

Essays on Information Structure and Corporate Policies

Alvin Chen

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Reading Committee:

Philip Bond, Chair

Yao Zeng

Mark Westerfield

Ed Rice

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University of Washington

Abstract

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Alvin Chen

Chair of the Supervisory Committee:

Philip Bond

Department of Finance, Foster School of Business

These essays explore the connection between the structure of information and corporate policies. The first essay, “Firm Performance Pay as Insurance for Workers,” studies a setting in which non-executive workers compete against each other for promotions into higher level positions. I show that when workers prefer early resolution of uncertainty, information about firm output generates promotion risk, against which firm performance pay insures. The second essay, “Fighting Fire with Fire: Mitigating Information Asymmetry with Open-Market Repurchase Program,” analyzes the effects of open-market repurchase programs on liquidity. I find that in the presence of other informed traders, share repurchases generate two competing effects on liquidity trading costs. I provide conditions under which repurchase programs mitigate the adverse selection problem, rather than exacerbate it.

Contents

Abstract	ii
Acknowledgements	vi
Introduction	1
1 Firm Performance Pay as Insurance for Workers	3
1.1 Introduction	3
1.2 Model	8
1.3 Optimal Contracts	13
1.3.1 Optimal Second-stage Contract	13
1.3.2 Optimal First-stage Contract	15
1.4 Comparative Statics	22
1.5 Empirical Predictions	25

1.5.1	Risk Aversion	25
1.5.2	Idiosyncratic Volatility and Employee Stock Options	26
1.5.3	Valuation of Stock Option Pay	27
1.5.4	Option-like Payoffs	28
1.5.5	Performance-based Vesting	28
1.5.6	Additional Testable Implications	29
1.6	Discussion	30
1.6.1	Tournament Incentives	30
1.6.2	Information Structure	31
1.6.3	Uniqueness of Equilibrium Outcome	31
1.6.4	Renegotiation	32
1.7	Conclusion	33
	Bibliography	35
2	Fighting Fire with Fire: Mitigating Information Asymmetry with Open-Market Repurchase Programs	40
2.1	Introduction	40
2.2	The Model	45

2.3	The Trading Game	47
2.3.1	Competition Effect	49
2.3.2	Dispersion Effect	51
2.4	Better-informed Manager	57
2.5	Patterns of Repurchases	60
2.6	Testable Empirical Predictions	61
2.7	Conclusion	63
	Bibliography	65
	A Appendix to Chapter 1	69
A.1	Parameter Restrictions for θ	69
A.2	Optimal Contracts with No Wages	70
A.3	Proofs	71
	B Appendix to Chapter 2	87
B.1	Fully Strategic Manager	87
B.2	Medium Repurchase Programs and Liquidity	88
B.3	Proofs	90

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Introduction

The first chapter studies a principal-agent model that accounts explicitly for the implicit incentives from internal promotions. I show that the optimal contract for a non-executive worker features firm performance pay even when firm output contains no information about his effort and type. The crux of this result is the timing of information. Because the firm learns about the workers' suitability for promotion by observing their individual outputs, a worker's likelihood of being promoted in the future depends on his current performance relative to those of others. In particular, his promotion probability is low when many workers do well. The worker learns about his promotion prospects from observing firm output, which is an aggregate of individual worker outputs. When workers prefer early resolution of uncertainty, this new information induces risk. Because firm performance tends to be high when the worker's promotion prospects are poor, firm performance pay helps insure the worker against poor promotion prospects. This insurance role of firm performance pay helps explain the prevalence of firm performance pay for non-executive workers as well as several puzzling observations in the compensation literature. Because the preference for early resolution of uncertainty is a critical ingredient behind the insurance role of firm performance pay, this work also suggests that further contracting studies with recursive preferences can help uncover interesting economic forces previously overlooked by conventional expected utility models.

The second chapter analyzes the effects of open-market repurchase programs on liquidity trading costs in a Kyle micro-structure model in which the firm competes against other informed traders. I find that an informed buyback program has two effects on liquidity trading costs. First, it intensifies the competition for trading profits. The increased competition makes the order flow more informative and reduces liquidity trading costs. However, at the same time, the repurchase program generates trading gains that make the firm's actual value per-share higher but only when its stock is undervalued. The increased dispersion of the firm's per-share value makes private information about the firm's fundamentals more valuable and raises liquidity trading costs. When buybacks are small, the second effect dominates. Intuitively, the informed trader responds strategically to small buyback programs by scaling back her informed trades. This accommodation limits the impact of small buybacks on the informativeness of the aggregate order flow. However, buybacks even at small magnitudes increase the dispersion of per-share payouts and make liquidity trading more costly. When high payoffs are more likely, trading profits from informed buys are low, which dampens the dispersion effect and make it more likely that buybacks reduce liquidity trading costs. The conditions under which buybacks mitigate the adverse selection problem help explain several stylized facts in the repurchase literature.

Chapter 1

Firm Performance Pay as Insurance for Workers

1.1 Introduction

The moral hazard literature highlights a tendency for risk-neutral firms to insure risk-averse workers against uncertain firm outcomes. Yet, non-executive workers in the United States routinely receive pay based on some measure of overall firm performance ([Blasi et al. 2008](#)). As [Bergman and Jenter \(2007\)](#) point out, the prevalence of firm performance pay for non-executive workers “is a puzzle for standard economic theory...[because] any positive incentive effects should be diminished by free-rider problems and overshadowed by the cost of imposing risk on employees.” This paper studies the non-executive firm performance pay puzzle in a framework that explicitly accounts for the implicit incentives from promotion tournaments that are common in the workplace.¹

¹See [Bognanno \(2001\)](#), [De Varo \(2006\)](#), [Cichello et al. \(2009\)](#) for empirical documentation of promotion tournaments in the workplace.

I show that the optimal contract for lower-level workers in this setting features firm performance pay because it insures them against uncertain promotion prospects. To understand the economic intuition behind this result, consider the career path of a non-executive worker. Because the company can promote only a limited number of people, this worker's ascent up the organization's hierarchy depends on his performance relative to others' at the same company. Because he does not know how other workers will perform, he is unsure about his prospects for promotion. In particular, his probability of being promoted is low when many workers do well. At the same time, firm performance is high because it is an aggregation of the workers' performances. Because firm performance tends to be high when the worker's promotion prospects are poor, firm performance pay provides the employee with a hedge against not being promoted. Thus, in this setting, firm performance pay is not indicative of inefficient risk-sharing; rather it *is* a form of insurance.

The structure of the optimal contract for lower-level workers stems from this insurance motive. To maintain incentives, the firm only insures high-performing workers. When there are fewer high performers than there are available higher-level positions, all high performers are promoted. When firm performance falls below the threshold that corresponds to the number of vacancies, promotion is certain; high performers receive a fixed wage in these instances. In contrast, when there are more high performers than there are available higher-level positions, the probability of being promoted declines with the total number of high performers. When firm performance exceeds the threshold, high performers receive additional wages to offset poor promotion prospects. Simply put, insurance against uncertain promotions for a high-performing worker takes the form of option-like payoffs in the firm's performance.

The key economic force underlying the insurance role of firm performance pay is the worker's preference for early resolution of uncertainty. In other words, it is the worker's

anxiety upon learning about tomorrow’s unfavorable career prospects that results in an insurance benefit of additional pay today. Absent this anxiety, the optimal contract would simply insure the worker by promising to pay him extra in the future if he is indeed passed over for promotion. I model this preference with Epstein-Zin utility functions.² This paper formally demonstrates that the preference for early resolution of uncertainty is a necessary condition for firm performance pay to play an insurance role in the optimal contract.

My model sheds light on three puzzling empirical findings in the broad-based firm performance pay literature. First, more risk-averse workers are more likely to receive stock-option pay and profit-sharing plans as part of their compensation (Blasi et al. 2008). Second, firms with more volatile stock returns are more likely to use broad-based stock option pay (Spalt 2013). Third, non-executive employees tend to overvalue stock option pay relative to fair-market prices (see Oyer and Schaefer 2005, Hallock and Olson 2006, Hodge et al. 2009), while executives tend to undervalue them (see Hall and Murphy 2002, Bettis et al. 2005).

The first two observations are surprising given the conventional view that risky pay results from an optimal trade-off between some benefit and the cost of imposing risk on workers. This perspective implies that an increase in the cost of imposing risk should discourage the use of firm performance pay. In contrast, firm performance pay provides a hedge against being passed over for promotion in my framework. As a result, an increase in the worker’s risk-aversion leads to greater demand for this insurance. In my model, increased uncertainty about the quality of workers at a firm makes that firm riskier and at the same time increases the uncertainty of promotion prospects. My analysis implies that some parts of a firm’s risk and its use of stock option pay are positively linked via optimal contracting.

²Epstein-Zin preferences routinely show up in the asset-pricing and macro-finance literature precisely because they captures how people respond to news about the future (see Lucas 2003, Bansal and Yaron 2004, Guvenen 2009, Ai and Bansal 2018). In particular, the usual parameterization of this utility function corresponds to a preference for early resolution of uncertainty.

My framework also helps explain the over- and under-valuation dichotomy of the third observation. Lower-level workers benefit from the insurance against uncertain promotion prospects provided by firm performance pay. This insurance benefit increases a lower-level worker's subjective valuation of stock option pay, possibly above fair-market price (e.g. Black-Scholes valuation). This insurance motive also helps explain why non-executive workers often voluntarily acquire equity in their employer at market prices ([Benartzi and Thaler 2001](#)). In contrast, the CEO derives no insurance benefit from stock option pay because she is already at the top of the hierarchy. She simply values stock option pay from the perspective of an under-diversified investor, which leads to a subjective valuation below fair-market price.

Separate explanations exist for each of these findings. For instance, [Spalt \(2013\)](#) posits that employees have “gambling preferences...for skewed, lottery like payoffs” that make stock options in riskier firms more valuable. [Hodge et al. \(2009\)](#) argue that non-executive employees overvalue the stock options in their pay because of bounded rationality. However, this paper provides unifying explanations for these findings within one framework with rational, risk-averse agents.

Tournaments play a crucial part in this framework. They generate uncertain promotion prospects, which lead to the demand for insurance via firm performance pay. In my model, the tournament arises endogenously because the firm wants to promote only high-type workers into a limited number of higher-level positions. Because the firm can only learn about a lower-level worker's type through his individual performance, the promotion process generates tournament-style competition among lower-level workers. Higher-level workers are harder to monitor so the firm has to pay them more to induce effort. The tournament prize corresponds to this pay increase due to a more severe moral hazard problem in higher-level positions. The workplace may feature tournaments for other reasons. Regardless of their

rationale, as long as tournament incentives exist, firm performance pay can still play an insurance role.

This paper primarily relates to the literature that considers firm performance pay for non-executive workers. Studies in this area almost always argue that firm performance pay provides a sufficiently large benefit to justify the cost of imposing risk on lower-level employees, who are presumably less diversified and more risk-averse than the firm.³ Some benefits include reduced financial constraint (Core and Guay 2001, Kim and Ouimet 2014), favorable accounting treatment (Hall and Murphy 2003), employee retention (Oyer 2004), employee sorting (Lazear 2004, Bergman and Jenter 2007), and improved product market competitiveness (Bova and Yang 2017). This paper contributes to this literature by considering the neglected side of the cost-benefit trade-off; it highlights how firm performance pay can actually *reduce* a worker’s payoff uncertainty.

The idea that non-executive employees prefer pay that covaries with overall firm performance because it insures them against increased competition for promotions is reminiscent of a series of papers on “keeping up with the Joneses” (DeMarzo et al. 2004, DeMarzo et al. 2007, and DeMarzo et al. 2008). They argue that investors may prefer payouts that correlate positively with aggregate wealth levels because such payouts hedge against increased competition over consumption and investment goods in the future. They investigate the asset-pricing implications of these relative wealth concerns. In contrast, I study the optimal contracting implications of workers’ relative performance concerns due to tournaments.

This paper also sits between broad literatures on optimal contracting and tournaments. Previous studies involving contracting and tournaments tend to consider each in isolation. Most dynamic contracting frameworks with career concerns do not feature tournament in-

³One exception is a recent working paper by Efung et al. (2018), which argues that bank employees accept bonuses in order to insure the firm against negative cash flow shocks.

centives (e.g. Gibbons and Murphy 1992, Gibbons and Waldman 1999, Holmstrom 1999, Axelson and Bond 2015). As a consequence, the insurance role that firm performance pay plays in my framework is absent in theirs. The tournament literature usually focuses on how best to set up such a contest (e.g. Lazear and Rosen 1981, Lazear 1989, O’Keeffe et al. 1984, Chen 2003, Boudreau et al. 2016). While some authors compare contracts to tournaments (e.g. Lazear and Rosen 1981, Nalebuff and Stiglitz 1983, Green and Stokey 1983), rarely do they consider how the two interact. This paper showcases the link between the firm’s organizational hierarchy and its compensation policy. A separate contribution of this paper is to study contracting under Epstein-Zin preferences. The optimal contract features insurance via firm performance pay only when workers prefer early resolution of uncertainty. This result suggests that further contracting studies under recursive preferences can help uncover interesting economic forces overlooked by conventional frameworks with expected utility.

1.2 Model

The model has three dates ($t = 0, 1, 2$) and features a risk-neutral firm that employs a unit-mass continuum of risk-averse workers. Production at the firm occurs in two stages. In the first, each worker, indexed by $i \in [0, 1]$, produces an output $q_i \in \{0, 1\}$ determined by his effort level $e_{1i} \in \{0, 1\}$ and his firm-specific talent x_i , which is ex-ante unknown to all parties. This type parameter is distributed identically and independently according to a Bernoulli distribution with unknown parameter θ :

$$x_i \stackrel{i.i.d.}{\sim} \text{Bernoulli}(\theta),$$

where θ takes values $\theta_1, \theta_2, \dots, \theta_n$ with probabilities p_1, p_2, \dots, p_n , respectively. The possible values of θ are indexed in order of increasing magnitude: $\theta_1 < \theta_2 < \dots < \theta_n$ and bounded

below by $\underline{\theta}$.⁴ If a worker is high-type ($x_i = 1$) and puts in effort ($e_{1i} = 1$), his output is high ($q_i = 1$) with certainty:

$$Prob(q_i = 1 | e_{1i} = 1 \text{ and } x_i = 1) = 1.$$

If he shirks ($e_{1i} = 0$) or is low-type ($x_i = 0$), then his output is low ($q_i = 0$) with certainty:

$$Prob(q_i = 1 | e_{2i} = 0 \text{ or } x_i = 0) = 1.$$

The firm's total first-stage production is a simple aggregation of the output of all workers:

$$Q = \int_0^1 q_i di. \tag{1.1}$$

In the second stage of production, the firm can employ up to a fraction $K < 1$ of the unit-mass of workers. Each of these promoted agents produces an output $y_i \in \{y_L, y_H\}$, where $y_L < 0 < y_H$. Low-type workers produce low output in the second-stage with certainty:

$$Prob(y_i = y_L | x_i = 0) = 1.$$

As in the first stage, high-type workers who exert effort produce high output with certainty:

$$Prob(y_i = y_H | e_{2i} = 1 \text{ and } x_i = 1) = 1.$$

However, in the second stage, even high-type workers who shirk produce high output with

⁴This lower bound simplifies the analysis of the optimal contract, but does not qualitatively affect the paper's main result on firm performance pay. Appendix A discusses the role of this parameter restriction in greater details.

some probability $\phi > 0$:

$$Prob(y_i = y_H | e_{2i} = 0 \text{ and } x_i = 1) = \phi.$$

The firm's total second-stage production is a simple aggregation of the output of all promoted workers:

$$Y = \int_0^1 \psi_i y_i di. \quad (1.2)$$

Because shirking by high-type workers during second-stage production cannot always be detected, the firm has to yield a rent to the worker to ensure incentive compatibility. Define P_0 to be the minimum wage the firm has to pay a promoted worker for high output during second-stage production to induce effort. I assume that y_L , y_H , P_0 , and the distribution of θ satisfy the following conditions:

1. $y_H - P_0 > \phi y_H + (1 - \phi)y_L$,
2. $y_H - P_0 > 0$,
3. $\bar{\theta}y_H + (1 - \bar{\theta})y_L - P_0 < 0$, where:

$$\bar{\theta} = \sum_{j=1}^n p_j \theta_j. \quad (1.3)$$

The first condition ensures that effort during second-stage production is optimal. The second means that hiring a high-type worker for second-stage production is NPV positive for the firm. The final indicates that filling a position for the second stage absent additional information about a candidate's type is NPV negative. These conditions imply that promoting the right workers to the higher-level positions provides value to the firm and motivate a role for tournaments within the organization. The tournament plays a crucial part in this model

because it generates the uncertain promotion prospects that drive the demand for insurance via firm performance pay. Section 1.6.1 discusses the limitations of these assumptions. Because inducing effort for first-stage production is also assumed to be optimal, the firm's profit-maximizing objective is equivalent to incentivizing effort at lowest expected cost.

A feasible wage contract for first-stage production specifies the worker's pay at $t = 1$ and $t = 2$ given verifiable variables. Wages at $t = 1$ for first-stage production ($W_1(q_i, Q)$) can only depend the worker's individual output q_i and total firm production Q because the promotion and second-stage production outcomes are not known.⁵ Wages at $t = 2$ for first-stage production ($W_2(q_i, Q, \psi)$) can depend on first-stage outputs (q_i, Q) and on the result of the promotion (ψ_i), which takes value 1 if the worker is promoted and 0 otherwise. A feasible wage contract for second-stage production specifies the promoted worker's pay at $t = 2$ given the production outcomes during the second stage ($W(y_i, Y)$). Workers are protected by limited liability so wages cannot be negative. The firm's discount factor is normalized to 1. Table 1.1 summarizes the timing of the model.

$t = 0$	$t = 1$	$t = 2$	$t = 3$
· Endowment	· Output (q, Q)	· Output (y, Y)	· Lifetime utility
· C_0	· $W_1(q, Q)$	· $W_2(q_i, Q, \psi_i), W(y_i, Y)$	· U_3
· Contract	· Promotion (ψ)	· Outside option	
· Effort: e_1	· Effort: e_2	· Effort: e_3	

Table 1.1. Model Timing

The workers have lifetime utility at time t given by the recursive formulation,

$$U_t = \left[c_t^{1-\rho} + s_t^{1-\rho} + \beta E_t [U_{t+1}^{1-\alpha}]^{\frac{1-\rho}{1-\alpha}} \right]^{\frac{1}{1-\rho}}, \quad (1.4)$$

⁵The assumption that the promotion decision comes after wage payment at $t = 1$ is meant to capture the idea that it takes time for the firm to determine the fit between a worker and a promoted position, but wages have to be paid to the worker in the interim period.

where c_t and s_t are the worker’s consumption and leisure at time t , respectively. The parameters $\alpha > 0$ ($\alpha \neq 1$), $\beta \in (0, 1]$, and $\rho \in (0, 1)$ capture the worker’s risk aversion, subjective discount rate, and intertemporal elasticity of substitution, respectively.⁶ The worker’s consumption at $t = 0$ is fixed at $c_0 > 0$. For simplicity, the worker can neither borrow nor save, consuming wages in the period in which the payments were made.⁷ If the worker shirks at time t , he enjoys s units of leisure that period. If he exerts effort at time t , $s_t = 0$. On the equilibrium path, where workers put forth effort, the utility function in (1.4) simplifies to the Epstein-Zin form.

Epstein-Zin utility is routinely used in asset-pricing and macro-finance models because it captures important properties of economic behavior. For example, [Ai and Bansal \(2018\)](#) argue that the macroeconomic announcement premium provides evidence of “a key aspect of investors’ preferences not captured by the time-separable expected utility.” Based on the large magnitude of these premiums in the data, they make a revealed preferences argument for non-expected utilities such as the recursive preferences of [Kreps and Porteus \(1978\)](#) and [Epstein and Zin \(1989\)](#).

In the context of this paper, using the common parameterization of $\alpha > \rho$, Epstein-Zin utility allows me to model workers with a preference for early resolution of uncertainty.⁸ This preference for early resolution of uncertainty motivates the demand for insurance against uncertain promotions prospects, which drives the firm performance pay as insurance mech-

⁶The model only has three periods. The lifetime utility at $t = 3$ is assumed to be $U_3 > 0$ for all workers and require effort $e_3 = 1$. This assumption keeps the worker’s utility well-defined when wages and shirking benefits are 0. The restriction $\rho < 1$ corresponds to the $IES > 1$, which implies that an increase in the real interest rate increases an worker’s savings. The paper’s main result on firm performance pay does not rely on this restriction. Because the firm’s discount factor is normalized to 1, one can think of β as the worker’s relative impatience; its exact magnitude also does not qualitatively alter the main result of the paper.

⁷Lower-level employees typically want to borrow because future earnings exceed the current wages; however, financial constraints due to moral hazard usually prevent this consumption smoothing.

⁸For concreteness, consider this example from [Kreps and Porteus \(1978\)](#): A lottery pays z_1 and z_2 at time 1 and 2, respectively. A coin flip determines the realized payoffs of this lottery. If the coin lands heads, $(z_1, z_2) = (5, 10)$. If it lands tails, $(z_1, z_2) = (5, 0)$. At time 0, an agent who prefers early resolution of uncertainty wants the coin flip to happen at time 1 rather than at time 2.

anism. A separate contribution of this paper is its study of optimal contracting under Epstein-Zin preferences.

1.3 Optimal Contracts

This section solves for the optimal contracts for first and second stage productions. I proceed with backward induction, first solving for the optimal second-stage contract, then solving for the optimal first-stage contract taking the tournament structure induced by the optimal second-stage contract as given. I assume that the firm is able to commit to the contracts for both stages at $t = 0$. Section 6.3 discusses the validity of this approach. I suppress the subscripts identifying the worker and focus on symmetric contracts in the remainder of the paper.

1.3.1 Optimal Second-stage Contract

I begin by identifying the optimal contract for promoted high-type workers in the second stage of production. A feasible contract specifies the promoted worker's wage $W(y, Y)$ conditional on his individual output and total firm output. On the equilibrium path, production outcomes during the first stage reveals the average quality of workers at the firm (θ), which also determines total firm production during the second stage when all promoted workers put in effort: $Y = \theta Y_H$.

To induce effort from promoted workers in all states of the world, the optimal second stage contract must satisfy incentive compatibility for every realization of θ . If a high-type worker exerts effort during second-stage production, his output is high ($y = y_H$) with certainty. If he shirks, he enjoys s units of leisure at $t = 1$, but generates high output at $t = 2$ only with probability $\phi < 1$. In either case, the worker's lifetime utility at $t = 3$ is a

constant $U_3 > 0$.

On the equilibrium path, the worker's incentive compatibility constraint is given by:

$$\begin{aligned} & \left[c_1^{1-\rho} + \beta W(y_H, \theta Y_H)^{1-\rho} + \beta^2 U_3^{1-\rho} \right]^{\frac{1}{1-\rho}} \\ \geq & \left[c_1^{1-\rho} + s^{1-\rho} + \beta \left(\phi [W(y_H, \theta Y_H)^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{1-\alpha}{1-\rho}} + (1-\phi) [W(y_L, \theta Y_H)^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{1-\alpha}{1-\rho}} \right)^{\frac{1-\rho}{1-\alpha}} \right]^{\frac{1}{1-\rho}}. \end{aligned} \quad (1.5)$$

Similarly, the worker's participation constraint can be expressed as:

$$\left[c_1^{1-\rho} + \beta W(y_H, Y_\theta)^{1-\rho} + \beta^2 U_3^{1-\rho} \right]^{\frac{1}{1-\rho}} \geq \bar{u}_1, \quad (1.6)$$

where \bar{u}_1 is the worker's reservation utility at $t = 1$. I assume that \bar{u}_1 is sufficiently small so that a contract that satisfies (1.5) also satisfies (1.6).

Lemma 1. *The optimal contract for second-stage production only depends on the worker's individual output. It pays the worker nothing for low-output ($W(y_L, \theta Y_H) = 0$) and P_0 for high-output ($W(y_H, \theta Y_H) = P_0$), where P_0 satisfies:*

$$P_0^{1-\rho} + \beta U_3^{1-\rho} = \frac{s^{1-\rho}}{\beta} + \left(\phi [P_0^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{1-\alpha}{1-\rho}} + (1-\phi) [\beta U_3^{1-\rho}]^{\frac{1-\alpha}{1-\rho}} \right)^{\frac{1-\rho}{1-\alpha}}.$$

The optimal second-stage contract does not depend on the firm's total output (Y) because it contains no information about the promoted worker's effort (e_2). Low output ($y_i = y_L$) only occurs when the worker shirks. Consequently, the optimal contract minimizes the payment to the worker given this outcome. Limited liability implies that $W_L = 0$. The payment to the worker given high output is the smallest W_H such that the worker's incentive compatibility constraint (1.5) is satisfied when $W_L = 0$. This wage payment P_0 plays the role of the prize for the promotion tournament for lower-level workers.

1.3.2 Optimal First-stage Contract

This section takes the implicit incentives from promotion as given and solves for the optimal first-stage contract. Up to a fraction K of workers with high-output in the first stage can advance to the second stage. Promoted workers receive a prize of P_0 at $t = 2$. The size of this prize is determined by the optimal second-stage contract of the previous section. Let $\{W_1(q, Q), W_2(q, Q, \psi)\}$ be the wage contract for first-stage production that specifies the $t = 1$ and $t = 2$ payments to the worker contingent on q , his individual output, Q , total firm production, and ψ , which takes value 1 if that worker was promoted and 0 otherwise.

A lower-level worker's promotion prospect depends on his individual output q and on total firm production Q . On the equilibrium path, the optimal contract induces effort from all workers. This fact implies that a worker is high-type if and only if $q = 1$. When $q = 0$, the firm does not promote this worker because doing so is NPV negative. In addition, when all workers exert effort, firm performance Q is a sufficient statistic for average employee quality. In particular, Q takes values $\theta_1, \dots, \theta_n$ with probabilities p_1, \dots, p_n , respectively. When $Q \leq K$, all high-type workers are promoted because there are enough upper-level positions available for all qualified candidates. When $Q > K$, high-type workers are promoted with the same probability $\frac{K}{Q}$. Promoted workers receive a prize of P_0 at $t = 2$.

Given a wage contract of the form $\{W_1(q, Q), W_2(q, Q, \psi)\}$, the worker's lifetime utility at $t = 2$ can be defined as a function of q , Q , and ψ :

$$U_2(q, Q, \psi) = \begin{cases} \left[(P_0 + W_2(q, Q, 1))^{1-\rho} + \beta U_3^{1-\rho} \right]^{\frac{1}{1-\rho}} & \psi = 1 \\ \left[W_2(q, Q, 0)^{1-\rho} + \beta U_3^{1-\rho} \right]^{\frac{1}{1-\rho}} & \psi = 0. \end{cases} \quad (1.7)$$

Let $\bar{U}_2(Q)$ be the high-output worker's $t = 1$ certainty equivalent $t = 2$ lifetime utility given

Q . When $Q \leq K$, a high-output worker is promoted with certainty. Hence, when $Q \leq K$:

$$\bar{U}_2(Q) = U_2(1, Q, 1).$$

When $Q > K$, a high-output worker is promoted with probability $\frac{K}{Q}$. Thus, when $Q > K$:

$$\bar{U}_2(Q) = \left(\frac{K}{Q} U_2(1, Q, 1)^{1-\alpha} + \left(1 - \frac{K}{Q} \right) U_2(1, Q, 0)^{1-\alpha} \right)^{\frac{1}{1-\alpha}}.$$

Given the above expressions for the worker's lifetime utility at $t = 2$, his $t = 1$ lifetime utility can be written as a function of q and Q :

$$U_1(q, Q) = \begin{cases} \left[W_1(1, Q)^{1-\rho} + \beta \bar{U}_2(Q)^{1-\rho} \right]^{\frac{1}{1-\rho}} & q = 1 \\ \left[W_1(0, Q)^{1-\rho} + \beta U_2(0, Q, 0)^{1-\rho} \right]^{\frac{1}{1-\rho}} & q = 0. \end{cases} \quad (1.8)$$

The incentive compatibility and participations constraints can be written using the expressions for the worker's $t = 1$ lifetime utility given in (1.8). Because there is a continuum of workers with mass one, total firm production Q takes value θ_j with probability p_j . Hence, the incentive compatibility (IC) constraint can be expressed as:

$$\begin{aligned} & \left[c_0^{1-\rho} + \beta \left(\sum_{j=1}^n p_j \theta_j U_1(1, \theta_j)^{1-\alpha} + \sum_{j=1}^n p_j (1 - \theta_j) U_1(0, \theta_j)^{1-\alpha} \right)^{\frac{1-\rho}{1-\alpha}} \right]^{\frac{1}{1-\rho}} \\ & \geq \left[c_0^{1-\rho} + s^{1-\rho} + \beta \left(\sum_{j=1}^n p_j U_1(0, \theta_j)^{1-\alpha} \right)^{\frac{1-\rho}{1-\alpha}} \right]^{\frac{1}{1-\rho}}, \end{aligned} \quad (1.9)$$

Let \bar{u}_0 be the worker's reservation utility at $t = 0$. The worker's participation (IR) constraint

is:

$$\left[c_0^{1-\rho} + \beta \left(\sum_{j=1}^n p_j \theta_j U_1(1, \theta_j)^{1-\alpha} + \sum_{j=1}^n p_j (1 - \theta_j) U_1(0, \theta_j)^{1-\alpha} \right)^{\frac{1-\rho}{1-\alpha}} \right]^{\frac{1}{1-\rho}} \geq \bar{u}_0. \quad (1.10)$$

The analysis in this section assumes that \bar{u}_0 is sufficiently small so that a contract that satisfies the IC constraint in (1.9) also satisfies the IR constraint in (1.10). Formally, the firms solves the following:

$$\begin{aligned} & \min_{\{W_1(q, Q), W_2(q, Q, \psi)\}} E[W_1(q, Q) + W_2(q, Q, \psi)], \\ & \text{subject to} \quad (1.9) \text{ and } (1.10). \end{aligned}$$

To begin, consider the contract that pays the worker nothing for first-stage production. Let k be the largest integer such that $\theta_k \leq K$. In this case, the incentive compatibility constraint simplifies to:

$$\begin{aligned} & \left(\sum_{j=1}^n p_j (1 - \theta) \beta^{\frac{2(1-\alpha)}{1-\rho}} U_3^{1-\alpha} + \sum_{j=1}^k p_j \theta_j \left[\beta P_0^{1-\rho} + \beta^2 U_3^{1-\rho} \right]^{\frac{1-\alpha}{1-\rho}} + \right. \\ & \left. \sum_{j=k+1}^n p_j \theta_j \left[\beta \left(\frac{K}{Q} [P_0^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{1-\alpha}{1-\rho}} + \left(1 - \frac{K}{Q} \right) [\beta U_3^{1-\rho}]^{\frac{1-\alpha}{1-\rho}} \right)^{\frac{1-\rho}{1-\alpha}} \right]^{\frac{1-\rho}{1-\alpha}} \right)^{\frac{1-\rho}{1-\alpha}} \quad (1.11) \\ & \geq \frac{s^{1-\rho}}{\beta} + \beta^2 U_3^{1-\rho}. \end{aligned}$$

When condition (1.11) is satisfied, the implicit incentives from the tournament alone is enough to induce effort from the worker during the first stage.⁹ Consequently, the optimal contract features no explicit wages (see [Terviö \(2009\)](#) for more details regarding such contracts). The lowest ranks of professional sports often feature such contracts. For instance,

⁹The tournament prize, P_0 , appears in this simplified incentive compatibility constraint only in the expression to the left of the inequality. In addition, this expression increases strictly in P_0 and condition (1.11) is satisfied in the limit when α is not too large. When the worker is very risk-averse, satisfying condition (1.11) requires positive wages.

players competing in open qualifiers for professional golf tournaments are not explicitly compensated. Instead, they vie for the opportunity to participate in the PGA Tour, which features a sizable purse. The same can be said of those college athletes competing for a roster spot on one of the major professional sports teams. The remainder of this paper focuses on the more interesting scenario in which the optimal contract features positive wages.

Lemma 2. *The optimal wage contract pays the worker no wages when his output is low ($q = 0$):*

$$W_1(0, Q) = 0 \text{ for all } Q \in \{\theta_1, \dots, \theta_n\},$$

$$W_2(0, Q, 0) = 0 \text{ for all } Q \in \{\theta_1, \dots, \theta_n\}.$$

When individual output is sufficiently informative ($\theta_1 \geq \underline{\theta}$), a worker's low output is much more likely the result of shirking than of low type. Consequently, the optimal contract pays the worker the lowest possible wages when his output is low. Limited liability implies that this lower-bound is zero.

To induce effort from the worker, the optimal contract concedes a certain amount of $t = 1$ lifetime utility to the worker for high output ($q=1$). Let $\bar{\mu}_1$ be this quantity. For a fixed $\bar{\mu}_1$, there is a unique optimal allocation of consumption across $t = 1$ and $t = 2$ that minimizes the required budget. Formally, this consumption plan (c_1^*, c_2^*) solves:

$$\begin{aligned} \min_{c_1, c_2} \quad & c_1 + c_2, \\ \text{subject to} \quad & [c_1^{1-\rho} + \beta c_2^{1-\rho} + \beta^2 u_3^{1-\rho}]^{\frac{1}{1-\rho}} = \bar{\mu}_1. \end{aligned}$$

Proposition 1. *When the tournament prize is sufficiently small ($P_0 \leq c_2^*$), the optimal contract fully insures the worker when his output is high ($q = 1$):*

$$W_1(1, Q) = c_1^* \text{ for all } Q \in \{\theta_1, \dots, \theta_n\},$$

$W_2(1, Q, 0) = c_2^*$ for all $Q \in \{\theta_1, \dots, \theta_n\}$, and

$W_2(1, Q, 1) = c_2^* - P_0$ for all $Q \geq K$.

When the tournament prize is sufficiently small ($P_0 \leq c_2^*$), the optimal contract sets wages so that the high-output worker consumes c_1^* at $t = 1$ and c_2^* at $t = 2$ in all states, fully insuring him. Notice that the contract above does not explicitly feature firm performance pay. Conditional on having high individual output, the worker's pay at $t = 1$ is independent of the firm's output and his wages at $t = 2$ increases with firm performance only in expectation. The result of this proposition shows that without additional frictions, the cheapest way to insure the worker against the uncertainty of promotion is to contract on this event directly. However, it may not always be feasible to fully and directly insure the worker against the possibility of being passed over for a promotion. In the remainder of this section, I focus on the case when the prize is too large to fully insure ($P_0 > c_2^*$).

Lemma 3. *When the prize is not too small ($P_0 > c_2^*$), the optimal contract does not fully insure the high-output ($q = 1$) worker against unfavorable promotion outcomes:*

$$W_2(1, Q, 0) < P_0 \text{ for all } Q \in \{\theta_1, \dots, \theta_n\}.$$

Intuitively, when $P_0 > c_2^*$, a contract that fully protects the worker against being passed over for a promotion either yields too much utility or provides an inefficiently low level of consumption at $t = 1$. In either case, a cheaper contract can be constructed by lowering wages at $t = 2$, which results in partial insurance.

Lemma 4. *When the optimal contract does not fully insure the worker's $t = 2$ consumption, it makes no additional payments to the worker at $t = 2$ if he is promoted:*

$$W_2(0, Q, 1) = 0 \text{ for all } Q \in \{\theta_1, \dots, \theta_n\}.$$

When the optimal contract does not fully insure, additional payments to the worker on top of the tournament prize makes his $t = 2$ utility more volatile, which is undesirable because of the worker's risk-aversion. In other words, because the contract does not fully insure, it must be that the worker's $t = 2$ lifetime utility is less than $[P_0^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{1}{1-\rho}}$ in some states of the world. Wages paid in addition to the tournament prize would provide more $t = 1$ lifetime utility to the worker if they were instead paid in one of the states in which his $t = 2$ utility falls below $[P_0^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{1}{1-\rho}}$. Consequently, the optimal contract for first-stage work does not feature additional pay at $t = 2$ for promoted workers when the tournament prize is not too small ($P_0 > c_2^*$).

It is worth noting that the results thus far in this section require only that the worker prefers more consumption to less and is risk-averse. They do not rely on a preference for early resolution of uncertainty. However, this preference plays a pivotal role in generating a demand for insurance via firm performance pay.

Proposition 2. *When the prize is too large to fully insure ($P_0 > c_2^*$) and workers prefer early resolution of uncertainty ($\alpha > \rho$), the optimal contract features option-like firm performance pay for the high-output worker:*

$$W_1(1, \theta_1) = \dots = W_1(1, \theta_k) < W_1(1, \theta_{k+1}) < \dots < W_1(1, \theta_n),$$

where k is the largest integer such that $\theta_k \leq K$.

The optimal contract uses explicit wages to offset the changes in the value of the implicit incentives from promotion. The firm's total production Q provides a sufficient statistic for the size of the pool of high-output workers. When $Q \leq K$, a high-output worker faces no additional risk because the firm promotes all qualified candidates. As a result, the optimal contract pays him a flat wage at $t = 1$ for $Q \leq K$. In contrast, when $Q > K$, a high-output worker's probability of receiving a promotion declines in Q . The optimal contract

compensates this worker for the lower promotion probability by paying him wages that increase in Q for $Q > K$. The crux of this argument is the worker's preference for early resolution of uncertainty.

Proposition 3. *The worker's preference for early resolution of uncertainty is a necessary condition for firm performance pay to play an insurance role.*

To see why this preference for early resolution of uncertainty is important for the firm performance pay result, it is useful to first consider the case when workers do not have this preference. For example, when $\alpha = \rho$, the Epstein-Zin formulation simplifies to time-separable utility and the worker is indifferent over the timing of uncertainty resolution. The firm can compensate the high-output worker for unfavorable promotion outcomes using wages at $t = 1$ and at $t = 2$. Wages at $t = 1$ can only provide an indirect form of insurance because the worker enjoys the benefits of those payments even if he subsequently receives a promotion at $t = 2$. In contrast, wages at $t = 2$ provides a direct form of insurance because those payments can be made contingent on the promotion outcome itself. When the worker is indifferent over the timing of uncertainty resolution, the optimal contract pays the high-output worker a fixed wage at $t = 1$ and promises to make another payment at $t = 2$ if he is not promoted. It features no insurance at $t = 1$ because wages at $t = 2$ are a superior form of insurance.

When the worker prefers early resolution of uncertainty, wages at $t = 2$ no longer dominate those at $t = 1$ as a superior form of insurance. As noted earlier, firm performance (Q) is a sufficient statistic for the proportion of qualified workers (θ). Thus, while firm performance pay only indirectly insures against unfavorable promotion outcomes, it directly insures against unfavorable promotion prospects. When the worker prefers early resolution of uncertainty, he derives an extra benefit from wages at $t = 1$ when he learns that promotion at $t = 2$ is less likely. In other words, he benefits from insurance not only against unfavorable

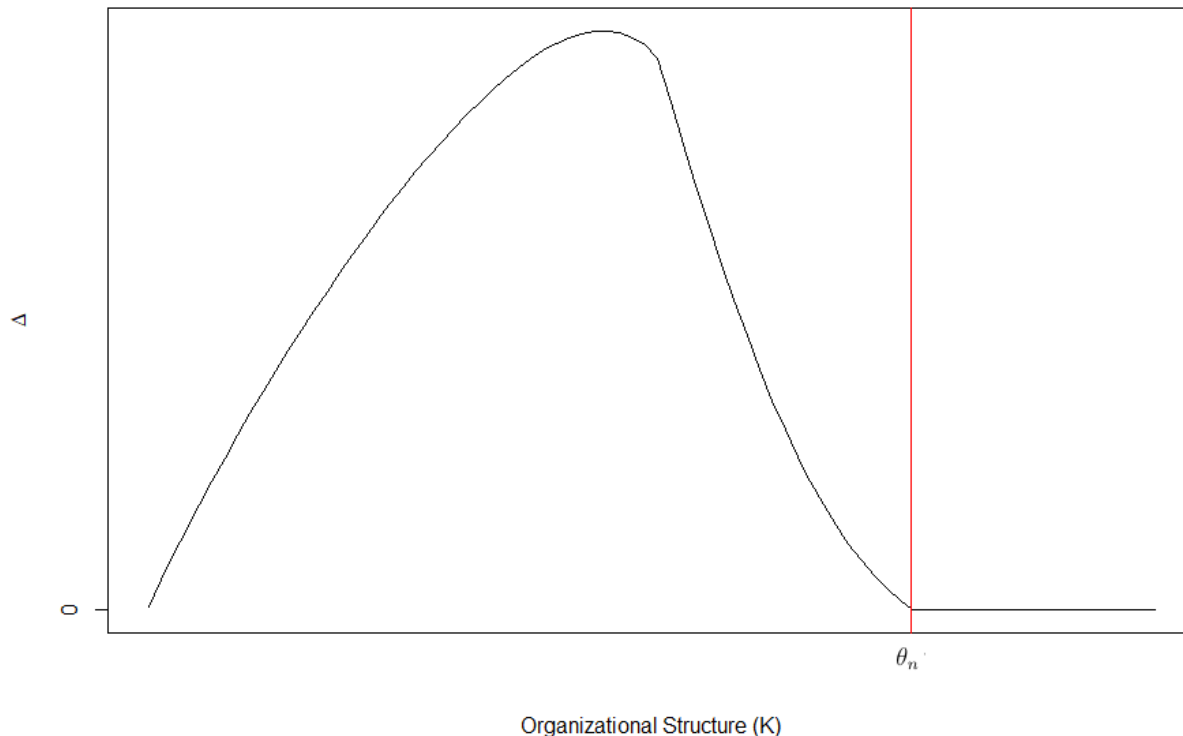


Figure 1.1. Δ vs organizational structure (K) This figure plots an example of Δ , defined as the difference between the value of the firm performance pay in the optimal contract to the work and its cost to the firm, against the organization structure of the firm, defined as the ratio of the number of higher-level positions to lower-level ones (K). The figure was generated using parameters $\alpha = 0.8$, $\rho = 0.5$, $\beta = 1$, $K = 0.3$, $s = 1$, $U_3 = 5$, $\theta \in \{0.2, 0.25, 0.3\}$ with equal probabilities, and $P_0 = 11$.

promotion outcomes, but also against unfavorable promotion prospects. In this way, the preference for early resolution of uncertainty drives the optimality of firm performance pay.

1.4 Comparative Statics

This section provides some intuition for how the optimality of firm performance pay varies in the cross-section using numerical results. Let V be the certainty equivalent value of the firm performance pay in the optimal contract; this quantity is the fixed wage that the worker

would accept in exchange for the firm performance pay in the optimal contract. Let C be the expected cost of the firm performance pay to the firm. A larger difference between the value of firm performance pay and its cost increases the likelihood that firm performance pay is employed. Let Δ be this difference:

$$\Delta = V - C.$$

The relationship between Δ and the organizational structure (K), defined as the ratio of the number of higher-level positions to lower-level ones, is non-monotonic. Changes in K exerts two forces on Δ . A larger value of K increases the value of implicit incentives from promotions. This increase in value means that uncertainty about promotion translates into greater risks and increases the value of insurance via firm performance pay. However, an increase in K also reduces the likelihood that the worker needs to be insured. The first effect dominates for smaller values of K , while the second dominates for larger ones. In particular, when K surpasses θ_n , no promotion uncertainty remains because all high-output workers can be promoted. In these instances, the optimal contract features no firm performance pay; hence, $V = C = 0$. This relationship is shown in Figure 1.1.

The optimality of firm performance pay (Δ) also increases in the firm's output volatility, defined as the variance of its first-stage output. In this model, the firm's output volatility stems entirely from uncertainty about the quality of its workers. An increase in the firm's output volatility corresponds to an increase in promotion uncertainty. Consequently, higher output volatility is associated with greater demand for insurance via firm performance pay. This relationship is shown in Figure 1.2.

The relationship between the degree of the worker's risk aversion (α) and the optimality of firm performance pay (Δ) is positive unlike in most traditional principal-agent models

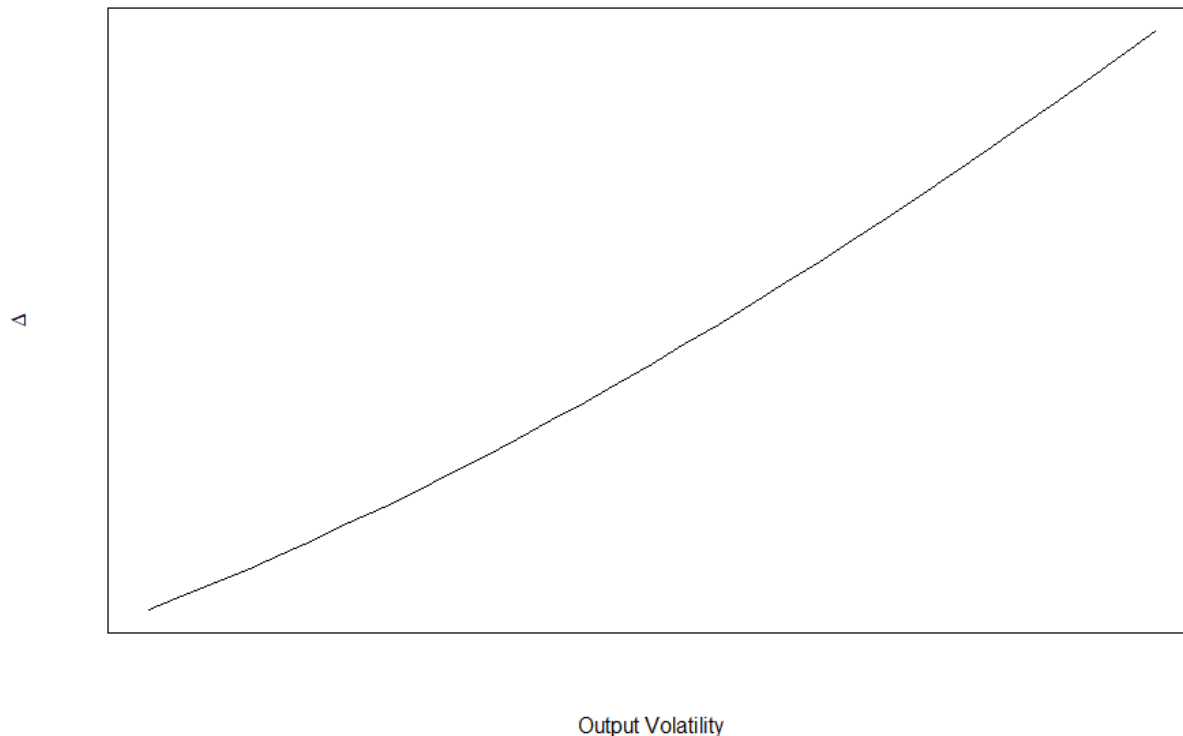


Figure 1.2. Δ vs volatility of firm's output This figure plots an example of Δ , which is the value of firm performance pay to the worker minus its cost to the firm, against the output volatility of the firm, defined as the variance of its first-stage output. The figure was generated using parameters $\alpha = 0.5$, $\rho = 0.3$, $\beta = 1$, $K = 0.1$, $s = 1$, $U_3 = 10$, and $P_0 = 20$. The parameter θ takes values 0.05, 0.1, and 0.15 with probabilities, z , $1 - 2z$, and z , respectively. Increasing output volatility corresponds to increasing the parameter $z \in (0, 0.5)$.

without tournament-style incentives.¹⁰ In this framework, firm performance pay does not induce additional risk; instead, it insures against the uncertainty of promotion. Hence, the more risk-averse the worker is, the more attractive this insurance instrument becomes. This relationship between the optimality of firm performance pay (Δ) and risk-aversion (α) is shown in Figure 1.3.

¹⁰[Inderst and Müller \(2003\)](#) is an exception to this rule. In those models, workers accept firm performance pay in order to stave off the possibility of unemployment due to firm bankruptcy. Hence, as long as increases in risk-aversion make unemployment more unpalatable than having risky pay, their models may also generate a positive relationship between risk-aversion and firm performance pay.

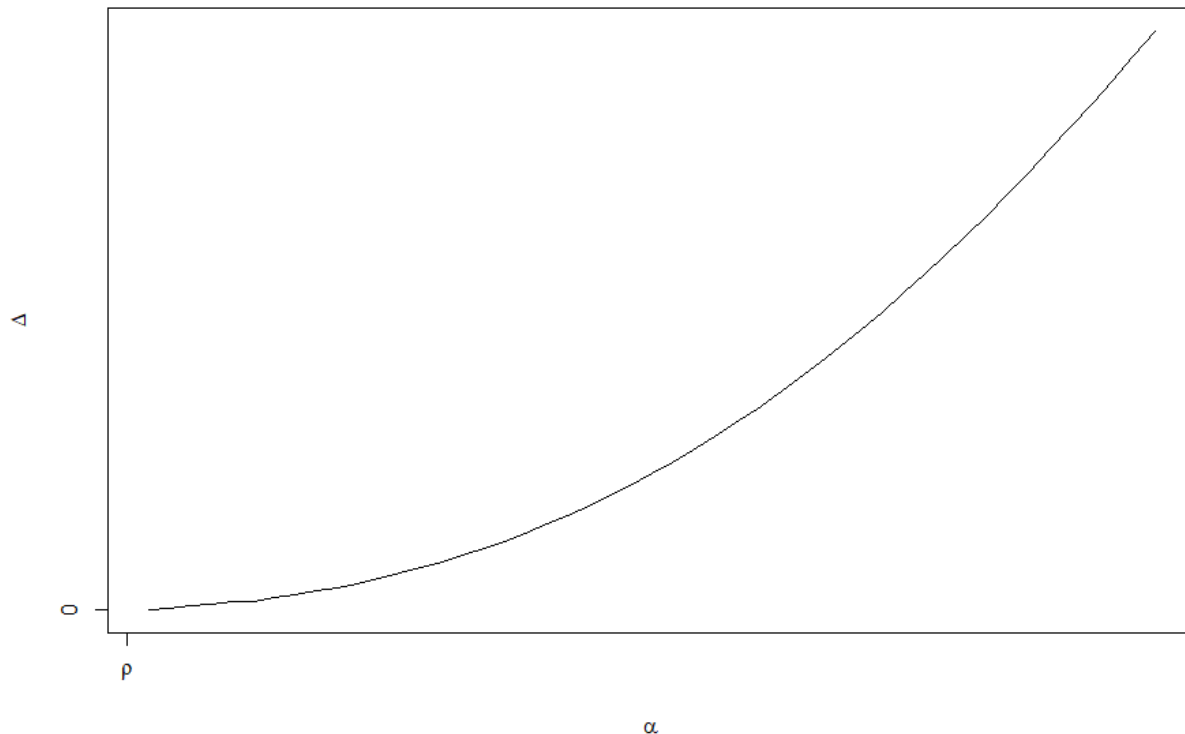


Figure 1.3. Δ vs risk-aversion This figure plots an example of Δ , which is the value of firm performance pay to the worker minus its cost to the firm, the risk-aversion of the worker (α). The figure was generated using parameters $\alpha > \rho = 0.5$, $\beta = 1$, $K = 0.2$, $s = 1$, $U_3 = 10$, $\theta \in \{0.25, 0.3, 0.35\}$ with equal probabilities, and $P_0 = 25$.

1.5 Empirical Predictions

This section relates the results of this model to stylized facts about firm performance pay. While other explanations exist for each of the following phenomena, this paper provides a single framework with unifying explanations for this set of stylized facts.

1.5.1 Risk Aversion

One surprising finding from the analysis of the NBER surveys on shared capitalism is a positive relationship between risk-aversion and the prevalence of firm performance pay. For example, employees who are more risk-averse are more likely to receive profit-sharing plans

and stock option pay than those who were not (Blasi et al. 2008). In my framework, firm performance pay plays an insurance role. The more risk-averse the workers are, the more valuable this insurance against uncertain promotion prospects becomes. Hence, the model predicts that more risk-averse employees are more likely to receive firm performance pay, consistent with the findings of Blasi et al. (2008).

1.5.2 Idiosyncratic Volatility and Employee Stock Options

Spalt (2013) document a puzzling empirical finding that firms with higher idiosyncratic volatility in their stock returns are more likely to use broad-based employee stock option plans (ESO). In general, standard models have difficulty with this observation because when employees are risk-averse and hold rational expectations, the use of ESOs is more costly when there is more risk (Lambert et al. 1991, Hall and Murphy 2002, Bettis et al. 2005, Oyer and Schaefer 2005). Consequently, explanations for this phenomenon typically relax one or both of the assumptions about risk-aversion and rationality. For instance, Spalt (2013) posits that employees have “gambling preferences...for skewed, lottery like payoffs” that make stock options in riskier firms more valuable.

In my model, the volatility of a firm’s total output stems from the uncertainty about the quality of its employees. The same uncertainty makes promotion prospects risky. Because firm performance pay provides a hedge against uncertainty promotions, such pay is more valuable at firms with more volatile outputs. If the firm’s stock price is an increasing function of its output, then my model predicts that the use of stock option pay and high levels of idiosyncratic risk in a firm’s stock returns are positively linked via optimal contracting.

1.5.3 Valuation of Stock Option Pay

Non-executive employees tend to value the stock options in their compensations above fair-market value. For instance, [Hodge et al. \(2009\)](#) find that the lower-level managers surveyed would require an additional \$38,688 in cash wages to forgo stock options in their pay package with a Black-Scholes value of \$30,000 (see [Oyer and Schaefer 2005](#), [Hallock and Olson 2006](#) for similar findings). Most explanations for this observation also rely on relaxed assumptions about rationality such as optimism ([Hallock and Olson 2006](#), [Oyer and Schaefer 2005](#), [Bergman and Jenter 2007](#)), loss aversion ([Devers et al. 2007](#)), and inexperience ([Hallock and Olson 2006](#), [Hodge et al. 2009](#)).

The insurance property of firm performance pay provides a rationale for why a non-executive worker's subjective valuation of stock option pay may exceed fair-market value. The portfolios of these non-executive employees contain an asset that represents the risk-adjusted present value of future promotions. Because stock options in the company correlates negatively with this asset, employees are willing to accept a lower risk premium than investors in the market are for this financial instrument. This insurance motive may also explain why employees voluntarily invest in their own company's stock. For example, [Benartzi and Thaler \(2001\)](#) find that employees often buy shares in their employer for their retirement plans; moreover, these workers do not view equity in other firms as a substitute for stock in their own, suggesting that company stock play a role that is distinct from other equities.

The absence of this hedging motive also helps explain the opposite observation for senior executives. In stark contrast to the findings for non-executive employees, [Bettis et al. \(2005\)](#) estimate that corporate insiders, such as the CEO and board members, value the stock options in their compensation at about 20% less than Black-Scholes value (see [Hall and Murphy 2002](#) for similar evidence). Unlike non-executive employees, these individuals at the top of the hierarchy have little need for hedging uncertain advancement prospects within the

firm. For example, the CEO cannot be promoted again at the same company. Consequently executives, who are under-diversified in firm-specific risks relative to the marginal investor, value stock option pay at less than fair market.

1.5.4 Option-like Payoffs

The model predicts an option-like payoff for high-output workers at $t = 1$. One common form of firm performance pay involves the use of employee stock options (ESOs), which certainly feature option-like payoffs.¹¹ This feature can also be seen in year-end bonus programs with minimum thresholds such as a revenue target.

1.5.5 Performance-based Vesting

As discussed in Section 2, one key feature of firm performance pay in this model is vesting based on individual production. This property matches common observations. For instance, equity-type compensation for non-executive employees routinely follow a vesting schedule over a period of four to five years. Workers lose this equity component of their pay if they are terminated. For example, DoubleClick, an Internet advertising agency, famously fired many of its software developers for poor performance prior to its IPO in the late 1990s; these employees lost their unvested stock options (“As Options Spread, So Too Do Suits From Workers Fired Before They Vest”, 2000). Because firms can fire bad workers, one can think of vesting periods as an implicit form of performance-based vesting. Contracts that contain explicit forms of performance-based vesting are also common. For instance, compensation plans for non-executive employees routinely include year-end bonuses based on two multipliers, one calculated from accounting measures at the firm level and the other

¹¹According to the National Center of Employee Ownership, 8.5 to 13.4 million U.S. employees receive stock options in the company they work at as part of their compensation. <https://www.nceo.org/articles/statistical-profile-employee-ownership>

from their individual evaluation.

1.5.6 Additional Testable Implications

My model also generates a number of testable implications. First, it predicts a connection between a firm's organizational hierarchy (K), and the usage of firm performance pay. Numerical calculations from Section 1.4 suggest an inverted-U shaped relationship between the prevalence of firm performance pay and K . To the best of my knowledge, this prediction is unique to my framework.

Second, uncertain promotions in this framework translates into risk for talented employees because their skills are not fully transferable to other firms. Thus, the model predicts more firm performance pay at companies where there are more frictions that limit labor mobility.¹²

Third, the optimal contract of Proposition 2 features fixed wages for the high-put worker when firm output is below K , which also measures the number of promotions available. Consequently, the model predicts that the threshold for firm-performance pay should be positively associated with the availability of higher-level positions.

Finally, my framework assumes a strong link between measures of firm performance and the quality of workers. In industries where firm performance is largely driven by other stochastic factors such as the productivity of capital or macro-economic conditions, the use of firm performance pay as a form of insurance is unattractive because it induces significant basis risk. This implies that the above predictions should hold more strongly in industries where human capital plays out-sized roles in determining firm performance. To my knowl-

¹²For example, according to U.S. Treasury Department, nearly 30 million American workers are subject to non-compete agreements, which represent a substantial obstacle to labor mobility. The heterogeneity in the enforceability of such agreements at the state level and across time provides one possible test of the model's predictions. I thank Mark Kamstra for pointing this out.

edge, no paper has empirically tested these predictions.

1.6 Discussion

1.6.1 Tournament Incentives

Firm performance pay insures against unfavorable promotion prospects associated with tournament-style incentives. In my framework, the tournament arises because the production technology fixes the number of higher-level positions available at the worker's current company. Because talent is firm-specific, the high-type worker who is not promoted misses out on a pay raise. In a scenario where firms can grow without limits to accommodate talent (e.g. [Gibbons and Waldman 1999](#)) or where skills are fully transferable without frictions (e.g. [Harris and Holmstrom 1982](#), [Gibbons and Murphy 1992](#), [Holmstrom 1999](#)), workers revealed to be high-types always receive higher pay. In such a scenario, firm performance pay does not play an insurance role because workers do not face tournament-style competition.

However, there are limits to the firm's ability to accommodate talent with new divisions and product lines. For example, [Miller and Friesen \(1984\)](#), in their seminal work on the organizational life cycle, point out that mature firms "are conservative...engage in very few efforts at diversification or acquisition...and fail to even make many incremental changes to the products or services being offered" (see also [Lester et al. 2003](#)). Even when firms do not face bureaucratic hurdles, there are economic rationales for a rigid organizational structure. For example, [Levin and Tadelis \(2005\)](#) point out that when the quality of labor inputs strongly affect the value of a firm's goods and services, a fixed organizational structure can serve as a signal of quality. Consequently, the career trajectory of a high-output worker within a firm can be uncertain.

There are also frictions that hinder labor mobility. For instance, according to the U.S.

Department of the Treasury, nearly 30 million American workers are covered by non-compete agreements.¹³ Consequently, workers may be contractually prevented from capitalizing on their talent at a different firm. Moreover, a worker's individual performance evaluation at one firm is typically not observable by other potential employers. As a result, a high-type worker, who seeks greener pastures after being passed over for a promotion, may be confronted by a labor market for lemons. As long as these limitations lead to tournament-style incentives, then there is scope for insurance via firm performance pay.

1.6.2 Information Structure

My model features a very simple information structure. It assumes a continuum of workers. As a result, the firm's total production provides no information about any individual worker's effort. The production technology implies that a high-output worker is high-type with certainty. Consequently, the firm's total production contains no information about a worker's type over and above that contained in his individual output. These features help highlight the insurance role of firm performance pay and distinguish it from the informational one of [Holmstrom \(1979\)](#).

1.6.3 Uniqueness of Equilibrium Outcome

The optimal contracts in Section 3 only specify the worker's pay on the equilibrium path. The incentive compatibility conditions in that section assume that all other workers put in effort. A contract that satisfies those conditions ensure that a worker does not have the incentive to unilaterally deviate from the equilibrium path. However, depending on how the contract is structured off the equilibrium path, alternative equilibria exist.¹⁴ For

¹³<https://www.treasury.gov/resource-center/economic-policy/Documents/UST%20Non-competes%20Report.pdf>

¹⁴I thank Jungsuk Han, Mariassunta Giannetti, and Marcus Opp for their insights regarding multiple equilibria.

example, suppose the optimal first-stage contract from Section 3 pays the worker nothing when $Q \notin \{\theta_1, \dots, \theta_n\}$. Then, all workers shirking can also be an outcome. To induce effort from all workers as a unique equilibrium outcome, the optimal contract must also properly specify wages off the equilibrium path.¹⁵

1.6.4 Renegotiation

The paper assumes that the firm can commit to contract for the second stage of production at $t = 0$. Allowing for renegotiations at $t = 1$ does not alter the paper's main results.

Let P_0 be as defined in Lemma 1. When $P_0 \leq c_2^*$, the firm is indifferent between offering any $W(y_H, Y) \in [P_0, c_2^*]$. In these instances, wages paid to promoted workers for second-stage production replaces wages that the firm would have paid those workers for first-stage production in a stand-alone contract dollar for dollar. In other words, the corresponding optimal first-stage contract fully extracts the rent the firm yields to promoted workers. Allowing for renegotiation only rules out second-stage contracts that pay $W(y_H, Y) > P_0$. The second-stage contract that pays $\{W(y_L, Y), W(y_H, Y)\} = \{0, P_0\}$ remains optimal at $t = 1$.

When $P_0 > c_2^*$, the firm will not commit to paying a promoted worker more than P_0 for high output during the second stage of production. Payment above P_0 does not improve production in the second stage; it simply relaxes the promoted worker's incentive compatibility constraint. Lemma 4 implies that it is inefficient to use additional payments above P_0 to incentivize workers during first-stage production. Consequently, when $P_0 > c_2^*$, the optimal second-stage contract at $t = 1$ is identical to the one that the firm commits to at $t = 0$: $\{W(y_L, Y), W(y_H, Y)\} = \{0, P_0\}$.

¹⁵For example, if the optimal first-stage contract from Section 3 pays a promoted worker an arbitrarily large prize when $Q \notin \{\theta_1, \dots, \theta_n\}$, then it induces effort from all workers as a unique outcome.

1.7 Conclusion

This paper analyzes a simple principal-agent problem with tournament-style incentives that are common in the workplace. Workers compete against each other for promotions. The firm uses the pay increase associated with winning a promotion as an implicit incentive to induce effort from lower-level workers. However, workers are risk-averse and dislike the additional risk associated with uncertain promotions. Consequently, the firm optimally sets wages in a way that offsets the fluctuation in the likelihood of promotion. By accounting explicitly for these implicit tournament-style incentives, the optimal contract features firm performance pay to insure workers against uncertain promotion prospects.

In this paper's setting, the technological and organizational structures of the firm endogenously generate tournament-style incentives for its lower-level workers, which lead the optimal contract to feature broad-based firm performance pay. Such pay, in its various forms, make lower-level employees residual claimants to a portion of the firm's output. Empirical studies in the finance literature document many effects of broad-based equity pay on the firm's capital structure choice ([McKeon 2015](#)), its innovation and investment decisions ([Chang et al. 2015](#), [Babenko et al. 2011](#)), and its payout policy ([Babenko 2009](#)). Consequently, this paper highlights an under-explored connection between organizational economics and finance.

This paper focused on emphasizing the insurance role of firm performance pay. In doing so, many simplifying assumptions were made about the workers and the firm. The production technologies are simple and exogenously fixed to induce a two-tiered organizational hierarchy. The workers are ex-ante identical, which eliminates adverse selection issues during contracting. Talent in this framework is firm-specific, which limits competition between firms. One natural extension would relax these assumptions and explore the macroeconomic

implications of this insurance motive in a general equilibrium framework. In particular, the use of firm performance pay as an insurance device may result in less dispersion of payoffs for employees within an organization but lead to greater income disparity between firms.

A separate contribution of this paper is its study of optimal contracting under Epstein-Zin preferences. Firm performance pay can play an insurance role in the optimal contract only when workers prefer early resolution of uncertainty. This result suggests that further work in contracting under recursive preferences can uncover interesting economic forces overlooked by conventional studies using expected utility.

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Chapter 2

Fighting Fire with Fire: Mitigating Information Asymmetry with Open-Market Repurchase Programs

2.1 Introduction

Repurchase programs represent the dominant payout policy of U.S. firms; S&P 500 companies are estimated to pay \$780 billion to shareholders via buybacks in 2017 alone (“The Ugly Truth Behind Stock Buybacks,” 2017). Open-market repurchase programs represent over 90% of total repurchases ([Stephens and Weisbach \(1998\)](#)). The announcement of an open-market repurchase program does not oblige the firm to buy back any shares; the authorization merely grants it the option to do so over a specified period of time. Managers exercise tremendous discretion over the actual implementation of these programs.¹ Evidence suggests

¹While factors such as earnings and the availability of free cash may drive the level of repurchases over a long window, managers decide the timing of actual buys within that window (Cooke et al (2004),

that managers use this discretion and private information to execute buybacks at favorable prices (see [Brockman and Chung \(2001\)](#), [Ikenberry et al. \(2000\)](#), [Cook et al. \(2004\)](#), [Dittmar and Field \(2015\)](#)). Traditionally, the literature asserts that these informed buybacks induce an adverse selection trading cost that hurts shareholders. For instance, [Barclay and Smith \(1988\)](#) argue that “the increased trading activity in the secondary market by better-informed managers...reduces the liquidity of the firm’s shares, and thereby increases the firm’s cost of capital” (see [Brockman and Chung \(2001\)](#), [Oded \(2005\)](#) for similar arguments). This perspective makes the popularity of such programs puzzling.

This paper reassesses the impact of repurchase programs on adverse selection trading costs and shareholder welfare in a parsimonious Kyle (1985) micro-structure model with two informed parties: a manager who executes buybacks for the firm and a speculator who trades on her own account. Previous works crucially assume that only the firm is informed. However, evidence suggests that the firm competes against other informed parties over the spoils of private information (see [Ben-David and Roulstone \(2005\)](#), [Dittmar and Field \(2015\)](#), [Chemmanur et al. \(2016\)](#)). I show that in the presence of other speculators, the firm’s informed buybacks can mitigate information asymmetry rather than exacerbate it.

This framework features a firm with assets in place that deliver a random payoff, either high or low. In the first period, shareholders can authorize a repurchase program allowing the manager to buy back shares at his discretion. In the second period, the manager and the speculator learn about future payoffs of the asset and submit orders based on their private information. These orders, along with those submitted by noise traders, are settled by

Skinner (2008). Median program completion rates in the U.S. are estimated to be less than 50% ([Stephens and Weisbach \(1998\)](#), [Chemmanur et al. \(2016\)](#)). In a quarter of cases, no shares are repurchased at all ([Bhattacharya and Jacobsen \(2015\)](#)). The completion rates internationally tend to be even lower. For instance, [Rau and Vermaelen \(2002\)](#) estimate the completion rate for U.K. firms to be 37%. [Ikenberry et al. \(2000\)](#) puts the figure for firms listed on the Toronto Stock Exchange at 28.6%

Kyle (1985) market-makers. In the final period, the payoff from underlying assets becomes public information, trading accounts are settled, and the firm's proceeds are distributed to remaining shareholders.

The repurchase program has two main effects on the liquidity of the firm's shares. First, it intensifies the competition for trading profits. Absent an authorization, the speculator exercises monopoly powers over private information and profits accordingly. Once a buyback is approved, competition between two informed parties leads to an increase in total informed trading. This increase results in a more revealing aggregate order flow, which mitigates the effects of information asymmetry.

Second, the firm's buyback alters the distribution of per-share payouts. Unlike speculators who trade on their own accounts, the manager repurchases stocks on behalf of the firm. The repurchase authorization limits the manager to the buy side of transactions and trading gains from these buybacks accrue to the remaining shareholders. When asset payoffs are low, the manager does not trade and the firm's per-share value remains low. When asset payoffs are high, the manager buys back undervalued shares and elevates the per-share value of the firm due to trading gains. This increased dispersion in the firm's per-share liquidation value across different payoffs states makes informed trading more profitable, dampening and at times overwhelming the mitigating effect of increased competition on information asymmetry.

This paper relates primarily to two strands in the literature. One deals with open-market repurchase programs as a payout policy. Many explanations have been offered for these programs' popularity such as favorable tax treatment ([Grullon and Michaely \(2002\)](#), [Moser \(2007\)](#)), financial flexibility ([Stephens and Weisbach \(1998\)](#), [Guay and Harford \(2000\)](#), [Jagannathan et al. \(2000\)](#), [Dittmar and Dittmar \(2008\)](#), [Bonaime et al. \(2013\)](#)), reduction

in agency problems (Oded (2011); Caton et al. (2016)), and signaling (Constantinides and Grundy (1989), Oded (2005), Bhattacharya and Jacobsen (2015)). Most of these works do not directly address the adverse selection cost, implicitly assuming that shareholders begrudgingly accept it in order to reap other benefits. One notable exception is Oded (2005), which relies on the differences in adverse selection costs across types as a signaling mechanism. In almost all instances, works in this area think about the impact of informed buybacks in a Barclay and Smith (1988) framework, in which the manager's private information worsens adverse selection. This paper contributes to this literature by rethinking the effect of informed buybacks on liquidity trading costs. With multiple informed parties, repurchases can soften the impact of existing informed trading on liquidity traders. This competition effect is intuitive but absent in previous papers on buybacks because they usually assume that only the manager is informed. A recent work by Bhattacharya and Jacobsen (2015) also features another informed trader in addition to the firm but describes a different mechanism. My analysis suggests that shareholders might authorize the manager to repurchase shares *because* he is informed, not in spite of it. Consistent with this idea, Singh et al. (1994) find that buyback authorizing firms experience an increase in adverse selection trading costs prior to the announcement and a decrease after.

The other related line of inquiry concerns informed trading in a Kyle (1985) framework with multiple informed players. These studies conclude additional informed traders mitigate information asymmetry in a variety of rich settings (see Admati and Pfleiderer (1988), Holden and Subrahmanyam (1992), Back et al. (2000)). This result stems from the increased competition between multiple informed traders. In these previous models, this competition effect completely captures the change in liquidity trading costs due to an additional speculator. Because my framework features the firm as an informed trader, there is also a dispersion effect, unique to this setting, that sometimes dominates the effects of competition. That the

addition of a second informed trader can increase liquidity trading costs stands in sharp contrast to these previous works with multiple informed traders.

The interplay between the competition and dispersion effects helps explain two stylized facts about repurchase programs. First, the size of buybacks tend to be substantial (see [Jagannathan and Stephens \(2003\)](#), [Hillert et al. \(2016\)](#)). For small repurchase programs, the dispersion effect dominates and adverse selection worsens. Intuitively, small buybacks have a negligible effect on the informativeness of the aggregate order flow because the speculator optimally adjusts her strategy in response to the increased informed trading by the firm. However, even small buybacks increase the dispersion of per-share payouts, making informed trading more profitable and liquidity trading more costly.

Second, aggregate repurchases tend to be pro-cyclical. This fact leads many to doubt that these programs improve shareholder welfare. For instance, a recent article in the Harvard Business Review counters the notion that “buybacks are investments in...undervalued shares” by noting that “major U.S. companies have tended to do buybacks in bull markets and cut back on them, often sharply, in bear markets” (“Profits Without Prosperity,” 2014). After all, firms are less likely to be undervalued in good times. However, this paper’s analysis suggests that it is precisely this fact that justifies repurchase programs in those periods. When trading profits from informed buys are low, the dispersion effect is dampened. Consequently, repurchase programs are more likely to reduce liquidity trading costs during booms.

Moreover, many works have empirically examined the impact of repurchase programs on liquidity with mixed results. Some find that adverse selection worsens after a buyback authorization (see [Barclay and Smith \(1988\)](#), [Brockman and Chung \(2001\)](#), [Ginglinger and Hamon \(2009\)](#)), while others document the opposite (see [Franz et al. \(1995\)](#), [Singh et al. \(1994\)](#), [Wiggins \(1994\)](#), [Hillert et al. \(2016\)](#)). The predictions of this model suggest that these

contradictory findings may be reconciled by considering the size of buybacks and aggregate market conditions in the samples of those studies.

2.2 The Model

Consider a world with three dates ($t = 0, t = 1, t = 2$), no inter-temporal discounting, no taxes, no transaction costs, and risk-neutral economic agents (shareholders, manager, speculator). The firm is entirely financed by equity and has one share. It has assets in place that generates a high cash flow A_H with $\rho \in [\frac{1}{2}, 1)$ and a low cash flow A_L with probability $1 - \rho$. For algebraic simplicity, these payoffs are normalized to 1 and 0, respectively. At $t = 0$, the firm's shareholders announce a repurchase program authorizing the manager to buy back α shares on behalf of the firm at his discretion.² The main body of the paper assumes that a contract in place incentivizes the manager to buy back exactly α shares when he knows they are under-priced and nothing otherwise. This assumption is consistent with the empirical evidence that in most cases, firms either buy up to the full authorized amount or not at all (Stephens and Weisbach (1998)). Appendix A discusses a modification in which the manager is fully strategic with the authorization serving only as an upper bound on buyback amount; the main results are qualitatively similar. An announcement with $\alpha = 0$ corresponds to having no repurchase program. In contrast to signaling models of buybacks, the authorization conveys no information about the fundamental value of the firm's assets because it takes place prior to the manager learning about future cash flows. This assumption is consistent with empirical findings. For instance, Grullon and Michaely (2002) find no evidence of future operational improvements, such as profitability, for buyback

²The paper abstracts away from issues stemming from financial constraints by assuming that firm has sufficient access to credit to complete any repurchase program. Alternatively, one can endow the firm with a sufficient amount of cash, i.e. A_L big enough before normalization, to carry out buybacks.

Table 1: Summary of Model Timing	
t=0	Firm announces repurchase program size α
t=1	Informed parties learn about the firm's t=2 cash flow Informed and noise traders simultaneously submit orders Market-makers set a price based on aggregate order flow and clear the market
t=2	The firm's t=2 cash flow becomes public information Accounts are settled The firm is liquidated and proceeds distributed to remaining shareholders

announcing firms (see [Jagannathan and Stephens \(2003\)](#) for similar findings).

At $t = 1$, the manager and a speculator learn about the firm's cash flow at $t = 2$. These informed parties condition the orders they submit on this information. For simplicity, the speculator is also limited to the buy side of the transaction.³ Noise traders submit orders of $-\frac{1}{2}$, 0, and $\frac{1}{2}$ with probabilities x , $1 - 2x$, and x , respectively. The parameter $x \in [\frac{1}{4}, \frac{1}{3})$ captures the volatility of noise trade.⁴ These noise trades are uncorrelated with the firm's cash flow at $t = 2$. The manager is prevented from trading on his own account due to restrictions on insider trading. All traders, noise and informed, submit market orders that are settled in a Kyle (1985) framework. Deep-pocketed market-makers competitively set a price, conditioned on the observed aggregate order flow q , that allows them to serve as the counter-party to any aggregate order imbalance and break even in expectation. At $t = 2$, the firm's final cash flow becomes public information. The market-makers then settle accounts and the firm is liquidated with proceeds going to its remaining shareholders. The timing of the model is summarized in Table 1.

As in Kyle (1985), the equilibrium of this trading game is characterized by a pricing rule $P(q)$ and a trading strategy \tilde{m} , which assigns probabilities over non-negative order quantities conditional on the speculator's private information such that the following conditions hold:

³Allowing the speculator to short-sell complicates the analysis because it introduces additional equilibriums, but does not qualitatively change the main results.

⁴The restriction on the parameter x ensures that the pricing rule is monotonic and that the equilibrium is unique.

1. *Market Efficiency*: The market-makers' pricing rule satisfies:

$$P(q) = E_1[V(q, \alpha)|q, \tilde{m}, \alpha]$$

where $V(q, \alpha)$ is the per-share liquidation value of the firm.

2. *Profit Maximization*: For any alternative trading strategy m' and for any realization of $\tilde{A} = A$:

$$E_1[a(V(q, \alpha) - P(q))|A, \tilde{m}, \alpha] \geq E_1[a(V(q, \alpha) - P(q))|A, m', \alpha]$$

These conditions imply that the market-makers set the price at the expected per share value of the firm at liquidation conditioned on the buyback authorization α , the observed aggregate order flow q , and the presumed trading strategy \tilde{m} . The trading strategy \tilde{m} maximizes the expected profits of the informed trader conditioned on the size of the buyback authorization α , the market-makers' pricing rule $P(q)$, and her private information about \tilde{A} .

2.3 The Trading Game

This section analyzes the effects of the firm's informed buys on the outcomes of the trading game at $t = 1$. I begin by describing this game's equilibrium given a repurchase authorization of α . Because neither the firm nor the speculator trades when the firm's payoffs are low, the possible order flows in those instances are $-\frac{1}{2}$, 0 , and $\frac{1}{2}$. As a result, a realized order flow $q \notin \{-\frac{1}{2}, 0, \frac{1}{2}\}$ shows that the firm's payoffs are high. Given an authorized buyback of size α , if the speculator submits an order $a \notin \{\frac{1}{2} - \alpha, 1 - \alpha\}$, the resulting order flow reveals her private information with certainty and she earns zero trading profits. Thus, $\{\frac{1}{2} - \alpha, 1 - \alpha\}$

represents the complete set of pure strategies that can lead to positive trading profits in expectation. Let m and $1 - m$ be the probabilities that the speculator submits an order of $1 - \alpha$ and $\frac{1}{2} - \alpha$, respectively, upon learning that the firm's payoffs are high. Because $a = \frac{1}{2} - \alpha$ and $a = 1 - \alpha$ are the only relevant order sizes for the speculator, m summarizes her equilibrium strategy.

Lemma 5. *There exists $\{\underline{\alpha}(\rho, x), \bar{\alpha}(\rho, x)\}$ with $0 < \underline{\alpha} < \bar{\alpha} < \frac{1}{2}$ such that the unique equilibrium of the trading games is given by a trading strategy⁵:*

$$m(\alpha) = \begin{cases} 0 & \alpha \leq \underline{\alpha} \\ \frac{(1-\alpha)(1-\rho)(1-2x)[\alpha(1-2x)-\frac{1}{2}(1-3x)]+\rho[\alpha(1-3x+x^2)-\frac{1}{2}(1-3x)]}{\rho[\alpha(1-x)(1-3x)-\frac{1}{2}(1-4x+2x^2)]} & \alpha \in (\underline{\alpha}, \bar{\alpha}) \\ 1 & \alpha \geq \bar{\alpha} \end{cases}$$

and a pricing rule

$$P(q, m(\alpha), \alpha) = \begin{cases} 0 & q = -\frac{1}{2} \\ \frac{\rho(1-m)x}{(1-\alpha)(1-\rho)(1-2x)+\rho(1-m)x} & q = 0 \\ \frac{\rho(1-2x-m(1-3x))}{(1-\alpha)(1-\rho)x+\rho(1-2x-m(1-3x))} & q = \frac{1}{2} \\ 1 & q \notin \{-\frac{1}{2}, 0, \frac{1}{2}\} \end{cases}$$

The case of $\alpha = 0$, corresponding to no repurchase authorization, serves as a benchmark. In this scenario, the speculator exercises monopoly power over informed trading, optimally buying $\frac{1}{2}$ shares when asset payoffs are high. When the repurchase authorization is sufficiently small, $\alpha \leq \underline{\alpha}$, the speculator accommodates the firm's entry with trading strategy $m = 0$, which involves submitting an order of $\frac{1}{2} - \alpha$. This strategic interaction keeps the

⁵Closed form expressions for $\underline{\alpha}(\rho, x)$ and $\bar{\alpha}(\rho, x)$ are given in the proof of this lemma. Uniqueness here refers to the speculator's strategy and the pricing rule on the equilibrium path. The market-makers may have other off-equilibrium beliefs that change the pricing rule for when $q \notin \{-\frac{1}{2}, 0, \frac{1}{2}, 1\}$ that still support these outcomes.

total amount of informed trading fixed at $\frac{1}{2}$ for small buyback programs. When the size of the buyback is large, $\alpha \geq \bar{\alpha}$, the speculator responds with a trading strategy of $m = 1$, which involves submitting an order of $1 - \alpha$ and increases the total amount of informed trading to 1. For a repurchase program of moderate size, $\alpha \in (\underline{\alpha}, \bar{\alpha})$, no pure strategy equilibrium exists. In these instances, the only equilibrium involves a mixed strategy in which the speculator plays $a = 1 - \alpha$ and $a = \frac{1}{2} - \alpha$ with probabilities m and $1 - m$, respectively.

2.3.1 Competition Effect

There are two main economic forces in this setting. The first involves competition. The following lemma shows that the speculator responds more aggressively to larger repurchase programs.

Lemma 6. *m is strictly increasing in α for $\alpha \in (\underline{\alpha}, \bar{\alpha})$*

Because in equilibrium the speculator submits an order of $1 - \alpha$ with probability m and $\frac{1}{2} - \alpha$ with probabilities $1 - m$, her strategy m provides a sufficient statistic for the expected amount of total informed trading in equilibrium.

$$E[a^* + \alpha] = \frac{1}{2}(1 - m) + m = \frac{1}{2} + \frac{1}{2}m \quad (2.1)$$

Therefore, Lemma 6 implies that when buybacks are sufficiently large, $\alpha > \underline{\alpha}$, the speculator, together with the firm, submits a larger order than she would alone. This increased trading pours additional information into the order flow. Let $\phi(m, \alpha)$ be the probability that the realized order flow fully reveals the firm's future cash flow. Lemma 5 implies that this

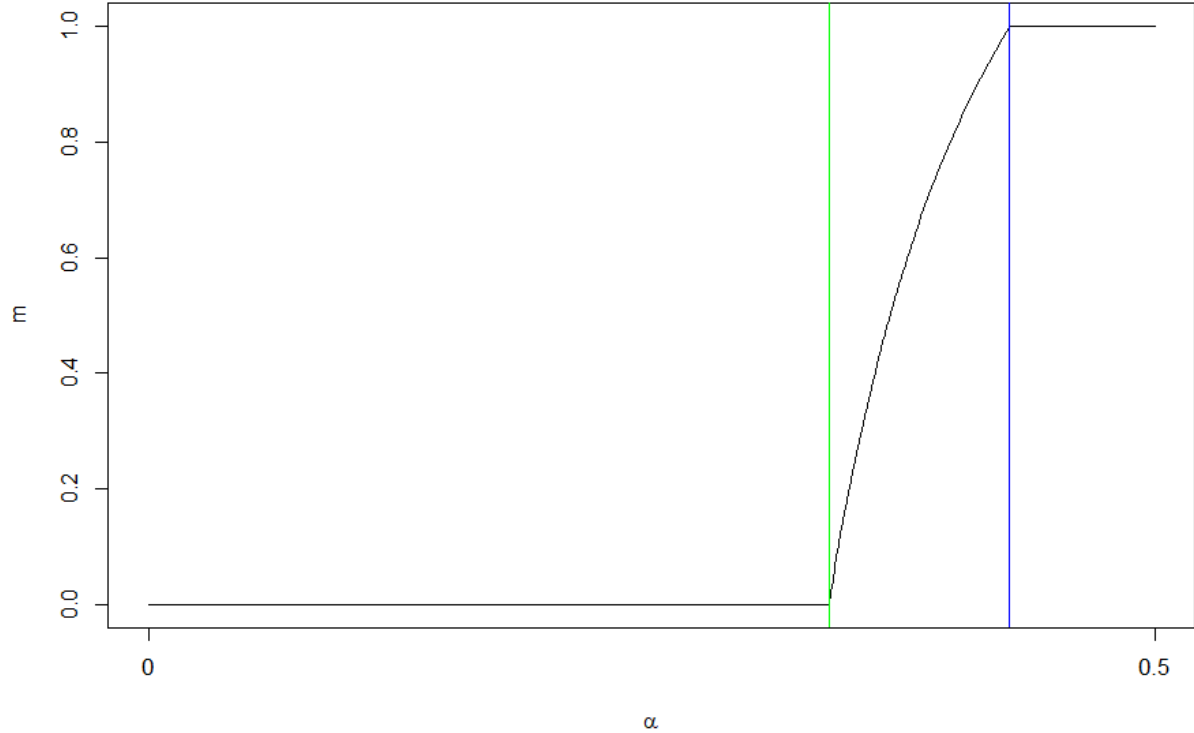


Figure 2.1. Speculator's strategy. This figure plots an example of the speculator's best response given a repurchase authorization of size α . When $m = 0$, the speculator's best response is the pure strategy $a^* = \frac{1}{2} - \alpha$. When $m = 1$, her best response is the pure strategy $a^* = 1 - \alpha$. The green and blue lines mark $\alpha = \underline{\alpha}$ and $\alpha = \bar{\alpha}$, respectively.

measure can be written as:

$$\phi(m, \alpha) = \begin{cases} x + \rho(1 - 2x)m & \alpha < \bar{\alpha} \\ (1 - x)m & \alpha \geq \bar{\alpha} \end{cases} \quad (2.2)$$

Equation (2.2) shows that the informativeness of the order flow increases in m , which is itself a weakly increasing function of α according to Lemma 6. Consequently, the authorization of a buyback weakly increases the informativeness of the order flow. The magnitude of this increase is given by:

$$\Delta\phi = \phi(m(\alpha), \alpha) - \phi(0, 0) = (1 - 2x)m \quad (2.3)$$

Equation (2.3) shows that the magnitude of the increase depends on the parameter x , which captures the volatility of noise trade. As x declines, non-information based trades becomes less noisy and makes additional informed trading more revealing. These features of the model are broadly consistent with previous papers studying imperfect competition between multiple informed traders in a Kyle (1985) framework (see [Admati and Pfleiderer \(1988\)](#), [Holden and Subrahmanyam \(1992\)](#), [Foster and Viswanathan \(1993\)](#), [Back et al. \(2000\)](#)).

2.3.2 Dispersion Effect

The second economic force in this setting deals with the firm's per-share value in different payoff states and is novel to this setting. To highlight this mechanism, consider the change in the per-share profitability of informed trading relative to the benchmark for a small α . Let $M(q, a, \alpha)$ be the equilibrium profit margin for a realized order flow q when the speculator and a second informed trader submit orders a and α , respectively.

$$M(q, a, \alpha) = V(q, a, \alpha | \tilde{A} = 1) - P(q, a, \alpha) \quad (2.4)$$

Let $\hat{\rho}(q)$ be the market-makers' posterior on the probability that $\tilde{A} = 1$ conditioned on observing total order flow q . Market efficiency implies that the profitability measure can be rewritten as:

$$M(q, a, \alpha) = [1 - \hat{\rho}(q)] * V(q, a, \alpha | \tilde{A} = 1) \quad (2.5)$$

When α is sufficiently small, strategic adjustments by the first speculator keeps total informed trading constant in equilibrium, negating the impact of the second informed party's trading on $\hat{\rho}(q)$. When both traders are non-firm speculators transacting using private accounts, as generally assumed in previous multi-trader Kyle models, trading activity alters the composition of shareholders, but does not directly affect $V(q, a, \alpha | \tilde{A})$, the per-share liq-

uidation value of the firm.⁶ Thus, small amounts of informed trading by a second non-firm speculator leaves the profitability of informed trading unchanged.

In contrast, a small stock buyback increases this margin. When $\tilde{A} = 0$, the manager refrains from repurchasing the overvalued shares. The firm's per-share value in this instance is simply 0. When $\tilde{A} = 1$, the manager buys back undervalued shares. The resulting trading gains accrue to remaining shareholders, increasing $V(q, a, \alpha | \tilde{A} = 1)$. This increase in the dispersion of the firm's per-share value across different payoff states makes informed trading more profitable.

It is worth noting that this dispersion effect does not depend on the manager being privately informed nor on restrictions against informed selling. Suppose buybacks were uninformed. When asset payoffs are high, the repurchase of undervalued shares still results in trading gains that increase per-share value. When asset payoffs are low, the program incurs trading losses from the buyback of overvalued shares, which further lowers per-share value. The implementation of a repurchase program without private information magnifies the dispersion effect and makes informed trading even more profitable. If firms were allowed to sell over-valued shares, via shelf registration for example, the model would still feature a dispersion effect as long as the firm realizes more trading profits per-share when asset payoffs are high than when they are low. This is likely because informed selling by the firm dilutes trading profits while informed buying concentrates them. Thus, the firm's trading does not need to be informed nor entirely one sided to generate a dispersion effect.

The main outcome of interest in this setting is the ease with which uninformed traders

⁶Financial feedback models are exceptions. For instance, in [Goldstein and Guembel \(2008\)](#), the manager extracts information from prices and uses that information to make real decisions. When misinformation becomes incorporated in prices, the manager makes suboptimal decisions, which lowers payouts. See [Bond et al. \(2012\)](#) for a survey of feedback models. The mechanism in feedback models differs from that in this analysis because the manager in this setting does not make any real investment decisions following the trading game. The change in the distribution of payouts to shareholders in this analysis stems entirely from realized trading profits.

can buy or sell shares during the trading game. The binary payoffs and discrete noise trades make this model tractable; however, these features do not yield a convenient measure of liquidity such as Kyle's λ in the Gaussian version of that model. Instead, I will focus on liquidity trading costs. In the static Gaussian version of the Kyle model with multiple informed traders, liquidity trading costs are positively related to Kyle's λ (see [Subrahmanyam \(1991\)](#)). Let $L(\alpha)$ be defined as the expected trading losses of noise traders due to adverse selection. Lemma 5 implies that this quantity can be expressed as:

$$L(\alpha) = \frac{1}{2}(1-m) \left[\frac{\rho(1-\rho)x(1-2x)}{(1-\alpha)(1-\rho)(1-2x) + \rho(1-m)x} + \frac{\rho(1-\rho)x(1-2x)}{(1-\alpha)(1-\rho)x + \rho(1-2x - m(1-3x))} \right] \\ + m \left[\frac{\rho(1-\rho)x^2}{(1-\alpha)(1-\rho)x + \rho(1-2x - m(1-3x))} \right] \quad (2.6)$$

where m is as defined in Lemma 5.

Proposition 4. *Liquidity trading costs are increasing in α for $\alpha < \underline{\alpha}$.*

When the repurchase program is small, the strategic response of the speculator keeps the informativeness of the aggregate order flow at the same level as in the benchmark, muting the competition channel.⁷ However, the dispersion effect still operates, increasing the profitability of informed trading. Consequently, total informed trading profits increase. Because market-makers break even in expectation, these additional trading gains must come at the expense of liquidity traders.

When buybacks are large, this dispersion effect is offset by the benefits of increased

⁷That the response of the speculator keeps the total informed trading constant for small repurchase programs is an artifact of discrete noise trading. However, the result of this proposition does not rely on this assumption. In general, in the absence of informed buybacks, the speculator's strategy exactly balances the benefits of a larger position against the drawbacks of a more informative order flow. Consequently, the increase in the informativeness of the aggregate order flow due to a small increase in total informed trading only has a negligible effect on trading profits.

competition. For ease of exposition, I focus on $\alpha = \frac{1}{2}$, the supremum of $\bar{\alpha}(\rho, x)$, in the following analysis. However, the results of this section hold qualitatively for any large repurchase program of size $\alpha \geq \bar{\alpha}(\rho, x)$. Appendix B discusses the impact on liquidity of a moderately sized repurchase program with $\alpha \in (\underline{\alpha}, \bar{\alpha})$. Let ΔL be the change in liquidity trading costs induced by a large repurchase program ($\alpha = \frac{1}{2}$) relative to the benchmark of no buybacks ($\alpha = 0$). Equation (2.6) implies that ΔL can be written as:

$$\Delta L = L(1/2) - L(0) = \frac{x\rho(1-\rho)}{2} \left[\frac{4}{1+\rho} - \frac{1-2x}{(1-\rho)(1-2x)+\rho x} - \frac{1-2x}{(1-\rho)x+\rho(1-2x)} \right] \quad (2.7)$$

The competition effect lowers liquidity trading costs while the dispersion effect increases them. The net effect of a large repurchase program depends on the relative magnitude of these two forces. The former acts through the informativeness of the aggregate order flow. The latter operates through trading profits from buybacks which increase the expected per-share value of the firm when asset payoffs are high. Let ΔV be the expected incremental per-share value due to the trading profits of a large repurchase program when asset payoffs are high. Lemma 5 implies that ΔV can be written as

$$\Delta V = \frac{2-\rho}{1+\rho}x \quad (2.8)$$

Equation (2.8) shows that this incremental value is increasing in x , the volatility of noise trade. As noted earlier, the parameter x proxies for the volatility of noise trades. Noisier non-information based trading provides additional camouflage for informed buybacks and increase their profitability. Additional trading gains make the expected per-share value of the firm higher. Equation (2.8) also implies that the increase in per-share value declines in ρ , the unconditional probability that firm payoffs are high. When the aggregate order is not fully revealing, the market-makers' posterior on the probability that $\tilde{A} = 1$ is strictly

increasing in ρ , implying that the cost of buybacks increases in ρ . Larger values of ρ are associated with lower trading profits for the firm in high payoff states. Because the dispersion effect stems from buyback profits increasing the per-share firm value when payoffs are high, the magnitude of this effect is positively related to the size of x and negatively related to that of ρ . These relationships allow for the characterization of the conditions under which large buybacks ($\alpha = \frac{1}{2}$) lower liquidity trading costs.

Proposition 5. *For a given value of ρ , the unconditional probability that firm payoffs are high, a large repurchase program lowers liquidity trading costs if and only if x , the volatility of noise trade, is sufficiently small. For a given value of x , a large repurchase program lowers liquidity trading costs if and only if ρ is sufficiently high.*

Proposition 5 presents necessary and sufficient conditions under which large buybacks reduce adverse selection during the trading game. It is important to indicate that these conditions are not always met. Figure 2.2 identifies the region where even a large repurchase program increases liquidity trading costs. The same rationale from the above discussion applies here. Smaller values of ρ are associated with greater trading profits for the firm in high-payoff states, increasing the dispersion effect. Larger values of x dampen the effects of competition by reducing the increase in informativeness due to additional informed trading. When ρ is small and x is large, the dispersion effect dominates. In these instances, the addition of another equally informed trader increases total trading profits.

This result stands in sharp contrast to previous works with multiple informed traders in a Kyle (1985) framework. For example, Back et al (2000) study competition among informed traders with imperfectly correlated signals and find that “in all cases [with different numbers of informed traders, and varying degrees of correlation], competing informed traders earn lower profits than would a monopolist.” [Admati and Pfleiderer \(1988\)](#) consider a similar scenario with equally informed parties and note that “the presence of more informed traders

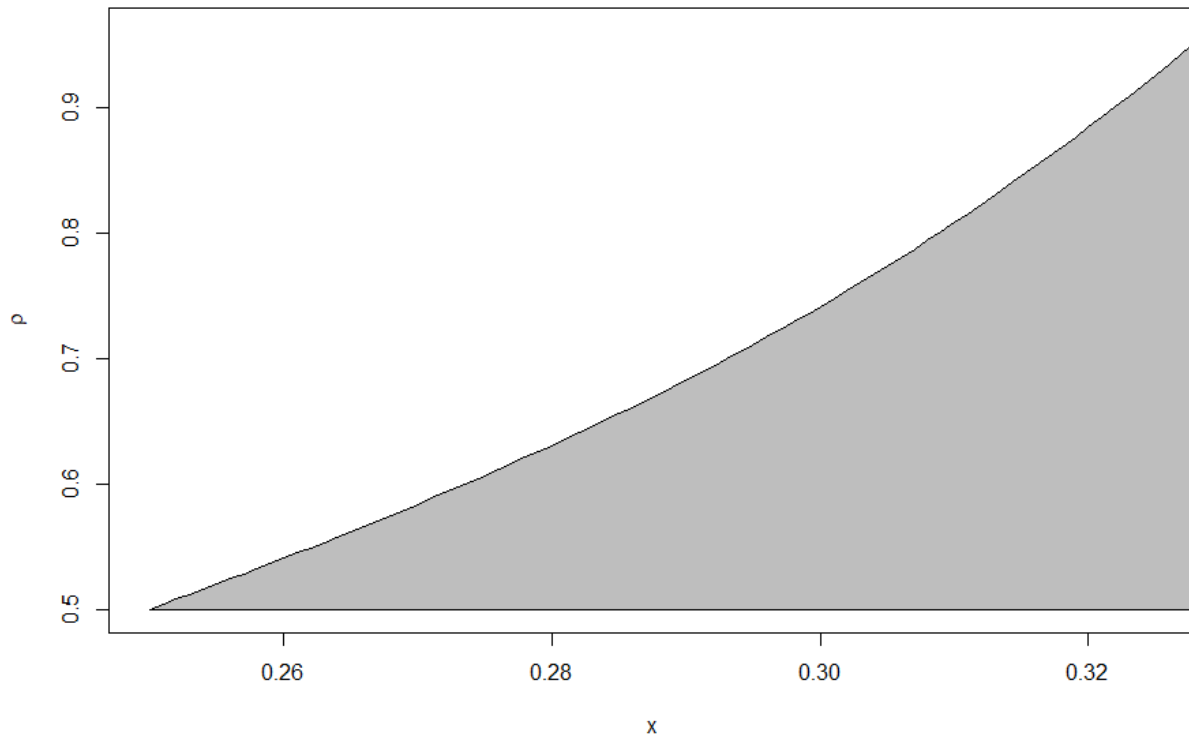


Figure 2.2. Region where buybacks worsen liquidity. This shaded area in this figure represents the parameter space over which even a large buyback program increase liquidity trading costs.

[when there is already at least one], actually *lowers* the liquidity traders’ cost of trading, intensifying the forces towards concentration of trading.” The main driver of this result in previous works is aggressive competition. As [Holden and Subrahmanyam \(1992\)](#) point out, when there are multiple informed traders, “the unique Nash equilibrium is an equilibrium in which imperfect competitors acting non-cooperatively choose larger quantities than a monopolist (or collusive agents) would choose.” As a result, too much private information is revealed through informed trading.⁸ In these previous works, this competition effect completely captures the change in total profits due to an additional informed trader. This

⁸These studies typically extend the dynamic version of the Kyle model in much richer settings; however, the conclusion that additional informed traders drive down total informed trading profits due to competition can be shown to also apply to the simple static version.

paper's framework, however, features informed trading by the firm, which adds another force that offsets and sometimes dominates the one from competition.

2.4 Better-informed Manager

The previous section studied a scenario in which the manager and the speculator are equally informed about the firm's future cash flow. In such a setting, informed buybacks induce more competition but also increases the dispersion of the firm's per-share payouts. However, one might expect the firm's management to have more precise information than the speculator. The relative imprecision of the speculator's information adds another aspect to this trade-off. When the manager knows more than the speculator does, the repurchase authorization also introduces new private information into the trading game, which may worsen adverse selection. This section analyzes the impact of this asymmetry.

I alter some assumptions from the previous section about the information structure. The manager retains his possession of perfect information about the firm's cash flow at $t=2$. However, when asset payoffs are high, the speculator only obtains this information with probability $1 - \gamma$. When asset payoffs are low, the speculator simply receives no signal. In this setup, the speculator does not always become informed, but is never wrong when she is. This parameter γ captures the manager's information advantage over the speculator. Because the speculator will only trade after receiving a signal of high payoffs, one can also think of $1 - \gamma$ as the probability of informed trading by non-firm actors. For simplicity, I limit the board to authorize buybacks of size $\alpha \in \{0, \frac{1}{2}\}$. The remainder of the model as well as the trading game's equilibrium concept are left intact. I begin by establishing the equilibrium of the trading game under this information structure.

Lemma 7. *The unique pure strategy equilibrium of the trading game when $\alpha = 0$ is given by $m = 0$ and a pricing rule:*

$$P(q) = \begin{cases} \frac{\rho\gamma}{\rho\gamma+1-\rho} & q = -\frac{1}{2} \\ \frac{\rho[(1-\gamma)x+\gamma(1-2x)]}{\rho[(1-\gamma)x+\gamma(1-2x)]+(1-\rho)(1-2x)} & q = 0 \\ \frac{\rho[(1-\gamma)(1-2x)+\gamma x]}{\rho[(1-\gamma)(1-2x)+\gamma x]+(1-\rho)x} & q = \frac{1}{2} \\ 1 & \text{otherwise} \end{cases}$$

The unique pure strategy equilibrium when $\alpha = \frac{1}{2}$ is given by $m = 1$ and a pricing rule:

$$P(q) = \begin{cases} 0 & q = -\frac{1}{2} \\ \frac{\rho\gamma x}{\frac{1}{2}(1-\rho)(1-2x)+\rho\gamma x} & q = 0 \\ \frac{\rho[(1-\gamma)x+\gamma(1-2x)]}{\frac{1}{2}(1-\rho)x+\rho[(1-\gamma)x+\gamma(1-2x)]} & q = \frac{1}{2} \\ 1 & \text{otherwise} \end{cases}$$

The equilibrium with $\alpha = 0$ under this information structure features a less revealing aggregate order flow than the one in the benchmark from Section 2 because the speculator is less informed. In particular, market-makers can no longer perfectly infer when asset payoffs are low. Lemma 7 implies that the liquidity trading costs can be written as:

$$L_\gamma(\alpha) = \begin{cases} \frac{1}{2}\rho(1-\rho)x(1-2x) \left[\frac{1-\gamma}{\rho[(1-\gamma)x+\gamma(1-2x)]+(1-\rho)(1-2x)} + \frac{1-\gamma}{\rho[(1-\gamma)(1-2x)+\gamma x]+(1-\rho)x} \right] & \alpha = 0 \\ \frac{1}{2}\rho(1-\rho)x \left[\frac{\gamma(1-2x)}{\frac{1}{2}(1-\rho)(1-2x)+\rho\gamma x} + \frac{\gamma(1-2x)+2x(1-\gamma)}{\frac{1}{2}(1-\rho)x+\rho[(1-\gamma)x+\gamma(1-2x)]} \right] & \alpha = \frac{1}{2} \end{cases} \quad (2.9)$$

Lemma 8. *In the absence of a buyback program, liquidity trading costs increase with the precision of the speculator's private information.*

Without a buyback authorization, the speculator is the only informed trader. An increase in the precision of her private information increases adverse selection during the trading game and results in higher liquidity trading costs.

Let ΔL_γ be the change in expected liquidity trading costs due to the firm's informed buybacks. Equation (2.9) implies that ΔL_γ can be expressed as:

$$\begin{aligned} \Delta L_\gamma &= L_\gamma(1/2) - L_\gamma(0) = \\ &\frac{1}{2}\rho(1-\rho)x \left[\frac{\gamma(1-2x)}{\frac{1}{2}(1-\rho)(1-2x) + \rho\gamma x} + \frac{\gamma(1-2x) + 2x(1-\gamma)}{\frac{1}{2}(1-\rho)x + \rho[(1-\gamma)x + \gamma(1-2x)]} \right. \\ &\left. - \frac{(1-\gamma)(1-2x)}{\rho[(1-\gamma)x + \gamma(1-2x)] + (1-\rho)(1-2x)} - \frac{(1-\gamma)(1-2x)}{\rho[(1-\gamma)(1-2x) + \gamma x] + (1-\rho)x} \right] \quad (2.10) \end{aligned}$$

Proposition 6. *The change in liquidity trading costs due to the firm's informed buybacks is increasing in γ .*

When the manager is more informed than the speculator, the repurchase authorization introduces new private information, which worsens the adverse selection problem. The amount of new private information added increases with the manager's information advantage, γ . Consequently, buybacks are more likely to reduce liquidity trading costs when γ is small.

Proposition 6 implies that when the manager holds a significant information advantage, repurchase programs increase liquidity trading costs, regardless of the probability of high payoff, buyback size, and volatility of noise trading. In these instances, the impact of a repurchase program on the trading game's outcome stems mainly from the introduction

of more private information, which worsens adverse selection. This result concurs with conventional thinking about informed buybacks. This implication is unsurprising given that the main innovation of this framework vis-a-vis others analyzing informed buybacks is the addition of another informed trader.

2.5 Patterns of Repurchases

Sections 2 and 3 analyze the outcomes of the trading game given a repurchase authorization of size α . This discussion relates the results of these previous sections with the observed patterns of buybacks, taking as a given that the firm acts to minimize the trading costs associated with adverse selection.⁹

Proposition 1 predicts that small buybacks increase liquidity trading costs. This proposition helps explain the substantial size of most repurchase programs, with the typical authorization allowing for the acquisition of 6 to 8 percent of shares outstanding (Jagannathan and Stephens (2003), Hillert et al. (2016)).

Proposition 5 predicts that large buybacks lower liquidity trading costs only when ρ , the probability that firm payoffs are high, is large enough. This proposition implies that the firm follows a simple cutoff rule: authorize a buyback if and only if ρ exceeds some threshold. In good times, the distribution of these probabilities shifts to the right, resulting in more firms meeting the cutoff. Thus, Proposition 5 predicts higher aggregate levels of authorizations

⁹For example, a survey of U.S. firms finds that "executives think that reduced liquidity can hurt their stock price...[and] a company would restrict repurchases if it feels that doing so would reduce liquidity below some critical level" (Brav et al. (2005)). Indeed, empirically, improved liquidity of a company's stock conveys many benefits such as lower costs of capital (Amihud and Mendelson (1986)), more efficient contracting (Fang et al. (2009)), and larger takeover premiums when acquired in M&A (Massa and Xu (2013)). As Pagano and Röell (1996) point out, empirical research suggests that adverse selection is a more important determinant of liquidity than other factors such as the market-makers' inventory holding costs. Section 5 provides a simple extension that links liquidity trading costs and ex-ante firm value.

during periods of economic expansion. Moreover, managers implement buybacks only when realized asset payoffs are high. Because the probability of high asset payoffs is greater in good times, this proposition also implies more actual buybacks during booms. These results are consistent with the pro-cyclical patterns of aggregate stock buybacks documented by [Jagannathan et al. \(2000\)](#) (see [Dittmar and Dittmar \(2008\)](#) for similar evidence).

Finally, Proposition 6 points out that a sufficiently informed speculator is a necessary condition for a buyback to lower liquidity trading costs. Consequently, this proposition predicts that surges in adverse selection trading costs, which signal the increased precision of the speculator's information according to Lemma 8, make buyback authorizations more likely. Consistent with this prediction, [Singh et al. \(1994\)](#), using repurchase announcements collected from *The Wall Street Journal* for NASDAQ firms between 1983 and 1990, find that the trading-risk component of the bid-ask spread for buyback-authorizing firms increase in the period leading up to the announcement and decline afterwards.

2.6 Testable Empirical Predictions

The model's predictions match stylized facts about the substantial size of most repurchase programs and about the pro-cyclical aggregate levels of authorizations and actual buybacks. In addition to matching these facts, the model also generates other testable implications. For small buybacks, the dispersion effect offsets the those of competition and increases the profitability of informed trading. For large buybacks, the opposite is true. These predictions are consistent with the empirical finding that the abnormal returns, net of brokerage fees, of institutional investors during periods of buyback programs is positive when the authorization is small and negative when it is large ([Chemmanur et al. \(2016\)](#)). A further test could involve examining the trades of retail accounts, which are presumably

uninformed. Some exchanges provide masked data for accounts classified as retail traders. The model predicts that the returns for these trades should worsen with small buybacks and improve with large ones.

The overall message of the paper is that buybacks can improve liquidity because it allows the manager to compete against other informed traders. However, this is not the only theory that predicts a positive relationship between buyback and liquidity. One alternative explanation, the competing market-maker hypothesis, involves a different sort of competition. That theory posits that market-makers use considerable monopoly power in the trading game to earn rents, which reduce liquidity. When a firm repurchases shares, it acts as a competing market-maker with buy orders and reduces the rents from market-making. However, as [Cook et al. \(2004\)](#) point out that “the NASDAQ operated as a pure dealer market, with the public unable to compete directly with market makers to directly effect a narrowing of the quoted spread.” Yet, [Franz et al. \(1995\)](#) and [Singh et al. \(1994\)](#) document a decrease in the spread for NASDAQ firms following repurchase announcements.

According to the competing market-maker hypothesis, improvements in liquidity following repurchase announcements are a result of the firm’s actual trades. In my model, adverse selection costs drop because the strategic interactions between the firm and the speculator make the order flow more informative. This distinction leads to a possible test. After a firm announces a repurchase program, the liquidity of its shares should improve even if it does not actually trade. To my knowledge, no study has examined this implication using a broad sample of firms.

2.7 Conclusion

The traditional view of informed buybacks is that they hurt shareholders by lowering the liquidity of the firm's stock. Yet, empirically, repurchase programs represent the dominant form of payout for U.S. firms. The growing significance of buyback programs is not merely a US phenomenon. The total value of repurchases has also surged internationally (Von Eije and Megginson (2008)). Under this conventional view, the popularity of these programs can only be explained by other benefits that make this cost tolerable.

This paper presents a different perspective of informed buybacks. It shows that when other informed traders are present, a repurchase program can mitigate the adverse selection problem by allowing the manager to use his private information against them. The increase in total informed trading makes the aggregate order flow more informative, improving liquidity. This framework recasts the manager's informed execution of buybacks as an asset rather than a liability. Because previous literature associates open-market stock buybacks with heightened adverse selection, the cost of such programs for shareholders may have been overstated. This reassessment helps explain the prevalence of open-market repurchase programs.

This analysis differs from other multi-trader Kyle models because it features the firm as an informed trader. Trading by the firm endogenously alters the per-share value of the firm. When asset payoffs are high, trading profits further increase the per-share liquidation value of the firm in those states of the world. The widening gap between the firm's per-share value in different payoff states makes informed trading more profitable. The existence of this dispersion effect suggests that efforts to limit the managerial discretion in executing buybacks may be counter-productive. It also suggests that the strategic interaction between the firm and other informed traders differs from that described by traditional multi-trader

models.

The net effect of informed buybacks on liquidity depends on the relative magnitudes of the effects of competition and dispersion. The model predicts that buybacks improve liquidity when the authorization is large and the probability of high payoffs is high. These conditions are consistent with stylized facts of repurchase programs.

When the speculator is less informed than the manager, the authorization of a buyback program introduces new private information into trading. As a result, a sufficiently informed speculator is a necessary condition for buybacks to improve liquidity. If the speculator's precision has increased over time due to improvements in information technology, this condition could help explain the upward trend in the time-series of aggregate buybacks.

The model in this paper represents a tractable framework that describes the main effects of open-market repurchase programs in an environment with multiple informed traders and heterogeneous liquidity traders. Several simplifying assumptions were made for tractability. For instance, I assumed that the firm's assets were already in place and ruled out potential managerial agency problems. An analysis under relaxed assumptions may paint a more complete picture of the effects of stock buybacks. One interesting extension might involve real investment decisions. Because buybacks allow shareholders, particularly long-term ones, to benefit from volatility in the firm's payoffs, the mechanism in this paper suggests a connection between a firm's payout policy and the risk of its investment projects.

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Appendix A

Appendix to Chapter 1

A.1 Parameter Restrictions for θ

The restriction on θ mainly affects the result of Lemma 2. The information structure featured in the model implies that overall firm performance contains no information about a high-output worker's effort or type. However, the same is not true of low-output workers. Firm performance corresponds to average worker quality, which affects the likelihood ratio of low output. In particular, firm performance is low when average worker quality is low. In these instances, a low output is less likely the outcome of shirking. Conversely, when firm performance is high, average worker quality is also high. Low output in these instances are more likely to be the result of shirking.

When $\theta_1 \geq \underline{\theta}$, the individual worker's output is sufficiently informative that even in the worst-case scenario for employee quality, the firm is reasonably sure that the worker shirked given the observation of low output. In this case, the optimal contract pays low-output workers nothing to maximize incentive provision.

When $\theta_1 < \underline{\theta}$, the low-output worker receives wages that weakly decline in firm performance. Because the low-output workers' types are positively correlated via parameter uncertainty, they receive relative performance pay as is standard in principal-agent frameworks.

The exact expression for $\underline{\theta}$ is given by:

$$\underline{\theta} = 1 - \left(\frac{U_3^{1-\rho}}{\frac{s^{1-\rho}}{\beta^3} + U_3^{1-\rho}} \right)^{\frac{\alpha-\rho}{1-\rho}}. \quad (\text{A.1})$$

It is worth noting that when $\alpha \leq \rho$, condition (A.1) is automatically satisfied because $\underline{\theta} \leq 0$.

A.2 Optimal Contracts with No Wages

When a first-stage contract offers no explicit wages, the incentive compatibility constraint simplifies to:

$$\begin{aligned} & \left(\sum_{j=1}^n p_j (1-\theta) \beta^{\frac{2(1-\alpha)}{1-\rho}} U_3^{1-\alpha} + \sum_{j=1}^k p_j \theta_j \left[\beta P_0^{1-\rho} + \beta^2 U_3^{1-\rho} \right]^{\frac{1-\alpha}{1-\rho}} + \right. \\ & \left. \sum_{j=k+1}^n p_j \theta_j \left[\beta \left(\frac{K}{\theta_j} [P_0^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{1-\alpha}{1-\rho}} + \left(1 - \frac{K}{\theta_j} \right) [\beta U_3^{1-\rho}]^{\frac{1-\alpha}{1-\rho}} \right)^{\frac{1-\rho}{1-\alpha}} \right]^{\frac{1-\alpha}{1-\rho}} \right)^{\frac{1-\rho}{1-\alpha}} \\ & \geq \frac{s^{1-\rho}}{\beta} + \beta^2 U_3^{1-\rho}. \end{aligned}$$

Note that the RHS of the inequality is a constant and that the LHS is a continuous increasing function of P_0 . When $P_0 = 0$, the inequality fails to hold. In the limit as $P_0 \rightarrow \infty$, the inequality holds strictly when $\alpha < 1$. Therefore, there must exist some \bar{P}_0 such that $P_0 \geq \bar{P}_0$ implies that the inequality holds when $\alpha < 1$. However, when $\alpha > 1$, the certainty

equivalent of the tournament prize is bounded, which implies that the LHS of the above IC constraint is bounded. The size of this boundary decreases in α . As a result, for α sufficiently large, the incentive compatibility constraint fails to hold for a contract with no explicit wages.

A.3 Proofs

Proof of Lemma 2. On the equilibrium path, an output of $q = 0$ reveals that the worker is low-type. Consequently, this worker has no prospect for a promotion and the only relevant wages at $t = 2$ are $W_2(0, Q, 0)$. For notation convenience, let $W_2(0, \theta_j) = w_{1j}$, $W_2(0, \theta_j, 0) = w_{2j}$, and $W_1(1, \theta_j) = W_{1j}$. Then the incentive compatibility constraint of the worker from (1.9) can be rewritten as:

$$\begin{aligned} & \left(\sum_{j=1}^n p_j \theta_j [W_{1j} j^{1-\rho} + \beta \bar{U}_2(\theta_j)^{1-\rho}]^{\frac{1-\alpha}{1-\rho}} + \sum_{j=1}^n p_j (1 - \theta_j) [w_{1j} j^{1-\rho} + \beta w_{2j}^{1-\rho} + \beta^2 U_3^{1-\rho}]^{\frac{1-\alpha}{1-\rho}} \right)^{\frac{1-\rho}{1-\alpha}} \\ & \geq \frac{s^{1-\rho}}{\beta} + \left(\sum_{j=1}^n p_j [w_{1j} j^{1-\rho} + \beta w_{2j}^{1-\rho} + \beta^2 U_3^{1-\rho}]^{\frac{1-\alpha}{1-\rho}} \right)^{\frac{1-\rho}{1-\alpha}}. \end{aligned}$$

When, $\alpha < 1$, the worker's IC constraint simplifies to:

$$\begin{aligned} & \sum_{j=1}^n p_j \theta_j [W_{1j} j^{1-\rho} + \beta \bar{U}_2(\theta_j)^{1-\rho}]^{\frac{1-\alpha}{1-\rho}} + \sum_{j=1}^n p_j (1 - \theta_j) [w_{1j} j^{1-\rho} + \beta w_{2j}^{1-\rho} + \beta^2 U_3^{1-\rho}]^{\frac{1-\alpha}{1-\rho}} \\ & - \left[\frac{s^{1-\rho}}{\beta} + \left(\sum_{j=1}^n p_j [w_{1j} j^{1-\rho} + \beta w_{2j}^{1-\rho} + \beta^2 U_3^{1-\rho}]^{\frac{1-\alpha}{1-\rho}} \right)^{\frac{1-\rho}{1-\alpha}} \right]^{\frac{1-\alpha}{1-\rho}} \geq 0. \end{aligned} \tag{A.2}$$

Define a function $G(w_{11}, \dots, w_{1n}, w_{21}, \dots, w_{2n})$ to be the LHS of the above IC constraint. Let μ_0 be defined as:

$$\mu_0 = \left(\sum_{j=1}^n p_j U_1(0, \theta_j)^{1-\alpha} \right)^{\frac{1}{1-\alpha}}. \quad (\text{A.3})$$

Then,

$$\frac{\partial G}{\partial w_{1j}} = (1 - \alpha) p_j [w_{1j}^{1-\rho} + \beta w_{2j}^{1-\rho} + \beta^2 U_3^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} (1 - \theta_j - [\frac{s^{1-\rho}}{\beta} + \mu_0^{\alpha-\rho}]^{\frac{\rho-\alpha}{1-\rho}}) w_{1j}^{-\rho}. \quad (\text{A.4})$$

The expression for $\frac{\partial G}{\partial w_{1j}}$ given by (A.4) is non-positive when:

$$1 - \theta_j \leq \frac{\mu_0^{\alpha-\rho}}{[\frac{s^{1-\rho}}{\beta} + \mu_0^{1-\rho}]^{\frac{\alpha-\rho}{1-\rho}}}. \quad (\text{A.5})$$

Note that condition (A.5) is automatically satisfied when $\alpha \leq \rho$. When $\alpha > \rho$, condition (A.5) is met for all j when θ_1 is not too small:

$$1 - \theta_j \leq 1 - \theta_1 \leq \frac{U_3^{\alpha-\rho}}{[\frac{s^{1-\rho}}{\beta^3} + U_3^{1-\rho}]^{\frac{\alpha-\rho}{1-\rho}}} \leq \frac{\mu_0^{\alpha-\rho}}{[\frac{s^{1-\rho}}{\beta} + \mu_0^{1-\rho}]^{\frac{\alpha-\rho}{1-\rho}}}.$$

The following expression for $\frac{\partial G}{\partial w_{2j}}$ is non-positive in similar fashion:

$$\frac{\partial G}{\partial w_{2j}} = (1 - \alpha) p_j [w_{1j}^{1-\rho} + \beta w_{2j}^{1-\rho} + \beta^2 U_3^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} (1 - \theta_j - [\frac{s^{1-\rho}}{\beta} + \mu_0^{\alpha-\rho}]^{\frac{\rho-\alpha}{1-\rho}}) \beta w_{2j}^{-\rho}. \quad (\text{A.6})$$

Because $\frac{\partial G}{\partial w_{1j}}$ and $\frac{\partial G}{\partial w_{2j}}$ are both non-positive, the optimal contract minimizes w_{1j} and w_{2j} for all $j \in \{1, \dots, n\}$. Limited liability implies that $w_{1j} = w_{2j} = 0$. The proof for the case when $\alpha > 1$ proceeds in an identical manner with the signs of (A.2), (A.4), and (A.6) flipped. ■

Proof of Proposition 1. Let \bar{P} be the expected cost of the prize paid to a high-output worker:

$$\bar{P} = \sum_{j=1}^k p_j \theta_j P_0 + \sum_{j=k+1}^n p_j \theta_j \frac{K}{\theta_j} P_0.$$

Let $\bar{\theta}$ be the ex-ante expected probability that a worker is high-type:

$$\bar{\theta} = \sum_{j=1}^n p_j \theta_j. \tag{A.7}$$

Full insurance of the worker's consumption at $t = 2$ implies that the prize paid to the worker reduces his first-stage wages at $t = 2$ on a dollar for dollar basis. Consequently, the expected wage cost of this full insurance contract is $\bar{\theta}(c_1^* + c_2^*) - \bar{P}$. Let $(c_1(Q), c_2(Q, \psi))$ be the consumption plan associated with an alternative contract that provides the high-output worker with a lifetime utility of $\bar{\mu}_1$ but does not fully insure his consumption at $t = 2$. By construction, the consumption plan (c_1^*, c_2^*) uniquely provides $\bar{\mu}_1$ units of lifetime utility to the high-output worker at lowest cost. Therefore,

$$\theta(c_1^* + c_2^*) < E[c_1(Q)] + E[c_2(Q, \psi)].$$

This implies that:

$$\theta(c_1^* + c_2^*) - \bar{P} < E[c_1(Q)] + E[c_2(Q, \psi)] - \bar{P}.$$

Thus, the expected wage cost of any other incentive compatible contract must be higher than the full insurance contract in Proposition 1. ■

Proof of Lemma 3. To begin, it is worth highlighting the optimality conditions that form the crux of most proofs in this paper. To reduce notational complexity, let $W_1(1, \theta_j) = W_{1j}$, $\bar{U}_2(\theta_j) = \bar{U}_{2j}$, $W_2(1, \theta_j, 0) = W_{2j}$, and $W_2(1, \theta_j, 1) = R_{2j}$. Let μ_1 be the lifetime utility that

a wage contract concedes to the high-output worker at $t = 1$:

$$\mu_1 = \left(\sum_{j=1}^n p_j \theta_j [W_{1j}^{1-\rho} + \beta \bar{U}_{2j}^{1-\rho}]^{\frac{1-\alpha}{1-\rho}} \right)^{\frac{1}{1-\alpha}}. \quad (\text{A.8})$$

First, the optimal contract must feature positive wages at $t = 1$ and positive wages at $t = 2$ when the worker is not promoted because:

$$\frac{\partial \mu_1}{\partial W_{1j}} = p_j \theta_j [W_{1j}^{1-\rho} + \beta \bar{U}_{2j}^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} W_{1j}^{-\rho} \rightarrow \infty \text{ as } W_{1j} \rightarrow 0$$

for all $j \in \{1, \dots, n\}$ and

$$\frac{\partial \mu_1}{\partial W_{2j}} = p_j \theta_j [W_{1j}^{1-\rho} + \beta \bar{U}_{2j}^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} \beta \bar{U}_{2j}^{\alpha-\rho} [W_{2j}^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} W_{2j}^{-\rho} \rightarrow \infty \text{ as } W_{2j} \rightarrow 0$$

for $j > k$ where k is the largest integer such that $\theta_k \leq K$.

Consequently, the optimal contract must satisfy the following conditions for some $\eta > 0$:

$$[W_{1j}^{1-\rho} + \beta \bar{U}_{2j}^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} W_{1j}^{-\rho} = \eta \quad (\text{A.9})$$

for all $j \in \{1, \dots, n\}$ and

$$[W_{1j}^{1-\rho} + \beta \bar{U}_{2j}^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} \beta \bar{U}_{2j}^{\alpha-\rho} [W_{2j}^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} W_{2j}^{-\rho} = \eta \quad (\text{A.10})$$

for all $j > k$.

Otherwise, a cheaper contract can be constructed by re-allocating wages appropriately.

For example, consider a contract \hat{W} such that for some $i \neq m$:

$$[W_{1i}^{1-\rho} + \beta \bar{U}_{2i}^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} W_{1i}^{-\rho} > [W_{1m}^{1-\rho} + \beta \bar{U}_{2m}^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} W_{1m}^{-\rho}.$$

Let f be a continuously differentiable function of (W_{1j}, \bar{U}_{2j}) :

$$f(W_{1j}, \bar{U}_{2j}) = [W_{1j}^{1-\rho} + \beta \bar{U}_{2j}^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} W_{1j}^{-\rho}. \quad (\text{A.11})$$

The continuity of f implies that exists some $\gamma \in (0, 1)$ such that:

$$f(W_{1m}, \bar{U}_{2m}) < \gamma f(W_{1i}, \bar{U}_{2i}).$$

Continuity also implies that there exists some $\bar{\epsilon} > 0$ such that:

$$f(W_{1m} - \epsilon, \bar{U}_{2m}) < \gamma f(W_{1m} + \gamma \frac{\gamma p_m \theta_m \epsilon}{p_i \theta_i}, \bar{U}_{2i})$$

for all $\epsilon < \bar{\epsilon}$. Consider an alternative contract W^* where:

$$W_1^*(0, \theta_j) = \hat{W}_1(0, \theta_j) \text{ for all } Q,$$

$$W_{2j}^* = \hat{W}_{2j} \text{ for all } j,$$

$$R_{2j}^* = \hat{R}_{2j} \text{ for all } j,$$

$$W_{1j}^* = \hat{W}_{1j} \text{ for all } j \neq i, m,$$

$$W_{1i}^* = \hat{W}_{1i} + \frac{\gamma p_m \theta_m \bar{\epsilon}}{p_i \theta_i},$$

and

$$W_{1m}^* = \hat{W}_{1m} - \bar{\epsilon}.$$

By construction, this alternative contract satisfies the IC constraint and results in lower expected wage costs.

With the above optimality conditions in mind, suppose for contradiction that the op-

timal contract fully insures high-output workers against uncertain promotions at $t = 2$. In other words, this contract specifies the same $t = 2$ consumption for the worker across all states. Let \hat{c}_2 be this quantity. Note that full insurance implies that $\hat{c}_2 \geq P_0 > c_2^*$. Full insurance at $t = 2$ and condition (A.9) imply that wages at $t = 1$ are flat, i.e. $W_{1j} = W_1 > 0$ for all j . Because (c_1^*, c_2^*) is the cost-minimizing consumption plan that delivers lifetime utility $\bar{\mu}_1$ and $\hat{c}_2 > c_2^*$, it must be that $W_1 < c_1^*$. Otherwise, this contract yields too much $t = 1$ lifetime utility to the high-output worker.

Conditions (A.9) and (A.10) imply that:

$$W_1^{-\rho} = \beta U_{2j}^{\alpha-\rho} [W_{2j}^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} W_{2j}^{-\rho}.$$

Full insurance of the high-output worker's consumption at $t = 1$ implies that the above equation can be simplified to:

$$W_1^{-\rho} = \beta \hat{c}_2^{-\rho}. \tag{A.12}$$

Equation (A.12) implies that:

$$W_1 = \beta^{-\frac{1}{\rho}} \hat{c}_2.$$

However, as noted earlier, $W_1 < c_1^*$ and $c_2^* < \hat{c}_2$. Together these inequalities imply:

$$W_1 < c_1^* = \beta^{-\frac{1}{\rho}} c_2^* < \beta^{-\frac{1}{\rho}} \hat{c}_2,$$

which is a contradiction. Hence by contradiction, when $P_0 > c_2^*$, the optimal contract cannot feature full insurance of $t = 2$ consumption for the high-output worker. ■

Proof of Lemma 4. This proof adopts the notational shorthand from the proof of Lemma 3.

There are three cases to consider: $\alpha > \rho$, $\alpha = 0$, and $\alpha < \rho$.

Case 1: $\alpha > \rho$.

Suppose for contradiction that the optimal contract features positive wages at $t = 2$ in some states in which the worker is promoted. In other words, there exists some m such that $R_{2m} > 0$. Optimality implies:

$$[W_{2m}^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} W_{2m}^{-\rho} = \beta \bar{U}_{2m}^{\alpha-\rho} [(P_0 + R_{2m})^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} (P_0 + R_{2m})^{-\rho},$$

which implies:

$$W_{2m} = P_0 + R_{2m}. \quad (\text{A.13})$$

Equation (A.13) implies that this contract provides full insurance in state m , i.e. $Q = \theta_m$:

$$[W_{1m}^{1-\rho} + \beta \bar{U}_{2m}^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} \beta (P_0 + R_{2k})^{-\rho} = \eta. \quad (\text{A.14})$$

Lemma 3 says that this contract does not fully insure the high-output worker's consumption at $t = 2$ across all states. Consequently, $W_{2i} < P_0 + R_{2m}$ for some i . $W_{2i} < P_0 + R_{2j} = W_{2m}$ implies $\bar{U}_{2i} < \bar{U}_{2m}$. Because $\bar{U}_{2i} < \bar{U}_{2m}$, condition (A.9) implies that $W_{1i} > W_{1m}$. Optimality of the contract implies:

$$[W_{1m}^{1-\rho} + \beta \bar{U}_{2m}^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} (P_0 + R_{2k})^{-\rho} = [W_{1i}^{1-\rho} + \beta P_i^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} \bar{U}_{2i}^{\alpha-\rho} [W_{2i}^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} W_{2i}^{-\rho} = \eta. \quad (\text{A.15})$$

Condition (A.9) implies that:

$$[W_{1m}^{1-\rho} + \beta \bar{U}_{2m}^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} = \eta W_{1m}^{\rho}$$

and

$$[W_{1i}^{1-\rho} + \beta \bar{U}_{2i}^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} = \eta W_{1i}^{\rho}.$$

Substituting the above into (A.15) yields

$$\begin{aligned} W_{1m}^\rho W_{2m}^{-\rho} &= W_{1i}^\rho \bar{U}_{2i}^{\alpha-\rho} [W_{2i}^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} W_{2i}^{-\rho} \\ \Rightarrow \underbrace{\left(\frac{W_{1m}}{W_{1i}}\right)^\rho}_{<1} \underbrace{\left(\frac{W_{2i}}{P_0 + R_{2m}}\right)^\rho}_{<1} &= \underbrace{\frac{\bar{U}_{2i}^{\alpha-\rho}}{[W_{2i}^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}}}}_{>1}, \end{aligned}$$

which is a contradiction. Thus, by contradiction, the optimal contract cannot feature positive $t = 2$ wages in states where the high-output worker is promoted when $\alpha > \rho$.

Case 2: $\alpha = \rho$.

When $\alpha = \rho$, the Epstein-Zin formulation simplifies to time-separable expected utility. In these instances, the expression for μ_1 becomes:

$$\mu_1 = \left(\sum_{j=1}^n p_j \theta_j [W_{1j}^{1-\rho} + \beta \bar{U}_{2j}^{1-\rho}] \right)^{\frac{1}{1-\alpha}},$$

where

$$\bar{U}_{2j} = \frac{K}{\theta_j} (P_0 + R_{2j})^{1-\rho} + \left(1 - \frac{K}{\theta_j}\right) W_{2j}^{1-\rho} + \beta U_3^{1-\rho}.$$

Suppose for contradiction that the optimal contract features positive wages at $t = 2$ in some states in which the worker is promoted so that $R_{2m} > 0$ for some m . Optimality implies that

$\frac{\partial \mu_1}{\partial R_{2m}} = \frac{\partial \mu_1}{\partial W_{2m}}$, which means that:

$$W_{2m} = P_0 + R_{2m}.$$

However, Lemma 3 implies that there exists some i such that $R_{2i} = 0$, which means that $W_{2i} \leq P_0$. Consequently, $\frac{\partial \mu_1}{\partial W_{2i}} > \frac{\partial \mu_1}{\partial W_{2m}}$, a contradiction. Hence, by contradiction, the optimal contract cannot feature positive $t = 2$ wages in states where the high-output worker is promoted when $\alpha = \rho$.

Case 3: $\alpha < \rho$.

Suppose for contradiction, the optimal contract features payment in some state of the world where the worker is promoted, i.e. $R_{2m} > 0$ for some m .

Optimality implies that $\frac{\partial \mu_1}{\partial R_{2m}} = \frac{\partial \mu_1}{\partial W_{2m}}$, which means that:

$$W_{2m} = P_0 + R_{2m} > P_0.$$

The above expression implies that the worker's $t = 2$ consumption in state m is $P_0 + R_{2m}$ regardless of the promotion outcome. Consequently, $\bar{U}_{2m} > [P_0^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{1}{1-\rho}}$. As before, Lemma 3 implies that there exists some i such that $R_{2i} = 0$. This implies that $W_{2i} \leq P_0$, which means that $\bar{U}_{2i} \leq [P_0^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{1}{1-\rho}}$. Condition (A.9) implies that $W_{1i} < W_{1j}$ because $\bar{U}_{2i} < \bar{U}_{2j}$. Recall that the μ_1 , the lifetime utility for the high-output worker at $t = 1$, satisfies:

$$\mu_1^{1-\alpha} = \sum_{j=1}^n p_j \theta_j \left([W_{1j}^{1-\rho} + \beta \bar{U}_{2j}^{1-\rho}]^{\frac{1}{1-\rho}} \right)^{1-\alpha}.$$

Consider the the power utility function g and the CES function h :

$$g(h) = h^{1-\alpha}$$

and

$$h(W_{1j}, \bar{U}_{2j}) = [W_{1j}^{1-\rho} + \beta \bar{U}_{2j}^{1-\rho}]^{\frac{1}{1-\rho}},$$

which are both concave and strictly increasing for $0 < \alpha < \rho < 1$. Consider the following deviating contract:

$$\begin{aligned} \bar{W}_1 &= \frac{p_i \theta_i W_{1i} + p_m \theta_m W_{1m}}{p_m \theta_m + p_i \theta_i}, \\ \bar{U}_2 &= \frac{p_i \theta_i \bar{U}_{2i} + p_m \theta_m \bar{U}_{2m}}{p_i \theta_i + p_m \theta_m}, \end{aligned}$$

$$\hat{W}_{1i} = \lambda \bar{W}_1 + (1 - \lambda)W_{1i} > W_{1i},$$

$$\hat{W}_{1m} = \lambda \bar{W}_1 + (1 - \lambda)W_{1m} < W_{1m},$$

$$\hat{U}_{2i} = \lambda \bar{U}_2 + (1 - \lambda)\bar{U}_{2i} > \bar{U}_{2i},$$

and

$$\hat{U}_{2m} = \lambda \bar{U}_2 + (1 - \lambda)\bar{U}_{2m} < \bar{U}_{2m},$$

where

$$\lambda \in (0, 1).$$

Let all other pieces of the original contract remain as they were. The concavity of the composition function $g(h(W_{1Q}, \bar{U}_{2Q}))$ implies that the above mean-preserving deviating contract preserves incentive compatibility. By construction, the $t = 1$ wage costs remain the same. To show that the $t = 2$ wage costs of this new contract is less, note that:

$$Z_W = \bar{U}_{2j}^\alpha [W_{2j}^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} W_{2j}^{-\rho}$$

is decreasing in W_{2j} and θ_j ,

$$Z_R = \bar{U}_{2j}^\alpha [(P_0 + R_{2j})^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} (P_0 + R_{2j})^{-\rho}$$

is decreasing in $(P_0 + R_{2j})$ and θ_j .

Moreover,

$$\frac{\partial \bar{U}_{2j}}{\partial W_{2j}} = \left(1 - \frac{K}{\theta_j}\right) Z_W,$$

$$\frac{\partial \bar{U}_{2j}}{\partial R_{2j}} = \left(\frac{K}{\theta_j}\right) Z_R.$$

Consequently, either $\frac{\partial \bar{U}_{2i}}{\partial W_{2i}} > \frac{\partial U_{2m}}{\partial W_{2m}}$ or $\frac{\partial \bar{U}_{2i}}{\partial W_{2i}} > \frac{\partial U_{2m}}{\partial R_{2m}}$. In the case of the former, the deviating contract achieves \hat{U}_{2i} and \hat{U}_{2m} by increasing W_{2i} and decreasing W_{2m} . In the latter, it does so by raising W_{2i} and lowering R_{2m} . This cheaper deviating contract contradicts the original's presumed optimality. Hence, by contradiction, the optimal contract cannot feature positive $t = 2$ wages in states where the high-output worker is promoted when $\alpha < \rho$. ■

Proof of Proposition 2. The first two properties of the proposition are the results of Lemma 2 and 4. This proof adopts the notational shorthand of Lemma 3. Let $\bar{\mu}_1$ be the lifetime utility at $t = 1$ that the optimal contract yields to the high-output worker:

$$\mu_1^{1-\alpha} = \sum_{j=1}^n p_j \theta_j [W_{1j}^{1-\rho} + \beta \bar{U}_{2j}^{1-\rho}]^{\frac{1-\alpha}{1-\rho}}.$$

Recall that optimality of the contract implies that exists some $\eta > 0$ such that:

1. $[W_{1j}^{1-\rho} + \beta \bar{U}_{2j}^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} W_{1j}^{-\rho} = \eta$ for all $j \in \{1, \dots, n\}$,
2. $[W_{1j}^{1-\rho} + \beta \bar{U}_{2j}^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} \beta \bar{U}_{2j}^{\alpha-\rho} [W_{2j}^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} W_{2j}^{-\rho}$ for all $j > k$.

As before, these condition imply that the marginal utility provided to the worker by the wages in the contract across different times and states are the same. If not, a cheaper contract could be constructed by re-allocating wages appropriately.

First, consider $i < m \leq k$. Because the number of qualified candidates in these instances do not exceed the number of higher-level positions, all high-output worker are promoted. Thus,

$$\bar{U}_{2i} = \bar{U}_{2m} = [P_0^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{1}{1-\rho}}.$$

Condition (A.9) implies that:

$$[W_{1i}^{1-\rho} + \beta \bar{U}_{2i}^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} W_{1i}^{-\rho} = [W_{1m}^{1-\rho} + \beta \bar{U}_{2m}^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} W_{1m}^{-\rho},$$

which can only be satisfied if $W_{1i} = W_{1m}$.

Second, consider $k \leq i < m$. Together, conditions (A.9) and (A.10) imply that for all $j > k$:

$$W_{1j}^\rho \bar{U}_{2j}^{\alpha-\rho} [W_{2j}^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} W_{2j}^{-\rho} = \frac{1}{\beta}. \quad (\text{A.16})$$

Recall that

$$U_2(1, \theta_j, \psi) = \begin{cases} \left[P_0^{1-\rho} + \beta U_3^{1-\rho} \right]^{\frac{1}{1-\rho}} & \psi = 1 \\ \left[W_{2j}^{1-\rho} + \beta U_3^{1-\rho} \right]^{\frac{1}{1-\rho}} & \psi = 0 \end{cases}$$

and

$$\bar{U}_{2j} = \left(\frac{K}{\theta_j} U_2(1, \theta_j, 1)^{1-\alpha} + \left(1 - \frac{K}{\theta_j} \right) U_2(1, \theta_j, 0)^{1-\alpha} \right)^{\frac{1}{1-\alpha}} \text{ for } j > k.$$

Define a function A as the LHS of (A.16):

$$A(W_{1j}, W_{2j}, \theta_j) = W_{1j}^\rho \bar{U}_{2j}^{\alpha-\rho} [W_{2j}^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} W_{2j}^{-\rho}.$$

Then,

$$\frac{\partial A}{\partial W_{1j}} = \rho W_{1j}^{\rho-1} \bar{U}_{2j}^{\alpha-\rho} [W_{2j}^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} W_{2j}^{-\rho} > 0 \quad (\text{A.17})$$

and

$$\frac{\partial A}{\partial \theta_j} = (\alpha - \rho) W_{1j}^\rho \bar{U}_{2j}^{\alpha-\rho-1} [W_{2j}^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} W_{2j}^{-\rho} * \frac{\partial \bar{U}_{2j}}{\partial \theta_j} < 0 \quad (\text{A.18})$$

because

$$\frac{\partial \bar{U}_{2j}}{\partial \theta_j} = \bar{U}_{2j}^\alpha \frac{K}{\theta_j^2} \left(\left[W_{2j}^{1-\rho} + \beta U_3^{1-\rho} \right]^{\frac{1}{1-\rho}} - \left[P_0^{1-\rho} + \beta U_3^{1-\rho} \right]^{\frac{1}{1-\rho}} \right) < 0.$$

For notation convenience, let $U_2(1, \theta_j, 0) = u_{2j}$. Then, A be rewritten as:

$$A = W_{1j}^\rho \bar{U}_{2j}^{\alpha-\rho} u_{2j}^{\rho-\alpha} W_{2j}^{-\rho}.$$

Then,

$$\begin{aligned} \frac{\partial A}{\partial W_{2j}} &= W_{1j}^\rho W_{2j}^{-\rho} \left[(\alpha - \rho) \bar{U}_{2j}^{\alpha-\rho-1} \frac{\partial \bar{U}_{2j}}{\partial u_{2j}} \frac{\partial u_{2j}}{\partial W_{2j}} u_{2j}^{\rho-\alpha} + (\rho - \alpha) \bar{U}_{2j}^{\alpha-\rho} u_{2j}^{\rho-\alpha-1} \frac{\partial u_{2j}}{\partial W_{2j}} \right] \\ &\quad - \rho W_{2j}^{-\rho-1} W_{1j}^\rho \bar{U}_{2j}^{\alpha-\rho} u_{2j}^{\rho-\alpha} \\ &= W_{1j}^\rho W_{2j}^{-\rho} (\alpha - \rho) \bar{U}_{2j}^{\alpha-\rho} \frac{\partial u_{2j}}{\partial W_{2j}} u_{2j}^{\rho-2\alpha} \left[\frac{1 - \frac{K}{\theta_j}}{\frac{K}{\theta_j} [P_0^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{1-\alpha}{1-\rho}} + (1 - \frac{K}{\theta_j}) u_{2j}^{1-\alpha}} - \frac{1}{u_{2j}^{1-\alpha}} \right] \\ &\quad - \rho W_{2j}^{-\rho-1} W_{1j}^\rho \bar{U}_{2j}^{\alpha-\rho} u_{2j}^{\rho-\alpha} \\ &< 0 \end{aligned} \tag{A.19}$$

because $\frac{\partial u_{2j}}{\partial W_{2j}} = u_{2j}^\rho W_{2j}^{-\rho} > 0$.

Condition (A.16) implies that:

$$A(W_{1i}, W_{2i}, \theta_i) = \frac{1}{\beta}.$$

Expression (A.18) implies that:

$$A(W_{1i}, W_{2i}, \theta_m) < \frac{1}{\beta}.$$

From (A.17) and (A.19), we know that $\frac{\partial A}{\partial W_{1q}} > 0$ and $\frac{\partial A}{\partial W_{2q}} < 0$. Thus, for (A.16) to hold for W_{1m} , W_{2m} , and θ_m , at least one of the following conditions must be true:

1. $W_{1m} > W_{1i}$,

2. $W_{2m} < W_{2i}$.

If the first condition is true, then the proposition is proved. If the second condition is true, then it must be that $\bar{U}_{2m} < \bar{U}_{2i}$. If $\bar{U}_{2m} < \bar{U}_{2i}$, then condition (A.9) implies that $W_{1m} > W_{1i}$. Thus, in either case, the proposition's assertion about W_{1j} is established. ■

Proof of Proposition 3. When $\alpha = \rho$, the Epstein-Zin formulation simplifies to time-separable utility. In these instances, the expression for μ_1 becomes:

$$\mu_1 = \left(\sum_{j=1}^n p_j \theta_j [W_{1j}^{1-\rho} + \beta \bar{U}_{2j}^{1-\rho}] \right)^{\frac{1}{1-\alpha}},$$

where

$$\bar{U}_{2Q} = \frac{K}{\theta_j} P_0^{1-\rho} + \left(1 - \frac{K}{\theta_j} \right) W_{2j}^{1-\rho} + \beta U_3^{1-\rho}.$$

The partial derivatives are:

$$\frac{\partial \mu_1}{\partial W_{1j}} = \mu_1^\alpha p_j \theta_j W_{1j}^{-\rho}$$

and

$$\frac{\partial \mu_1}{\partial W_{2j}} = \mu_1^\alpha p_j \theta_j W_{2j}^{-\rho}.$$

Equating the margins across the states θ_j , $t = 1$, and $t = 2$ imply that:

$$W_{1i} = W_{1m}$$

and

$$W_{2i} = W_{2m}$$

for all i, m . Hence, when $\alpha = \rho$, the optimal contract does not feature firm performance pay.

When $\alpha < \rho$, the expression for μ_1 is:

$$\mu_1 = \left(\sum_{j=1}^n p_j \theta_j [W_{1j}^{1-\rho} + \beta \bar{U}_{2j}^{1-\rho}]^{\frac{1-\alpha}{1-\rho}} \right)^{\frac{1}{1-\alpha}},$$

where

$$\bar{U}_{2j} = \left(\frac{K}{\theta_j} [P_0^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{1-\alpha}{1-\rho}} + \left(1 - \frac{K}{\theta_j} \right) [W_{2j}^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{1-\alpha}{1-\rho}} \right)^{\frac{1}{1-\alpha}}.$$

Suppose for contradiction that for some $m > i > K$, the optimal contract features $W_{1i} < W_{1m}$. Condition (A.9) implies that $\bar{U}_{2i} > \bar{U}_{2m}$.

Consider the the power utility function g and the CES function h :

$$g(h) = h^{1-\alpha}$$

and

$$h(W_{1j}, \bar{U}_{2j}) = [W_{1j}^{1-\rho} + \beta \bar{U}_{2j}^{1-\rho}]^{\frac{1}{1-\rho}},$$

which are both concave and strictly increasing for $0 < \alpha < \rho < 1$. Consider the following deviating contract:

$$\bar{W}_1 = \frac{p_i \theta_i W_{1i} + p_m \theta_m W_{1m}}{p_i \theta_i + p_m \theta_m},$$

$$\bar{U}_2 = \frac{p_i \theta_i \bar{U}_{2i} + p_m \theta_m \bar{U}_{2m}}{p_i \theta_i + p_m \theta_m},$$

$$\hat{W}_{1i} = \lambda \bar{W}_1 + (1 - \lambda) W_{1i} > W_{1i},$$

$$\hat{W}_{1m} = \lambda \bar{W}_1 + (1 - \lambda) W_{1m} < W_{1m},$$

$$\hat{U}_{2i} = \lambda \bar{U}_2 + (1 - \lambda) \bar{U}_{2i} > \bar{U}_{2i},$$

$$\hat{U}_{2m} = \lambda \bar{U}_2 + (1 - \lambda) \bar{U}_{2m} < \bar{U}_{2m},$$

where

$$\lambda \in (0, 1).$$

Let all other pieces of the original contract remain as they were. The concavity of the composition function $g(h(W_{1j}, \bar{U}_{2j}))$ implies that the above mean-preserving deviating contract preserves incentive compatibility. By construction, the $t = 1$ wage costs remain the same. To show that the $t = 2$ wage costs of this new contract is less, note that:

$$Z_W = \bar{U}_{2j}^\alpha [W_{2j}^{1-\rho} + \beta U_3^{1-\rho}]^{\frac{\rho-\alpha}{1-\rho}} W_{2j}^{-\rho},$$

is decreasing in W_{2j} and θ_j . Moreover,

$$\frac{\partial \bar{U}_{2j}}{\partial W_{2j}} = \left(1 - \frac{K}{\theta_j}\right) Z_W.$$

The assumption that $i < m$ and $W_{2i} < W_{2m}$ imply that $\frac{\partial \bar{U}_{2i}}{\partial W_{2i}} > \frac{\partial \bar{U}_{2m}}{\partial W_{2m}}$. Consequently, for some small λ , the deviating contract can achieve \hat{U}_{2i} and \hat{U}_{2m} at lower costs than the original contract by increasing W_{2i} and decreasing W_{2m} . This cheaper deviating contract contradicts the original's optimality. Hence, by contradiction, the optimal contract features $t = 1$ wages that decrease with firm performance. ■

Appendix B

Appendix to Chapter 2

B.1 Fully Strategic Manager

The model has to specify an objective for the manager when he is fully strategic. Let $\hat{\alpha} \leq \alpha$ be the manager's strategy, where α is the maximum authorized buyback amount. Suppose the manager chooses $\hat{\alpha}$ to maximize the expected per-share value of the firm at $t = 2$ because he is paid with equity that he has to hold until the last period. Allowing the manager to buy back any amount up to the authorization does not invalidate the equilibriums identified in Section 2. However, because trade is discrete, this modification introduces an infinite number of other equilibriums. For instance, consider an authorization of size $\hat{\alpha} \leq \underline{\alpha}$. Then, any $\alpha \leq \hat{\alpha}$ can be part of an equilibrium with the speculator trading $a = \frac{1}{2} - \alpha$ when payoffs are high and the market-makers adopting the pricing rule from the first part of Lemma 5. The discreteness of noise trade makes deviations by the manager to an alternative trading size less than the authorized amount $\hat{\alpha}$ renders the resulting order flow fully revealing. In these equilibriums, the result of Proposition 4 continues to hold as long as $\alpha > 0$. The

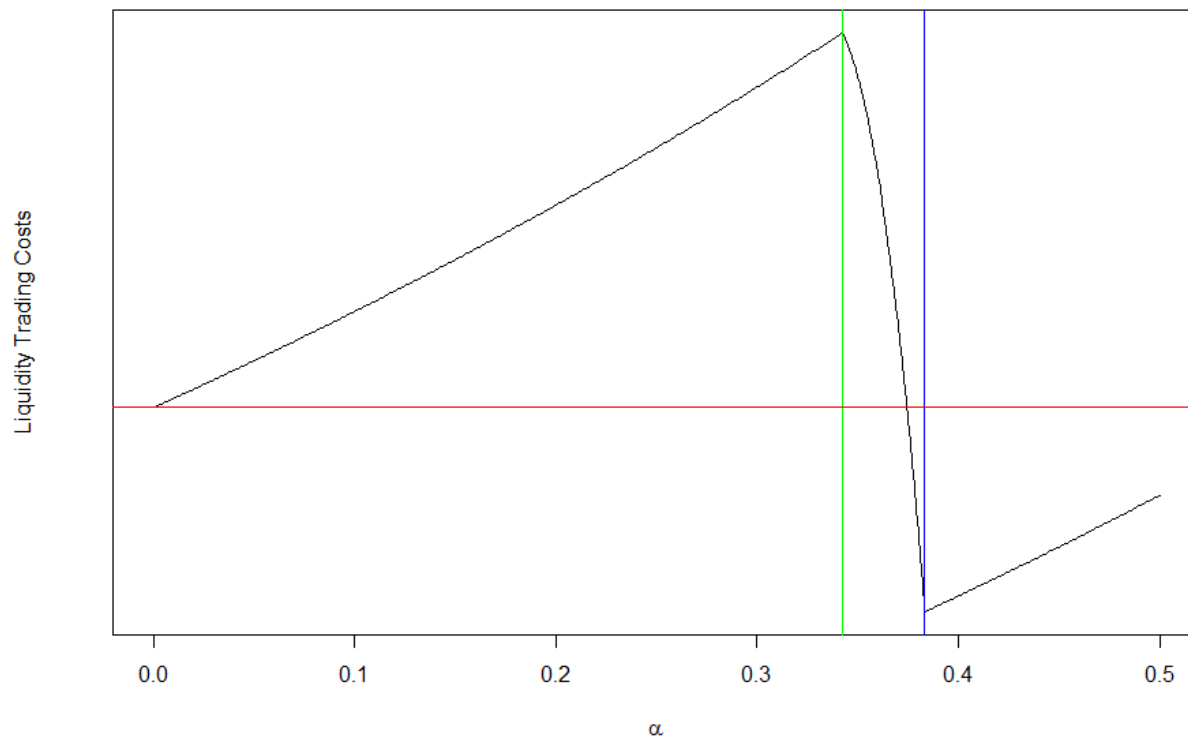


Figure B.1. Case 1: Large buyback lowers liquidity trading costs. This figure plots an example of liquidity against the buyback size when a program of size $\alpha = \bar{\alpha}$ decreases liquidity trading costs.

same is true when the authorized amount $\hat{\alpha}$ is above $\bar{\alpha}$. Any strategy $\alpha \in [\bar{\alpha}, \hat{\alpha}]$ forms an equilibrium with $a = 1 - \hat{\alpha}$ and the pricing rule from the second of Lemma 5. In these equilibriums, the result of Proposition 5 continues to hold qualitatively.

B.2 Medium Repurchase Programs and Liquidity

Liquidity is a continuous function of repurchase program size, α because m is a continuous function of α . As a result, there are two main cases to consider. The first is when a repurchase program of size $\alpha = \bar{\alpha}$ improves liquidity. Figure B.1 shows an example of how

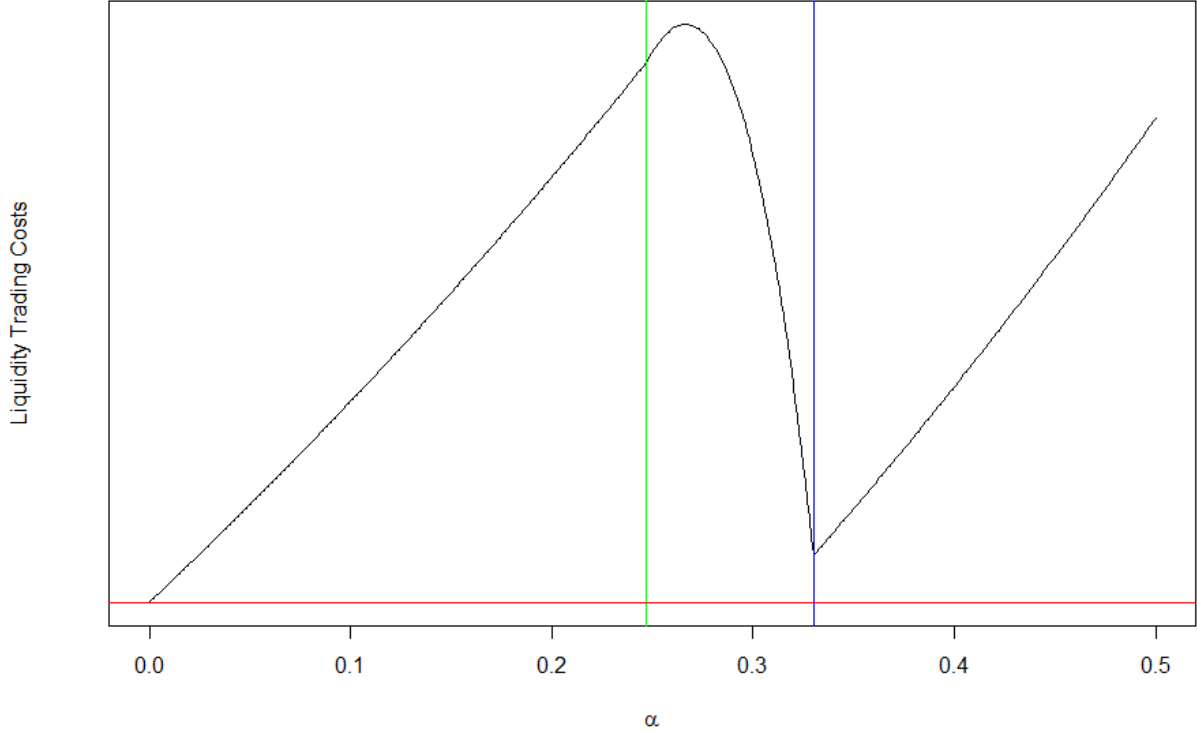


Figure B.2. Case 2: Large buyback raises liquidity trading costs. This figure plots an example of liquidity against the buyback size when a program of size $\alpha = \bar{\alpha}$ increases liquidity trading costs.

liquidity responds to buyback size in this instance. Because liquidity is a continuous function of α , $\Delta L(\bar{\alpha}) > 0$ implies that there exists a range of medium sized buyback programs that also improve liquidity. Furthermore, numerical calculations suggest that liquidity achieves a unique minimum on $[\underline{\alpha}, \bar{\alpha}]$ and that it monotonically increases from this trough until $\bar{\alpha}$. These numerical results are consistent with the proposition that a sufficiently large buyback size is a necessary condition for liquidity improvement. The second case is when buybacks of size $\alpha = \bar{\alpha}$ reduces liquidity. Figure B.1 shows an example of how liquidity responds to buyback size in this scenario. As in case 1, numerical calculations suggest that liquidity achieves a unique minimum on $[\underline{\alpha}, \bar{\alpha}]$ and that it monotonically increases from this trough until $\bar{\alpha}$. Continuity of liquidity as a function of α and $\Delta L(\bar{\alpha}) < 0$ imply that a buyback

program of any size $\alpha < x$ consumes liquidity. These numerical results are consistent with the idea that not all firms can improve the liquidity of its shares via a repurchase program.

B.3 Proofs

Proof of Lemma 5. Because of the discrete distribution for noise trade, the only viable candidates for a pure strategy equilibrium involves total informed trading of $\frac{1}{2}$ or 1. All other trading strategies lead to a perfectly revealing order flow and zero informed trading profits. Let $1 - m$ and m be the probability that the speculator submits an order of $\frac{1}{2} - \alpha$ and $1 - \alpha$, respectively.

Suppose the market-makers believe that the speculator's strategy is m . Then, by Bayes' rule, their posterior given an observed order flow q is given by:

$$\hat{\rho}(q) = \begin{cases} 0 & q = -\frac{1}{2} \\ \frac{\rho(1-m)x}{\rho(1-m)x+(1-\rho)(1-2x)} & q = 0 \\ \frac{\rho(1-2x-m(1-3x))}{\rho(1-2x-m(1-3x))+(1-\rho)x} & q = \frac{1}{2} \\ 1 & q \in \{1, \frac{3}{2}\} \end{cases} \quad (\text{B.1})$$

To help support the equilibrium, I set off-equilibrium posteriors at 1. Let $P(q)$ be the equilibrium price set by market-makers after observing an order flow q . When asset payoffs are low, the manager does not buy back any stock and the firm's per-share value is simply 0. When asset payoffs are high, the manager repurchases α shares of the firm at price $P(q)$. In this case, the per-share value of the firm is:

$$V(q) = \frac{1 - \alpha P(q)}{1 - \alpha} \quad (\text{B.2})$$

By the efficient market condition, the equilibrium stock price must be the expected per-share value conditioned on an order flow of q .

$$P(q) = \hat{\rho}(q)V(q) = \hat{\rho}(q)\frac{1 - \alpha P(q)}{1 - \alpha}$$

$$\Rightarrow P(q) = \frac{\hat{\rho}(q)}{1 - \alpha + \alpha\hat{\rho}(q)} \quad (\text{B.3})$$

Substituting (B.1) into (B.3) with the appropriate value of m yields the expressions for the equilibrium price in the lemmas. Similarly, the per-share value of the firm can be derived by substituting the equilibrium price into (B.2). To complete the construction of the equilibriums, one simply has to show that it is indeed optimal for the speculator to follow the strategy m .

Let $E(m, \alpha) = \frac{1}{\rho(1-\rho)x}(\Pi_1 - \Pi_0)$, where Π_0 and Π_1 are the expected trading profits of the speculator for following the strategy $m = 0$ and $m = 1$, respectively, given the above pricing rule.

$$E(m, \alpha) = \frac{(2x - \frac{1}{2}) + \alpha(1 - 3x)}{(1 - \rho)x + \rho(1 - 2x - m(1 - 3x))} - \frac{(\frac{1}{2} - \alpha)(1 - 2x)}{(1 - \alpha)(1 - \rho)(1 - 2x) + \rho(1 - m)x} \quad (\text{B.4})$$

When $E(m, \alpha) \leq 0$, the optimal response for the speculator is the pure strategy $m = 0$. When $E(m, \alpha) = 0$ for $m \in (0, 1)$, the optimal response is a mixed strategy. When $E(m, \alpha) \geq 0$, the optimal response is the pure strategy $m = 1$.

First, note that $E(m, \alpha)$ is a continuous function of α and m for $\alpha \in [0, \frac{1}{2}]$ and $m \in [0, 1]$. Moreover, for any value of $m \in [0, 1]$, $E(m, \alpha)$ is strictly increasing in α .

$$\frac{\partial E}{\partial \alpha} = \frac{1 - 3x}{(1 - \alpha)(1 - \rho)x + \rho(1 - 2x - m(1 - 3x))} + \frac{[2x - \frac{1}{2} + \alpha(1 - 3x)](1 - \rho)x}{[(1 - \alpha)(1 - \rho)x + \rho(1 - 2x - m(1 - 3x))]^2}$$

$$+ \frac{1-2x}{(1-\alpha)(1-\rho)(1-2x) + \rho(1-m)x} \underbrace{\left[1 - \frac{2(\frac{1}{2}-\alpha)(1-\rho)(1-2x)}{(1-\alpha)(1-\rho)(1-2x) + \rho(1-m)x} \right]}_{>0} > 0 \quad (\text{B.5})$$

Next, observe that for any value $m \in [0, 1]$:

$$E(m, 0) = \frac{x - \frac{1}{2}(1-2x)}{(1-\rho)x + \rho(1-2x - m(1-3x))} - \frac{\frac{1}{2}(1-2x)}{(1-\rho)(1-2x) + \rho(1-m)x} < 0 \quad (\text{B.6})$$

and

$$E(m, 1/2) = \frac{\frac{1}{2}x}{(1-\rho)x + \rho(1-2x - m(1-3x))} > 0 \quad (\text{B.7})$$

Thus, by the intermediate value theorem, there always exist some $\alpha(m)$ such that $E(m, \alpha(m)) = 0$. Moreover, by the strict monotonicity of $E(m, \alpha)$ in α , $E(m, \alpha) < 0$ for $\alpha < \alpha(m)$ and $E(m, \alpha) > 0$ for $\alpha > \alpha(m)$. This implies that when $\alpha \leq \alpha(0) = \underline{\alpha}$, the only pure strategy equilibrium is $m = 0$ and that when $\alpha \geq \alpha(1) = \bar{\alpha}$, the only pure strategy equilibrium is $m = 1$.

To show that $\underline{\alpha} < \bar{\alpha}$, suppose for contradiction that $\underline{\alpha} \geq \bar{\alpha}$. Then, there must exist some $\hat{\alpha}$ such that $\hat{\alpha} \geq \bar{\alpha}$ and $\hat{\alpha} \leq \underline{\alpha}$.

$$\hat{\alpha} \leq \underline{\alpha} \Rightarrow E(0, \hat{\alpha}) \leq 0 \Rightarrow \frac{(1-\hat{\alpha})x - (\frac{1}{2}-\hat{\alpha})(1-2x)}{(1-\hat{\alpha})(1-\rho)x + \rho(1-2x)} \leq \frac{(\frac{1}{2}-\hat{\alpha})(1-2x)}{2(1-\hat{\alpha})(1-\rho)(1-2x) + \rho x} \quad (\text{B.8})$$

$$\hat{\alpha} \geq \bar{\alpha} \Rightarrow E(1, \hat{\alpha}) > 0 \Rightarrow \frac{(1-\hat{\alpha})x - (\frac{1}{2}-\hat{\alpha})(1-2x)}{(1-\hat{\alpha})(1-\rho)x + \rho x} \geq \frac{1}{(1-\hat{\alpha})(1-\rho)}$$

$$\Rightarrow \frac{(1-\hat{\alpha})(1-\rho)x + \rho x}{(1-\hat{\alpha})x - (\frac{1}{2}-\hat{\alpha})(1-2x)} \leq (1-\hat{\alpha})(1-\rho) \quad (\text{B.9})$$

Inequalities (B.8) and (B.9) together imply that:

$$\begin{aligned}
& \frac{(1 - \hat{\alpha})(1 - \rho)x + \rho x}{(1 - \hat{\alpha})(1 - \rho)x + \rho(1 - 2x)} \leq \frac{(1 - \hat{\alpha})(\frac{1}{2} - \hat{\alpha})(1 - 2x)(1 - \rho)}{(1 - \hat{\alpha})(1 - \rho)(1 - 2x) + \rho x} \\
\Rightarrow 0 & \leq (1 - \hat{\alpha})^2(1 - \rho)^x(1 - 2x) \underbrace{(\hat{\alpha} - \frac{1}{2})}_{\leq 0} \underbrace{-(1 - \hat{\alpha})(1 - \rho)\rho x^2 - \rho^2 x^2}_{< 0} \\
& + (1 - \hat{\alpha})(1 - \rho)(1 - 2x)[\rho(1 - 2x)(\frac{1}{2} - \hat{\alpha}) - x] \\
& \Rightarrow \rho(1 - 2x)(\frac{1}{2} - \hat{\alpha}) - x > 0
\end{aligned}$$

The last inequality implies that $\hat{\alpha} < 0$, which is a contradiction because $\hat{\alpha}$ cannot be negative because the firm does not short sell. Thus, by contradiction, $\underline{\alpha} < \bar{\alpha}$.

To construct the mixed strategy equilibrium, I first point out that no mixed strategy equilibriums exist for $\alpha < \underline{\alpha}$ and $\alpha > \bar{\alpha}$. Suppose for contradiction that there exist a mixed strategy equilibrium for some $\hat{\alpha} < \underline{\alpha}$. Then, there exist some $m^* \in (0, 1)$ such that:

$$\begin{aligned}
E(m^*, \hat{\alpha}) = 0 & \Leftrightarrow \frac{(2x - \frac{1}{2}) + \hat{\alpha}(1 - 3x)}{(1 - \hat{\alpha})(1 - \rho)x + \rho(1 - 2x - m^*(1 - 3x))} = \frac{(\frac{1}{2} - \hat{\alpha})(1 - 2x)}{(1 - \hat{\alpha})(1 - \rho)(1 - 2x) + \rho(1 - m^*)x} \\
& \Leftrightarrow \frac{(1 - \hat{\alpha})(1 - \rho)x + \rho(1 - 2x - m^*(1 - 3x))}{(2x - \frac{1}{2}) + \hat{\alpha}(1 - 3x)} = \frac{(1 - \hat{\alpha})(1 - \rho)(1 - 2x) + \rho(1 - m^*)x}{(\frac{1}{2} - \hat{\alpha})(1 - 2x)}
\end{aligned} \tag{B.10}$$

$$\hat{\alpha} \leq \underline{\alpha} \Rightarrow E(0, \hat{\alpha}) \leq 0 \Rightarrow \frac{(2x - \frac{1}{2}) + \hat{\alpha}(1 - 3x)}{(1 - \hat{\alpha})(1 - \rho)x + \rho(1 - 2x)} \leq \frac{(\frac{1}{2} - \hat{\alpha})(1 - 2x)}{(1 - \hat{\alpha})(1 - \rho)(1 - 2x) + \rho x} \tag{B.11}$$

Equation (B.10) and inequality (B.11) imply that:

$$(1 - \hat{\alpha})(1 - \rho)[(1 - 2x)(1 - 3x) - x^2] \geq \rho x^2 \quad (\text{B.12})$$

The inequality (B.12) is false for all $\rho \leq \frac{1}{2}$, $\hat{\alpha} \in [0, \frac{1}{2}]$, and $x \in [\frac{1}{4}, \frac{1}{3}]$. Therefore, by contradiction, a mixed strategy does not exist for $\alpha < \underline{\alpha}$. A similar argument can be used to show that no mixed strategy equilibriums exist when $\alpha > \bar{\alpha}$.

Let $\hat{\alpha} \in (\underline{\alpha}, \bar{\alpha})$. $\hat{\alpha} \in (\underline{\alpha}, \bar{\alpha})$ implies that $E(0, \hat{\alpha}) > 0$ and $E(1, \hat{\alpha}) < 0$. As noted earlier, $E(m, \alpha)$ is a continuous function of m . Thus, by the intermediate value theorem, there exists some $m^* \in (0, 1)$ such that $E(m^*, \hat{\alpha}) = 0$. Equation B.10 implies that this m^* can be expressed as:

$$\begin{aligned} m^* &= \frac{(1 - \hat{\alpha})(1 - \rho)(1 - 2x)[2x - \frac{1}{2} + \hat{\alpha}(1 - 3x) - (\frac{1}{2} - \hat{\alpha})x] + \rho[x(2x - \frac{1}{2}) + \hat{\alpha}x(1 - 3x) - \frac{1}{2}(1 - 2x)^2]}{\rho[x(2x - \frac{1}{2}) + \hat{\alpha}x(1 - 3x) - (1 - 2x)(1 - 3x)(\frac{1}{2} - \hat{\alpha})]} \\ &= \frac{(1 - \hat{\alpha})(1 - \rho)(1 - 2x)[\hat{\alpha}(1 - 2x) - \frac{1}{2}(1 - 3x)] + \rho[\hat{\alpha}(1 - 3x + x^2) - \frac{1}{2}(1 - 3x)]}{\rho[\hat{\alpha}(1 - x)(1 - 3x) - \frac{1}{2}(1 - 4x + 2x^2)]} \end{aligned}$$

For completeness, I also note that the expressions for $\underline{\alpha}$ and $\bar{\alpha}$ can be derived by solving for the roots of the quadratic equations associated with $E(0, \underline{\alpha}) = 0$ and $E(1, \bar{\alpha}) = 0$. Discarding the extraneous roots yields the expressions:

$$\underline{\alpha} = \frac{-\underline{b} + \sqrt{\underline{b}^2 - 4\underline{a}\underline{c}}}{2\underline{a}} \quad (\text{B.13})$$

where

$$\begin{aligned} \underline{a} &= -(1 - \rho)(1 - 2x)^2 \\ \underline{b} &= \frac{1}{2}(1 - \rho)(1 - 2x)(3 - 7x) \end{aligned}$$

$$\underline{c} = -\frac{1}{2}(1-3x)[(1-\rho)(1-2x) + \rho]$$

and

$$\bar{\alpha} = \frac{-\bar{b} + \sqrt{\bar{b}^2 - 4\bar{a}\bar{c}}}{2\bar{a}} \quad (\text{B.14})$$

where

$$\begin{aligned} \bar{a} &= -(1-\rho)(1-2x)^2 \\ \bar{b} &= \frac{1}{2}(1-\rho)(1-2x)(3-7x) + \rho x(1-2x) \\ \bar{c} &= -(1-2x)\left[\frac{1}{2}(1-3x) + \rho\left(2x - \frac{1}{2}\right)\right] \end{aligned}$$

■

Proof of Lemma 6. This proof begins by establishing a lower bound for $\underline{\alpha}$. Let $\alpha_x = \frac{1-3x}{2-5x}$.

$$E(0, \alpha_x) = \frac{(2x - \frac{1}{2}) + \alpha_x(1-3x)}{(1-\alpha_x)(1-\rho)x + \rho(1-2x)} - \frac{(\frac{1}{2} - \alpha_x)(1-2x)}{(1-\alpha_x)(1-\rho)(1-2x) + \rho x} \quad (\text{B.15})$$

When $\alpha_x = \frac{1-3x}{2-5x}$, the numerators of the fractions in (B.15) are equal. Moreover, $\rho(1-3x) > (1-\alpha_x)(1-\rho)(1-3x)$. Taken together these facts imply that $E(0, \alpha_x) < 0$. The monotonicity of E in α implies that:

$$\underline{\alpha}(x, \rho) > \frac{1-3x}{2-5x}$$

$$\begin{aligned} E(m^*, \alpha) = 0 &\Leftrightarrow \frac{(2x - \frac{1}{2}) + \alpha(1-3x)}{(1-\alpha)(1-\rho)x + \rho(1-2x - m^*(1-3x))} = \frac{(\frac{1}{2} - \alpha)(1-2x)}{(1-\alpha)(1-\rho)(1-2x) + \rho(1-m^*)x} \\ &\Leftrightarrow [(2x - \frac{1}{2}) + \alpha(1-3x)][(1-\alpha)(1-\rho)(1-2x) + \rho(1-m^*)x] = \\ &\quad \frac{1}{2}(1-\alpha)(1-2x)[(1-\alpha)(1-\rho)x + \rho(1-2x - m^*(1-3x))] \end{aligned}$$

Let $G(m^*, \alpha) \equiv 0$ be defined as:

$$G(m^*, \alpha) = [(2x - \frac{1}{2}) + \alpha(1 - 3x)][(1 - \alpha)(1 - \rho)(1 - 2x) + \rho(1 - m^*)x] - (\frac{1}{2} - \alpha)(1 - 2x)[(1 - \alpha)(1 - \rho)x + \rho(1 - 2x - m^*(1 - 3x))] \quad (\text{B.16})$$

$$\begin{aligned} \frac{\partial G}{\partial m^*} &= -\rho x(2x - \frac{1}{2} + \alpha(1 - 3x)) + \rho(1 - 3x)(\frac{1}{2} - \alpha)(1 - 2x) \\ &= \rho(\frac{1}{2}(2x^2 - 4x + 1) - \alpha(1 - 3x)(1 - x)) < 0 \end{aligned} \quad (\text{B.17})$$

because $\alpha \geq \underline{\alpha} > \frac{1-3x}{2-5x} \geq \frac{2x^2-4x+1}{2(1-3x)(1-x)}$

$$\begin{aligned} \frac{\partial G}{\partial \alpha} &= (1 - 3x)[(1 - \alpha)(1 - \rho)(1 - 2x) + \rho(1 - m)x] - [2x - \frac{1}{2} + \alpha(1 - 3x)](1 - \rho)(1 - 2x) \\ &\quad + (1 - 2x)[(1 - \alpha)(1 - \rho)x + \rho(1 - 2x - m(1 - 3x))] + (\frac{1}{2} - \alpha)(1 - 2x)(1 - \rho)x > 0 \end{aligned} \quad (\text{B.18})$$

Implicit differentiation implies that:

$$\frac{\partial m^*}{\partial \alpha} = -\frac{\frac{\partial G}{\partial \alpha}}{\frac{\partial G}{\partial m^*}} > 0$$

■

Proof of Proposition 4. When $\alpha < \underline{\alpha}$, $m^* = 0$ and total informed trading is $\frac{1}{2}$. Thus, liquidity

trading losses as a function of α can be written as:

$$L(\alpha) = \frac{1}{2} \left[\frac{\rho(1-\rho)x(1-2x)}{(1-\alpha)(1-\rho)(1-2x) + \rho x} + \frac{\rho(1-\rho)x(1-2x)}{(1-\alpha)(1-\rho) + \rho(1-2x)} \right]$$

Taking the derivative of $L(\alpha)$ with respect to α for $\alpha < \bar{\alpha}$ yields:

$$\frac{dL}{d\alpha} = \frac{x(1-2x)\rho(1-\rho)}{2} \left[\frac{(1-\rho)(1-2x)}{[(1-\alpha)(1-\rho)(1-2x) + \rho x]^2} + \frac{(1-\rho)x}{[(1-\alpha)(1-\rho) + \rho(1-2x)]^2} \right] > 0$$

■

Proof of Proposition 5. The change in expected liquidity trading costs due to a repurchase authorization of size $\alpha \geq \bar{\alpha}$ is:

$$\Delta L(\alpha) = \frac{\rho(1-\rho)x}{1-\alpha + \alpha\rho} - \frac{1}{2}\rho(1-\rho)x(1-2x) \left[\frac{1}{(1-\rho)(1-2x) + \rho x} + \frac{1}{(1-\rho)x + \rho(1-2x)} \right] \quad (\text{B.19})$$

Let $A(x, \rho) = \frac{2}{1-\alpha + \alpha\rho} - (1-2x) \left[\frac{1}{(1-\rho)(1-2x) + \rho x} + \frac{1}{(1-\rho)x + \rho(1-2x)} \right]$. A has the same sign as $\Delta L(\alpha)$.

Thus, a large repurchase program lowers liquidity trading costs if and only if $A(x, \rho) < 0$.

$$\frac{\partial A}{\partial x} = \frac{4(1-\rho)(1-2x) + \rho(4x-1)}{[(1-\rho)(1-2x) + \rho x]^2} + \frac{4\rho(1-2x) + (1-\rho)(4x-1)}{[(1-\rho)x + \rho(1-2x)]^2} > 0 \quad (\text{B.20})$$

$$A(1/4, \rho) = \frac{2}{1-\alpha + \alpha\rho} - \frac{2}{2-\rho} - \frac{2}{1+\rho} \leq 0$$

$$A(1/3, \rho) = \frac{2}{1-\alpha + \alpha\rho} - 2 > 0$$

These two inequalities, along with the monotonicity of A , establish the first result of this proposition.

As noted earlier, a large repurchase program lowers liquidity trading costs if and only

if $A(x, \rho) < 0$, where

$$A(x, \rho) = \frac{2}{1 - \alpha + \alpha\rho} - (1 - 2x) \left[\frac{1}{(1 - \rho)(1 - 2x) + \rho x} + \frac{1}{(1 - \rho)x + \rho(1 - 2x)} \right]$$

$$\frac{\partial A}{\partial \rho} = \frac{-2\alpha}{[1 - \alpha(1 - \rho)]^2} + (1 - 2x)(1 - 3x) \left[\frac{1}{[(1 - \rho)(1 - 2x) + \rho x]^2} - \frac{1}{[(1 - \rho)x + \rho(1 - 2x)]^2} \right] > 0 \quad (\text{B.21})$$

$$A(x, 1) = 3 - \frac{1}{x} < 0$$

$$A(x, 1/2) \Big|_{\alpha=1/2} = \frac{8}{3} - \frac{4(1 - 2x)}{1 - x} \geq 0$$

These two inequalities, along with the monotonicity of A , establish the second result of this proposition in the main body of the paper. When $\alpha \in [\bar{\alpha}, \frac{1}{2})$, there still exists a cutoff such that $A(x, \rho) > 0$. However, this cutoff may be so small for some values of x that it is always satisfied by $\rho \in [\frac{1}{2}, 1)$. ■

Proof of Lemma 7. The proof of this lemma follows closely that of Lemma 5. ■

Proof of Lemma 8. When $\alpha = 0$, the expected liquidity trading cost is:

$$L_\gamma^0 = \frac{1}{2} \left[\frac{\rho(1 - \rho)x(1 - 2x)(1 - \gamma)}{\rho[(1 - \gamma)x + \gamma(1 - 2x)] + (1 - \rho)(1 - 2x)} + \frac{\rho(1 - \rho)x(1 - 2x)(1 - \gamma)}{\rho[(1 - \gamma)(1 - 2x) + \gamma x] + (1 - \rho)x} \right]$$

Taking the derivative of L_γ^0 with respect to γ yields:

$$\frac{dL_\gamma^0}{d\gamma} = \frac{1}{2} \left[\underbrace{-\frac{\rho(1 - \rho)x(1 - 2x)^2}{[\rho[(1 - \gamma)x + \gamma(1 - 2x)] + (1 - \rho)(1 - 2x)]^2}}_{<0} - \underbrace{\frac{\rho(1 - \rho)x^2(1 - 2x)}{[\rho[(1 - \gamma)(1 - 2x) + \gamma x] + (1 - \rho)x]^2}}_{<0} \right] < 0 \quad (\text{B.22})$$

■

Proof of Proposition 6. The change in expected liquidity trading costs due to a repurchase authorization of $\alpha = \frac{1}{2}$ is:

$$\Delta L_\gamma = \frac{1}{2}\rho(1-\rho)x \left[\frac{\gamma(1-2x)}{\frac{1}{2}(1-\rho)(1-2x) + \rho\gamma x} + \frac{\gamma(1-2x) + 2x(1-\gamma)}{\frac{1}{2}(1-\rho)x + \rho[(1-\gamma)x + \gamma(1-2x)]} \right. \\ \left. - \frac{(1-\gamma)(1-2x)}{\rho[(1-\gamma)x + \gamma(1-2x)] + (1-\rho)(1-2x)} - \frac{(1-\gamma)(1-2x)}{\rho[(1-\gamma)(1-2x) + \gamma x] + (1-\rho)x} \right]$$

For notational convenience, let $\psi = (1-\gamma)x + \gamma(1-2x)$.

$$\frac{d\Delta L_\gamma}{d\gamma} = \frac{1}{2}\rho(1-\rho)x \left[\frac{\frac{1}{2}(1-\rho)(1-2x)^2}{[\frac{1}{2}(1-\rho)(1-2x) + \rho\gamma x]^2} + \frac{\frac{1}{2}(1-\rho)x(1-2x) - (1-\rho)x^2}{[\frac{1}{2}(1-\rho)x + \rho\psi]^2} \right. \\ \left. - \frac{\rho x(1-2x)}{[\frac{1}{2}(1-\rho)x + \rho\psi]^2} \right] \\ + \frac{1}{2} \frac{\rho(1-\rho)x(1-2x)^2}{[\rho\psi + (1-\rho)(1-2x)]^2} + \frac{1}{2} \frac{\rho(1-\rho)x^2(1-2x)}{[\rho[(1-\gamma)(1-2x) + \gamma x] + (1-\rho)x]^2} \quad (\text{B.23})$$

The sum of the first two terms of (B.23) is bounded below by:

$$\frac{\frac{1}{2}(1-\rho)(1-3x)}{[\frac{1}{2}(1-\rho)x + \rho\psi]^2} > 0$$

The sum of the third and fourth is bounded below by 0 because:

$$\frac{\sqrt{\rho x}}{\frac{1}{2}(1-\rho)x + \rho\psi} \leq \frac{\sqrt{1-2x}}{(1-\rho)(1-2x) + \rho\psi}$$

The final term of (B.23) is a positive fraction. Thus, $\frac{d\Delta L_\gamma}{d\gamma} > 0$. ■