in the answer options for the speed calculation task (multiple-choice) and their corresponding answers for each isomorphic task..... 70
Table 5.14. Student performance on the original and modified SB context on pretests... 77

Table 5.15. Selected student explanations on the original and modified CR contexts on pretest version 1. The student responses in each row are from the same student. 83
Table 5.16. Selected student explanations on the original and modified CR contexts on pretest version 2. The student responses in each row are from the same student. 84
Table 5.17. Students' responses for version 3 of the modified CR context. 85
Table 5.18. Correlation table between students' speed-time graphs and speed calculations in a pretest in the Wi'17 quarter. 90
Table 5.19. Results of the resolution stage for the question "Are your answers consistent?". 91
Table 5.20. Results from the resolution stage – identification of mistakes. 93
Table 5.21. Results from the resolution stage – selected students' responses. 94
Table 5.22. Correlation table between students' speed-time graphs and speed calculations in a pretest in the Sp'17 quarter. 97
Table 5.23. Results from the resolution stage (modified) – identification of mistakes. ... 98
Table 5.24. Results from the resolution stage (modified) – selected students' responses. 99
Table 5.25. Posttest results of the homework modifications in the Wi'19 and Au'19 quarters. 106
Table 6.26. Angular quantities and their operational definitions. 118
Table 6.27. Relationships between linear and angular kinematical quantities. *r* is the distance from the origin to the point of interest, and r is the position vector. 120
Table 6.28. Summary of student difficulties with rolling motion and relative motion reported in prior research. 132
Table 7.29. Summary of the student responses on the RMT-Direction question on the pretest **before** lecture instruction. 140
Table 7.30. Summary of the student responses on the RMT-Direction question on the pretest **after** lecture instruction. 140
Table 7.31. Summary of student responses on the RMT-Direction task in the final exam. The published results from RRMCS are included for comparison. 141
Table 7.32. Summary of the student responses on the RMT-Magnitude task on the pretest **before** lecture instruction. 142

Table 7.33. Summary of the student responses on the RMT-Magnitude task on the pretest **after** lecture instruction. 142
Table 7.34. Summary of student responses on the RMT-Magnitude question in the final exam. The published results from RRMCS are included for comparison. 143
Table 7.35. Correlation table (aggregated, N = 744) between the RMT-Direction and RMT-Magnitude tasks in the final exam of the Au'17 quarter. 145
Table 7.36. Student explanations on the RMT-Direction task in the pretest. The results are aggregated from the pretest responses in Table 7.29 and Table 7.30 (from Wi'18 to Sp'19 quarters). 147
Table 7.37. Common student difficulties in the dynamics of rolling motion problems. 161
Table 8.38. Selected student explanations on Task 1A (T1A) and Task 2 (T2). The student responses in each row are from the same student. 171
Table 8.39. Selected student explanations on Task 1B (T1B) and Task 2 (T2). The student responses in each row are from the same student. 172
Table 8.40. Correlation table of student responses (N = 191) on the modified Rolling Motion Task (with displacement) on the pretest **after** lecture instruction in the Au'18 quarter. 177
Table 8.41. Correlation table of student responses (N = 454) on the modified Rolling Motion Task (with displacement) on the pretest **before** lecture instruction in the Wi'19 quarter. 177
Table 8.42. Correlation table of student responses (N = 203) on the modified RMT-Direction (trajectory) on the final exam of the Wi'19 quarter. 179
Table 8.43. Summary of the student responses on the RMT-Direction task with different rigid bodies in pretests from pre-lecture and post-lecture instruction. 184
Table 8.44. Summary of the student responses on the RMT-Magnitude task with different rigid bodies in pretests. 186
Table 8.45. Correlation table between the RMT-Direction and RMT-Magnitude tasks involving different rigid bodies in pretests. 187
Table 8.46. Student responses on the ball-boat context in pretests. 192

Table 8.47. Students' pretest responses for the direction of the velocity of the boat in the ground frame on the boat-water context. 195
Table 8.48. Students' pretest responses for the vector diagram involving the three velocities on the boat-water context. 196
Table 8.49. Students' pretest responses for Version 1 (observer's frame only) of the ball-block contexts. 201
Table 8.50. Student's pretest responses (N = 150) for Version 2 (observer's vs. block's frame) of the ball-block contexts. 202
Table 8.51. Correlation between student performance in the ball-boat/circular-track contexts and that of the boat-water context. 204
Table 9.52. Student responses on the cylinder-on-board task on pretests. 210
Table 9.53. Student responses to the board-on-cylinder task on pretests. 217
Table 9.54. Student responses to the mBoC task in clicker question. 224
Table 9.55. Student responses to the mBoC task on pretests before lecture instruction. 226
Table 9.56. Student responses for the mBoC task on pretests. 227
Table 9.57. Student performance of the mBoC task on pretests and in-class clicker questions and the BoC task on pretests in Au'18, Sp'19, and Au'19 quarters. In each cell, the first number is the percentage correct of the students who responded at that time. The second number is the percentage correct of students who completed all the administered tasks in that quarter. 229
Table 10.58. Pretest and posttest results for the Rolling Motion Task (Direction) from Wi'18 to Au'18 quarters. In all cases, students are required to determine the direction of point 2 on the rolling wheel (Frisbee for posttest in Sp'18) in the ground frame. 246
Table 10.59. Pretest and posttest results for the Rolling Motion Task (Magnitude) from Wi'18 to Au'18 quarters. In all cases, students are required to rank the speeds of points 1, 2, and 3 on the rolling wheel (Frisbee for posttest in Sp'18) in the ground frame. 246
Table 10.60. Percentage correct for the board-on-cylinder task in the pretest and posttest of Au'18 quarter. Different pretest vs. posttest combinations are presented. 248
Table 10.61. Student responses for the cylinder-on-board task during the pretest and the ball-on-moving-belt task on the final exam. 252

Table 10.62. Student responses for the friction on rolling cylinder problem in the final exams before and after tutorial implementation. 254
Table 10.63. Student responses to the fish-in-river problem in the final exams before and after tutorial implementation. 257
Table 10.64. Pretest and posttest results for the RMT-Direction in Sp'19 and Au'19 quarters. In all cases, students are required to determine the direction of point 2 in the ground frame. 274
Table 10.65. Pretest and posttest results for the RMT-Magnitude in Sp'19 and Au'19 quarters. In all cases, students are required to rank the speeds of points 1, 2, and 3 in the ground frame. 274
Table 10.66. Student responses for the BoC task in the final exams after implementations of preliminary and modified versions of tutorial. 276
Table 10.67. Student responses for the FoC problem in the final exams after implementations of preliminary and modified versions of tutorial. 278
Table 10.68. Student responses for the modified RRMCS friction question on the effects of friction on rolling objects. 280
Table 11.69. Summary of student difficulties with fluid dynamics reported in prior research. 301
Table 12.70. Summary of student responses and explanations on the HN2W question. 307
Table 12.71. Selected student explanations on the HN2W question. 308
Table 12.72. Summary of student responses and explanations on the VDN2W question.310
Table 12.73. Selected student explanations on the VDN2W question. 312
Table 12.74. Summary of student responses and explanations on the VUW2N task..... 314
Table 12.75. Selected student explanations on the VDN2W question. 314
Table 12.76. Summary of pretest results on the HN2W, VDN2W, and the VUW2N. Results from the University of Cape Town (UCT) [19] and Washington State University (WSU) [18] are listed for comparison. Note that the sample sizes are not the same. 315
Table 12.77. Student responses to the fluid task in Figure 12.5. 319
Table 12.78. Student responses to the fluid task in Figure 12.6. 321

Table 13.79. Student responses to the modified HN2W question. The percentage correct of the correct combination is in bold. 328
Table 13.80. Student explanations for the modified HN2W question (speed comparison). The percentage correct of the correct combination is in bold. 329
Table 13.81. Student explanations for the modified HN2W question (pressure comparison). The percentage correct of the correct combination is in bold. 329
Table 13.82. Student responses to the modified VDN2W question. 330
Table 13.83. Student explanations for the modified VDN2W question (speed comparison). The percentage correct of the correct combination is in bold. 331
Table 13.84. Student explanations for the modified VDN2W question (pressure comparison). The percentage correct of the correct combination is in bold. 331
Table 13.85. Student responses to the modified VUW2N question. The percentage correct of the correct combination is in bold. 333
Table 13.86. Student explanations for the modified VUW2N question (speed comparison). The percentage correct of the correct combination is in bold. 334
Table 13.87. Student explanations for the modified VUW2N question (pressure comparison). The percentage correct of the correct combination is in bold. 334
Table 13.88. Examples of student explanations for the modified VUW2N question (pressure comparison). 335
Table 13.89. Student responses to the VDU question. The percentage correct of the correct combination is in bold. 337
Table 13.90. Student explanations for the VDU question (speed comparison). The percentage correct of the correct combination is in bold. 337
Table 13.91. Student explanations for the VDU question (pressure comparison). The percentage correct of the correct combination is in bold. 338
Table 13.92. Student responses to the VDU-with-context question. The percentage correct of the correct combination is in bold. 341
Table 13.93. Student explanations for the VDU-with-context question (speed comparison). The percentage correct of the correct combination is in bold. 342

Table 13.94. Student explanations for the VDU-with-context question (pressure comparison). The percentage correct of the correct combination is in bold. 342
Table 13.95. Student responses to the VUW2N-with-context question. The percentage correct of the correct combination is in bold. 344
Table 13.96. Student explanations for the speed comparison task for the VUW2N-with-context question. The percentage correct of the correct combination is in bold. 345
Table 13.97. Student explanations for the VUW2N-with-context. The percentage correct of the correct combination is in bold. 346
Table 13.98. Student responses on the Continuity (Traffic) task versions A and B. 350
Table 13.99. Student explanations for the Continuity (Traffic) task versions A and B. 350
Table 13.100. Student responses and explanations for the Continuity (Traffic) task version C. 351
Table 13.101. Student responses and explanations for the Continuity (Traffic) task version D. 352
Table 13.102. Selected student explanations for the Continuity (Traffic) task. 353
Table 13.103. Student responses on the 1C2C-Speed task. 355
Table 13.104. Student responses on the 1C2C-Pressure task. 356
Table 14.105. Number of sections receiving intervention and the weeks that students were presented the vector superposition tasks in PHYS 114 in each quarter. 381
Table 14.106. Comparisons of different generalized linear mixed models. The numbers under "p-value" are the *p*-values of a likelihood ratio test comparing with the model in the previous row. For example, the number in M2 is the *p*-value of the likelihood ratio test comparing with M1. The symbol "*" encompasses individual factors and their interaction. 400
Table 14.107. Point estimates, standard errors, 95% confidence intervals, and p-values of a Wald-test of parameters in Model 5. The symbol ":" means interaction. 401
Table 14.108. Point estimates, standard errors, 95% confidence intervals and *p*-values of a Wald-test of parameters in Model 5, where the missing data are imputed as "correct" for pretests 1 and 2, and "incorrect" for pretests 3 and 4. 404

Table 14.109. Point estimates, standard errors, 95% confidence intervals, and *p*-values of a Wald-test of parameters in Model 5, where the missing data in the no-intervention group are imputed as “correct,” and the missing data in the intervention group are imputed as “incorrect.” 405
Table 14.110. Pretest results (percentage correct) for the original and modified 2v-E tasks in the calculus-based and algebra-based students in the Wi’20 quarter. 413
Table 14.111. Differences between our study and the study by Mikula and Heckler. ... 415
Table 15.112. An example of how textual data looks like before and after processed and cleaned 429
Table 15.113. An example of a document-feature matrix (DFM). 430
Table 15.114. An example of a feature-cooccurrence matrix (FCM)..... 430
Table 15.115. The two-word phrases that are removed in our computational text analysis.437
Table 15.116. Pairwise cosine similarity values with two-word phrases that are used in the explanations to the selected answers in Tasks 1A, 1B, and 2. The values are rounded to the closest 0.05. 441
Table 15.117. The similarity of student explanations between qualitative methods and cosine similarity. (T1 indicates Task 1A/B and T2 indicates Task 2.) 443

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DEDICATION

To my family and Chin Yein

Chapter 1. INTRODUCTION

This dissertation consists of two primary studies. The first presents results from a broad investigation of student understanding of the motion of different kinds of objects – particle, rigid body, and fluid. The second study presents preliminary results from two studies: one consisting of an effort to improve student ability to do vector superposition in physics courses, and the second from preliminary use of natural language processing techniques to examine student explanations in physics education research. All these studies took place primarily in the introductory algebra-based and calculus-based physics courses at the University of Washington. However, some data were collected from high school students in the USA.

These efforts are part of an ongoing investigation of student understanding of physics and physical science conducted by the Physics Education Group at the University of Washington (UWPEG). Prior related research by the group and by others has identified a variety of student difficulties with the specific topic examined in this dissertation: particle kinematics [1-7], rigid body dynamics [8-15], and fluid motion [16-22]. The goals of the study are: (1) to corroborate some of the reported student difficulties with both known and with new tasks, (2) to fill in specific gaps in the existing research, (3) to develop relevant curriculum to address specific student difficulties. Also, the last component of this dissertation has had the goal (4) to explore alternative methods for analyzing student response data and identifying implications for instruction.

The research and curriculum development included in this dissertation span a wide range of topics covered in introductory physics. In the first part, we focus on the motion of a single particle in one dimension in the context of the ability of students to calculate the speed of the object during different portions of its motion. The second part is on the kinematics and dynamics of rigid

bodies in the context of rolling motion. The third is on the ability of students to apply the equations for continuity and Bernoulli's equation in fluid flow through pipes. (The last two of these topics have had relatively little prior research.) In each of these three contexts, we examine student conceptual understanding and problem-solving ability. These investigations began using questions that our group or other researchers had developed, but we supplemented these with additional conceptual and calculational questions to probe student understanding in greater depth. The relevant concepts and reasoning for different parts of these investigations are described in detail in these chapters: Chapter 3 for particle motion, Chapter 6 for rigid body motion, and Chapter 11 for fluid motion.

The last part of this dissertation presents results from two different explorations in the use of technology to teach conceptual understanding and to conduct physics education research. The first involves the design of online weekly teaching quizzes to help students learn vector superposition. Their efficacies were assessed through the longitudinal tracking of student performance on this topic throughout the algebra-based physics sequence. The second involves the use of natural language processing (NLP) techniques to compare student explanations on two specific physics tasks. Both methods expand the usage of currently available tools in physics education research and suggest that they can give us greater insights into student learning. The details on these were applied can be found in Chapter 14 and Chapter 15.

1.1 MOTIVATION FOR RESEARCH

There are two primary motivations for the choice of topics for the investigations discussed in this dissertation.

Parts I – III. The study of the motion of particles, be it causes or description, is part of the core of physics. Students learn about particle kinematics and dynamics, rigid body dynamics, and, sometimes, fluid motion as they progress through introductory physics courses. The UWPEG has performed extensive research and created relevant curricula to address specific, known student difficulties across all of the areas. However, the results from prior research and our ongoing investigation continue to give us insights into particular gaps in student understanding that have not been addressed before. We can then develop new or modify the present curriculum to improve physics instruction. Detailed descriptions of the motivations for each of these three specific topics can be found in these chapters: Chapter 3 for particle motion, Chapter 6 for rigid body motion, and Chapter 11 for fluid motion.

Part IV. The second motivation arises from reflections on the process of performing physics education research. As technologies and tools change, we want to explore new and different methods of data analysis and instruction. Two are described in this dissertation.

The UWPEG often conducts research using a pretest-posttest study design [23]. This study design is used to evaluate the effectiveness of interventions in general. For example, the tutorial curriculum [24] is assessed by testing students after lecture instruction (pretest) and then again after tutorial instruction (posttest). More details of the design can be found in Chapter 2. In this dissertation, we explore the use of a series of short online exercises to track and improve student ability to do vector superposition over a period of time. We chose vector superposition because it is a commonly used skill in physics, especially in the discussion of vector quantities such as forces

and electric fields. In this study, the intervention is the modification to the short online exercises. Besides that, this kind of investigation involves a longitudinal study, which allows us to probe student learning and the effect of the intervention over time [25]. This study is described in Chapter 14.

In the context of exploring new and different methods of data analysis, we also perform a preliminary study into using natural language processing (NLP) techniques to compare the similarity of student explanations to two physics tasks. The description of NLP and details of this study can be found in Chapter 15. We hope this exploration leads to new research ideas for our future graduate students.

1.2 CONTEXT OF INVESTIGATION

Most of the data described in this dissertation were collected in the regular (non-honors), algebra-based, and calculus-based, introductory physics sequences at the University of Washington. In a few cases, data also came from high school students from the USA. (These are described where appropriate.) The data, for the most part, comes from written or typed responses to questions administered at various stages of instruction. The questions have been in multiple-choice or long-answer format. The latter is used to provide insights into student reasoning. Observations of students in the classroom have supplemented this data.

Both the algebra-based and calculus-based physics sequences at the UW are made up of three, one-quarter (10 weeks) courses. The algebra-based physics courses (PHYS 11X) are Physics 114 (Mechanics), 115 (Thermal physics, fluid mechanics, and electricity & magnetism), and 116 (Waves, optics, and modern physics). The calculus-based physics courses (PHYS 12X) are Physics 121 (Mechanics), 122 (Electricity & Magnetism), and 123 (Waves, optics, modern physics,

thermal physics, and fluid mechanics). The course structure and mode of instruction in Physics 12X and Physics 11X courses are summarized in Table 1.1.

Table 1.1. Summary of the course structure and mode of instruction in Physics 11X and 12X courses.

	Physics 11X (algebra-based)	Physics 12X (calculus-based)
Total enrollment in the sequence each quarter	500-800	900-1300
Number of lecture sections each quarter	1-3	2-4
Lecture instruction	• 50 minutes, 3 times a week • Pre-lecture: Reading quizzes (MasteringPhysics) • In-class: Learning Catalytics • Post-lecture: Online homework (MasteringPhysics)	
Textbooks used during this investigation	*College Physics* by Knight, Jones, and Field	*Physics for Scientists and Engineers with Modern Physics* by Tipler and Mosca *Principles and Practice of Physics* by Eric Mazur
Tutorial instruction	• 50 minutes, once a week. • Interactive tutorial lecture (combining all sections) • One lecturer with several teaching assistants	• 50 minutes, once a week. • Small sections (22-24 students) • Two teaching assistants
Lab sessions	Separate course (optional with separate grade)	110 minutes, once a week (required)

The enrollment in the Physics 11X and 12X sequences range from 100-700 students each quarter. Each course is usually divided into several sections, with the size of each section typically

around 100-220 students. Each has three components: lectures, tutorials, and labs. The labs are optional in PHYS 11X.

Lectures

In both 11X and 12X, the courses have a 'flipped' mode of instruction, which is different from traditional instruction. In traditional instruction, students are introduced to the content during the lecture, and then they do homework later. In flipped instruction, students are introduced to the course materials outside of class, and in-class time is repurposed for inquiry, application, and assessment in order to meet the needs of individual learners.

The students attend 50-minute lectures three times each week. They are assigned reading quizzes on an online system (Mastering Physics) [26] before attending lectures, in-class quizzes (administered by the lecturers through Learning Catalytics [27]) during lectures, and weekly online homework assignments on Mastering Physics. During the study reported in this dissertation, the textbook used by the Physics 11X students is *College Physics* by Knight, Jones, and Field [28]. For the Physics 12X students, two textbooks have been used, *Physics for Scientists and Engineers with Modern Physics* by Tipler and Mosca (Au'14-Sp'16) [29], and *Principles and Practice of Physics* by Eric Mazur (Au'17-present) [30].

Tutorials

Besides lecture instruction, both groups of students attend a 50-minute tutorial session once a week. However, the tutorials for Physics 11X and 12X are quite different in the mode of instruction and class size. Physics 11X has an interactive tutorial lecture that includes students from all the sections of each course (e.g., students in PHYS 114A and B). The students meet in

one lecture hall. To the contrary, the Physics 12X tutorial sessions have a small class size of 22-24 students. In the Physics 11X interactive tutorial lectures, there is one lecturer with several graduate student teaching assistants (TAs). The lecturer can be the instructor of the course or an experienced graduate student teaching assistant. Learning Catalytics is used in the interactive tutorial lecture, but not in the 12X tutorials. For the small tutorial sections in Physics 12X, two TAs (usually graduate students) are assigned in most cases. The TAs prepare for tutorials by attending weekly meetings to work through the tutorial and discuss issues related to teaching the tutorial.

Before the students attend the tutorial in both PHYS 11X and PHYS 12X, they attempt an online pretest that is related to the upcoming tutorial (more details about the pretests can be found in Chapter 2). They are typically given after lecture instruction. The tutorial worksheets and homework that students use in both the algebra-based and calculus-based physics are published in the *Tutorials in Introductory Physics (TiIP)* [31]. However, if student understanding of a particular physics topic is undergoing research, then the associated tutorial(s) that students use may not be the version found in *TiIP*, but a revised version, which is printed and given to the students as a handout. For a discussion of the tutorials and the role of the TAs, see the article written by McDermott [32].

Labs

Labs are typically part of an introductory physics course. However, the 11X labs are not part of the Physics 11X courses, and not all students in the algebra-based physics course are required to take the matching lab course concurrently. (The choice often is related to the students' major or intended major.) The students in the Physics 12X courses, however, attend a weekly 110-

minute lab that is part of the course. They need to attend a minimum of 6 (out of 8) labs to pass the course.

Teaching assistants

We also collect data from the graduate students who are working as teaching assistants in the associated introductory courses. These include all first-year graduate student TAs, who are required to sign up for a graduate teaching seminar (Physics 501-3) as preparation for teaching in the tutorials in the Physics 12X sequence. The enrollment in the teaching seminar is usually around 25 students. In each weekly meeting, the TAs take a pretest that is either identical or similar to that taken by undergraduate students. After finishing the pretest, TAs work through the tutorial in groups of 5-6, with the guidance of an experienced TA, who sits with them (instead of roaming around the room as in the actual tutorials). After the TAs have completed the tutorial, the instructor for the seminar distributes pretest responses from current undergraduates to the TAs. The TAs inspect the responses, looking for trends in the reasoning, and offer their observations in a large class discussion. In some cases, the instructor presents some numerical evidence that the tutorial is both appropriately targeted to the students' initial understanding and effective in helping the students perform better on some non-trivial tasks.

During the tutorials, the TAs walk around in the class, monitor student progress on the tutorial worksheet, and assist students if necessary. The TAs do not lecture or give answers but pose questions to guide students through the necessary reasoning.

1.3 ITERATION OF RESEARCH AND CURRICULUM DEVELOPMENT

Curriculum development, as typically conducted by the Physics Education Group at the University of Washington (UWPEG), is an iterative process [32]. It involves three components: (1) researching to probe student understanding and identify difficulties; (2) using the results to develop curriculum; and (3) assessing student learning. These are illustrated in Figure 1.1. These three components interact and reinforce each other. Based on this process, the group has published two curriculum materials: *Physics by Inquiry* [33] and *Tutorials in Introductory Physics* [31], which are widely used in various institutions worldwide.

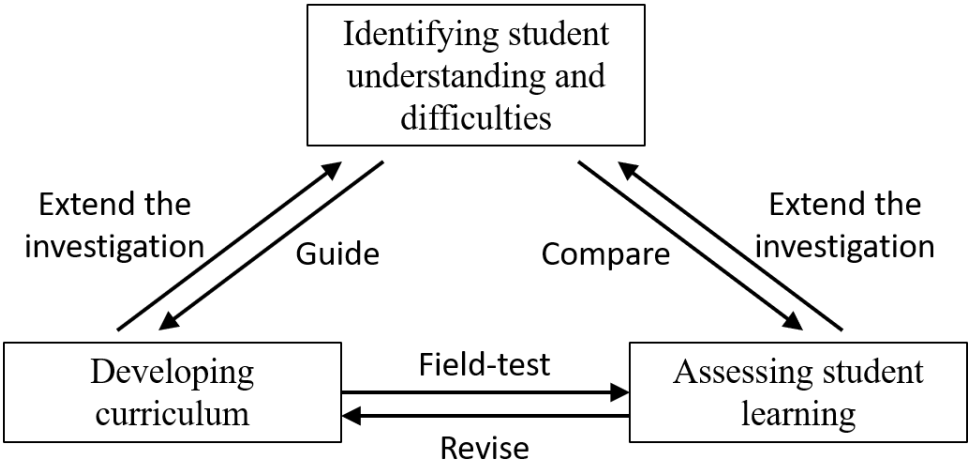


Figure 1.1. The iterative process of curriculum development in the UWPEG.

As described by Heron (one of the physics education researchers at UW), the perspective on learning that prevails in the UWPEG can be characterized as ***pragmatic constructivism*** [34]. It is pragmatic because we try to address student difficulties we found in our investigations. It is constructivist because we assume students construct new knowledge from existing knowledge that came from prior instruction or real-life experience [35]. Heron also describes the instructional approach used in the group as ***intentional teaching***: "each step in an instructional sequence is

carefully chosen with specific goals in mind and with a view to the likely thought processes of the learner." Since curriculum development goes hand-in-hand with learning and instruction, taking the perspective and instructional approach in mind, we describe how we follow the iterative process to develop the curriculum in detail.

We investigate student conceptual understanding and reasoning skills by presenting students with qualitative (and sometimes quantitative) tasks. Students are asked to explain their answers, which allows us to find out how they approach the tasks. We characterize and categorize typical student approaches as we look through student responses. Through this process, we try to identify the conceptual and reasoning difficulties among students. Using this information, we sometimes modify the existing tasks or generate new tasks that further probe student understanding from different angles. For example, we may change some features in the given diagrams, use different representations, and/or different descriptions in the questions to find out how the changes affect student performance and explanations. All these findings are used to guide curriculum development.

Each curriculum we have designed consists of in-class worksheets and homework. They are meant to help students build coherent concepts, inculcate reasoning skills, and relate the physics content learned in class to the real world [32, 36]. We choose the goals such that most students can complete the worksheet within the class time. We use different instructional strategies in our curricula, such as *apply-reflect-generalize* [32], *elicit-confront-resolve* [37], and *observe-recognize-apply* [36]. The homework helps to reinforce the ideas developed during the tutorial.

The next step of curriculum development is assessment. It involves examining data obtained in three ways: in-class observations, homework responses, and student performance on exams. For the first, we observe how students interact with the material and note down any

misunderstandings that students had with the worksheet in the classroom or interactions that suggest ideas that might be useful to probe in other ways. We look at students' responses to the homework problems and make necessary amendments if students were confused by the wording of the questions, or if we gain additional insights through their responses. Student performance on relevant conceptual questions included on exams informs the assessment of the effectiveness of the curriculum. We then use all these results to modify or revise the curriculum and assess it again.

The iteration cycle described above is an ongoing one. A common benchmark for success is when student posttest performance matches the performance of physics graduate TAs on pretests. Even so, we often make further investigations to probe student thinking in more detail or to explore other possible intellectual or instructional approaches that may be more effective.

1.4 ORGANIZATION OF THE DISSERTATION

The dissertation is organized into four main parts. These are preceded by Chapter 2, which provides a detailed overview of the research methodology and statistical analysis used in our work. Each of the remaining four parts is composed of multiple chapters. Part I (Chapter 3 to Chapter 5) describes the research on student understanding of particle motion. It focuses on student ability to determine speeds based on given information that comes from written or visual descriptions of motion. Student ability to make connections between kinematics graphs and numerical answers are also investigated. Part II (Chapter 6 to Chapter 10) illustrates the research on student understanding of rolling motion and the development of curriculum based on the research results. We describe in detail the assessment of the curriculum and how the results and additional research guided the modification of the curriculum. Part III (Chapter 11 to Chapter 13) documents a brief

study of student difficulties with fluid motion. The research expands investigations by others on this topic by using new populations and tasks that extend the prior findings. All of the chapters in parts I to III have a similar organizational structure. Each starts by discussing the relevant physics background and prior research, followed by preliminary investigations of the topic, and then in-depth investigations. There is an additional chapter in Part II, which describes the curriculum development and assessment in detail.

Part IV (Chapter 14 and Chapter 15) describes two other investigations that we conducted. The first is an ongoing effort to improve student understanding of vector superposition. The original investigation started in the early 2000s with the students in calculus-based courses and continued in the years 2014-2016. Here, we document a continuation with the algebra-based physics students that expands the scope of this effort. The second investigation involves the use of natural language processing (NLP) techniques to analyze student explanations. We believe this technique has potential applications in physics education research. Chapter 16 is the conclusion of the dissertation.

Chapter 2. RESEARCH METHODOLOGY & STATISTICAL ANALYSIS

In this chapter, we present a broad overview of the research methods and statistical analyses used in this dissertation. First, the formed methods of data collection and experimental designs are described. These include data consisting of student responses to surveys and written questions. Next, the statistical methods that we used to analyze the survey data are illustrated. Lastly, we discuss some limitations and concerns about the investigations and data collection. More detailed information appears in the individual chapters.

2.1 DATA COLLECTION

This section describes how our methods for collecting relevant qualitative and quantitative data during the iterative process of curriculum development. Both types of data are collected through web-based surveys (pretests), midterms, and final exams.

2.1.1 Pretests

Each week, students do a web-based survey before the weekly tutorial instruction. In most cases, students will do the pretest after they have seen all the relevant content in lectures. We call these "pretests" since they come before relevant tutorial instruction. Cases in which the pretest was given before lecture instruction will be indicated explicitly.

The purposes of the pretests are: (1) to test student understanding of the content after relevant lecture instruction; (2) to identify the areas in which students encounter difficulty; (3) to provide students a "preview" of the content to be presented in the in-class tutorial in the following week.

We use an online survey tool called Catalyst, developed by the University of Washington, to deliver the pretest questions. The Catalyst system allows for both multiple-choice and free-response questions so that we can obtain both answers and reasoning from students. These both provide relevant data for our research. Student responses on the multiple-choice questions inform us about student understanding of the content after relevant lecture instruction and the prevalence of certain answers. The student explanations allow us to find out how well students apply the correct physics concepts and their incorrect approaches. The complete versions of the pretests administered to students are included in appendices B, C, D, and E.

Students have 15 minutes (in Physics 12X courses) and 30 minutes (in Physics 11X courses) to complete the online pretests. We set the time limit to reduce student collaboration and limit students' ability to find relevant references (class notes, textbooks, internet) to answer the questions. A major goal of the pretests is to probe student thinking of the concepts when they are not trying to use memorized equations or formalism. Students in Physics 11X are given more time to reduce the pressure they may feel in answering since they are mostly in life sciences programs with a limited physics background. All students are awarded points for completing the pretests, not for the correctness of their answers to the questions.

2.1.2 Posttests

We assess our curriculum by putting questions on midterms or final examinations of the courses after all the relevant material (lecture and tutorial) has been presented. The posttest questions are usually free-response questions on the midterms in the Physics 11X and 12X sequences, and usually multiple-choice questions in the final exams. Occasionally, we administer free-response questions on the final exams.

The data analysis from multiple-choice questions usually focus on the proportion of students who answered correctly, and the percentage of students who selected the most common incorrect answer(s). In order to improve student understanding on a topic, the most common answer serves as a good starting point, as that answer can inform us of the area in which most students need assistance. Since there are no explanations for the multiple-choice questions, we are unable to know the approach(es) that students are using to arrive at their answers. Likewise, the proportion of correct answers is an upper bound on the fraction of students correctly arrive at the answers with the correct concepts and reasoning.

Students are asked to explain their reasoning to the free-response questions. Based on their responses, we analyze the written explanations accompanying student answers to determine how students approach the questions. These data inform us about two changes from pretest to posttest—first, the change in the proportion of students with both the correct answer and reasoning; second, the change in the incorrect approaches used by students for a particular incorrect answer.

The posttest questions we used for this research can be found in the appropriate chapters and appendices of this dissertation.

2.1.3 In-class observations

The in-class tutorial sessions are structured in a way that allows students to interact with each other and teaching assistants. The setting allows us to make informal observations of student conversations. Although the observations tend to be anecdotal and not systematic, they often provide insights into student thinking that might guide curriculum development and instruction. The observations may reveal lines of student reasoning or student difficulties not apparent through the analysis of student responses. Also, we can observe how students interpret the statements or exercises in the tutorial material in the classroom.

2.2 EXPERIMENTAL DESIGN

In this section, we describe three primary experimental designs that were used in this dissertation: cross-sectional designs, pretest-posttest studies, and longitudinal studies.

2.2.1 Cross-sectional design

In cross-sectional designs, as described by Cummings, all of the observed data are collected at a single point in time [38]. They are often used to examine and compare single variables across multiple subgroups that are similar in other characteristics. For example, in Chapter 4 of this dissertation, we compare the student performance on one physics task in high school, algebra-based physics course, and calculus-based physics course. The results inform us about how students perform on that task at different educational levels and majors.

As mentioned, the data in cross-sectional studies are collected at one point in time, and they are best used to identify patterns and correlations. So, most surveys are cross-sectional.

However, the results are limited, and no causal relationship can be drawn. Sometimes, repeated cross-sectional designs are used to help establish a causal time-order. The repeated measurements observe different samples of the same population over time. These results can help to establish trends, patterns, and rates of charge on a subject of interest to researchers.

Studies with cross-sectional designs have several benefits and weaknesses. The benefits are that these studies are easy to conduct and tend to be cheaper to implement. There is no issue with missing data. As for the weaknesses, these studies have difficulty in providing specific information about causal relationships.

2.2.2 Design of pretest-posttest studies

The pretest-posttest comparisons are used extensively in our investigation. The basic premise involves a measurement outcome in the pre-treatment stage (pretest), followed by the same measurement outcome in post-treatment (posttest). (Recall, the pretest is usually given after lecture instruction, so it represents what students would have learned if tutorial instruction were not part of the course) We can use this design with or without control groups, and the main difference lies in the strength of the conclusions. In our case, the curriculum the UWPEG creates can be counted as a treatment during the field-testing (see Figure 1.1) stage. The measurement outcome of interest is student performance.

According to Bell, pretest-posttest designs can be characterized as one-group pretest-posttest design, two-group pretest-posttest design, two-group pretest-posttest design with a double pretest, and so on [23]. Bell also describes the kinds of quantitative analysis that can be used for each design. In this dissertation, the ‘group’ usually refers to the students in all sections of a course

in one quarter. We describe the designs that we used in our investigation using Bell's classification below.

The simplest design is the one-group pretest-posttest design. It involves measuring student performance on one physics task using a pretest, administering a treatment (tutorial), and then assessing student performance on the posttest. This design allows us to determine the effect of the treatment on student performance. There are several underlying assumptions. First, one might have gotten the same student posttest performance without using the treatment. Moreover, the time lapse between pretest and posttest might affect posttest performance, as a short time difference might reduce the opportunity for confounders to appear. However, a change in performance from pretest to posttest that occurs over a short time-lapse can then be strongly associated with the treatment.

To address the first assumption, one can include a non-treated control group in the investigation. This inclusion can allow us to determine the effect of treatment better, as long as the pretest performance of the students in the treatment and control groups are about the same. This design with a control group is called the two-group pretest-posttest design. In our case, we can first do a control group study in one quarter and then introduce the tutorial curriculum in the subsequent quarter to find out if the treatment is effective.

2.2.3 Longitudinal design

According to Hofer and Piccinin [25], a longitudinal design "measures the characteristics of the same individuals on at least two, but ideally more, occasions over time." The purpose is to study individual change and variation directly. It allows the "direct assessment of within-person change over time." It is generally costly and labor-intensive to conduct such experimental design,

for example, measuring the change in the blood lead levels of 100 children for four weeks in a clinical trial. However, such a design can help in understanding developmental change processes better.

The pretest-posttest design above can sometimes be counted as a longitudinal design. For example, one measures student performance on a particular skill with the same group of students repeatedly. In this dissertation, we conducted a longitudinal study on the student ability with vector superposition of PHYS 114 students, which we described in greater detail in Chapter 14.

2.3 DATA ANALYSIS METHODS

We have used quantitative and qualitative methods to analyze our data. The quantitative methods refer to the statistical methods we used to compare student performance on pretests and posttests. These methods include the chi-square test for association, McNemar's test, and effect size. The results from the longitudinal study are analyzed using a generalized linear mixed model method. The qualitative methods refer to the approach we used for categorizing student explanations. All of the methods used in this dissertation are described below.

2.3.1 Quantitative methods

The pretests and posttests results in this dissertation are typically presented using frequency tables or bar charts (e.g., showing % correct or % of students selecting particular answers). Sometimes, contingency tables are used to compare the results with and without the treatment, or pretest vs. posttest.

In this work, the dependent variable is student performance on one task on pretests or posttests, which is considered a categorical variable (typically has a few outcomes such as correct or incorrect). Another kind of measuring student performance is the test score that usually goes from 0 to 100, which is considered a continuous variable. (We are not comparing test scores in our work.) The independent variable is the treatment (also a categorical variable). The word 'treatment' here can mean with or without tutorial instruction, before and after a question or curriculum modification, or modification on the same task.

The statistical methods used to test the association between the change in student performance with the treatment in our study are Fisher's exact test/chi-square test for association and McNemar's test [39].

Chi-square (χ^2) test of association

Consider a situation in our work, whereby the pretest data are collected before or after lecture instruction in different quarters. Suppose we are interested in finding out if the difference in the pretest results is associated to lecture instruction, then we have two samples – the pretest results *before* lecture instruction and *after* lecture instruction. We arrange the data in a 2 × 2 contingency table. Then, we perform chi-square (χ^2) test to answer the above question.

This test is a two-sample test for binomial proportions for a large sample size (> 100) [39]. For studies with small sample sizes (< 100), Fisher's exact test is used.

Back to the above example, in the 2 × 2 contingency table, the row variable is correct/incorrect, and the column variable is before/after lecture instruction, with the entry in each cell (4 cells in total) as the number of students satisfying the description. The null hypothesis is

that the proportions of correct answers are the same before and after lecture instruction. The test statistic is then calculated using statistical software, and the *p*-value is reported.

These tests of association can also be used to compare the results of the same posttest question from two different quarters. In this case, the null result is that the proportions of correct answers are the same in both quarters. In our study, we use the χ^2 test since our sample size is usually large. This test is used extensively in our work in a few ways. They are (1) to compare results of the same pretest over two quarters, (2) to compare pretest/posttest results after treatment (with/without tutorial or change of wording in question), and (3) to compare results of an exam question with and without tutorial instruction.

McNemar's test

McNemar's test is used for one-group pretest-posttest study designs. We used it for studies in which we track student performance by students, *i.e.*, students from the same course who took the pretest and posttest in the same quarter. In this case, the pretest and posttest data are not independent but paired. The difference between the McNemar's test and the previous tests is in the independence of the two samples. That is, χ^2 test is used for two different groups of students while the McNemar's test is for the same group of students. This test informs whether the gain in the posttest performance in the same group of students is associated with the tutorial instruction. If the gain is strongly associated with the tutorial instruction, then we say that the curriculum is effective. The basic idea of this test is to compare the number of students who shift from incorrect (IC) in pretest to correct (C) in posttest ($N_{IC(\mathrm{pre})\to C(\mathrm{post})}$) with the number of students who change from correct in pretest to incorrect in posttest ($N_{C(\mathrm{pre})\to IC(\mathrm{post})}$). The null hypothesis is that the difference between the two numbers ($N_{IC(\mathrm{pre})\to C(\mathrm{post})} - N_{C(\mathrm{pre})\to IC(\mathrm{post})}$) equals zero, while the

alternative hypothesis is that the difference is not zero. This test is used in Chapter 10, where we assess the effectiveness of the tutorial curriculum to address student difficulties in rolling motion.

Effect size

In addition to reporting the test statistics mentioned above, we also report the effect size [40]. The effect size measures the degree to which a null hypothesis is wrong. One important purpose of effect size is to convey, if possible, the meaning of research results in the most understandable form for persons who have no or little knowledge of statistics. According to Rosenthal, effect size informs the readers "the impact of treatment intervention on an outcome variable" [41]. Most of the outcomes in this dissertation are categorical variables, and they are presented in proportions. The most straightforward estimate of the effect size is the difference between the percentage correct on one task before or after treatment. We follow the Rosenthal's size conventions proposal, which applies only when the percentages in both groups are in the 15–85% range. Here are the categories of the effect size (difference in the pretest and posttest): *small* (~7%), *medium* (~18%), *large* (~30%), and *very large* (~45% or more) [41]. Note that this method provides a quick estimate of the strength of the association. If precision is needed or when the proportions for one or both groups are outside the range stated above (15–85%), then we need to calculate the effect size by using odds ratio, phi-coefficient, or Cramer's V [41].

In this dissertation, we use the two-tailed *p*-value instead of one-tailed since students who received the intervention can perform worse than students who did not. The *p*-values are calculated using the website VassarStats [42]. When the calculated *p*-value is less than 0.05, we reject the null hypothesis; otherwise, we do not reject the null hypothesis and conclude that the difference between the proportions is not statistically significant at the 5% level.

Method for analyzing longitudinal data – generalized linear mixed model

Next, we discuss the statistical methods for analyzing longitudinal data. We use Theobald's suggestion to use generalized linear mixed model to analyze the longitudinal data in our vector superposition study. According to Theobald, the students in discipline-based education research are usually grouped in sections, terms, years, or other categories [43]. They have different instructors, different lecture materials, assessment schemes, and hence, different learning experiences. Furthermore, the treatments (interventions) in education research could be administered at different quarters in the experimental phase. To evaluate the effectiveness of different interventions, one can use regression analysis to compare them quantitatively [44]. However, simple linear regression requires all observations to be independent of each other. This condition will not hold for educational studies when one conducts multiple observations on the same student. Repeated measures make the observations not truly independent, so regression models with random effects should be used to "account for the nonindependence of responses by the same individual student" [43]. We use R to fit our longitudinal data using generalized linear mixed models [45].

The definitions of the terminologies and additional details of the statistical tests can be found in Appendix A.

2.3.2 Qualitative analysis

The approach we have used for categorizing student explanations collected from pretests and posttests is mainly qualitative. In this research, we have used a generative coding approach to categorize student explanations [46]. By this, we mean that we do not pre-determine the categories we will use to group the explanations. We devise and revise the categories as we read through

student explanations. There are multiple ways to interpret the results. Still, often certain statements by students stand out as being in contrast to the reasoning being presented in a course or as used by physicists. We try to interpret these in ways that we think may be useful for curriculum development. In the end, we summarize and make general observations from our analysis by reporting the frequencies of the categories.

There are different methods to integrate quantitative and qualitative analysis. Chi summarized four different approaches to do so [47]. These are (1) to use the qualitative data to help interpret the quantitative results, (2) to use some quantitative measures along with the qualitative criteria, (3) to use the qualitative analysis as a backdrop for generating hypotheses, which are then tested by experimental methods, and (4) to compare the frequencies of the codes generated by one's subjective impression.

2.4 POSSIBLE SOURCES OF THREATS TO VALIDITY

This section discusses some of the possible sources of errors that could affect the validity of our conclusion. They include (1) missing data, (2) lack of randomization, and (3) variations of student performance from quarter to quarter.

2.4.1 Missing data

Not all students who register in a given course participated in both the pretests and posttests. As a result, we tend to look at the responses from students who completed both the pretest and posttest, which is known as the complete-case analysis. However, there is a danger that the data obtained may not be representative of the full population, so the reported learning

gains may be biased, especially when the probability of missingness is associated with outcomes. For example, students who missed the second to fourth pretest are those who answered the first pretest incorrectly. The missing data problem is potentially more severe in longitudinal data analysis. For example, if the number of complete cases (students who participated in all pretests) is 50% of the students who participated in the first pretest, then the results from the longitudinal analysis using the complete cases are highly unlikely to be representative of the population or have good statistical power.

For longitudinal data, the way of classifying missing data, according to statisticians, is based on whether the observed or unobserved outcomes are predictive of missing data [48, 49]. There are three types of missing data: missing completely at random (MCAR), missing at random (MAR), and not missing at random (NMAR). We describe each of them using the descriptions by Fitzmaurice, Laird, and Ware [49] below.

If the missing data is purely unrelated to the outcomes, then the missing data are MCAR. For example, the probability that the responses from pretest 3 are missing is unrelated to the responses obtained from pretests 1 to 3. The missing data are called MAR when the probability that responses are missing depends on the set of observed responses but is conditionally unrelated to the specific missing values that, in principle, should have been obtained. For example, the probability of a pretest 2 response is missing depends on the responses on pretest 1. However, given one particular response from pretest 1, the probability of the pretest 2 response is missing does not depend on the response on pretest 2. Data that are not missing at random (NMAR) when the probability that responses are missing is related to the specific values that should have been obtained. We illustrate this using an example stated in the book by Fitzmaurice *et al.* [49], the Muscatine Coronary Risk Factor (MRCF) study. Missing data were NMAR when parents of

children who were obese were either more or less likely to sign the consent form than parents of children who were not obese. The missingness on weight and height data is related to the obesity status of the child.

If the data are MCAR, then both the missing and the observed outcomes are representative of the student population, so the reported summaries based on the complete cases are valid. If the data are MAR or NI, then summaries based on the available cases may be biased.

Jayson *et al*. suggested performing multiple imputations to handle the missing data problem in pretest-posttest studies [50]. Multiple imputations generally mean substitute or fill in the values that were not recorded with imputed values multiple times, and perform complete-case analyses. Based on their simulation studies, they reported that the bias was reduced with a high participation rate. They also claimed that the bias from complete-case analysis at a participation rate of more than 75% was only slightly higher than that of multiple imputations. Their claim implies that if the number of complete cases is more than 80% of the number of students registered to the course, it is generally fine to report the learning gains from complete-case analysis as the bias to the actual result is low. As for the longitudinal studies we conducted, we attempted some multiple imputations schemes, which we discuss in detail in Part 4.

2.4.2 Lack of randomization

One of the fundamental assumptions in applying statistical methods to our investigation is that students are sampled at random from the population. However, we are unable to randomize the sampling of students in our case. In our two-group pretest-posttest study designs, we are not able to choose the students in the control group and the experimental group at random. When the treatment is related to tutorials, we cannot pick control and experimental groups in a quarter. For

example, the control that receives no tutorial instruction may not take place during the same quarter as the experimental groups, which receives tutorial instruction. Even if the treatment is related to an online intervention, we may assign one lecture section to undergo the intervention and the other as the control group in the same quarter. However, we are unable to select students in the sections randomly because they self-select into lecture sections that fit their timetable before the start of the quarter. Moreover, sometimes the control and experimental groups have different instructors. These factors all produce confounding variables in our investigation.

While this is a potential issue, we, in general, do not find any significant variations in student performance in the tasks we administered in pretests between lecture sections and quarters. In that case, we aggregate the results from different sections and quarters and report them as one value. Sometimes significant variations do exist, and that is usually attributed to different lecture instructions.

2.4.3 Variations in student performance

One may ask whether the pretest results collected over only several quarters are a good representation of the base average performance of the population since there are not enough data points to provide a good variation. Heron has investigated the historical changes in student performance on 20 different pretest questions (spanning 15 years) designed by UWPEG members [51]. These students were taught by many different instructors and with a variation of textbooks. Out of the 20 questions, she found that "the average performance of the "after lecture" classes was significantly better than that of the "before lecture" classes for 11 questions, significantly worse for two questions, and indistinguishable for the remaining seven." She reported that the standard deviation in performance for most questions was between 5-10%. The result suggests that

differences in the pretest and posttest performance greater than 20% are not due to normal variation but instead to the effect of tutorial instruction.

2.5 SUMMARY

The quantitative and qualitative methods we described in this chapter are consistently used in the dissertation. One can come back to this chapter or, in some cases, Appendix A, to refer to the discussions and calculations of specific statistical tests.

PART I
INVESTIGATION OF STUDENT UNDERSTANDING OF PARTICLE MOTION

Student understanding of kinematics is arguably one of the most examined areas in physics education research. The study of motion begins as early as elementary or middle-school and is covered, with increasing sophistication through the university level. It is often the first topic covered in a course on physics or physical science and lays a foundation for the study of other, more advanced, topics. Typically, students begin by examining the motion of idealized, point objects in one-dimension using formal terms such as position, displacement, velocity, and acceleration. The case of uniform motion is examined first, and students are then introduced to more complex motions in which the speed is changing. The mathematical definitions and relationships between the kinematical quantities are quickly introduced, and students are given practice in applying these concepts and equations to systems of increasing complexity.

The study of kinematics is rich, in part, because it involves abstractions based on the readily accessible behaviors of real-world objects. Prior research on student thinking has included examining ability to: distinguishing between related concepts (e.g., velocity and acceleration), recognize how the formal definitions for various kinematical quantities differ from their informal use, create and interpret motion graphs, and relate graphical and mathematical representations to the physical motions of objects. Students are typically expected to apply a range of formulae and definitions to solve problems in kinematics, and many instructors/researchers have focused on student problem-solving ability as well.

The research presented in Part I of this dissertation began with an examination of results from a question administered to a high-school class. The students were given position and time

data from the motion of an object and asked to find the speeds at various points. The motion had two parts – the object initially moves with increasing speed and then travels with constant motion. The performance was surprisingly low (on the order of 10% correct). Some of the errors made by students revealed difficulties with the underlying concepts that have been reported in the literature, while others suggested a tendency to apply formulae indiscriminately. However, the approaches taken by students suggested that the task could also give insight into student problem-solving ability.

The original and modified versions of the question have since been given across multiple quarters to hundreds of students in the introductory calculus-based mechanics course at the University of Washington. These tasks have included graphs and ticker-tape records of the motion as well as isomorphic variations given in other contexts. The results have allowed us to probe, over extended periods of time, various aspects of student reasoning, and the extent to which students relate some of the common ways in which motion is represented in physics.

Part I begins with Chapter 3, which contains a discussion of prior relevant research into student understanding of kinematics. Chapter 4 contains a description of the original research and a summary of the results when it was presented to students at the high school and university levels. In Chapter 5, we present alternative versions of the questions of the question designed to probe student thinking in greater detail. It also contains results from cross-sectional studies in which we examined how students' ability to answer the questions changed, in the aggregate, throughout the course as well as results from longitudinal students in which we tracked the results from individual students. Chapter 5 concludes with results from a preliminary attempt at curriculum development, although the results show little or no impact on student performance.

Chapter 3. BACKGROUND & PRIOR RESEARCH

This chapter reviews the relevant physics definitions that physicists use to describe the linear motion of point particles. We also describe some relevant prior research on student understanding of velocity in one-dimension, rates of change, and student problem-solving ability. This investigation on linear motion mainly revolves around student ability to calculate instantaneous velocity under non-constant acceleration situations.

3.1 REVIEW OF KINEMATICAL QUANTITIES RELEVANT TO THIS INVESTIGATION

This section reviews operational definitions of the kinematical quantities that allow people to analyze particle motion and are relevant to our investigation. We limit our discussion to one-dimensional motion in this chapter. The relevant quantities are described in Table 1.1.

Before describing the motion of a particle, the location of the particle relative to a chosen origin is identified. This location is called the position of the particle. When the particle moves, its position changes from one location to another. This change in position is the displacement of the particle. In one-dimensional motion, the particle can move left or right. Here we adopt the standard convention of a positive sign for rightward motion and a negative sign for leftward motion.

Average velocity and instantaneous velocity are vector quantities used to describe how fast and in which direction the particle changes its position. Similarly, average acceleration and instantaneous acceleration are used to describe how fast and in which direction the particle changes its instantaneous velocity. If the particle speeds up during its motion, then the directions of velocity

and acceleration are the same. Conversely, if the particle slows down, the directions of velocity and acceleration are opposite to each other.

Table 3.2. Operational definitions of linear kinematical quantities.

Linear Kinematical Quantity	**Definition**	**Operational definition**
Position	Location with respect to a chosen origin	x
Displacement (Δx)	Change in position	$x_f - x_i$
Average velocity (v_{avg})	Change in position in a unit time interval	$\frac{\Delta x}{\Delta t}$
Instantaneous velocity (v)	Rate of change in position	$\frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$
Average acceleration (a_{avg})	Change in velocity in a unit time interval	$\frac{\Delta v}{\Delta t}$
Instantaneous acceleration (a)	Rate of change in velocity	$\frac{dv}{dt} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$

There are several representations used to describe motion, such as pictorial, graphical, and mathematical. Pictorial representations visually indicate the position of a particle at different times. The recorded positions can be plotted into a position-time graph (graphical representation), and an equation can be written to describe it. Graphically, the instantaneous velocity at an instant is the slope of a position-time graph at that instant. Similarly, instantaneous acceleration is the slope of the velocity-time graph. In this investigation, we used all the above representations to evaluate students' ability to calculate instantaneous speeds (magnitudes of instantaneous velocity) of an object at different locations under non-constant acceleration.

If a particle is in uniform motion, the instantaneous speed and average speed are the same. In a graphical representation, the position-time graph of the particle in uniform motion is linear.

So, the slope of the graph and the average velocity are the same. If the particle is undergoing acceleration, the position-time graph can curve upward or downward. The slope of the graph changes with time. In that case, the instantaneous velocity of the particle is not the average velocity over a time interval.

3.2 PRIOR RESEARCH

There has been extensive research on student understanding of kinematics. Most of this had focused on conceptual understanding. In our investigation, we focused on the students' ability to calculate instantaneous speeds under non-constant acceleration situations. So, in addition to reviewing research articles on student conceptual understanding of kinematics, we also review prior works on student understanding of rates and student problem-solving in this section.

3.2.1 Research on student conceptual understanding of kinematics

The first Ph.D. student in the UW physics education research group, David E. Trowbridge, conducted his investigation on the student understanding of the concept of velocity and acceleration in one dimension under the guidance of L. C. McDermott [3, 4]. They replicated and extended some Piagetian motion tasks [52] to explore how introductory physics students thought about motion [3]. They developed two speed-comparison tasks: one that involved two moving objects passing, and one that involved two objects not passing each other. They conducted interviews with students and found that students' mistake was related to their improper use of position to determine relative velocity. One such example is that objects at the same position at the same instant must have the same instantaneous speed.

McDermott *et al.* documented some common errors students made in interpreting kinematic graphs [1]. They investigated two categories of student difficulties involving graphs in physics: difficulties in connecting graphs to physical concepts and the real world. They highlighted five specific difficulties in each category. The difficulties they found that are relevant to our investigation are (1) matching narrative information with relevant features of a graph, (2) interpreting the area under a graph, and (3) separating the shape of a graph from the path of the motion. The results of the investigations by Trowbridge and McDermott *et al.* were used in the curriculum development and published in *Physics by Inquiry* [33] and *Tutorials in Introductory Physics* [31].

Halloun and Hestenes surveyed and analyzed the commonsense beliefs college students have about motion and its causes [6]. In the context of the description of motion, they found that students did not differentiate between these three sets of quantities, namely interval and instant of time, average velocity and instantaneous velocity, and lastly, distance, velocity, and acceleration. Their summary about how students related motion and its causes is also insightful, but not relevant to our scope of the investigation.

Beichner developed the Test of Understanding Graphs in Kinematics (TUG-K), an inventory to test student interpretation of kinematics graphs [53]. He found several student difficulties such as graph-as-picture errors, slope/height confusion, variable confusion, non-origin slope errors, area ignorance, and area/slope/height confusion. To help students with the difficulties, he suggested that instructors should give students opportunities to consider their ideas about kinematic graphs and then help them to modify those ideas when necessary. There are other physics inventories, such as Force Concept Inventory (FCI) [7] and Force and Motion Conceptual

Evaluation (FMCE) [5], with motion-related questions. They focus more on the conceptual understanding of forces, which are less relevant to our investigation.

3.2.2 Research on student understanding of rates

Herbert and Pierce investigated whether students' expressions of their conceptions of rates are affected by either context or mathematical representation [54]. They interviewed twenty Australian high school students, asking questions in distance-time and area-height contexts, in numerical, graphical, and symbolic representations. Their work on the distance-time context is of interest here. They found that students could obtain more information about rates from the numerical representation, followed by graphical, and then the symbolic representation. They also found that students' familiarity with speed did not help them understand the idea of rates in the area-height context.

Hauger documented conceptions and knowledge about the rate of change in high school and college students [55]. The investigation looked at three different types of rate of change: global, interval, and pointwise. The global rate of change describes the general trend of a function. The function can increase or decrease at steady, increasing, or decreasing rates. The interval rate of change refers to the rise over run of two points on a given function, for example, the average velocity or the average acceleration in the context of kinematics. The pointwise rate of change refers to the instantaneous rate of change, such as instantaneous velocity and instantaneous acceleration. Hauger found that students answered the instantaneous rate of change questions using the interval rate of change approach. This finding corroborates what Halloun and Hestenes had found - students did not distinguish between average velocity and instantaneous velocity.

3.2.3 Research on student problem-solving

Problem-solving is a broad and vital research area in physics education research (PER) since it is how physics instructors assess student learning. Maloney gave an overview of physics education research on problem-solving [56] in which he summarized some approaches other people used to investigate problem-solving in physics. Among these are comparing problem-solving strategies between experts and novices [57, 58], comparing the categorizations of physics problems by experts and novices [59], using isomorphic problems to uncover factors that affect student ability to transfer problem-solving skills from one task to another [60-62], and instructing students step-by-step problem-solving strategies and monitoring their performance [63]. He also reviewed other aspects of problem-solving research, such as how individuals functioned when they solved problems, the effectiveness of problem-solving instructions, assessing students' problem-solving skills, faculties attitudes and approaches to assessing problem assignments, mathematics in physics problem-solving, problem-solving and conceptual understanding, and solving ill-defined problems.

In our investigation, we used isomorphic problems to track student ability to calculate instantaneous speeds as students progressed through a mechanics course. We review the relevant prior research in the next few paragraphs.

Lin and Singh investigated college students' ability to perform analogical reasoning between two isomorphic problems [60]. Two problems are isomorphic if they can be solved by the same physics principles but differ in surface features. They asked students to learn from a solved problem, and then use what they had learned from it to solve a quiz problem that was isomorphic to the earlier version. The questions required the use of conservation of energy and Newton's laws, or multiple physics concepts to get the answer. Interventions were introduced to

help students identify and take advantage of the underlying similarities of both problems. The authors found that students could transfer what they had learned from the solved problem to the quiz problem to some extent but could not apply the correct physics principles necessarily. They suggested that students needed more scaffolding for better results. They also noted that students found multiple-concept questions challenging based on prior research.

In an earlier paper, Singh reported the effect of some potential factors on problem-solving and transfer when she assessed students using isomorphic problems [62]. Her goal of the study is to "evaluate whether giving both (isomorphic) quantitative and conceptual questions together improves performance on each type of question compared to the case when only the conceptual or the quantitative question was given." She used 25 pairs of problems to assess student problem-solving skills, and she was interested in the student performance in one of the questions in each pair. She compared the performance of the target question in two groups of students, with one group given the isomorphic problem and the other not receiving the intervention. She found that when quantitative and conceptual questions were paired and given back to back, students who received two isomorphic problems often performed better in the conceptual questions than those who answered the corresponding conceptual questions only. However, there was not much difference when the target question was quantitative. She highlighted that solving paired isomorphic problems could help students reflect on the problem-solving process and improve metacognitive skills. Singh also mentioned that instructors could use the isomorphic problems to help students identify the underlying similarities of the problems so that students could develop expertise in the subject matter.

3.2.4 Commentary

We reviewed several papers on students' conceptual understanding of kinematics, student understanding of rates, and problem-solving ability. We focused on several previously reported student difficulties, such as not distinguishing time interval and instant of time, not distinguishing average velocity and instantaneous velocity, and some difficulties in motion graphs. These errors were found in our preliminary investigation.

We discussed several methods to investigate student problem-solving ability but decided on the use of isomorphic problems in our work. We tracked the student ability and persistence of student difficulties using isomorphic problems at different times in the quarter. More details can be found in Chapter 5.

In Part I of the dissertation, we use speed instead of velocity in most descriptions since we are interested in the magnitudes only.

3.3 MOTIVATION AND CONTEXT OF RESEARCH

This research on student understanding of particle motion started by looking at some student responses to a velocity task (more details in Chapter 4). Our group used this task in the 1990s on the first tutorial pretest. It served two purposes: (1) to assess prior student knowledge of one-dimensional motion and (2) to assess the effectiveness of lecture instruction. In addition, we collaborated with some local high school teachers. They used the same velocity task for the same purposes as ours. By collecting additional data from students in PHYS 114 and PHYS 121, we could compare the results from different educational levels at different times (high school vs. college, pretest vs. post-instruction). We were interested in the similarities and differences

between the population groups and the trends in the student difficulties between the population groups.

One aspect we explored here was student problem-solving ability. Prior research tended to look at how students solved complex problems. In much of this work, it seems that there was an implicit assumption that students could solve more straightforward physics problems. Simple plug-and-chug problems are undoubtedly easy for students to do, provided they do not need to check the relevant assumptions or conditions. However, there were some simple physics problems from an expert physicist point of view, yet the students achieved low to moderate performance levels. The task we used within our investigation appears to fit into this category of physics problems. We wanted to investigate student problem-solving ability over time.

Next, we investigated the correlation of student responses between graphical and numerical representations. The design feature in the Catalyst survey allowed us to collect such data easily. Furthermore, we could utilize the feature to get students to compare their answers and check whether they were consistent with each other. This motivation arose because of the availability of the feature, and we wanted to explore its additional utilities for our research purposes.

Chapter 4. PRELIMINARY INVESTIGATION OF STUDENT UNDERSTANDING OF SPEED AND RELATED TASKS

This chapter first describes the speed calculation task that started our investigation into the student understanding of particle motion. We discuss the results based on written data collected from high school and college students. These two groups of populations represent two stages of a physics learning journey: the start of physics learning during high school, and its continuation at the college level. The results are interpreted as the evolution of student problem-solving ability on this particular task. We have a first glance at how the difficulties associated with the task persisted from high school to college and how student approaches differ over this period.

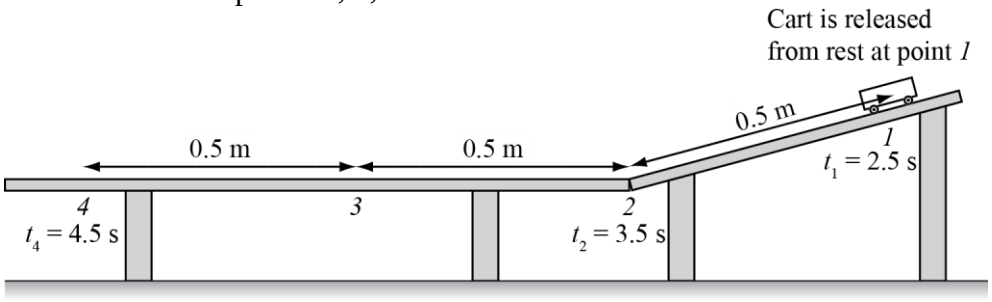
Next, we used the written responses to create the multiple-choice version so that we can administer online for ease of analysis. We compare the results between the written version (collected from the year 1993) and the multiple-choice version (collected from the year 2015) to determine the efficacy of the multiple-choice version and the student knowledge level at the start of the college level physics instruction after 22 years.

Third, we look at the students' kinematic graphs (position-time and speed-time graphs) in the same context on written pretests. The student difficulties with those graphs are discussed. We then compare students' speed-time graphs with their calculated speeds to check their consistency in the graphical and numerical representations. We repeat this procedure to create the multiple-choice versions of some questions to put on online pretests, and then compare the results from written and online pretests from algebra-based and calculus-based mechanics students.

4.1 STUDENT RESPONSES ON THE SPEED CALCULATION TASK

The speed calculation task describes a cart that is released from rest on an incline. It speeds up as it moves down the incline, and then moves with constant speed on a level track. Four points are labeled on the track, and the times when the cart is located at specific points are shown. Students need to find the speed of the cart at points 1-4 and briefly explain how they arrive at the answer. The task is shown in Figure 6.1, and we describe it as the cart-on-ramp context (CR).

A cart is released from rest, speeds up as it moves down an incline, and then moves with constant speed on the level segment of track as shown in the diagram below. Four points are labeled on the track. Point *3* is located halfway along the level track segment. The times when the cart is located at points *1*, *2*, and *4* are shown.



Use the information that you have been given to find the speed of the cart, if possible, at each of the points *1-4*. Briefly explain. If you do not have enough information to find the speed at one or more of the points, state what information you would need.

Figure 4.1. The speed calculation task – cart-on-ramp (CR) context.

The speed of the cart at point 1 is zero because it is released from rest. The cart moves at constant speed along the level track segment, so the speeds of the cart at points 2-4 are equal, and the magnitude can be determined by using the average speed of the cart on the level track segment, which in this case is 1.0 m/s. In terms of concepts involved in this task from the experts' point of view, students need to recognize that the cart is released from rest, so v_1 is 0 m/s. The cart travels at a constant speed from point 2 onwards, since the average speed and instantaneous speed are the

same during uniform motion. However, not all students view physics tasks the same way as experts, as demonstrated by Chi *et al.* [59].

We first present the results on the written version of this task that we administered to high school and college students in different years. We use the top few common responses from the written task on a midterm to generate the choices for the multiple-choice format of the above question. We then compare the results between the written format and the multiple-choice format.

4.1.1 Results and discussions from the written format

The CR task was administered on a written pretest to 138 high school (HS) students and 280 PHYS 121 students before they had any instruction on mechanics in their schools. It was also given as a free-response question in PHYS 114 and PHYS 121 midterms after students had relevant lecture instruction. The pretest data from high school and PHYS 121 students were collected in the year 2014 and the year 1993 (Sp'93 quarter), respectively. The midterm data from PHYS 114 and PHYS 121 students were collected in the Wi'18 and Sp'93 quarters, respectively. The students' responses to the speeds at each point are tabulated in Table 4.3.

Before describing what the results from each population represent, we illustrate who PHYS 114 students generally are. They are mostly from life science or health sciences majors. Most of them have not taken high school physics (We surveyed 478 PHYS 114 students in Au'19 and found that 44% of them did some high school physics.), so in terms of the stages of the physics learning journey, the high school and PHYS 114 students are equivalent. However, the PHYS 114 students are mostly juniors and seniors, so they have more years of science (non-physics) instruction than the high school students.

Table 4.3. HS and introductory students' responses for the CR task on the written pretest and written exam. The correct answer and the corresponding percentages are in bold.

Speed	Answer (m/s)	**High school (HS) & PHYS 114**		**PHYS 121 (Sp'93)**	
		HS pretest (*N* = 138, 2014)	114 midterm (*N* = 261, Wi'18)	Pretest (*N* = 280)	Midterm (*N* = 262)
v_1	**0**	**69%**	**87%**	**91%**	**98%**
	No answer	21%	-	9%	-
	0.5	-	5%	-	-
v_2	**1.0**	**12%**	**31%**	**19%**	**50%**
	0.5	43%	49%	40%	39%
	No answer	14%	3%	21%	-
	0.14 (0.5m/3.5s)	12%	3%	13%	4%
v_3	**1.0**	**53%**	**55%**	**39%**	**72%**
	0.5	1%	16%	4%	8%
	No answer	29%	7%	41%	-
	0.67 (1.0m/1.5s)	1%	3%	2%	8%
	0.75 (1.5m/2.0s)	1%	1%	1%	2%
v_4	**1.0**	**60%**	**59%**	**48%**	**71%**
	0.5	2%	15%	4%	8%
	No answer	9%	5%	19%	-
	0.75 (1.5m/2.0s)	7%	5%	11%	10%
All 4 (v_1, v_2, v_3, v_4)	**(0, 1, 1, 1)**	**7%**	**26%**	**17%**	**49%**
	(0, 0.5, 1, 1)	17%	22%	13%	19%
	(0, 0.5, 0.5, 0.5)	1%	12%	3%	8%
	(0, 0.5, 0.67, 0.75)	1%	3%	2%	7%

We assume that these different groups of students we had sampled are representative of the student population at that stage of physics instruction. The results in Table 4.3 represent the students' ability to calculate speeds at various stages of physics learning. We define the stages of the physics learning journey as follows: high school (HS) physics is the journey from beginner to novice, the whole undergraduate physics program is the novice to the intermediate stage, and the graduate physics program is intermediate to the expert stage.

Now we illustrate how we associate the results we found to the student's ability to find speeds at different stages of physics learning. The HS pretest and PHYS 114 midterm results represent the student ability at the start of and after *some* HS physics instruction. The PHYS 121 pretest and midterm results signify the ability at the start of and after some physics instruction at the college level. The PHYS 121 pretest result also represents the ability after *all* HS physics instruction. One thing to note is that the 114-midterm results cannot be used to assess the HS physics instruction, but the assessment stage is the same as the posttest for HS students.

The percentage correct for v_1 increases from 69% at the start of physics instruction to 98% at PHYS 121 midterm. Students who did not answer v_1 correctly on the pretest mentioned they needed to know the distance traveled by the cart before it reached point 1.

For v_2, the percentage correct increases from 12% at the start to 31% at posttest, then decreases to 19% after HS physics instruction, increases to 50% after lecture instruction at the college level. Less than 20% of all populations answered point 2 correctly on the pretest. Most students (around 40%-50% of students from all populations) answered 0.5 m/s, which was the average speed for the motion along the inclined track. Some students did not distinguish the instant of time and time interval, so they used the given t_2 to calculate v_2 and arrived at 0.14 m/s. This error can also be interpreted as assuming the motion started when the stopwatch is 0 s.

The results for v_3 and v_4 are harder to interpret, since there is no difference between the HS and PHYS 114 students. However, the performance of PHYS 121 students for both questions improved after lecture instruction. Among the students who did not answer v_3 on the pretest incorrectly, most of them needed additional information to find the speed. We believe these students wanted to do plug-and-chug with the average speed formula, but the time at point 3 was not given in the question. They were not aware that they could utilize the given phrase “constant

speed" to arrive at the answer. However, most students answered the PHYS 114 and PHYS 121 midterms. The most common incorrect answer in the midterm was 0.5 m/s, which was the average speed of the cart along the incline. Some students who wrote 0.5 m/s had utilized the "constant speed" information and said that $v_2 = v_3 = v_4$.

Some students made an error in finding the instantaneous speed at point 4 by using the average speed between points 1 and 4, which is 0.75 m/s. In the PHYS 121 midterm, some students used the same approach to find v_3 by using the average speed between points 1 and 3, which is 0.67 m/s.

Now we consider the four speeds altogether. We interpret the 7% correct in the HS pretest to 26% correct in PHYS 114 midterm as the increase in students' abilities in finding speeds in the beginning stage of physics learning. The decrease from 26% correct on the PHYS 114 midterm to 17% correct on the PHYS 121 pretest represents some retention of that ability. The increase in percentage correct from 17% to 49% in PHYS 121 was the result of the effect after lecture and tutorial instruction.

The most common incorrect speed combination for points 1 to 4 was (0, 0.5, 1, 1) m/s. This approach is using the average speed of the prior motion as the final speed of that motion. That is, the speed of point 2 is the average speed between points 1 and 2, the speed of point 3 is the average speed between points 2 and 3, and so on. We also found the speed combinations (0, 0.5, 0.5, 0.5) and (0, 0.5, 0.67, 0.75) to be quite common in the PHYS 114 and PHYS 121 midterms.

We also scored the task by assigning 1 for each correct speed and 0 for each incorrect speed. The percentage of score distribution for the task is tabulated in Table 4.4. The proportion of students who answered all speeds correctly follows the same pattern as v_2, so students who

answered v_2 correctly were more likely to score full points. The score distribution on the midterm for both PHYS 114 and PHYS 121 students was the same, with most students scored 1, 3, and 4 points. Students who scored 3 points usually answered v_2 incorrectly and students who scored 1 point only answered v_1 correctly.

Table 4.4. Percentage of score distribution for the CR task (written format) for high school and introductory physics students.

Score	**High school (HS) and PHYS 114**		**PHYS 121 (Sp'93)**	
	HS pretest (*N* = 138, 2014)	114 midterm (*N* = 261, Wi'18)	Pretest (*N* = 280)	Midterm (*N* = 262)
0	12%	7%	6%	-
1	25%	33%	44%	28%
2	26%	7%	13%	1%
3	30%	26%	20%	22%
4	7%	26%	17%	49%

One thing to note when we compare the difference in the distribution is the year in which the data were collected, especially the results from college students in 1993. The students in both populations had different learning experiences, especially in the technological aspect. However, the results from the PHYS 114 and PHYS 121 midterms suggest that the student difficulties do not vary too much after lecture instruction.

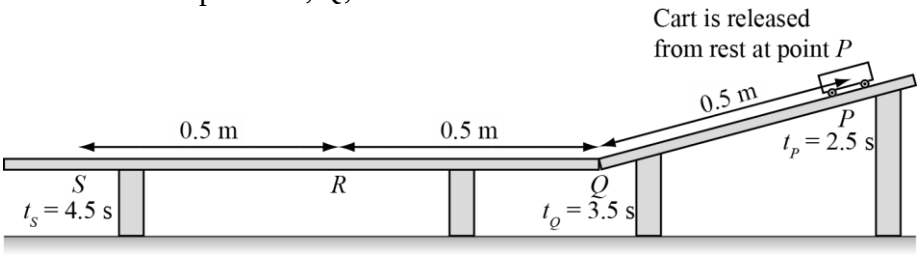
We can also see the effect of lecture and tutorial instruction in Table 4.4. The mean of the score increases. Around 70% of the class answered three or four speeds correctly on the posttest, compared to 30% on the pretest. In 1993, the PHYS 121 students did the preliminary version of the *Velocity* and *Acceleration in one-dimension* tutorials [31]. More recently, they only use the finalized, published *Acceleration in one-dimension* tutorial. We administer the same task on

pretest and midterm/final to compare the learning gains then and now. The results are discussed next.

4.1.2 Results from the multiple-choice format

In the written format of the speed calculation task, we saw many variations of answers. Each of them represents one way of student thinking. However, it does not make the analysis of results easier. The multiple-choice format allows faster analysis of results if we use the top few written responses, which represent at least 80% of the population surveyed. In this case, we used the top few written responses from the PHYS 121 midterm in 1993. The common incorrect answers reflect several approaches students used. The multiple-choice format of the speed calculation task is shown in Figure 4.2.

A cart is released from rest, speeds up as it moves down an incline, and then moves with constant speed on the level segment of track as shown in the diagram below. Four points are labeled on the track. Point *R* is located halfway along the level track segment. The times when the cart is located at points *P*, *Q*, and *S* are shown.



Which of the following best shows the speeds at points *P*, *Q*, *R*, and *S* (in m/s)?

	v_P	v_Q	v_R	v_S
A	0	1.0	1.0	1.0
B	0	0.50	1.0	1.0
C	0	0.50	0.75	0.75
D	0	0.50	0.67	0.75
E	0	0.50	0.50	0.50

Figure 4.2. The CR context in the multiple-choice format.

Option A is the correct answer. Option B is obtained by using the distance of the segment before the point divided by the corresponding time interval, for example, $v_2 = \frac{x_2 - x_1}{t_2 - t_1}$, $v_3 = \frac{x_3 - x_2}{t_3 - t_2}$, and so on. The value of v_4 in option C is calculated by $\frac{x_4 - x_1}{t_4 - t_1}$ and $v_3 = v_4$ because of constant speed. The speeds in option D is calculated by taking the initial time and position to be point 1, and the final time and position to be the point of interest. Option E is using the fact that the cart is undergoing uniform motion from points 2 to 4, but with v_2 as the average velocity between points 1 and 2. The errors in options B to E are consistent with indiscriminate use of average velocity and instantaneous velocity, with options C and D also containing errors consistent with incorrect initial values. If we score these options as we did previously, option B is 3 points, and the rest is 1 point.

We administered this new version to PHYS 121 students on a pretest in Sp'16 and final exam in the Sp'15 quarter. We surveyed the PHYS 114 students on a pretest in Au'17 and final exam in the Au'18 quarter. The students had lecture instruction on one-dimensional motion before attempting this modified task on the pretest. The results are tabulated in Table 4.5.

Table 4.5. Student responses for the CR task (multiple-choice version) on the pretest and final exam.

Answer	PHYS 114 (Algebra-based)		PHYS 121 (Calculus-based)	
	Pretest (Au'17, *N* = 477)	Final exam (Au'18, *N* = 353)	Pretest (Sp'16, *N* = 474)	Final exam (Sp'15, *N* = 519)
A (Correct)	**20%**	**25%**	**25%**	**53%**
B	33%	35%	20%	19%
C	14%	13%	7%	4%
D	10%	5%	10%	2%
E	23%	22%	37%	22%

First, we compare the performance of PHYS 121 students in the written format in the midterm and the multiple-choice format in the final. Despite the difference of 22 years, there is no difference in the student performance and score distribution in the exams (midterm and final) between the years 1993 and 2015. That is, 49% in 1993 and 53% in 2015 scored 4 points, 22% in 1993 and 19% in 2015 scored 3 points, and 28% of students scored 1 point in both 1993 and 2015. This result suggests that although we have changed in the instructional strategies and educational technologies over the 20 years, students continue to struggle with calculating instantaneous speeds when only half the class can answer correctly after lecture instruction. Comparing the results between the pretest in Sp'16 and the final exam in Sp'15, it suggests an improvement in the performance over the quarter.

Next, we compare the performance of the PHYS 114 students on the pretest and final exams. Our results show that there is no significant improvement in student performance. This result suggests that algebra-based lecture instruction does not improve student performance on this task the same way that the calculus-based lecture instruction does. However, this does not show that the lecture instruction in the algebra-based course is not as effective as the calculus-based course. Possible reasons for the differences include that students in both courses do not have the same preparation, and the areas of improvement among the students in different courses could be different. The difference in the areas of improvement may be something to investigate so that we have a proper perspective on the learning progression. If we compare the score distribution in the written format on the midterm and the multiple-choice format on the final exam for the PHYS 114 students, there is not much difference, just like the case of PHYS 121.

Assuming there is not much difference in the performance of the student population across years, by combining the results from the PHYS 121 students in Table 4.4 and Table 4.5, we can

see a progression of student performance from before actual instruction to assessment. The percentage correct starts from 15% before lecture instruction, increases to 25% after lecture instruction and then increases to about 50% in midterms or exams. It also suggests that students can achieve the same level of performance without doing the *Velocity* tutorial.

Table 4.6 shows some explanations students gave on pretests. The approaches students used for each answer in the algebra-based and calculus-based mechanics classes are the same. Some students described their methods in detail, similar to what we have described. One way to look at this data is that 75%-80% of the students were taking the average speed of the cart between points 1 and 2 to be the instantaneous speed of the cart at point 2 on the pretest.

We also asked 325 PHYS 121 students the approach (in symbolic form) they would use to calculate v_2 on a pretest in the Wi'16 quarter. 34% of the class used the average speed of the cart between points 2 and 3 (correct approach), while 44% used the average speed of the cart between points 1 and 2. These results show that students had difficulty identifying the appropriate average speeds to calculate when asked about instantaneous speeds.

The responses on the multiple-choice version also suggest that there may be some persistence in student difficulties, as 40% of the class selected options B and E. We study the persistence of student difficulties through a series of isomorphic tasks and track the student performance as they progressed through the introductory mechanics course. We discuss the results in the next chapter.

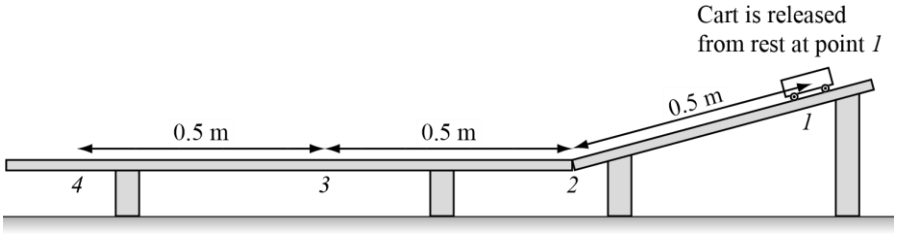
Table 4.6. Selected student explanations for the CR task on pretests.

Answer	PHYS 114 Pretest	PHYS 121 Pretest
A (Correct)	*Because before the cart is released, the v is 0. At horizontal track, the car moves at a constant speed with x = 1m, t = 1s. So* $v_Q = v_R = v_S = 1/1$ *= 1m/s.*	*When the cart rests on the incline, its speed is zero. When it moves down on the incline, its speed rises up. When the cart arrives on the horizontal segment, its speed remains constant which is equal to 1m/1s = 1m/s.*
B	*Because from point P-Q the velocity is 0.5m/1sec. Then at Q-R-S, the cart moves 1m/1sec. Because the cart is moving at a constant speed at Q-R-S, Q-R and S-R has the same velocity.*	*The distance between P and Q is 0.5m, and the time elapsed is 1.0s. 0.5m/1.0s = 0.5m/s. Therefore,* v_Q *is 0.5.* *The distance between Q and S is 1.0m, and the time elapsed is 1.0s. 1.0m/1.0s = 1.0m/s. Therefore,* v_S *is 1.0.*
C	v_P *= 0 because the cart is at rest.* v_Q *= 0.50 because the cart moves 0.5 m in 1 s.* v_S *= 0.75 because the cart moves 1.5 m in 2 s.* v_R *= 0.75 because the cart is moving at a constant speed on the level track segment so* $v_R = v_S$.	*When the car reaches point Q 1 s has passed and it has traveled 0.5 m, so the speed at that time is 0.5 m/s. At point S, the car has traveled 1.5 m and 2 s have passed so the cars speed is 0.75 m/s. On the flat surface the speed is the same, so at R the speed is also 0.75 m/s.*
D	*Speed is total distance over time. P is starting point so it's 0 m/s. Q is 0.5m/1s = 0.5 m/s. R is 1.0m/1.5s = 0.67 m/s. S is 1.5m/2s = 0.75 m/s.*	*The formula for velocity is v = distance/time therefore to simply find the velocity at a certain point you would simply find the total distance traveled to the specific point over the time (in this case x-2.5 seconds).*
E	*The car travels 0.5 m in 1 second down the ramp, accumulates a speed due to gravity then continues at this constant speed for the duration of the time.*	*The cart accelerates down the incline from a start of 0 m/s so P = 0 and the point Q = change in distance over time from the incline so 0.5m/1s = 0.5m/s and QRS are the same.*

4.2 STUDENT RESPONSES ON KINEMATICS GRAPH TASK

On the same written pretest in the Au'93 quarter, we asked students to sketch the position-time and speed-time graphs for the situation described in Figure 6.1 before calculating the speeds. The kinematics graph task is shown in Figure 4.3. Students need to indicate the approximate times of the cart's location. We first summarize how students sketched the graphs. Next, we look at how students labeled their time instants to infer how they ranked the time intervals for the cart to travel between consecutive points, which we call the time interval ranking task. We also find the correlation between students' graphs and their calculated speeds by comparing whether their graphical and numerical representations described uniform motion.

A cart is released from rest, speeds up as it moves down an incline, and then moves with constant speed on the level segment of track as shown in the diagram below. Four points are labeled on the track. Point *3* is located halfway along the level track segment.



Sketch graphs of the position and the velocity of the cart vs. time on the axes provided. Label the approximate times when the cart is at each of the points 1, 2, 3, and 4.

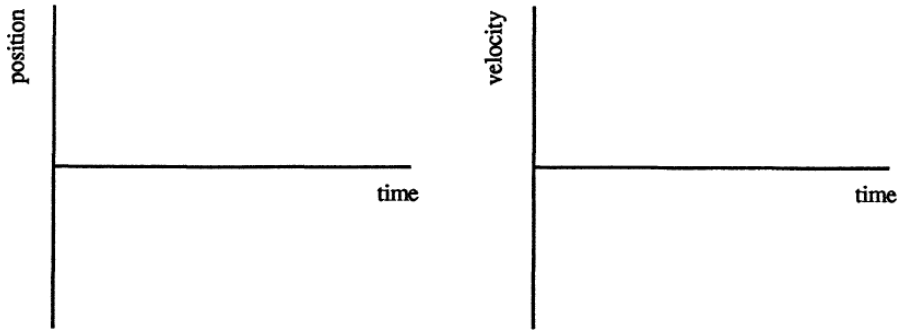


Figure 4.3. The kinematics graph task.

We continue our investigation with the current students (from 2015-2019) in the time interval ranking task. The data come from online pretest and written responses from midterms. The students surveyed are from the algebra-based (PHYS 114) and calculus-based (PHYS 121) mechanics courses. Their results can be interpreted as a progression of student understanding of speeds as they progress from beginner to novice.

4.2.1 Response from the written pretest

The correct graphs are shown in Figure 4.4. The cart speeds up as it moves down the incline and the acceleration of the cart is constant (while not stated, but it is the simplest non-zero acceleration model to work with), so the position-time graph is a parabola from t_1 to t_2, and the speed-time graph is a straight line starting from zero to the maximum speed at point 2. The cart moves at a constant speed from points 2 to 4, so the position-time graph is a straight line with a positive gradient, and the speed-time graph is a horizontal line from t_2 to t_4. One thing to note in the position-time graph is that the curve at point 2 should be smooth; that is, there is no drastic change in the gradient at point 2. A drastic change in the gradient would imply that the speed of the cart at point 2 has two values, which does not make sense.

Another thing to note is the time interval between each consecutive point. The correct ranking of the time interval is $\Delta t_{12} > \Delta t_{23} = \Delta t_{34}$. This ranking can be seen from the velocity-time graph easily. The area under the speed-time graph is the distance traveled by the object. It is evident that $\Delta t_{23} = \Delta t_{34}$ because the speed is constant, and the distance between each consecutive point is the same. If $\Delta t_{12} = \Delta t_{23}$, then the area under the graph from t_1 to t_2 is less than that from t_2 to t_3. So, only when $\Delta t_{12} > \Delta t_{23}$ is it possible for the area under the graph from

t_1 to t_2 to be equal to that from t_2 to t_3. Another way to arrive at the correct ranking of the time interval is to consider average speed. The average speed of the cart along the incline is half of that along the horizontal track. In order to travel the same distance along the incline and the horizontal track, the time taken along the incline must be twice that along the leveled surface, hence the relationship of the consecutive time intervals.

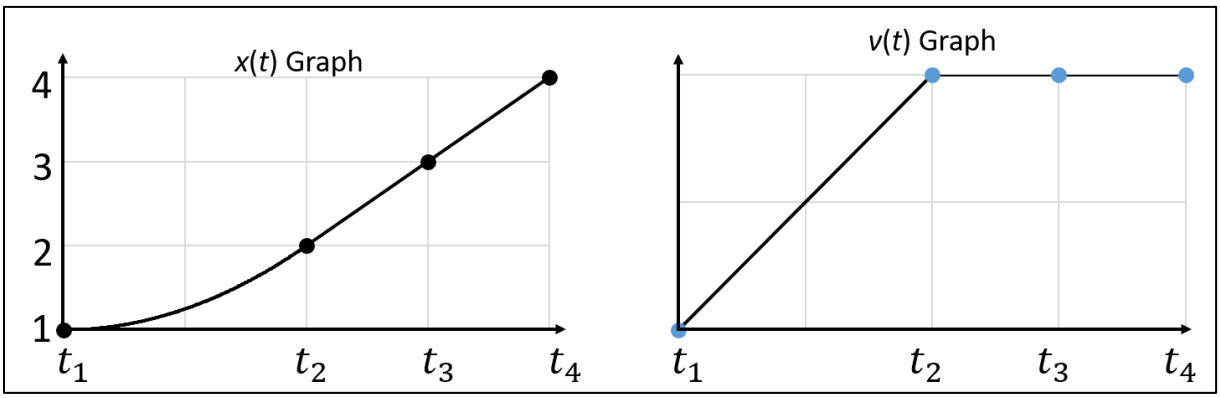


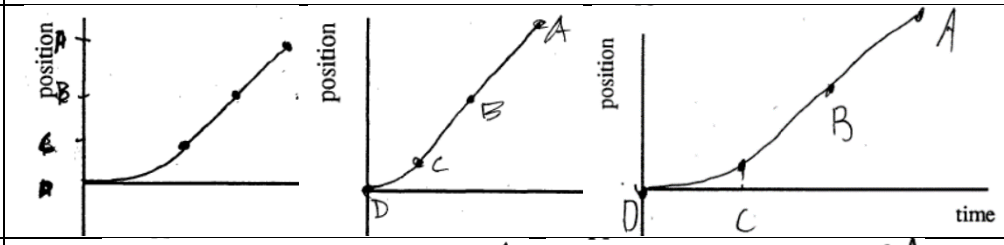
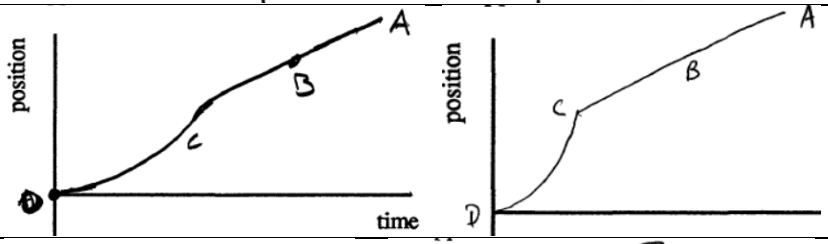
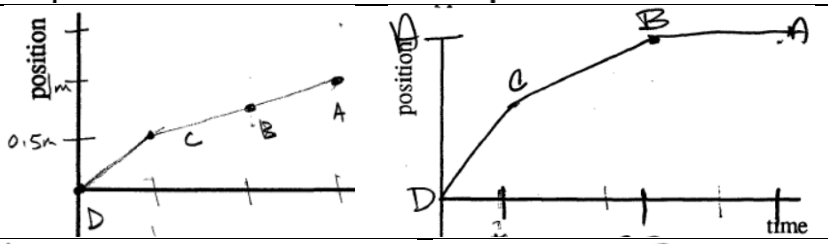
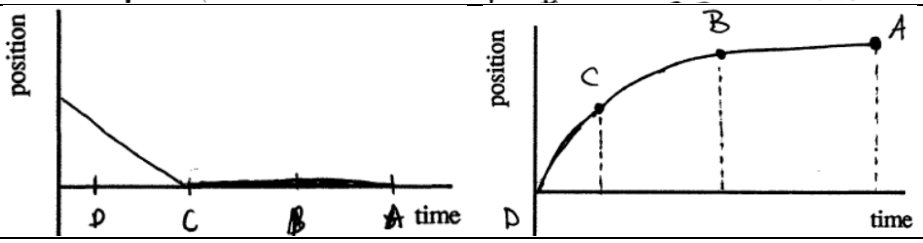
Figure 4.4. The correct position-time and speed-time graphs for the kinematics graph task.

We analyzed around 400 students' responses from the Au'93 quarter. The students attempted the pretest before the start of the first lecture. Most of them sketched both graphs; a small group of students did not sketch one of them or both. However, not all the students who sketched their curves labeled their time intervals accurately. We categorized the shape of the curves in three different time intervals and the ranking of the time intervals. The categorizations of the students' curves are shown in Table 4.7.

Only 15% of the students sketched the position-time graph with the correct features. The correct graph starts with a concaving upward curve, followed by a straight line without a change in gradient. Next, 19% of the students sketched curves with a change in the slope at point 2 (point

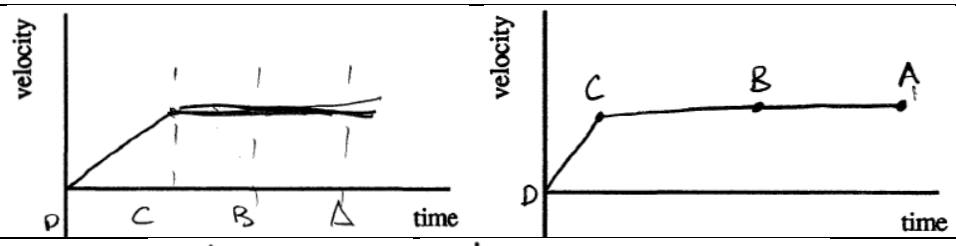
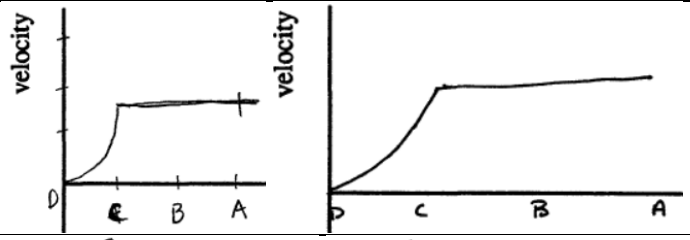
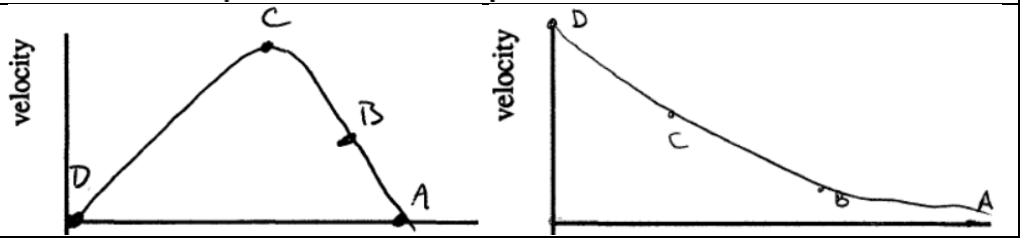
C on sketched curves). Some of them knew that the cart's speed at point 2 was single-valued but did not represent that in their curve correctly. Students who sketched straight lines did not represent the motion of the cart along the incline accurately. Some students (15% of the class) drew straight lines with kinks at point 2, implying that v_2 had two values. The remaining group of students sketched a large variety of curves.

Table 4.7. Categorizations of students' position-time graphs. Students used A–D as labels because those were given on the written pretest, with D as the starting point. The labels were changed to 1–4 to prevent confusion with the answer options A–E on a multiple-choice exam.

Category (N = 375)	Example graphs
Correct (15%)	
Speeding up initially, with kink (19%)	
Straight lines 2 or more (15%) 1 only (14%)	
Others (37%)	

In some cases, the whole curve took the shape of the track. In others, students drew horizontal lines, meaning that the cart stopped moving. They could have been treating the position-time graph as the speed-time graph in that a horizontal line would represent constant speed motion. These difficulties were described by McDermott *et al.* [1].

Table 4.8. Categorizations of students' speed-time graphs. Students used A-D as labels because those were given on the written pretest, with D as the starting point. We changed the labels to 1 – 4 to prevent confusion with the answer options A-E on a multiple-choice exam.

Category (N = 394)	Example graphs
Correct (44%)	
Increasing acceleration, constant speed (12%)	
Others 43 kinds (44%)	

We summarize the students' speed-time graphs in Table 4.8. Among the 394 students who sketched the curve, 44% of them drew the graph with correct features. The correct features include a straight line with a positive slope when the cart moves down the incline and a horizontal line when the cart moves along the horizontal track at a constant speed. Some students (12%) sketched

a concaving upward curve initially, followed by a horizontal line. These students did not think that the acceleration was constant. The concaving upward curve suggests that students might have conflated velocity and acceleration, that the acceleration increased as the speed increased. As for the remaining graphs, most students did not draw graphs to describe the uniform motion along the leveled track.

We infer students' time interval ranking by their time labels and tabulate the results in Table 4.9. Some students did not indicate the instants on their graphs. The proportion of student responses indicated the correct time interval ranking for the position-time and speed-time graphs are 29% and 15%, respectively. The most common incorrect answer was that all the time intervals were of the same size. This result suggests that students may use the distance between consecutive points to infer the time interval. Since the distances between consecutive points are the same, so are the time intervals. Around 13% of the responses indicated that $\Delta t_{12} < \Delta t_{23} = \Delta t_{34}$. These students might be thinking that since the cart was speeding up along the incline, the speed was increasing, so it traveled along the incline in a shorter time.

Table 4.9. Students' time interval ranking on the kinematic graphs.

Time interval ranking	Position-time graph (Au'93, *N* = 375)	Speed-time graph (Au'93, *N* = 398)
$\Delta t_{12} > \Delta t_{23} = \Delta t_{34}$ (Correct)	29%	15%
$\Delta t_{12} = \Delta t_{23} = \Delta t_{34}$	27%	27%
$\Delta t_{12} < \Delta t_{23} = \Delta t_{34}$	11%	13%
No time labels	14%	23%
Others	19%	22%

Combining students' graphs and time interval ranking, we found that 11% of the students sketched the correct position-time graph with the correct time intervals, and 8% did so for the speed-time graph.

Next, we look at how students' speed-time graphs are correlated to their answers in the speed calculation task. Based on results from the Sp'93 quarter, only 15% of the students answered the speed calculation task correctly. In the Au'93 quarter, only 19% of the students surveyed answered correctly. There are many combinations of speed calculation answers, so we narrow our analysis by only looking at whether students were able to represent uniform motion graphically and numerically. The results are shown in Table 4.10.

Table 4.10. Correlation table between students' speed-time graphs and speed calculations.

		Speed calculations (N = 394)	
		Constant $v_2 = v_3 = v_4$	Non-constant v_2, v_3, and v_4
Speed-time graph	Constant $v_2 = v_3 = v_4$	22%	41%
	Non-constant v_2, v_3, and v_4	3%	34%

More than half of the respondents (63%) drew horizontal lines to represent uniform motion from points 2 to 4, but only 25% of the students did that numerically. Two-third of the students who represented uniform motion on the graph correctly did not do so numerically. Note that not all the students who represented uniform motion numerically answered the speed calculation task correctly, some of them indicated that they needed additional information. Students who did not represent the uniform motion on the graph correctly did not necessarily calculate speeds to reflect the trend that they sketched on the graph. These results show that students had difficulty in reconciling graphical and numerical representations on the pretest.

In summary, when we analyzed the kinematic graph and speed calculation tasks together, we found that there was a lack of consistency in students' graphical and numerical representations. Students did not realize that when an object transitioned from accelerated motion to non-accelerated motion, the speed was the same at the transition, the slope at the point would not change. The students also did not rank the time intervals properly. The most common error was that the time intervals were the same, which could be related to equal distances between consecutive points. Note that the students did the pretest on the first day of class, which represented their knowledge state sometime after high school physics instruction. Some students did not label the time instants on their graphs, which could be due to the time limit on the pretest. It is possible that their attention was on the shape of the graph, but not on other details.

4.2.2 Student responses on the time-interval ranking task

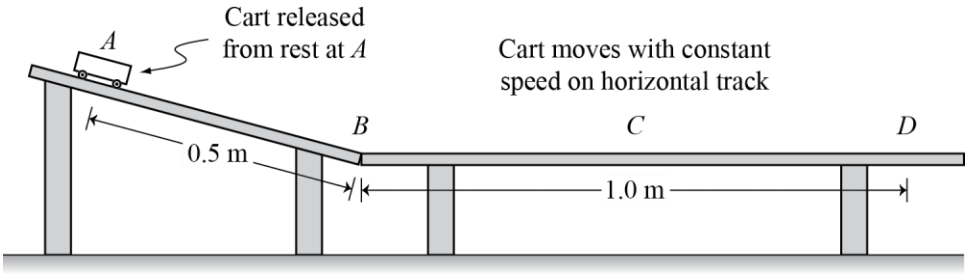
In this section, we present the results on how students compared the time intervals as the cart travels the same distances along the incline and the horizontal track. The time ranking tasks we used on pretest and midterm are shown in Figure 4.5. We modified the context for the midterm, but the nature of the task was the same as the pretest version. We administered this task to the PHYS 114 and PHYS 121 students in several quarters.

The correct answer is the time taken to travel along the incline is greater than the time taken to travel the same distance along the horizontal track. We can explain by comparing the average speeds along the incline and the horizontal track. The cart starts from rest at the top of the incline, speeds up and reaches the maximum speed at the bottom, then travels at that speed along the horizontal track without slowing down. The average speed along the incline is half that along the horizontal track. We can then compare the time taken to travel the same distance to travel along

the incline and horizontal track by using $\Delta t = d/v_{\text{avg}}$. Alternatively, the speed of the cart along the incline is less than that along the horizontal track, then the cart should spend more time to travel the same distance. The students' responses are summarized in Table 4.11.

Pretest version:

A cart is released from rest, speeds up as it moves down an incline, and then moves with constant speed on the level segment of track as shown in the diagram below. Four points (*A*, *B*, *C*, and *D*) are labeled on the track. Point *C* is located halfway along the level track segment.



Question: Is the time taken for the cart to travel from *A* to *B greater than*, *less than*, or *equal to* the time taken for the cart to travel from *B* to *C*? Explain your reasoning.

Midterm version:

Cart P is released from rest on a frictionless incline at time t_1 as shown. Between times t_1 and t_2, it travels a distance *d*, reaching the end of the incline. Between times t_2 and t_3, it moves a distance *d* across a frictionless horizontal surface.

Is $t_2 - t_1$ *greater than*, *less than*, or *equal to* $t_3 - t_2$? Explain.

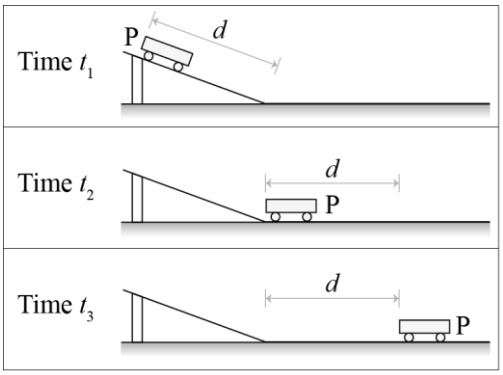


Figure 4.5. The time ranking task on pretest and midterm.

Table 4.11. Students' responses for the time ranking task on pretest and midterm.

Answer	PHYS 114 (Algebra-based)		PHYS 121 (Calculus-based)	
	Pretest Au'19 (N = 462)	Midterm Au'19 (N = 462)	Pretest Wi'16 (N = 190)	Midterm Au'18 (N = 519)
Greater than (Correct)	40%	45%	65%	71%
Less than	51%	38%	24%	21%
Equal to	9%	17%	11%	8%

We administered the task to PHYS 114 students in the pretest and midterm of the Au'19 quarter. The responses from students who answered in both instances are compared. The percentages correct in the pretest and midterm are 40% and 45%, respectively. Around 20% of the class answered on both the pretest and midterm correctly. The 5% gain in the percentage correct is not significant, and the effect size is small.

For the PHYS 121 students, most students (65-71%) answered the time ranking task on the pretest and midterm correctly. Most students (60% of the class) correctly explained their correct answers on the pretest and midterm. This result suggests that most students understood the requirements of the question and applied the relevant ideas to answer the task on pretest and midterm; however, there was no significant improvement from lecture and tutorial instruction.

Students who answered correctly tended to explain using average speeds or comparing the instantaneous speeds of the cart along the incline and horizontal track. Students who chose "less than" usually emphasized the non-zero acceleration along the incline and zero acceleration along the horizontal track. They thought that the initial speeding up implies the cart's speed along the incline to be greater than that along the track. Students who chose "equal to" tended to say equal distances traveled between consecutive points.

The results from PHYS 114 pretest to PHYS 121 midterm may represent the progression of a class of beginners to novices in physics. Unlike the results of the speed calculation task shown in Table 4.5, where there is no difference in the percentage correct between the PHYS 114 final and PHYS 121 pretest, there is a big difference in the percentage correct between the PHYS 114 midterm and PHYS 121 pretest.

Here are some possible reasons to explain this significant difference. First, the time-ranking task may easier than the speed calculation task because there are fewer details to work within the former task. Second, the students from both courses are different in different areas, such as their prior knowledge, majors, and interest levels in the subject. Most PHYS 121 students have prior knowledge and are more familiar with the ideas. In contrast, the PHYS 114 students are taught the content the first time (some of them did physics in high school but may have forgotten the relevant ideas), which could explain some of the 20% difference between the PHYS 114 midterm and the PHYS 121 pretests. The third possible explanation for the difference is to imagine the kind of students who would continue to do physics after having some high school / algebra-based physics instruction. These students (for example, the PHYS 121 students), on average, may have a better understanding of the underlying physics concepts and, thus, a higher probability of answering some tasks correctly, and the time-ranking task may be such a task.

4.3 SUMMARY OF THE PRELIMINARY INVESTIGATION

In this chapter, we described the preliminary investigation on student understanding of average speed and instantaneous speed. The task we used involves a cart that moves down a ramp from rest and then moves along a horizontal track with constant speed. The data collected were

from high school and college students, in both written and multiple-choice formats, on pretests and midterms/exams.

We looked at students' responses in the kinematics graphs and speed calculation tasks. The student difficulties related to the kinematics graph task were drawing kinks on a position-time graph – implying that the speed of an object to be multi-valued at an instant, and unable to represent uniform motion accurately on the position-time and speed-time graphs. The kinematics graph task gave rise to the time-interval ranking task. Students sometimes thought that the speed of the cart when it was speeding up from rest along the incline was faster than the speed of the cart along the horizontal track. Some students assumed that traveling equal distances implies the same traveling time. On the speed calculation task, students had difficulties distinguishing average speed and instantaneous speed. They tended to use the average speed of the prior motion as the instantaneous speed of the endpoint without checking if that motion is uniform.

We collected data from students that can be represented as the various stages of physics learning. That is, we treated high school and algebra-based students as beginners and calculus-based students as novice learners. On the time interval ranking task, we saw a progression from 40% to around 70% in the calculus-based course. For the speed calculation task, we also saw some progression, and the impact of lecture instruction on student performance was higher in the calculus-based course than the algebra-based course.

Chapter 5. IN-DEPTH INVESTIGATION OF STUDENT UNDERSTANDING OF SPEEDS AND RELEVANT IDEAS

This chapter describes the in-depth investigations we conducted in the student understanding of average speed and instantaneous speed. The first study involved the use of isomorphic problems to look at how student understanding of speeds progresses through the introductory mechanics course. We were interested to know if students' ability to solve such problems improves over time within a single course. The goal of this study is to get a better picture of the persistence of students' errors so that we can find better methods to improve student learning.

The second study looks at the students' ability to find speeds when there is a small change in the context. The previous contexts involved objects starting from rest, speeding up to maximum speed, and moving uniformly after that. The context was changed into objects starting with an initial speed, moving with constant speed over some distance, then slowing down until they reach the turnaround point. The difference in the student performance in finding speeds between these two setups would suggest some context-dependence.

The third study was about the students' ability to perform consistency checks on their answers on online pretests. One reason for doing so comes from the elicit-confront-resolve method that is used in the design of our tutorial curriculum. We were curious if the same method could be used online and what its effectiveness would be. Lastly, we described one small modification we made to the tutorial homework to address student difficulties in average speed and instantaneous speed. Its effectiveness in addressing the difficulties was assessed.

5.1 INVESTIGATING STUDENT UNDERSTANDING OF SPEED THROUGH ISOMORPHIC PROBLEMS

In the previous chapter, we found that student performance on the CR context improves from pretest to the final exam. We attributed this improvement to instruction. However, the results did not come from the same group of students. A better assessment would be tracking how individual student responses change using a set of isomorphic problems in one quarter. So, we conducted an in-depth investigation with PHYS 121 students in the Sp'16 quarter.

We used isomorphic problems to track students' performance through the course. According to Singh, isomorphic problems are problems that can be solved with the same physics concepts [61, 62]. The problem descriptions do not need to have the same template necessarily. The reason to use isomorphic problems is that instructors would not think that it is reasonable to test students on the same problem more than once within a quarter. Furthermore, each midterm assesses different chapters, so we designed problems that are of relevant contexts for the test.

5.1.1 The isomorphic problems – description and administration

We administered four isomorphic problems in our study, as shown in Table 5.12. The four problems were of different contexts: cart-on-ramp (CR), hand-on-block (HB), spring-block system (SB), and pulley-block system (PB). In each context, an object started from rest and sped up uniformly or non-uniformly, then it continued with uniform motion. We standardized the problem description, question of interest, and the answer options for each context so that the surface features of the problems and physics concepts involved were the same. All isomorphic questions asked for speeds except for the SB context. Students were asked to calculate kinetic energies in the SB context, which were dependent on instantaneous speeds.

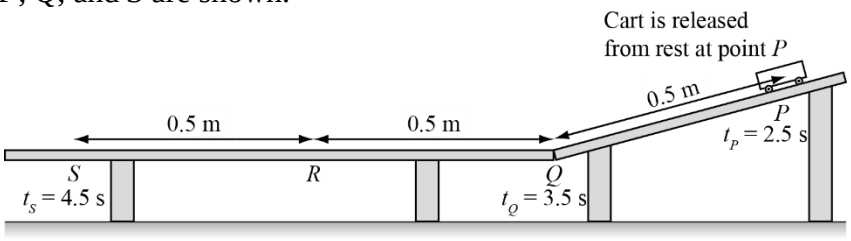
In each context, the answer options mirrored the common approaches students used in the multiple-choice version described in the previous chapter (see section 4.1.2). In the preliminary investigation, we found that students tended to use the average speed of the prior motion as the instantaneous speed of the endpoint without checking if that motion is uniform. For example, in the CR context, all incorrect options have $v_Q = 0.50\ m/s$. Students who answered incorrectly used the average speed of the cart between points P and Q as the instantaneous speed of the cart at point Q. This idea is incorrect because the average speed is equal to the instantaneous speed only when the object is in uniform motion, but the cart is accelerating from P to Q. This tendency was reflected in all options, but some options had additional errors. The errors in each option are summarized in Table 5.13.

We can look at students' performance across all four isomorphic problems by their correctness and approaches. That allows us to track students' progress throughout the quarter and the frequency with which they use incorrect approaches.

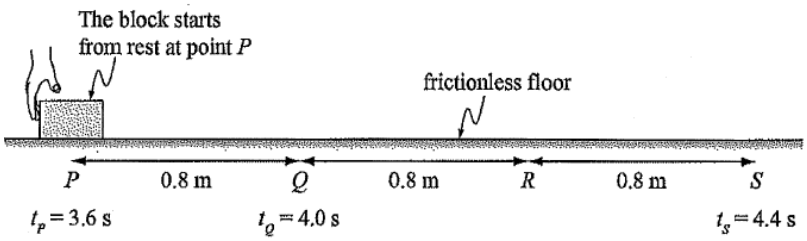
The problems were administered at different times of the quarter. The students did the CR context in the first pretest at the end of week 1, the HB context in midterm 1 in week 3, the SB context in midterm 2 in week 6, and the PB context in midterm 3 in week 9.

Table 5.12. Isomorphic problems on average speed and instantaneous speed.

<table>
<tr><th>Context</th><th>Problem Description</th></tr>
<tr><td>Task 1
(T1)
Pretest

Cart-on-ramp
(CR)</td><td>
A cart is released from rest, speeds up as it moves down an incline, and then moves with constant speed on the level segment of track as shown in the diagram below. Four points are labeled on the track. Point R is located halfway along the level track segment. The times when the cart is located at points P, Q, and S are shown.


Which of the following best shows the speeds at points P, Q, R, and S (in m/s)?
<table>
<tr><th></th><th>v_P</th><th>v_Q</th><th>v_R</th><th>v_S</th></tr>
<tr><td>A</td><td>0</td><td>1.0</td><td>1.0</td><td>1.0</td></tr>
<tr><td>B</td><td>0</td><td>0.50</td><td>1.0</td><td>1.0</td></tr>
<tr><td>C</td><td>0</td><td>0.50</td><td>0.75</td><td>0.75</td></tr>
<tr><td>D</td><td>0</td><td>0.50</td><td>0.67</td><td>0.75</td></tr>
<tr><td>E</td><td>0</td><td>0.50</td><td>0.50</td><td>0.50</td></tr>
</table>
</td></tr>
<tr><td>Task 2
(T2)
Midterm 1

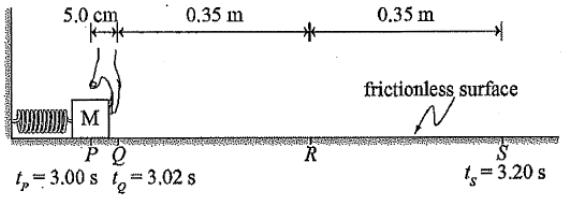
Hand-on-block
(HB)</td><td>
(5 points) A student pushes a wooden block from rest on a frictionless floor at point P as shown below. The block accelerates uniformly as it moves from point P to point Q. The student stops pushing the block when it reaches point Q and it continues to move on the floor. Point R is halfway between points Q and S. The block is located at points P, Q, and S at times 3.6 s, 4.0 s, and 4.4 s, respectively.


Which of the following best shows the speeds at points P, Q, R, and S (in m/s)?
<table>
<tr><th></th><th>v_P</th><th>v_Q</th><th>v_R</th><th>v_S</th></tr>
<tr><td>A)</td><td>0</td><td>2.0</td><td>2.0</td><td>2.0</td></tr>
<tr><td>B)</td><td>0</td><td>2.0</td><td>4.0</td><td>4.0</td></tr>
<tr><td>C)</td><td>0</td><td>4.0</td><td>4.0</td><td>4.0</td></tr>
<tr><td>D)</td><td>0</td><td>2.0</td><td>2.7</td><td>3.0</td></tr>
<tr><td>E)</td><td>0</td><td>2.0</td><td>3.0</td><td>3.0</td></tr>
</table>
</td></tr>
</table>

Task 3 (T3)
Midterm 2

Spring-block system (SB)

(5 points) Block M, with a mass of 1.00 kg, is placed next to a spring on a frictionless surface. It is pushed to the left, compressing the spring by 5.0 cm until the center of the block reaches point P. The block is then released. When the center of the block is at point Q, the block leaves the spring. It moves along the frictionless surface past point S. Point R is halfway between point Q and point S.



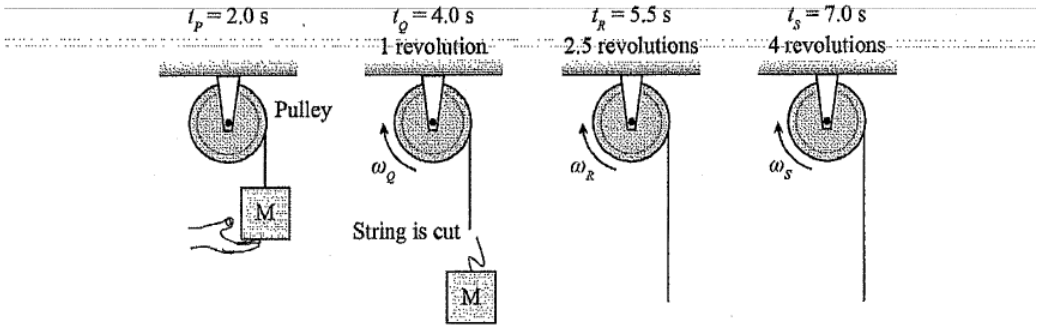
Which of the following best describes the kinetic energies of the block at points P, Q, R, and S (in J)?

	K_P	K_Q	K_R	K_S
A)	0	3.13	3.13	3.13
B)	0	3.13	6.61	7.03
C)	0	3.13	7.03	7.03
D)	0	3.13	7.56	7.56
E)	0	7.56	7.56	7.56

Task 4 (T4)
Midterm 3

Pulley-block system (PB)

(5 points) Block M is attached to a pulley as shown. A timer is started at $t_0 = 0.0$ s. When the timer reads $t_P = 2.0$ s, the hand holding the block is removed. At $t_Q = 4.0$ s, the string is cut when the pulley has made exactly 1 revolution. The pulley continues to rotate at constant angular velocity. It has made exactly 2.5 revolutions at $t_R = 5.5$ s, and has made exactly 4 revolutions at $t_S = 7.0$ s.



Assume there is no friction in the axle of the pulley. Which of the following best describes the angular velocities of the pulley at the 4 instants (in rad/s)?

	ω_P	ω_Q	ω_R	ω_S
A	0	1.0π	1.0π	1.0π
B	0	1.0π	2.0π	2.0π
C	0	2.0π	2.0π	2.0π
D	0	1.0π	1.4π	1.6π
E	0	1.0π	1.6π	1.6π

Table 5.13. Approaches used in the answer options for the speed calculation task (multiple-choice) and their corresponding answers for each isomorphic task.

Approach	Answers for each context			
	T1-CR	**T2-HB**	**T3-SB**	**T4-PB**
Correct $v_Q = v_R = v_S = v_{QS}$	A	C	E	C
Speed Misconception 1 (Error 1) $v_Q = v_R = v_S = v_{PQ}$	E	A	A	A
Speed Misconception 2 (Error 2) $v_Q = v_{PQ}$, $v_R = v_S = v_{QS}$	B	B	D	B
Speed Misconception (Others) $v_Q = v_{PQ}$, $v_R = v_{PR}$, $v_S = v_{PS}$ $v_Q = v_{PQ}$, $v_R = v_S = v_{PS}$	CD	DE	BC	DE

5.1.2 Results and discussions

The results of the study are presented in Figure 5.1. The percentage correct starts at 23% for the CR context in the pretest, increases to 58% for the HB context in midterm 1, then decreases to 46% for the SB context in midterm 2, further decreases to 34% for the PB context. Only 8% of the class answered everything correctly. If we consider students' performance in midterms only, then 16% of the class answered Tasks 2, 3, and 4 correctly.

Around 20% of the class did not answer any of the tasks correctly. All the incorrect options contain $v_Q = v_{PQ}$. This finding means that by the end of PHYS 121, these students thought that the instantaneous speed at the end of the motion (point Q) is the average speed of the prior time interval (between points P and Q.)

Figure 5.1 also shows the proportion of students moving from either correct or incorrect option on one task to another on the next task. Some students moved from incorrect to correct and stay correct on the subsequent tasks. From T1 to T2, more students moved from incorrect to correct than the opposite direction. One possible reason for the difference is that students had

more time thinking about the relevant ideas for the midterm but not for the pretest. Students did the pretest immediately after instruction while the midterm was two weeks later.

The percentage correct decreases by 12% in each midterm from midterms 1 to 3 can be explained by the difference in the contexts, student retention, and lecture instruction. From T2 to T3, more students move from correct to incorrect than from incorrect to correct. The same pattern is also found from T3 to T4. We interpret this as some indication of retention of the correct interpretation of average and instantaneous speeds. On midterms 2 and 3, the content tested were energy, momentum, and rotational motion. Most instructors would have assumed that students have understood the difference between average and instantaneous speeds. They wanted to focus on teaching the new content, so they would not devote time to the difference between average and instantaneous speeds as much as when they taught linear kinematics.

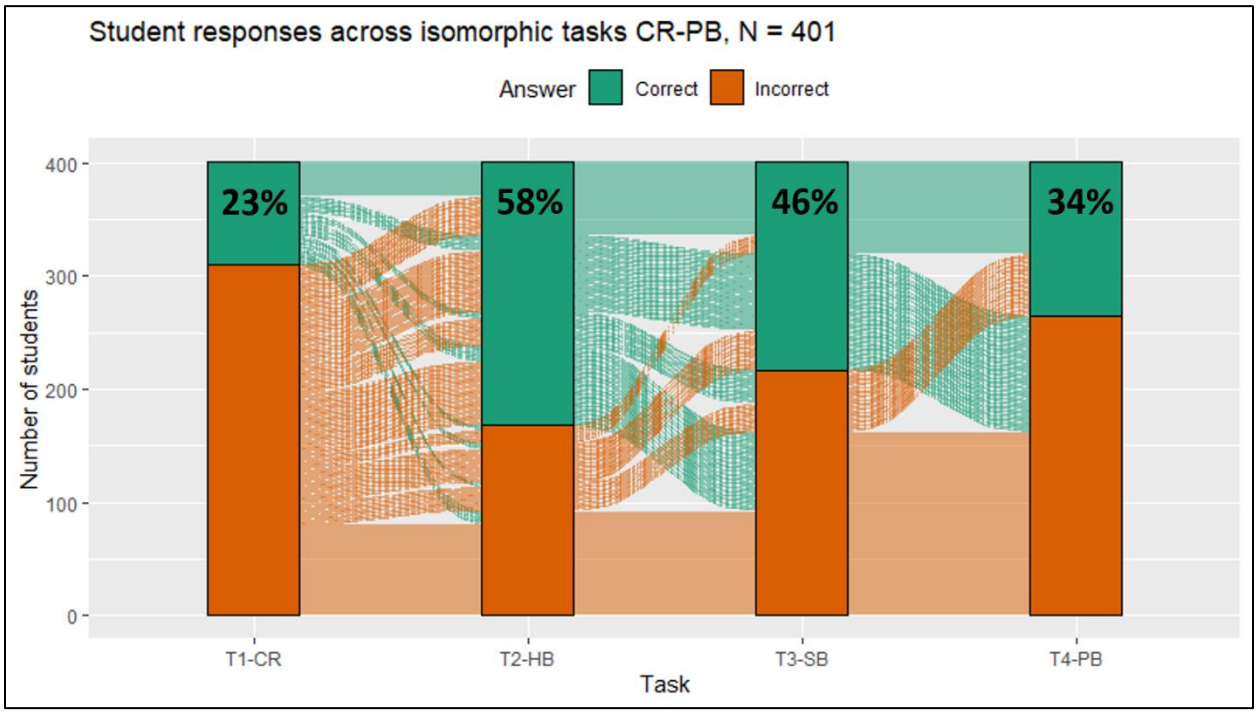


Figure 5.1. Student responses across all the speed isomorphic tasks. The contexts for the isomorphic tasks are described in Table 5.12.

The SB and PB contexts may be more difficult for students. In the SB context, the decompression of spring causes the block to move with non-uniform acceleration. Students may try to calculate the speed of the block at point Q (end of accelerated motion) using kinematics equation involving constant acceleration, but unable to do so because they are not given the spring constant. If the spring constant is provided, more students may get the correct answer, as they can solve the problem using conservation of energy instead of the kinematics approach.

The PB context is about rotational kinematics. Some students may not be familiar with such contexts as typical rotational kinematics questions tend to use turntables instead of pulleys. The turntable questions tend to involve accelerated motion only, so students can apply the relevant kinematics equations to find the answer. The motions involved in the PB context are motion with constant angular acceleration followed by uniform angular velocity. Another possible reason for the lower performance is the minimal transfer of the understanding of the average and instantaneous speeds from linear kinematics to the rotational kinematics.

We present the detailed breakdown of student errors on all the speed isomorphic tasks in Figure 5.2. It displays everyone's responses. Students make the same kind of error on all tasks consistently or flip-flop between correct and incorrect approaches. This result continues to suggest that students have difficulties in distinguishing average and instantaneous speeds as they progress through the quarter, and some of them do not have a consistent approach in calculating instantaneous speeds. It is possible that these students are doing plug-and-chug but do not have a method to check the validity of their answers.

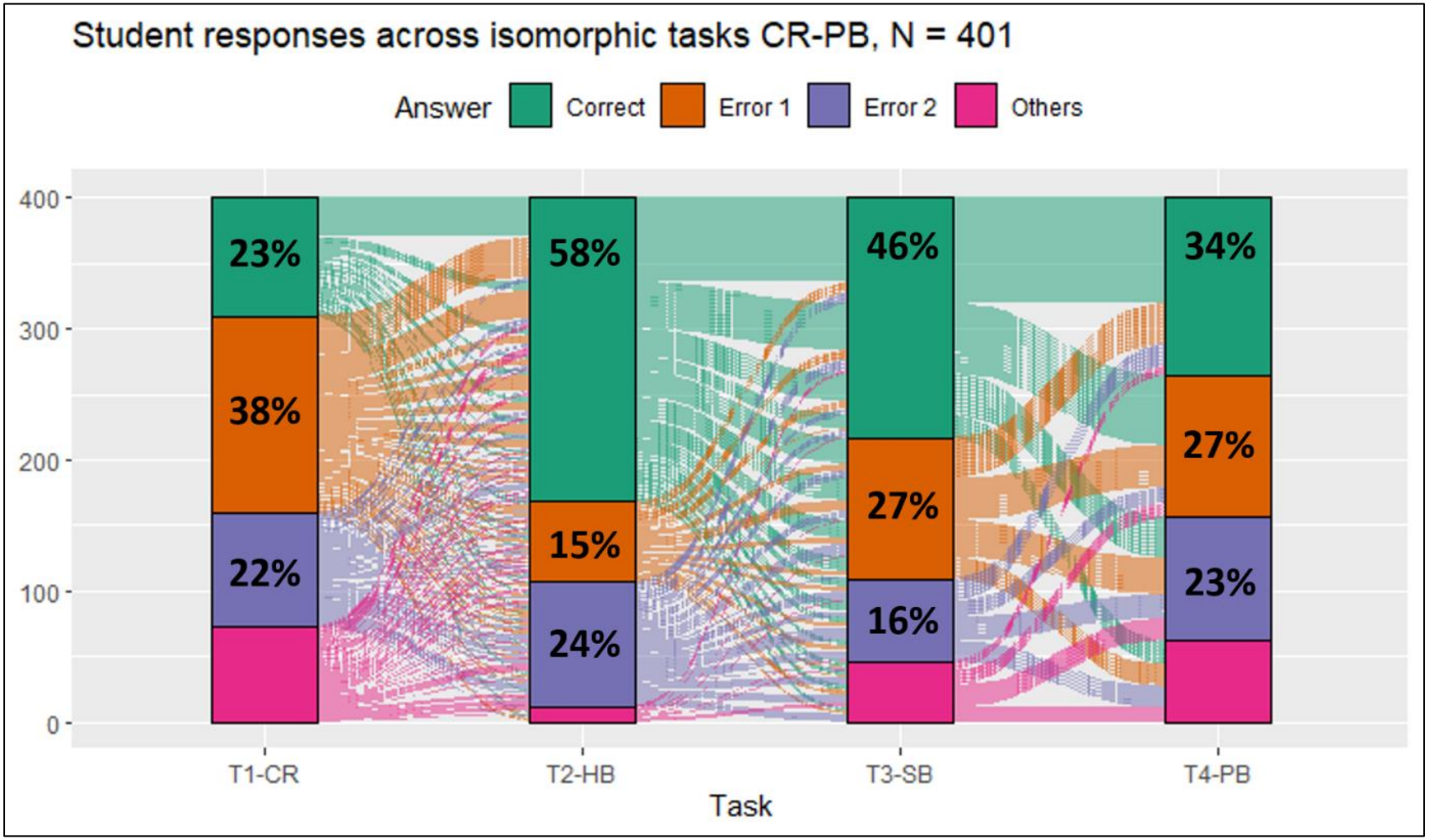


Figure 5.2. Students' responses across all the speed isomorphic tasks in greater detail.

More students chose the Error 1 option ($v_Q = v_R = v_S = v_{PQ}$) than the Error 2 option ($v_Q = v_{PQ}$, $v_R = v_S = v_{QS}$) on the CR, SB, and PB contexts. The HB context was different from the rest, where 9% more students chose the Error 2 option than the Error 1 option. One possible reason is that the concepts tested in the HB context were closely related to the topics covered in the first midterm.

Another way to interpret the result is to find out if students choose numerical answers that match the physical situation. That is, the speeds/kinetic energies of points Q, R, and S are constant in all four contexts. So, the relevant options are Correct and Error 1. Based on this interpretation, at least 60% of the students (CR & PB – 61%, HB & SB – 73%) chose the correct numerical representation in each context.

5.1.3 Commentary

Four isomorphic tasks were used to probe student understanding of average and instantaneous speeds through PHYS 121. The performance increased from pretest to midterm 1 at first, then decreased in each subsequent midterm steadily. Students who answered incorrectly tended to use the average speed of the prior motion as the instantaneous speed of the endpoint without checking if that motion is uniform. Around 20% of the students did not answer any tasks correctly, indicating that the difficulty persisted until the end of the course. We also presented student errors on each task in greater detail. Many students did not have a consistent approach to solve such problems correctly. Lastly, based on the student responses in each task, at least 60% of the students recognized that the speeds/kinetic energies of points Q, R, and S were constant.

5.2 ADDITIONAL INVESTIGATION INVOLVING SPRING-BLOCK & CART-ON-RAMP SYSTEMS

This section describes the additional investigation we conducted involving the spring-block (SB) and cart-on-ramp (CR) contexts. In previous investigations, students tend to use the average speed of the prior motion as the instantaneous speed of the endpoint, not checking if that motion is uniform. That is, students find v_Q by calculating the average speed between PQ when the object accelerates from P to Q (see Table 4.6). We want to find out if the results turn out differently if we reverse the motion of the block in the SB and CR contexts.

We first describe the modifications to the SB context and discuss the results. Then, we extend this method to the CR context. The results and implications of this work are discussed.

5.2.1 Modifications and results for the SB context

In the original SB context as described in Table 5.12, the block is released from rest after the spring is compressed at P, then detaches from the spring at the equilibrium position (point Q) and travels at a constant speed from points Q to S. In the modified SB context, the motion of the block is reversed, as shown in Figure 5.3. The block moves from P to R with uniform speed, then is slowed down by the spring at R until it reaches zero speed momentarily at point S. In order to find out if surface features of the question would affect student performance, we prepare two different versions: (a) the block at point S (turnaround point) and (b) the block with an initial speed at point P (starting point).

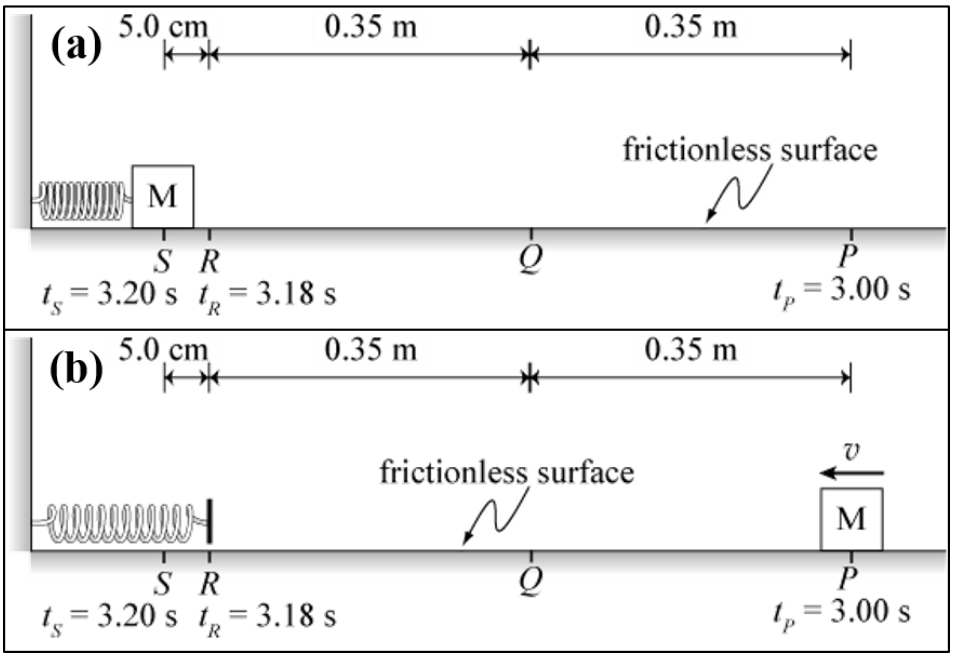


Figure 5.3. The modifications on the SB context – (a) block ends at point S, (b) block starts with speed v.

Which of the following options best represents the kinetic energies of the block at points *P*, *Q*, *R*, and *S* (in J)?

	K_P	K_Q	K_R	K_S
A	3.1	3.1	3.1	0
B	7.0	6.6	3.1	0
C	7.0	7.0	3.1	0
D	7.6	7.6	3.1	0
E	7.6	7.6	7.6	0

Figure 5.4. Options for the modified SB context on pretests.

We also modified the answer choices to reflect the possible approaches students would use, although we did not solicit written answers. The answer choices are shown in Figure 5.4, with option E being the correct answer. All the incorrect options have the same K_R, the kinetic energy of the block at point R, which depends on the speed of the block at point R, v_R. The value of v_R in the incorrect options is calculated using the distance and time interval from R to S. The other kinetic energies are calculated using the average speeds described in section 4.1.2.

We administered the original SB context, modified version (a), and modified version (b) to three groups of PHYS 121 students through online pretests in the Wi'18 quarter. The student performance on these three versions is summarized in Table 5.14. Students who chose option A are categorized under Error 1, and option D under Error 2, according to what we have presented in Table 5.13. The categorization of errors between the original and modified versions are based on the student approaches and pattern of answers produced.

Only 36% of the students who received the original version chose the correct answer, while 55-60% of the students did so for the modified version. We performed 2×4 χ^2 test (two versions, 4 approaches) to compare the student performances between the two different SB contexts. The difference between the two modified versions ($\chi^2\ 3\ = 2.64$, *p*-value = 0.45) was not statistically significant, which meant that the position of the block on the diagram did not affect student performance. However, the differences between the original and aggregated results for the modified versions ($\chi^2\ 3\ = 42$, *p*-value < 0.1%) are statistically significant. The difference in the percentage correct between the original and modified versions is around 20%, which suggests that reversing the direction of motion of the block has a medium effect on the results.

Table 5.14. Student performance on the original and modified SB context on pretests.

Approach (Answer)	Original (*N* = 140)	Modified version A Block at point S (*N* = 153)	Modified version B Block at point P (*N* = 129)	Aggregate (Modified version) (*N* = 282)
Correct (E)	36%	60%	55%	58%
Error 1 (A)	35%	9%	12%	10%
Error 2 (D)	12%	15%	21%	18%
Others (B,C)	17%	16%	12%	14%

The top two common errors in the original and modified versions are Error 1 and Error 2, respectively. The proportion of students choosing option A (Error 1) in the original version is about 25% greater than the modified versions. This difference is quite significant, and here is the reason. Point Q in the original version and point R in the modified version are points at which the block transitions from uniform motion to non-uniform motion or vice versa. Students who did $v_Q = v_{PQ}$ (option A) in the original version and $v_R = v_{QR}$ in the modified version used the same approach in determining the speeds of points of interest. In both cases, students were using the information about the prior motion to calculate the speed at the endpoint. These students would answer the original version incorrectly but not so in the modified version using the same approach.

In summary, reversing the motion of the block in the SB context has a medium effect on student performance. The same effect may appear in other contexts. So, we repeat this study with the CR context in another quarter.

5.2.2 Results from the CR context

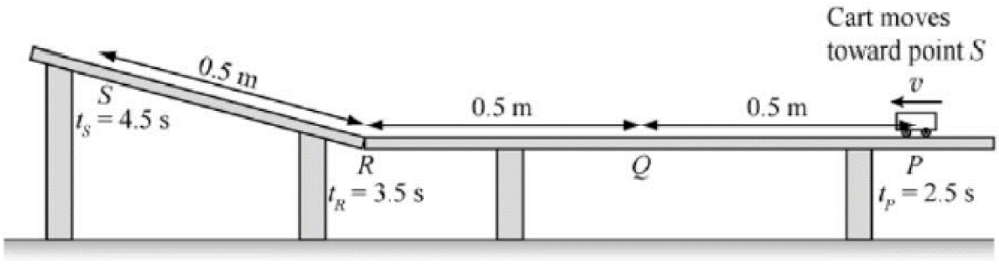
The modified CR context is shown in Figure 5.5. The answer options are prepared in the same manner as the SB context. Three versions of pretests were created. In version 1, students did the original CR context first, followed by the modified CR context. We reversed the order in version 2. In the third version, students did a free response question to give the speeds for the modified CR context. The options in versions 1 and 2 of the modified CR context are categorized in the same manner as the modified SB context (see Table 5.14).

The purpose of version 1 is to find out how students' answer changes from the original to the modified versions. For version 2, we can find out the effect of the modified version on the student performance of the original version. Lastly, we can use the written responses from version

3 to verify our options on the multiple-choice version and change some of them for future work if necessary.

We administered these three versions to three groups of PHYS 121 students in the Wi'19 quarter. There was a difference in terms of how we collect the data in the modified SB context and the modified CR context. In the modified SB context, we were comparing results between groups of students. In the modified CR context, we were comparing results within the same group of students in versions 1 and 2.

A cart, with an initial velocity, moves across a frictionless, horizontal segment of track, and then moves up the incline and turns around at point *S* as shown in the diagram below. Four points (*P*, *Q*, *R*, and *S*) are labeled on the track. Point *Q* is located halfway along the level track segment.



Question 1.

Which of the following options best represents the instantaneous speeds at points *P*, *Q*, *R*, and *S* (in m/s)?

	v_P	v_Q	v_R	v_S
A	0.50	0.50	0.50	0
B	0.75	0.67	0.50	0
C	0.75	0.75	0.50	0
D	1.0	1.0	0.50	0
E	1.0	1.0	1.0	0

Figure 5.5. The modified CR context on a pretest.

The results for versions 1 and 2 are shown in Figure 5.6. In version 1, 37% of the students chose the correct answer in the original version, and 61% did so in the modified CR context. From Figure 5.6a, 34% of students who selected the correct answer in the original version also answered the modified version correctly. The other 24% mostly came from students who selected the options Error 1 and Error 2 in the original version. This increase in percentage correct from the original CR context to modified CR context is also consistent with what we found in the previous context. From Table 5.14, the increase in percentage correct from the original SB context to the modified SB context ranges from 19%-24%.

In version 2 of the pretest (Figure 5.6b), 42% of the students answered both the modified and original CR contexts correctly. The percentage correct for the original context went up to 52% after students did the modified context, as compared 37% correct without seeing the modified context. Combining the results from versions 1 and 2, we found that around 35%-40% of the students were using the correct method to get the answer.

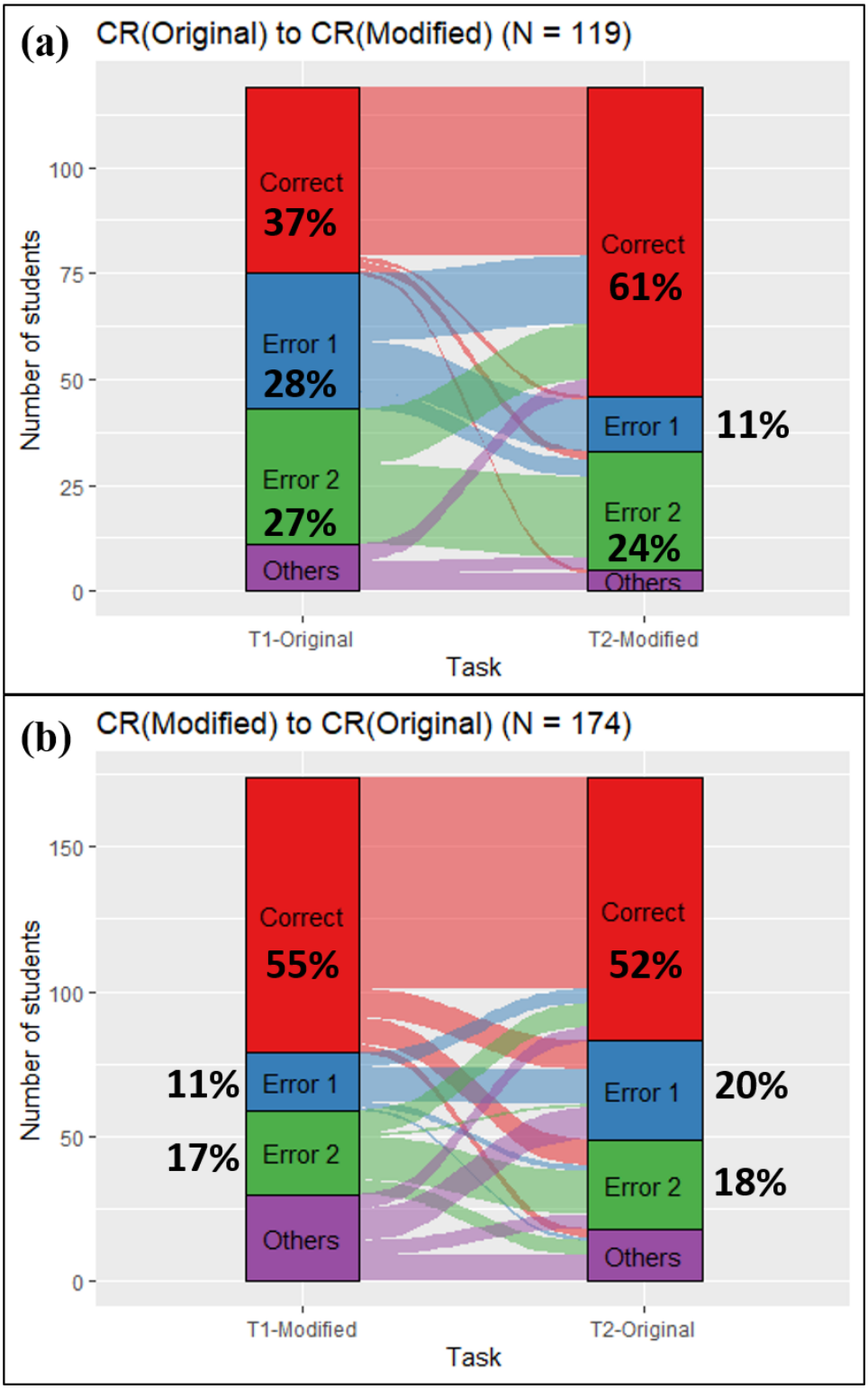


Figure 5.6. Results involving the original and modified CR contexts on pretests – (a) version 1 and (b) version 2.

Next, we examine the student explanations in both the original and modified contexts when they went from wrong to correct or vice versa in both versions of the pretest. Selected student responses for versions 1 and 2 are tabulated in Table 5.15 and Table 5.16, respectively. We select those student responses that explicitly describe how they arrive at their answers in both contexts. All the student responses highlight here appear to have a common approach, which is to use the information about the motion in the previous time interval to determine the speed at the final instant. For example in version 2, one student wrote, "*During the time t=2.5 s to t=3.5 s, the cart moves 1.0 meters down the horizontal track (1 m/s)*" for the modified CR context and "*Then, it takes one second (t=2.5 s to t=3.5 s) to travel 0.5 m, meaning that the velocity at Q is 0.5*" for the original CR context. The student answered the modified CR context correctly but not for the original context. There are some exceptions. For example, in version 2, another student wrote, "*The distance over time from R to S is .5 and the time is 1 sec, so the velocity [at R] is .5 m/s.*" In this case, the student uses the motion in the latter time interval to determine the initial speed. The responses show that some students are consistent in their methods of calculating speeds, but they do not always check the validity of their methods.

In version 3, we asked students to write down their answers in a text box. The results are tabulated in Table 5.17. About 45% of the students wrote the correct speeds. The difference in the percentage correct between the multiple-choice and written version is around 10-15%, which is a small effect. More than half of the students who wrote the incorrect speeds (37% of the class) found the speed at the turnaround point (v_S) to be 0.5 m/s. Lastly, 19% of the class wrote the initial speed of the cart to be 0 m/s.

Table 5.15. Selected student explanations on the original and modified CR contexts on pretest version 1. The student responses in each row are from the same student.

T1 approach ↓ T2 approach	T1 - Original CR context	T2 - Modified CR context
Error 1 ↓ Correct	*It takes one second for the cart to travel from P to Q, which is a distance of 0.5m. The speed at Q must be 0.5m/s. Then, because the surface is frictionless and there are no other forces acting on the cart, the speed remains constant for points R and S.*	*The cart moves from point P to R in 1 s and travels 1 m. Thus, the speed for those points must be a constant 1m/s because there are no other forces acting on the cart. The cart will not begin to slow down until it reaches the incline. Thus, the instantaneous speed and the beginning of the incline, point R, but be the same as before: 1m/s.*
	The car will increase in velocity as it goes down the track. v = dx/dt, so the velocity is .5/1 at point Q. Since the car is then on a level track, it will not speed up.	*It took 1 s for the cart to traverse from point P to point R, which is 1 m. Since v = dx/dt, the velocity at point R is 1.0. That means, since the cart had no acceleration that the velocity at points P and Q are the same as point R.*
Error 2 ↓ Correct	*The cart covers 0.5 m in 1 sec at point Q. The cart then covers 1 m in 1 sec for point R and point S.*	*The cart covers 1 m in 1 sec for point Q and point R. The cart starts at point P moving at 1 m/s.*
	The cart is released at P, so the speed is 0 m/s at P. From P to Q, the distance is 0.5m, it takes 1 second, therefore, the speed is 0.5 m/s. From Q to R is 0.5m, and it takes 0.5s, so the speed is 1 m/s. From R to S is also 0.5m, and it takes 0.5s, so the speed is 1 m/s too.	*Because it is a frictionless, horizontal track, the cart has same instantaneous speed at P, Q and R. The speed at Q is 1 m/s, therefore, the speed of P, Q and R are all 1 m/s. For the point S, because the cart turns around at S, therefore, the speed at the point S is 0 m/s.*

Table 5.16. Selected student explanations on the original and modified CR contexts on pretest version 2. The student responses in each row are from the same student.

T1 approach ↓ T2 approach	T1 - Modified CR context	T2 - Original CR context
Error 1 ↓ Correct	*While the cart is on the level part of the frictionless track, it has zero acceleration in the x-direction. Therefore, the velocities at points Q, R, and S are equal. However, moving up the incline, its velocity decreases to zero. The distance over time from R to S is .5 and the time is 1 sec, so the velocity is .5 m/s.*	*The cart starts at rest from P.* *The cart travels from Q to S (1 m) in 1 s so the velocity is 1 m/s.* *Because there is no x-direction acceleration on the level part, the velocities of Q, R, and S are the same.*
Error 2 ↓ Correct	*Instantaneous velocity is delta x/delta t. For P, delta x = 1 and delta t = 1, then Vp = 1. The same goes for Vq, while Vr is 0.5/1, and at S because it is turning around and not moving, it is 0.*	*At point P, the car is not moving so its velocity is 0. At point Q, R and S, the car is said to be moving with constant speed, so they all must be the same. Between points Q and S, the car covers 1 m in 1 s, so the velocity is 1.*
Correct ↓ Error 1	*We know that the velocities at point R, Q, and S are the same because there is no friction. It is 0 at point S because that is when it turns around. We know that it is 1 m/s because between point P and R is 1m and it took 1s.*	*At P velocity is 0 because it is at rest. It accelerates until it reaches point Q then travels at a constant speed through R and S - so those values are the same. We know that it is 0.5 m/s because it travels 0.5 meters in 1 second.*
Correct ↓ Error 2	*During the time t=2.5 s to t=3.5 s, the cart moves 1.0 meters down the horizontal track (1 m/s). There is no friction on the track, so the velocity remains constant until point R, at which the cart enters the incline. At point S, the cart turns around, meaning the velocity is 0.*	*The cart is released from rest, meaning that the velocity at point P is 0. Then, it takes one second (t=2.5 s to t=3.5 s) to travel 0.5 m, meaning that the velocity at Q is 0.5. From Q to S, the cart travels 1 meter in 1 second meaning that the velocity at both R and Q is 1.0 m/s.*

The students' written answers differ a lot from our answer options. The distractors on the multiple-choice versions do not reflect the written options accurately. However, the students' responses for the incorrect answers continued to show the use of the average speed of the former motion as the speed at the final instant. The fact that some students said that v_P is at rest also supports this finding.

Table 5.17. Students' responses for version 3 of the modified CR context.

Speed answers (v_P, v_Q, v_R, and v_S)	Percentage (N = 124)	Selected student explanations
Correct - (1, 1, 1, 0) m/s	45%	*The cart moves uniformly from P to R, so the velocity at those points are the same, which is equal to the average velocity on PR: (0.5+0.5)/(3.5-2.5)=1 (m/s) It turns around at S so the velocity at S must be 0.*
(1, 1, 1, 0.5) m/s	20%	*I used the velocity function, V= d/t , and knew from the picture it took 0.5 seconds to go 0.5m for all P, Q, and R points. S was similar but the cart was going 0.5m in 1.0 second.*
(0, 1, 1, 0.5) m/s	10%	*instantaneous speeds are the speed at the time given. At P, it starts moving which means it does not have distance moved. Thus it is 0. At Q, it has moved 0.5m for about 0.5s; it is 1m/s. So does at R. However, at S, it has moved for 0.5m from R to S for 1s, which gives 0.5m/s for its instantaneous speed.*
Others (24 combinations)	24%	-

5.2.3 Implications of this study

We make several observations based on the results of this study. They are (1) the student performance in calculating speeds at different locations depends on whether the non-uniform motion is at the start or the end of the whole motion, (2) the student exposure on the modified context has some effect on the performance of the original context, and (3) the students use the average speed of the prior motion as the speed at the final instant. Sometimes students can get the

correct answer by using the incorrect approach. The third observation somewhat explains the first two observations.

The third observation suggests that some students do not examine their approach to calculating speeds carefully. Previous studies reported that students do not distinguish average and instantaneous speeds. This study shows that they tend to use the average speed of the previous time interval to calculate the speed at the final instant without checking whether the motion in that time interval is uniform. In some cases, students may say that the initial speed of the object is zero when it is stated in the question that the object has an initial speed. We think the reason is that the object is not shown to have moved some distance into the given diagram.

On the other hand, it is perhaps possible to use the original and modified tasks to find out students' physics intuition. The reversibility in the motions of the objects involved in both the CR and SB contexts can be used for checking answers if used appropriately.

5.3 STUDENTS' ABILITY TO PERFORM CONSISTENCY CHECK IN ONLINE PRETESTS

In this section, we describe our work to find out students' ability to perform consistency checks on online pretests. The motivation is to have data on how well students can perform consistency checks individually. This data can be helpful in justifying the group activities we do in the tutorial. In our curriculum, we often ask students to resolve their inconsistencies as part of the *elicit-confront-resolve* (ECR) pedagogy [36]. In the ECR approach, students make predictions or a choice between different statements in a specific situation. Next, students confront another related situation that challenges their beliefs and answers, usually through experiments or demonstrations. If students make incorrect predictions or choices, then they realize they need to learn the relevant concepts properly [64]. The UWPEG believes that students are then more willing to resolve in a group work setting with some help from the tutorial instructors.

This work is an extension of the tasks of the CR context on the written pretest in the preliminary investigation. We are interested in finding out the effect of students' speed-time graphs on their speed calculations and whether they can detect and resolve inconsistencies in their answers. We first describe the method we used to conduct this investigation and report the preliminary results. We then present additional results after some modifications. Since this study is conducted online, there is some difference as compared to when the ECR approach is used in in-class curriculum.

5.3.1 Methodology

We used the CR context in this study. The schematic of how the ECR approach is set up is shown in Figure 5.7. In the elicit phase, we ask students to select the speed-time graphs first (see Figure 5.8), followed by the speed calculation task (CR context), as shown in Table 5.12. When students are asked to calculate the speeds, we present their speed-time graph to see if students choose numerical options that represent the situation correctly.

We set up the confront and resolve phase on the next page of the pretest. The students' answers from the speed-time graph and speed calculation tasks are placed side-by-side, with another graph using their calculated speeds (see Figure 5.9). The purpose is to let students compare their chosen speed-time graph and their calculated speeds visually. We ask students to identify and explain whether their graph and speed calculations are consistent with each other.

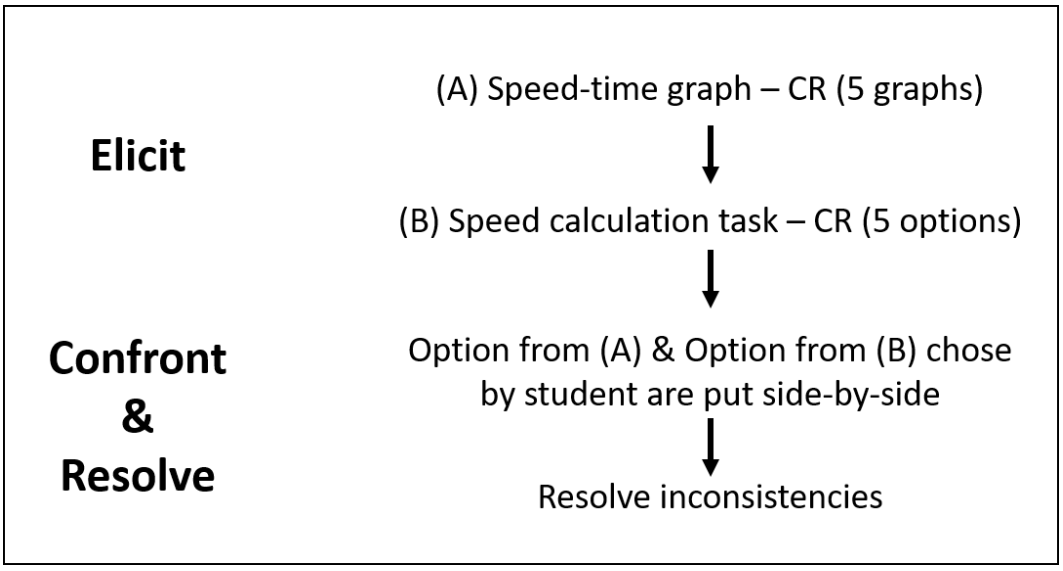
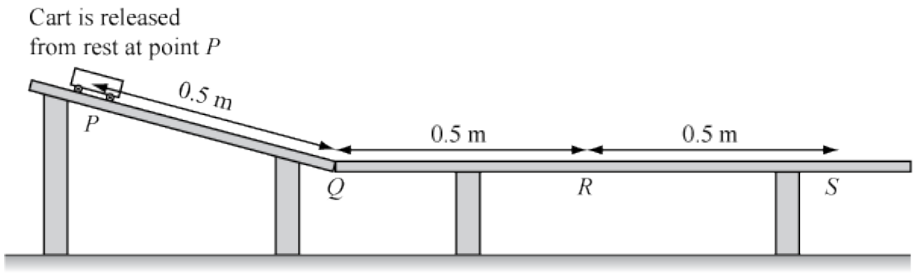


Figure 5.7. Schematic diagram of the study to investigate students' ability to perform a consistency check on online pretests.

A cart is released from rest, speeds up as it moves down an incline, and then moves with constant speed on the level segment of track as shown in the diagram below. Four points are labeled on the track. Point *R* is located halfway along the level track segment.



Which of the following speed-time graphs best describes the motion of the cart as it travels from point *P* to point *S*?

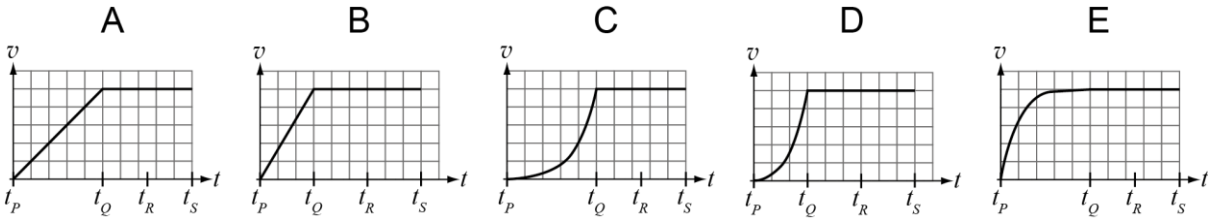
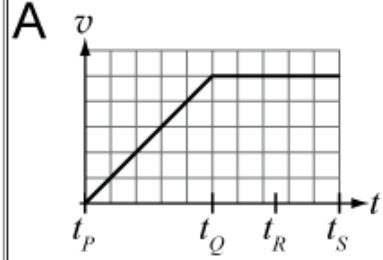
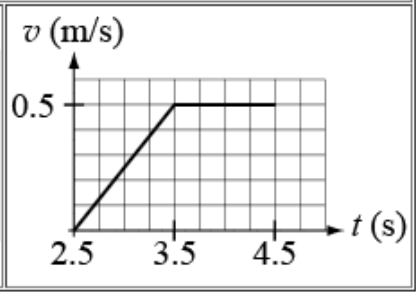


Figure 5.8. The speed-time graph task (elicit stage).

You have selected the following answers on pages 1 and 2:

Page 1	Page 2	
A 	v_P 0 v_Q 0.50 v_R 0.50 v_S 0.50	

Are your answers to page 1 and page 2 consistent? If yes, why? If no, how would you resolve the inconsistencies?

Figure 5.9. Example of the confrontation and resolution stage in the pretest.

5.3.2 Discussion of preliminary results

We administered this version of the pretest to 100 PHYS 121 students in the Wi'16 quarter. The correct speed-time (v-t) graph is option A, in which the areas under the curve between consecutive time intervals match the displacement of the cart. The students' responses to the graphing and speed-calculation tasks are presented in Table 5.18.

Table 5.18. Correlation table between students' speed-time graphs and speed calculations in a pretest in the Wi'17 quarter.

		Speed calculations (N = 100)	
		Correct	Incorrect
Speed-time graph	Correct	26%	13%
	Incorrect	23%	38%

The proportions of students who answered the v-t graph and speed-calculation tasks correctly are 39% and 49%, respectively. About a quarter of the class answered both tasks correctly. The most common incorrect error in the v-t graph task is option B, which is chosen by 40% of the class. Option B describes equal time intervals between each consecutive point, which students explain through equal distances between consecutive points. Another way to look at this result is that 79% of the class choose options A and B, indicating that most students think that the cart is undergoing constant acceleration along the incline.

The percentage correct of the speed-calculation task on pretests without doing the v-t graph was around 25% (see Table 4.5 and Figure 5.2), and 49% after choosing the v-t graph. The 24% difference could be due to the prior attempt of the graphing task and the inclusion of students' choice of the v-t graph in the speed-calculation task. However, this difference could not be

determined from the written pretest, because not all students actively used their sketched graphs for speed calculations on the written version.

Next, we discuss the results from the resolution stage. We focus on three different aspects as we read the students' responses: (1) how students think about the consistency in their answers, (2) whether they have performed their consistency check correctly, and (3) whether students know the kinds of mistakes they made if they have performed their consistency check correctly.

We look at the student responses to the question, "Are your answers to pages 1 and 2 consistent?" to address the first two aspects. Ideally, students who answered both tasks correctly should find their answers to be consistent with each other, and students who answered either task incorrectly should find otherwise. Since most students are novices, we expect to see a mix in the results, which are presented in Table 5.19.

Table 5.19. Results of the resolution stage for the question "Are your answers consistent?".

		"Are your answers consistent?" (N = 100)	
		Yes	**No**
Speed-time graph – Speed calculation	Correct-Correct	21%	5%
	Correct-Incorrect	10%	3%
	Incorrect-Correct	7%	16%
	Incorrect-Incorrect	14%	24%

This finding is interesting – students who chose the correct speed-time graph tended to find their answers to be consistent, while those who chose the incorrect graph tended to find inconsistencies in their answers. Students in the Correct-Incorrect category usually chose the Error 1 option, which is shown in Figure 5.9. These students were focusing on the shapes and time axes of the graphs that were presented to them. Students thought that their answers were consistent when the two axes and shapes matched up nicely. On the other hand, students who chose the

incorrect graphs and indicated that both of their answers were inconsistent discussed the differences in the time axes and the shapes of the graphs. Selected students' responses are tabulated in Table 5.21.

The second aspect is to find out if students did their consistency checks correctly. From Table 5.19, 64% of the class performed a correct consistency check. We consider a correct consistency check if a student who answered both tasks correctly answers "Yes," or if a student who answered either task incorrectly answers "No." More than half the class could perform consistency checks correctly.

The last aspect is the most important of all, which is to see if students can identify their mistakes when their answers are inconsistent. The results are tabulated in Table 5.20. For the students who chose the correct answers (26% of students), most (21% of students) found no mistakes, but the rest found inconsistencies. Next, 19% of the students found inconsistencies in their answers and identified the mistakes. The rest of the students could not identify the mistake or identified the wrong mistake in their answers. We categorized those student responses in Table 5.21 and determined whether the mistakes were identified correctly or not.

Table 5.20. Results from the resolution stage – identification of mistakes.

		Mistakes identified? (N = 100)		
		No mistake	Identified mistake	Could not identify mistake / wrong mistake identified
Speed-time graph – Speed calculation	Correct-Correct	21%	-	5%
	Correct-Incorrect	-	-	13%
	Incorrect-Correct	-	5%	18%
	Incorrect-Incorrect	-	14%	21%

Table 5.21. Results from the resolution stage – selected students' responses.

Speed-time graph – Speed calculation	Selected student responses
Correct-Correct	• *My answers are consistent because the velocity is 0 at the first time value of both graphs. After the second time value, velocity remains constant at 1 m/s for page 2. Since page 1 does not have a scale for velocity, it is only important that this arbitrary value remains constant, which it does.* (Consistent, no mistake) • *They are consistent as the time intervals represented by 2.5, 3.5 and 4.5 (t_P, t_Q, t_S) are equally spaced on both graphs and the shape of the curve is also the same.* (Consistent, no mistake) • *Not consistent. velocity at Q, R, S should be smaller than 1.* (Inconsistent, wrong mistake identified)
Correct-Incorrect	• *Yes they are! Both have the cart starting at speed 0 time 0, speeding up at constant acceleration, and reaching the highest velocity, which it remains at for the same amount of time it took to reach that speed.* (Consistent, could not identify mistake) • *No, I would have chosen a different graph to represent the motion. The slope is greater than I thought it would be and the time intervals were shorter between Q and S.* (Inconsistent, wrong mistake identified)
Incorrect-Correct	• *No, I forgot and need to take the time differences into consideration. Even though the points P, Q, R, and S are evenly spaced distance-wise, they are not spaced by time. The time between P and Q is greater than the rest.* (Inconsistent, mistake identified) • *Yes the answers are consistent. The time intervals are equal, the table has the same values as the graph.* (Consistent, could not identify mistake) • *No, the speed between t_P and t_Q should be a parabola rather than a line.* (Inconsistent, wrong mistake identified)
Incorrect-Incorrect	• *No they are not consistent. I think the inconsistency came about with the confusion between position and speed. The position is changing in the shape of a upward facing parabola but the speed should be linear I think.* (Inconsistent, mistake identified) • *Yes, because the time representations on the first graph match the given values displayed on the second graph. The shape of both graphs matches as well.* (Consistent, could not identify mistake)

All these results suggest that most students can do consistency checks themselves. Most students determine the consistency of their answers by the time axes and shapes of the given graphs. A small proportion of students who find their answers to be inconsistent can identify their mistakes. We extrapolate this result to a classroom setting where most students are sitting in groups of four working through the tutorial curriculum. In many situations, at most, one student in each group can identify the mistakes if their answers are inconsistent. Some may not notice that their inconsistencies at all. This situation is not productive for student learning.

What this means is that the effectiveness of the ECR teaching approach is low if students work through the tutorial curriculum individually. It is essential to encourage students to discuss their answers among themselves actively and seek help from instructors if necessary, during tutorial sessions.

In this preliminary study, we allow students to use any approach to determine whether their answers are consistent. We are interested in finding out if students can use the area under the speed-time graph to determine the consistency of their answers. This approach is most applicable to students who choose options A and E on the speed calculation task (which the speeds of points Q, R, and S are constant.) We hypothesized that if students understand that the area under the speed-time graph is the displacement, then they can compare the areas under consecutive intervals with the distance traveled by the cart and decide whether it matches with the given situation. Besides that, we also inform students that there is inconsistency when both of their answers are incorrect and ask them to figure out the mistakes involved. The results are discussed next.

5.3.3 Results and discussions from further investigation

The purpose of this additional investigation is to find out if students can resolve inconsistencies between their answers using the area-under-graph approach. For students who answered both tasks correctly, we asked them the question, "Are your answers consistent?". Students who picked options A and E on the speed calculation task were asked to determine the consistency of their answers by considering the area under the graph, as shown in Figure 5.10. Students who picked speed options that did not demonstrate $v_Q = v_R = v_S$ were told that their answers are inconsistent, and asked to resolve the inconsistency (see Figure 5.11).

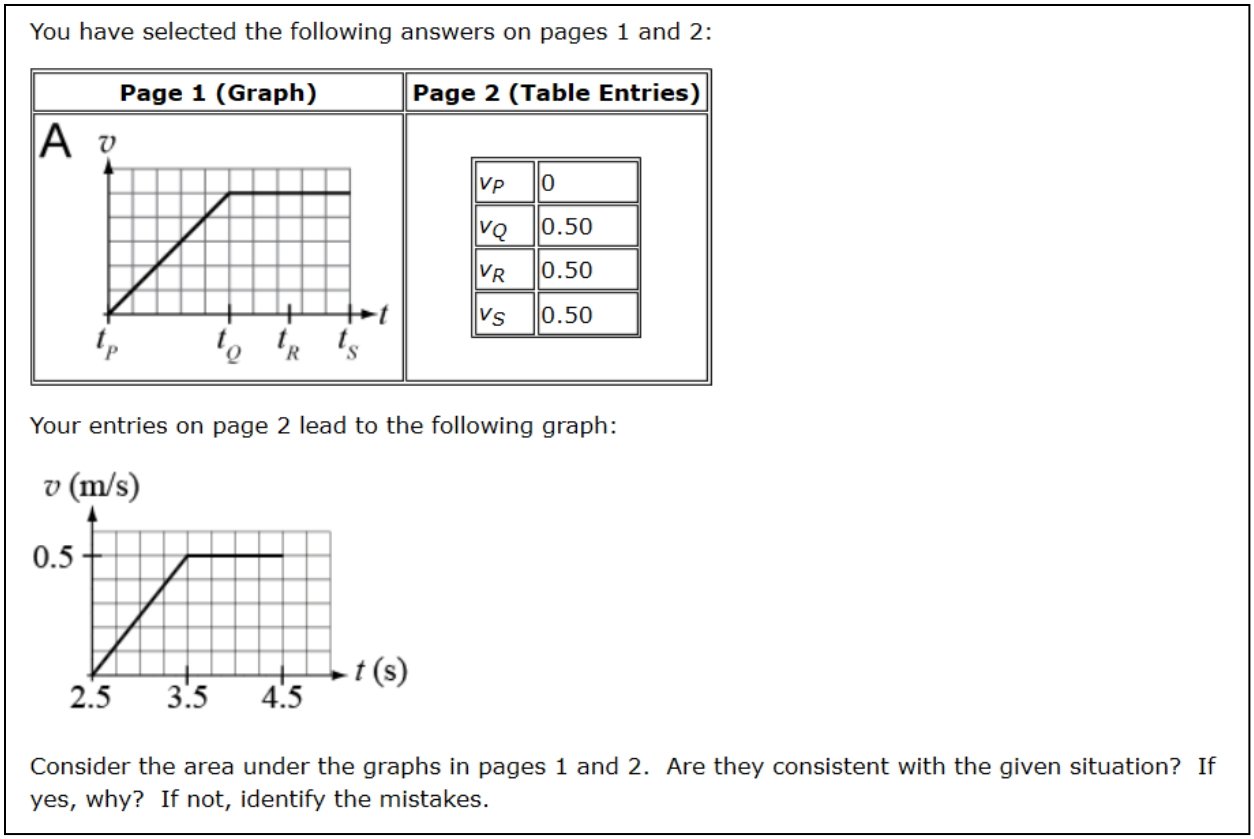


Figure 5.10. Modification of the confrontation and resolution stage by prompting students to consider the area under graphs in determining the consistency of answers.

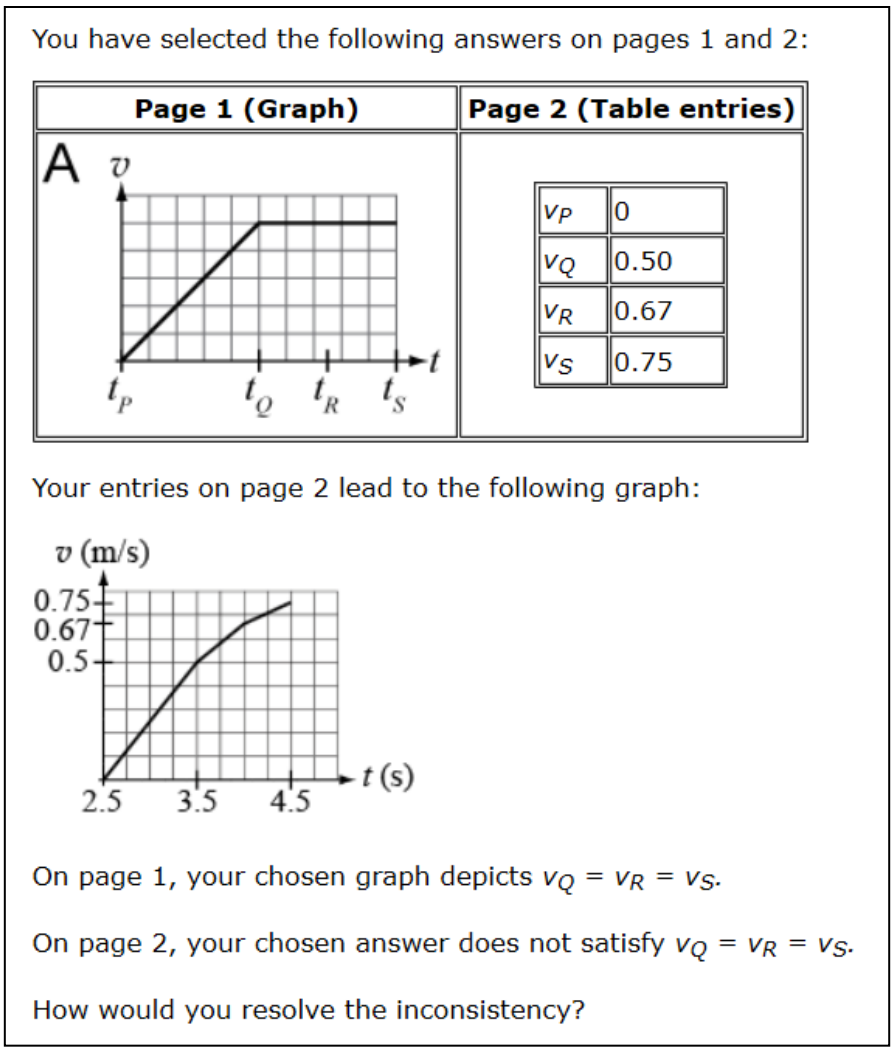


Figure 5.11. Modification of the confrontation and resolution stage in pretest by highlighting the constancy and non-constancy of speeds at points Q, R, and S in both graphs.

Table 5.22. Correlation table between students' speed-time graphs and speed calculations in a pretest in the Sp'17 quarter.

		Speed calculations (N = 180)	
		Correct	Incorrect
Speed-time graph	Correct	24%	11%
	Incorrect	26%	39%

We administered the modified pretest to 180 PHYS 121 students in the Sp'17 quarter. The results are summarized in Table 5.22, and they are about the same as the Wi'17 (Table 5.18) quarter. Graph B is still the most common incorrect option, with 32% of the students choosing it. Compared to the Wi'17 quarter, 40% of the students chose graph B. The results suggest that the students in these two quarters are not much different.

We summarize the students' ability to identify mistakes in the modified pretest in Table 5.23. On the pretest, the percentage of students who chose options A and E (correct answer) on the speed calculation task were 34% and 50%, respectively. Twenty-four percent of the students answered both the v-t graph and speed calculation tasks correctly, and most of them found their answers to be consistent.

Next, we look at the students' ability to identify inconsistencies. Sixty percent of the students were asked to determine the consistency of their answers using the area-under-graph approach. Only 16% of the students could use this approach to recognize the inconsistency in their answers and identify their mistakes.

Lastly, 16% of the students who chose options which v_Q, v_R, and/or v_S are not equal were informed that their answers were inconsistent. Only a few identified their mistakes.

Table 5.23. Results from the resolution stage (modified) – identification of mistakes.

		Mistakes identified? (N = 180)		
		No mistake	Identified mistake	Could not identify mistake / wrong mistake identified
Speed-time graph – Speed calculation	Correct-Correct	22%	-	2%
	Correct-Incorrect	-	4%	7%
	Incorrect-Correct	-	4%	21%
	Incorrect-Incorrect	-	13%	26%

Table 5.24. Results from the resolution stage (modified) – selected students' responses.

Speed-time graph – Speed calculation	Selected student responses
Correct-Correct	• *Yes it is consistent. The car takes a longer time to reach it's constant velocity, and from there it moves at a constant velocity of 1 m/s.* (Consistent, no mistake) • *They are not consistent because at t = 2.5 s, the velocity is not zero. The time interval should be changed so that t = 2.5 s is not at the origin.* (Inconsistent, wrong mistake identified)
Correct-Incorrect	• *Yes they are, as the time passes from 2.5 - 3.5 the velocity increasingly changes. as it continues to increase the area and distance covered is larger over a period of time since v is larger.* (Area - Consistent, could not identify mistake) • *No, they are not consistent. The area under the graph gives me 0.5m when it should give me 1.0m from Q to S. This means that the correct answer on page 2 was letter E, which would give me the appropriate area under the graph.* (Inconsistent, mistake identified)
Incorrect-Correct	• *We know the area under the graph represents the total distance traveled. Since the total distance traveled from Point Q to point P to S is 1.5m and the area under the graph is also 1.5m, our answers should be correct.* (Chose graph B. Area - Consistent, could not identify mistake) • *From P to Q, the area should be 0.5. However, my chosen graph is less than that. I identified the wrong graph.* (Chose graph C. Area - Consistent, mistake identified) • *The graph on page two is not consistent with the given situation because it implies constant acceleration while the acceleration is actually increasing due to gravity.* (Inconsistent, wrong mistake identified)
Incorrect-Incorrect	• *The velocity at Q is actually 1 m/s, as the cart has reached full velocity once it is at the bottom of the incline. Therefore, the velocities for page 2 should be:* v_P*: 0,* v_Q*:1,* v_R*:1,* v_S*:1.* (Non-constancy of speed, mistake identified) • *No the area under the curve is not consistent. The cart actually travels 1.5 m, when the area under my velocity graph shows it travelling only 0.75 m. My error comes by calculating the speed of the cart at the last three points. The speed should be 1 m/s.* (Area - Inconsistent, mistake identified)

Selected students' responses from the modified version are tabulated in Table 5.24. Some students demonstrated that they knew the area under the speed-time graph was the displacement of the cart, and they used this knowledge to justify their answers. We surveyed 107 PHYS 121 students on an earlier pretest and found that 80% of them said that the area under the v-t graph is the displacement, so we expected most students could use the idea of area-under-graph to resolve inconsistencies when instructed. However, only 16% of the students demonstrated that.

Here are some possible reasons for the low result. First, the instruction to ask students to check the consistency of their answers through the area-under-graph approach may be vague to students. Students who understand the approach may not necessarily know how to apply it in this context. Second, some students were not careful with the proportions of areas between regions, especially those who chose graph B and correct speed combinations.

Another possible source of confusion among students is the absence of time labels on the second graph (from speed calculations) when the labels are on the first graph. We prepared another pretest version with the labels t_P, t_Q, and t_S on the second graph. The results were almost identical to those in Table 5.22 and Table 5.23, suggesting that students were still able to interpret the graphs correctly, even though the time labels P, Q, R, and S were not shown in both graphs.

5.3.4 Commentary

Based on our results, there is some positive effect on the performance of the speed-calculation task after students attempted the v-t graph task. The percentage correct is greater by around 20% compared with students who did not attempt the graphing task. Students tend to compare the time axes and the shape of the graphs to decide whether their answers are consistent.

Most students who answered both tasks correctly found their answers to be consistent. Students recognize their answers to be inconsistent at twice the rate when they choose the wrong graphs. However, not all students can identify their mistakes. This finding is not unexpected, because if they were able to identify their mistakes, they would not be choosing those incorrect answers. There is also no difference in identifying mistakes when we suggest the area-under-graph approach. These results suggest that physics educators not only need to teach students the relevant concepts; but also to provide opportunities for students to learn how to identify mistakes and resolve inconsistencies.

There are a few things to note when we interpret these results. First, these data were collected at the beginning of the quarter. Students were asked to check the consistency of their answers for the first time online. Second, students were given 15 minutes to complete the whole pretest; some of them might need more time to process the information to arrive at the correct conclusion. Third, this pretest was administered online, and students were not able to go back and forth to look at their answers, unlike the written version. These factors could contribute to the low proportion of students identifying their mistakes.

That said, it may be relevant to find out if the students' ability to perform consistency check improves over the quarter or after three-quarters of physics courses. Physics educators hope to see this ability to improve over time. In most cases, it is manifested in the grades since students who are more experienced in doing consistency checks tend to score better than those who are less experienced. It could be a potential area for somebody to work on in the future if they want to improve student problem-solving ability.

5.4 MODIFICATION OF CURRICULUM TO ADDRESS SPEED MISCONCEPTION

In this section, we describe one modification of the curriculum to address the speed misconception we found in our investigation. The aim is to help students to distinguish average and instantaneous speeds better. We discuss the assessment of the effectiveness of the curriculum.

5.4.1 Description of the curriculum

We first give some context before describing the actual modification.

The tutorial *Acceleration in one-dimension* has been used as the first PHYS 121 tutorial in the lesson plan for more than ten years. This tutorial is meant to help students to distinguish between velocity and acceleration. In the homework, the problems are designed for students to practice on a few skills, such as finding the change in velocity vectors, applying the definition of average acceleration, and representing motions using written description, graphical and pictorial representations. The published version of this tutorial and its homework can be found in Appendix B.

We also have a tutorial titled *Velocity*, which is meant to help students to understand average and instantaneous velocity. However, this tutorial is not used in the PHYS 121 due to logistical constraints. So, if we were to address student misconceptions on average and instantaneous velocity, we can only make small changes to the *Acceleration in one-dimension* homework. The homework curriculum we designed is shown in Figure 5.12. We call this new question as Vel-HW from then onward.

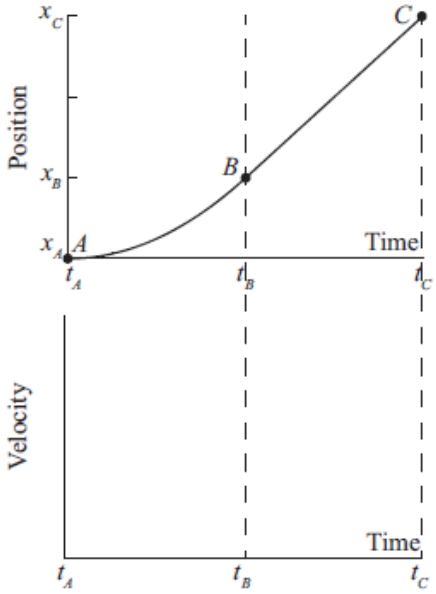
In the following question, assume any acceleration to be constant.

A block undergoes one-dimensional motion from point *A* to point *C*. An *x* vs *t* graph for the motion is shown below.

a. Is the block *speeding up*, *slowing down*, or *moving with constant speed* when:

i. it moves from x_A to x_B? Explain.

ii. it moves from x_B to x_C? Explain.



b. On the *v* vs *t* axes at right, sketch the corresponding velocity-time graph. Explain your reasoning.

c. In terms of x_A, x_B, x_C, t_A, t_B, and t_C, write an expression for the following quantities:

i. the average velocity of the block between x_A and x_B, $v_{avg,\,AB}$.

ii. the average velocity of the block between x_B and x_C, $v_{avg,\,BC}$.

d. One of the average velocities in part c above represents the *instantaneous* velocity of the block at point B. Which is it? Explain.

Does the other average velocity in part c represent the *instantaneous* velocity of the block at any instant? If so, mark the point on the *x* vs *t* graph above and label it as point X. Explain briefly.

Figure 5.12. Homework curriculum (Vel-HW) to address students' misconceptions on average and instantaneous velocity.

We designed the Vel-HW question to be different from the other contexts we used (CR, HB, and SB contexts). We want to use those contexts to serve as posttests so that we can compare student performance after students complete the Vel-HW question.

In this question, we give students a position-time graph that describes the motion of a block. The block starts from rest at point A, speeds up as it moves to B, then moves uniformly from B to C. We ask students to identify whether the block is speeding up, slowing down, or moving at a constant speed in different time intervals. Then they sketch the corresponding velocity-time graph for the motion. In part c, we ask students to write down expressions that represent the average velocity at different time intervals. For part d, we ask students to identify the expression that represents the instantaneous velocity of the block at point B. The last question in part d is related to the mean value theorem in calculus.

5.4.2 Assessment of curriculum

The posttest question, the hand-on-block (HB) context, is used to assess the efficacy of the new homework question, as shown in Figure 5.13. In this context, a student pushes a block on a level, frictionless floor. The hand pushes the block from t_1 to t_5, and stops pushing from t_5 onward with the block continuing to move across the floor. Instead of asking students for the speeds of the block at different times, we ask students to select the correct expressions that represent the magnitude of the instantaneous velocity of the block at t_5, v_5. The correct answer is $(x_7 - x_5)/(t_7 - t_5)$ since the block moves at a constant speed after t_5 because the hand is not pushing it. The average speed is equal to the instantaneous speed when the object is in uniform motion. The value of v_5 can be calculated using the average speed from t_5 to t_7.

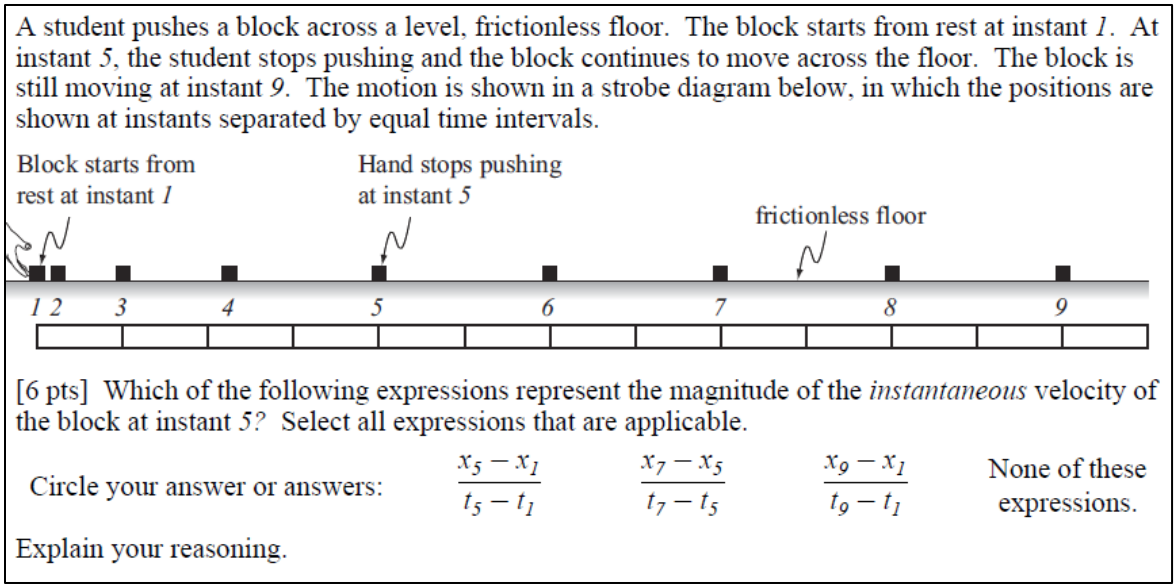


Figure 5.13. HB context on written posttest.

Since the modification is made in the homework, we assess its efficacy by comparing posttest performance for students who were assigned the problem and those who were not. We assigned the Vel-HW problem (together with other questions) as homework to students in the Wi'19 quarter, but not in the Au'19 quarter. The teaching assistants did not grade Vel-HW in the Wi'19 quarter, so students did not receive any feedback. Although we did not assign Vel-HW to students in the Au'19 quarter, it was still available to students so that students might practice on Vel-HW for the exam.

The posttest results of Wi'19 and Au'19 quarters are tabulated in Table 5.25. The percentage correct in both quarters are around 70%, with no significant difference. We can interpret the result in two ways: (1) the posttest question is not able to assess the efficacy of the question, or (2) the new homework problem has no or little effect in improving student understanding of average and instantaneous speed. We are not able to test the first interpretation as it takes several iterations to find the most suitable posttest question, so we leave that for now.

Table 5.25. Posttest results of the homework modifications in the Wi'19 and Au'19 quarters.

Answer	Wi'19 (N = 548) Problem assigned in homework	Au'19 (N = 507) Problem not assigned in homework
$\frac{x_7 - x_5}{t_7 - t_5}$ (Correct)	72%	67%
None of these expressions	14%	25%
$\frac{x_5 - x_1}{t_5 - t_1}$	9%	5%
Others	5%	3%

To examine the second interpretation, we asked another variant of the posttest question in the Sp'18 quarter, with the answer options written as $\Delta x_{15}/\Delta t_{15}$ and so on. The Vel-HW was not assigned to students in that quarter. We found that 66% of the 516 students answered correctly in aggregate, though the percentage correct ranged from 51% to 77% across three lecture sections. The reason for the big spread was due to the exam conditions. There were two exam rooms. The notation "Δx_{15}" as $x_5 - x_1$ was clarified in one room, but not the other. On average, students who were notified did better compared to students who were not. There were also three lecture sections of students in the Au'19 quarter, and the percentage correct ranged from 63% to 70%. The two aggregate results in the Sp'18 and Au'19 quarters suggest that the Vel-HW did not improve student performance significantly since the difference between the two groups who were assigned and not assigned the new homework problem is small.

Next, the proportion of students choosing "None of these expressions" in the Au'19 quarter is greater than that in the Wi'19 quarter. However, the difference and the effect size are small. Students who chose this option tended to say that all the given expressions are average speeds, not the actual definition of instantaneous speed. They defined the instantaneous speed by its the proper

mathematical definition or the slope of the tangent of the position-time curve. They have the right ideas but may have misunderstood the question. Some of them may not know that the average speed is equal to the instantaneous speed when the object is in uniform motion.

5.4.3 Additional discussions

We also asked a similar task using the CR context in one pretest in the Wi'16 quarter and found 34% of the students answered correctly. Comparing the pretest and posttest results, we found that the percentage correct doubled in the mid-term, with or without students attempting the new homework problem.

If we compare the result from this study (~ 70% correct) with the percentage correct (58% correct) of the HB context in our isomorphic problems study (see Figure 5.1), we do not see a significant difference. We designed part of the homework problem in the Sp'16 quarter when we did the investigation with isomorphic problems, so the student performance in the study was somewhat affected by our intervention. Another reason for the difference could be due to the nature of the tasks. This modified HB posttest task requires students to find an expression of v_5, while in the isomorphic problems study, students need to find the speeds at four different locations.

It is possible that students who were not assigned to do the Vel-HW problem also did it for exam preparation. They might do so by going through all the unassigned tutorial homework problems for additional practice. So, for a better-controlled study of the efficacy of the Vel-HW problem, it is necessary to exclude it in the homework.

This study also presents the opportunity to examine the efficacy of homework problems on student learning. It is not easy to tell if our homework problems serve the intended pedagogical purpose. However, we tend to assess our curriculum (tutorial + homework) most of the time. If

we were to improve the curriculum, it is the in-class tutorial worksheet that receives the most attention. However, homework is also a critical component. So, in principle, one could improve the homework exercises successively by conducting similar investigations.

5.5 SUMMARY AND CONCLUSION

We summarize the results from the additional investigations about student understanding of average and instantaneous speed.

We designed additional problems that are isomorphic to each other so that we can track student performance over the mechanics course. The percentage correct started at around 20% on a pretest immediately after lecture instruction, increased to around 60% in the first midterm, then decreased by 12% in each test in the subsequent two midterms. This study shows some information about students' retention of knowledge and how the context of isomorphic problems affects student performance. It also shows how persistent student errors are, especially in using the average speed of the object during non-uniform motion as the instantaneous speed at the end of that motion.

Our second study shows that the student performance in finding speeds depends on whether the object is released from rest or is moving with an initial velocity. However, the options of the multiple-choice version were not entirely compatible with the written responses given by students, so the most common error on the multiple-choice version is not representative. Additional work can be conducted to improve the multiple-choice version to reflect student difficulties better.

The third investigation is on the students' ability to resolve inconsistencies in their answers individually. We asked students to compare their speed-time graphs and speed calculations on the

cart-on-ramp context through online pretests. We found that around 20% of the students could identify their inconsistencies and mistakes. Around 25% of the students answered both tasks correctly, and most of them found their answers to be consistent. These results show that not all students could resolve the inconsistencies in their answers on our assigned tasks individually. Though our results are not quite generalizable to all students at different times of the quarters, we believe that it is necessary to have instructors to help students resolve inconsistencies in tutorials as the content gets more difficult as the quarter progresses.

We added a new homework question to improve student understanding of average and instantaneous speed. We assessed its efficacy by comparing the student posttest performance between students who were assigned and not assigned to complete the new homework problem. In both groups of students, around 70% answered correctly, with no significant difference between them. Possible reasons are that the posttest question may not be a suitable one, or the study itself is not well-designed. A better-controlled study needs to be designed to find out the efficacy of the new homework question accurately.

In conclusion, our additional investigation shows that student's ability to find instantaneous speeds in contexts that involve both uniform and non-uniform motions decreases over the quarter. The performance seems to be affected by whether the motion starts from rest or at an initial speed. The most common error is that students are calculating the speed at the final instant by using the average speed of the prior motion without checking whether the object is speeding up or slowing down or reaching to a stop. Students generally need help in resolving inconsistencies during class, since not many students can do that individually. Lastly, our newly designed homework question does not appear to help students to improve their understanding of average and instantaneous speed.

CONCLUSION FOR PART I

We began the research discussed in Part I of this dissertation motivated, in part, by the poor performance of students on what seemed to be a relatively simple task. A question in which students were asked to find velocities at certain points given positions and times for an object rolling down a ramp and then across a horizontal, frictionless track. (See Figure 4.1). Note that there are two segments to the motion: in the first, the object moves from rest with increasing speed. In the second, the object moves with constant speed. It thus represents one of the simplest examples of a two-part motion.

After instruction in high school, only about 10% answered this task correctly, and at the start of university instruction, only about 15% answered correctly. Although the percentage correct increased after university instruction, the percentage who answered correctly at all the points of interest was at most only about 50%.

Some of the errors made by students suggested difficulties with underlying concepts that have been reported in the literature (for example, a failure to distinguish between instantaneous and average speed and difficulty with the concept of limits). However, the consistency of the results suggested that the task could serve to give insights into student problem-solving ability.

We designed several variations of this question to probe student thinking and reasoning in greater detail. These involved changing the order of the motions (so that the object first moves with constant velocity on a level track and then with changing speed up an incline), changing the context for the motion (a block that is accelerated by a compressed spring on a level surface, then leaves the spring with constant speed); and asking students to graph the motion. We also asked this question with different populations (algebra as well as calculus-based physics students) and at

different points in instruction, which formed the basis for both cross-sectional and longitudinal studies.

All the variations of the question also proved to be difficult for students. Some of the more relevant findings were:

- Student difficulty in distinguishing average and instantaneous velocity appears to remain throughout instruction in introductory physics courses
- There is a strong tendency of students to reason sequentially (from earlier to later times), and many do not reflect on the motion during each stage of the motion (local rather than global reasoning).
- Not many students can graph the motion in each stage of the motion, and for those who can, they often fail to relate the graphical representation to their numerical calculations.
- For many students, there seemed to be a general tendency to apply formulae indiscriminately without comparing the results to the corresponding motion of the object.

Around 40% of the students were able to draw or choose the correct speed-time graphs of the motion might be interpreted as indicating that they understand the way in which the speed changes with time. However, the fact that many fail to relate the graphs they draw to the calculations they make (even when these are done one-after-the-other) indicates that either they do not relate this information across the two representations or that the graphs are part of a memorized response.

We made only one attempt to design a task to help students recognize the discrepancy between their graphical and numerical representations. That was administered as part of homework for the tutorials. The posttest results, however, did not indicate any improvement in

student performance. In some ways, this is not surprising. Many attempts have been made to improve student problem-solving ability and learn to reflect critically on their answers. A more sustained effort is probably needed, together with additional research to document effective ways to improve these skills.

PART II
INVESTIGATION OF STUDENT UNDERSTANDING OF ROLLING MOTION

In a typical introductory physics course, students are taught to describe the motion of rigid bodies after studying the kinematics, dynamics, energy, and momentum of point objects in linear motion and after having been taught basic ideas about rotational motion, including the concepts and definitions of angular position, velocity, and acceleration. The study of rigid-body motion provides a chance for students to extend what they have learned to real-world situations beyond the idealized case of point-particle motion. The description of the motion of extended bodies is more complicated because such bodies can both translate and rotate (about a fixed or moving axis) at the same time. The kinematics of rolling motion also requires the application of ideas about relative motion.

Part II of this dissertation presents results from our in-depth investigation into student understanding of rolling motion. Rolling motion is one of the simpler cases of rigid body motion – the case in which the planes of the translational and rotational motions are the same. Rolling motion is usually taught in the introductory calculus-based physics course, but not in the algebra-based physics course. This likely is because this topic serves as an introduction to general plane motion for mechanical and civil engineering students, who will study this topic in more detail in engineering courses on statics and dynamics.

There has been relatively little prior research on rolling motion and essentially no research-based curriculum that has been developed on this topic. This provided some of the motivation for this research.

In Chapter 6, we review the underlying physics concepts required to describe rolling motion and then give an overview of the past research on this topic, together with relevant research on relative motion. In the process, we compile a list of student difficulties and tasks that have been used to assess student knowledge about rolling motion.

Chapter 7 describes a set of replication studies that were then conducted to confirm what others have found and to establish baseline performance at the UW before any interventions were developed. Student responses on past examination questions are discussed and how these corroborate difficulties reported in the literature.

The results from Chapters 6 and 7 suggested research questions that we decided to pursue in order to understand student thinking about rolling motion in greater detail. In Chapter 8, we discuss the development of new tasks that were developed to conduct in-depth investigations and present results from these studies.

In Chapter 9, we collect data on tasks that were illustrated in the literature to demonstrate how one can use relative motion to rigid body motion but did not report any student responses.

Finally, in Chapter 10, we describe the development of the preliminary version of a tutorial that was designed based initially on suggestions for instruction that had been proposed by others based on prior research. We found some improvement in student performance after the preliminary version, but these were modest. The assessment result prompted us to review the results from the in-depth investigation and preliminary assessment and to modify the curriculum to take additional research findings into account. We discuss the changes and the improved posttest results after using the modified tutorial.

Chapter 6. BACKGROUND & PRIOR RESEARCH

In a typical physics course, after students have studied how to describe the linear motion of point particles, they are introduced to methods for addressing the motions of real-world objects. Such objects cannot be modeled as point particles as they have extended shapes and sizes. In most introductory physics classes, the discussion is limited to rigid bodies of uniform shapes and densities. Students are first taught how to describe the rotational motion of a rigid body about a fixed axis, and then how to combine the ideas from translational and rotational motion to describe rolling motion. They usually encounter this content in weeks 8 and 9 in the 10-week PHYS 121 course at the University of Washington.

In this chapter, we review the physics behind rotational and rolling motion and some prior research on student understanding of this topic. Since the analysis of rolling motion involves relative motion concepts, we also include the relevant literature on student ideas related to Galilean relativity. We restrict our attention to rigid bodies of uniform density unless otherwise stated.

6.1 OVERVIEW OF RELEVANT PHYSICS UNDERLYING ROTATIONAL AND ROLLING MOTION

This section first gives an overview of how physicists describe the rotational motion of a point object and a rigid body about a fixed axis. We present operational definitions of the angular quantities and how some linear and angular kinematical quantities are related to each other. Lastly, we describe rolling motion as a special case of general rigid body motion.

6.1.1 Rotational motion of a point object

Physicists describe rotational motion analogously to linear motion. They use displacement, velocity, and acceleration to describe the motion of a point particle along a curve. When the curve is circular, besides using the previously mentioned quantities, they also use angular displacement, angular velocity, and angular acceleration to describe the motion. Polar coordinates can be used to describe rotational motion mathematically. The relevant angular quantities, as seen by most students, are defined in Table 1.1.

Table 6.26. Angular quantities and their operational definitions.

Quantity	**Definition**	**Operational definition**
Angular position (θ)	Angle as measured from the positive *x*-axis	-
Angular displacement ($\Delta\theta$)	Change in angular position	$\theta_f - \theta_i$
Average angular velocity (ω_{avg})	Change in angular position in a unit time interval	$\dfrac{\Delta\theta}{\Delta t}$
Instantaneous angular velocity (ω)	Rate of change of angular position	$\dfrac{d\theta}{dt} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta\theta}{\Delta t}$
Average angular acceleration (α_{avg})	Change in angular velocity in a unit time interval	$\dfrac{\Delta\omega}{\Delta t}$
Instantaneous angular acceleration (α)	Rate of change of angular velocity	$\dfrac{d\omega}{dt} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta\omega}{\Delta t}$

6.1.2 Rotation of a rigid body about a fixed axis

A rigid body can be treated as a collection of point particles arranged in some fixed shape. It is modeled as not deforming under the forces that are exerted on its surface. Consider one such object that is confined on the horizontal plane of the page shown in Figure 6.1, and rotating counter-clockwise about a fixed axis along the *z*-direction (pointing out of the page) that passes

through the origin. Point P is on the rigid body and its angular positions at t_i and t_f are shown. (The angular position of point P is defined to be the angle between the position vector r_{P} and the positive *x*-axis.) The operational definitions in Table 1.1 are used to find the rotational kinematical quantities of interest. One thing to note is that when the object is rotating about a fixed axis, every particle on the rigid body rotates through the same angle in the given time interval and has the same angular velocity and angular acceleration.

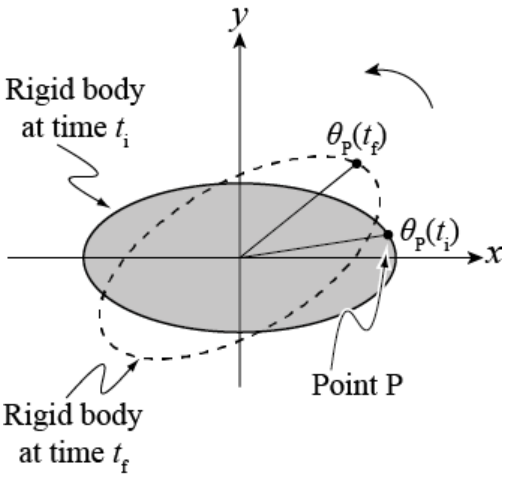


Figure 6.1. Rotation of rigid body about a fixed axis passing through the origin along the *z*-direction.

Students are presented with a way to determine the direction of rotation by using the right-hand rule. By wrapping the four fingers of the right hand around the fixed rotation axis in the direction of rotation, the thumb points in the direction of rotation. In this example, the direction of the angular displacement ($\Delta\theta$) and angular velocity ($\vec{\omega}$) are in the z direction (pointing out of the page). The direction of angular acceleration (α) is then determined by the direction of the change in angular velocity. If $\vec{\omega}$ is increasing in magnitude, then α and $\vec{\omega}$ point in the same direction. Conversely, α and $\vec{\omega}$ point in the opposite directions when the magnitude of $\vec{\omega}$ is

decreasing in time. The way the directions of $\vec{\omega}$ and α relate to each other during speeding up and slowing down in rotational motion is analogous to one-dimensional motion.

Table 6.27. Relationships between linear and angular kinematical quantities. *r* is the distance from the origin to the point of interest, and r is the position vector.

Linear quantities	**Angular quantities**	**Relationship**
Displacement along circular arc (*s*)	Angular displacement ($\Delta\theta$)	$s = r\Delta\theta$
Tangential velocity (v)	Angular velocity ($\vec{\omega}$)	$v = \vec{\omega} \times r$
Tangential acceleration (a_t)	Angular acceleration (α)	$a_t = \alpha \times r$

The relationships between the linear and angular quantities for rigid body rotation are presented in Table 6.27. Some important points that students are expected to know or use are described below. The magnitudes of the tangential velocity and tangential acceleration (the acceleration that accounts for the change in the magnitude of the velocity) are $r\omega$ and $r\alpha$, respectively. At any instant, there is only one unique value of the linear and angular quantities for a point particle in rotational motion. However, each point on the rigid body has different linear quantities since the distance from the origin or angle to each point is different. Furthermore, the total acceleration of each point on a rigid body is the vector sum of its tangential acceleration and centripetal acceleration (the acceleration that accounts for the change in the direction of the velocity). In short, when rotational motion is involved, the discussion of linear kinematical quantities of each point on the rigid body becomes more complicated.

On a side note, the definitions of tangential velocity and tangential acceleration are also applicable even when a point particle is moving along a straight line. However, this is not typically covered in the introductory physics class at the University of Washington.

6.1.3 Rolling motion

Rolling motion is a special case of general rigid body motion. A rigid body can translate and rotate about some moving axis at the same time. While such motion is complex, one can analyze the translational motion and rotational motion independently. The analysis is similar to that for projectile motion, in which the horizontal and vertical motions are analyzed separately.

In typical introductory physics textbooks, the discussion of rolling motion mainly involves rolling without slipping of cylinders or spheres. When a uniform cylinder of radius *R* rolls without slipping on a horizontal surface, the center-of-mass (CM) moves a distance *s*, and the cylinder rotates about the CM through an angle θ, where the distance *s* and the angle θ are related by $s = R\theta$. In most introductory textbooks, the condition for rolling without slipping, $v_{\mathrm{CM}} = R\omega$, is derived by differentiating the equation $s = R\theta$ with respect to time and then relating the quantity $\frac{ds}{dt}$ to the speed of the CM of the cylinder.

In Figure 6.2, a cylinder rolls to the right without slipping on a horizontal surface. The trajectory of a point (black dot) on the rim of the rolling cylinder is a cycloid (in red), whereas the path of the CM is a straight line (dashed). The direction of the velocity for the point on the rim changes as the cylinder rolls across the surface, while the direction of v_{CM} is always to the right.

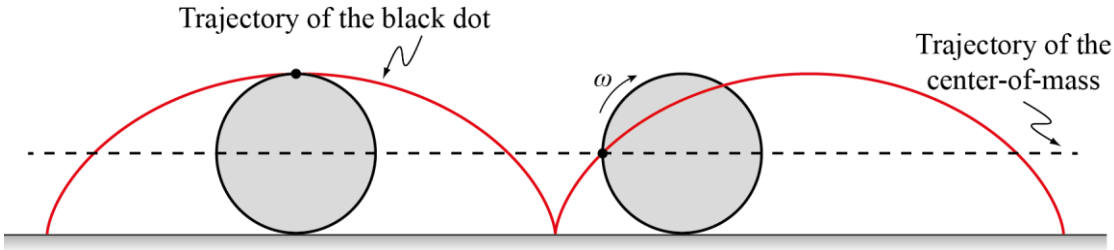


Figure 6.2. The trajectory of a point on the rim of a cylinder (black dot) and the center-of-mass of the cylinder when it rolls without slipping.

The direction of the velocity of the black dot at any instant in the ground frame can be obtained by drawing the tangent line of the red trajectory at that instant. Note, however, there is a cusp in the red line, which poses a difficulty in using the tangent line to determine the direction of the velocity when the black dot touches the horizontal surface. So, we need another approach.

Since the rotational motion is about the CM, and the CM is moving relative to the ground, we can apply relative motion ideas to determine the direction of the velocity of the black dot (B) relative to the ground (G) at any instant for rolling without slipping. That is, $v_{\mathrm{B}}^{(\mathrm{G})} = v_{\mathrm{B}}^{(\mathrm{CM})} + v_{\mathrm{CM}}^{(\mathrm{G})}$, where $v_{\mathrm{B}}^{(\mathrm{CM})}$ is the tangential velocity of B relative to the CM, and $v_{\mathrm{CM}}^{(\mathrm{G})}$ is the velocity of the CM relative to the ground. When the black dot touches the horizontal surface, $v_{\mathrm{B}}^{(\mathrm{CM})}$ is pointing to the left with magnitude $R\omega$. Since $v_{\mathrm{CM}}^{(\mathrm{G})}$ is pointing to the right with a magnitude v_{CM}, we find that $v_{\mathrm{B}}^{(\mathrm{G})}$ has zero magnitude. Similar reasoning demonstrates that when the black dot is at the top of the cylinder, it has magnitude $2v_{\mathrm{CM}}$. This chain of reasoning is summarized in Figure 6.3. In general, the relative motion approach allows us to determine the magnitude and direction of the velocity of any point on a rolling object.

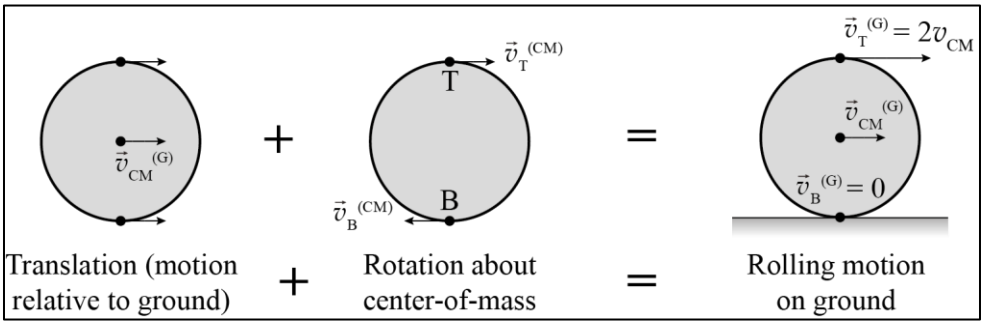


Figure 6.3. Applying relative motion ideas to illustrate rolling motion.

6.1.4 Commentary on the traditional approach to instruction on rolling without slipping

A review of common textbooks [29, 30, 65, 66] reveals that many describe rolling motion as a combination of translational motion and rotational motion, but it may not be evident to students that the underlying concept is relative motion [12]. Furthermore, the presentation of the equation of rolling without slipping ($v_{\mathrm{CM}} = R\omega$) in textbooks may lead students to overgeneralize its applicability in all rolling motion problems, be it slipping or no-slipping. However, it is possible to arrive at the equation by using relative motion and the no-slip condition.

The formal operational definition of the no-slip condition is such that there is no relative velocity at the point of contact between the two objects. In the context of the rolling cylinder, when the black dot touches the ground, $v_{\mathrm{CM}}^{(\mathrm{G})}$ is pointing right, $v_{\mathrm{B}}^{(\mathrm{CM})}$ is pointing left, so the magnitude of $v_{\mathrm{B}}^{(\mathrm{G})}$ is $v_{\mathrm{CM}} - R\omega$. Since the cylinder does not slip on the ground, in the frame of the ground, the black dot is at zero velocity, $v_{\mathrm{B}}^{(\mathrm{G})} = v_{\mathrm{CM}} - R\omega = 0$, hence we arrive at $v_{\mathrm{CM}} = R\omega$.

The textbook authors may have chosen a more straightforward approach to introduce rolling motion (*i.e.*, just introduce the situation and relevant equations), but it is not generalizable to all situations. The relative motion approach presents a systematic procedure to combine the equations of motions from linear and angular kinematics to analyze general rigid body motion. The lack of emphasis on relative motion ideas and the definition of the no-slip condition in rolling motion in textbooks affects how instructors deliver the content to students, which may lead to some student difficulties presented in prior research and our investigations.

6.2 PRIOR RESEARCH ON STUDENT UNDERSTANDING OF ROTATIONAL AND ROLLING MOTION

Many of the physics education research articles in mechanics focus on student understanding of linear motion and forces. There are not many on student understanding of rotational and rolling motion. In this section, we review articles by Rimoldini and Singh, López, and Fang and Uziak on student understanding related to rotational and rolling motion. They reported on the responses from relevant tasks among undergraduate physics and engineering majors. We also describe the work by Ambrosis *et al.* as they developed an instructional approach to improve student understanding of rolling motion. Based on the published articles, several educators have suggested several approaches to improve student understanding of rolling motion.

Note: The terms "kinematics" and "dynamics" are used interchangeably in this section as some authors did not distinguish them in their articles. In later sections, we use the term "kinematics" for motion-related questions and the term "dynamics" for force-related questions. Readers can look at the original papers if they have any confusion.

6.2.1 Student difficulties with rotational dynamics and rolling motion

Rimoldini and Singh identified common difficulties students had with ideas related to rotational and rolling motion, such as moment of inertia, torque, angular acceleration, angular velocity, relative motion, rolling motion, and friction [12]. They conducted think-aloud demonstration-based interviews with sixteen students (ten students from introductory physics courses and six physics juniors) after students had completed the relevant lecture instruction. Based on the findings from the interviews, they developed a 30-item multiple-choice test that was related to the essential concepts tested in the tasks. The test was titled the Rotational and Rolling

Motion Conceptual Survey (RRMCS). Students were asked to explain their answers in each item. About 3000 students were involved in the subsequent design iteration of the test. Out of the 30 questions in RRMCS, eight questions are on rolling motion. Four of them involved relative motion in rolling motion, while the other four were about the role of friction in rolling motion.

One of the interview tasks involved the context of a wheel that is rolling without slipping (see Figure 1.1 in Chapter 7). Students were asked to explain the velocity of the top and bottom points relative to the ground, but none of them could do so correctly. Based on the interviews, the authors found that "students were not comfortable with (linear) relative motion concepts" and "they did not understand the meaning of rolling without slipping." The authors also mentioned that "some students kept confusing the velocities of the top and bottom points relative to the CM of the wheel with the velocities with respect to the ground."

In another example of the interview, the researcher first explained to a student step-by-step why the bottom of the rolling wheel should have zero velocity to the ground. Then the researcher asked the student about the velocity of the top point with respect to the ground. The student replied that the speed of the top of the wheel was v_{CM} with respect to the ground. The authors inferred that students were uncomfortable with relative motion concepts. The authors then further illustrated that students were used to thinking in terms of one preferred frame of reference and that a different frame of reference represented a change of their acquired "common sense." In this example, the student's preferred frame of reference was the CM frame of the wheel, and the student was unable to apply the newly introduced approach (researcher's explanation) to the top point.

In the same rolling wheel context, students were asked to compare the speeds at different points on the rim of a rolling wheel. Students tended to say, "The speed of all points should be the same with respect to the ground because they are all on the same wheel which is rolling." Next,

students were asked for the direction of the velocity of a point on the rim of the wheel. Students tended to explain, “The instantaneous velocity with respect to the ground is always tangent to the rolling circle.” Based on the findings, the authors commented that future instruction on rolling motion should consider student difficulties with relative motion concepts.

Besides examining student difficulties with rolling motion, the authors also investigated the role of friction in rolling without slipping on a rigid horizontal floor. They found that many students believed that a frictional force was present on a wheel rolling without slipping at uniform speed on a flat surface. Their students claimed that “a constant force must be applied to keep a wheel rolling. Otherwise friction would slow it down.” Many of their students believed that “friction must slow any kind of motion (including rolling).”

López also reported student misconceptions about rolling motion, but her paper did not present any student responses [13]. Hers is more anecdotal. She observed that many students solved rolling-without-slipping problems by rote and used the formula $a_{\mathrm{CM}} = R\alpha$, which says the linear acceleration of the CM of a rolling cylinder on the ground is equal to the product of its angular acceleration and radius. However, the equation is not applicable if the surface below the rolling cylinder is accelerating.

After reviewing how introductory physics textbooks tend to introduce rolling without slipping motion, López presented a general approach to this topic that applies to every rigid body that rolls without slipping. She defined no-slip motion between the surfaces of two rigid bodies to be “the relative velocity of the contact points is zero” and used relative motion concepts to present her general approach. She then illustrated how her method is used in some cases, such as a uniform disk rolling without slipping on an accelerated flat surface and a horizontal bar that is moving without slipping on uniform cylinders, with the cylinders rolling without slipping on the ground.

Fang and Uziak conducted a review of some student misconceptions of general plane motion (GPM) in rigid-body kinematics among mechanical and civil engineering students [15]. GPM is an essential idea in engineering dynamics. They summarized four methods that are typically used in analyzing and solving GPM problems: absolute motion analysis, relative-motion analysis using translating axes, instantaneous center of zero velocity, and relative motion using rotating axes. Out of the four methods, the first two are more relevant to our current work.

Based on previous research the authors reviewed, the authors highlighted that students had difficulties with the following four physics concepts. The first difficulty is, "Different points on a rigid body in motion have different velocities and accelerations, which vary continuously." Second, "angular velocities and angular accelerations are properties of a body as a whole and can vary with time." Third, "points on an object that is rolling without slip[ping] have velocities and accelerations that depend on the rolling without slip[ping] condition." The last difficulty is "an object can have nonzero acceleration and zero velocity or nonzero velocity and zero acceleration."

The authors suggested five reasons for student difficulties with GPM: lack of fundamental understanding of basic motion concepts, lack of necessary mathematics skills, lack of knowledge transfer skills, lack of spatial skills, and lack of hand-sketching skills.

Lack of fundamental understanding of basic motion concepts and lack of necessary mathematics skills. The authors commented that students confused related quantities such as position with velocity, velocity with acceleration, and linear velocity with angular velocity. They also found that students could not convert a vector equation into a set of two or three scalar equations and calculate the relevant components using the geometrical properties of triangles.

Lack of knowledge transfer skills. In knowledge transfer, they found students could answer simple relative motion problems (determining absolute and relative velocities of a passenger

walking in a train) but were unable to apply the same ideas to the problem of two points on a rigid body.

Lack of spatial skills. Fang and Uziak defined spatial skills using the definition provided by Uttal *et al.* [67]. While there is no specific definition of spatial skills among the researchers (mainly psychologists), Uttal *et al.* stated that the researchers agreed on the existence of spatial visualization skills, "which involves the ability to imagine and mentally transform spatial information." Another paper authored by Ha and Fang [68] provided a review on "the role of spatial ability in solving physics problems and makes connections to relevant problems in engineering mechanics." Ha and Fang cited a report from ENGAGE: Engaging Students in Engineering, a National Science Foundation-funded project. The project has a spatial visualization course, and students who enrolled in the course got better grades in first year mathematics, engineering and science courses.

Lack of hand-sketching skills. Fang and Uziak stated that the underlying reason was due to the extensive use of modern computer-aided tools in classroom teaching. The authors claimed that proper sketching of objects undergoing GPM "is highly required to visualize and consequently understand GPM."

Fang and Uziak also emphasized the importance of the instructors' role because both the curriculum and instruction could contribute to student misconceptions. They recommended demonstrating fundamental hand-sketching skills in the design and implementation of engineering curricula.

6.2.2 Curriculum development and instructional strategies for rolling motion

The studies discussed in the prior section reported on student difficulties with rolling motion. Each paper presented suggestions for changing instruction to improve student understanding. However, no results based on their recommendations were reported. We found one other paper that tried a different instructional approach from those suggested and reported on the findings.

Ambrosis *et al.* cited the findings by Rimoldini, Singh, and López. They designed an instructional sequence for undergraduates based on experiments and simulations to investigate the role of sliding friction in rolling motion [14]. The activities were meant to help students understand the difference between static and kinetic friction and how each affects the motion of rolling objects on a horizontal or inclined surface. They reported success in improving twenty student teachers' (the trial population) understanding of rolling motion after instruction. Although it is not possible to implement their teaching sequence (6 hours in total) in our tutorials (50 minutes only), some of the learning objectives have been incorporated in the creation of the new tutorial curriculum to address student difficulties in rolling motion.

6.3 PRIOR RESEARCH ON STUDENT UNDERSTANDING OF GALILEAN RELATIVITY

As discussed previously, relative motion concepts are applicable in reasoning about rolling motion. Rimoldini and Singh, in their studies, also mentioned that students failed to realize that they could apply relative motion ideas to arrive at the answers to some rolling motion tasks. The approach López presented in her paper was also based on relative motion concepts. Other research on student understanding of Galilean relativity was conducted by Panse *et al.*, O'Brien Pride, and

Boudreaux. The work by O'Brien Pride and Boudreaux are most relevant to our research, and we describe them below.

O'Brien Pride and other members of the University of Washington Physics Education Group (UWPEG) investigated student difficulties with relativity in introductory mechanics [69]. They examined student performance on an exam question involving a two-body collision in one dimension in the CM frame. The researchers found that few students could answer the momentum or kinetic energy of colliding blocks in the CM frame correctly. The most common incorrect responses were those in which students reported laboratory frame values. The researchers concluded that traditional instruction was inadequate for most students to gain a basic understanding of reference frames that are related to the CM frame.

O'Brien Pride and the UWPEG also interviewed students on measurement ideas and transformations of simple motions between reference frames. The context was two observers, one stationary and the other on a moving train, observing an apple falling from a tree. Students were asked to determine the velocity or acceleration vectors or the trajectories of moving objects in multiple frames of reference. Students were also asked to describe procedures for the speed measurement in a reference frame that was moving relative to the laboratory. Through the interviews, the researchers found students had difficulty in visualizing a given situation in different reference frames, interpreting questions regarding observers in different frames, and transferring specific measurement procedures to different inertial reference frames.

Boudreaux and the UWPEG extended O'Brien Pride's research on relative motion [70]. The researchers investigated student understanding of displacement, velocity, and acceleration in multiple frames of reference. They used several contexts, such as two spaceships moving toward each other, two cars traveling side-by-side, etc. In those contexts, students were asked to determine

displacements, velocities, or accelerations of objects of interest. Through the student responses, the researchers found several student difficulties. They are:

- Preferred frame reasoning,
- Tendency to confuse position and displacement,
- Failure to recognize that displacement depends on frame of reference,
- Failure to recognize that observers in the same frame agree on displacement and velocity.
- Tendency to sum initial and final positions when calculating displacements,
- Failure to measure position relative to a fixed point on an extended medium,
- Tendency to confuse relative position and relative velocity,
- Tendency to confuse changes in relative position and changes in relative velocity,
- Failure to relate relative velocity to changes in relative position,
- Failure to recognize that the acceleration has the same value in two inertial frames.

Out of all the difficulties they documented, the idea of 'preferred frame reasoning' is most relevant to our investigation. Boudreaux wrote, "Students tend to associate the values of quantities and the description of motion directly with the object under consideration, rather than with the systematic application of formalism in any desired frame." So, in the rolling wheel context, for example, students might associate the velocity of the point on the rim of the wheel to the rotational motion of the wheel in its own reference frame. They should, however, be applying relative motion ideas in this context to determine the direction of velocity in the ground frame. Based on the results, Boudreaux designed the *Relative Motion* tutorial [31] to address the following student difficulties: (1) failure to recognize that displacement and velocity depend on the frame of

reference, and (2) failure to relate relative velocity to changes in relative position. The context used in the tutorial involves multiple objects moving in one-dimension.

6.4 COMMENTARY ON PRIOR RESEARCH

The studies described above have documented some common student difficulties with the dynamics of rolling motion and with Galilean relativity more broadly. We summarize the student difficulties most relevant to this investigation in Table 6.28. In the preliminary stages of our research (Chapter 7), we tried to confirm some of the prevalence reported by previous researchers in order to construct a baseline in our investigation. We also developed additional research tasks to probe these difficulties in greater detail. As a result, other difficulties were observed.

Table 6.28. Summary of student difficulties with rolling motion and relative motion reported in prior research.

Topic	Student difficulties
Rolling Motion	• Difficulty with relative motion concepts and rolling without slipping. • Tendency to confuse the ground frame with the center-of-mass frame in rolling motion questions. • Tendency to think that a frictional force is needed for a wheel that is rolling without slipping on a horizontal surface. • Indiscriminate use of the rolling without slipping formula $a_{\mathrm{CM}} = R\alpha$ in all related contexts. • Difficulty in realizing that the points on the objects that are rolling without slipping have velocities and accelerations that rely on the rolling without slipping condition.
Relative Motion	• Difficulty in visualizing a given situation in different reference frames. • Difficulty in interpreting questions regarding observers in different frames. • Difficulty in transferring specific measurement procedures to different inertial reference frames. • Tendency to use preferred frame reasoning.

6.5 MOTIVATION AND CONTEXT OF INVESTIGATION

One motivation for this research is developing a research-based and research-validated curriculum on this topic. Two of the studies described in this chapter (López and Ambrosis *et al.*) outlined some suggestions for the development of the curriculum to improve student understanding of rolling motion. Our efforts to develop a tutorial allowed us to test their suggestions and compare the efficacy in addressing the difficulties. We then used our results from the investigation to guide the subsequent design of instructional materials. Testing the tutorial in multiple classes across many quarters allowed us to examine the persistence of identified difficulties and guide further research to find additional difficulties and probe the persistence. This research and curriculum development are presented in the next four chapters.

This investigation was conducted with students from the introductory calculus-based physics course, PHYS 121.

Chapter 7. INVESTIGATION OF STUDENT DIFFICULTIES WITH ROLLING MOTION I – REPLICATION STUDIES & RESULTS FROM PAST EXAMINATIONS

This chapter describes the results of investigations of student difficulties with rolling motion at the University of Washington (UW) before any intervention through tutorial instruction. We used research tasks that were found in published conceptual surveys and final exam questions administered at UW early in this study. The questions presented involve relative motion concepts and the role of friction in rolling motion. The objectives are: (1) to find out the extent at which the published results are reproducible, (2) to investigate whether the student responses are context-dependent by making minor changes of the published tasks, (3) to assess the effectiveness of lecture instruction of rolling motion without any tutorial instruction, and (4) uncover the student difficulties in the role of friction in rolling motion from the student responses in past exams.

In section 7.1, we describe the results from two rolling wheel questions (relative-motion related) from the Rotational and Rolling Motion Survey (RRMCS) [12]. These were administered on pretests and final exams in the introductory calculus-based physics courses at the UW before any intervention through tutorial instruction. Section 7.1 also contains a comparison of our results with those reported by Rimoldini and Singh. Then in section 7.2, we describe the student explanations to the rolling wheel questions and compare them to those reported by Rimoldini and Singh. In section 7.3. we investigate student difficulties related to the role of friction in rolling motion by looking at the student performance in related problems from RRMCS and past exam questions.

7.1 ROLLING WHEEL QUESTION IN RRMCS

In this section, we present results from UW students on two questions from the RRMCS that involve a rolling wheel context. The rolling wheel context involves a typical example illustrated in all physics textbooks describing the kinematics of a rolling cylinder. There are two questions: one about the direction and the other about the magnitude of the velocity of a point on the rim. In the subsequent discussion of this question, we call the pair of questions the Rolling Motion Task (RMT).

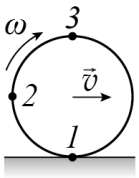
We administered the RMT to UW students on online pretests and a final exam, and then we compare the results with those reported in the literature. The results represent student performance at UW before and after traditional lecture instruction, and before any intervention through tutorial instruction. They show how ubiquitous the errors are among introductory physics students in different institutions.

7.1.1 Description of the Rolling Motion Task (RMT)

The RMT involves a rigid wheel that is rolling to the right without slipping on a road surface (ground). Three points, 1, 2, and 3, are situated on the wheel at the instant given, as shown in Figure 1.1.

Students were asked (1) to find the direction of the instantaneous velocity of point 2 in the ground frame (the “Direction task” or RMT-Direction) and (2) to rank the speeds of 1, 2, and 3 (the “Magnitude task” or RMT-Magnitude). Several variations of the task (change from rolling right to rolling left, switch the position of point 2 to the other side) were used in administrations of the question on different pretests and exams.

A rigid wheel of radius *R* rolls without slipping on a horizontal road. The linear velocity of the center of the wheel with respect to the road is $\vec{v}$ and the angular speed is ω.



Direction task: What is the direction of the instantaneous velocity of point *2* with respect to the road?

Magnitude task: Rank the speeds of points *1*, *2*, and *3* at the rim of the wheel with respect to the road.

Figure 7.1. Rolling Motion Task in the Rotational and Rolling Motion Conceptual Survey (RRMCS) [12].

In our initial investigation, we asked the published version of the RMT-Direction task on both pretests and exams. The RMT-Magnitude task was slightly modified on the pretest version, but the exam version was identical to the published version. On the pretest, we asked students to compare the speeds of points 1, 2, 3, and the center-of-mass (CM) on the pretests. We did so because we expected similar results as reported by others, but we anticipated that this modification could probe student understanding further.

7.1.2 Correct response

Students can use the Galilean transformation of velocities to obtain the correct answer to both the RMT-Direction and RMT-Magnitude tasks. Students also need to realize that rolling without slipping implies that the velocity of point 1 with respect to the road is zero.

The velocity transformation equation for point 2 is $v_2^{(\mathrm{R})} = v_2^{\ \mathrm{C}} + v_{\mathrm{C}}^{(\mathrm{R})}$, where $v_2^{(\mathrm{R})}$ is the velocity vector of point 2 with respect to the road, $v_2^{\ \mathrm{C}}$ is the velocity vector of point *2* with respect to the center-of-mass (CM) of the wheel, and $v_{\mathrm{C}}^{(\mathrm{R})}$ is the velocity vector of the CM of the wheel

with respect to the road. Applying the same rule to point 1, rolling without slipping implies that $v_1^{(\mathrm{R})} = 0$, so the magnitude of $v_1^{(\mathrm{C})}$ is equal to the magnitude of $v_{\mathrm{C}}^{(\mathrm{R})}$. Since the magnitudes of ${v_2}^{\mathrm{C}}$ and ${v_1}^{\mathrm{C}}$ are equal (same distance from the axis of rotation), the direction of ${v_2}^{\mathrm{R}}$ will be 45° above the horizontal. Other variations will have different answers, but the approach is still the same.

The same approach can be used to find the speeds of points 1, 2, and 3 with respect to the road, and hence rank the speeds accordingly. As mentioned, rolling without slipping implies $v_1^{(\mathrm{R})} = 0$. Then, the magnitudes of ${v_2}^{\mathrm{R}}$ and ${v_3}^{\mathrm{R}}$ are $\sqrt{2}v$ and $2v$, respectively. The speed ranking is then $3 > 2 > 1$. Another method for ranking the speeds is to treat point 1 as the point through which the axis of rotation passes through since it has zero speed in the ground frame. The angular velocity is still ω, and the speeds can be calculated using the distances from point 1 to points 2 and 3. The distances are $\sqrt{2}R$ and $2R$, respectively. This method gives the same speed ranking.

Alternatively, students can sketch out the trajectory of a point on the wheel as the wheel travels across the road. The result is a cycloid. However, this approach only allows students to determine the direction of instantaneous velocity for points 2 and 3, but not point 1. In addition, the trajectory has no information about time, so the speeds cannot be ranked easily.

7.1.3 Administration of the RMT

The RMT, or its variations, were administered in the calculus-based introductory mechanics course on the last tutorial pretest in Wi'17, Sp'17, Wi'18, Sp'18, Au'18, and Sp'19 quarters and the final exams in Au'17 and Su'18 quarters. The relevant content was usually covered in lectures during week 9 or week 10 (the last week of the quarter). Students completed some reading questions before that lecture, participated in clicker questions during lectures, and

had some homework exercises related to rolling motion. However, none of the exercises completed by the students looked like the RMT.

Some online pretests were administered before lecture instruction, some after because there were variations in the teaching schedules in different quarters. We indicate whether the pretest results were collected before or after lecture instruction as we present them.

7.1.4 Results for the RMT-Direction task

The student responses for the RMT-Direction task on the pretests before and after lecture instruction on rolling motion are shown in Table 7.29 and Table 7.30, respectively. The percentage correct ranges from 13% to 24%. The pretest results after lecture instruction are within the range of percentage correct before lecture instruction, which suggests that the lecture instruction has no immediate effect on student performance. We use the aggregate result of all pretests, 18%, as the percentage correct for the RMT-Direction task at the pretest. This aggregate result is for convenience, readers who are interested in the pairwise χ^2 test *p*-values can refer to Appendix C.

Table 7.31 shows the student responses to the RMT-Direction question on final exams in Au'17 and Su'18 quarters. The percentage correct for similar contexts ranges from 25% to 36%, and they are within 10% of what has been published [12]. In both the pretest and exams, the most common incorrect answer is that the direction of v_2^{C} is vertical, which corresponds to the instantaneous velocity of point 2 in the center-of-mass (CM) frame.

Table 7.29. Summary of the student responses on the RMT-Direction question on the pretest **before** lecture instruction.

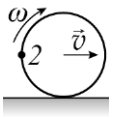
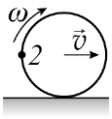
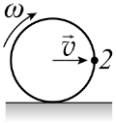
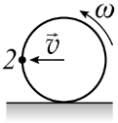
	Pretest results **before** lecture instruction				
	 (N = 399) Wi'17	 (N = 264) Wi'18	 (N = 141) Sp'18	 (N = 119) Sp'18	All (N = 924)
Correct (Relative Motion)	**13%**	**24%**	**21%**	**17%**	**18%**
Incorrect (CM frame)	75%	66%	65%	65%	70%
Other incorrect	13%	10%	14%	18%	12%

Table 7.30. Summary of the student responses on the RMT-Direction question on the pretest **after** lecture instruction.

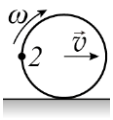
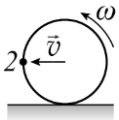
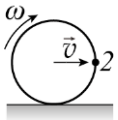
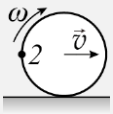
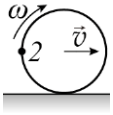
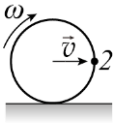
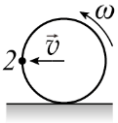
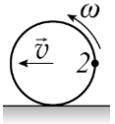
	Pretest results **after** lecture instruction			
	 (N = 279) Sp'17	 (N = 160) Au'18	 (N = 295) Sp'19	All (N = 734)
Correct (Relative Motion)	**18%**	**15%**	**20%**	**18%**
Incorrect (CM frame)	73%	73%	65%	70%
Other incorrect	18%	12%	15%	12%

Table 7.31. Summary of student responses on the RMT-Direction task in the final exam. The published results from RRMCS are included for comparison.

	Results for the RMT-Direction task in the final exam					
	Published RRMCS	University of Washington (N = 747)				
	 (N = 559)	 (N = 389) Au'17	 (N = 130) Au'17	 (N = 133) Au'17	 (N = 96) Su'18	All
Correct (Relative Motion)	**33%**	**34%**	**36%**	**32%**	**25%**	**33%**
Incorrect (CM frame)	48%	56%	50%	55%	62%	56%
Other incorrect	19%	10%	14%	13%	13%	11%

7.1.5 Results for the RMT-Magnitude task

As mentioned, the RMT-Magnitude task on the pretest is different than that in the exam. So, there are some differences in the responses presented in this section.

The student responses for the RMT-Magnitude task on the pretest before and after lecture instruction on rolling motion are shown in Table 7.32 and Table 7.33, respectively. The percentage correct ranges from 18% to 27%. Similar to the RMT-Direction task, our results in the Magnitude task show no significant difference from before to after lecture instruction, which continues to suggest that the lecture instruction has no immediate effect on student performances. We use the aggregate result of all pretests, about 22%, as the percentage correct for the Magnitude task.

Table 7.34 shows how students responded to the RMT-Magnitude question in the final exam. The percentage correct for similar contexts ranges from 36% to 43%, and they are lower than the published RRMCS result, which was 54%.

Table 7.32. Summary of the student responses on the RMT-Magnitude task on the pretest **before** lecture instruction.

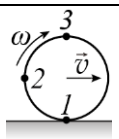
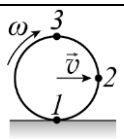
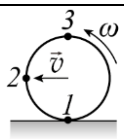
	Pretest results **before** lecture instruction			
	 (*N* = 264) Wi'18	 (*N* = 141) Sp'18	 (*N* = 119) Sp'18	All (*N* = 524)
Correct $v_3 > v_2 > v_{\mathrm{CM}} > v_1$ (Relative Motion)	**21%**	**18%**	**18%**	**20%**
$v_3 = v_2 = v_1 > v_{\mathrm{CM}}$	39%	40%	44%	$v_3 = v_2 = v_1$ 78%
$v_3 = v_2 = v_1 = v_{\mathrm{CM}}$	28%	35%	34%	
$v_3 = v_2 = v_1 < v_{\mathrm{CM}}$	9%	9%	3%	

Table 7.33. Summary of the student responses on the RMT-Magnitude task on the pretest **after** lecture instruction.

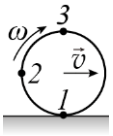
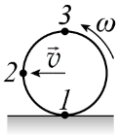
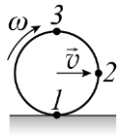
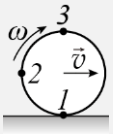
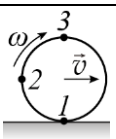
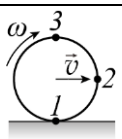
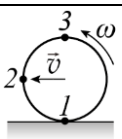
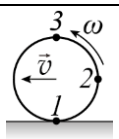
	Pretest results **after** lecture instruction			
	 (*N* = 279) Sp'17	 (*N* = 160) Au'18	 (*N* = 295) Sp'19	All (*N* = 734)
Correct $v_3 > v_2 > v_{\mathrm{CM}} > v_1$ (Relative Motion)	**27%**	**23%**	**22%**	**24%**
$v_3 = v_2 = v_1 > v_{\mathrm{CM}}$	26%	37%	35%	$v_3 = v_2 = v_1$ 68%
$v_3 = v_2 = v_1 = v_{\mathrm{CM}}$	19%	37%	38%	
$v_3 = v_2 = v_1 < v_{\mathrm{CM}}$	9%	10%	4%	

Table 7.34. Summary of student responses on the RMT-Magnitude question in the final exam. The published results from RRMCS are included for comparison.

	Results for the RMT-Magnitude task in the final exam					
	Published RRMCS	University of Washington (N = 744)				
	 (N = 559)	 (N = 386) Au'17	 (N = 130) Au'17	 (N = 133) Au'17	 (N = 95) Su'18	All
Correct $v_3 > v_2 > v_1$ (Relative Motion)	**54%**	**42%**	**41%**	**43%**	**36%**	**41%**
$v_3 = v_2 = v_1$ (CM frame)	32%	48%	51%	42%	45%	47%
Other incorrect	14%	10%	8%	15%	19%	12%

The most common incorrect answer our students selected in all contexts is related to the option $v_3 = v_2 = v_1$, which is the same as the published results. The result in the Sp'17 quarter is 23% lower than the later quarters, which is likely because we asked students to provide a written response on the online pretest while the later versions were multiple-choice. There were more variations in the written responses, and we used the top four responses ($v_3 > v_2 > v_{\text{CM}} > v_1$, $v_3 = v_2 = v_1 > v_{\text{CM}}$, $v_3 = v_2 = v_1 = v_{\text{CM}}$, and $v_3 = v_2 = v_1 < v_{\text{CM}}$) in the written response version as the options for the multiple-choice version on pretests in later quarters. Among the students who selected $v_3 = v_2 = v_1$, many students thought that the speeds of the three points were either greater than or equal to v_{CM}.

7.1.6 Discussion

The results presented in this section suggests that lecture instruction does not have an immediate impact on student performance on the RMT-Direction and RMT-Magnitude tasks on the pretest. However, comparing the averages over all versions, the gains in the Direction and Magnitude tasks from pretest to exam are 15% (pretest: 18%, exam: 33%) and 19% (pretest: 22%, exam: 41%), respectively. One possible reason for these gains is that students revised for the exam and/or that more of them thought through the ideas behind, but they did not do so for the pretests. We tend to believe that students took the pretests seriously, as most students explained their answers properly on the pretests. (Note: students who take the pretests seriously do not imply that they apply the concepts properly.) To compare the performance in both tasks on pretests and exams, we performed the χ^2 test ($\chi^2\ 1\ = 65$, $p < 0.01\%$ for RMT-Direction task, and $\chi^2\ 1\ = 78$, $p < 0.01\%$ for RMT-Magnitude task) and found that the difference is statistically significant, and with a medium effect size.

The most common incorrect answer in both the pretests and exams is one that is consistent with the CM frame. Our results corroborated what was reported by Rimoldini and Singh. Our results also showed how ubiquitous the common errors are across different institutions and among students after relevant lecture instruction in the introductory physics class.

From the student explanations in the Magnitude task on pretests, students who selected $v_3 = v_2 = v_1 > v_{\mathrm{CM}}$ were more likely to reason based on the CM frame because they would say that $v_{\mathrm{CM}} = 0$. Students who selected $v_3 = v_2 = v_1 = v_{\mathrm{CM}}$ could be reasoning based on the CM and the Earth frames. Some determined the speeds of points 1, 2, and 3 in the CM frame, and used the equation $v_{\mathrm{CM}} = R\omega$ to arrive at the answer. We discuss the explanations in detail in the next section.

Rimoldini and Singh did not report on how consistent the students' approaches were in both the Direction and Magnitude tasks in their paper. We did so in Table 7.35. It shows a comparison of the approach students used in the Direction task and that used in Magnitude task on the Au'17 final exam. At least 20% of the students were able to apply relative motion ideas to determine the direction of $v_2^{(\mathrm{R})}$ and rank the speeds properly. Around 60% of the students selected an answer that is consistent with the CM frame at least once on both tasks. The comparison for pretests is not presented because the pretest results suggest that fewer students can apply the correct concepts on both tasks.

Table 7.35. Correlation table (aggregated, *N* = 744) between the RMT-Direction and RMT-Magnitude tasks in the final exam of the Au'17 quarter.

		Magnitude task		
		Correct $v_3 > v_2 > v_1$ (Relative Motion)	Incorrect $v_3 = v_2 = v_1$ (CM frame)	Other incorrect
Direction task	Correct (Relative Motion)	22%	8%	3%
	Incorrect (CM frame)	18%	32%	6%
	Other incorrect	4%	5%	3%

Rimoldini and Singh suggested that students tended to think in a preferred frame (CM frame in this case). Boudreaux also found students sometimes demonstrated preferred frame reasoning in his investigation of student understanding of relative motion. If we interpret the CM frame as the students' preferred frame in this context, then the results show a high prevalence of preferred frame reasoning among our students. We describe student reasoning on pretests in the next section, which gives more insight into student approaches.

Finally, on both the pretests and exam, the percentages correct on both the Direction and Magnitude tasks of variations of the same contexts are around 10%, which suggests that the direction of rolling motion and the location of point 2 do not affect how students approach both tasks. These results indicate that we do not need to use the same context on pretests and posttests to assess the effectiveness of the newly developed tutorial curriculum. What we then need to determine is how far the curriculum can stretch students' understanding within the limited time in the tutorial sessions.

7.2 STUDENT EXPLANATIONS ON THE RMT ON PRETESTS

In the previous section, we presented results from both the RMT-Direction and RMT-Magnitude tasks from RRMCS on the pretests and final exams. The results provided an overview of the level of student understanding of rolling without slipping before and after traditional lecture instruction. They suggest that at least 60% of the students could not apply relative motion concepts on either the Direction or Magnitude task. As mentioned in our literature review in section 6.2.1, Rimoldini and Singh found that students tended to say "The instantaneous velocity with respect to the ground is always tangent to the rolling circle" and "The speed of all points should be the same with respect to the ground because they are all on the same wheel which is rolling" for the Direction and Magnitude tasks, respectively.

In this section, we present our categorizations of student explanations from both tasks when administered on online pretests. The explanations provide more insight into how students interpret the tasks, some of which are similar to those reported by Rimoldini and Singh described above. We also formulate one hypothesis through our interpretations of the student explanations, which we describe later.

7.2.1 Student reasoning in the RMT-Direction task

In this sub-section, we describe the reasoning students used to explain their answers in the RMT-Direction task. We categorized 699 explanations from the pretest responses in Table 7.29 and Table 7.30 (from Wi'18 to Sp'19 quarters). The explanations used by students for their correct and most common incorrect answers are summarized in Table 7.36.

Table 7.36. Student explanations on the RMT-Direction task in the pretest. The results are aggregated from the pretest responses in Table 7.29 and Table 7.30 (from Wi'18 to Sp'19 quarters).

		Student explanations (N = 699)			
		Relative motion	Tangential velocity	Others	Total
Student answers	Correct (Relative Motion)	16%	1%	3%	20%
	Incorrect (CM frame)	-	48%	17%	65%

On the RMT-Direction task, students were asked to determine the direction of velocity of a point on the rim of a rolling wheel at a specific instant. About 16% of the students gave correct explanations using ideas that are consistent with the Galilean transformation of velocities or combining translational motion and rotational motion. An example is given in the student statement below:

> *[The] velocity of a point on a circle moving at constant speed is tangent and in the same direction as the motion. Since the sphere is rolling, [its] center of mass has an instantaneous velocity pointing to the left. So, the instantaneous velocity of the point is the combination of the velocity of the circle and the tangential velocity, which would be B [down and to the left].*

About 65% of the students chose an incorrect answer that is consistent with the CM frame option. Some of the explanations given by these students were consistent with the "preferred frame reasoning," as described by Boudreaux [70], Rimoldini, and Singh [12]. These students only seemed to consider the rotational motion of the cylinder about its CM, and ignore the contribution from translational motion, as in the student statement given in Sp'18 below:

> *The hoop is spinning counter-clockwise, so point [2] at this point is moving straight down.*

Among the students who chose the CM frame answer, about 70% explained using the ideas of tangential velocity or circular motion, a tendency that was also reported by Rimoldini and Singh. Some of the explanations could be interpreted as "preferred frame reasoning." However, it is also possible that students are using the shape of the rolling circle as the trajectory of the point of interest. Consider the student statements below:

- *The velocity would be tangential to the slope of the point.*
- *Instantaneous velocity is always tangent to the curve.*
- *The instantaneous velocity is tangential to the motion of the curve.*

The use of "tangential velocity" or equivalent phrases in explanations suggests that students may not know the actual trajectory of the point in a rolling motion situation. It is also possible that students only rely on the given diagram to arrive at their answers. In any case, such prevalence prompts us to investigate how students apply their prior knowledge of motion in two-dimension in the context of rolling motion readily.

7.2.2 Student reasoning in the RMT-Magnitude task

On the RMT-Magnitude tasks, students were also asked to compare the speeds of points 1, 2, 3, and the CM. See Figure 1.1. We discuss the reasoning students used to arrive at the responses presented in Table 7.32 and Table 7.33.

Most students who answered correctly that $v_3 > v_2 > v_{\mathrm{CM}} > v_1$ (about 22% of the class) used a relative motion approach. This approach takes into account both the translational and rotational motions of the wheel, as demonstrated in the two explanations below:

> *3 is the largest because not only is it rotating to the right at the fastest rate, but the movement of the hoop to the right makes it seem to go even faster from the earth reference frame. 1 is the smallest because it is moving left while the hoop is moving right, so its velocity is small. 2 is larger than CM because 2 is both rotating and being translated to the right, while CM is just being translated to the right without rotating.*

> *3 has the largest velocity because its tangential and rotational components point in the same direction. CM is smaller than 2 because 2 does not have a rotational velocity. 1 has the smallest velocity because its translational and rotational components point in opposite directions.*

Some students explicitly reasoned that point 1 has zero velocity because the wheel is rolling without slipping. For example:

> *In the earth reference frame, point 1's instantaneous velocity is zero because the hoop rolls without slipping, meaning the contact point of the hoop does not move relative to the surface. 3's instantaneous velocity is to the right relative to the center of mass and the center of mass is moving to the right relative to the earth. So the magnitude of 3's instantaneous velocity must be greater than CM. 2's instantaneous velocity points up relative to CM, and CM moves right relative to the earth. So 2 has a greater magnitude of instantaneous velocity compared to CM but not as great as 3 because it's velocity relative to CM and CM's velocity relative to the earth are not in the same direction.*

Although correct, only a few students arrived at the correct ranking of the speeds by treating point 1 as the point of rotation and considered the relative distances between point 1 and other points. One student, for example, stated, "*effectively rank by distance from point of rotation, that is, rank by distance from 1.*"

The correct explanations students provided described the necessary steps to arrive at the answer. These students had demonstrated their understanding of the topic. However, only 10%-15% of the class were able to do so.

Now, we discuss the reasoning students used to justify their incorrect answers.

Students who chose answers that contained $v_3 = v_2 = v_1$ would generally use the equation $v_t = R\omega$ to explain that the tangential speeds of points 1, 2, and 3 are equal. Many explicitly noted that the three points are equidistant from the center of the wheel and that the angular velocities, ω, are the same.

The inclusion of the CM in the speed comparison provided more insights about student approaches than could be obtained in the previous studies.

$v_3 = v_2 = v_1 > v_{\mathrm{CM}}$: About 40% of the students chose this answer. We found a few approaches students used to obtain this answer. The most common approach is related to the CM frame. These students would compare the tangential velocities and then say that $v_{\mathrm{CM}} = 0$ explicitly. The explanations below demonstrate this idea clearly.

> *The center of mass velocity is zero, but the rest of the points all encounter the same magnitude of velocity because the rotational speed is constant.*
>
> *CM's velocity is zero, as it is the center. 1, 2, 3 all have an equal distance away from the center, meaning equal velocities.*
>
> *1, 2, and 3 have tangential velocities but CM has no lever arm therefore no circular velocity.*

The second common approach that led to the answer $v_3 = v_2 = v_1 > v_{\mathrm{CM}}$, suggested a lack of understanding of the rolling without slipping condition. Students would mention that the tangential velocities are larger than v_{CM}, because points 1, 2, and 3 were traveling along the circumference while the CM was moving in a straight line. In some cases, their explanations suggested that they were thinking that $v_{\mathrm{CM}} < R\omega$. Below are some explanations that describe the second approach.

> *The velocity of 1, 2 and 3 are the same because their velocities are tangent to the object. CM is smaller than they because when the object roll a quarter of the circle, CM only moved the radius of the circle.*
>
> *The instantaneous velocity on the hoop goes farther than the velocity of center of mass of the hoop during the same time interval.*
>
> *All points on the hoop (1, 2, 3) are at the same radius away from the center on the same circular object, meaning that they experience the same tangential magnitude from the rotation of the object, whereas point CM is located at the center of mass, which does not move rotational, only translational to the center of mass of the hoop. This is less than the points outward on the hoop in magnitude of velocity.*

In the third approach yielding $v_3 = v_2 = v_1 > v_{\mathrm{CM}}$, students appeared to know that they needed to add the center-of-mass velocity and tangential velocity due to rotational motion. However, they did not add the two velocities vectorially. They seemed to add the two magnitudes of the velocity vectors, and since there is no tangential velocity due to rotation for the CM, hence $v_3 = v_2 = v_1 > v_{\mathrm{CM}}$. Below are some examples of the student explanations.

> *1, 2, 3 all have rotational velocity along with translational velocity. CM only has translational.*

1, 2, 3 are all rolling so they have tangential or rotating velocity as well as center of mass/translational velocity so they all are equal but more than C which is just the center of mass velocity.

The object has translational and rotational velocity. Assuming there is constant rotational velocity, the points 1, 2, and 3 will appear to have great velocity. Translational velocity + rotational velocity.

$v_3 = v_2 = v_1 = v_{\mathrm{CM}}$: Next, we describe the explanations students gave for the answer $v_3 = v_2 = v_1 = v_{\mathrm{CM}}$. Most of these students tended to combine the two equations, $v_t = R\omega$ and $v_{\mathrm{CM}} = R\omega$, to arrive at their answers. The explanations below demonstrate the ideas clearly.

I think they would all be moving with the same velocity because the hoop moves at constant speed and rolls without slipping so when one point on the outside of the hoop is displaced one full circle (the circumference of the circle), the point in the center would also have to move forward by that much.

The hoop rolls without spinning and the points along the edge of the hoop all move with the same magnitude of the velocity, and the speed of the edge of the circle is the same as the center.

Since points 1, 2, 3 are all on the rim of the hoop, their magnitudes of V always same. Since this hoop rolls without slipping, the V of point CM is equal to the V of center of mass, which is equal to the V of point 1, 2 and 3.

$v_3 = v_2 = v_1 < v_{\mathrm{CM}}$: Lastly, a small group of students chose $v_3 = v_2 = v_1 < v_{\mathrm{CM}}$. Their answers suggested that the students thought that $v_{\mathrm{CM}} > R\omega$. Some of them may have confused tangential velocity with angular velocity. Here are some explanations.

The center of mass is translating, but since the rest of the points are on the rim, and the hoop is not slipping, they have an instantaneous velocity of 0.

Because points 1, 2, 3 are equidistant from the center of mass of the hoop, so they will be moving at the same speed. However, CM will be moving at a speed greater

than the other points because $v_{\mathrm{CM}} = R\omega$, so the rotational speed of the other points will be equal to the velocity of the center of mass divided by the radius, meaning cm velocity must be larger.

$v = R\omega$, so velocities of 1, 2, 3 will be equal because radius is constant and rotational velocity is also constant. the translational velocity of the center of mass will be the greatest as it is undergoing only translational velocity.

In summary, students who gave the correct answer generally demonstrated that they applied the concepts correctly. For students who selected the incorrect answers that contained $v_3 = v_2 = v_1$, some of them explained using the CM frame and did not consider the translational motion of the wheel. Some students in all three incorrect options demonstrated that they did not understand the rolling without slipping situation and the formula $v_{\mathrm{CM}} = R\omega$. Lastly, some students did consider both v_{CM} and the tangential velocity due to rotational motion, but they did not add the velocities vectorially. Our first two difficulties corroborated the findings of Rimoldini and Singh. We found additional difficulty (the last one) that Rimoldini and Singh did not report.

7.2.3 Discussion

Based on the results on the RMT-Direction task, students' uses of phrases such as "tangential velocity" and "tangent to the curve" suggests that many were using the shape of the given circle to arrive at their incorrect answers. We hypothesized that students might incorrectly be using a "tangent to the curve" approach in situations that require students to determine the direction of velocity when a curve is given in the problem. Based on this, we performed an in-depth investigation to probe this hypothesis. The details are described in the next chapter.

The last difficulty that we found in the RMT-Magnitude task (students add vectors as if they are scalars) was common in vector superposition, especially when the two vectors are of the

same magnitude. We investigated this difficulty among students in the algebra-based physics courses. More details can be found in Part IV.

7.3 STUDENT UNDERSTANDING OF FRICTION IN ROLLING MOTION

This section describes the student responses to the various questions involving the role of friction in rolling motion in pretests and final exams at the University of Washington. We also included one question from the RRMCS for comparison (details in 7.3.1). Those responses allow us to establish a baseline to illustrate the level of student understanding in this topic after traditional lecture instruction. All the questions are related to the frictional force acting on the rolling or slipping cylinder/sphere either on a horizontal surface or an inclined plane.

7.3.1 Friction on a rolling object (horizontal)

The first question (Figure 7.2) is a question from the RRMCS. It involves the effect of different surfaces on the motion of objects that are rolling without slipping on a horizontal surface. In this question, students need to understand that the marbles are rolling without slipping, the frictional forces acting on them are zero, so the correct answer is B.

Among the 559 students who participated in Rimoldini and Singh's investigation, only 20% answered correctly. The most common incorrect answer (46%) is option C, which coincides with the rule that "friction opposes motion." Students also need to apply the $f_s = \mu_s N$ relation. However, the relation is only applicable for maximum static friction.

Two identical rigid marbles roll without slipping across rigid horizontal floors. One rolls on a stone floor with coefficient of static friction $\mu_s = 0.80$, and the other rolls on a glass floor with $\mu_s = 0.40$. Which marble is slowed down more by friction, and why? Ignore air-resistance.

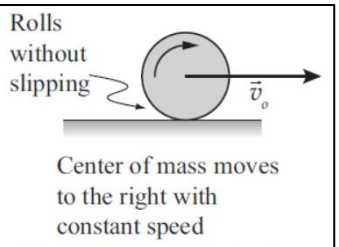
A. Both marbles are slowed equally because the marbles themselves are identical.
B. Neither marble is slowed by friction because both roll without slipping.
C. The marble rolling on stone is slowed more, because the greater μ_s makes the force of friction on it greater.
D. The marble rolling on glass is slowed more, because the slippery nature of the glass impedes rolling.
E. It is impossible to answer without knowing the coefficient of kinetic friction μ_k because the marbles are moving.

Figure 7.2. Friction on rolling objects in RRMCS. The correct answer is B.

For students to know that the friction acting on the marble is zero, they could either observe the motion and remember the conclusion or go through some form of reasoning that goes like this:

- Suppose the friction opposes the motion, the center-of-mass velocity (v_{CM}) decreases.
- The friction generates a torque that increases the angular velocity (ω) of the marble.
- The magnitude of the velocity at the point of contact (point P) in the ground frame follows the equation $v_{\mathrm{P}}^{(\mathrm{G})} = v_{\mathrm{CM}} - R\omega$, so when v_{CM} decreases and ω increases, $v_{\mathrm{P}}^{(\mathrm{G})}$ is not zero.
- Any friction in this situation will cause the velocity at the point of contact not to be zero, which contradicts with the fact that the marble continues to roll without slipping. So, the only possibility is that friction must be zero.

A rigid cylinder rolls to the right without slipping (*i.e.*, there is no relative movement between the surface and the lowest point on the ball). The coefficient of static friction between the cylinder and flat surface is $\mu_{static} > 0$.



It is observed that the center of mass of the cylinder moves with constant speed (*i.e.*, **it does not slow down**).

22. [5 pts] While the cylinder is rolling with constant speed without slipping, which of the following best describes the direction of the frictional force on the cylinder by the flat surface?

A. The frictional force on the cylinder by the flat surface is **to the left.**

B. The frictional force on the cylinder by the flat surface is **to the right.**

C. The frictional force on the cylinder by the flat surface is **upward.**

D. The frictional force on the cylinder by the flat surface is **zero.**

Figure 7.3. Friction on rolling cylinder (horizontal). The correct answer is D.

The second question (Figure 7.3) describes a rigid cylinder that is rolling without slipping on a rough horizontal surface. The cylinder is moving with constant speed. Students need to determine the direction of the frictional force on the cylinder while the cylinder is rolling with constant speed on the horizontal surface.

Students need to recognize that the acceleration of the cylinder is zero since the motion is uniform. By Newton's 2nd law, the net force is zero, so there is no horizontal force acting on the cylinder. Therefore, the frictional force on the cylinder by the flat surface is zero.

Only 20% of the 167 students who responded in Au'12 answered correctly. The most common incorrect answer (45%) is option A, which indicates that the frictional force is to the left. Option A coincides with the rule that "friction opposes motion," so when the cylinder is moving right, the frictional force points left. This finding suggests that students tend to use a memorized rule, rather than using the changes in motion given in the question to arrive at their answer. We also asked this question to 184 students in the pretest of the Au'19 quarter. Only 11% of them answered correctly and 68% chose option A. Clearly, students were unable to apply Newton's laws to objects on rolling motion after lecture instruction.

Although the two questions and the populations sampled are different, the percentages in the correct answer and the most common incorrect answer are about the same. The tendency for students to apply the "friction opposes motion" rule is quite strong.

7.3.2 Friction on a rolling object (inclined plane)

The first question (Figure 7.4) describes a uniform sphere that is rolling down a ramp without slipping. Students need to pick the correct extended free body diagram of the sphere when it is rolling down the ramp. The gravitational force by the earth and the normal force by the ramp are present in all options; students need to decide whether there is a frictional force acting on the sphere, and if there is, they need to decide the direction and the type of friction.

A uniform sphere of mass 0.60 kg rolls down a ramp without slipping. The sphere is released from rest from a height of 0.70 m above the bottom of the ramp.

Which of the following is the best extended free body diagram for this problem? F_N is the normal force of the ramp on the sphere, F_W is the gravitational force of the earth on the sphere, f_k is the kinetic friction between the ramp and the sphere, and f_s is the static friction between the ramp and the sphere. Note diagrams A and B are different from D and E.

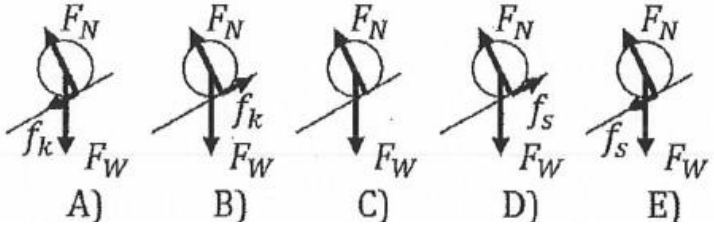


Figure 7.4. Sphere rolling down the ramp. The correct answer is D.

To get the correct answer, students need to recognize there is no slipping, so the friction can only be static. The sphere is moving with increasing speed as it goes down the ramp, so its

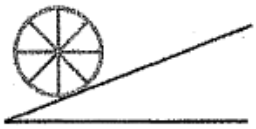
v_{CM} and ω are increasing. In order to have an angular acceleration and hence a net torque to cause ω to increase, the frictional force must point up the ramp. Therefore, the answer is D.

Out of the 501 students who responded in Sp'15, 62% of them answered correctly. The most common incorrect answer (25%) was B. Thus, around a quarter of the class did not make a distinction between static and kinetic friction. This question is not able to distinguish whether students apply the rule "friction opposes motion" or not. However, the next question can do so.

The second question (Figure 7.5) is asking students to determine the type and the direction of the frictional force acting on the wheel when it rolls without slipping up an incline.

A wagon wheel consists of a uniform outer ring of mass 7.0 kg with negligible thickness and eight uniform spokes that extend from the center to the outer ring. The wheel has radius 0.60 m and moment of inertia about the center of mass of 4.0 kg m^2. It rolls without slipping up an incline. The initial angular speed of the wheel about the center of mass is 3.0 rad/s. The magnitude of the angular acceleration of the wheel about the center of mass is 4.9 rad/s^2.

While the wheel rolls up the incline what is the type of the friction force and in what direction?



A. Kinetic friction directed up the ramp
B. Kinetic friction directed down the ramp
C. Static friction directed up the ramp
D. Static friction directed down the ramp
E. There is no friction.

Figure 7.5. Wheel rolling up the incline. The correct answer is C.

The correct answer is obtained by considering how the motion changes when the wheel rolls up the incline and how that relates to the forces acting on the wheel. The wheel is slowing down, so its center of mass velocity decreases in magnitude. Its angular velocity decreases as well. There is a linear acceleration, and therefore the net force directed downward along the incline. The direction of angular acceleration is pointing out of the page, as is the net torque. The normal

force and the gravitational force are not generating torque about the center of mass, so the only way for the net torque about the center of mass to point out of the page is for the friction to point up the ramp. The frictional force is static because the wheel rolls without slipping, so the answer is C.

Among the 429 students who answered the question in Wi'16, 35% answered correctly. The most common incorrect answer (47%) is option D. This shows that most of the students (82%) recognized that the type of frictional force is static friction. Students who chose options B and D were consistent in applying the rule "friction opposes motion," and around 55% of the students did so. The prevalence of the indiscriminate use of the rule is high.

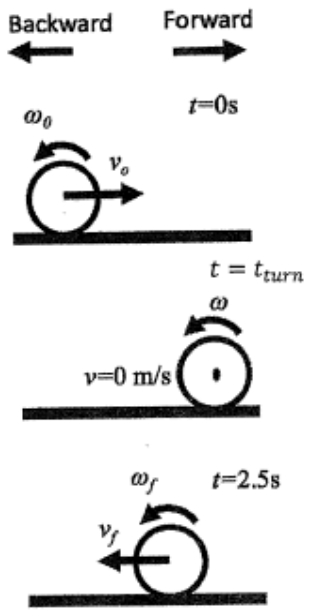
7.3.3 Friction on a slipping object (horizontal)

The question in Figure 7.6 describes how the motion of a slipping ball that moves rightward initially, turns around and moves leftward after some time. Students need to determine the type and direction of the frictional force that is acting on the ball while it is rolling with slipping.

Students can arrive at the correct answer by considering the changes in the motion of the ball. From $t = 0$ s to $t = t_{\text{turn}}$, the ball is slowing down, so the acceleration is to the left, the net force is to the left. From $t = t_{\text{turn}}$ to $t = 2.5$ s, the ball is speeding up to the left, so the acceleration and the net force are to the left. The net force on the ball is to the left throughout. The ball is slipping throughout, so the friction is kinetic, the answer is A.

In the Wi'16 quarter, 48% of 222 students answered correctly. The most common incorrect response (36%) is option B. Most students (84%) recognized that the friction is kinetic. Students who may use the rule "friction opposes motion" would select B and D, which consists of 40% of the class. This question demonstrates a high prevalence of the indiscriminate use of the rule.

As shown, at time $t = 0$ s, a uniform ball with mass $m = 0.16$ kg and radius $R = 0.030$ m is spinning at an angular speed ω_0 and has center-of-mass speed of $v_0 = 0.20$ m/s. Note that the ball is initially slipping on the ground. As shown at t_{turn}, the ball reverses its direction of travel while still slipping. At $t = 2.5$ s the ball stops slipping, and reaches a final center-of-mass speed of $v_f = 0.05$ m/s in the opposite direction to the initial motion. The friction force between the ball and the table is 0.016 N.



What type of frictional force acts on the ball while it is rolling with slipping, and what is the direction of this force?

A. Kinetic friction directed to the left throughout
B. Kinetic friction directed to the left when the ball moves forward and to the right when the ball moves backward
C. Static friction directed to the left throughout
D. Static friction directed to the left when the ball moves forward and to the right when the ball moves backward
E. None of the above

Figure 7.6. Slipping ball. The correct answer is A.

7.3.4 Commentary

Table 7.37 summarizes the common student difficulties found in the five questions presented previously. The tendency among students to apply the rule "friction opposes motion" ranges from 40% to 55%, which corroborated what Rimoldini and Singh had found. We found 25% of the students were unable to distinguish static and kinetic friction in the question involving a sphere rolling down the ramp. In sum, these two difficulties are prevalent among students.

Table 7.37. Common student difficulties in the dynamics of rolling motion problems.

Problem	Difficulties	Prevalence
Rolling cylinder (Figure 7.3)	Indiscriminate use of the rule "friction opposes motion."	45%
Rolling marbles (Figure 7.2)	Indiscriminate use of the rule "friction opposes motion."	46%
Sphere rolling down incline (Figure 7.4)	Failure to distinguish static or kinetic friction	25%
Wheel rolling up incline (Figure 7.5)	Indiscriminate use of the rule "friction opposes motion."	55%
Slipping ball (Figure 7.6)	Indiscriminate use of the rule "friction opposes motion."	40%

7.4 SUMMARY

We investigated student difficulties with the kinematics of rolling motion using the Rolling Motion Task from the RRMCS. Results from post-lecture instruction (without tutorial) indicated that students tended to analyze the motion from the CM frame of the wheel. The student performance on the online pretests showed the same tendency.

Our investigation generally corroborates with what Rimoldini and Singh had reported. However, there is a strong tendency for students to justify their directions of velocities by using the given shape of the object in the task as the trajectory of the point of interest. This finding suggests that students may not be using preferred frame reasoning, as reported by previous literature. We decided to investigate further by looking at the tendency of students in using the given shape to justify their answers using different tasks, which we discussed in the next chapter. We also found that some students added vectors as if they are scalar quantities.

The questions on the role of frictional force in rolling motion in the past PHYS 121 exams served as a good baseline for how students decided on the types of friction acting on the rolling

object and its direction after lecture instruction. In most cases, students tended to overgeneralize the “friction opposes motion” rule in all situations that involve frictional forces, especially when static friction was involved. We also found that students are unable to distinguish static or kinetic friction in one instance. It might be beneficial to students if the tutorial curriculum also addresses this difficulty.

Chapter 8. INVESTIGATION OF STUDENT DIFFICULTIES WITH ROLLING MOTION II – FURTHER INVESTIGATIONS RELATED TO ROLLING MOTION TASKS

The previous chapter (Chapter 7) presents results from replication studies that we conducted on student understanding of rolling motion. These involved giving students in our introductory calculus-based course questions from the Rotational and Rolling Motion Conceptual Survey [12] developed at the University of Pittsburg. The questions we used were both based on the Rolling Motion Task (RMT) that involves a disk rolling without slipping on a level surface. One question (RMT-Direction) asked students to determine the direction of the velocity of a point on the rim of the wheel at a given instant. The other (RMT-Magnitude) asked students to rank the speeds for three different points on the rim of the wheel.

The authors of the test and others had interpreted student responses in terms of a tendency of students to use the center-of-mass motion of the wheel when thinking about the motion of a point on the translating and rotating wheel. We observed, however, that students who answered the RMT-Direction task incorrectly tended to explain using "tangent to curve" ideas. We hypothesized that these students could be using the shape of the wheel given in the question to arrive at their answer or that difficulties with the idea of instantaneous velocity might be part of the problem. Both suggested it might not be sufficient to think of the students as simply neglecting the translational motion of the wheel. This observation led us to develop a series of questions to probe student thinking about rolling without slipping in greater detail. This 4-part investigation is the subject of this chapter.

In the first investigation, discussed in section 8.1, we were interested in the extent to which students could think about the motion of the point on the wheel over an extended period of time

and not just the motion as reflected by the instantaneous velocity. We also wanted to examine how students were consistent in using the "tangent of curve" idea. We presented students with the cycloid that corresponds to the trajectory of a point on the rim of a cylinder that is rolling without slipping. We then asked students about the instantaneous velocity and compared the results to those from the RMT-Direction task.

In section 8.2, we describe a task to probe student ability to think about the motion of the point over a short time and use that knowledge to determine the direction of instantaneous velocity. In order to do this, we modified the RMT-Direction task by asking students to consider the displacement of the point on the rim of the rolling cylinder first, then determine the direction of velocity. Our results showed that students did better on the Direction task under this modification. We also asked a similar version in one final exam that students would sketch the trajectory and determine the direction of the velocity of the point on the rim at one instant. Students also performed better in the latter version.

In section 8.3, we present results from another modification to the RMT. This version was intended to find out how students responded to the Direction and Magnitude tasks with rigid bodies of different shapes, such as frisbee, rod, and plus-shaped object. The results would inform us about how student responses may change according to the given shape and circumstance (e.g., absence of the ground or side-view vs. top-view), assuming the lecture instruction was about the same.

Finally, in section 8.4, we were interested in verifying a claim made by Fang and Uziak. They said that "students could answer simple relative motion problems (e.g., determining absolute and relative velocities of a passenger walking in a train), but were unable to apply the same ideas to the problem of two points on a rigid body." We did this by looking at the correlation of student performance between some typical relative motion problems with the RMT-Direction task.

8.1 INVESTIGATION 1 – TENDENCY OF STUDENTS TO USE THE TANGENT TO THE SHAPE OF AN OBJECT TO DETERMINE THE VELOCITY OF A POINT ON THE OBJECT

This portion of the investigation was motivated by our observations in Chapter 7 that on the RMT-Direction task, many students stated the instantaneous velocity of a point on the rolling disk was tangent to the disk and explained their reasoning using phrases like "the velocity is tangent to the curve." This could simply have been a memorized response based on the definition of instantaneous velocity developed for point objects. It could also have been a result of a tendency to reason based on the center-of-mass motion of the wheel and to ignore the translational motion of the wheel. We were interested, however, whether students understood the motion of the point over an extended period of time.

We decided to pursue this question by providing students with the overall trajectory of the point on the wheel during an extended period for the motion of the wheel. This curve is a cycloid for a wheel rolling and translating without slipping.

8.1.1 Study design and task administration

The study design is shown in Figure 8.1. We administered two different tasks for two groups of students in a given quarter (administered according to their lecture sections). These were given as part of a pretest given to the students 1 week (sometimes 2 weeks) before they attempted the Direction task described in section 7.1.

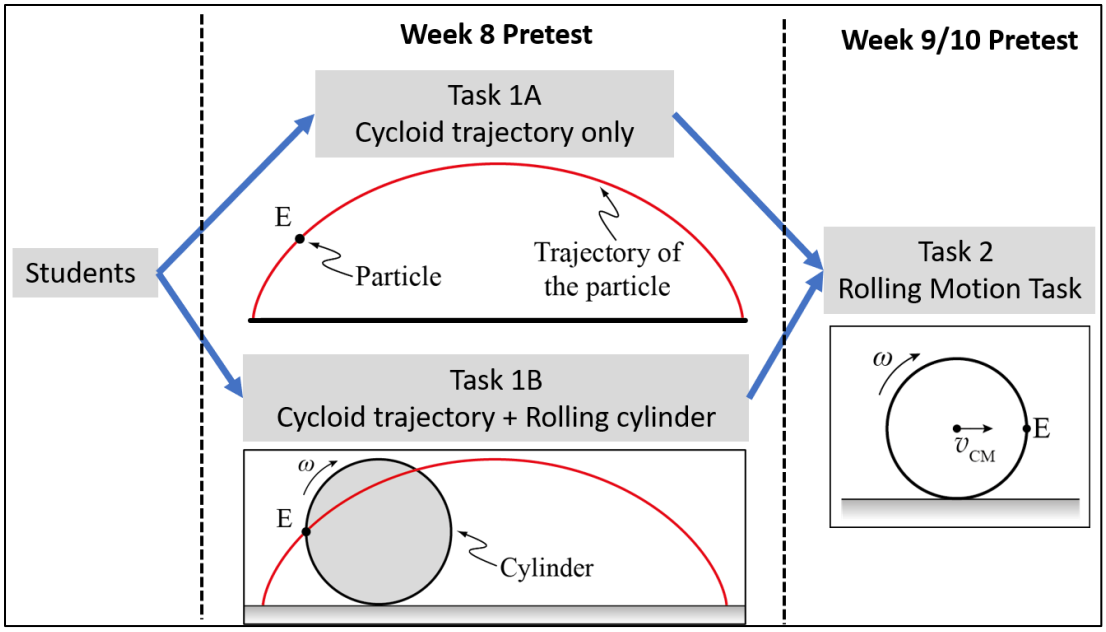


Figure 8.1. The schematics of the study design to find out the prevalence of students using the given curve to determine the direction of velocity.

In the first pretest (week 8), students from lecture section A were given Task 1A, and students from lecture section B were given Task 1B. Task 1A consists of a picture of a cycloid and told that it represents the motion of a point particle. Students from lecture section B were shown the rolling cylinder from the Direction Task with the cycloid that results from the motion of a point on it. Both groups were shown the same point E on the cycloid and asked to find the direction of the instantaneous velocity at point E. The following week or after two weeks (week 9 or 10), both groups were administered the Direction Task (called Task 2 in this study). In Task 2, they are given a cylinder that is rolling without slipping and asked to find the direction of the instantaneous velocity of a point on the rim of the cylinder.

The correct answer for both Tasks 1A and 1B is a rightward arrow pointing 45° above the horizontal. The correct answer for Task 2 is a rightward arrow pointing 45° below the horizontal. This answer can be obtained using relative motion, as described in Chapter 7.

There were several comparisons made possible by this study design.

First, by comparing the results from Task 1A and Task 1B, we can determine how showing the wheel (the object on which the point is located) affects student ability to identify the direction of the instantaneous velocity of the point. Second, by comparing the results from Task 1B and Task 2, we can determine the extent to which showing students the trajectory of point E on the cylinder affects their ability to determine the direction of instantaneous velocity at point E. Third, by examining the student responses on the same two tasks (Task 1A and Task 2, Task 1B, and Task 2), we can tell the extent to which students use the "tangent to curve" idea to arrive at their answers on both tasks.

We conducted this study in both the Sp'18 and Sp'19 quarters. In Sp'18, the students attempted both pretests **before** lecture instruction on rolling motion. We repeated this study in the Sp'19 quarter to see if the results were replicable. The only difference between the two quarters was that the students in Sp'19 attempted Task 2 **after** lecture instruction. Based on our preliminary investigation (see section 7.2.1), we did not expect the results in Task 2 to change significantly.

8.1.2 Results and discussion

The results obtained in both the Sp'18 and Sp'19 quarters are summarized in Figure 8.2. In the Sp'18 quarter, 92% of the students answered Task 1A correctly, while only 59% of the students answered Task 1B correctly. Around 30% of the students chose the upward arrow on Task 1B. In these three groups of students, most of them (~70%) explained their answer using "tangent to curve." In Task 1B, students who answered correctly mostly used the cycloid as the curve, while students who chose the upward arrow used the circle as the curve. We labeled the upward arrow as the answer that corresponds to the direction of v_{E} in the CM frame.

In Task 2, around 20% of the students answered correctly, and 65% of the students answered downward direction, an answer that is consistent with the CM frame. Most students who chose the downward arrow in Task 2 also chose the answer that was tangent to the cycloid in Tasks 1A and 1B, and the answer that was tangent to the circle in Task 1B.

We obtained similar results to the Sp'18 quarter when we repeated our study in the Sp'19 quarter. The only difference was in the results obtained for Task 1B. The percentage correct was 47% in the Sp'19 quarter, a 12% decrease from the Sp'18 quarter. We performed a 2×3 χ^2 test (2 quarters, 3 answer categories) to determine whether the students who attempted Task 1B in the Sp'18 quarter was different from that of the Sp'19 quarter. The χ^2 statistic is 0.38, with a *p*-value of 0.83, which suggests that the students in both quarters are not significantly different.

Based on our results, we performed a few statistical tests to determine the extent to which showing the wheel or the trajectory (cycloid) affect student ability to identify the direction of the instantaneous velocity of the point?

We determined the effect of the circle on Task 1 by conducting χ^2 test with a 2 × 3 contingency table (2 groups of students, 3 answers choices in Tasks 1A and 1B). The χ^2 statistic is 35, and the *p*-value is nearly zero, which suggests that the inclusion of the circle influences the student performance significantly. The McNemar test statistic is calculated with the responses from Task 1B and Task 2 to determine the effect of the cycloid, and the *p*-value was also nearly zero. This result suggests that the cycloid strongly affects student performance on the same task. In summary, the features on the given diagram, be it the circle or cycloid, affects the student performance to determine the direction of instantaneous velocity significantly.

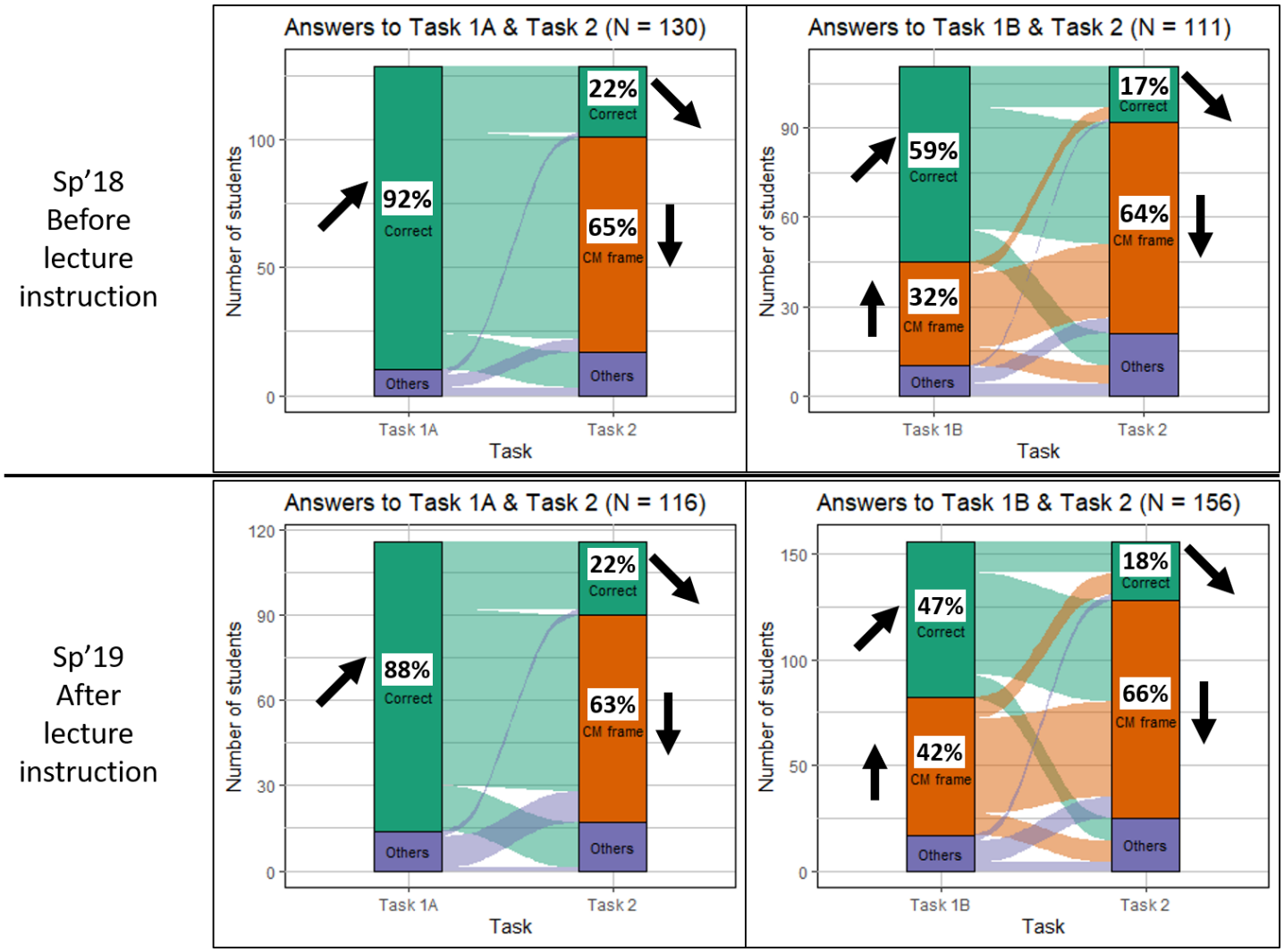


Figure 8.2. The results of the study in both the Sp'18 and Sp'19 quarters. In the Sp'18 quarter, students attempted Task 2 **before** they had lecture instruction. Students attempted Task 2 **after** they had lecture instruction in the Sp'19 quarter. The arrows refer to the answers chosen by the students. The percentages represent the sample proportion.

To determine the extent to which students use the "tangent to curve" idea to arrive at their answers on Tasks 1A and 2 and Tasks 1B and 2, we selected a few student explanations for Tasks 1A, 1B, and 2 from the Sp'18 quarter. Table 8.38 lists the student explanations in the Task 1A to Task 2 sequence, and Table 8.39 lists the explanations in the Task 1B to Task 2 sequence. The explanations are categorized according to students' answers. The explanations in each row are from the same student.

In the Task 1A to Task 2 sequence, students who answered both tasks correctly used different ideas to explain their answers. They used "tangent to the curve" to explain their answer in Task 1A and the addition of velocities in Task 2. However, for students who answered Task 1A correctly and chose the downward arrow (answer associated with the CM frame) in Task 2, they used "tangent to the curve" to explain their answers in both tasks.

In the Task 1B to Task 2 sequence, some student explanations exhibited similar patterns as in the previous questioning sequence. That was true for students who answered both tasks correctly and students who answered Task 1B correctly but chose the downward arrow in Task 2. Students who chose the upward arrow in Task 1B and chose the downward arrow in Task 2 also used "tangent to the curve" to explain their answers.

Based on the students' explanations, most students who answered both Task 1 and Task 2 correctly used different approaches to explain their answers on both tasks. In comparison, around 50% of students who chose the downward arrow in Task 2 explained their answers using "tangent to curve" in both Tasks 1 and 2. The latter result suggests that there is a tendency for students to use the given curve to explain their choice of the direction of instantaneous velocity. Lastly, we found no significant difference in the types of reasoning students used in the Sp'19 quarter.

Table 8.38. Selected student explanations on Task 1A (T1A) and Task 2 (T2). The student responses in each row are from the same student.

T1A answer ↓ T2 answer	Task 1A – Cycloid only	Task 2 – Cylinder only
Correct ↓ Correct	*It is moving upward to the right so the instantaneous velocity must point upward and to the right.*	*Because the instantaneous velocities are in the Earth's reference frame the velocity would be pointing forward and down.*
	The direction of its instantaneous velocity is the tangent line of the curve	*Because the hoop is rolling towards the right side and E is getting closer to the ground as the hoop rolls.*
	The direction is up and to the right because the instantaneous velocity direction represents the tangent to the position curve, so it is the direction it is traveling at that instant.	*For a spinning object that is rotating clockwise, the point E would be going down. However, there is also a velocity to the right from the translational motion of the object, so it would be down and to the right.*
Correct ↓ CM frame	*Instantaneous velocity is tangent to the path of the particle.*	*Velocity is tangent to the spinning object and in the direction of the motion so the velocity will point directly down for point E.*
	The velocity is the tangent line of the curve	*The velocity is the tangent to the point*
	The velocity is tangential to the trajectory	*The instantaneous velocity is tangential to the rotation.*
	Instantaneous velocity is tangent to the trajectory path at any given point.	*The instantaneous linear velocity is tangential to the position relative to the path.*
	The instantaneous velocity is tangent to arc or rotation	*Tangent to the circle at that point when accounting for the rotation direction as well*

Table 8.39. Selected student explanations on Task 1B (T1B) and Task 2 (T2). The student responses in each row are from the same student.

T1B answer ↓ T2 answer	Task 1B – Cycloid and cylinder	Task 2 – Cylinder only
Correct ↓ Correct	*Tangent to the path*	*The instantaneous direction of its motion is dependent on the translational and rotational velocity*
	The instantaneous velocity of a point is tangent to its path.	*The rotational component of the velocity points down while the translational component of the velocity points to the left.*
Correct ↓ CM frame	*Tangent to the path drawn out by the red line.*	*Instantaneous velocity of circular motion is tangent to the circle.*
	I took the tangent line	*Its velocity is tangent to the circle*
	Since the velocity should be tangent to the path of motion.	*The hoop is rolling to the left and the instantaneous velocity would be tangent to the rotational velocity so at E its direction would be straight down.*
CM frame ↓ CM frame	*Instantaneous velocity is tangent to the edge of the circle rotating so it would be pointing straight up*	*The instantaneous velocity is tangential to the motion of the curve*
	The instantaneous velocity should be a tangent line of the circle.	*The line velocity at any instant should point to the direction of the tangent line*
	I think the instantaneous velocity is [~~the~~] on the same line as the tangent line.	*The instantaneous velocity is parallel to the tangent line at a point. At point E, the tangent line is vertical. Since the ball is rolling to the left, the velocity is downward.*

8.1.3 Commentary

We designed a study to find out the prevalence of students using the given curve to determine the direction of velocity from Sp'18 and Sp'19 quarters. We found that students tended to explain their chosen direction of velocity by using tangential velocity, suggesting that they were using the given curve to arrive at the answer. When two curves (circle and cycloid) were given in the question, the results showed that students tended to use the cycloid to determine the answer. The results obtained on Task 2 were about the same before and after students had lecture instruction.

Based on the student explanations, it was apparent that some students were using similar ideas to explain their answers. We were curious if we could use computer algorithms to compare student explanations. So, we used natural language processing (NLP) to analyze student explanations on both tasks. More details can be found in Chapter 15. The preliminary results appeared to be promising, which suggested that there might be a potential of applying NLP in physics education research.

Lastly, students tend to use the given curve to determine the direction of velocity. They do not apply relative motion concepts to rolling motion readily. One can design activities that help students bridge motion in two-dimension ideas with relative motion concepts together to strengthen their understanding of either topic.

8.2 INVESTIGATION 2 – THE MODIFIED RMT-DIRECTION TASK

The results from the previous section show that students tend to use the given curve to determine the direction of velocity. The subsequent investigation is to determine whether students could figure out the trajectory or displacement of a point on the rim of a rolling wheel. Two additional questions are raised: (1) how well can students imagine the displacement of a point on the rim of a rolling wheel? And (2) is there a correlation between the ability to draw the trajectory (cycloid) and do the Rolling Motion Task (Direction) correctly?

To probe these two questions, the questioning sequence is changed. For the first question, we asked students to think of the direction of displacement of a point on the rim of the rolling hoop first, before the actual RMT-Direction task on pretests. For the second question, we asked them to sketch the trajectory for the point on the rim of a rolling wheel on a final exam. These two questions are called the modified RMT-Direction (with displacement / trajectory) tasks.

8.2.1 Student responses to the modified RMT-Direction (displacement) task on pretest

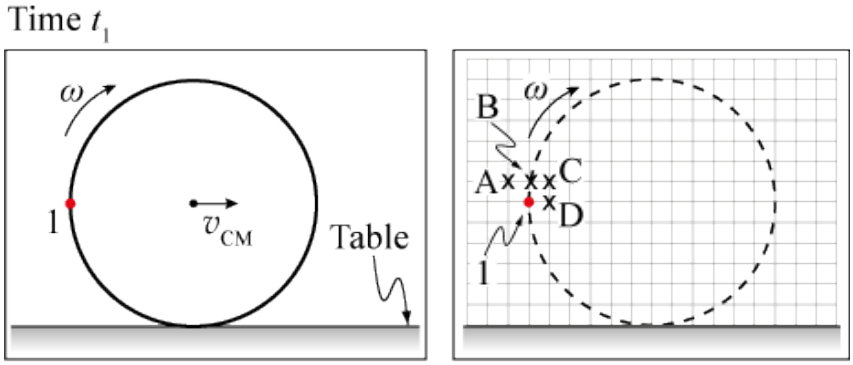
The modified RMT (displacement) on the pretest is shown in Figure 8.3. Point 1 (on the rim of the rolling hoop) was labeled on the diagram. In the displacement task, we asked students to select the position point 1 would be a short time after t_1. The options were chosen, such that there would be a matching arrow in the RMT-Direction task. This questioning sequence allows us to find out if students are consistent in their answers, and how the performance in the Direction task is affected by having a displacement task before that.

The correct answers for the displacement and the Direction task through the relative motion approach are options C and B, respectively. In contrast, the most common incorrect answers are

options B and D, respectively. These incorrect answers are consistent if students were to consider the direction of velocity in the center-of-mass frame.

We administered this sequence of questions in the Au'18 and Wi'19 quarters. Students did these questions *after* lecture instruction in the Au'18 quarter; *before* lecture instruction in the Wi'19 quarter.

A hoop rolls without slipping to the right on a stationary table as shown below. The center of the hoop moves with constant speed. A small red spot is marked on the hoop (point 1). The diagram shows the red spot at time t_1.



Displacement task:
A grid is marked on the wall behind the loop. Which point on the wall best corresponds to the location of the red spot a short time later?

RMT-Direction task:
For the instant shown, which arrow below best shows the direction of instantaneous velocity of point 1 on the hoop in the Earth's reference frame?

A. B. C. D. E. Zero vector

Figure 8.3. The modified RMT-Direction (displacement) task. Students are first asked to think about the direction of the displacement before the direction of velocity on the pretest.

The results from the Au'18 quarter are summarized in Table 8.40. Out of the 191 students who responded, the correct proportions who answered the displacement and RMT-Direction tasks

correctly were 71% and 64%, respectively. Fifty-seven percent of the class answered both tasks correctly. The proportion of students who answered the velocity task correctly in this sequence of questioning was about three times higher than when only the velocity task was administered in the same setting (see section 7.1).

The results from the Wi'19 quarter is tabulated in Table 8.41. The results from this quarter inform us whether lecture instruction influences student performance when we compare with the results from the Au'18 quarter. The proportions of the students who answered the displacement and velocity tasks correctly were 72% and 56%, respectively. Around 50% of the students answered both tasks correctly.

We performed χ^2 test using the results for students who answered both tasks correctly in both quarters. The test statistic ($\chi^2\ 1\ = 3.27$, *p*-value = 0.07) suggests that there is no significant difference in the percentage correct in the Wi'19 (before lecture instruction) and the Au'18 (after lecture instruction) quarters. It appears that students had some intuition on how the point should move in the ground frame when they were asked about the displacement before or after lecture instruction. However, when they were given the circle alone, they tended to select the direction of velocity that was consistent with the direction of the point were to move in the CM frame.

Another observation we can make from these results is that students who answered the displacement task correctly are more likely to answer the RMT-Direction task correctly. The performance on the RMT-Direction task significantly improved after students did the displacement task.

Table 8.40. Correlation table of student responses (N = 191) on the modified Rolling Motion Task (with displacement) on the pretest **after** lecture instruction in the Au'18 quarter.

		RMT-Direction task		
		Correct (Relative Motion)	Incorrect (CM frame)	Other incorrect
Displacement task	Correct (Relative motion)	57%	8%	6%
	Incorrect (CM frame)	5%	17%	3%
	Other incorrect	2%	2%	1%

Table 8.41. Correlation table of student responses (N = 454) on the modified Rolling Motion Task (with displacement) on the pretest **before** lecture instruction in the Wi'19 quarter.

		RMT-Direction task		
		Correct (Relative Motion)	Incorrect (CM frame)	Other incorrect
Displacement task	Correct (Relative motion)	49%	19%	4%
	Incorrect (CM frame)	6%	12%	3%
	Other incorrect	1%	3%	3%

8.2.2 Student responses to the modified RMT-Direction (trajectory) task on exam

The modified RMT-Direction (trajectory) task is shown in Figure 8.4. Instead of asking students to determine the final position of a point on the rim, we asked students to sketch the trajectory of the point as the cylinder rolls across the surface.

We administered this task in the final exam of the Wi'19 quarter. Due to some snow days in that quarter, the school was closed, and the tutorial schedule was revised. Students were unable to do the tutorial curriculum we designed for improving the understanding of rolling motion. The

lecture instruction for all PHYS 121 classes in the quarter ended on the rolling motion. We asked this sequence of questions to find out the effect of lecture instruction.

A cylinder rolls to the right without slipping on a level surface with the constant center-of-mass velocity shown. Point *E* (the black dot) is a point on the rim of the cylinder.

Trajectory task: On the diagram below, sketch the trajectory of point *E* as the cylinder rolls across the surface.

RMT-Direction task:
At some instant, point *E* is located as shown at right. Draw an arrow on the diagram to represent the direction of instantaneous velocity of point E in the ground frame. Explain.

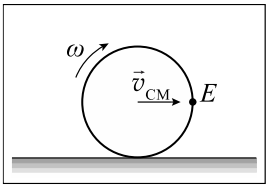


Figure 8.4. Modified RMT-Direction (trajectory) on the final exam.

The correct curve for the trajectory sketching task is a cycloid. However, not all students would draw a perfect cycloid during exams. Moreover, not everyone knows the correct curve is a cycloid, and some students drew a sinusoid instead. We considered both the cycloid and sinusoid as correct answers for the trajectory task. In the RMT-Direction task, the direction for v_{E} is an arrow pointing in the south-west direction.

We summarized the responses of 203 students in Table 8.42. Fifty percent of the students sketched the cycloid, and 25% of them sketched a sinusoid, summing up to 75% correct on the trajectory task. On the RMT-Direction task, 65% answered correctly. Overall, 53% of them answered both tasks correctly. These results were comparable to what we obtained from the pretest, both on the level of individual tasks and combining the two. There were apparent

differences in the tasks on the pretest and final exam at the surface level, but the tasks in both sequences were of the same nature. Most students explained their direction of v_{E} using relative motion ideas, and less than ten students referenced their sketched trajectory to explain their answers.

Table 8.42. Correlation table of student responses (N = 203) on the modified RMT-Direction (trajectory) on the final exam of the Wi'19 quarter.

		RMT-Direction task		
		Correct (Relative Motion)	Incorrect (CM frame)	Other incorrect
Trajectory task	Correct (Cycloid or sinusoid)	53%	20%	2%
	Incorrect	12%	10%	3%

We observed the same trend as in the previous task. That is, on this modified RMT-Direction (trajectory) task, students who answered the trajectory task correctly were more likely to answer the RMT-Direction task correctly. The performance on the RMT-Direction task improved after students did the trajectory task.

However, on the RMT-Direction task, a higher percentage of students explained their answers using relative motion but not the curve. We believe it is due to a combination of these reasons: (1) students had their last lecture instruction on rolling motion, (2) students revise for the final exam, (3) the question was asked in a free-response format instead of a multiple-choice format, and (4) students did the trajectory task before the RMT-Direction task.

The first two reasons are possible as students tend to focus on topics that were last taught before the final exam. For the third and fourth reasons, as students are asked to write down their explanations on the RMT-Direction task after doing the trajectory task, students may be more readily to apply the relative motion concepts in rolling motion contexts.

8.2.3 Commentary

On both modified RMT-Direction tasks, the student performances on the pretest and the final exam were about the same, where 50%-60% of the students answered both the displacement/trajectory and RMT-Direction tasks correctly. Moreover, the percentages correct on the RMT-Direction task in this questioning sequence were two to three times higher than that when the students were asked to respond to the task itself. It appeared that the displacement/trajectory task helped the students to answer the RMT-Direction task. These results suggested that it might be productive for students to think about the displacement or trajectory of the point during the in-class instruction. The details of how we incorporate this finding into the modified tutorial curriculum can be found in section 10.4.

One point to note is that we are not able to compare the effect of the displacement or trajectory tasks on the RMT-Direction task directly. The displacement task was administered on the pretest, while the trajectory task was given to students on the final exam. Based on our experience, students view the pretest and final exam differently; they prepare more for the final exam than the pretest. However, if we were to ask the trajectory task on the pretest online, it may not be feasible. The reason is that we were unable to ask students to submit sketches online, and choosing a sketch from a list of options is not the same from sketching it directly.

8.3 INVESTIGATION 3 – RMT INVOLVING DIFFERENT RIGID BODIES

The RMT in the RRMCS only involves a cylinder rolling on a horizontal surface. Our lecture instruction introduces the rolling motion in the context of cylinders, spheres, or hoops moving on a horizontal or inclined surface. The idea of combining translational and rotational motions can be extended to any rigid body motion. However, there are no reported results on how students apply the idea of combining motions to new contexts such as rod and hoop.

Our results previously have shown a high tendency for students to use the curve to explain their chosen direction of velocity. Another area of interest of this investigation is to find out how students would respond if the given rigid body is not drawn as a circle.

In this section, we expanded the scope of our investigation by having different rigid bodies (hoop or rod) moving in different manners (translating and spinning in the air or on a frictionless surface). We asked the Direction and Magnitude tasks involving these objects. The results inform us about how different contexts affect student performances as compared with the actual RMT.

8.3.1 RMT-Direction task involving different rigid bodies

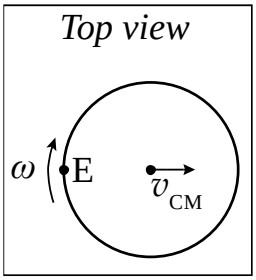
We chose four different rigid bodies in this study: a hoop, a rod, an L-shaped, and a 'plus'-shaped object, as shown in Figure 8.5. The hoop context is to compare with the rolling cylinder context to see if the presence of the surface affects student performance. We compare the student responses from the hoop and rod contexts to see the effect of different shapes of the rigid bodies. The same goes for the L-shaped and 'plus'-shaped objects. These contexts were administered in pretests from Wi'18 to Au'19 quarters. The students answered the rod and hoop contexts before they had lecture instruction on rolling motion but not for the L-shaped rod and 'plus'-shaped object. Note that no students did all four contexts in one quarter.

(a) Hoop

A hoop of radius *R* spins horizontally with an angular velocity of ω in the air as shown at right. The speed of the center of mass, v_{CM}, is equal to $R\omega$.

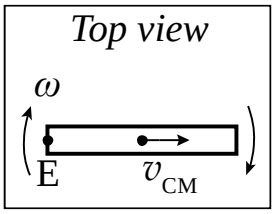
For the instant shown, which arrow below best shows the direction of v_E in the Earth's reference frame?

A. ↘ B. ↗ C. → D. ↑ E. Zero vector



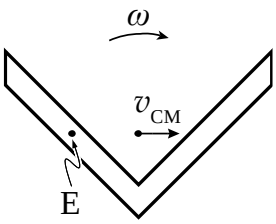
(b) Rod

A uniform rod of length 2*R* is moving to the right and rotating on a level, frictionless table. The rod rotates clockwise about its center-of-mass with an angular velocity ω. The speed of the center of mass, v_{CM}, is equal to $R\omega$.



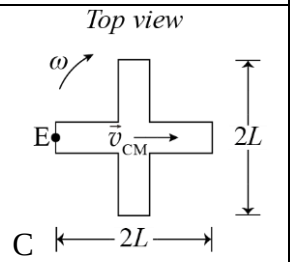
(c) L-shaped rod

A uniform L-shaped rod is moving to the right and rotating on a level, frictionless table. The rod rotates clockwise about its center-of-mass with an angular velocity ω.



(d) 'Plus'-shaped object

An object of dimensions $2L$ by $2L$ is moving to the right and rotating on a level, frictionless table. The object rotates clockwise about its center-of-mass with an angular velocity ω. The speed of its center-of-mass, v_{CM}, is equal to $L\omega$.



Question for the rod, L-shaped rod, and the 'plus'-shaped object:
For an observer standing at rest next to the table, which arrow below best represents the direction of v_E at the instant shown?

A. ↖ B. ↗ C. → D. ↑ E. Zero vector

Figure 8.5. Rolling Motion Task (Direction) with different rigid bodies: (a) hoop, (b) rod, (c) L-shaped rod, and (d) 'plus'-shaped object.

The results on the RMT-Direction question with different rigid bodies on pretests are tabulated in Table 8.43. Around one-third of the class answered the contexts with flying hoop and

sliding rod, and two-thirds of the class answered the task with an L-shaped rod correctly. The most common incorrect answer for both the hoop and rod contexts was those that were consistent with the observer at the CM of the object, but the rod context was 20% lower than the hoop context. For the L-shaped rod context, the most common incorrect answer (18%) was the direction of v_{CM}.

The percentage correct of the RMT-Direction question with cylinder context on pretest is around 20% on average, while it is around 33% for the flying hoop context. We did the χ^2 test ($\chi^2\ 1\ = 11, p < 0.01$), which shows that the presence of the surface is statistically significant with a small effect size.

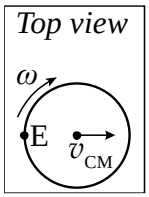
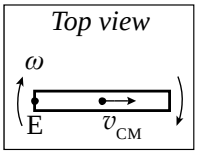
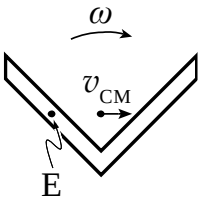
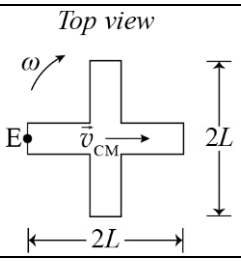
Comparing the student responses for the hoop and rod contexts, the percentages correct is the same, but the proportions of students choosing the vertical arrow (up or down) differ by 20%. It seems that the shape of the given object not only affects the percentage correct in some cases (see section 8.1); it can also affect the proportion of the most common incorrect answer.

Analysis of student explanations in the hoop and rod contexts showed students using similar ideas when the context is rolling cylinder. Most students who answered correctly explained using relative motion ideas, and students who chose upward or downward arrow tended to mention tangent to the circle (hoop) / sketched circle (rod) in their explanations.

Surprisingly, students performed the best in the L-shaped context out of all the rolling-motion related questions. Not many students chose the upward arrow, probably because there is no visible circle that can be drawn from the center-of-mass of the object. A brief analysis of 152 student explanations from Au'18 quarter revealed that 31% of them explained their correct answer using relative motion, and the other 20% of them explained their correct answer by stating that the rod is rotating about the tip. This finding suggested that some students thought that the rod is not rotating about its center-of-mass. If a circle was drawn from the center of the tip, then the tangent

line to that circle at point E is parallel to the direction of the velocity. The result of 67% correct may not be representative that students could attempt this context better than other contexts since some students were arriving at the correct answer with the incorrect reasoning.

Table 8.43. Summary of the student responses on the RMT-Direction task with different rigid bodies in pretests from pre-lecture and post-lecture instruction.

	Results from pre-lecture instruction		Results from post-lecture instruction	
	Hoop (N = 291) Wi'18, Sp'18	Rod (N = 324) Su'18, Sp'19, and Su'19	L-shaped rod (N = 313) Au'18, Sp'19	'Plus'-shaped (N = 230) Au'19
				
Correct (Relative Motion)	↗ 33%	↗ 34%	↗ 67%	↗ 55%
Incorrect (CM frame)	↑ 58%	↑ 38%	↑ 12%	↑ 28%
Incorrect ($\vec{v}_{CM}$ only)	→ 4%	→ 15%	→ 18%	→ 12%
Other incorrect	5%	13%	3%	5%

In the Sp'19 quarter, students were asked to determine the location of the center-of-mass of the L-shaped rod before choosing the direction of v_E. Among the 161 students, 42% found both the position of the center-of-mass (61% correct only) and the direction of v_E (64% correct only) correctly. We can estimate around 30-40% of the students could determine the direction of v_E with the correct reasoning on this L-shaped rod context.

Lastly, we asked students to consider the 'plus'-shaped object in the Au'19 quarter. Fifty-five percent of the students answered correctly; most of them explained using the addition of the tangential and center-of-mass velocities. Students who chose the upward arrow tended to explain using tangential velocity in the context of circular motion. These students only focused on the rotational motion or the motion in the center-of-mass frame, but not the translational motion of the object. In the same quarter, we found that around 50% of the students answered the RMT-Direction task correctly in the pretest. This result suggested that there were gains from the lecture instruction, unlike the results obtained from earlier quarters, as presented in section 7.1. It could also be due to improvement in the lecture instruction over the quarters. We included this observation because the results did not suggest that students tended to perform better on RMT-Direction tasks with different rigid bodies. One possible takeaway was that the continuous improvement in the lecture instruction led to better student performances on pretests.

8.3.2 RMT-Magnitude task involving different rigid bodies

The rigid bodies we used for the RMT-Magnitude task were the hoop and the 'plus'-shaped object, as shown in Figure 8.6. We asked students to rank the magnitudes of the velocities of points D, E, and F. These two rigid bodies were comparable to the original RMT-Magnitude task that the speeds of the points of interests could be ranked at the same instant. We administered these questions in the Wi'18, Sp'18, and Au'19 quarters. The correct answer is the same as that in the original RMT-Magnitude task, which is $v_D > v_E > v_F$.

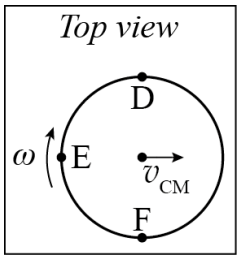
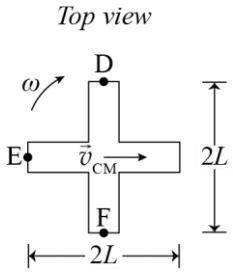
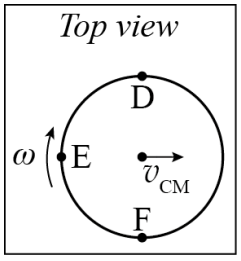
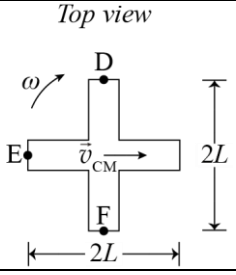
(a) Hoop A hoop of radius *R* spins horizontally with an angular velocity of ω in the air as shown at right. The speed of the center of mass, v_{CM}, is equal to $R\omega$. **Task:** Rank the magnitudes of the velocities of points D, E, and F on the hoop in the **Earth's reference frame**, from ***largest*** to ***smallest***.	
(b) 'Plus'-shaped An object of dimensions $2L$ by $2L$ is moving to the right and rotating on a level, frictionless table. The object rotates clockwise about its center-of-mass with an angular velocity ω. The speed of its center-of-mass, v_{CM}, is equal to $L\omega$. **Task:** Rank the magnitudes of the velocities of points D, E, and F on the hoop in the **Earth's reference frame**, from ***largest*** to ***smallest***.	

Figure 8.6. Rolling Motion Task (Magnitude) with different rigid bodies: (a) hoop and (b) plus-shaped object.

Table 8.44. Summary of the student responses on the RMT-Magnitude task with different rigid bodies in pretests.

	Pre-lecture instruction	Post-lecture instruction
	Hoop (*N* = 291) Wi'18, Sp'18	'Plus'-shaped (*N* = 230) Au'19
		
Correct $v_D > v_E > v_F$	32%	45%
CM frame $v_D = v_E = v_F$	66%	50%
Other incorrect	2%	5%

The student responses are summarized in Table 8.44. The percentage correct for the hoop and 'plus'-shaped object contexts are 32% and 45%, respectively. These results were greater than that in the original RMT-Magnitude task (~20% correct, see section 7.1). The most common incorrect error for both contexts is $v_D = v_E = v_F$, and it was selected by at least half the class.

Table 8.45 shows the correlation table for the student responses between the RMT-Direction and RMT-Magnitude tasks involving the hoop and 'plus'-shaped object in pretests. We found that student approaches to both tasks were mostly consistent. If students used relative motion concepts to answer the Direction task, then they were highly likely to answer the Magnitude task correctly. The same observation can be found for students who only consider the motion of the object in its center-of-mass (CM) frame.

Table 8.45. Correlation table between the RMT-Direction and RMT-Magnitude tasks involving different rigid bodies in pretests.

			Magnitude task		
			Correct $v_D > v_E > v_F$ (Relative Motion)	Incorrect $v_D = v_E = v_F$ (CM frame)	Other incorrect
Direction task	Hoop	Correct (Relative Motion)	25%	4%	4%
		Incorrect (CM frame)	7%	54%	5%
		Other incorrect	-	1%	-
	'Plus'-shaped	Correct (Relative Motion)	38%	13%	3%
		Incorrect (CM frame)	2%	26%	-
		Other incorrect	4%	10%	2%

As mentioned previously, the pretest results for the RMT-Direction (wheel) task obtained in the Au'19 quarter were much better than other quarters. It was also true for the RMT-Magnitude task. The percentage correct for the corresponding RMT-Magnitude (wheel) task was 45%, which was not different than what we found for the 'plus'-shaped object. Based on what we found so far, there was some evidence of a difference in student performance with different rigid bodies. However, the continuous improvement of lecture instruction can improve student performance in the RMT-Direction and RMT-Magnitude tasks, both in the wheel context and other rigid bodies.

8.3.3 Commentary

In summary, students tend to perform better on the RMT-Direction task with different rigid bodies other than the rolling cylinder contexts in pretests. For the case of the flying hoop, the difference could be associated with the absence of a surface. A smaller proportion of students chose the upward arrow in the rod and L-shaped rod context. This difference could be associated with the shape of the rigid body. Since people tend to relate circular or rotational motions to circles, so it is possible that students do not put themselves into the CM frame when analyzing the direction of velocity for non-circular rigid bodies that translate and rotate at the same time.

Better results were also seen in the RMT-Magnitude task. Additional investigations revealed that the continuous improvements in the lecture instruction lead to an improvement in student ability in applying relative motion concepts in general rigid body motions, as observed in the better student performance in the pretest.

8.4 INVESTIGATION 4 – CORRELATION BETWEEN RELATIVE MOTION AND RMT

In the previous sections, we have presented how student performances on the Rolling Motion Task (Direction) are affected by the curves given on the diagram, and by answering a displacement task or sketching the trajectory of the same question. In this section, we turn our attention to the student understanding of relative motion, which is the underlying idea for determining the velocity of a point on a moving rigid body.

When students are introduced to relative motion concepts, the working examples usually involve two moving objects and two reference frames. The two reference frames involved tend to be stationary and moving frames. For example, two cars, A and B, are moving toward each other at speed v (ground/stationary frame), then the speed of A relative to B (B's/moving frame) is $2v$. However, when it comes to rolling motion, students usually treat this situation to be of one moving object (rolling cylinder) and one reference frame (ground frame). According to what Chi *et al.* have found, novice students tended to rely on surface features to answer physics questions, as compared to physics experts [59]. We posit that our students, being novices, are relying on the surface features in the context. If that is the case, then we may expect that the student performance on relative motion questions that involve two moving objects and two reference frames to be better than the rolling motion task. That would also verify what Fang and Uziak had claimed, as they said that students were unable to transfer what they learned in simple relative motion situations to rolling motion problems.

We first looked at student performance in typical relative motion problems that involve two moving objects and two reference frames. Then, we designed some tasks that were analogous to the RMT-Direction task but also involved two moving objects and two reference frames. The student performances in the new task and the RMT-Direction task are compared and discussed.

8.4.1 Student performance in typical relative motion problems

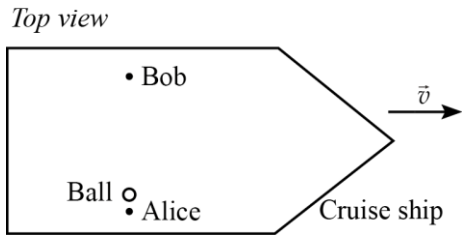
We discuss student performance in two typical relative motion problems. They are typical because they can be found in the end-of-chapter problems in textbooks.

The first relative motion problem we used to find out student understanding of relative motion is called the moving-ball-on-moving-boat problem (ball-boat context), shown in Figure 8.7. Alice and Bob are on a cruise ship that is moving to the right in the water. Alice throws a ball directly to Bob. Chris observes the motion of the ball in a helicopter hovering in the air. Students are asked to select the option that best describes the direction of the velocity of the ball in the reference frame of Chris. The two moving objects in this question are the ball and the boat, while the two reference frames are of Alice's and Chris's (the ground).

According to the Galilean transformation of velocity, the velocity of the ball in Chris's frame is the sum of the velocity of the ball in Alice's frame and the velocity of Alice in Chris's frame, *i.e.*, $v_B^{(C)} = v_B^{(A)} + v_A^{(C)}$. The velocity of the ball in Alice's frame is upward, and the velocity of Alice in Chris's frame is rightward, so the velocity of the ball in Chris's frame is pointing rightward and upward (option 2). Since the magnitudes of the velocities are not known, option 2 is the best option among the four. We administered this question in the pretests in the Sp'18 and Sp'19 quarters.

Alice and Bob are on a cruise ship in still water as shown. The cruise ship is moving to the right in the ground frame. Alice throws a ball directly to Bob.

Chris, who is in a helicopter hovering in the air (not moving relative to the ground) above the cruise ship, observes the motion of the ball.



Which of the following best describes the direction of the velocity of the ball as observed by Chris?

(1) (2) (3) (4)

Figure 8.7. The moving-ball-on-moving-boat problem (ball-boat context).

The results of the ball-boat context are presented in Table 8.46. Seventy-two percent of the students answered correctly, and most of them explained using velocity addition. Some students chose the upward arrow did not recognize that the ball was already moving to the right in Chris's frame before Alice threw the ball to Bob. Some students who chose the arrow that points upward and leftward (arrow 1) described the motion of the ball as if Chris was also moving rightward.

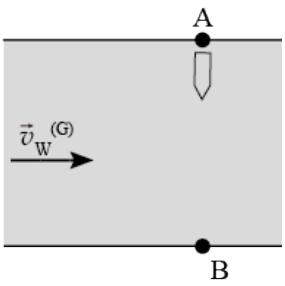
This relative motion problem involves the direct addition of two velocities. The results suggest that most students can answer this conceptual relative motion problem correctly. However, it is not enough to conclude with one instance. We look at the student performance in another typical relative motion problem.

Table 8.46. Student responses on the ball-boat context in pretests.

Answer	% class (N = 303)	Selected student explanations
(Correct)	72%	• *Since the ball is moving vertically, [it's] velocity is also upward. Since the entire ball and ship system is also moving to the right in reference to Chris, he would perceive it to be moving up but also to the right.* • *cruise ship + throw of ball = resultant vector* • *The ball will look like it travels to the right because the entire boat is moving. However, it is actually traveling straight from Alice to Bob in the boat frame of reference.*
	15%	• *I think it will still be a vertical trajectory because even though Chris is not moving relative to the ground frame, Alice, the ball and Bob are all moving to the right in the ground frame. Thus, the ball, Bob, Alice should be on the same line no matter what. So, the ball should travel in a straight line in Chris's eyes.* • *Alice throws the ball, applying only force in the +y direction (the direction running between Bob and Alice, with positive being defined as up towards bob). Ignoring air resistance, the ball has no reason to accelerate laterally (in either x-direction). Therefore, from the perspective of Chris, the ball will move straight up in y-direction.*
Others	13%	• *Since the ship is moving to the right, in Chris' perspective the ball would be left behind by the ship, therefore it must move towards the left.* (Answer: upward and leftward arrow, 9%) • *Although in Alice's and Bob's reference frame the ball moves straight from one to the other, because they are moving to right relative to Chris's reference frame, Chris observes the ball as going to the right with the same horizontal velocity as the cruise ship is moving at.* (Answer: rightward arrow, 4%)

The second relative motion problem we used is called the moving-boat-in-moving-water problem (boat-water context), shown in Figure 8.8. In this problem, a boat travels from point A to point B (vertically downward of A) directly in a river that is moving rightward. Students need to determine the direction of the velocity of the boat in the ground frame, and then the vector triangle that describes the Galilean transformation of velocity correctly. The two moving objects in this problem are the boat and the river, while the two reference frames involved are the ground frame and the water frame.

A boat is located at point A on the riverbank shown at right. The water in the river flows to the right at a speed of 3.0 m/s relative to the ground. The boat travels at a speed of 5.0 m/s relative to the water.



Q1: Suppose the boat needs to travel directly from point A to point B. Which of the following best describes the direction of the velocity of the boat in the **ground** frame?

(1) ↘ (2) ↙ (3) ← (4) ↓

Consider these 3 vectors:

$v_B^{(G)}$: velocity of the boat in the ground frame

$v_B^{(W)}$: velocity of the boat in the water frame

$v_W^{(G)}$: velocity of the water in the ground frame

Q2: As before, the boat needs to travel from point A to point B directly. Which of the following vector diagrams correctly illustrates the relationship of these 3 vectors?

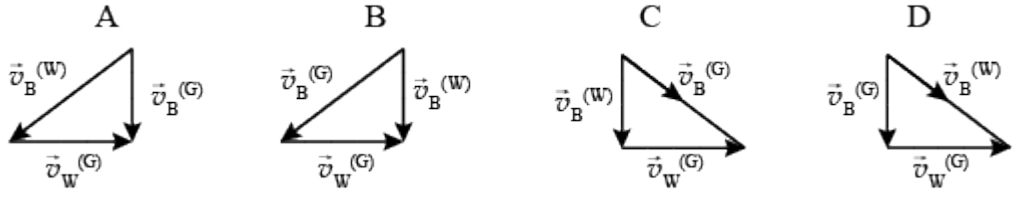


Figure 8.8. The moving-boat-in-moving-water problem (boat-water context).

The direction of the velocity of the boat in the ground frame, $v_B^{(G)}$, when it travels from A to B, is represented by a downward arrow. The answer is obvious since the vertical line is the trajectory the boat will take to go from point A to point B directly. The relationship between $v_B^{(G)}$, the velocity of the boat in the water frame, $v_B^{(W)}$, and the velocity of the water in the ground frame, $v_W^{(G)}$, would be $v_B^{(G)} = v_B^{(W)} + v_W^{(G)}$. The correct vector triangle that describes the velocity transformation equation is option A. The student responses were collected from the Sp'18 and Sp'19 quarters.

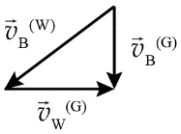
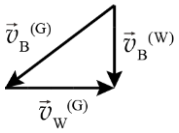
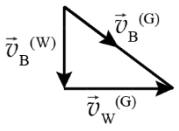
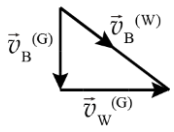
The results for the direction of $v_B^{(G)}$ are summarized in Table 8.47. Only 17% of the class chose the correct arrow. Most students chose the arrow that is pointing left and downward, which is the possible direction of $v_B^{(W)}$. Some of the explanations suggested that students were putting themselves in the boat and imagining how they would steer the boat to get the boat from A to B, not realizing that this direction was the velocity of the boat in the water's frame. Twenty-seven percent of the students thought that the boat was directed downward, and because of the action of the water, the boat ended up pointing right and downward in the ground frame.

The results for the vector diagram involving the three velocities on the moving-boat-in-moving-water problem are summarized in Table 8.48. Thirty-six percent of the students answered correctly. Thirty percent chose option B, and 24% chose option C. Here is how we interpreted students' answers. Those who chose A most likely understood the situation and the Galilean transformation of velocity. Students who chose B or C conflated the meaning of $v_B^{(G)}$ and $v_B^{(W)}$, but only option C followed the correct velocity addition equation. Finally, students who chose D knew that $v_B^{(G)}$ had to point downward but did not understand the velocity addition equation.

Table 8.47. Students' pretest responses for the direction of the velocity of the boat in the ground frame on the boat-water context.

Answer	% class (N = 304)	Selected student explanations
(Correct)	17%	• *In the ground frame, the boat will be moving straight down if it goes directly from point A to point B.* • *If it travels directly, then the velocity vector must point directly from A to B.* • *Since the ground frame is not affected by the moving water, and since B is directly underneath A, the velocity vector should just be pointing straight downwards.*
	55%	• *The current will push the boat downstream, therefore the boat must point upriver of point B to hit point B.* • *Since the current will push the boat to the right, the boat must propel itself towards the left, so it reaches point B.* • *It needs to travel downwards with a slight deviation to the left to compensate for the horizontal right vector of the river.* • *There is the velocity of the water pointing to the right so the boat must be moving in some direction left to counter that motion. And there must be some velocity directed downward to travel to point B.*
	27%	• *Vector addition points to the southeast direction because the boat is heading south but a force going to the east is affecting its trajectory.* • *It is the addition of the river vector and the boat vector of velocity since the river affects the motion of the boat. The boat goes down towards point B but is also being pushed to the right by the river at the same time, so its velocity is down and to the left.* • *because the boat is traveling towards B and the current is also pushing the boat to the right, the boat will move both down and to the right.*

Table 8.48. Students' pretest responses for the vector diagram involving the three velocities on the boat-water context.

Answer	% class (N = 304)	Selected student explanations
A  (Correct)	36%	• *from the perspective of the ground, the boat is traveling directly to B, but from the perspective of the water, the boat is going against the current at an angle* • *The velocity of the boat would look like it's moving straight from the ground frame, but the velocity with respect to the water would be moving towards the left.* • *A makes the most sense, it has all the parts, and the two vectors make that resultant of velocity of the boat in the ground direction in the correct path*
B 	30%	• *The velocity of the water in ground frame will always push towards the right. The boat, in order to compensate for this push, will travel more to the left in the ground frame. When these two vectors work together, the final location is point B.* • *The ground frame takes into account for both the water and the boat in the water. It will see the boat traveling straight, but the boat to the water it will look diagonal.* • *The vector of velocity of the in relation to the boat and water is vertically down because that is the velocity of the boat. But in relation to the ground the boat is moving to the left.*
C 	24%	• *Vector addition points to the southeast direction because the boat is heading south but a force going to the east is affecting its trajectory.* • *The velocity vectors of the water and boat have to add up to equal the final motion of the boat from the ground's reference frame, so it's C.* • *The boat's vector sum is equal to the sum of both component vectors. The boat velocity is down, and the water velocity is right so the boat with respect to ground is down and right.*
D 	10%	• *The ground velocity is heading straight down, the velocity of the boat in the water frame is travelling slightly sideways, and the velocity of the water in ground frame is travelling to the right at a 90 degree angle to the velocity of the boat in ground frame.*

The direction of $v_B^{(G)}$ and the vector diagram questions are related. For example, the arrow (4) is matched with vector diagrams A and D, the arrow (1) is matched with vector diagram C, and arrow (2) is matched with vector diagram B. We found that the answers given by around half the class were consistent. That is, 10% of the students answered both questions correctly, 19% chose arrow (1) and vector diagram C, and 26% of them chose arrow (2) and option B. The explanations given by students with consistent answers (not all) were consistent with each other as well.

Twenty-three percent of the students chose arrow (2) and the correct vector diagram. Their answers were not consistent with each other; the inconsistencies were also reflected in some of their explanations. They did not realize that they conflated the meaning of $v_B^{(G)}$ and $v_B^{(W)}$. Only one student realized the inconsistencies and indicated that in their explanations quoted below:

> Direction of $v_B^{(G)}$: *I am not very sure of this answer, but in the ground frame, it would appear as if the boat is going in the upstream direction in order to maintain a straight line to reach point B.*
>
> Vector diagram: *Now I am realizing my answer to the previous question is probably incorrect. It seems more accurate that in the water reference frame, the boat would be traveling upstream (in order to maintain the straight path against the river flow). Meanwhile, in the ground reference frame (for instance, someone standing stationary outside of the water), it would just look like the boat moves straight across. Finally, the velocity of the water is to the right in all frames.*

We presented results from two typical relative motion problems: the ball-boat and boat-water contexts. The student performances are of two extremes. Around 70% of the students answered the moving-ball-on-moving-boat context correctly, while only 10% did so for the moving-boat-in-moving-water context (Note that these are not the same students). Based on these two results, we could say that the percentage correct for any typical relative motion problem lies between 10% to 70%.

Even though we claim that these two problems are similar in terms of two moving objects and two reference frames, there are some differences between them. The first difference is in the starting and ending points. The two points are moving in the ground frame in the ball-boat context, but not moving in the boat-water context. The second difference is in the pictorial representation of the moving frame, *i.e.*, there is a shape that represents the boat in the ball-boat context, but a shaded space to represent the water in the boat-water context.

The third difference lies in the expected question to be asked in the two problems. In the ball-boat context, the most reasonable question to be asked is to find both the magnitude and direction of the velocity of the ball in Chris's frame. In the boat-water context, more questions can be asked. For example, (i) find the direction that the boat should travel in water so that it can travel from A to B directly, or (ii) suppose the boat is directed vertically downward, find the velocity of the boat in the ground frame. The second question in the boat-water context is similar to that in the ball-boat context. In our case, the tasks in the boat-water context are related to the steps that help to answer question (i).

Based on the student responses, we did not see many explanations from both contexts that suggested students misunderstood the motion of the start and endpoints in the ground frame. The pictorial representation of the moving frame did not appear to affect students' interpretation of the context either. The third difference might be able to explain why some students chose arrow (1) and option C if they were answering question (ii) instead of question (i).

However, we found little conflation of $v_{\mathrm{Ball}}^{(G)}$ and $v_{\mathrm{Ball}}^{(\mathrm{Boat})}$ in the ball-boat context but a higher prevalence of the conflation of $v_B^{(G)}$ and $v_B^{(W)}$ in the boat-water context. We are unable to tell whether this error can be counted as preferred-frame reasoning, because students may treat the

water and ground frames to be the same. More investigation may be necessary to find out the actual reason.

In summary, we collected student responses from two typical relative motion problems. We found that the range of percentage correct was wide, which suggested that the student performance in applying relative motion ideas was highly dependent on the contexts.

8.4.2 Student performance in relative motion tasks analogous to RMT-Direction task

We designed two relative motion tasks that we thought to be analogous to the RMT-Direction task. Both tasks are of moving-ball-on-moving-block contexts, with the motion of the ball confined in a circular track in one context, and in a straight track for the other. They are shown in Figure 8.9. We refer them as circular-track and straight-track contexts from now on.

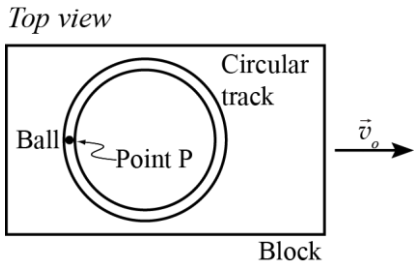
The setup of both tasks is the same. A ball is moving on a block that is moving at a constant speed on a table. The motion of the ball is confined by either a circular track or a straight track. We ask students to choose the arrow that best represents the direction of the ball at point P in the stationary observer's and the block's frame. In the observer's frame, the direction of the ball at point P is pointing right and upward, while in the block's frame, the direction is upward. These two contexts are analogous to the RMT-Direction task in the approach and physics concepts.

We created two versions of these questions. Version 1 consists of the observer's frame only while version 2 consists of both the observer's and block's frames. In both versions, students did the circular-track context first, followed by the straight-track context. We administered both versions in the Au'18 quarter.

(a) Moving-ball-on-moving-block (circular track)

A circular track is mounted on a rectangular block. A ball is moving clockwise along the circular track with constant speed v_o. The block is then pulled to the right with constant speed v_o across a level table.

At time t_o, a stationary observer sees the ball at point P as shown in the top-view diagram.



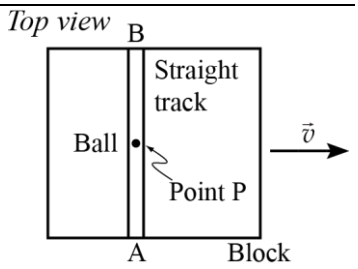
Which of the following options best represents the direction of the instantaneous velocity of the ball when it is at point P according to the observer? According to the block's frame?

A. ↘ B. ↗ C. → D. ↑ E. Zero vector

(b) Moving-ball-on-moving-block (straight track)

A large block is pulled at constant speed across a table. The block has a ball on it that rolls along a straight track from point A to point B.

At time t_o, a stationary observer sees the ball at point P as shown in the top-view diagram.



Which of the following options best represents the direction of the instantaneous velocity of the ball when it is at point P according to the observer? According to the block's frame?

(1) ↖ (2) ↗ (3) → (4) ↑

Figure 8.9. The moving-ball-on-moving-block problem – (a) circular track and (b) straight track.

The results from version 1 are summarized in Table 8.49. The percentages correct for the circular-track and straight-track contexts are 61% and 69%, respectively. 52% of the class answered both correctly. The most common error in the circular-track context is the upward direction, with students using "tangent to the track" to explain their answers. In the straight-track context, students explained their upward arrow either by thinking that the reference frames of the observer and the block were the same or ignoring the motion of the block. Some students chose the rightward arrow, and their explanations seemed to suggest that they were treating small bodies (ball) that were located on a larger body (block) as 'part of the frame' of the larger body.

Table 8.49. Students' pretest responses for Version 1 (observer's frame only) of the ball-block contexts.

Task	Answer	% class (N = 370)	Selected student explanations
Circular track	(Correct)	61%	• *The ball's velocity is straight up in the track but the whole track moves to the right so we would combine both vectors to obtain the new vector from the observer.* • *The observer, being stationary, would observe the ball's circular motion along with the motion of the block, resulting in a vector sum of B.*
		26%	• *the instantaneous velocity at this point will be tangent to the curve of the track and this is the vertical arrow shown by D* • *The observer sees the ball traveling on the track, so the velocity is tangent to the curve at point P.*
		7%	• *The ball is traveling to the right while also being pulled to the right*
Straight track	(Correct)	69%	• *the ball would continue moving straight up with an added x component of movement towards the right caused by the moving block.* • *The ball is on the block which is moving to the right and the ball itself is moving upward on the block, so a NE direction*
		13%	• *As the observer is at the same reference frame as the block, it only sees the ball going up as they are both experiencing the velocity to the right. Thus, the resultant vector is upwards.* • *It is moving in a straight line at constant velocity, so the velocity vector is in same direction as motion.*
		13%	• *Only velocity in the equation is the velocity of the block and the ball itself is not moving in relation to the block* • *Because the ball is "trapped" inside the track, we know it must stay on a straight path because the block's velocity vector is pointing to the right.*

Table 8.50. Student's pretest responses (N = 150) for Version 2 (observer's vs. block's frame) of the ball-block contexts.

	Observer's frame			Block's frame			
	↗ (Correct)	↑	→	↑ (Correct)	→	↗	Zero
Circular track	66%	20%	13%	66%	9%	11%	13%
Straight track	77%	17%	4%	78%	9%	9%	-

The results for version 2 are tabulated in Table 8.50. The percentage correct ranges from 66% to 78%. Fifty-nine percent of the students chose the right arrows in the circular-track context for both the observer's and block's frames, while 71% of them did so in the straight-track context. Fifty-four percent of the students answered all four questions correctly. It appears that the students did better in the straight-track context than the circular-track context, but the difference is not significant. The better performance in the straight-track context might be due to the exposure of the circular-track context before it. These results suggest that around half the class can answer the assigned tasks correctly, and students appear to understand the difference between reference frames in these contexts.

In summary, we designed the circular-track and straight-track contexts that are analogous to the RMT-Direction tasks. Their setups are also the same as the ball-boat context, and the results are also comparable. In that case, we can group the ball-boat, circular-track, and straight track contexts as isomorphic problems.

8.4.3 Correlation with the RMT-Direction Task

This sub-section covers the additional data we collected from students who attempted the ball-boat, boat-water, circular-track, and straight-track contexts. We asked some of them the RMT-Direction task on pretests two weeks after they answered those four contexts on earlier pretests. We wanted to find out how the student performance on the typical relative motion problems was correlated to that in the RMT-Direction task.

Not all students who did the four contexts also did the RMT-Direction task, so the number of students in the following data was less than that was presented in the previous two sub-sections. We have presented that the student performance on the ball-boat, circular-track, and the straight-track contexts was about the same on pretests, including in their performances on the RMT-Direction tasks, so we aggregated those results. The correlation table is presented in Table 8.51.

Among the students who did the ball-boat context and RMT-Direction task, 71% answered the ball-boat context correctly, while 14% did so in the RMT-Direction task. As for the other group of students who did the boat-water context and RMT-Direction task, 13% answered the boat-water context correctly, while 22% did so in the second task. Only 5-10% of the students answered both the typical relative motion tasks and the RMT-Direction task correctly. Based on the percentages correct, the RMT-Direction task is slightly easier than the boat-water context but more difficult than the ball-boat context.

The results suggest that students do not approach all three tasks the same way as physicists, although the concepts involved are the same. The questions in the boat-water context are different from the rest. The direction of the velocity of the boat in the ground frame is given in the boat-water context. However, such information is not given in the ball-boat context or the RMT-Direction task. The results also show that 3% of students answered the ball-boat context

incorrectly, and the RMT-Direction task correctly, but 17% of the students were under the same category in the boat-water context. So, it may not be suitable to compare the results from the boat-water context with that from the RMT-Direction task.

Table 8.51. Correlation between student performance in the ball-boat/circular-track contexts and that of the boat-water context.

		RMT-Direction Task	
		Correct	Incorrect
Ball-boat / Circular-track contexts (N = 413)	Correct	11%	60%
	Incorrect	3%	26%
Boat-water context (N = 115)	Correct	5%	8%
	Incorrect	17%	70%

The difference in the percentages correct between the ball-boat context, and the RMT-Direction task is 57%. We think this large difference could be due to the surface features of these questions. There is more than one object in motion in the ball-boat/circular-track contexts, but there is only one moving object in the RMT-Direction task. The setup of the ball-boat context makes it clear to students that there is more than one reference frame to work with, but not in the RMT-Direction task. Previous investigations indicated that students might treat the given shape as the trajectory of the point of interest. We are unable to tell which surface feature affects student performance more.

8.4.4 Commentary

We investigated the student performance in typical relative motion problems and compared that with how they did in the RMT-Direction task. We found that students could do those relative

motion problems involving direct addition of velocities in different frames, such as the ball-boat, circular-track, and straight-track contexts.

Students did poorly in the boat-water context. They tended to treat the velocity of the boat in the water's frame as the velocity of the boat in the ground frame. We were unable to compare the student performance in the boat-water context to that in the RMT-Direction task because of the setup of these questions were not the same. Another reason is that most students who answered the RMT-Direction task correctly answered the boat-water context incorrectly, which was inconsistent with the other group of students who attempted a different set of tasks.

Several reasons could explain why students did much worse in the RMT-Direction task than in the ball-boat context. It is evident to students that there is more than one reference frame to consider in the ball-boat context, but not so in the RMT-Direction task. The given curve that represents the shape of the object can be treated as the trajectory, which we had discussed earlier. We are unable to tell which factor affects the most.

Rimoldini and Singh concluded that students "in general had difficulty applying the principle of conservation of energy and relative velocity concepts. Instructional strategies that focus on improving student understanding of rotational and rolling motion concepts should take into account these difficulties" in their article [12]. Based on what we found, students need help in applying relative motion concepts to various situations. It is not clear that student difficulties in relative motion necessarily lead to poor performance in the RMT-Direction task.

8.5 SUMMARY

In this chapter, we presented results for four additional investigations involving the Rolling Motion Task in RRMCS. First, we found that students tend to use the given trajectory to determine

the direction of velocity, especially in contexts in which only a simple curve (cycloid or circle) is given. Second, we found that students can determine the direction of velocity better after students were asked to determine the displacement or the trajectory first. These results suggest that most students are used to imagine or use the trajectory to determine the direction of velocity.

Third, we changed the object in the Rolling Motion Task to different rigid bodies that are translating and spinning at the same time. The results suggest that student performance may have some context-dependence. Besides, we found that some improvement in the pretest results could be related to discussing the trajectory of a point on a rigid body that was translating and spinning freely in the air during lecture instruction.

Lastly, we verified the claim made by Fang and Uziak. They said that "students could answer simple relative motion problems (in our case, problems that are almost analogous to RMT), but were unable to apply the same ideas to the problem of two points on a rigid body." These results suggest that students need more exposure in applying relative motion ideas in some contexts to determine the correct direction of velocity. These findings can help in the design of the curriculum. One possible strategy is to rely on what most students can do best to learn a new technique to determine the direction of velocity in new contexts. Students can determine the direction of velocity through displacement or trajectory, but we hope that they can also do so through a relative motion approach whenever possible. We implement this strategy in our curriculum, and more details can be found in Chapter 10.

Chapter 9. INVESTIGATION OF STUDENT DIFFICULTIES WITH ROLLING MOTION III – OTHERS STUDIES

In the previous two chapters, we investigated student difficulties in rolling motion using tasks related to the RMT. In this chapter, we describe the results of such investigations using different tasks. These tasks are examples presented in López's paper. They are: (1) a uniform disk rolling without slipping on an accelerated flat surface (referred to as cylinder-on-board) and (2) a horizontal bar that is moving without slipping on uniform cylinders, with the cylinders rolling without slipping on the ground (referred as board-on-cylinder). She used these examples to demonstrate the utility of relative motion concepts and the no-slip condition in situations involving a rolling object and a moving surface that are in contact with each other [13]. Since López did not include any student responses in her paper, and we did not find anyone reported them, so we collected these data.

As mentioned in section 6.1, the no-slip condition is such that there is no relative velocity at the point of contact between the two objects. We have not seen any research conducted on student understanding of the no-slip condition before our investigation. Since students need to apply the no-slip condition in rolling motion, it would be relevant to find out their level of understanding.

In López's paper, she made a concluding remark that "another possible source of confusion is thinking that the condition of non-slipping means zero absolute velocity at the contact point instead of zero relative velocity." Applying this remark to the RMT, it means that students may know the velocity of the contact point (the point on the wheel that is in contact with the ground) is zero for the wrong reason. That also means that if we were to investigate student understanding

of the no-slip condition, besides using the RMT, the cylinder-on-board and board-on-cylinder tasks should also be used. The point of contact is not moving at absolute zero velocity in the ground frame, so these tasks provide more insight into student understanding of the no-slip condition.

In this chapter, section 9.1 describes the student responses on the cylinder-on-board task on pretests. Section 9.2 describes the student responses on the board-on-cylinder task on pretests. Then in section 9.3, we describe the additional investigations involving the board-on-cylinder task.

9.1 STUDENT RESPONSES ON THE CYLINDER-ON-BOARD (COB) TASK

The cylinder-on-board (CoB) task is about a uniform disk rolling without slipping on an accelerated flat surface. It is one example presented in López's paper to demonstrate the utility of relative motion concepts and the no-slip condition. We administered the question on pretests. The results and student explanations are presented.

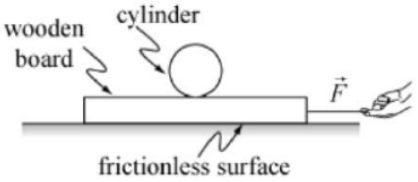
9.1.1 Description and administration of the CoB task

The cylinder-on-board (CoB) task is shown in Figure 9.1. A cylinder is placed on top of a wooden board. When the board is pulled to the right by a constant horizontal force, the cylinder is observed to roll counterclockwise without slipping on the board. Students are asked to choose the equation that correctly describes the relationship between the speed of the board (v_{b}), the speed of the center-of-mass of the cylinder (v_{c}), the radius of the cylinder (*R*), and the angular velocity of the cylinder (ω) at an instant when all objects are in motion due to the external force.

Students need to go through a series of reasoning to arrive at the correct equation. Since the cylinder rolls counterclockwise without slipping when the board is pulled, the cylinder must

experience a torque that points out of the page. For an out-of-page torque acting on the cylinder, there must be a horizontal force acting on it directed to the right. This rightward horizontal force will cause the cylinder to accelerate to the right, hence v_c will be pointing right. Using the relative motion ideas, let the point on the cylinder that is in contact with the board be *P*, the velocity of point *P* in the ground frame is $v_c + R\omega$ to the right. The no-slip condition requires the speed of the board to be equal to the speed of point *P*, so $v_b = v_c + R\omega$.

A cylinder is placed on top of a wooden board. The board is pulled to the right by a constant horizontal force of magnitude *F* on a frictionless surface. The cylinder is observed to roll counterclockwise without slipping on the board.



At the instant that the board has speed v_b, the center of mass of the cylinder has speed v_c, and the cylinder is rotating with a rotational speed of ω.

Question 5.

Which of the following equations correctly describes the relationship between v_b, v_c, R (radius of cylinder), and the rotational speed of the cylinder, ω?

Note: Treat all of the above quantities as having positive values.

- $v_b = v_c$
- $v_b = v_c + R\omega$
- $v_b = v_c - R\omega$
- $v_b = v_c + 2R\omega$
- Not enough information
- Other:

Figure 9.1. Cylinder-on-board task on a pretest.

Alternatively, one could expect that the cylinder will roll off the left end of the board eventually, so the board must have traveled faster than the cylinder in the ground frame. However, that does not mean students can arrive at the correct equation to describe the relationship correctly.

Through this task, we can explore how students reason out the difference between the speeds of the two objects without going through the lengthy reasoning. Students attempted the cylinder-on-board task on pretests in the Sp'18, Au'18, and Sp'19 quarters.

9.1.2 Results

The results are summarized in Table 9.52. About 50% of the students answered correctly. There is no significant change in student performance, whether they attempted the question before or after lecture instruction. This result is not surprising because there is no explicit mention of the no-slip condition and how to apply it in lectures. Among the students who answered incorrectly, about equal proportion, students thought that the board and the cylinder should be moving at the same speed, or the cylinder would be moving faster than the board.

Table 9.52. Student responses on the cylinder-on-board task on pretests.

	Sp'18 *Before* instruction (N = 420)	Au'18 *After* instruction (N = 499)	Sp'19 *After* instruction (N = 453)	Aggregate (N =1372)
(Correct) $v_b = v_c + R\omega$	**49%**	**54%**	**49%**	**51%**
$v_b = v_c$	18%	15%	18%	17%
$v_b = v_c - R\omega$	16%	18%	19%	18%
$v_b = v_c + 2R\omega$	12%	12%	11%	12%
Others	5%	1%	3%	2%

9.1.3 Student explanations

Not all students explained their answers, and some may be out of time because this is the last question on the pretest. For those who explained, some students clearly expressed the no-slip condition in their explanations. Some expressed that they did not know how to approach the problem, which suggests that they did not see the application of relative motion concepts on this task. They also lacked the operational definition of no-slip motion to approach the task. Some just explained by restating their answers.

Approaches for the correct answer ($v_b = v_c + R\omega$). Students who chose the correct answer explained through two main approaches. The first approach is through the addition of velocities (relative motion) and the no-slip condition. Below are some examples:

> *Since there rotational and translation velocity involved with the cylinder and there is no slipping at the point of contact between plank and cylinder. The net velocity of the point of contact is sum rotational + translational velocity and that should equal to velocity of the plank to prevent slipping*

> *I'm not quite sure. My best guess is that the board's movement causes a frictional force that leads to a torque and rotation of the cylinder. Some of this force is translated into rotation and some of it is translated into rotational movement. Thus, the two objects cannot have the same velocity;* v_b *must be a sum of two different quantities.*

> *Because the cylinder rolls counterclockwise without slipping on the board, the speed of the cylinder and the speed of the board are equal. The velocity of the cylinder is calculated by the sum of the speed at its center of mass and the instantaneous velocity. The instantaneous velocity is calculated by angular velocity times radius, which is* $R\omega$*. So* $v_b = v_c + R\omega$*.*

The second approach is energy-related reasoning. However, it is an incorrect approach. Some students compared the total kinetic energies of the board and cylinder to arrive at the answer.

These students seemed to assume that the kinetic energy is linearly proportional to speed, which is clearly incorrect. The student explanations below demonstrate

> *Since the kinetic energy of the center of mass of the board is equal to the rotational kinetic energy and translational kinetic energy of the cylinder, velocity of center of mass of the board is equal to the velocity of center of mass of the cylinder and the radius of the cylinder times the rotational speed of the cylinder.*
>
> *We know that some of the KE of the cylinder is going to be contributing to its rotation speed ω, so it'd have to be both the sum of the velocity center of mass of the cylinder along with the rotational speed in order to equal the block's velocity.*

Very few students mentioned that the CM of the cylinder had zero velocity in the Earth's frame. They might be confusing the Earth's frame with the cylinder's CM frame. For example, one student said, "*Cylinder doesn't move, hence v_c is 0 and speed of the board is equal to rotational speed of outermost point on the cylinder.*" Another small group of students cited conservation of momentum. One student, for example, wrote, "*Conservation of momentum, since the masses remain constant, the overall velocities must remain constant. All the velocity given to the cylinder by the board must equal the boards velocity. This means adding the rotational velocity to the center of mass velocity.*" This student assumed both objects had the same masses, and by doing so and citing the conservation of momentum, a relationship between different velocities was obtained.

Now we discuss the student explanations for the incorrect answers.

Approaches for $v_b = v_c$. Students who chose $v_b = v_c$ usually stated the no-slip condition. These students might know that the no-slip condition meant that there is zero relative velocity, instead of applying it at the point-of-contact, they applied it at the CM. Some of them thought that the no-slip condition implied equal displacements. Below are some student explanations.

> *Since the cylinder is rolling without slipping on the board, the distances travels by the border of the cylinder and the board are the same, which means their linear velocities are the same.*
>
> *Rolling without slipping refer to motion with no relative velocity between the point of contact of two objects, although the cylinder is rolling, the center of mass of the cylinder stays at the same speed with the board, so* $v_b = v_c$.

Other approaches include force/energy reasoning and conservation of momentum. Example of the former approach is, "*The cylinder is exerted the same force as the block, so in order to conserve the energy and momentum, it must be nearly the same as the speed as the block.*" For the latter approach, one student wrote, "*The velocity of the board should be equal to the velocity of the cylinder. They should be equal and opposite of each other. If the system is closed, then momentum should be conserved to the velocities should be equal.*" The rest of the students tended to restate the answer.

Approaches for $v_b = v_c - R\omega$. Students who chose the answer, $v_b = v_c - R\omega$, had several approaches. There were no prevalent ones, so we highlighted two different approaches. The first approach is using relative motion idea and no-slip condition, but some students did not recognize that the tangential velocity due to rotation of the cylinder at the point of contact was in the same direction as the v_{CM}. Below are some student explanations.

> $v = R\omega$*, so the linear velocity component of* ω *will be* $R\omega$*. Since the center of the cylinder is still moving to the right, then the difference between it and the linear component* $R\omega$ *should be equal to the net linear velocity of it which will be equal to* v_b *since there is no slipping between them.*
>
> *The velocity of the cylinder and the tangential velocity are in opposite direction to roll without slipping so they must be subtracted to find the velocity of the board.*

In the second approach, some students recognized that the board should be moving faster than the cylinder, or the cylinder would roll off the left side of the board, so the $R\omega$ was subtracted from v_c. However, they did not realize that their answer meant otherwise, which suggested that they did not understand the equation. Here are the student explanations.

> *The more the cylinder rolls, the more it goes to the left and has a smaller center of mass velocity than the wooden board.*

> *Because the cylinder rolls counterclockwise (move left),* $|v_b| > |v_c|$ *and* $R\omega$ *should be negative.*

In summary, students who gave incorrect explanations exhibited several difficulties. First, some misinterpreted the no-slip condition and thought that the two objects had the same v_{CM}. Second, some students used momentum conservation to approach the problem. They did not realize that there was an external force on the system, which suggested that they did not understand when to apply momentum conservation. Third, some students were not able to interpret the relevant equations correctly. They either chose the wrong answer or thought that the kinetic energy and speed were linearly dependent on each other.

9.1.4 Discussions

Students performed better in the CoB task than the RMTs on pretests. Based on the results alone, students appeared to add velocities readily than in the RMTs. However, it may be due to the way we set up the options for the question. The performance might be worse if this question were asked in a written format since the results from the previous chapter have shown that students do not apply relative motion concepts readily in rolling motion problems.

The student explanations gave insights to student approaches. One thing we did not find was that many students using $v_{\mathrm{CM}} = R\omega$ by rote. While this finding is different from what López has written, we believe it is due to the way we ask the question. We did not consider asking this question in written format, because they might perform poorly, and their explanations might not be useful for the investigation. The options may have prompted some of them to think in terms of relative motion and no-slip condition. However, one may want to find out by asking the written format in the future.

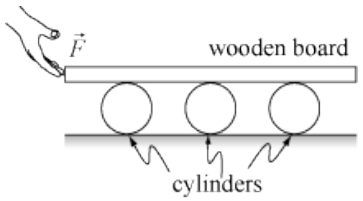
9.2 STUDENT RESPONSES ON THE BOARD-ON-CYLINDER (BOC) TASK

In this section, we describe the student responses from the board-on-cylinder (BoC) task. The boC task is another example that López used to illustrate how one can use relative motion and no-slip condition to analyze the situation. Similar to the previous section, we first describe the task, then we discuss the results and explanations on pretests.

9.2.1 Description and administration of the BoC task

The BoC task involves a board that is placed on top of three identical solid cylinders, as shown in Figure 9.2. A hand pushes the board with a constant force to the right. During the push, the cylinders roll without slipping on the ground. There is also no slipping between the cylinders and the board. Students need to find the correct equation that describes the relationship between $\Delta x_{\mathrm{board}}$ and Δx_{cyl} when the board is pushed to the right, and all objects are in motion. We generated other versions of the same task that asked students to find the correct relationship that relates the displacements or the accelerations between the board and the cylinder.

A wooden board is placed on top of three identical solid cylinders. The board is pushed to the right by a constant horizontal force of magnitude *F* applied to the end of the board. The cylinders roll without slipping on a flat surface. There is also no slipping between the cylinders.



Consider the time interval when the horizontal force is still exerted on the board and the board is still in contact with all of the cylinders.

Let Δx_b be the magnitude of the horizontal displacement of the board and Δx_{cyl} be the magnitude of the horizontal displacement of the center-of-mass of each cylinder.

In the reference frame of the Earth, which of the following correctly describes the relationship between Δx_b and Δx_{cyl}?

A. $\Delta x_b = 0.5\Delta x_{\text{cyl}}$	B. $\Delta x_b = \Delta x_{\text{cyl}}$	C. $\Delta x_b = 1.5\Delta x_{\text{cyl}}$
D. $\Delta x_b = 2\Delta x_{\text{cyl}}$	E. Not enough information	

Figure 9.2. The BoC task (displacement) on a pretest.

The correct answer can be obtained by using the same approach as in the RMT-Magnitude task. The no-slip condition means that the point of contact between the cylinder and the ground is moving at zero velocity at any instant. The point of contact between the cylinder and the board (point 3 in the Magnitude task) moves at twice the magnitude of v_{cyl}. The board moves at twice of v_{cyl}, so the correct equation is $v_{\text{b}} = 2v_{\text{cyl}}$. The same relationship is also applicable for the displacements and accelerations after considering some time intervals during the motion. This relationship is independent of the number of cylinders below the board.

We administered the BoC task to students on online pretests from Wi'18 to Au'19 quarters, but we only present results from the Wi'18 and Sp'18 quarters. Students attempted it before they had relevant lecture instructions on the rolling motion. We gave most students the version that asked for the correct equation that relates to the displacements of the board and cylinder. In the Sp'18 quarter, we also asked students to compare the velocities and accelerations of both objects.

9.2.2 Results of the BoC task on pretests

The student responses to this task are summarized in Table 9.53. In both quarters, 24% of the students answered this task correctly. When we asked students for the correct equations that relate the displacements, velocities, and accelerations of the board and the cylinder in the Sp'18 quarter, the results did not differ from each other significantly, so we combined the results.

The most common incorrect answer is that both objects have the same displacement. About 40% of the students chose that answer. We can expect some students would explain the same way as in the CoB task. As for the remaining students, they chose the remaining answers at equal proportions.

Table 9.53. Student responses to the board-on-cylinder task on pretests.

	Wi'18 *Before* instruction (N = 395)	Sp'18 *Before* instruction (N = 420)	Aggregate *Before* instruction (N = 815)
$\Delta x_b = 0.5\Delta x_{cyl}$	12%	12%	12%
$\Delta x_b = \Delta x_{cyl}$	35%	42%	39%
$\Delta x_b = 1.5\Delta x_{cyl}$	17%	12%	14%
(Correct) $\boldsymbol{\Delta x_b = 2\Delta x_{cyl}}$	**24%**	**24%**	**24%**
Others	11%	10%	11%

9.2.3 Student explanations

Around 8% of students gave the correct answer ($\Delta x_{\mathrm{b}} = 2\Delta x_{\mathrm{cyl}}$) and explanations. We found three approaches students used to arrive at the correct answer. The first approach is the correct one, which involves relative motion and the no-slip condition. Below are some student explanations:

> *The speed of the board is twice that of the speed of the center of mass of each cylinder because the top of the cylinder is moving twice as fast as the center of mass relative to the earth. This is because the top of the cylinder has angular velocity in addition to the velocity of its center of mass.*

> *There is no slipping between the cylinders and the board. In the time it takes for the cylinders to make one full rotation, they will have moved relative to the ground one circumference of distance. In that same time, the board will have moved relative to the cylinders one circumference of distance. Therefore, the board moves twice as far in the frame of the Earth.*

> *When rolling without slipping, the speed of the centers of the cylinders are Rω, and the speed of the board, which is the same as the speed of the top point of the cylinder, is 2Rω.*

The second approach is by comparing the motion of the board with the motion of the cylinder. Some students would say that the board only translates while the cylinder translates and rotates. Some did not distinguish between net force and net torque. Some compared translational kinetic energy of the board with translational and rotational kinetic energies of the cylinder. Here are some student explanations:

> *I am not so sure about the exact times that the displacement of the board is greater than the displacement of the cylinder, but I think that the board will travel farther than the cylinder because the board has purely translational KE while the cylinder has rotational and translational KE, so the board will travel farther.*

The board moves farther then the cylinder because the rolling motion of the cylinder pushes the board added to the horizontal force of the push would make the board move further to the right at twice the magnitude of as the horizontal motion of the board.

The board would move faster than each of the cylinders over the time interval, meaning it would have a greater displacement, because the circles have to turn while the board does not. The turning of the cylinders would also contribute to the displacement of the board.

The third approach that was expressed by very few students is through experimentation. These students experimented when they attempted the question on pretests. We highlighted this approach because it is unique. Here are some student explanations.

I rolled a tube of chap-stick on my desk and looked at each displacement and my hand moved about twice the distance as the tube.

Because I just tried it with my pencil and I got around half, and there is no way in my head that I could understand it other than the object is rotating so it travels twice the distance since it touches two objects.

For the remaining students, some arrived at the answer by saying that the board would roll off the cylinders eventually. However, they did not explain why they chose $\Delta x_{\mathrm{b}} = 2\Delta x_{\mathrm{cyl}}$ but not $\Delta x_{\mathrm{b}} = 1.5\ \Delta x_{\mathrm{cyl}}$, since both equations express that the board moves faster than the cylinders.

Now, we describe the student explanations for the incorrect answers.

$\Delta x_{\mathrm{b}} = \Delta x_{\mathrm{cyl}}$: Most students thought that the displacement of the board is equal to that of the cylinder. They stated that because there is no slipping between the board and the cylinder, then the two objects traveled the same distance. The students thought that “no-slip” or “in contact” in this context is the same as two stacked blocks that are moving together. Here are some student statements.

Because there is no slipping between the cylinders and the board, so for the amount that the cylinder moved should equal to the amount the board moved.

They all have to move the same amount since they are all in contact with each other.

$\Delta x_{\mathrm{b}} = 0.5\Delta x_{\mathrm{cyl}}$: About 12% of students who chose the answer $\Delta x_{\mathrm{b}} = 0.5\Delta x_{\mathrm{cyl}}$. Some students recognized that the cylinders would end up behind the board during the motion, which implies that the cylinder will travel less distance than the board over the time interval, so they chose the equation with a "0.5" in front of the displacement of the cylinder. These students might have some difficulty in interpreting the equation. The explanations below illustrate this idea.

I know the board moves more than the cylinders do because eventually the board rolls past the cylinders. I didn't calculate anything but knew that delta x board is > delta x cylinder so the coefficient has to be .5.

The displacement of the board is equal to the displacement of the arc length of the circle which changes faster than its center of mass. As a result, the displacement of the cylinders would be less than the displacement of the board itself.

The board has full kinetic energy while the balls have both kinetic energy and rotational kinetic energy. So, the balls will move at a slower speed.

The board is on the top of the cylinders, so the velocity at the point of contact of the cylinders is twice as much as the board, so the CM of each cylinder is going half as fast as the board.

Some explanations given for the answer $\Delta x_{\mathrm{b}} = 0.5\Delta x_{\mathrm{cyl}}$ were correct, especially the last one. Some students thought that both objects have the same total kinetic energies, which is a line of (incorrect) reasoning we have presented in the *Dynamics of rigid bodies* tutorial [24].

$\Delta x_{\mathrm{b}} = 1.5\Delta x_{\mathrm{cyl}}$: Students chose $\Delta x_{\mathrm{b}} = 1.5\Delta x_{\mathrm{cyl}}$ for various reasons. Some of them recognized that the board is moving faster than the cylinder but picked the wrong equation. While others indicated that there are three cylinders below the board, so the coefficient should be a

number that is divisible by three. The former has some correct intuition, but the latter seems to suggest that the motion of the top object could be the sum of the motions of the objects below it. Here are some student explanations.

> *The board moves more than the cylinders so it must be* $\Delta x_{\mathrm{b}} = 1.5\Delta x_{\mathrm{cyl}}$ *or* $\Delta x_{\mathrm{b}} = 2\Delta x_{\mathrm{cyl}}$. *I chose* $\Delta x_{\mathrm{b}} = 1.5\Delta x_{\mathrm{cyl}}$ *because it did not make sense that the board would move exactly twice as much as the cylinders because there are three different cylinders.*
>
> *I don't have good reasoning - intuitively, I feel that the board will move further than the cylinders, and I feel that the board's movement will be less than two times that of the cylinders. This leaves the displacement of the board being 1.5 times the displacements of the cylinders.*
>
> *The board is moving faster than the cylinders. I also assumed the displacement of the cylinder would be a multiple of three because there are three cylinders. Thus, I believe that* $\Delta x_{\mathrm{b}} = 1.5\Delta x_{\mathrm{cyl}}$ *is the answer.*

In summary, the incorrect student explanations reveal several student difficulties. Some are similar to those found in the CoB task, such as the no-slip implies equal displacements or velocities and the misinterpretation of equation. Another common difficulty we found is to compare the translational motion of the board with the translational and rotational motion of the cylinder. In some cases, students compare the total kinetic energies of the board and cylinder, a reasoning that was presented in the *Dynamics of rigid bodies* tutorial.

9.2.4 Discussions

From both the CoB and BoC tasks, we found that many students thought that the no-slip condition implied that the displacements and velocities of objects that were in contact were equal.

One possible reason is that they do not have an operational definition of no-slip motion that they could apply. So, if the curriculum were to address this difficulty, it is necessary to provide students the operational definition and give students opportunities to apply it in the homework.

The tendency for students to compare the translational motion of the board with the translational and rotational motion of the cylinder to reason their answers was quite common. This tendency had a similar nature to some of the difficulties reported by Close *et al.* when the authors investigated the student understanding of the application of Newton's second law to rotating rigid bodies [71]. The relevant difficulties reported were (1) tendency to treat force as partially or completely "used up" in rotational motion and (2) tendency to treat the translational/rotational partition of kinetic energy as having direct causal influence on translational motion. The authors created and modified the *Dynamics of rigid bodies* (DRB) curriculum based on the investigation.

We usually schedule the DRB tutorial before students did the BoC task on pretests. The student exposure to the tutorial may affect students' answers on the BoC task. The students in the Sp'18 quarter did the BoC task before the DRB tutorial. We found some student explanations that were similar to those reported by Close *et al.*, though less than the Wi'18 quarter. In other words, some students might use what they saw in the DRB tutorial on the BoC task, which also suggested that students had difficulties analyzing rigid bodies that translate and rotate at the same time.

9.3 FURTHER INVESTIGATIONS INVOLVING THE BOARD-ON-CYLINDER (BOC) TASK

In this section, we describe the additional investigations conducted on the board-on-cylinder (BoC) task. The reason for doing so is that instructors were using the modified board-on-cylinder (mBoC) task (see Figure 9.3) as an in-class clicker question during the lecture instruction of rolling motion in the Wi'18 quarter, which affected our interpretation of the gain from pretest to posttest. The in-class activity appears to interfere in determining the effectiveness of the tutorial curriculum. However, it broadens our scope of the investigation revolving the BoC task.

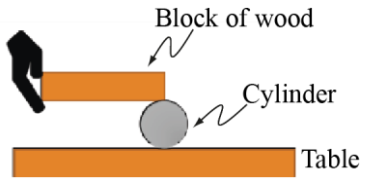
The arrangement of the section is as follows. We first describe the mBoC task and the results from the in-class clicker during lecture instruction. Then, we describe the student results and explanations on pretests. The results for the mBoC and BoC tasks on pretests are compared.

9.3.1 Results for the modified board-on-cylinder (mBoC) task during lectures

The modified version of the board-on-cylinder task, as shown in Figure 9.3. It was created by one of the PHYS 121 instructors. He administered this as a clicker question during class in some quarters. He asked students to determine how far the cylinder traveled after the board moved a distance *L*. The intention was to let students realized that there was a difference in the speeds between the center-of-mass and the top point of the cylinder when it rolled without slipping. Another purpose is to demonstrate how to apply relative motion ideas in rolling motion.

The approach to get the correct answer to this task is the same as the BoC task. From the equation $\Delta x_{\mathrm{b}} = 2\Delta x_{\mathrm{cyl}}$, when the board moves a distance *L*, the cylinder moves *L*/2.

A cylinder rests on a flat table. A block of wood is placed horizontally with one end resting on the cylinder and the other end supported by a person. The person then pushes the block so that it moves to the right by a distance *L*. The block rolls without slipping on the cylinder and the cylinder rolls without slipping on the table. Assume the block remains horizontal and in contact with the cylinder throughout the motion.



How far does the cylinder move?

A. 0 B. $L/2$ C. L D. $2L$

Figure 9.3. The mBoC task on a pretest. It is also used as a clicker question during the lecture.

The mBoC task was administered as an in-class clicker question during lectures in both Wi'18 and Sp'18 quarters. Students did the BoC task on Catalyst before they did the mBoC task. One disadvantage of the in-class clicker question was that we were unable to know how students explained their answers.

The student performance on the modified board-on-cylinder task during the in-class instruction is summarized in Table 9.54. The percentage correct was around 67%. The most common incorrect response was that both objects traveled the same distance, which was analogous to students saying $\Delta x_{\mathrm{b}} = \Delta x_{\mathrm{cyl}}$ in the original board-on-cylinder task.

Table 9.54. Student responses to the mBoC task in clicker question.

	Wi'18 (*N* = 398)	Sp'18 (*N* = 416)	Aggregate (*N* = 814)
(Correct) *L*/2	**66%**	**68%**	**67%**
L	29%	26%	28%
Others (0, 2*L*)	5%	6%	5%

Students performed better on the mBoC task compared to the BoC task. One possible reason is that the mBoC task does not require students to write an equation, while the BoC task does. Another reason could be that $L/2$ is the most intuitive answer: they knew that the cylinder would move forward, but it would not move as far as the block, so they picked the only answer that was between 0 and L.

Based on the data that we have collected from the online pretest and the clicker question in both quarters, most students who chose L in the mBoC task during the lecture selected $\Delta x_{\mathrm{b}} = \Delta x_{\mathrm{cyl}}$ in the BoC task on pretests. The result suggests that they do not have an operational definition of no-slip motion. So, it may be essential to include the operational definition in the curriculum.

9.3.2 Student responses for the mBoC task on pretests

We administered the mBoC task on pretests in the Au'18, Sp'19, and Au'19 quarters. The students did the task before they had lecture instruction on rolling motion, so these pretest results were collected before the in-class clicker session.

The pretest results are presented in Table 9.55. The percentage correct ranges from 47% to 56%. The aggregated percentage correct (N = 1443) was 53%. It was lower than that from the in-class clicker question because not all students had learned how to analyze such situations. The difference in the aggregated percentage correct between the pretest and in-class clicker was 14%, probably because students could still guess the correct answer if they knew that the cylinder should move a distance not as far as L.

Table 9.55. Student responses to the mBoC task on pretests before lecture instruction.

	Au'18 (N = 538)	Sp'19 (N = 471)	Au'19 (N = 434)	Aggregate (N = 1443)
(Correct) $L/2$	**56%**	**55%**	**47%**	**53%**
L	37%	40%	42%	40%
Others (0 or $2L$)	7%	5%	11%	7%

Next, we discuss the student explanations on pretests. The student explanations are presented in Table 9.56. In general, the approaches students used in the mBoC task are about the same as in the BoC task.

Most students who answered correctly did not give the correct explanation. Given the options 0, $L/2$, L, and $2L$, some students knew that the cylinder could not travel 0 and L, so the only possible answer was $L/2$. Some students explained by thinking that the force was distributed equally to the block and the cylinder or by considering translational and rotational kinetic energies. Such lines of reasoning were also seen in the *Dynamics of rigid bodies* tutorial. Some students explained by describing a small experiment they conducted when they saw the question. All these explanations suggested that most students did not use relative motion concepts to approach the problem. Besides that, students did not have an operational definition for no-slip motion.

The most common incorrect response is that both the block and the cylinder move the same distance, as students tend to relate the no-slip condition to imply equal displacement.

In summary, students did not apply relative motion concepts readily in the mBoC task. The higher percentage correct in this task in pretests and in-class clicker questions can be mostly attributed to the given options.

Table 9.56. Student responses for the mBoC task on pretests.

Answer	% class (N = 1443)	Selected student explanations
$L/2$ (Correct)	53%	• *Since the cylinder does not slip at any point it must move, but it cannot have moved as far as the wood did.* • *I think that some of the work done to the cylinder goes into rotational KE rather than linear KE and therefore it does not go as far.* • *It moves less than the block because it is rotating and moving, but it does not remain in place, and the only answer between 0 and L is L/2.* • *The cylinder is moving with the table and the block so it will only go half the distance. I also tested it with an orange.* • *The top of the cylinder moves just as much as the bottom, but the block moves twice as fast as the cylinder because the cylinder moves along the block as if moving backwards.* • *Half of the force goes to rotational which then only half goes into translational.* • *The cylinder rolls without slipping, so it must travel some distance. But it has a rotational component of movement as well as a linear displacement, so the distance it travels must be less than that of the block, which only has a linear displacement.*
L	40%	• *Because there is no slipping, the motion of the block a distance L causes any point on the ball's edge to move one L's distance.* • *They stay in contact, so they move the same displacement.* • *Since there is no slippage, the amount that the block moves the cylinder should also be the amount it travels on the ground.* • *The cylinder does not slide on the wood, and they are in contact all the time.*

9.3.3 Results from additional investigations

In this discussion, we present the correlation of student responses of the mBoC and BoC tasks on pretests. These data were collected in the Au'18, Sp'19, and Au'19 quarters. Students first did the mBoC task on the 8th pretest over the weekend before the 9th week, went through the lecture instruction of rolling motion in the 9th week, and did the BoC task on the 9th pretest during the weekend of that week. In other words, the pretest results from the mBoC task were collected before lecture instruction. The pretest results from the BoC task were collected after lecture instruction. Not all students in these three quarters did the mBoC task as an in-class clicker question during lecture instruction. This data collection scheme is summarized in Figure 9.4.

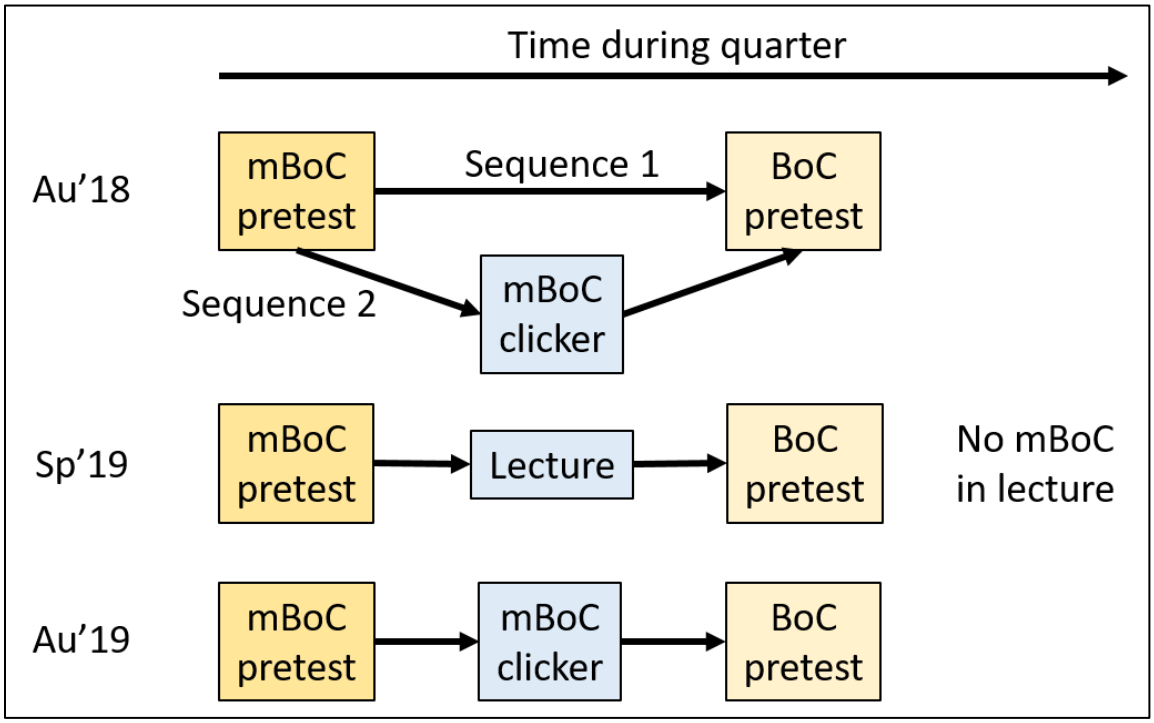


Figure 9.4. The sequence of administration of the BoC and mBoC tasks in pretests and in-class clicker questions in the Au'18, Sp'19, and Au'19 quarters.

This sequence of administration is interesting, because we can compare the pretest results on the BoC task with and without the influence of using the mBoC task as one of the in-class clicker questions during lecture instruction. Another point to note is that the BoC task in the Au'19 quarter is different from previous quarters. Instead of asking students to select the correct equation that relates the displacements of the board and cylinder, we gave options such as (A) Between 0 and $L/2$, (B) $L/2$, (C) Between $L/2$ and L, (D) L, (E) Between L and $2L$, and (F) $2L$. These options were modeled from the mBoC task.

Table 9.57. Student performance of the mBoC task on pretests and in-class clicker questions and the BoC task on pretests in Au'18, Sp'19, and Au'19 quarters. In each cell, the first number is the percentage correct of the students who responded at that time. The second number is the percentage correct of students who completed all the administered tasks in that quarter.

Quarter	Percentage correct in the mBoC task		Percentage correct in the BoC task
	Pretest (Before lecture instruction)	In-class clicker	Pretest (After lecture instruction)
Au'18 Seq 1	55% (N = 367) 55% (N = 317)	-	27% (N = 339) 29% (N = 317)
Au'18 Seq 2	56% (N = 171) 60% (N = 119)	57% (N = 143) 56% (N = 119)	47% (N = 159) 50% (N = 119)
Sp'19	55% (N = 471) 56% (N = 419)	-	25% (N = 452) 25% (N = 419)
Au'19	47% (N = 434) 49% (N = 282)	71% (N = 356) 71% (N = 282)	46% (N = 412) 52% (N = 282)

The percentages correct for each administration are presented in Table 9.57. The percentages correct of the mBoC task on pretests and in-class clicker range from 47% to 56% and from 57% to 71%, respectively. The results for the in-class clicker in the Au'18 and Au'19 quarters are within 10% of what we found in the Wi'18 and Sp'19 quarters. The results of the mBoC tasks on pretests are quite consistent across quarters.

The percentage correct for the BoC task on pretests was about 25% when students did not do the in-class clicker. It was about 50% when students saw the mBoC task during lecture instruction. The 25% increase in percentage correct can be attributed to the presentation of the mBoC task during lecture instruction, which is not surprising since more exposure and familiarity with the context tends to improve the performance. Although the student performance in the Au'18 quarter was lower than in the Au'19 quarter, the effects on the BoC task on pretests were the same.

There was no significant difference between the results from students who did all the administered tasks and those who responded to each task. This finding suggested that students who missed either one of the tasks selected answers the same way as from the students who responded.

The next question would be whether more students could explain their answer on the BoC task correctly after they saw the mBoC task during lecture instruction. Around 10% of the students explained correctly. Many did not mention the no-slip condition at all. Those who explained correctly mainly stated that the top of the wheel moves twice the v_{CM}. Some students knew the answer based on what they saw during lectures, but they did not know the correct reasoning. Besides, we found similar types of explanations as described in subsection 9.2.3, which we presented the student explanations for the BoC task.

9.3.4 Commentary

We conducted additional investigations related to the BoC task using the mBoC task. The results from the pretests and in-class clicker for the mBoC task were better than the BoC task primarily because of the options given in the mBoC task. We also found that after students did the mBoC task on pretests, the student performance on the BoC task on the subsequent pretest

improved after students did the mBoC task again during lecture instruction. However, there was not much improvement in student explanations in the BoC task. Lastly, the student explanations in the mBoC task were about the same as in the BoC task. Students did not apply relative motion concept and the no-slip condition readily. The incorrect approaches include (1) comparing the translational motion in the board with the translational and rotational motions of the cylinder, (2) thinking that no-slip condition implies equal displacement, and (3) thinking that the force is split into translational and rotational parts.

9.4 SUMMARY

In this chapter, we investigated student understanding of the no-slip condition using the examples illustrated in López's paper. These examples were the cylinder-on-board (CoB) and board-on-cylinder (BoC) tasks. They involved two objects moving relative to each other, and one of them is rolling without slipping. We collected student responses on these two tasks because no one has reported them at the time we performed this investigation. We also included results from the modified BoC task on pretests and during in-class clicker questions.

The percentage correct for the different tasks on different settings were found to be: (1) about 50% correct on the CoB task on pretests, (2) about 25% correct on the BoC task on pretests, (3) about 55% correct on the mBoC task on pretests, and (4) about 65% correct on the mBoC task during in-class clicker. We believe that students did better on the mBoC task than the BoC task because of the options given in the question.

Based on the student explanations, we found that students did not have the operational definition of the no-slip condition. They tended to relate the no-slip condition to imply equal displacements. Students also did not apply relative motion concepts in the CoB and BoC contexts

readily. When asked to compare kinematical quantities between two rigid bodies, some students used reasons that were reported by Hunter *et al.*, such as (1) tendency to treat force as partially or completely "used up" in rotational motion and (2) tendency to treat the translational/rotational partition of kinetic energy as having direct causal influence on translational motion. Lastly, we found that some students misinterpreted equations. For example, in the BoC task, these students knew that the board would travel further than the cylinder (or the cylinder would travel less distance than the board), so they selected $\Delta x_{\mathrm{b}} = 0.5\Delta x_{\mathrm{cyl}}$, because there is a coefficient of '0.5' in front of the term representing the displacement of the cylinder.

Chapter 10. ADDRESSING STUDENT DIFFICULTIES WITH ROLLING MOTION

In the previous three chapters, we have discussed results from our investigation into aspects of student understanding of rolling motion. First, we verified the reported student difficulties and reproduced results that were reported in the literature. Second, we extended our scope of the investigation to gather additional insights into student approaches to similar problems. We also correlated student performance on both relative motion and rolling motion problems. Third, we gathered student responses to contexts that were not yet reported in the literature. Through these investigations, we identified several additional student difficulties related to rolling motion as well as some subtleties related to those that had been reported. This chapter describes the curriculum development process to address specific student difficulties.

The structure of this chapter is as follows. In section 10.1, we review the student difficulties with rolling motion based on the results of the investigations presented in the previous three chapters. The preliminary version of the tutorial curriculum is discussed in section 10.2, and the results from the of that version are presented in section 10.3. The curriculum was then modified according to the results from the preliminary assessment and some additional investigations. The modifications are described in section 10.4, and the corresponding assessment results presented in section 10.5. The last section, 10.6, summarizes this chapter.

10.1 STUDENT DIFFICULTIES WITH ROLLING MOTION AND LEARNING OBJECTIVES OF THE TUTORIAL CURRICULUM

Before we outline the curriculum development to address the student difficulties, we first review the difficulties we have found based on the results presented in the previous three chapters. We then outline the learning objectives for the tutorial curriculum.

10.1.1 Difficulty in applying relative motion concepts in rolling motion

Students had difficulty in applying relative motion concepts in rolling motion. This difficulty had been documented by previous research such as Rimoldini and Singh [12], and Fang and Uziak [15]. We used tasks of rolling wheel context (the Rolling Motion Task in this dissertation) published by Rimoldini and Singh in our replication studies. Our results in Chapter 7 showed that this difficulty is ubiquitous among our introductory physics students.

Students tended to choose answers that were consistent with the center-of-mass (CM) frame. The CM frame appeared to be the students' preferred frame, as described by Rimoldini and Singh. The preferred frame reasoning is one of the student difficulties in relative motion described in Boudreaux's thesis. In Boudreaux's study, he wrote that students tended to select answers in the laboratory (ground) frame when students were asked about the kinematical quantities in a frame that was moving relative to the ground. In the case of rolling motion, on the contrary, many students chose an answer that was in the CM frame, which was moving relative to the ground. Students were unable to distinguish the ground frame and the CM frame clearly.

From the explanations, students tended to use "tangential velocity" to explain their incorrect answers. We interpreted that students might be using the curve that was given in the question to arrive at their answers. In Chapter 8, we found a few results. First, the approach

students used to determine the direction of velocity in the Rolling Motion Task (RMT) was consistent with the approach they used in determining that direction when the cycloid trajectory was provided. Second, students performed better in the Rolling Motion Task after students were asked to think about the trajectory of a point of the rim of the rolling cylinder. Third, when students determined the direction of the velocity of a point on other rigid bodies (such as rod) that translated and rotated at the same time, they did not always use the tangent of the trajectory to arrive at the answer. Lastly, we correlated student performance in simple relative motion questions with that in the RMT. We found that students could do some simple relative motion questions but unable to apply relative motion concepts in the rolling motion questions.

The results of our investigation suggested that introductory physics students tended to use the tangent of the circular trajectory to determine the direction of velocity if students were given a curve or prompted to think about the trajectory. However, students did not use the relative motion approach readily in the rolling motion context.

10.1.2 Difficulty in thinking of the dynamics (forces) involved in rolling motion

Rimoldini and Singh highlighted that students had difficulty in thinking of the frictional force involved in rolling motion. In Chapter 7, we presented results from pretests and exams to show that our students displayed similar difficulties. In most cases, students tended to think that the direction of the frictional force would be opposite to the direction of motion. This reasoning would be correct for kinetic friction, but not necessarily true for static friction. In one instance, some students were unable to distinguish static and kinetic friction.

10.1.3 Difficulty in applying the no-slip condition in situations involving rolling without slipping

This difficulty was primarily investigated in Chapter 9. We used the cylinder-on-board and board-on-cylinder contexts to investigate student ability in applying the no-slip condition. Our results showed that many students thought that the no-slip condition meant equal displacement. This idea would be true if the two objects that were in contact with each other have the same center-of-mass velocities (v_{CM}). However, if one of the objects rolled without slipping, then the v_{CM} of both objects would not be the same. We believe that many students do not have an operational definition for the no-slip condition because it is not explicitly mentioned in the lecture instruction. The no-slip condition is a specific situation that can be expressed in terms of relative motion concept. The students' inability to apply the no-slip condition could also be classified as another student difficulty in applying relative motion.

10.1.4 Learning objectives for the tutorial curriculum

Based on the prior research and our investigations, many students needed help in understanding rolling motion. Some researchers have suggested a few instructional strategies, namely (1) take account on student difficulties in relative motion [12], (2) use relative motion approach to teach rolling motion and emphasize its utility in different physics contexts [13], and (3) in-class experimentation of the motion of rolling objects, using motion analysis to determine the direction of velocity, acceleration, and sometimes forces [14].

After reviewing the literature and results from our investigation, we decided on four learning objectives for the tutorial. We hope that after the tutorial, students can: (1) apply relative motion concepts to rolling motion or rigid body motion in general, (2) define the no-slip condition

and show how it applies to different situations, (3) recognize the role of friction in rolling without slipping and rolling with slipping situations, and lastly, (4) perform better in questions that involve relative motion ideas. The first three objectives are related to rolling motion, and the ultimate objective is to help students reinforce fundamental concepts. The challenge is to design a 50-minute in-class curriculum and homework problems that could fulfill the above learning objectives.

10.2 THE PRELIMINARY VERSION OF THE TUTORIAL CURRICULUM

This section describes how we incorporated the four learning objectives in the preliminary version of the tutorial *Rolling and slipping: A relative motion approach*. We then present results from the posttest to assess student learning after the tutorial in the following section. The preliminary versions of the tutorial can be found in Appendix C.

The preliminary version was used in four successive academic quarters, from Au'17 to Au'18 quarters (not used in the Su'18 quarter). There were minor variations to the same primary sequence of the instructional activities. The tutorial is usually scheduled in the last instructional week of the quarter. Due to some unforeseen circumstances in the Sp'18 quarter, we modified the tutorial schedule so that we were able to test the tutorial without interruption. The teaching assistants were then able to give feedback to students' homework. This arrangement affected some posttest results in that quarter, but we had the opportunity to test the effectiveness of both the tutorial and homework on specific tasks. We present some posttest results to highlight the difference. Rather than describing all variations of the tutorial sequence, we present the version used in the Au'18 quarter. A description of the version used in earlier quarters was published in a conference proceeding [72].

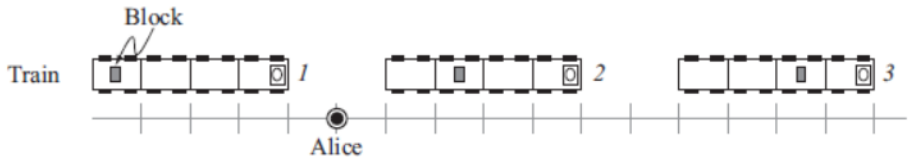
10.2.1 Description of the tutorial sequence

The first section of the tutorial guides students through a review of the Galilean transformation of velocities for the case when all the motions are collinear, as shown in Figure 10.1. The purpose of this section is meant for students to review relative motion. The students had lecture instruction on relative motion in the middle of the quarter while rolling motion is taught near the end of the quarter. Some students may not remember relative motion when they enter the classroom, so to get students up to speed for the tutorial, we start the tutorial worksheet with a review of relative motion.

In section I, students are shown a ground-based observer who is observing a box being pushed across a moving train car. They are asked to determine the three separate displacements and to relate these vectors, both diagrammatically and mathematically. They then relate the displacements with the corresponding velocities and express the results in terms of velocity.

In Section II, we want students to apply relative motion ideas to a rigid body that is translating and spinning in the air. This question is related to the first learning objective (students can apply relative motion concepts to rolling motion or rigid body motion in general). Students consider the motion of a hoop of radius *R* that is moving to the right through the air while spinning clockwise with an angular velocity ω, shown in Figure 10.2. Students first draw a vector to represent the velocity of the center-of-mass of the hoop in the ground frame ($v_{\mathrm{CM}}^{(G)}$). Next, they are asked to find the following velocities for the center-of-mass frame: (1) the center-of-mass of the hoop, (2) the lowest point on the rim, and (3) the highest point on the rim. They apply the Galilean transformation of velocities in two dimensions to find the possible magnitudes and directions of the instantaneous velocities of all three points in the ground frame (Figure 10.3).

Alice, a stationary observer, observes a train moving to the right. A block on the train is pushed so that it moves at constant velocity relative to the train. The positions of the train and the block are shown at instants *1*–*3*, separated by equal time intervals.

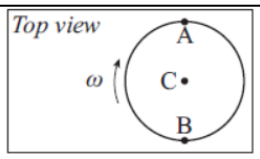


A. In the spaces provided below, draw vectors to represent the following displacements over the interval from instant *1* to instant *3*:

- the displacement of the center of the train *in Alice's frame*, $\Delta\vec{x}_{T}^{(A)}$
- the displacement of the block *in the frame of the center of the train*, $\Delta\vec{x}_{B}^{(T)}$
- the displacement of the block *in Alice's frame*, $\Delta\vec{x}_{B}^{(A)}$

Figure 10.1. Tutorial exercise in which students review Galilean transformation of velocity in a one-dimensional context.

A hoop of radius R (shown at right) is thrown in the air horizontally such that it spins with angular velocity ω and moves to the right.



A. Draw a vector on the diagram to represent the direction of the instantaneous velocity of the center of mass of the hoop in the reference frame of the ground.

B. Consider the instantaneous velocities of points A, C, and B in the reference frame of the *center-of-mass* of the hoop (point C), *i.e.*, $\vec{v}_{A}^{(C)}$, $\vec{v}_{C}^{(C)}$, and $\vec{v}_{B}^{(C)}$, respectively. In the table below,

1. write down their magnitudes in terms of R and ω, and
2. draw vectors to represent their directions

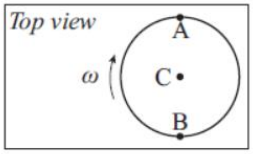
	$\vec{v}_{A}^{(C)}$	$\vec{v}_{C}^{(C)}$	$\vec{v}_{B}^{(C)}$
Magnitude			
Direction			

Figure 10.2. Tutorial exercise in which students analyze the direction of velocities of points A, B, and C in the center-of-mass frame for a rotating hoop moving through the air.

The section concludes by leading students to recognize that without additional information (*i.e.*, the relationship between the magnitudes of $v_{\mathrm{CM}}^{(G)}$ and ω), it is not possible to determine whether, in the ground frame, the lowest point on the hoop has a velocity to the right, to the left, or zero.

E. Consider the instantaneous velocities of points A, C, and B in the *ground* frame for the instant shown at right. In the spaces below,

1. write down their magnitudes in terms of v_{CM}, R, and ω (the positive direction is chosen to be to the right), and
2. draw vectors to show all their *possible* directions using your answer in part D.



	$\vec{v}_{\mathrm{A}}^{(G)}$	$\vec{v}_{\mathrm{C}}^{(G)}$	$\vec{v}_{\mathrm{B}}^{(G)}$
1. Magnitude			
2. Possible directions			

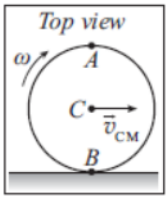
Figure 10.3. Tutorial exercise in which students apply Galilean relativity to determine the possible magnitudes and directions of the velocities of points A, B, and C in the ground frame.

In the final section of the tutorial, we want students to apply the no-slip condition in rolling without slipping situation and recognize the role of frictional force in rolling motion. Section 3 starts by asking students to predict how the motion of a cylinder changes with time for three different initial conditions: $v_{\mathrm{CM}} = R\omega$, $v_{\mathrm{CM}} > R\omega$, and $v_{\mathrm{CM}} < R\omega$. Students need to draw possible free-body diagrams for the cylinders and use the diagrams to explain their predicted changes in motion. After that, the teaching assistants perform a demonstration so that students can check their predictions. Students record their observations, revise their free-body diagrams for the cylinders to account for the observed changes in motion in the table shown in Figure 10.4.

The section continues with a description of how rolling with slipping compares to rolling without slipping for a cylinder on a table. Students apply the definitions of rolling with slipping and rolling without slipping to describe the initial motion of the cylinders in those three cases.

They then arrive at the condition for pure rolling motion (*i.e.*, $v_{\mathrm{CM}} = R\omega$). Lastly, they compare the direction of the velocity of point B in the ground frame with the direction of the frictional force acting on the cylinder when the force exists.

III. Rolling and slipping



A. A cylinder moves across a horizontal surface. The surface is *not* frictionless.

Consider the motion of the cylinder for three different initial conditions: (i) $v_{\mathrm{CM}} = R\omega$, (ii) $v_{\mathrm{CM}} > R\omega$, and (iii) $v_{\mathrm{CM}} < R\omega$.

For each case, complete the table below:

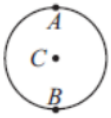
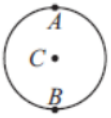
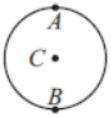
	Initial condition of the motion		
	$v_{\mathrm{CM}} = R\omega$	$v_{\mathrm{CM}} > R\omega$	$v_{\mathrm{CM}} < R\omega$
Indicate the initial directions of $\vec{v}_A^{(\mathrm{G})}$, $\vec{v}_C^{(\mathrm{G})}$, and $\vec{v}_B^{(\mathrm{G})}$.			
Draw a possible extended free-body diagram of the cylinder.			
Would v_{CM} *increase*, *decrease*, or *remain the same* as the motion continues?	Prediction:		
	Observation:		
Would ω *increase*, *decrease*, or *remain the same* as the motion continues?	Prediction:		
	Observation:		

Figure 10.4. Tutorial exercise in which students predict and observe how the motion of the cylinder changes with time for three different initial conditions: $v_{\mathrm{CM}} = R\omega$, $v_{\mathrm{CM}} > R\omega$, and $v_{\mathrm{CM}} < R\omega$.

10.2.2 Description of the homework sequence

In the first question of the tutorial homework, students apply what they have learned to determine the magnitudes and directions of the velocities of all three points D, E, and F on the cylinder shown in Figure 10.5. The purpose of this question is for students to apply the no-slip motion and relative motion concepts in rolling motion.

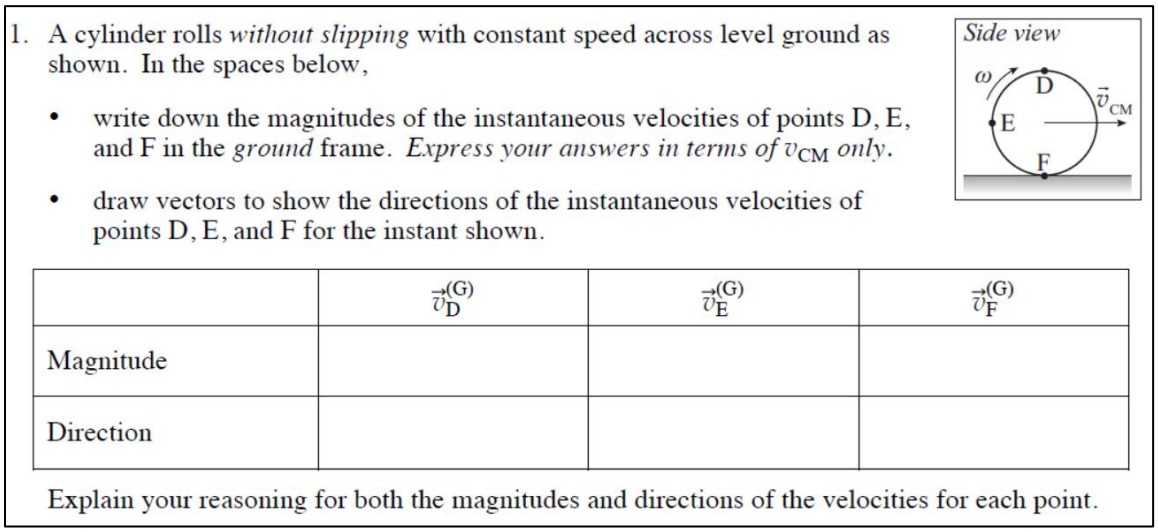
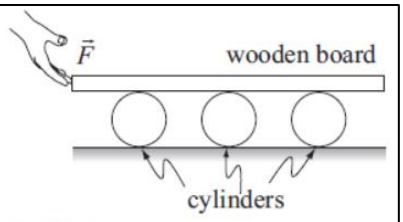


Figure 10.5. Homework exercise in which students determine the answers to the RMT-Direction and Magnitude tasks.

The second question is the board-on-cylinder (BoC) task that was described in section 9.2, as shown in Figure 10.6. Similar to the previous question, they apply the ideas on Galilean relativity and no-slip condition to determine the magnitudes of the velocities of three different points on the cylinder. They are then asked to relate the velocities and accelerations of the board and the center-of-mass of the cylinder.

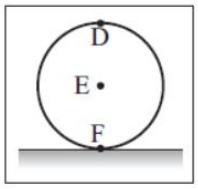
In the preliminary version of the tutorial homework, there are no questions for students to practice what they learned about the role of friction in rolling motion. More homework questions are added as we continue to improve the curriculum. The details are seen in section 10.4.

A wooden board is placed on top of three identical solid cylinders. The board is pushed to the right by a constant horizontal force of magnitude F. The cylinders roll without slipping on a flat surface. There is also no slipping between the cylinders and the board.



a. What is the direction of center-of-mass velocity of the cylinders when the horizontal force is applied to the board? Explain.

b. Consider an instant while the force is being applied to the board. Rank the magnitudes of the instantaneous velocities of points D, E, and F on the cylinder shown at right in the **reference frame of the ground**. If any of the magnitudes is zero, state so explicitly. Explain.



c. In the ground frame, express the magnitude of the instantaneous velocity of the board, v_{board}, in terms of the magnitude of the instantaneous velocity of the center-of-mass of the cylinder, $v_{cylinder}$. Explain.

d. In the ground frame, express the acceleration of the board, a_{board}, in terms of the acceleration of the center-of-mass of the cylinder, $a_{cylinder}$. Explain.

Figure 10.6. Homework exercise in which students determine the relationship between the velocity/acceleration of the board and the velocity/acceleration of the CM of the cylinder in the board-on-cylinder task.

10.3 ASSESSMENT OF PRELIMINARY VERSION OF THE TUTORIAL

Several tasks were used to assess student learning after they worked through the tutorial. In most cases, the teaching assistants (TAs) were not able to give students feedback to their homework responses because students did the tutorial in the last week of the quarter, and they took their final exams in the following week. We assessed the effectiveness of the tutorial according to our learning objectives. The objectives are met when the student performance on posttests match with how graduate student TAs performed on written pretests. In some cases, we compare the student posttest results with the results when there was no tutorial intervention.

10.3.1 Learning objective 1 – students can apply relative motion concepts to rolling motion or rigid body motion

We used several variations of the Rolling Motion Task (RMT) to assess this learning objective. In the tutorial worksheet, students worked with a cylinder that rolls without slipping to the right. Students need to determine the direction of the velocity of some points on the rim of the cylinder in the ground frame. In the posttest, we asked students to find the direction of the velocity of points that they did not do in the homework on the rim of the rolling cylinder (refer to Table 7.31). In some cases, we modified the rolling wheel context into a Frisbee that is both translating and spinning in the air, with $v_{\mathrm{CM}} = R\omega$.

The posttest questions were mostly multiple-choice, as this was the usual format in the final exam. We cannot find out how students explained their answers from multiple-choice questions. However, based on our preliminary investigations, most students chose two answers (the correct and the most common incorrect). We also know the reasoning students used behind their answers so we can ask our posttest questions in multiple-choice to capture most of the information.

We had administered the Rolling Motion Task (Direction & Magnitude) and its variation (Frisbee) in both the pretest and posttest from Wi'18 to Au'18 quarter. The results are presented together with the context shown in Table 10.58 and Table 10.59. During the quarters when the tutorial is implemented, the gain from pretest to posttest ranges from 22% to 51% in the RMT-Direction, and from 50% to 66% in the RMT-Magnitude. In both tasks, the most common incorrect answer is consistent with the CM frame approach. In all comparisons, the McNemar test *p*-values were less than 5%, which implied that the gains were highly associated with the tutorial.

The tutorial is usually scheduled in the last week of the quarter after students had the relevant lecture instruction. In the Sp'18 quarter, as there might be a TA strike in the last week of the quarter, we changed the schedule so that students did the tutorial a week before the last week. This arrangement would mean that students had to attend the tutorial before they had lecture instruction. However, it allowed us to assess the impact of homework feedback on student learning. From Table 10.58 and Table 10.59, we saw that the gains in the percentage correct for the RMT-Direction and RMT-Magnitude in the Sp'18 quarter were typically higher than in other quarters when students did not get homework feedback. These results suggest that homework feedback is helpful in student learning, which is consistent with our expectations.

Now we compare the posttest results with the TA's pretest results. Out of the 43 graduate TAs we surveyed during our investigation, 77% of them answered the RMT-Direction task correctly, and 81% of them answered the RMT-Magnitude task correctly. On the RMT-Direction task, the students' posttest results, and the TAs pretest results were comparable after the tutorial instructors gave students feedback on homework. On the RMT-Magnitude task, the results from both groups were comparable with or without the feedback. In conclusion, our posttest results suggested that students could apply relative motion concepts to rolling motion contexts. However, one may note that the students did similar tasks on pretests and posttests, so students may remember answers instead of truly understand the concepts.

Table 10.58. Pretest and posttest results for the Rolling Motion Task (Direction) from Wi'18 to Au'18 quarters. In all cases, students are required to determine the direction of point 2 on the rolling wheel (Frisbee for posttest in Sp'18) in the ground frame.

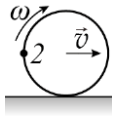
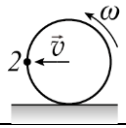
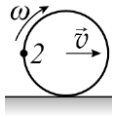
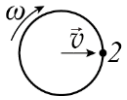
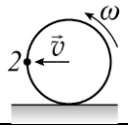
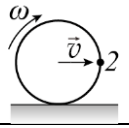
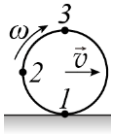
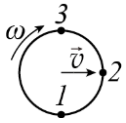
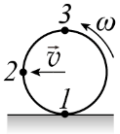
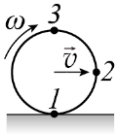
		Wi'18 quarter (*N* = 262) **No** tutorial HW feedback		**Sp'18 quarter (*N* = 256)** **With** tutorial HW feedback		**Au'18 quarter (*N* = 156)** **No** tutorial HW feedback	
		Pretest Before any instruction 	**Posttest** Final exam 	**Pretest** Before any instruction 	**Posttest** Final exam (Frisbee) 	**Pretest** After lecture instruction 	**Posttest** Final Exam 
Approach	Relative Motion (Correct)	24%	46%	19%	70%	15%	51%
	CM frame	65%	44%	65%	23%	73%	37%

Table 10.59. Pretest and posttest results for the Rolling Motion Task (Magnitude) from Wi'18 to Au'18 quarters. In all cases, students are required to rank the speeds of points 1, 2, and 3 on the rolling wheel (Frisbee for posttest in Sp'18) in the ground frame.

		Sp'18 quarter (*N* = 256) **With** tutorial HW feedback		**Au'18 quarter (*N* = 156)** **No** tutorial HW feedback	
		Pretest Before any instruction 	**Posttest** Final exam (Frisbee) 	**Pretest** After lecture instruction 	**Posttest** Final Exam 
Approach	Relative Motion (Correct)	18%	84%	24%	74%
	CM frame	81%	13%	73%	19%

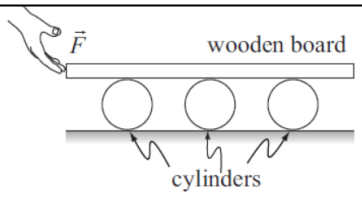
10.3.2 Learning objective 2 – students can define the no-slip condition and show how it applies to different situations

We used two different tasks to assess this learning objective in the final exam. They were the board-on-cylinder task and the ball-on-moving-belt task. Students saw the exact setup of the board-on-cylinder problem twice: once in the pretest, and then in the final exam after they finished the tutorial. The ball-on-moving-belt task was the posttest question that matched with the cylinder-on-board task since they required the same concepts to arrive at the correct answer.

(A) Board-on-cylinder (BoC) task

The BoC task in the final exam (Figure 10.7) was a free-response question. Students were asked to determine the correct equation that relates to the acceleration of the board, a_b, and the center-of-mass acceleration of the cylinder, a_{cyl}, and explained their reasoning. We used several versions of the BoC tasks, and they involved equations that relate to other kinematical quantities (displacements and velocities).

A wooden board is placed on top of three identical solid cylinders. The board is pushed to the right by a constant horizontal force of magnitude F applied to the end of the board. The cylinders roll without slipping on a flat surface. There is also no slipping between the cylinders and the board.



Consider an instant when the horizontal force is still exerted on the board and the board is still in contact with **all** of the cylinders.

Let a_b be the magnitude of the acceleration of the board and a_{cyl} be the magnitude of the center-of-mass acceleration of each cylinder.

[8 pts] Which of the following correctly describes the relationship between a_b and a_{cyl}? **Circle** the correct answer below. **Explain how you determined the answer**.

$a_b = 0.5\, a_{\text{cyl}}$	$a_b = a_{\text{cyl}}$	$a_b = 1.5\, a_{\text{cyl}}$	$a_b = 2\, a_{\text{cyl}}$	$a_b = 3\, a_{\text{cyl}}$	Not enough information

Figure 10.7. Board-on-cylinder task in the final exam.

This task was administered as a free-response question in the final exam of the Wi'18 and Au'18 quarter. We presented the results from the Au'18 quarter because the posttest results obtained in the Wi'18 quarter were affected by the lecture instruction. In the Wi'18 quarter, the students had discussions on the modified board-on-cylinder task during lectures after they took the pretest in the last week of the quarter. Around 60-70% of the students answered the posttest correctly, which was similar to the percentage correct as of the in-class clicker question, so we suspected the lecture-discussion had affected the percentage correct to some extent, and this result did not help evaluate the impact of the tutorial. The results obtained in the Au'18 quarter are presented in Table 10.60.

Table 10.60. Percentage correct for the board-on-cylinder task in the pretest and posttest of Au'18 quarter. Different pretest vs. posttest combinations are presented.

Instructor	**Pretest task vs. posttest task**	***N***	**% correct (Pre to Post)**	**Gain**	**% correct on both pretest and posttest**
A	Displacement vs Acceleration	220	29% → 37%	8%	17%
	Displacement vs Velocity	118	25% → 41%	16%	18%
B	Acceleration vs Displacement	157	46% → 61%	15%	34%

We asked students different kinematical relationships between the board and the cylinder in pretests and posttests in the Au'18 quarter. The percentage correct in the pretest of instructor B's students was higher than that of instructor A's. Students who taught by instructor B saw the modified board-on-cylinder problem in-class clicker question the day before they attempted the pretest, while students taught by instructor A did not. The increase in the percentage correct in the pretest is highly associated with the in-class activity using the χ^2 test ($\chi^2\ 1\ = 17.1, p < 0.01\%$).

The difference in the percentage correct in the pretest is around 20%, so the effect size of the in-class clicker question is medium.

The gain in percentage correct from pretest to posttest ranged from 8-16%. We computed the McNemar's test *p*-value for each population, and all three increments were statistically significant. We also used the percentage correct in pretest and posttest to compute the χ^2 association test *p*-value for the two student groups under instructor A to determine whether the effect of types of tasks on the percentage correct. We found no significant difference between them ($\chi^2\ 1\ = 0.34$, $p = 0.6$). The percentages among the two groups of students under instructor A who answered both the pretest and posttest correctly were about the same, which suggested that there was no difference between them. All these results suggested that the gain in the percentage correct was associated with the tutorial curriculum.

Among the instructor A's students who answered correctly, two-thirds of them (~25% of the class) provided correct reasoning. The remaining of them explained using forces/energy-related concepts, such as "the board has only force/translational kinetic energy while the cylinder has both the force and torque/translational and rotational kinetic energy." Students who compared the motion of the board (translational) and the motion of the cylinder (translational and rotational) are categorized under force/energy-related concepts.

The most common incorrect answer chose in the posttest was $a_{\mathrm{b}} = 3a_{\mathrm{cyl}}$. Students who chose this answer thought that the board's acceleration was related to the number of cylinders below the board. Since there were three cylinders below the board, the acceleration of the board was three times the acceleration of the cylinder. This line of reasoning was also seen in the pretest (see sub-section 9.2.3).

The percentage correct of instructor B's students in the posttest is 61%, which is within the range of what we found in the Wi'18 quarter. The result shows that the sequence of the in-class activity and the tutorial curriculum can improve the student understanding of the board-on-cylinder problem. We also managed to obtain some knowledge on how each of the instructional activity (in-class clicker question and tutorial curriculum) affect the student success on this task.

Around 40% of instructor B's students gave the correct answer and explanation on the posttests. Many students who answered correctly but gave incorrect explanations explained using force/energy-related concepts, same as instructor A's students. The most common incorrect answer instructor B's students chose in the posttest is $a_{\mathrm{b}} = 0.5a_{\mathrm{cyl}}$ (~15% of class). Some students recognized that the board moved faster than the cylinder, but they chose the wrong equation. These students associated the coefficient "0.5" in front of the a_{cyl} to mean that the cylinder was moving at half the distance moved by the board.

Comparing with results from graduate TAs on written pretests, 35% of 43 TAs answered correctly. The students' posttest results were comparable or better than the graduate TAs after the tutorial instruction. The results from the TAs suggested that the graduate students had difficulty dealing with the BoC context. Nonetheless, the posttest results suggested that the tutorial curriculum helped students to understand the situation better.

(B) Ball-on-moving-belt (BoM) task

This posttest question (Figure 10.8) is administered as a multiple-choice question in the final exam of the Sp'18 quarter. A ball rolls without slipping to the right on a conveyor belt that is also moving to the right. Students need to choose the correct equation that relates to the speed of the conveyor belt in the ground frame (v_{CG}), the speed of the ball in the ground frame (v_{BG}),

the radius of the ball (R), and the angular velocity of the ball (ω). The purpose of this task is to find out whether students can relate what they learned in the board-on-cylinder task in homework to a new context that uses the same underlying concepts.

The correct answer requires the use of the no-slip condition, the same as the cylinder-on-board task (see Section 9.1). At the point of contact between the ball and the conveyor belt, its speed in the ground frame is $v_{\mathrm{BG}} - R\omega$. By the no-slip condition, the speed of the conveyor belt is the same as the speed of the point on the ball in the same (ground) frame, so the correct answer is $v_{\mathrm{CG}} = v_{\mathrm{BG}} - R\omega$.

[6 pts] A ball rolls on a conveyor belt without slipping to the right as shown. A stationary observer (ground frame) observes the motion of the ball.

Let v_{CG} be the speed of the conveyor belt in the ground frame, v_{BG} be the speed of the center-of-mass of the ball in the ground frame, R and ω be the radius and the rotational speed of the ball, respectively.

How should the stationary observer relate the above quantities (v_{CG}, v_{BG}, R, and ω)?

A. $v_{CG} = v_{BG} = R\omega$

B. $v_{CG} = v_{BG} \neq R\omega$

C. $v_{CG} = v_{BG} - R\omega$

D. $v_{CG} = v_{BG} + R\omega$

E. $v_{CG} < v_{BG} - R\omega$

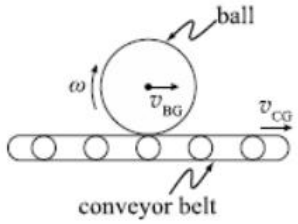
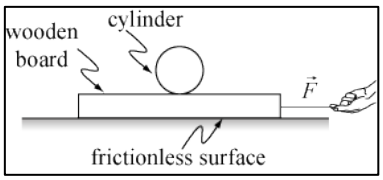
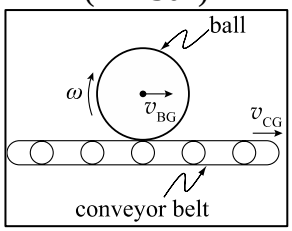


Figure 10.8. The ball-on-moving-belt task on the final exam.

The student responses on the posttest question are summarized in Table 10.61. 50% of the class answered the question correctly. The most common incorrect response is $v_{\mathrm{CG}} = v_{\mathrm{BG}} + R\omega$. Students who selected this answer could be confused by the application of the relative motion concept in this task. They could be confused by the notation, but we had defined the quantities clearly in the question and indicated them on the diagram. Around 15% of the class chose the

answer $v_{\mathrm{CG}} = v_{\mathrm{BG}} = R\omega$. This result suggests that some students were applying the rolling without slipping rule indiscriminately. They also think that if there is no slipping, then the ball and the cylinder must have the same speed.

Table 10.61. Student responses for the cylinder-on-board task during the pretest and the ball-on-moving-belt task on the final exam.

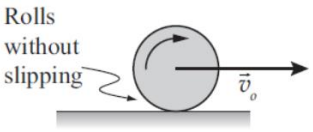
Types of responses	**Sp'18 Pretest** Cylinder-on-board (N = 420) 	**Sp'18 Posttest** Ball-on-moving-belt (N = 501) 
Correct relative motion relation	49%	50%
Incorrect relative motion relation	16%	25%
Indiscriminate use of $v_{\mathrm{CM}} = R\omega$ and/or no-slip = equal CM speed rule	18%	17%
Others	17%	8%

Among the 412 students who did both the pretest and the posttest, only 20-25% of the class answered both tasks correctly. There is no significant improvement in how students apply relative motion ideas and the no-slip condition to new contexts. The reason for the minimal gain is the lack of adequate practice or relevant questions in the tutorial homework. Most students do not see the underlying connection between the board-on-cylinder problem and the cylinder-on-board/ball-on-moving-belt tasks. So, we decided to include the cylinder-on-board problem in the tutorial homework to let students apply the relevant concepts.

10.3.3 Learning objective 3 – students can recognize the role of friction in rolling without slipping and rolling with slipping situations

We use the problem in Figure 10.9 to assess whether students can recognize the role of friction in rolling without slipping situation. We call this posttest problem the friction-on-cylinder (FoC) problem. This problem was also used in past exams, so we believe it is a fair assessment of the effectiveness of the tutorial curriculum. We are not ambitious to assess student knowledge of friction in rolling with slipping situations since the curriculum is in the preliminary stage of development.

A rigid cylinder rolls to the right without slipping (*i.e.*, there is no relative movement between the surface and the lowest point on the ball). The coefficient of static friction between the cylinder and flat surface is $\mu_{\text{static}} > 0$.



It is observed that the center of mass of the cylinder moves with constant speed (*i.e.*, **it does not slow down**).

Center of mass moves to the right with constant speed

22. [5 pts] While the cylinder is rolling with constant speed without slipping, which of the following best describes the direction of the frictional force on the cylinder by the flat surface?

A. The frictional force on the cylinder by the flat surface is **to the left.**

B. The frictional force on the cylinder by the flat surface is **to the right.**

C. The frictional force on the cylinder by the flat surface is **upward.**

D. The frictional force on the cylinder by the flat surface is **zero.**

Figure 10.9. The friction on rolling cylinder (FoC) problem. The correct answer is D.

The problem describes a cylinder that is rolling without slipping to the right on a horizontal surface. Students need to determine the direction of friction acting on the cylinder by the surface. In the problem statement, students are told that the cylinder moves at a constant speed, and it does not slow down. Students can use Newton's 2nd law to find the answer. The cylinder moves at a constant speed, so it has zero acceleration. From $F_{\text{net}} = ma$, the net force on the cylinder is zero.

Since the ground is the only object which can exert a horizontal force on the cylinder, the only possible answer is zero friction.

The student responses to the question before and after the implementation of the tutorial is summarized in Table 10.62. The percentage correct before tutorial implementation is around 15%. It increases to around 20-25% in Wi'18 and Au'18 after the tutorial is implemented. Since the students in all quarters are independent and mutually exclusive from each other, we compute the χ^2 values for both implementations and the *p*-values are less than 5%. The result suggests that the increase in percentage correct on this task is highly associated with the tutorial implementation.

We surveyed 19 graduate TAs on the FoC problem on a written pretest. Only 32% of them answered correctly. The students' posttest results were only a few percentage points lower than that from the TAs. The results from the posttest and TA's pretest showed that many had difficulty in determining the direction of the frictional force on a cylinder that rolled without slipping on a horizontal surface.

Table 10.62. Student responses for the friction on rolling cylinder problem in the final exams before and after tutorial implementation.

Answer	**Before tutorial implementation**	**After tutorial implementation**	
	Au'12 & Au'17 (aggregated) (N = 820)	Wi'18 (N = 479)	Au'18 (N = 597)
Correct (D)	16%	21%	27%
Opposed direction of motion (A)	49%	48%	41%
Others (B, C)	28%, 8%	27%, 4%	30%, 2%

Despite the statistical significance in the χ^2 test of association (due to large sample size), more than half the class continue to exhibit difficulties in determining the direction of the frictional

force on a rolling cylinder after the tutorial instruction. We think that it takes time for students to change their heuristic in determining the direction of the frictional force on moving objects.

Furthermore, students do the tutorial in the last week of the quarter. They are unable to receive feedback on their homework and revise their incorrect ideas before the final exam. Based on the significant improvement found in the RMT-Direction & Magnitude after students received feedback from homework, we believe similar improvements could be achieved as well.

10.3.4 Learning objective 4 – students can perform better in questions that involve relative motion ideas

We assess this learning objective by using a standard relative motion in a two-dimension problem given in the final exam. We compare the exam results in quarters before the tutorial is introduced in the curriculum to the results after the students did the tutorial.

The problem is about the motion of a fish in a flowing river, as shown in Figure 10.10. Students need to determine the direction of velocity the fish needs to move in the river so that the fish can move along path A. They need to create a vector triangle that describes this velocity transformation relation: $v_{\mathrm{F}}^{(\mathrm{G})} = v_{\mathrm{F}}^{(\mathrm{W})} + v_{\mathrm{W}}^{(\mathrm{G})}$. The vector students need to determine the direction of the velocity of the fish in the frame of the water, $v_{\mathrm{F}}^{(\mathrm{W})}$. The other two vectors, $v_{\mathrm{F}}^{(\mathrm{G})}$ and $v_{\mathrm{W}}^{(\mathrm{G})}$, represent the velocity of the fish and the river in the ground frame, respectively.

To obtain the correct answer, students need to realize that the velocity of the fish in the ground frame is downward, and the velocity of the river in the ground frame is to the right. The vector triangle that satisfies the relation is shown in Figure 10.11. The diagram shows that the direction of the velocity of the fish in the frame of the water is downward and to the left, and the

angle that is required by the question is θ shown in the diagram below. Using the given values of the speed of $v_{\mathrm{F}}^{(\mathrm{G})}$ and $v_{\mathrm{W}}^{(\mathrm{G})}$ in the question, and some trigonometry, the answer is 124°, option C.

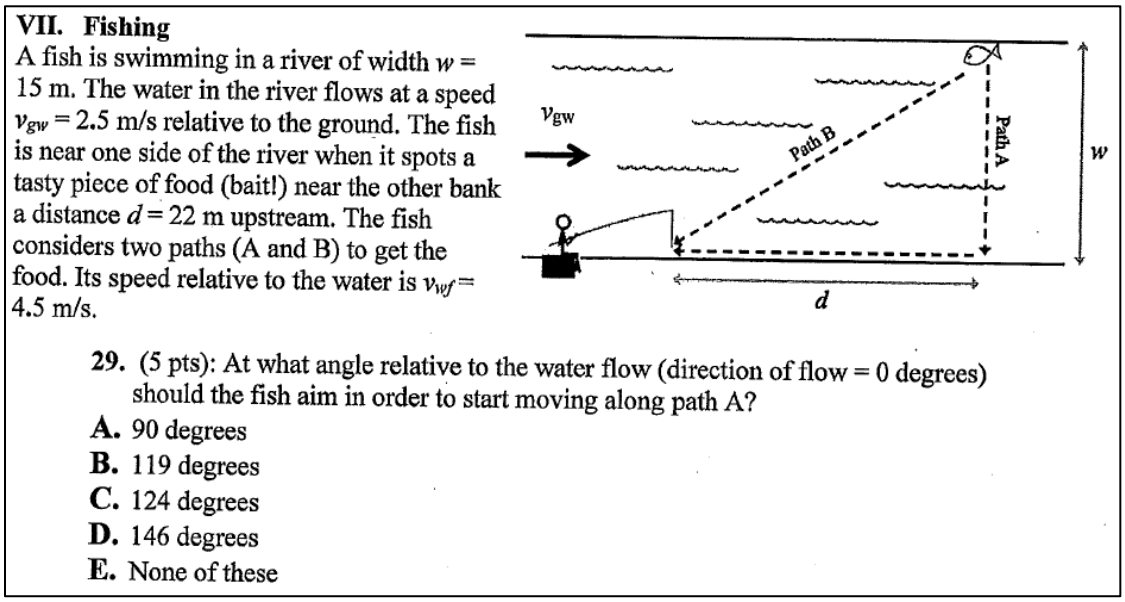


Figure 10.10. Relative motion in 2D problem (fish-in-river) in the final exam.

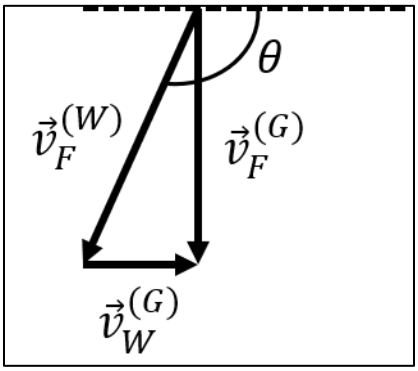


Figure 10.11. Vector triangle for the fish-in-river problem in Figure 10.10.

The results are shown in Table 10.63. The percentage correct drops by 10% after the implementation of the tutorial. The three distractors (A, B, and D) corresponds to different mistakes students make when they solve the problem. Students who chose option A (90°) seemed

to ignore the effects of the moving river. Students who chose option B confused the directions of $v_{\mathrm{F}}^{(\mathrm{G})}$ and $v_{\mathrm{F}}^{(\mathrm{W})}$ on the diagram. They thought that $v_{\mathrm{F}}^{(\mathrm{W})}$ was downward and $v_{\mathrm{F}}^{(\mathrm{G})}$ was downward and to the left, and the output of the calculation (using $90° + \tan^{-1} 2.5/4.5$) would be 119°. Students who chose D (146°) might have used the wrong trigonometric relations (using cosine instead of sine) to determine the value of θ.

Table 10.63. Student responses to the fish-in-river problem in the final exams before and after tutorial implementation.

Answer	**Before tutorial implementation** Au'16 & Au'17 (aggregated) (*N* = 317)	**After tutorial implementation** Au'18 (*N* = 258)
Correct (C)	48%	38%
Incorrect calculations (B, D)	19%, 17%	19%, 17%
Others (A, E)	11%, 5%	20%, 6%

The results suggest that the tutorial does not help students in performing better in standard relative motion in 2D problems in exams significantly. Students may be confused with the labels or the given information. They may not realize that the motion of the fish along path A is in the ground frame but not in the frame of water. The other reason could be the failure to apply the relevant trigonometric relations in this question.

10.3.5 Commentary of assessment of preliminary version of the tutorial

Based on our assessment results, we saw significant improvement in the student performance in the Rolling Motion Task, especially after the students received feedback for their

tutorial homework. There was an increase in the percentage correct on the board-on-cylinder and the friction on rolling cylinder tasks after the tutorial implementation. The posttest results were generally comparable or sometimes better than the results from the TAs' written pretests. However, we did not see a significant gain in the ball-on-moving-belt and relative motion tasks. The results suggested that our preliminary version of the tutorial met the learning objectives that are related to rolling motion and friction on the rolling cylinder, but not on applying the no-slip condition and relative motion.

It should be evident that the activities proposed in the curriculum were meant to address student difficulties in Rolling Motion Task directly. The next apparent student difficulty we tried to address is the friction on the rolling cylinder through in-class demonstration. The application of no-slip condition in rolling motion and its underlying concept – relative motion – appeared to be secondary in the tutorial. The assessment of the preliminary version of the tutorial was modest at best, but there was room for improvement, especially in the application of no-slip condition in rolling motion. As for improving student understanding of relative motion, it may appear to be something unattainable through this curriculum. Students were able to answer certain types of relative motion tasks at ease, but not the question that we used for our assessment. We describe the modifications made in the next section.

10.4 THE MODIFIED VERSION OF THE TUTORIAL CURRICULUM

From our preliminary and in-depth investigation of student difficulties in rolling motion, we found many students tended to use tangential velocity to explain their incorrect answers (see section 8.1). We also found that when students were prompted to sketch the trajectory of a point on the rolling cylinder before attempting the Rolling Motion Task (Direction), the percentage

correct increased significantly as compared to what they would have performed in the pretest. Based on all these findings, we decided to design an activity that would allow students to sketch the cycloid trajectory during the tutorial session.

The assessment of the preliminary version of the tutorial suggests that it would be more useful to focus on the first three learning objectives, namely (1) students can apply relative motion concepts to rolling motion or rigid body motion, (2) students can define the no-slip condition and show how it applies to different situations, and (3) students can recognize the role of friction in rolling without slipping and rolling with slipping situations. The improvement of student understanding of relative motion is placed as a secondary learning objective.

The modified version of the tutorial was used in two quarters, the Sp'19 and Au'19 quarters. Same as the preliminary version, there were minor changes from the first quarter to the last quarter of implementation. We describe the modified version of the tutorial curriculum that was used in the Au'19 quarter.

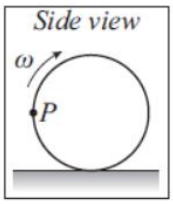
10.4.1 Description of the modified tutorial

We changed the first section of the preliminary version of the tutorial from reviewing of the Galilean transformation of velocity to something that directly dives into rolling motion. The first three questions of section I is shown in Figure 10.12.

In part A, students are asked to predict the direction of the velocity of point *P* (a point on the rolling cylinder) in the ground frame when the cylinder rolls to the right at the given instant. Then in part B, they predict and sketch the actual path of point *P* as the cylinder rolls across the horizontal surface. These two questions are meant to *elicit* students' correct and incorrect responses. The common responses in part A are the upward arrow and an arrow that is 45^{o} above

the horizontal (correct answer). In part B, students tend to draw sinuisoid. Not many students know the cycloid, and some of them draw helical trajectories.

I. Rolling motion



A cylinder rolls without slipping to the right with constant speed on a horizontal surface. A point, P, on the rim of the cylinder is marked. The diagram at right shows the cylinder and point P at an instant $t = t_o$.

A. Draw an arrow on the diagram to indicate the direction of the instantaneous velocity of point P at $t = t_o$ in the ground frame. Briefly explain.

B. Predict the path that point P will follow over time. Sketch the trajectory below and label it as your prediction.

C. Ask an instructor to provide you with a set of equipment and use the equipment to sketch a curve on the diagram above. Label the curve as your experimental result.

On the experimental curve, draw an arrow to indicate the direction of the instantaneous velocity of point P when it is at the location shown in part A.

Compare the direction of $\vec{v}_P$ obtained from the curve with your answer in part A. Discuss your answers with your partners and resolve any inconsistencies.

Figure 10.12. Section I of the modified tutorial (parts A, B, and C) – students predict the direction of $\vec{v}_P$ and the path taken by point *P*, and then sketch the cycloid using a gear rack set.

In part C, students are given a spirograph-like tool for them to sketch the cycloid. After that, they are asked to draw an arrow to indicate the direction of the instantaneous velocity of point P when it is at the location shown in part A. Students are then asked to compare their velocity arrows in parts A and C and resolve any inconsistencies. This phase is the confront and resolve of the tutorial. This instructional strategy is called *elicit-confront-resolve*, an effective method for securing the mental engagement of students [32].

The sketched cycloid is shown in Figure 10.13. Although the cycloid is not perfect (not a cusp when point *P* reaches the ground), but it gives a clear idea to students that the direction of the velocity of point *P* is not always tangential to the given circle, it should be the tangent of the sketched curve.

The sketching activity in part C was made possible by creating gear and rack set using a 3D printer. This idea was originated from a spirograph toy, suggested by our post-doctorate Alexis Olsho. The gear has a pitch circle diameter of 28 mm with a pressure angle of 20°, 28 teeth, and a height of 2.5 mm. The gear rack has a length of 146 mm, a width of 13 mm, and a height of 2.5 mm. There is a hole in the gear that allows students to trace out the cycloid like a spirograph. The center of the hole is 10.3 mm away from the center of the gear, and the hole has a diameter of 2.8 mm. The hole is at a distance near the edge of the gear, but not compromising the fidelity of the gear at the same time. The design of the gear and rack set is done on Onshape [73], an online, free, computer-aided design (CAD) tool. The set can be printed using any 3D printer. Additional details of the gear and rack can be found in Appendix C.

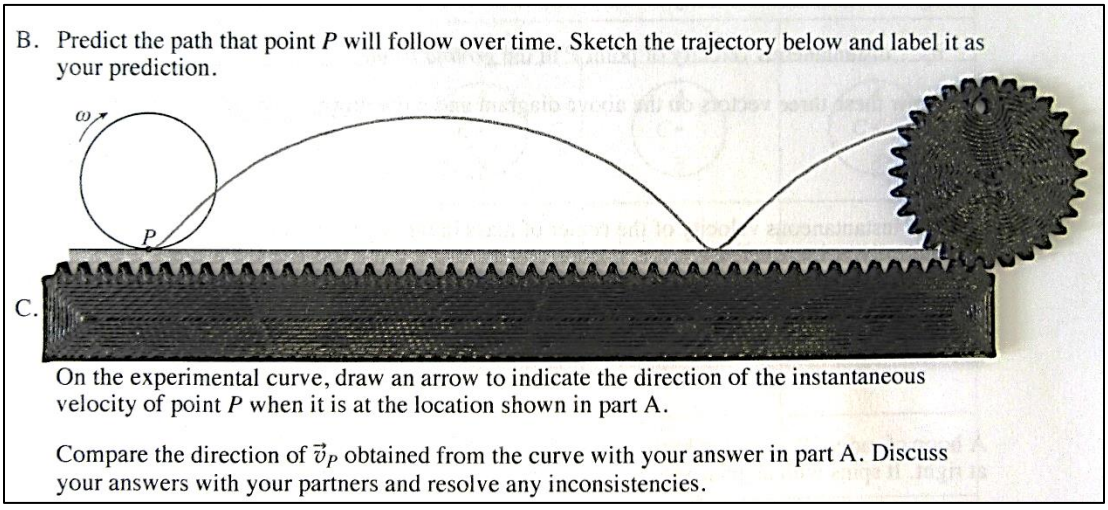


Figure 10.13. The cycloid trajectory is sketched using the 3D-printed gear rack set. Note that this trajectory is not a perfect cycloid.

Parts D and E of section I of the modified tutorial is shown in Figure 10.14. Students need to select the correct reasoning out of the two given statements, which were constructed based on explanations students gave on the pretest. Statement 2 is the correct statement, and it implies that there is some addition of velocities involved. The purpose is to bring students' attention so that they can write down the velocity addition equation quickly in section 2. Part E is a recap of what they did in parts C and D. Students can use the trajectory sketched in part C or statement 2 in part D to arrive at the correct answer. The instructors then check their reasoning to make sure the students understand the content on the first page before they proceed to the next section.

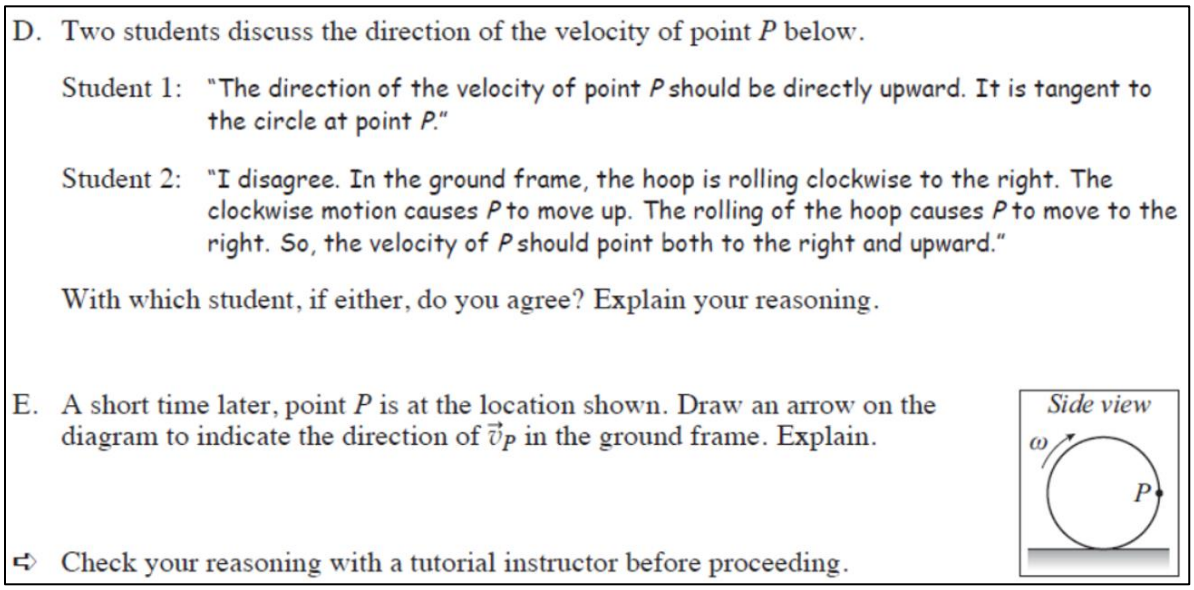


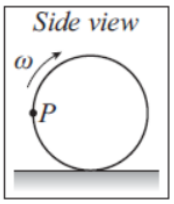
Figure 10.14. Parts D and E of section I of the modified tutorial – student interact with two discussion statements and determine the direction of v_P after the cylinder rolls for some time.

The first two parts of section II of the modified tutorial (Figure 10.15) aims to get students to arrive at the velocity addition equation based on what they have completed in section I. Students need to draw the three velocities ($v_{\mathrm{P}}^{(\mathrm{C})}$, $v_{\mathrm{C}}^{(\mathrm{G})}$, and $v_{\mathrm{P}}^{(\mathrm{G})}$) on the rolling cylinder. We hope that by

doing so, students can arrive at the Galilean transformation of velocity equation for rolling motion on their own. After that, they proceed to complete the task, as shown in Figure 10.3, which students need to apply relative motion ideas to determine the magnitudes and directions of three different points on a hoop that is flying horizontally in the air. We kept most parts of section II in the preliminary version but incorporated what students did in the previous section to link the Galilean relativity ideas in rolling motion.

II. Combining translational and rotational motion

In the previous section, the velocity of point P on the rim of a rolling cylinder in the ground frame involves adding two velocities.



A. Consider the following three vectors:

$\vec{v}_P^{(C)}$: instantaneous velocity of point P in the *center of mass frame*

$\vec{v}_C^{(G)}$: instantaneous velocity of the center of mass in the *ground frame*

$\vec{v}_P^{(G)}$: instantaneous velocity of point P in the *ground frame*

Draw these three vectors on the above diagram and write down an equation that relates them.

The instantaneous velocity of the center of mass in the reference frame of the ground is typically written as $\vec{v}_{CM}$.

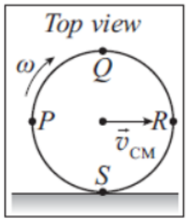
B. Rewrite the equation in part A using $\vec{v}_{CM}$.

Figure 10.15. Parts A and B of section II of the modified tutorial – students determine the velocity addition equation based on the exercise in section I.

We kept most parts of the last section of the preliminary version, such as the table shown in Figure 10.4 and the description in section 10.2.1. To get students to recap what they have learned from the tutorial, we added questions for students to revisit the case in which the cylinder rolls without slipping, as shown in Figure 10.16. Part F is a repeat to the last part of in section II

but expressed the magnitudes in terms of v_{CM} only. This question is taken from the first question of the preliminary version of the tutorial homework. Part G gets students to look at what they did in part A and recognize that the frictional force acting on the cylinder is zero. We wanted students to understand the frictional force acting on a cylinder that rolls without slipping better so that we can meet one of our learning objectives.

F. A cylinder rolls *without slipping* with constant speed across level ground as shown. In the spaces below,



1. write down the magnitudes of the instantaneous velocities of points P, Q, R, and S in the *ground* frame. *Express your answers in terms of* v_{CM} *only*.

2. draw vectors to show the directions of the instantaneous velocities of points P, Q, R, and S for the instant shown.

	$\vec{v}_P^{(\mathrm{G})}$	$\vec{v}_Q^{(\mathrm{G})}$	$\vec{v}_R^{(\mathrm{G})}$	$\vec{v}_S^{(\mathrm{G})}$
Magnitude				
Direction				

Explain your reasoning for both the magnitudes and directions of each velocities above.

G. What is the direction of the frictional force acting on the cylinder? Explain.

Figure 10.16. The last two questions of the last section of the modified tutorial – students revisit the case in which the cylinder rolls without slipping on the horizontal surface.

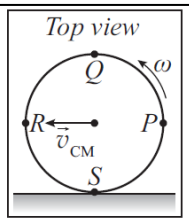
10.4.2 Description of the modified homework

The tutorial is designed to meet two learning objectives: (1) students can apply relative motion concepts to rolling motion, or rigid body motion, and (2) students can recognize the role of friction in rolling without slipping and rolling with slipping situations. There are no questions

for students to apply the no-slip condition in the tutorial. So, in the homework, questions are designed to meet all three learning objectives. The modified version of the homework can be found in Appendix C.

In the modified version of the homework, we kept the two questions in the preliminary version, as shown in Figure 10.5 (determine the speeds at different points on the rim of a rolling cylinder) and Figure 10.6 (the board-on-cylinder task). Since the problem in Figure 10.5 has moved to the tutorial, we modified the question to make the cylinder to roll to the left (shown in Figure 10.17), so that students have more opportunity to practice what they learned from the tutorial.

1. A cylinder rolls *without slipping* with constant speed across level ground as shown. In the spaces below,



   a. write down the magnitudes of the instantaneous velocities of points P, Q, R, and S in the *ground* frame. *Express your answers in terms of* v_{CM} *only*.

   b. draw vectors to show the directions of the instantaneous velocities of points P, Q, R, and S for the instant shown.

	$\vec{v}_P^{(G)}$	$\vec{v}_Q^{(G)}$	$\vec{v}_R^{(G)}$	$\vec{v}_S^{(G)}$
Magnitude				
Direction				

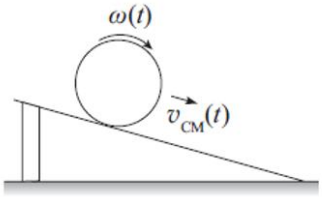
Explain your reasoning for both the magnitudes and directions of each velocities above.

Figure 10.17. The first question of the modified homework – students revisit the case in which the cylinder rolls without slipping on the horizontal surface, the cylinder rolls to the left.

Questions 3 and 4 of the homework focus on helping students to determine the direction of frictional force acting on the cylinder when it rolls up or down the incline, as shown in Figure 10.18 and Figure 10.19. In these questions, students will determine the direction of the frictional force by considering the changes in v_{CM} and ω. The frictional force that is drawn on the free-body

diagram of the cylinder must allow the angular velocity of the cylinder to increase as it rolls down or decrease as it rolls up the incline. If students go through the exercise correctly, they should realize that the direction of the frictional force on the cylinder is directed upward as it rolls up or down the incline.

3. A sphere rolls without slipping down an incline. There is friction between the sphere and the incline.

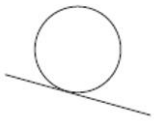


a. Describe briefly how v_{CM} and ω change with time when the sphere is rolling down the incline.

b. Indicate the direction of the acceleration and the direction of the angular acceleration of the sphere.

c. In the space at right, draw an extended free-body diagram for the sphere. Clearly label each force to indicate (1) the type of force, (2) the object exerting the force, and (3) the object on which the force is exerted.

Extended free-body diagram for the sphere



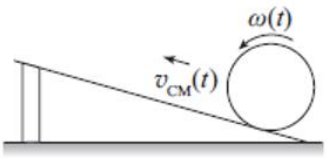
- Which of the forces you have drawn contribute to the net force acting on the sphere?

- Which of the forces you have drawn contribute to the net torque about the center-of-mass of the sphere?

d. Is your answer to part c consistent with your description in part a? Resolve any inconsistencies if necessary.

Figure 10.18. Question 3 of the modified homework – a guided exercise for students to determine the direction of frictional force acting on the cylinder as it rolls down an incline.

4. A sphere rolls without slipping **up** a rough incline as shown.

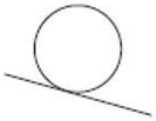


   a. Describe briefly how v_{CM} and ω change with time when the sphere is rolling up the incline.

   b. Indicate the direction of the acceleration and the direction of the angular acceleration of the sphere.

   c. In the space at right, draw an extended free-body diagram for the sphere. Clearly label each force to indicate (1) the type of force, (2) the object exerting the force, and (3) the object on which the force is exerted.

Extended free-body diagram for the sphere



      - Which of the forces you have drawn contribute to the net force acting on the sphere?

      - Which of the forces you have drawn contribute to the net torque about the center-of-mass of the sphere?

   d. Is your answer to part c consistent with your description in part a? Resolve any inconsistencies if necessary.

   e. Is the magnitude of the acceleration of the sphere when it is rolling down the incline *greater than*, *less than*, or *equal to* the magnitude of the acceleration of the sphere when it is rolling up the incline? Explain.

Figure 10.19. Question 4 of the modified homework – a guided exercise for students to determine the direction of frictional force acting on the cylinder as it rolls up an incline.

The next question is a revisit of the class demonstration, as shown in Figure 10.20. After students indicate how the v_{CM} and ω changes in each case, we ask them to find the directions of the center-of-mass acceleration and angular acceleration. Then, students are asked to draw the proper extended free-body diagram of the cylinder for each case. This question is similar to the previous two questions. Students draw the correct free-body diagram by considering the changes in v_{CM} and ω of the cylinder.

5. This problem revisits the demonstration at the end of the tutorial *Rolling and Slipping*. That demonstration showed the motion of a cylinder with three different initial conditions.

Consider a cylinder that moves across a horizontal surface. The surface is **not** frictionless. Imagine the motion of the cylinder for three different initial conditions: (i) $v_{CM} = R\omega$, (ii) $v_{CM} > R\omega$, and (iii) $v_{CM} < R\omega$.

For each case, complete the table below.

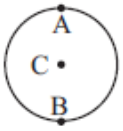
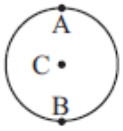
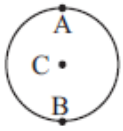
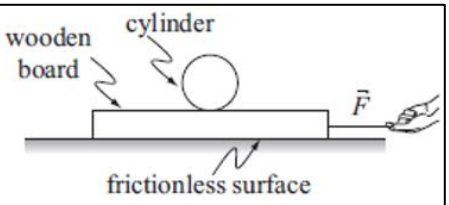
	Initial condition of the motion		
	$v_{CM} = R\omega$	$v_{CM} > R\omega$	$v_{CM} < R\omega$
Indicate the initial directions of $\vec{v}_A^{(G)}$, $\vec{v}_C^{(G)}$, and $\vec{v}_B^{(G)}$.			
Does the v_{CM} *increase*, *decrease*, or *remain the same* as the motion continues?			
Indicate the direction of the center-of-mass acceleration.			
Does the ω *increase*, *decrease*, or *remain the same* as the motion continues?			
Indicate the direction of the angular acceleration.			
Draw an extended free-body diagram of the cylinder.			

Figure 10.20. Question 5 of the modified homework – students revisit the observations from the class demonstration and determine the directions of center-of-mass acceleration and angular acceleration.

Questions 3, 4, and 5 represent a set of exercises to guide students to determine the correct free-body diagrams through the changes in motion. We are aware that students tend to draw the direction of the frictional force to be in the opposite direction of the v_{CM}, as shown in our preliminary investigation. If they use that rule in some of the cases highlighted in these three questions, they will obtain the incorrect direction of frictional force for the cylinder that is rolling

up the incline or on the horizontal surface. So, these three questions are designed to help address this student difficulty.

6. A cylinder of radius R is placed on top of a wooden board. They are initially at rest. The board is then pulled to the right by a constant horizontal force of magnitude F on a frictionless surface. The cylinder is observed to roll counterclockwise without slipping on the board.



At the instant that the board has speed v_b, the center-of-mass of the cylinder has speed v_c, and the cylinder is rotating with angular velocity ω.

For the following questions, consider the time interval when the cylinder is still moving on the board.

a. The cylinder is observed to roll counterclockwise without slipping on the board. Based on this observation, what do you know about the direction of horizontal force(s) that are acting on the cylinder? Explain.

b. Based on your answer in part a, what is the direction of acceleration of the center-of-mass of the cylinder in the ground frame? Explain.

c. What is the direction of the velocity of the center-of-mass of the cylinder in the ground frame? Explain.

d. Consider the point on the cylinder that is in contact with the wooden board at some instant t. At that instant, in the ground frame, what is the magnitude of the velocity of that point in terms of v_c, R, and ω? Explain.

e. Write an equation that relates the quantities v_b, v_c, R, and ω. Explain.

Figure 10.21. Question 6 of the modified homework – students practice applying what they learned from the previous questions on the cylinder-on-board context.

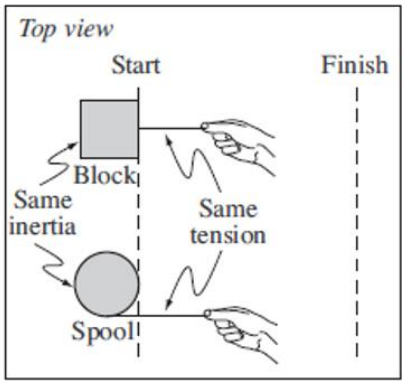
We designed a sequence of questions to guide students to understand the cylinder-on-board context in problem 6. In the previous five questions, students had some practice to (1) apply relative motion ideas to determine the magnitude and direction of velocities at different points on the rolling cylinder, (2) apply no-slip conditions to determine the relationship of the velocities between two objects that are moving together, and (3) determine the direction of forces acting on a rolling object through changes in motion. The sequence of questions walks students through

those learning points again. If students manage to proceed through the questions correctly, then they will arrive at the final equation, $v_b = v_c + R\omega$. Intuitively, the board will travel faster than the cylinder. The cylinder will move backward relative to the board. The final equation describes this intuition correctly.

The last question in the homework is a revisit to the block-spool experiment from the *Dynamics of rigid bodies* tutorial [31]. In that tutorial homework, students use energy analysis and conclude that the hand that is pulling the spool travels farther than the hand that is pulling the block. This result also implies the hand that pulls the spool moves at a greater speed than the hand that pulls the block. We can also prove this result by using the no-slip condition since there is no slipping when the spool unwinds. When the hand that is pulling the block moves at v_o, the hand that is pulling the spool moves at $v_o + R\omega$. The no-slip condition approach provides an alternative to the energy analysis presented in the *Dynamics of rigid bodies* homework to compare the speeds of the hands.

7. This problem revisits the "block-and-spool problem" from the tutorial *Dynamics of rigid bodies*.

A block and a spool are each pulled across a level, frictionless surface by a string, as illustrated in the top-view diagram at right.



The string pulling the block is tied to a hook (not shown) at the center of the front of the block. The string pulling the spool is wrapped many times around the spool and may unwind as it is pulled.

The block and the spool have the same inertias. The strings start pulling with the same constant force at the same instant. Assume that the inertias of the strings and the hook are negligible.

a. At some instant, the block is moving to the right at speed v_o. At that instant, is the speed of the center-of-mass of the spool *greater than*, *less than*, or *equal to* v_o? Explain.

b. At some instant, the hand pulling the block is moving at speed v_o. At that instant, what is the speed of the hand pulling the spool? Express your answer in terms of v_o, the radius of the spool, R, and the angular velocity ω_o of the spool at the same instant. Explain.

Figure 10.22. Question 7 of the modified homework – students apply the no-slip condition to the block-spool experiment from the *Dynamics of rigid bodies* tutorial.

10.5 ASSESSMENT OF THE MODIFIED TUTORIAL

We assessed the modified tutorial according to the three learning objectives. They are: (1) students can apply relative motion concepts to rolling motion, or rigid body motion, (2) students can define the no-slip condition and show how it applies to different situations, and (3) students can recognize the role of friction in rolling without slipping and rolling with slipping situations. The data for the assessment of the modified curriculum were collected from Sp'19 and Au'19 quarters. We discuss the results by the learning objectives.

10.5.1 Learning objective 1 – students can apply relative motion concepts to rolling motion or rigid body motion

We used the RMT-Direction and Magnitude tasks in our pretests and posttests to assess this learning objective. We also used the 'plus'-shaped object (see Figure 8.6) in our assessment. All the pretest data were collected after students had relevant lecture instruction. In both quarters, the TAs did not give feedback to students' homework as the tutorial was scheduled in the last week of the quarter. The pretest and posttest results are presented in Table 10.64 and Table 10.65.

The posttest results range from 49% to 80% for the RMT-Direction task, from 72% to 82% for the RMT-Magnitude task. The range of percentage correct for the RMT-Direction task is greater than the RMT-Magnitude task. We are unable to account for the difference in the range of the posttest results. However, in most instances, the posttest results are above 60%. The posttest results for the RMT-Direction in the preliminary version were around 50% of students were not assigned to do the homework. So, based on these posttest results alone, the modified version is better than the preliminary version of the tutorial.

In the RMT-Direction task, the gains from pretest to posttest in the Sp'19 quarter range from 30% to 59%. But, the gains in the Au'19 quarter are lower, they range from 7% to 19%. The difference in the gains comes from the difference in the pretest results in both quarters. The pretest results in the Au'19 quarter are better than that in the Sp'19 quarter. We found that the lectures in the Au'19 quarter had a few slides discussing the motion of a hammer that is flying and spinning through the air. Three points on the hammer were chosen, and students were asked to select the curves that describe the trajectory of those points. We believed that this additional discussion contributed to the gain in the pretest performance, and we found similar results when we asked students to think of displacements or trajectories in our in-depth investigations (see Section 8.3).

In the RMT-Magnitude task, the gains from pretest to posttest range from 26%-61% in both quarters. The datasets exhibit trends that are similar to the RMT-Direction task.

We conducted McNemar's test for each matched dataset. We found that all the *p*-values for the matched datasets were less than 1%, except for the 7% gain. The McNemar's test results suggest that the gains in most matched datasets are highly associated with the tutorial instruction, with medium to large effect sizes.

Now we compare the posttest results with the TA's pretest results. As mentioned in subsection 10.3.1, we collected data from 43 graduate TAs. We found 77% of them answered the RMT-Direction task correctly, and 81% of them answered the RMT-Magnitude task correctly. On the RMT-Direction task, the students' posttest results and the TAs pretest results were comparable without the tutorial instructors giving students feedback on homework. On the RMT-Magnitude task, the results from both groups were comparable with or without the feedback.

In conclusion, more than 50% of the students can answer the RMT-Direction and RMT-Magnitude tasks after they did the modified tutorial. The student performance is comparable with TA's pretest performance. The posttest results from the modified version are generally better than that from the preliminary version, with better effect sizes.

Table 10.64. Pretest and posttest results for the RMT-Direction in Sp'19 and Au'19 quarters. In all cases, students are required to determine the direction of point 2 in the ground frame.

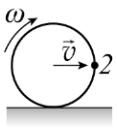
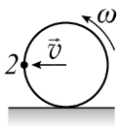
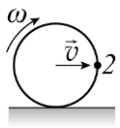
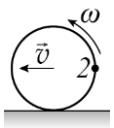
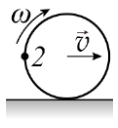
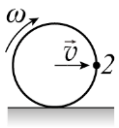
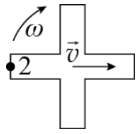
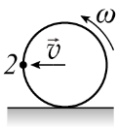
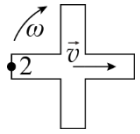
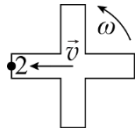
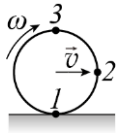
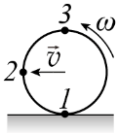
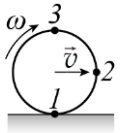
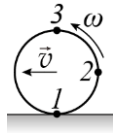
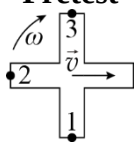
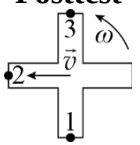
Approach	Sp'19 (N = 161)		Sp'19 (N = 127)		Au'19 (N = 146)		Au'19 (N = 112)		Au'19 (N = 115)	
	Pretest	**Posttest**	**Pretest**	**Posttest**	**Pretest**	**Posttest**	**Pretest**	**Posttest**	**Pretest**	**Posttest**
										
Relative Motion (Correct)	19%	49%	21%	80%	53%	60%	54%	71%	57%	76%
CM frame	65%	25%	66%	17%	38%	25%	29%	16%	27%	17%

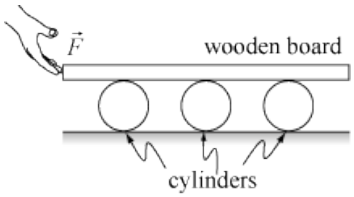
Table 10.65. Pretest and posttest results for the RMT-Magnitude in Sp'19 and Au'19 quarters. In all cases, students are required to rank the speeds of points 1, 2, and 3 in the ground frame.

Approach	Sp'19 (N = 161)		Sp'19 (N = 127)		Au'19 (N = 115)	
	Pretest	**Posttest**	**Pretest**	**Posttest**	**Pretest**	**Posttest**
						
Relative Motion (Correct)	21%	82%	24%	75%	46%	72%
CM frame	78%	8%	75%	17%	48%	21%

10.5.2 Learning objective 2 – students can define the no-slip condition and show how it applies to different situations

We compare the student performance in the BoC task in pretest and posttest in the Au'19 quarter. Some of the results have been presented in Section 9.2. The BoC task in pretests and final exams in Au'19 are shown in Figure 10.23. The results are tabulated in Table 10.66. We included the results from the Au'18 quarter to compare the effectiveness of the tutorial before and after the modification.

A wooden board is placed on top of three identical solid cylinders. The board is pushed to the right by a constant horizontal force of magnitude *F* applied to the end of the board. The cylinders roll without slipping on a flat surface. There is also no slipping between the cylinders.



Consider the instant when the horizontal force is still exerted on the board and the board is still in contact with all of the cylinders.

Let v_b be the magnitude of the horizontal displacement of the board and v_{cyl} be the magnitude of the horizontal displacement of the center-of-mass of each cylinder.

Pretest in Au'19

When the board moves a distance *L*, which of the following best describes the distance each cylinder moves?

A. Between 0 and $L/2$	B. $L/2$	C. Between $L/2$ and L
D. L	E. Between L and $2L$	F. $2L$

Final exams in Au'19

In the reference frame of the Earth, which of the following correctly describes the relationship between v_b and v_{cyl}?

A. $v_b = 0.5v_{\text{cyl}}$	B. $v_b = v_{\text{cyl}}$	C. $v_b = 1.5v_{\text{cyl}}$
D. $v_b = 2v_{\text{cyl}}$	E. $v_b = 3v_{\text{cyl}}$	

Figure 10.23. The BoC task on the pretests and final exams in Au'19 quarters.

In the Au'18 pretest, students were asked to select the correct equation that described the relationship between the displacements of the block and cylinders. The posttest in Au'18 was a free-response question, and the task was the same as the pretest. The Au'19 pretest was different from Au'18 that we asked a qualitative version of the BoC task. In the posttest of the Au'19 quarter, we asked students for the correct equation without asking them to explain.

The pretest results from both Au'18 and Au'19 quarters are comparable, but the posttest result from Au'18 quarter was better than that from Au'19 quarter. We believe the difference in performance is due to the retesting effect. These students in both the Au'18 and Au'19 quarters did the mBoC task in pretest and in-class clicker. These students did different versions of the BoC task on the pretest. The BoC task on the Au'18 pretest had similar options as the mBoC task and the posttest, but not in the Au'19 quarter. So, students in the Au'18 quarter did similar versions of the problem three times before they did the posttest, but not for the students in the Au'19 quarter. Since there was no discussion of the answers to the BoC task during tutorial explicitly, and if students did not do the homework problems after the tutorial, then we would expect minimal improvement in the posttest, as seen in the Au'19 result.

Table 10.66. Student responses for the BoC task in the final exams after implementations of preliminary and modified versions of tutorial.

Approach / Answer	**Preliminary version** Au'18 (N = 157)		**Modified version** Au'19 (N = 256)		**TA pretest**	
	Pretest	Posttest	Pretest	Posttest	Au'18 (N = 24)	Au'19 (N = 19)
Correct	**46%**	**61%**	**42%**	**44%**	**38%**	**32%**
Less than $L/2$	-	-	25%	-	-	21%
No-slip = equal displacement	21%	4%	21%	21%	38%	26%
$v_b = 0.5v_{\text{cyl}}$	22%	15%	3%	14%	-	-
Others	10%	20%	9%	21%	24%	21%

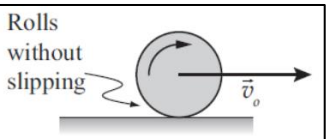
An equal proportion of students thought that the no-slip condition meant that the objects would have equal displacements in the pretest and posttest of the Au'19 quarter. This result shows that student participation in the tutorial alone is not enough to get students to understand the implications of the no-slip condition under different situations. The tutorial only introduces the no-slip condition, but students only get to apply it in the homework. As of now, we are unable to assess the impact of the homework because of scheduling issues. For other errors, we have discussed them in Section 9.2.

The posttest results after the modified tutorial are 12% better than the TAs' written pretest results. The students' pretest results were also better than the TAs' results because students did the in-class clicker question before they did the BoC task on pretests. After comparing the results from both groups, it seemed that the improvement due to the tutorial instruction was less than the in-class clicker question.

10.5.3 Learning objective 3 – students can recognize the role of friction in rolling without slipping and rolling with slipping situations

We continued to use the FoC problem shown in Figure 10.24 for the assessment of this learning objective. The results from the preliminary and modified versions of the tutorial are shown in Table 10.67. We did not assess students on the role of friction in rolling with slipping situations because of lack of time.

A rigid cylinder rolls to the right without slipping (*i.e.*, there is no relative movement between the surface and the lowest point on the ball). The coefficient of static friction between the cylinder and flat surface is $\mu_{static} > 0$.



It is observed that the center of mass of the cylinder moves with constant speed (*i.e.*, **it does not slow down**).

Center of mass moves to the right with constant speed

22. [5 pts] While the cylinder is rolling with constant speed without slipping, which of the following best describes the direction of the frictional force on the cylinder by the flat surface?

A. The frictional force on the cylinder by the flat surface is **to the left.**

B. The frictional force on the cylinder by the flat surface is **to the right.**

C. The frictional force on the cylinder by the flat surface is **upward.**

D. The frictional force on the cylinder by the flat surface is **zero.**

Figure 10.24. The friction on the rolling cylinder (FoC) problem.

The results from the modified curriculum were comparable to or better than the preliminary version of the tutorial. We did not modify the part on the curriculum that addresses student difficulties on friction in rolling motion much, so we did not expect a decrease in the percentage correct. In the Au'19 quarter, we asked the same question on the pretest, and 10% of the respondents answered correctly. We perform the McNemar's test on the data we collected from the Au'19 quarter, and the calculated *p*-value is less than 1%. The gain is 16%, so the effect size is medium.

Table 10.67. Student responses for the FoC problem in the final exams after implementations of preliminary and modified versions of tutorial.

Approach	**Preliminary version**	**Modified version**		**TA pretest**
	Wi'18 & Au'18 (*N* = 1076)	Sp'19 (*N* = 396)	Au'19 (*N* = 168)	Au'19 (*N* = 19)
Correct (D)	25%	36%	26%	32%
Opposed direction of motion (A)	44%	42%	42%	31%
Others	31%	22%	32%	37%

The results continued to suggest that students have difficulties in determining the direction of the frictional force. Most of them tended to use the direction of motion to decide the direction of frictional force because frictional force pointed in the direction that opposed the motion. They did not use the changes in motion to determine the direction of frictional force when the question mentioned that the cylinder moves with constant speed. We believe it is necessary to help students to understand that there are instances that they need to use changes in motion to determine the directions of forces that are acting on the object.

The aggregated students' posttest results and the TAs' pretest results were comparable. The posttest performance in the Au'19 is less than that in the Sp'19 quarter. The instructors in those quarters were teaching PHYS 121 the first time, but the instructor in Sp'19 spent more time to discuss the role of friction in affecting rolling motion than the instructor in Au'19. We think this difference in lecture instruction may contribute to the difference in posttest performance.

We also asked another question that was related to friction in rolling motion to assess the effectiveness of the tutorial. This question was modified slightly from the Rotation and Rolling Motion Conceptual Survey (RRMCS) [12]. The original and modified questions are presented in Figure 10.25. The modifications made are in the question statement and option B, because in the original option B, the fact that the marbles roll without slipping does not explain why they are not slowed down. The change in option B leads to the change into a more general question statement. Option B is the correct answer in both versions.

We administered this new friction question in the final exams of Sp'19 and Au'19. We did not have any pretest results from our students. So, we compared the posttest results with the results reported in the literature. The result from the literature and posttest are tabulated in Table 10.68.

Original

Two identical marbles roll without slipping across horizontal floors. One rolls on a stone floor with coefficient of static friction $\boldsymbol{\mu_s} = 0.80$, and the other rolls on a glass floor with $\boldsymbol{\mu_s} = 0.40$. Which marble is slowed down more by friction, and why? Ignore air-resistance.

A. Both marbles are slowed equally since the marbles themselves are identical.

B. Neither marble is slowed by friction because both roll without slipping.

C. The marble rolling on stone is slowed more. The stone floor has greater μ_s, which makes the force of friction on it greater.

D. The marble rolling on glass is slowed more. The slippery nature of the glass impedes rolling.

E. It is impossible to answer without knowing the coefficient of kinetic friction $\boldsymbol{\mu_k}$ because the marbles are moving.

Two identical marbles roll without slipping across horizontal floors. One rolls on a stone floor with coefficient of static friction $\boldsymbol{\mu_s} = 0.80$, and the other rolls on a glass floor with $\boldsymbol{\mu_s} = 0.40$. Ignore air-resistance. Which of the following statements is correct?

A. Both marbles are slowed equally since the marbles themselves are identical.

B. Neither marble is slowed by friction. *Both marbles roll without slipping.*

C. The marble rolling on stone is slowed more. The stone floor has greater μ_s, which makes the force of friction on it greater.

D. The marble rolling on glass is slowed more. The slippery nature of the glass impedes rolling.

E. It is impossible to answer without knowing the coefficient of kinetic friction μ_k because the marbles are moving.

Figure 10.25. The original and modified RRMCS question on the effects of friction on rolling objects. The changes in the modified RRMCS question is italicized.

Table 10.68. Student responses for the modified RRMCS friction question on the effects of friction on rolling objects.

	RRMCS results (N = 559)	Results from UW		
		Sp'19 (N = 142)	Au'19 (N = 177)	Aggregate (N = 319)
Correct (B)	19%	44%	45%	45%
Opposed direction of motion (C)	46%	37%	41%	39%
Others	35%	19%	14%	16%

The reported RRMCS result was 19%, and it represented the student performance after traditional lecture instruction. Our posttest result was 45%, a 26% gain after tutorial instruction. We performed the χ^2 test ($\chi^2\ 1\ = 64$, $p < 0.1\%$), and the improvement of the performance were highly associated with our tutorial demonstration. The effect size is large.

The most common error students made in this question is consistent with what we had found in the previous friction on rolling cylinder question. Many students think that the cylinder on the stone floor is slowed down more because the coefficient of static friction of the stone surface is greater than that on the grass surface. This error is the same as thinking there is a frictional force opposing the motion of the rolling cylinder.

10.5.4 Commentary

We modified the curriculum to meet the three learning objectives we set out. We managed to improve student performance in the RMT-Direction and Magnitude tasks and the friction-on-cylinder task, but not the BoC task. We discussed some reasons to explain the results we obtained.

The student performance on the BoC task can be improved if students can do the homework, and TAs can provide feedback to them. One possible future work is to assess the effectiveness of the curriculum on the BoC task if the logistical constraints are removed.

Some students continue to have difficulty in determining the frictional force acting on the rolling cylinder. This finding shows that the tutorial demonstration may not be enough to change students' idea of friction easily. It may be necessary to have an exercise to get students to examine the effect of friction on a rolling cylinder. Students can examine the changes in the center-of-mass velocity, v_{CM}, and the angular velocity, ω, and then decide if those changes are consistent with $v_{CM} = R\omega$ or not.

The students' posttest results were compared with graduate TAs' pretest results. They were found to be comparable. Some difficulties found among the introductory physics students were also found among the graduate TAs. Such trends suggested that those difficulties were not addressed when the graduate students were in their introductory physics courses. It might also be that the graduate students did not spend much time on such content in their undergraduate years.

10.6 SUMMARY

In this chapter, we describe how we designed a research-based and validated curriculum on rolling and slipping. Based on prior research and our investigations, the student difficulties that we targeted include (1) difficulty in applying relative motion concepts in rolling motion, (2) difficulty in thinking of the dynamics involved in rolling motion, and (3) difficulty in applying the no-slip condition in situations involving rolling without slipping.

The learning objectives of the tutorial curriculum are drafted to address the difficulties above. The preliminary version of the curriculum has four learning objectives. They are (1) apply relative motion concepts to rolling motion or rigid body motion in general, (2) define the no-slip condition and show how it applies to different situations, (3) recognize the role of friction in rolling without slipping and rolling with slipping situations, and lastly, (4) perform better in typical relative motion questions in 2-dimension. In developing the preliminary version, we drew on suggestions from prior research and suggestions made by other researchers. The preliminary version of the tutorial focused on the application of relative motion concepts in rolling motion situations and the effects of friction on the changes of motion of a moving cylinder.

Our initial assessment showed some improvement in student ability to apply relative motion concepts to determine the directions and magnitudes of the velocities of various points on

an object undergoing rolling motion, as well as on student ability in deciding the direction of the friction acting on a cylinder that is rolling without slipping. There is also some improvement in the application of the no-slip condition in the board-on-cylinder task. (See Table 10.60.) However, the tutorial did not seem to have an impact on improving student ability to solve typical relative motion problems. On reflection, we decided that expecting transfer to relative motion contexts, in general, was too ambitious. So, we decided to focus on the first three learning objectives in our modified tutorial.

We modified the tutorial to employ an elicit-confront-resolve strategy. The revised version starts by asking students to complete a task that we knew was difficult for many: using relative motion concepts to determine the direction and magnitudes of the velocity of a point on a cylinder that rolls without slipping. We then asked students to use a geared wheel (see Figure 10.13) to find the trajectory of a point on the wheel and use the trajectory to check their previous answer. This task is one that most students appeared to be able to do well (determining the direction of the velocity using the idea of the tangent to the trajectory).

The assessment of the modified tutorial gave promising, but mixed results. Regarding the first objective (students can apply relative motion concepts to rolling motion in general), we found that the tutorial alone, without the homework, improved student posttest performance. We were not able to test for the second objective (students can define and apply the no-slip condition) since in the quarter in which the tutorial was used, the homework was not used, due to unforeseen circumstances. (The homework was the only place where this idea was addressed.) For the third objective (recognizing the role of friction in rolling without slipping and rolling with slipping situations), the results from the modified tutorial were comparable or slightly better than the preliminary version.

A different way to assess the effectiveness of the tutorial is to compare the students' posttest results on some tasks to the results from physics graduate teaching assistants (TAs) on a pretest on those same tasks. In this regard, the results were comparable, which suggested that the curriculum helps students to achieve the performance level of the graduate students on some tasks.

In summary, we designed a tutorial curriculum through a process of ongoing research to address the student difficulties in rolling motion that we found from our preliminary and in-depth investigations. The modified version was generally better than the preliminary version, based on the posttest results and content coverage. We were unable to assess the impact of the modified tutorial homework, and that component of the tutorial needs to be done in a quarter when students turn in the homework and can receive feedback from their TAs.

CONCLUSION FOR PART II

Part II of this dissertation describes the application of the iterative process of research-curriculum development-instruction discussed in Chapter 1. We applied this in the context of addressing student difficulties in rolling motion.

Prior research on rolling motion had revealed that many students have difficulties in these areas: (1) applying relative motion to rolling motion, (2) reasoning about the rolling without slipping condition, and (3) reasoning about the frictional forces involved in rolling without slipping situations. We began by replicating some of these studies and found that similar difficulties existed with our population in similar, but not identical percentages. We also developed tasks and collected data that had not previously been reported, such as on the board-on-cylinder context.

Our additional investigations showed that most students were able to visualize the trajectory of a point on a rolling object in the ground frame even though they were unable to determine the direction of the instantaneous velocity of the point. Many failed to apply relative motion ideas to determine the direction of the velocity of the same point. We also found that students can answer simple relative motion problems but were unable to apply the same ideas to rolling motion. Through our in-depth investigations, we discovered that students often rely on some ideas better than others, which we took advantage of when we developed a new tutorial curriculum to improve student understanding of rolling motion.

The curriculum development of the Rolling Motion tutorial can be described in two phases. The first phase based on recommendations made by previous researchers and the second phase incorporated the results from our in-depth investigations. The objectives of the latest version of the tutorial are to assist students in (1) applying relative motion concepts to rolling motion, and rigid body motion in general, (2) defining the no-slip condition and understanding how it applies

to different situations, and (3) recognizing the role of friction in rolling without and with slipping situations. In the modified tutorial, we designed activities that apply the *elicit-confront-resolve* cycle so that the student difficulties can be addressed more effectively.

The assessment of the preliminary and revised tutorial showed the following results: (1) on the first learning objective, the modified tutorial achieved similar gains as the preliminary version without providing feedback to students on their homework, (2) the feedback to students on their appears to have an impact on student learning gains, (3) student posttest performance sometimes matched that of graduate TAs on a pretest on the same material, and (4) there were mixed learning gains on the second and third learning objectives.

One possible reason for the mixed learning gains from the use of the modified tutorial is that during our study, TAs did not give feedback to students on their tutorial homework. (The modified tutorial takes place in the last week of the quarter and homework in that week is not collected or turned in.) The student difficulties related to the second and third learning objectives are addressed in the homework. Thus, if there are opportunities to reassess the curriculum again, we hope that the schedule can be arranged such that students can receive feedback from TAs.

Although many instructors may not regard the topic of rolling and slipping on its own as a critical, important topic, it represents an opportunity for students to pull together several disparate ideas (instantaneous velocity, relative motion, friction, etc.). In other words, it allows instructors to test whether or not students have developed a functional understanding of the material covered in mechanics courses. Moreover, rolling motion is a topic that is important for a subset of our students in introductory calculus-based physics courses, especially those who would be majoring in mechanical and civil engineering. This research and that by others indicate that many students are unable to transfer ideas they have learned earlier in the course to the basic case of rolling with

and without slipping. The goal of the research in this part is to identify the problems that students encounter and develop strategies that can be used to help students with the short time available on this topic. Then, students can leave the introductory courses with a meaningful understanding of basic physics that they can apply to more complicated cases.

PART III
INVESTIGATION OF STUDENT UNDERSTANDING IN FLUID MOTION

Fluids are materials that do not have a fixed shape and size; they change to accommodate the container in which they are placed, such as liquids and gases. Fluids are all around us, so it is interesting and necessary to analyze their motion and behavior mathematically. In the introductory physics sequence at the University of Washington, the students are taught the physics of fluids after studying particle and rigid body motion. Fluid mechanics was taught in the introductory calculus-based sequence at the end of the first course, PHYS 121, prior to the Sp'17 quarter and was later moved to the last course in the sequence, PHYS 123, from Sp'17 onward. It is also covered in the second course of the algebra-based sequence, PHYS 115.

There is relatively little research on student understanding of fluid motion. Most of the work on this topic has been done in engineering departments, usually in sophomore or junior courses. In this part of the dissertation, we present results from an examination of student thinking about fluids at the introductory physics level.

The research discussed in this part of the dissertation follows the same format as in Parts I and II, we first discuss the physics behind fluid motion in Chapter 11. Two main concepts of fluid motion are described: the continuity and Bernoulli's equations. We then review previous research on student understanding of these two concepts and summarize the student difficulties reported in the literature. We used questions from the Fluid Mechanics Concept Survey and found those reported difficulties in our students in our preliminary investigation. The results from students in both the calculus-based and algebra-based courses are presented in Chapter 12.

In Chapter 13, we describe some of our additional investigations into fluid motion. These were based on suggestions made by prior researchers to probe student difficulties in greater detail.

As this study progressed, the research tasks were modified, and some changes in the student performance were observed. We then developed additional tasks to probe student understanding of the continuity equation. Due to the limited coverage of this topic in these courses, no tutorial curriculum was developed.

Chapter 11. BACKGROUND & PRIOR RESEARCH

After students are taught how to describe the motion of particles and rigid bodies, they are briefly introduced to the physics of fluids. Fluids are non-rigid objects such as liquids and gases, which made up of point particles not fixed in space. The physics of fluids discusses the properties of the fluid at rest (hydrostatics/fluid statics) and fluid in motion (hydrodynamics/fluid dynamics). In this chapter, we review the fluid dynamics content at the introductory physics level and some prior research on student understanding of fluid dynamics in physics and engineering students. The motivation and context of investigation for investigating student difficulties in fluid dynamics are also discussed.

11.1 PHYSICS OF FLUID MECHANICS

The study of fluid mechanics itself is a rich subject and highly relevant to our daily lives. The existence of atmospheric pressure, the reason why we feel lighter in the water, the nature of the flow of water in pipes, and phenomena that involve water or gases in motion can be modeled using the equations in fluid mechanics. In this section, we review the fluid mechanics content presented at the introductory physics level. The advanced content is highly mathematical, and it can be a course by itself at the upper-division level, which we will not discuss here.

In college physics, the fluid statics content focuses on pressure due to fluids at rest, while the fluid dynamics content discusses the speed and pressure in the context of fluid flow in pipes.

11.1.1 Fluid statics

Fluid statics mainly discusses the pressure due to fluids at rest. Since fluids are unable to sustain tensile and shear forces, they can only exert forces that tend to compress an object that is submerged in a static fluid on all sides. The force the fluid exerts on the object is always perpendicular to the surfaces of the object. If the force acting on the surface is *F*, and *A* is the area of the surface, then the pressure exerted by the fluid on the surface is defined as $P = F/A$.

The pressure due to fluid is different at different vertical positions. For example, the water pressure increases with increasing depth, while atmospheric pressure decreases with increasing altitude. In the introductory physics level, the focus is on the pressure due to liquids. The discussion about the height dependence of atmospheric pressure involves the ideal gas law and differential equations, which is not typically discussed in the introductory physics courses.

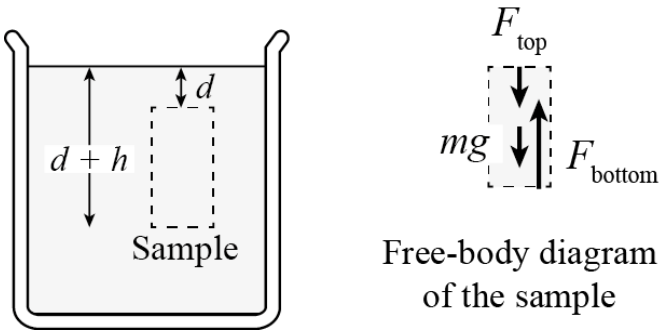


Figure 11.1. A sample of liquid with density ρ and the forces acting on it. The arrows represent the forces exerted on the fluid sample, but the lengths do not represent the magnitudes.

Consider a liquid of uniform density ρ that is at rest, as shown in Figure 6.1, and a sample of the liquid contained within an imaginary cylinder of cross-sectional *A* extending from depth *d* to depth $d + h$. The liquid external to the sample exerts forces at all points on the surface of the sample, perpendicular to the surface. The forces on the curved surface cancel each other out, so

the relevant forces acting on the sample are the downward force acting on the top surface, the weight of the sample, and the upward force acting on the bottom surface. Since the sample is at rest, the net force is zero, the equation is $F_{\text{bottom}} = F_{\text{top}} + mg$, where $m = \rho V = \rho Ah$. Divide the equation by *A* throughout, and we arrive at:

$$P_{\text{bottom}} = P_{\text{top}} + \rho gh \tag{11.1}$$

Equation 1.1 tells us that the pressure difference between the top and the bottom surface of the sample depends on the height *h*. The bottom surface has a higher pressure than the pressure at the top surface. This relationship also shows that the pressure in a liquid is the same at the same depth. It does not depend on where the imaginary cylinder is placed if it has the same P_{top}. The relationship is independent of the shape and size of the containers.

The above equation implies that a submerged cylindrical object experiences imbalanced forces if the density of the object is not equal to the density of the fluid. The difference between the forces exerted by the fluid on the top and bottom surfaces is called the "buoyant force" acting on the cylinder. Archimedes' principle says that the magnitude of the buoyant force acting on the immersed object is equal to the magnitude of the weight of the fluid displaced, which is the force differential between the top and bottom surfaces. If the buoyant force is less than the weight of the object, then the object sinks. If the buoyant force is greater than the weight of the object, then the object moves up to the liquid surface and eventually comes to rest when it displaces the amount of fluid equal to its weight. The buoyant force does not depend on the shape of the submerged object. It solely depends on the volume of the fluid displaced by the object.

Next, we look at the physics of moving fluids in pipes.

11.1.2 Fluid dynamics

The description of moving fluids tends to be highly mathematical. So, some simplifications and characterizations of moving fluids are needed at the introductory level. Fluid flow can be characterized into two types: laminar (steady) and turbulent. Laminar flow means each fluid particle follows a smooth path, such that the paths of different particles never cross each other. The velocity of fluid particles passing any point remains constant in time. Fluid flow becomes turbulent above a certain critical speed (depends on the density and viscosity of fluid).

Since the flow of real fluid is complex and not fully understood, the discussion of fluid flow is limited to the ideal fluid flow model. It has four assumptions [74], which are:

- The fluid is **non-viscous**, which means there is no internal friction.
- The flow is **laminar**; the velocity of the fluid at each point remains constant.
- The fluid is **incompressible**, which means the density is constant throughout the fluid.
- The flow is **irrotational**. The fluid has no angular momentum at any point.

There are two crucial equations for ideal fluid flow in pipes: the continuity equation and Bernoulli's equation. The former is a statement of mass conservation, while the latter describes the conservation of energy.

Continuity equation. Consider an incompressible, ideal fluid of density ρ flowing through a pipe segment of the non-uniform size shown in Figure 11.2. Since there is no inflow and outflow of fluid in between, the mass of fluid entering the left end and exiting the right end are the same in the same time interval. In a time interval Δt, the mass of fluid entering point 1 is $\rho A_1 v_1 \Delta t$, where A_1 and v_1 are the cross-sectional area and the speed of the fluid flow at point 1, respectively. The mass of fluid leaving point 2 in the interval Δt is the same as point 1, *i.e.*, $\rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t$, where A_2 and v_2 are the cross-sectional area and the speed of the flow at point 2, respectively.

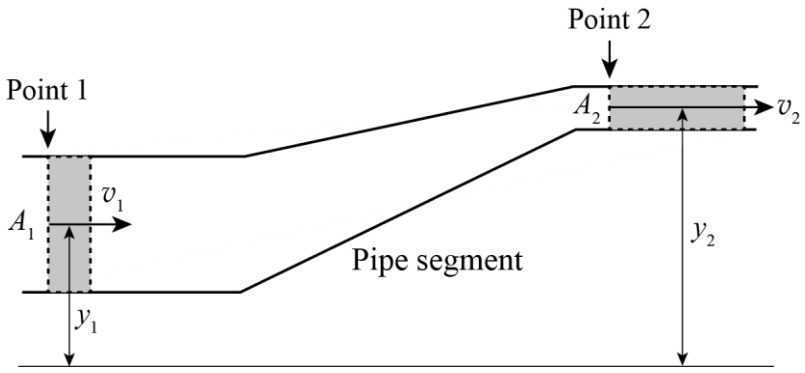


Figure 11.2. Fluid flow in a pipe segment that is filled with fluid completely. The shaded part shows the parcels of fluid that are flowing in the time interval Δ*t* past points 1 and 2.

Simplifying the above equation yields the continuity equation, $A_1 v_1 = A_2 v_2$. The product of the cross-sectional area and fluid speed at all points along a pipe is **constant** in time for an incompressible fluid. The quantity Av is known as the flow rate. The continuity equation can be interpreted as the constancy of the flow rate.

Bernoulli's equation. The equation is written as the following:

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \tag{11.2}$$

where P_1 and P_2 are the pressures at points 1 and 2, y_1 and y_2 are the vertical distances of points 1 and 2 from a reference height, respectively (see Figure 11.2). This equation is derived from the conservation of energy, which states $W_{\text{net}} = \Delta E_{\text{tot}}$. The external forces acting on the parcels at points 1 and 2 appear in the term net work done, and kinetic energy and potential energy appear in the changes in total energy. The term $\frac{1}{2}\rho v^2$ can be interpreted as the kinetic energy density in the fluid, and the term $\rho g y$ can be interpreted as the potential energy density. The pressure terms are part of the net work done per unit volume. This equation also implies that the quantity $P + \frac{1}{2}\rho v^2 + \rho g y$ is constant along a streamline (the path taken by a fluid particle under steady flow).

One thing to note is that the continuity equation is used in the derivation of Bernoulli's equation. So, for any pipe flow problems, one needs to consider the continuity equation first before one applies Bernoulli's equation.

We have discussed the physics of liquids at rest and moving liquids in a typical introductory physics course. Our investigation focuses on student understanding of fluid dynamics in introductory physics courses. The relevant prior research are discussed in the next section.

11.2 PRIOR RESEARCH ON STUDENT UNDERSTANDING IN FLUID MECHANICS

The research on student understanding in fluid mechanics at the introductory physics level is relatively new and recent. Smith described the misuse of Bernoulli's equation among science students to explain the existence of lift in the airfoil and curving baseball situations in 1972 [75]. His description was anecdotal, rather than careful documentation.

Recent research has been performed with mechanical engineering and civil engineering students, and much of what might inform instruction is either intended for engineering students or not research-based. Martin J. *et al.* developed a concept inventory to probe the understanding of fluid mechanics in mechanical and civil engineering students [20]. Fraser D. M. *et al.* and Brown *et al.* used the concept inventory for their investigations, either to find out the effectiveness of their curriculum [19] or in-depth study of student difficulties in fluid dynamics [18]. Suarez *et al.* reported on the conceptual difficulties in fluid dynamics among introductory physics students [17].

11.2.1 Misinterpretation of Bernoulli's equation

Smith commented that Bernoulli's equation was misused in explaining the dynamic lift in the airfoil and curving baseball situations. From his contact with hundreds of teachers in the aerospace workshop, he found that teachers had an inadequate understanding of Bernoulli's theorem. He noted that Bernoulli's equation could be used to describe the changes in pressures or velocities in moving fluids. However, the same equation should not be used to explain the presence of a net force on an object that is moving relative to the fluid. Instead of using Bernoulli's equation, he used Newton's 3rd law to account for the dynamic lift in the airfoil and curving baseball situations.

In our investigation, we only focus on student difficulties in applying Bernoulli's equation for liquid in pipe flow contexts, since our students usually learn about liquids in lectures. This article is to highlight that students can misuse Bernoulli's equation to explain some daily life phenomena.

11.2.2 Research on student difficulties in fluid dynamics

Martin J. *et al.* published a 30-item, multiple-choice Fluid Mechanics Concept Inventory (FMCI) for engineering students [20]. The questions were based on the key concepts in a typical fluid course. Some of the concepts investigated are conservation of mass, Bernoulli's equation, laminar and turbulent flows, ideal and viscous fluids, and the Navier-Stokes equation. These concepts were identified by experienced faculty members and students who just completed the fluid mechanics course. The authors assessed the reliability of the inventory and modified some questions [21].

Watson *et al.* revisited FMCI with two objectives: (1) collect and analyze experts' opinions on the applicability of the questions in civil engineering courses, (2) use student performance and opinions to validate existing questions so that the authors can determine what to retain or omit [22]. The authors used three methods. First, they used a modified, two-phase Delphi process to gather opinions on the appropriateness of FMCI items from four civil engineering fluid mechanics content experts. Second, they analyzed the performance of 35 civil engineering students in a fluid mechanics course via the pretest-posttest. Third, they collected insights from a high performing civil engineering student who just completed a fluid mechanics course. Through their analysis, they proposed to retain 14 questions in FMCI for the civil engineering fluid mechanics course.

Fraser D. M. *et al.* reported on how computer simulations could impact engineering students' understanding of fluid mechanics in South Africa [19]. The authors assessed the impact of the computer simulations on student learning using various questions from the FMCI in a pretest-posttest setting. They looked at the gain in performance in five different concept categories: pressure-measuring instruments, flow through a pipe, velocity profiles, force balance, and general fluid mechanics theory. The results of fluid flow through a pipe were relevant to our investigation. The authors found that computer simulations have a significant effect on student learning.

Brown *et al.* investigated student understanding of pressurized pipeline concepts [18]. They conducted interviews with civil engineering students using three FMCI questions and pressurized pipe systems. One of the questions is shown in Figure 11.3. The students consisted of sophomores before the fluid mechanics course, juniors, and seniors after completing the fluid mechanics course. The concepts involved in their study were the conservation of mass and Bernoulli's equation. The authors found that students have two misconceptions: (1) Water is a

compressible fluid; (2) Using hydrostatic pressure to reason about the changes in water pressure for flows in vertical pipelines.

The authors found that the student approaches in vertical pipe flow problems were different from horizontal pipe flow situations. They hypothesized that the students might have misinterpreted the boundary conditions of the problems. Besides, the authors found that some students wanted to select responses that were not given in the question (only four options for the question in Figure 11.3). So, they suggested that if these FMCI questions were to be used as part of the investigation, there should be two questions: one for comparing speeds, the other for comparing pressures. This arrangement allows for every possible response, thus providing a more nuanced understanding of students' misconceptions.

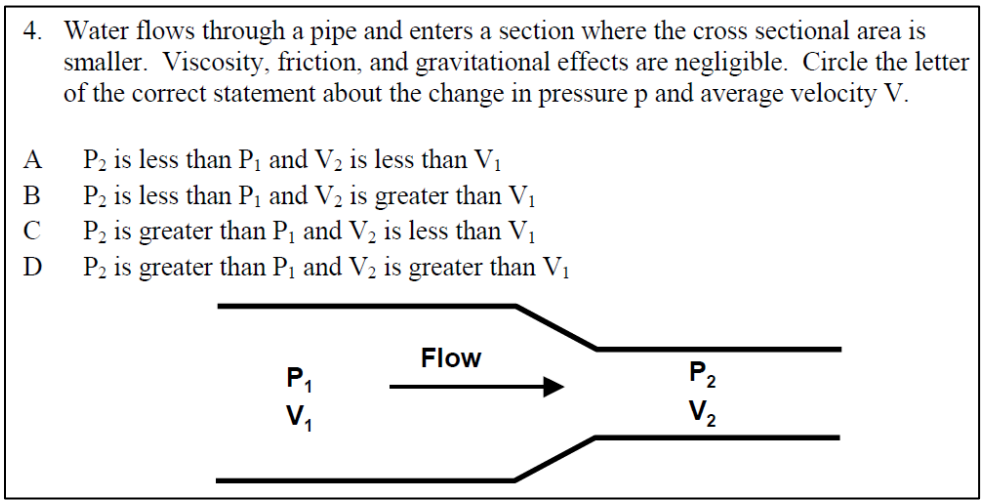


Figure 11.3. Fluid flow in a horizontal pipe segment, from a region of a wide cross-section to a region with a narrow cross-section (from Martin J. *et al.* [20])

Suarez *et al.* reported on the conceptual difficulties in fluid dynamics among the students who enrolled in introductory physics courses in Uruguay [17]. They found the difficulties by analyzing student responses of specific fluid problems in written exams. The reported errors are:

- Error 1: The pressure of a fluid in motion is the same as the pressure of a fluid at rest.

- Error 2: In vertical pipes of uniform cross-section, fluid velocity increases due to gravitational acceleration.
- Error 3: For a fluid to be at rest in a vertical pipe, the pressure difference between its extremities must be zero.
- Error 4: Applying Bernoulli's equation to two points of fluid, one of which is at rest.

Errors 1, 3, and 4 are related to Bernoulli's equation, and error 2 is related to the continuity equation. If we look at equation 1.1, when the cross-section of the pipe is uniform, the fluid velocity stays the same. This relationship is true for horizontal and vertical pipes. Errors that are related to Bernoulli's equation can sometimes be traced back to ignoring its applicability since the equation is only applicable for two points along the same streamline. Students apply the Bernoulli's equation without checking whether the two points in consideration are on the same streamline.

11.3 MOTIVATION AND CONTEXT OF THE INVESTIGATION

The studies described above have documented some student difficulties with the continuity and Bernoulli's equation. The difficulties are summarized in Table 6.28. To date, we only found one reported findings from the students in the introductory physics courses. The rest were from engineering students. In short, researching student difficulties in fluid dynamics in introductory physics courses is new, and documenting the difficulties is relevant for developing suitable curriculum materials.

Our investigation mainly covers two areas: (1) to replicate previous studies to find out if the reported difficulties are also present (Table 6.28) in our students and (2) to find out additional difficulties that have not yet reported.

The data we collected in our investigation came from students who are enrolled in PHYS 115, PHYS 121 (before Sp'17 quarter), and PHYS 123 courses. These three courses introduce some basic fluid dynamics content to students, which discusses the application of the continuity equation and Bernoulli's equation in the fluid flow in pipes. Viscosity is discussed in PHYS 115 in the context of human blood, but it is not investigated.

We discuss the results of our investigations in detail in the next two chapters.

Table 11.69. Summary of student difficulties with fluid dynamics reported in prior research.

Topic	Student difficulties
Continuity equation	• In vertical pipes of uniform cross-section, fluid velocity increases due to gravitational acceleration.
Bernoulli's equation	• The pressure of a fluid in motion is the same as the pressure of a fluid at rest. • For a fluid to be at rest in a vertical pipe, the pressure difference between its extremities must be zero. • Applying Bernoulli's equation to two points of fluid, one of which is at rest. • Using hydrostatic pressure to reason about the changes in water pressure for flows in vertical pipelines.
Others	• Water is a compressible fluid.

Chapter 12. INVESTIGATION OF STUDENT DIFFICULTIES IN FLUID DYNAMICS I – REPLICATION STUDIES AND OBSERVATIONS FROM EXAMS

This chapter describes the results of preliminary investigations of student difficulties in fluid dynamics at the University of Washington (UW). As mentioned in the previous chapter, the objectives of our study are to examine the presence of reported difficulties in our students and find out additional difficulties that have not yet reported. We did so by examining the student responses to selected questions from the Fluid Mechanics Concept Inventory (FMCI) on pretests after lecture instruction and final exams. The results of other fluid dynamics questions on final exams are also examined. In short, this chapter looks at student performance and documents the common difficulties in fluid dynamics after lecture instruction (pretests and final exams).

Section 12.1 describes the student responses to three FMCI questions on pretests. These questions test students' understanding of continuity and Bernoulli's equation in pipe flow situations. These concepts are related to what students learned in the introductory physics courses. The questions were administered on pretests after lecture instruction. Our results are compared with those reported by Fraser D. M. *et al.* [19] and Brown *et al.* [18]. Section 12.2 describes the student performance of some fluid dynamics questions and the same three FMCI questions on final exams. The exam results inform the persistence of student difficulties. Lastly, we summarize the results from the preliminary investigation.

12.1 STUDENT RESPONSES TO SELECTED FMCI QUESTIONS ON PRETESTS

This section describes the students' responses to selected FMCI questions on pretests. Three questions were selected because they were related to the continuity and Bernoulli's equations, which were what students learned from lectures. In all these questions, they are about water flowing in horizontal or vertical pipes with changing cross-sectional areas. Students are asked to compare the speeds and pressures of water flow at two different points with different cross-sectional areas. Each question is described in detail in 12.1.2.

We administered these questions to students on online pretests after lecture instruction. The student responses and explanations for each question are presented in subsections 12.1.2 to 12.1.4. We found similarities and differences in the common difficulties with those reported. Lastly, our pretest results are compared with those reported by Fraser D. M. *et al.* [19] and Brown *et al.* [18], and our results were generally better.

12.1.1 The selected FMCI questions in our investigation

We selected three questions from the FMCI that are related to the continuity and Bernoulli's equations, shown in Figure 12.1. The three questions are: (a) water flowing through a horizontal pipe from a region of narrow cross-section to a region of wide cross-section (labeled as the HN2W question), (b) water flowing downward through a vertical pipe from a region of narrow cross-section to a region of wide cross-section (labeled as the VDN2W question), and (c) water flowing upward through a vertical pipe from a region of wide cross-section to a region of narrow cross-section (labeled as the VUW2N question). Students are asked to compare the speeds and pressures of points 1 and 2, where those points are at regions with different cross-sectional areas.

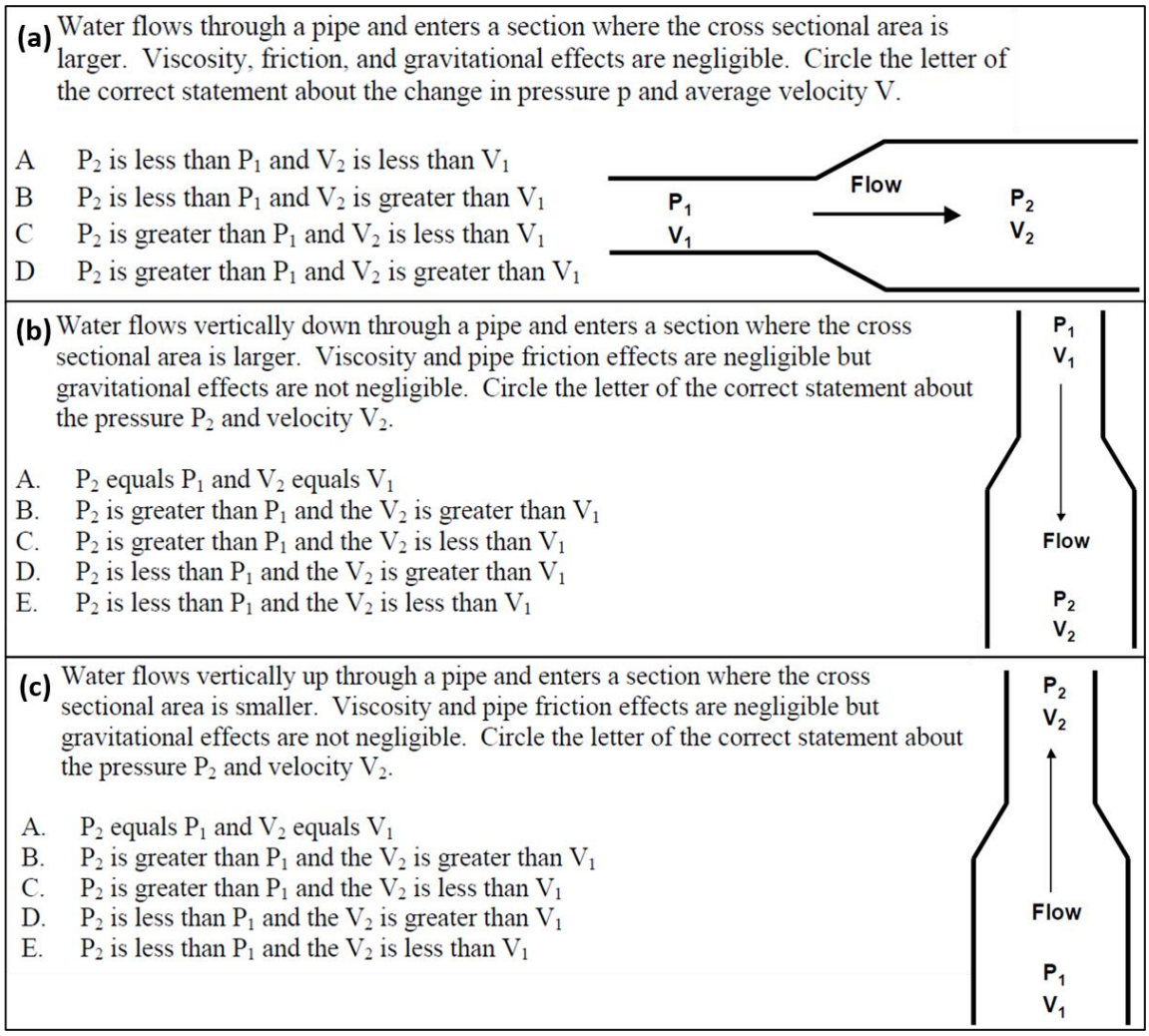


Figure 12.1. The three FMCI questions used in our investigation [20]. (a) HN2W question: Fluid flow in a horizontal pipe segment, from a region of narrow cross-section to a region of wide cross-section. (b) VDN2W question: Downward flow in a vertical pipe segment, from a region of narrow cross-section to a region of wide cross-section. (c) VUW2N question: Upward flow in a vertical pipe segment, from a region of wide cross-section to a region of narrow cross-section.

In all three questions, the viscosity of water is negligible, so the fluid is ideal. The water is also incompressible, so Bernoulli's equation can be used.

The approach to get the correct answer is to apply the continuity equation to compare v_2 with v_1 first, and then apply the Bernoulli's equation to compare P_2 with P_1. It is important not to mix up the order of applying the equations, which seems to be suggested by how the choices

are written in the question. In the tasks where vertical flows are involved, one needs to consider the height difference between points 1 and 2 to compare the pressures correctly.

These questions were administered to PHYS 123 students in thermal physics pretests in the Sp'17 quarter as there were no fluids related tutorials in PHYS 123. The questions were posted on pretests near the end of the quarter after students had lecture instruction and the second midterm, which fluid dynamics were tested. The pretest results are discussed in the next few sub-sections.

12.1.2 Student responses on the HN2W question

The HN2W question, shown in Figure 12.2, is about water flowing through a pipe from a region of narrow cross-section to a region of wide cross-section. Students are asked to compare the speeds and pressures of points 1 and 2 of the water flow.

To answer the HN2W question correctly, students need to apply the continuity equation $A_1 v_1 = A_2 v_2$ to compare v_2 with v_1 first. Since A_2 is greater than A_1, so v_2 is less than v_1. Then, using Bernoulli's equation at constant height ($y_1 = y_2$), $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$, with v_2 less than v_1, P_2 is greater than P_1. The correct option is C, $P_2 > P_1$ and $v_2 < v_1$.

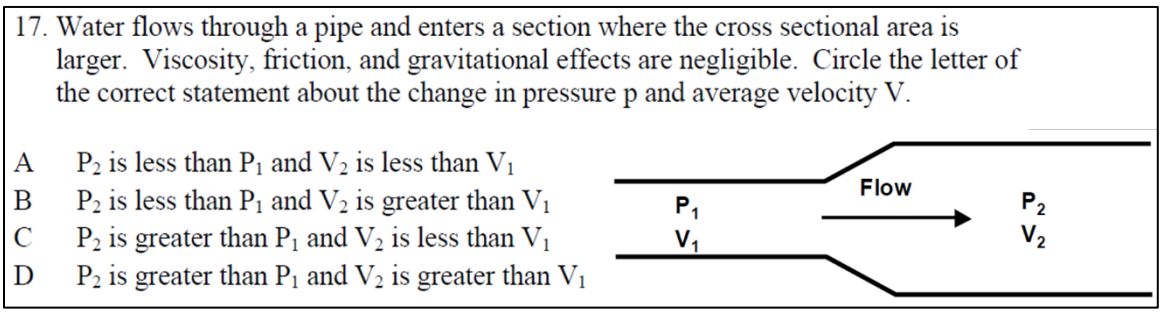


Figure 12.2. The HN2W question in FMCI. Fluid flow in a horizontal pipe segment, from a region of narrow cross-section to a region of wide cross-section [20].

The student responses (answers and explanations) to the HN2W question in the Sp'17 quarter are summarized in Table 7.34. Two-thirds of the students chose the correct speed and pressure comparisons. We found 83% and 72% of the students chose the correct comparisons of speeds and pressures, respectively.

Table 12.70. Summary of student responses and explanations on the HN2W question.

		Answers (N = 375)			Total
		Correct ($P_2 > P_1$ & $v_2 < v_1$)	$P_2 < P_1$ & $v_2 < v_1$	Others	
Categories of student explanation	Continuity and Bernoulli equation	28%	-	2%	30%
	Pressure-related	9%	7%	7%	23%
	Continuity only	9%	2%	-	11%
	Bernoulli only	6%	1%	1%	8%
	No explanations & Others	14%	7%	7%	28%
Total		67%	16%	17%	100%

Selected student explanations of the HN2W question are presented in Table 12.71. We found 28% of the students gave correct answers and explanations. They usually mentioned both the continuity and Bernoulli's equations, or a combination of the next two categories. Students who wrote about the constancy of flow rate or continuity principle only were categorized under "Continuity only." Explanations involving "low-speed high-pressure" or "high-speed low-pressure" only were categorized under "Bernoulli only."

Table 12.71. Selected student explanations on the HN2W question.

Category	Example of student explanations
Continuity and Bernoulli equation	Correct ($P_2 > P_1$ & $v_2 < v_1$): • *According to the continuity principle, the larger the cross-section of a pipe, the slower the liquid will move compared to a smaller cross-section of the same pipe with the same liquid moving through. Area 1 is smaller to v_1 is larger. According to Bernoulli's principle, the slower the liquid is moving the higher the pressure is, so pressure is higher in area 2.* • *Since $A_1 v_1 = A_2 v_2$, if $A_1 < A_2$, then $v_1 > v_2$. Also, due to the Bernoulli equation, if $v_1 > v_2$, then $P_1 < P_2$.*
Pressure-related	Correct ($P_2 > P_1$ & $v_2 < v_1$): • *Velocity is proportional to the cross-sectional area of the pipe. Larger area means greater speed. Pressure is also proportional to the cross-sectional area of the pipe. Greater area means greater pressure.* • *The pressure is smaller in P_1 as the pipe has a smaller diameter, and the velocity is greater in P_1 as the pressure is smaller.* $P_2 < P_1$ & $v_2 < v_1$: • *The pressure must be lower in section 2 because the flow is being driven from 1 to 2, and since the cross-sectional area is greater in 2 the volume is greater* • *Greater pressure and velocity in 1 because smaller cross-sectional area of pipe.* • *The pressure and velocity at point 1 is higher because the diameter of the pipe is smaller.* Others - $P_2 < P_1$ & $v_2 > v_1$: • *The pipe with a larger diameter will have a lower pressure, and therefore a lower pressure will cause the water flow to be faster.*
Continuity only	Correct ($P_2 > P_1$ & $v_2 < v_1$): • *because P_2 has a greater amount of water push on the area than P_1 and due to the continuity principle if P_2 is greater than P_1, then v_2 must be less v_1*
Bernoulli only	Correct ($P_2 > P_1$ & $v_2 < v_1$): • *Pressure and velocity have an inverse relationship.*

Some students explained their correct answers using pressure-related arguments such as the definition of pressure, pressure difference, or stating that the pressure is proportional to the cross-sectional area. Students who explained by pressure difference tended to choose answers that

correspond to $P_2 < P_1$. Students may be demonstrating the "motion implies a force" preconception, as documented by Clement [76]. Since the water is flowing from left to right, there is a force/net force acting on the water in the direction of motion. The force on the left is greater than that on the right. Since the pressure is proportional to force, so $P_2 < P_1$.

Students who used the definition of pressure ($P = F/A$) tended to choose answers that correspond to $P_2 < P_1$ as well. The preconception mentioned above could also explain it. Since the fluid moved from left to right, then a constant force pointing from left to right was needed to sustain the motion. Since $A_2 > A_1$, the definition of pressure gave $P_2 < P_1$.

Several students stated that the pressure is proportional to the cross-sectional area when they chose $P_2 > P_1$. The reasoning is incorrect. After analyzing the situation using the continuity and Bernoulli's equation, one can get $P_2 - P_1 = \frac{1}{2}\rho v_2^2 \left[\left(\frac{A_2}{A_1}\right)^2 - 1\right]$. When $A_2 > A_1$, the right-hand side is positive, so the pressure at region 2 is greater than the pressure at region 1. Another possibility is that some students did not go through the reasoning properly. They learned that $P_2 > P_1$ from the lecture, and saw that $A_2 > A_1$ from the diagram, then they formed a preliminary but incorrect pressure-area relationship.

12.1.3 Student responses on the VDN2W question

The VDN2W question (Figure 12.3) looks like the HN2W question except that the pipe is oriented vertically. The correct speed comparison is $v_2 < v_1$ by using the continuity equation and recognizing that $A_2 > A_1$. The change in height cannot be ignored when we apply Bernoulli's equation in this case. However, that does not affect the conclusion that $P_2 > P_1$ since $y_2 < y_1$ and $v_2 < v_1$. So, the correct option is C, $P_2 > P_1$ and $v_2 < v_1$.

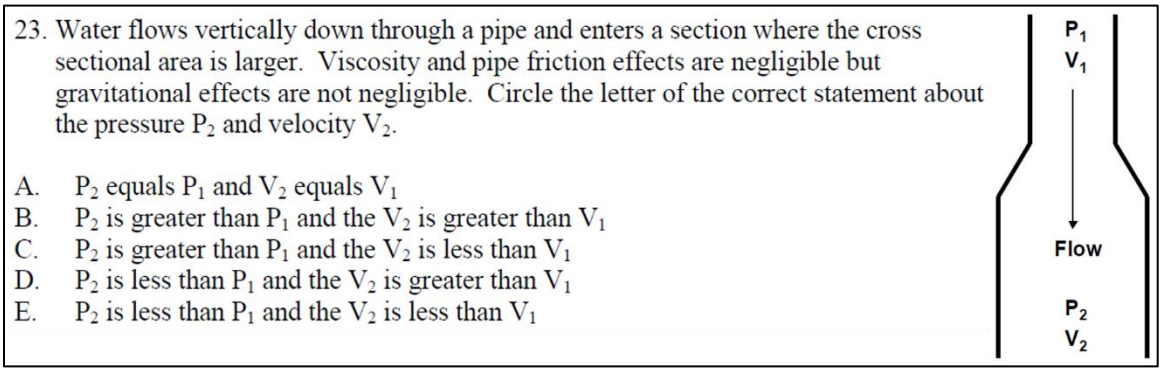


Figure 12.3. The VDN2W question in FMCI. Downward fluid flow in a vertical pipe segment, from a region of narrow cross-section to a region with wide cross-section [20].

The student responses are summarized in Table 12.72. Around 40% of the students chose the correct answer. Considering the speed and pressure comparisons individually, 57% of the students did the former, and 63% did the latter correctly. Around one-third of the students chose the incorrect speed ranking $v_2 > v_1$, suggesting that students had some difficulties with vertical pipe flow.

Table 12.72. Summary of student responses and explanations on the VDN2W question.

		Answers (N = 183)				Total
		Correct ($P_2 > P_1$ & $v_2 < v_1$)	$P_2 > P_1$ & $v_2 > v_1$	$P_2 < P_1$ & $v_2 > v_1$	Others	
Categories of student explanation	Continuity and Bernoulli	17%	2%	1%	-	20%
	Free fall-related	-	10%	7%	5%	22%
	Pressure-related	9%	3%	1%	3%	16%
	Continuity or Bernoulli only	9%	-	-	1%	10%
	No explanations & Others	7%	5%	7%	13%	32%
Total		42%	20%	16%	22%	100%

Selected student explanations are presented in Table 12.73. Only 17% of the students answered and explained correctly. The student explanations were categorized the same way as in the HN2W question. Around 20% of the student explanations were related to free fall. Most students who chose options with $v_2 > v_1$ cited "acceleration due to gravity" or conversion from gravitational potential energy to kinetic energy in their explanation. This finding, although different from what Suarez *et al.* reported (see section 11.2.2), shows that some students use gravitational acceleration to explain why the water speed increases as it moves down a vertical pipe.

We found 10% of the students used the formula of hydrostatic pressure ($P = \rho g h$) to explain $P_2 > P_1$. This reasoning was reported by Brown *et al.* [18]. These students were able to arrive at the correct answer by considering the difference in height only, without considering the difference in speeds. We cannot tell whether the students were treating the fluid as moving fluid or static fluid since the formula $P = \rho g h$ is for static fluid, and the Bernoulli's equation is used for moving fluid.

Table 12.73. Selected student explanations on the VDN2W question.

Category	Example of student explanations
Continuity and Bernoulli equation	Correct ($P_2 > P_1$ & $v_2 < v_1$): • *According to continuity equation, the cross-sectional area of 1 is less than 2, thus the velocity at 1 is larger than 2. The pressure at the higher velocity place is less than at lower velocity place.* • *The velocity of a liquid is inversely proportional to the cross-section of the pipe, so since part 2 has a bigger area, then the velocity is smaller there. According to Bernoulli's relation, then the pressure goes down as velocity of the fluid increases. on top of this there is more liquid in the second part of the pipe and thus pressure would increase.*
Pressure-related	Correct ($P_2 > P_1$ & $v_2 < v_1$): • *In terms of height, the second pressure still has greater depth than pressure 1. Hence, pressure 2 is bigger than pressure 1.* • *P = P(surface) $+\rho g d$, here we observe that P_1 is at a higher location than P_2, such that P_1 must be smaller than of P_2. Viscosity [Velocity] will heavily depend on gravitation effects, such that the force at a higher location will have much more potential energy, thus more energy transferred to kinetic energy* • *Pressure increases with a decrease in height.* $P_2 > P_1$ & $v_2 > v_1$: • *The pipe with a larger diameter will have a lower pressure, and therefore a lower pressure will cause the water flow to be faster.* • *Gravity will keep the pressure on the bottom higher than on the top because the weight of the water on top will crush the water below it, giving it more pressure.* • *$\rho g h$ for pressure*
Free fall related	$P_2 > P_1$ & $v_2 > v_1$: • *Larger pressure must be in larger area to accelerate fluid when the passage gets smaller, but for velocity, it now has less potential energy at 2 than 1, so it must have more kinetic energy.* • *Because they are flowing due to gravity, v_2 will be greater, because gravity has been acting longer on the fluid at that point and thus P_2 is also greater.* • *Velocity is greater in this situation because of acceleration caused by gravity*

12.1.4 Student responses on the VUW2N question

In the VUW2N question (Figure 12.4), the water is flowing in the opposite direction to what is presented in the VDN2W question. So, the conclusions obtained by using the continuity equation and Bernoulli's equation for this question are the opposite of the VDN2W question. The correct answer is option B, which $P_2 < P_1$ and $v_2 > v_1$. We present the results and some student explanations in Table 12.74 and Table 12.75, respectively

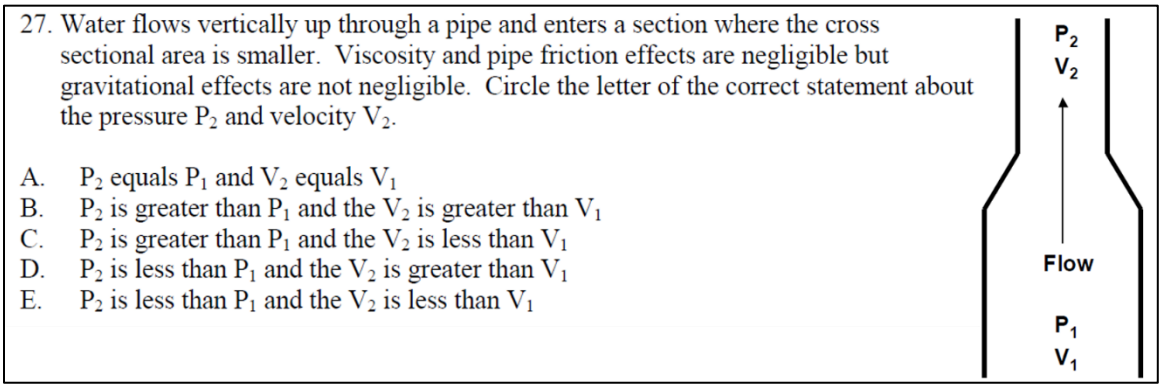


Figure 12.4. The VUW2N question. Upward fluid flow in a vertical pipe segment, from a region of wide cross-section to a region with narrow cross-section [20].

Our results show that 57% of the students answered this task correctly. Considering the speed and pressure comparisons individually, 67% of the students did the former, and 72% did the latter correctly. Students performed 15% better in the VUW2N question (57% correct) than in the VDN2W question (42% correct).

Fourteen percent of the students answered and explained correctly. Not many students used free-fall reasoning to explain why $v_2 < v_1$ on this task. This result showed that students tended to use free-fall to compare the speeds when the water was flowing down. The proportion of students using pressure-related arguments on this task was greater than the VDN2W task.

Table 12.74. Summary of student responses and explanations on the VUW2N task.

		Answers (N = 192)			
		Correct ($P_2 < P_1$ & $v_2 > v_1$)	$P_2 < P_1$ & $v_2 < v_1$	Others	Total
Categories of student explanation	Continuity and Bernoulli	14%	-	2%	16%
	Pressure-related	26%	4%	5%	35%
	Continuity or Bernoulli only	6%	1%	3%	10%
	Free fall related	1%	7%	1%	9%
	No explanations & Others	9%	4%	17%	30%
Total		57%	16%	27%	100%

Table 12.75. Selected student explanations on the VDN2W question.

Category	Example of student explanations
Continuity and Bernoulli equation	Correct ($P_2 < P_1$ & $v_2 > v_1$): • *First since pipe 2 has less cross-section area, it should be faster. Then we should know $P + \rho v^2/2 + \rho gh$ is a constant, since v_1 and h_1 is lower for pipe 1, P_1 should be larger than P_2.*
Pressure-related	Correct ($P_2 < P_1$ & $v_2 > v_1$): • *Gravitational force has a greater effect on the pressure.* • *In this case, the gravity affects the pressure of the water flows. The lower the water, the higher the pressure.* • *Pressure increases with a decrease in height.*
Free fall related	$P_2 < P_1$ & $v_2 < v_1$: • *because of gravity, v_1 is larger than v_2.* • *P_1 is obviously going to have higher pressure because it is lower and has larger cross-sectional area. I don't really know about velocity, however. As the water flows upwards, it loses kinetic energy and gains gravitational potential energy. If point 2 is located at a sufficiently tall height, then it is possible for the velocity at point 2 to be less than the velocity at point 1.* • *Because pressure depends on height, the pressure at P_1 is still greater than the pressure at P_2. However, the velocity at P_2 is now also less, due to gravitational effects.*

12.1.5 Discussions

In the Sp'17 quarter, we administered the HN2W and the VDN2W questions to one group of PHYS 123 students (group 1, $N = 183$), the HN2W, and the VUW2N questions to another group of students (group 2, $N = 192$). We found that 35% of students in group 1 and 44% of students in group 2 answered both their assigned questions correctly. On average, around 40% of the class could answer them correctly after lecture instruction.

The pretest results on each task are summarized in Table 12.76 and compared with the published results from the University of Cape Town (UCT) [19] and Washington State University (WSU) [18]. Comparing the results from UW and UCT, the consistent pattern is that students performed better in horizontal pipe flow questions than vertical pipe flow questions. However, the results from WSU do not demonstrate the trend. More results from different institutions are needed to confirm the trend.

Table 12.76. Summary of pretest results on the HN2W, VDN2W, and the VUW2N. Results from the University of Cape Town (UCT) [19] and Washington State University (WSU) [18] are listed for comparison. Note that the sample sizes are not the same.

Task	% Correct		
	UW	UCT	WSU
Horizontal flow, narrow to wide (HN2W)	67%	47%	41%
Vertical downward flow, narrow to wide (VDN2W)	42%	30%	Not reported
Vertical upward flow, wide to narrow (VUW2N)	57%	34%	49%

The results in Table 12.76 were obtained after students had relevant lecture instructions. The UW results were collected immediately after students had a midterm on fluid mechanics. The UCT results were posttest results after the students had computer simulations in fluid mechanics

lessons. At the same time, the WSU results were obtained from student interviews after completing the fluid mechanics class. The UW students generally performed better than students from other institutions, which could be attributed to students' revision of fluid mechanics for the midterm. The relevant ideas were retained when the UW students attempted these questions on pretests.

The student explanations from these three questions can be categorized generally into these categories: a correct explanation involving both the continuity and Bernoulli's equation, Bernoulli's equation or continuity equation only, pressure-related arguments, and free-fall-related arguments (in vertical pipe flow). Not all students explained both the speed and pressure comparisons on pretests. Some students only explained one of the comparisons, so the explanation was incomplete. Brown *et al.* suggested that in the future administration of these three FMCI questions, the speed and pressure comparison tasks should be asked as individual questions so that researchers can analyze the explanations for each task independently. The authors also mentioned that the pipe segments presented in those tasks are not given a physical context, which may affect the student responses.

We found the student difficulties that were reported by Brown *et al.* and Suarez *et al.* from our preliminary investigation of vertical fluid flow situations.

Brown *et al.* found students using the definition of the hydrostatic pressure to explain the pressure difference, which was present in our student responses for the VDN2W and VUW2N questions. Brown *et al.* also reported that their students wrote water is a compressible fluid in horizontal pipe flow situations, but that was not found in our investigation.

Suarez *et al.* found students using gravitational acceleration to explain the changes in fluid speed in vertical pipes with uniform cross-section. This reasoning is present in both the VDN2W and VUW2N tasks, with the former in greater proportion than the latter. Although these two tasks

do not involves pipes with uniform cross-section, but we believe that some students may demonstrate this difficulty in vertical fluid flow in pipes with uniform cross-section. The results for the new context (vertical fluid flow in uniform pipes) are described in the next chapter.

12.1.6 Commentary

In our preliminary investigation, we selected three FMCI questions (HN2W, VDN2W, and VUW2N) to investigate student understanding of continuity and Bernoulli's equations in the introductory physics course. These questions involve water flowing in horizontal or vertical pipes with changing cross-sections. They were administered on pretests after students had lecture instruction and midterm. Our results were generally better than those reported, which could be attributed to that students' correct ideas on fluid dynamics somewhat remained fresh after the midterm.

The student explanations in those FMCI questions were categorized into these categories: a correct explanation involving both the continuity and Bernoulli's equation, Bernoulli's equation or continuity equation only, pressure-related arguments, and free-fall-related arguments (in vertical pipe flow). Some explanations were consistent with the reported difficulties involving fluid flow in vertical pipes, namely (1) using the formula of hydrostatic pressure to compare pressures and (2) using gravitational acceleration to compare speeds. We did not find our students mentioning water being a compressible fluid. Several additional investigations to probe student difficulties in fluid dynamics were briefly described.

12.2 STUDENT RESPONSES TO FLUID DYNAMICS QUESTIONS ON EXAMS

Besides using the pretest responses from selected FMCI questions to investigate student difficulties in fluid dynamics, we also examine the student responses on some fluid dynamics questions (including the selected FMCI questions) on exams. Based on the responses, we can infer the difficulties students had after all instructions. Some of these difficulties may be consistent with what we found in the previous section, and there may be more. The FMCI questions (HN2W, VDN2W, VUW2N) are asked in final exams to find out if certain student difficulties persist.

The student responses to the exam questions are presented according to the concepts involved. We first discuss the responses to a question involving the continuity equation only. The next question involves both the continuity and Bernoulli's equations. Lastly, we present the results to the three FMCI questions on final exams.

12.2.1 Exam question involving continuity equation only

The greatest-speed task, shown in Figure 12.5, involves a pipe system that is connected to a water tank, with water flowing from a wide cross-section region to a narrow cross-section horizontally. Then the water flows down to another horizontal pipe section at a lower height, from a narrow cross-section to a wide cross-section. Students are asked to choose the points where the speed of the water is the greatest.

This question was administered to the PHYS 121 students in Wi'16 and Sp'16 quarters. The correct answer is obtained by using the continuity equation. The speed of the water is the greatest at points in which the cross-sectional area is the smallest, which are points 2 and 3.

(5 points) Water is discharged from a tank from an opening at the right of the tank as shown. At which point(s) is the speed of the water the greatest?

A) 1
B) 2
C) 3
D) 4
E) 2 and 3

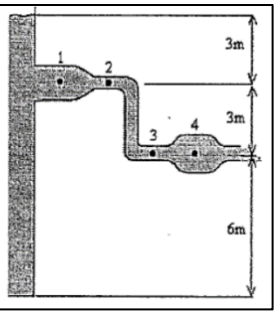


Figure 12.5. The greatest-speed task for fluid flow in a pipe system. The correct answer is E.

Another variant of this question is to ask for the points where the water speed is the least (least-speed task), which was asked in the Sp'16 quarter. In this case, the correct answer is points 1 and 4 with the continuity equation. The student responses to this task (points with the greatest speed) and its variant (points with least speed) are summarized in Table 12.77.

Table 12.77. Student responses to the fluid task in Figure 12.5.

Approach (answer)	Greatest-speed task Wi'16, Sp'16 (N = 418)	Least-speed task Sp'16 (N = 197)	Aggregate (N = 615)
Correct (Greatest: points 2 and 3 Least: points 1 and 4)	29%	23%	27%
Partially correct (Greatest: point 2 only Least: point 4 only)	28%	26%	27%
Gravity-related reasoning (Greatest: point 3 only Least: point 1 only)	39%	39%	39%

The percentage correct for the greatest-speed task and the least-speed task are 29% and 23%, respectively. To see if students responded to these two questions differently, we performed χ^2 test ($\chi^2\ 1\ = 2.06$, p-value = 0.15) and found that there was no significant difference. The χ^2 test result suggests that we can aggregate the responses from both tasks. We found that 39% of the students chose points 3 and 1 with the greatest and least water speed, respectively. These

students could be taking gravitational acceleration or gravitational potential energy into account. Using the greatest-speed task, for example, since point 3 is lower than point 2, so the potential energy in the water at point 2 is converted into its kinetic energy at point 3, so the speed at point 3 is greater than that at point 2.

The approach for the most common error in this question is consistent with the free-fall reasoning in the VDN2W task in the previous section. The direction of the flow of water in the VDN2W task is downward, the same as this situation. The similarities in the most common error and contexts in these two tasks suggest that students tend to use free-fall reasoning to compare water speeds in downward flows.

12.2.2 Exam questions involving both the continuity and Bernoulli's equations

The question in Figure 12.6 is about a fluid flowing in a horizontal pipe system. It has two variations. In Case 1, the fluid is flowing from a pipe with wide channel to a pipe with narrow channel, while it is the reverse in case 2. To find the pressure difference between points P and Q, students need to use the continuity equation to find the fluid speed at the wide channel first, then use the Bernoulli's equation to find the pressure difference. The correct answers for cases 1 and 2 are E and D, respectively.

These two variations were administered in two different lecture sections of PHYS 123 in the Au'17 quarter. The results are summarized in Table 12.78.

The percentages correct for Cases 1 and 2 are 56% and 63%, respectively. The χ^2 test result ($\chi^2\ 1\ = 1.3$, *p*-value = 0.25) shows that there is no significant difference in how students respond to these two questions. So, we aggregate the results from both cases.

A non-viscous fluid with a density of 750 kg/m^3 flows through a horizontal pipe that gets narrower (Case 1) / wider (Case 2) by a factor of two as shown. The flow is laminar and at point Q (Case 1) / P (Case 2) the fluid is flowing with a speed of 1.0 m/s. What is the pressure difference between points P and Q?

A. $P_P - P_Q = 0$ Pa
B. $P_P - P_Q = -281$ Pa
C. $P_P - P_Q = 281$ Pa
D. $P_P - P_Q = -352$ Pa
E. $P_P - P_Q = 352$ Pa

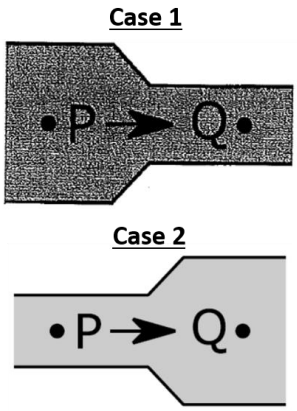


Figure 12.6. Fluid flow in a horizontal pipe system. Case 1 is from a region of wide cross-section to a region of narrow cross-section. Case 2 is from narrow cross-section to wide cross-section.

The most common incorrect answers for Cases 1 and 2 are options C and B, respectively. Students who chose these answers did not square the radius when they applied the continuity equation to find the speed of the fluid flow in the wide channel. Using case 1 as an example, when the fluid flows from P to Q, the diameter decreases by a factor of two, so the cross-sectional area decreases by four times. These students ignored that area is proportional to the square of the radius/diameter. Instead of getting $v_Q = 4v_P$ from the continuity equation, they obtained the incorrect relationship $v_Q = 2v_P$. We categorized this error as a scaling error or proportional reasoning error.

Table 12.78. Student responses to the fluid task in Figure 12.6.

Approach	Case 1 (N = 174)	Case 2 (N = 170)	Aggregate (N = 344)
Correct (Case 1: E, Case 2: D)	56%	63%	60%
Ignoring proportionality between area and diameter/radius (Case 1: C, Case 2: B)	18%	24%	20%
Others (3 options)	26%	13%	20%

12.2.3 Student responses to the FMCI questions on exams

Here, we present the student responses to the three FMCI questions (HN2W, VDN2W, and VUW2N tasks) on final exams. These results inform us whether the student difficulties found in the previous section persist through the end of the quarter. If some difficulties persist, then there is motivation to design a tutorial curriculum to address them.

The HN2W task was asked in the finals of the Wi'17 quarter. The VDN2W and VUW2N tasks were asked in the PHYS 115 and PHYS 123 finals in the Wi'19 quarter. The results are summarized in Figure 12.7. Students generally perform better in the final exams than in the pretest, and they also perform better than the results reported from the literature [18, 19].

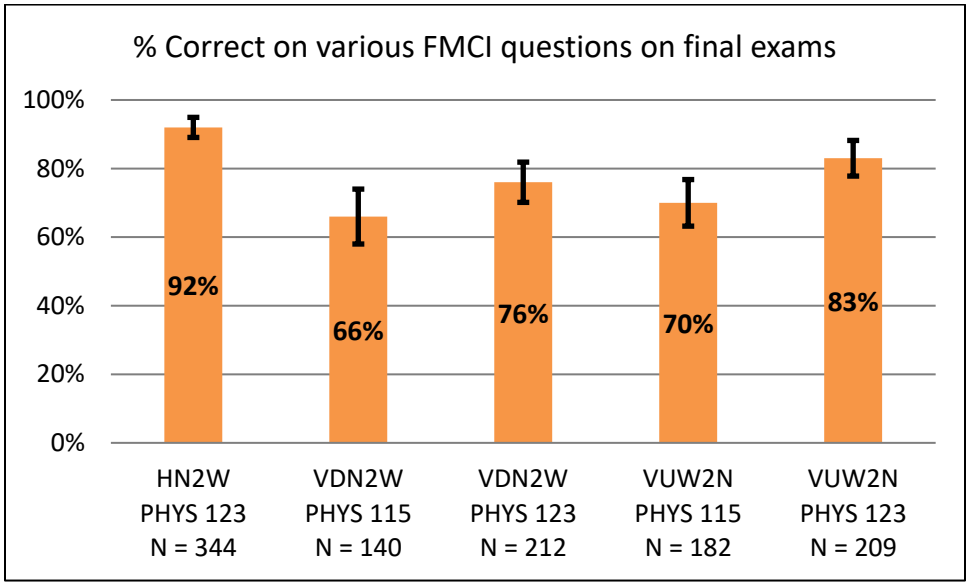


Figure 12.7. Results on the FMCI questions (HN2W, VDN2W, and VUW2N) in PHYS115 and PHYS 123 final exams. The error bars are twice the standard error.

The HN2W is the easiest of all, with more than 90% of the class answering correctly. The result is 30% better than the quantitative version (case 2 in Figure 12.6). One possible reason could be students do not pay attention to the mathematical details when calculations are involved.

The results for questions involving vertical pipe flows are generally lower than horizontal pipe flows. The exam results in the VUW2N task is slightly higher than that in the VDN2W task, although not significant, in both the PHYS 115 and PHYS 123 classes. These trends are also observed on pretests in the previous section.

The PHYS 123 students performed 10%-13% better than the PHYS 115 students in the vertical pipe flow situations (VDN2W and VUW2N). We performed χ^2 test for both VDN2W and VUW2N tasks to determine if the difference in the results are statistically significant. The statistical test results are $\chi^2\ 1\ = 3.87$, *p*-value = 0.049 and $\chi^2\ 1\ = 7.82$, *p*-value = 0.005 for the VDN2W and VUW2N tasks, respectively. Since both *p*-values are below 5%, the differences between the results from PHYS 123 and PHYS 115 are statistically significant.

Next, we discuss the implications of these exam results in curriculum development. As described in Chapter 1, the benchmark for success is when student posttest performance matches the performance of physics graduate TAs on pretests. However, the TA responses on these three FMCI questions were not collected. So, we need to rely on another benchmark.

According to Hestenes *et al.* (the creators for the Force Concept Inventory, or FCI) [7], they claimed that their FCI data suggested the existence of "a kind of conceptual threshold near 60% on the inventory" for efficient problem-solving. The FCI is a 30-item inventory testing students' ideas on Newtonian mechanics. The authors' claim could be interpreted as every student answered more than 18 questions correctly or a correct rate of 60% on each item. The former interpretation is the intended one. If we were to use the latter interpretation, then our exam results would have met the threshold, and it would not be necessary to develop a tutorial curriculum to address the student difficulties. However, using the threshold for a 30-item inventory for specific

questions appears to be a misuse of the benchmark. So, the need to develop a tutorial curriculum to address student difficulties in fluid dynamics is still up for discussion.

Lastly, since the final exams were multiple-choice, we were unable to find out student approaches. Even though the results from the final exams were above 60%, it did not mean that these students could answer and explain correctly. So, it is possible that the prevalence of specific difficulties to not change after the pretests. One such example is the use of hydrostatic pressure to explain the changes in pressure in vertical flow contexts. One can arrive at the correct answer by using either the hydrostatic pressure formula or Bernoulli's equation in those contexts.

12.3 SUMMARY

We reported the student responses to three FMCI questions on pretests and reviewed the student performances of fluid mechanics questions on final exams. The proportion of students who answered the tasks correctly ranged from 40% to 90%. The student difficulties found were pressure-related, gravity-related in the context of vertical fluid flow, and difficulty in proportional reasoning (not realizing that the cross-sectional area is proportional to the square of the radius/diameter). The difficulties that were consistent with those reported were (1) using hydrostatic pressure to explain the changes in pressures in vertical flow contexts and (2) using gravitational acceleration to explain the changes in speeds in vertical flow contexts.

More than 60% of the algebra-based and calculus-based students were able to answer the selected FMCI questions on final exams. These results suggest that it may not be necessary to design a curriculum to address specific student difficulties. Since the student approaches were not directly available in the finals, some student difficulties may still be prevalent, especially when students arrive at the correct answers with incorrect reasoning.

Chapter 13. INVESTIGATION OF STUDENT DIFFICULTIES IN FLUID DYNAMICS II – ADDITIONAL STUDIES

In the previous chapter, we compared the student difficulties in fluid dynamics in our introductory physics students to those reported in the literature. The concepts involved were continuity and Bernoulli's equations. The student responses from three Fluid Mechanics Concept Inventory (FMCI) questions and other fluid dynamics questions on pretests and exams were examined. The reported difficulties that were found in our students were related to vertical flow contexts. The difficulties are (1) using hydrostatic pressure to explain the changes in pressures and (2) using gravitational acceleration to explain the changes in speeds.

In this chapter, we expand the scope of investigation in a few ways. First, Brown *et al.* suggested to change the way the selected FMCI questions are being asked [18]. Students are asked to compare pressures and speeds in one question in the original FMCI version, but not all combinations are available for students. So, the authors suggested splitting the original task into speed and pressure comparison tasks. The authors claimed that this way of questioning could provide a "more nuanced understanding of where exactly misconceptions exist." Other advantages are that students can give complete explanations (assuming that they do so), and researchers do not need to guess whether their explanations are for the speed or pressure comparison tasks or both. The results are described in section 13.1.

Second, we included fluid flow in vertical pipes with uniform cross-sections. This task is to confirm what Suarez *et al.* had reported in their work [17], that is, to find out whether students use gravity to explain why the fluid speed increases as it flows vertically down along a pipe. We have already seen students using gravity to explain in the VUW2N and VDN2W tasks.

Third, Brown *et al.* also suggested giving context to the selected FMCI questions [18]. In their investigation, some of their students struggled that "the vertical pipeline segment was part of a larger pressurized pipeline system." The authors suggested to "alter the drawings of the pipe segments to be explicitly obvious that the pipe segment is 'broken off' from a larger pipe system." They claimed that the change in the representation (picture) might change students' conceptions. We made the suggested changes, and the student responses obtained were analyzed and compared with the original versions.

Fourth, from the previous chapter, we found that in vertical pipe flow contexts, students used hydrostatic pressure to explain the changes in pressures and arrived at the correct answer. To distinguish whether students are using hydrostatic pressure or Bernoulli's equation to arrive at their answers, we rewrote the answer options using equations in the pressure comparison tasks.

The student responses to the above four changes are described in section 13.1. All the changes are modified from existing FMCI, so they are called the modified FMCI questions.

Section 13.2 describes the student responses to the additional tasks to probe student understanding in fluid dynamics, such as the application of the continuity equation in the traffic flow context and one-channel to two-channel fluid flow. The responses in those tasks gave additional information about what students could or could not do after lecture instruction.

After summarizing the student difficulties in fluid dynamics in introductory physics courses we found so far, we conclude the investigation.

13.1 STUDENT RESPONSES TO THE MODIFIED FMCI QUESTIONS

In this section, we describe all the student responses to the modified FMCI questions.

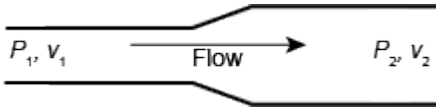
There are four modifications. The first modification is about changing the original versions of HN2W, VUN2W, and VDW2N tasks into two-part question, the speed, and pressure comparison tasks. The results are discussed in subsections 13.1.1 to 0. The responses on the modified versions are compared to the original versions. The second modification is about adding a question about water flowing down a vertical pipe of uniform cross-section, and its results are discussed in 13.1.4. This result is to confirm the difficulty that Suarez *et al.* reported, as we have summarized in section 11.2, one of the errors is that "in vertical pipes of uniform cross-section, fluid velocity increases due to gravitational acceleration."

Third, we draw the pipe segment that is in the original version as part of a larger pipe system. Fourth, we rewrite the answer options into (more complicated) equations in the pressure comparison task to see if students are using hydrostatic pressure or Bernoulli's equation to arrive at the correct ranking. The results of the last two modifications are described in subsection 13.1.5. We then summarize the findings of this section. In all tasks, we administered the modified versions on pretests after students had relevant lecture instructions.

13.1.1 Student responses to the modified HN2W question

The modified HN2W question is illustrated in Figure 13.1. We administered the modified HN2W question to 210 PHYS 123 students in the Au'17 and Au'19 quarters. The correct speed comparison is $v_2 < v_1$, and the correct pressure comparison is $P_2 > P_1$.

Water flows through a pipe and enters a section where the cross-sectional area is larger. Viscosity, friction, and gravitational effects are negligible.



Speed comparison task: Is the speed of the fluid flow at point 2, v_2, *greater than*, *less than*, or *equal to* the speed of the fluid flow at point 1, v_1?

Pressure comparison task: Is the fluid pressure at point 2, P_2, *greater than*, *less than*, or *equal to* the speed of the fluid flow at point 1, P_1?

Figure 13.1. The modified HN2W question.

The results of the modified HN2W question are summarized in Table 13.79. We found 86%, and 67% of the students chose the correct answers on the speed and pressure comparison tasks, respectively.

Around two-thirds of the students answered both tasks correctly in the modified version. From Table 7.34, 67% of the students answered the original HN2W question correctly. So, the results of the original and modified versions are about the same.

Table 13.79. Student responses to the modified HN2W question. The percentage correct of the correct combination is in bold.

		Pressure comparison task (N = 210)			
		$P_2 > P_1$	$P_2 < P_1$	$P_2 = P_1$	Total
Speed comparison task	$v_2 > v_1$	2%	8%	1%	11%
	$v_2 < v_1$	**65%**	14%	7%	**86%**
	$v_2 = v_1$	-	3%	-	3%
	Total	**67%**	25%	8%	100%

On another occasion, 174 PHYS 115 students attempted the modified HN2W question in the Au'19 quarter (results not shown). The results for the speed and pressure comparison tasks are 82% and 56%, respectively. Around 52% of the students answered both tasks correctly, which

is lower than the PHYS 123 students. The χ^2 test result is $\chi^2\ 1\ = 7.23$, *p*-value = 0.007, so the 15% difference between the algebra-based and calculus-based students is statistically significant.

Now we consider the student explanations for each task. The categories are presented in Table 13.80 and Table 13.81. We found that 67% of the students gave the correct answer and explanation on the speed comparison task; 30% did so on the pressure comparison task.

The proportions of students who answered and explained correctly in the original and modified HN2W tasks are 28% and 26%, respectively. There is no significant difference in student performance in the original and modified versions.

Table 13.80. Student explanations for the modified HN2W question (speed comparison). The percentage correct of the correct combination is in bold.

Categories	Speed comparison task (*N* = 210)		
	$v_2 < v_1$	Others	Total
Continuity	**67%**	2%	69%
Bernoulli	4%	1%	5%
No explanation & Others	15%	11%	26%
Total	86%	14%	100%

Table 13.81. Student explanations for the modified HN2W question (pressure comparison). The percentage correct of the correct combination is in bold.

Categories	Pressure comparison task (*N* = 210)		
	$P_2 > P_1$	Others	Total
Bernoulli	**30%**	5%	35%
Greater area	12%	-	12%
Pressure related	5%	10%	15%
No explanation & Others	20%	18%	38%
Total	74%	26%	100%

From Table 13.81, 12% of the students cited "greater area" to explain $P_2 > P_1$. They seemed to imply that the pressure is proportional to the cross-sectional area. In the original version,

7% of the students said so, while 12% did so in the modified HN2W question. These students may have correlated greater fluid pressure with the greater cross-sectional area after lecture instruction.

13.1.2 Student responses to the modified VDN2W question

We administered the modified VDN2W question to the PHYS 123 students in the Au'17 and Au'19 quarters. The correct speed and pressure comparisons are $v_2 < v_1$ and $P_2 > P_1$, respectively. The results of the modified VDN2W question are summarized in Table 13.82.

Out of the 210 students, 77% and 69% of them chose the correct answers on the speed and pressure comparison tasks, respectively. Around 60% of them answered both tasks correctly. From Table 12.72, the result for the original VDN2W question is 42%, which is 18% lower than the modified version. The results between the original and modified VDN2W questions are compared using the χ^2 test, and the result is $\chi^2\ 1\ = 10.9$, *p*-value < 0.001. The test statistic suggests that the improved results from the modification are statistically significant with a medium effect.

Table 13.82. Student responses to the modified VDN2W question.

		Pressure comparison task (*N* = 173)			
		$P_2 > P_1$	$P_2 < P_1$	$P_2 = P_1$	Total
Speed comparison task	$v_2 > v_1$	5%	9%	-	14%
	$v_2 < v_1$	**60%**	12%	5%	**77%**
	$v_2 = v_1$	4%	3%	2%	9%
	Total	**69%**	24%	7%	100%

The increase in student performance could be due to better lecture instruction. In the Sp'17 quarter, the instructor was using a new textbook written by Mazur [30] the first time. In the later quarters, the instructors (not necessarily the same) had more experience in using the book to teach,

which may contribute to better student learning. One way to check this claim is to administer the original VDN2W task to the PHYS 123 students again.

The student explanations for each task are categorized and presented in Table 13.83 and Table 13.84. We found that 61% of the students gave the correct answer and explanation on the speed comparison task; 31% did so on the pressure comparison task. Seventeen percent answered and explained correctly on the original VDN2W question, compared to 24% correct on the modified version. The χ^2 test result is $\chi^2\ 1\ = 2.5$, *p*-value is 0.11, so the difference in the results for students who answered and explained correctly in both versions is not statistically significant.

Table 13.83. Student explanations for the modified VDN2W question (speed comparison). The percentage correct of the correct combination is in bold.

Categories	Speed comparison task (*N* = 173)		
	$v_2 < v_1$	Others	Total
Continuity	**61%**	1%	62%
Bernoulli	1%	3%	4%
Gravity	-	6%	6%
No explanation & Others	15%	13%	28%
Total	77%	23%	100%

Table 13.84. Student explanations for the modified VDN2W question (pressure comparison). The percentage correct of the correct combination is in bold.

Categories	Pressure comparison task (*N* = 173)		
	$P_2 > P_1$	Others	Total
Bernoulli	**31%**	4%	35%
Greater area	10%	-	10%
Depth (ρgh)	8%	2%	10%
Inversely proportional to v	7%	1%	8%
No explanation & Others	13%	24%	37%
Total	69%	31%	100%

Now we discuss the student explanations in each task. In the speed comparison task, 61% of the students answered and explained correctly. Few students who chose $v_2 > v_1$ and explained with free-fall reasoning.

In the pressure comparison task, some students cited greater area (point 2 has a greater cross-sectional area) or the formula $P = \rho gh$ to explain why $P_2 > P_1$. The greater area reasoning is the same as what we found in the HN2W task. The usage of the formula $P = \rho gh$ is consistent with what Brown *et al.* had reported, which is using the hydrostatic pressure to explain the changes in pressures in vertical flows.

Some students stated that pressure is inversely proportional to velocity. For example, one student wrote, "Pressure is inversely proportional to the speed of the fluid." These students were similar to those who cited greater area in their reasoning. One possible reason comes from the implication of Bernoulli's equation, which is "high-speed low-pressure." Students may get the wrong impression that the pressure and speed are inversely proportional to each other.

13.1.3 Student responses to the modified VUW2N question

We administered the modified VUW2N question to the PHYS 123 students in Au'17, Wi'18, Sp'18, and Su'18 quarters after they had relevant lecture instructions. The correct speed comparison is $v_2 > v_1$, and the correct pressure comparison is $P_2 < P_1$. The results of the speed and pressure comparison tasks are summarized in Table 13.85.

Out of the 515 students, 81% and 69% of them chose the correct answers on the speed and pressure comparison tasks, respectively. Around 60% of them answered both tasks correctly. The percentage correct in the original VUW2N question is 57% (see Table 12.74), which is 4% lower than the modified version. The modification in VUW2N has little effect on the performance.

Summing up the effect of the modification on the performance in HN2W, VDN2W, and VUW2N tasks, the effect was not significant except on the VDN2W task. A closer look at the results for the modified speed and pressure comparison tasks, we found that they were about the same. For example, the results for the speed comparison task in HN2W, VDN2W, and VUW2N contexts were 86%, 77%, and 81%, respectively. The results for the pressure comparison task in those three contexts were about 70%. The results for students who answered both comparison tasks correctly were about 60% for each context. So, there is no significant difference in student performance across these modified tasks. As explained previously, these results could be the effect of better lecture instruction. One needs to collect the student responses on these three original versions to examine the claim.

Table 13.85. Student responses to the modified VUW2N question. The percentage correct of the correct combination is in bold.

		Pressure comparison task (N = 515)			
		$P_2 > P_1$	$P_2 < P_1$	$P_2 = P_1$	Total
Speed comparison task	$v_2 > v_1$	14%	**61%**	6%	**81%**
	$v_2 < v_1$	5%	5%	3%	13%
	$v_2 = v_1$	1%	3%	2%	6%
	Total	20%	**69%**	11%	100%

Now we discuss the student explanations of the speed and pressure comparison tasks for the VUW2N context. They are categorized and summarized in Table 13.86 and Table 13.87, respectively. On the speed comparison task, 62% of the students gave both the correct answer and explanation, but only 23% did so on the pressure comparison task. Overall, 19% of the students answered and explained both tasks correctly, which is 5% higher than the original question (see Table 12.74).

Table 13.86. Student explanations for the modified VUW2N question (speed comparison). The percentage correct of the correct combination is in bold.

Categories	Speed comparison task (N = 515)		
	$v_2 > v_1$	Others	Total
Continuity	**62%**	2%	64%
Bernoulli	5%	1%	6%
Free-fall related	1%	4%	5%
No explanation & Others	13%	12%	25%
Total	81%	19%	100%

Table 13.87. Student explanations for the modified VUW2N question (pressure comparison). The percentage correct of the correct combination is in bold.

Categories	Pressure comparison task (N = 515)		
	$P_2 < P_1$	Others	Total
Bernoulli	**23%**	2%	25%
Pressure difference	10%	-	10%
Depth (ρgh)	9%	1%	10%
Force / Area	1%	8%	9%
Proportional to area	9%	-	9%
No explanation & Others	17%	19%	36%
Total	69%	31%	100%

In the speed comparison task, only a few students applying free-fall related reasoning to explain their answers. There were more categories for the pressure comparison task, and selected explanations are shown in Table 13.88. Some students explained their correct answers using pressure difference. Students who used the formula $P = \rho gh$ to explain their correct answers failed to include the difference in the speeds. Nine percent of the students said that $P_2 < P_1$ because $A_2 < A_1$, suggesting some proportionality between pressure and area.

Some students (6%) explained their correct answers by saying, “pressure and velocity are inversely proportional to each other.” These students might have taken the results of the Bernoulli’s equation for horizontal pipe flow with changing cross-sections and changed it into a

simplified inverse relationship. Students who said that $P_2 > P_1$, excluding those who did not explain their answers, tended to use force/area to explain. These students might be thinking that the force that was pushing the fluid to flow upward was constant. These types of explanations were also found in the original HN2W question (see subsection 12.1.2).

Table 13.88. Examples of student explanations for the modified VUW2N question (pressure comparison).

Category	Example of student explanations
Bernoulli equation	Correct ($P_2 < P_1$): • *As a fundamental law of fluid dynamics, faster flowing liquids in a pipe must have lower pressure to be forced to flow. additionally, v_2 is above v_1.*
Pressure difference	Correct ($P_2 < P_1$): • *$P_2 < P_1$ because that's the condition necessary for fluid to flow from the direction of P_1 to P_2. There has to be a pressure differential.* • *Fluids will always flow from high pressure to lower pressure. Since the flow is upward, we know that P_2 is low pressure.*
Depth (ρgh)	Correct ($P_2 < P_1$): • *There is less hydrostatic pressure at that location due to the rise in the z direction.* • *Point 1 is at a greater depth than point 2 so the hydrostatic pressure is greater for point 1.*
Force / Area	Others ($P_2 > P_1$): • *Smaller area in P_2.* • *The cross-sectional area decreases, the pressure increases.*
Proportional to area	Correct ($P_2 < P_1$): • *Smaller diameter smaller pressure.* • *Pressure is greater in the pipe where the radius is larger.*

13.1.4 Student responses to vertical pipe flow of uniform cross-section context

From the modified FMCI questions that involve vertical pipe flow, we did not see many students use gravity to explain the changes in the speed of fluid flow. Most of them used the equation of continuity to explain their answers. Also, we wanted to verify the difficulty reported by Suarez *et al.*, which was students use gravity to explain why the fluid speed increases as it flows vertically down along a pipe of uniform cross-section.

To find out if students would use gravity to account for the changes in speeds, we designed a new task that involves a vertical pipe with a uniform cross-section, as shown in Figure 13.2. The written description of the question was modeled from the FMCI. Changes were made in the context and diagram. This new task, or the VDU question, was administered to PHYS 123 students after relevant lecture instructions in Su'17 and Au'17 quarters.

Water flows through a pipe and enters a section where the cross-sectional area is larger. Viscosity and pipe friction effects are negligible but gravitational effects are not negligible.

Speed comparison task: Is the speed of the fluid flow at point 2, v_2, *greater than*, *less than*, or *equal to* the speed of the fluid flow at point 1, v_1?

Pressure comparison task: Is the fluid pressure at point 2, P_2, *greater than*, *less than*, or *equal to* the fluid pressure at point 1, P_1?

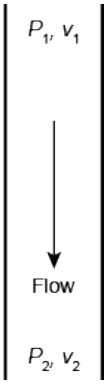


Figure 13.2. The VDU question. Fluid flows downward along a vertical pipe with uniform cross-section.

In the VDU question, the speed at point 2 is equal to that at point 1 because of the continuity equation. The pressure at point 2 is greater than that at point 1 because of Bernoulli's equation. Since the speeds of both points 1 and 2 are equal, Bernoulli's equation is reduced to the simple

formula $\Delta P = \rho g \Delta y$. So, explanations that quoted the previous formula or referenced the difference in height in the correct way are categorized as correct.

The results of the VDU question are summarized in Table 13.89. Out of the 126 students responded, 33% and 48% of them chose the correct speed and pressure comparisons, respectively. Seventeen percent of the students answered both comparison tasks correctly.

The results of this task were worse than the other FMCI questions (original and modified) we have administered. About 47% of the students chose $v_2 > v_1$ for the speed comparison task. Most of these students (37% of them) thought that the water would speed up due to gravity as it flowed down the pipe, as shown in Table 13.90. This error was reported by Suarez *et al.* [17].

Table 13.89. Student responses to the VDU question. The percentage correct of the correct combination is in bold.

		Pressure comparison task (*N* = 126)			
		$P_2 > P_1$	$P_2 < P_1$	$P_2 = P_1$	Total
Speed comparison task	$v_2 > v_1$	17%	20%	10%	47%
	$v_2 < v_1$	14%	4%	2%	20%
	$v_2 = v_1$	**17%**	3%	13%	**33%**
	Total	**48%**	27%	25%	100%

Table 13.90. Student explanations for the VDU question (speed comparison). The percentage correct of the correct combination is in bold.

Categories	Speed comparison task (*N* = 126)			
	$v_2 > v_1$	$v_2 < v_1$	$v_2 = v_1$	Total
Continuity	-	-	**17%**	17%
Bernoulli	2%	10%	2%	14%
Gravity related	37%	2%	-	39%
No explanation & Others	9%	8%	13%	30%
Total	47%	20%	33%	100%

Table 13.91. Student explanations for the VDU question (pressure comparison). The percentage correct of the correct combination is in bold.

Categories	Pressure comparison task (N = 126)			
	$P_2 > P_1$	$P_2 < P_1$	$P_2 = P_1$	Total
Depth (ρgh)	**37%**	3%	-	40%
Bernoulli	-	14%	1%	15%
No explanation & Others	10%	10%	25%	45%
Total	47%	27%	26%	100%

Now we discuss the student explanations. We found 17% of the students answered and explained the speed comparison task correctly, and 37% did so for the pressure comparison task. Around 10% of the students gave the correct answers and explanations for both tasks. Some students explained their correct answer by citing that gaps would appear in between the water column if the speed at the lower point is greater than the speed at the higher point.

From Table 13.90, some students used Bernoulli's equation to explain why $v_2 < v_1$ on the speed comparison task. Here is their approach we infer based on their explanations. For them to use Bernoulli's equation to compare speeds, they need to determine pressure first. They knew that $P_2 > P_1$ and then used "higher-pressure lower-speed" to arrive at $v_2 < v_1$. So, these students did not consider the continuity equation in comparing speeds. Since students were asked to compare speed first, the use of Bernoulli's equation on this task suggested that these students did not understand the proper order of continuity and Bernoulli's equation. Since Bernoulli's equation is derived from the continuity equation, one should use the latter before using the former.

From Table 13.91, 37% of the students gave the correct answer and explanation on the pressure comparison task. Fourteen percent of the students used Bernoulli's equation to explain why $P_2 < P_1$. They chose $v_2 > v_1$ on the speed comparison task, cited free fall as their reasoning. This is another example of the "higher-speed lower-pressure" idea from Bernoulli's equation.

Some students who answered $P_2 = P_1$ used "pressure is inversely proportional to velocity" to explain since the speeds were equal. There were students using pressure difference to explain why $P_2 < P_1$. These types of explanations are commonly found in the pressure comparison tasks for all contexts we described so far.

13.1.5 Student responses on modified fluid questions (mathematical)

During our analysis of student explanations, we saw a few students referencing atmospheric pressure to explain their answers in the pressure comparison task. This finding seemed to echo what Brown *et al.* suggested. The authors suggested that future researchers could give context to the given pipe flow situation so that students would not think that the pipe segment is open to the atmosphere.

Besides that, we also found there were multiple ways students could explain their correct answers to the pressure comparison task, namely the hydrostatic pressure formula and Bernoulli's equation. The given options did not prompt students to think about Bernoulli's equation. We interpreted this finding in two ways: (1) the students had some gap in their understanding of Bernoulli's equation, or (2) they just found the easiest way to explain the answer. One solution to this limitation is to change the options such that they can prompt students to apply Bernoulli's equation. Our approach is to change the existing three options into seven mathematical expressions, which we will describe later.

In this section, we incorporated these two modifications (context and mathematical expressions) in the VDU and VUW2N questions. There is no particular reason why we chose these two. The results are reported next.

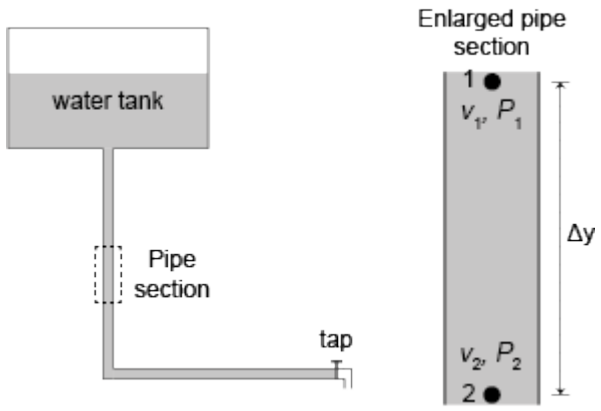
The VDU-with-context question

The VDU-with-context question is shown in Figure 13.3. Water flows from a tank into a pipe section. Students are asked to compare the speeds and pressures between points 1 and 2 when the tap is opened. We did not indicate the direction of the flow of water on this version, because we assumed that students were able to infer the direction of the flow when the tap is opened.

The diagram at right shows a tap that is connected to a water tank through a pipe system.

Consider the flow of water in the pipe section when the tap is open. An enlarged view of the pipe section is shown at far right.

Viscosity and pipe friction effects are negligible.



Speed comparison task: Is the speed of the fluid flow at point 2, v_2, *greater than, less than,* or *equal to* the speed of the fluid flow at point 1, v_1?

Pressure comparison task: Which of the following describes the relationship between the pressure in the fluid at point 2, P_2, and the pressure in the fluid at point 1, P_1? Note: Δy is a positive quantity and ρ_w is density of water.

$P_2 > P_1$ (Greater than)	$P_2 = P_1$ (Equal to)	$P_2 < P_1$ (Less than)
• $P_2 > P_1 + \rho_w g \Delta y$ • $P_2 = P_1 + \rho_w g \Delta y$ • $P_1 < P_2 < P_1 + \rho_w g \Delta y$	• $P_2 = P_1$	• $P_1 - \rho_w g \Delta y < P_2 < P_1$ • $P_2 = P_1 - \rho_w g \Delta y$ • $P_2 < P_1 - \rho_w g \Delta y$

Figure 13.3. The VDU-with-context question. Water flowing down from a tank along a vertical pipe with a uniform cross-section. The pressure comparison task is modified to contain options (7 bullet points) that are expressed mathematically.

The options for the pressure comparison task have changed from three options (greater than, less than, equal to) to 7 options that were expressed mathematically. The correct answer for the pressure comparison task is $P_2 = P_1 + \rho_w g \Delta y$. We administered this version to a total of 570

PHYS 123 students in Wi'18, Au'18, and Sp'19 quarters, after relevant lecture instruction. The data collected are summarized in Table 13.92.

Our results showed 49% of the students answered the speed comparison task correctly, and 61% did so for the pressure comparison task. Around 1/3 of the students surveyed answered both the speed and pressure comparison tasks correctly. Compared to the results of the VDU question presented previously, there was an increase of 15% who answered both tasks correctly.

Table 13.92. Student responses to the VDU-with-context question. The percentage correct of the correct combination is in bold.

			Speed comparison task (N = 570)			
			$v_2 > v_1$	$v_2 < v_1$	$v_2 = v_1$	Total
Pressure comparison task	$P_2 > P_1$	$P_2 = P_1 + \rho_w g \Delta y$	21%	8%	**32%**	76%
		Others	6%	4%	5%	
	$P_2 < P_1$	$P_2 = P_1 - \rho_w g \Delta y$	2%	2%	4%	13%
		Others	2%	2%	1%	
	$P_2 = P_1$		2%	1%	8%	11%
	Total		33%	17%	49%	100%

Now we discuss the student explanations for the speed comparison task presented in Table 13.93. About half the class answered the speed comparison task correctly, and many of them applied the continuity equation to explain their answer. The most common incorrect answer is still $v_2 > v_1$, with around half of these students explained using gravitational acceleration.

Some students said that $v_2 > v_1$ because of $P_2 > P_1$. They may be conflating force with velocity, which is a common difficulty found among students in mechanics [6, 7]. The pressure at point 2 is greater than at point 1. Since $F = PA$, the force at point 2 is greater than at point 1 as $A_2 = A_1$, then $v_2 > v_1$ under the force-velocity conflation.

Some student who chose $v_2 < v_1$ explained using Bernoulli's equation. The idea of "lower-speed higher-pressure" in a horizontal pipe situation is used by these students to justify their answers. This reasoning was also described in the VDU task previously.

Table 13.93. Student explanations for the VDU-with-context question (speed comparison). The percentage correct of the correct combination is in bold.

Categories	Speed comparison task (*N* = 570)			
	$v_2 > v_1$	$v_2 < v_1$	$v_2 = v_1$	Total
Continuity	-	-	**32%**	32%
Gravity related	15%	-	-	15%
Higher pressure	11%	-	-	11%
Bernoulli	2%	8%	-	10%
No explanation & Others	5%	9%	16%	31%
Total	33%	17%	49%	100%

Table 13.94. Student explanations for the VDU-with-context question (pressure comparison). The percentage correct of the correct combination is in bold.

Categories	Pressure comparison task (*N* = 570)			
	$P_2 > P_1$		$P_2 < P_1$ $P_2 = P_1$	Total
	$P_2 = P_1 + \rho_w g \Delta y$	Others		
Depth (ρgh)	**39%**	6%	4%	49%
Bernoulli	**8%**	1%	4%	13%
No explanation & Others	14%	8%	17%	38%
Total	61%	15%	24%	100%

Now we discuss the student explanations for the pressure comparison task, as presented in Table 13.94. Around 47% of the students surveyed gave the correct answer and explanation. Most of them said that the vertical position of point 2 is lower than point 1 and so the chosen expression correctly reflected that. Some of them stated "Bernoulli" in their explanation or described how

the Bernoulli's equation is reduced into the given correct expression after considering the correct speed comparison.

Some students chose incorrect expressions that are consistent with $P_2 > P_1$ and explained using the difference in height. They were not incorporating the difference in speeds properly, or they had the wrong answer for the speed comparison task.

Few students chose $P_2 = P_1 - \rho_w g \Delta y$ and justified using the change in height. They could be thinking that pressure increases with increasing height or misinterpreting the equation.

Another small group of students mentioned $v_1 = v_2$ to justify $P_2 = P_1$. There are a few interpretations. These students could be thinking, "pressure is inversely proportional to velocity," Bernoulli's equation (higher-speed, lower-pressure), or conflating force with velocity.

Among the 570 students surveyed, 19% of them answered and explained both tasks correctly. This result was significantly better ($\chi^2\ 1\ = 6.62$, *p*-value = 0.01) than the no-context VDU version, but with a small effect size. The options may have prompted students to use the proper equations for both the speed and pressure comparison tasks.

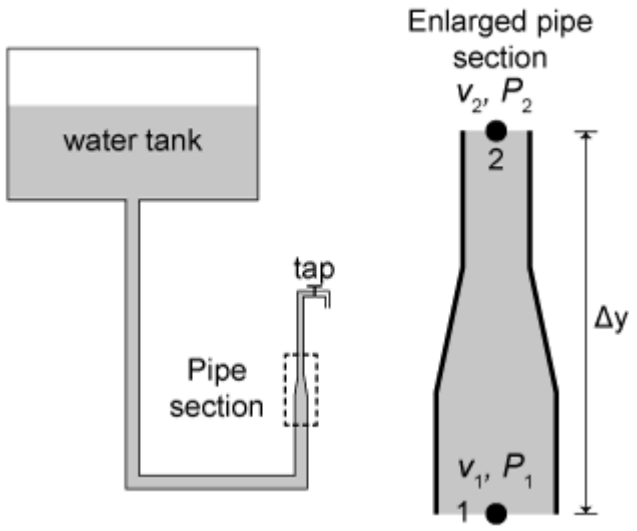
The VUW2N-with-context question

The VUW2N-with-context question is shown in Figure 13.4. The answer to the speed comparison task is $v_2 > v_1$ from the continuity equation. The correct answer to the pressure comparison task in this case is $P_2 < P_1 - \rho_w g \Delta y$. From subsection 12.1.4, we know that $P_2 < P_1$ from Bernoulli's equation. If the pipe were of uniform cross-section, then $P_2 < P_1 - \rho_w g \Delta y$ as the speed of the fluid flow is unchanged. When $v_2 > v_1$, then P_2 is smaller than $P_1 - \rho_w g \Delta y$. This question was administered to 188 students in Wi'18 and Au'18 quarters after relevant lecture instructions. The results are tabulated in Table 13.95.

The diagram at right shows a tap that is connected to a water tank through a pipe system.

Consider the flow of water in the pipe section when the tap is open. An enlarged view of the pipe section is shown at far right.

Viscosity and pipe friction effects are negligible.



Speed comparison task: Is the speed of the fluid flow at point 2, v_2, *greater than, less than*, or *equal to* the speed of the fluid flow at point 1, v_1?

Pressure comparison task: Which of the following describes the relationship between the pressure in the fluid at point 2, P_2, and the pressure in the fluid at point 1, P_1? Note: Δy is a positive quantity and ρ_w is density of water.

$P_2 > P_1$ (Greater than)	$P_2 = P_1$ (Equal to)	$P_2 < P_1$ (Less than)
• $P_2 > P_1 + \rho_w g \Delta y$ • $P_2 = P_1 + \rho_w g \Delta y$ • $P_1 < P_2 < P_1 + \rho_w g \Delta y$	• $P_2 = P_1$	• $P_1 - \rho_w g \Delta y < P_2 < P_1$ • $P_2 = P_1 - \rho_w g \Delta y$ • $P_2 < P_1 - \rho_w g \Delta y$

Figure 13.4. The VUW2N-with-context question. Water flowing upward along a vertical pipe from a wide channel to a narrow channel.

Table 13.95. Student responses to the VUW2N-with-context question. The percentage correct of the correct combination is in bold.

			Speed comparison task (N = 188)			
			$v_2 > v_1$	$v_2 < v_1$	$v_2 = v_1$	Total
Pressure comparison task	$P_2 > P_1$	$P_2 = P_1 + \rho_w g \Delta y$	16%	1%	3%	37%
		Others	14%	2%	2%	
	$P_2 < P_1$	$P_2 < P_1 - \rho_w g \Delta y$	**22%**	1%	-	56%
		Others	27%	3%	3%	
	$P_2 = P_1$		4%	2%	1%	7%
	Total		83%	11%	9%	100%

From Table 13.95, 83% of the students answered the speed comparison task correctly, and 56% did so in the pressure comparison task. Around 22% of the students answered both tasks correctly. Comparing these results with the VUW2N-no-context question in subsection 0, the results for the speed comparison task remained unchanged, but dropped from 69% to 56% for the pressure comparison task. The proportion of students who answered both tasks correctly also dropped from 61% to 49%. This 12% decrease is statistically significant ($\chi^2\ 1\ = 7.52$, *p*-value < 0.01). One possible reason for the decrease in performance is the increase in cognitive load to interpret the options for the pressure comparison task.

The student explanations for the speed comparison task are summarized in Table 13.96. Most students (61% of them) who chose the correct answer explained correctly, similar results in the VUW2N-no-context speed comparison task (see Table 13.86).

Table 13.96. Student explanations for the speed comparison task for the VUW2N-with-context question. The percentage correct of the correct combination is in bold.

Categories	Speed comparison task (*N* = 188)			
	$v_2 > v_1$	$v_2 < v_1$	$v_2 = v_1$	Total
Continuity	**61%**	1%	-	62%
Bernoulli	4%	1%	-	5%
Higher pressure	3%	1%	-	4%
No explanation & Others	15%	6%	8%	29%
Total	83%	9%	8%	100%

The types of student explanations for the pressure comparison task are tabulated in Table 13.97. Around 1/3 of the students (36%) restated their answers or did not explain. Only 15% gave the correct answer and explanation for the pressure comparison task, as compared to 23% on the no-context version (see Table 13.87). We found that 18% of the students mentioned height

difference when they selected $P_2 = P_1 + \rho_w g \Delta y$ or $P_2 = P_1 - \rho_w g \Delta y$ as their answers. Clearly, these students were treating the moving fluid as if the fluid is static. Lastly, we still found a few students mentioning atmospheric pressure in their explanations, which suggests that these students were focusing more on the enlarged pipe segment to determine their answers.

Table 13.97. Student explanations for the VUW2N-with-context. The percentage correct of the correct combination is in bold.

Categories	Pressure comparison task (*N* = 188)				
	$P_2 < P_1$		$P_2 > P_1$	$P_2 = P_1$	Total
	$P_2 < P_1 - \rho_w g \Delta y$	Others			
Depth ($\rho g h$)	3%	13%	11%	-	27%
Bernoulli	**15%**	6%	4%	-	25%
No explanation & Others	5%	14%	22%	7%	48%
Total	23%	33%	37%	7%	100%

13.1.6 Commentary

We summarize the findings of the four modifications we made to the FMCI questions.

The first modification is about changing the original versions of HN2W, VUN2W, and VDW2N tasks into two-part question, the speed, and pressure comparison tasks. Our results showed a few things. The students performed about the same in both the original and modified HN2W and VUW2N questions, but not the VDN2W question. Based on the percentage correct on both the speed and pressure comparison tasks, the students performed about the same on all three modified tasks. The modified FMCI questions allowed us to obtain the task-specific explanations, and these explanations we categorized were both found in the original and modified FMCI questions. So, in sum, for the first modification, we saw some changes in the student

performance, obtained task-specific explanations, but no major change in the student approaches to both tasks.

The second modification is about a question involving water flowing down a vertical pipe of uniform cross-section, which is called the VDU question. Students did not perform well in this context. Students who answered that the speed of the fluid increases as it flowed down a vertical uniform pipe tended to explain using gravity, which was a student error reported by Suarez *et al*.

The third modification is about providing a physical context to the original FMCI questions. The last modification is about expanding the options for the pressure comparison task to distinguish whether students are using hydrostatic pressure or Bernoulli's equation to determine the answers. We probed the extent to which these modifications affect student performance on the VDU-with-context and VUW2N-with-context tasks. There was a small increase or a significant decrease in the proportion of students who answered both comparison tasks correctly. The significant decrease in the performance could be due to the increase in the cognitive load to interpret the seven options in the pressure comparison task.

Two observations were made from these additional investigations using the modified FMCI task. The first observation was that the types of student explanations to the speed and pressure comparison tasks in the modified tasks were about the same as the original version. The explanations were task-specific and, in some cases, easier to categorize. The second observation was that students did poorly on the VDU task. These students tended to use gravitational acceleration to explain the changes in speed of water as it flowed down a vertical uniform pipe. This difficulty is the most prevalent in vertical pipe flows, and more prevalent in pipes with uniform cross-sections than non-uniform pipes.

13.2 STUDENT RESPONSES TO ADDITIONAL FLUID MECHANICS TASKS

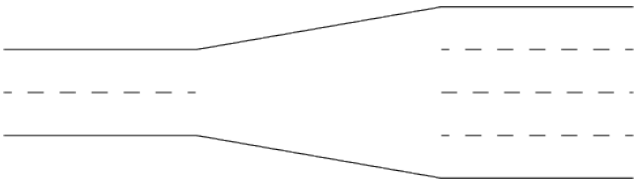
In the previous section, we have presented student responses from questions that were created by other researchers, as well as the results from modifications suggested by others and ourselves. In this section, we report on the data collected from additional fluid mechanics tasks we had administered. These tasks were meant to probe student ability in applying the continuity and Bernoulli's equations in different contexts. The first task is related to the continuity equation but in a traffic context. The second task is about water flowing from one channel to two channels, in which students need to apply the continuity and Bernoulli's equation to determine speed and pressure difference at a point along a channel.

13.2.1 Continuity equation in a traffic context

The flow of water in a channel can be described as water molecules flowing in from one end and going out at another end. Suppose we treat the channel as a road and the water molecules as identical cars, the continuity equation would mean that the number of cars coming in from one end must be equal to the number of cars leaving the other end. For the continuity equation to hold, all the traffic lanes must always be filled with cars.

The setup, as described above, is shown in Figure 13.5. We created two versions, A and B, to test the effect of the statement, "All the lanes are fully occupied with cars" on student performance. Version A did not include the statement, while Version B did. We administered Versions A and B to different lecture sections of the PHYS 121 students in the Au'17 quarter. The reason why we asked PHYS 121 students was to find out the students' initial ideas of the continuity equation in the mechanics context before learning fluid mechanics in PHYS 123. There were 281 students responded to Version A, 314 students responded to Version B.

Cars move from **left to right** on the section of road shown. The section of left has 2 lanes; the section of right has 4 lanes.



Assumptions:

- All the cars are identical (i.e., same length), and are equally spaced.
- All cars move with the same speed on the left section and on the right section, but the speeds on the two section may not be the same.
- The cars enter the left end of the section with speed v. In every time interval, the number of cars entering the left end of the section is ***equal to*** the number of cars exiting the right end of the section.
- All the lanes are fully occupied with cars. (Not in Version A, but in Version B)

What is the speed of the cars when they exit the right end? Explain.

A. v B. $v/2$ C. $v/4$ D. $v/8$

Figure 13.5. Continuity equation in a traffic context (Versions A and B).

To answer this task correctly, students need to understand the assumptions. Since the number of cars going in and out of the given road segment is equal and the lanes are fully occupied with vehicles, the continuity equation means that the cars must exit the right end at speeds lower than they enter. In this case, we have $v_{\text{right}} = v_{\text{left}}/2$, the correct answer is (B).

The results of this task are tabulated in Table 13.98. The proportions of students who answered Versions A and B correctly were 35% and 60%, respectively. The most common incorrect answer in both versions was that there is no change in the speed when the cars exit the right end of the segment. The test of association ($\chi^2\ 1\ = 37$, $p < 0.001$) shows that the difference associated with the inclusion of the statement "All the lanes are fully occupied with cars" is statistically significant with a medium effect.

Table 13.98. Student responses on the Continuity (Traffic) task versions A and B.

Answers	Version A (N = 281)	Version B (N = 314)
$v/2$ (Correct)	**35%**	**60%**
v	58%	32%
Others	7%	8%

Student explanations are tabulated in Table 13.99. Most students who answered correctly gave reasoning consistent with the continuity equation. Students who answered $v_{\text{right}} = v_{\text{left}}$ tended to cite the statement (assumption 3), "In every time interval, the number of cars entering the left end of the section is ***equal to*** the number of cars exiting the right end of the section." The percentage in version A was greater than version B. The reason was in the 4th assumption – it was not on version A, but version B. Without assumption 4 (All the lanes are fully occupied with cars), two cars could enter and exit the road segment with the same speed.

Table 13.99. Student explanations for the Continuity (Traffic) task versions A and B.

Approach	Version A (N = 281)			Version B (N = 314)		
	$v/2$	v	Others	$v/2$	v	Others
Continuity	**29%**	-	-	**52%**	-	-
Equal flow rate = equal speed	-	40%	-	-	18%	-
No explanations & Others	6%	18%	7%	8%	14%	8%

Besides asking the traffic context with the cars moving from left to right, we also asked students with the cars moving from right to left (version C). We want to find out whether there is a difference in student performance between going from fewer lanes to more lanes or vice versa. When the cars enter the four-lane section with speed v, to ensure the number of cars entering and

exiting the given road segment, the cars must exit the two-lane section with speed $2v$. We administered version C of this task to 423 PHYS 121 students in the Sp'18 quarter. The student responses and explanations are tabulated in Table 13.100.

Table 13.100. Student responses and explanations for the Continuity (Traffic) task version C.

Approach	Version C (N = 423)			
	$2v$ (Correct)	$v/2$	v	Others
Continuity	**42%**	-	-	-
Proportional to the number of lanes	1%	25%	-	-
Equal flow rate = equal speed	-	-	11%	-
No explanations & Others	2%	4%	11%	4%

Around 40% of the class answered and explained correctly. The most common error is $v/2$, and many students explained by saying that since the number of lanes decreases from four lanes to two lanes, the speed of the cars should be reduced by half. Some students relied on real-life experience since drivers tend to slow down when they travel from a road segment with more lanes to another segment with fewer lanes. This error was not found in the version B of the traffic task, because there was not an option that was proportional to the number of lanes. We continued our investigation by administering version D of the traffic task, which was the same as version B, but with an option that was proportional to the number of lanes, *i.e.,* $2v$.

Table 13.101 shows the results for version D, which was administered to PHYS 121 students in the Au'19 quarter. The proportion of students who explained correctly is about the same as that of version B. The proportion of students using "equal flow rate = equal speed" to explain their answers in both versions B and D were the same at 18%. 15% of the students

explained their answer $2v$ by saying the speed is proportional to the number of lanes. Some student explanations of these tasks are presented in Table 13.102.

Table 13.101. Student responses and explanations for the Continuity (Traffic) task version D.

Approach	Version D (N = 446)			
	$v/2$ (Correct)	$2v$	v	Others
Continuity	**50%**	-	-	2%
Proportional to the number of lanes	-	15%	-	-
Equal flow rate = equal speed	-	-	18%	-
No explanations & Others	2%	4%	8%	-

All these results and some student explanations suggest that students have some ideas about continuity at the start of the calculus-based physics course. If we compare these results with what we found on all the FMCI speed comparison tasks, there is a 30%-40% increase after relevant PHYS 123 lecture instruction.

Table 13.102. Selected student explanations for the Continuity (Traffic) task.

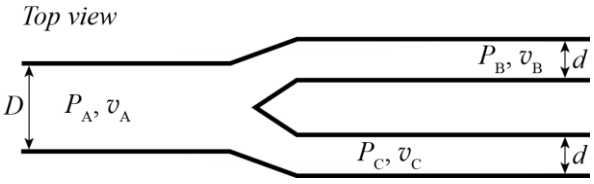
Category	Example of student explanations
Continuity	Wide to narrow (version C): • *If the lanes are always fully occupied with cars, then in a time interval two sets of two cars would need to get through the two left lanes as a new car enters each of the right lanes with speed v. If two sets need to get through the left side, then they need to go twice as fast, $2v$.* • *Cars in = cars out, so (2 lanes)($2v$) = (4 lanes)(v)* Narrow to wide (version D): • *Since all lanes must be fully occupied with cars (like gas in a tube), the cars must slow down to occupy all 4 lanes. Thus, since the number of cars entering is the same as cars exiting, the cars must slow down by half in order for there to be enough cars to occupy the lanes. This can be explained though the volume flow rate with the equation $A_1 v_1 = A_2 v_2$.* • *If cars are exiting and entering the road at the same rate while the right section has twice the amount of lanes means they are moving twice as slow.*
Proportional to number of lanes	Wide to narrow (version C): • *Because the number of lanes in the left section is half as many as the number of lanes in the right section, causing the cars to decrease their speed by 1/2 (v/2) when there are four lanes merging into two lanes.* • *Because the width of the lane reduced by 2* Narrow to wide (version D): • *The cars are initially at v, starting on the left. When they enter the right, the number of cars will be split from the two initial lanes to the four final lanes. Thus, the cars will travel at double the velocity because there will be the same number of cars but more lanes in which to travel.* • *Because 2 lanes become 4 lanes and the number of cars exiting and entering are the same, the exiting speed should be twice the entering speed*
Equal flow rate = equal speed	Wide to narrow (version C): • *Since the cars enter and leave at the same rate I am assuming they both have the same speed as they enter from the right and leave from the left.* Narrow to wide (version D): • *The number of cars entering the left end of the section is equal to the number of cars exiting the right end of the section. It can only be the case if the speed of cars entering these lanes are the same of those exiting it.*

13.2.2 One-channel to two-channel context

The second task (labeled as 1C2C Task) we designed is about water flowing from one channel into two channels, as shown in Figure 13.6. The diameter of the one-channel section is twice that of the diameter in the two-channel section. We asked students two questions: (1) rank the speeds of the fluid flow in A, B, and C and (2) find the magnitude of the pressure difference between points A and B. Students need to apply the continuity and Bernoulli's equations to answer both questions correctly. The two problems are labeled as 1C2C-Speed and 1C2C-Pressure tasks from now on.

The method to answer 1C2C-Speed correctly is to use the continuity equation. Students need to recognize that the mass flow rate remains constant, so the mass flow rate in the wider channel is equal to the sum of the mass flow rate in both narrow channels. The equation gives $A_A v_A = A_B v_B + A_C v_C$. Students also need to recognize that $v_B = v_C$ since the cross-sectional areas of channels B and C are the same. After substituting the correct cross-sectional areas, the correct ranking is $v_A < v_B = v_C$, and specifically, $v_B = \sqrt{2} v_A$.

A horizontal cylindrical tube has water running steadily through it. The water reaches a junction where the tube splits as shown. The diameter of the pipe at A is twice that of B, *i.e.*, $D = 2d$. Treat water as a non-viscous, incompressible fluid.



(1) Rank the speeds of the fluid flow, v_A, v_B, and v_C, from largest to smallest. If any of the speeds is zero, state so explicitly. Explain your reasoning.
(2) Find the magnitude of the pressure difference between A and B in terms of the density of water, ρ, and v_A.

Figure 13.6. The 1C2C task. Water flowing from one channel to two-channel context in midterm 2 of the Sp'17 quarter.

For the 1C2C-Pressure task, students need to use the relationship between v_A and v_B obtained in the 1C2C-Speed task and apply the Bernoulli's equation to find the pressure difference between A and B. Since the tube is horizontal, there is no need to consider height difference. Applying the Bernoulli's equation will arrive at the answer $\Delta P = \frac{3}{2}\rho v_A^2$.

We administered this question to PHYS 123 students on midterm 2 in the Sp'17 quarter. We analyzed 455 student responses, and the results for the 1C2C-Speed task are tabulated in Table 13.103. Most students (87%) gave the correct ranking, and 77% of the class also explained correctly. However, not all students who ranked the speeds correctly obtained the correct equation for v_A and v_B. For students who answered incorrectly, some thought that channels B and C were at different heights, and some used compensation reasoning (2 branches, half diameter) to say that the speeds were all equal.

Table 13.103. Student responses on the 1C2C-Speed task.

Approach	Answers (N = 455)	
	Correct	Incorrect
Continuity	77%	1%
Others	10%	12%

The 1C2C-Pressure task is more mathematical and not easy, so not many students can answer correctly. Some students may not be clear on all the quantities in the formula, such as confusing the height term in the Bernoulli's equation as diameter or confuse density with pressure. Some students may make algebraic errors or use the wrong formula as they solve the problem. The student responses for the 1C2C-Pressure task were analyzed and presented in Table 13.104.

Table 13.104. Student responses on the 1C2C-Pressure task.

Approach	Answers (N = 455)	
	Correct	Incorrect
Correct (Continuity & Bernoulli)	14%	-
Bernoulli, wrong continuity	-	37%
Continuity only	-	9%
Bernoulli only	-	8%
Others & No explanation	2%	30%
Total	16%	84%

16% of the class answered the 1C2C-Pressure task correctly, with most of them used the correct approach. A tiny group of students obtained the correct answer by having some math errors. Around 1/3 of the class (37%) obtained the incorrect answer because they did not apply the continuity equation to obtain the correct relationship between v_A and v_B. Some students applied the continuity equation or Bernoulli's equation, but not both to solve this problem. The remaining 30% of the class made some of the errors mentioned earlier. The most common error on this task is the incorrect application of the continuity equation to one branch of the split pipe, *i.e.*, $A_A v_A = A_B v_B$, instead of $A_A v_A = A_B v_B + A_C v_C$.

In the same quarter, we also administered another question on the one-channel to the two-channel context in the final exam, as shown in Figure 13.7. The diameters of the one-channel and two-channel pipes are d_1 and d_2, respectively. Instead of asking students to rank the speeds, students are asked to compare the diameters of the channels. In this case, the speed of the fluid flow remains constant. The approach to solving this question is the same as in the midterm 2, $A_1 v_1 = 2A_2 v_2$. The answer is option C, $d_2 = d_1/\sqrt{2}$.

We collected responses from 230 students in the final exam. Around 70% of the class answered correctly, and 20% chose option B, $d_2 = d_1/2$. Students who chose B thought that the

cross-sectional area is directly proportional to the diameter, not the square of the diameter. This error was reported in subsection 12.2.2.

Water is flowing through a pipe of diameter d_1 at a speed v. The pipe then branches off into 2 identical pipes of diameter d_2. The water continues to move at speed v through the two identical pipes. Which of the following correctly relates d_1 and d_2? Treat water as an ideal fluid.

A. $d_2 = d_1/4$ **B.** $d_2 = d_1/2$ **C.** $d_2 = d_1/\sqrt{2}$ **D.** $d_2 = 2d_1$ **E.** $d_2 = 4d_1$

Figure 13.7. Water flowing from one channel to two-channel context in the final exam of Sp'17.

In summary, the fluid flow in the one-channel to two-channel context appeared to be difficult for students if we asked students to answer questions that involve both the continuity and Bernoulli's equation. This context is also different from what is presented in traditional lecture instruction (single channel situation), although the approach to solving the problem is based on the conservation of mass. However, more information will be needed if the two-channel sections are not identical, so we think it is not necessary to investigate further.

The result of the 1C2C context in the final exam was better than midterm 2. Some students might have learned from their mistakes in the midterm. One student difficulty was prevalent in this case, which is using the incorrect proportionality between area and diameter.

13.3 SUMMARY OF STUDENT DIFFICULTIES IN FLUID DYNAMICS IN OUR INVESTIGATION

In this section, we summarize all the student difficulties we had found so far based on our investigations. This summary provides a setting for future work if one were to create a tutorial curriculum to address student difficulties in fluid dynamics. The results are relevant for either introductory physics level or in a course that solely focuses on fluid mechanics. We categorize the student difficulties into three types: (i) gravity-related reasoning, (ii) pressure-related reasoning, and (iii) proportional reasoning.

13.3.1 Gravity-related reasoning

Students tend to use gravitational acceleration to explain why the speed of the water increases as it flows down a vertical pipe of uniform cross-section on the VDU task. They were applying concepts they learned from kinematics instead of the continuity equation. Suarez *et al.* reported this error, and it was prevalent among our students. They did not consider that the flow of water is a continuous medium that cannot be broken into pieces. We found some students explained their correct answer by citing that gaps would appear in between the water column if the speed at the lower point is greater than the speed at the higher point.

13.3.2 Pressure-related reasoning

First, students do not distinguish between hydrostatic and dynamic pressure in fluids. Brown *et al.* reported that students were using hydrostatic pressure to reason for the changes in water pressure for flows in vertical pipelines. This difficulty had been consistently found in the

VDN2W and VUW2N questions, even after we implemented suggestions made by Brown *et al.* to administer two-part tasks (speed and pressure comparison tasks) instead of the original version and provide context to the problem. Again, we found the same difficulty reported by the previous study, and it was something persistent among students, even with additional modifications to the questions.

Second, students oversimplify the results of Bernoulli's equation. We found students mentioned that pressure is proportional to the area or inversely proportional to velocity in their explanations for all the tasks we took from the FMCI. Some students mentioned that the pressure of the fluid at the wide channel is greater than that at the narrow channel, since this is the outcome by applying Bernoulli's equation. Students just correlate the greater pressure in the channel to the greater cross-sectional area of the channel. For the latter, Suarez *et al.* also reported that their students tended to say, "higher velocity always means lower pressure."

Next, some students do not use Bernoulli's equation to explain the difference in pressures. These students brought in real-life experience to compare the pressures at two different points. They mentioned that the water is flowing from a high-pressure region to a low-pressure region. The pressure-difference induced fluid flow explanation is consistent with a viscous fluid such as blood, in which the beating heart causes it to circulate throughout the whole body. However, it does not work in ideal fluid flow in pipes all the time, especially in situations in which the fluid flows from narrow channel to wide channel.

The last type of error involves the use of the definition of pressure ($P = F/A$). These students thought that the force acting on the fluid is constant, and so the pressure depends on the cross-sectional area of the channel.

13.3.3 Proportional reasoning

The student difficulty related to proportional reasoning can be found in the calculational questions involving the continuity equation. Some students thought that the cross-sectional area is linearly proportional to the diameter/radius of the channel instead of its square.

In the traffic context question, some students misunderstood the meaning of an equal rate of entering or leaving the road segment under the assumption that the lanes are filled with cars. They thought that this equal rate implied equal speed for the cars entering and leaving the segment.

13.4 INSTRUCTIONAL IMPLICATIONS

With the student difficulties highlighted in the previous section, we make the following suggestions to instructions. We hope that they can help instructors address some of the difficulties.

For students who cited gravity to explain the difference in the fluid speeds in a vertical pipe or connected pipes at different heights, one may get them to consider dialogues from fictitious students involving two contrasting reasonings – one correct and the other incorrect. The correct reasoning may involve something along the lines of "gaps may appear if the speeds are not the same" (then the fluid would not be continuous) since we found some students used that in their pretest explanation. Instructors may consider emphasizing that the fluid speeds are the same in the tubes when they illustrate how a siphon works during lectures.

The student difficulties categorized under pressure-related reasoning are multi-faceted. We think that it may not be easy for students to distinguish hydrostatic and dynamic pressure in a short course. Most difficulties can be addressed by emphasizing the proper steps because most errors involve oversimplification or overgeneralization.

The last student difficulty in fluid mechanics involves proportional reasoning. We think that this difficulty may also be present in other contexts that involve length, area, and volume. For example, students may think that the volume of the ball is linearly proportional to its radius even though they know the formula. They may not be sensitive to the effect of the exponent that is present in the formula. Instructors who are interested in addressing this difficulty may need to create additional opportunities for students to pay attention to the effect of the exponent. It may be too late to address this difficulty when students learn fluid mechanics; it can be addressed earlier in the physics sequence.

CONCLUSION FOR PART III

In part III of this dissertation, we investigated student difficulties with fluid motion, in particular related to the ideas of continuity and Bernoulli's equations. We used three relevant questions from a test developed to probe student understanding of fluids more generally: the Fluid Mechanics Concept Inventory (FMCI). For the most part, this test had been used in upper-division courses in engineering.

We found that students have similar difficulties, as reported in the literature. They are: (1) students use hydrostatic pressure to explain for the changes in water pressure for flows in vertical pipelines, and (2) in vertical pipes of uniform cross-section, students assume that fluid velocity increases due to gravitational acceleration. However, we did not find students mention that water is compressible in their explanations on the selected FMCI questions. Some difficulties were not yet reported, such as the indiscriminate use of $P = F/A$ and overgeneralization or oversimplification of the results from Bernoulli's equation.

We also modified the three FMCI questions based on the suggestions by Brown *et al.* and the results from our preliminary investigation to better probe student reasoning. The modifications were (1) changing the original versions of those FMCI tasks into two-part question, the speed, and pressure comparison tasks, (2) administering a question involving water flowing down a vertical pipe of a uniform cross-section to investigate students' usage of gravitational acceleration to compare speeds, (3) providing a physical context to the original FMCI questions, and (4) rewriting the answer options into mathematical equations on the pressure comparison task to distinguish whether students are using hydrostatic pressure or Bernoulli's equation to compare pressures.

In the first modification, the two-part tasks (speed and pressure comparison tasks) allowed us to obtain task-specific explanations. From the second modification, the student difficulty, as

described by Suarez *et al.*, was prevalent, which was, "in vertical pipes of uniform cross-section, fluid velocity increases due to gravitational acceleration."

In the third and fourth modifications, the introduction of a context to the original FMCI questions did not seem to improve student performance, especially in the pressure comparison task. The use of more complicated mathematical equations in the options increased the cognitive load for online pretests, which could explain the results. One possible improvement is to administer the third and fourth modification individually, instead of combining them.

We also investigated the student difficulties by looking at student performance in past fluid mechanics questions in final exams and tasks that were designed by us. Additional difficulties were found, such as incorrect proportionality between area and diameter/radius and inability to apply the principle of mass conservation properly in one-channel to two-channel pipe flow situations.

Part of our motivation in this study was to determine the extent to which known difficulties were addressed by standard physics instruction. The pretest results at UW were comparable or sometimes better than those reported in the literature (e.g., 42%-67% vs. 30%-49%, respectively, see Table 12.76). Moreover, student performance on final exams was better than on pretests in general (from 42%-67% to 76%-92% correct in calculus-based physics courses). These results suggested that the students in our courses could perform reasonably well after they had prepared for the final exams. These results are puzzling, and there is a need for additional research. It may be a case in which more advanced knowledge or ideas that students have studied in a fluid mechanics course interfere with their ability to apply the core ideas of continuity and Bernoulli's equation – but determining the reasons requires further study.

After considering the student performance on final exams, we concluded that there is no urgency to develop tutorials as of now. However, for institutions or departments that require students to enroll in fluid mechanics courses for their degree requirements, our findings can serve as a basis for improving student understanding. We also made some suggestions to address specific student difficulties.

PART IV
OTHER INVESTIGATIONS

Part IV of this dissertation presents results from two different preliminary explorations into the use of technology to teach conceptual understanding and to conduct physics education research. Although online instruction is becoming more prevalent, there are still relatively few studies into the impact of online teaching and how it can be used effectively. There is even less research into how the use of technology can supplement, enhance, and promote physics education research.

Chapter 14 describes the design of online weekly teaching quizzes to help students learn vector superposition. The efficacies of these quizzes were assessed through the longitudinal tracking of student performance on this topic throughout several quarters of instruction in the algebra-based physics sequence. The results showed some improvement as the students progressed through the course. We believe the strategies used in this intervention can inspire future efforts to improve students' basic skills that can help them do well in physics courses.

In Chapter 15, we describe the use of natural language processing (NLP) techniques to compare student explanations on two specific physics tasks. There are three reasons to try this approach. (1) When physics education researchers examine student explanations of reasoning to specific tasks, a goal is often to identify different reasoning patterns. By examining many different explanations, researchers may identify several such patterns and use specific cases to illustrate their belief in the differences. We were interested in whether NLP could provide corroborating evidence of these patterns other than just presenting or summarizing students' quotes. (2) We were also curious about the extent to which an analysis of student explanations by human and

machine would match and whether such an analysis might be used to corroborate the findings from a single researcher. (3) We were also interested in exploring the possibility of using NLP in physics education research to identify differences in student reasoning patterns and, thus, if it could be used as a first step in identifying student difficulties. The findings from our study suggest that comparing student explanations using the NLP technique can be comparable to the qualitative analysis performed by humans and thus, methods in NLP might be used to generate corroborating evidence to support the claims made by researchers. As a result, we think that there is a huge potential in applying NLP in physics education research.

Chapter 14. IMPROVING STUDENT ABILITY TO APPLY VECTOR SUPERPOSITION IN ALGEBRA-BASED PHYSICS THROUGH ONLINE INSTRUCTION

Vectors are useful mathematical tools for describing important physical quantities in mechanics (*e.g.,* force, momentum, displacement) and electromagnetism (*e.g.,* electric force, magnetic force). Thus, knowing how to manipulate vectors becomes one of the essential skills students need to have so that they can understand physics better [77].

The rules for adding and subtracting vectors, however, are usually covered quickly in a standard introductory physics course. An implicit assumption is made that students will learn these techniques relatively quickly. However, there is extensive evidence that this is not the case. Student difficulties related to vector math and the extent to which these are not addressed by standard instruction have been documented by a number of instructors (See, for example, Knight [78], Kanim [79], Flores *et al.* [80], Nguyen and Meltzer [81], and Heckler and Scaife [82].) Previous graduate students from our group (Stephen Kanim [79] and Paul Emigh [83]) have also examined student facility with vector superposition in the calculus-based physics sequence (PHYS 12X).

Given the relatively small amount of time devoted to this topic in a typical course, we were interested in whether or not online tools could be used to promote student learning of vectors. It seemed like a good area to use since many of the relevant difficulties had been well-documented. Thus, in this investigation, we did not probe additional difficulties, although we did focus the investigation in the algebra-based sequence at the UW, which involves a population that prior members of our group had not examined on this topic.

We begin this part of the dissertation by first reviewing the previous research on student difficulties with vector superposition and prior attempts to improve student ability to manipulate vectors. This is followed by a discussion of the motivation and research questions used in this study. We then discuss prior results obtained from students in calculus-based physics courses as a background of our investigation. The results from algebra-based physics courses – before any interventions – is then presented. After that, we describe the online interventions we tried with the algebra-based students and analyze the effectiveness using longitudinal data analysis. The results and instructional implications are discussed.

14.1 LITERATURE REVIEW, MOTIVATION, AND RESEARCH QUESTIONS

This work focuses on improving student understanding of addition of vectors, or in other words, vector superposition, in algebra-based physics courses. In this section, we first describe the prior investigations reported by other researchers, such as Knight [78], Kanim [79], Flores *et al.* [80], Nguyen and Meltzer [81], and Heckler and Scaife [82]. In these studies, they mainly focused on the student difficulties in vector superposition using the arrow representation. We also highlighted the study by Mikula and Heckler [77], in which they reported a framework for improving students' vector math skills through computer-based instruction. Lastly, we describe the motivations and the research questions in our study.

14.1.1 Literature review of student difficulties in vector superposition

The first student difficulty in vector addition is student inability to add vectors in arrow (graphical) representation. It was reported by Knight [78], Barniol and Zavala [84], and Nguyen

and Meltzer [81]. Knight investigated the vector knowledge of beginning physics students by using the *Vector Knowledge Test*. The test probed student ability to recognize and use vector components, assess the magnitude and direction of a vector, add two vectors graphically, and add two vectors using components. He found that 43% of the students could add vectors graphically, and the most common error was to draw the resultant by connecting the tips of both vectors when the tails of those two vectors were placed at the same point. Barniol and Zavala also reported on this error. They found that student responses can be somewhat sensitive to the placement of the vectors represented as arrows on the page. In another study, Nguyen and Meltzer administered a seven-item quiz to around 2000 students in algebra-based and calculus-based introductory physics courses. They investigated student understanding of vector addition, magnitude, and directions for problems presented in graphical representation. They found that 30%-40% of calculus-based physics students and 60%-80% of algebra-based physics students were unable to add two vectors that were drawn on a two-dimensional grid after one semester of physics course. The authors found that the student difficulties are related to the incorrect algorithm to find the sum of two vectors.

A related study to the first difficulty was reported by Heckler and Scaife. They investigated student ability with adding and subtracting vectors in arrow and mathematical representations [82]. The authors found that students performed better in the mathematical representation than in the arrow representation. Their results suggested that most students struggled with simple vector problems given in arrow representations. The authors suggested that teaching both the arrow and mathematical representations of vectors concurrently may be helpful to improve student performance and help students to learn the relevant physics concepts.

The second student difficulty in vector superposition is that students tend to treat vectors as if they are scalars. Kanim, a previous graduate student of the UWPEG, found that students had difficulties in vector superposition in electrostatic problems [79]. The other student difficulties he documented include: (1) use vectors to reason about the direction of the net force but not the magnitude, (2) associate vector magnitudes with components in one direction only, (3) confuse components of force vectors with distances, and (4) believe that vectors completely cancel if some of the components canceled. Some of Kanim's findings were published in Flores *et al.* [80].

14.1.2 Literature review on instructions to improve vector superposition skill

Previously, Heckler and Scaife suggested that teaching both the arrow and mathematical representations of vectors concurrently may be helpful to improve student performance and help students to learn the relevant physics concepts.

Heckler and Mikula reported another approach. They tried to improve calculus-based physics students' vector math through computer-based practice [85]. They focused on improving calculus-based student ability to do dot and cross products. The factors that they had analyzed were training time, feedback complexity, prior knowledge, physics course grade, and student beliefs about both their proficiency in and the importance of vector math.

According to Roper, there are five types of computerized feedback [86]. They are (in increasing complexity): (1) no feedback; (2) knowledge of results (KR), in which user is informed of whether the answer is right or wrong; (3) knowledge of correct response (KCR), in which user is informed of the correct answer; (4) knowledge of correct response and elaborated feedback (EF), in which user is informed of the correct answer, and further information such as an explanation is given; and (5) knowledge of correct response and interactive teaching.

The general finding that Mikula and Heckler found was "computer-based training with static question sequences and immediate elaborated feedback in the form of simple and general explanations can be an effective way to improve student performance on a physics essential skill (vector math), especially for less prepared and low-performing students." Mikula and Heckler tried a combination of KR, KCR, and EF in their work. Two schemes have produced better outcomes, which are (1) the combination of KR and EF and (2) the combination of KR, KCR, and EF.

14.1.3 Motivations and research questions

We first describe the motivations of our research.

From the literature review, most research were conducted with calculus-based physics students. Emigh, another graduate student from the UWPEG, created more vector superposition tasks in arrow representation (with and without physics contexts) and administered them to different groups of calculus-based physics students. Since we have not yet collected any data from algebra-based physics students at the University of Washington, we extended the study of student ability with vector supreposition to this population.

The framework for computer-based instruction to improve students' vector math, as reported by Mikula and Heckler, piqued our interest. The weekly online pretests are administered to students in algebra-based physics courses through the *Catalyst* website. We were interested if one could design some interventions on online pretests to improve student ability with vector addition. Besides, since the students are doing weekly pretests throughout the quarter, we can track the performance longitudinally. Readers can refer to Chapter 2 for more details about longitudinal study.

Next, here are the research questions (RQs) in this investigation:

- *RQ1: How does the performance in vector superposition in the calculus-based students compare to that in the algebra-based students?*
- *RQ2: What is the progression of the student performance in vector superposition in algebra-based physics courses?*
- *RQ3: How does the prior knowledge in vectors affect the progression of the student performance in an algebra-based physics course?*
- *RQ4: Suppose we implement interventions through online pretests to improve students' vector superposition skills in algebra-based physics course. How can it be done? How effective is it?*

The purpose of RQ1 is to compare the data Emigh had collected with the data collected during this investigation. In RQ2, we present the results of the longitudinal study. For RQ3, around 30%-50% of the students self-reported that they had not learned vectors at the start of algebra-based physics sequence, so we are interested to find the effect of prior knowledge in vectors on the performance in vector superposition tasks in the first algebra-based physics course (PHYS 114). RQ4 is the main focus of this chapter, as it demonstrates how we incorporate results from prior research to improve current instruction.

14.1.4 Commentary and organization of the chapter

We reviewed past research on the student difficulties in vector superposition. Two main difficulties are highlighted. They are (1) students have difficulty adding vectors in the arrow/graphical representation and (2) students tend to add vectors as scalars. We also discussed the use of computer-based training to improve student ability in vector superposition. The

motivations and research questions in this study are outlined. Our study involves collecting data from students in algebra-based physics courses longitudinally. Such data allow us to track student progress in vector superposition to find out the effects of usual lecture instruction and interventions through online pretests.

The chapter is organized as follows. We first provide a description of the vector superposition tasks used in the study, a description of the student participants, and an overview of the methodology to answer the research questions. Next, we present and discuss the results. The results of additional investigation are also presented. Lastly, we summarize the entire study and propose some future work.

14.2 OVERVIEW OF RESEARCH METHODOLOGY

This section describes the vector superposition tasks used in the investigation, the participants from which the data were collected, and an overview of the research methods we used to answer the research questions. The statistical method to analyze our longitudinal data is also described.

14.2.1 The vector superposition tasks used in the investigation

The vector superposition tasks that were used in our investigation are shown in Figure 14.1. They were mostly designed by Emigh (except the 2v4v-NC task), and modeled based on Kanim's work [79, 80]. In most tasks, students need to compare the magnitudes of the resultant vectors in cases/objects A and B.

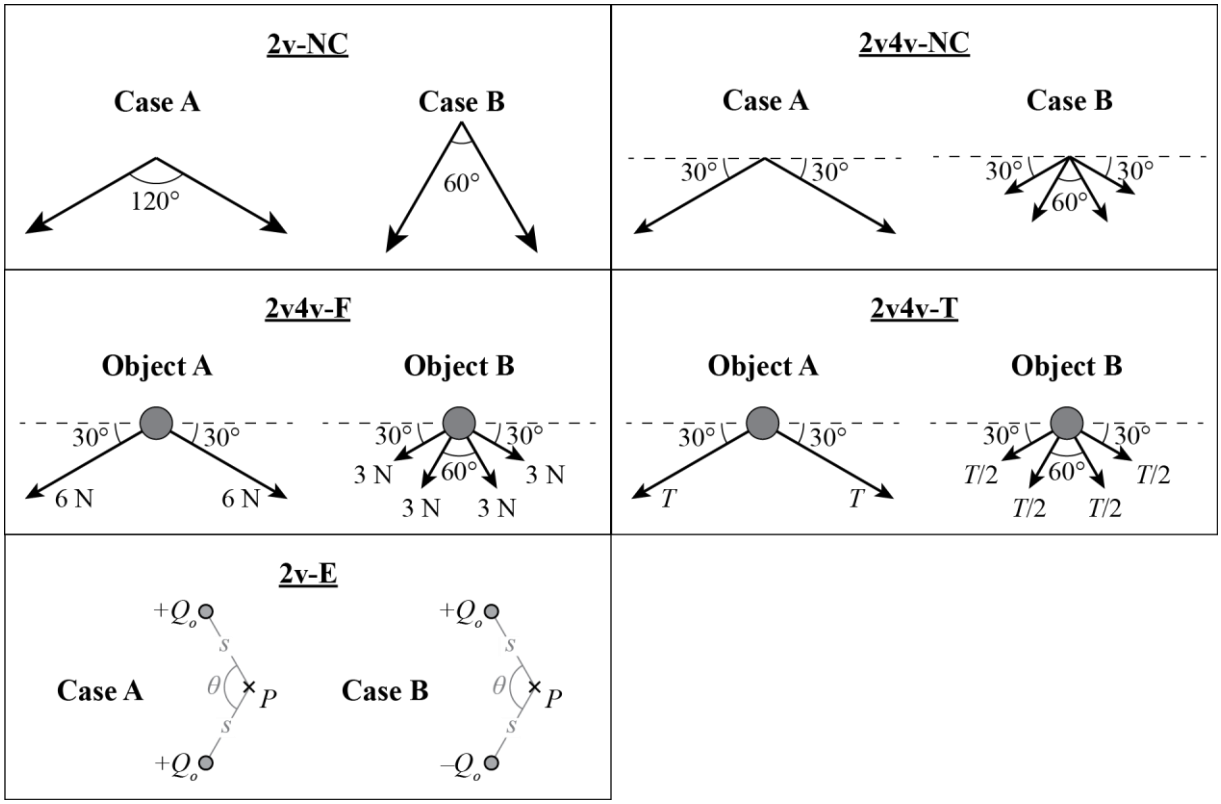


Figure 14.1. The vector superposition tasks used in our investigation. 2v-NC stands for two-vectors-no-context. 2v4v-NC stands for two-vectors-four-vectors-no-context. The F, T, and E in the next three refer to force, tension, and electric field contexts, respectively.

Here is the detailed description of the five vector superposition tasks. These tasks are labeled based on their surface features and physical contexts. For example, two of the tasks are comparing the magnitudes of the resultant of the sum of two vectors. They are labeled with "2v" to represent two-vectors. In these tasks, the two vectors have the same magnitudes but have different angles between them.

In the other three tasks, there are two vectors in case A, and four vectors in case B. The magnitudes of the vectors in case B are half of that in case A. The arrows are arranged such that case B has a greater resultant than case A. We call these three tasks "2v4v" to represent two-vectors-versus-four-vectors.

The physical contexts of the tasks are represented by the capitalized letters. For example, "NC" stands for no-context, "F," "T," and "E" refer to force, tension, and electric field contexts, respectively.

The angles in the vector superposition tasks are arbitrary. We used symbolic angles when we first collected the data from the calculus-based and algebra-based students. Starting from the Sp'19 quarter, we switched to numerical angles for the algebra-based students, so the convenient angles like 30º, 60º, and 120º were used. We did not investigate whether the symbolic or numerical angles affect students' performance in the algebra-based physics course. We assume they are comparable.

In each task, the magnitude of the resultant vector in case B is greater than that in case A. The correct answer can be obtained multiple ways, including the tip-to-tail method (see Figure 14.2) and adding the vector components. In each case, the horizontal components of the vectors cancel out, and the sum of the vertical components of case B is greater than that of case A. The 2v-E task is different from the rest. Students need to draw the individual electric field vectors due to each charge at point *P* in each case before comparing the magnitudes of the net electric fields at that point.

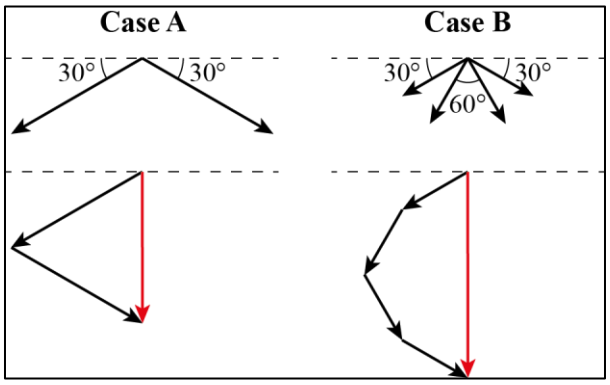


Figure 14.2. The tip-to-tail method to compare the magnitude of the resultant (in red) of the vectors in the 2v4v-NC task.

14.2.2 Student populations and data collection

The data presented in our investigation were from the calculus-based physics courses (PHYS 121 and PHYS 122) and algebra-based physics courses (PHYS 114 and PHYS 115.) The content in PHYS 114 and PHYS 121 is mechanics. In PHYS 122, it is mainly electromagnetism, and PHYS 115 covers thermodynamics, fluid dynamics, and electromagnetism.

The data collected in the calculus-based physics courses were from the year 2013 to the year 2016; while the data collected in the algebra-based physics courses were from the Au'18 quarter to the Wi'20 quarter.

As described in Chapter 2, the student responses are collected from online pretests, midterms, and final exams. Questions asked on online pretests and final exams are usually in multiple-choice format, while those on midterms are in free-response format. In this work, most data were collected from online pretests. However, the data for the 2v-E task were collected from online pretests, midterms, and final exams.

In the calculus-based courses, most vector superpositions tasks were asked in PHYS 121, except the electric field context, which was asked in PHYS 122. In the algebra-based courses, the four tasks (2v-NC, 2v4v-NC, 2v4v-F, and 2v4v-T) were asked in both PHYS 114 and PHYS 115, while the 2v-E task was asked in PHYS 115 only. We have data for all seven vector superposition tasks from students in the calculus-based physics courses, but we only collected responses for five vector superposition tasks in the algebra-based physics courses.

The reason for the discrepancy is that we conduct longitudinal studies in the algebra-based physics courses, and students are doing seven pretests in a quarter. As the vector superposition tasks are only one of the questions on the pretests, we do not want to overload students with questions of similar nature, so we only select five of them to be used in the study.

14.2.3 Procedures for research questions (RQs) 1, 2, and 3

The procedures we used to answer research questions 1, 2, and 3 are described here. The procedure to answer research question 4 is long, so we describe it separately.

RQ1: How does the performance in vector superposition in the calculus-based students compare to that in the algebra-based students?

As mentioned previously, the data for the calculus-based physics courses (PHYS 121 and PHYS 122) were collected from different groups of students in different quarters from the year 2013 to the year 2016, except for the 2v-E task. The 2v-E task was administered as a free-response question in a PHYS 122 midterm. Each question was administered after students had relevant lecture instruction.

The data from PHYS 114 were collected from the Au'18 quarter. We assigned these vector superposition tasks, 2v4v-NC, 2v-NC, 2v4v-F, and 2v4v-T, to students on the first four pretests. We also asked a question similar to the 2v4v-F context on the final exam of PHYS 114.

The data from PHYS 115 were collected from the Wi'19 quarter. Students did the tasks in the same order on the first four pretests. The 2v-E task was asked on the final exam of PHYS 115 in the Wi'19 quarter. The relevant results from the algebra-based physics students were then compared to that from the calculus-based physics students.

RQ2: What is the progression of the student performance in vector superposition in an algebra-based physics course?

The answer to this research question can be obtained from the results from PHYS 114 in the Au'18 quarter and PHYS 115 in the Wi'19 quarter. These are the no-intervention group results.

RQ3: How does the prior knowledge in vectors affect the progression of the student performance in an algebra-based physics course?

As mentioned in the previous section, this question is only relevant to the PHYS 114 students. Students who register for PHYS 115 must pass PHYS 114, so we treat the PHYS 115 students to have equipped with some basic ideas on vector addition.

The information about whether students had prior knowledge of vectors was obtained on the first pretest. The progression of student performance in vector superposition is compared between those who had prior knowledge and those who had not.

14.2.4 Procedure for the research question (RQ) 4

RQ4 is about how we implement the interventions through online pretests to improve student ability with vector superposition and their efficacies. As more questions are introduced in interventions, we call the questions in Figure 14.1, the actual vector superposition tasks.

Two intervention schemes, A and B, were tried in the PHYS 114 course. Intervention A was tried in the Wi'19 quarter, while intervention B was tried in the Au'19 quarter. The implementations of the interventions in PHYS 114 are summarized in Table 14.105.

The success of the intervention is determined in two ways. A successful intervention should lead to better performance on the final exam, as well as a better trendline of progression compared to the no-intervention group.

In PHYS 115, only intervention B was implemented in the Au'19 quarter. Intervention A was not implemented in PHYS 115 because it did not meet the criterion for a successful intervention.

Table 14.105. Number of sections receiving intervention and the weeks that students were presented the vector superposition tasks in PHYS 114 in each quarter.

Quarter	# of sections receiving no intervention	# of sections receiving intervention	Weeks that students see the pretests
Au'18	3	0	2, 3, 5, 6
Wi'19	1	1 (Intervention A)	2, 3, 4, 6
Au'19	0	3 (Intervention B)	

One difference between the no-intervention group and the intervention group is in the order of questions. In the no-intervention and intervention-A group, students did these four tasks in this order, 2v4v-NC, 2v-NC, 2v4v-F, and 2v4v-T, in the first four pretests. However, in the intervention-B group, the first two tasks were swapped, *i.e.*, the order became: 2v-NC, 2v4v-NC, 2v4v-F, and 2v4v-T.

Next, we describe each intervention scheme in detail.

Intervention A (Limiting case approach)

Intervention A is called the limiting case approach, which is one of the problem-solving methods physicists used to check their answers. In some instances, it can be a useful method to check the validity of the solution. In the case of adding two vectors of equal magnitude, the limiting case would be when they are pointing in the same direction (maximum magnitude) or when they are pointing in the opposite direction (zero magnitudes). There is no previous research that we could find, so this approach is a trial-and-error attempt in our case.

In this intervention, we designed helper questions that included a case that gave zero resultant or maximum resultant. Students did the helper question before they attempt the actual task on a pretest. An example of such implementation is shown in Figure 14.3. We administered

the intervention in 2v4v-NC, 2v4v-F, and 2v4v-T tasks. The 2v-NC task was modified into a helper task.

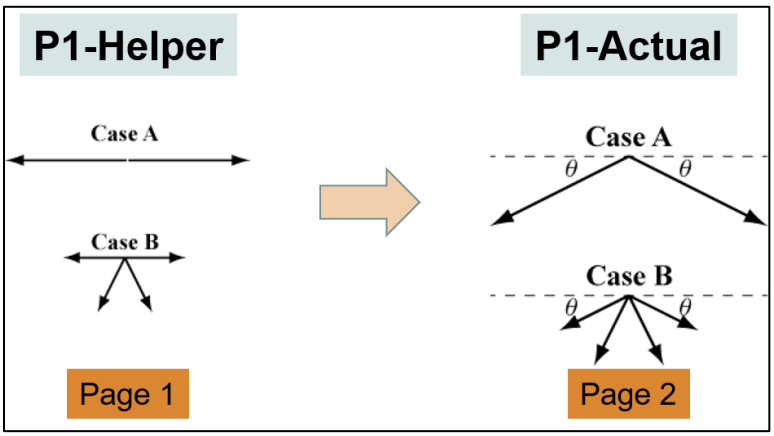


Figure 14.3. An example of the implementation of limiting case intervention in improving student vector superposition ability on pretests. Students see the helper question on page 1 and the actual question on page 2 of the pretest.

Intervention B (Targeted intervention with elaborated feedback)

Intervention B is called the targeted intervention with elaborated feedback. It is targeted because only students who answered the actual vector superposition tasks incorrectly received the intervention. These students would receive some form of elaborated feedback on pretests so that they learn how to add vectors using the tip-to-tail method. In the elaborated feedback, we were encouraged by the good results reported by Mikula and Heckler. So, we also used a combination of computerized feedbacks, such as knowledge of result (KR), knowledge of correct response (KCR), and elaborated feedback (EF), in intervention B.

Figure 14.4 shows the outline of the sequence of questions in intervention B. We illustrate the sequence of questions with actual examples of what students see on pretests. Figure 14.5 shows the page students will see if students answered the actual task incorrectly. They are informed that

their answer is wrong. Then we present them the tip-to-tail vector addition method pictorially, ask them to comment on the relationship between the magnitude of the resultant of the two vectors and the angle between the two vectors.

The students redo the actual task again, as shown in Figure 14.6. We inform students whether their retest answer is correct or incorrect on a separate webpage. If students choose the incorrect answer the second time, they will be brought to the page, as shown in Figure 14.7. Students are shown how the vectors in the actual task are added using the tip-to-tail approach. After they are given the correct answer, they redo the actual task the third time.

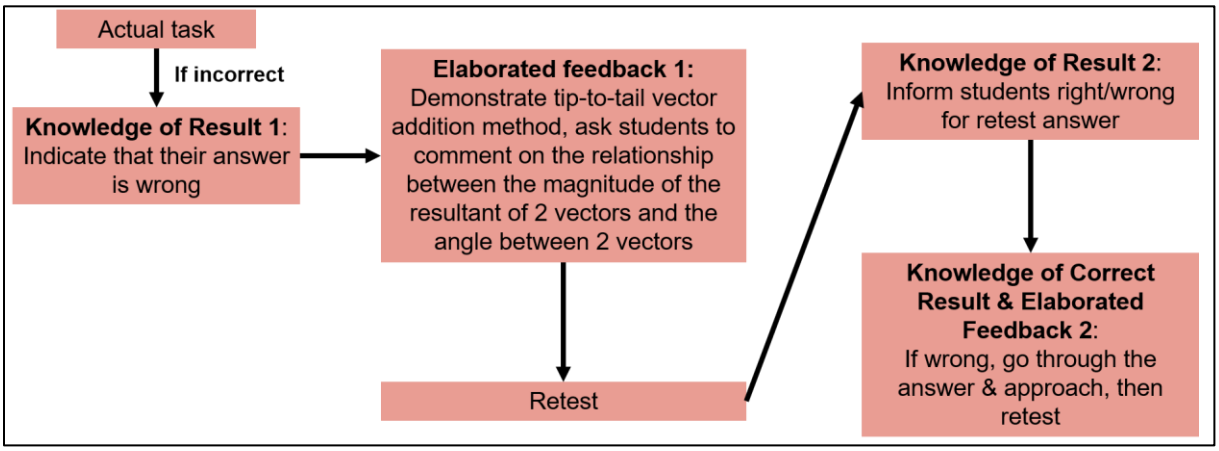


Figure 14.4. The outline of the sequence of questions in intervention B.

One thing to note is that the sequence of questions in intervention B was the product of some experimentation with KR, KCR, and EF in the Sp'19 quarter. The results of the intermediate stages can be found in Appendix E.

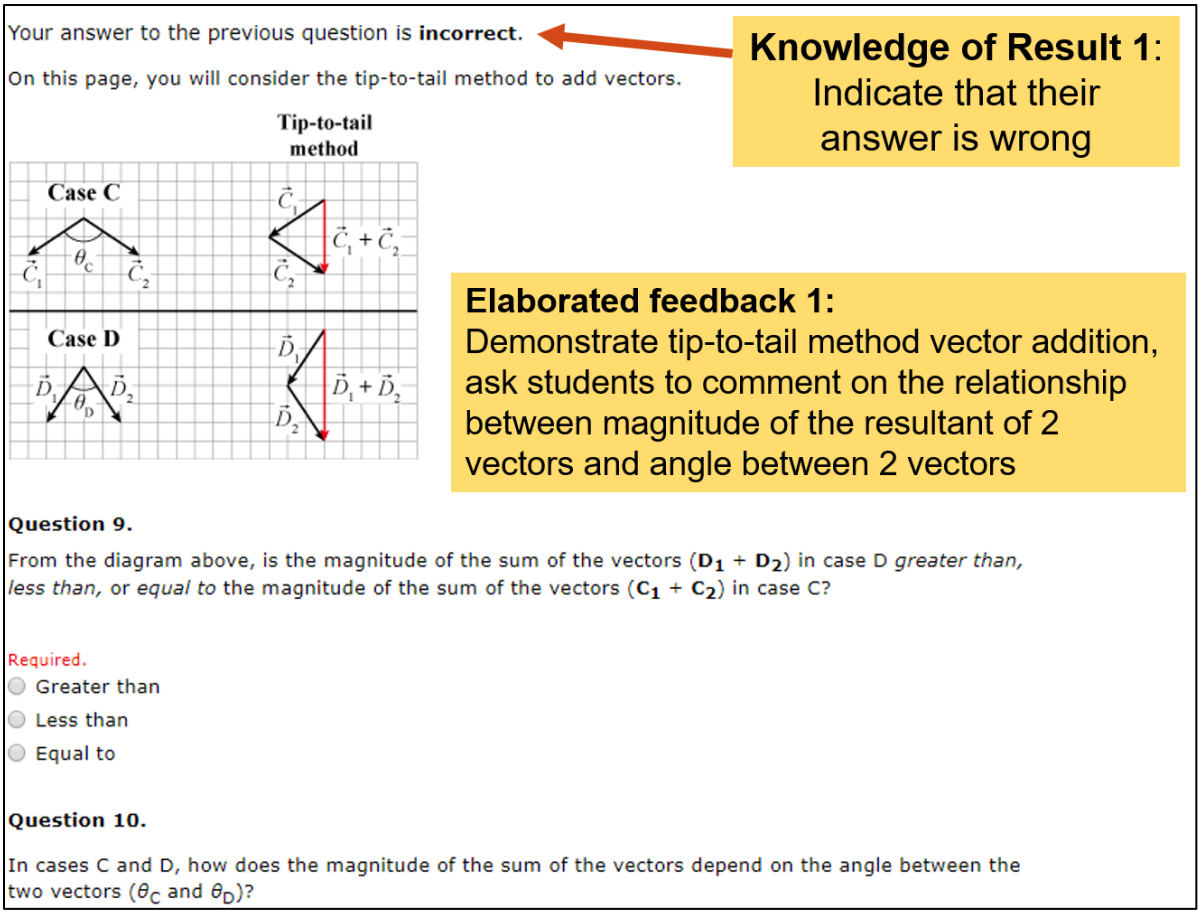


Figure 14.5. The sequence of questions when students answered the actual task incorrectly. Students are informed that their answer is incorrect, then they are demonstrated the tip-to-tail vector addition method. Students are also asked to comment on the relationship between the magnitude of the resultant of two vectors and the angle between the two vectors.

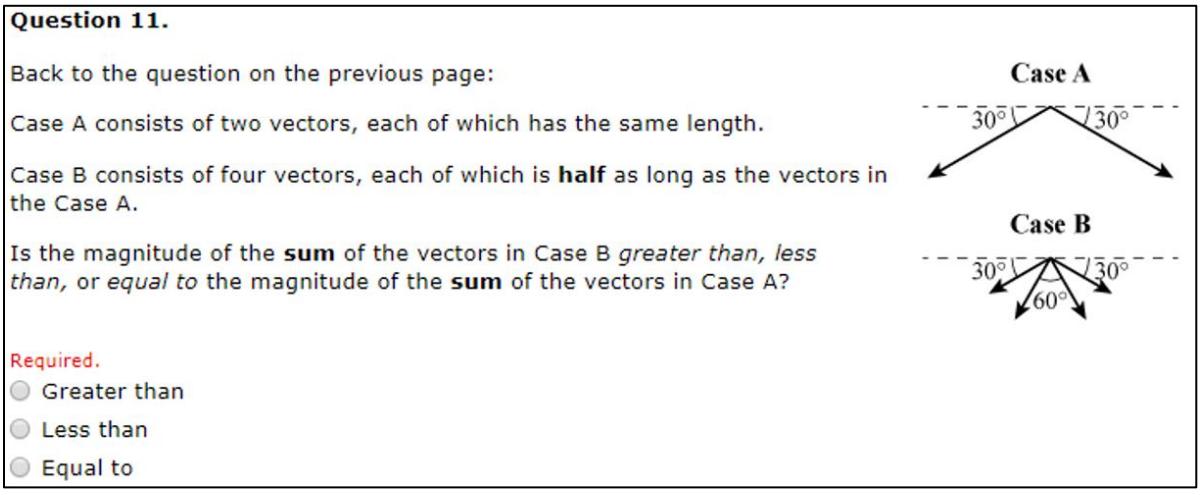


Figure 14.6. The retest in which students redo the actual task the second time.

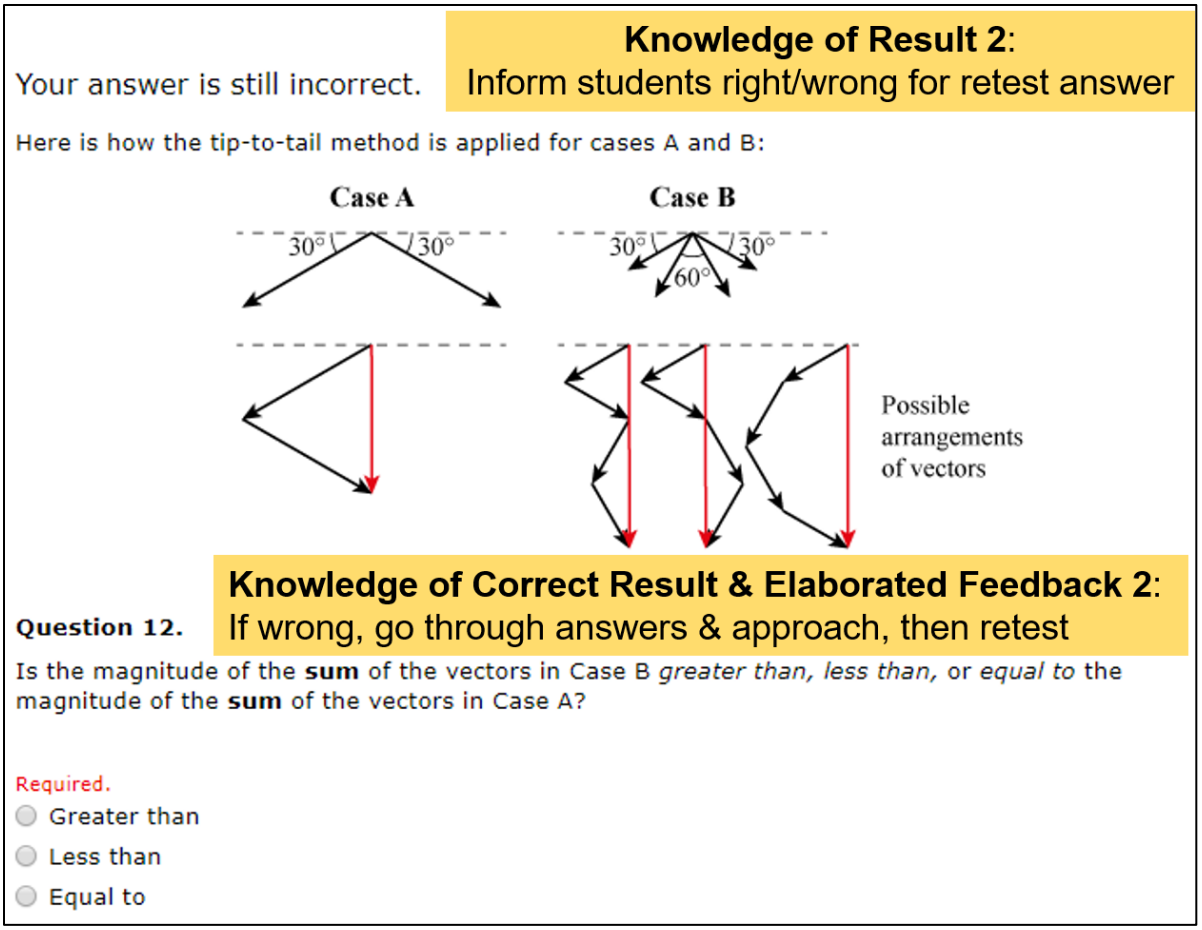


Figure 14.7. The page in which students see if they did the retest incorrectly. Students are informed that their answer is incorrect, then they are given the correct answer and retest for the third time.

14.2.5 Statistical method for analyzing longitudinal data

In Chapter 2, we briefly described the statistical method for analyzing longitudinal data, which is the generalized linear mixed model (GLMM). We also explained the rationale of using GLMM. In this section, we describe how we model our data in greater detail.

The GLMM is only used to analyze the data we collected from PHYS 114, based on what was summarized in Table 14.105.

We have described several minor differences during the implementation of the tasks between quarters, which means our study is not well-controlled. We may not be able to draw good conclusions from the results of statistical analysis. To simplify our data analysis, we make two assumptions.

The first assumption is related to the order of the first two tasks. In Au'18, students did 2v4v-NC task before the 2v-NC task, while the students in Au'19 did the 2v-NC task before the 2v4v-NC task. We found that there was no significant difference in the results whether students did the 2v-NC task or 2v4v-NC task first when we compared the results in Au'18, Sp'19, and Au'19 quarters. So, to make the results from Au'18 comparable to the results in Au'19, we switch the order of the first two pretests for the data taken in the Au'18 quarter. That is, in the Au'18 quarter, the result for the 2v-NC task is under pretest 1 (originally pretest 2), and the result for the 2v4v-NC task is under pretest 2 (originally pretest 1) for the statistical analysis.

The second assumption is related to the weeks the students did the third pretest. The students did pretest 3 in week 5 of the Au'18 quarter, while the students in the Wi'19 quarter did so in week 4. We did not see a significant difference in the results between those two quarters. So, the results of pretest 3 in the Au'18 quarter were treated as if it had been collected in week 4.

We followed Theobald's flowchart to determine the fixed and random effects in our study [43]. In our analysis, the fixed effects are correctness of pretest 1 (PT1), interventions, pretest number (a proxy for time), and students' background with vectors. The random effects are sections and students. The outcome is whether the student is correct or wrong in pretests 2, 3, and 4.

As mentioned, we used GLMM to assess the students' correct rate in pretests 2, 3, and 4 with a logit link function. We considered five models:

- Model 1: Regressing the outcome on correctness of PT1;

- Model 2: Regressing the outcome on students' background and correctness of PT1;
- Model 3: Regressing the outcome on students' background, correctness of PT1, and pretests (2, 3, and 4, coded as a categorical variable);
- Model 4: Regressing the outcome on intervention (coded as a categorical variable, 1 as intervention A, 2 as intervention B), students' background, and pretests, and the interaction of previous factors, and correctness of PT1;
- Model 5: Regressing the outcome on intervention, pretests and the interaction of the previous two factors, students' background, and correctness of PT1

We explain the rationale behind some fixed effects. The pretest 1 factor acts as the "control" for comparing students from different quarters. While in previous chapters, we have demonstrated that the student performance is largely consistent across quarters under traditional lecture instructions, it may not be generalizable in all situations. We want to find out how student performance depends on students' background knowledge on vectors, the types of intervention, and time. The factor pretest number serves as a proxy for time in these models. In some cases, there may be interactions between the types of intervention and time.

In each model, we include a student-specific Gaussian random intercept to account for the within-subject correlation of survey results. Models were compared using likelihood ratio tests. For each model, we report point estimates (log odds ratio), standard errors, 95% Wald-type confidence intervals, and *p*-values for testing against no effects. Then, based on the log-likelihood and Akaike information criterion (AIC), we select the best model. The AIC provides a means of model selection. Given that we have five models to work with, one should select the model with the lowest AIC [49].

Next, some students missed some pretests, so there was missing data. To evaluate the impact of missing data on the results of the analysis, we performed a sensitivity analysis by repeating the analysis with two different imputation strategies under "worst-case scenarios" using the best model from the model selection:

- Strategy 1: Impute "correct" for the missing results in pretests 1 and 2 and "incorrect" for the missing results in pretests 3 and 4;
- Strategy 2: Impute "correct" for the missing results of students who did not receive the intervention and "incorrect" for those who did.

14.3 RESULTS AND DISCUSSIONS

In this section, we summarize and discuss the results according to the research questions. Some results lead to additional investigations which we describe in the next section.

14.3.1 Research question 1 – How does the performance in vector superposition in the calculus-based students compare to that in the algebra-based students?

We first describe the results obtained from students in the calculus-based physics and algebra-based physics courses, then we compare them.

Results from the calculus-based physics courses. The results (percentage correct) from the students in the calculus-based physics courses from each task are shown in Figure 14.8. The results of the two-vectors comparison problems (2v-NC and 2v-E) are 56%-64%, while that of the two-vector-four-vector comparison problems (2v4v-NC, 2v4v-F, and 2v4v-T) are 21%-44%.

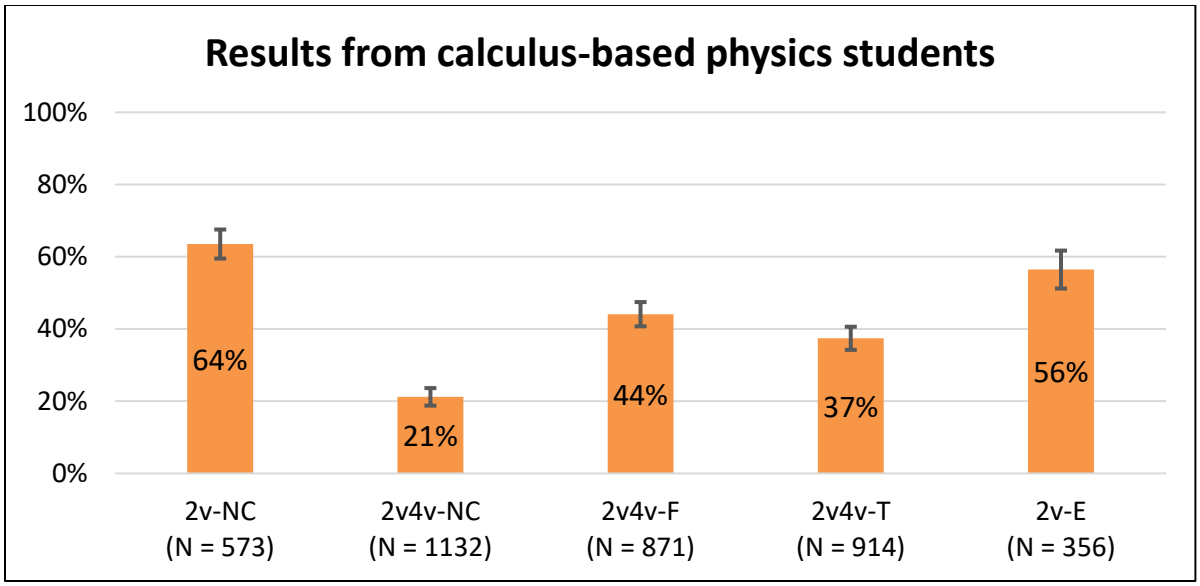


Figure 14.8. The results of each vector superposition task from the calculus-based physics students. Error bars indicate ± 2 SE.

Students did better in the two-vectors comparison problems than the two-vectors-four-vectors comparison problems when the no-context (2v-NC and 2v4v-NC) tasks were administered at the same time during the quarter. The results suggest that the surface features of the problems have a large effect.

The difference in the results from both tasks could be explained by cognitive load theory [87]. The cognitive load in adding four vectors is greater than that in adding two vectors, which may lower the performance in the 2v4v-NC task.

If we compare the results of all the two-vectors-four-vectors problems, the no-context (2v4v-NC) is around 20% lower than problems with physics contexts (2v4v-F and 2v4v-T). These questions had the same cognitive load as they had the same surface features. Students did the force/tension-context tasks one week after the no-context task in the two-vectors-four-vectors problems.

The difference in the performance could be explained by retesting effect or context-dependence. The retesting effect is that students did better with more practice or exposure. Some researchers found that the student performance on vector subtraction depended on the math or physics contexts [2, 88], which suggested that similar dependence may be present in vector addition. However, we could not tell how these factors influenced the performance from our results. It may be the interaction between the retesting effect or context-dependence and the required cognitive load of the question that gives rise to the difference in the results. More investigations are needed to test this hypothesis. While we are unable to confirm the reasons, the effect sizes in the two-vectors comparison problems are smaller than that in the two-vectors-four-vectors comparison problems.

One thing to note in these data were aggregated over several quarters. Some students did some of them in the same quarter, so the results may not truly represent the student progression in vector superposition at the start of the quarter.

Results from the algebra-based physics courses. Figure 14.9 shows the results of each vector superposition task from around 550-600 PHYS 114 students in the Au'18 and Wi'19 quarters. The results in both quarters do not differ significantly, so we present the aggregate. We present the results in two ways: one is the actual percentage correct of those who did the task (represented by the black line), and the other is according to students' prior knowledge on vectors (represented by the bar chart).

The percentage correct started from 13% in the 2v4v-NC task, increased to and plateaued at around 35% in the 2v-NC, 2v4v-F, and 2v4v-T tasks. Students who learned vectors before registering for PHYS 114 did better than those who had not generally.

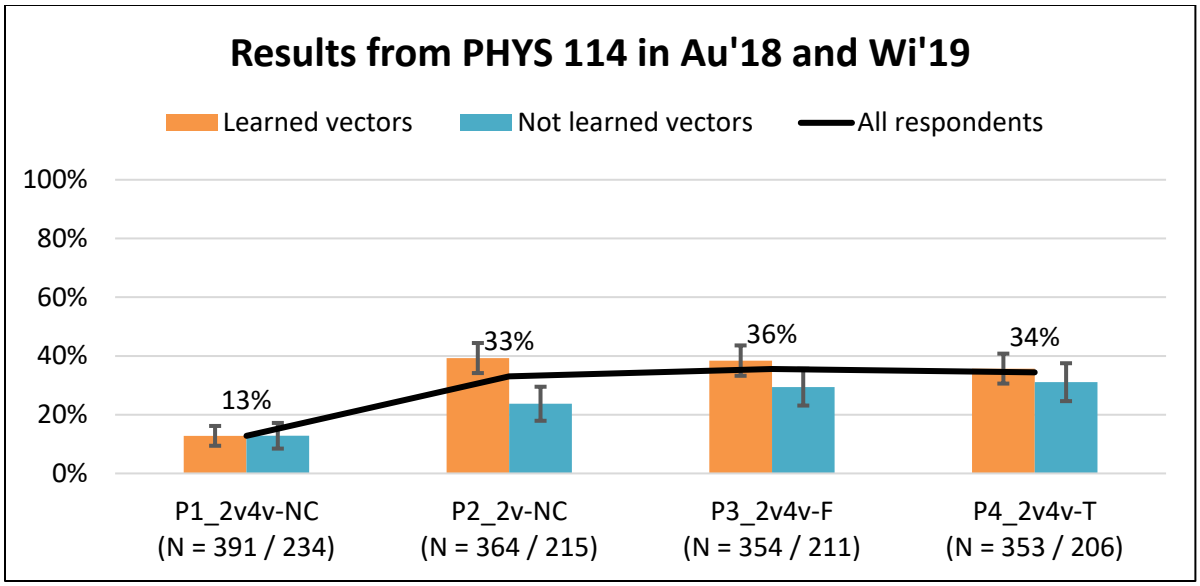


Figure 14.9. Results of each vector superposition task from PHYS 114 in the Au'18 and Wi'19 quarters. N = 391 / 234 refers to there are 391 students and 234 students who learned/did not learned vectors before registering for PHYS 114. 'All respondents' refers to the results of all students who responded to that task. Error bars indicate ± 2 SE.

Next, we discuss the results from the PHYS 115 students in the Wi'19 quarter (shown in Figure 14.10). Some of these students also took the PHYS 114 course in the Au'18 quarter. The 2v-E task was asked in their final exams so that the results can be compared with those in the calculus-based students.

The trend of the results of the PHYS 115 students is similar to that of the PHYS 114 students. It starts from 26% in the 2v4v-NC task, increases to around 40%-45% in 2v-NC, 2v4v-F, and 2v4v-T tasks, and reaches 49% in the 2v-E task. So, the PHYS 115 students performed better than the PHYS 114 students in the first four vector superposition tasks.

The results of PHYS 115 students in the second, third, and fourth tasks were higher than the first task. This trend was also found in PHYS 114, which could be attributed to the lecture instruction on vectors early in the quarter. However, there was no instruction on vectors in the first four weeks of PHYS 115 lectures. So, the trend in PHYS 115 is more likely due to retesting.

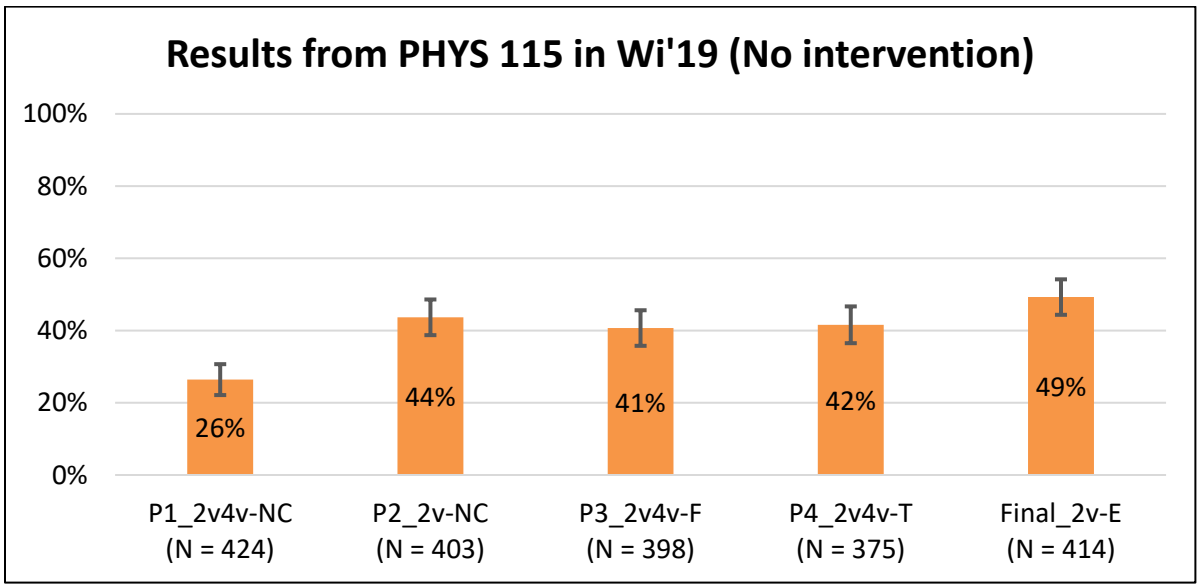


Figure 14.10. Results of each vector superposition task from the PHYS 115 students in the Wi'19 quarter. Error bars indicate ± 2 SE.

Now we compare the results of the algebra-based students with that of the calculus-based students. First, the algebra-based students were not as good as the calculus-based students in the 2v4v-NC and 2v-NC tasks, but within 10% of the calculus-based students in the 2v4v-F and 2v4v-T tasks. Although the algebra-based students were less prepared as the calculus-based students in vector superposition, they were able to perform almost as well as the calculus-based students after three to four weeks of lecture instruction.

Second, the gain in percentage correct from 2v4v-NC task to 2v4v-F/2v4v-T tasks in the algebra-based students was around 20%, which is also seen in the calculus-based students. The PHYS 114 students learned about vector additions in week 2, which was between the 2v4v-NC and 2v4v-F tasks, which might explain part of the gain. Third, the results of the 2v-NC, 2v4v-F, and 2v4v-T tasks were the same in the algebra-based students, but the result of 2v-NC was 20% better than 2v4v-F and 2v4v-T tasks in the calculus-based students. This finding suggests that the

effect of the factors that affect student performance (surface features, lecture instruction, context-dependence, and retesting) and their interactions in those two populations are different. To examine how these factors and their interactions affect student performance in both groups, one may need to plan a proper longitudinal study with these two student populations in the future.

Fourth, if we compare the results from all the five tasks for both the PHYS 115 and calculus-based students, we see that all the results are comparable except the 2v-NC task. The algebra-based students can perform as well as the calculus-based students in most vector superposition problems after one quarter of physics instruction.

Lastly, in all tasks, the most common error students made is that the magnitudes of the resultant vector are equal. Their explanations suggested that students were adding the vectors as if they were scalars. They did not take the directions of the vectors into account. This difficulty was documented by Kanim in his dissertation [79] and described by Flores *et al.* [80].

14.3.2 Research question 2 – What is the progression of the student performance in vector superposition in an algebra-based physics course?

In PHYS 114 (see Figure 14.9), the percentage correct started from 13% in the 2v4v-NC task, increased to and plateaued at around 35% in the 2v-NC, 2v4v-F, and 2v4v-T tasks. Students had vector instruction before they attempted pretest 3. This result suggested that additional lecture instruction did not lead to an improvement in the performance, which was possible for the following reasons. Not every lecture involves the discussion of vector addition, and students did not retain the knowledge of vector addition from the lectures for answering pretest questions.

The trend of the results of the PHYS 115 students is similar to that of the PHYS 114 students. It starts from 26% in the 2v4v-NC task, increases to around 40%-45% in 2v-NC, 2v4v-

F, and 2v4v-T tasks, and reaches 49% in the 2v-E task. So, the PHYS 115 students performed better than the PHYS 114 students in the first four vector superposition tasks. We also analyzed the results from 228 students who took PHYS 114 in Au'18 and PHYS 115 in Wi'19. Their results for all tasks had improved in the second course.

14.3.3 Research question 3 – How does the prior knowledge in vectors affect the progression of the student performance in an algebra-based physics course?

From Figure 14.9, students who learned vectors before registering for PHYS 114 generally did better than those who had not. However, as everyone progressed through the course, the gap between the two groups became smaller, as seen from the overlapping error bars in the 2v4v-F and 2v4v-T tasks. The percentage correct in all four tasks did not exceed 40%, which suggested that the algebra-based students needed help in doing vector addition.

14.3.4 Research question 4 – The effectiveness of interventions through online pretests

In this section, we first present the results from interventions A and B, and then the results from the generalized linear mixed model.

Results from Intervention A (limiting case approach)

The results of intervention A are shown in Figure 14.11. Students generally did better in the helper question than in the actual question. The results in the helper question appeared to increase linearly, but the results in the actual question appeared to reach a plateau at around 50%. The results in the actual question after the students did the helper question were also better than

that from PHYS 114 students who did not do the helper questions (Figure 14.9). The results of 2v4v-F and 2v4v-T tasks after students did the helper questions were also better than that from the calculus-based students (Figure 14.8). In summary, our results showed that students could add vectors slightly better after intervention A in pretests.

We asked a vector superposition task in the PHYS 114 final exam to find out how student perform after one quarter of physics instruction. Forty-five percent of the students who did not see the helper question and 50% of the students who saw the helper question answered the vector superposition task correctly. The difference in the results with or without the intervention was not significant, which suggested that the limiting case approach did not help students to add vectors.

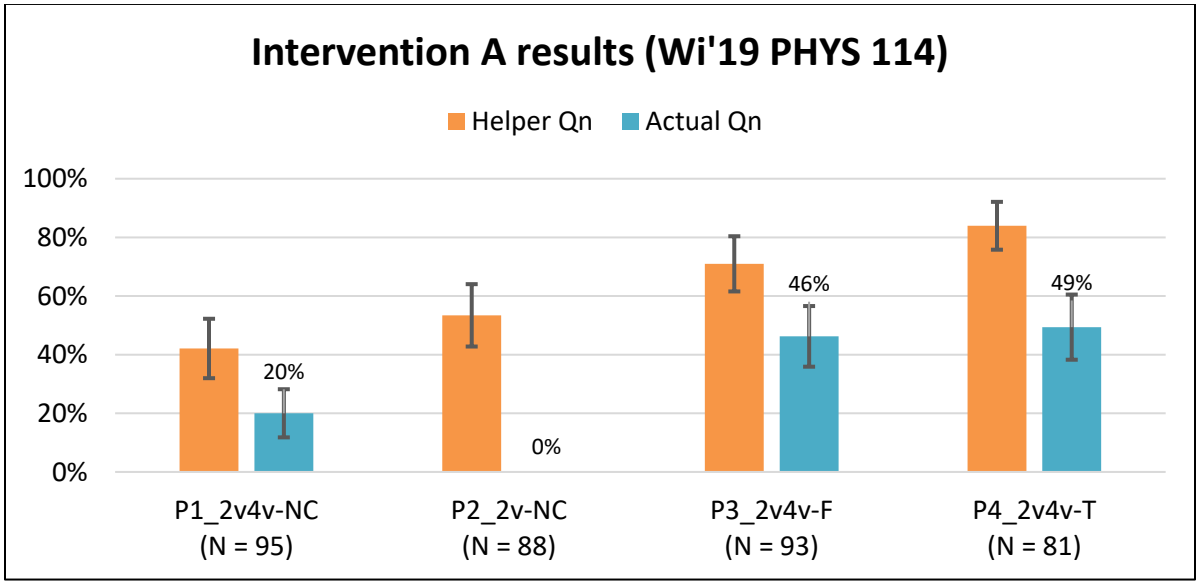


Figure 14.11. Results of intervention A from PHYS 114 class in the Wi'19 quarter.

Results from Intervention B (targeted intervention with elaborated feedback)

Figure 14.12 shows the results of intervention B in the PHYS 114 course. The percentage correct starts around 30% in the first two tasks, increases steadily as students attempted more tasks in the quarter, and reaches around 85% in the fifth pretest. We also gave students a similar vector superposition task in the final exam, and around 75% of them answered correctly. The students who did not learn vectors before taking PHYS 114 also improved from the intervention. The performance gap between the two groups became less significant as they progressed through the quarter. The progression and retention of performance in PHYS 114 show that intervention B is a viable approach to improve students' ability to add vectors qualitatively. In some tasks, the algebra-based students perform as well as or better than the calculus-based students.

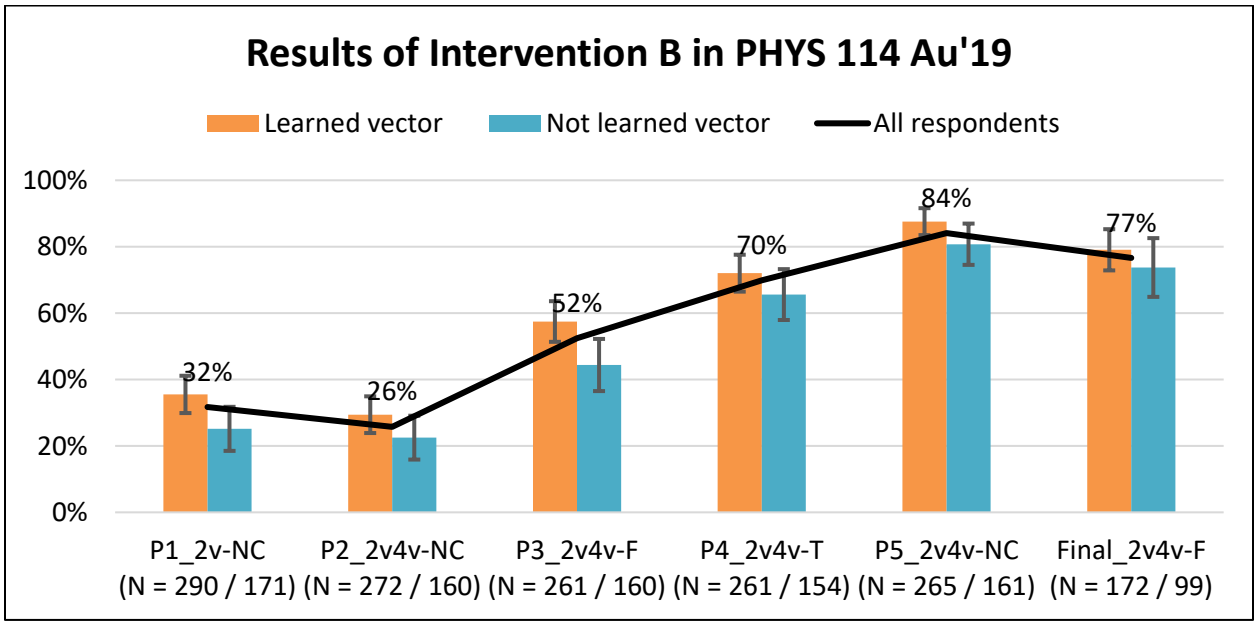


Figure 14.12. The results of intervention B for PHYS 114 in the Au'19 quarter. Vector Y/N refers to students who have/have not learned about vectors before attending PHYS 114. N = 290 / 171 refers to there are 290 students and 171 students who learned/did not learned vectors before registering for PHYS 114. 'All respondents' refers to the results of the students who responded to that task. Error bars indicate ± 2 SE.

Figure 14.13 shows the results of intervention B from PHYS 115 in the Au'19 quarter. The percentage correct starts at 48% in the first task and increases to 80% in the third and fourth tasks. In the fifth pretest (2v-E task), the percentage correct decreases to 30%. It increases to 50% for the same 2v-E task in the final exam.

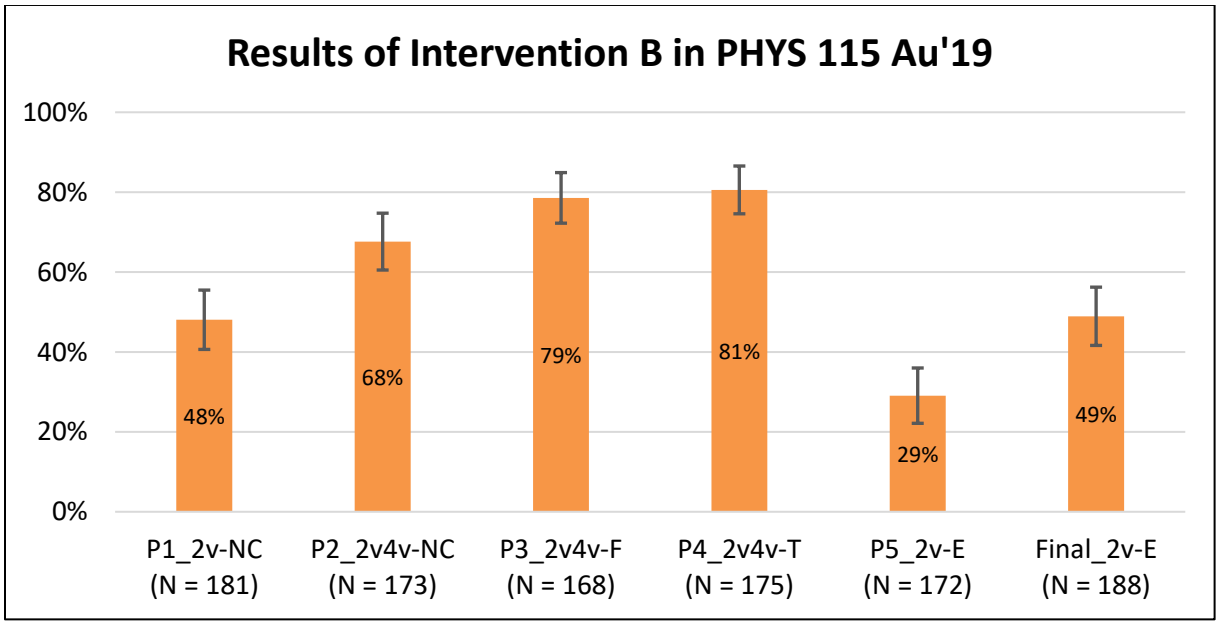


Figure 14.13. The results of intervention B for PHYS 115 in the Au'19 quarter. Error bars indicate ± 2 SE.

The results in the first four tasks in PHYS 115 are better than the students in the same course who did not receive any intervention (see Figure 14.10). The PHYS 115 students in Au'19 quarter took PHYS 114 in the Sp'19 and Su'19 quarters. They participated in previous intervention schemes, which could explain the good performance in the 2v-NC task. We also did not see a decrease in percentage correct from the 2v-NC task to the 2v4v-NC task in PHYS 115, as compared to PHYS 114. Overall, the PHYS 115 students had more exposures than the PHYS 114 students, so their results were generally better.

The 2v-E task in the fifth pretest of PHYS 115 is not a posttest for any previous tasks. The students did significantly worse in the 2v-E than the previous task on pretest. There are two possible reasons that led to this low performance. First, the electric field vectors were not provided on the diagram, unlike the previous four vector superposition tasks. Second, students just learned about electric fields from lectures, so not everyone was fluent in imagining the electric field vectors at point *P* on a web-based pretest. Students performed worse when they had to generate and add those vectors themselves.

Comparing the PHYS 115 results with the results from the calculus-based students (see Figure 14.8), the former did better than the latter in 2v4v-NC, 2v4v-F, and 2v4v-T tasks. The algebra-based students performed as well in the 2v-E task in the final exam, but not as well in the 2v-NC task. The better results in the PHYS 115 students were attributed to the additional practices on pretests.

Results from GLMM analysis

The results from pretests 1 to 4 are shown in Figure 14.14. We have described the trend of the results in previous sections. The percentages correct were higher later in the quarter and higher among the students who received either intervention A or B. However, the results for intervention B were better.

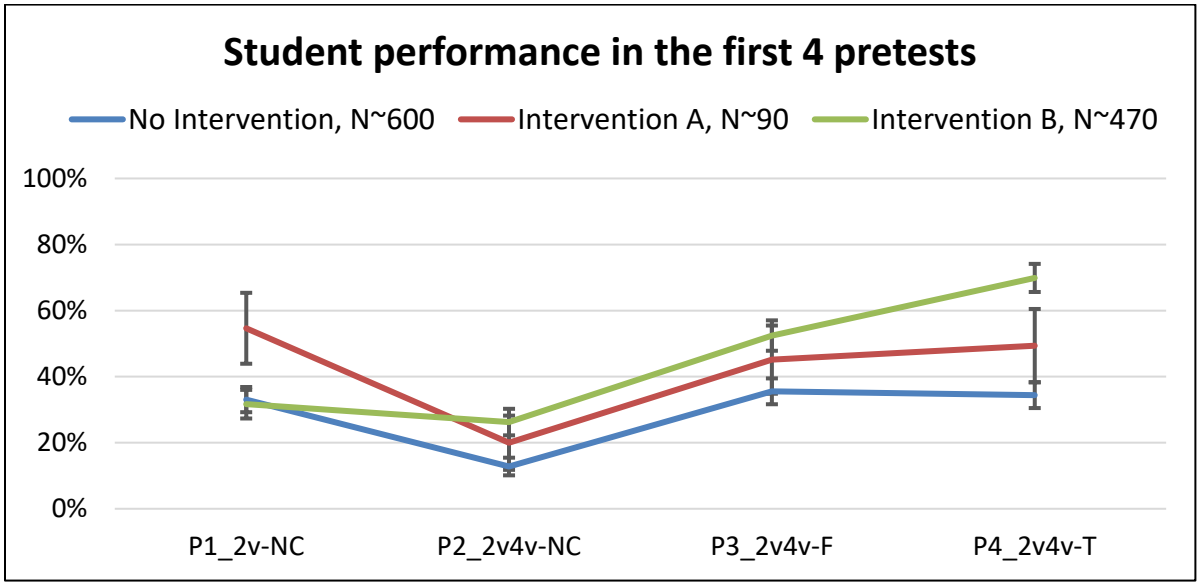


Figure 14.14. The student performance of vector superposition in the first four pretests of students in the no-intervention, intervention-A, and intervention-B groups. Error bars indicate ± 2 SE.

Table 14.106 shows the result of the likelihood ratio tests comparing the five models with different sets of covariates. Adjusting for other factors, students' percentage correct is significantly associated with students' background knowledge on vectors, pretest number (a proxy for time, labeled as PT in Table 14.106), and the presence of the intervention. This is consistent with the trend in Figure 14.14. Out of the five models, model 5 has the lowest AIC. Although the log-likelihood of model 4 is greater than that of model 5, model 4 has more covariates (21 degrees of freedom compared to 13 degrees of freedom in model 5). The AIC puts a penalty on model

with a higher number of covariates, so model 5 has a lower AIC than model 4. In other words, if two models have a similar number of fixed effects and log-likelihood, then the AIC tends to favor the model with fewer covariates.

Table 14.106. Comparisons of different generalized linear mixed models. The numbers under "p-value" are the *p*-values of a likelihood ratio test comparing with the model in the previous row. For example, the number in M2 is the *p*-value of the likelihood ratio test comparing with M1. The symbol "*" encompasses individual factors and their interaction.

Model	**Log likelihood**	**AIC**	***p*-value**
M1: Outcome ~ Pretest1	–1965	3938	-
M2: Outcome ~ Pretest1 + BG	–1962	3935	0.017
M3: Outcome ~ Pretest1 + BG + PT	–1808	3630	< 0.001
M4: Outcome ~ BG * PT * Intervention + Pretest1	–1719	3481	< 0.001
M5: Outcome ~ BG + PT * Intervention + Pretest1	–1724	3474	< 0.001

Table 14.107 shows the detailed results of Model 5. The point estimate for the intercept is the log odds for pretest 2 when students have no background on vectors prior to the course and students who answered incorrectly in PT1. (More details on the mathematics behind the logistic regression can be found in Appendix A.) The other factors do not contribute to the intercept. The individual point estimates for Pretest 3 and Pretest 4 represent the difference in the log odds between Pretest 2 and these two factors while holding other covariates constant. The positive values mean that the odds for students answering correctly in later pretests are higher than at pretest 2. This result is consistent with Figure 14.14.

Table 14.107. Point estimates, standard errors, 95% confidence intervals, and p-values of a Wald-test of parameters in Model 5. The symbol ":" means interaction.

	Estimate	**Std. Err.**	**95% CI**	***p*-value**
(Intercept)	-2.97	0.19	(-3.34, -2.61)	< 0.001
Pretest 3	1.62	0.18	(1.28, 1.97)	< 0.001
Pretest 4	1.56	0.18	(1.21, 1.90)	< 0.001
Intervention A	0.49	0.36	(-0.20, 1.19)	0.16
Intervention B	1.19	0.20	(0.80, 1.57)	< 0.001
Background on vectors (BG)	0.28	0.11	(0.06, 0.50)	0.01
Correctness of Pretest 1	1.15	0.12	(0.92, 1.38)	< 0.001
Pretest 3:Intervention A	-0.17	0.45	(-1.05, 0.70)	0.70
Pretest 4:Intervention A	0.05	0.46	(-0.85, 0.94)	0.91
Pretest 3:Intervention B	-0.23	0.24	(-0.70, 0.23)	0.33
Pretest 4:Intervention B	0.72	0.24	(0.24, 1.19)	0.003

Based on Model 5, the coefficients for both factors, students' background on vectors and correctness of PT1, are statistically significant. The coefficients are positive, which suggests that these factors play significant roles in increasing the odds of students answering pretests 3 and 4.

Both coefficients for the factors intervention A and intervention B are positive. This result is consistent with Figure 14.14 that the percentages correct are generally greater with interventions. However, the coefficient for the factor intervention B is greater than that for the factor intervention A. The former is statistically significant, but the latter is not. This result suggests that intervention B is better than intervention A in improving student performance in vector superposition.

Comparing the coefficients for the interaction factors (Pretest 3:Intervention A, Pretest 4:Intervention A, Pretest 3:Intervention B, Pretest 4:Intervention B), only the factor for Pretest 4:Intervention B is statistically significant. We interpret this result in this way. It takes time for students to realize the correct approach. The results suggest that the percentage correct would

reach above 60% after students did three pretests. (Note that students do not see the intervention in pretest 4 if they answered the actual task correctly.)

As described in the previous section, there is a drastic difference in sample sizes of students who received intervention A ($N \sim 90$), students who received intervention B ($N \sim 470$), and students did not receive any intervention ($N \sim 600$). Also, not all students we surveyed did all the pretests. Most incomplete cases missed only one pretest. The disparity in sample sizes and missingness of data reduces the power of inferring the difference in the percentage correct between intervention groups.

Now, we discuss the missing data in our study. The results of the GLMM analysis are valid if the missing observations are missing at random (MAR) [89]. From Figure 14.14 and Table 14.107, we see that the percentages correct in pretests 3 and 4 are significantly higher than that in pretest 2, and tend to be higher among those who received the intervention. To assess the extent to which missing observations influence the results of the analysis, we consider two scenarios where the "true value" of the missing observations, if observed, would reverse our conclusions as much as possible. In other words, we are performing sensitivity analysis.

The first scenario is to impute "correct" for every missing result in the pretests 1 and 2 and "incorrect" for every missing result in pretests 3 and 4. The reason we did so is that Model 5 compares the differences in the percentage correct between pretest 2 and pretest 3, pretest 2 and pretest 4, subjected to other covariates. This imputation method is meant to downplay the effects of time, with or without the effect of the intervention. So, we can expect the point estimates for the factors Pretest 3 and Pretest 4 to be lower than those shown in Table 14.107.

Figure 14.15 and Table 14.108 show the results of the first imputation method. As expected, the point estimates for the factors Pretest 3 and Pretest 4 are lower than those presented

in Table 14.107. They are still statistically significant. Similarly, the point estimates for the factors intervention B, students' background on vectors, and correctness on PT1 continue to be statistically significant. This result means that the difference between the log odds of the results of pretests 2 and 3 is significantly associated with the three factors mentioned. The conclusions derived using the imputed dataset are the same as the collected dataset.

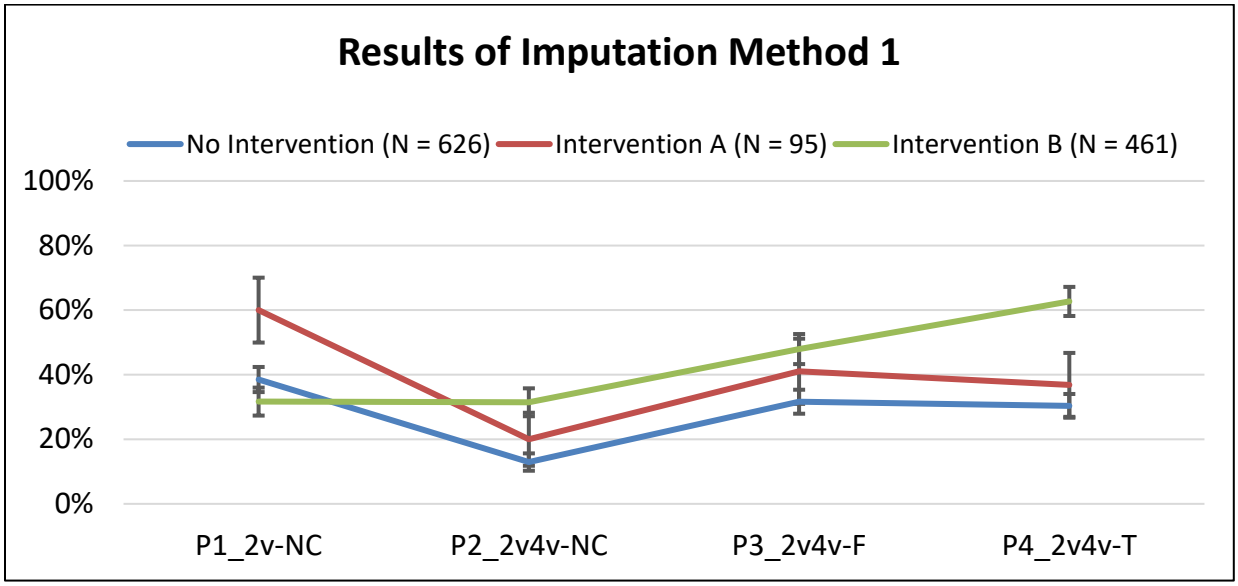


Figure 14.15. Results among students with missing data imputed as "correct" for pretests 1 and 2, and "incorrect" for pretests 3 and 4. Error bars indicate ± 2 SE.

Table 14.108. Point estimates, standard errors, 95% confidence intervals and *p*-values of a Wald-test of parameters in Model 5, where the missing data are imputed as "correct" for pretests 1 and 2, and "incorrect" for pretests 3 and 4.

	Estimate	**Std. Err.**	**95% CI**	***p*-value**
(Intercept)	-2.82	0.17	(-3.15, -2.49)	< 0.001
Pretest 3	1.34	0.16	(1.02, 1.65)	< 0.001
Pretest 4	1.26	0.16	(0.95, 1.58)	< 0.001
Intervention A	0.37	0.32	(-0.26, 1.01)	0.25
Intervention B	1.42	0.18	(1.06, 1.77)	< 0.001
Background on vectors (BG)	0.26	0.10	(0.06, 0.46)	0.01
Correctness of Pretest 1	0.95	0.10	(0.75, 1.16)	< 0.001
Pretest 3:Intervention A	-0.09	0.40	(-0.87, 0.68)	0.81
Pretest 4:Intervention A	-0.24	0.40	(-1.02, 0.54)	0.55
Pretest 3:Intervention B	-0.50	0.22	(-0.93, -0.07)	0.02
Pretest 4:Intervention B	0.30	0.22	(-0.13, 0.73)	0.17

The second method is imputing "correct" for missing survey results of students who did not receive the intervention and "incorrect" for those who did. This method is to downplay the effects of the interventions. So, we expect the point estimates for the factors involving the interventions to be lower than those presented in Table 14.107.

Figure 14.16 and Table 14.109 show the results of the second imputation method. The point estimate for intervention A, 0.51, is about the same as before imputation (0.49), and not statistically significant. The point estimate for intervention B is less than the value calculated using the collected data, which is expected. It is still statistically significant. Similarly, the coefficients for the interaction terms with interventions are also lower than the values obtained from the collected data. In other words, the effects of interventions are reduced under the second imputation strategy.

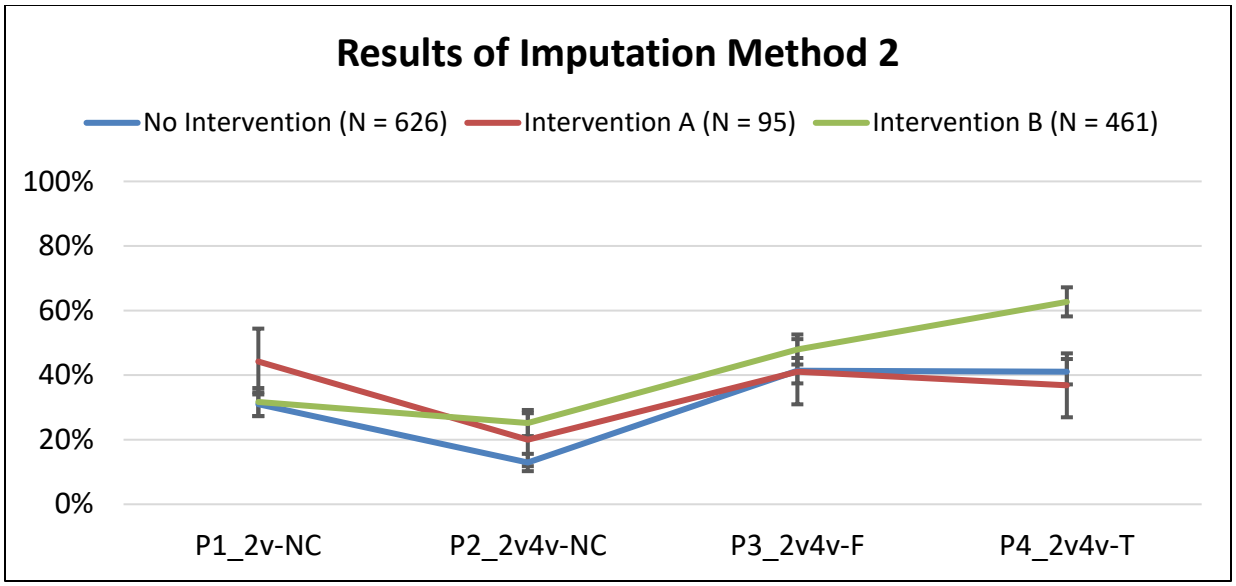


Figure 14.16. Results among students with missing data in the no-intervention group imputed as "correct," and missing data in the intervention group imputed as "incorrect." Error bars indicate ± 2 SE.

Table 14.109. Point estimates, standard errors, 95% confidence intervals, and *p*-values of a Wald-test of parameters in Model 5, where the missing data in the no-intervention group are imputed as "correct," and the missing data in the intervention group are imputed as "incorrect."

	Estimate	**Std. Err.**	**95% CI**	***p*-value**
(Intercept)	-2.77	0.17	(-3.10, -2.45)	< 0.001
Pretest 3	1.84	0.16	(1.53, 2.16)	< 0.001
Pretest 4	1.82	0.16	(1.51, 2.14)	< 0.001
Intervention A	0.51	0.32	(-0.12, 1.14)	0.11
Intervention B	1.03	0.18	(0.68, 1.39)	< 0.001
Background on vectors (BG)	0.23	0.10	(0.04, 0.43)	0.02
Correctness of Pretest 1	0.95	0.10	(0.75, 1.15)	<0.001
Pretest 3:Intervention A	-0.60	0.40	(-1.37, 0.18)	0.13
Pretest 4:Intervention A	-0.80	0.40	(-1.58, -0.02)	0.043
Pretest 3:Intervention B	-0.64	0.22	(-1.07, -0.21)	0.004
Pretest 4:Intervention B	0.10	0.22	(-0.34, 0.53)	0.65

14.3.5 Additional results – intervention B in the Wi'20 quarter

The results from intervention B was promising, so we repeated intervention B in both PHYS 114 and PHYS 115 in the Wi'20 quarter. Since only 73% of the PHYS 115 students took PHYS 114 in the Au'19 quarter, not everyone was receiving intervention B the second time. One thing to note is that there was a global pandemic caused by the Covid-19 that hit the Washington state near the end of the quarter. The final exam was then canceled due to the inability to ensure its integrity, and no data was collected.

First, we discuss the motivations for the changes made and the corresponding results in PHYS 114. The results in the Au'19 quarter (see Figure 14.14), especially the 70% correct in pretest 4, and 84% correct in pretest 5, appeared to be too good to be true. The correct answer ("Greater than") in the pretests were always the same. We did so to simplify the data analysis. We suspected that some students might remember the correct answers through repeated exposures of the earlier pretests and interventions, so they might not read the question and arrive at the correct answer through proper reasoning in later pretests. In the Wi'20 quarter, minor changes were made to the questions in pretests 4 and 5 to ensure students answered correctly through proper reasoning so that the answers were different from before.

The results of intervention B in PHYS 114 in the Wi'20 quarter is presented in Figure 14.17. The results for the first three pretests had a similar trend as in the Au'19 quarter (see Figure 14.12). In the fourth and fifth pretests, the results in the Wi'20 quarter were 11%-13% lower than that in the previous quarter. We found some students (less than ten students, ~5% of the class) indicated that they misread the question on the fourth pretest. This finding confirmed our suspicion, so it was necessary to adjust the questions on the pretests to make sure students arrived at the correct answer after going through the proper reasoning.

On the other hand, the results of pretest 4 in the Au'19 quarter and the results of pretest 5 in the Wi'20 quarter were about the same. We believed the gain from pretest 4 to pretest 5 in the Wi'20 quarter was more realistic than the gain from pretest 3 to pretest 4 in the Au'19 quarter because some students selected answers based on pattern recognition. While we were unable to find out the student performance of vector superposition in the final exam of the Wi'20 quarter, based on the result from the previous quarter, we believed at least 60% of the students could answer correctly.

In sum, intervention B continues to improve the performance of PHYS 114 students in adding vectors in the Wi'20 quarter. It may be necessary to make minor changes in the questions in later pretests to ensure that students use the proper reasoning to arrive at the correct answer.

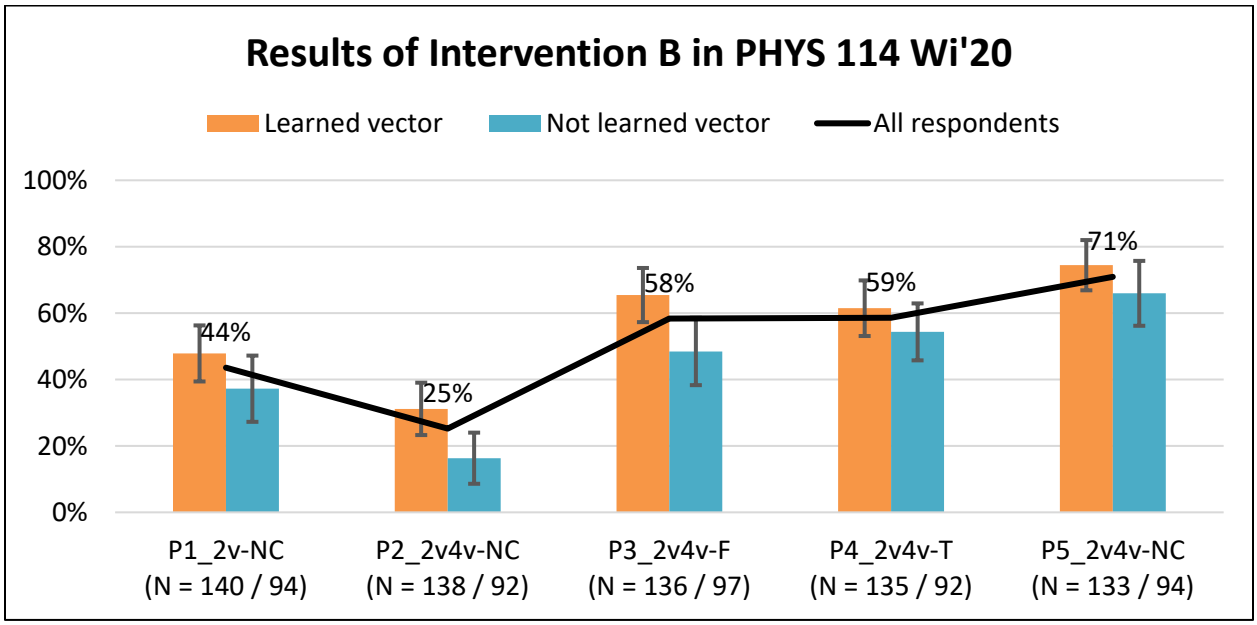


Figure 14.17. The results of intervention B for PHYS 114 in the Wi'20 quarter. Vector Y/N refers to students who have/have not learned about vectors before attending PHYS 114. N = 140 / 94 refers to there are 140 students and 94 students who learned/did not learned vectors before registering for PHYS 114. 'All respondents' refers to the results of the students who responded to that task. Error bars indicate ± 2 SE.

Next, we discuss the changes made and the corresponding results in PHYS 115. No changes were made for the first two pretests, and the first two tasks were 2v-NC and 2v4v-NC. On the third pretest (2v4v-F), we changed the correct answer to see if students were choosing the correct answer based on previous pretests. We found that the results on the first three pretests were above 60% (see Figure 14.19), so we did not give students the 2v4v-T task on the fourth pretest. The 2v-E task was administered without the intervention instead. Then, we asked the 2v-E and 2v4v-E tasks in midterm 2 (MT2), as shown in Figure 14.18. No final exam was administered in PHYS 115 as well.

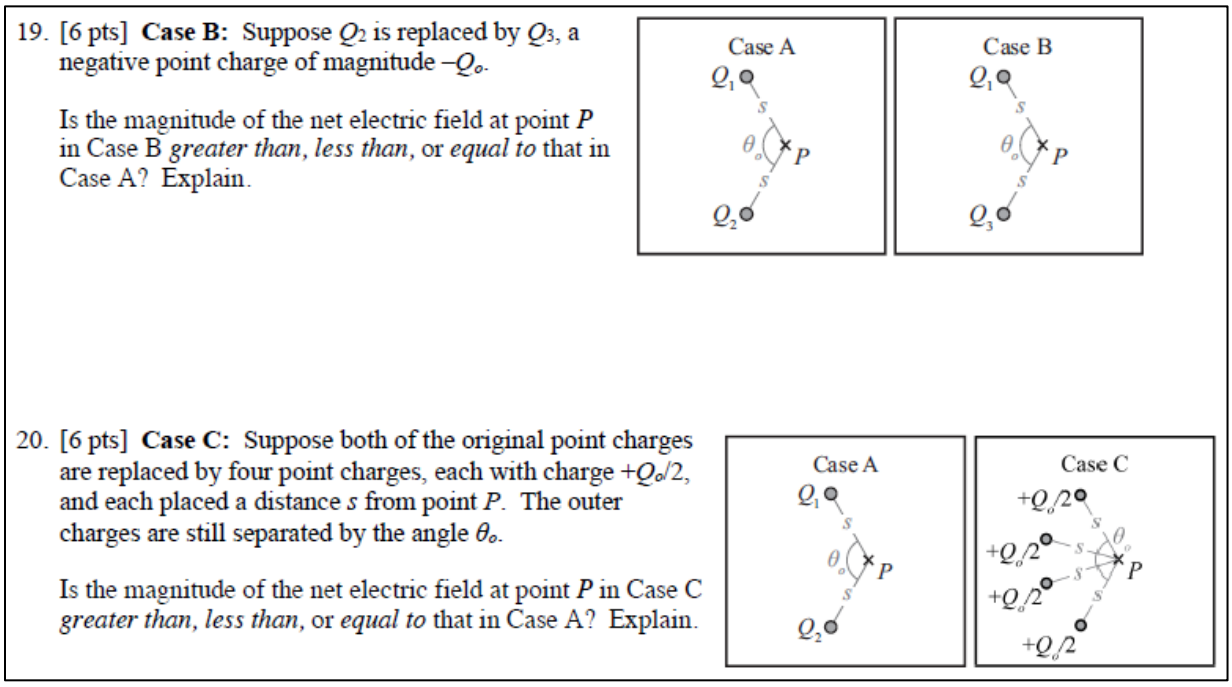


Figure 14.18. The 2v-E and 2v4v-E tasks in PHYS 115 midterm 2 of the Wi'20 quarter. Q_1 and Q_2 are positive charges of magnitude Q_o.

The results from PHYS 115 in the Wi'20 quarter are shown in Figure 14.19. More than 60% of the students answered the first three pretests correctly. Around 25% of the students did the 2v-E on the fourth pretest correctly. The pretest results for the 2v-E task were about the same

(6% difference, $\chi^2\ 1\ = 2.5$, *p*-value = 0.11) in the Au'19 and Wi'20 quarters. This result was not surprising, as explained in the previous section.

The results for the 2v-E and 2v4v-E tasks on the second midterm of PHYS 115 in the Wi'20 quarter are 38% and 34%, respectively. Comparing the result for the 2v-E task (49% correct) on the final exam without any intervention (see Figure 14.10), the 38% correct could be interpreted as the student performance two weeks after tutorial instruction, and the 49% correct on the final exam would be the performance after additional revision.

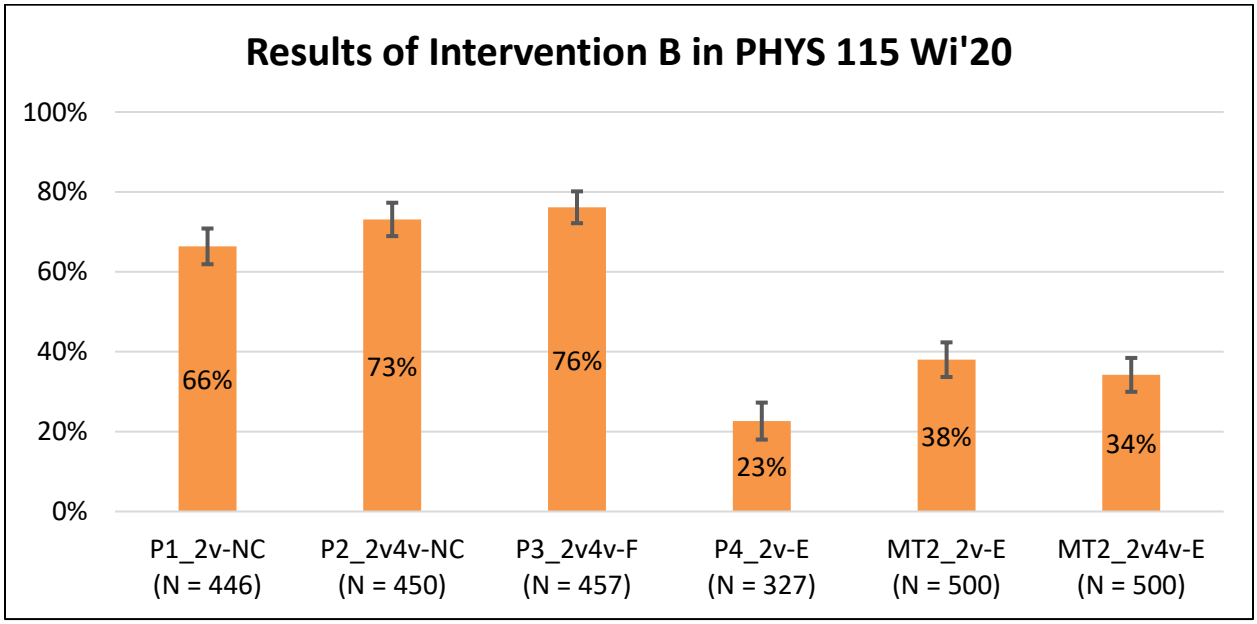


Figure 14.19. The results of intervention B for PHYS 115 in the Wi'20 quarter. Error bars indicate ± 2 SE.

Lastly, the student performance on the 2v4v-E task on MT2 was about the same as the 2v-E task. Only 19% of the students answered both the 2v-E and 2v4v-E tasks correctly.

On both tasks, 44% of the students wrote "Equal to" as their answer. There were three general approaches in arriving at the incorrect answer. The first approach was adding the electric field vectors by individual charges like scalars. Some students arrived at "Equal to" when they

only considered the magnitudes of the vectors. Some students arrived at "Less than" when they included the negative sign from Q_3 (see Figure 14.18), and they found the net electric field in Case B to be 0. The second approach was ignoring the angles between the vectors. These students tended to say the horizontal and vertical components in both cases (cases A and B in Qn 19, cases A and C in Qn 20) were the same. In the third approach, some students thought that there was a negative test charge at point P (it was present in Qn 17 of MT2) and used that test charge to determine the net electric field. These students did not understand that the net electric field is independent of the test charge.

In summary, the results of PHYS 115 in the Wi'20 quarter suggest that most students did not need intervention B to arrive at the correct answers for the 2v-NC, 2v4v-NC, and 2v4v-F tasks (recall that students only receive help if they answered the question correctly). However, intervention B did not appear to improve student performance in the 2v-E task.

14.3.6 Discussions

Two areas are discussed. First, we compare the effectiveness of interventions A and B. Then, we discuss some new results we collected that were related to the 2v-E task.

Comparing the effectiveness of interventions A and B

Two metrics were used to compare the effectiveness of the interventions in PHYS 114. They are: (1) the student performance of the vector superposition task administered in the final exam and (2) the trend of the performance as students progress through pretests.

The students' final exam results in the no-intervention group, intervention-A group, and intervention-B group were 47%, 50%, and 77%, respectively. These results showed that there was no significant difference in performance between the students in the no-intervention group and the students in the intervention-A group. The difference is significant if we compare the students in the no-intervention group and the intervention-B group. Based on the final exam results, intervention B stood out.

Using the second metric, we plotted the student performance in the first four pretests in the three groups (no intervention, intervention A, intervention B) of students in Figure 14.14. We made some assumptions when we plotted the graph.

The first assumption is related to the first two tasks. In the no-intervention and intervention-A groups, the first two tasks were 2v4v-NC and 2v-NC, while the tasks were in the opposite order in the intervention-B group. We assumed that there was no difference in the order of the first two tasks in the no-intervention and the intervention-A groups so that we could reverse the order. The reason for this assumption was that the students did not have enough lecture instruction on vectors at the beginning of the quarter, so whether students did the 2v-NC task or 2v4v-NC task first, it had a negligible effect on the student performance on the subsequent task.

The second assumption is related to the 2v-NC task in the intervention-A group. The helper task and the actual task for the 2v-NC context were about the same (see Appendix E), it made no sense to students if they did the actual task again. So, we used the results of the helper task as the results for the actual 2v-NC task.

From Figure 14.14, the result of the 2v-NC task in the intervention-A group was better than the other two. However, starting from the second pretest, the results from the intervention-B group were generally better than the other two. The results appeared to plateau at around 35%-50% in

the no-intervention and intervention-A groups but continued to increase in the intervention-B group. The results from intervention B had a better trendline than the other two.

However, it may not be fair to compare the two interventions because of the small sample size in intervention A. We did not continue to use intervention A with a larger population because the final exam result of the intervention-A and no-intervention groups were about the same. If the sample size of intervention A is larger, assuming the trend line stays the same, then the error bars will become smaller. That would further strengthen the case for intervention B. While it is difficult to tell whether the assumption is valid without performing the investigation, the results of intervention B in both the Au'19 and Wi'20 quarters suggest that the assumption is most likely to be true. So, instead of running an intervention that does not produce significant improvements in the final exam, we searched for better intervention and found intervention B.

Finally, based on the two metrics we set out, students under intervention B had a steady increase in pretest performance and better final exam results. That said, when comparing the results from the Au'19 quarter with that from the Wi'20 quarter, one needs to be mindful of how students arrived at the answers as students might follow the pattern of past correct answers.

Additional results related to the 2v-E task

When the 2v-E task was asked on pretests in the Au'19 and Wi'20 quarters, 23%-29% of the algebra-based students answered correctly. The reasons for the low performance were (1) the electric field concepts were new to these students and (2) there were no arrows drawn in the 2v-E task, but arrows were provided in the past vector superposition tasks. Kanim had reported the student difficulties related to electric fields in his dissertation. (We believe the student difficulties in electric field concepts among the calculus-based and algebra-based students are about the same

because the calculus-based students were once algebra-based students.) Then, to investigate the second reason, we modified the 2v-E task such that the electric field vectors at point *P* by individual charges were drawn, as shown in Figure 14.20.

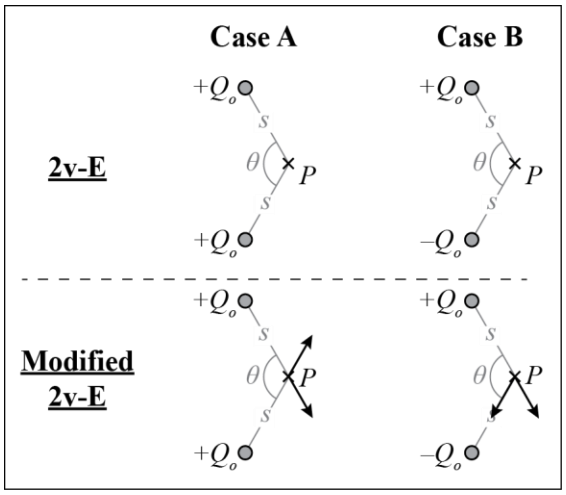


Figure 14.20. The original and modified 2v-E tasks. The electric field vectors at point *P* due to each individual charge were drawn on the modified 2v-E task.

We administered the original and modified 2v-E tasks on pretests to the calculus-based (PHYS 122) and algebra-based students (PHYS 115) in the Wi'20 quarter. The results are presented in Table 14.110.

Table 14.110. Pretest results (percentage correct) for the original and modified 2v-E tasks in the calculus-based and algebra-based students in the Wi'20 quarter.

	Original 2v-E task (no arrows)	Modified 2v-E task (with arrows)	χ^2 test
Calculus-based (PHYS 122)	50% (N = 304)	63% (N = 114)	6.1 (p-value = 0.01)
Algebra-based (PHYS 115)	23% (N = 327)	45% (N = 132)	23.7 (p-value < 0.01%)

The performance of the calculus-based students was about 20% better than that of the algebra-based students because most students in the former group had learned about the electric field ideas while the latter group encountered field ideas the first time. Among the calculus-based and algebra-based students we surveyed, those who saw the modified 2v-E task performed better than students who saw the original task. We performed χ^2 tests for each group, and each difference was found to be statistically significant. The difference in the performance ranged from 13% to 22%, so the presence of the electric field vectors had a medium effect.

There are a few possible reasons to account for the results between the original and modified 2v-E tasks. First, some students might not realize that the angles between the two electric field vectors in both cases were not the same because it might not easy to see without drawing out the arrows. Second, students might not remember that the electric field was a vector quantity. The presence of the arrows reminded students of that fact. All these reasons contributed to the low pretest performance of the algebra-based students on the original 2v-E task, despite undergoing intervention B in the Wi'20 quarter.

14.4 INSTRUCTIONAL IMPLICATIONS AND LIMITATIONS OF OUR STUDY

In this section, we discuss the instructional implications of intervention B and limitations of our study.

14.4.1 Instructional implications of intervention B

Our results from intervention B are consistent with the improvement in students' vector math performance that was reported by Mikula and Heckler [77]. Although they reported a

minimal gain in vector addition, they achieved substantial gains in other vector math skills such as vector components, dot and cross products. The differences between them and our studies are presented in Table 14.111.

Table 14.111. Differences between our study and the study by Mikula and Heckler.

Areas	**Study by Mikula and Heckler [77]**	**Our study**
Student population	Calculus-based students	Algebra-based students
Skills focused	Dot product, cross product	Vector addition with arrow representation only
Method of implementation	Online application that is constructed for mastery practice	Online tutorial pretest that is part of the course assessment

The differences are student population, the targeted essential skills, and method of implementation. Mikula and Heckler performed their investigation with calculus-based students while we did our study with algebra-based students. We focused only on vector addition with arrow representation instead of the relevant vector math required for calculus-based physics courses. Lastly, we implemented the computer-based training with feedback using online tutorial pretests instead of creating an online application for mastery training purposes.

The algebra-based students do not need to learn dot and cross products in the physics course. So, it is possible to implement intervention B to improve student ability in vector subtraction and vector component through online pretests. It is also possible to improve other physics essential skills too. Mikula and Heckler defined the characteristics of an essential skill to be procedural, relatively simple, constitutes some steps necessary to solve usual physics problems, and largely automated by experts. So, other essential skills such as understanding and interpreting

graphs, and proportional reasoning can be improved through intervention B in our institution. The instructors and researchers can follow the framework suggested by Mikula and Heckler to decide the essential skills to improve on before implementing intervention B [77].

14.4.2 Limitations to our study

Our study has several limitations. (1) The intervention was assigned with one section as a unit, any section-level factor that impacts students' survey results is a confounder for the analysis, including students' average commitment to class, the instructor's effectiveness in teaching the material, quarterly variation, etc. We attempted to address the confounding issue by adjusting for students' background knowledge, but there may be other potential confounders, such as the students' SAT scores and high school GPAs. (2) We focused on vector superposition, but the conclusion may not generalize to the overall performance in the class. (3) The analysis treats the survey results as binary variables. Since the survey questions are multiple-choice, an alternative analysis is to record the number of correctly selected choices for each survey question and treat the result (number of correctly selected choices) as an ordinal categorical variable. The analysis of the association between the correct rates and time/intervention can be performed using a proportional odds model [90]. (4) In this study, we assumed the survey questions about vector superposition were comparable throughout the quarter. If they are drastically different in terms of difficulty either throughout the quarter or between different quarters, the variety in difficulty will confound the association between students' survey results and time/intervention.

14.5 SUMMARY, FUTURE WORK, AND CONCLUSIONS

We conclude this chapter by summarizing our findings and outline some possible future work that arises from this study.

14.5.1 Summary of findings

RQ1: How does the performance in vector superposition in the calculus-based students compare to that in the algebra-based students?

A set of vector superposition tasks (2v4v-NC, 2v-NC, 2v4v-F, and 2v4v-T) were used to investigate student ability to compare the magnitudes of resultant vectors in calculus-based physics courses. Students generally did better in two-vectors comparison problems than two-vectors-four-vectors comparison problems, which could be explained by the cognitive load theory. The difference in the results between no-context and physics-context in the two-vectors-four-vectors comparison tasks was greater than that in the two-vectors comparison tasks, but we were unable to account for it.

Comparing the results of the calculus-based students with the algebra-based students, the PHYS 114 students (Figure 14.9) did not do as well as the calculus-based students (Figure 14.8) in both the 2v-NC (40% vs. 60%) and 2v4v-NC (15% vs. 20%) tasks. However, after a few weeks of instruction, they performed at about the same level (38% vs. 40%) as the calculus-based students in the 2v4v-F and 2v4v-T tasks. Our results were about the same in different quarters and with different instructors. Comparing the results between the PHYS 115 and calculus-based students, the former group (Figure 14.10) did less well than the latter group in the 2v-NC task, but their results were comparable in the 2v4v-NC, 2v4v-F, and 2v4v-T tasks.

RQ2: What is the progression of the student performance in vector superposition in algebra-based physics courses?

We used four vector superposition tasks (2v4v-NC, 2v-NC, 2v4v-F, and 2v4v-T) to investigate the student ability to do vector addition in algebra-based physics courses. The results in PHYS 114 started low at around 15%, increased to 35%, and plateaued. In PHYS 115, the results improved by 5%-10% and matched the performance of the calculus-based students in some tasks. The factors that might account for the difference in the performance are the surface features, context dependence, lecture instruction, and retesting effect. These factors may play different roles in different groups of students at different times.

RQ3: How does the prior knowledge in vectors affect the progression of the student performance in algebra-based physics courses?

In PHYS 114, the students who had prior knowledge in vector performed better than those without prior knowledge generally. Their performances progressed similarly under usual lecture instruction (Figure 14.9) and intervention B (Figure 14.12). The performance gap narrowed after a few weeks of instruction, but students who learned vector before PHYS 114 still performed better.

RQ4: Suppose we implement interventions through online pretests to improve students' vector superposition skills in algebra-based physics course. How can it be done? How effective is it?

Two intervention schemes were tested. They were (A) the limiting case approach and (B) targeted intervention with elaborated feedback. In intervention A, students received a helper task that presented a limiting case before the actual task. In intervention B, students received additional

scaffolding questions when they answered the actual task incorrectly. The sequence of questions informed students about whether their answers were correct (knowledge of result, KR), showed students specific steps to arrive at the correct answer (elaborated feedback, EF), and in some cases, informed students about the correct answer (knowledge of correct response, KCR).

Two metrics were used to evaluate the effectiveness of the interventions in the PHYS 114 courses. They were (1) comparing the student performance in the final exams (no intervention vs. intervention A and B) and (2) comparing the progression of student performance through the pretests under each intervention. Under intervention B, students progressed steadily on pretests during the quarter and performed better in the final exams. So, based on the metrics, intervention B produced better results.

Next, the results in the PHYS 114 and PHYS 115 from the Wi'20 quarter suggested that the trend in the improvement under intervention B was replicable.

The data from PHYS 114 in our vector superposition study were analyzed using the generalized linear mixed model (GLMM). We found that the increase in student performance from pretest 2 to pretests 3 and 4 is strongly associated with students' background in vectors, students' performance in pretest 1, and intervention B. These additional factors were not evident from the trendlines in each subpopulation. The results of the sensitivity analysis indicated that the conclusion for the association between the difference in the percentage correct between pretests and time was robust to the missing values. Similarly, when we imputed the missing observations in the intervention group to be "incorrect," the coefficients that were related to interventions decreased, showing that the model was adequate in responding to the missing values. Finally, the GLMM analysis showed that intervention B was effective in improving student ability to do vector

superposition. This result is consistent with the conclusion based on the students' final exam performance.

Other findings

Students who undergone intervention B did not perform better than the no-intervention group on the 2v-E task on midterms and final exams. On pretests, students who were given electric field vectors performed better than those who were not. Three reasons were proposed to attribute for this result. First, students might not realize that the angles between the two electric field vectors in both cases were not the same. Second, students might not remember that the electric field was a vector quantity. The presence of the arrows reminded students of that fact. Third, some students might not realize that there was a change in the direction of one of the electric field vectors when one positive charge was changed to a negative charge.

14.5.2 Future work

Intervention B brought promising results in improving students' physics essential skills. These essential skills, as defined by Mikula and Heckler, were procedural, relatively simple, constituted some steps necessary to solve usual physics problems, and largely automated by experts. Instructors and future graduate students can decide on the essential skills to be improved on, design sets of practice questions to be put on online pretests, then measure the effectiveness. Modifications can be made along the way to improve student performance of those essential skills, including vector superposition.

14.5.3 Conclusion

In this chapter, we examined the ability of students in algebra-based courses to apply vector superposition. We did this by administering a series of online pretests at various points during the quarter for several classes. In some cases, these included an intervention. This longitudinal study examined student growth in this area, both with and without these online interventions.

The student performance in PHYS 114 started low – around 15% on the first pretest before any relevant lecture instruction. In the course without any interventions, there was some improvement a few weeks into the quarter, but the results then stayed flat at about 35% until the end of the quarter, when performance rose to around 50% on the final exam. Since the task was one that most people regarded as a relatively simple application of vector superposition, we thought that there was a need for helping students learn to reason about vectors qualitatively.

Over several quarters, we designed and tested several online intervention schemes to improve student performance. The success of the intervention was determined by a better trend of progression and better results on the final exam. The intervention scheme that produced the best results involved a sequence of questions given over multiple weeks, each of which incorporated several different kinds of feedback to students. These include knowledge of results (user is informed of whether the answer is correct or incorrect), knowledge of correct response (user is informed of the correct answer), and elaborated feedback (detailed explanation). Students received the intervention when they answered the vector superposition tasks incorrectly. The students' ability to add vectors improved significantly through successive practices. On the final exam, around 75% of the students answered the vector superposition task correctly, as compared to 50% in the no-intervention case.

As part of the analysis of the results, we used generalized linear mixed models to analyze the longitudinal data. We tested five different models. Based on our best model, we found that the intervention with elaborated feedback, students' initial background on vectors, and the correctness of the first pretest increases the odds of students answering the vector superposition tasks later in the quarter significantly.

That said, the intervention involving elaborated feedback did not appear to help students in the task involving electric field context, as shown by their lower performance on pretests and midterms. Several reasons could explain the differences, such as not realizing that the electric field is a vector quantity and did not draw out the relevant electric field vectors to arrive at the correct answer.

Ultimately, we believe that the relative success in the intervention that provides elaborated feedback to students suggests that this approach has a great deal of potential. The use of these techniques opens additional opportunities to improve students' other essential skills in physics, such as graphing and scaling / proportional reasoning.

Chapter 15. THE USE OF COMPUTATIONAL TEXT ANALYSIS IN PHYSICS EDUCATION RESEARCH

This chapter describes a preliminary study into the application of computational text analysis in conducting physics education research. Computational text analysis refers to the use of programming language to analyze text. It is also called natural language processing (NLP).

We first give a brief overview of relevant aspects of NLP and describe some of the motivations we had for applying NLP in our research. Next, we review some prior work in which NLP techniques have been applied to physics education research. Then, we outline the basic methodology in NLP as applied in this study. After that, we illustrate how we use NLP to compare the similarity of student explanations in the context of motion in two-dimension and rolling motion. The results are compared with analysis by humans.

(Note: Many specific terms have been developed related to linguistics and natural language processing techniques. In this chapter, we only used the terms that are most relevant to this investigation. Interested readers can use the references to find more extensive discussion of the techniques and additional terminology.)

15.1 BACKGROUND AND MOTIVATION

The use of NLP and the relevant research started in the early 1950s. At first, people used NLP for machine translation [91, 92]. After the birth of internet in the 1980s, more and more information became machine-readable, NLP branches out into different areas such as information retrieval [93, 94], text summarization [95, 96], question answering [97, 98], information extraction

[99, 100], topic modeling [101, 102], and opinion mining [103, 104]. The emergence of social media such as Facebook and Twitter in the early 2000s generates huge amounts of information (in the order of 10^{15} bytes) weekly. More people start to create and share content on the internet, and data can also be collected easily via the internet.

Since the information on the internet is mainly unstructured (produced explicitly for human consumption), it is not directly machine-processable. Algorithms are written so that information can be extracted efficiently. The early processing techniques focus on retrieving texts, splitting them into parts, checking the spelling, and counting the number of words. The current NLP techniques are trying to interpret sentences, extract meaningful information, or predict upcoming texts. However, the current algorithms are limited to the information that people provided. The gap between the algorithms and human text processors is that humans can go beyond that by considering the contexts and relevant experiences to interpret the written texts deeper and quicker. In order to model how the human brain processes natural language, the algorithms need to take in a large amount of information to generate plausible models. The algorithms are constantly being improved so that they can analyze large amounts of information on the web accurately and efficiently.

To date, many methods have been devised in NLP to fulfill different tasks. We are interested in using some of the existing algorithms to analyze student explanations in physics education research. Here are some of the motivations that lead to this preliminary study.

First, the introductory physics courses at the UW have a high enrolment, with an average of 300-600 students in each course every quarter. Student responses are collected through online pretests. Some questions are asked on pretests repeatedly every quarter, which translates into thousands of responses in those questions – a large dataset. It would be a difficult task if one were

to read and categorize each student's response. So, it may be necessary to look for some tools that can analyze and summarize the student explanations in a shorter time. If possible, NLP can help to categorize the explanations too.

The second motivation is in the method of presenting student explanations in physics education research. There are different ways of representing textual data in NLP, such as word cloud, lists of frequently used words/phrases, or co-occurrence of words/phrases. These representations can be used to represent the student explanations in the physics tasks, in addition to providing quotes of student explanations. If most students are using one concept/approach to explain their answers, then it may be beneficial to represent those explanations through frequently used phrases or word clouds to convey that message objectively. This way of reporting can strengthen the findings, and it makes the reporting of the percentages more credible.

The third motivation is the potential research directions that NLP can bring in analyzing student explanations. There are many existing packages and algorithms that can be used to analyze textual data. Each of them has its pros and cons; each has its advantages and limitations. One needs to find time to explore the different approaches and their efficacies. There are numerous opportunities to be explored, which may lead to some interdisciplinary research projects for future graduate students. NLP is also an ongoing research area in data science, which is a skill that future employers may consider. So, we think this new area is worth exploring, and it will open more employment opportunities for graduate students.

15.2 LITERATURE REVIEW OF NLP WORK RELATED TO PHYSICS EDUCATION

As described previously, people used NLP for many purposes. In this section, we discuss some prior research on the use of NLP in projects related to physics education. These articles

discuss the use of NLP to analyze student interview data and in the development of intelligent tutoring systems for physics learning.

These articles we reviewed involve the use of latent semantic analysis (LSA) to categorize student explanations. We do not use LSA in our research, but it is an area that could be explored in the future. LSA is a statistical technique to extract relations among words using their contexts of use in documents, passages, or sentences. Dumais gave a review on LSA [105], and interested readers can check Deerwester *et al*. for the mathematical details [106].

Dam and Kaufmann used LSA to assess interview data from 34 7th grade students [107]. The data were about student explanations of the causes of Earth's seasons, which were collected by the Conceptual Dynamics Project at Northwestern University. Among the thirty-four student interviews, 21 of them were collected before instruction, the remaining after instruction. The authors used their LSA-based technique on three basic tasks: (1) the classification of student interviews by comparing with three human-coded documents, (2) the comparison of pre- and post-instruction interviews to assess differences due to instruction, and (3) the classification of smaller passages to detect conceptual change in the interview itself. The results from LSA were compared to the independent coding performed by four human raters, and the accuracy was 90%. The authors commented that the most promising practical application would "aid in the management and annotation of interview data." It can support research on a larger amount of data. The authors planned to scale this technique up to analyze larger and diverse data sets.

Sherin extended the work by Dam and Kaufmann described above [108]. His goal of the research was to explore the use of computational methods to increase the confidence of traditional methods of coding clinical interview data. Sherin described the traditional method to code interview data is to have a team of researchers deriving a coding scheme. Then multiple

researchers apply the scheme to assess the reliability of the scheme, improving the confidence of the results obtained along the way. Sherin suggested that computational methods can be used as an independent means of confirming coding schemes derived through traditional methods. He explored the use of LSA, combined with cluster analysis, to automate both the creation and application of a coding scheme. The data he used was 54 interviews from the Conceptual Dynamics Project, in which 7th or 8th-grade students were asked about issues related to earth's climate and seasons. He used the LSA technique to analyze the student interviews (after removing texts by the interviewers) and then used cluster analysis to generate 3-10 clusters of similar ideas. Each cluster has its list of words, and the idea represented by each cluster can be inferred (by human) from the top 10 frequent words. His results suggested that it was plausible to implement computational techniques to analyze interview data without the creation of codes by researchers.

Graesser *et al.* described the use of NLP in an intelligent tutoring system for physics learning [109]. The system is called the AutoTutor. They developed AutoTutor for Newtonian qualitative physics and computer literacy. The design of AutoTutor was inspired by "explanation-based constructivist theories of learning, intelligent tutoring systems that adaptively respond to student knowledge, and empirical research on dialogue patterns in tutorial discourse." AutoTutor is a natural language dialogue tutor that poses questions or problems that require a paragraph of information to answer. After students respond to a question, the AutoTutor uses LSA to analyze the student responses. Then, it can reply in several ways, such as providing positive, neutral, and feedback to students, asking students for more information, prompting the learner to fill in missing words, giving hints, filling in missing information with assertions, identifying and correcting wrong answers, answering learners' questions, and summarizing answers. Students often articulate misconceptions and false assertions in this back and forth process. If these

misconceptions are anticipated and incorporated into the program, AutoTutor provides the learner with information to correct the misconceptions. The authors experimented to determine the efficacy of Autotutor through pretest-treatment-posttest study. Three groups of students were under these treatments: AutoTutor, textbook, and read nothing. The pretest and posttest were multiple-choice questions or conceptual physics problems that required essay answers. The results from the AutoTutor group had the highest gain from pretest to posttest.

In summary, we reviewed articles that utilized the LSA technique to analyze student interview data and in intelligent tutoring systems. Our work is more aligned with Sherin's goals. That is, using computational methods as independent means of confirming coding schemes derived through traditional methods. In our preliminary work, we used NLP to determine the similarity of student explanations in two physics tasks, and then we compare that with the analysis conducted by humans. The details are described in the next few sections.

15.3 METHODOLOGY

In this section, we describe the procedures and analysis we used to compare student explanations. First, we describe how the data is processed after we collected the student responses. Next, we describe several the different types of exploratory data analyses that can be performed on the processed data. After that, we describe the mathematical approach to determine the similarity of explanations.

As and when necessary, we illustrate the procedures and outputs using some student explanations we have collected in the study described in section 15.4.

15.3.1 Data processing

The typical processing pipeline involves preparing data, cleaning data, generating a readable format, and lastly, performing exploratory and quantitative analysis. We prepare the student explanations to be looked at and put them in a format that is readable by R programming language [45]. The cleaning of data involves making all the words into lowercase, removing digits, punctuations, and symbols. We also remove stopwords, such as "a," "an," and "the." They are commonly used words that help to build ideas but do not convey the key ideas [110]. Students may have some spelling mistakes when they type their explanations in the survey. For that, we look at the word list generated in the exploratory analysis and correct the misspelled words in the original file. The data cleaning step is vital as we want to represent students' ideas accurately through the words they used.

Table 15.112 shows an example of how textual data looks like before and after being processed and cleaned. Three original student explanations are presented. After cleaning and removing stopwords, the remaining words represent the main ideas expressed by each student.

Table 15.112. An example of how textual data looks like before and after processed and cleaned

Document	**Original version**	**After cleaning and removing stopwords**
1	because the instantaneous velocity would be the tangent line	instantaneous velocity tangent line
2	the velocity is the tangent to the point	velocity tangent point
3	tangential plus translational velocity 3	tangential plus translational velocity

After data cleaning, we need to transform the data into a readable format for analysis. The readable format is called the corpus, which means a collection of documents. By the researcher's discretion, each document can be one student explanation, a group of explanations which students

provided when they select one answer, or an interview transcript. In a corpus, each row represents a document; each entry is the content of the document after data cleaning.

The corpus can then be transformed into a document-feature matrix (DFM) or feature-cooccurrence matrix (FCM). A DFM is a matrix in which the rows are made up of documents, and the columns are made up of words found in the document. Each entry represents the number of times that word appears in one document. The columns can also be two-word phrases or three-word phrases, depending on the needs of the analysis. The three student explanations shown in Table 15.112 are represented in a DFM in Table 15.113.

Table 15.113. An example of a document-feature matrix (DFM).

Doc	instantaneous	velocity	tangent	line	point	tangential	plus	translational
1	1	1	1	1	0	0	0	0
2	0	1	1	0	1	0	0	0
3	0	1	0	0	0	1	1	1

An FCM is a matrix in which both the rows and columns are made up of the words found in all the documents. Each entry in the matrix represents the number of times when word *m* appears with word *n*. If some word exists more than once in a document, then the value will be multiplicative. An example of an FCM based on the example shown in Table 15.112 is shown in Table 15.114. We can use the generated DFM or FCM for exploratory data analysis or quantitative data analysis.

Table 15.114. An example of a feature-cooccurrence matrix (FCM).

	instantaneous	velocity	tangent	line	point	tangential	plus	translational
instantaneous	0	1	1	1	0	0	0	0

velocity	0	0	2	1	1	1	1	1
tangent	0	0	0	1	1	0	0	0
line	0	0	0	0	0	0	0	0
point	0	0	0	0	0	0	0	0
tangential	0	0	0	0	0	0	1	1
plus	0	0	0	0	0	0	0	1
translational	0	0	0	0	0	0	0	0

15.3.2 Exploratory data analysis

Exploratory data analysis is a quick-and-dirty approach to data analysis. It is mainly used to summarize the main characteristics of the data, often with visual displays. We can calculate word frequency (term frequency) from a DFM and use the word frequency table to generate a word cloud. The term frequency is defined as the number of times the word appears in the document, divided by the total number of words considered (after data cleaning) in the document. A word cloud displays commonly found words used in the document, with the size of the word proportional to its frequency. It informs the audience what words or phrases are commonly used by students when they explain their answers. It can be used to corroborate the claims made from the qualitative analysis.

We can generate a network of co-occurrence of words from the FCM. The network allows us to find out which two words or two phrases tend to appear together. It can be used to represent an overview of students' ideas. The network has words as vertices and non-zero cooccurrences as links. The higher the co-occurrence, the thicker the line. Unlike networks that describe social interactions, the network of co-occurrence of words is undirected. Directed networks can show the interactions between two or more parties, be it one-sided or two-sided.

An example of a word cloud and a network of co-occurrence of words is shown in Figure 15.1. The student explanations came from the Rolling Motion Task (RMT) presented in part II,

Chapter 6. The RMT is about a wheel rolling without slipping on a horizontal surface. Students are asked to determine the direction of the velocity of a point on the rim of the wheel. Figure 15.1 is based on the two-word phrases in the student explanations for the direction that is tangent to the circle. The word cloud and network of two-word phrases show students' ideas that are consistent with the answer.

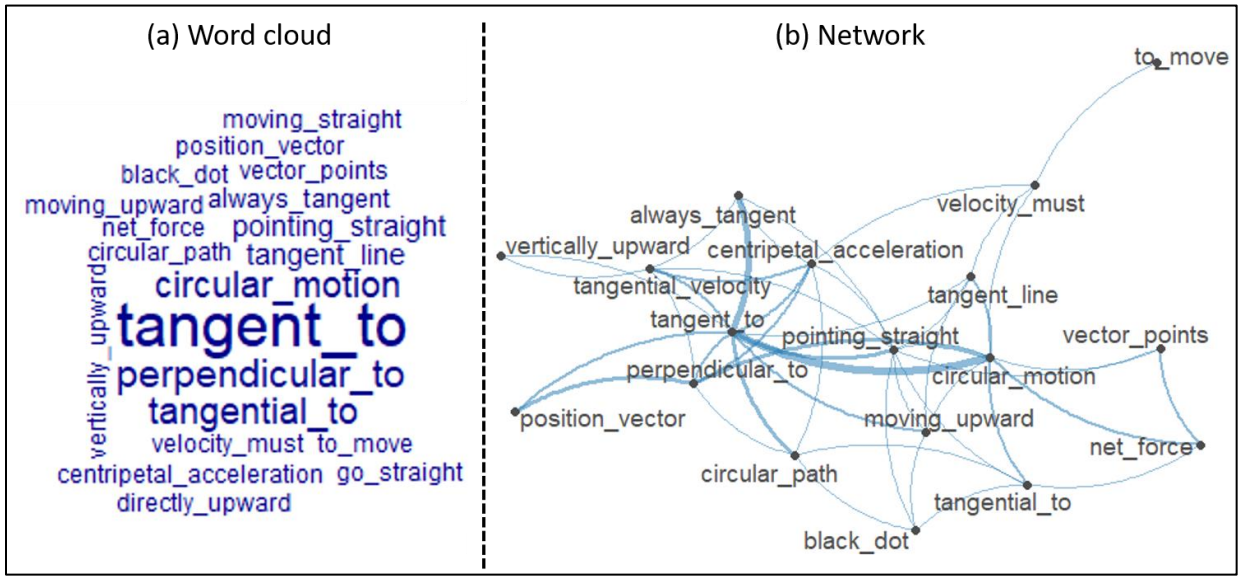


Figure 15.1. An example of (a) a word cloud and (b) a network of co-occurrence of words. On the word cloud, the phrase "tangent to" is the most common. On the network, the phrases "tangent to" and "circular motion" tend to appear together in student explanations.

In this dissertation, we use word frequency tables to show the common words or phrases used by students in their explanations. It is enough to represent the key ideas students used.

15.3.3 Quantitative analysis in NLP

The quantitative analysis in NLP we used is called cosine similarity. It is one method to compare student explanations. The definition of cosine similarity is:

$$\cos\theta = \frac{\boldsymbol{A}\cdot\boldsymbol{B}}{\|\boldsymbol{A}\|\|\boldsymbol{B}\|} = \frac{\sum_{i=1}^{n} A_i B_i}{\sqrt{\sum_{i=1}^{n} A_i^2}\sqrt{\sum_{i=1}^{n} B_i^2}}$$

where vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ are called document vectors which the basis vectors are the words or phrases used in the document and the coefficients A_i and B_i are the term frequencies, which is the number of times the word appears in the document. The value of $\cos\theta$ ranges from 0 to 1, with 0 means the two documents are not the same at all, and 1 means the two documents are the same. In the context of comparing student explanations, suppose one student answers and explains in two different tasks P and Q, then vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ are made up of the words used in the explanations for tasks P and Q, respectively.

The value of cosine similarity is dependent on the words that appear in both vectors. If the words are commonly found in both vectors, then the cosine similarity value will be closer to 1. If those common words represent certain physics concepts, and if two groups of explanations have high cosine similarity value (0.7-1), then it is highly probable that students are using the same physics ideas to explain their answers. Conversely, if the cosine similarity value is low (0-0.3), and the commonly used words/phrases represent different physics concepts. The two groups of students we are comparing are using different ideas to explain their answers. Note: These ranges are arbitrary that we defined, so more studies may be needed to justify them.

15.4 USING NLP TO DETERMINE SIMILARITY OF STUDENT EXPLANATIONS

In this section, we describe how we use NLP to determine the similarity of student explanations. The context of this work is based on the additional investigation in part II, Chapter

8. In that investigation, the student tendency to use the tangent of the shape of an object to determine the velocity of a point on the object was probed. The details can be found in section 8.1. Through the investigation, the student approaches, as interpreted from their explanations, showed some similarities. We figured that NLP techniques might help to corroborate our observations.

In this section, we first describe and summarize the results from that investigation. Then we proceed to describe the NLP procedures that allow us to determine the similarity of explanations and present the results.

15.4.1 Summary of results of the investigation

The three tasks that we administered were shown in Figure 15.2. They are Task 1A, Task 1B, and Task 2 (RMT-Direction). Task 1A and Task 1B were administered to students from two different lecture sections in week 8. Task 1A consists of a picture of a cycloid that represents the motion of a point particle. Task 1B consists of both a rolling cylinder and a cycloid that represents the motion of a point on it. Both groups were asked to find the direction of the instantaneous velocity at point E. These two groups of students were administered the Task 2 one or two weeks later, in which they saw a cylinder that is rolling without slipping and asked to find the direction of the instantaneous velocity point E at that instant shown. These tasks were administered to PHYS 121 students in both Sp'18 and Sp'19 quarters.

As discussed in section 8.1, the results in both quarters were similar, so the combined results are shown in Figure 15.3. Our results showed that most students were consistent in their approaches to these tasks. For example, most of the students who did Task 1A and Task 2 chose answers that were tangent to the cycloid (rightward arrow pointing 45° above the horizontal) on

Task 1A and tangent to the circle (downward arrow) on Task 2. Then for students who did Task 1B and Task 2, we found most students who chose the downward arrow (CM frame answer) on Task 2 chose answers that were either tangent to the cycloid or to the circle on Task 1B. We examined the student explanations in two tasks (Task 1A vs. Task 2 and Task 1B vs. Task 2) and found that the approaches were consistent.

We used "CM frame" to label the arrow that is tangent to the circle at the point of interest for convenience. This label does not imply that students are indeed using the CM frame to arrive at the answer.

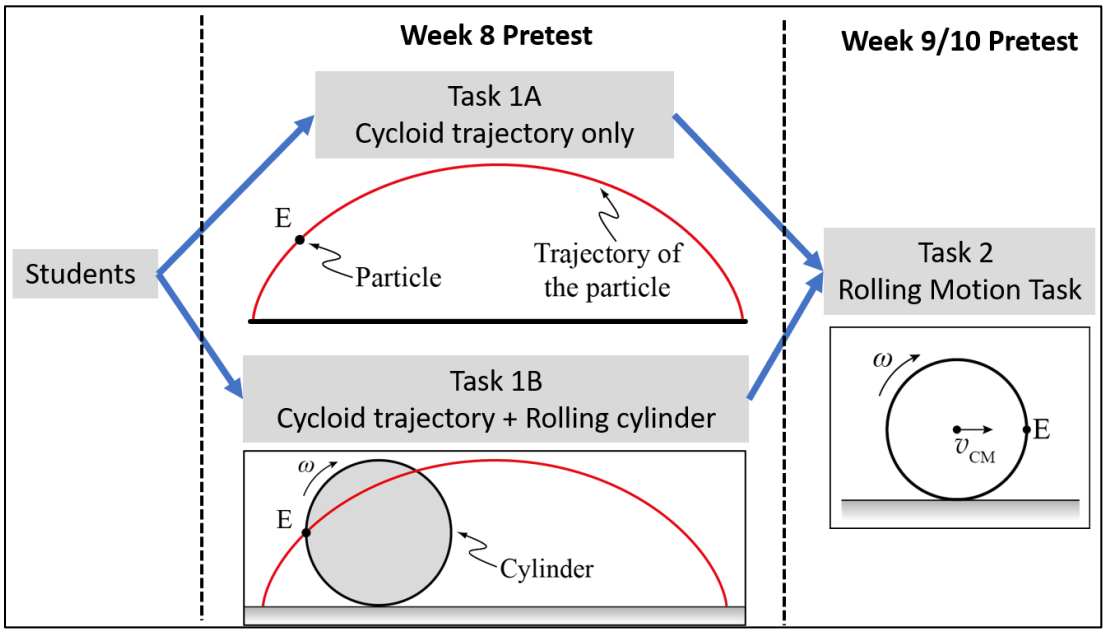


Figure 15.2. The schematics of the study design to find out the prevalence of students using the given curve to determine the direction of velocity.

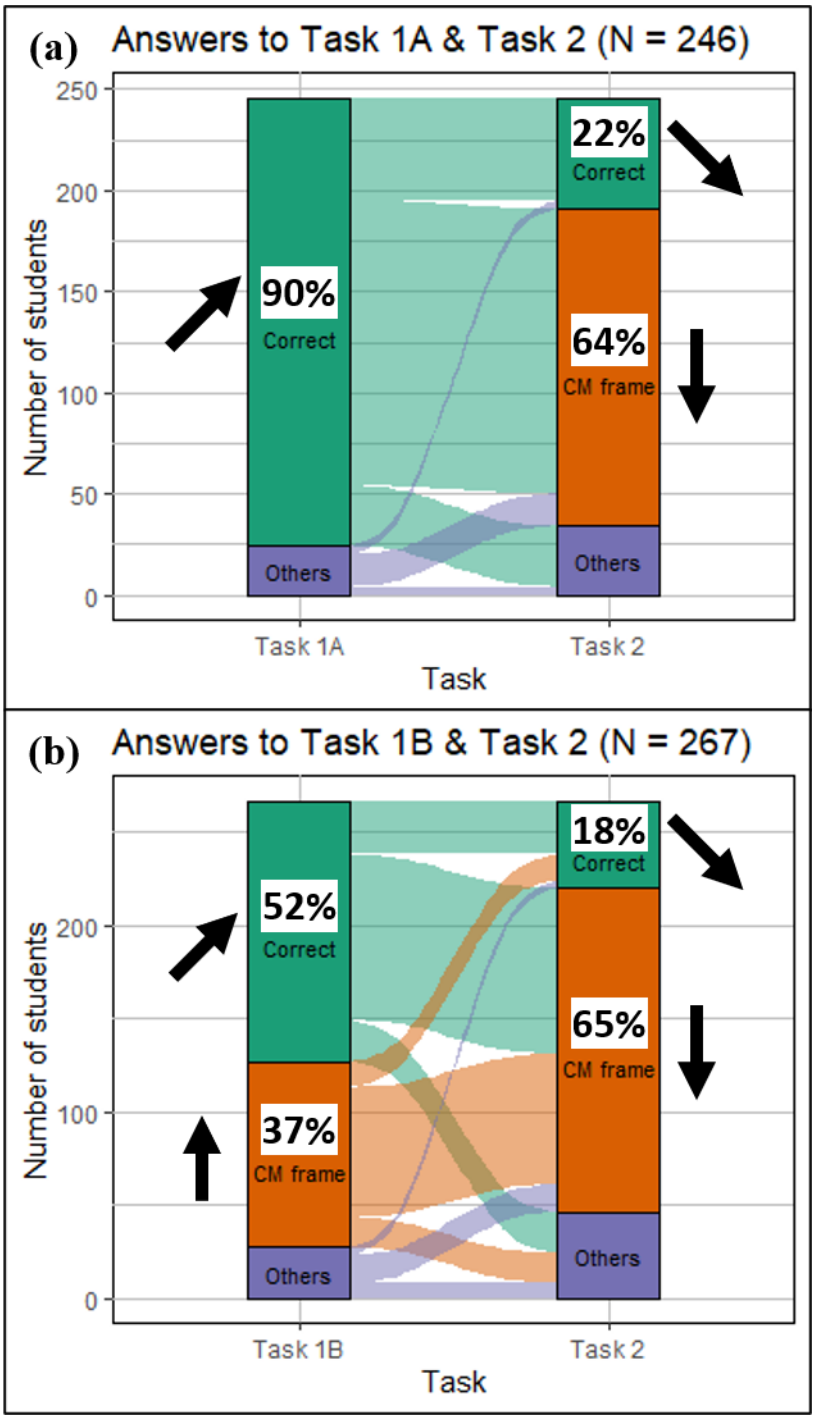


Figure 15.3. The results of investigating student tendency to use the tangent to the shape of the given object to determine the velocity of a point. The results from the Sp'18 and Sp'19 quarters are combined.

15.4.2 Methodology

The procedures on how we apply NLP in our investigation have been described in the previous section. The NLP metric to determine the similarity of student explanations is the cosine similarity method. We were interested in determining the extent to which the cosine similarity method would corroborate the results from the manual analysis. Based on the results presented in Figure 15.3, we performed text analysis on the student explanations to the correct option on Task 1A, as well as the correct and CM frame options on both Task 1B and Task 2.

For each group of student explanations, we first compiled the list of two-word phrases that were used. Two-word phrases were chosen because they represented student ideas better than single words. Some phrases, such as "instantaneous velocity" and "point E," were removed because they appeared in the question setup, and some students were using them in their explanations. The two-word phrases that were removed and the question texts used in each task are presented in Table 15.115.

Table 15.115. The two-word phrases that are removed in our computational text analysis.

Task	**Question text**	**2-word phrases removed**
Task 1A (Cycloid only)	When the particle is at the position shown on the diagram (point E), which of the following options best represents the direction of its instantaneous velocity?	Instantaneous velocity, point E, velocity vector
Task 1B (Cycloid and cylinder)	When the black dot is at point E shown on the diagram, which of the following options best represents the direction of its instantaneous velocity?	Instantaneous velocity, point E, velocity vector
Task 2 (Cylinder only)	For the instant shown, which arrow at right best shows the direction of the instantaneous velocity of point E on the hoop in the **Earth's reference frame**?	Instantaneous velocity, point E, reference frame, velocity vector

Then, we calculated the pairwise cosine similarity of explanations using the list of two-word phrases compiled in the previous step. For example, the cosine similarity between Task 1A (Correct) and Task 2 (Correct) is calculated using the two-word phrases given by students in their explanations. This step is comparing the similarity of the two-word phrases used in the explanations; some students who were in Task 1A (Correct) might not be in Task 2 (Correct).

The next step is to compare the similarity of the two-word phrases used in the explanations according to students' answers on both tasks. We performed both manual analysis and cosine similarity calculations. We focused our attention on three groups of students, which are: (1) students who answered Tasks 1 and 2 correctly, (2) those who answered Task 1 correctly but chose the "CM frame" answer on Task 2, and (3) those who answered "CM frame" answer on both Tasks 1 and 2. The manual, qualitative comparison was based on whether the researcher (we) noted the idea of "tangent" in the student explanations for both tasks.

One thing to note is that the manual comparison is made at the individual level, while the cosine similarity method is done at the group level. For example, we compare student A's explanations manually by looking at whether that student is using the idea of "tangent" in both tasks. We do not calculate the cosine similarity of the student A's explanations to both tasks. Suppose 20 students answered both Tasks 1 and 2 correctly, we calculate the cosine similarity of the two-word phrases used by these 20 students in their explanations on both tasks.

We included the R code that we used to analyze the similarity of student explanations in Appendix E.

15.4.3 Results and discussions

Based on our methodology, there are three separate results to be presented. They are: (1) the commonly used two-word phrases in each explanation, (2) the pairwise cosine similarity values of the two-word phrases used in each explanation, and (3) the comparison between manual analysis and cosine similarity values.

First, the top six commonly used two-word phrases in the explanations for the selected answers to Tasks 1A, 1B, and 2 are shown in Figure 15.4. The phrases "tangent to," "tangent line" and "tangential to" are commonly used by students in Task 1A (Correct), Task 1B (Correct), Task 1B (CM frame), and Task 2 (CM frame). These phrases are relevant for explaining Task 1A (Correct) and Task 1B (Correct) when students use the cycloid to find the direction of velocity at point E. For Task 1B (CM frame) and Task 2 (CM frame), the phrases mentioned above, as well as "circular motion" and "perpendicular to," suggests that students are using the tangent of the circle to determine the direction of instantaneous velocity, which corroborates what we observed in student explanations through manual reading. The use of the phrase "circular motion" in Task 1A (Correct) suggests that some students may view the cycloid as a circle.

For Task 2 (Correct), the top two commonly used phrases are "tangential velocity" and "translational velocity." This result supports our qualitative interpretations of student explanations that students who answered Task 2 correctly are adding the translational velocity of the wheel and the tangential velocity of point E at the center-of-mass frame to arrive at the answer.

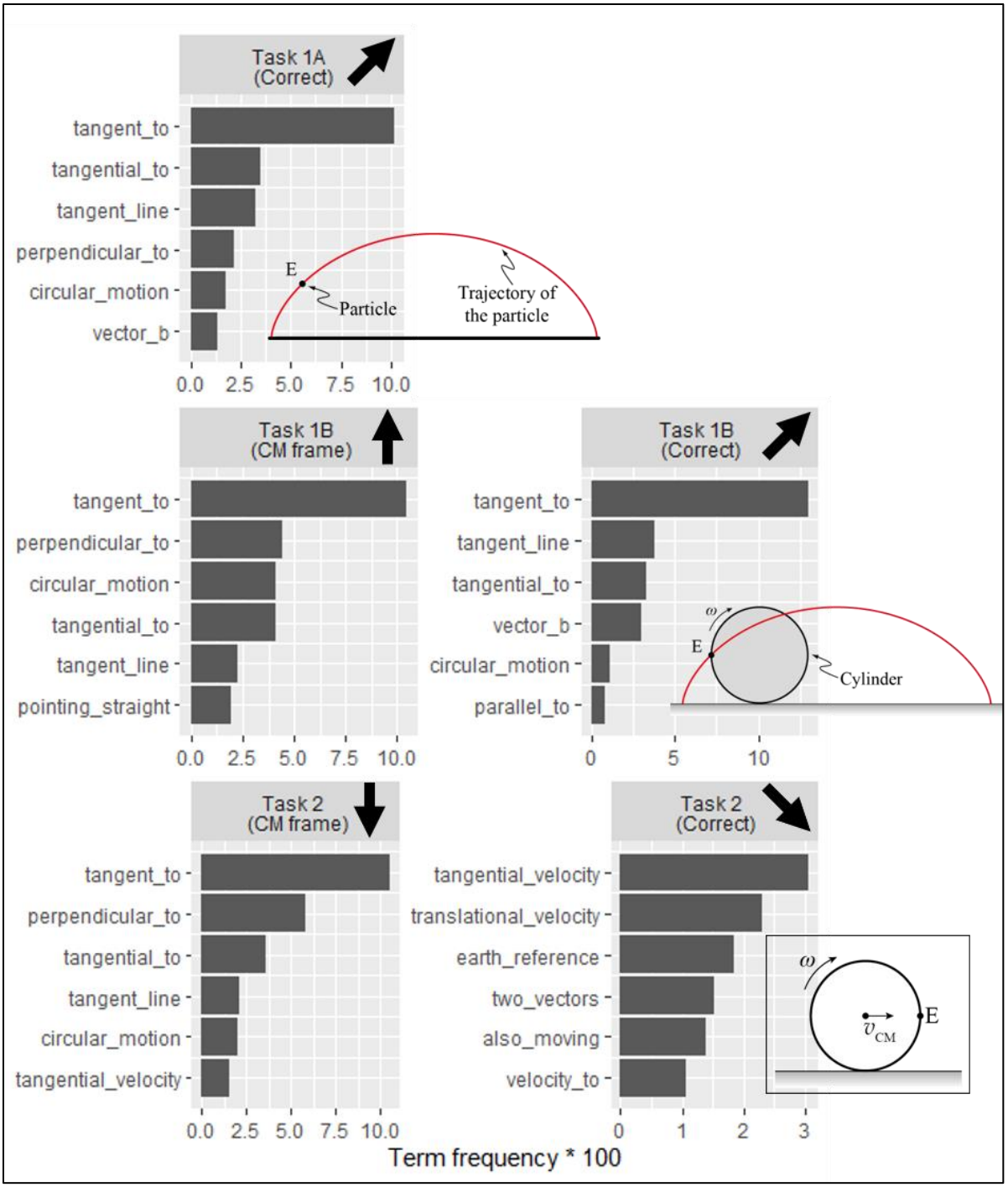


Figure 15.4. The top six commonly used two-word phrases in the explanations for selected answers in Task 1A, Task 1B, and Task 2.

Next, the pairwise cosine similarity values are computed to find out how similar the explanations are in different answers. The pairwise similarity values for two-word phrases are tabulated in Table 15.116. The values are rounded to the closest 0.05. In our investigation, we think that cosine similarity values of 0.70 and above have a higher likelihood to have the *same* ideas or concepts, and cosine similarity values of 0.30 and below have a higher likelihood to have *different* ideas or concepts. Cosine similarity values that are between 0.30 and 0.70 are not easy to discuss. So, we only discuss pairwise similarity values that are higher than 0.70 or lower than 0.30.

Table 15.116. Pairwise cosine similarity values with two-word phrases that are used in the explanations to the selected answers in Tasks 1A, 1B, and 2. The values are rounded to the closest 0.05.

	Task 1B (CM frame)	Task 1B (Correct)	Task 2 (CM frame)	Task 2 (Correct)
Task 1A (Correct)	0.85	0.90	0.85	0.25
Task 1B (CM frame)	-	0.80	0.85	0.25
Task 1B (Correct)	-	-	0.80	0.20
Task 2 (CM frame)	-	-	-	0.35

From Table 15.116, six out of ten of the pairwise cosine similarity values are equal to or greater than 0.80. They are calculated using the two-word phrases found in the explanations from Task 1A (Correct), Task 1B (Correct), Task 1B (CM frame), and Task 2 (CM frame). These values are expected since the explanations exhibit similar ideas, such as “tangent to curve,” “perpendicular to,” and “circular motion,” as seen from Figure 15.4.

The other four pairwise cosine similarity values ranged from 0.20 to 0.35. These are calculated using the two-word phrases found in the explanations from Task 2 (Correct) with the other four lists of two-word phrases. These values are lower than the other six values. The low values can be interpreted in two ways: (1) the ideas demonstrated in the explanations from Task 2 (Correct) are different from the other explanations or (2) there are more ideas illustrated in the explanations from Task 2 (Correct) than the other explanations. From Figure 15.4, we conclude that it is the latter that contributes to the low pairwise cosine similarity values.

Third, we compare how well the text analysis corroborates the findings from the manual comparison. In this case, we looked at the explanations from three groups of students in the study: students who answered both tasks correctly, students who answered Task 1 (A and B) correctly and chose the downward arrow in Task 2, and students who chose answers that correspond to the CM frame.

The manual comparison was based on whether we noted the idea of "tangent" in the student explanations for both tasks. The cosine similarity is calculated using the two-word phrases given by students in their explanations for both tasks. The results are shown in Table 15.117. The manual, qualitative analysis, suggests that about 50% of the students who chose the answer consistent with CM frame in either Task 1 or 2 used similar ideas to explain their answers. However, about 20% of students answered both tasks correctly used similar ideas. The cosine similarity values indicate the same trend; the value for students who answered both tasks correctly is lower than that of students who chose the CM frame answer in either task.

Table 15.117. The similarity of student explanations between qualitative methods and cosine similarity. (T1 indicates Task 1A/B and T2 indicates Task 2.)

Category	*N*	Manual analysis	Cosine similarity
T1 (Correct) → T2 (Correct)	80	0.20	0.15
T1 (Correct) → T2 (CM)	230	0.50	0.85
T1 (CM) → T2 (CM)	70	0.50	0.80

15.5 SUMMARY AND FUTURE WORK

In this study, we compared the results of a manual coding scheme and a computer-based text analysis to assess student explanations on two related physics tasks involving rolling motion. The manual coding had been conducted early in the study before we came up with the idea of using natural language processing. That analysis had suggested that many students use the idea of "tangent to a curve" inappropriately to find the direction of motion of a point on an object that is both rotating and translating. The answers and explanations suggested that students are often unable to apply the concept of relative motion to rolling motion and inappropriately use ideas from two-dimensional motion.

The computer-based text analysis was coded to look at the overall use of terms by students in their explanations. Differences in student responses on the various questions were probed by looking at the frequency of the two-word phrases used written by students. The results from this computer-based text analysis were quite similar to those from the manual coding. This finding supports the idea that the effect found by manual coding is a real signal that does not reflect an implicit bias by the researcher.

If the similarity between the two methods is borne-out in future studies, it would suggest that computer-based textual analysis is a tool that can support other forms of qualitative analyses.

It can also help researchers who are working alone at their institutions obtain independent corroboration of their interpretations of student explanations.

We plan to continue exploring the use of computer-based text analysis to assess student explanations in the future. Some fundamental question about the method remains. For example, to what extent do the results depend on whether 1, 2, or 3-word phrases are used? We are also interested in the possibility of using the method to 'mine' large repositories of student explanations. The method could allow for quick comparisons of student explanations to related questions and help identify tasks that probe student thinking in ways that have not previously been identified.

We do not anticipate that computational text analysis will become a substitute for manual, qualitative text analysis in PER. Certain aspects of student-written responses are likely not readily captured by algorithms. Also, a current limitation is the inability of this method to interpret student problem-solving strategies when equations are involved. Nonetheless, the method appears to have promise in analyzing large quantities of student explanations and, if applied carefully, can be used as a tool to help inform instructors and researchers about student conceptual understanding.

CONCLUSION FOR PART IV

In part IV of this dissertation, we explored two different uses of technology in physics education research. The first is about the use of online quizzes to help students learn vector superposition in algebra-based physics courses. The second is about the use of natural language processing (NLP) to compare student explanations on two related physics tasks.

Study 1: In the first study, we extended a previous investigation of student ability to conduct vector superposition from calculus-based physics courses to include algebra-based physics courses. We performed a longitudinal study by asking students several vector superposition questions on online quizzes through the quarter. With the usual lecture and tutorial instruction in PHYS 114, student performance started low (around 15% on the first pretest), improved somewhat, and stayed flat for much of the quarter at 35%, then increased to around 50% on the final exam. The results on some vector superposition tasks from the algebra-based courses were comparable than the results from the calculus-based courses, but lower on others.

Several online interventions were tried over several quarters to help the students in algebra-based physics courses improve their ability to reason about vector superposition. The relative success of each intervention was determined by looking for a better trend of progression and better results on the final exam. After three quarters in which different interventions were used, one was created that seems to work relatively well. This particular intervention provides elaborated feedback to students each time they answer a vector superposition task incorrectly. This intervention scheme was based on previous work by Mikula and Heckler. They reported that sequences of questions using a combination of different kinds of feedback, such as knowledge of results (user is informed of whether the answer is correct or incorrect), knowledge of correct

response (user is informed of the correct answer), and elaborated feedback (detailed explanation), had the largest, positive impact. The results were replicable across several quarters.

Besides, we used generalized linear mixed models to analyze our longitudinal data. We tested five different models. Based on the best model, we found that the intervention with elaborated feedback, students' initial background on vectors, and the correctness of the first pretest increases the odds of students answering the vector superposition tasks later in the quarter significantly.

The results are encouraging. They suggest that this intervention method may be useful in improving students' other essential skills such as graphing and scaling.

Study 2: In the second study, we were interested in determining whether Natural Language Processing (NLP), which has been used in science education research, could be used in conducting physics education research. We applied NLP to compare student explanations on two related physics tasks, which were in the context of motion in two-dimension and rolling motion. We summarized the student explanations by presenting the commonly used 2-word phrases. The similarity of student explanations was determined by using the cosine similarity of the 2-word phrases used in the explanations. We found that the computed values were quite similar to those from the manual coding.

The potential to use NLP in physics education research is vast. For example, many research groups at the UW and elsewhere have collected a great deal of student textual data (e.g., through online pretests). In addition to the NLP techniques described here, many others have been developed that could analyze these textual data. We believe NLP can help physics education researchers to analyze this large amounts of textual data, reducing the time they needed to go

through the texts one by one, and generating graphical representations that researchers can use to summarize the relevant features in the data and look for patterns in student thinking and learning.

Chapter 16. CONCLUSION AND FUTURE WORK

The study of the motion of objects is central to physics. Students learn how to describe the motions of point particles, rigid bodies, and (sometimes) fluids in a typical introductory physics course. The early physics education research focuses on student difficulties in linear motion, while relatively little research documenting the student difficulties in rigid body motion and fluid motion. This dissertation revisits and reproduces the student difficulties that have been reported, and in some cases, the difficulties persist through the course. Additional insights and difficulties are discovered through further investigations. The insights obtained from the investigation of student difficulties in rolling motion have led to the development of a tutorial curriculum to address the difficulties.

Besides that, this dissertation includes a study on student ability with vector superposition in algebra-based physics courses. The findings from prior research guided the development of a series of online pretests to help students learn vector superposition in algebra-based physics courses. Lastly, this dissertation documents a preliminary exploration of the use of natural language processing for comparing the similarity of student explanations.

In this chapter, we summarize the crucial findings and discuss how the results have guided the development of instructional materials. Some future works are proposed.

16.1 SUMMARY OF SPECIFIC DIFFICULTIES

One major area in the study of physics involves describing motions of different kinds of objects, such as point particles, rigid bodies, and fluids. The first three parts (out of four) of the dissertation are dedicated to documenting the specific difficulties in particle motion, rigid body motion, and fluid motion, respectively. In part IV, we address student difficulties in vector superposition.

I Student difficulties in particle motion

In the first part of the study, students have difficulty in distinguishing average and instantaneous velocity in contexts in which the acceleration of the entire motion is not constant. The common mistake students made is to calculate the instantaneous speed at the final instant of an accelerated motion using the average speed of the accelerated motion. Furthermore, additional investigations reveal that students tend to reason sequentially (from earlier to later times), and many do not reflect on the motion during each stage of the motion. This difficulty persists in introductory mechanics courses, based on the results from isomorphic tasks given throughout the quarter.

The students' ability to represent the motion graphically and relate both the graphical and mathematical representations are also investigated. Not many students can graph the motion in each stage of the motion, and for those who can, they often fail to relate the graphical representation to their numerical calculations. The results from this investigation suggest that students tend to apply formulae indiscriminately without comparing the results to the corresponding motion of the object.

II Student difficulties in rolling motion

Several student difficulties in rigid body motion have been reported by prior research. First, students have difficulty applying relative motion concepts to rolling motion situations. In a rolling wheel context, when students are asked to determine the direction of the velocity of a point on the rim of the wheel, they tend to choose vectors that are tangential to the circle. One interpretation of this difficulty is the preferred frame reasoning, as described by Boudreaux. Since the direction of velocity in the center-of-mass (CM) frame is tangential to the circle, the student tendency to select that answer suggests that students are viewing the motion of the rolling wheel in its CM frame. Our findings show that this difficulty is ubiquitous among students who first encounter rolling motion. We have also found that students can answer simple relative motion problems but unable to apply the same ideas to rolling motion.

The second difficulty is related to the frictional forces that are involved in rolling without slipping situations. Students tend to choose the direction of frictional force such that it is in the opposite direction to the direction of motion. They do not consider the changes in the motion of the rolling object to reason out the direction of the net force and use that to determine the direction of the frictional force. For example, a wheel rolls to the right with constant speed on a surface, students tend to say the frictional force acting on the wheel is to the left. This difficulty is also prevalent in graduate student teaching assistants.

The third student difficulty is with the rolling without slipping condition. For example, in the board-on-cylinder context, as the board is pushed and the cylinder rolls without slipping with the board and the ground, many students think that both objects will move at the same speed. The board moves twice the speed of the cylinder because of the rolling without slipping condition. Other difficulties are also found in this context by analyzing students' explanations. They are (1)

tendency to treat force as partially or entirely "used up" in rotational motion and (2) tendency to treat the translational/rotational partition of kinetic energy as having a direct causal influence on the translational motion. The last two difficulties have been reported by Hunter *et al.* in the context of the dynamics of rigid bodies.

III Student difficulties in fluid motion

Several difficulties are identified in the investigation into student difficulties in fluid motion. We only focus on the difficulties involved when students apply continuity and Bernoulli's equations. Some are consistent with what had been reported, and others had not been reported.

Two reported difficulties were found among our students. They are (1) the tendency to use hydrostatic pressure to explain for the changes in water pressure for flows in vertical pipelines, and (2) the tendency to assume that fluid velocity increases due to gravitational acceleration in vertical pipes of uniform cross-section. These two difficulties are prevalent among our students. Our results suggest that students find vertical fluid flow contexts more difficult than horizontal flow contexts.

The other difficulties that have not yet reported on literature are mostly related to when students are asked to use Bernoulli's equation to compare pressures of two points in a pipe with changing cross-sectional areas. For example, some students use the pressure formula $P = F/A$ indiscriminately. They tend to assume the force in the formula to be constant. Another kind of difficulty involves oversimplification or overgeneralization of the results obtained from Bernoulli's equation. For instance, after using Bernoulli's equation, the pressure is greater at regions with the greater cross-sectional area, some students just correlate greater pressure with greater area without going through the proper steps.

IV Student difficulties in vector superposition

The student difficulties in vector superposition have been well-documented by Kanim, Nguyen and Meltzer, Knight, Heckler and Scaife, and Flores. The difficulty we focus on is that students tend to add vectors just like scalars. They do not consider the directions of the vectors when they add vectors.

16.2 IMPLICATIONS FOR INSTRUCTION AND THE DEVELOPMENT OF CURRICULUM

The prior research and our investigation into the student understanding of motions and vector superposition have guided the design of instructional materials. Most difficulties have been highlighted by prior research. Our work verifies what has been reported and identifies additional difficulties in areas such as rolling motion and fluid motion. The persistence of difficulties is also investigated, for example, failure to distinguish between average and instantaneous speeds.

We used two different approaches to address student difficulties in particle motion, rolling motion, and vector superposition. The first approach is through the development of a tutorial curriculum to address student difficulties in particle motion and rolling motion. The second approach is using a series of online pretests to improve student ability with vector superposition.

In the context of particle motion, we developed one homework question to help students distinguish between average speed and instantaneous speed. In the context of rolling motion, we have developed a tutorial curriculum (50-minute in-class worksheet and tutorial homework) to address the specific difficulties described previously. The learning objectives of the rolling motion tutorial are designed based on the specific difficulties that are identified. Our additional investigation reveals that students do better on the rolling wheel context after students were given the trajectory or asked to consider the displacement or trajectory of the point on the rim of the

wheel. This finding has been incorporated in the modification of the tutorial curriculum to help students make connections between concepts developed from motion in two-dimension and relative motion.

In the context of vector superposition, we developed a series of online pretests (four of them) in the algebra-based physics courses. Based on the results reported from prior research, the intervention questions on the pretests were designed by incorporating different kinds of computerized feedback. The feedback focuses on teaching students how to add vectors using the tip-to-tail method, and it is targeted to students who answered the vector superposition tasks incorrectly. They receive more guidance from more pretests weekly if they need it.

16.3 ASSESSMENT OF STUDENT LEARNING

We have tested the instructional materials in the classroom with the populations for whom they are intended.

For the rolling motion tutorial, the effects on student learning have been assessed. Identical examination questions were used in posttests so that we can compare the results after all instruction that includes the tutorial with that without the tutorial. In both the preliminary and modified versions of the rolling motion tutorial, the student performance in some posttest questions is significantly better than that after traditional instruction alone. In some cases, the performance of introductory students after tutorial instruction is comparable to that of physics graduate students. In other cases, the gain in student learning is mixed. Based on the posttest results obtained, we only managed to address one (out of three) learning objective successfully.

In the vector superposition context, students in the intervention group performed better on the given tasks as they progressed through the course. They also did better on the final exam as

compared to the no-intervention group. We believe this is a small success in improving student ability with vector superposition.

16.4 QUESTIONS FOR FURTHER RESEARCH

The study of motion is central to physics. Students encounter difficulties when they are taught how to describe them. Some of those difficulties have been documented, while some are identified in this investigation. This dissertation also explores the persistence of some reported student difficulties. In some cases, we designed instructional strategies to address them. However, not all the difficulties we described in this dissertation were addressed.

Another aspect of this dissertation is the use of technology in physics education research. First, we explore the use of online pretests to improve student ability with vector superposition. Second, we explore the use of natural language processing to compare the similarity of student explanations. These two areas have the potential to spur further research.

Here are some additional research questions that could be explored. They are: (1) What instructional strategies can promote student ability to perform consistency checks between graphical and numerical representations of motion? (2) How can one assess the effectiveness of tutorial homework questions? (3) What kind of instructional strategies can be used to address student difficulties in fluid mechanics in introductory physics? (4) To what extent can online pretests with targeted feedback improve students' essential physics skills such as graphing and scaling? (5) To what extent can natural language processing help in analyzing textual data in physics education research?

This dissertation describes two main studies. The first is an effort to investigate student understanding of the motion of different kinds of objects – particle, rigid body, and fluid. The

second study presents preliminary results from two areas: one consisting of an effort to improve student ability with vector superposition in physics courses and the second from preliminary use of natural language processing techniques to examine student explanations in physics education research. On the one hand, we conduct research on student difficulties, develop and test instructional materials, and assess the effects on student learning. On the other hand, we also explore different methods to analyze our textual data. We hope that this work can highlight the persistence of specific student difficulties in physics courses, contribute to the continuous improvement in how physics is taught, as well as open up more data analysis methods of the relevant data.

BIBLIOGRAPHY

1. McDermott, L.C., M.L. Rosenquist, and E.H. Van Zee, *Student difficulties in connecting graphs and physics: Examples from kinematics.* American Journal of Physics, 1987. **55**(6): p. 503-513.

2. Shaffer, P.S. and L.C. McDermott, *A research-based approach to improving student understanding of the vector nature of kinematical concepts.* American Journal of Physics, 2005. **73**(10): p. 921-931.

3. Trowbridge, D.E. and L.C. McDermott, *Investigation of student understanding of the concept of velocity in one dimension.* American Journal of Physics, 1980. **48**(12): p. 1020-1028.

4. Trowbridge, D.E. and L.C. McDermott, *Investigation of student understanding of the concept of acceleration in one dimension.* American Journal of Physics, 1981. **49**(3): p. 242-253.

5. Thornton, R.K. and D.R. Sokoloff, *Assessing student learning of Newton's laws: The force and motion conceptual evaluation and the evaluation of active learning laboratory and lecture curricula.* american Journal of Physics, 1998. **66**(4): p. 338-352.

6. Halloun, I.A. and D. Hestenes, *The initial knowledge state of college physics students.* American journal of Physics, 1985. **53**(11): p. 1043-1055.

7. Hestenes, D., M. Wells, and G. Swackhamer, *Force concept inventory.* The Physics Teacher, 1992. **30**(3): p. 141-158.

8. Mashood, K. and V.A. Singh, *An inventory on rotational kinematics of a particle: unravelling misconceptions and pitfalls in reasoning.* European Journal of Physics, 2012. **33**(5): p. 1301.

9. Mashood, K. and V.A. Singh, *Rotational kinematics of a particle in rectilinear motion: Perceptions and pitfalls.* American Journal of Physics, 2012. **80**(8): p. 720-723.

10. Mashood, K. and V. Singh, *Variation in angular velocity and angular acceleration of a particle in rectilinear motion.* European Journal of Physics, 2012. **33**(3): p. 473.

11. Mashood, K. and V.A. Singh, *Rotational kinematics of a rigid body about a fixed axis: development and analysis of an inventory.* European Journal of Physics, 2015. **36**(4): p. 045020.

12. Rimoldini, L.G. and C. Singh, *Student understanding of rotational and rolling motion concepts.* Physical Review Special Topics-Physics Education Research, 2005. **1**(1): p. 010102.

13. López, M.-L., *Angular and linear acceleration in a rigid rolling body: students' misconceptions.* European Journal of Physics, 2003. **24**(6): p. 553.

14. De Ambrosis, A., et al., *Investigating the role of sliding friction in rolling motion: a teaching sequence based on experiments and simulations.* European Journal of Physics, 2015. **36**(3): p. 035020.

15. Fang, N. and J. Uziak, *Student Misconceptions of General Plane Motion in Rigid-Body Kinematics*. 2018, American Society of Civil Engineers.

16. Bedford, D. and R. Lindsay, *A misinterpretation of Bernoulli's theorem.* Physics Education, 1977. **12**(5): p. 311.

17. Suarez, A., et al., *Students' conceptual difficulties in hydrodynamics.* Physical Review Physics Education Research, 2017. **13**(2): p. 020132.

18. Brown, S., et al., *Engineering Students' Fluid Mechanics Misconceptions: A Description and Theoretical Explanation.* International Journal of Engineering Education, 2017. **33**(4): p. 1149-1162.

19. Fraser, D.M., et al., *Enhancing the learning of fluid mechanics using computer simulations.* Journal of Engineering Education, 2007. **96**(4): p. 381-388.

20. Martin, J., J. Mitchell, and T. Newell. *Development of a concept inventory for fluid mechanics*. in *Frontiers in Education, 2003. FIE 2003 33rd Annual*. 2003.

21. Martin, J.K., J. Mitchell, and T. Newell, *Work in progress: Analysis of reliability of the Fluid Mechanics Concept Inventory*. 2004.

22. Mary Katherine, W., et al., *Refinement of a Concept Inventory to Assess Conceptual Understanding in Civil Engineering Fluid Mechanics*. ASEE Conferences: Seattle, Washington.

23. Bell, B.A., *Pretest-posttest design.* Encyclopedia of research design, 2010: p. 1087-1092.

24. McDermott, L. and P. Shaffer, *the Physics Education Group at the University of Washington, Tutorials in Introductory Physics, Custom Edition: Updated Preliminary 2nd ed., 2011–2012*. 2012, Prentice-Hall, Upper Saddle River.

25. Salkind, N.J., *Encyclopedia of research design*. Vol. 1. 2010: Sage.

26. *MasteringPhysics*. 03/20/2019]; Available from: https://www.pearsonmylabandmastering.com/northamerica/masteringphysics/.

27. *Learning Catalytics*. 03/20/2019]; Available from: https://www.pearson.com/us/higher-education/products-services-teaching/learning-engagement-tools/learning-catalytics.html.

28. Knight, R.D., B. Jones, and S. Field, *College Physics*. 2015: Pearson Education UK.

29. Tipler, P.A. and G. Mosca, *Physics for scientists and engineers*. 2007: Macmillan.

30. Mazur, E., et al., *Principles & practice of physics*. 2015: Pearson.

31. McDermott, L.C. and P.S. Shaffer, *the Physics Education Group at the University of Washington, Tutorials in Introductory Physics*. 2002.

32. McDermott, L.C., *Oersted medal lecture 2001:"Physics Education Research—the key to student learning".* American Journal of Physics, 2001. **69**(11): p. 1127-1137.

33. McDermott, L.C. and others, *Physics by inquiry: An introduction to physics and the physical sciences*. Vol. 1. 1995: Wiley.

34. Heron, P.R.L., *Identifying and Addressing Difficulties: Reflections on the empirical and theoretical basis of an influential approach to improving physics education.* Reviews in PER Vol. 2, 2018.

35. Wadsworth, B.J., *Piaget's theory of cognitive and affective development: Foundations of constructivism*. 1996: Longman Publishing.

36. McDermott, L.C., *Millikan Lecture 1990: What we teach and what is learned—Closing the gap.* American Journal of Physics, 1991. **59**(4): p. 301-315.

37. Shaffer, P.S. and L.C. McDermott, *Research as a guide for curriculum development: An example from introductory electricity. Part II: Design of instructional strategies.* American Journal of Physics, 1992. **60**(11): p. 1003-1013.

38. *The SAGE Encyclopedia of Communication Research Methods.* 2017.

39. Rosner, B., *Fundamentals of biostatistics*. 2015: Nelson Education.

40. Grissom, R.J. and J.J. Kim, *Effect sizes for research: A broad practical approach*. 2005: Lawrence Erlbaum Associates Publishers.

41. Rosenthal, J.A., *Qualitative descriptors of strength of association and effect size.* Journal of social service Research, 1996. **21**(4): p. 37-59.

42. *VassarStats: Website for Statistical Computation*. Available from: http://vassarstats.net/tab2x2.html.

43. Theobald, E., *Students are rarely independent: When, why, and how to use random effects in discipline-based education research.* CBE—Life Sciences Education, 2018. **17**(3): p. rm2.

44. Theobald, R. and S. Freeman, *Is it the intervention or the students? Using linear regression to control for student characteristics in undergraduate STEM education research.* CBE—Life Sciences Education, 2014. **13**(1): p. 41-48.

45. Team, R.C., *R: A language and environment for statistical computing.* 2013.

46. Otero, V.K. and D.B. Harlowe, *Getting started in qualitative physics education research.* 2009.

47. Chi, M.T., *Quantifying qualitative analyses of verbal data: A practical guide.* The journal of the learning sciences, 1997. **6**(3): p. 271-315.

48. Laird, N.M., *Missing data in longitudinal studies.* Statistics in medicine, 1988. **7**(1‐2): p. 305-315.

49. Fitzmaurice, G.M., N.M. Laird, and J.H. Ware, *Applied longitudinal analysis*. Vol. 998. 2012: John Wiley & Sons.

50. Nissen, J., R. Donatello, and B. Van Dusen, *Missing data and bias in physics education research: A case for using multiple imputation.* arXiv preprint arXiv:1809.00035, 2018.

51. Heron, P.R.L., *Effect of lecture instruction on student performance on qualitative questions.* Physical Review Special Topics-Physics Education Research, 2015. **11**(1): p. 010102.

52. Piaget, J., *Child's conception of movement and speed*. 2013: Routledge.

53. Beichner, R.J., *Testing student interpretation of kinematics graphs.* American journal of Physics, 1994. **62**(8): p. 750-762.

54. Herbert, S. and R. Pierce, *What is rate? Does context or representation matter?* Mathematics education research journal, 2011. **23**(4): p. 455-477.

55. Hauger, G.S., *Rate of Change Knowledge in High School and College Students.* 1995.

56. Maloney, D.P., *An overview of physics education research on problem solving.* getting started in PER, 2011. **2**.

57. Larkin, J., et al., *Expert and novice performance in solving physics problems.* Science, 1980. **208**(4450): p. 1335-1342.

58. Singh, C., *When physical intuition fails.* American Journal of Physics, 2002. **70**(11): p. 1103-1109.

59. Chi, M.T., P.J. Feltovich, and R. Glaser, *Categorization and representation of physics problems by experts and novices.* Cognitive science, 1981. **5**(2): p. 121-152.

60. Lin, S.-Y. and C. Singh, *Using isomorphic problems to learn introductory physics.* Physical Review Special Topics-Physics Education Research, 2011. **7**(2): p. 020104.

61. Singh, C., *Assessing student expertise in introductory physics with isomorphic problems. I. Performance on nonintuitive problem pair from introductory physics.* Physical Review Special Topics-Physics Education Research, 2008. **4**(1): p. 010104.

62. Singh, C., *Assessing student expertise in introductory physics with isomorphic problems. II. Effect of some potential factors on problem solving and transfer.* Physical Review Special Topics-Physics Education Research, 2008. **4**(1): p. 010105.

63. Heller, P., R. Keith, and S. Anderson, *Teaching problem solving through cooperative grouping. Part 1: Group versus individual problem solving.* American journal of physics, 1992. **60**(7): p. 627-636.

64. Wenning, C.J., *Dealing more effectively with alternative conceptions in science.* Journal of Physics Teacher Education Online, 2008. **5**(1): p. 11-19.

65. Halliday, D.R. and R. Resnick, *R. & Walker. J.(2007). Fundamentals of Physics*. Wiley. New York. USA.

66. Young, D., A. Freedman, and A.L. Ford, *University Physics with Modern Physics, 13th.* San Frasisco: Pearson Education, 2011.

67. Uttal, D.H., et al., *The malleability of spatial skills: A meta-analysis of training studies.* Psychological bulletin, 2013. **139**(2): p. 352.

68. Ha, O. and N. Fang, *Spatial ability in learning engineering mechanics: Critical review.* Journal of Professional Issues in Engineering Education and Practice, 2016. **142**(2): p. 04015014.

69. O'Brien Pride, T.E., *An investigation of student difficulties with two dimensions, two-body systems, and relativity in introductory mechanics.* 1997.

70. Boudreaux, A., *An investigation of student understanding of Galilean relativity*. 2002.

71. Close, H.G., L.S. Gomez, and P.R. Heron, *Student understanding of the application of Newton's second law to rotating rigid bodies.* American Journal of Physics, 2013. **81**(6): p. 458-470.

72. Sheh Lit, C. and S. Peter. *Curriculum development to improve student understanding of rolling motion*. in *Physics Education Research Conference 2018*. 2018. Washington, DC.

73. *Onshape*. 08/31/2019]; Available from: www.onshape.com.

74. Serway, R.A. and J.W. Jewett, *Physics for scientists and engineers with modern physics*. 2018: Cengage learning.

75. Smith, N.F., *Bernoulli and Newton in fluid mechanics.* The Physics Teacher, 1972. **10**(8): p. 451-455.

76. Clement, J., *Students' preconceptions in introductory mechanics*. American Journal of physics, 1982. **50**(1): p. 66-71.

77. Mikula, B.D. and A.F. Heckler, *Framework and implementation for improving physics essential skills via computer-based practice: Vector math.* Physical Review Physics Education Research, 2017. **13**(1): p. 010122.

78. Knight, R.D., *The vector knowledge of beginning physics students.* The physics teacher, 1995. **33**(2): p. 74-77.

79. Kanim, S.E., *An investigation of student difficulties in qualitative and quantitative problem solving: Examples from electric circuits and electrostatics.* 2000.

80. Flores, S., S.E. Kanim, and C.H. Kautz, *Student use of vectors in introductory mechanics.* American Journal of Physics, 2004. **72**(4): p. 460-468.

81. Nguyen, N.-L. and D.E. Meltzer, *Initial understanding of vector concepts among students in introductory physics courses.* American journal of physics, 2003. **71**(6): p. 630-638.

82. Heckler, A.F. and T.M. Scaife, *Adding and subtracting vectors: The problem with the arrow representation.* Physical Review Special Topics-Physics Education Research, 2015. **11**(1): p. 010101.

83. Emigh, P.J., *Investigating and improving student understanding of key ideas in quantum mechanics throughout instruction*. 2015.

84. Barniol, P. and G. Zavala. *Vector addition: Effect of the context and position of the vectors*. in *Aip conference proceedings*. 2010. AIP.

85. Heckler, A.F. and B.D. Mikula, *Factors affecting learning of vector math from computer-based practice: Feedback complexity and prior knowledge.* Physical Review Physics Education Research, 2016. **12**(1): p. 010134.

86. Roper, W., *Feedback in computer assisted instruction.* Programmed learning and educational technology, 1977. **14**(1): p. 43-49.

87. Sweller, J., *Cognitive load theory, learning difficulty, and instructional design.* Learning and instruction, 1994. **4**(4): p. 295-312.

88. Van Deventer, J. and M.C. Wittmann. *Comparing student use of mathematical and physical vector representations*. in *AIP Conference Proceedings*. 2007. AIP.

89. McCulloch, C.E. and J.M. Neuhaus, *Generalized linear mixed models.* Encyclopedia of biostatistics, 2005. **4**.

90. Brant, R., *Assessing proportionality in the proportional odds model for ordinal logistic regression.* Biometrics, 1990: p. 1171-1178.

91. Hutchins, W.J., *Machine translation: past, present, future*. 1986: Ellis Horwood Chichester.

92. Young, T., et al., *Recent trends in deep learning based natural language processing.* ieee Computational intelligenCe magazine, 2018. **13**(3): p. 55-75.

93. Lewis, D.D. and K.S. Jones, *Natural language processing for information retrieval.* Communications of the ACM, 1996. **39**(1): p. 92-101.

94. Mayr, P., et al., *Introduction to the special issue on bibliometric-enhanced information retrieval and natural language processing for digital libraries (BIRNDL).* International Journal on Digital Libraries, 2018. **19**(2-3): p. 107-111.

95. Mishra, R., et al., *Text summarization in the biomedical domain: a systematic review of recent research.* Journal of biomedical informatics, 2014. **52**: p. 457-467.

96. Nenkova, A. and K. McKeown, *A survey of text summarization techniques*, in *Mining text data*. 2012, Springer. p. 43-76.

97. Andrenucci, A. and E. Sneiders. *Automated question answering: Review of the main approaches*. in *Third International Conference on Information Technology and Applications (ICITA'05)*. 2005. IEEE.

98. Moldovan, D., et al., *Performance issues and error analysis in an open-domain question answering system.* ACM Transactions on Information Systems (TOIS), 2003. **21**(2): p. 133-154.

99. Faletti, J., et al., *Systems and methods for pairing of a semantic network and a natural language processing information extraction system*. 2013, Google Patents.

100. Piskorski, J. and R. Yangarber, *Information extraction: Past, present and future*, in *Multi-source, multilingual information extraction and summarization*. 2013, Springer. p. 23-49.

101. Hagen, L., et al. *Understanding citizens' direct policy suggestions to the federal government: a natural language processing and topic modeling approach*. in *2015 48th Hawaii International Conference on System Sciences*. 2015. IEEE.

102. Jelodar, H., et al., *Latent Dirichlet Allocation (LDA) and Topic modeling: models, applications, a survey.* Multimedia Tools and Applications, 2019. **78**(11): p. 15169-15211.

103. Liu, B., *Opinion mining and sentiment analysis*, in *Web Data Mining*. 2011, Springer. p. 459-526.

104. Sun, S., C. Luo, and J. Chen, *A review of natural language processing techniques for opinion mining systems.* Information fusion, 2017. **36**: p. 10-25.

105. Dumais, S.T., *Latent semantic analysis.* Annual review of information science and technology, 2004. **38**(1): p. 188-230.

106. Deerwester, S., et al., *ImprovingInformation-Retrieval with Latent Semantic Indexing.* 1988.

107. Dam, G. and S. Kaufmann, *Computer assessment of interview data using latent semantic analysis.* Behavior Research Methods, 2008. **40**(1): p. 8-20.

108. Sherin, B., *A computational study of commonsense science: An exploration in the automated analysis of clinical interview data.* Journal of the Learning Sciences, 2013. **22**(4): p. 600-638.

109. Graesser, A.C., et al., *AutoTutor: A tutor with dialogue in natural language.* Behavior Research Methods, Instruments, & Computers, 2004. **36**(2): p. 180-192.

110. Rajaraman, A. and J.D. Ullman, *Mining of massive datasets*. 2011: Cambridge University Press.

111. Kulik, C.-L.C. and J.A. Kulik, *Effectiveness of computer-based instruction: An updated analysis.* Computers in human behavior, 1991. **7**(1-2): p. 75-94.

112. Niemiec, R. and H.J. Walberg, *Comparative effects of computer-assisted instruction: A synthesis of reviews.* Journal of Educational Computing Research, 1987. **3**(1): p. 19-37.

113. Tamim, R.M., et al., *What forty years of research says about the impact of technology on learning: A second-order meta-analysis and validation study.* Review of Educational research, 2011. **81**(1): p. 4-28.

114. Van der Kleij, F.M., R.C. Feskens, and T.J. Eggen, *Effects of feedback in a computer-based learning environment on students' learning outcomes: A meta-analysis.* Review of educational research, 2015. **85**(4): p. 475-511.

115. Bangert-Drowns, R.L., et al., *The instructional effect of feedback in test-like events.* Review of educational research, 1991. **61**(2): p. 213-238.

116. Hattie, J. and H. Timperley, *The power of feedback.* Review of educational research, 2007. **77**(1): p. 81-112.

117. Shute, V.J., *Focus on formative feedback.* Review of educational research, 2008. **78**(1): p. 153-189.

118. Hake, R.R., *Interactive-engagement versus traditional methods: A six-thousand-student survey of mechanics test data for introductory physics courses.* American journal of Physics, 1998. **66**(1): p. 64-74.

APPENDIX A. NOTES ON THE STATISTICS USED IN THE DISSERTATION

Appendix A describes the details of the statistics used in the dissertation.

The statistical methods we discuss are: the chi-square test, the McNemar's test, and the generalized linear mixed model. We summarize the key points and interpretations of the results in each method. More details can be found in these books: (1) *Fundamentals of Biostatistics* by Bernard Rosner and (2) *Applied Longitudinal Analysis* by Fitzmaurice, Laird, and Ware.

Readers may need to have some definitions in statistics side-by-side as they read this appendix.

Table of contents

A1. Chi-square ($\chi 2$) test 466

A2. McNemar's test 469

A3. Generalized linear mixed model (GLMM) 471

A1. Chi-square (χ^2) test

The χ^2 test is used to test relationships between two categorical variables. For example, one may ask whether the improvement in the student performance in a question (correct/incorrect) depends on the presence of interactive lecture in two different classes (Yes/No). The two categorical variables here are student performance and presence of interactive lecture in class. Another way of saying is to find out whether the proportion of correct responses are the same in the two independent samples.

It is a method of testing hypothesis. The null hypothesis of the χ^2 test is that there is no relationship between the two variables. Using the example above, the null hypothesis is that the student performance does not change under the presence of interactive lecture. The χ^2 test is also called a test of independence or a test of association between the two characteristics.

The χ^2 test compares the observed values to the pattern that would be in the expected values if the variables were truly independent of each other. Starting with the observed data in a 2 × 2 contingency table shown in Table A.1, the corresponding table with the expected values are calculated. The expected value for the cell in the first row and first column, E_{11}, can be calculated using $(a+b)(a+c)/N$. The meaning of the formula is the expected number of students who had interactive lecture and answered the question correctly. The other cells can be calculated the same way, using different row totals and column totals. The expected value in the (i, j) cell, denoted by E_{ij}, is the product of the row total of the ith row and column total of jth row, divided by the grand total, N.

Table A.1. An example of how observed data can be arranged in a 2 × 2 contingency table.

		Student performance		Total
		Correct	Incorrect	
Interactive lecture	Yes	$a = O_{11}$	$b = O_{12}$	$a + b$
	No	$c = O_{21}$	$d = O_{22}$	$c + d$
Total		$a + c$	$b + d$	$N = a + b + c + d$

Next, we calculate the test statistic, X^2.

$$X^2 = \sum_{i,j=1}^{2} \frac{(\ O_{ij} - E_{ij}\ - 0.5)^2}{E_{ij}}$$

When the observed and expected values are about the same, the test statistic gives 0, which means that the two categorical variables are not associated to each other. When these two sets of values are different, the test statistic, X^2, is compared to the critical value from the χ^2 distribution. The researchers can assess whether the difference is statistically significant.

The 0.5 in the formula is the continuity correction, which is a result of approximating the binomial distribution to the normal distribution when the grand total is large. The formula above is called the Yates-corrected test statistic. Because of the approximation, the rule of thumb is that the expected values for each cell need to be greater than 5. The Yates-corrected test statistic yields more accurate *p*-values than the uncorrected when approximated by a chi-square distribution. The *p*-values reported in this dissertation are obtained through the Yates-corrected χ^2 test.

The chi-square distribution depends on the degree of freedom. For a 2 × 2 table, the degree of freedom is 1. This is because the whole table can be constructed by specifying one of the four cells when the row total, column total, and grand total are fixed.

The *p*-value from the chi-square test is the area-under-curve of the chi-square distribution curve for values that are greater than the test statistic. One such example is shown in Figure A.1.

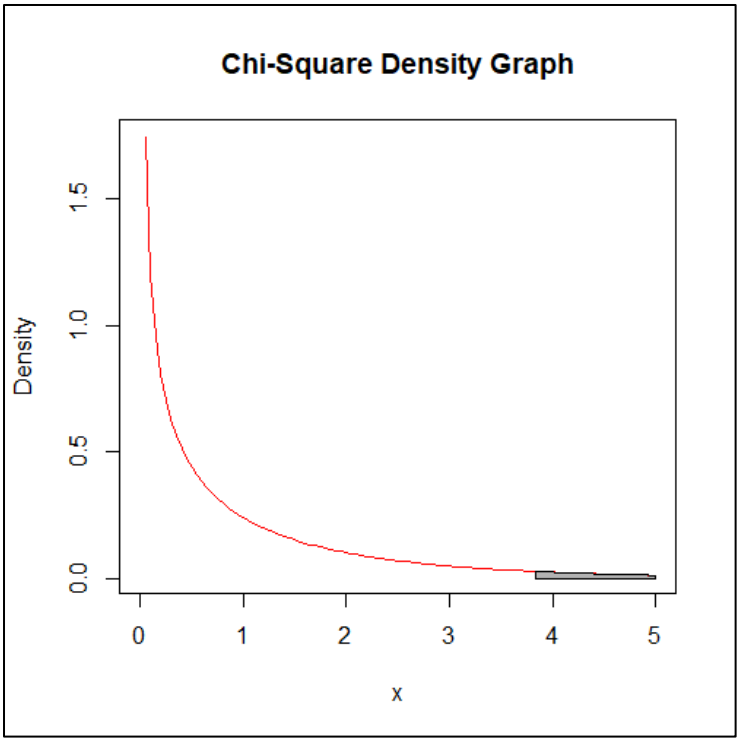


Figure A.1. The χ^2 distribution curve for degree of freedom equals to 1. The shaded area corresponds to *p*-value equals to 0.05.

The *p*-value is 0.05 when the test statistic is equal to 3.84 under the null hypothesis. This means, if the calculated test statistic is greater than 3.84, the null hypothesis is rejected, showing that the one variable is significantly associated to the other variable.

It is possible to do χ^2 test for tables larger than 2 × 2, the calculations are similar, the degrees of freedom and so the chi-square distribution curve will change, the interpretation of the results are similar.

A2. McNemar's test

The McNemar's test is a hypothesis testing method for two sets of data that are correlated to each other. In our case, it refers to the same group of students who did the same pretest and posttest. We use this test to find out the effectiveness of tutorial curriculum. It follows the same chi-square distribution, but the calculation of the test statistic is not the same.

Suppose we have the students' pretest and posttest data arranged in a 2 × 2 contingency table as shown in Table A.2. If the tutorial instruction is effective, the number of students answering the posttest correctly should be greater than the number of students answering the pretest correctly. In other words, the number of students in the pretest-incorrect-posttest-correct cell (c) should be greater than the number of students in the pretest-correct-posttest-incorrect cell (*b*). If the two numbers are equal, *i.e.*, $b = c$, then the curriculum is not doing anything in improving the whole class overall.

Table A.2. The students' pretest and posttest data arranged in a 2 × 2 contingency table.

		Posttest		Total
		Correct	Incorrect	
Pretest	Correct	a	b	$a+b$
	Incorrect	c	d	$c+d$
Total		$a+c$	$b+d$	$N=a+b+c+d$

Assuming large sample size, so the normal approximation to the binomial distribution holds, and using a continuity correction for a better approximation. As a rule of thumb, this approximation will be valid if $b + c \geq 20$.

The test statistic is calculated using: $X^2 = \ |b-c|-1\ ^2/(b+c)$.

The test statistic is compared in the chi-square distribution curve, and the *p*-value is the area under the distribution curve for values greater than the test statistic. This step is the same as described in the previous section.

If the sample size is small such that $b + c < 20$ (excluding the case which the sum of the diagonal cells is almost equal to the grand total), the approximation cannot be used, and a test based on exact binomial probabilities is required. However, as the computation power increases, it is possible to calculate the exact binomial probabilities with $b + c < 1000$.

A3. Generalized linear mixed model (GLMM)

This section gives a brief description of generalized linear mixed model. It is a method we used to analyze the longitudinal data we collected in Chapter 14.

There are several methods to model longitudinal data, such as linear models, linear mixed effects models, generalized estimating equations, generalized linear mixed effect models, and multilevel models. The details for each method can be found in *Applied Longitudinal Analysis* by Fitzmaurice, Laird, and Ware.

As mentioned in Chapter 2 and Chapter 14, when repeated measurements are collected from the same student, there is within-subject association among the measurements. That is, the third measurement of one student is associated with the first and second measurements of the same student. So, in analyzing longitudinal data, one needs to account for the fixed and random effects. The fixed effects are the predictors (or independent variables), and the random effects are to account for the within-subject association.

We first describe the meaning of some terminologies, which we will indicate the different possible statistical models that could be used. In our work, we are using the logistic regression model, which we give an in-depth description. Lastly, we discuss the introduction of random effects in the model.

Note: I am not an expert in this area, I am just doing my best to put down what I know over here. Some phrasing were taken from the reference book as I am unable to describe better than them.

Terminologies

We first describe what each word means in the name.

Linear models are simple regression models that follow a simple equation $y = mx + c$. In this model, y is the response variable, x is the independent variable, m is the coefficient, and c is the vertical intercept. This model is the simplest model to use, and the coefficient is interpreted as the change in y when there is a change in 1 unit in x. The determination of the equation is through the ordinary least squares method.

Generalized linear models are a collection of regression models, such as the linear models previously described, logistic regression models, and Poisson regression models. Each model are for different kinds of data. For example, linear models are for response data that are normally distributed, logistic regression models are for binary responses (Yes/No or Correct/Incorrect), and Poisson regression models are for count data. In addition, the models allow more than one predictor. The word 'linear' means that the coefficients are raised to the power of 1 only.

The word 'mixed' refers to the combination of fixed and random effects in the model.

The generalized linear mixed model (GLMM) is then the combination of all the features described above. In our case, the response variable is binary (correct/incorrect). So, we are using logistic regression models to analyze our data.

Logistic Regression Model for Binary Response

The logistic regression model is used to describe the relationship between a binary response variable (correct/incorrect) and a set of covariates. In this section, we describe the relevant mathematics behind the model and the interpretation of the coefficients and intercepts.

We use the longitudinal data described in Chapter 14 as an example. Let Y_i denote a binary response variable, representing the responses students gave in the vector superposition questions

on pretests 2 to 4. The subscript refers to the *i*th student. The two possible outcomes are "correct" and "incorrect." Denote Y_i by 1 (for "correct") and 0 (for "incorrect"), the probability distribution of Y_i is Bernoulli, with $\Pr\ Y_i = 1\ = \mu_i$ and correspondingly, $\Pr\ Y_i = 0\ = 1 - \mu_i$, where μ_i is the mean of Y_i. The covariates (predictors) are denoted using X_i. In our case, the predictors are prior knowledge on vectors, correctness on pretest 1, pretest number, types of intervention and interaction between pretest and intervention.

The goal is to investigate the relationship between μ_i and X_i. In logistic regression, the simplest model only involves one covariate, so it looks like:

$$\log\left(\frac{\mu_i}{1-\mu_i}\right) = \beta_1 + \beta_2 X_i.$$

We will discuss the model used in our context later.

The reason for the left-hand-side (LHS) is that μ_i is a number between 0 to 1, which has limited range. By transforming into what is seen on the LHS, then the range of allowed values of LHS goes from $-\infty$ to ∞. The term in the parenthesis, $\mu_i/(1-\mu_i)$ is known as the the odds of success. As the probability of correct increases, so does the odds and the log odds of success. The relationship for the other direction is also true, the odds and the log odds of success decreases as μ_i decreases. The LHS, $\log\left(\frac{\mu_i}{1-\mu_i}\right)$, is also called the logit function, $\text{logit}\ \mu_i$.

Now we interpret the coefficients. Consider X_i as another binary variable, taking values of 0 and 1, then β_1 is the log odds of correct when $X_i = 0$. The probability of correct for $X_i = 0$ can be obtained using the following formula:

$$\Pr\ Y_i = 1 | X_i = 0\ = \frac{\exp\ \beta_1}{1 + \exp\ \beta_1}$$

The other coefficient, β_2, is the log odds ratio. It is obtained by comparing the log odds of correct when $X_i = 1$ to the log odds of correct when $X_i = 0$. That is,

$$\text{logit } \mu_i|X_i = 1 \ - \text{logit } \mu_i|X_i = 0 \ = \ \beta_1 + \beta_2 \ - \beta_1 = \beta_2$$

The $\exp \ \beta_2$ is the odds ratio of the response for the two possible values of the predictor.

If X_i is a continuous variable, then β_2 is interpreted as the change in the log odds of correct for a unit change in X_i. In other words, a unit change in X_i increases or decreases the odds of success multiplicatively by a factor of $\exp \ \beta_2$.

If there are more than one predictor in the logistic model (let say *p* predictors), the equation becomes

$$\log\left(\frac{\mu_i}{1-\mu_i}\right) = \beta_1 X_{i1} + \beta_2 X_{i2} + \cdots + \beta_p X_{ip},$$

where $X_{i1} = 1$ for all $i = 1, \ldots, N$. The intercept, β_1, is interpreted as the log odds of correct when all the predictors are set to zero. That is,

$$\Pr(Y_i = 1|X_{i2} = \cdots = X_{ip} = 0) = \frac{\exp \ \beta_1}{1 + \exp \ \beta_1}.$$

As for the other coefficients, β_k (for $k = 2, \ldots, p$), each of them represents the change in the log odds of correct for a unit change in X_{ik} given that all of the other predictor variables remain constant.

In the models we tried, some has only one predictor, some have more than ten predictors.

Generalized Linear Mixed Models (GLMM)

In longitudinal studies, the data has within-subject association. To account for this, one method is to introduce random effects. This model is called the generalized linear mixed models (GLMM).

There are several assumptions in GLMM. One assumption is that a subset of regression coefficients are assumed to vary across individuals according to some statistical distribution. For convenience, we assume the random effects have a multivariate normal distribution. Then, conditional on the random effects, we assume that the responses for any particular individual are independent observations from a distribution.

An example of a logistic regression model with randomly varying intercepts is:

$$\log\left(\frac{\mu_{ij}}{1-\mu_{ij}}\right) = \beta_1 X_{ij1} + \beta_2 X_{ij2} + \cdots + \beta_p X_{ijp} + b_i$$

where μ_{ij} represents the probability of correct response for the *i*th student and *j*th pretest, X_{ijk} represents the covariates, and b_i is the random intercepts term, assumed to have a univariate normal distribution, with zero mean and variance being the between-subject variation in the mean response of baseline. The coefficients, β_k, is in terms of changes in any given individual's log odds of response for a unit within-subject change in the corresponding covariate.

The interpretations for the coefficients are not quite the same as in the usual logistic regression models, because the coefficients have subject-specific interpretation. So, GLMM is most useful when the main objective is to make inferences about individuals rather than the population averages.

If X_{ijk} is a between-subject covariate, for example, gender or treatment, it is misleading to give a subject-specific interpretation in terms of the change in the log odds of response for a unit increase in X_{ijk}. To give a proper interpretation, one needs to compare between two different

individuals who happen to have the same value of random effects, but differ by one unit in the covariate.

For example, in our case, one of the covariates is prior knowledge on vectors. The coefficient related to that is positive, which means that the odds of correct answer increases when students have prior knowledge on vectors. The caveat on the interpretation of the coefficient is that the two students we are comparing need to have the same random effect. However, this information is not available to us since they are unobserved.

On last note, we are still learning the details on this model, so it is possible that this model may not be the best model to analyze the data. However, that does not diminish our attempt to find a proper statistical method to analyze the data.

APPENDIX B. PRETESTS, POSTTESTS, AND CURRICULUM FOR PART I

In Appendix B, we include the pretests, tutorial homework curriculum, and exam questions for the investigation of student understanding of particle motion. The area we focused on is student understanding of average speed and instantaneous speed.

Table of contents

B1. Pretests used in Part I's investigation..... 478

B2. Tutorial homework curriculum for Part I..... 528

B3. Posttests used in the investigation of Part I..... 529

B1. Pretests used in Part I's investigation

In this section, we present the online pretests that we used in the investigation of student understanding of average speed and instantaneous speed. These pretests were used in the PHYS 114 and PHYS 121 courses. The results are presented in Chapter 4 and Chapter 5 of the dissertation. We arranged the pretests according to chapters. There are questions that were administered in both courses.

The pretests presented here are not the way the students see online. This is because some had conditional logic involved (e.g., students who answered A in question 2 see question 4 subsequently, students who answered B in question 2 are directed to question 5 etc.). The originals occupy a lot of pages, so only the relevant questions are cropped and presented.

(A) The pretests (written and online) of which the results were reported in Chapter 4

Physics 121A
Autumn 1993

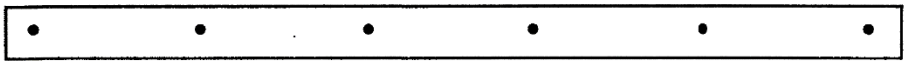
Pretest #1

Name _____
Section _____

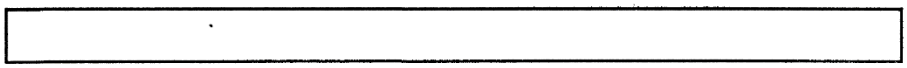
The following questions concern the motion demonstrated at the front of the room. A diagram of the apparatus is shown below in part 2.

The cart is released from rest, speeds up as it moves down the incline, and then moves with constant speed on the level segment of track. A marker records the motion by marking dots at equal time intervals on the long strip of paper attached to the cart.

1. One portion of the paper record is reproduced below. This portion represents the motion on the final half of the level track segment (from B to A), a distance of 0.5 meters.

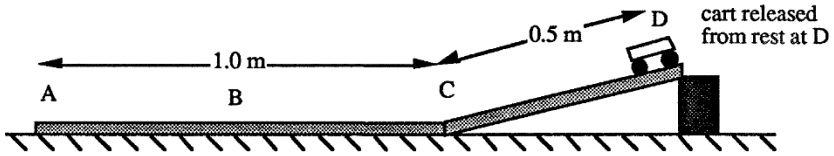


In the space below, sketch the pattern of dots that would correspond to the motion of the cart on the incline (from D to C), also a distance of 0.5 meters.

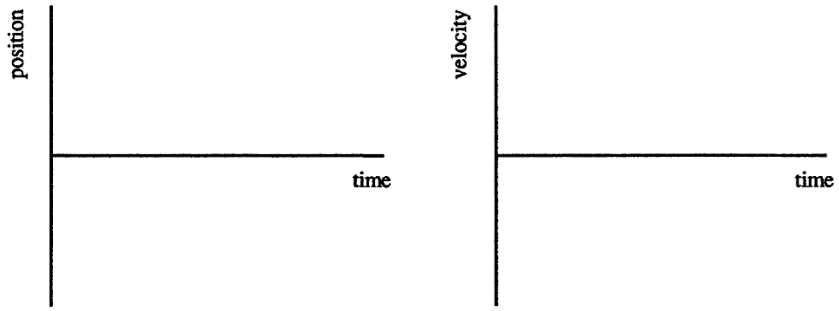


Explain in words why you chose to arrange the dots as you did.

2. A diagram of the apparatus is shown below. The length of each track segment is also shown. Point B is located halfway along the level track segment.



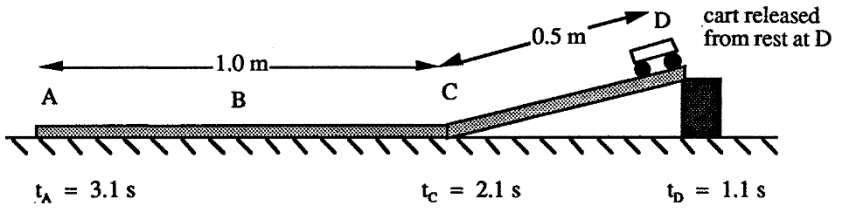
Sketch graphs of the position and the velocity of the cart *vs.* time on the axes provided. Label the approximate times when the cart is at each of the points A, B, C, and D.



(continued on other side)

Figure B.1. The *Velocity* tutorial pretest (written) given to PHYS 121 students in the Au'93 quarter (2 pages).

3. The diagram of the apparatus is reproduced below. Also shown are the times when the cart was located at points A, C and D.



Use this information to find the speed of the cart, if possible, at each of the points A - D. If you do not have enough information to find the speed at one or more of the points, state what information you would need.

Speed at point A:

Speed at point C:

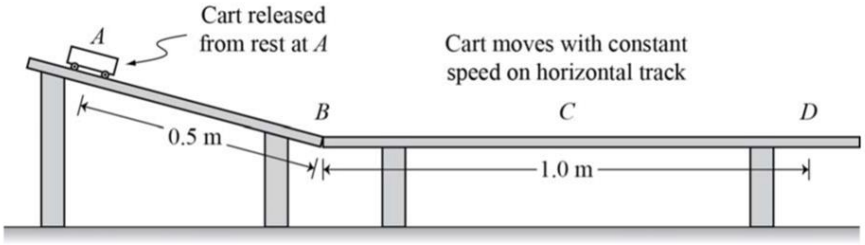
Speed at point B:

Speed at point D:

(A1D)U8a_Superposition

University of Washington

A cart is released from rest, speeds up as it moves down an incline, and then moves with constant speed on the frictionless, horizontal segment of track as shown in the diagram below. Four points (*A*, *B*, *C* and *D*) are labeled on the track. Point *C* is located halfway along the track segment.



Question 3.

Which of the following expressions represents the instantaneous speed at point *B*?

Define s_{AB} as the distance between point A and point B and so on, Δt_{AB} as the time interval between point A and point B and so on.

- $s_{AB}/\Delta t_{AB}$
- $s_{BC}/\Delta t_{BC}$
- $s_{AC}/\Delta t_{AC}$
- $s_{AD}/\Delta t_{AD}$
- Other:

Question 4.

Is the time taken for the cart to travel from *A* to *B* *greater than*, *less than*, or *equal to* the time taken for the cart to travel from *B* to *C*?

- The time taken for the cart to travel from *A* to *B* is *greater than* the time taken for the cart to travel from *B* to *C*.
- The time taken for the cart to travel from *A* to *B* is *less than* the time taken for the cart to travel from *B* to *C*.
- The time taken for the cart to travel from *A* to *B* is *equal to* the time taken for the cart to travel from *B* to *C*.

Question 5.

Explain the reasoning for your answer in the previous question.

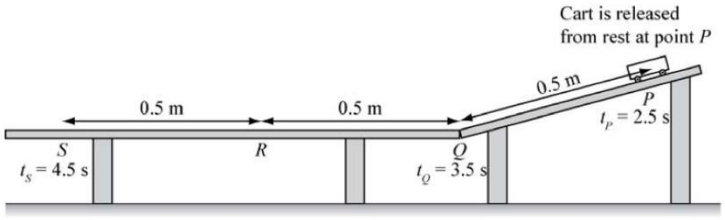
Figure B.2. The Cart on Ramp (CR) context on pretest given to PHYS 121 students in the Wi'16 quarter.

(A1D)U9a_Superposition

University of Washington

Part II

A cart is released from rest, speeds up as it moves down an incline, and then moves with constant speed on a frictionless, horizontal segment of track as shown in the diagram below. Four points (*P*, *Q*, *R*, and *S*) are labeled on the track. Point *R* is located halfway along the level track segment.



Question 3.

Which of the following options best represents the instantaneous speeds at points *P*, *Q*, *R*, and *S* (in m/s)?

	v_P	v_Q	v_R	v_S
A	0	0.50	0.50	0.50
B	0	0.50	0.67	0.75
C	0	0.50	0.75	0.75
D	0	0.50	1.0	1.0
E	0	1.0	1.0	1.0

- ○ A
- ○ B
- ○ C
- ○ D
- ○ E
- ○ Other:

Question 4.

Briefly explain the reasoning you used to answer the previous question.

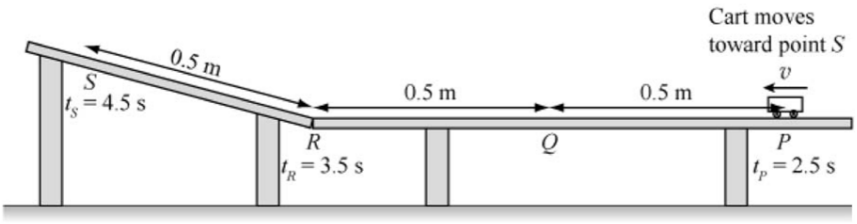
Figure B.3. The Cart on Ramp (CR) context on pretest given to PHYS 121 students in the Sp'16 quarter.

(B) The online pretests of which the results were reported in section 5.2

(A1D_M)U8c
University of Washington

Part I

A cart, with an initial velocity, moves across a frictionless, horizontal segment of track, and then moves up the incline and turns around at point *S* as shown in the diagram below. Four points (*P*, *Q*, *R*, and *S*) are labeled on the track. Point *Q* is located halfway along the level track segment.



Question 1.

Which of the following options best represents the instantaneous speeds at points *P*, *Q*, *R*, and *S* (in m/s)?

	v_P	v_Q	v_R	v_S
A	0.50	0.50	0.50	0
B	0.75	0.67	0.50	0
C	0.75	0.75	0.50	0
D	1.0	1.0	0.50	0
E	1.0	1.0	1.0	0

- A
- B
- C
- D
- E
- Other:

Question 2.

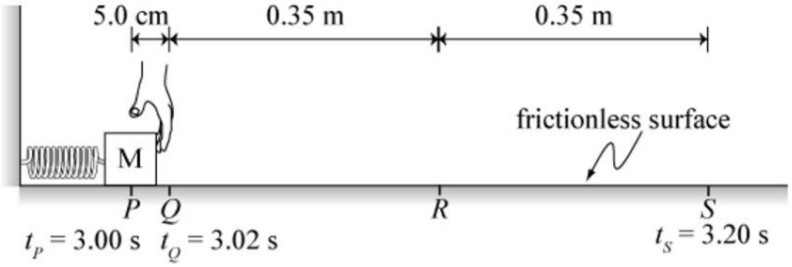
Explain the reasoning you used to answer the previous question. Show your work briefly.

Figure B.4. The Cart on Ramp (CR) context on pretest given to PHYS 121 students in the Wi'18 quarter. The cart is starting from the horizontal track with an initial speed.

(WCE_M)U4a
University of Washington

Part I

Block M, with a mass of 1.0 kg, is placed next to a spring on a frictionless surface. It is pushed to the left, compressing the spring by 5.0 cm until the center of the block reaches point *P*. The block is then released. When the center of the block is at point *Q*, the block leaves the spring. It moves along the frictionless surface past point *S*. Point *R* is halfway between point *Q* and point *S*.



Question 1.

Which of the following options best represents the kinetic energies of the block at points *P*, *Q*, *R*, and *S* (in J)?

	K_P	K_Q	K_R	K_S
A	0	3.1	3.1	3.1
B	0	3.1	6.6	7.0
C	0	3.1	7.0	7.0
D	0	3.1	7.6	7.6
E	0	7.6	7.6	7.6

- A
- B
- C
- D
- E

Question 2.

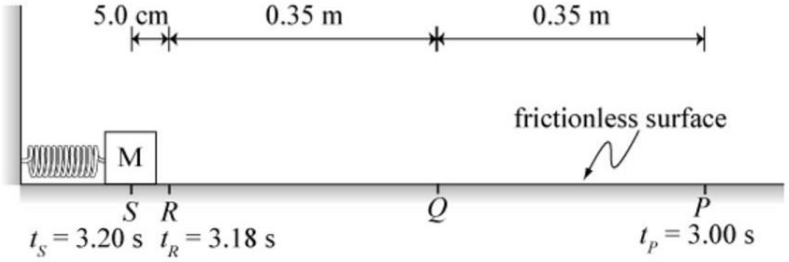
Explain your reasoning for your answer to the previous question.

Figure B.5. The spring-block system (SB) context on pretest given to PHYS 121 students in the Wi'18 and Au'18 quarters.

(WCE_M)U4b
University of Washington

Part I

Block M, with a mass of 1.0 kg, travels from point *P* to point *S* on a frictionless surface. When it reaches point *R*, it encounters a spring and compresses the spring by 5.0 cm before turnaround. Point *Q* is halfway between point *P* and point *R*.



Question 1.

Which of the following options best represents the kinetic energies of the block at points *P*, *Q*, *R*, and *S* (in J)?

	K_P	K_Q	K_R	K_S
A	3.1	3.1	3.1	0
B	7.0	6.6	3.1	0
C	7.0	7.0	3.1	0
D	7.6	7.6	3.1	0
E	7.6	7.6	7.6	0

- A
- B
- C
- D
- E

Question 2.

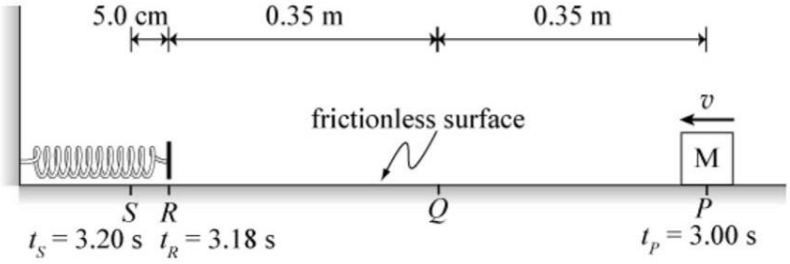
Explain your reasoning for your answer to the previous question.

Figure B.6. The spring-block system (SB) context on pretest given to PHYS 121 students in the Au'18 quarter. The block is moving from the right initially. The diagram shows the final position of the block.

(WCE_M)U4c
University of Washington

Part I

Block M, with a mass of 1.0 kg, travels from point *P* to point *S* on a frictionless surface. When it reaches point *R*, it encounters a spring and compresses the spring by 5.0 cm before turnaround. Point *Q* is halfway between point *P* and point *R*.



Question 1.

Which of the following options best represents the kinetic energies of the block at points *P*, *Q*, *R*, and *S* (in J)?

	K_P	K_Q	K_R	K_S
A	3.1	3.1	3.1	0
B	7.0	6.6	3.1	0
C	7.0	7.0	3.1	0
D	7.6	7.6	3.1	0
E	7.6	7.6	7.6	0

- A
- B
- C
- D
- E

Question 2.

Explain your reasoning for your answer to the previous question.

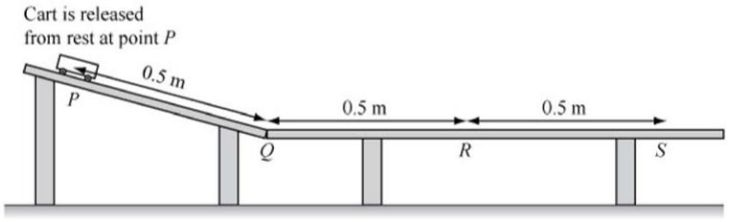
Figure B.7. The spring-block system (SB) context on pretest given to PHYS 121 students in the Au'18 quarter. The block is moving from the right initially. The diagram shows the initial position of the block.

(C) The online pretests of which the results were reported in section 5.3

(A1D_M)U6a
University of Washington

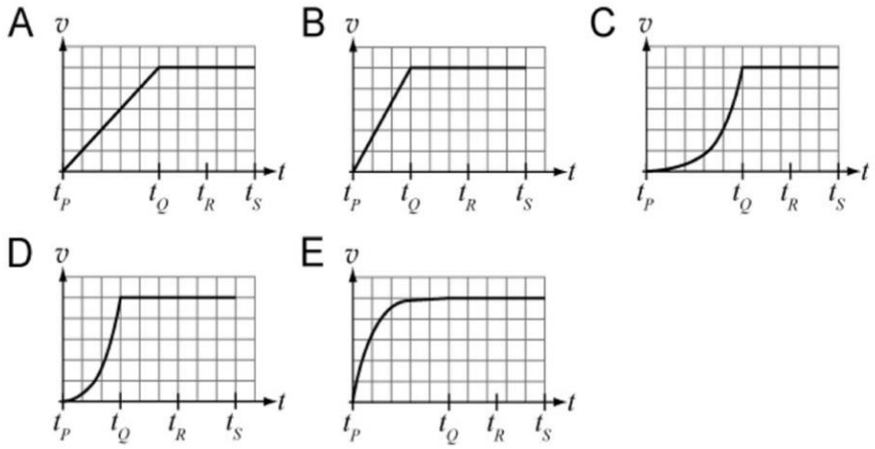
Part I

A cart is released from rest, speeds up as it moves down an incline, and then moves with constant speed on a frictionless, horizontal segment of track as shown in the diagram below. Four points (*P*, *Q*, *R,* and *S*) are labeled on the track. Point *R* is located halfway along the level track segment.



Question 1.

Which of the following speed-time graphs best describes the motion of the cart as it travels from point *P* to point *S*?



- A → Proceed to Qn 2 and 3
- B → Proceed to Qn 2 and 10
- C → Proceed to Qn 2 and 17
- D → Proceed to Qn 2 and 24
- E → Proceed to Qn 2 and 31

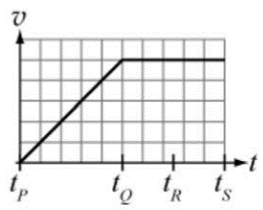
Question 2.

Explain the reasoning you used to answer the previous question.

Figure B.8. The online pretest that was used to investigate student ability to do consistency check given to PHYS 121 students in the Wi'17 quarter (11 pages).

Question 3.

You have selected the following speed-time graph:

A 

Which of the following options best represents the instantaneous speeds at points *P*, *Q*, *R*, and *S* (in m/s)?

	v_P	v_Q	v_R	v_S
A	0	0.50	0.50	0.50
B	0	0.50	0.67	0.75
C	0	0.50	0.75	0.75
D	0	0.50	1.0	1.0
E	0	1.0	1.0	1.0

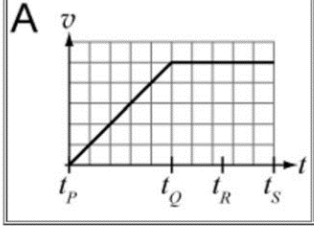
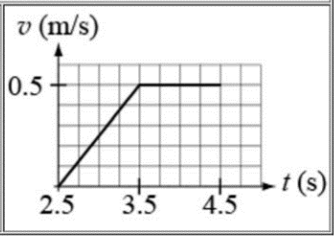
- A → Proceed to Qn 4 and 5
- B → Proceed to Qn 4 and 6
- C → Proceed to Qn 4 and 7
- D → Proceed to Qn 4 and 8
- E → Proceed to Qn 4 and 9
- Other:

Question 4.

Explain the reasoning you used to answer the previous question. Show your work briefly.

Question 5.

You have selected the following answers on pages 1 and 2:

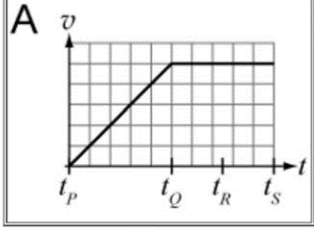
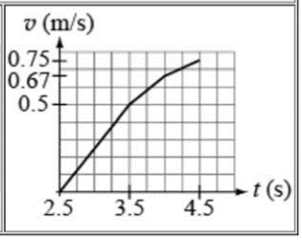
Page 1	Page 2	
A 	v_P 0 v_Q 0.50 v_R 0.50 v_S 0.50	

Note:
Page 1 = Qn 1
Page 2 = Qn 3

Are your answers to page 1 and page 2 consistent? If yes, why? If no, how would you resolve the inconsistencies?

Question 6.

You have selected the following answers on pages 1 and 2:

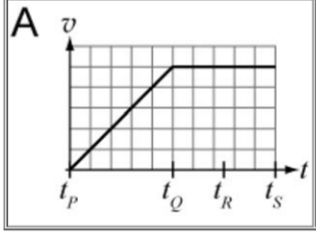
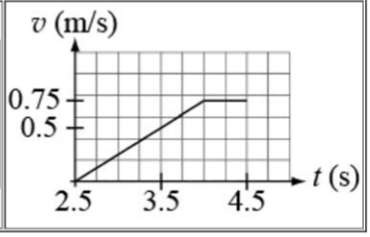
Page 1	Page 2	
A 	v_P 0 v_Q 0.50 v_R 0.67 v_S 0.75	

Note:
Page 1 = Qn 1
Page 2 = Qn 3

Are your answers to page 1 and page 2 consistent? If yes, why? If no, how would you resolve the inconsistencies?

Question 7.

You have selected the following answers on pages 1 and 2:

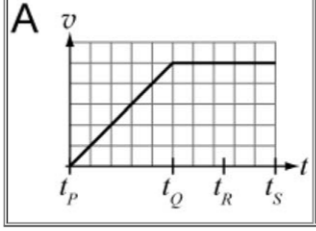
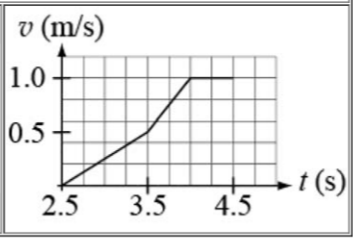
Page 1	Page 2	
	v_P 0 v_Q 0.50 v_R 0.75 v_S 0.75	

Note:
Page 1 = Qn 1
Page 2 = Qn 3

Are your answers to page 1 and page 2 consistent? If yes, why? If no, how would you resolve the inconsistencies?

Question 8.

You have selected the following answers on pages 1 and 2:

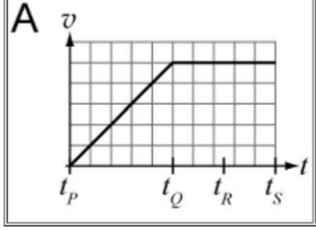
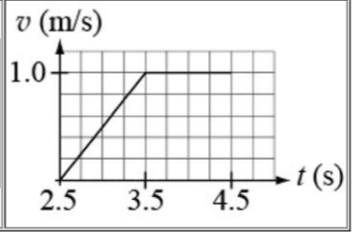
Page 1	Page 2	
	v_P 0 v_Q 0.50 v_R 1.0 v_S 1.0	

Note:
Page 1 = Qn 1
Page 2 = Qn 3

Are your answers to page 1 and page 2 consistent? If yes, why? If no, how would you resolve the inconsistencies?

Question 9.

You have selected the following answers on pages 1 and 2:

Page 1	Page 2	
	v_P 0 v_Q 1.0 v_R 1.0 v_S 1.0	

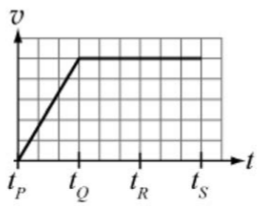
Note:
Page 1 = Qn 1
Page 2 = Qn 3

Are your answers to page 1 and page 2 consistent? If yes, why? If no, how would you resolve the inconsistencies?

Question 10.

You have selected the following speed-time graph:

B



Which of the following options best represents the instantaneous speeds at points *P*, *Q*, *R*, and *S* (in m/s)?

	v_P	v_Q	v_R	v_S
A	0	0.50	0.50	0.50
B	0	0.50	0.67	0.75
C	0	0.50	0.75	0.75
D	0	0.50	1.0	1.0
E	0	1.0	1.0	1.0

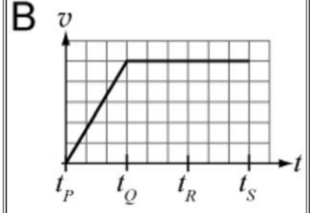
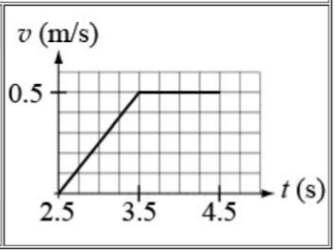
- ○ A → Proceed to Qn 11 and 12
- ○ B → Proceed to Qn 11 and 13
- ○ C → Proceed to Qn 11 and 14
- ○ D → Proceed to Qn 11 and 15
- ○ E → Proceed to Qn 11 and 16
- ○ Other:

Question 11.

Explain the reasoning you used to answer the previous question. Show your work briefly.

Question 12.

You have selected the following answers on pages 1 and 2:

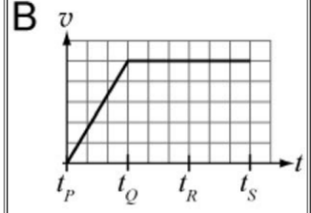
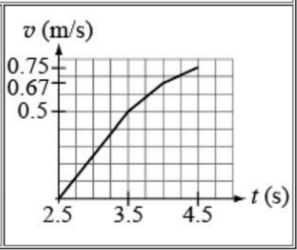
Page 1	Page 2	
B 	v_P 0 v_Q 0.50 v_R 0.50 v_S 0.50	

Note:
Page 1 = Qn 1
Page 2 = Qn 10

Are your answers to page 1 and page 2 consistent? If yes, why? If no, how would you resolve the inconsistencies?

Question 13.

You have selected the following answers on pages 1 and 2:

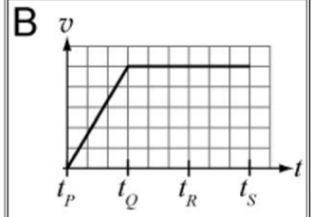
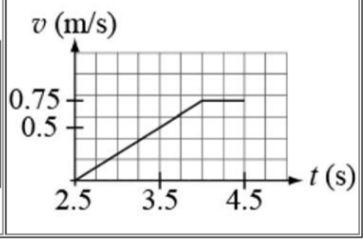
Page 1	Page 2	
B 	v_P 0 v_Q 0.50 v_R 0.67 v_S 0.75	

Note:
Page 1 = Qn 1
Page 2 = Qn 10

Are your answers to page 1 and page 2 consistent? If yes, why? If no, how would you resolve the inconsistencies?

Question 14.

You have selected the following answers on pages 1 and 2:

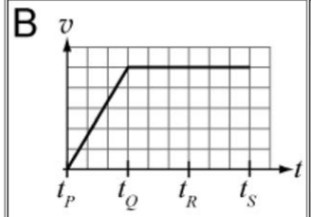
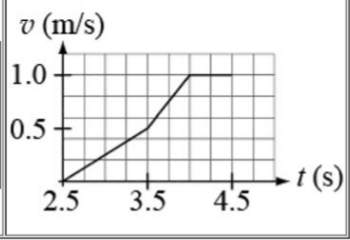
Page 1	Page 2	
	v_P 0 v_Q 0.50 v_R 0.75 v_S 0.75	

Note:
Page 1 = Qn 1
Page 2 = Qn 10

Are your answers to page 1 and page 2 consistent? If yes, why? If no, how would you resolve the inconsistencies?

Question 15.

You have selected the following answers on pages 1 and 2:

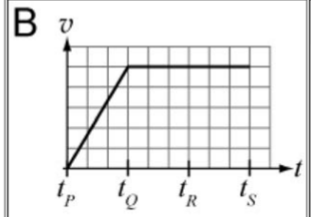
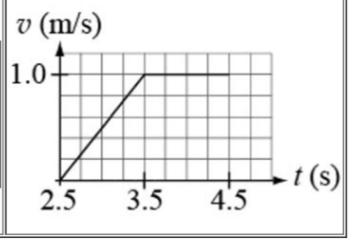
Page 1	Page 2	
	v_P 0 v_Q 0.50 v_R 1.0 v_S 1.0	

Note:
Page 1 = Qn 1
Page 2 = Qn 10

Are your answers to page 1 and page 2 consistent? If yes, why? If no, how would you resolve the inconsistencies?

Question 16.

You have selected the following answers on pages 1 and 2:

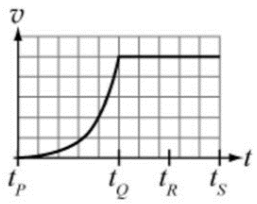
Page 1	Page 2	
	v_P 0 v_Q 1.0 v_R 1.0 v_S 1.0	

Note:
Page 1 = Qn 1
Page 2 = Qn 10

Are your answers to page 1 and page 2 consistent? If yes, why? If no, how would you resolve the inconsistencies?

Question 17.

You have selected the following speed-time graph:

C 

Which of the following options best represents the instantaneous speeds at points *P*, *Q*, *R*, and *S* (in m/s)?

	v_P	v_Q	v_R	v_S
A	0	0.50	0.50	0.50
B	0	0.50	0.67	0.75
C	0	0.50	0.75	0.75
D	0	0.50	1.0	1.0
E	0	1.0	1.0	1.0

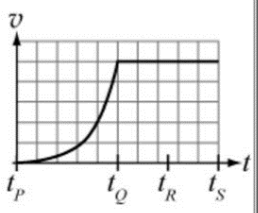
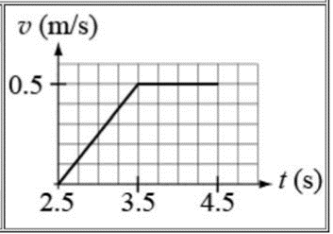
- ○ A → Proceed to Qn 18 and 19
- ○ B → Proceed to Qn 18 and 20
- ○ C → Proceed to Qn 18 and 21
- ○ D → Proceed to Qn 18 and 22
- ○ E → Proceed to Qn 18 and 23
- ○ Other:

Question 18.

Explain the reasoning you used to answer the previous question. Show your work briefly.

Question 19.

You have selected the following answers on pages 1 and 2:

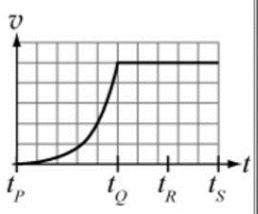
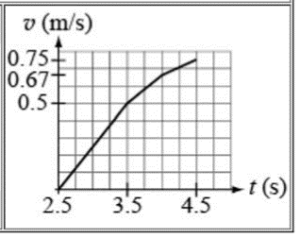
Page 1	Page 2	
C 	v_P 0 v_Q 0.50 v_R 0.50 v_S 0.50	

Note:
Page 1 = Qn 1
Page 2 = Qn 17

Are your answers to page 1 and page 2 consistent? If yes, why? If no, how would you resolve the inconsistencies?

Question 20.

You have selected the following answers on pages 1 and 2:

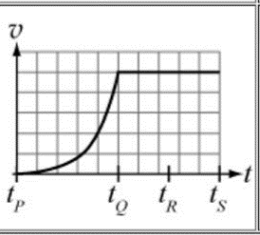
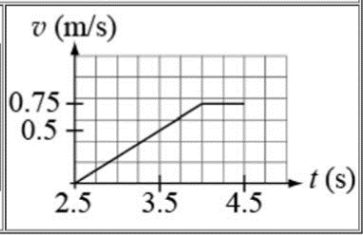
Page 1	Page 2	
C 	v_P 0 v_Q 0.50 v_R 0.67 v_S 0.75	

Note:
Page 1 = Qn 1
Page 2 = Qn 17

Are your answers to page 1 and page 2 consistent? If yes, why? If no, how would you resolve the inconsistencies?

Question 21.

You have selected the following answers on pages 1 and 2:

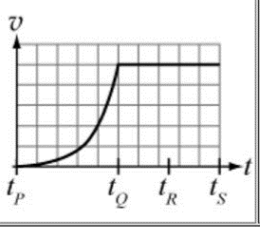
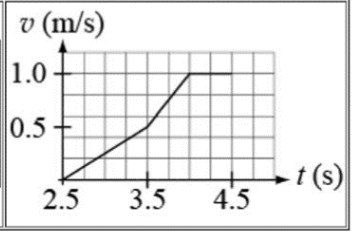
Page 1	Page 2	
C 	v_P 0 v_Q 0.50 v_R 0.75 v_S 0.75	

Note:
Page 1 = Qn 1
Page 2 = Qn 17

Are your answers to page 1 and page 2 consistent? If yes, why? If no, how would you resolve the inconsistencies?

Question 22.

You have selected the following answers on pages 1 and 2:

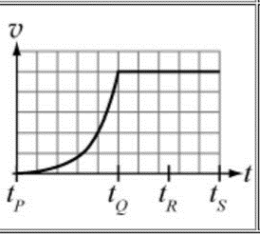
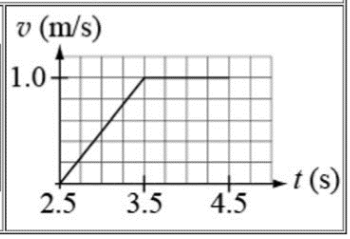
Page 1	Page 2	
C 	v_P 0 v_Q 0.50 v_R 1.0 v_S 1.0	

Note:
Page 1 = Qn 1
Page 2 = Qn 17

Are your answers to page 1 and page 2 consistent? If yes, why? If no, how would you resolve the inconsistencies?

Question 23.

You have selected the following answers on pages 1 and 2:

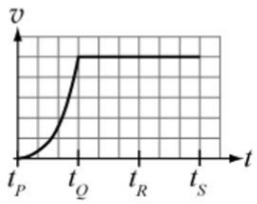
Page 1	Page 2	
C 	v_P 0 v_Q 1.0 v_R 1.0 v_S 1.0	

Note:
Page 1 = Qn 1
Page 2 = Qn 17

Are your answers to page 1 and page 2 consistent? If yes, why? If no, how would you resolve the inconsistencies?

Question 24.

You have selected the following speed-time graph:

D 

Which of the following options best represents the instantaneous speeds at points *P*, *Q*, *R*, and *S* (in m/s)?

	v_P	v_Q	v_R	v_S
A	0	0.50	0.50	0.50
B	0	0.50	0.67	0.75
C	0	0.50	0.75	0.75
D	0	0.50	1.0	1.0
E	0	1.0	1.0	1.0

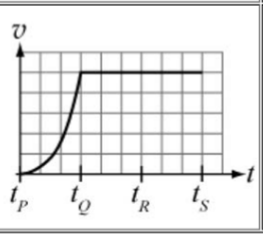
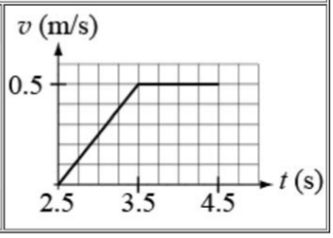
- ○ A → Proceed to Qn 25 and 26
- ○ B → Proceed to Qn 25 and 27
- ○ C → Proceed to Qn 25 and 28
- ○ D → Proceed to Qn 25 and 29
- ○ E → Proceed to Qn 25 and 30
- ○ Other:

Question 25.

Explain the reasoning you used to answer the previous question. Show your work briefly.

Question 26.

You have selected the following answers on pages 1 and 2:

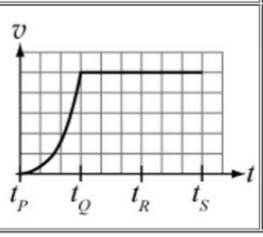
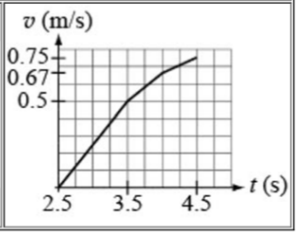
Page 1	Page 2	
D 	v_P 0 v_Q 0.50 v_R 0.50 v_S 0.50	

Note:
Page 1 = Qn 1
Page 2 = Qn 24

Are your answers to page 1 and page 2 consistent? If yes, why? If no, how would you resolve the inconsistencies?

Question 27.

You have selected the following answers on pages 1 and 2:

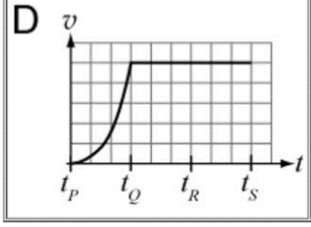
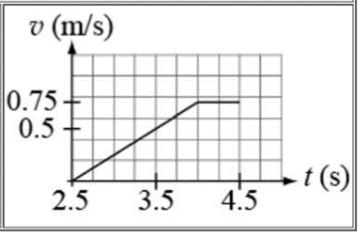
Page 1	Page 2	
D 	v_P 0 v_Q 0.50 v_R 0.67 v_S 0.75	

Note:
Page 1 = Qn 1
Page 2 = Qn 24

Are your answers to page 1 and page 2 consistent? If yes, why? If no, how would you resolve the inconsistencies?

Question 28.

You have selected the following answers on pages 1 and 2:

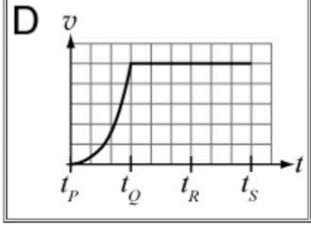
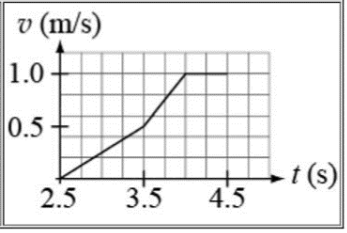
Page 1	Page 2	
D 	v_P 0 v_Q 0.50 v_R 0.75 v_S 0.75	

Note:
Page 1 = Qn 1
Page 2 = Qn 24

Are your answers to page 1 and page 2 consistent? If yes, why? If no, how would you resolve the inconsistencies?

Question 29.

You have selected the following answers on pages 1 and 2:

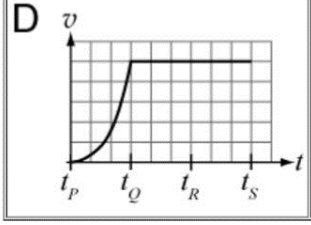
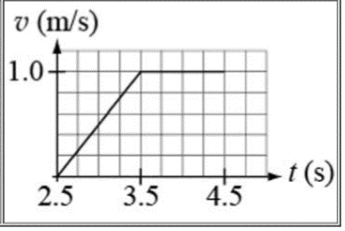
Page 1	Page 2	
D 	v_P 0 v_Q 0.50 v_R 1.0 v_S 1.0	

Note:
Page 1 = Qn 1
Page 2 = Qn 24

Are your answers to page 1 and page 2 consistent? If yes, why? If no, how would you resolve the inconsistencies?

Question 30.

You have selected the following answers on pages 1 and 2:

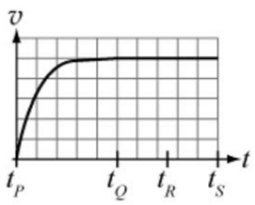
Page 1	Page 2	
D 	v_P 0 v_Q 1.0 v_R 1.0 v_S 1.0	

Note:
Page 1 = Qn 1
Page 2 = Qn 24

Are your answers to page 1 and page 2 consistent? If yes, why? If no, how would you resolve the inconsistencies?

Question 31.

You have selected the following speed-time graph:

E 

Which of the following options best represents the instantaneous speeds at points *P*, *Q*, *R*, and *S* (in m/s)?

	v_P	v_Q	v_R	v_S
A	0	0.50	0.50	0.50
B	0	0.50	0.67	0.75
C	0	0.50	0.75	0.75
D	0	0.50	1.0	1.0
E	0	1.0	1.0	1.0

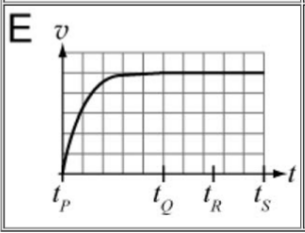
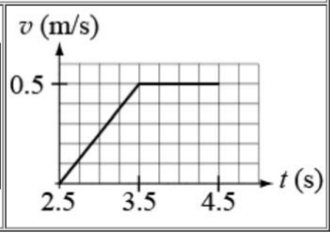
- ○ A → Proceed to Qn 32 and 33
- ○ B → Proceed to Qn 32 and 34
- ○ C → Proceed to Qn 32 and 35
- ○ D → Proceed to Qn 32 and 36
- ○ E → Proceed to Qn 32 and 37
- ○ Other:

Question 32.

Explain the reasoning you used to answer the previous question. Show your work briefly.

Question 33.

You have selected the following answers on pages 1 and 2:

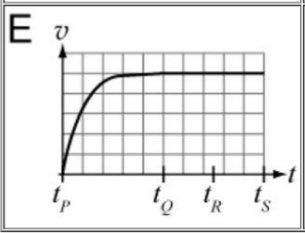
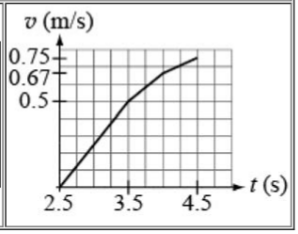
Page 1	Page 2	
E 	v_P 0 v_Q 0.50 v_R 0.50 v_S 0.50	

Note:
Page 1 = Qn 1
Page 2 = Qn 31

Are your answers to page 1 and page 2 consistent? If yes, why? If no, how would you resolve the inconsistencies?

Question 34.

You have selected the following answers on pages 1 and 2:

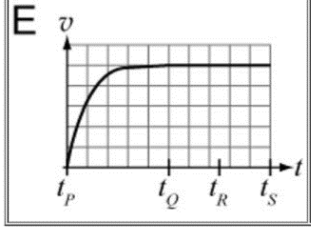
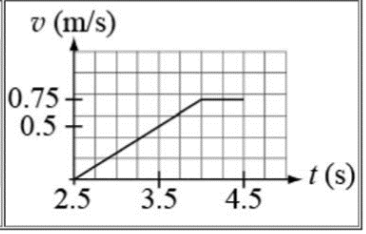
Page 1	Page 2	
E 	v_P 0 v_Q 0.50 v_R 0.67 v_S 0.75	

Note:
Page 1 = Qn 1
Page 2 = Qn 31

Are your answers to page 1 and page 2 consistent? If yes, why? If no, how would you resolve the inconsistencies?

Question 35.

You have selected the following answers on pages 1 and 2:

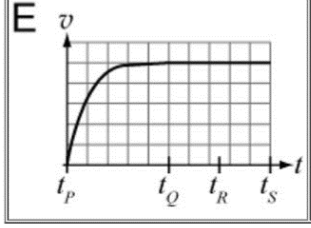
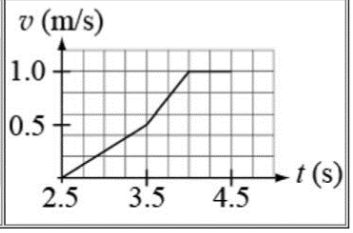
Page 1	Page 2	
E 	v_P 0 v_Q 0.50 v_R 0.75 v_S 0.75	

Note:
Page 1 = Qn 1
Page 2 = Qn 31

Are your answers to page 1 and page 2 consistent? If yes, why? If no, how would you resolve the inconsistencies?

Question 36.

You have selected the following answers on pages 1 and 2:

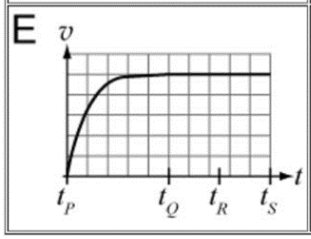
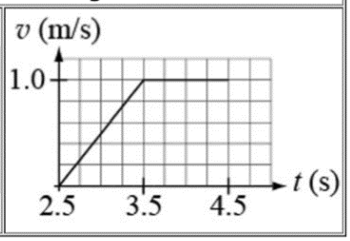
Page 1	Page 2	
E 	v_P 0 v_Q 0.50 v_R 1.0 v_S 1.0	

Note:
Page 1 = Qn 1
Page 2 = Qn 31

Are your answers to page 1 and page 2 consistent? If yes, why? If no, how would you resolve the inconsistencies?

Question 37.

You have selected the following answers on pages 1 and 2:

Page 1	Page 2	
E 	v_P 0 v_Q 1.0 v_R 1.0 v_S 1.0	

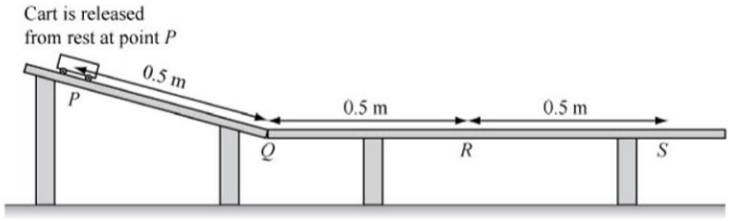
Note:
Page 1 = Qn 1
Page 2 = Qn 31

Are your answers to page 1 and page 2 consistent? If yes, why? If no, how would you resolve the inconsistencies?

(A1D_M)U7a
University of Washington

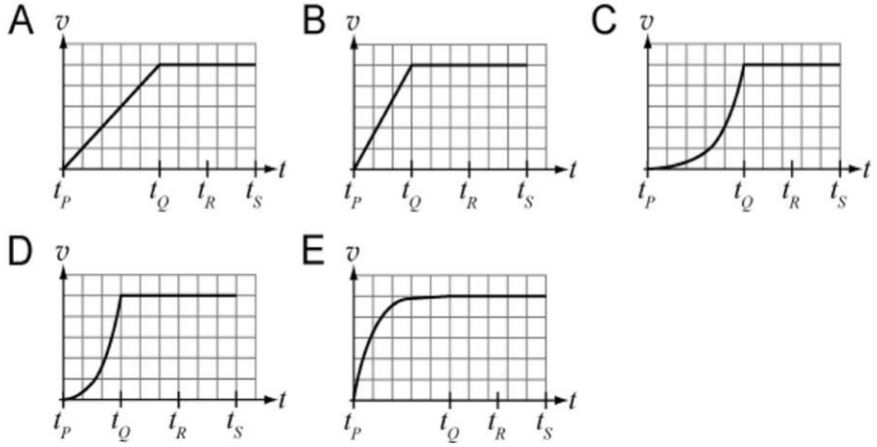
Part I

A cart is released from rest, speeds up as it moves down an incline, and then moves with constant speed on a frictionless, horizontal segment of track as shown in the diagram below. Four points (*P*, *Q*, *R*, and *S*) are labeled on the track. Point *R* is located halfway along the level track segment.



Question 1.

Which of the following speed-time graphs best describes the motion of the cart as it travels from point *P* to point *S*?



- A → Proceed to Qn 2 and 3
- B → Proceed to Qn 2 and 10
- C → Proceed to Qn 2 and 17
- D → Proceed to Qn 2 and 24
- E → Proceed to Qn 2 and 31

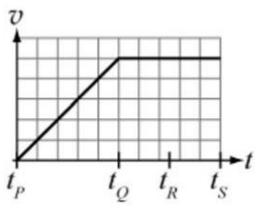
Question 2.

Explain the reasoning you used to answer the previous question.

Figure B.9. The online pretest that was used to investigate student ability to do consistency check given to PHYS 121 students in the Sp'17 quarter (15 pages).

Question 3.

You have selected the following speed-time graph:

A 

Which of the following options best represents the instantaneous speeds at points *P*, *Q*, *R*, and *S* (in m/s)?

	v_P	v_Q	v_R	v_S
A	0	0.50	0.50	0.50
B	0	0.50	0.67	0.75
C	0	0.50	0.75	0.75
D	0	0.50	1.0	1.0
E	0	1.0	1.0	1.0

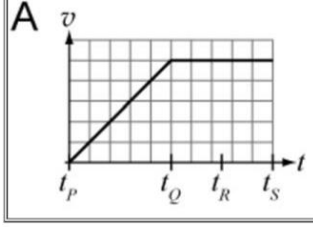
- ○ A → Proceed to Qn 4 and 5
- ○ B → Proceed to Qn 4 and 6
- ○ C → Proceed to Qn 4 and 7
- ○ D → Proceed to Qn 4 and 8
- ○ E → Proceed to Qn 4 and 9
- ○ Other:

Question 4.

Explain the reasoning you used to answer the previous question. Show your work briefly.

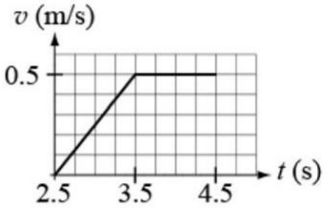
Question 5.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table Entries)
A 	v_P 0 v_Q 0.50 v_R 0.50 v_S 0.50

Note:
Page 1 = Qn 1
Page 2 = Qn 3

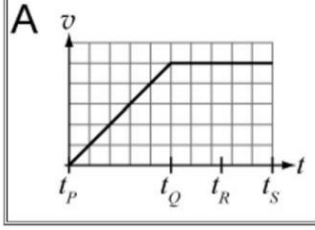
Your entries on page 2 lead to the following graph:



Consider the area under the graphs in pages 1 and 2. Are they consistent with the given situation? If yes, why? If not, identify the mistakes.

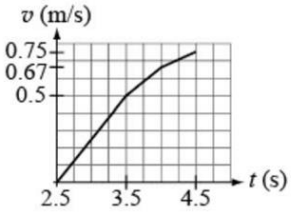
Question 6.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
A 	v_P 0 v_Q 0.50 v_R 0.67 v_S 0.75

Note:
Page 1 = Qn 1
Page 2 = Qn 3

Your entries on page 2 lead to the following graph:



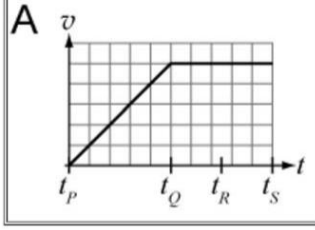
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

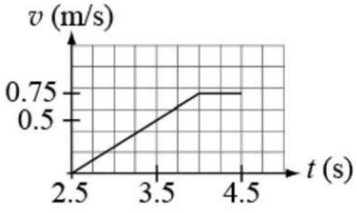
Question 7.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
A 	v_P 0 v_Q 0.50 v_R 0.75 v_S 0.75

Note:
Page 1 = Qn 1
Page 2 = Qn 3

Your entries on page 2 lead to the following graph:



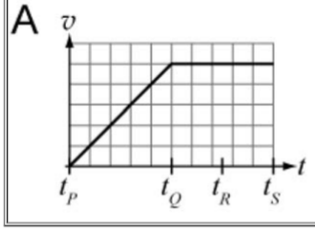
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

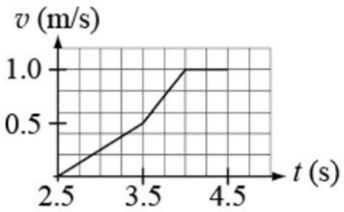
Question 8.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
A 	v_P 0 v_Q 0.50 v_R 1.0 v_S 1.0

Note:
Page 1 = Qn 1
Page 2 = Qn 3

Your entries on page 2 lead to the following graph:



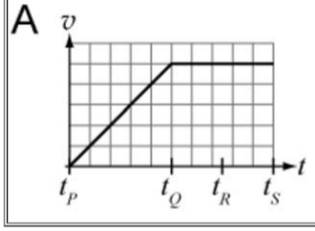
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

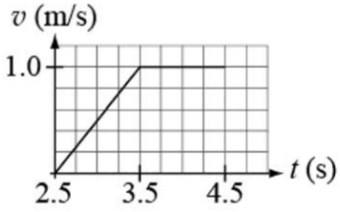
Question 9.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
A 	v_P 0 v_Q 1.0 v_R 1.0 v_S 1.0

Note:
Page 1 = Qn 1
Page 2 = Qn 3

Your entries on page 2 lead to the following graph:

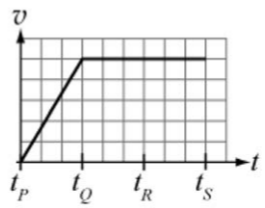


Are your answers to page 1 and page 2 consistent? If yes, why? If no, how would you resolve the inconsistencies?

Question 10.

You have selected the following speed-time graph:

B



Which of the following options best represents the instantaneous speeds at points *P*, *Q*, *R*, and *S* (in m/s)?

	v_P	v_Q	v_R	v_S
A	0	0.50	0.50	0.50
B	0	0.50	0.67	0.75
C	0	0.50	0.75	0.75
D	0	0.50	1.0	1.0
E	0	1.0	1.0	1.0

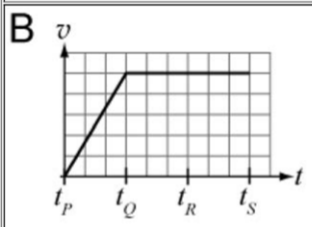
- A → Proceed to Qn 11 and 12
- B → Proceed to Qn 11 and 13
- C → Proceed to Qn 11 and 14
- D → Proceed to Qn 11 and 15
- E → Proceed to Qn 11 and 16
- Other:

Question 11.

Explain the reasoning you used to answer the previous question. Show your work briefly.

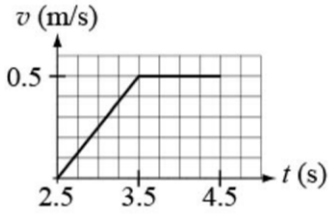
Question 12.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
B 	v_P 0 v_Q 0.50 v_R 0.50 v_S 0.50

Note:
Page 1 = Qn 1
Page 2 = Qn 10

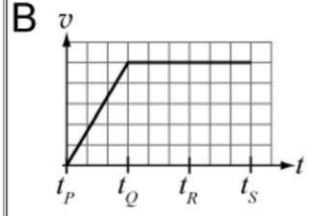
Your entries on page 2 lead to the following graph:



Consider the area under the graphs on pages 1 and 2. Are they consistent with the given situation? If yes, why? If not, identify the mistakes.

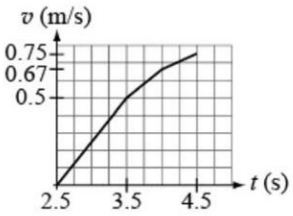
Question 13.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
B 	v_P 0 v_Q 0.50 v_R 0.67 v_S 0.75

Note:
Page 1 = Qn 1
Page 2 = Qn 10

Your entries on page 2 lead to the following graph:



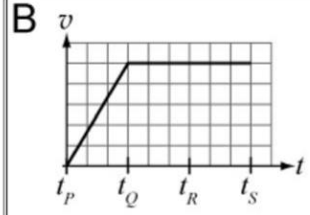
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

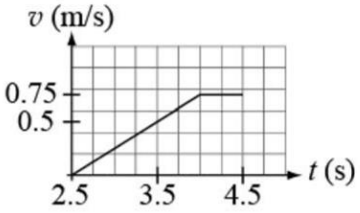
Question 14.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
B 	v_P 0 v_Q 0.50 v_R 0.75 v_S 0.75

Note:
Page 1 = Qn 1
Page 2 = Qn 10

Your entries on page 2 lead to the following graph:



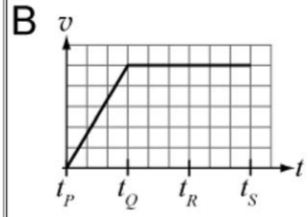
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

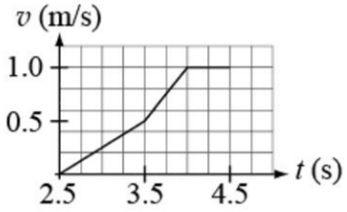
Question 15.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
	v_P 0 v_Q 0.50 v_R 1.0 v_S 1.0

Note:
Page 1 = Qn 1
Page 2 = Qn 10

Your entries on page 2 lead to the following graph:



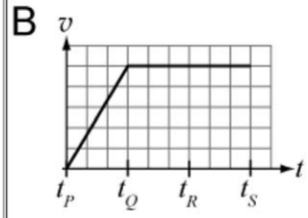
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

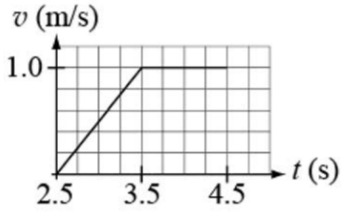
Question 16.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
	v_P 0 v_Q 1.0 v_R 1.0 v_S 1.0

Note:
Page 1 = Qn 1
Page 2 = Qn 10

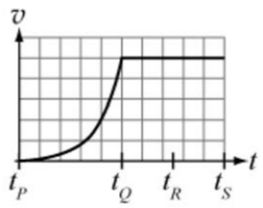
Your entries on page 2 lead to the following graph:



Consider the area under the graphs on pages 1 and 2. Are they consistent with the given situation? If yes, why? If not, identify the mistakes.

Question 17.

You have selected the following speed-time graph:

C 

Which of the following options best represents the instantaneous speeds at points *P*, *Q*, *R*, and *S* (in m/s)?

	v_P	v_Q	v_R	v_S
A	0	0.50	0.50	0.50
B	0	0.50	0.67	0.75
C	0	0.50	0.75	0.75
D	0	0.50	1.0	1.0
E	0	1.0	1.0	1.0

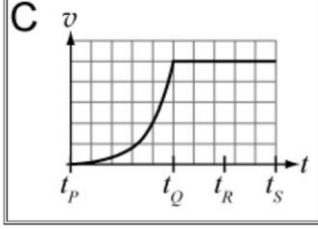
- ○ A → Proceed to Qn 18 and 19
- ○ B → Proceed to Qn 18 and 20
- ○ C → Proceed to Qn 18 and 21
- ○ D → Proceed to Qn 18 and 22
- ○ E → Proceed to Qn 18 and 23
- ○ Other:

Question 18.

Explain the reasoning you used to answer the previous question. Show your work briefly.

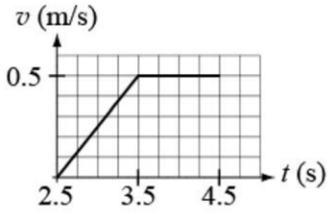
Question 19.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
C 	v_P 0 v_Q 0.50 v_R 0.50 v_S 0.50

Note:
Page 1 = Qn 1
Page 2 = Qn 17

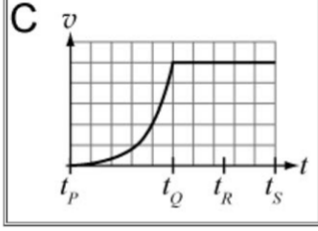
Your entries on page 2 lead to the following graph:



Consider the area under the graphs on pages 1 and 2. Are they consistent with the given situation? If yes, why? If not, identify the mistakes.

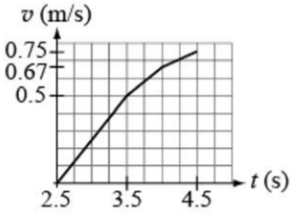
Question 20.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
C 	v_P 0 v_Q 0.50 v_R 0.67 v_S 0.75

Note:
Page 1 = Qn 1
Page 2 = Qn 17

Your entries on page 2 lead to the following graph:



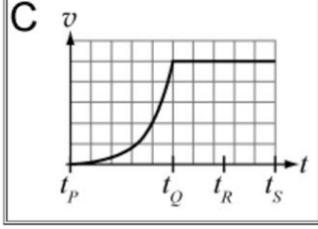
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

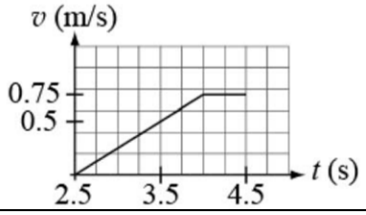
Question 21.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
C 	v_P 0 v_Q 0.50 v_R 0.75 v_S 0.75

Note:
Page 1 = Qn 1
Page 2 = Qn 17

Your entries on page 2 lead to the following graph:



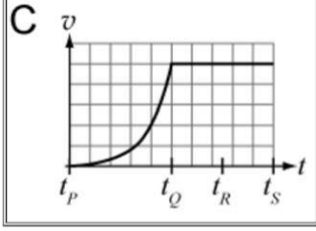
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

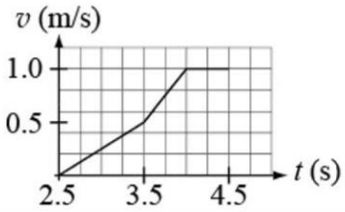
Question 22.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
C 	v_P 0 v_Q 0.50 v_R 1.0 v_S 1.0

Note:
Page 1 = Qn 1
Page 2 = Qn 17

Your entries on page 2 lead to the following graph:



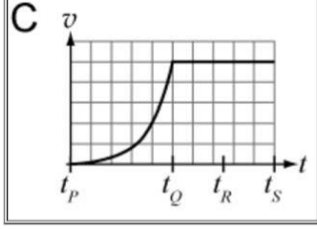
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

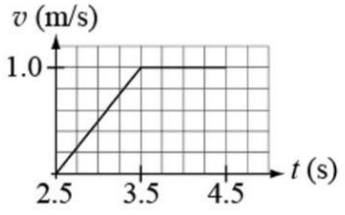
Question 23.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
C 	v_P 0 v_Q 1.0 v_R 1.0 v_S 1.0

Note:
Page 1 = Qn 1
Page 2 = Qn 17

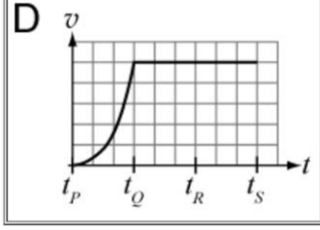
Your entries on page 2 lead to the following graph:



Consider the area under the graphs on pages 1 and 2. Are they consistent with the given situation? If yes, why? If not, identify the mistakes.

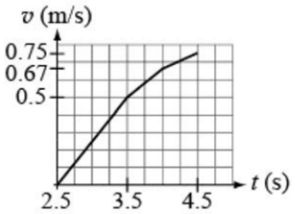
Question 27.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
D 	v_P 0 v_Q 0.50 v_R 0.67 v_S 0.75

Note:
Page 1 = Qn 1
Page 2 = Qn 24

Your entries on page 2 lead to the following graph:



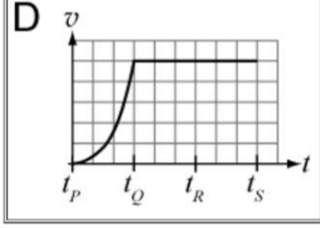
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

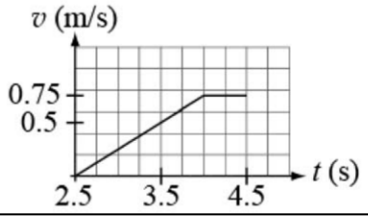
Question 28.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
D 	v_P 0 v_Q 0.50 v_R 0.75 v_S 0.75

Note:
Page 1 = Qn 1
Page 2 = Qn 24

Your entries on page 2 lead to the following graph:



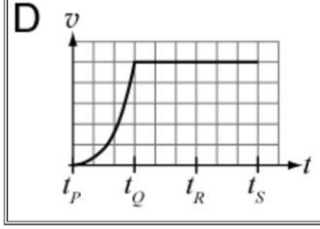
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

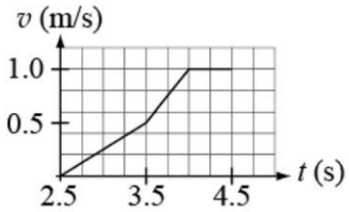
Question 29.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
D 	v_P 0 v_Q 0.50 v_R 1.0 v_S 1.0

Note:
Page 1 = Qn 1
Page 2 = Qn 24

Your entries on page 2 lead to the following graph:



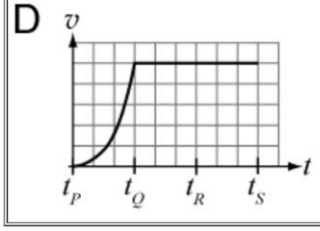
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

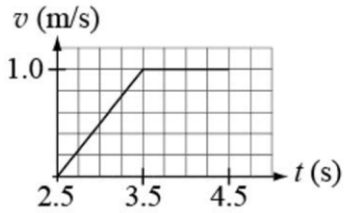
Question 30.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
D 	v_P 0 v_Q 1.0 v_R 1.0 v_S 1.0

Note:
Page 1 = Qn 1
Page 2 = Qn 24

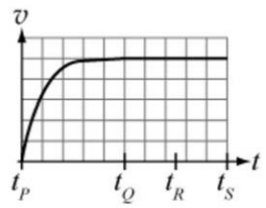
Your entries on page 2 lead to the following graph:



Consider the area under the graphs on pages 1 and 2. Are they consistent with the given situation? If yes, why? If not, identify the mistakes.

Question 31.

You have selected the following speed-time graph:

E 

Which of the following options best represents the instantaneous speeds at points *P*, *Q*, *R*, and *S* (in m/s)?

	v_P	v_Q	v_R	v_S
A	0	0.50	0.50	0.50
B	0	0.50	0.67	0.75
C	0	0.50	0.75	0.75
D	0	0.50	1.0	1.0
E	0	1.0	1.0	1.0

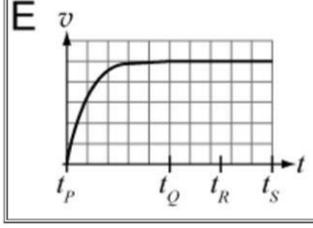
- ○ A → Proceed to Qn 32 and 33
- ○ B → Proceed to Qn 32 and 34
- ○ C → Proceed to Qn 32 and 35
- ○ D → Proceed to Qn 32 and 36
- ○ E → Proceed to Qn 32 and 37
- ○ Other:

Question 32.

Explain the reasoning you used to answer the previous question. Show your work briefly.

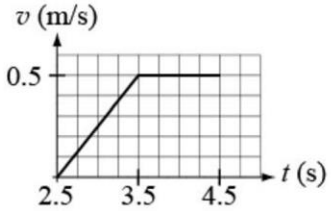
Question 33.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
E 	v_P 0 v_Q 0.50 v_R 0.50 v_S 0.50

Note:
Page 1 = Qn 1
Page 2 = Qn 31

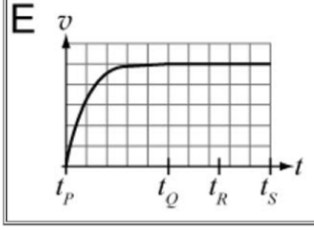
Your entries on page 2 lead to the following graph:



Consider the area under the graphs on pages 1 and 2. Are they consistent with the given situation? If yes, why? If not, identify the mistakes.

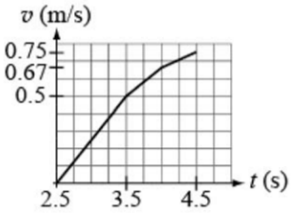
Question 34.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
	v_P 0 v_Q 0.50 v_R 0.67 v_S 0.75

Note:
Page 1 = Qn 1
Page 2 = Qn 31

Your entries on page 2 lead to the following graph:



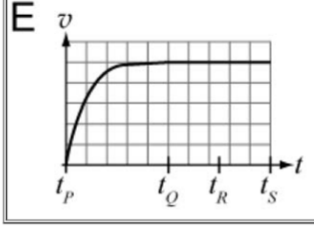
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

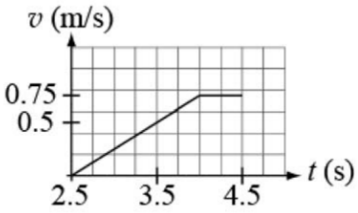
Question 35.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
	v_P 0 v_Q 0.50 v_R 0.75 v_S 0.75

Note:
Page 1 = Qn 1
Page 2 = Qn 31

Your entries on page 2 lead to the following graph:



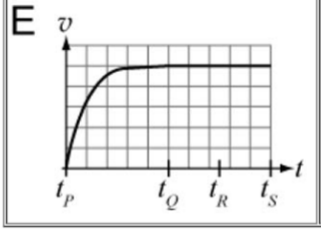
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

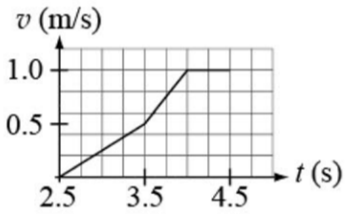
Question 36.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
	v_P 0 v_Q 0.50 v_R 1.0 v_S 1.0

Note:
Page 1 = Qn 1
Page 2 = Qn 31

Your entries on page 2 lead to the following graph:



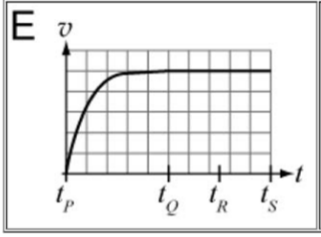
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

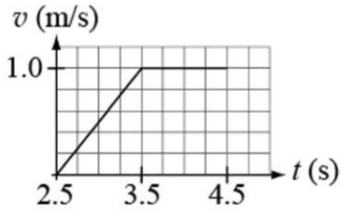
Question 37.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
	v_P 0 v_Q 1.0 v_R 1.0 v_S 1.0

Note:
Page 1 = Qn 1
Page 2 = Qn 31

Your entries on page 2 lead to the following graph:

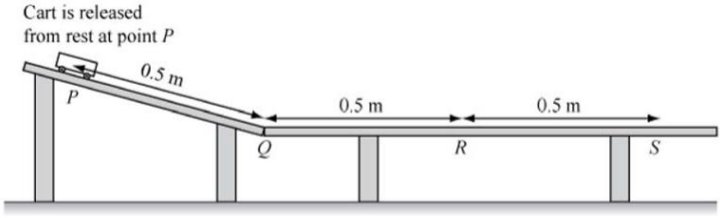


Consider the area under the graphs on pages 1 and 2. Are they consistent with the given situation? If yes, why? If not, identify the mistakes.

(A1D_M)U7b
University of Washington

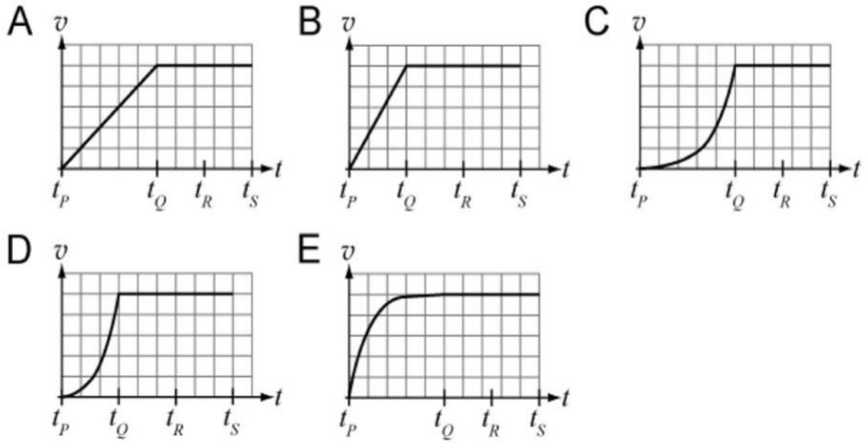
Part I

A cart is released from rest, speeds up as it moves down an incline, and then moves with constant speed on a frictionless, horizontal segment of track as shown in the diagram below. Four points (*P*, *Q*, *R*, and *S*) are labeled on the track. Point *R* is located halfway along the level track segment.



Question 1.

Which of the following speed-time graphs best describes the motion of the cart as it travels from point *P* to point *S*?



- A → Proceed to Qn 2 and 3
- B → Proceed to Qn 2 and 10
- C → Proceed to Qn 2 and 17
- D → Proceed to Qn 2 and 24
- E → Proceed to Qn 2 and 31

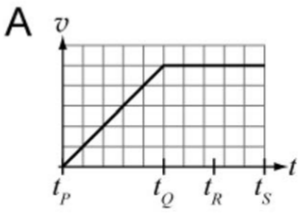
Question 2.

Explain the reasoning you used to answer the previous question.

Figure B.10. The online pretest (version U7b) that was used to investigate student ability to do consistency check given to PHYS 121 students in the Sp'17 quarter (15 pages).

Question 3.

You have selected the following speed-time graph:



Which of the following options best represents the instantaneous speeds at points *P*, *Q*, *R*, and *S* (in m/s)?

	v_P	v_Q	v_R	v_S
A	0	0.50	0.50	0.50
B	0	0.50	0.67	0.75
C	0	0.50	0.75	0.75
D	0	0.50	1.0	1.0
E	0	1.0	1.0	1.0

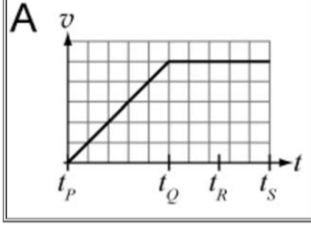
- ○ A → Proceed to Qn 4 and 5
- ○ B → Proceed to Qn 4 and 6
- ○ C → Proceed to Qn 4 and 7
- ○ D → Proceed to Qn 4 and 8
- ○ E → Proceed to Qn 4 and 9
- ○ Other:

Question 4.

Explain the reasoning you used to answer the previous question. Show your work briefly.

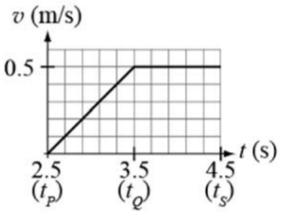
Question 5.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table Entries)
	v_P (t = 2.5s) = 0 v_Q (t = 3.5s) = 0.50 v_R = 0.50 v_S (t = 4.5s) = 0.50

Note:
Page 1 = Qn 1
Page 2 = Qn 2

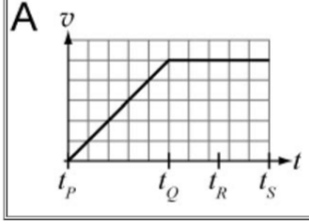
Your entries on page 2 lead to the following graph:



Consider the area under the graphs on pages 1 and 2. Are they consistent with the given situation? If yes, why? If not, identify the mistakes.

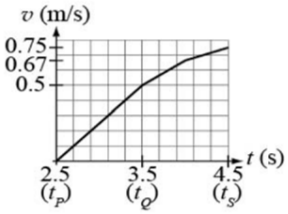
Question 6.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
A 	v_P (t = 2.5s) | 0 v_Q (t = 3.5s) | 0.50 v_R | 0.67 v_S (t = 4.5s) | 0.75

Note:
Page 1 = Qn 1
Page 2 = Qn 2

Your entries on page 2 lead to the following graph:



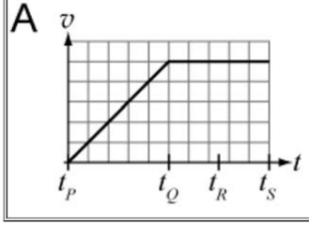
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

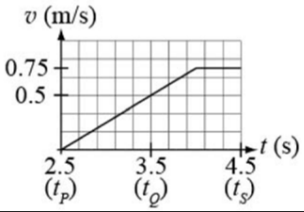
Question 7.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
A 	v_P (t = 2.5s) | 0 v_Q (t = 3.5s) | 0.50 v_R | 0.75 v_S (t = 4.5s) | 0.75

Note:
Page 1 = Qn 1
Page 2 = Qn 2

Your entries on page 2 lead to the following graph:



On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

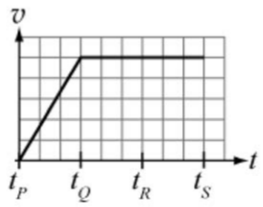
On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

Question 10.

You have selected the following speed-time graph:

B



Which of the following options best represents the instantaneous speeds at points *P*, *Q*, *R*, and *S* (in m/s)?

	v_P	v_Q	v_R	v_S
A	0	0.50	0.50	0.50
B	0	0.50	0.67	0.75
C	0	0.50	0.75	0.75
D	0	0.50	1.0	1.0
E	0	1.0	1.0	1.0

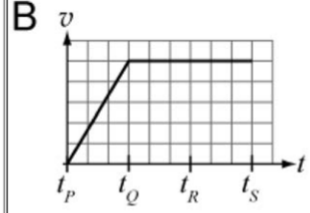
- ○ A → Proceed to Qn 11 and 12
- ○ B → Proceed to Qn 11 and 13
- ○ C → Proceed to Qn 11 and 14
- ○ D → Proceed to Qn 11 and 15
- ○ E → Proceed to Qn 11 and 16
- ○ Other:

Question 11.

Explain the reasoning you used to answer the previous question. Show your work briefly.

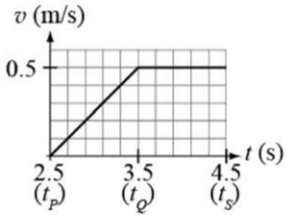
Question 12.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
B 	v_P (t = 2.5s) 0 v_Q (t = 3.5s) 0.50 v_R 0.50 v_S (t = 4.5s) 0.50

Note:
Page 1 = Qn 1
Page 2 = Qn 10

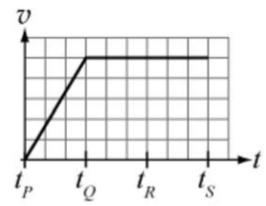
Your entries on page 2 lead to the following graph:



Consider the area under the graphs on pages 1 and 2. Are they consistent with the given situation? If yes, why? If not, identify the mistakes.

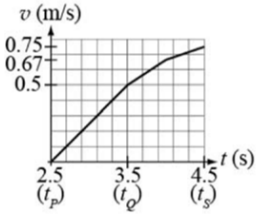
Question 13.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
B 	v_P (t = 2.5s) | 0 v_Q (t = 3.5s) | 0.50 v_R | 0.67 v_S (t = 4.5s) | 0.75

Note:
Page 1 = Qn 1
Page 2 = Qn 10

Your entries on page 2 lead to the following graph:



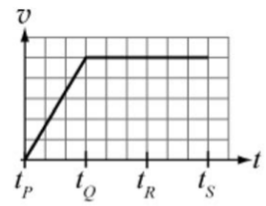
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

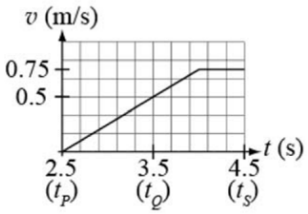
Question 14.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
B 	v_P (t = 2.5s) | 0 v_Q (t = 3.5s) | 0.50 v_R | 0.75 v_S (t = 4.5s) | 0.75

Note:
Page 1 = Qn 1
Page 2 = Qn 10

Your entries on page 2 lead to the following graph:



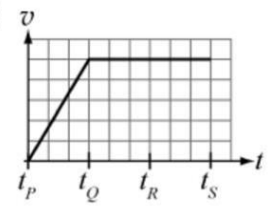
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

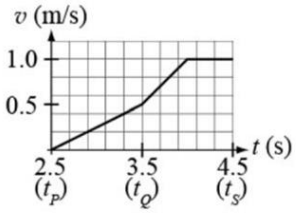
Question 15.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
B 	v_P (t = 2.5s) = 0 v_Q (t = 3.5s) = 0.50 v_R = 1.0 v_S (t = 4.5s) = 1.0

Note:
Page 1 = Qn 1
Page 2 = Qn 10

Your entries on page 2 lead to the following graph:



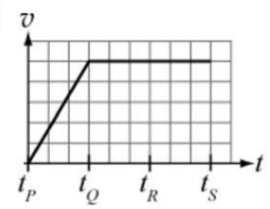
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

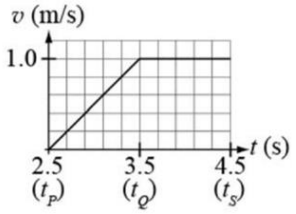
Question 16.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
B 	v_P (t = 2.5s) = 0 v_Q (t = 3.5s) = 1.0 v_R = 1.0 v_S (t = 4.5s) = 1.0

Note:
Page 1 = Qn 1
Page 2 = Qn 10

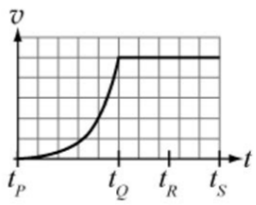
Your entries on page 2 lead to the following graph:



Consider the area under the graphs on pages 1 and 2. Are they consistent with the given situation? If yes, why? If not, identify the mistakes.

Question 17.

You have selected the following speed-time graph:

C 

Which of the following options best represents the instantaneous speeds at points *P*, *Q*, *R*, and *S* (in m/s)?

	v_P	v_Q	v_R	v_S
A	0	0.50	0.50	0.50
B	0	0.50	0.67	0.75
C	0	0.50	0.75	0.75
D	0	0.50	1.0	1.0
E	0	1.0	1.0	1.0

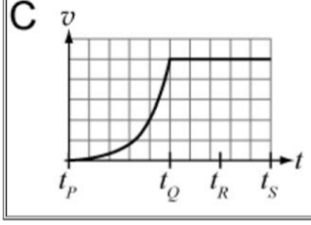
- A → Proceed to Qn 18 and 19
- B → Proceed to Qn 18 and 20
- C → Proceed to Qn 18 and 21
- D → Proceed to Qn 18 and 22
- E → Proceed to Qn 18 and 23
- Other:

Question 18.

Explain the reasoning you used to answer the previous question. Show your work briefly.

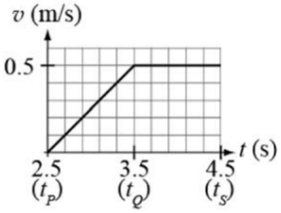
Question 19.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
C 	v_P (t = 2.5s) 0 v_Q (t = 3.5s) 0.50 v_R 0.50 v_S (t = 4.5s) 0.50

Note:
Page 1 = Qn 1
Page 2 = Qn 17

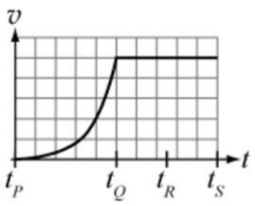
Your entries on page 2 lead to the following graph:



Consider the area under the graphs on pages 1 and 2. Are they consistent with the given situation? If yes, why? If not, identify the mistakes.

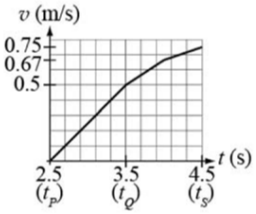
Question 20.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
C 	v_P (t = 2.5s) 0 v_Q (t = 3.5s) 0.50 v_R 0.67 v_S (t = 4.5s) 0.75

Note:
Page 1 = Qn 1
Page 2 = Qn 17

Your entries on page 2 lead to the following graph:



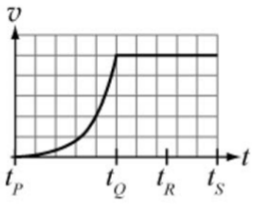
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

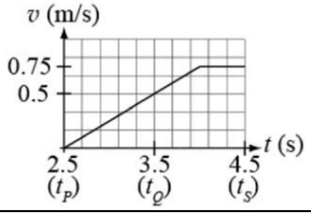
Question 21.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
C 	v_P (t = 2.5s) 0 v_Q (t = 3.5s) 0.50 v_R 0.75 v_S (t = 4.5s) 0.75

Note:
Page 1 = Qn 1
Page 2 = Qn 17

Your entries on page 2 lead to the following graph:



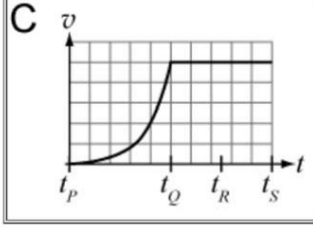
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

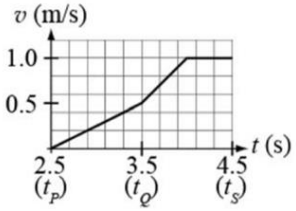
Question 22.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
C 	v_P (t = 2.5s) 0 v_Q (t = 3.5s) 0.50 v_R 1.0 v_S (t = 4.5s) 1.0

Note:
Page 1 = Qn 1
Page 2 = Qn 17

Your entries on page 2 lead to the following graph:



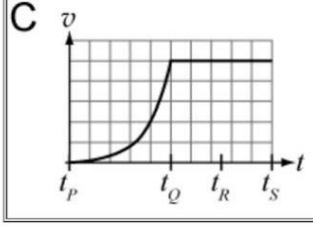
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

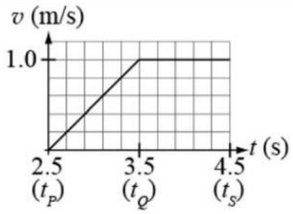
Question 23.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
C 	v_P (t = 2.5s) 0 v_Q (t = 3.5s) 1.0 v_R 1.0 v_S (t = 4.5s) 1.0

Note:
Page 1 = Qn 1
Page 2 = Qn 17

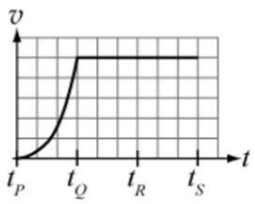
Your entries on page 2 lead to the following graph:



Consider the area under the graphs on pages 1 and 2. Are they consistent with the given situation? If yes, why? If not, identify the mistakes.

Question 24.

You have selected the following speed-time graph:

D 

Which of the following options best represents the instantaneous speeds at points *P*, *Q*, *R*, and *S* (in m/s)?

	v_P	v_Q	v_R	v_S
A	0	0.50	0.50	0.50
B	0	0.50	0.67	0.75
C	0	0.50	0.75	0.75
D	0	0.50	1.0	1.0
E	0	1.0	1.0	1.0

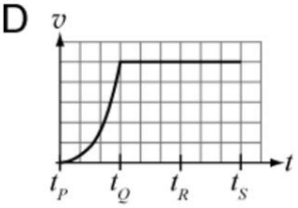
- A → Proceed to Qn 25 and 26
- B → Proceed to Qn 25 and 27
- C → Proceed to Qn 25 and 28
- D → Proceed to Qn 25 and 29
- E → Proceed to Qn 25 and 30
- Other:

Question 25.

Explain the reasoning you used to answer the previous question. Show your work briefly.

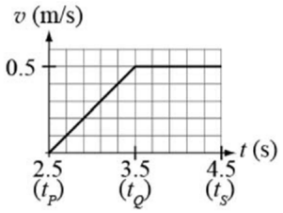
Question 26.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
D 	v_P (t = 2.5s) = 0 v_Q (t = 3.5s) = 0.50 v_R = 0.50 v_S (t = 4.5s) = 0.50

Note:
Page 1 = Qn 1
Page 2 = Qn 24

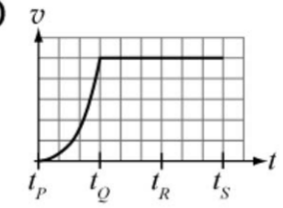
Your entries on page 2 lead to the following graph:



Consider the area under the graphs on pages 1 and 2. Are they consistent with the given situation? If yes, why? If not, identify the mistakes.

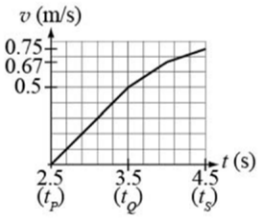
Question 27.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
D 	v_P (t = 2.5s) | 0 v_Q (t = 3.5s) | 0.50 v_R | 0.67 v_S (t = 4.5s) | 0.75

Note:
Page 1 = Qn 1
Page 2 = Qn 24

Your entries on page 2 lead to the following graph:



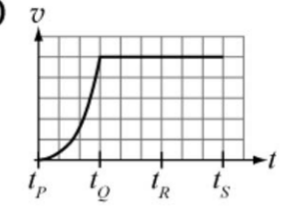
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

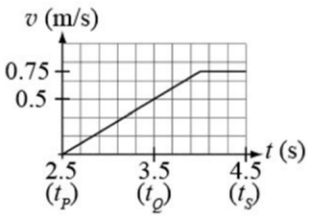
Question 28.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
D 	v_P (t = 2.5s) | 0 v_Q (t = 3.5s) | 0.50 v_R | 0.75 v_S (t = 4.5s) | 0.75

Note:
Page 1 = Qn 1
Page 2 = Qn 24

Your entries on page 2 lead to the following graph:



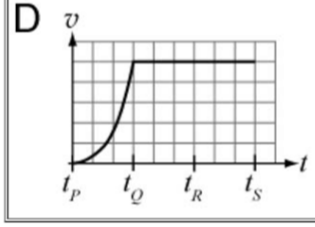
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

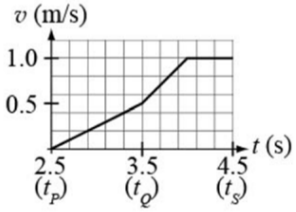
Question 29.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
D 	v_P (t = 2.5s) | 0 v_Q (t = 3.5s) | 0.50 v_R | 1.0 v_S (t = 4.5s) | 1.0

Note:
Page 1 = Qn 1
Page 2 = Qn 24

Your entries on page 2 lead to the following graph:



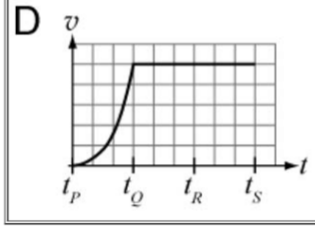
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

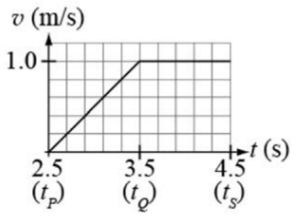
Question 30.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
D 	v_P (t = 2.5s) | 0 v_Q (t = 3.5s) | 1.0 v_R | 1.0 v_S (t = 4.5s) | 1.0

Note:
Page 1 = Qn 1
Page 2 = Qn 24

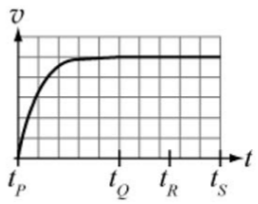
Your entries on page 2 lead to the following graph:



Consider the area under the graphs on pages 1 and 2. Are they consistent with the given situation? If yes, why? If not, identify the mistakes.

Question 31.

You have selected the following speed-time graph:

E 

Which of the following options best represents the instantaneous speeds at points *P*, *Q*, *R*, and *S* (in m/s)?

	v_P	v_Q	v_R	v_S
A	0	0.50	0.50	0.50
B	0	0.50	0.67	0.75
C	0	0.50	0.75	0.75
D	0	0.50	1.0	1.0
E	0	1.0	1.0	1.0

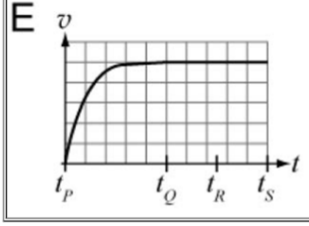
- A → Proceed to Qn 32 and 33
- B → Proceed to Qn 32 and 34
- C → Proceed to Qn 32 and 35
- D → Proceed to Qn 32 and 36
- E → Proceed to Qn 32 and 37
- Other:

Question 32.

Explain the reasoning you used to answer the previous question. Show your work briefly.

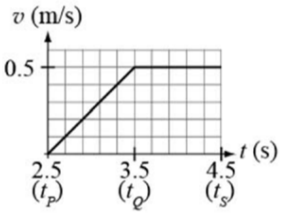
Question 33.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
E 	v_P (t = 2.5s) 0 v_Q (t = 3.5s) 0.50 v_R 0.50 v_S (t = 4.5s) 0.50

Note:
Page 1 = Qn 1
Page 2 = Qn 31

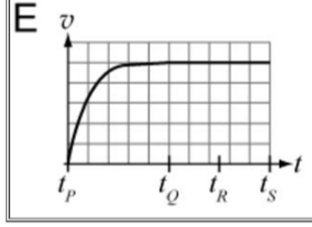
Your entries on page 2 lead to the following graph:



Consider the area under the graphs on pages 1 and 2. Are they consistent with the given situation? If yes, why? If not, identify the mistakes.

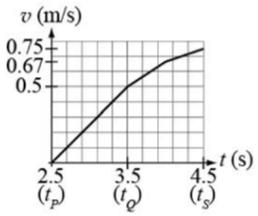
Question 34.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
E 	v_P (t = 2.5s) | 0 v_Q (t = 3.5s) | 0.50 v_R | 0.67 v_S (t = 4.5s) | 0.75

Note:
Page 1 = Qn 1
Page 2 = Qn 31

Your entries on page 2 lead to the following graph:



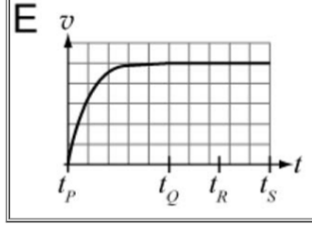
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

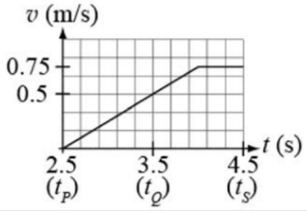
Question 35.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
E 	v_P (t = 2.5s) | 0 v_Q (t = 3.5s) | 0.50 v_R | 0.75 v_S (t = 4.5s) | 0.75

Note:
Page 1 = Qn 1
Page 2 = Qn 31

Your entries on page 2 lead to the following graph:



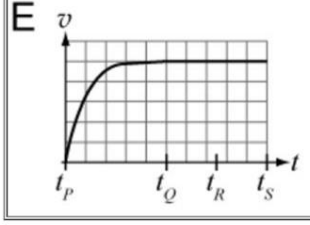
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

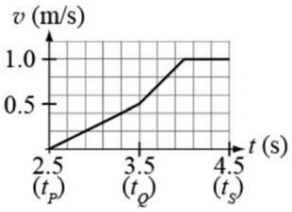
Question 36.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
E 	v_P (t = 2.5s) = 0 v_Q (t = 3.5s) = 0.50 v_R = 1.0 v_S (t = 4.5s) = 1.0

Note:
Page 1 = Qn 1
Page 2 = Qn 31

Your entries on page 2 lead to the following graph:



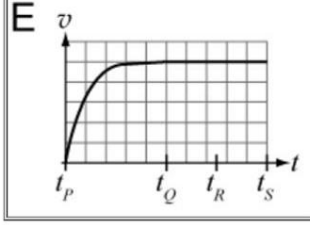
On page 1, your chosen graph depicts $v_Q = v_R = v_S$.

On page 2, your chosen answer does not satisfy $v_Q = v_R = v_S$.

How would you resolve the inconsistency?

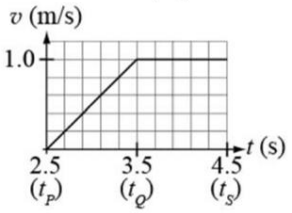
Question 37.

You have selected the following answers on pages 1 and 2:

Page 1 (Graph)	Page 2 (Table entries)
E 	v_P (t = 2.5s) = 0 v_Q (t = 3.5s) = 1.0 v_R = 1.0 v_S (t = 4.5s) = 1.0

Note:
Page 1 = Qn 1
Page 2 = Qn 31

Your entries on page 2 lead to the following graph:



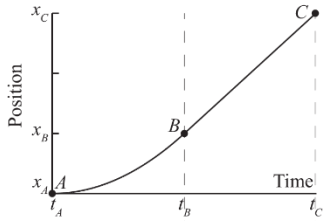
Consider the area under the graphs on pages 1 and 2. Are they consistent with the given situation? If yes, why? If not, identify the mistakes.

B2. Tutorial homework curriculum for Part I

Acceleration in one dimension Name _____ Mech HW–3

In the following question, assume any acceleration to be constant.

5. A block undergoes one-dimensional motion from point *A* to point *C*. An *x* vs *t* graph for the motion is shown below.

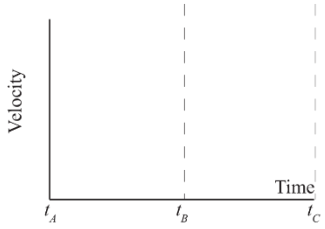


a. Is the block *speeding up*, *slowing down*, or *moving with constant speed* when:

i. it moves from x_A to x_B? Explain.

ii. it moves from x_B to x_C? Explain.

b. On the v vs t axes at right, sketch the corresponding velocity-time graph. Explain your reasoning.



c. In terms of x_A, x_B, x_C, t_A, t_B, and t_C, write an expression for the following quantities:

i. the average velocity of the block between x_A and x_B, $v_{avg,\,AB}$.

ii. the average velocity of the block between x_B and x_C, $v_{avg,\,BC}$.

d. One of the average velocities in part c above represents the *instantaneous* velocity of the block at point B. Which is it? Explain.

Does the other average velocity in part c represent the *instantaneous* velocity of the block at any instant? If so, mark the point on the *x* vs *t* graph above and label it as point X. Explain briefly.

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Figure B.11. The homework in *Acceleration in one-dimension* tutorial designed to help students to distinguish average speed and instantaneous speed.

B3. Posttests used in the investigation of Part I

In this section, we present the exam questions for the investigation of student understanding of average speed and instantaneous speed. The exam questions were administered on midterms and final exams in PHYS 114 and PHYS 121. The questions on PHYS 114 are presented first, followed by those on PHYS 121. Some questions were administered in both courses and in multiple quarters.

We do not show the original versions as there are other questions that are unrelated to the research on the exam.

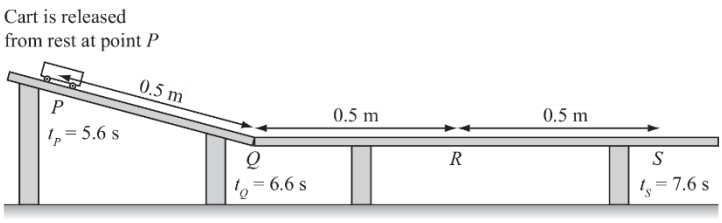
(A) Exam questions administered on PHYS 114

Posttest and exam questions used in PHYS 114

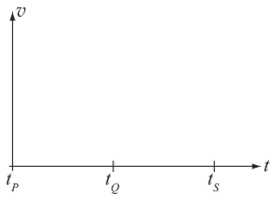
Wi'18 quarter

Midterm 1

[15 points total] A cart is released from rest at point P on an incline, as shown below. The cart speeds up as it moves down the incline, and then moves with constant speed on a horizontal frictionless track. Point R is halfway between point Q and point S. The cart is located at points P, Q, and S at times 5.6 s, 6.6 s, and 7.6 s, respectively.



A. [4 pts] In the space at right, sketch a qualitatively correct speed-time graph to represent the motion of the cart on the track above. Explain clearly how you drew your sketch.
(Note: You do not need to do any calculations.)



B. [8 pts] Calculate the speed of the cart at points P, Q, R, and S. Briefly explain your calculations.

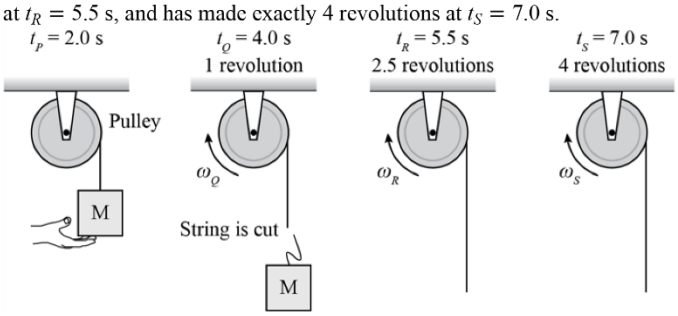
[2 pts] Speed of the cart at point P:	[2 pts] Speed of the cart at point Q:
[2 pts] Speed of the cart at point R:	[2 pts] Speed of the cart at point S:

C. [3 pts] What is the average acceleration of the cart during the time interval from P to Q? Briefly explain your calculations.

Figure B.12. The exam questions used in the investigation of student understanding of average speed and instantaneous speed. They were administered in PHYS 114 in different quarters (3 pages).

Midterm 2

Block M is attached to a pulley as shown. A timer is started at $t = 0.0$ s. When the timer reads $t_P = 2.0$ s, the hand holding the block is removed. At $t_Q = 4.0$ s, the string is cut when the pulley has made exactly 1 revolution. The pulley continues to rotate at constant angular velocity. It has made exactly 2.5 revolutions at $t_R = 5.5$ s, and has made exactly 4 revolutions at $t_S = 7.0$ s.

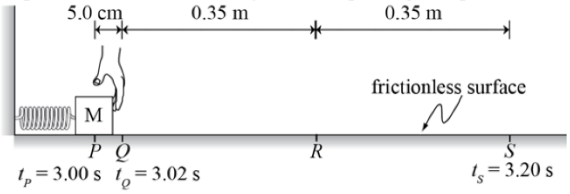


Assume there is no friction in the axle of the pulley. Which of the following best describes the angular velocities of the pulley at the 4 instants (in rad/s)?

	ω_p	ω_Q	ω_R	ω_S
A	0	1.0π	1.0π	1.0π
B	0	1.0π	2.0π	2.0π
C	0	2.0π	2.0π	2.0π
D	0	1.0π	1.4π	1.6π
E	0	1.0π	1.6π	1.6π

Final Exam

Block M, with a mass of 1.00 kg, is placed next to a spring on a frictionless surface. It is pushed to the left, compressing the spring by 5.0 cm until the center of the block reaches point *P*. The block is then released. When the center of the block is at point *Q*, the block leaves the spring. It moves along the frictionless surface past point *S*. Point *R* is halfway between point *Q* and point *S*.



Which of the following best describes the kinetic energies of the block at points *P*, *Q*, *R*, and *S* (in J)?

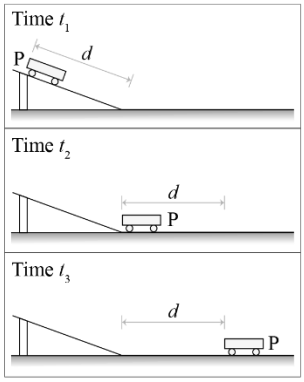
	K_P	K_Q	K_R	K_S
A	0	3.13	3.13	3.13
B	0	3.13	6.61	7.03
C	0	3.13	7.03	7.03
D	0	3.13	7.56	7.56
E	0	7.56	7.56	7.56

Wi'19 quarter

Midterm 1

Cart P is released from rest on a frictionless incline at time t_1 as shown. Between times t_1 and t_2, it travels a distance d, reaching the end of the incline. Between times t_2 and t_3, it moves a distance d across a frictionless horizontal surface. The time intervals may or may not be equal.

[6 pts] Is $t_3 - t_2$ *greater than*, *less than*, or *equal to* $t_2 - t_1$? Explain.

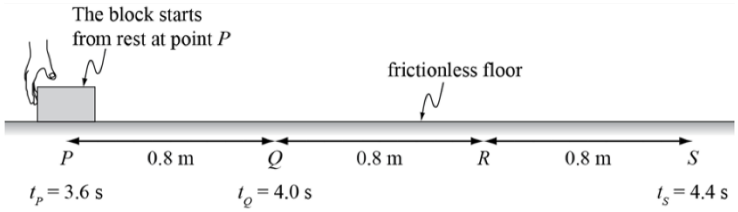


Posttest and questions used in PHYS 121

Sp'16 quarter

Midterm 1

A student pushes a wooden block from rest on a frictionless floor at point *P* as shown below. The block accelerates uniformly as it moves from point *P* to point *Q*. The student stops pushing the block when it reaches point *Q* and it continues to move on the floor. Point *R* is halfway between points *Q* and *S*. The block is located at points *P*, *Q*, and *S* at times 3.6 s, 4.0 s, and 4.4 s, respectively.

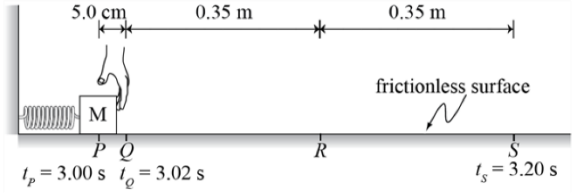


Which of the following best shows the speeds at points *P*, *Q*, *R*, and *S* (in m/s)?

	v_P	v_Q	v_R	v_S
A	0	2.0	2.0	2.0
B	0	2.0	4.0	4.0
C	0	4.0	4.0	4.0
D	0	2.0	2.7	3.0
E	0	2.0	3.0	3.0

Midterm 2

Block M, with a mass of 1.00 kg, is placed next to a spring on a frictionless surface. It is pushed to the left, compressing the spring by 5.0 cm until the center of the block reaches point *P*. The block is then released. When the center of the block is at point *Q*, the block leaves the spring. It moves along the frictionless surface past point *S*. Point *R* is halfway between point *Q* and point *S*.



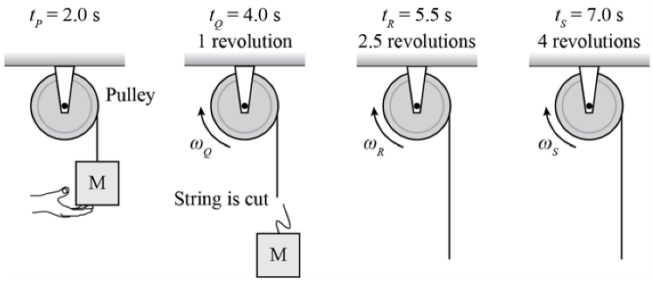
Which of the following best describes the kinetic energies of the block at points *P*, *Q*, *R*, and *S* (in J)?

	K_P	K_Q	K_R	K_S
A	0	3.13	3.13	3.13
B	0	3.13	6.61	7.03
C	0	3.13	7.03	7.03
D	0	3.13	7.56	7.56
E	0	7.56	7.56	7.56

Figure B.13. The exam questions used in the investigation of student understanding of average speed and instantaneous speed. They were administered in PHYS 121 in different quarters (3 pages).

Midterm 3

Block M is attached to a pulley as shown. A timer is started at $t = 0.0$ s. When the timer reads $t_P = 2.0$ s, the hand holding the block is removed. At $t_Q = 4.0$ s, the string is cut when the pulley has made exactly 1 revolution. The pulley continues to rotate at constant angular velocity. It has made exactly 2.5 revolutions at $t_R = 5.5$ s, and has made exactly 4 revolutions at $t_S = 7.0$ s.



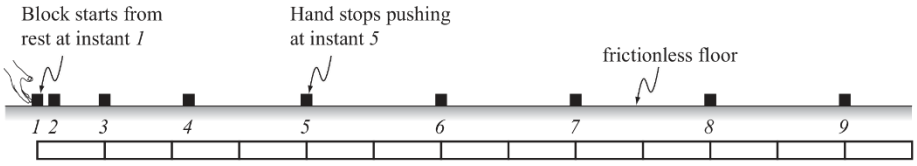
Assume there is no friction in the axle of the pulley. Which of the following best describes the angular velocities of the pulley at the 4 instants (in rad/s)?

	ω_P	ω_Q	ω_R	ω_S
A	0	1.0π	1.0π	1.0π
B	0	1.0π	2.0π	2.0π
C	0	2.0π	2.0π	2.0π
D	0	1.0π	1.4π	1.6π
E	0	1.0π	1.6π	1.6π

Sp'18

Midterm 1

D. A student pushes a block across a frictionless floor. The block starts from rest at instant *1*. At instant *5*, the student stops pushing and the block continues to move across the floor. The block is still moving at instant *9*. The motion is shown in a strobe diagram below, in which the positions are shown at instants separated by equal time intervals.



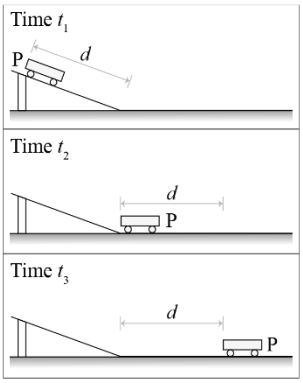
[6 pts] Which of the following expressions represent the magnitude of the *instantaneous* velocity of the block at instant *5*? You may choose more than one expression. Explain.

Circle your answer or answers: $\Delta x_{15}/\Delta t_{15}$ $\Delta x_{57}/\Delta t_{57}$ $\Delta x_{19}/\Delta t_{19}$ None

Au'18 quarter

Midterm 1

Cart P is released from rest on a frictionless incline at time t_1 as shown. Between times t_1 and t_2, it travels a distance d, reaching the end of the incline. Between times t_2 and t_3, it moves a distance d across a frictionless horizontal surface. The time intervals may or may not be equal.

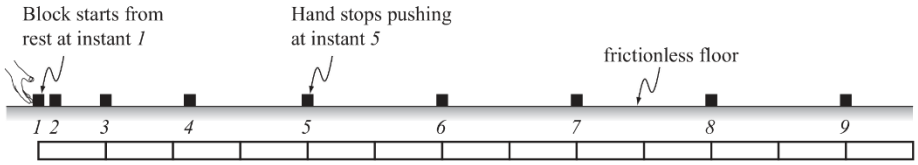


E. [6 pts] Is $t_3 - t_2$ *greater than*, *less than*, or *equal to* $t_2 - t_1$? Explain.

Wi'19 and Au'19 quarters

Midterm 1

F. A student pushes a block across a level, frictionless floor. The block starts from rest at instant *1*. At instant *5*, the student stops pushing and the block continues to move across the floor. The block is still moving at instant *9*. The motion is shown in a strobe diagram below, in which the positions are shown at instants separated by equal time intervals.



[6 pts] Which of the following expressions represent the magnitude of the *instantaneous* velocity of the block at instant *5*? Select all expressions that are applicable.

Circle your answer or answers: $\frac{x_5 - x_1}{t_5 - t_1}$ $\frac{x_7 - x_5}{t_7 - t_5}$ $\frac{x_9 - x_1}{t_9 - t_1}$ None of these expressions.

Explain your reasoning.

APPENDIX C. PRETESTS, POSTTESTS, CURRICULUM, AND ADDITIONAL NOTES FOR PART II

In Appendix C, we include the pretests, tutorial curriculum, and exam questions for the investigation of student understanding of rolling motion.

Table of contents

C1. Pretests used in Part II 538

C2. Tutorial curriculum for Rolling Motion 568

C3. Posttests and exam questions for Part II 604

C4. Miscellaneous items that are relevant for Part II 635

C1. Pretests used in Part II

This section presents the Catalyst (online) pretests we used during the investigation of student understanding of rolling motion. It includes pretests that were used before we included the *Rolling and Slipping* tutorial in the tutorial schedule and when the curriculum was being tested. Pretests during the Wi'17 and Sp'17 quarters were administered before the *Rolling and Slipping* tutorial curriculum was included in the tutorial schedule. The tutorial curriculum was tested in Wi'18, Sp'18, Au'18, Sp'19, and Au'19 quarters.

Multiple versions of pretests were created and some problems were reused in many versions. We present the unique questions that we used on pretests. For those questions that were reused many times, we indicate the subsequent versions that included them.

The questions that we asked on pretests include Rolling Motion Task (Direction & Magnitude) using wheel, frisbee, and rod, relative motion problems, motion in 2D problems, board-on-cylinder task, and cylinder-on-board task.

The tasks are arranged in a chronological order in general, but we only present unique questions from the versions to save pages. The list of pretests used in our investigation is shown in Table C.1.

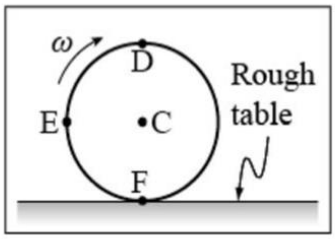
Table C.1. The list of pretests used in the investigation of student understanding of rolling motion. The quarters and lecture sections that the pretests were administered are included.

Quarter	**Pretest versions**	**Lecture sections**
Wi'17	DRB_M U2a	121ABC (Before lecture instruction on rolling motion)
Sp'17	CNL_M U2a	121AC (After lecture instruction on rolling motion)
	CNL_M U2b	121B (After lecture instruction on rolling motion)
Wi'18	M2D_M U4a	121A
	M2D_M U4b	121B
	RnS_M U1b	121A (Before lecture instruction)
	RnS_M U1c	121B (Before lecture instruction)
	RnS_M U1d	121C (Before lecture instruction)
Sp'18	M2D_M U4c	121A
	M2D_M U4d	121B
	M2D_M U4e	121C
	RnS_M U2a	121A (Before lecture instruction)
	RnS_M U2b	121B (Before lecture instruction)
	RnS_M U2c	121C (Before lecture instruction)
Au'18	M2D_M U4g	121AD
	M2D_M U4h	121C
	DRB_M U3f	121ACD (Before lecture instruction)
	RnS_M U2c	121D (After lecture instruction)
	RnS_M U2d	121A (After lecture instruction)
	RnS_M U2e	121C (After lecture instruction)
Wi'19	DRB_M U3g	121ABC (Before lecture instruction)
Sp'19	M2D_M U4d	121C
	M2D_M U4e	121B
	M2D_M U4f	121A
	DRB_M U3f	121ABC (Before lecture instruction)
	RnS_M U2a	121BC (After lecture instruction)
	RnS_M U2f	121A (After lecture instruction)
Au'19	DRB_M U3f	121ABC (Before lecture instruction)
	RnS_M U2g	121A (After lecture instruction)
	RnS_M U2h	121CE (After lecture instruction)

(DRB_M)U2a
University of Washington

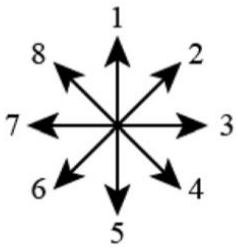
Part I

A hoop rolls without slipping on a rough table as shown at right. The center of the hoop moves with constant speed.



In the following questions, consider the direction of the velocities of different points on the hoop in the **Earth's reference frame**.

Select the directions of the velocities using the diagram below:



9: zero

Question 1.

Determine the directions of the velocities of the center of mass of the hoop, and of points D, E, and F in the **Earth's reference frame**.

	1	2	3	4	5	6	7	8	9 (zero)
Center of mass of the hoop	○	○	○	○	○	○	○	○	○
Point D	○	○	○	○	○	○	○	○	○
Point E	○	○	○	○	○	○	○	○	○
Point F	○	○	○	○	○	○	○	○	○

Question 2.

Explain your reasoning for your previous response.

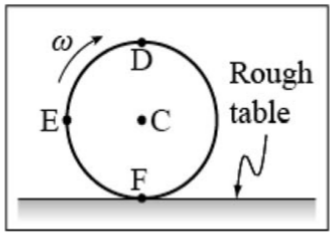
Figure C.1. The Rolling Motion Task-Direction (wheel, earth's frame).

(CNL_M)U2a
University of Washington

Part II

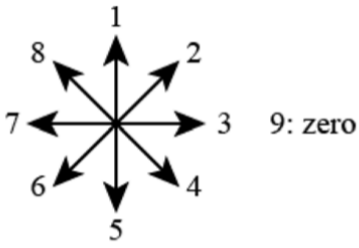
A hoop rolls without slipping on a rough table as shown at right. The center of the hoop moves with constant speed.

In the following questions, consider the velocities of different points on the hoop in the **Earth's reference frame**.



Question 9.

Which arrow at right best shows the direction of the velocity of each labelled point on the hoop in the **Earth's reference frame**?



	1	2	3	4	5	6	7	8	9 (zero)
Center of mass of the hoop, C	○	○	○	○	○	○	○	○	○
Point D	○	○	○	○	○	○	○	○	○
Point E	○	○	○	○	○	○	○	○	○
Point F	○	○	○	○	○	○	○	○	○

Question 10.

Explain your reasoning for your answer to the previous question.

Question 11.

Rank the magnitudes of the velocities of the following points on the hoop in the **Earth's reference frame**, from ***greatest*** to ***smallest***. If any of the magnitudes are equal, state so explicitly.

C: Center of mass of the hoop
D: Point D on the hoop
E: Point E on the hoop
F: Point F on the hoop

Question 12.

Explain your reasoning for your answer to the previous question.

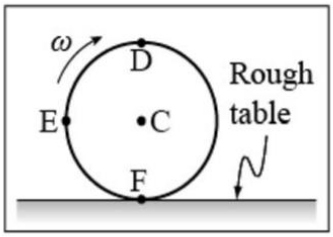
Figure C.2. The Rolling Motion Task (Wheel, earth frame) on a pretest.

(CNL_M)U2b
University of Washington

Part II

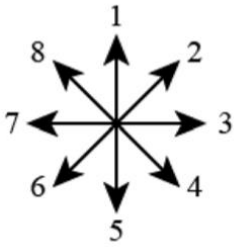
A hoop rolls without slipping on a rough table as shown at right. The center of the hoop moves with constant speed.

In the following questions, consider the velocities of different points on the hoop in the **reference frame of the center of mass of the hoop**.



Question 9.

Which arrow at right best shows the direction of the velocity of each labelled point on the hoop in the **reference frame of the center of mass of the hoop**?



	1	2	3	4	5	6	7	8	9 (zero)
Center of mass of the hoop, C	○	○	○	○	○	○	○	○	○
Point D	○	○	○	○	○	○	○	○	○
Point E	○	○	○	○	○	○	○	○	○
Point F	○	○	○	○	○	○	○	○	○

Question 10.

Explain your reasoning for your answer to the previous question.

Question 11.

Rank the magnitudes of the velocities of the following points on the hoop in the **reference frame of the center of mass of the hoop**, from ***greatest*** to ***smallest***. If any of the magnitudes are equal, state so explicitly.

C: Center of mass of the hoop
D: Point D on the hoop
E: Point E on the hoop
F: Point F on the hoop

Question 12.

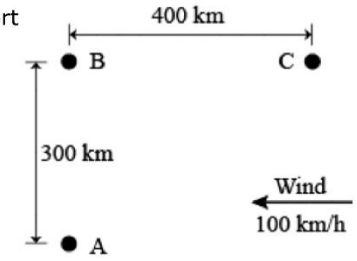
Explain your reasoning for your answer to the previous question.

Figure C.3. The Rolling Motion Task (Wheel, CM frame) on a pretest.

(M2D_M)U4a

Part II

Locations A, B, and C are airports. A plane is currently located at airport A. The wind is blowing to the west at 100 km/h relative to the ground. The plane moves with an average speed of 300 km/h relative to the wind.



Consider these 3 vectors:

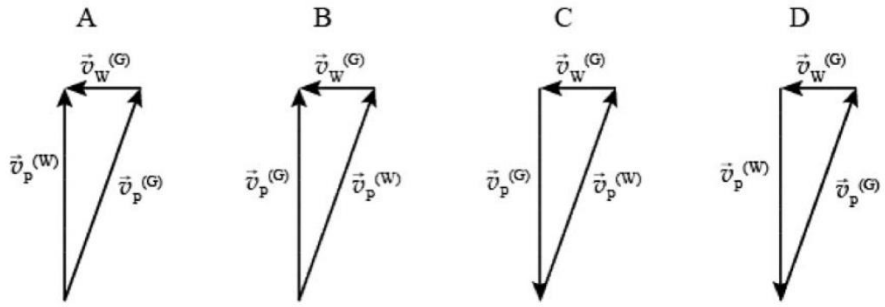
$\vec{v}_P^{(G)}$: instantaneous velocity of the plane in the ground frame

$\vec{v}_P^{(W)}$: instantaneous velocity of the plane in the wind frame

$\vec{v}_W^{(G)}$: instantaneous velocity of the wind in the ground frame

Question 7.

Suppose the plane needs to travel from airport A to airport B directly. Which of the following vector diagrams correctly illustrate the relationship of these 3 vectors?



- A
- B
- C
- D
- Other:

Question 8.

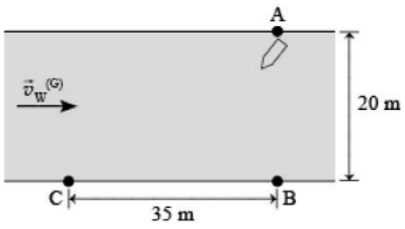
Explain your response for the previous question.

Figure C.4. The relative motion task (airplane context). This question was also asked in RnS_M U1d.

(M2D_M)U4b

Part II

A boat is located at point A on the riverbank shown at right. The water in the river flows at a speed of 3.0 m/s relative to the ground. The boat travels at a speed of 5.0 m/s relative to the water.



Consider these 3 vectors:

$\vec{v}_B^{(G)}$: instantaneous velocity of the boat in the ground frame

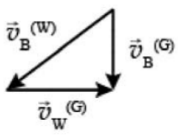
$\vec{v}_B^{(W)}$: instantaneous velocity of the boat in the water frame

$\vec{v}_W^{(G)}$: instantaneous velocity of the water in the ground frame

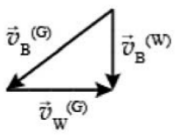
Question 7.

Suppose the boat needs to travel from point A to point B directly. Which of the following vector diagrams correctly illustrate the relationship of these 3 vectors?

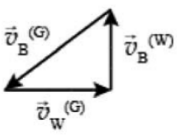
A



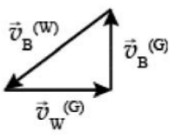
B



C



D



- A
- B
- C
- D

Question 8.

Explain your response for the previous question.

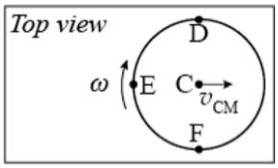
Figure C.5. The relative motion task (boat-in-river context).

(RnS_M)U1b

University of Washington

Part I

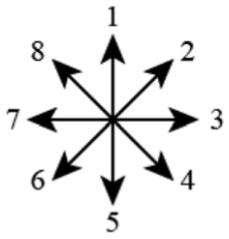
A hoop of radius R spins horizontally with an angular velocity of ω in the air as shown at right. The speed of the center of mass, v_{CM}, is equal to $R\omega$.



In the following questions, consider the instantaneous velocities of different points on the hoop in the **Earth's reference frame**.

Question 1.

For the instant shown, which arrow at right best shows the direction of the instantaneous velocity of each labeled point on the hoop in the **Earth's reference frame**?



9: zero

	1	2	3	4	5	6	7	8	9 (zero)
Point D	○	○	○	○	○	○	○	○	○
Point E	○	○	○	○	○	○	○	○	○
Point F	○	○	○	○	○	○	○	○	○

Question 2.

Explain your reasoning for your answers to the previous question.

Question 3.

Rank the magnitudes of the velocities of the following points on the hoop in the **Earth's reference frame**, from ***largest*** to ***smallest***.

C: Center of mass of the hoop
D: Point D on the hoop
E: Point E on the hoop
F: Point F on the hoop

- ○ D = E = F > C
- ○ C = D = E = F
- ○ D > E > C > F
- ○ C > D = E = F
- ○ Other:

Question 4.

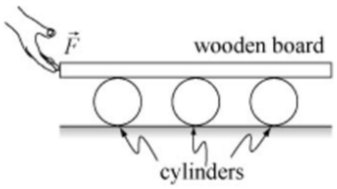
Explain your reasoning for your answer to the previous question.

Figure C.6. The Rolling Motion Task-Direction (Frisbee).

(RnS_M)U1b
University of Washington

Part II

A wooden board is placed on top of three identical solid cylinders. The board is pushed to the right by a constant horizontal force of magnitude *F* applied to the end of the board. The cylinders roll without slipping on a flat surface. There is also no slipping between the cylinders and the board.



Question 5.

Consider the time interval when the horizontal force is still exerted on the board and the board is still in contact with **all** the cylinders.

Let Δx_{board} be the magnitude of the horizontal displacement of the board and $\Delta x_{cylinder}$ be the magnitude of the horizontal displacement of the center-of-mass of each cylinder.

In the reference frame of the Earth, which of the following correctly describes the relationship between Δx_{board} and $\Delta x_{cylinder}$?

- $\Delta x_{board} = 0.5\ \Delta x_{cylinder}$
- $\Delta x_{board} = \Delta x_{cylinder}$
- $\Delta x_{board} = 1.5\ \Delta x_{cylinder}$
- $\Delta x_{board} = 2\ \Delta x_{cylinder}$
- Not enough information
- Other:

Question 6.

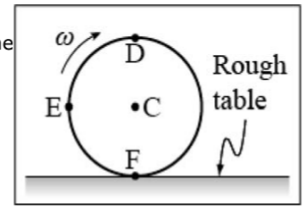
Explain your reasoning for your answer to the previous question.

Figure C.7. The board-on-cylinder. This question was also used in these versions: RnS_M U1c, RnS_M U1d, RnS_M U2a, RnS_M U2d, RnS_M U2e, and RnS_M U2f.

(RnS_M)U1c
University of Washington

Part I

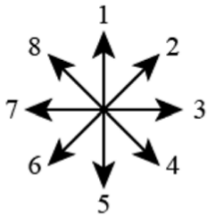
A hoop rolls without slipping on a rough table as shown at right. The center of the hoop moves with constant speed. The table is at rest in the Earth's reference frame.



In the following questions, consider the instantaneous velocities of different points on the hoop in the **Earth's reference frame**.

Question 1.

For the instant shown, which arrow at right best shows the direction of the instantaneous velocity of each labeled point on the hoop in the **Earth's reference frame**?



9: zero

	1	2	3	4	5	6	7	8	9 (zero)
Center of mass of the hoop, C	○	○	○	○	○	○	○	○	○
Point D	○	○	○	○	○	○	○	○	○
Point E	○	○	○	○	○	○	○	○	○
Point F	○	○	○	○	○	○	○	○	○

Question 2.

Explain your reasoning for your answers to the previous question.

Question 3.

Rank the magnitudes of the velocities of the following points on the hoop in the **Earth's reference frame**, from ***largest*** to ***smallest***.

C: Center of mass of the hoop
D: Point D on the hoop
E: Point E on the hoop
F: Point F on the hoop

- ○ D = E = F > C
- ○ C = D = E = F
- ○ D > E > C > F
- ○ C > D = E = F
- ○ Other:

Question 4.

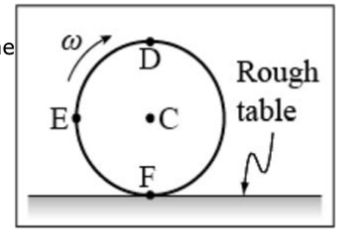
Explain your reasoning for your answer to the previous question.

Figure C.8. The Rolling Motion Task (Wheel). The speed-ranking task has changed.

(RnS_M)U1d
University of Washington

Part II

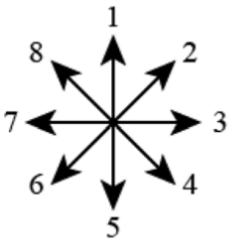
A hoop rolls without slipping on a rough table as shown at right. The center of the hoop moves with constant speed. The table is at rest in the Earth's reference frame.



In the following questions, consider the instantaneous velocities of different points on the hoop in the **Earth's reference frame**.

Question 3.

For the instant shown, which arrow at right best shows the direction of the instantaneous velocity of point E on the hoop in the **Earth's reference frame**?



	1	2	3	4	5	6	7	8	9 (zero)
Point E	○	○	○	○	○	○	○	○	○

Question 4.

Explain your reasoning for your answers to the previous question.

Question 5.

Rank the magnitudes of the velocities of the following points on the hoop in the **Earth's reference frame**, from ***largest*** to ***smallest***.

C: Center of mass of the hoop
D: Point D on the hoop
E: Point E on the hoop
F: Point F on the hoop

- ○ D = E = F > C
- ○ C = D = E = F
- ○ D > E > C > F
- ○ C > D = E = F
- ○ Other:

Question 6.

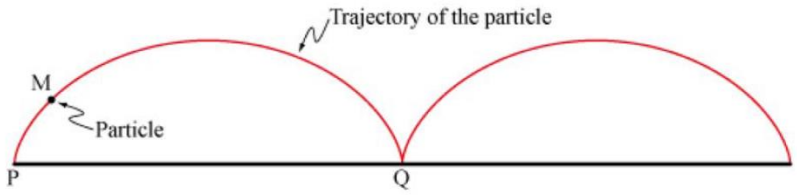
Explain your reasoning for your answer to the previous question.

Figure C.9. The Rolling Motion Task (wheel). Only direction of point E was asked.

(M2D_M)U4c
University of Washington

Part II

A particle travels from point P to point Q along a curved trajectory (red line) shown below.



Question 5.

When the particle is at the position shown on the diagram (point M), which of the following options best represents the direction of its instantaneous velocity?

A. B. C. D. E. Zero vector

- A
- B
- C
- D
- E

Question 6.

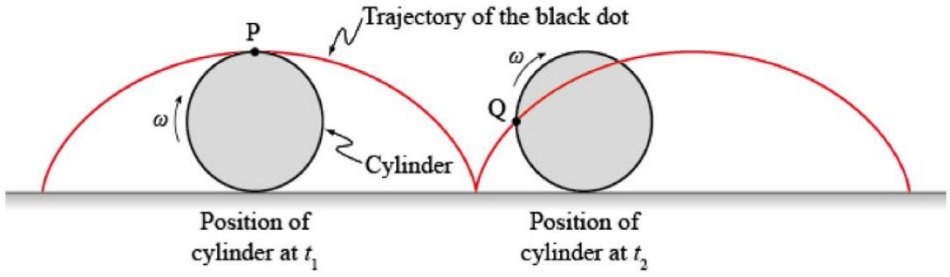
Explain your response for the previous question.

Figure C.10. The cycloid task. Students are asked to determine the velocity of point M. This question was also used in the M2D_M U4d pretest.

(M2D_M)U4e
University of Washington

Part II

A cylinder rolls without slipping on a surface as shown below. The trajectory of a point on the rim of the cylinder (marked by the black dot) is traced-out in red.



Question 5.

When the black dot is at point Q (at time t_2) shown on the diagram, which of the following options best represents the direction of its instantaneous velocity?

A. ↘ B. ↗ C. → D. ↑ E. Zero vector

- A
- B
- C
- D
- E

Question 6.

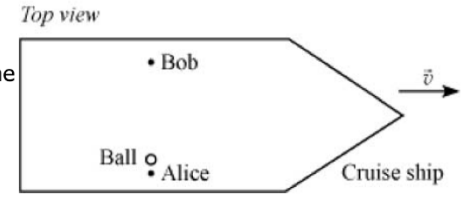
Explain your response for the previous question.

Figure C.11. The rolling-wheel-with-cycloid task Students are asked to determine the velocity of point Q.

(M2D_M)U4e
University of Washington

Part III

Alice and Bob are on a cruise ship in still water as shown. The cruise ship is moving to the right in the ground frame. Alice throws a ball directly to Bob.



Chris, who is in a helicopter hovering in the air (not moving relative to the ground) above the cruise ship, observes the motion of the ball.

Question 7.

Which of the following best describes the direction of the velocity of the ball as observed by Chris?

(1) (2) (3) (4)

- 1
- 2
- 3
- 4
- Other:

Question 8.

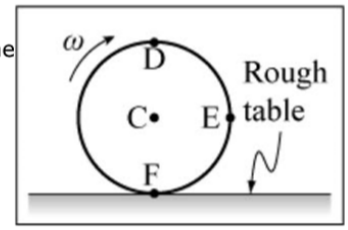
Explain your response for the previous question.

Figure C.12. The ball-on-moving-boat task.

(RnS_M)U2a
University of Washington

Part I

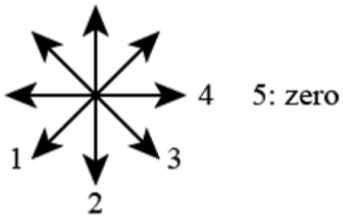
A hoop rolls without slipping on a rough table as shown at right. The center of the hoop moves with constant speed. The table is at rest in the Earth's reference frame.



In the following questions, consider the instantaneous velocities of different points on the hoop in the **Earth's reference frame**.

Question 1.

For the instant shown, which arrow at right best shows the direction of the instantaneous velocity of point E on the hoop in the **Earth's reference frame**?



	1	2	3	4	5 (zero)
Point E	○	○	○	○	○

Question 2.

Explain your reasoning for your answer to the previous question.

Question 5.

Rank the magnitudes of the velocities of the following points on the hoop in the **Earth's reference frame**, from ***largest*** to ***smallest***.

C: Center of mass of the hoop
D: Point D on the hoop
E: Point E on the hoop
F: Point F on the hoop

- ○ D = E = F > C
- ○ C = D = E = F
- ○ D > E > C > F
- ○ C > D = E = F
- ○ Other:

Question 6.

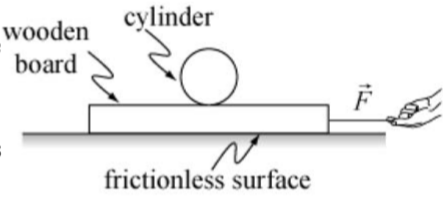
Explain your reasoning for your answer to the previous question.

Figure C.13. The Rolling Motion Task (wheel). The position of point E has changed.

(RnS_M)U2a
University of Washington

Part III

A cylinder is placed on top of a wooden board. The board is pulled to the right by a constant horizontal force of magnitude F on a frictionless surface. The cylinder is observed to roll counterclockwise without slipping on the board.



At the instant that the board has speed v_b, the center of mass of the cylinder has speed v_c, and the cylinder is rotating with a rotational speed of ω.

Question 7.

Which of the following equations correctly describes the relationship between v_b, v_c, R (radius of cylinder), and the rotational speed of the cylinder, ω?

Note: Treat all of the above quantities as having positive values.

- $v_b = v_c$
- $v_b = v_c + R\omega$
- $v_b = v_c - R\omega$
- $v_b = v_c + 2R\omega$
- Not enough information
- Other:

Question 8.

Explain your reasoning for your answer to the previous question.

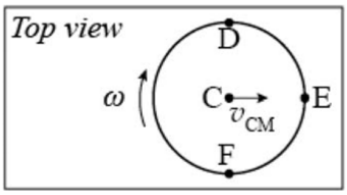
Figure C.14. The cylinder-on-board task. The question can also be found in these pretest versions: RnS_M U2b, RnS_M U2c, RnS_M U2d, RnS_M U2e, and RnS_M U2f.

(RnS_M)U2b

University of Washington

Part I

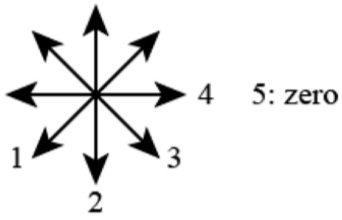
A hoop of radius *R* moves horizontally through the air. It spins with an angular velocity of ω and the speed of its center of mass, v_{CM}, is equal to $R\omega$.



For the following questions, consider the instantaneous velocities of different points on the hoop in the **Earth's reference frame**.

Question 1.

For the instant shown, which arrow at right best shows the direction of the instantaneous velocity of point E on the hoop in the **Earth's reference frame**?



	1	2	3	4	5 (zero)
Point E	○	○	○	○	○

Question 2.

Explain your reasoning for your answer to the previous question.

Question 3.

Rank the magnitudes of the velocities of points C, D, E, and F on the hoop in the **Earth's reference frame**, from ***largest*** to ***smallest***.

- ○ D = E = F > C
- ○ C = D = E = F
- ○ D > E > C > F
- ○ C > D = E = F
- ○ Other:

Question 4.

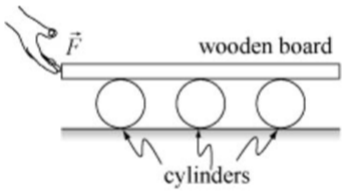
Explain your reasoning for your answer to the previous question.

Figure C.15. The Rolling Motion Task-Direction (Frisbee). The position of point E has changed.

(RnS_M)U2b
University of Washington

Part II

A wooden board is placed on top of three identical solid cylinders. The board is pushed to the right by a constant horizontal force of magnitude *F* applied to the end of the board. The cylinders roll without slipping on a flat surface. There is also no slipping between the cylinders and the board.



Question 5.

Consider an instant when the horizontal force is still exerted on the board and the board is still in contact with **all** of the cylinders.

Let v_{board} be the magnitude of the velocity of the board and $v_{cylinder}$ be the magnitude of the velocity of the center-of-mass of each cylinder.

In the reference frame of the Earth, which of the following correctly describes the relationship between v_{board} and $v_{cylinder}$?

- $v_{board} = 0.5\ v_{cylinder}$
- $v_{board} = v_{cylinder}$
- $v_{board} = 1.5\ v_{cylinder}$
- $v_{board} = 2\ v_{cylinder}$
- Not enough information
- Other:

Question 6.

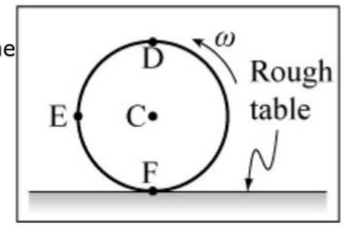
Explain your reasoning for your answer to the previous question.

Figure C.16. The board-on-cylinder task. Students are asked to find the correct equation for velocities.

(RnS_M)U2c
University of Washington

Part I

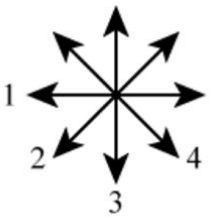
A hoop rolls without slipping on a rough table as shown at right. The center of the hoop moves with constant speed. The table is at rest in the Earth's reference frame.



For the following questions, consider the instantaneous velocities of different points on the hoop in the **Earth's reference frame**.

Question 1.

For the instant shown, which arrow at right best shows the direction of the instantaneous velocity of point E on the hoop in the **Earth's reference frame**?



5: zero

	1	2	3	4	5 (zero)
Point E	○	○	○	○	○

Question 2.

Explain your reasoning for your answer to the previous question.

Question 3.

Rank the magnitudes of the velocities of points C, D, E, and F on the hoop in the **Earth's reference frame**, from ***largest*** to ***smallest***.

- ○ D = E = F > C
- ○ C = D = E = F
- ○ D > E > C > F
- ○ C > D = E = F
- ○ Other:

Question 4.

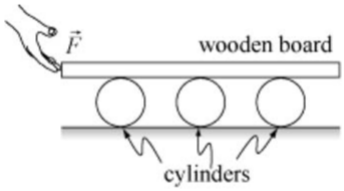
Explain your reasoning for your answer to the previous question.

Figure C.17. The Rolling Motion Task (wheel). The direction of rolling has changed.

(RnS_M)U2c
University of Washington

Part II

A wooden board is placed on top of three identical solid cylinders. The board is pushed to the right by a constant horizontal force of magnitude *F* applied to the end of the board. The cylinders roll without slipping on a flat surface. There is also no slipping between the cylinders and the board.



Question 5.

Consider an instant when the horizontal force is still exerted on the board and the board is still in contact with **all** of the cylinders.

Let a_{board} be the magnitude of the acceleration of the board and $a_{cylinder}$ be the magnitude of the acceleration of the center-of-mass of each cylinder.

In the reference frame of the Earth, which of the following correctly describes the relationship between a_{board} and $a_{cylinder}$?

- $a_{board} = 0.5\ a_{cylinder}$
- $a_{board} = a_{cylinder}$
- $a_{board} = 1.5\ a_{cylinder}$
- $a_{board} = 2\ a_{cylinder}$
- Not enough information
- Other:

Question 6.

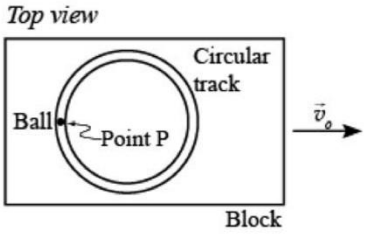
Explain your reasoning for your answer to the previous question.

Figure C.18. The board-on-cylinder. Students are asked to find the correct equation for accelerations.

(M2D_M)U4g
University of Washington

Part II

A circular track is mounted on a rectangular block. A ball is moving clockwise along the circular track with constant speed v_o. The block is then pulled to the right with constant speed v_o across a level table.



Question 5.

At time t_o, a stationary observer sees the ball at point P as shown in the top-view diagram above.

Which of the following options best represents the direction of the instantaneous velocity of the ball when it is at point P according to the observer?

A. ↘ B. ↗ C. → D. ↑ E. Zero vector

- A
- B
- C
- D
- E

Question 6.

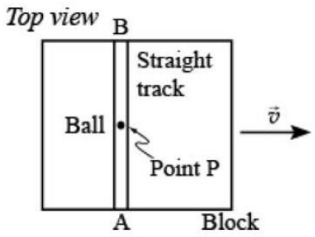
Explain your response for the previous question.

Figure C.19. The ball-in-circular-track-on-moving-block task.

(M2D_M)U4g
University of Washington

Part III

A large block is pulled at constant speed across a table. The block has a ball on it that rolls along a straight track from point A to point B.



Question 7.

At time t_o, a stationary observer sees the ball at point P as shown in the top-view diagram above.

Which of the following options best represents the direction of the instantaneous velocity of the ball when it is at point P according to the observer?

- 1
- 2
- 3
- 4

Question 8.

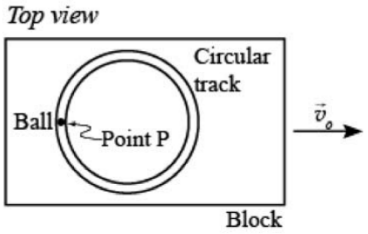
Explain your response for the previous question.

Figure C.20. The ball-in-straight-track-on-moving-block task.

(M2D_M)U4h
University of Washington

Part II

A circular track is mounted on a rectangular block. A ball is moving clockwise along the circular track with constant speed v_o. The block is then pulled to the right with constant speed v_o across a level table.



Question 5.

At time t_o, a stationary observer sees the ball at point P as shown in the top-view diagram above.

Which of the following options below best represents the direction of the instantaneous velocity of the ball when it is at point P in the (i) **observer's frame** and the (ii) **moving block's frame**?

A. ↘ B. ↗ C. → D. ↑ E. Zero vector

	A	B	C	D	E
(i) Observer's frame	○	○	○	○	○
(ii) Block's frame	○	○	○	○	○

Question 6.

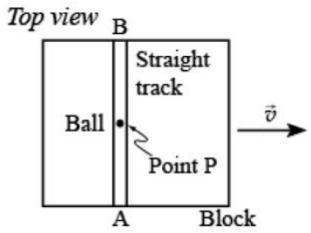
Explain your response for the previous question.

Figure C.21. The ball-in-circular-track-on-moving-block task. Two reference frames (the observer's and the moving block's frames) are considered.

(M2D_M)U4h
University of Washington

Part III

A large block is pulled at constant speed across a table. The block has a ball on it that rolls along a straight track from point A to point B.



Question 7.

At time t_o, a stationary observer sees the ball at point P as shown in the top-view diagram above.

Which of the following options below best represents the direction of the instantaneous velocity of the ball when it is at point P in the (i) **observer's frame** and the (ii) **moving block's frame**?

(1) (2) (3) (4)

	1	2	3	4
(i) Observer's frame	○	○	○	○
(ii) Moving block's frame	○	○	○	○

Question 8.

Explain your response for the previous question.

Figure C.22. The ball-in-straight-track-on-moving-block task. Two reference frames (the observer's and the moving block's frames) are considered.

(DRB_M)U3f_Video
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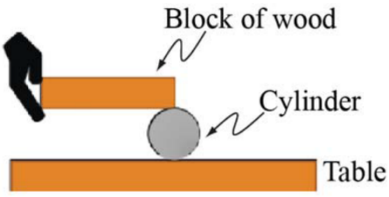
Page **5** of 5

Question 11.

Part IV

A cylinder rests on a flat table. A block of wood is placed horizontally with one end resting on the cylinder and the other end supported by a person. The person then pushes the block so that it moves to the right by a distance *L*. The block rolls without slipping on the cylinder and the cylinder rolls without slipping on the table. Assume the block remains horizontal and in contact with the cylinder throughout the motion.



How far does the cylinder move?

- 0
- L/2
- L
- 2L

Question 12.
Explain your reasoning for your previous response.

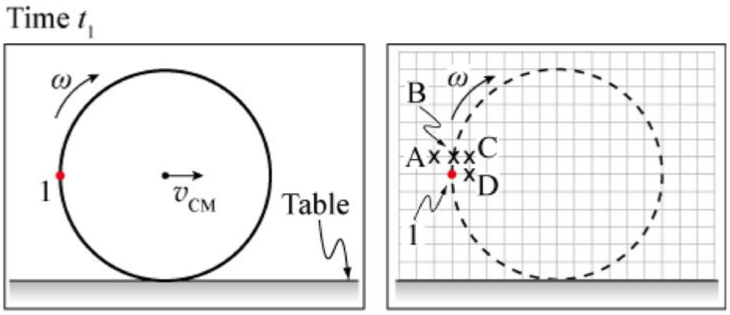
Figure C.23.The modified board-on-cylinder task.

(RnS_M)U2d

University of Washington

Part I

A hoop **rolls without slipping** to the right on a stationary table as shown below. The center of the hoop moves with constant speed. A small red spot is marked on the hoop (point 1). The diagram shows the red spot at t_1.



Question 1.

A grid is marked on the wall behind the hoop. Which point on the wall best corresponds to the location of the red spot a short time later?

- A
- B
- C
- D
- None of the above
- Other (please describe):

Question 2.

Explain your reasoning for your answer to the previous question.

Question 3.

For the instant shown, which arrow below best shows the direction of the instantaneous velocity of point 1 on the hoop in the **Earth's reference frame**?

A. B. C. D. E. Zero vector

- A
- B
- C
- D
- E

Question 4.

Explain your reasoning for your answer to the previous question.

Figure C.24. The modified Rolling Motion task. Students are asked to think of displacement first before answering the direction of velocity. This question is also asked in DRB_M U3g pretest.

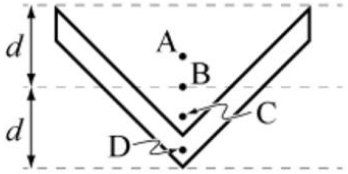
(RnS_M)U2f
University of Washington

Question 1.

Part I

For the uniform L-shaped rod shown at right, which of the points best represent the location of the center-of-mass of the rod?

If you think it should be between 2 of the given consecutive points (*e.g.,* between C and D), state so explicitly.



- A
- B
- C
- D
- Other:

Question 2.

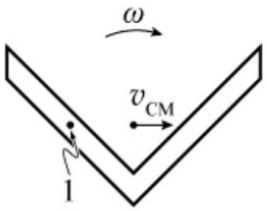
Explain your reasoning for your answer to the previous question.

Question 3.

Part I (cont'd)

A uniform L-shaped rod is moving to the right and rotating on a level, frictionless table. The rod rotates clockwise about its center-of-mass with an angular velocity ω.

For an observer standing at rest next to the table, which of the following options best represents the direction of the instantaneous velocity of point 1 at the instant shown?



A. B. C. D. E. Zero vector

- A
- B
- C
- D
- E

Question 4.

Explain your reasoning for your answer to the previous question.

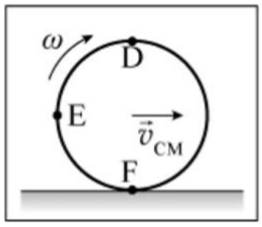
Figure C.25. The Rolling Motion Task with L-shaped rod. Question 3 is also asked in RnS_M U2e pretest.

(RnS_M)U2g

University of Washington

Part I

A rigid cylinder rolls to the right without slipping on a leveled surface as shown. The coefficient of friction of the surface is not zero. It is observed that the cylinder moves with constant speed.



Question 1.

For the instant shown, which arrow below best shows the direction of the instantaneous velocity of point E on the cylinder in the **Earth's reference frame**?

A. B. C. D. E. Zero vector

- A
- B
- C
- D
- E

Question 2.

Explain your reasoning for your answer to the previous question.

Question 3.

Rank the magnitudes of the velocities of points D, E, and F on the cylinder in the **Earth's reference frame**, from ***largest*** to ***smallest***.

- D = E = F
- F > E > D
- D > E > F
- E > D = F

Question 4.

Explain your reasoning for your answer to the previous question.

Question 5.

What is the direction of the frictional force acting on the cylinder?

- Left
- Right
- Zero
- Not enough information

Question 6.

Explain your reasoning for your answer to the previous question.

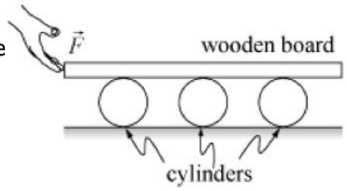
Figure C.26. The Rolling Motion Task in RnS_U2g version.

(RnS_M)U2g
University of Washington

Page **2** of 2

Part II

A wooden board is placed on top of three identical solid cylinders. The board is pushed to the right by a constant horizontal force of magnitude *F* applied to the end of the board. The cylinders roll without slipping on a flat surface. There is also no slipping between the cylinders and the board.



Question 7.

Consider the time interval when the horizontal force is still exerted on the board and the board is still in contact with **all** of the cylinders.

When the board moves a distance L, which of the following best describes the distance each cylinder moves?

- Between 0 to L/2
- L/2
- Between L/2 to L
- L
- Between L to 2L
- 2L
- Other:

Question 8.

Explain your reasoning for your answer to the previous question.

Figure C.27. The board-on-cylinder task with different answer options. This question is also found in RnS_M U2h pretest.

(RnS_M)U2h
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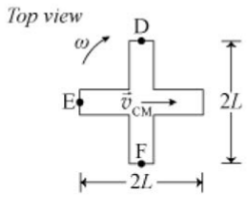
Part I

An object of dimensions 2*L* by 2*L* is moving to the right and rotating on a level, frictionless table. The object rotates clockwise about its center-of-mass with an angular velocity ω. The speed of its center-of-mass, v_{CM}, is equal to $L\omega$.



Question 1.

For the instant shown, which arrow below best shows the direction of the instantaneous velocity of point E on the object in the **Earth's reference frame**?

A. ↘ B. ↗ C. → D. ↑ E. Zero vector

- A
- B
- C
- D
- E

Question 2.

Explain your reasoning for your answer to the previous question.

Question 3.

Rank the magnitudes of the velocities of points D, E, and F on the object in the **Earth's reference frame**, from ***largest*** to ***smallest***.

- D = E = F
- F > E > D
- D > E > F
- E > D = F
- Other:

Question 4.

Explain your reasoning for your answer to the previous question.

Figure C.28. The Rolling Motion Task with plus-shaped object.

C2. Tutorial curriculum for Rolling Motion

This section of Appendix C presents the tutorial curriculum (in-class worksheet, followed by homework) from Wi'18 to Au'19 quarters. They are arranged according to quarters. The preliminary version of the tutorial curriculum is used in the Wi'18, Sp'18, and Au'18 quarters. The modified version of the curriculum is used in the Sp'19 and Au'19 quarters.

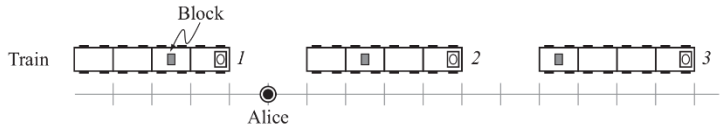
(A) Wi'18 and Sp'18 quarter

ROLLING AND SLIPPING: A RELATIVE MOTION APPROACH

Mech 45

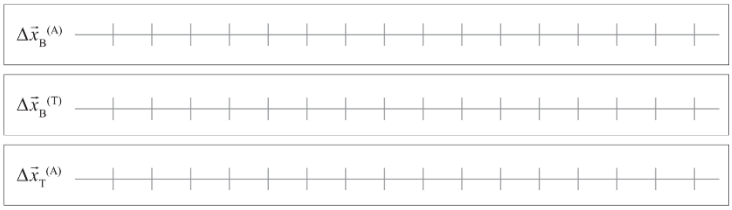
I. Review of Galilean transformation of velocities

Alice, a stationary observer, observes a train moving to the right. A block is pushed so that it moves backward on the train. The positions of the train and the block are shown at instants 1–3, separated by equal time intervals.



A. In the spaces provided below, draw vectors to represent the following displacements over the interval from instant *1* to instant *3:*

- the displacement of the block *in Alice's frame*, $\Delta\vec{x}_B^{(A)}$
- the displacement of the block *in the frame of the center of the train*, $\Delta\vec{x}_B^{(T)}$
- the displacement of the center of the train *in Alice's frame*, $\Delta\vec{x}_T^{(A)}$



B. In the space below, draw and label a vector diagram to show how one of these three displacement vectors is the sum of the other two.

Express the relationship between the three vectors as an algebraic equation.

In mechanics, relative velocities in two frames can be expressed using *the Galilean transformation of velocities*. In this case, the relationship is written as $\vec{v}_B^{(A)} = \vec{v}_B^{(T)} + \vec{v}_T^{(A)}$.

C. Check that the above equation is consistent with your result above for the displacements. Explain how these two equations are related.

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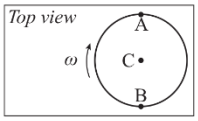
Figure C.29. Preliminary version of in-class tutorial worksheet administered to all sections of PHYS 121 in the Wi'18 and Sp'18 quarters (4 pages).

II. Combining translational and rotational motion

A hoop of radius R (shown at right) is thrown in the air horizontally such that it spins with angular velocity ω and moves to the right.



A. Draw a vector on the diagram to represent the direction of the instantaneous velocity of the center of mass of the hoop in the reference frame of the ground. Explain.

B. Consider the instantaneous velocities of points A, C, and B in the reference frame of the *center of mass* of the hoop (point C), *i.e.*, $\vec{v}_{\mathrm{A}}^{(\mathrm{C})}$, $\vec{v}_{\mathrm{C}}^{(\mathrm{C})}$, and $\vec{v}_{\mathrm{B}}^{(\mathrm{C})}$ respectively. In the table below,

1. write down their magnitudes in terms of R and ω, and
2. draw vectors to represent their directions

	$\vec{v}_{\mathrm{A}}^{(\mathrm{C})}$	$\vec{v}_{\mathrm{C}}^{(\mathrm{C})}$	$\vec{v}_{\mathrm{B}}^{(\mathrm{C})}$
Magnitude			
Direction			

C. Consider the following three vectors:

$\vec{v}_{\mathrm{M}}^{(\mathrm{C})}$: instantaneous velocity of point A in the *center of mass frame*

$\vec{v}_{\mathrm{C}}^{(\mathrm{G})}$: instantaneous velocity of the center of mass in the *ground frame*

$\vec{v}_{\mathrm{M}}^{(\mathrm{G})}$: instantaneous velocity of point A in the *ground frame*

Write down an equation to describe how these three vectors are related. Provide a written description of the equation.

The instantaneous velocity (vector) of the center of mass in the reference frame of the ground is typically written as $\vec{v}_{\mathrm{CM}}$.

D. Rewrite the equation in part C using $\vec{v}_{\mathrm{CM}}$.

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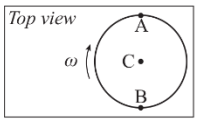
E. Consider the instantaneous velocities of points A, C, and B in the *ground* frame for the instant shown at right. In the spaces below,



1. write down their magnitudes in terms of v_{CM}, R, and ω (the positive direction is chosen to be to the right), and
2. draw vectors to show all their *possible* directions using your answer in part D.

	$\vec{v}_A^{(G)}$	$\vec{v}_C^{(G)}$	$\vec{v}_B^{(G)}$
1. Possible directions			
2. Magnitude			

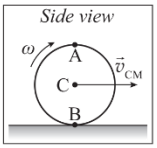
➪ Check your reasoning with a tutorial instructor before proceeding.

If a rigid body is simultaneously undergoing both translational motion and rotational motion, we can use the above method to find the magnitude and direction of the instantaneous velocity of any point on the rigid body in the ground/Earth's frame.

III. Rolling and slipping

Objects can move on surfaces with or without slipping. *Non-slipping motion* refers to motion in which there is *no relative velocity* between the point of contact of the object on the surface, and the surface itself. *Motion with slipping* refers to motion in which there is a *non-zero relative velocity* between the point of contact of the object on the surface, and the surface itself.

A cylinder rolls *without slipping* with constant speed on a leveled ground. The ground remains at rest.



A. What is the magnitude of the velocity of point B in the reference frame of the ground, $\vec{v}_B^{(G)}$? Explain.

B. Write down an equation that relates v_{CM}, R, and ω using your answer above.

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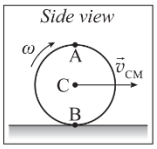
C. A cylinder moves across a horizontal surface. The surface is not frictionless.



Consider the motion of the cylinder for three different initial conditions: (i) $v_{CM} = R\omega$, (ii) $v_{CM} > R\omega$, and (iii) $v_{CM} < R\omega$.

For each case, complete the table below:

	Initial condition of the motion		
	$v_{CM} = R\omega$	$v_{CM} > R\omega$	$v_{CM} < R\omega$
Indicate the initial directions of $\vec{v}_A^{(G)}$, $\vec{v}_C^{(G)}$, and $\vec{v}_B^{(G)}$.			
Indicate whether the cylinder is *rolling without slipping* or *rolling with slipping* initially.			
Draw a possible extended free-body diagram of the cylinder.			
Would v_{CM} *increase*, *decrease*, or *remain the same* as the motion continues?	Prediction:		
	Observation:		
Would ω *increase*, *decrease*, or *remain the same* as the motion continues?	Prediction:		
	Observation:		

D. Ask a tutorial instructor for a demonstration for the motion of the cylinder with three different initial conditions. Observe how the motion of the cylinder changes over time.

On the table above, write down your observations and revise your extended free-body diagrams if necessary.

Does the cylinder eventually roll without slipping on the ground for each case?

E. Based on your observations for each case, what can you say about the direction of the horizontal force acting on the cylinder and the direction of $\vec{v}_B^{(G)}$?

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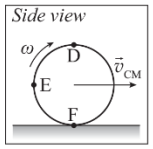
ROLLING AND SLIPPING: A RELATIVE MOTION APPROACH

Name:_____ Section:_____

Mech HW–45

1. A cylinder rolls *without slipping* with constant speed on a leveled ground as shown. In the spaces below,



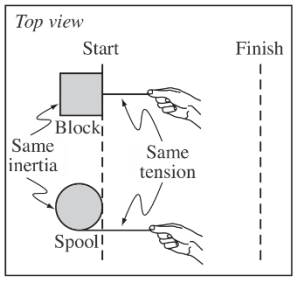
   a. write down the magnitudes of the instantaneous velocities of points D, E, and F in the *ground* frame in terms of v_{CM} only. Take right as the positive direction, and

   b. draw vectors to show the directions of the instantaneous velocities of points D, E, and F for the instant shown at right. Explain.

	$\vec{v}_D^{(G)}$	$\vec{v}_E^{(G)}$	$\vec{v}_F^{(G)}$
c. Magnitude			
d. Direction			

2. This problem revisits the "block-and-spool problem" from the tutorial *Dynamics of rigid bodies*.

A block and a spool are each pulled across a level, frictionless surface by a string, as illustrated in the top-view diagram at right.



The string pulling the block is tied to a hook (not shown) at the center of the front of the block. The string pulling the spool is wrapped many times around the spool and may unwind as it is pulled.

The block and the spool have the same inertias. The strings start pulling with the same constant force at the same instant. Assume that the inertias of the strings and the hook are negligible.

   a. When the block is moving to the right at speed v, is the speed of the center of mass of the spool *greater than*, *less than*, or *equal to* v? Explain.

   b. When the hand pulling the block is moving at speed v, find the speed of the hand pulling the spool in terms of v, the radius of the spool, R, and the angular velocity of the spool at the same instant, ω. Explain your reasoning.

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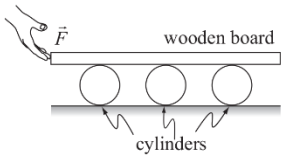
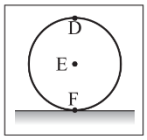
Figure C.30. Preliminary version of tutorial homework administered to all sections of PHYS 121 in the Sp'18 quarter (2 pages).

3. A wooden board is placed on top of two identical solid cylinders. The board is pushed to the right by a constant horizontal force of magnitude F. The cylinders roll without slipping on a flat surface. There is also no slipping between the cylinders and the board.



a. What is the direction of center-of-mass velocity of the cylinders when the horizontal force is applied to the slab? Explain.

b. Consider an instant when the force is applied to the slab. Rank the magnitudes of the instantaneous velocities of points D, E, and F on the cylinder shown at right in the **reference frame of the ground**. If any of the magnitudes is zero, state so explicitly. Explain.



c. In the ground frame, express the magnitude of the instantaneous velocity of the slab, v_{slab}, in terms of the magnitude of the instantaneous velocity of the center-of-mass of the cylinder, v_{cylinder}. Explain.

d. In the ground frame, express the acceleration of the slab, a_{slab}, in terms of the acceleration of the center-of-mass of the cylinder, a_{cylinder}. Explain.

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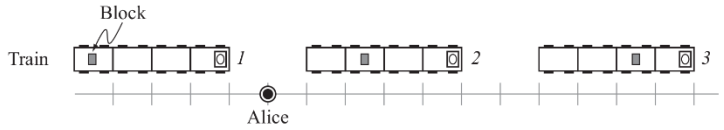
(B) Au'18 quarter

ROLLING AND SLIPPING: A RELATIVE MOTION APPROACH

Mech 45

I. Review of Galilean transformation of velocities

Alice, a stationary observer, observes a train moving to the right. A block on the train is pushed so that it moves at constant velocity relative to the train. The positions of the train and the block are shown at instants *1–3*, separated by equal time intervals.



A. In the spaces provided below, draw vectors to represent the following displacements over the interval from instant *1* to instant *3:*

- the displacement of the center of the train *in Alice's frame,* $\Delta\vec{x}_{\mathrm{T}}^{(\mathrm{A})}$
- the displacement of the block *in the frame of the center of the train,* $\Delta\vec{x}_{\mathrm{B}}^{(\mathrm{T})}$
- the displacement of the block *in Alice's frame,* $\Delta\vec{x}_{\mathrm{B}}^{(\mathrm{A})}$

$\Delta\vec{x}_{\mathrm{T}}^{(\mathrm{A})}$

$\Delta\vec{x}_{\mathrm{B}}^{(\mathrm{T})}$

$\Delta\vec{x}_{\mathrm{B}}^{(\mathrm{A})}$

B. In the space below, draw and label a vector diagram to show how one of these three displacement vectors is the sum of the other two.

Express the relationship between the three vectors as an algebraic equation.

C. Suppose instead that the block started at the front of the train and was pushed toward the back of the train. How would your answers to parts A and B change? Would your algebraic equation in part B still hold? Explain.

In mechanics, relative velocities in two frames can be expressed using *the Galilean transformation of velocities*. In this case, the relationship is written as $\vec{v}_{\mathrm{B}}^{(\mathrm{A})} = \vec{v}_{\mathrm{B}}^{(\mathrm{T})} + \vec{v}_{\mathrm{T}}^{(\mathrm{A})}$.

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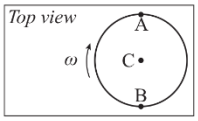
Figure C.31. Preliminary version of in-class tutorial worksheet administered to all sections of PHYS 121 in the Au'18 quarter (4 pages).

II. Combining translational and rotational motion

A hoop of radius R (shown at right) is thrown in the air horizontally such that it spins with angular velocity ω and moves to the right.



A. Draw a vector on the diagram to represent the direction of the instantaneous velocity of the center of mass of the hoop in the reference frame of the ground.

B. Consider the instantaneous velocities of points A, C, and B in the reference frame of the *center-of-mass* of the hoop (point C), *i.e.*, $\vec{v}_{\mathrm{A}}^{(\mathrm{C})}$, $\vec{v}_{\mathrm{C}}^{(\mathrm{C})}$, and $\vec{v}_{\mathrm{B}}^{(\mathrm{C})}$, respectively. In the table below,

1. write down their magnitudes in terms of R and ω, and
2. draw vectors to represent their directions

	$\vec{v}_{\mathrm{A}}^{(\mathrm{C})}$	$\vec{v}_{\mathrm{C}}^{(\mathrm{C})}$	$\vec{v}_{\mathrm{B}}^{(\mathrm{C})}$
Magnitude			
Direction			

Describe the motion of the hoop for an observer in the reference frame of the *center-of-mass* of the hoop. Check that your description is consistent with your answers in the table.

C. Consider the following three vectors:

$\vec{v}_{\mathrm{A}}^{(\mathrm{C})}$: instantaneous velocity of point A in the *center of mass frame*

$\vec{v}_{\mathrm{C}}^{(\mathrm{G})}$: instantaneous velocity of the center of mass in the *ground frame*

$\vec{v}_{\mathrm{A}}^{(\mathrm{G})}$: instantaneous velocity of point A in the *ground frame*

Write down an equation to describe how these three vectors are related.

Provide a written description of the equation.

The instantaneous velocity (vector) of the center of mass in the reference frame of the ground is typically written as $\vec{v}_{\mathrm{CM}}$.

D. Rewrite the equation in part C using $\vec{v}_{\mathrm{CM}}$.

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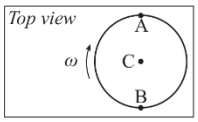
E. Consider the instantaneous velocities of points A, C, and B in the *ground* frame for the instant shown at right. In the spaces below,



1. write down their magnitudes in terms of v_{CM}, R, and ω (the positive direction is chosen to be to the right), and
2. draw vectors to show all their *possible* directions using your answer in part D.

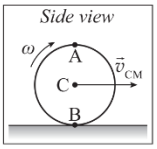
	$\vec{v}_A^{(G)}$	$\vec{v}_C^{(G)}$	$\vec{v}_B^{(G)}$
1. Magnitude			
2. Possible directions			

➪ Check your reasoning with a tutorial instructor before proceeding.

The hoop in this section is undergoing both translational motion and rotational motion simultaneously. As you have found above, for a rigid body that is translating and rotating, we can use a relative motion approach to find the magnitude and direction of the instantaneous velocity of any point on the rigid body in the ground/Earth's frame.

III. Rolling and slipping

A. A cylinder moves across a horizontal surface. The surface is not frictionless.



Consider the motion of the cylinder for three different initial conditions: (i) $v_{CM} = R\omega$, (ii) $v_{CM} > R\omega$, and (iii) $v_{CM} < R\omega$.

For each case, complete the table below:

	Initial condition of the motion		
	$v_{CM} = R\omega$	$v_{CM} > R\omega$	$v_{CM} < R\omega$
Indicate the initial directions of $\vec{v}_A^{(G)}$, $\vec{v}_C^{(G)}$, and $\vec{v}_B^{(G)}$.			
Draw a possible extended free-body diagram of the cylinder.			

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	Initial condition of the motion		
	$v_{CM} = R\omega$	$v_{CM} > R\omega$	$v_{CM} < R\omega$
Would v_{CM} *increase*, *decrease*, or *remain the same* as the motion continues?	Prediction:		
	Observation:		
Would ω *increase*, *decrease*, or *remain the same* as the motion continues?	Prediction:		
	Observation:		

B. Ask a tutorial instructor for a demonstration of the motion of the cylinder with all three different initial conditions. Observe how the motion of the cylinder changes over time in each case.

On the table above, write down your observations and revise your extended free-body diagrams according to your observations if necessary.

Objects can move on surfaces with or without slipping. *Non-slipping motion* refers to motion in which there is *no relative velocity* between the point of contact of the object on the surface, and the surface itself. *Motion with slipping* refers to motion in which there is a *non-zero relative velocity* between the point of contact of the object on the surface and the surface itself.

C. For each of the above case, determine whether the cylinder is initially rolling without slipping or slipping. Explain.

D. Does the cylinder eventually roll without slipping on the ground for each case? If so, what is the magnitude of $\vec{v}_B^{(G)}$ then?

E. Based on your observations for each case, what can you say about the initial direction of the horizontal force acting on the cylinder and the initial direction of $\vec{v}_B^{(G)}$?

➪ Check your answers to this section with a tutorial instructor.

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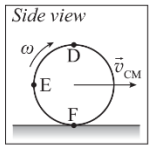
ROLLING AND SLIPPING: A RELATIVE MOTION APPROACH

Name:_____
Section:

Mech
HW–45

1. A cylinder rolls *without slipping* with constant speed across level ground as shown. In the spaces below,

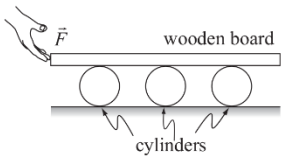


- write down the magnitudes of the instantaneous velocities of points D, E, and F in the *ground* frame. *Express your answers in terms of* v_{CM} *only*.
- draw vectors to show the directions of the instantaneous velocities of points D, E, and F for the instant shown.

	$\vec{v}_D^{(G)}$	$\vec{v}_E^{(G)}$	$\vec{v}_F^{(G)}$
Magnitude			
Direction			

Explain your reasoning for both the magnitudes and directions of the velocities for each point.

2. A wooden board is placed on top of three identical solid cylinders. The board is pushed to the right by a constant horizontal force of magnitude *F*. The cylinders roll without slipping on a flat surface. There is also no slipping between the cylinders and the board.



a. What is the direction of center-of-mass velocity of the cylinders when the horizontal force is applied to the board? Explain.

b. Consider an instant while the force is being applied to the board. Rank the magnitudes of the instantaneous velocities of points D, E, and F on the cylinder shown at right in the **reference frame of the ground**. If any of the magnitudes is zero, state so explicitly. Explain.

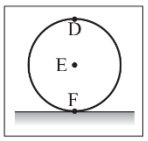


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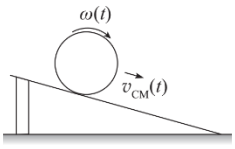
Figure C.32. Preliminary version of tutorial homework administered to all sections of PHYS 121 in the Au'18 quarter (5 pages).

c. In the ground frame, express the magnitude of the instantaneous velocity of the board, v_{board}, in terms of the magnitude of the instantaneous velocity of the center-of-mass of the cylinder, v_{cylinder}. Explain.

d. In the ground frame, express the acceleration of the board, a_{board}, in terms of the acceleration of the center-of-mass of the cylinder, a_{cylinder}. Explain.

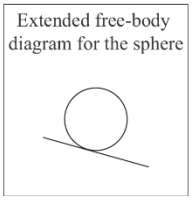
3. A sphere rolls without slipping down an incline. There is friction between the sphere and the incline.



a. Describe briefly how v_{CM} and ω change with time when the sphere is rolling down the incline. Base your answer on your intuition for the motion.

b. Indicate the direction of the acceleration and the direction of the angular acceleration of the sphere.

c. In the space at right, draw an extended free-body diagram for the sphere. Clearly label each force to indicate (1) the type of force, (2) the object exerting the force, and (3) the object on which the force is exerted.



- Which of the forces you have drawn have a component in the direction of the net force acting on the sphere?

- Which of the forces you have drawn have a non-zero contribution to the net torque about the center of mass of the sphere?

d. Is your answer to part c consistent with your description in part a? Resolve any inconsistencies if necessary.

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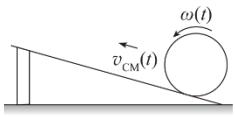
4. A sphere rolls without slipping **up** a rough incline as shown.



   a. Describe briefly how v_{CM} and ω change with time when the sphere is rolling up the incline. Base your answer on your intuition for the motion.

   b. Indicate the direction of the acceleration and the direction of the angular acceleration of the sphere.

   c. In the space at right, draw an extended free-body diagram for the sphere. Clearly label each force to indicate (1) the type of force, (2) the object exerting the force, and (3) the object on which the force is exerted.

Extended free-body diagram for the sphere

      - Which of the forces you have drawn have a component in the direction of the net force acting on the sphere?

      - Which of the forces you have drawn have a non-zero contribution to the net torque about the center of mass of the sphere?

   d. Is your answer to part c consistent with your description in part a? Resolve any inconsistencies if necessary.

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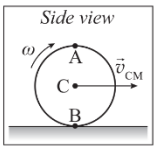
5. This problem revisits the demonstration at the end of the tutorial *Rolling and Slipping*. That demonstration showed the motion of a cylinder with three different initial conditions.

Consider a cylinder that moves across a horizontal surface. The surface is **not** frictionless. Imagine the motion of the cylinder for three different initial conditions: (i) $v_{CM} = R\omega$, (ii) $v_{CM} > R\omega$, and (iii) $v_{CM} < R\omega$.



For each case, complete the table below.

	Initial condition of the motion		
	$v_{CM} = R\omega$	$v_{CM} > R\omega$	$v_{CM} < R\omega$
Indicate the initial directions of $\vec{v}_A^{(G)}$, $\vec{v}_C^{(G)}$, and $\vec{v}_B^{(G)}$.			
Does v_{CM} *increase*, *decrease*, or *remain the same* as the motion continues?			
Indicate the direction of the center-of-mass acceleration.			
Does ω *increase*, *decrease*, or *remain the same* as the motion continues?			
Indicate the direction of the angular acceleration.			
Draw an extended free-body diagram of the cylinder.			

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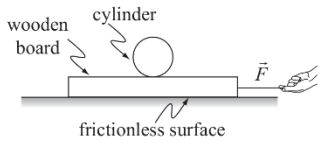
6. A cylinder of radius R is placed on top of a wooden board. They are initially at rest. The board is then pulled to the right by a constant horizontal force of magnitude F on a frictionless surface. the cylinder is observed to roll counterclockwise without slipping on the board.



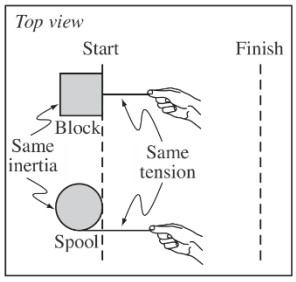
At the instant that the board has speed v_b, the center of mass of the cylinder has speed v_c, and the cylinder is rotating with angular velocity ω.

a. Consider the point on the cylinder that is in contact with the wooden board at some instant t. At that instant, in the ground frame, what is the magnitude of the velocity of that point in terms of v_c, R, and ω? Explain.

b. Write an equation that relates the quantities v_b, v_c, R, and ω. Explain.

7. This problem revisits the "block-and-spool problem" from the tutorial *Dynamics of rigid bodies*.

A block and a spool are each pulled across a level, frictionless surface by a string, as illustrated in the top-view diagram at right.



The string pulling the block is tied to a hook (not shown) at the center of the front of the block. The string pulling the spool is wrapped many times around the spool and may unwind as it is pulled.

The block and the spool have the same inertias. The strings start pulling with the same constant force at the same instant. Assume that the inertias of the strings and the hook are negligible.

a. At some instant, the block is moving to the right at speed v_o. At that instant, is the speed of the center of mass of the spool *greater than*, *less than*, or *equal to* v_o? Explain.

b. At some instant, the hand pulling the block is moving at speed v_o. At that instant, what is the speed of the hand pulling the spool? Express your answer in terms of v_o, the radius of the spool, R, and the angular velocity ω_o of the spool at the same instant. Explain.

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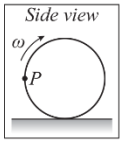
(C) Sp'19 quarter

ROLLING AND SLIPPING: A RELATIVE MOTION APPROACH

Mech 45

I. Rolling motion



A cylinder rolls without slipping to the right with constant speed on a horizontal surface. A point, P, on the rim of the cylinder is marked. The diagram at right shows the cylinder and point P at an instant $t = t_o$.

A. Draw an arrow on the diagram to indicate the direction of the instantaneous velocity of point P at $t = t_o$ in the ground frame. Briefly explain.

B. Predict the path that point P will follow over time. Sketch the trajectory below and label it as your prediction.

C. Ask an instructor to provide you with a set of equipment and use the equipment to sketch a curve on the diagram above. Label the curve as your experimental result.

On the experimental curve, draw an arrow to indicate the direction of the instantaneous velocity of point P when it is at the location shown in part A.

Compare the direction of $\vec{v}_P$ obtained from the curve with your answer in part A. Discuss your answers with your partners and resolve any inconsistencies.

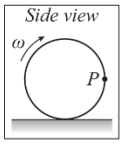
D. Two students discuss the direction of the velocity of point P below.

Student 1: "The direction of the velocity of point P should be directly upward. It is tangent to the circle at point P."

Student 2: "I disagree. In the ground frame, the hoop is rolling clockwise to the right. The clockwise motion causes P to move up. The rolling of the hoop causes P to move to the right. So, the velocity of P should point both to the right and upward."

With which student, if either, do you agree? Explain your reasoning.

E. A short time later, point P is at the location shown. Draw an arrow on the diagram to indicate the direction of $\vec{v}_P$ in the ground frame. Explain.



➪ Check your reasoning with a tutorial instructor before proceeding.

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Figure C.33. Modified version of in-class tutorial worksheet administered to all sections of PHYS 121 in the Sp'19 quarter (4 pages).

II. Combining translational and rotational motion

In the previous section, the velocity of point *P* on the rim of a rolling cylinder in the ground frame involves adding two velocities.

A. Consider the following three vectors:

$\vec{v}_P^{(C)}$: instantaneous velocity of point *P* in the *center of mass frame*

$\vec{v}_C^{(G)}$: instantaneous velocity of the center of mass in the *ground frame*

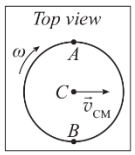
$\vec{v}_P^{(G)}$: instantaneous velocity of point *P* in the *ground frame*

Write down an equation that relates these three vectors.

The instantaneous velocity of the center of mass in the reference frame of the ground is typically written as $\vec{v}_{CM}$.

B. Rewrite the equation in part A using $\vec{v}_{CM}$.

A hoop of radius *R* is thrown horizontally through the air as shown at right. It spins with angular velocity ω and moves to the right.



C. Consider the instantaneous velocities of points *A*, *C*, and *B* in the *ground* frame for the instant shown at right. In the spaces below,

1. write down their magnitudes in terms of v_{CM}, *R*, and ω (the positive direction is chosen to be to the right), and
2. draw vectors to show all their *possible* directions.

	$\vec{v}_A^{(G)}$	$\vec{v}_C^{(G)}$	$\vec{v}_B^{(G)}$
Magnitude			
Possible directions			

➪ Check your reasoning with a tutorial instructor before proceeding.

The hoop in this section is undergoing both translational motion and rotational motion simultaneously. As you found above, for a rigid body that is translating and rotating, we can use a relative motion approach to find the magnitude and direction of the instantaneous velocity of any point on the rigid body in the ground/Earth's frame.

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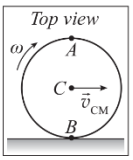
III. Rolling and slipping



A. A cylinder moves across a horizontal surface. The surface is *not* frictionless.

Consider the motion of the cylinder for three different initial conditions: (i) $v_{CM} = R\omega$, (ii) $v_{CM} > R\omega$, and (iii) $v_{CM} < R\omega$.

For each case, complete the table below:

	Initial condition of the motion		
	$v_{CM} = R\omega$	$v_{CM} > R\omega$	$v_{CM} < R\omega$
Indicate the initial directions of $\vec{v}_A^{(G)}$, $\vec{v}_C^{(G)}$, and $\vec{v}_B^{(G)}$.			
Draw a possible extended free-body diagram of the cylinder.			
Would v_{CM} *increase*, *decrease*, or *remain the same* as the motion continues?	Prediction:		
	Observation:		
Would ω *increase*, *decrease*, or *remain the same* as the motion continues?	Prediction:		
	Observation:		

B. Ask a tutorial instructor for a demonstration of the motion of the cylinder with all three different initial conditions. Observe how the motion of the cylinder changes over time in each case.

Record your observations on the table above. Revise your extended free-body diagrams according to your observations if necessary.

Objects can move on surfaces with or without slipping. *Non-slipping motion* refers to motion in which there is *no relative velocity* between the point of contact of the object on the surface, and the surface itself. *Motion with slipping* refers to motion in which there is a *non-zero relative velocity* between the point of contact of the object on the surface and the surface itself.

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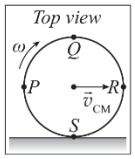
C. For each of the above cases, is the cylinder initially slipping or rolling without slipping? Explain.

D. Does each cylinder eventually roll without slipping on the ground? If so, what is the final magnitude of $\vec{v}_B^{(G)}$ in each case?

E. Based on your observations of each case, what can you say about the initial direction of the horizontal force acting on the cylinder and the initial direction of $\vec{v}_B^{(G)}$?

F. A cylinder rolls *without slipping* with constant speed across level ground as shown. In the spaces below,



1. write down the magnitudes of the instantaneous velocities of points P, Q, R, and S in the *ground* frame. *Express your answers in terms of* v_{CM} *only*.
2. draw vectors to show the directions of the instantaneous velocities of points P, Q, R, and S for the instant shown.

	$\vec{v}_P^{(G)}$	$\vec{v}_Q^{(G)}$	$\vec{v}_R^{(G)}$	$\vec{v}_S^{(G)}$
Magnitude				
Direction				

Explain your reasoning for both the magnitudes and directions of each velocities above.

G. What is the direction of the frictional force acting on the cylinder? Explain.

➪ Check your answers to this section with a tutorial instructor.

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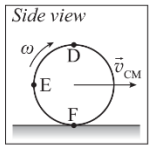
ROLLING AND SLIPPING: A RELATIVE MOTION APPROACH

Name:_____
Section:

Mech
HW–45

1. A cylinder rolls *without slipping* with constant speed across level ground as shown. In the spaces below,

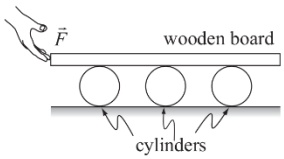


- write down the magnitudes of the instantaneous velocities of points D, E, and F in the *ground* frame. *Express your answers in terms of* v_{CM} *only*.
- draw vectors to show the directions of the instantaneous velocities of points D, E, and F for the instant shown.

	$\vec{v}_D^{(G)}$	$\vec{v}_E^{(G)}$	$\vec{v}_F^{(G)}$
Magnitude			
Direction			

Explain your reasoning for both the magnitudes and directions of the velocities for each point.

2. A wooden board is placed on top of three identical solid cylinders. The board is pushed to the right by a constant horizontal force of magnitude *F*. The cylinders roll without slipping on a flat surface. There is also no slipping between the cylinders and the board.



a. What is the direction of center-of-mass velocity of the cylinders when the horizontal force is applied to the board? Explain.

b. Consider an instant while the force is being applied to the board. Rank the magnitudes of the instantaneous velocities of points D, E, and F on the cylinder shown at right in the **reference frame of the ground**. If any of the magnitudes is zero, state so explicitly. Explain.

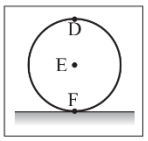


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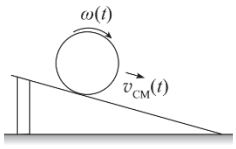
Figure C.34. Modified version of tutorial homework administered to all sections of PHYS 121 in the Sp'19 quarter (6 pages).

c. In the ground frame, express the magnitude of the instantaneous velocity of the board, v_{board}, in terms of the magnitude of the instantaneous velocity of the center-of-mass of the cylinder, v_{cylinder}. Explain.

d. In the ground frame, express the acceleration of the board, a_{board}, in terms of the acceleration of the center-of-mass of the cylinder, a_{cylinder}. Explain.

3. A sphere rolls without slipping down an incline. There is friction between the sphere and the incline.



a. Describe briefly how v_{CM} and ω change with time when the sphere is rolling down the incline.

b. Indicate the direction of the acceleration and the direction of the angular acceleration of the sphere.

c. In the space at right, draw an extended free-body diagram for the sphere. Clearly label each force to indicate (1) the type of force, (2) the object exerting the force, and (3) the object on which the force is exerted.

Extended free-body diagram for the sphere

- Which of the forces you have drawn contribute to the net force acting on the sphere?

- Which of the forces you have drawn contribute to the net torque about the center-of-mass of the sphere?

d. Is your answer to part c consistent with your description in part a? Resolve any inconsistencies if necessary.

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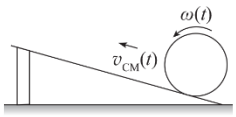
4. A sphere rolls without slipping **up** a rough incline as shown.



   a. Describe briefly how v_{CM} and ω change with time when the sphere is rolling up the incline.

   b. Indicate the direction of the acceleration and the direction of the angular acceleration of the sphere.

   c. In the space at right, draw an extended free-body diagram for the sphere. Clearly label each force to indicate (1) the type of force, (2) the object exerting the force, and (3) the object on which the force is exerted.

Extended free-body diagram for the sphere

   - Which of the forces you have drawn contribute to the net force acting on the sphere?

   - Which of the forces you have drawn contribute to the net torque about the center-of-mass of the sphere?

   d. Is your answer to part c consistent with your description in part a? Resolve any inconsistencies if necessary.

   e. Is the magnitude of the acceleration of the sphere when it is rolling down the incline *greater than*, *less than*, or *equal to* the magnitude of the acceleration of the sphere when it is rolling up the incline? Explain.

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5. This problem revisits the demonstration at the end of the tutorial *Rolling and Slipping*. That demonstration showed the motion of a cylinder with three different initial conditions.

   Consider a cylinder that moves across a horizontal surface. The surface is **not** frictionless. Imagine the motion of the cylinder for three different initial conditions: (i) $v_{CM} = R\omega$, (ii) $v_{CM} > R\omega$, and (iii) $v_{CM} < R\omega$.

   For each case, complete the table below.

	Initial condition of the motion		
	$v_{CM} = R\omega$	$v_{CM} > R\omega$	$v_{CM} < R\omega$
Indicate the initial directions of $\vec{v}_A^{(G)}$, $\vec{v}_C^{(G)}$, and $\vec{v}_B^{(G)}$.			
Does the v_{CM} *increase, decrease,* or *remain the same* as the motion continues?			
Indicate the direction of the center-of-mass acceleration.			
Does the ω *increase, decrease,* or *remain the same* as the motion continues?			
Indicate the direction of the angular acceleration.			
Draw an extended free-body diagram of the cylinder.			

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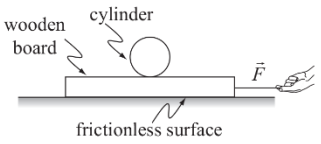
6. A cylinder of radius R is placed on top of a wooden board. They are initially at rest. The board is then pulled to the right by a constant horizontal force of magnitude F on a frictionless surface. The cylinder is observed to roll counterclockwise without slipping on the board.



At the instant that the board has speed v_b, the center-of-mass of the cylinder has speed v_c, and the cylinder is rotating with angular velocity ω.

For the following questions, consider the time interval when the cylinder is still moving on the board.

a. The cylinder is observed to roll counterclockwise without slipping on the board. Based on this observation, what do you know about the direction of horizontal force(s) that are acting on the cylinder? Explain.

b. Based on your answer in part a, what is the direction of acceleration of the center-of-mass of the cylinder in the ground frame? Explain.

c. What is the direction of the velocity of the center-of-mass of the cylinder in the ground frame? Explain.

d. Consider the point on the cylinder that is in contact with the wooden board at some instant t. At that instant, in the ground frame, what is the magnitude of the velocity of that point in terms of v_c, R, and ω? Explain.

e. Write an equation that relates the quantities v_b, v_c, R, and ω. Explain.

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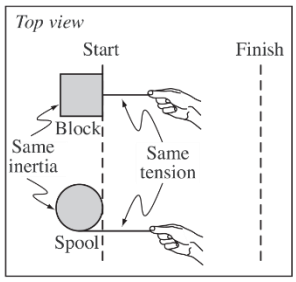
7. This problem revisits the "block-and-spool problem" from the tutorial *Dynamics of rigid bodies*.

A block and a spool are each pulled across a level, frictionless surface by a string, as illustrated in the top-view diagram at right.



The string pulling the block is tied to a hook (not shown) at the center of the front of the block. The string pulling the spool is wrapped many times around the spool and may unwind as it is pulled.

The block and the spool have the same inertias. The strings start pulling with the same constant force at the same instant. Assume that the inertias of the strings and the hook are negligible.

a. At some instant, the block is moving to the right at speed v_o. At that instant, is the speed of the center-of-mass of the spool *greater than*, *less than*, or *equal to* v_o? Explain.

b. At some instant, the hand pulling the block is moving at speed v_o. At that instant, what is the speed of the hand pulling the spool? Express your answer in terms of v_o, the radius of the spool, R, and the angular velocity ω_o of the spool at the same instant. Explain.

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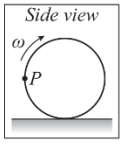
(D) Au'19 quarter

ROLLING AND SLIPPING: A RELATIVE MOTION APPROACH

Mech 45

I. Rolling motion



A cylinder rolls without slipping to the right with constant speed on a horizontal surface. A point, P, on the rim of the cylinder is marked. The diagram at right shows the cylinder and point P at an instant $t = t_o$.

A. Draw an arrow on the diagram to indicate the direction of the instantaneous velocity of point P at $t = t_o$ in the ground frame. Briefly explain.

B. Predict the path that point P will follow over time. Sketch the trajectory below and label it as your prediction.

C. Ask an instructor to provide you with a set of equipment and use the equipment to sketch a curve on the diagram above. Label the curve as your experimental result.

On the experimental curve, draw an arrow to indicate the direction of the instantaneous velocity of point P when it is at the location shown in part A.

Compare the direction of $\vec{v}_P$ obtained from the curve with your answer in part A. Discuss your answers with your partners and resolve any inconsistencies.

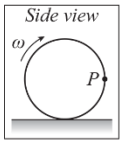
D. Two students discuss the direction of the velocity of point P below.

Student 1: "The direction of the velocity of point P should be directly upward. It is tangent to the circle at point P."

Student 2: "I disagree. In the ground frame, the hoop is rolling clockwise to the right. The clockwise motion causes P to move up. The rolling of the hoop causes P to move to the right. So, the velocity of P should point both to the right and upward."

With which student, if either, do you agree? Explain your reasoning.

E. A short time later, point P is at the location shown. Draw an arrow on the diagram to indicate the direction of $\vec{v}_P$ in the ground frame. Explain.



➪ Check your reasoning with a tutorial instructor before proceeding.

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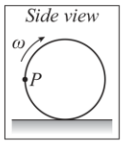
Figure C.35. Modified version of in-class tutorial worksheet administered to all sections of PHYS 121 in the Au'19 quarter (4 pages).

II. Combining translational and rotational motion

In the previous section, the velocity of point P on the rim of a rolling cylinder in the ground frame involves adding two velocities.



A. Consider the following three vectors:

$\vec{v}_P^{(C)}$: instantaneous velocity of point P in the *center of mass frame*

$\vec{v}_C^{(G)}$: instantaneous velocity of the center of mass in the *ground frame*

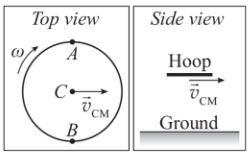
$\vec{v}_P^{(G)}$: instantaneous velocity of point P in the *ground frame*

Draw these three vectors on the above diagram and write down an equation that relates them.

The instantaneous velocity of the center of mass in the reference frame of the ground is typically written as $\vec{v}_{CM}$.

B. Rewrite the equation in part A using $\vec{v}_{CM}$.

A hoop of radius R is thrown horizontally through the air as shown at right. It spins with angular velocity ω and moves to the right.



C. Consider the instantaneous velocities of points A, C, and B in the *ground* frame for the instant shown. In the spaces below,

1. write down their magnitudes in terms of v_{CM}, R, and ω (the positive direction is chosen to be to the right), and
2. draw vectors to show all their *possible* directions.

	$\vec{v}_A^{(G)}$	$\vec{v}_C^{(G)}$	$\vec{v}_B^{(G)}$
Magnitude			
Possible directions			

➪ Check your reasoning with a tutorial instructor before proceeding.

The hoop in this section is undergoing both translational motion and rotational motion simultaneously. As you found above, for a rigid body that is translating and rotating, we can use a relative motion approach to find the magnitude and direction of the instantaneous velocity of any point on the rigid body in the ground/Earth's frame.

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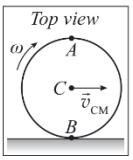
III. Rolling and slipping



A. A cylinder moves across a horizontal surface. The surface is *not* frictionless.

Consider the motion of the cylinder for three different initial conditions: (i) $v_{CM} = R\omega$, (ii) $v_{CM} > R\omega$, and (iii) $v_{CM} < R\omega$.

For each case, complete the table below:

	Initial condition of the motion		
	$v_{CM} = R\omega$	$v_{CM} > R\omega$	$v_{CM} < R\omega$
Indicate the initial directions of $\vec{v}_A^{(G)}$, $\vec{v}_C^{(G)}$, and $\vec{v}_B^{(G)}$.			
Draw a possible extended free-body diagram of the cylinder.			
Would v_{CM} *increase*, *decrease*, or *remain the same* as the motion continues?	Prediction:		
	Observation:		
Would ω *increase*, *decrease*, or *remain the same* as the motion continues?	Prediction:		
	Observation:		

B. Ask a tutorial instructor for a demonstration of the motion of the cylinder with all three different initial conditions. Observe how the motion of the cylinder changes over time in each case.

Record your observations on the table above. Revise your extended free-body diagrams according to your observations if necessary.

Objects can move on surfaces with or without slipping. *Non-slipping motion* refers to motion in which there is *no relative velocity* between the point of contact of the object on the surface, and the surface itself. *Motion with slipping* refers to motion in which there is a *non-zero relative velocity* between the point of contact of the object on the surface and the surface itself.

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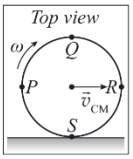
C. For each of the above cases, is the cylinder initially slipping or rolling without slipping? Explain.

D. Does each cylinder eventually roll without slipping on the ground? If so, what is the final magnitude of $\vec{v}_B^{(G)}$ in each case?

E. Based on your observations of each case, what can you say about the initial direction of the horizontal force acting on the cylinder and the initial direction of $\vec{v}_B^{(G)}$?

F. A cylinder rolls *without slipping* with constant speed across level ground as shown. In the spaces below,



1. write down the magnitudes of the instantaneous velocities of points P, Q, R, and S in the *ground* frame. *Express your answers in terms of* v_{CM} *only*.
2. draw vectors to show the directions of the instantaneous velocities of points P, Q, R, and S for the instant shown.

	$\vec{v}_P^{(G)}$	$\vec{v}_Q^{(G)}$	$\vec{v}_R^{(G)}$	$\vec{v}_S^{(G)}$
Magnitude				
Direction				

Explain your reasoning for both the magnitudes and directions of each velocities above.

G. What is the direction of the frictional force acting on the cylinder? Explain.

➪ Check your answers to this section with a tutorial instructor.

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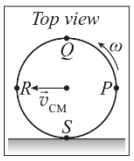
ROLLING AND SLIPPING: A RELATIVE MOTION APPROACH

Name:_____ Mech
Section: HW–45

1. A cylinder rolls *without slipping* with constant speed across level ground as shown. In the spaces below,



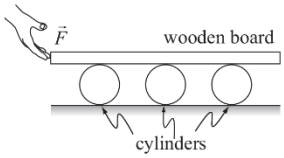
   a. write down the magnitudes of the instantaneous velocities of points P, Q, R, and S in the *ground* frame. *Express your answers in terms of* v_{CM} *only*.

   b. draw vectors to show the directions of the instantaneous velocities of points P, Q, R, and S for the instant shown.

	$\vec{v}_P^{(G)}$	$\vec{v}_Q^{(G)}$	$\vec{v}_R^{(G)}$	$\vec{v}_S^{(G)}$
Magnitude				
Direction				

Explain your reasoning for both the magnitudes and directions of each velocities above.

2. A wooden board is placed on top of three identical solid cylinders. The board is pushed to the right by a constant horizontal force of magnitude F. The cylinders roll without slipping on a flat surface. There is also no slipping between the cylinders and the board.



   a. What is the direction of center-of-mass velocity of the cylinders when the horizontal force is applied to the board? Explain.

   b. Consider an instant while the force is being applied to the board. Rank the magnitudes of the instantaneous velocities of points D, E, and F on the cylinder shown at right in the **reference frame of the ground**. If any of the magnitudes is zero, state so explicitly. Explain.

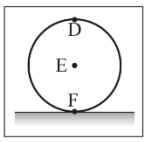


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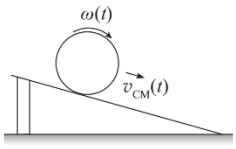
Figure C.36. Modified version of tutorial homework administered to all sections of PHYS 121 in the Au'19 quarter (6 pages).

c. In the ground frame, express the magnitude of the instantaneous velocity of the board, v_{board}, in terms of the magnitude of the instantaneous velocity of the center-of-mass of the cylinder, v_{cylinder}. Explain.

d. In the ground frame, express the acceleration of the board, a_{board}, in terms of the acceleration of the center-of-mass of the cylinder, a_{cylinder}. Explain.

3. A sphere rolls without slipping down an incline. There is friction between the sphere and the incline.

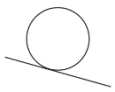


a. Describe briefly how v_{CM} and ω change with time when the sphere is rolling down the incline.

b. Indicate the direction of the acceleration and the direction of the angular acceleration of the sphere.

c. In the space at right, draw an extended free-body diagram for the sphere. Clearly label each force to indicate (1) the type of force, (2) the object exerting the force, and (3) the object on which the force is exerted.

Extended free-body diagram for the sphere



- Which of the forces you have drawn contribute to the net force acting on the sphere?

- Which of the forces you have drawn contribute to the net torque about the center-of-mass of the sphere?

d. Is your answer to part c consistent with your description in part a? Resolve any inconsistencies if necessary.

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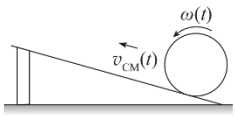
4. A sphere rolls without slipping **up** a rough incline as shown.



   a. Describe briefly how v_{CM} and ω change with time when the sphere is rolling up the incline.

   b. Indicate the direction of the acceleration and the direction of the angular acceleration of the sphere.

   c. In the space at right, draw an extended free-body diagram for the sphere. Clearly label each force to indicate (1) the type of force, (2) the object exerting the force, and (3) the object on which the force is exerted.

Extended free-body diagram for the sphere

   - Which of the forces you have drawn contribute to the net force acting on the sphere?

   - Which of the forces you have drawn contribute to the net torque about the center-of-mass of the sphere?

   d. Is your answer to part c consistent with your description in part a? Resolve any inconsistencies if necessary.

   e. Is the magnitude of the acceleration of the sphere when it is rolling down the incline *greater than*, *less than*, or *equal to* the magnitude of the acceleration of the sphere when it is rolling up the incline? Explain.

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5. This problem revisits the demonstration at the end of the tutorial *Rolling and Slipping*. That demonstration showed the motion of a cylinder with three different initial conditions.

   Consider a cylinder that moves across a horizontal surface. The surface is **not** frictionless. Imagine the motion of the cylinder for three different initial conditions: (i) $v_{CM} = R\omega$, (ii) $v_{CM} > R\omega$, and (iii) $v_{CM} < R\omega$.

   For each case, complete the table below.

	Initial condition of the motion		
	$v_{CM} = R\omega$	$v_{CM} > R\omega$	$v_{CM} < R\omega$
Indicate the initial directions of $\vec{v}_A^{(G)}$, $\vec{v}_C^{(G)}$, and $\vec{v}_B^{(G)}$.			
Does the v_{CM} *increase*, *decrease*, or *remain the same* as the motion continues?			
Indicate the direction of the center-of-mass acceleration.			
Does the ω *increase*, *decrease*, or *remain the same* as the motion continues?			
Indicate the direction of the angular acceleration.			
Draw an extended free-body diagram of the cylinder.			

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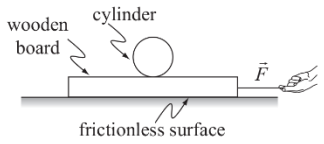
6. A cylinder of radius R is placed on top of a wooden board. They are initially at rest. The board is then pulled to the right by a constant horizontal force of magnitude F on a frictionless surface. The cylinder is observed to roll counterclockwise without slipping on the board.



At the instant that the board has speed v_b, the center-of-mass of the cylinder has speed v_c, and the cylinder is rotating with angular velocity ω.

For the following questions, consider the time interval when the cylinder is still moving on the board.

a. The cylinder is observed to roll counterclockwise without slipping on the board. Based on this observation, what do you know about the direction of horizontal force(s) that are acting on the cylinder? Explain.

b. Based on your answer in part a, what is the direction of acceleration of the center-of-mass of the cylinder in the ground frame? Explain.

c. What is the direction of the velocity of the center-of-mass of the cylinder in the ground frame? Explain.

d. Consider the point on the cylinder that is in contact with the wooden board at some instant t. At that instant, in the ground frame, what is the magnitude of the velocity of that point in terms of v_c, R, and ω? Explain.

e. Write an equation that relates the quantities v_b, v_c, R, and ω. Explain.

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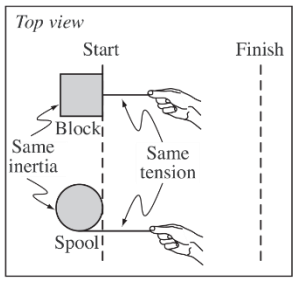
7. This problem revisits the "block-and-spool problem" from the tutorial *Dynamics of rigid bodies*.

A block and a spool are each pulled across a level, frictionless surface by a string, as illustrated in the top-view diagram at right.



The string pulling the block is tied to a hook (not shown) at the center of the front of the block. The string pulling the spool is wrapped many times around the spool and may unwind as it is pulled.

The block and the spool have the same inertias. The strings start pulling with the same constant force at the same instant. Assume that the inertias of the strings and the hook are negligible.

a. At some instant, the block is moving to the right at speed v_o. At that instant, is the speed of the center-of-mass of the spool *greater than*, *less than*, or *equal to* v_o? Explain.

b. At some instant, the hand pulling the block is moving at speed v_o. At that instant, what is the speed of the hand pulling the spool? Express your answer in terms of v_o, the radius of the spool, R, and the angular velocity ω_o of the spool at the same instant. Explain.

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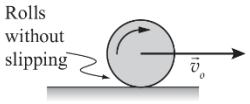
C3. Posttests and exam questions for Part II

This section presents the exam questions that we administered to PHYS 121 students during the investigation of student understanding of rolling motion. The questions include the Rolling Motion Task, board-on-cylinder, and cylinder-on-board tasks. They are arranged according to the quarters administered, from Au'17 to Au'19, and lecture sections.

Name _____ **Student ID** _____ **Score** _____
last *first*

Problems 31 and 32 involve the following situation.

A rigid cylinder rolls to the right without slipping (*i.e.,* there is no relative movement between the surface and the lowest point on the ball). The coefficient of static friction between the cylinder and flat surface is $\mu_{static} > 0$.



It is observed that the center of mass of the cylinder moves to the right with constant speed (*i.e.,* **it does not slow down**).

Center of mass moves to the right with constant speed

31. [5 pts] While the cylinder is rolling with constant speed without slipping, which of the following best describes the direction of the frictional force on the cylinder by the flat surface?

    A. The frictional force on the cylinder by the flat surface is **to the left.**
    B. The frictional force on the cylinder by the flat surface is **to the right.**
    C. The frictional force on the cylinder by the flat surface is **upward.**
    D. The frictional force on the cylinder by the flat surface is **zero.**

32. [5 pts] Later, while the cylinder is still rolling to the right, the cylinder encounters a *frictionless* region.

    Which of the following best describes the rotational motion of the cylinder while on the frictionless surface?

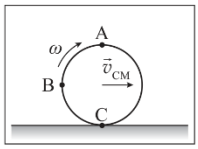
    A. The cylinder rolls without slipping.
    B. The cylinder rotates and slips on the surface.
    C. The cylinder stops rotating**.**
    D. The answer **depends** on the moment of inertia of the cylinder.

Figure C.37. Exam questions administered to PHYS 121 students in the Au'17 quarter (8 pages). The sections to which the questions were administered can be found at the footer of the figure.

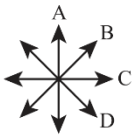
Name _____ **Student ID** _____ **Score** _____
last *first*

Problems 34 and 35 involve the following situation.

A rigid wheel of radius R rolls without slipping on a horizontal road shown at right. The linear velocity of the center of the wheel with respect to the road is $\vec{v}_{CM}$ and the angular speed is ω.



34. [5 pts] Which of the following best describes the direction of the instantaneous velocity of point B **with respect to the road**?



E. There is no direction since the instantaneous speed of point B is zero with respect to the road.

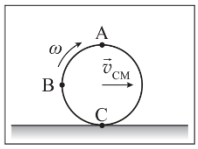
35. [5 pts] Rank the magnitudes of the speeds of points A, B, and C at the rim of the wheel **with respect to the road**, from largest to smallest.

A. $v_A = v_B = v_C$
B. $v_A > v_B > v_C$
C. $v_B > v_C > v_A$
D. $v_C > v_A > v_B$
E. $v_C > v_B > v_A$

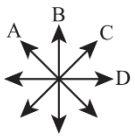
Name _____ **Student ID** _____ **Score** _____
last *first*

Problems 22 and 23 involve the following situation.

A rigid wheel of radius R rolls without slipping on a horizontal road shown at right. The linear velocity of the center of the wheel with respect to the road is $\vec{v}_{CM}$ and the angular speed is ω.



22. [5 pts] Which of the following best describes the direction of the instantaneous velocity of point B **with respect to the road**?



E. There is no direction since the instantaneous speed of point B is zero with respect to the road.

23. [5 pts] Rank the magnitudes of the speeds of points A, B, and C at the rim of the wheel **with respect to the road**, from largest to smallest.

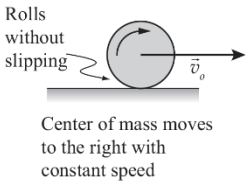
A. $v_C > v_A > v_B$
B. $v_C > v_B > v_A$
C. $v_B > v_C > v_A$
D. $v_A > v_B > v_C$
E. $v_A = v_B = v_C$

Name _____ **Student ID** _____ **Score** _____
last *first*

Problems 34 and 35 involve the following situation.

A rigid cylinder rolls to the right without slipping (*i.e.,* there is no relative movement between the surface and the lowest point on the ball). The coefficient of static friction between the cylinder and flat surface is $\mu_{static} > 0$.

It is observed that the center of mass of the cylinder moves to the right with constant speed (*i.e.,* **it does not slow down**).



34. [5 pts] While the cylinder is rolling with constant speed without slipping, which of the following best describes the direction of the frictional force on the cylinder by the flat surface?

    A. The frictional force on the cylinder by the flat surface is **to the left.**
    B. The frictional force on the cylinder by the flat surface is **to the right.**
    C. The frictional force on the cylinder by the flat surface is **upward.**
    D. The frictional force on the cylinder by the flat surface is **zero.**

35. [5 pts] Later, while the cylinder is still rolling to the right, the cylinder encounters a *frictionless* region.

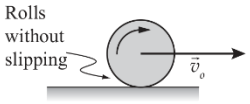
    Which of the following best describes the rotational motion of the cylinder while on the frictionless surface?

    A. The cylinder rolls without slipping.
    B. The cylinder rotates and slips on the surface.
    C. The cylinder stops rotating.
    D. The answer **depends** on the moment of inertia of the cylinder.

Name _____ **Student ID** _____ **Score** _____
last *first*

Problems 10 and 11 involve the following situation.

A rigid cylinder rolls to the right without slipping (*i.e.,* there is no relative movement between the surface and the lowest point on the ball). The coefficient of static friction between the cylinder and flat surface is $\mu_{static} > 0$.



It is observed that the center of mass of the cylinder moves to the right with constant speed (*i.e.,* **it does not slow down**).

Center of mass moves to the right with constant speed

10. [5 pts] While the cylinder is rolling with constant speed without slipping, which of the following best describes the direction of the frictional force on the cylinder by the flat surface?

    A. The frictional force on the cylinder by the flat surface is **to the left.**

    B. The frictional force on the cylinder by the flat surface is **to the right.**

    C. The frictional force on the cylinder by the flat surface is **upward.**

    D. The frictional force on the cylinder by the flat surface is **zero.**

11. [5 pts] Later, while the cylinder is still rolling to the right, the cylinder encounters a *frictionless* region.

    Which of the following best describes the rotational motion of the cylinder while on the frictionless surface?

    A. The cylinder rolls without slipping.

    B. The cylinder rotates and slips on the surface.

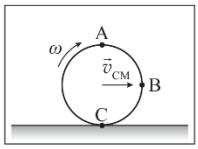
    C. The cylinder stops rotating**.**

    D. The answer **depends** on the moment of inertia of the cylinder.

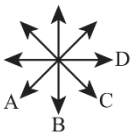
Name _____ **Student ID** _____ **Score** _____
last *first*

Problems 12 and 13 involve the following situation.

A rigid wheel of radius R rolls without slipping on a horizontal road shown at right. The linear velocity of the center of the wheel with respect to the road is $\vec{v}_{CM}$ and the angular speed is ω.



12. [5 pts] Which of the following best describes the direction of the instantaneous velocity of point B **with respect to the road**?



E. There is no direction since the instantaneous speed of point B is zero with respect to the road.

13. [5 pts] Rank the magnitudes of the speeds of points A, B, and C at the rim of the wheel **with respect to the road**, from smallest to largest.

A. $v_C < v_A < v_B$
B. $v_C < v_B < v_A$
C. $v_B < v_C < v_A$
D. $v_A < v_B < v_C$
E. $v_A = v_B = v_C$

A block of mass m is **at rest** on a perfectly horizontal table. The coefficient of static friction between the block and table is $\mu_{static} > 0$.

Block at rest on horizontal table

14. [5 pts] Which of the following best describes the magnitude of the static frictional force f_{static} on the block by the table?

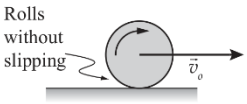
A. $f_{static} = 0$
B. $0 < f_{static} < \mu_{static} mg$
C. $f_{static} = \mu_{static} mg$
D. $f_{static} > \mu_{static} mg$
E. Not enough information to answer

Name _____ **Student ID** _____ **Score** _____
last *first*

Problems 10 and 11 involve the following situation.

A rigid cylinder rolls to the right without slipping (*i.e.,* there is no relative movement between the surface and the lowest point on the ball). The coefficient of static friction between the cylinder and flat surface is $\mu_{static} > 0$.

It is observed that the center of mass of the cylinder moves to the right with constant speed (*i.e.,* **it does not slow down**).



Center of mass moves to the right with constant speed

10. [5 pts] While the cylinder is rolling with constant speed without slipping, which of the following best describes the direction of the frictional force on the cylinder by the flat surface?

    A. The frictional force on the cylinder by the flat surface is **to the left.**

    B. The frictional force on the cylinder by the flat surface is **to the right.**

    C. The frictional force on the cylinder by the flat surface is **upward.**

    D. The frictional force on the cylinder by the flat surface is **zero.**

11. [5 pts] Later, while the cylinder is still rolling to the right, the cylinder encounters a *frictionless* region. Which of the following best describes the rotational motion of the cylinder while on the frictionless surface?

    A. The cylinder rolls without slipping.

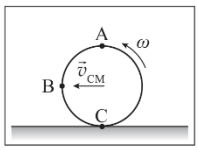
    B. The cylinder rotates and slips on the surface.

    C. The cylinder stops rotating**.**

    D. The answer **depends** on the moment of inertia of the cylinder.

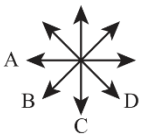
Name _____ **Student ID** _____ **Score** _____
last *first*

Problems 12 and 13 involve the following situation.



A rigid wheel of radius R rolls without slipping on a horizontal road shown at right. The linear velocity of the center of the wheel with respect to the road is $\vec{v}_{CM}$ and the angular speed is ω.

12. [5 pts] Which of the following best describes the direction of the instantaneous velocity of point B **with respect to the road**?



E. There is no direction since the instantaneous speed of point B is zero with respect to the road.

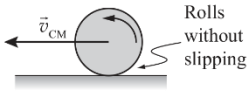
13. [5 pts] Rank the magnitudes of the speeds of points A, B, and C at the rim of the wheel **with respect to the road**, from smallest to largest.

A. $v_C < v_A < v_B$
B. $v_C < v_B < v_A$
C. $v_B < v_C < v_A$
D. $v_A < v_B < v_C$
E. $v_A = v_B = v_C$

Name _____ **Student ID** _____ **Score** _____
last *first*

Problems 29 and 31 involve the following situation.

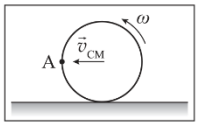
A rigid cylinder rolls to the left on a flat surface without slipping (*i.e.,* there is no relative movement between the surface and the lowest point on the ball). The coefficient of static friction between the cylinder and surface is $\mu_{static} > 0$.

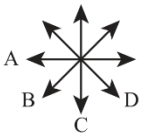


It is observed that the center of mass of the cylinder moves to the left with constant speed.

Center of mass moves to the left with constant speed

29. [6 pts] While the cylinder is moving to the left, which of the following best describes the direction of the instantaneous velocity of point A **with respect to the surface**? Select E if the instantaneous speed of point A with respect to the surface is zero.





30. [6 pts] While the cylinder is rolling with constant speed without slipping, which of the following best describes the direction of the frictional force on the cylinder by the flat surface?

A. The frictional force on the cylinder by the flat surface is **to the left.**
B. The frictional force on the cylinder by the flat surface is **to the right.**
C. The frictional force on the cylinder by the flat surface is **upward.**
D. The frictional force on the cylinder by the flat surface is **zero.**

31. [6 pts] Later, while the cylinder is still rolling to the left, the cylinder encounters a *frictionless* region.

Which of the following best describes the rotational motion of the cylinder while on the frictionless surface?

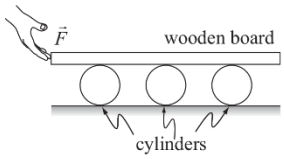
A. The cylinder rolls without slipping.
B. The cylinder rotates and slips on the surface.
C. The cylinder stops rotating.
D. The answer **depends** on the moment of inertia of the cylinder.

Figure C.38. Exam questions administered to PHYS 121 students in the Wi'18 quarter (6 pages). The sections to which the questions were administered can be found at the footer of the figure.

Name _____ **Student ID** _____ **Score** _____
last *first*

IV. Free-Response Questions [12 points total]

A wooden board is placed on top of three identical solid cylinders. The board is pushed to the right by a constant horizontal force of magnitude F applied to the end of the board. The cylinders roll without slipping on a flat surface. There is also no slipping between the cylinders and the board.



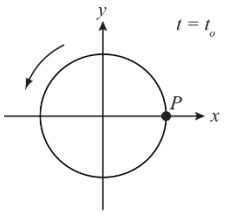
Consider an instant when the horizontal force is still exerted on the board and the board is still in contact with **all** the cylinders.

Let a_{board} be the magnitude of the acceleration of the board and a_{cylinder} be the magnitude of the center-of-mass acceleration of each cylinder.

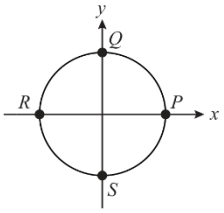
32. [6 pts] Which of the following correctly describes the relationship between a_{board} and a_{cylinder}? **Circle** the correct answer below. **Explain how you determined the answer**.

$a_{\text{board}} = 0.5\, a_{\text{cylinder}}$	$a_{\text{board}} = a_{\text{cylinder}}$	$a_{\text{board}} = 1.5\, a_{\text{cylinder}}$	$a_{\text{board}} = 2\, a_{\text{cylinder}}$	Not enough information

33. [6 pts] A wheel with a spot on its rim is rotating counter-clockwise with a constant rotational speed ω_o. At $t_o = 0$, the spot is at point P as shown at right. The spot completes one full revolution in one period.



The wheel is then slowed down until it rotates at a steady rotational speed $\omega_o/2$. On the diagram below, **circle** the location of the spot after one *new* period of this slower rotation. **Explain how you determined your answer.**

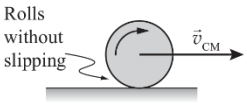


Name _____ **Student ID** _____ **Score** _____
last *first*

III. Tutorial Questions [30 points total]

Problems 25 and 27 involve the following situation.

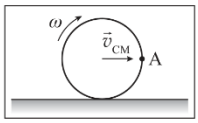
A rigid cylinder rolls to the right on a flat surface without slipping (*i.e.,* there is no relative movement between the surface and the lowest point on the ball). The coefficient of static friction between the cylinder and surface is $\mu_{static} > 0$.

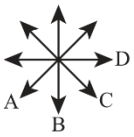


It is observed that the center of mass of the cylinder moves to the right with constant speed.

Center of mass moves to the right with constant speed

25. [6 pts] While the cylinder is moving to the right, which of the following best describes the direction of the instantaneous velocity of point A **with respect to the surface**?





E. There is no direction since the instantaneous speed of point A is zero with respect to the surface.

26. [6 pts] While the cylinder is rolling with constant speed without slipping, which of the following best describes the direction of the frictional force on the cylinder by the flat surface?

A. The frictional force on the cylinder by the flat surface is **to the left.**

B. The frictional force on the cylinder by the flat surface is **to the right.**

C. The frictional force on the cylinder by the flat surface is **upward.**

D. The frictional force on the cylinder by the flat surface is **zero.**

27. [6 pts] Later, while the cylinder is still rolling to the left, the cylinder encounters a *frictionless* region.

Which of the following best describes the rotational motion of the cylinder while on the frictionless surface?

A. The cylinder rolls without slipping.

B. The cylinder rotates and slips on the surface.

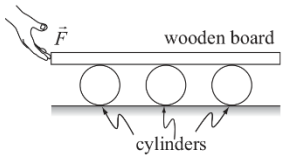
C. The cylinder stops rotating**.**

D. The answer **depends** on the moment of inertia of the cylinder.

Name _____ **Student ID** _____ **Score** _____
last *first*

IV. Free Response Questions [12 pts total]

30. [6 pts] A wooden board is placed on top of three identical solid cylinders. The board is pushed to the right by a constant horizontal force of magnitude F applied to the end of the board. The cylinders roll without slipping on a flat surface. There is also no slipping between the cylinders and the board.



Consider the time interval when the horizontal force is still exerted on the board and the board is still in contact with **all** the cylinders.

Let Δx_b be the magnitude of the horizontal displacement of the board and Δx_{cyl} be the magnitude of the horizontal displacement of the center-of-mass of each cylinder.

Which of the following correctly describes the relationship between Δx_b and Δx_{cyl}? **Circle** the correct answer below. **Explain how you determined the answer**.

$\Delta x_b = 0.5\ \Delta x_{cyl}$	$\Delta x_b = \Delta x_{cyl}$	$\Delta x_b = 1.5\ \Delta x_{cyl}$	$\Delta x_b = 2\ \Delta x_{cyl}$	Not enough information

31. [6 pts] A wheel with a spot on its rim is rotating about its center with a constant rotational speed ω_o. The spot makes one full revolution in one period, T.

The wheel is then slowed down until it rotates at a new constant rotational speed $\omega_o/2$.

What is the number of revolutions the spot makes in a time interval $\Delta t = T'$, where T' is the *new* period? **Circle** the correct answer below.

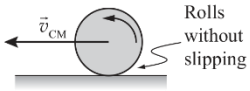
1/2 of a full revolution	1 full revolution	2 full revolutions

Explain how you determined your answer.

Name _____ **Student ID** _____ **Score** _____
last *first*

Problems 28 and 30 involve the following situation.

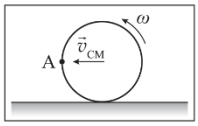
A rigid cylinder rolls to the left on a flat surface without slipping (*i.e.,* there is no relative movement between the surface and the lowest point on the ball). The coefficient of static friction between the cylinder and surface is $\mu_{static} > 0$.

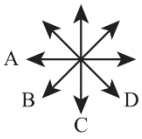


It is observed that the center of mass of the cylinder moves to the left with constant speed.

Center of mass moves to the left with constant speed

28. [6 pts] While the cylinder is moving to the left, which of the following best describes the direction of the instantaneous velocity of point A **with respect to the surface**? Select E if the instantaneous speed of point A with respect to the surface is zero.





29. [6 pts] While the cylinder is rolling with constant speed without slipping, which of the following best describes the direction of the frictional force on the cylinder by the flat surface?

A. The frictional force on the cylinder by the flat surface is **to the left.**

B. The frictional force on the cylinder by the flat surface is **to the right.**

C. The frictional force on the cylinder by the flat surface is **upward.**

D. The frictional force on the cylinder by the flat surface is **zero.**

30. [6 pts] Later, while the cylinder is still rolling to the left, the cylinder encounters a *frictionless* region.

Which of the following best describes the rotational motion of the cylinder while on the frictionless surface?

A. The cylinder rolls without slipping.

B. The cylinder rotates and slips on the surface.

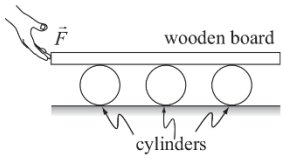
C. The cylinder stops rotating**.**

D. The answer **depends** on the moment of inertia of the cylinder.

Name _____ **Student ID** _____ **Score** _____
last *first*

IV. Free Response Questions [12 points total]

A wooden board is placed on top of three identical solid cylinders. The board is pushed to the right by a constant horizontal force of magnitude F applied to the end of the board. The cylinders roll without slipping on a flat surface. There is also no slipping between the cylinders and the board.



Consider an instant when the horizontal force is still exerted on the board and the board is still in contact with **all** the cylinders.

Let v_{board} be the magnitude of the instantaneous velocity of the board and v_{cylinder} be the magnitude of the center-of-mass instantaneous velocity of each cylinder.

31. [6 pts] Which of the following correctly describes the relationship between v_{board} and v_{cylinder}? **Circle** the correct answer below. **Explain how you determined the answer**.

$v_{\text{board}} = 0.5\, v_{\text{cylinder}}$	$v_{\text{board}} = v_{\text{cylinder}}$	$v_{\text{board}} = 1.5\, v_{\text{cylinder}}$	$v_{\text{board}} = 2\, v_{\text{cylinder}}$	Not enough information

32. [6 pts] A wheel with a spot on its rim is rotating about its center with a constant rotational speed ω_o. At $t = 0$, the rotational coordinate θ_o of the spot is zero. The rotational coordinate at a later time can be found by $\theta = \theta_o + \omega_o t$. The spot makes one complete revolution in one period.

The wheel is then slowed down until it rotates at a steady rotational speed $\omega_o/2$. Determine the change in rotational coordinate of the spot after one *new* period of this slower rotation. **Circle** the correct answer below.

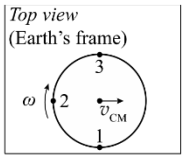
π	2π	4π

Explain how you determined your answer.

Name _____ **Student ID** _____ **Score** _____
last *first*

Problems 28 and 29 involve the following situation.

A hoop of radius R spins horizontally with a rotational velocity of ω in the air as shown at right. The speed of the center of mass, $\vec{v}_{CM}$, is equal to $R\omega$.



28. [6 pts] Which of the following best describes the direction of the instantaneous velocity of point 2 **in the reference frame of the Earth**?

A. ↘ B. ↗ C. → D. ↑

E. There is no direction since the instantaneous speed of point 2 is zero in the Earth's frame.

29. [6 pts] Rank the magnitudes of the speeds of points 1, 2, and 3 at the rim of the hoop **in the Earth's reference frame**, from largest to smallest.

A. $v_3 > v_1 > v_2$

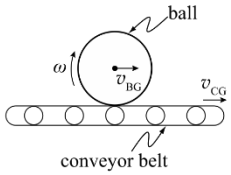
B. $v_3 > v_2 > v_1 = 0$

C. $v_3 > v_2 > v_1 \neq 0$

D. $v_1 > v_2 > v_3$

E. $v_1 = v_2 = v_3$

30. [6 pts] A ball rolls on a conveyor belt without slipping to the right as shown. A stationary observer (ground frame) observes the motion of the ball.



Let v_{CG} be the speed of the conveyor belt in the ground frame, v_{BG} be the speed of the center-of-mass of the ball in the ground frame, R and ω be the radius and the rotational speed of the ball, respectively.

How should the stationary observer relate the above quantities (v_{CG}, v_{BG}, R, and ω)?

A. $v_{CG} = v_{BG} = R\omega$

B. $v_{CG} = v_{BG} \neq R\omega$

C. $v_{CG} = v_{BG} - R\omega$

D. $v_{CG} = v_{BG} + R\omega$

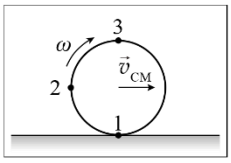
E. $v_{CG} < v_{BG} - R\omega$

Figure C.39. Exam questions administered to PHYS 121 students in the Sp'18 quarter (3 pages). The sections to which the questions were administered can be found at the footer of the figure.

Name _____ **Student ID** _____ **Score** _____
last *first*

Problems 4 and 5 involve the following situation.

A rigid wheel of radius R rolls without slipping on a horizontal road shown at right. The linear velocity of the center of the wheel with respect to the road is $\vec{v}_{CM}$ and the rotational speed is ω.



4. [6 pts] Which of the following best describes the direction of the instantaneous velocity of point 2 **with respect to the road**?

   A. ↘ B. ↗ C. → D. ↑

   E. There is no direction since the instantaneous speed of point 2 is zero with respect to the road.

5. [6 pts] Rank the magnitudes of the speeds of points 1, 2, and 3 at the rim of the wheel **with respect to the road**, from largest to smallest.

   A. $v_3 > v_1 > v_2$

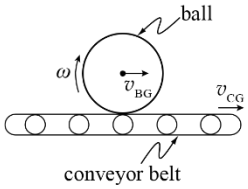
   B. $v_3 > v_2 > v_1 = 0$

   C. $v_3 > v_2 > v_1 \neq 0$

   D. $v_1 > v_2 > v_3$

   E. $v_1 = v_2 = v_3$

6. [6 pts] A ball rolls on the conveyor belt without slipping to the right as shown. A stationary observer (ground frame) observes the motion of the ball.



   Let v_{CG} be the speed of the conveyor belt in the ground frame, v_{BG} be the speed of the center-of-mass of the ball in the ground frame, R and ω be the radius and the rotational speed of the ball, respectively.

   How should the stationary observer relate the above quantities (v_{CG}, v_{BG}, R, and ω)?

   A. $v_{CG} = v_{BG} = R\omega$

   B. $v_{CG} = v_{BG} \neq R\omega$

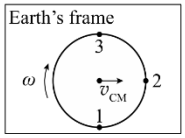
   C. $v_{CG} = v_{BG} - R\omega$

   D. $v_{CG} = v_{BG} + R\omega$

   E. $v_{CG} < v_{BG} - R\omega$

Problems 21 and 22 involve the following situation.

A frisbee of radius R is spinning in a **horizontal plane** about an axis (into the page) passing through its center-of-mass as shown at right. The speed of the rotation about its center-of-mass is ω. The translational speed of the center of mass, $\vec{v}_{CM}$, is equal to $R\omega$.



21. [6 pts] Which of the following best describes the direction of the instantaneous velocity of point 2 **in the reference frame of the Earth**?

A. ↘ B. ↙ C. → D. ↓

E. There is no direction since the instantaneous speed of point 2 is zero in the Earth's frame.

22. [6 pts] Rank the magnitudes of the speeds of points 1, 2, and 3 at the rim of the frisbee **in the Earth's reference frame**, from largest to smallest.

A. $v_3 > v_1 > v_2$

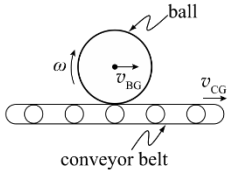
B. $v_1 > v_2 > v_3$

C. $v_1 = v_2 = v_3$

D. $v_3 > v_2 > v_1 = 0$

E. $v_3 > v_2 > v_1 \neq 0$

23. [6 pts] A ball rolls without slipping on a conveyor belt to the right as shown. A stationary observer (ground frame) observes the motion of the ball.



Let v_{CG} be the speed of the conveyor belt in the ground frame, v_{BG} be the speed of the center-of-mass of the ball in the ground frame, R and ω be the radius and the speed of rotation about the center-of-mass of the ball, respectively.

Which of the following relationships is true from the perspective of the stationary observer?

A. $v_{CG} = v_{BG} + R\omega$

B. $v_{CG} = v_{BG} - R\omega$

C. $v_{CG} = v_{BG} \neq R\omega$

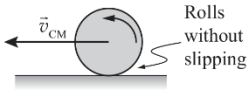
D. $v_{CG} = v_{BG} = R\omega$

E. $v_{CG} < v_{BG} - R\omega$

Name _____ **Student ID** _____ **Score** _____
last *first*

Problems 10 and 12 involve the following situation.

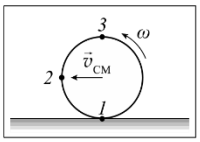
A rigid cylinder rolls to the left on a flat surface without slipping (*i.e.,* there is no relative movement between the surface and the lowest point on the ball). The coefficient of static friction between the cylinder and surface is $\mu_{static} > 0$.



It is observed that the center of mass of the cylinder moves to the left with constant speed.

Center of mass moves to the left with constant speed

10. [6 pts] Which of the following best describes the direction of the instantaneous velocity of point *2* (at the instant shown on diagram) **in the reference frame of the surface**?



A. ↘ B. ↙ C. ← D. ↓ E. Zero

11. [6 pts] For the instant shown on the diagram, rank the magnitudes of the speeds of points *1, 2,* and *3* at the rim of the cylinder **in the reference frame of the surface**. Give your answer from largest to smallest.

A. $v_3 > v_1 > v_2$
B. $v_3 > v_2 > v_1$
C. $v_2 > v_3 > v_1$
D. $v_1 > v_2 > v_3$
E. $v_1 = v_2 = v_3$

12. [6 pts] While the cylinder is rolling with constant speed without slipping, which of the following best describes the direction of the frictional force on the cylinder by the flat surface?

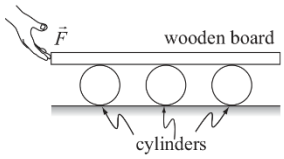
A. The frictional force on the cylinder by the flat surface is **to the left.**
B. The frictional force on the cylinder by the flat surface is **to the right.**
C. The frictional force on the cylinder by the flat surface is **upward.**
D. The frictional force on the cylinder by the flat surface is **zero.**

Figure C.40. Exam questions administered to PHYS 121 students in the Au'18 quarter (6 pages). The sections to which the questions were administered can be found at the footer of the figure.

Name _____ **Student ID** _____ **Score** _____
last *first*

IV. Free-Response Question [8 points total]

A wooden board is placed on top of three identical solid cylinders. The board is pushed to the right by a constant horizontal force of magnitude F applied to the end of the board. The cylinders roll without slipping on a flat surface. There is also no slipping between the cylinders and the board.



Consider an instant when the horizontal force is still exerted on the board and the board is still in contact with **all** of the cylinders.

Let a_b be the magnitude of the acceleration of the board and a_{cyl} be the magnitude of the center-of-mass acceleration of each cylinder.

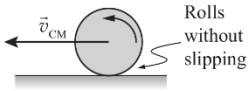
[8 pts] Which of the following correctly describes the relationship between a_b and a_{cyl}? **Circle** the correct answer below. **Explain how you determined the answer**.

$a_b = 0.5\, a_{cyl}$	$a_b = a_{cyl}$	$a_b = 1.5\, a_{cyl}$	$a_b = 2\, a_{cyl}$	$a_b = 3\, a_{cyl}$	Not enough information

Name _____ **Student ID** _____ **Score** _____
last *first*

Problems 10 and 12 involve the following situation.

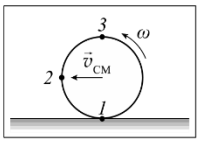
A rigid cylinder rolls to the left on a flat surface without slipping (*i.e.,* there is no relative movement between the surface and the lowest point on the ball). The coefficient of static friction between the cylinder and surface is $\mu_{static} > 0$.



It is observed that the center of mass of the cylinder moves to the left with constant speed.

Center of mass moves to the left with constant speed

10. [6 pts] Which of the following best describes the direction of the instantaneous velocity of point *2* (at the instant shown on diagram) **in the reference frame of the surface**?



A. ↘ B. ↙ C. ← D. ↓ E. Zero

11. [6 pts] For the instant shown on the diagram, rank the magnitudes of the speeds of points *1, 2,* and *3* at the rim of the cylinder **in the reference frame of the surface**. Give your answer from largest to smallest.

A. $v_3 > v_1 > v_2$
B. $v_1 > v_2 > v_3$
C. $v_2 > v_3 > v_1$
D. $v_3 > v_2 > v_1$
E. $v_1 = v_2 = v_3$

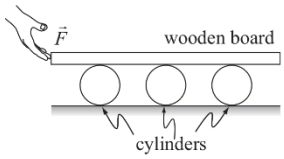
12. [6 pts] While the cylinder is rolling with constant speed without slipping, which of the following best describes the direction of the frictional force on the cylinder by the flat surface?

A. The frictional force on the cylinder by the flat surface is **to the left.**
B. The frictional force on the cylinder by the flat surface is **to the right.**
C. The frictional force on the cylinder by the flat surface is **upward.**
D. The frictional force on the cylinder by the flat surface is **zero.**

Name _____ **Student ID** _____ **Score** _____
last *first*

IV. Free Response Question [8 points total]

A wooden board is placed on top of three identical solid cylinders. The board is pushed to the right by a constant horizontal force of magnitude F applied to the end of the board. The cylinders roll without slipping on a flat surface. There is also no slipping between the cylinders and the board.



Consider an instant when the horizontal force is still exerted on the board and the board is still in contact with **all** of the cylinders.

Let v_b be the magnitude of the instantaneous velocity of the board and v_{cyl} be the magnitude of the center-of-mass instantaneous velocity of each cylinder.

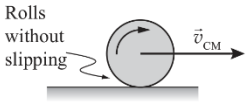
[8 pts] Which of the following correctly describes the relationship between v_b and v_{cyl}? **Circle** the correct answer below. **Explain how you determined the answer**.

$v_b = 0.5\, v_{cyl}$	$v_b = v_{cyl}$	$v_b = 1.5\, v_{cyl}$	$v_b = 2\, v_{cyl}$	$v_b = 3\, v_{cyl}$	Not enough information

Name _____ **Student ID** _____ **Score** _____
last *first*

Problems 34 and 36 involve the following situation.

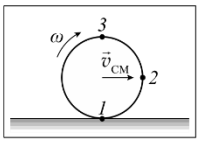
A rigid cylinder rolls to the right on a flat surface without slipping (*i.e.,* there is no relative movement between the surface and the lowest point on the ball). The coefficient of static friction between the cylinder and surface is $\mu_{static} > 0$.



It is observed that the center of mass of the cylinder moves to the right with constant speed.

Center of mass moves to the right with constant speed

34. [6 pts] Which of the following best describes the direction of the instantaneous velocity of point *2* (at the instant shown on diagram) **in the reference frame of the surface**?



A. ↘ B. ↙ C. → D. ↓ E. Zero

35. [6 pts] For the instant shown on the diagram, rank the speeds of points *1, 2,* and *3* at the rim of the cylinder **in the reference frame of the surface**. Give your answer from largest to smallest.

A. $v_3 > v_2 > v_1$
B. $v_3 > v_1 > v_2$
C. $v_1 = v_2 = v_3$
D. $v_1 > v_2 > v_3$
E. $v_2 > v_3 > v_1$

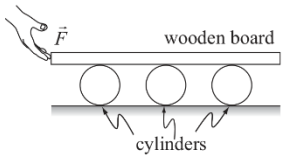
36. [6 pts] While the cylinder is rolling with constant speed without slipping, which of the following best describes the direction of the frictional force on the cylinder by the flat surface?

A. The frictional force on the cylinder by the flat surface is **to the left.**
B. The frictional force on the cylinder by the flat surface is **to the right.**
C. The frictional force on the cylinder by the flat surface is **upward.**
D. The frictional force on the cylinder by the flat surface is **zero.**

Name _____ **Student ID** _____ **Score** _____
last *first*

IV. Free Response Question [8 pts total]

A wooden board is placed on top of three identical solid cylinders. The board is pushed to the right by a constant horizontal force of magnitude F applied to the end of the board. The cylinders roll without slipping on a flat surface. There is also no slipping between the cylinders and the board.



Consider the time interval when the horizontal force is still exerted on the board and the board is still in contact with **all** of the cylinders.

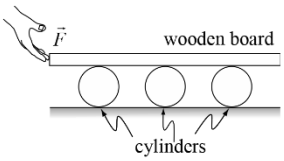
Let Δx_b be the magnitude of the horizontal displacement of the board and Δx_{cyl} be the magnitude of the horizontal displacement of the center-of-mass of each cylinder.

[8 pts] Which of the following correctly describes the relationship between Δx_b and Δx_{cyl}? **Circle** the correct answer below. **Explain how you determined the answer**.

$\Delta x_b = 0.5\,\Delta x_{cyl}$	$\Delta x_b = \Delta x_{cyl}$	$\Delta x_b = 1.5\,\Delta x_{cyl}$	$\Delta x_b = 2\,\Delta x_{cyl}$	$\Delta x_b = 3\,\Delta x_{cyl}$	Not enough information

Name _____ **Student ID** _____ **Score** _____
last *first*

Lecture Multiple Choice



A wooden board is placed on top of three identical solid cylinders. The board is pushed to the right by a constant horizontal force of magnitude F applied to the end of the board. The cylinders roll without slipping on a flat surface. There is also no slipping between the cylinders and the board.

Consider the time interval when the horizontal force is still exerted on the board and the board is still in contact with **all** of the cylinders.

Let Δx_b be the magnitude of the horizontal displacement of the board and Δx_{cyl} be the magnitude of the horizontal displacement of the center-of-mass of each cylinder.

1. Which of the following correctly describes the relationship between the magnitude of the acceleration of the board and the magnitude of the center-of-mass acceleration of the cylinders?

   A. $\Delta x_b = 0.5\,\Delta x_{cyl}$
   B. $\Delta x_b = \Delta x_{cyl}$
   C. $\Delta x_b = 1.5\,\Delta x_{cyl}$
   D. $\Delta x_b = 2\,\Delta x_{cyl}$
   E. $\Delta x_b = 3\,\Delta x_{cyl}$

Lecture Free Response

A cylinder rolls to the right without slipping on the level surface with an initial velocity shown. Point E (the black dot) is a point on the rim of the cylinder.

[4 pts] On the diagram below, sketch the trajectory of point E as the cylinder rolls across the surface. Explain briefly.

[4 pts] On the diagram at right, draw an arrow to represent the direction of instantaneous velocity of point E in the ground frame. Explain.

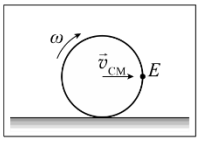
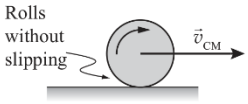


Figure C.41. Exam questions administered to PHYS 121 (section B) students in the Wi'19 quarter.

Name _____ **Student ID** _____ **Score** _____
last *first*

Problems 30 to 32 involve the following situation.

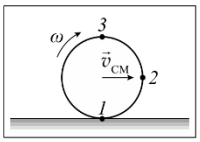
A rigid cylinder rolls to the right on a flat surface without slipping (*i.e.,* there is no relative movement between the surface and the lowest point on the ball). The coefficient of static friction between the cylinder and surface is $\mu_{static} > 0$.



It is observed that the center of mass of the cylinder moves to the right with constant speed.

Center of mass moves to the right with constant speed

30. [6 pts] In the reference frame of the surface, which of the following best describes the direction of the instantaneous velocity of point *2* (at the instant shown on diagram at right)?



A. ↘ B. ↙ C. → D. ↓ E. Zero

31. [6 pts] In the reference frame of the surface, rank the speeds of points *1, 2,* and *3* at the rim of the cylinder for the instant shown on the diagram. Give your answer from largest to smallest.

A. $v_3 > v_2 > v_1$
B. $v_3 > v_1 > v_2$
C. $v_1 = v_2 = v_3$
D. $v_1 > v_2 > v_3$
E. $v_2 > v_3 > v_1$

32. [6 pts] While the cylinder is rolling with constant speed without slipping, which of the following best describes the direction of the frictional force on the cylinder by the flat surface?

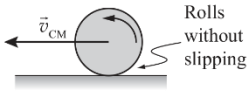
A. The frictional force on the cylinder by the flat surface is **to the left.**
B. The frictional force on the cylinder by the flat surface is **to the right.**
C. The frictional force on the cylinder by the flat surface is **upward.**
D. The frictional force on the cylinder by the flat surface is **zero.**

Figure C.42. Exam questions administered to PHYS 121 students in the Sp'19 quarter (3 pages). The sections to which the questions were administered can be found at the footer of the figure.

Name _____ **Student ID** _____ **Score** _____
last *first*

Problems 33 and 35 involve the following situation.

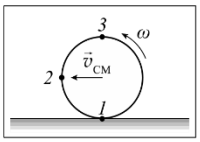
A rigid cylinder rolls to the left on a flat surface without slipping (*i.e.,* there is no relative movement between the surface and the lowest point on the ball). The coefficient of static friction between the cylinder and surface is $\mu_{static} > 0$.



It is observed that the center of mass of the cylinder moves to the left with constant speed.

Center of mass moves to the left with constant speed

33. [6 pts] Which of the following best describes the direction of the instantaneous velocity of point *2* (at the instant shown on diagram) **in the reference frame of the surface**?



A. ↘ B. ↙ C. ← D. ↓ E. Zero

34. [6 pts] For the instant shown on the diagram, rank the magnitudes of the speeds of points *1, 2,* and *3* at the rim of the cylinder **in the reference frame of the surface**. Give your answer from largest to smallest.

A. $v_3 > v_1 > v_2$
B. $v_3 > v_2 > v_1$
C. $v_2 > v_3 > v_1$
D. $v_1 > v_2 > v_3$
E. $v_1 = v_2 = v_3$

35. [6 pts] While the cylinder is rolling with constant speed without slipping, which of the following best describes the direction of the frictional force on the cylinder by the flat surface?

A. The frictional force on the cylinder by the flat surface is **to the left.**
B. The frictional force on the cylinder by the flat surface is **to the right.**
C. The frictional force on the cylinder by the flat surface is **upward.**
D. The frictional force on the cylinder by the flat surface is **zero.**

Name _____ **Student ID** _____ **Score** _____
last *first*

30. [6 pts] Two identical marbles roll without slipping across horizontal floors. One rolls on a stone floor with coefficient of static friction $\mu_s = 0.80$, and the other rolls on a glass floor with $\mu_s = 0.40$. Ignore air-resistance.

    Which of the following statements is correct?

    A. Both marbles are slowed equally since the marbles themselves are identical.

    B. Neither marble is slowed by friction. Both marbles roll without slipping.

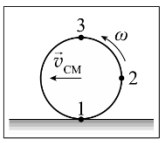
    C. The marble rolling on stone is slowed more. The stone floor has greater μ_s, which makes the force of friction on it greater.

    D. The marble rolling on glass is slowed more. The slippery nature of the glass impedes rolling.

    E. It is impossible to answer without knowing the coefficient of kinetic friction μ_k because the marbles are moving.

Problems 31 and 32 involve the following situation.

A rigid wheel of radius R rolls without slipping on a horizontal road shown at right. The linear velocity of the center of the wheel with respect to the road is $\vec{v}_{CM}$ and the rotational speed is ω.



31. [6 pts] In the reference frame of the road, which of the following best describes the direction of the instantaneous velocity of point 2?

    A. ↖ B. ↙ C. ← D. ↑

    E. There is no direction since the instantaneous speed of point 2 is zero with respect to the road.

32. [6 pts] In the reference frame of the road, rank the magnitudes of the speeds of points 1, 2, and 3 on the rim of the wheel, from largest to smallest.

    A. $v_1 = v_2 = v_3$

    B. $v_3 > v_1 > v_2$

    C. $v_2 > v_3 > v_1$

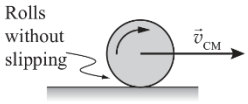
    D. $v_3 > v_2 > v_1$

    E. $v_1 > v_2 > v_3$

Name _____ **Student ID** _____ **Score** _____
last *first*

Problems 28 to 29 involve the following situation.

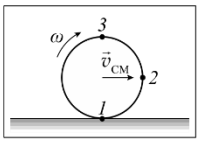
A rigid cylinder rolls to the right on a flat surface without slipping (*i.e.,* there is no relative movement between the surface and the lowest point on the ball). The coefficient of static friction between the cylinder and surface is $\mu_{static} > 0$.



It is observed that the center of mass of the cylinder moves to the right with constant speed.

Center of mass moves to the right with constant speed

28. [6 pts] In the reference frame of the surface, which of the following best describes the direction of the instantaneous velocity of point *2* (at the instant shown on diagram at right)?

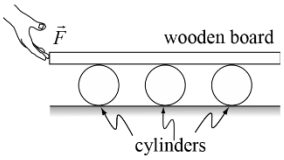


A. ↘ B. ↙ C. → D. ↓ E. Zero

29. [6 pts] While the cylinder is rolling with constant speed without slipping, which of the following best describes the direction of the frictional force on the cylinder by the flat surface?

A. The frictional force on the cylinder by the flat surface is **to the left.**
B. The frictional force on the cylinder by the flat surface is **to the right.**
C. The frictional force on the cylinder by the flat surface is **upward.**
D. The frictional force on the cylinder by the flat surface is **zero.**

A wooden board is placed on top of three identical solid cylinders. The board is pushed to the right by a constant horizontal force of magnitude F applied to the end of the board. The cylinders roll without slipping on the ground. There is also no slipping between the cylinders and the board.



Consider the instant when the horizontal force is being exerted on the board and the board is in contact with **all** of the cylinders.

Let a_b be the magnitude of the acceleration of the board and a_{cyl} be the magnitude of the acceleration of the center-of-mass of each cylinder.

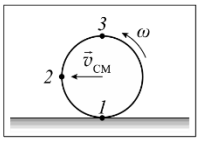
30. Which of the following correctly describes the relationship between a_b and a_{cyl}?

A. $a_b = 0.5\, a_{cyl}$
B. $a_b = a_{cyl}$
C. $a_b = 1.5\, a_{cyl}$
D. $a_b = 2\, a_{cyl}$
E. $a_b = 3\, a_{cyl}$

Figure C.43. Exam questions administered to PHYS 121 students in the Au'19 quarter (3 pages). The sections to which the questions were administered can be found at the footer of the figure.

Name _____ **Student ID** _____ **Score** _____
last *first*

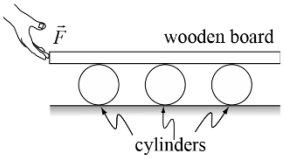
A rigid cylinder rolls to the left on a flat surface without slipping (*i.e.,* there is no relative movement between the surface and the lowest point on the ball).



34. [6 pts] Which of the following best describes the direction of the instantaneous velocity of point *2* (at the instant shown on diagram) **in the reference frame of the surface**?

A. ↘ B. ↙ C. ← D. ↓ E. Zero

A wooden board is placed on top of three identical solid cylinders. The board is pushed to the right by a constant horizontal force of magnitude F applied to the end of the board. The cylinders roll without slipping on the ground. There is also no slipping between the cylinders and the board.



Consider the instant when the horizontal force is being exerted on the board and the board is in contact with **all** of the cylinders.

Let v_b be the magnitude of the velocity of the board and v_{cyl} be the magnitude of the velocity of the center-of-mass of each cylinder.

35. Which of the following correctly describes the relationship between v_b and v_{cyl}?

A. $v_b = 0.5\, v_{cyl}$
B. $v_b = v_{cyl}$
C. $v_b = 1.5\, v_{cyl}$
D. $v_b = 2\, v_{cyl}$
E. $v_b = 3\, v_{cyl}$

36. [6 pts] Two identical marbles roll without slipping across horizontal floors. One rolls on a stone floor with coefficient of static friction $\mu_s = 0.80$, and the other rolls on a glass floor with $\mu_s = 0.40$. Ignore air-resistance.

Which of the following statements is correct?

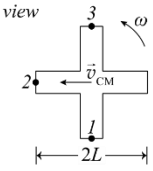
A. Both marbles are slowed equally since the marbles themselves are identical.
B. Neither marble is slowed by friction. Both marbles roll without slipping.
C. The marble rolling on stone is slowed more. The stone floor has greater μ_s, which makes the force of friction on it greater.
D. The marble rolling on glass is slowed more. The slippery nature of the glass impedes rolling.
E. It is impossible to answer without knowing the coefficient of kinetic friction μ_k because the marbles are moving.

Name _____ **Student ID** _____ **Score** _____
last *first*

Problems 10 and 11 involve the following situation.

A plus-shaped object of dimension $2L$ is moving to the left and rotating on a level, frictionless table. The object rotates counterclockwise about its center-of-mass with an angular velocity ω. The speed of its center-of-mass, v_{CM}, is equal to $L\omega$.

Top view



10. [6 pts] To an observer standing at rest next to the table, which of the following best describes the direction of the instantaneous velocity of point 2?

A. ↘ B. ↙ C. ← D. ↓ E. Zero

11. [6 pts] In the reference frame of the table, rank the magnitudes of the speeds of points 1, 2, and 3, from largest to smallest.

A. $v_1 = v_2 = v_3$
B. $v_3 > v_1 > v_2$
C. $v_2 > v_3 > v_1$
D. $v_3 > v_2 > v_1$
E. $v_1 > v_2 > v_3$

C4. Miscellaneous items that are relevant for Part II

This section covers the miscellaneous items such as statistical calculations and the specifications of the gear rack drawing for 3D printing.

(A) Statistical calculations

We perform χ^2 test of independence calculations for the pretest results of Rolling Motion Task (RMT)-Direction and Magnitude presented in Chapter 7. The calculation is to show that whether it is appropriate to aggregate the results from all pretests, even though the conditions when the students answered the questions were not the same. The results are shown in Table C.2 and Table C.3. (See subsections 7.1.4 and 7.1.5 for details of the

Table C.2. The *p*-values of the pairwise χ^2 test of independence for the RMT-Direction task.

	Sp'17 (N = 279)	Wi'18 (N = 264)	Sp'18 (N = 141)	Sp'18 (N = 119)	Au'18 (N = 160)	Sp'19 (N = 295)
Wi'17 (N = 399)	0.10	0.0005***	0.03*	0.37	0.63	0.02*
Sp'17 (N = 279)	-	0.11	0.49	0.88	0.51	0.60
Wi'18 (N = 264)	-	-	0.64	0.16	0.04*	0.32
Sp'18 (N = 141)	-	-	-	0.45	0.21	0.86
Sp'18 (N = 119)	-	-	-	-	0.81	0.54
Au'18 (N = 160)	-	-	-	-	-	0.23

Table C.3. The *p*-values of the pairwise χ^2 test of independence for the RMT-Magnitude task.

	Wi'18 (N = 264)	Sp'18 (N = 141)	Sp'18 (N = 119)	Au'18 (N = 160)	Sp'19 (N = 295)
Sp'17 (N = 279)	0.11	0.046*	0.06	0.42	0.19
Wi'18 (N = 264)	-	0.54	0.56	0.66	0.81
Sp'18 (N = 141)	-	-	0.89	0.31	0.36
Sp'18 (N = 119)	-	-	-	0.33	0.39
Au'18 (N = 160)	-	-	-	-	0.89

In both tables, the results showed that the pretest results before the Wi'18 quarter tended to be different from other quarters. One possible reason for such results is that the instructors were new to the textbook because the new textbook was introduced in the Au'16 quarter.

While this may pose some issues in the validity of aggregating all pretest data, the results are aggregated for easier comparison and discussion.

(B) Specifications of the gear rack for 3D printing

We describe the specifications for the gear and rack setup (Figure C.44) that are used the modified curriculum. Readers who are interested can use OnShape to create the equipment. There are some online tutorials on YouTube for creating the gear and rack. OnShape is quite easy to use, readers just need to go through the available tutorials to familiarize with the functions and the equipment can be created.

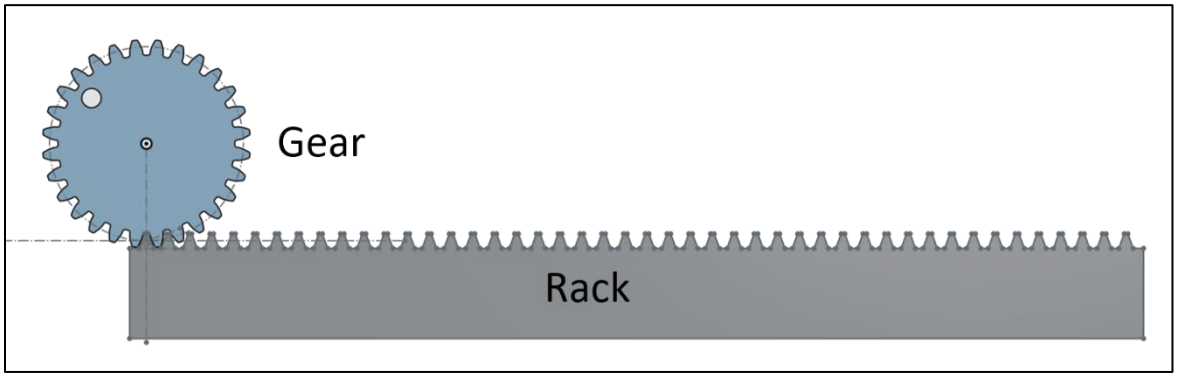


Figure C.44. The 2D sketch gear and rack used in the modified curriculum.

The specifications of the gear and rack are shown in Figure C.45 and Figure C.46, respectively. The numbers are mostly in mm. The numbers “2x” and “45x” are the number of units to be repeated. Readers who are using OnShape should be able to interpret those numbers.

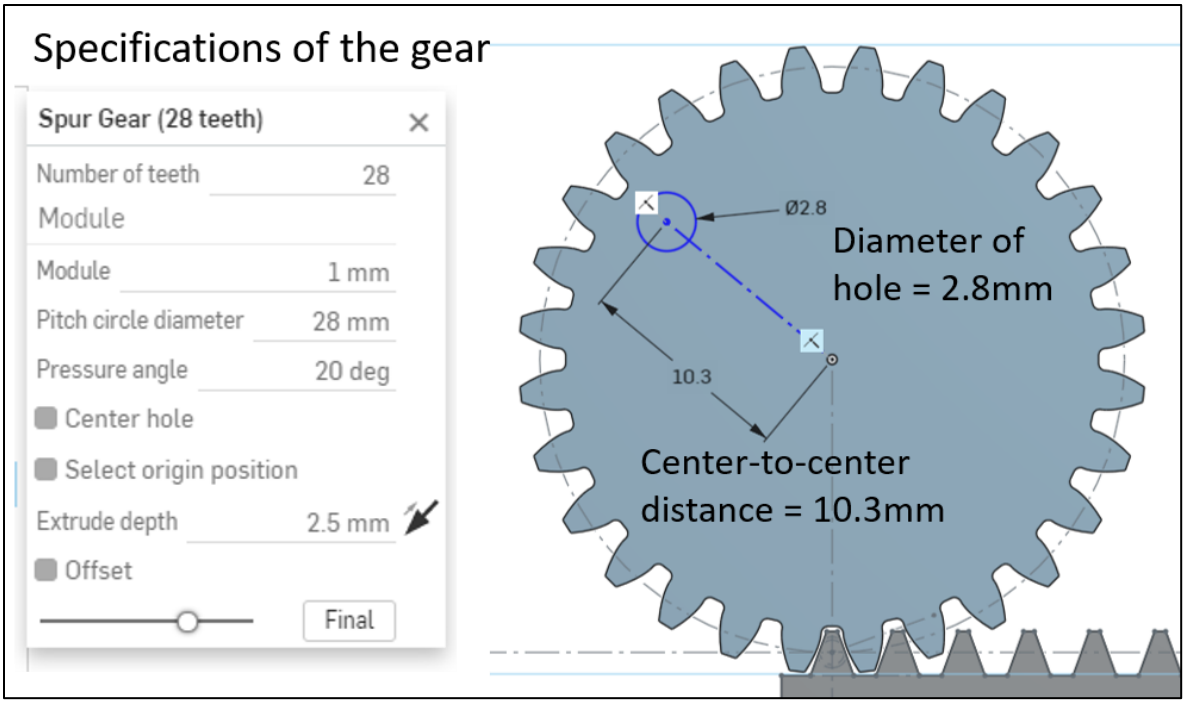


Figure C.45. The specifications of the gear.

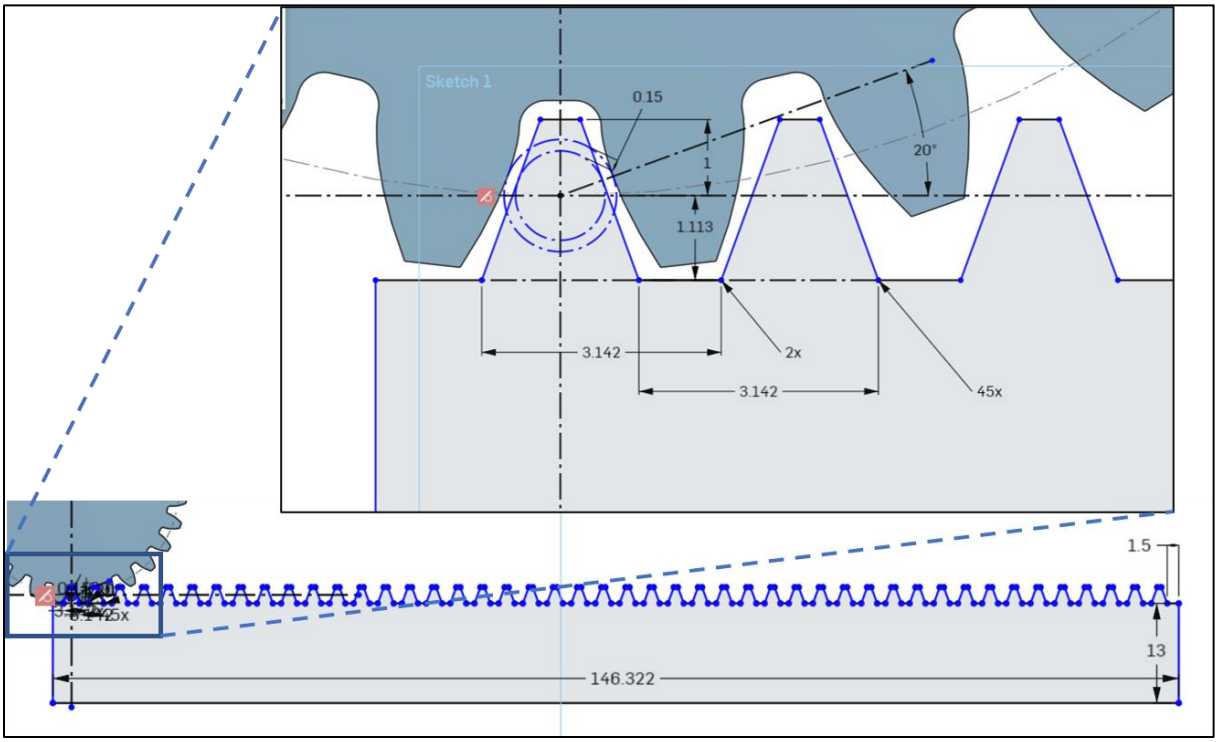


Figure C.46. The specifications of the rack. The measurements are in millimeters.

APPENDIX D. PRETESTS AND POSTTESTS FOR PART III

In Appendix D, we include all pretests and exam questions for the investigation of student understanding of fluid motion.

Table of contents

D1. Pretests questions for Part III 640

D2. Exam questions for Part III 652

D1. Pretests questions for Part III

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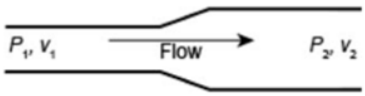
Page **2** of 2

Part III

Water flows through a pipe and enters a section where the cross-sectional area is larger. Viscosity, friction, and gravitational effects are negligible.



Question 9.

Is P_2 *greater than* or *less than* P_1? Is v_2 *greater than* or *less than* v_1?

- $P_2 < P_1$ and $v_2 < v_1$
- $P_2 < P_1$ and $v_2 > v_1$
- $P_2 > P_1$ and $v_2 < v_1$
- $P_2 > P_1$ and $v_2 > v_1$

Question 10.

Explain the reasoning that you used to answer the previous question.

Water flows vertically down through a pipe and enters a section where the cross sectional area is larger. Viscosity and pipe friction effects are negligible but gravitational effects are not negligible.

Question 11.

Is P_2 *greater than*, *less than*, or *equal to* P_1? Is v_2 *greater than*, *less than*, or *equal to* v_1?

- $P_2 < P_1$ and $v_2 < v_1$
- $P_2 < P_1$ and $v_2 > v_1$
- $P_2 > P_1$ and $v_2 < v_1$
- $P_2 > P_1$ and $v_2 > v_1$
- $P_2 = P_1$ and $v_2 = v_1$

Question 12.

Explain the reasoning that you used to answer the previous question.

Figure D.1. The HN2W and VDN2W tasks on PHYS 123 pretest.

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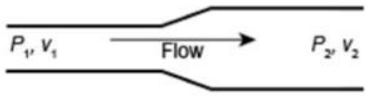
Part II

Water flows through a pipe and enters a section where the cross-sectional area is larger. Viscosity, friction, and gravitational effects are negligible.



Question 5.

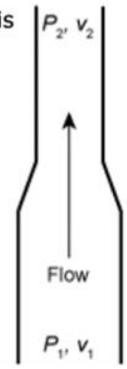
Is P_2 *greater than* or *less than* P_1? Is v_2 *greater than* or *less than* v_1?

- $P_2 < P_1$ and $v_2 < v_1$
- $P_2 < P_1$ and $v_2 > v_1$
- $P_2 > P_1$ and $v_2 < v_1$
- $P_2 > P_1$ and $v_2 > v_1$

Question 6.

Explain the reasoning that you used to answer the previous question.

Water flows vertically up through a pipe and enters a section where the cross-sectional area is smaller. Viscosity and pipe friction effects are negligible but gravitational effects are not negligible.



Question 7.

Is P_2 *greater than*, *less than*, or *equal to* P_1? Is v_2 *greater than*, *less than*, or *equal to* v_1?

- $P_2 < P_1$ and $v_2 < v_1$
- $P_2 < P_1$ and $v_2 > v_1$
- $P_2 > P_1$ and $v_2 < v_1$
- $P_2 > P_1$ and $v_2 > v_1$
- $P_2 = P_1$ and $v_2 = v_1$

Question 8.

Explain the reasoning that you used to answer the previous question.

Figure D.2. The HN2W and VUW2N tasks on PHYS 123 pretest.

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Part III

Water flows vertically down through a pipe in a steady manner. Viscosity and pipe friction effects are negligible but gravitational effects are not negligible.

Question 9.
Is the speed of the fluid flow at point 2, v_2, *greater than*, *less than*, or *equal to* that at point 1, v_1?

- $v_2 > v_1$
- $v_2 < v_1$
- $v_2 = v_1$
- Not enough information

Question 10.
Explain the reasoning that you used to answer the previous question.

Question 11.
Is the pressure in the fluid at point 2, P_2, *greater than*, *less than*, or *equal to* that at point 1, P_1?

- $P_2 > P_1$
- $P_2 < P_1$
- $P_2 = P_1$
- Not enough information

Question 12.
Explain the reasoning that you used to answer the previous question.

Figure D.3. The VDU task on PHYS 123 pretest.

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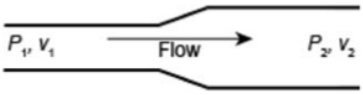
Part III

Water flows through a pipe and enters a section where the cross-sectional area is larger. Viscosity, friction, and gravitational effects are negligible.



Question 9.

Is the speed of the fluid flow at point 2, v_2, *greater than, less than* or *equal to* the speed of the fluid flow at point 1, v_1?

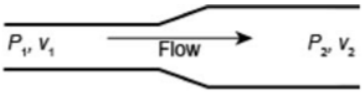
- $v_2 > v_1$
- $v_2 < v_1$
- $v_2 = v_1$
- Not enough information

Question 10.

Explain the reasoning that you used to answer the previous question.

Question 11.

Is the fluid pressure at point 2, P_2, *greater than, less than* or *equal to* the fluid pressure at point 1, P_1?



- $P_2 > P_1$
- $P_2 < P_1$
- $P_2 = P_1$
- Not enough information

Question 12.

Explain the reasoning that you used to answer the previous question.

Figure D.4. The HN2W speed and pressure tasks on PHYS 123 pretest.

(FLT_M)U3c
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Part III

Water flows vertically down through a pipe and enters a section where the cross sectional area is larger. Viscosity and pipe friction effects are negligible but gravitational effects are not negligible.

Question 9.
Is the speed of the fluid flow at point 2, v_2, *greater than, less than* or *equal to* the speed of the fluid flow at point 1, v_1?

- $v_2 > v_1$
- $v_2 < v_1$
- $v_2 = v_1$
- Not enough information

Question 10.
Explain the reasoning that you used to answer the previous question.

Question 11.
Is the fluid pressure at point 2, P_2, *greater than, less than* or *equal to* the fluid pressure at point 1, P_1?

- $P_2 > P_1$
- $P_2 < P_1$
- $P_2 = P_1$
- Not enough information

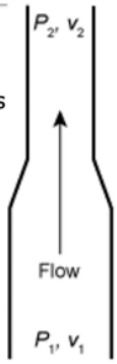
Question 12.
Explain the reasoning that you used to answer the previous question.

Figure D.5. The VDN2W speed and pressure tasks on PHYS 123 pretest.

(FLT_M)U3d
University of Washington

Part III

Water flows vertically up through a pipe and enters a section where the cross-sectional area is smaller. Viscosity and pipe friction effects are negligible but gravitational effects are not negligible.



Question 5.
Is the speed of the fluid flow at point 2, v_2, *greater than, less than* or *equal to* the speed of the fluid flow at point 1, v_1?

- $v_2 > v_1$
- $v_2 < v_1$
- $v_2 = v_1$
- Not enough information

Question 6.
Explain the reasoning that you used to answer the previous question.

Question 7.
Is the fluid pressure at point 2, P_2, *greater than, less than* or *equal to* the fluid pressure at point 1, P_1?

- $P_2 > P_1$
- $P_2 < P_1$
- $P_2 = P_1$
- Not enough information

Question 8.
Explain the reasoning that you used to answer the previous question.

Figure D.6. The VUW2N speed and pressure tasks on PHYS 123 pretest.

(FLT_M)U3e
University of Washington

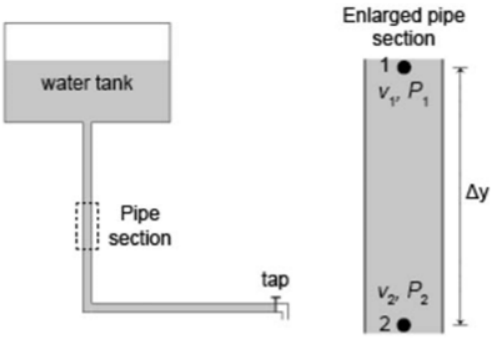
Page **2** of 2

Part III

The diagram at right shows a tap that is connected to a water tank through a pipe system.

Consider the flow of water in the pipe section when the tap is open. An enlarged view of the pipe section is shown at far right.

Viscosity and pipe friction effects are negligible.



Question 9.
Is the speed of the fluid flow at point 2, v_2, *greater than*, *less than*, or *equal to* that at point 1, v_1?

- $v_2 > v_1$
- $v_2 < v_1$
- $v_2 = v_1$
- Not enough information

Question 10.
Explain the reasoning that you used to answer the previous question.

Question 11.
Which of the following correctly describes the relationship between the pressure in the fluid at point 2, P_2, and the pressure in the fluid at point 1, P_1? Note: Δy is a positive quantity and ρ_w is density of water.

- $P_2 > P_1 + \rho_w g \Delta y$
- $P_2 = P_1 + \rho_w g \Delta y$
- $P_1 < P_2 < P_1 + \rho_w g \Delta y$
- $P_2 = P_1$
- $P_1 - \rho_w g \Delta y < P_2 < P_1$
- $P_2 = P_1 - \rho_w g \Delta y$
- $P_2 < P_1 - \rho_w g \Delta y$
- Not enough information

Question 12.
Explain the reasoning that you used to answer the previous question.

Figure D.7. The modified VDU speed and pressure tasks on PHYS 123 pretest.

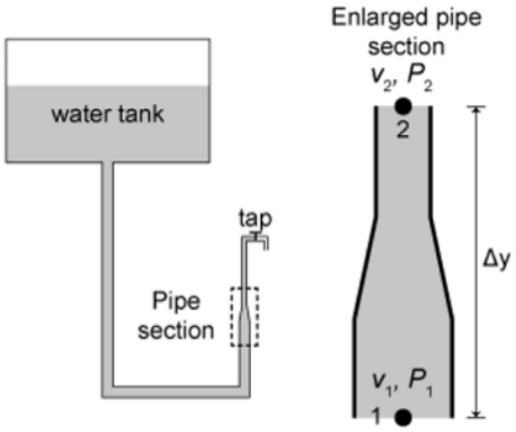
(FLT_M)U3f
University of Washington

Part II

The diagram at right shows a tap that is connected to a water tank through a pipe system.

Consider the flow of water in the pipe section when the tap is open. An enlarged view of the pipe section is shown at far right.

Viscosity and pipe friction effects are negligible.



Question 5.
Is the speed of the fluid flow at point 2, v_2, *greater than, less than* or *equal to* the speed of the fluid flow at point 1, v_1?

- $v_2 > v_1$
- $v_2 < v_1$
- $v_2 = v_1$
- Not enough information

Question 6.
Explain the reasoning that you used to answer the previous question.

Question 7.
Which of the following correctly describes the relationship between the pressure in the fluid at point 2, P_2, and the pressure in the fluid at point 1, P_1? Note: Δy is a positive quantity and ρ_w is density of water.

- $P_2 > P_1 + \rho_w g \Delta y$
- $P_2 = P_1 + \rho_w g \Delta y$
- $P_1 < P_2 < P_1 + \rho_w g \Delta y$
- $P_2 = P_1$
- $P_1 - \rho_w g \Delta y < P_2 < P_1$
- $P_2 = P_1 - \rho_w g \Delta y$
- $P_2 < P_1 - \rho_w g \Delta y$
- Not enough information

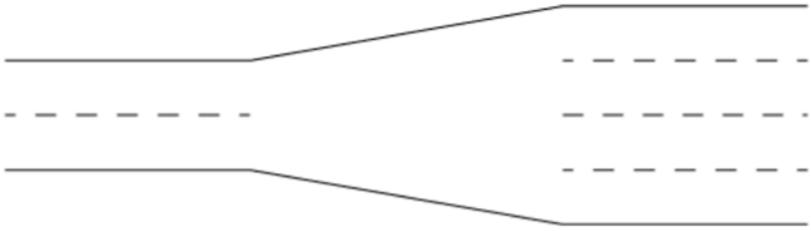
Question 8.
Explain the reasoning that you used to answer the previous question.

Figure D.8. The modified VUW2N speed and pressure tasks on PHYS 123 pretest.

(A1D_M)U8a
University of Washington

Part I

Cars move from left to right on the section of road shown below. The section of left has 2 lanes; the section of right has 4 lanes.



Assumptions:

1. All the cars are identical (i.e., same length), and are equally spaced.
2. All cars move with the same speed on the left section and on the right section, but the speeds on the two section may not be the same.

The cars enter the left end of the section with speed *v*. In every time interval, the number of cars entering the left end of the section is ***equal to*** the number of cars exiting the right end of the section.

Question 1.

What is the speed of the cars when they exit the right end?

- ○ v
- ○ v/2
- ○ v/4
- ○ v/8
- ○ Other:

Question 2.

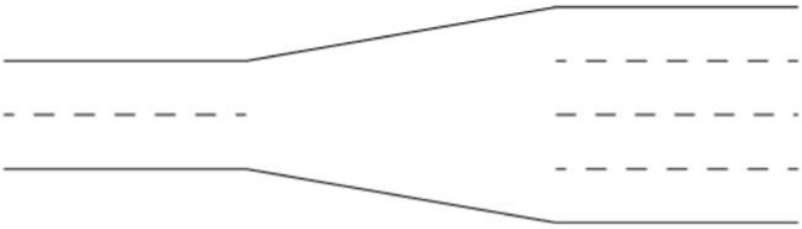
Explain the reasoning you used to answer the previous question.

Figure D.9. The continuity equation in traffic context (version 1) on PHYS 121 pretest.

(A1D_M)U8b
University of Washington

Part I

Cars move from left to right on the section of road shown below. The section of left has 2 lanes; the section of right has 4 lanes.



Assumptions:

1. All the cars are identical (i.e., same length), and are equally spaced.
2. All cars move with the same speed on the left section and on the right section, but the speeds on the two section may not be the same.
3. All the lanes are fully occupied with cars.

The cars enter the left end of the section with speed *v*. In every time interval, the number of cars entering the left end of the section is ***equal to*** the number of cars exiting the right end of the section.

Question 1.

What is the speed of the cars when they exit the right end?

- v
- v/2
- v/4
- v/8
- Other:

Question 2.

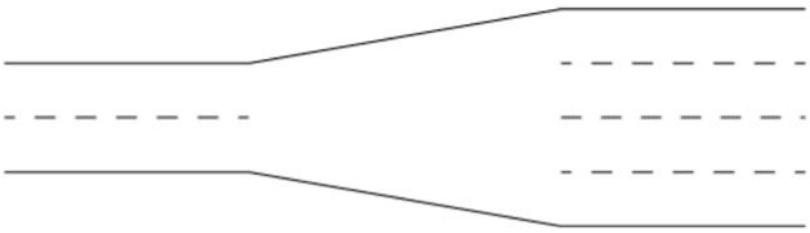
Explain the reasoning you used to answer the previous question.

Figure D.10. The continuity equation in traffic context (version 2a) on PHYS 121 pretest.

(A1D_M)U8b2
University of Washington

Part I

Cars move from left to right on the section of road shown below. The section of left has 2 lanes; the section of right has 4 lanes.



Assumptions:

1. All the cars are identical (i.e., same length), and are equally spaced.
2. All cars move with the same speed on the left section and on the right section, but the speeds on the two section may not be the same.
3. All the lanes are fully occupied with cars.

The cars enter the left end of the section with speed *v*. In every time interval, the number of cars entering the left end of the section is ***equal to*** the number of cars exiting the right end of the section.

Question 1.

What is the speed of the cars when they exit the right end?

- 2v
- v
- v/2
- v/4

Question 2.

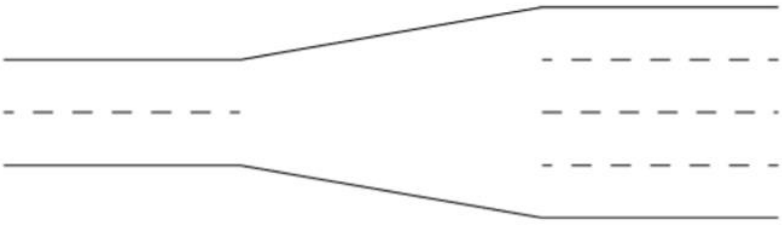
Explain the reasoning you used to answer the previous question.

Figure D.11. The continuity equation in traffic context (version 2b) on PHYS 121 pretest. The choices are slightly different from version 2a.

(A1D_M)U8c
University of Washington

Part I

Cars move **from right to left** on the section of road shown below. The section of right has 4 lanes; the section of left has 2 lanes.



Assumptions:

1. All the cars are identical (i.e., same length), and are equally spaced.
2. All cars move with the same speed on the right section and on the left section, but the speeds on the two section may not be the same.
3. All the lanes are fully occupied with cars.

The cars enter the right end of the section with speed *v*. In every time interval, the number of cars entering the right end of the section is ***equal to*** the number of cars exiting the left end of the section.

Question 1.

What is the speed of the cars when they exit the left end?

- 2v
- v
- v/2
- v/4
- Other:

Question 2.

Explain the reasoning you used to answer the previous question.

Figure D.12. The continuity equation in traffic context (version 3) on PHYS 121 pretest.

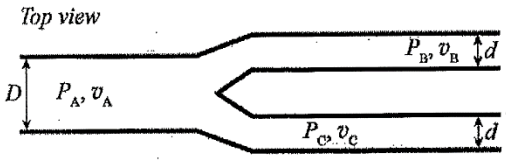
D2. Exam questions for Part III

Name_____ **Student ID** _____ **Score** _____
last *first*

III Lecture free response (26 points total)

Show enough work to get partial credit.

A. A horizontal cylindrical tube has water running steadily through it. The water reaches a junction where the tube splits as shown. The diameter of the pipe at A is twice that of B, *i.e.*, $D = 2d$. Treat water as a non-viscous, incompressible fluid.



Note that the image is a top view and all the points are at the same height.

i. (6 points) Rank the speeds of the fluid flow, v_A, v_B, and v_C, from largest to smallest. If any of the speeds is zero, state so explicitly. Explain your reasoning.

ii. (7 points) Find the magnitude of the pressure difference between A and B in terms of the density of water, ρ, and v_A?

Figure D.13. The one-channel to two-channel pipeflow (Question A) on PHYS 123 Midterm 2 in the Sp'17 quarter.

Name_____ **Student ID** _____ **Score** _____
last *first*

16. (6 points) Water is flowing through a pipe of diameter d_1 at a speed v. The pipe then branches off into two identical pipes of diameter d_2. The water continues to move at speed v through the two identical pipes. Which of the following correctly relates d_1 and d_2? Treat water as an ideal fluid.

A) $d_2 = d_1/4$
B) $d_2 = d_1/2$
C) $d_2 = d_1/\sqrt{2}$
D) $d_2 = 2d_1$
E) $d_2 = 4d_1$

Figure D.14. The one-channel to two-channel pipeflow (Question 16) on PHYS 123A final exam in the Sp'17 quarter.

APPENDIX E. PRETESTS, POSTTESTS, AND ADDITIONAL NOTES FOR PART IV

Appendix E presents the pretests and exam questions that we administered in the investigation of student ability with vector superposition in algebra-based physics courses (PHYS 114 and PHYS 115). It also includes the R codes that we used in Chapter 14 (generalized linear mixed model) and Chapter 15 (computational text analysis).

Table of contents

E1. Pretests used in the investigation of student ability in applying vector superposition in algebra-based physics course 654

E2. Posttests used in the investigation of student ability in applying vector superposition in algebra-based physics course 738

E3. Additional results for Chapter 14 745

E4. The R codes used in Part IV of the dissertation 753

E1. Pretests used in the investigation of student ability in applying vector superposition in algebra-based physics course

This section presents the Catalyst (online) pretests we used during the investigation of student ability in applying vector superposition in algebra-based physics course. The pretests were used in PHYS 114 and PHYS 115 from the Au'18 to Wi'20 quarters.

This investigation is a longitudinal study, so students in one lecture section in a particular quarter did a sequence of pretests. As readers go through the pretests, the names are usually shown on top. If there was a page without the names on top, it means that it follows the preceding name.

The sequence of pretests are arranged in a chronological order. The lists of pretests we used PHYS 114 and PHYS 115 are shown in Table E.1 and Table E.2, respectively.

Table E.1. The list of pretests used in the investigation of student ability in applying vector superposition in PHYS 114. The quarters and lecture sections that the pretests were administered are included.

Quarter	Lecture sections	Pretests names on Catalyst
Au'18	114A/B/C	(SCL)114_U1b_2v4vNC (A1D)114_U6a(2vNC) (N23) 114_U13c_2v4vF (TNS)114_U3a_2v4vT
Wi'19	114A	(SCL)114_U1c_2v4vNC (A1D)114_U7a_ 2vNC (N23) 114_U13b_2v4vF (TNS)114_U3a_2v4vT
	114B	(SCL)114_U1d_2v4vNC (A1D)114_U7b_2vNC (N23) 114_U13e_2v4vF (TNS)114_U3b_2v4vT
Sp'19	114A	(SCL)114_U1e_NoVec (RPM)114_U2c_2vNC (A1D)114_U7c_2v4vNC (N23) 114_U13f_2v4vF (TNS)114_U3c_2v4vT
	114B	(SCL)114_U1e_NoVec (RPM)114_U2d_2vNC (A1D)114_U7d_2v4vNC (N23) 114_U13g_2v4vF (TNS)114_U3d_2v4vT
Au'19	114A/B/C	(SCL)114_U2a(114A) / U2b(114B) / U2c(114C) (A1D)114_U7e_2v4vNC (FRC)114_U3a_2v4vF (TNS)114_U3d_2v4vT (ERB)114_U2h_2v4vNC
Wi'20	114A/B	(SCL)114_U1e_NoVec (RPM)114_U2e_2vNC (A1D)114_U7f_2v4vNC (N23) 114_U13h_2v4vF (TNS)114_U3f_2v4vT (ERB)114_U2j_2v4vNC

Table E.2. The list of pretests used in the investigation of student ability in applying vector superposition in PHYS 115. The quarters and lecture sections that the pretests were administered are included.

Quarter	Lecture sections	Pretests names on Catalyst
Wi'19	115A/B/C	(IGL)115_U2a_2v4vNC (FLT)115_U2c_ 2vNC (PRS) 115_U1c_2v4vF (CHG)115_U3c_2v4vT
Sp'19	115A/B	(IGL)115_U2e_2v4vNC (FLT)115_U2d_ 2vNC (PRS) 115_U1d_2v4vF (CHG)115_U3d_2v4vT
Au'19	115A	(FLT)115_U2e_2vNC (IGL)115_U2f_ 2v4vNC (PRS) 115_U1e_2v4vF (BNC)115_U3b_Bernoulli_2v4vT (CHG)115_U3e_2vE
Wi'20	115A/B/C	(IGL)115_U2g_2vNC (FLT)115_U2f_ 2v4vNC (PRS) 115_U1f_2v4vF (CHG)115_U3e_2vE / U3f (115C)

(A) Pretests used in PHYS 114

(SCL)114_U1b
University of Washington

Question 1.

Have you studied vectors prior to this course?

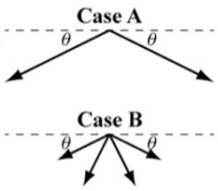
- Yes
- No

Question 2.

List down the math courses you have taken at UW.

Part II

Consider the vectors shown at right. Case A consists of two vectors, each of which has the same length. Case B consists of four vectors, each of which is half as long as the vectors in the Case A.



Question 9.

Is the magnitude of the sum of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the sum of the vectors in Case A?

- Greater than
- Less than
- Equal to

Question 10.

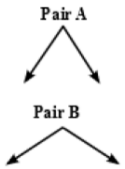
Explain the reasoning you used to answer the previous question.

Figure E.1. The series of pretests used to collect data on student ability to do vector superposition in the Au'18 quarter (3 pages).

(A1D)114_U6a
University of Washington

Part III

Consider the vectors shown at right. Each vector shown has the same length. Pair A consists of the two vectors at the top and Pair B consists of the two vectors at the bottom.



Question 11.

Is the magnitude of the sum of the vectors in Pair A *greater than, less than,* or *equal to* the magnitude of the sum of the vectors in Pair B?

- Greater than
- Less than
- Equal to

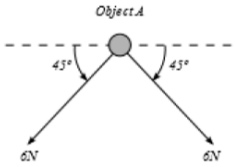
Question 12.

Explain your reasoning for the previous question.

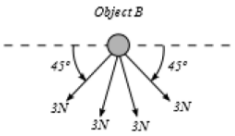
(N23)114_U13c_VecSp_2v4vF
University of Washington

Part I

Object A has two forces of magnitude 6 N each acting on it, as shown in the free-body diagram at right.



Object B has four forces of magnitude 3 N acting on it, as shown in the free-body diagram shown at right.



Question 1.

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

- The magnitude of the net force on object B is *greater than* the magnitude of the net force on object A.
- The magnitude of the net force on object B is *less than* the magnitude of the net force on object A.
- The magnitude of the net force on object B is *equal to* the magnitude of the net force on object A.
- There is not enough information to answer this question

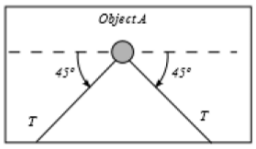
Question 2.

Explain the reasoning you used to answer the previous question.

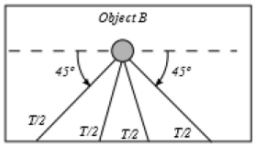
(TNS)U3a
University of Washington

Part II

Object A is shown at right. Two ropes are attached to object A as shown, each pulling with tension *T*.



Object B is acted on by four ropes, as shown at right. The tension in each of these ropes is *T*/2.



Question 9.

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

- The magnitude of the net force on object B is *greater than* the magnitude of the net force on object A.
- The magnitude of the net force on object B is *less than* the magnitude of the net force on object A.
- The magnitude of the net force on object B is *equal to* the magnitude of the net force on object A.
- Not enough information to answer

Question 10.

Explain the reasoning you used to obtain your answer to the previous question.

(SCL)114_U1b
University of Washington

Question 1.

Have you studied vectors prior to this course?

- ○ Yes
- ○ No

Question 2.

Select the math courses you have taken at UW before taking PHYS 114.

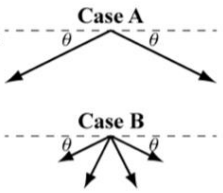
Please select more than one if you have taken more math courses.

If the math courses you have taken are not listed, please select "Other" and write down the course code (e.g., QSCI 381).

- ☐ MATH 103 Introduction to Elementary Functions
- ☐ MATH 111 Algebra with Applications
- ☐ MATH 112 Application of Calculus to Business and Economics
- ☐ MATH 120 Precalculus
- ☐ MATH 124 Calculus with Analytic Geometry I
- ☐ MATH 125 Calculus with Analytic Geometry II
- ☐ MATH 126 Calculus with Analytic Geometry III
- ☐ None at UW
- ☐ Other:

Part II

Consider the vectors shown at right. Case A consists of two vectors, each of which has the same length. Case B consists of four vectors, each of which is half as long as the vectors in the Case A.



Question 9.

Is the magnitude of the sum of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the sum of the vectors in Case A?

- ○ Greater than
- ○ Less than
- ○ Equal to

Question 10.

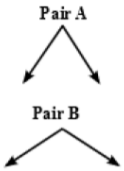
Explain the reasoning you used to answer the previous question.

Figure E.2. The series of pretests used to collect data on student ability to do vector superposition in PHYS 114A in the Wi'19 quarter (3 pages).

(A1D)114_U7a
University of Washington

Part III

Consider the vectors shown at right. Each vector shown has the same length. Pair A consists of the two vectors at the top and Pair B consists of the two vectors at the bottom.



Question 7.

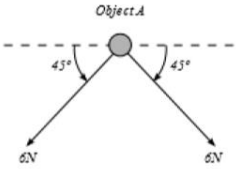
Is the magnitude of the sum of the vectors in Pair A *greater than, less than,* or *equal to* the magnitude of the sum of the vectors in Pair B?

- Greater than
- Less than
- Equal to

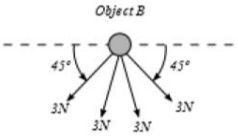
Question 8.
Briefly explain your reasoning for your previous response.

(N23)U13b
University of Washington

Object A has two forces of magnitude 6 N each acting on it, as shown in the free-body diagram at right.



Object B has four forces of magnitude 3 N acting on it, as shown in the free-body diagram shown at right.



Question 1.

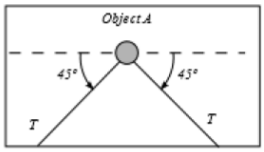
Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

- The magnitude of the net force on object B is *greater than* the magnitude of the net force on object A.
- The magnitude of the net force on object B is *less than* the magnitude of the net force on object A.
- The magnitude of the net force on object B is *equal to* the magnitude of the net force on object A.
- There is not enough information to answer this question

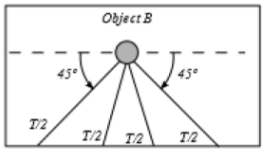
Question 2.
Explain the reasoning you used to answer the previous question.

Part II

Object A is shown at right. Two ropes are attached to object A as shown, each pulling with tension *T*.



Object B is acted on by four ropes, as shown at right. The tension in each of these ropes is *T*/2.



Question 9.

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

- The magnitude of the net force on object B is *greater than* the magnitude of the net force on object A.
- The magnitude of the net force on object B is *less than* the magnitude of the net force on object A.
- The magnitude of the net force on object B is *equal to* the magnitude of the net force on object A.
- Not enough information to answer

Question 10.

Explain the reasoning you used to obtain your answer to the previous question.

(SCL)114_U1d
University of Washington

Question 1.

Have you studied vectors prior to this course?

- ○ Yes
- ○ No

Question 2.

Select the math courses you have taken at UW before taking PHYS 114.

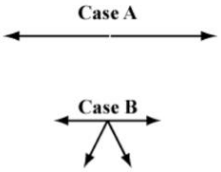
Please select more than one if you have taken more math courses.

If the math courses you have taken are not listed, please select "Other" and write down the course code (e.g., QSCI 381).

- ☐ MATH 103 Introduction to Elementary Functions
- ☐ MATH 111 Algebra with Applications
- ☐ MATH 112 Application of Calculus to Business and Economics
- ☐ MATH 120 Precalculus
- ☐ MATH 124 Calculus with Analytic Geometry I
- ☐ MATH 125 Calculus with Analytic Geometry II
- ☐ MATH 126 Calculus with Analytic Geometry III
- ☐ None at UW
- ☐ Other:

Part II

Consider the vectors shown at right. Case A consists of two vectors, each of which has the same length. Case B consists of four vectors, each of which is half as long as the vectors in the Case A.



Question 9.

Is the magnitude of the sum of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the sum of the vectors in Case A?

- ○ Greater than
- ○ Less than
- ○ Equal to

Question 10.

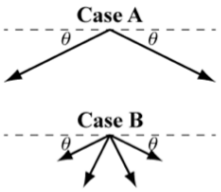
Explain the reasoning you used to answer the previous question.

Figure E.3. The series of pretests used to collect data on student ability to do vector superposition in PHYS 114B in the Wi'19 quarter (6 pages).

(SCL)114_U1d
University of Washington

Part III

Consider the vectors shown at right. Case A consists of two vectors, each of which has the same length. Case B consists of four vectors, each of which is half as long as the vectors in the Case A.



Question 11.

Is the magnitude of the sum of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the sum of the vectors in Case A?

- Greater than
- Less than
- Equal to

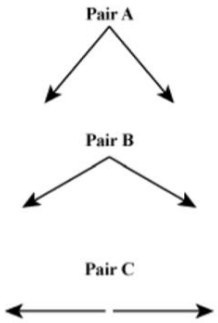
Question 12.

Explain the reasoning you used to answer the previous question.

(A1D)114_U7b
University of Washington

Part III

Consider the three pairs of vectors shown at right. Each vector (arrow) shown has the same length.



Question 7.

Which of the following best describes the ranking of the magnitudes of the **sum** of the vectors in each pair?

- A = B = C
- A > B > C
- A > B = C
- A = B > C
- Other:

Question 8.

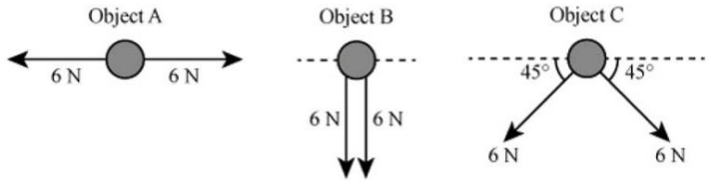
Briefly explain your reasoning for your previous response.

(N23)114_U13e_2v4vF

University of Washington

Part I

On objects A, B, and C, each of them has two forces of magnitude 6 N acting on it with different directions, as shown in the free-body diagrams below.



Question 1.

Which of the following best describes the ranking of the magnitudes of net force acting on each object?

- A = B = C
- A > B > C
- B > C > A
- C > B > A
- B = C > A
- Other:

Question 2.

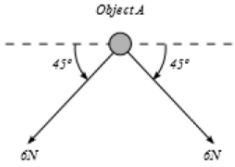
Explain your reasoning for your previous response.

(N23)114_U13e_2v4vF

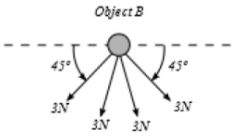
University of Washington

Part II

Object A has two forces of magnitude 6 N each acting on it, as shown in the free-body diagram at right.



Object B has four forces of magnitude 3 N acting on it, as shown in the free-body diagram shown at right.



Question 3.

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

- The magnitude of the net force on object B is *greater than* the magnitude of the net force on object A.
- The magnitude of the net force on object B is *less than* the magnitude of the net force on object A.
- The magnitude of the net force on object B is *equal to* the magnitude of the net force on object A.
- There is not enough information to answer this question

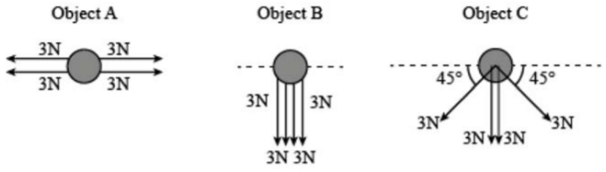
Question 4.

Explain your reasoning for your previous response.

(TNS)U3b
University of Washington

Part I

On objects A, B, and C, each of them has four tensions of magnitude 3 N acting on it with different directions, as shown in the free-body diagrams below.



Question 1.

Which of the following best describes the ranking of the magnitudes of net force acting on each object?

- A = B = C
- A > B > C
- B > C > A
- B = C > A
- Other:

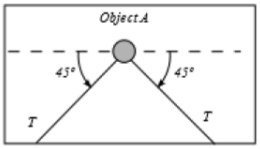
Question 2.

Explain the reasoning you used to obtain your answer to the previous question.

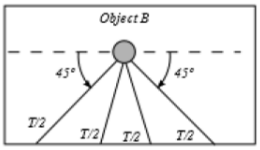
(TNS)U3b
University of Washington

Part II

Object A is shown at right. Two ropes are attached to object A as shown, each pulling with tension *T.*



Object B is acted on by four ropes, as shown at right. The tension in each of these ropes is *T*/2.



Question 3.

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

- The magnitude of the net force on object B is *greater than* the magnitude of the net force on object A.
- The magnitude of the net force on object B is *less than* the magnitude of the net force on object A.
- The magnitude of the net force on object B is *equal to* the magnitude of the net force on object A.
- Not enough information to answer

Question 4.

Explain the reasoning you used to obtain your answer to the previous question.

(RPM)114_U2c_2vNC
University of Washington

Part II

Consider the pair of vectors in cases A and B shown at right. Each vector shown has the same length.

In Case A, the two vectors are 120° apart.

In Case B, the two vectors are 60° apart.

Question 7.

Is the magnitude of the sum of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the sum of the vectors in Case A?

Required.

- Greater than
- Less than
- Equal to

Question 8.

Explain the reasoning you used to answer the previous question.

Figure E.4. The series of pretests used to collect data on student ability to do vector superposition in PHYS 114A in the Sp'19 quarter (13 pages).

(RPM)114_U2c_2vNC

University of Washington

Part II (continued)

Consider the pair of vectors in cases A and B shown at right. Each vector shown has the same length.

In Case A, the two vectors are 120° apart.

In Case B, the two vectors are 60° apart.

Question 9.

Is the magnitude of the sum of the **horizontal components** of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the sum of the **horizontal components** of the vectors in Case A?

- Greater than
- Less than
- Equal to

Question 10.

Is the magnitude of the sum of the **vertical components** of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the sum of the **vertical components** of the vectors in Case A?

- Greater than
- Less than
- Equal to

Question 11.

Is the magnitude of the sum of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the sum of the vectors in Case A?

- Greater than
- Less than
- Equal to

Question 12.

Your answer on the previous page is "The sum of vectors in Case B is **less than** the sum of vectors in case A".

Is your answer to the previous question different from that? If so, explain why. If not, explain why not.

(RPM)114_U2c_2vNC

University of Washington

Part II (continued)

Consider the pair of vectors in cases A and B shown at right. Each vector shown has the same length.

In Case A, the two vectors are 120° apart.

In Case B, the two vectors are 60° apart.

Question 13.

Is the magnitude of the sum of the **horizontal components** of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the sum of the **horizontal components** of the vectors in Case A?

- Greater than
- Less than
- Equal to

Question 14.

Is the magnitude of the sum of the **vertical components** of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the sum of the **vertical components** of the vectors in Case A?

- Greater than
- Less than
- Equal to

Question 15.

Is the magnitude of the sum of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the sum of the vectors in Case A?

- Greater than
- Less than
- Equal to

Question 16.

Your answer on the previous page is "The sum of vectors in Case B is **equal to** the sum of vectors in case A".

Is your answer to the previous question different from that? If so, explain why. If not, explain why not.

(A1D)114_U7c

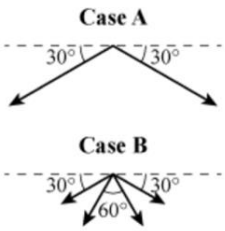
University of Washington

Part III

Consider the vectors shown at right.

Case A consists of two vectors, each of which has the same length.

Case B consists of four vectors, each of which is **half** as long as the vectors in the Case A.



Question 7.

Is the magnitude of the **sum** of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the **sum** of the vectors in Case A?

Required.

- ○ Greater than → End of pretest
- ○ Less than → Proceed to Questions 9, 10, 11, and 12
- ○ Equal to → Proceed to Questions 13, 14, 15, and 16

Question 8.

Briefly explain your reasoning for your previous response.

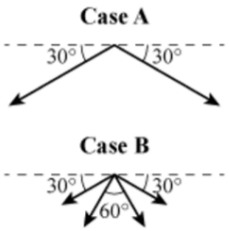
(A1D)114_U7c
University of Washington

Part III (continued)

Consider the vectors shown at right.

Case A consists of two vectors, each of which has the same length.

Case B consists of four vectors, each of which is **half** as long as the vectors in the Case A.



Question 9.

Is the magnitude of the **sum** of the **horizontal components** of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the **sum** of the **horizontal components** of the vectors in Case A?

- Greater than
- Less than
- Equal to

Question 10.

Is the magnitude of the **sum** of the **vertical components** of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the **sum** of the **vertical components** of the vectors in Case A?

- Greater than
- Less than
- Equal to

Question 11.

Is the magnitude of the **sum** of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the **sum** of the vectors in Case A?

- Greater than
- Less than
- Equal to

Question 12.

Your answer on the previous page is:
"The sum of the vectors in Case B is **less than** the sum of the vectors in case A".

Is your answer to the previous question different from that? If so, explain why. If not, explain why not.

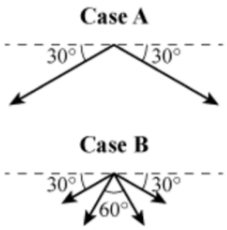
(A1D)114_U7c
University of Washington

Part III (continued)

Consider the vectors shown at right.

Case A consists of two vectors, each of which has the same length.

Case B consists of four vectors, each of which is **half** as long as the vectors in the Case A.



Question 13.

Is the magnitude of the **sum** of the **horizontal components** of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the **sum** of the **horizontal components** of the vectors in Case A?

- Greater than
- Less than
- Equal to

Question 14.

Is the magnitude of the **sum** of the **vertical components** of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the **sum** of the **vertical components** of the vectors in Case A?

- Greater than
- Less than
- Equal to

Question 15.

Is the magnitude of the **sum** of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the **sum** of the vectors in Case A?

- Greater than
- Less than
- Equal to

Question 16.

Your answer on the previous page is:
"The sum of the vectors in Case B is **equal to** the sum of the vectors in case A".

Is your answer to the previous question different from that? If so, explain why. If not, explain why not.

(N23)114_U13f_2v4vF

University of Washington

Part I

In the free-body diagram below, object A has two forces of magnitude 6 N each acting on it, and object B has four forces of magnitude 3 N acting on it.

Question 1.

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

Required.

- ○ Greater than → End of pretest
- ○ Less than → Proceed to Questions 3, 4, 5, 6, and 7
- ○ Equal to → Proceed to Questions 3, 4, 5, 6, and 7

Question 2.

Explain your reasoning for your previous response.

(N23)114_U13f_2v4vF
University of Washington

Part I (continued)

Your answer to the previous question is **incorrect**.

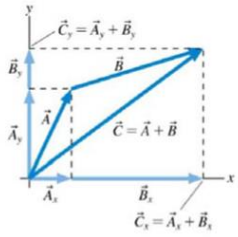
On this page, you will consider the sum of the *x*-components of the forces (*x*-component of the net force) and the sum of the *y*-components of the forces (*y*-component of the net force) to determine the correct answer.

A reminder from your lecture notes:

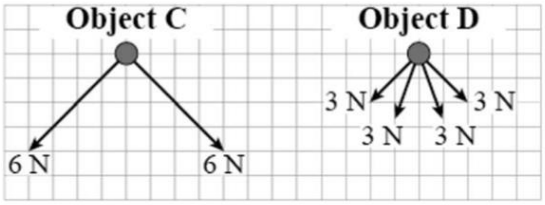
3.2 Working with components

- If $\vec{C} = \vec{A} + \vec{B} + \cdots$

$$C_x = A_x + B_x + \cdots$$
$$C_y = A_y + B_y + \cdots$$



Consider the free-body diagrams of objects C and D below,



Question 3.

Is the magnitude of the ***x*-component** of the **net force** on object D *greater than, less than,* or *equal to* the magnitude of the ***x*-component** of the **net force** on object C?

Required.

- Greater than
- Less than
- Equal to

Question 4.

Is the magnitude of the ***y*-component** of the **net force** on object D *greater than, less than,* or *equal to* the magnitude of the ***y*-component** of the **net force** on object C?

Required.

- Greater than
- Less than
- Equal to

Question 5.

Is the magnitude of the net force on object D *greater than, less than,* or *equal to* the magnitude of the net force on object C?

Required.

- Greater than
- Less than
- Equal to

Question 6.

Back to the question on the previous page:

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

- Greater than
- Less than
- Equal to

Question 7.

Are your answers to all the questions on this page consistent with each other?

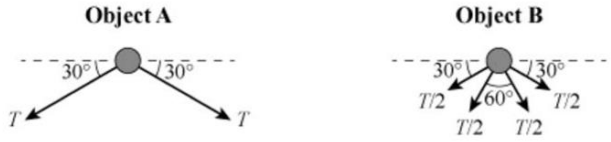
Required.

- Yes
- No

(TNS)U3c
University of Washington

Part I

In the free-body diagrams below, object A has two forces of magnitude *T* each acting on it, and object B has four forces of magnitude *T*/2 acting on it.



Question 1.

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

Required.

- Greater than → End of pretest
- Less than → Proceed to Questions 3, 4, 5, and 6
- Equal to → Proceed to Questions 3, 4, 5, and 6

Question 2.

Explain the reasoning you used to obtain your answer to the previous question.

(TNS)U3c
University of Washington

Part I (continued)

Your answer to the previous question is **incorrect**.

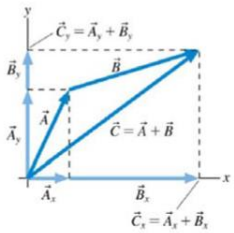
On this page, you will consider the sum of the *x*-components of the forces (*x*-component of the net force) and the sum of the *y*-components of the forces (*y*-component of the net force) to determine the correct answer.

A reminder from your lecture notes:

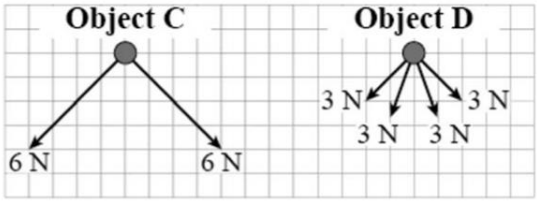
3.2 Working with components

- If $\vec{C} = \vec{A} + \vec{B} + \cdots$

$$C_x = A_x + B_x + \cdots$$
$$C_y = A_y + B_y + \cdots$$



Consider the free-body diagrams of objects C and D below,



Question 3.

Is the magnitude of the ***x*-component** of the **net force** on object D *greater than, less than,* or *equal to* the magnitude of the ***x*-component** of the **net force** on object C?

- Greater than
- Less than
- Equal to

Question 4.

Is the magnitude of the ***y*-component** of the **net force** on object D *greater than, less than,* or *equal to* the magnitude of the ***y*-component** of the **net force** on object C?

Required.

- Greater than
- Less than
- Equal to

Question 5.

Is the magnitude of the net force on object D *greater than, less than,* or *equal to* the magnitude of the net force on object C?

Required.

- Greater than
- Less than
- Equal to

Question 6.

Back to the question on the previous page:

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

Required.

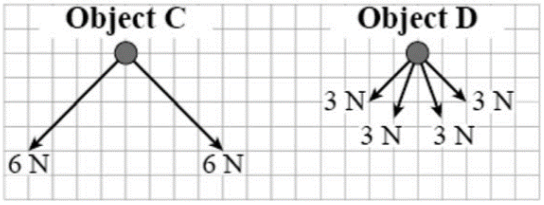
- Greater than → "Great. Your answer is correct!" → End of pretest
- Less than → Proceed to Questions 7 and 8
- Equal to → Proceed to Questions 7 and 8

(TNS)U3c
University of Washington

Great. Your answer is correct!

Your answer is still **incorrect**.

Let us look at the answers to these questions:



Compare the **x-component** of the net forces on objects C and D:
Answer: Both are **equal to zero** as the x-components in each case cancel out each other

Compare the **y-component** of the net forces on objects C and D:
Answer: D is **greater than** C, the two 3N forces in the middle have longer vertical components than the outer two 3N forces

Compare the net forces on objects C and D:
Answer: D is **greater than** C since the x-component of the net force is zero, and the y-component of the net force in D is greater than that of C

Method to arrive at the answer: Find the x and y components of each force using the given grids and sum them up by taking into account the directions

Question 7.

Based on the answers above, let's answer this question again:

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

- Greater than
- Less than
- Equal to

Question 8.

Do you still have any questions?

(RPM)114_U2d_2vNC
University of Washington

Part II

Consider the pair of vectors in cases A and B shown at right. Each vector shown has the same length.

In Case A, the two vectors are 120° apart.

In Case B, the two vectors are 60° apart.

Question 7.

Is the magnitude of the sum of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the sum of the vectors in Case A?

Required.

- ○ Greater than → End of pretest
- ○ Less than → Proceed to Questions 9 and 10
- ○ Equal to → Proceed to Questions 11 and 12

Question 8.

Explain the reasoning you used to answer the previous question.

Figure E.5. The series of pretests used to collect data on student ability to do vector superposition in PHYS 114B in the Sp'19 quarter (13 pages).

(RPM)114_U2d_2vNC

University of Washington

Part III

Consider the pair of vectors in cases A, B, and C shown below. Each vector shown has the same length.

In Case A, the two vectors are 120° apart.

In Case B, the two vectors are 60° apart.

In Case C, the two vectors are 180° apart.

Question 9.

Which of the following best describes the ranking of the magnitudes of the **sum** of the vectors in each pair?

- B = A = C
- B > A > C
- B = A > C
- A > B > C

Question 10.

Your answer on the previous page is "The sum of vectors in Case B is **less than** the sum of vectors in case A".

Is your answer to the previous question different from that? If so, explain why. If not, explain why not.

(RPM)114_U2d_2vNC

University of Washington

Part III

Consider the pair of vectors in cases A, B, and C shown below. Each vector shown has the same length.

In Case A, the two vectors are 120° apart.

In Case B, the two vectors are 60° apart.

In Case C, the two vectors are 180° apart.

Question 9.

Which of the following best describes the ranking of the magnitudes of the **sum** of the vectors in each pair?

- B = A = C
- B > A > C
- B = A > C
- A > B > C

Question 10.

Your answer on the previous page is "The sum of vectors in Case B is **less than** the sum of vectors in case A".

Is your answer to the previous question different from that? If so, explain why. If not, explain why not.

(A1D)114_U7d

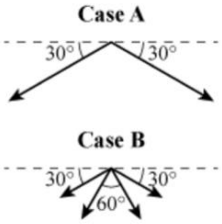
University of Washington

Part III

Consider the vectors shown at right.

Case A consists of two vectors, each of which has the same length.

Case B consists of four vectors, each of which is **half** as long as the vectors in the Case A.



Question 7.

Is the magnitude of the **sum** of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the **sum** of the vectors in Case A?

Required.

- Greater than → End of pretest
- Less than → Proceed to Questions 9 and 10
- Equal to → Proceed to Questions 11 and 12

Question 8.

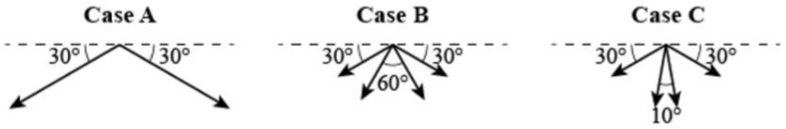
Briefly explain your reasoning for your previous response.

(A1D)114_U7d

University of Washington

Part III (continued)

Consider the vectors shown below.



Case A consists of two vectors, each of which has the same length.

Case B consists of four vectors, each of which is **half** as long as the vectors in the Case A.

Case C is similar to case B, with different arrangement of vectors.

Question 9.

Which of the following best describes the ranking of the magnitudes of the **sum** of the vectors in each case?

- A = B = C
- C > B > A
- C > B = A
- A < B = C
- Other:

Question 10.

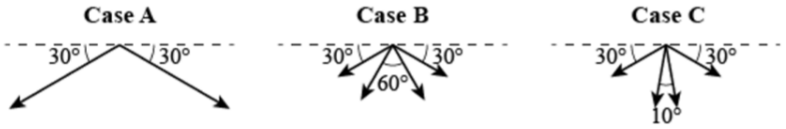
Your answer on the previous page is:
"The sum of the vectors in Case B is **less than** the sum of the vectors in case A".

Is your answer to the previous question consistent to your answer on the previous page? If so, explain why. If not, explain why not.

(A1D)114_U7d
University of Washington

Part III (continued)

Consider the vectors shown below.



Case A consists of two vectors, each of which has the same length.

Case B consists of four vectors, each of which is **half** as long as the vectors in the Case A.

Case C is similar to case B, with different arrangement of vectors.

Question 11.

Which of the following best describes the ranking of the magnitudes of the **sum** of the vectors in each case?

- A = B = C
- C > B > A
- C > B = A
- A < B = C
- Other:

Question 12.

Your answer on the previous page is:
"The sum of the vectors in Case B is **equal to** the sum of the vectors in case A".

Is your answer to the previous question consistent to your answer on the previous page? If so, explain why. If not, explain why not.

(N23)114_U13g_2v4vF

University of Washington

Part I

In the free-body diagram below, object A has two forces of magnitude 6 N each acting on it, and object B has four forces of magnitude 3 N acting on it.

Question 1.

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

Required.

- Greater than → End of pretest
- Less than → Proceed to Questions 3, 4, 5, 6, and 7
- Equal to → Proceed to Questions 3, 4, 5, 6, and 7

Question 2.

Explain your reasoning for your previous response.

(N23)114_U13g_2v4vF
University of Washington

Part I (continued)

Your answer to the previous question is **incorrect**.

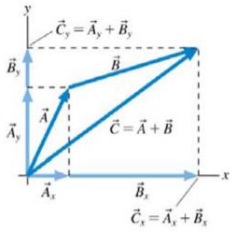
On this page, you will consider the tip-to-tail method to add vectors.

A reminder from your lecture notes:

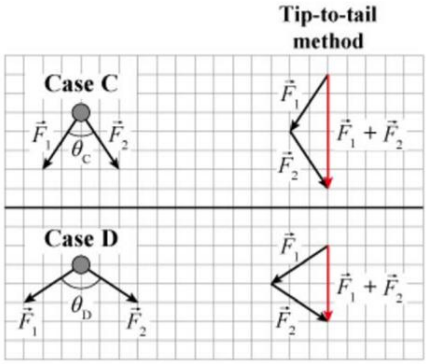
3.2 Working with components

- If $\vec{C} = \vec{A} + \vec{B} + \cdots$

$$C_x = A_x + B_x + \cdots$$
$$C_y = A_y + B_y + \cdots$$



Consider the free-body diagrams in cases C and D below. In each case, the forces F_1 and F_2 have the same magnitude.



Question 3.

Is the angle θ_C *greater than, less than,* or *equal to* the angle θ_D ?

Required.

- Greater than
- Less than
- Equal to

Question 4.

Is the magnitude of the net force ($\vec{F}_1 + \vec{F}_2$) in case C *greater than, less than,* or *equal to* the magnitude of the net force in case D?

Required.

- Greater than
- Less than
- Equal to

Question 5.

In the context of cases C and D, you are given two forces of the same magnitude.

How does the magnitude of the net force depend on the angle between the two force vectors?

Question 6.

Back to the question on the previous page:

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

Required.

- Greater than
- Less than
- Equal to

Question 7.

Are your answers to all the questions on this page consistent with each other?

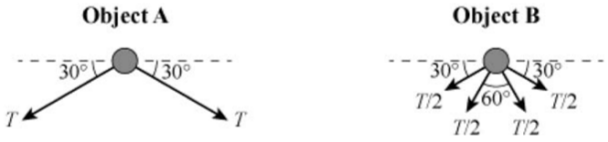
Required.

- Yes
- No

(TNS)U3d
University of Washington

Part I

In the free-body diagrams below, object A has two forces of magnitude *T* each acting on it, and object B has four forces of magnitude *T*/2 acting on it.



Question 1.

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

Required.

- Greater than → End of pretest
- Less than → Proceed to Questions 3, 4, 5, and 6
- Equal to → Proceed to Questions 3, 4, 5, and 6

Question 2.

Explain the reasoning you used to obtain your answer to the previous question.

(TNS)U3d
University of Washington

Part I (continued)

Your answer to the previous question is **incorrect**.

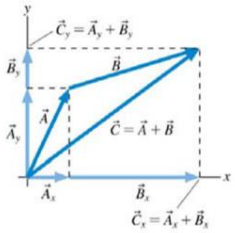
On this page, you will consider the tip-to-tail method to add vectors.

A reminder from your lecture notes:

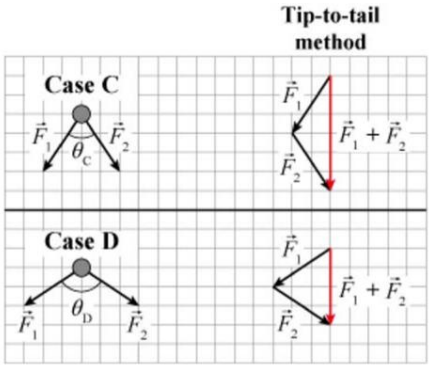
3.2 Working with components

- If $\vec{C} = \vec{A} + \vec{B} + \cdots$

$$C_x = A_x + B_x + \cdots$$
$$C_y = A_y + B_y + \cdots$$



Consider the free-body diagrams in cases C and D below. In each case, the forces F_1 and F_2 have the same magnitude.



Question 3.

Is the angle θ_C *greater than, less than,* or *equal to* the angle θ_D ?

Required.

- Greater than
- Less than
- Equal to

Question 4.

Is the magnitude of the net force ($\vec{F}_1 + \vec{F}_2$) in case C *greater than, less than,* or *equal to* the magnitude of the net force in case D?

Required.

- Greater than
- Less than
- Equal to

Question 5.

In the context of cases C and D, you are given two forces of the same magnitude.

How does the magnitude of the net force depend on the angle between the two force vectors?

Question 6.

Back to the question on the previous page:

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

Required.

- Greater than → "Great. Your answer is correct!" → End of pretest
- Less than → Proceed to Questions 7 and 8
- Equal to → Proceed to Questions 7 and 8

(TNS)U3d
University of Washington

Great. Your answer is correct!

(TNS)U3d
University of Washington

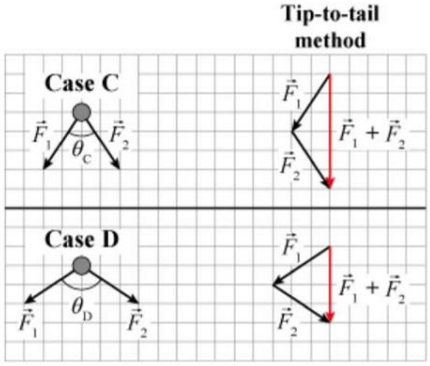
Your answer is still **incorrect**.

Let us look at the answers to these questions:

Is the angle θ_C *greater than, less than,* or *equal to* the angle θ_D ?
Answer: Less than

Is the magnitude of the net force ($\vec{F}_1 + \vec{F}_2$) in case C *greater than, less than,* or *equal to* the magnitude of the net force in case D?
Answer: Greater than

Method to arrive at the answer: Use the given diagram.



Question 7.

Based on the answers above, let's answer this question again:

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

Required.

- Greater than
- Less than
- Equal to

Question 8.

Do you still have any questions?

(SCL)114_U2a

University of Washington

Question 1.

Have you studied vectors prior to this course?

- ○ Yes
- ○ No

Question 2.

Select the math courses you have taken at UW before taking PHYS 114.

Please select more than one if you have taken more math courses.

If the math courses you have taken are not listed below, please select "Other" and write down the course code (e.g., QSCI 381).

- ☐ MATH 103 Introduction to Elementary Functions
- ☐ MATH 111 Algebra with Applications
- ☐ MATH 112 Application of Calculus to Business and Economics
- ☐ MATH 120 Precalculus
- ☐ MATH 124 Calculus with Analytic Geometry I
- ☐ MATH 125 Calculus with Analytic Geometry II
- ☐ MATH 126 Calculus with Analytic Geometry III
- ☐ None at UW
- ☐ Other:

Consider the pair of vectors in cases A and B shown at right. Each vector shown has the same length.

In Case A, the two vectors are 120° apart.

In Case B, the two vectors are 60° apart.

Question 10.

Is the magnitude of the sum of the vectors in Case B greater than, less than, or equal to the magnitude of the sum of the vectors in Case A?

Required.

- ○ Greater than → End of pretest
- ○ Less than → Proceed to Questions 12, 13, and 14 → End of pretest
- ○ Equal to → Proceed to Questions 12, 13, and 14 → End of pretest

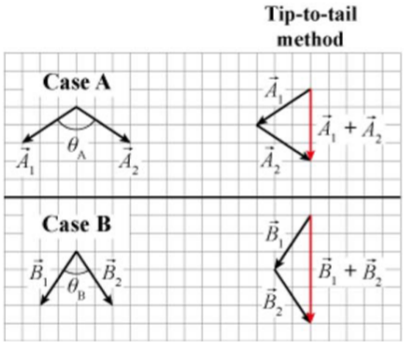
Question 11.

Explain the reasoning you used to answer the previous question.

Figure E.6. The series of pretests used to collect data on student ability to do vector superposition in PHYS 114 in the Au'19 quarter (12 pages).

Your answer to the previous question is **incorrect**.

On this page, you will consider the tip-to-tail method to add vectors.



Question 12.

From the diagram above, is the magnitude of the sum of the vectors ($\mathbf{B_1}$ + $\mathbf{B_2}$) in case B *greater than, less than,* or *equal to* the magnitude of the sum of the vectors ($\mathbf{A_1}$ + $\mathbf{A_2}$) in case A?

Required.

- Greater than
- Less than
- Equal to

Question 13.

In cases A and B, how does the magnitude of the sum of the vectors depend on the angle between the two vectors?

Question 14.

Do you have any other questions on the tip-to-tail method?

(A1D)114_U7e

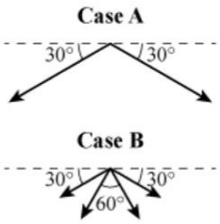
University of Washington

Part III

Consider the vectors shown at right.

Case A consists of two vectors, each of which has the same length.

Case B consists of four vectors, each of which is **half** as long as the vectors in the Case A.



Question 7.

Is the magnitude of the **sum** of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the **sum** of the vectors in Case A?

Required.

- ○ Greater than → End of pretest
- ○ Less than → Proceed to Questions 9, 10, and 11
- ○ Equal to → Proceed to Questions 9, 10, and 11

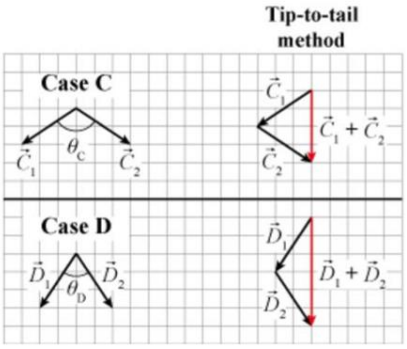
Question 8.

Briefly explain your reasoning for your previous response.

(A1D)114_U7e
University of Washington

Your answer to the previous question is **incorrect**.

On this page, you will consider the tip-to-tail method to add vectors.



Question 9.

From the diagram above, is the magnitude of the sum of the vectors ($\mathbf{D_1}$ + $\mathbf{D_2}$) in case D *greater than, less than,* or *equal to* the magnitude of the sum of the vectors ($\mathbf{C_1}$ + $\mathbf{C_2}$) in case C?

Required.

- ○ Greater than
- ○ Less than
- ○ Equal to

Question 10.

In cases C and D, how does the magnitude of the sum of the vectors depend on the angle between the two vectors (θ_C and θ_D)?

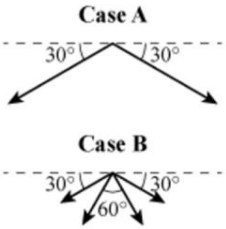
Question 11.

Back to the question on the previous page:

Case A consists of two vectors, each of which has the same length.

Case B consists of four vectors, each of which is **half** as long as the vectors in the Case A.

Is the magnitude of the **sum** of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the **sum** of the vectors in Case A?



Required.

- ○ Greater than → "Great. Your answer is correct!" → End of pretest
- ○ Less than → Proceed to Questions 12 and 13 → End of pretest
- ○ Equal to → Proceed to Questions 12 and 13 → End of pretest

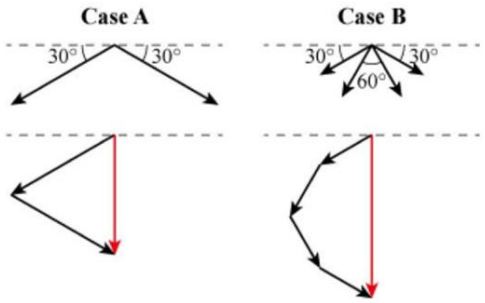
(A1D)114_U7e
University of Washington

Great. Your answer is correct! The magnitude of the resultant vector in Case B is greater than that in Case A.

(A1D)114_U7e
University of Washington

Your answer is still incorrect.

Here is how the tip-to-tail method is applied for cases A and B:



Question 12.

Is the magnitude of the **sum** of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the **sum** of the vectors in Case A?

Required.

- Greater than
- Less than
- Equal to

Question 13.

Do you still have any questions?

(FRC)114_U3a

University of Washington

Part II

In the free-body diagram below, object A has two forces of magnitude 6 N each acting on it, and object B has four forces of magnitude 3 N acting on it.

Question 7.

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

Required.

- Greater than → End of pretest
- Less than → Proceed to Questions 12, 13, and 14
- Equal to → Proceed to Questions 12, 13, and 14

Question 8.

Explain your reasoning for your previous response.

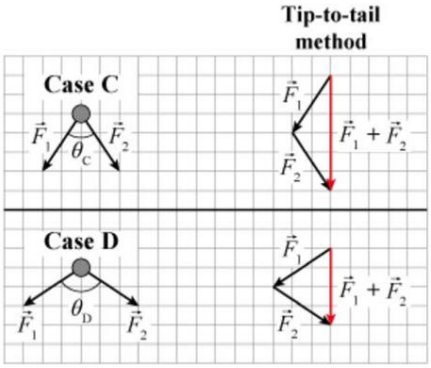
(FRC)114_U3a
University of Washington

Part II (continued)

Your answer to the previous question is **incorrect**.

On this page, you will consider the tip-to-tail method to add vectors.

Consider the free-body diagrams in cases C and D below. In each case, the forces F_1 and F_2 have the same magnitude.



Question 9.

Is the magnitude of the net force ($\vec{F}_1 + \vec{F}_2$) in case C *greater than, less than,* or *equal to* the magnitude of the net force in case D?

Required.

- Greater than
- Less than
- Equal to

Question 10.

In the context of cases C and D, you are given two forces of the same magnitude.

How does the magnitude of the net force depend on the angle between the two force vectors?

Question 11.

Back to the question on the previous page:

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

Required.

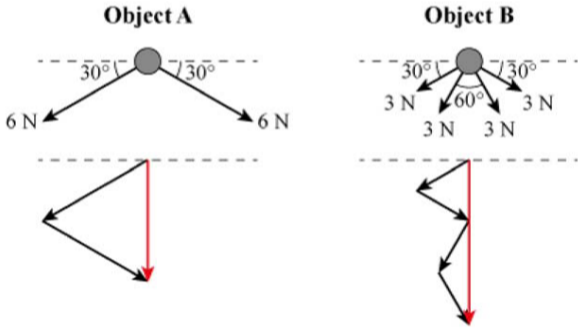
- Greater than
- Less than
- Equal to

(FRC)114_U3a
University of Washington

Great. Your answer is correct! The magnitude of the net force on object B is greater than that in object A.

Your answer is still incorrect.

Here is how the tip-to-tail method is applied to find the net forces on objects A and B:



Question 12.

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

Required.

- Greater than
- Less than
- Equal to

Question 13.

Do you still have any questions?

(TNS)U3e

University of Washington

Part I

In the free-body diagrams below, object A has two forces of magnitude *T* each acting on it, and object B has four forces of magnitude *T*/2 acting on it.

Question 1.

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

Required.

- Greater than → End of pretest
- Less than → Proceed to Questions 3, 4, and 5
- Equal to → Proceed to Questions 3, 4, and 5

Question 2.

Explain the reasoning you used to obtain your answer to the previous question.

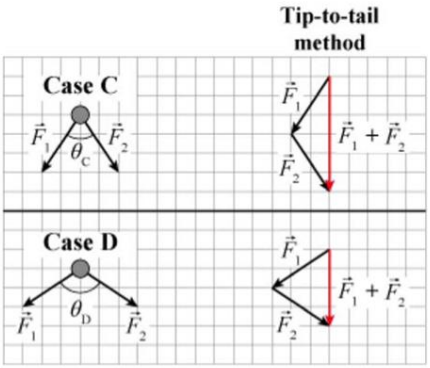
(TNS)U3e
University of Washington

Part I (continued)

Your answer to the previous question is **incorrect**.

On this page, you will consider the tip-to-tail method to add vectors.

Consider the free-body diagrams in cases C and D below. In each case, the forces F_1 and F_2 have the same magnitude.



Question 3.

Is the magnitude of the net force ($\vec{F}_1 + \vec{F}_2$) in case C *greater than, less than,* or *equal to* the magnitude of the net force in case D?

Required.

- Greater than
- Less than
- Equal to

Question 4.

In the context of cases C and D, you are given two forces of the same magnitude.

How does the magnitude of the net force depend on the angle between the two force vectors?

Question 5.

Back to the question on the previous page:

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

Required.

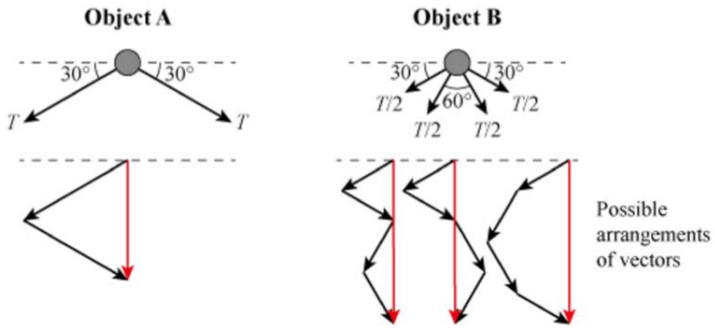
- Greater than → "Great. Your answer is correct!" → End of pretest
- Less than → Proceed to Questions 6 and 7 → End of pretest
- Equal to → Proceed to Questions 6 and 7 → End of pretest

(TNS)U3e
University of Washington

Great. Your answer is correct!

Your answer is still **incorrect**.

Here is how the tip-to-tail method is applied to find the net forces on objects A and B:



Question 6.

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

Required.

- Greater than
- Less than
- Equal to

Question 7.

Do you still have any questions?

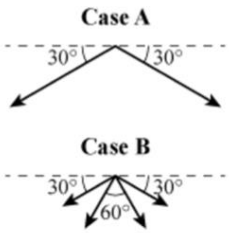
(ERB)U2h

Part I

Consider the vectors shown at right.

Case A consists of two vectors, each of which has the same length.

Case B consists of four vectors, each of which is **half** as long as the vectors in the Case A.



Question 1.

Is the magnitude of the **sum** of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the **sum** of the vectors in Case A?

- ○ Greater than
- ○ Less than
- ○ Equal to

Question 2.

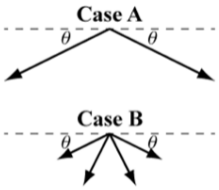
Explain the reasoning you used to obtain your answer to the question above.

(B) Pretests used in PHYS 115

(IGL)115_U2a_2v4vNC

Part III

Consider the vectors shown at right. Case A consists of two vectors, each of which has the same length. Case B consists of four vectors, each of which is half as long as the vectors in the Case A.



Question 8.

Is the magnitude of the sum of the vectors in Case A *greater than, less than,* or *equal to* the magnitude of the sum of the vectors in Case B?

- Greater than
- Less than
- Equal to

Question 9.

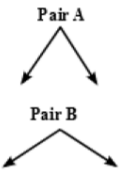
Briefly explain the reasoning that you used to answer the previous question.

(FLT)115_U2c_2vNC

University of Washington

Part III

Consider the vectors shown at right. Each vector shown has the same length. Pair A consists of the two vectors at the top and Pair B consists of the two vectors at the bottom.



Question 10.

Is the magnitude of the sum of the vectors in Pair A *greater than, less than,* or *equal to* the magnitude of the sum of the vectors in Pair B?

- Greater than
- Less than
- Equal to

Question 11.

Briefly explain the reasoning that you used to answer the previous question.

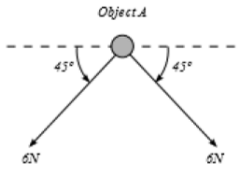
Figure E.7. The series of pretests used to collect data on student ability to do vector superposition in PHYS 115 in the Wi'19 quarter (3 pages).

(PRS)U1c_2v4vF

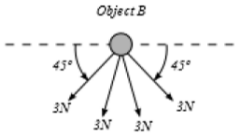
University of Washington

Part I

Object A has two forces of magnitude 6 N each acting on it, as shown in the free-body diagram at right.



Object B has four forces of magnitude 3 N acting on it, as shown in the free-body diagram shown at right.



Question 1.

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

- The magnitude of the net force on object B is *greater than* the magnitude of the net force on object A.
- The magnitude of the net force on object B is *less than* the magnitude of the net force on object A.
- The magnitude of the net force on object B is *equal to* the magnitude of the net force on object A.
- There is not enough information to answer this question

Question 2.

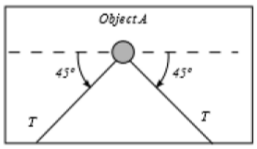
Explain the reasoning you used to answer the previous question.

(CHG)115_U3c_2v4vT

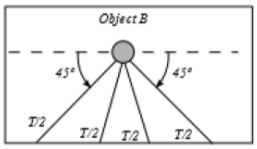
University of Washington

Part I

Object A is shown at right. Two ropes are attached to object A as shown, each pulling with tension *T*.



Object B is acted on by four ropes, as shown at right. The tension in each of these ropes is *T*/2.



Question 1.

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

- The net force on object B is greater
- The net force on object B is less
- The net force is equal
- There is not enough information to answer this question

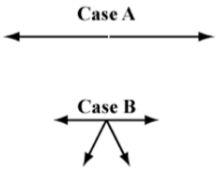
Question 2.

Explain the reasoning you used to obtain your answer to the question above.

(IGL)115_U2e_2v4vNC

Part II

Consider the vectors shown at right. Case A consists of two vectors, each of which has the same length. Case B consists of four vectors, each of which is half as long as the vectors in the Case A.



Question 7.

Is the magnitude of the sum of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the sum of the vectors in Case A?

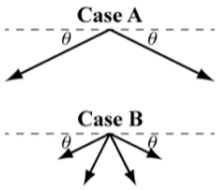
- Greater than
- Less than
- Equal to

Question 8.

Briefly explain the reasoning that you used to answer the previous question.

Part III

Consider the vectors shown at right. Case A consists of two vectors, each of which has the same length. Case B consists of four vectors, each of which is half as long as the vectors in the Case A.



Question 9.

Is the magnitude of the sum of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the sum of the vectors in Case A?

- Greater than
- Less than
- Equal to

Question 10.

Briefly explain the reasoning that you used to answer the previous question.

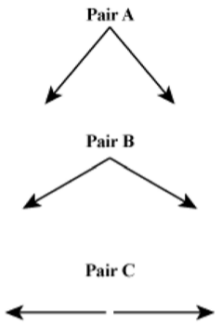
Figure E.8. The series of pretests used to collect data on student ability to do vector superposition in PHYS 115 in the Sp'19 quarter (6 pages).

(FLT)115_U2d_2vNC

University of Washington

Part II

Consider the three pairs of vectors shown at right. Each vector (arrow) shown has the same length.



Question 9.

Which of the following best describes the ranking of the magnitudes of the **sum** of the vectors in each pair?

- A = B = C
- A > B > C
- A > B = C
- A = B > C

Question 10.

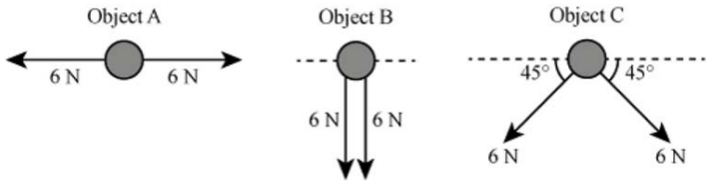
Briefly explain the reasoning that you used to answer the previous question.

(PRS)115_U1d_2v4vF

University of Washington

Part I

On objects A, B, and C, each of them has two forces of magnitude 6 N acting on it with different directions, as shown in the free-body diagrams below.



Question 1.

Which of the following best describes the ranking of the magnitudes of net force acting on each object?

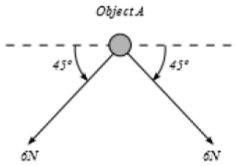
- A = B = C
- A > B > C
- B > C > A
- C > B > A
- B = C > A

Question 2.

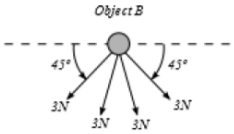
Explain the reasoning you used to answer the previous question.

Part II

Object A has two forces of magnitude 6 N each acting on it, as shown in the free-body diagram at right.



Object B has four forces of magnitude 3 N acting on it, as shown in the free-body diagram shown at right.



Question 3.

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

- The magnitude of the net force on object B is *greater than* the magnitude of the net force on object A.
- The magnitude of the net force on object B is *less than* the magnitude of the net force on object A.
- The magnitude of the net force on object B is *equal to* the magnitude of the net force on object A.
- There is not enough information to answer this question

Question 4.

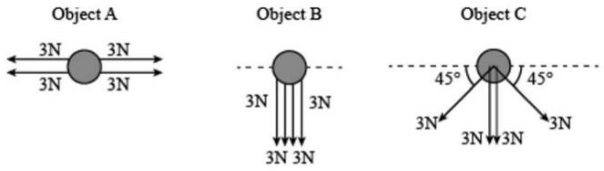
Explain the reasoning you used to answer the previous question.

(CHG)115_U3d_2v4vT

University of Washington

Part I

On objects A, B, and C, each of them has four tensions of magnitude 3 N acting on it with different directions, as shown in the free-body diagrams below.



Question 1.

Which of the following best describes the ranking of the magnitudes of net force acting on each object?

- A = B = C
- A > B > C
- B > C > A
- B = C > A

Question 2.

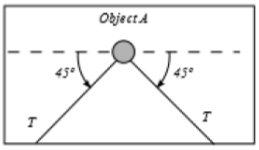
Explain the reasoning you used to obtain your answer to the question above.

(CHG)115_U3d_2v4vT

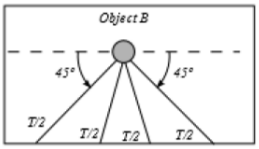
University of Washington

Part II

Object A is shown at right. Two ropes are attached to object A as shown, each pulling with tension *T*.



Object B is acted on by four ropes, as shown at right. The tension in each of these ropes is *T*/2.



Question 3.

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

- The net force on object B is greater
- The net force on object B is less
- The net force is equal
- There is not enough information to answer this question

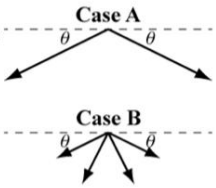
Question 4.

Explain the reasoning you used to obtain your answer to the question above.

(IGL)115_U2f_2v4vNC

Part II

Consider the vectors shown at right. Case A consists of two vectors, each of which has the same length. Case B consists of four vectors, each of which is half as long as the vectors in the Case A.



Question 8.

Is the magnitude of the sum of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the sum of the vectors in Case A?

Required.

- Greater than → End of pretest
- Less than → Proceed to Questions 10, 11 and 12
- Equal to → Proceed to Questions 10, 11, and 12

Question 9.

Briefly explain the reasoning that you used to answer the previous question.

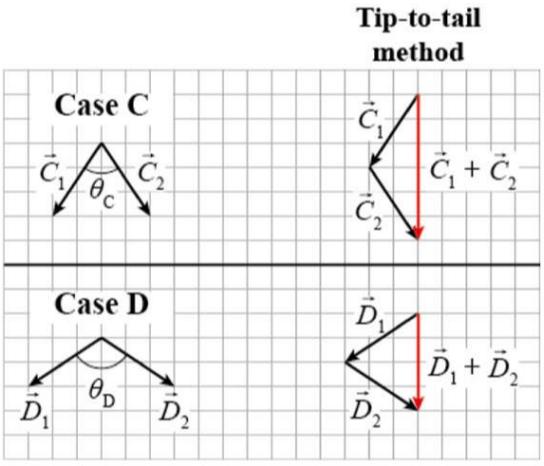
Figure E.9. The series of pretests used to collect data on student ability to do vector superposition in PHYS 115 in the Au'19 quarter (13 pages).

Part II (continued)

Your answer to the previous question is **incorrect**.

On this page, you will consider the tip-to-tail method to add vectors.

In cases C and D below, all the vectors ($\mathbf{C_1}$, $\mathbf{C_2}$, $\mathbf{D_1}$, and $\mathbf{D_2}$) have the same magnitude.



Question 10.

Is the magnitude of the sum of the vectors ($\mathbf{C_1}$ + $\mathbf{C_2}$) in case C greater than, less than, or equal to the magnitude of the sum of the vectors ($\mathbf{D_1}$ + $\mathbf{D_2}$) in case D?

Required.

- ○ Greater than
- ○ Less than
- ○ Equal to

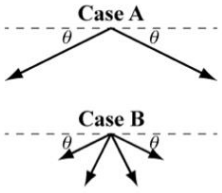
Question 11.

In cases C and D, how does the magnitude of the sum of the vectors depend on magnitude of the angle between the two vectors (θ_C & θ_D)?

Question 12.

Back to the question on the previous page:

Is the magnitude of the sum of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the sum of the vectors in Case A?



Required.

- ○ Greater than → "Great. Your answer is correct!" → End of pretest
- ○ Less than → Proceed to Questions 13 and 14 → End of pretest
- ○ Equal to → Proceed to Questions 13 and 14 → End of pretest

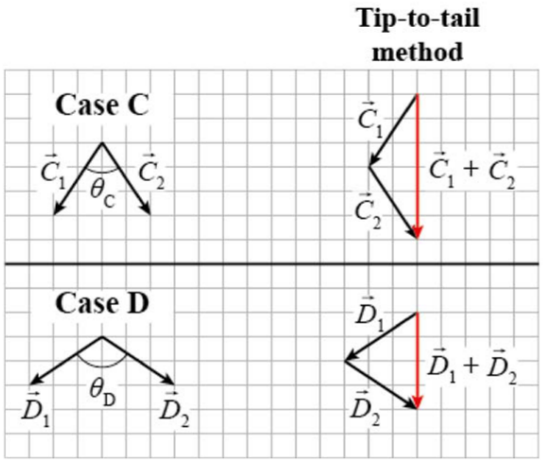
(IGL)115_U2f_2v4vNC

Great. Your answer is correct!

(IGL)115_U2f_2v4vNC

Your answer is still **incorrect**.

Let us look at the answers to these questions:



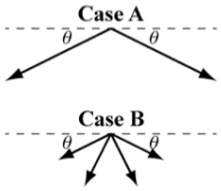
Is the magnitude of the sum of the vectors (**C_1** + **C_2**) in case C *greater than, less than,* or *equal to* the magnitude of the sum of the vectors (**D_1** + **D_2**) in case D?

Answer: Greater than

The magnitude of the sum of the vectors is greater when the angle between the vectors is smaller.

Question 13.

Based on the answers above, let's answer this question again:



Is the magnitude of the sum of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the sum of the vectors in Case A?

Required.

- ○ Greater than
- ○ Less than
- ○ Equal to

Question 14.

Do you still have any questions?

(FLT)115_U2e_2vNC

University of Washington

Part II

Consider the pair of vectors in cases A and B shown at right. Each vector shown has the same length.

In Case A, the two vectors are 120° apart.

In Case B, the two vectors are 60° apart.

Question 9.

Is the magnitude of the sum of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the sum of the vectors in Case A?

Required.

- ○ Greater than → End of pretest
- ○ Less than → Proceed to Questions 11, 12 and 13
- ○ Equal to → Proceed to Questions 11, 12, and 13

Question 10.

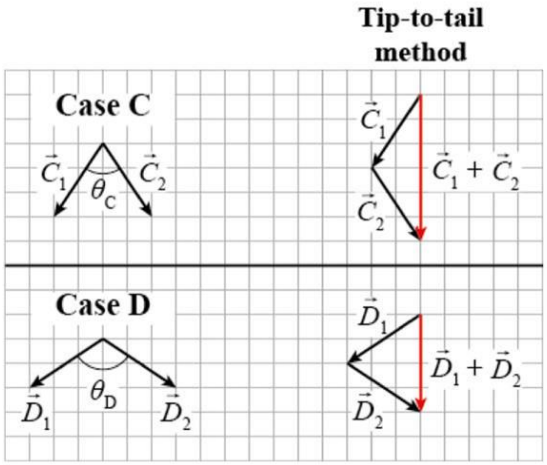
Explain the reasoning that you used to answer the previous question.

Part II (continued)

Your answer to the previous question is **incorrect**.

On this page, you will consider the tip-to-tail method to add vectors.

In cases C and D below, all the vectors (**C_1**, **C_2**, **D_1**, and **D_2**) have the same magnitude.



Question 11.

Is the angle θ_C *greater than, less than,* or *equal to* the angle θ_D ?

Required.

- Greater than
- Less than
- Equal to

Question 12.

Is the magnitude of the sum of the vectors (**C_1** + **C_2**) in case C *greater than, less than,* or *equal to* the magnitude of the sum of the vectors (**D_1** + **D_2**) in case D?

- Greater than
- Less than
- Equal to

Question 13.

In cases C and D, how does the magnitude of the sum of the vectors depend on the angle between the two vectors?

Question 14.

Back to the question on the previous page:

Is the magnitude of the sum of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the sum of the vectors in Case A?

Required.

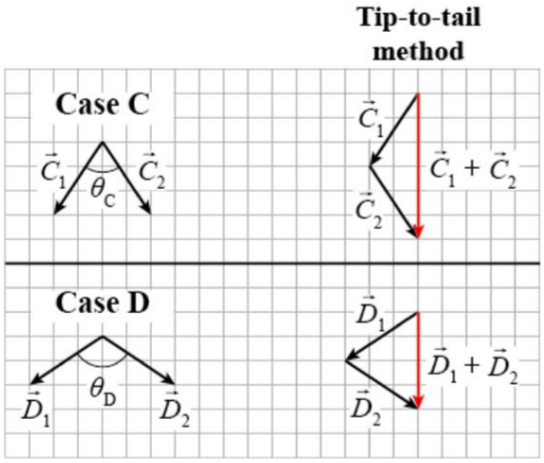
- Greater than → "Great. Your answer is correct!" → End of pretest
- Less than → Proceed to Questions 15 and 16 → End of pretest
- Equal to → Proceed to Questions 15 and 16 → End of pretest

(FLT)115_U2e_2vNC
University of Washington

Great. Your answer is correct!

Your answer is still **incorrect**.

Let us look at the answers to these questions:



Is the angle θ_C *greater than, less than,* or *equal to* the angle θ_D ?
Answer: Less than

Is the magnitude of the sum of the vectors (**C_1** + **C_2**) in case C *greater than, less than,* or *equal to* the magnitude of the sum of the vectors (**D_1** + **D_2**) in case D?
Answer: Greater than

Method to arrive at the answer: Use the given diagram.

Question 15.

Based on the answers above, let's answer this question again:

Is the magnitude of the sum of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the sum of the vectors in Case A?

- Greater than
- Less than
- Equal to

Question 16.

Do you still have any questions?

(PRS)115_U1e_2v4vF

University of Washington

Part II

In the free-body diagram below, object A has two forces of magnitude 6 N each acting on it, and object B has four forces of magnitude 3 N acting on it.

Question 5.

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

Required.

- ○ Greater than → End of pretest
- ○ Less than → Proceed to Questions 7, 8 and 9
- ○ Equal to → Proceed to Questions 7, 8, and 9

Question 6.

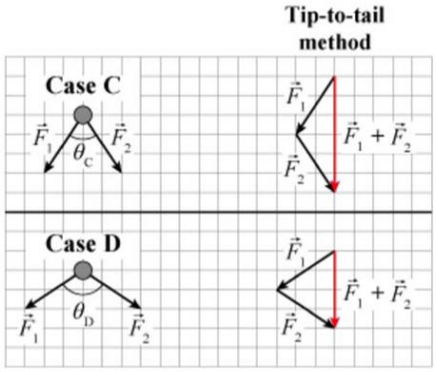
Explain the reasoning you used to answer the previous question.

Part II (continued)

Your answer to the previous question is **incorrect**.

On this page, you will consider the tip-to-tail method to add vectors.

Consider the free-body diagrams in cases C and D below. In each case, the forces F_1 and F_2 have the same magnitude.



Question 7.

Is the magnitude of the net force ($\vec{F}_1 + \vec{F}_2$) in case C *greater than, less than,* or *equal to* the magnitude of the net force in case D?

Required.

- Greater than
- Less than
- Equal to

Question 8.

In cases C and D, how does the magnitude of the sum of the vectors depend on magnitude of the angle between the two vectors (θ_C & θ_D)?

Question 9.

Back to the question on the previous page:

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

Required.

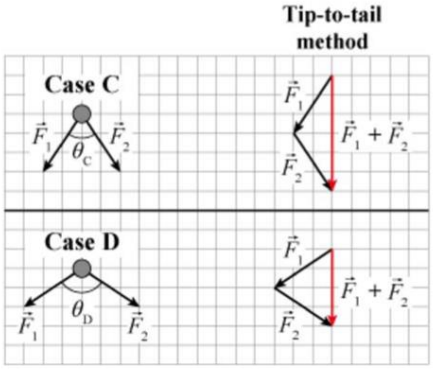
- Greater than → "Great. Your answer is correct!" → End of pretest
- Less than → Proceed to Questions 10 and 11 → End of pretest
- Equal to → Proceed to Questions 10 and 11 → End of pretest

(PRS)115_U1e_2v4vF
University of Washington

Great. Your answer is correct!

Your answer is still **incorrect**.

Let us look at the answers to these questions:



Is the magnitude of the net force in case C *greater than, less than,* or *equal to* the magnitude of the net force in case D?

Answer: Greater than

Note: The magnitude of the sum of the vectors is represented by the length of the vector.
Note: The magnitude of the resultant is greater when the angle between the vectors are smaller.

Question 10.

Based on the answers above, let's answer this question again:

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

Required.

- Greater than
- Less than
- Equal to

Question 11.

Do you still have any questions?

(BNC)115_U3b

University of Washington

Part III

In the free-body diagrams below, object A has two forces of magnitude *T* each acting on it, and object B has four forces of magnitude *T*/2 acting on it.

Question 9.

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

Required.

- Greater than → End of pretest
- Less than → Proceed to Questions 11, 12 and 13
- Equal to → Proceed to Questions 11, 12, and 13

Question 10.

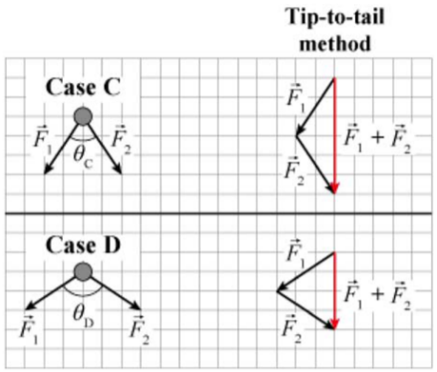
Explain the reasoning you used to answer the previous question.

Part III (continued)

Your answer to the previous question is **incorrect**.

On this page, you will consider the tip-to-tail method to add vectors.

Consider the free-body diagrams in cases C and D below. In each case, the forces F_1 and F_2 have the same magnitude.



Question 11.

Is the magnitude of the net force ($\vec{F}_1 + \vec{F}_2$) in case C *greater than, less than,* or *equal to* the magnitude of the net force in case D?

Required.

- Greater than
- Less than
- Equal to

Question 12.

In cases C and D, how does the magnitude of the sum of the vectors depend on magnitude of the angle between the two vectors (θ_C & θ_D)?

Question 13.

Back to the question on the previous page:

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

Required.

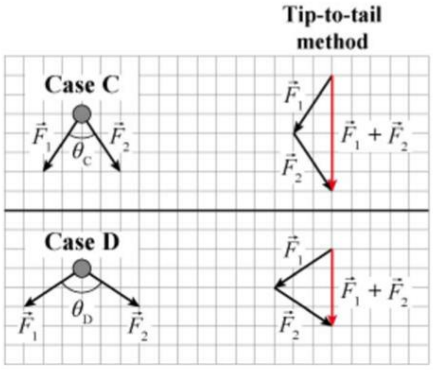
- Greater than → "Great. Your answer is correct!" → End of pretest
- Less than → Proceed to Questions 14 and 15 → End of pretest
- Equal to → Proceed to Questions 14 and 15 → End of pretest

(BNC)115_U3b
University of Washington

Great. Your answer is correct!

Your answer is still **incorrect**.

Let us look at the answers to some questions:



Is the magnitude of the net force in case C *greater than, less than,* or *equal to* the magnitude of the net force in case D?

Answer: Greater than

Note: The magnitude of the sum of the vectors is represented by the length of the vector.
Note: The magnitude of the resultant is greater when the angle between the vectors are smaller.

Question 14.

Based on the answers above, let's answer this question again:

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

Required.

- ○ Greater than
- ○ Less than
- ○ Equal to

Question 15.

Do you still have any questions?

(CHG)115_U3e_2vE

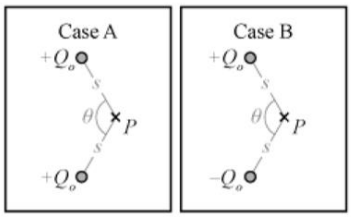
University of Washington

Part II

Case A: Point *P* is a distance *s* away from each of the two identical positive point charges $+Q_o$.

Case B: One of the positive charges in Case A is replaced by a negative point charge of magnitude $-Q_o$.

In both cases, the angle θ is greater than 120°.



Question 5.

Is the magnitude of the net electric field at point *P* in Case B *greater than*, *less than*, or *equal to* that in Case A?

- Greater than
- Less than
- Equal to

Question 6.

Explain the reasoning you used to obtain your answer to the question above.

(IGL)115_U2g_2vNC

Part II

Consider the pair of vectors in cases A and B shown at right.

Each vector shown has the same length.

Question 7.

Is the magnitude of the sum of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the sum of the vectors in Case A?

Required.

- Greater than → End of pretest
- Less than → Proceed to Questions 9, 10 and 11
- Equal to → Proceed to Questions 9, 10, and 11

Question 8.

Briefly explain the reasoning that you used to answer the previous question.

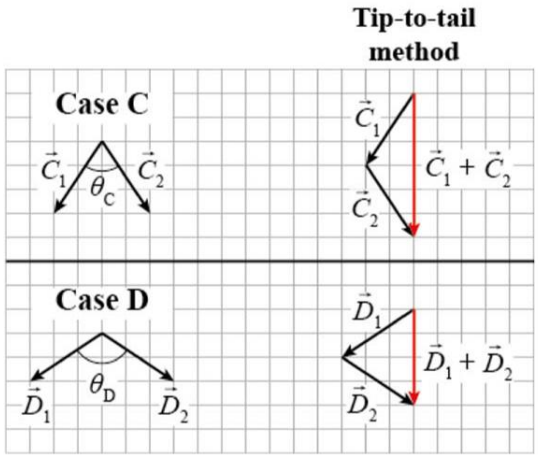
Figure E.10. The series of pretests used to collect data on student ability to do vector superposition in PHYS 115 in the Wi'20 quarter (9 pages).

Part II (continued)

Your answer to the previous question is **incorrect**.

On this page, you will consider the tip-to-tail method to add vectors.

In cases C and D below, all the vectors (**C_1**, **C_2**, **D_1**, and **D_2**) have the same magnitude.



Question 9.

Is the magnitude of the sum of the vectors (**C_1** + **C_2**) in case C *greater than, less than,* or *equal to* the magnitude of the sum of the vectors (**D_1** + **D_2**) in case D?

Required.

- Greater than
- Less than
- Equal to

Question 10.

In cases C and D, how does the magnitude of the sum of the vectors depend on magnitude of the angle between the two vectors (θ_C & θ_D)?

Question 11.

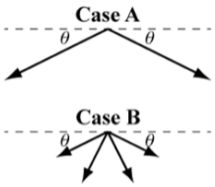
Do you have any other questions?

(FLT)115_U2f_2v4vNC

University of Washington

Part II

Consider the vectors shown at right. Case A consists of two vectors, each of which has the same length. Case B consists of four vectors, each of which is half as long as the vectors in the Case A.



Question 9.

Is the magnitude of the sum of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the sum of the vectors in Case A?

Required.

- ○ Greater than → End of pretest
- ○ Less than → Proceed to Questions 11, 12 and 13
- ○ Equal to → Proceed to Questions 11, 12, and 13

Question 10.

Explain the reasoning that you used to answer the previous question.

(FLT)115_U2f_2v4vNC

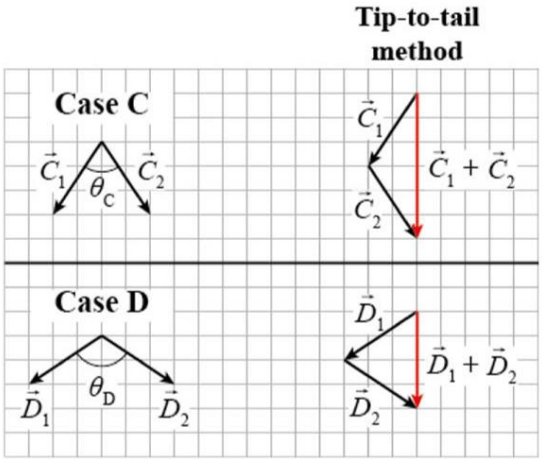
University of Washington

Part II (continued)

Your answer to the previous question is **incorrect**.

On this page, you will consider the tip-to-tail method to add vectors.

In cases C and D below, all the vectors ($\mathbf{C_1}$, $\mathbf{C_2}$, $\mathbf{D_1}$, and $\mathbf{D_2}$) have the same magnitude.



Question 11.

Is the magnitude of the sum of the vectors ($\mathbf{C_1}$ + $\mathbf{C_2}$) in case C *greater than, less than,* or *equal to* the magnitude of the sum of the vectors ($\mathbf{D_1}$ + $\mathbf{D_2}$) in case D?

Required.

- Greater than
- Less than
- Equal to

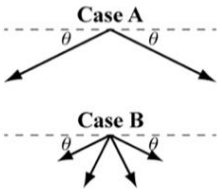
Question 12.

In cases C and D, how does the magnitude of the sum of the vectors depend on the angle between the two vectors?

Question 13.

Back to the question on the previous page:

Is the magnitude of the sum of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the sum of the vectors in Case A?



Required.

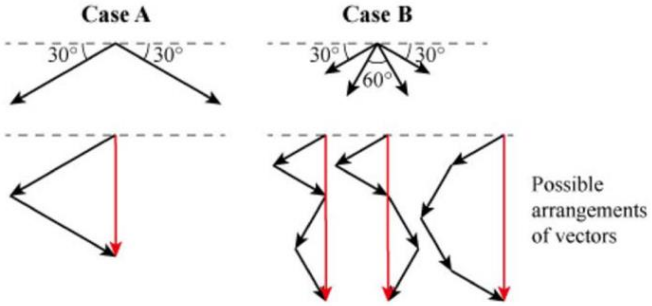
- Greater than → "Great. Your answer is correct!" → End of pretest
- Less than → Proceed to Questions 14 → End of pretest
- Equal to → Proceed to Questions 14 → End of pretest

(FLT)115_U2f_2v4vNC
University of Washington

Great. Your answer is correct!

Your answer is still incorrect.

Here is how the tip-to-tail method is applied for cases A and B:



Question 14.

Is the magnitude of the **sum** of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the **sum** of the vectors in Case A?

Required.

- Greater than
- Less than
- Equal to

(PRS)U1f_2v4vF
University of Washington

Part II

In the free-body diagram below, object A has two forces of magnitude 6 N each acting on it, and object B has four forces of magnitude 3 N acting on it.

Question 5.

Is the magnitude of the net force on object A *greater than, less than,* or *equal to* the magnitude of the net force on object B?

Required.

- Greater than → End of pretest
- Less than → Proceed to Questions 7, 8 and 9
- Equal to → Proceed to Questions 7, 8, and 9

Question 6.

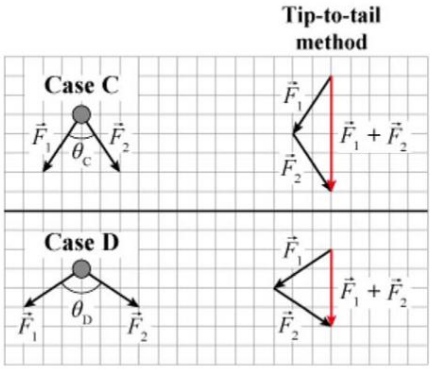
Explain the reasoning you used to answer the previous question.

Part II (continued)

Your answer to the previous question is **incorrect**.

On this page, you will consider the tip-to-tail method to add vectors.

Consider the free-body diagrams in cases C and D below. In each case, the forces F_1 and F_2 have the same magnitude.



Question 7.

Is the magnitude of the net force ($\vec{F}_1 + \vec{F}_2$) in case C *greater than, less than,* or *equal to* the magnitude of the net force in case D?

Required.

- Greater than
- Less than
- Equal to

Question 8.

In cases C and D, how does the magnitude of the sum of the vectors depend on magnitude of the angle between the two vectors (θ_C & θ_D)?

Question 9.

Back to the question on the previous page:

Is the magnitude of the net force on object A *greater than, less than,* or *equal to* the magnitude of the net force on object B?

Required.

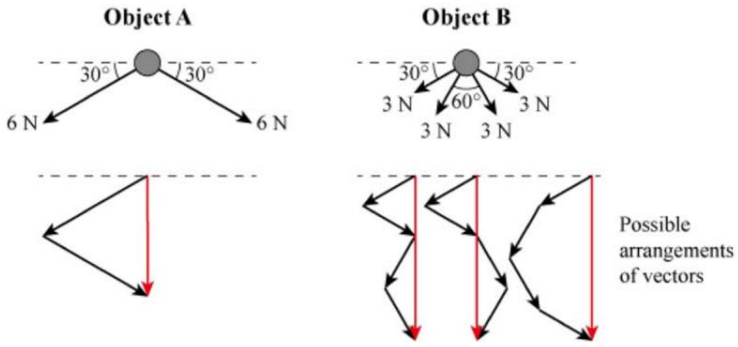
- Greater than → "Great. Your answer is correct!" → End of pretest
- Less than → Proceed to Questions 10 and 11 → End of pretest
- Equal to → Proceed to Questions 10 and 11 → End of pretest

(PRS)U1f_2v4vF
University of Washington

Great. Your answer is correct!

Your answer is still **incorrect**.

Here is how the tip-to-tail method is applied for objects A and B:



Question 10.

Based on the answers above, let's answer this question again:

Is the magnitude of the net force on object A *greater than, less than,* or *equal to* the magnitude of the net force on object B?

Required.

- ○ Greater than
- ○ Less than
- ○ Equal to

Question 11.

Do you still have any questions?

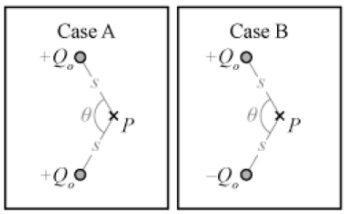
(CHG)115_U3e_2vE
University of Washington

Part II

Case A: Point *P* is a distance *s* away from each of the two identical positive point charges $+Q_o$.

Case B: One of the positive charges in Case A is replaced by a negative point charge of magnitude $-Q_o$.

In both cases, the angle θ is greater than 120°.



Question 5.

Is the magnitude of the net electric field at point *P* in Case B *greater than*, *less than*, or *equal to* that in Case A?

- Greater than
- Less than
- Equal to

Question 6.
Explain the reasoning you used to obtain your answer to the question above.

(CHG)U3f
University of Washington

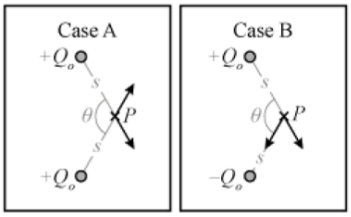
Part II

Case A: Point *P* is a distance *s* away from each of two identical positive point charges $+Q_o$.

Case B: One of the positive charges in Case A is replaced by a negative point charge of magnitude $-Q_o$.

The black arrows represent the electric fields at point *P* due to the point charges.

In both cases, the angle θ is greater than 120°.



Question 5.

Is the magnitude of the net electric field at point *P* in Case B *greater than*, *less than*, or *equal to* that in Case A?

- Greater than
- Less than
- Equal to

Question 6.
Explain the reasoning you used to obtain your answer to the question above.

E2. Posttests used in the investigation of student ability in applying vector superposition in algebra-based physics course

In this section, we present the exam questions for the investigation of student ability in applying vector superposition in algebra-based physics course. The exam questions were administered on midterms and final exams in PHYS 114 and PHYS 115. The questions on PHYS 114 are presented first, followed by those on PHYS 115. Some questions were administered in both courses and in multiple quarters.

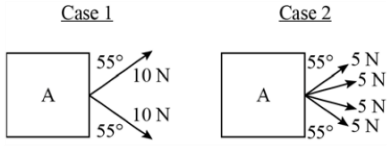
We do not show the original versions as there are other questions that are unrelated to the research on the exam.

Name _____ **Student ID** _____ **Score** _____
last *first*

Vector Superposition Posttest Questions in PHYS 114

Au'18 Final Exam (PHYS 114B)

On the diagram at right, in case 1, block A has two forces of magnitude 10 N each acting on it. In case 2, block A has four forces of magnitude 5 N acting on it.



Is the magnitude of the net force on block A in Case 2 *greater than*, *less than*, or *equal to* the magnitude of the net force on block A in Case 1?

A. $F_{net,2}$ is greater than $F_{net,1}$
B. $F_{net,2}$ is less than $F_{net,1}$
C. $F_{net,2}$ is equal to $F_{net,1}$
D. Not enough information

Wi'19 Final Exam (PHYS 114A and 114B)

Sp'19 Final Exam (PHYS 114B)

Au'19 Final Exam (PHYS 114C)

On the diagram below right, in case 1, block A has two forces of magnitude 10 N each acting on it. In case 2, block A has four forces of magnitude 5 N acting on it.

Is the magnitude of the net force on block A in Case 2, $F_{net,2}$, *greater than*, *less than*, or *equal to* the magnitude of the net force on block A in Case 1, $F_{net,1}$?

A. $F_{net,2}$ is greater than $F_{net,1}$
B. $F_{net,2}$ is less than $F_{net,1}$
C. $F_{net,2}$ is equal to $F_{net,1}$
D. Not enough information

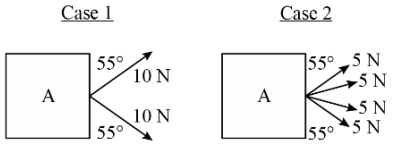
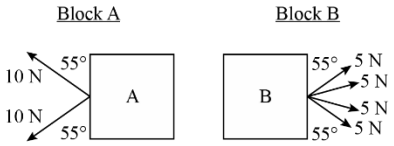


Figure E.11. The posttest questions used to collect data on student ability to do vector superposition in PHYS 114 from Au'19 to Au'19 quarters. The quarters that the questions were administered were labeled (2 pages).

Name _____ **Student ID** _____ **Score** _____
last *first*

Sp'19 Final Exam (PHYS 114A)

On the diagram at right, block A has two forces of magnitude 10 N acting on it. Block B has four forces of magnitude 5 N acting on it.

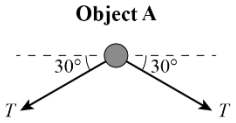


Is the magnitude of the net force on block B, $F_{net,B}$, *greater than*, *less than*, or *equal to* the magnitude of the net force on block A, $F_{net,A}$?

A. $F_{net,B}$ is greater than $F_{net,A}$
B. $F_{net,B}$ is less than $F_{net,A}$
C. $F_{net,B}$ is equal to $F_{net,A}$
D. Not enough information

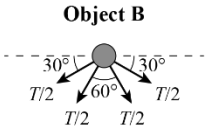
Au'19 Final Exam (PHYS 114B)

On the diagram at right, object A has two forces of magnitude T acting on it. Object B has four forces of magnitude $T/2$ acting on it.



Is the magnitude of the net force on object B, $F_{net,B}$, *greater than*, *less than*, or *equal to* the magnitude of the net force on object A, $F_{net,A}$?

A. $F_{net,B}$ is greater than $F_{net,A}$
B. $F_{net,B}$ is less than $F_{net,A}$
C. $F_{net,B}$ is equal to $F_{net,A}$
D. Not enough information



Name _____ **Student ID** _____ **Score** _____
last *first*

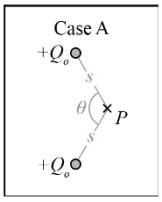
Vector Superposition Posttest Questions in PHYS 115

Wi'19 Final Exam (PHYS 115ABC)

Au'19 Final Exam (PHYS 115A)

The next 3 problems involve the following situation.

Case A: Point P is a distance s away from each of the two identical positive point charges $+Q_o$. The angle θ is greater than 120°.



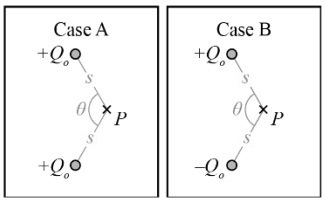
1. Suppose a *negative* test charge were at point P. Which of the following arrows best represent the direction of the **net electric force** on the negative test charge?

A. ↘ B. ↗ C. → D. ↑ E. ←

2. Which of the following arrows best represent the direction of the **net electric field** acting on the negative test charge?

A. ↘ B. ↗ C. → D. ↑ E. ←

Case B: The test charge is now removed. Suppose one of the positive charges in Case A is replaced by a negative point charge of magnitude $-Q_o$ as shown at right.



3. Is the magnitude of the net electric field at point P in Case B *greater than, less than,* or *equal to* that in Case A?

   A. Greater than
   B. Less than
   C. Equal to
   D. Not enough information

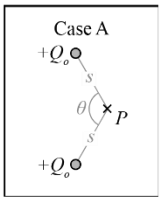
Figure E.12. The posttest questions used to collect data on student ability to do vector superposition in PHYS 115 from Wi'19 to Wi'20 quarters. The quarters that the questions were administered were labeled (3 pages).

Name _____ **Student ID** _____ **Score** _____
last *first*

Sp'19 Final Exam (PHYS 115A)

The next two problems involve the following situation.

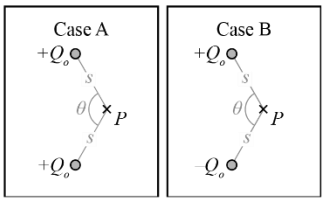
Case A: Point P is a distance s away from each of the two identical positive point charges $+Q_o$. The angle θ is greater than 120°.



4. Which of the following arrows best represent the direction of the **net electric field** at point P?

A. ↘ B. ↗ C. → D. ↑ E. ←

Case B: The test charge is now removed. Suppose one of the positive charges in Case A is replaced by a negative point charge of magnitude $-Q_o$ as shown at right.



5. Is the magnitude of the net electric field at point P in Case B *greater than, less than,* or *equal to* that in Case A?

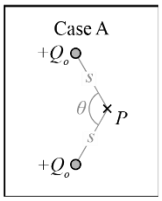
   A. Greater than
   B. Less than
   C. Equal to
   D. Not enough information

Name _____ **Student ID** _____ **Score** _____
last *first*

Sp'19 Final Exam (PHYS 115A)

The next two problems involve the following situation.

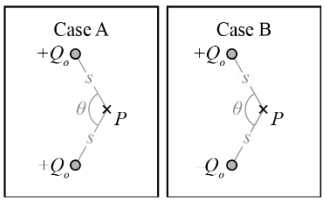
Case A: Point P is a distance s away from each of the two identical positive point charges $+Q_o$. The angle θ is greater than 120°.



4. Which of the following arrows best represent the direction of the **net electric field** at point P?

A. ↘ B. ↗ C. → D. ↑ E. ←

Case B: The test charge is now removed. Suppose one of the positive charges in Case A is replaced by a negative point charge of magnitude $-Q_o$ as shown at right.



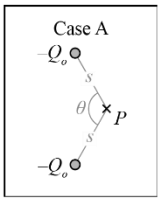
5. Is the magnitude of the net electric field at point P in Case B *greater than, less than,* or *equal to* that in Case A?

A. Greater than
B. Less than
C. Equal to
D. Not enough information

Sp'19 Final Exam (PHYS 115B)

The next two problems involve the following situation.

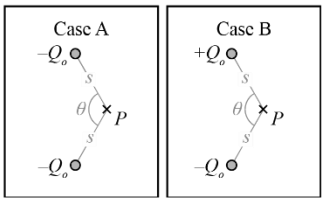
Case A: Point P is a distance s away from each of the two identical negative point charges $-Q_o$. The angle θ is greater than 120°.



6. Which of the following arrows best represent the direction of the **net electric field** at point P?

A. ↘ B. ↗ C. → D. ↑ E. ←

Case B: Suppose one of the negative charges in Case A is replaced by a positive point charge of magnitude $+Q_o$ as shown at right.



7. Is the magnitude of the net electric field at point P in Case B *greater than, less than,* or *equal to* that in Case A?

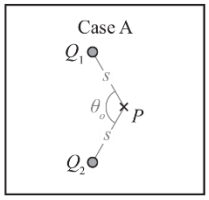
A. Greater than
B. Less than
C. Equal to
D. Not enough information

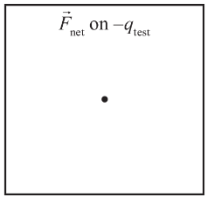
Name: _____ Student ID: _____ Score: _____

PRINT Last First

Tutorial questions: In all questions below, the angle θ_o is the same and is between 90° and 180°.

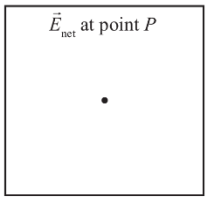
Case A: Point P is a distance s away from two positive point charges, Q_1 and Q_2, as shown. Both charges have magnitude $+Q_o$.





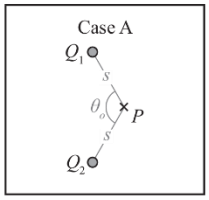
17. [4 pts] Suppose a *negative* test charge $-q_{test}$ were at point P. In the box at far right, draw an arrow showing the net electric force on $-q_{test}$. Draw a sketch to support your answer, labeling each part.

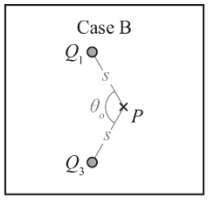
18. [4 pts] In the box at right, draw an arrow representing the net electric field at point P. Explain your reasoning.



19. [6 pts] **Case B:** Suppose Q_2 is replaced by Q_3, a negative point charge of magnitude $-Q_o$.

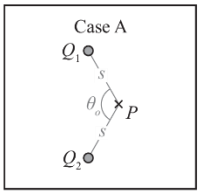
    Is the magnitude of the net electric field at point P in Case B *greater than*, *less than*, or *equal to* that in Case A? Explain.

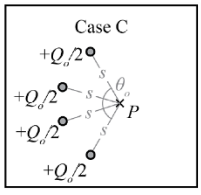




20. [6 pts] **Case C:** Suppose both of the original point charges are replaced by four point charges, each with charge $+Q_o/2$, and each placed a distance s from point P. The outer charges are still separated by the angle θ_o.

    Is the magnitude of the net electric field at point P in Case C *greater than*, *less than*, or *equal to* that in Case A? Explain.





E3. Additional results for Chapter 14

This section describes the additional results for Chapter 14. It describes the experimentation we did with different types of computerized feedback in the Sp'19 quarter.

Results obtained in the Sp'19 quarter

Background

Prior research has shown that computer-based instruction was effective [111-113]. There were plenty of reviews discussing the effectiveness of different methods of feedback [114-117]. The effectiveness of the feedback depends on the type and level of knowledge or skill, complexity of feedback, prior knowledge of topic, interest in topic, self-efficacy and other attitudinal factors.

Based on the insights from the previous research, Heckler and Mikula investigated the factors that affect the learning of vector math from computer-based practice [85]. They focused on improving calculus-based student ability to do dot and cross product. The factors that they had analyzed were training time, feedback complexity, prior knowledge, physics course grade, and student beliefs about both their proficiency in and the importance of vector math. They adopted a framework in which feedback can be broken into five types of increasing complexity: (1) no feedback, (2) knowledge of results (KR), (3) knowledge of correct response (KCR), (4) knowledge of correct response and elaborated feedback (EF), and (5) knowledge of correct response and interactive teaching [86]. The general finding they found was "computer-based training with static question sequences and immediate elaborated feedback in the form of simple and general explanations can be an effective way to improve student performance on a physics essential skill (vector math), especially for less prepared and low-performing students." As for the results from specific investigations, they found that there were two schemes that produced better outcomes.

They are the combination of KR (user is informed of whether the answer is correct or wrong) and EF and the combination of KR, KCR (user is informed of the correct answer), and EF.

Based on the findings by Heckler and Mikula, we tested several static question sequences according to the framework on pretests to find out which approach gave the best result. Students saw the sequence of questions when they gave an incorrect answer on the vector superposition tasks, but not when they answered the tasks correctly. We evaluated the effectiveness of the sequence by considering the proportion of students who answered the second time correctly.

Experimental design

We conducted this investigation in the Sp'19 quarter. There were two sections of PHYS 114, A and B, so we ran different sequences of static questions in different pretests in both sections to find out their effectiveness. The approaches we used to help students to add vectors in both sections were different. In PHYS 114A, we asked students to add vectors by considering the sum of the horizontal and vertical components. In PHYS 114B, we asked students to consider the limiting case approach and the tip-to-tail method. The four vector superposition tasks the PHYS 114 students were (in chronological order) 2v-NC, 2v4v-NC, 2v4v-F, and 2v4v-T.

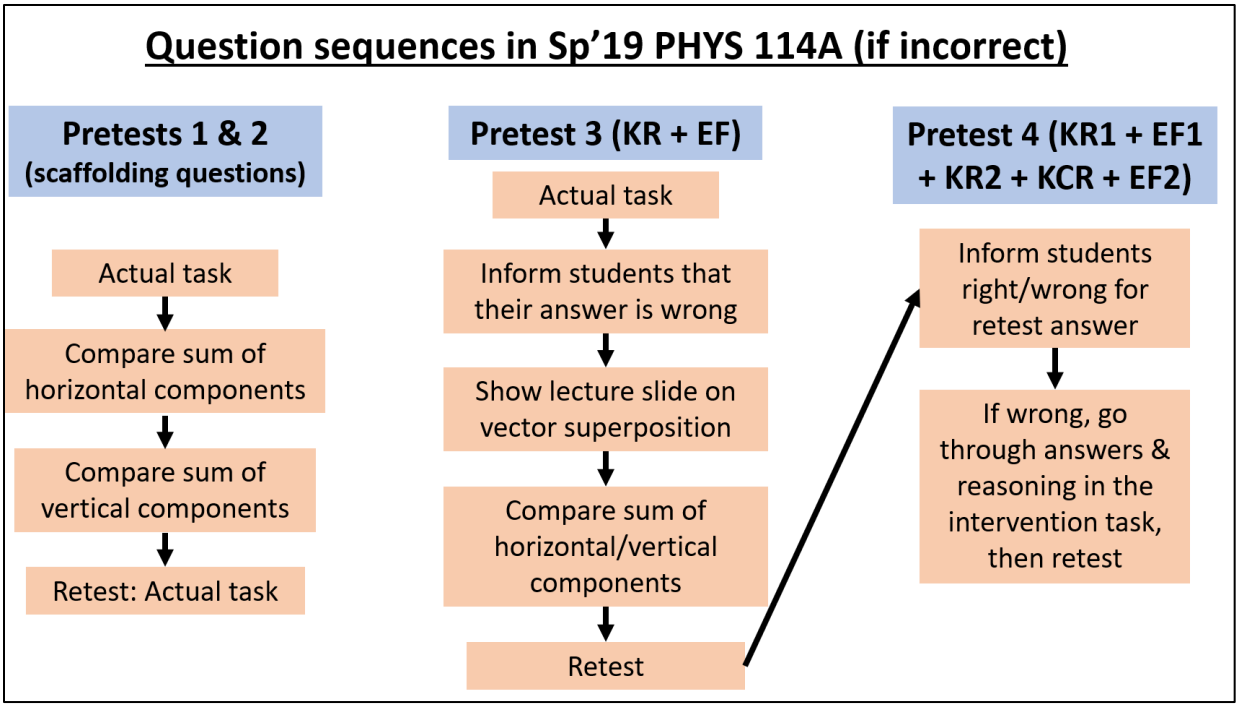


Figure E.13. Question sequences in different pretests (vector components) for PHYS 114A in the Sp'19 quarter. The question sequence in pretest 3 is included in pretest 4.

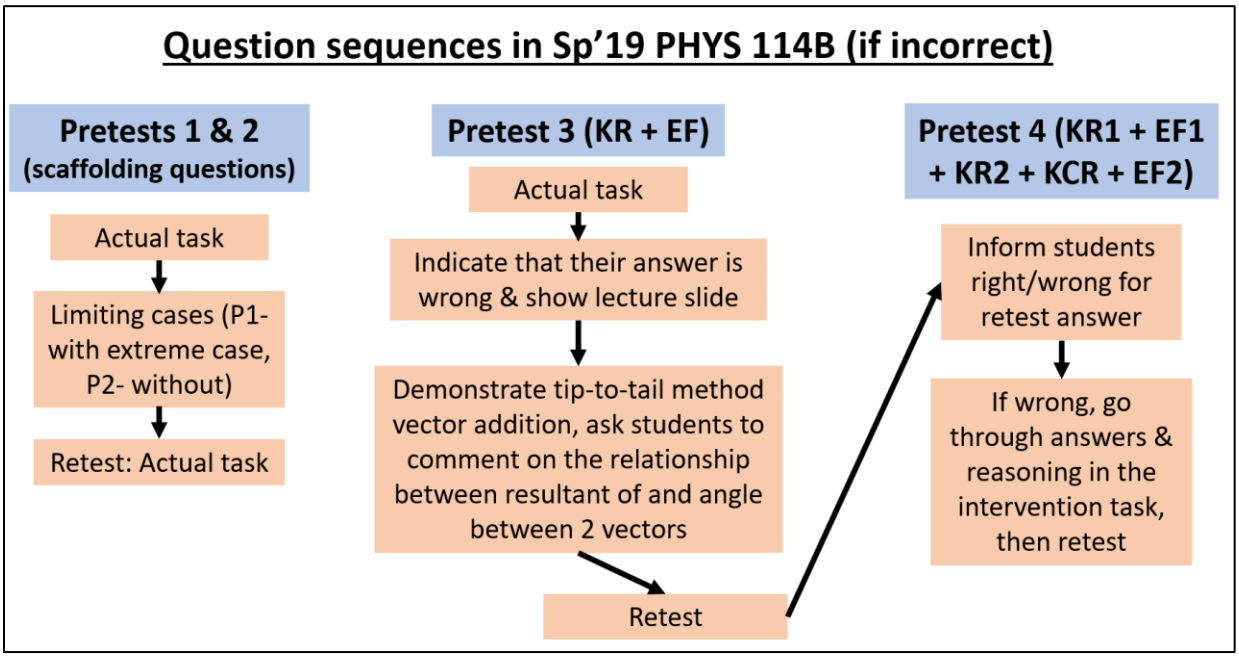


Figure E.14. Question sequences in different pretests (limiting case approach and tip-to-tail approach) for PHYS 114B class in the Sp'19 quarter. The question sequence in pretest 3 is included in pretest 4.

We summarize the question sequences in PHYS 114A and PHYS 114B in each pretest in Figure E.13 and Figure E.14. The question sequence after students answered the actual task incorrectly in each pretest was different. For example, in pretests 1 and 2 for PHYS 114A, students did not receive any feedback on their answers (Figure E.15), we asked them to sum horizontal and vertical components as scaffolding questions. In pretest 3, we informed students that their answers were wrong (KR), showed them the lecture slide on vector addition (EF), asked them several questions, and retested them the actual task (Figure E.16). In the fourth pretest, the sequence started with that in pretest 3, then informed them if their answers of the retests were correct or incorrect (KR2). Students who were incorrect the second time saw the correct answers and reasoning in the intervention tasks and attempted the actual task the third time (Figure E.17). The actual pretests in both sections are in Appendix E1.

Consider the pair of vectors in cases A and B shown at right. Each vector shown has the same length.

In Case A, the two vectors are 120° apart.

In Case B, the two vectors are 60° apart.

Question 13.

Is the magnitude of the sum of the **horizontal components** of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the sum of the **horizontal components** of the vectors in Case A?

Question 14.

Is the magnitude of the sum of the **vertical components** of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the sum of the **vertical components** of the vectors in Case A?

Question 15.

Is the magnitude of the sum of the vectors in Case B *greater than, less than,* or *equal to* the magnitude of the sum of the vectors in Case A?

Question 16.

Your answer on the previous page is "The sum of vectors in Case B is **equal to** the sum of vectors in case A".

Is your answer to the previous question different from that? If so, explain why. If not, explain why not.

Figure E.15. Question sequence (no feedback) in pretest 1 for PHYS 114A students if they answered the 2v-NC task incorrectly.

Your answer to the previous question is **incorrect**.

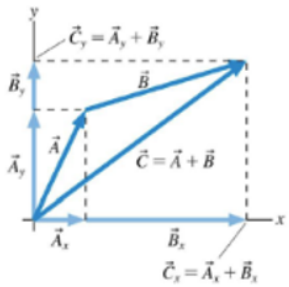
On this page, you will consider the tip-to-tail method to add vectors.

A reminder from your lecture notes:

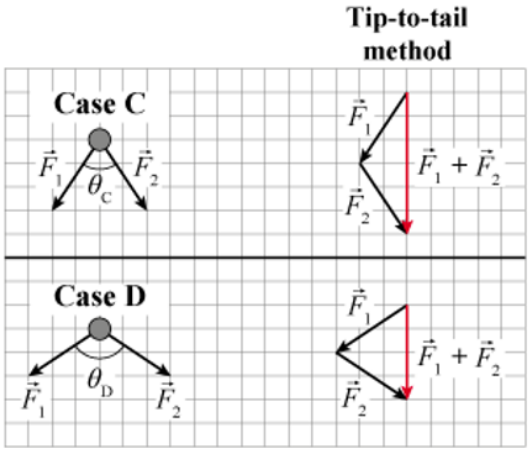
3.2 Working with components

- If $\vec{C} = \vec{A} + \vec{B} + \cdots$

$$C_x = A_x + B_x + \cdots$$
$$C_y = A_y + B_y + \cdots$$



Consider the free-body diagrams in cases C and D below. In each case, the forces F_1 and F_2 have the same magnitude.



Question 3.

Is the angle θ_C *greater than, less than,* or *equal to* the angle θ_D ?

Question 4.

Is the magnitude of the net force ($\vec{F}_1 + \vec{F}_2$) in case C *greater than, less than,* or *equal to* the magnitude of the net force in case D?

Question 5.

In the context of cases C and D, you are given two forces of the same magnitude.

How does the magnitude of the net force depend on the angle between the two force vectors?

Figure E.16. Question sequence (KR1 + EF1) in pretests 3 and 4 for PHYS 114B students if they answered the 2v4v-T task incorrectly. Students redo the actual task in Question 6.

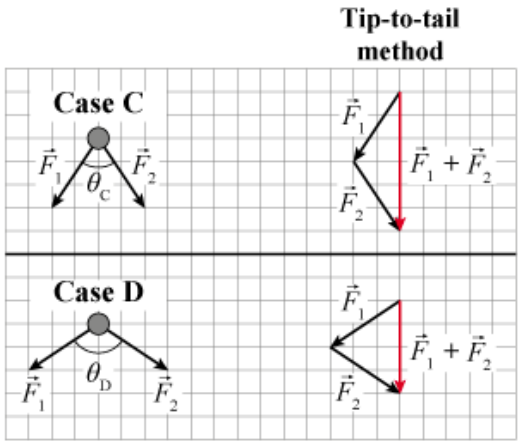
Your answer is still **incorrect**.

Let us look at the answers to these questions:

Is the angle θ_C *greater than, less than,* or *equal to* the angle θ_D ?
Answer: Less than

Is the magnitude of the net force ($\vec{F}_1 + \vec{F}_2$) in case C *greater than, less than,* or *equal to* the magnitude of the net force in case D?
Answer: Greater than

Method to arrive at the answer: Use the given diagram.



Question 7.

Based on the answers above, let's answer this question again:

Is the magnitude of the net force on object B *greater than, less than,* or *equal to* the magnitude of the net force on object A?

Figure E.17. Question sequence (KR2 + KCR + EF2) in pretest 4 for PHYS 114B students if they answered the 2v4v-T task incorrectly the second time.

Results and discussion

The results from both lecture sections (PHYS 114A & B) in the Sp'19 quarter are shown in Figure E.18. We present the results from the first and final attempts of the actual task. The trends in the first attempt for both sections follow similar patterns in the Au'18 and Wi'19 quarter (Figure 14.9), which suggest that there is no significant difference in students from different quarters. The results from the final attempt in each pretest are generally higher than the first attempt, which mean that the sequences of static question do help students who had an incorrect initial response to answer correctly. The gains from the first attempt to final attempt are generally greater in the last two tasks (2v4v-F and 2v4v-T) than in the first two tasks (2v-NC and 2v4v-NC).

The greater gains mean that the question sequences that contain KR + EF improve student performance more effectively, which corroborated with what previous researchers have found. Lastly, if we compare the third and fourth tasks, the gain from the first attempt in 114A is less than that in 114B, which suggests that giving elaborated feedback through the tip-to-tail method produces better outcomes than asking students to add horizontal and vertical components. One reason is that we can present the explanation using the tip-to-tail method better than the vector components method. It may be possible that if we put in more effort to present the vector components method better, similar effectiveness can be achieved.

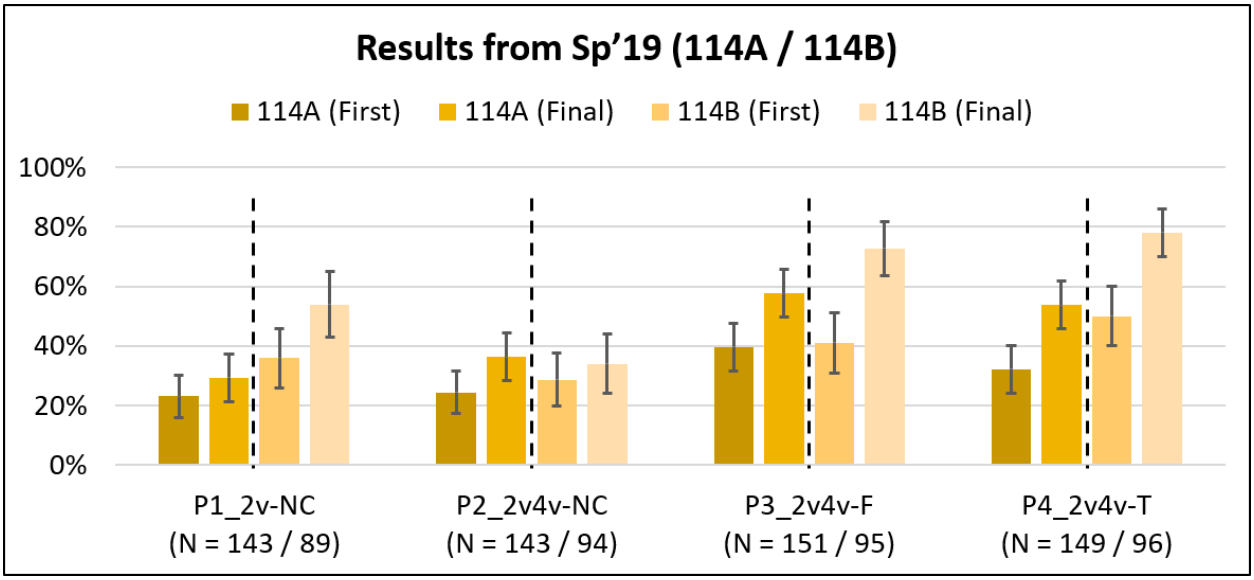


Figure E.18. Results from the Sp'19 quarter. In each pretest, the first two bars are results from 114A, and the next two bars are results from 114B. In each two-bar set, they are the results from the first and final attempts of the same task. Error bars indicate 2 SE.

We also tabulate the results and calculate the normalized gains for each question sequence in Table E.3. Normalized gain is defined as gain (result of final attempt – result of first attempt) divided by the maximum possible gain from first attempt [118]. Since students who answered the first attempt incorrectly receive the intervention, the normalized gain also represents how helpful

the question sequence is in guiding students to the right track. As discussed before, the KR + EF feedback approach produces higher normalized gains than the no-feedback approach. The explanation using the tip-to-tail method produces higher normalized gains than the vector components method.

Based on these results, our second intervention is the feedback approach in pretest 4 (KR1 + EF1 + KCR + KR2 + EF2) with the tip-to-tail explanation.

Table E.3. Results of the first and final attempts and normalized gains for question sequence in each pretest.

Pretest # (Feedback approach)	Results in 114A (Vector components)			Results in 114B (Limiting case & tip-to-tail)		
	First attempt	Final attempt	Normalized gain	First attempt	Final attempt	Normalized gain
P1 (no feedback)	23%	29%	0.08	36%	54%	0.28
P2 (no feedback)	24%	36%	0.16	29%	34%	0.07
P3 (KR + EF)	40%	58%	0.30	41%	73%	0.54
P4 (KR1 + EF1 + KR2 + KCR + EF2)	32%	54%	0.32	50%	78%	0.56

E4. The R codes used in Part IV of the dissertation

In this section, we present the R codes used in part IV of the dissertation. We used R for two purposes: (1) to perform statistical analysis in the longitudinal studies (generalized linear mixed model) and (2) to perform text analysis in comparing student explanations.

(A) The R code for the statistical analysis for the vector superposition studies in Chapter 14

11X Vector Superposition GLMM notes

Introduction

This document is to describe the longitudinal data analysis I have conducted in R. It includes the R codes I used to perform the statistical analysis and generate the figures presented in my dissertation. Future readers can refer to this document to get a headstart.

Loading relevant packages and datafiles

Packages

```
library(xlsx)
library(ggalluvial)
library(dplyr)
library(lme4)
library(optimx)
library(xtable)
library(ggplot2)
library(knitr)
```

The details of what each package can do can be found on the internet.

Here are the short descriptions of what these packages can do:

Package	Short description
ggalluvial	Generate Sankey plots
dplyr	Tools to manipulate dataframes in r
lme4	Statistical package to perform longitudinal data analysis
ggplot2	Plot graphics
xtable	Generate tables
optimx	Optimizer for longitudinal data analysis

Datafiles

```
setwd("D:/Dropbox/UW Physics/Research/Thesis/Word Docs/R Markdown/Vectors")
phys114 <- read.csv("longiv7.csv")
phys114_time <- read.csv("longiv8-time.csv")
phys114_int <- read.csv("longiv9-int.csv")
```

Context of investigation and study design

The context of investigation and study design can be found in Chapter 14.

Descriptive Statistics

Here are some Sankey plots for the results obtained in the Au'19 quarter. They show how student responses change from one question to another. It only shows students who participated in all tasks.

1

Figure E.19. The R code for the statistical analysis (generalized linear mixed model) in Chapter 14 (11 pages).

The first two plots are for all students. The third and fourth plots are for students who learned vectors prior to PHYS 114, and the last two are for students who have not learned vectors before registering for the course.

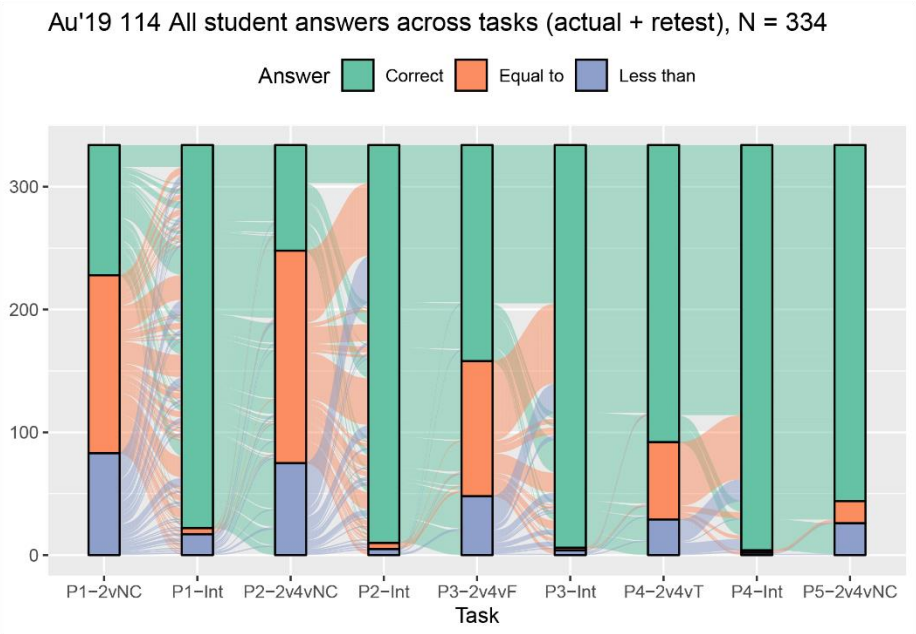
In each pair of plots, the first plot shows student responses on the actual task and the retest after feedback. Students who are correct on the actual task are assumed to be correct on the retest task. The second plot shows the student responses change from one actual task to another actual task.

The actual and the retest tasks are the same. Those tasks are the vector superposition tasks presented in Chapter 14. Readers may want to read this part with Chapter 14 concurrently.

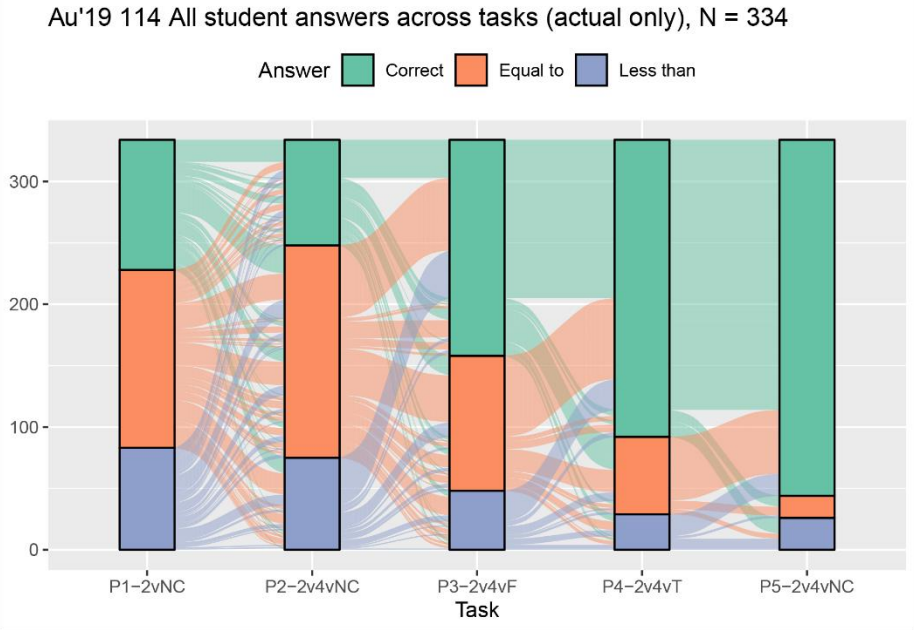
```
a1 <- read.xlsx("194 Vector alluvial.xlsx", sheetName = "114All")
a2 <- read.xlsx("194 Vector alluvial.xlsx", sheetName = "114All2")
a3 <- read.xlsx("194 Vector alluvial.xlsx", sheetName = "114VecY")
a4 <- read.xlsx("194 Vector alluvial.xlsx", sheetName = "114VecY2")
a5 <- read.xlsx("194 Vector alluvial.xlsx", sheetName = "114VecN")
a6 <- read.xlsx("194 Vector alluvial.xlsx", sheetName = "114VecN2")
```

Sankey plots for Au'19 PHYS 114 students (all)

```
ggplot(a1,
       aes(x = Task, stratum = Answer, alluvium = Student,
           fill = Answer, label = Answer)) +
  scale_fill_brewer(type = "alluvium", palette = "Set2") +
  geom_flow(stat = "alluvium", lode.guidance = "rightleft") +
  geom_stratum() +
  theme(legend.position = "top") +
  ggtitle("Au'19 114 All student answers across tasks (actual + retest), N = 334")
```

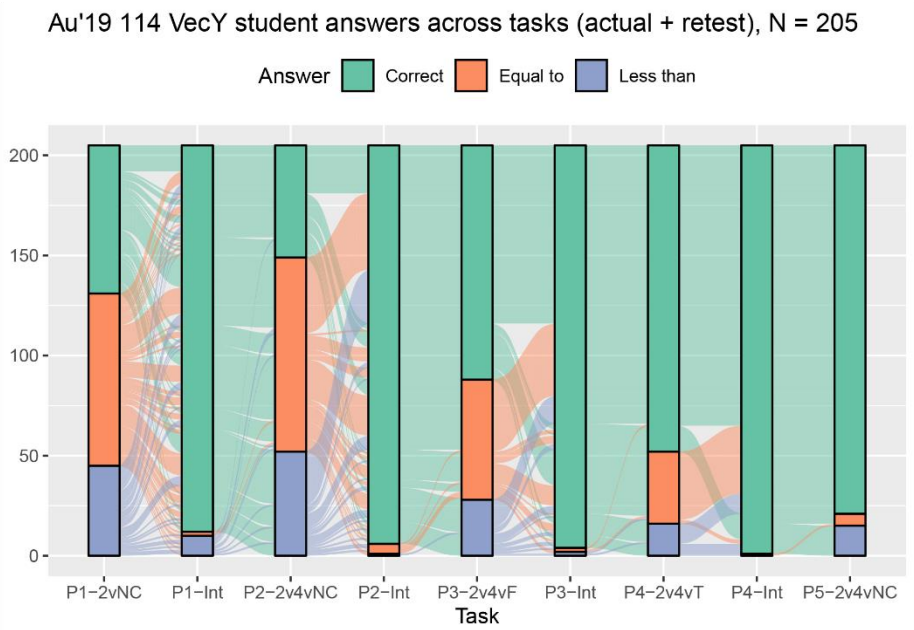


```
ggplot(a2,
       aes(x = Task, stratum = Answer, alluvium = Student,
           fill = Answer, label = Answer)) +
  scale_fill_brewer(type = "alluvium", palette = "Set2") +
  geom_flow(stat = "alluvium", lode.guidance = "rightleft") +
  geom_stratum() +
  theme(legend.position = "top") +
  ggtitle("Au'19 114 All student answers across tasks (actual only), N = 334")
```

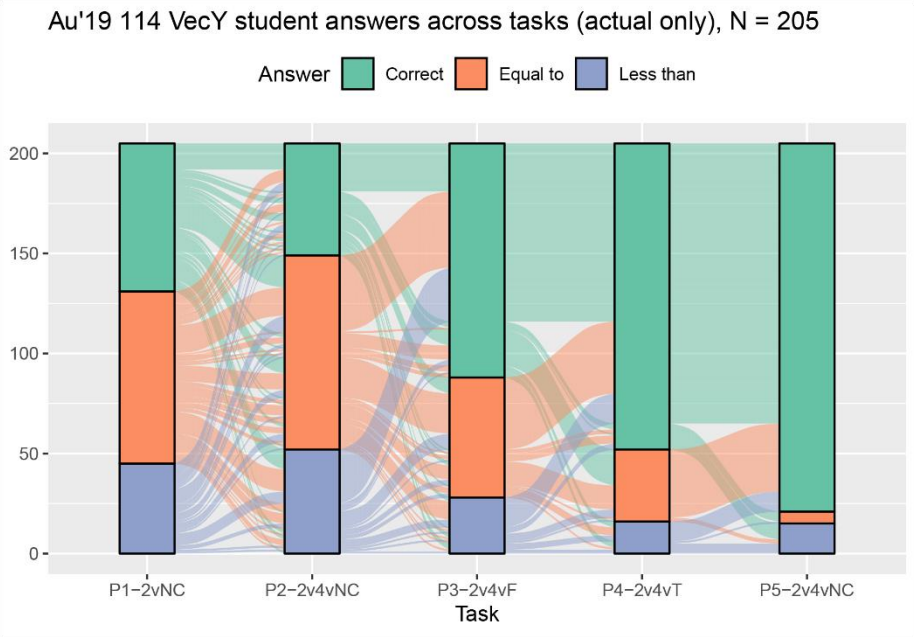


Sankey plots for Au'19 PHYS 114 students (Learned vectors before PHYS 114)

```
ggplot(a3,
       aes(x = Task, stratum = Answer, alluvium = Student,
           fill = Answer, label = Answer)) +
  scale_fill_brewer(type = "alluvium", palette = "Set2") +
  geom_flow(stat = "alluvium", lode.guidance = "rightleft") +
  geom_stratum() +
  theme(legend.position = "top") +
  ggtitle("Au'19 114 VecY student answers across tasks (actual + retest), N = 205")
```

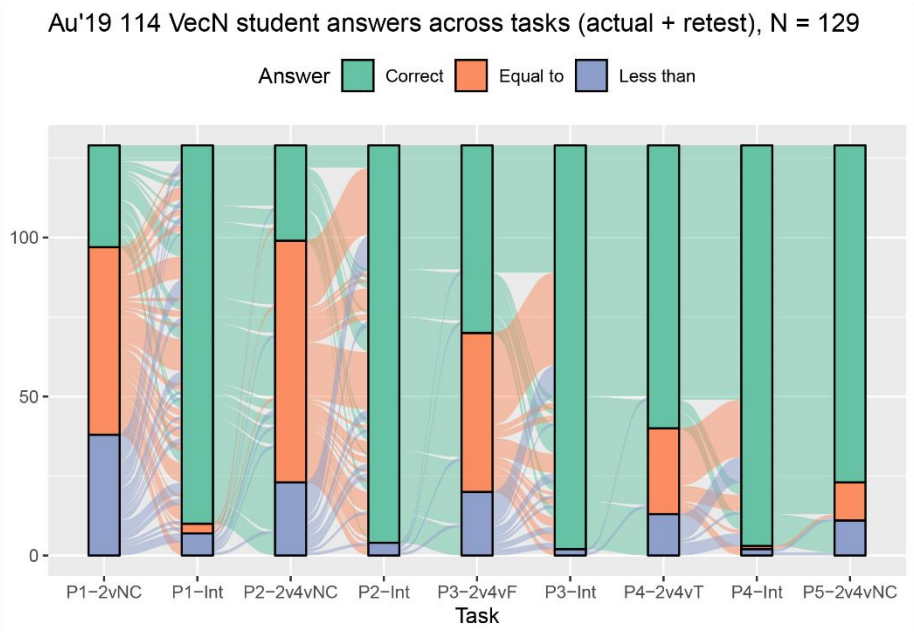



```
ggplot(a4,
       aes(x = Task, stratum = Answer, alluvium = Student,
           fill = Answer, label = Answer)) +
  scale_fill_brewer(type = "alluvium", palette = "Set2") +
  geom_flow(stat = "alluvium", lode.guidance = "rightleft") +
  geom_stratum() +
  theme(legend.position = "top") +
  ggtitle("Au'19 114 VecY student answers across tasks (actual only), N = 205")
```

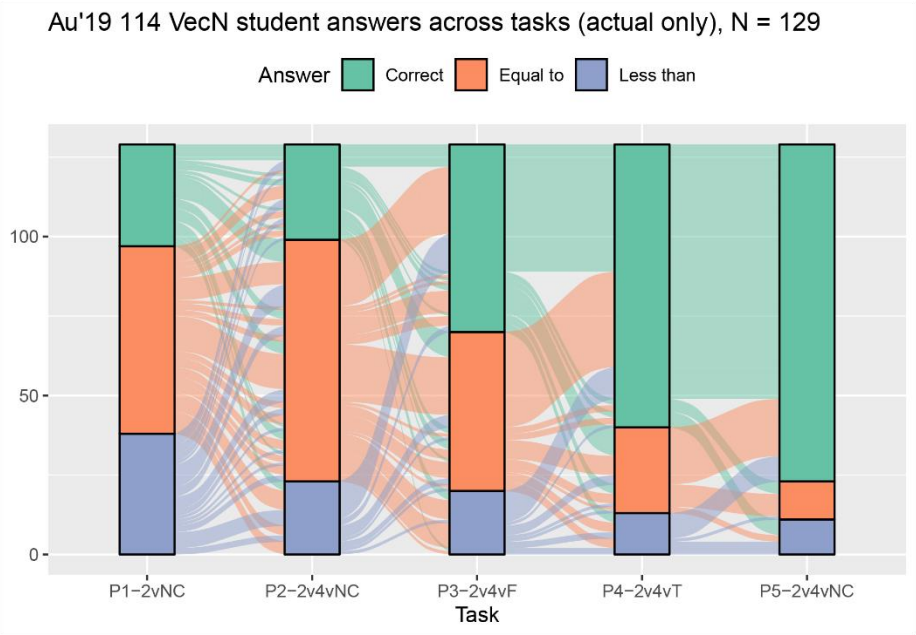


Sankey plots for Au'19 PHYS 114 students (Not learned vectors before PHYS 114)

```
ggplot(a5,
       aes(x = Task, stratum = Answer, alluvium = Student,
           fill = Answer, label = Answer)) +
  scale_fill_brewer(type = "alluvium", palette = "Set2") +
  geom_flow(stat = "alluvium", lode.guidance = "rightleft") +
  geom_stratum() +
  theme(legend.position = "top") +
  ggtitle("Au'19 114 VecN student answers across tasks (actual + retest), N = 129")
```




```
ggplot(a6,
       aes(x = Task, stratum = Answer, alluvium = Student,
           fill = Answer, label = Answer)) +
  scale_fill_brewer(type = "alluvium", palette = "Set2") +
  geom_flow(stat = "alluvium", lode.guidance = "rightleft") +
  geom_stratum() +
  theme(legend.position = "top") +
  ggtitle("Au'19 114 VecN student answers across tasks (actual only), N = 129")
```



Statistical modeling

In this section, we outline the code for the statistical modeling (generalized linear mixed model) we used.

Some description of the generalized linear mixed model can be found in Appendix A.

We tried these models in our analysis:

• Model 1: Regressing the outcome on result of pretest 1 (PT1); • Model 2: Regressing the outcome on students' background and correctness of PT1; • Model 3: Regressing the outcome on students' background, correctness of PT1, and pretests (2, 3, and 4, coded as categorical variable); • Model 4: Regressing the outcome on intervention (coded as categorical variable, 1 as intervention A, 2 as intervention B), students' background, and pretests, and the interaction of previous factors, and correctness of PT1; • Model 5: Regressing the outcome on intervention, pretests and the interaction of the previous two factors, students' background, and correctness of PT1

```
#Models tried

glmm_mod1 <- glmer(Answer ~ PT1 + (1|Number/Section),
                   data = phys114,
                   family = binomial)

glmm_mod2 <- glmer(Answer ~ PT1 + VectorYN + (1|Number/Section),
                   data = phys114,
                   family = binomial)

glmm_mod3 <- glmer(Answer ~ as.factor(Pretest) + PT1 + VectorYN + (1|Number/Section),
                   data = phys114,
```

```
                  family = binomial)

glmm_mod4 <- glmer(Answer ~ as.factor(Intervention) * VectorYN * as.factor(Pretest) +
                     PT1 + (1|Number/Section),
                   data = phys114,
                   family = binomial)

glmm_mod5 <- glmer(Answer ~ VectorYN + as.factor(Pretest) * as.factor(Intervention) +
                     PT1 + (1|Number/Section),
                   data = phys114,
                   family = binomial,
                   control = glmerControl(optimizer = "optimx",
                                          optCtrl = list(method = "nlminb")))
```

We use the 'anova' command to compare models.

```
anova(glmm_mod1, glmm_mod2, glmm_mod3, glmm_mod4, glmm_mod5)
```

```
## Data: phys114
## Models:
## glmm_mod1: Answer ~ PT1 + (1 | Number/Section)
## glmm_mod2: Answer ~ PT1 + VectorYN + (1 | Number/Section)
## glmm_mod3: Answer ~ as.factor(Pretest) + PT1 + VectorYN + (1 | Number/Section)
## glmm_mod5: Answer ~ VectorYN + as.factor(Pretest) * as.factor(Intervention) +
## glmm_mod5:     PT1 + (1 | Number/Section)
## glmm_mod4: Answer ~ as.factor(Intervention) * VectorYN * as.factor(Pretest) +
## glmm_mod4:     PT1 + (1 | Number/Section)
##           npar    AIC    BIC  logLik deviance    Chisq Df Pr(>Chisq)
## glmm_mod1    4 3938.3 3962.5 -1965.2   3930.3
## glmm_mod2    5 3934.6 3964.8 -1962.3   3924.6   5.7296  1    0.01668 *
## glmm_mod3    7 3630.5 3672.8 -1808.3   3616.5 308.0607  2    < 2e-16 ***
## glmm_mod5   13 3473.9 3552.5 -1724.0   3447.9 168.6132  6    < 2e-16 ***
## glmm_mod4   21 3480.6 3607.5 -1719.3   3438.6   9.2678  8    0.32021
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The way to judge how well the models are is to look at the AIC, BIC, logLik, and deviance. The rule of thumb is better models should have lower AIC, BIC, and deviance. For logLik, the higher the better. The Pr(>chisq) is p-value of the likelihood ratio test when comparing the consecutive models. The models are arranged in ascending order of degrees of freedom.

In our case, Model 5 has the lowest AIC and BIC. While Model 4 has higher logLik and lower deviance than Model 5, the likelihood ratio test shows that Model 4 is not significantly different from Model 5, so we choose Model 5 since it has less degrees of freedom.

Here is the table of the coefficients for model 5.

```
mod_coef <- summary(glmm_mod5)$coefficients %>% as.data.frame

mod_coef$CI <- paste("(", round(mod_coef[, 1] - qnorm(0.975) * mod_coef[, 2], 3),
                     ", ", round(mod_coef[, 1] + qnorm(0.975) * mod_coef[, 2], 3), ")",
                     sep = "")
mod_tab <- data.frame(Estimate = round(mod_coef$Estimate, 3),
                      Std.Err = round(mod_coef$`Std. Error`, 3),
                      "95% CI" = mod_coef$CI,
                      "P-value" = round(mod_coef$`Pr(>|z|)`, 3))
```

```
row.names(mod_tab) <- row.names(mod_coef)

kable(mod_tab, digits = 3)
```

	Estimate	Std.Err	X95..CI	P.value
(Intercept)	-2.972	0.185	(-3.335, -2.61)	0.000
VectorYN	0.282	0.113	(0.06, 0.503)	0.013
as.factor(Pretest)3	1.621	0.176	(1.275, 1.967)	0.000
as.factor(Pretest)4	1.555	0.177	(1.209, 1.902)	0.000
as.factor(Intervention)2	0.495	0.355	(-0.201, 1.191)	0.164
as.factor(Intervention)3	1.185	0.195	(0.803, 1.567)	0.000
PT1	1.148	0.117	(0.918, 1.378)	0.000
as.factor(Pretest)3:as.factor(Intervention)2	-0.174	0.448	(-1.051, 0.704)	0.698
as.factor(Pretest)4:as.factor(Intervention)2	0.048	0.457	(-0.848, 0.944)	0.916
as.factor(Pretest)3:as.factor(Intervention)3	-0.233	0.237	(-0.697, 0.231)	0.325
as.factor(Pretest)4:as.factor(Intervention)3	0.717	0.243	(0.241, 1.193)	0.003

The intercept represents the log odds of pretest 2. The negative sign means that the probability of getting pretest 2 correct is less than 0.5. The point estimates represent the log odds ratio.

For example, the point estimate for the variable VectorYN is 0.282 and it is statistically significant (p-value = 0.013), while holding all other variables constant. The log-odds-ratio of a student getting pretest 2 correct versus incorrect if the student learned vector before PHYS 114 versus not learned vector before PHYS 114 is 0.282. In pretest 2, out of the 1090 students, 688 learned vectors before PHYS 114. Among the 688 students, 139 answered the actual task on pretest 2 correctly. Among the 402 students who did not learn vectors, 66 of them answered the actual task correctly. The odds ratio is then (139/549)/(66/336) = 1.29. The point estimate, after exponentiate, e^(0.282) = 1.33.

The rest of the point estimates are interpreted the same way as well.

Missing data

Here is the code to deal with missing data in our longitudinal data.

To evaluate the impact of missing data on the results of the analysis, we performed sensitivity analysis by repeating the analysis with two different imputation strategies under "worst-case scenarios" using the best model from the model selection:

• Strategy 1: Impute "correct" for the missing results in pretests 1 and 2 and "incorrect" for the missing results in pretests 3 and 4;

• Strategy 2: Impute "correct" for the missing results of students who did not receive the intervention and "incorrect" for those who did.

Results from Strategy 1

```
phys114_time <- read.csv("longiv8-time.csv")

glmm_mi1 <- glmer(as.factor(Answer) ~ VectorYN + PT1 + (1|Number/Section) +
                    as.factor(Pretest) * as.factor(Intervention) ,
                  data = phys114_time,
                  family = binomial,
                  control = glmerControl(optimizer = "optimx",
                                         optCtrl = list(method = "nlminb")))
```

```r
mi1_coef <- summary(glmm_mi1)$coefficients %>% as.data.frame

mi1_coef$CI <- paste("(", round(mi1_coef[, 1] - qnorm(0.975) * mi1_coef[, 2], 3),
                     ", ", round(mi1_coef[, 1] + qnorm(0.975) * mi1_coef[, 2], 3), ")",
                     sep = "")
mi1_tab <- data.frame(Estimate = round(mi1_coef$Estimate, 3),
                      Std.Err = round(mi1_coef$`Std. Error`, 3),
                      "95% CI" = mi1_coef$CI,
                      "P-value" = round(mi1_coef$`Pr(>|z|)`, 3))

row.names(mi1_tab) <- row.names(mi1_coef)

kable(mi1_tab, digits = 3)
```

	Estimate	Std.Err	X95..CI	P.value
(Intercept)	-2.815	0.168	(-3.145, -2.485)	0.000
VectorYN	0.261	0.102	(0.06, 0.462)	0.011
PT1	0.951	0.104	(0.746, 1.155)	0.000
as.factor(Pretest)3	1.337	0.161	(1.021, 1.654)	0.000
as.factor(Pretest)4	1.264	0.162	(0.947, 1.581)	0.000
as.factor(Intervention)2	0.372	0.323	(-0.261, 1.005)	0.250
as.factor(Intervention)3	1.415	0.181	(1.061, 1.769)	0.000
as.factor(Pretest)3:as.factor(Intervention)2	-0.093	0.397	(-0.87, 0.684)	0.814
as.factor(Pretest)4:as.factor(Intervention)2	-0.241	0.399	(-1.022, 0.54)	0.546
as.factor(Pretest)3:as.factor(Intervention)3	-0.500	0.219	(-0.928, -0.071)	0.022
as.factor(Pretest)4:as.factor(Intervention)3	0.301	0.221	(-0.132, 0.733)	0.173

Results from Strategy 2

```r
phys114_int <- read.csv("longiv9-int.csv")

glmm_mi2 <- glmer(as.factor(Answer) ~ VectorYN + PT1 + (1|Number/Section) +
                    as.factor(Pretest) * as.factor(Intervention) ,
                  data = phys114_int,
                  family = binomial,
                  control = glmerControl(optimizer = "optimx",
                                         optCtrl = list(method = "nlminb")))

mi2_coef <- summary(glmm_mi2)$coefficients %>% as.data.frame

mi2_coef$CI <- paste("(", round(mi2_coef[, 1] - qnorm(0.975) * mi2_coef[, 2], 3),
                     ", ", round(mi2_coef[, 1] + qnorm(0.975) * mi2_coef[, 2], 3), ")",
                     sep = "")
mi2_tab <- data.frame(Estimate = round(mi2_coef$Estimate, 3),
                      Std.Err = round(mi2_coef$`Std. Error`, 3),
                      "95% CI" = mi2_coef$CI,
                      "P-value" = round(mi2_coef$`Pr(>|z|)`, 3))

row.names(mi2_tab) <- row.names(mi2_coef)

kable(mi2_tab, digits = 3)
```

	Estimate	Std.Err	X95..CI	P.value
(Intercept)	-2.772	0.165	(-3.096, -2.448)	0.000
VectorYN	0.230	0.100	(0.035, 0.426)	0.021
PT1	0.947	0.102	(0.747, 1.146)	0.000
as.factor(Pretest)3	1.840	0.161	(1.525, 2.156)	0.000
as.factor(Pretest)4	1.824	0.161	(1.509, 2.139)	0.000
as.factor(Intervention)2	0.513	0.322	(-0.117, 1.143)	0.111
as.factor(Intervention)3	1.033	0.182	(0.676, 1.39)	0.000
as.factor(Pretest)3:as.factor(Intervention)2	-0.599	0.395	(-1.374, 0.175)	0.129
as.factor(Pretest)4:as.factor(Intervention)2	-0.802	0.397	(-1.581, -0.024)	0.043
as.factor(Pretest)3:as.factor(Intervention)3	-0.642	0.220	(-1.073, -0.211)	0.004
as.factor(Pretest)4:as.factor(Intervention)3	0.099	0.221	(-0.335, 0.533)	0.654

(B) The R code for the computational text analysis to compare student explanations in Chapter 15

R code for text analysis

Setting up

This chunk of code loads the packages, imports the relevant files, and merge some datasets.

```
library(xlsx)
library(quanteda)
library(stringr)
library(stringi)
library(dplyr)
library(data.table)
library(ggplot2)
library(reshape2)
library(widyr)
library(tidytext)

#Change this line to after you download the files for your own purpose
setwd("D:/Dropbox/UW Physics/Research/Rolling/AY1819 Q3 Spring/For R")

m2d <- read.xlsx("M2DRnSAll.xlsx", 1, sheetName = "M2D", stringsAsFactors = FALSE)
rns <- read.xlsx("M2DRnSAll.xlsx", 1, sheetName = "RnS", stringsAsFactors = FALSE)

m2d$M2DType <- as.factor(m2d$M2DType)
m2d$M2D_Cat <- as.factor(m2d$M2D_Cat)
rns$RnS_Cat <- as.factor(rns$RnS_Cat)
m2drns <- merge(m2d,rns,by = "Username")

m2drns_noC <- subset(m2drns, m2drns$M2DType == "No Cyl")
m2drns_wC <- subset(m2drns, m2drns$M2DType == "With Cyl")

m2d_nocylB <- m2d[which(m2d$M2D_Cat == 'Correct' &
                        m2d$M2DType == 'No Cyl'),]
m2d_wcylB <- m2d[which(m2d$M2D_Cat == 'Correct' &
                       m2d$M2DType == 'With Cyl'),]
m2d_wcylD <- m2d[which(m2d$M2D_Cat == 'CM frame' &
                       m2d$M2DType == 'With Cyl'),]
rnsRM <- rns[which(rns$RnS_Cat == 'Correct'),]
rnsCM <- rns[which(rns$RnS_Cat == 'CM frame'),]

nocylB_rnsRM <- m2drns_noC[which(m2drns_noC$M2D_Cat == 'Correct' &
                                 m2drns_noC$RnS_Cat == 'Correct'),]
nocylB_rnsCM <- m2drns_noC[which(m2drns_noC$M2D_Cat == 'Correct' &
                                 m2drns_noC$RnS_Cat == 'CM frame'),]
wcylB_rnsRM <- m2drns_wC[which(m2drns_wC$M2D_Cat == 'Correct' &
                               m2drns_wC$RnS_Cat == 'Correct'),]
wcylB_rnsCM <- m2drns_wC[which(m2drns_wC$M2D_Cat == 'Correct' &
                               m2drns_wC$RnS_Cat == 'CM frame'),]
wcylD_rnsCM <- m2drns_wC[which(m2drns_wC$M2D_Cat == 'CM frame' &
```

1

Figure E.20. The R code for computational text analysis to compare student explanations (8 pages).

```
                                m2drns_wC$RnS_Cat == 'CM frame'),]
c1 <- rbind(nocylB_rnsRM, wcylB_rnsRM)
c2 <- rbind(nocylB_rnsCM, wcylB_rnsCM)

# New stopwords for 2-gram and higher
stpwd2 <- stopwords()
stpwd2 <- stpwd2[!stpwd2 %in% c("to")]
```

Defining some functions

The codes below define two functions we used in this text analysis. The first one is for cleaning text, the second one is to generate text frequency.

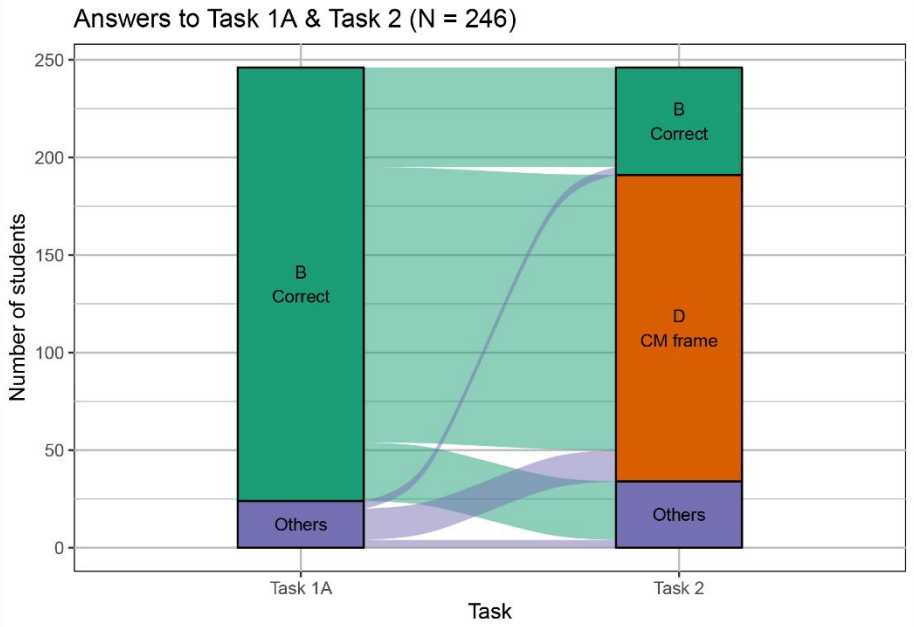
```
#Cleaning the text files, modify if necessary
Clean_String <- function(string){

  #To standardize all apostrophes
  temp <- gsub("\u2019", "'", string)

  #Remove numbers, punctuations except apostrophe, non-graphical characters
  temp <- str_replace_all(temp, "[^[a-zA-Z]['-]\\s]", "")

  temp <- tolower(temp)
  temp <- str_replace_all(temp, "[\\s]+", " ")
  return(temp)
}
```

```
#Generate text frequency
mywordfreq <- function(dfm){

  #Generate text frequency
  temp <- textstat_frequency(dfm) %>%
    mutate(tf = frequency/sum(frequency)) %>%
    mutate(df = docfreq/nrow(dfm)) %>%
    mutate(idf = log(1/df)) %>%
    mutate(tfidf = round(tf*idf*100,2))

  return(temp)
}
```

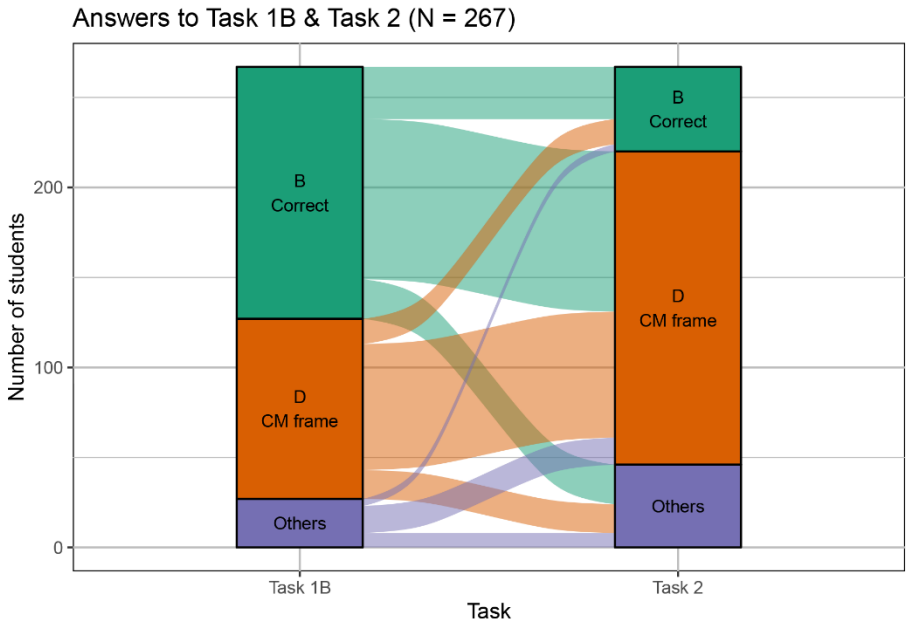
Results presented in Sankey diagrams

```
library(networkD3)
library(ggalluvial)

a1 <- read.xlsx("M2DRnSAll.xlsx", sheetName = "Task1A-2")
a2 <- read.xlsx("M2DRnSAll.xlsx", sheetName = "Task1B-2")

ggplot(a1,
       aes(x = Task, stratum = Answer, alluvium = Student,
           fill = Answer, label = Answer)) +
  ylab("Number of students") +
  scale_fill_brewer(type = "qual", palette = "Dark2") +
  geom_flow() +
```

```
  theme(panel.background = element_rect(fill = "white", colour = "black"),
        panel.grid.major = element_line(colour="grey"),
        panel.grid.minor = element_line(colour="grey")) +
  geom_stratum() +
  geom_text(stat = "stratum", size = 3) +
  theme(legend.position = "none") +
  ggtitle("Answers to Task 1A & Task 2 (N = 246)")
```




```
ggplot(a2,
       aes(x = Task, stratum = Answer, alluvium = Student,
           fill = Answer, label = Answer)) +
  ylab("Number of students") +
  scale_fill_brewer(type = "qual", palette = 2) +
  geom_flow() +
  theme(panel.background = element_rect(fill = "white", colour = "black"),
        panel.grid.major = element_line(colour="grey"),
        panel.grid.minor = element_line(colour="grey")) +
  geom_stratum() +
  geom_text(stat = "stratum", size = 3) +
  theme(legend.position = "none") +
  ggtitle("Answers to Task 1B & Task 2 (N = 267)")
```

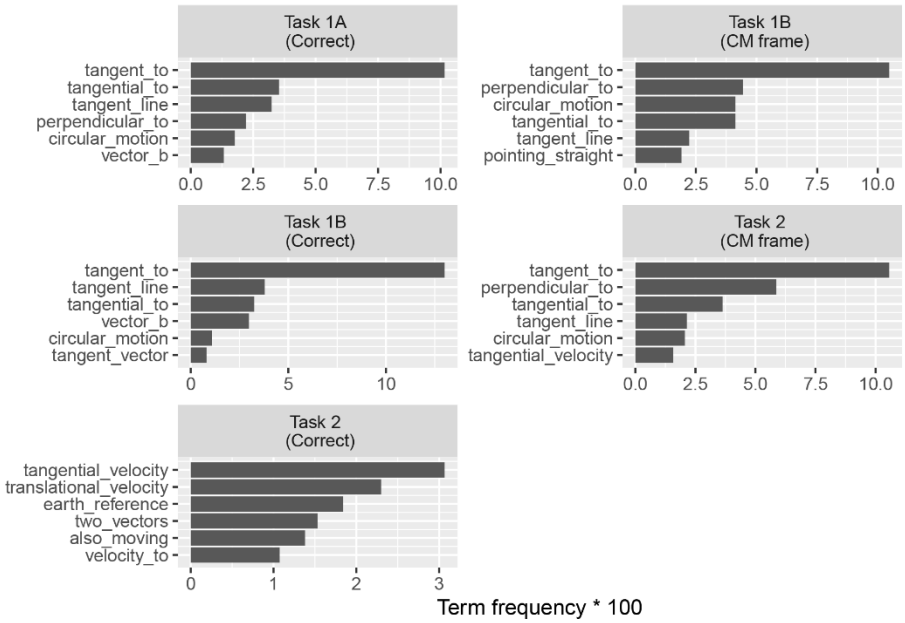


Top 6 frequently used 2-word phrases

This chunk of code shows how we generate the top 6 frequently used 2-word phrases in student explanations to the answers.

```
#Generate corpus
nocylB_corpus <- corpus(Clean_String(m2d_nocylB$M2DExp))
wcylB_corpus <- corpus(Clean_String(m2d_wcylB$M2DExp))
wcylD_corpus <- corpus(Clean_String(m2d_wcylD$M2DExp))
rnsCM_corpus <- corpus(Clean_String(rnsCM$RnSExp))
rnsRM_corpus <- corpus(Clean_String(rnsRM$RnSExp))

#Finding frequently used 2-word phrase (Top 6) by tf
nocyl_2gdfm <- tokens(nocylB_corpus) %>%
  tokens_remove(c(stpwd2), padding = TRUE) %>%
  tokens_ngrams(n = 2) %>%
  dfm(remove = c("instantaneous_velocity", "point_m", "velocity_vector"))
nocylB_2gfreq <- mywordfreq(nocyl_2gdfm) %>%
  mutate(source = "Task 1A \n (Correct)")
nocylB_2gfreq2 <- head(nocylB_2gfreq[order(-nocylB_2gfreq$df),], 6)

wcylB_2gdfm <- tokens(wcylB_corpus) %>%
  tokens_remove(c(stpwd2, "red", "black"), padding = TRUE) %>%
  tokens_ngrams(n = 2) %>%
  dfm(remove = c("instantaneous_velocity", "point_q", "velocity_vector", "line_to",
                 "time_t", "option_b"))
```

```
wcylB_2gfreq <- mywordfreq(wcylB_2gdfm) %>%
  mutate(source = "Task 1B \n (Correct)")
wcylB_2gfreq2 <- head(wcylB_2gfreq[order(-wcylB_2gfreq$df),], 6)

wcylD_2gdfm <- tokens(wcylD_corpus) %>%
  tokens_remove(c(stpwd2), padding = TRUE) %>%
  tokens_ngrams(n = 2) %>%
  dfm(remove = c("instantaneous_velocity", "point_q", "velocity_vector"))
wcylD_2gfreq <- mywordfreq(wcylD_2gdfm) %>%
  mutate(source = "Task 1B \n (CM frame)")
wcylD_2gfreq2 <- head(wcylD_2gfreq[order(-wcylD_2gfreq$df),], 6)

rnsCM_2gdfm <- tokens(rnsCM_corpus) %>%
  tokens_remove(c(stpwd2), padding = TRUE) %>%
  tokens_ngrams(n = 2) %>%
  dfm(remove = c("instantaneous_velocity", "point_e", "velocity_vector",
                 "reference_frame"))
rnsCM_2gfreq <- mywordfreq(rnsCM_2gdfm) %>%
  mutate(source = "Task 2 \n (CM frame)")
rnsCM_2gfreq2 <- head(rnsCM_2gfreq[order(-rnsCM_2gfreq$df),], 6)

rnsRM_2gdfm <- tokens(rnsRM_corpus) %>%
  tokens_remove(c(stpwd2, "due"), padding = TRUE) %>%
  tokens_ngrams(n = 2) %>%
  dfm(remove = c("instantaneous_velocity", "point_e", "velocity_vector", "moving_to",
                 "reference_frame"))
rnsRM_2gfreq <- mywordfreq(rnsRM_2gdfm) %>%
  mutate(source = "Task 2 \n (Correct)")
rnsRM_2gfreq2 <- head(rnsRM_2gfreq[order(-rnsRM_2gfreq$df),], 6)

#To plot top 6 common 2-word phrase by tf
tbl2 <- rbind(nocylB_2gfreq2, wcylB_2gfreq2, wcylD_2gfreq2, rnsCM_2gfreq2, rnsRM_2gfreq2)
ggplot(data = tbl2, aes(x = nrow(tbl2):1, y = tf*100)) +
  geom_col(show.legend = FALSE) +
  facet_wrap(~ source, scales = "free", ncol = 2) +
  coord_flip() +
  scale_x_continuous(breaks = nrow(tbl2):1,
  labels = tbl2$feature) +
  labs(x = NULL, y = "Term frequency * 100")
```



Computing cosine similarity between 2-word phrases used in student explanations

There are two ways we compute the cosine similarity.

The first way is to look at the 2-word phrases used in the explanations to the answers. For example, we compare the 2-word phrases used in the explanations in answer A in question 1 to the 2-word phrases used in the explanations in answer A in question 2.

The second way is to first select students who choose answer A in question 1 and answer B in question 2. Then, we compare the two-word phrases they used in their explanations to answer A in question 1 and answer B in question 2.

First way

```
#To calculate cosine similarity between 2-word phrases used in the explanations to each answer
tbl4 <- rbind(nocylB_2gfreq, wcylB_2gfreq, wcylD_2gfreq, rnsRM_2gfreq, rnsCM_2gfreq)
closest2g <- pairwise_similarity(tbl4, source, feature, frequency) %>% as.data.frame()
closest2g$similarity <- round(closest2g$similarity,3)
simi2g <- reshape(closest2g, direction="wide", idvar="item1", timevar="item2")
simi2g <- simi2g[order(simi2g$item1),]
simi2g <- simi2g[,order(names(simi2g))]
colnames(simi2g) <- gsub('similarity.', '', colnames(simi2g))
simi2g$item1 <- gsub('\n', '', simi2g$item1)
simi2g
```

```
##                 item1 Task 1A \n (Correct) Task 1B \n (CM frame)
```

```
## 5  Task 1A  (Correct)                  NA                0.844
## 2 Task 1B  (CM frame)               0.844                   NA
## 1  Task 1B  (Correct)               0.892                0.778
## 4  Task 2  (CM frame)               0.864                0.873
## 3   Task 2  (Correct)               0.246                0.262
##   Task 1B \n (Correct) Task 2 \n (CM frame) Task 2 \n (Correct)
## 5                0.892               0.864                0.246
## 2                0.778               0.873                0.262
## 1                   NA               0.804                0.211
## 4                0.804                  NA                0.352
## 3                0.211               0.352                   NA
```

Second way

```
#Method to calculate similarity between 2-word phrases used in the explanations by
#students who selected the same combination of answers
T1Co_T2Co_m2d <- corpus(Clean_String(c1$M2DExp))
T1Co_T2Co_rns <- corpus(Clean_String(c1$RnSExp))
T1BCM_T2CM_m2d <- corpus(Clean_String(wcylD_rnsCM$M2DExp))
T1BCM_T2CM_rns <- corpus(Clean_String(wcylD_rnsCM$RnSExp))
T1Co_T2CM_m2d <- corpus(Clean_String(c2$M2DExp))
T1Co_T2CM_rns <- corpus(Clean_String(c2$RnSExp))

T1CoT2Com2d_2gdfm <- tokens(T1Co_T2Co_m2d) %>%
  tokens_remove(c(stpwd2), padding = TRUE) %>%
  tokens_ngrams(n = 2) %>%
  dfm(remove = c("instantaneous_velocity", "point_m", "velocity_vector", "point_q"))
T1CoT2Com2d_2gfreq <- mywordfreq(T1CoT2Com2d_2gdfm) %>%
  mutate(source = "T1-Correct")

T1CoT2Corns_2gdfm <- tokens(T1Co_T2Co_rns) %>%
  tokens_remove(c(stpwd2), padding = TRUE) %>%
  tokens_ngrams(n = 2) %>%
  dfm(remove = c("instantaneous_velocity", "point_e", "reference_frame",
                 "velocity_vector", "earth_reference"))
T1CoT2Corns_2gfreq <- mywordfreq(T1CoT2Corns_2gdfm) %>%
  mutate(source = "T2-Correct")
T1CoT2Co <- rbind(T1CoT2Com2d_2gfreq, T1CoT2Corns_2gfreq)
T1CoT2Co2g_cossim <- pairwise_similarity(T1CoT2Co, source, feature, frequency)

T1CoT2Co2g_cossim
```

```
## # A tibble: 2 x 3
##   item1      item2      similarity
##   <chr>      <chr>           <dbl>
## 1 T2-Correct T1-Correct      0.154
## 2 T1-Correct T2-Correct      0.154
```

```
T1BCMT2CMm2d_2gdfm <- tokens(T1BCM_T2CM_m2d) %>%
  tokens_remove(c(stpwd2), padding = TRUE) %>%
  tokens_ngrams(n = 2) %>%
  dfm(remove = c("instantaneous_velocity", "point_q", "velocity_vector"))
T1BCMT2CMm2d_2gfreq <- mywordfreq(T1BCMT2CMm2d_2gdfm) %>%
  mutate(source = "T1B-CM")
```

```
T1BCMT2CMrns_2gdfm <- tokens(T1BCM_T2CM_rns) %>%
  tokens_remove(c(stpwd2), padding = TRUE) %>%
  tokens_ngrams(n = 2) %>%
  dfm(remove = c("instantaneous_velocity", "point_e", "reference_frame",
                 "velocity_vector", "earth_reference"))
T1BCMT2CMrns_2gfreq <- mywordfreq(T1BCMT2CMrns_2gdfm) %>%
  mutate(source = "T2-CM")
T1BCMT2CM <- rbind(T1BCMT2CMm2d_2gfreq, T1BCMT2CMrns_2gfreq)
T1BCMT2CM2g_cossim <- pairwise_similarity(T1BCMT2CM, source, feature, frequency)

T1BCMT2CM2g_cossim
```

```
## # A tibble: 2 x 3
##   item1  item2  similarity
##   <chr>  <chr>       <dbl>
## 1 T2-CM  T1B-CM      0.782
## 2 T1B-CM T2-CM       0.782
```

```
T1CoT2CMm2d_2gdfm <- tokens(T1Co_T2CM_m2d) %>%
  tokens_remove(c(stpwd2), padding = TRUE) %>%
  tokens_ngrams(n = 2) %>%
  dfm(remove = c("instantaneous_velocity", "point_m", "velocity_vector", "point_q"))
T1CoT2CMm2d_2gfreq <- mywordfreq(T1CoT2CMm2d_2gdfm) %>%
  mutate(source = "T1-Correct")

T1CoT2CMrns_2gdfm <- tokens(T1Co_T2CM_rns) %>%
  tokens_remove(c(stpwd2), padding = TRUE) %>%
  tokens_ngrams(n = 2) %>%
  dfm(remove = c("instantaneous_velocity", "point_e", "reference_frame",
                 "velocity_vector", "earth_reference"))
T1CoT2CMrns_2gfreq <- mywordfreq(T1CoT2CMrns_2gdfm) %>%
  mutate(source = "T2-CM")
T1CoT2CM <- rbind(T1CoT2CMm2d_2gfreq, T1CoT2CMrns_2gfreq)
T1CoT2CM2g_cossim <- pairwise_similarity(T1CoT2CM, source, feature, frequency)

T1CoT2CM2g_cossim
```

```
## # A tibble: 2 x 3
##   item1      item2      similarity
##   <chr>      <chr>           <dbl>
## 1 T2-CM      T1-Correct      0.841
## 2 T1-Correct T2-CM           0.841
```

VITA

Sheh Lit Chang was born in Penang, Malaysia. He spent the first 12 years of his life in Malaysia, then went to Singapore for his middle school and high school education. He completed his double degree in Physics & Materials Science & Engineering from the National University of Singapore in 2010. He worked as a teaching assistant from 2011 to 2014 and discovered physics education research. He then applied to University of Washington to embark his research journey with the physics education group in 2014. He graduated in 2020.