Modeling Social Network Change over Time: A Comparison of Methods

Elizabeth A. Dietrich

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Committee:

Elizabeth A. Sanders, Chair

Robert D. Abbott

Min Li

Dagmar Amtmann

Thomas J. Halverson

Program Authorized to Offer Degree:

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University of Washington

Abstract

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Elizabeth A. Dietrich

Chair of the Supervisory Committee:
Associate Professor Elizabeth A. Sanders
Measurement and Statistics, Educational Psychology
College of Education

This study contributes to the gap in educational research methods knowledge by 1) assessing prevalence of using analytical social network methods in educational research, 2) being the first to directly compare four possible approaches for modeling longitudinal stochastic social networks: temporal exponential random graph model with bootstrapping (TERGM-B), temporal exponential random graph model with Markov Chain Monte Carlo maximum likelihood estimation (TERGM-M), separable temporal exponential random graph model (STERGM), and stochastic actor-based model (SABM), and 3) providing a demonstration of each method analyzing a small-sample, low-density network of professional development participants who were followed for four years.

The field of education has been slower to adopt social network analysis (SNA) as a research tool compared to the health and social sciences. To date, few studies employ quantitative SNA, and none have reported using longitudinal SNA models. The present study examines the prevalence of stochastic network models in educational research and compares options for conducting stochastic models for longitudinal data using both simulated and observed network data. The simulated network data included two and four time points, with and without linear and quadratic change in density. The observed network data involved a small sample size, a sparse network, and four time points. For both kinds of datasets, exponential random graph model (ERGM) approaches, including temporal and separable temporal ERGM (TERGM and STERGM), as well as stochastic actor-based model (SABM) were applied. Results of the simulated datasets has shown that the TERGM appears to perform closest to the simulation parameter settings, with STERGM performing second best. The STERGM and SABM presents challenges in practice. Advantages and disadvantages of each approach are discussed.

Keywords: social network analysis, longitudinal, teacher advice network, temporal exponential random family graph model (TERGM), separable temporal exponential random family graph model (STERGM), stochastic actor-based model (SABM)

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Chapter 1. INTRODUCTION

Social networks are social systems in which people are connected and have the potential to influence others (Knoke & Yang, 2011). In many areas of the social sciences, such as demography studies, epidemiology, political science, and sociology, researchers use social network analysis (SNA) models to investigate disease spread, drug use, physical and emotional health outcomes, political party affiliation, political influence, job attainment, jury consensus, trade patterns, kinship, and social media relations (e.g., Granovetter, 1973; Valente, Gallaher, & Mouttapa, 2004; White, 2014). Across all of these disciplines, the goal of using SNA is to better understand the structure of the social system under study, including the direct and indirect linkages among individuals in the network.

1.1 Disciplinary Roots of Social Network Analysis

Social Networks vs. Social Network Analysis

Importantly, the term **social network analysis** (SNA) can mean different things in different contexts. While a *social network* refers to individuals who relate to others in a certain relational context (e.g., a network of individuals that teachers seek advice from), or how one interacts with others on social media platforms (e.g., which individuals communicate with other individuals about trending news topics), the term *social network analysis* has a more specific meaning. Freeman (2004) outlined four characteristics of a SNA; specifically, that it: 1) focuses on relationships ("ties") between individuals (also known as "nodes" and "actors") as being important, rather than the individual attributes; 2) is based on relationships/ties; 3) relies on graphic imagery to illustrate patterns of relationships/ties; and 4) uses mathematical or computation tools to evaluate patterns of relationships/ties (p. 3). In education, the individuals in a social network could be students within a school, teachers participating in a district professional development program, school administrators participating in leadership activities, and even schools or institutions themselves. In contrast to most typical research scenarios, however, it is the linkages between the individuals (i.e., the "ties") that would be the focal unit of analysis in SNA.

As one example, a teacher professional development project may have a goal of increasing the "density" (or cohesiveness) of a network of teacher collaborators in an effort to improve teaching practice (e.g., Thompson & Richards, 2016; Windschitl, Thompson, Braaten, & Stroupe, 2012). Friendship networks in schools can evaluate how immigrant youth are in integrating socially into schools and if the racial or ethnic or generational composition of their friendship may put them at risk for marginalization (Reynolds & Crea, 2017). As another example, network interventions in schools can have positive effects on students' health, in particular after school programs can help increase children's physical activity in schools where the obesity rate is high (Zhang, Shoham, Tesdahl, & Gesell, 2015). In each of these scenarios, a researcher could use SNA as a tool to: 1) quantify social network characteristics pre- or post-intervention/policy change, 2) test the probability of any individual in the network in forming a new connection within that network, 3) test whether individual characteristics are associated with the probability of forming a new connection, and 4) test whether the network itself changes over a period of time.

Historical Roots of Social Network Analysis

As mentioned above, SNA as it is known today is a method used to analyze the relationships (i.e., "ties") among individuals (i.e., "nodes" or "actors"), rather than individuals' particular characteristics (Wasserman & Faust, 1994). The earliest version of SNA is sociometry, the study of measuring social relationships, which first came about in the 1930s with psychologist Jacob Moreno's study of social structures and well-being. While there was some interest in the topic it did not gain much traction at the time. Nevertheless, pockets of researchers studied social networks, but mostly in silos within their respective fields of communication, geography, mathematical biology, linguistics, political science, psychology, and sociology (Freeman, 2014). It was in the 1970s that SNA emerged in its more present day form, in particular because of the universal recognition of social networks that resulted from the theoretical and empirical contributions to the literature made by Harrison White and his students at Harvard. In 1978, the *Social Networks* journal was founded by Linton C. Freeman, which publishes substantive and theoretical work dealing with social networks and behavior in social science disciplines (Bernard, 2005).

Social Network Analysis in the Health Sciences

Social relations influence what we do, who we do it with, and may have long term effects on our health, as well as affect where we go and how we interact with the people and space around us (Berkman & Glass, 2000; Berkman & Syme, 1979). For example, health science researchers have used SNA to evaluate how diseases such as HIV spread, as well as to assess the spread of information to inform health related policies (Kebede, 2012; Krivitsky & Morris, 2017; Paxton, 2010; Shearer, Dion, & Lavis, 2014; Ssali, Wagner, Tumwine, Nannungi, & Green, 2012; Young, Halgin, DiClemente, Sterk, & Havens, 2014). In addition, SNA has been used to investigate aspects of drug use such as influence of peers, risk patterns, needle sharing, the role of gender, being well-liked, and geography by looking at types of ties that exist and the structure of these networks (Dombrowski et al., 2013; Jacobs, Goodson, Barry, & McLeroy, 2016; Valente et al., 2004; Van Ryzin, DeLay, & Dishion, 2016; Wylie, Shah, & Jolly, 2007). Finally, SNA has also been used to study happiness levels among networks of people (Chang, Lin, & Chen, 2012; Fowler & Christakis, 2009), depression and suicide (Berkman, Glass, Brissette, & Seeman, 2000), as well as general health levels (Berkman & Glass, 2000). In each of these studies, friends, parents, and geographic areas comprising social networks were taken into consideration.

Social Network Analysis in the Social Sciences

Our interactions with what is around us can include working relations, political relations, personal relations, and even those between entities such as businesses or countries (Yang, Keller, & Zheng, 2017). The social sciences have used SNA to investigate aspects of kinship relations, corporate work relations, trade relations, and migration patterns. For example, anthropologists have studied structural patterns of and linkages inherent to kinship relations and class (White, 2014), as well as the evidence of homophily (the tendency to relate to those with similar characteristics) in friendship networks and the position of certain types of folks in networks such as happy people befriending happy people and being more central in the network (Fowler & Christakis, 2009). In business and communications research, it is well known that one's social network is an important part of finding a job (Granovetter, 1973). Studies have found that social networks play an important role in human behavior, in particular with regard to political beliefs, attitudes, and voting patterns, finding that individuals behave similarly to those around them and exhibit similar behaviors (Anderson & Heath, 2002; Johnston & Pattie, 2014; Settle, Bond, & Levitt, 2011). Violent relationships and the relation between characteristics of individuals in different groups of homeless people have been evaluated (Petering, Rice, & Rhoades, 2015). The applications of social network analysis extend as far as there are social relations.

Social Network Analysis in Social Media

With the advent and increased use of social media tools that connect individuals with each other such as Facebook and Twitter, large amounts of data are becoming available for analyses, even prior to any research hypotheses forming. This secondary data may have some potential for explaining behavioral (Fischer & Rueber, 2011), social (Butler & Matook, 2015), or even health outcomes (Lefebvre & Bornkessel, 2013). Additionally, although individuals may be of interest in some research applications (e.g., what is the likelihood that a new "friend" will be made in the network of people who "like" a certain webpage?), websites (such as chat room forums) themselves may be the focal unit of analysis. For example, the number of "tags" or "likes" on a given website might be indicative of whether another website within the network will be "liked."

1.2 Prevalence of Social Network Analysis in Education

Although other disciplines have used SNA for some time, the spread of SNA as a tool for educational research appeared at face value to be slower. To determine the prevalence of SNA in educational research, a review of SNA use was conducted within the University of Washington's library databases. The review began with a search in the *Education Source* database for articles from academic peer-reviewed journals published in 2014-2016 (in English) that included any of the search terms "social network analysis" or "social network modeling" anywhere in the article. That search returned 700 articles, and after evaluating the types of journals in which these articles were published, the search was further narrowed to educational journals based in the United States, including: *American Journal of Education*, *Computers and Education*, *Educational Psychologist*, *Journal of Educational Technology and Society, Journal of Higher Education*, *The International Journal of Higher Education Research*, *The Internet and Higher*

Education, *Online Learning Journal*, and *Teaching & Teacher Education*. Of these journals, 58 articles were identified and screened to determine whether they sought to investigate social networks. Of these, 37 articles (64%) were found to use SNA as the primary research focus. Those 37 articles were then further evaluated for the types of SNA reported.

Results showed three clear patterns (see Table 1): first, 76% of SNA-focused articles reported social network **visualizations** (or "sociograms") based solely on descriptive statistics of network properties (e.g., number of ties per node). Nearly half (46%) did at least report some form of network descriptive statistics (i.e., measures of connectedness that will be further described shortly). Second, it was clear was a sizable portion (46%) employed a qualitative form of "social network analysis," dealing solely with verbally describing underlying processes/phenomena of social networks. Third, because the present study's interest is in probabilistic applications of SNA (e.g., being able to empirically predict the formation of a relationship/tie between individuals/nodes in a network), it was interesting that only five of the articles (14%) employed such methods. Two of these used a probabilistic (also known as "stochastic") model to study networks that were followed over time, but neither implemented a longitudinal SNA model. Additionally, one used an approach to estimating tie formation that is non-model-based (it suffers from non-independence within network; it will be further described in the next section). Overall, the review showed that probabilistic SNA has largely not been in use in recent educational research. It may be that educational research questions differ from those of other fields, or it may be that the method is just now making its way into the educational research sphere.

Table 1. *Prevalence of Social Network Analysis in Educational Research*

Journal	Count	Count Used SNA	Descriptives	Sociograms	Stochastic SN Model	Social Online	Qualitative
Journal of Educational Technology and Society	11	8	5	5	0	6	1
The International Journal of Higher Education Research	9	7	5	5	2	1	2
Online Learning Journal	8	2	1	2	0	2	2
The Internet and Higher Education	6	6	4	5	0	6	3
American Journal of Education	5	3	1	3	1	0	2
Computers and Education	5	4	1	3	1	3	2
Educational Psychologist	5	1	0	1	0	0	1
Journal of Higher Education	5	3	1	2	0	0	2
Teaching & Teacher Education	4	3	2	2	1	1	2
Total	58	37	20	28	5	19	17
Percent of Articles Using SNA		64%	54%	76%	14%	51%	46%

1.3 Probabilistic Approaches for Social Network Analysis

Social network analyses that are grounded in inferential statistics, rather than descriptive statistics, are the focus of the present study. Importantly, the term "stochastic model" can be used interchangeably with the term "probabilistic model" to indicate a parameterized model that can be evaluated using probability theory, thereby affording quantitative predictions/inferences with some level of confidence or certainty. A brief overview of social network analysis models is provided below to demonstrate the variety of approaches possible as the basis for the present study; however, these methods are explained in greater detail in Chapter 3 (Methods).

1.3.1 *Modeling a Social Network at a Single Point in Time*

There are two major quantitative methods from which to choose when evaluating a social network measured at a single time point (i.e., cross-sectional data): one is a non-model based approach (relying on an empirically constructed distribution) and one is model-based (relying on theoretical sampling distributions from probability theory). The first type is called the Quadratic Assignment Procedure (QAP); it is implemented using *UCINET* software (Borgatti, Everett, & Freeman, 2002). Specifically, the software computes a correlation/regression between matrices, and then the correlation/regression is re-applied but with one of the original matrices' rows and columns permuted. This process is then repeated until the correlations comprise a distribution of some fixed value specified by the researcher; the default is 5,000 permutations, 50,000 is suggested (Borgatti, Everett, & Johnson, 2013). Then the first correlation matrix, based on observed data, is compared to the distribution to obtain a *p*-value (Yang et al., 2017). However, this method does not account for dependence that is characteristic of network data and is therefore not ideal when analyzing network data.

The second type of SNA approach is called the exponential family random graph model (ERGM); it is the dominant approach for estimating SNA. The ERGM arose from the original *p** models proposed by Frank and Strauss (1986) and popularized by Wasserman and Pattison (1996). The ERGM model includes the Markov random graphs in addition to the dyadic independence model (p_1 model; (Holland & Leinhardt, 1981). It allows one to conduct a logistic regression predicting the existence of a tie, based on the possible permutations of the network, or matrix of dependent data. In explaining the network formation, ERGM allows exogenous (node-level) and endogenous (network descriptors such as transitivity and popularity) factors to be represented in the model (Yang et al., 2017), however this model can suffer from degeneracy issues, meaning that the algorithm converges to networks that are complete or empty (Handcock, 2003a). ERGM can be run in packages using *R* software (CRAN, 2013; Handcock, Hunter, Butts, Goodreau, & Morris, 2008).

1.3.2 *Longitudinal Social Network Modeling*

When a social network is measured at multiple time points, there is a greater number of model choices for SNA and it is not clear which method may be preferable, and under what circumstances. Robins and Pattison (2001) proposed the **temporal exponential random graph model** (TERGM), which includes a *temporal* component to the basic exponential family random graph model (ERGM). Like the ERGM, the focus of the TERGM approach is estimating the probability of a tie formation. For the TERGM, the estimate is for the formation of a tie at the last time point measured, conditioned on status of the network's ties at a prior time point(s) (Hanneke, Fu, & Xing, 2010). Of relevance to the current study, the TERGM can be estimated with two different likelihood algorithms, called TERGM-B and TERGM-M (more details in the methods section).

Another longitudinal SNA approach, called the **separable temporal random graph model** (STERGM; (Krivitsky & Handcock, 2014)), is similarly based on the ERGM but includes a *separable temporal* component. This model is able to estimate both the probability of a tie formation and the probability of a tie dissolution, given the ties at a prior time point(s).

The **stochastic actor-based model** (SABM) model (Snijders, van de Bunt, & Steglich, 2010b) originally termed "stochastic actor-oriented model" (SAOM; (Snijders, 1996; Snijders, 2001b)) is the third approach to modeling SNA over time and grew out of the reciprocity model (Wasserman, 1980). The SABM is an *actor-based* model focusing on predicting the probability of a *tie change* in the network, and taking the perspective that actors (nodes) make their own decisions (not necessarily consciously) about to whom they connect with, based on the structure of the network. The SABM considers each actor's potential to form, maintain, or dissolve a tie with every other node in the network, called "ministeps" such that only one change in the network is possible at each of these "ministeps," but each alters the network state for the next "ministep." Thus, network changes are organized by the nodes in the network (Ripley, Snijders, Boda, Voros, & Preciado, 2017). This actor-based approach is different than the ERGM-based approaches which assess how likely the observed network is to occur, given the possible permutations of the network that retain its structure.

In all of these longitudinal SNA approaches, measurement of the complete network at least two time points is required. However, to date no one has evaluated which of these longitudinal approaches may be preferred based on known parameters (contextual factors). Thus, a researcher might choose one method based simply on the one they have been exposed to within their discipline. Alternatively, the researcher may know of two or more of these approaches and decide that one is best for the kind of conclusions they would like to draw. TERGM would be preferred if the interest is in total change in ties over time, whereas STERGM would allow one to examine the formation and dissolution of ties over time (Krivitsky & Handcock, 2014). Importantly, Desmarais and colleagues (2012) recommend that SABM be used instead of ERGM when: 1) a single node changing one tie at a time changes the network, 2) behavior and network co-evolve at the same time, 3) the network is suspected to be non-stationary, and 4) the ERGM exhibits degeneracy.

1.3.3 *Software for Social Network Analyses*

For modeling a social network measured at a single time point, the QAP method is typically conducted using commercially available *UCINET* software (Borgatti et al., 2002), whereas the ERGM approach (Handcock et al., 2008) is modeled using freely available packages in *R* (CRAN, 2013). All of the longitudinal SNA approaches, including TERGM (Leifeld, Cranmer, & Desmarais, 2017a), STERGM (Krivitsky & Handcock, 2016), and SABM (Ripley, Boitmanis, & Snijders, 2013), can be estimated in freely available *R* packages (CRAN, 2013).

1.4 The Current Study

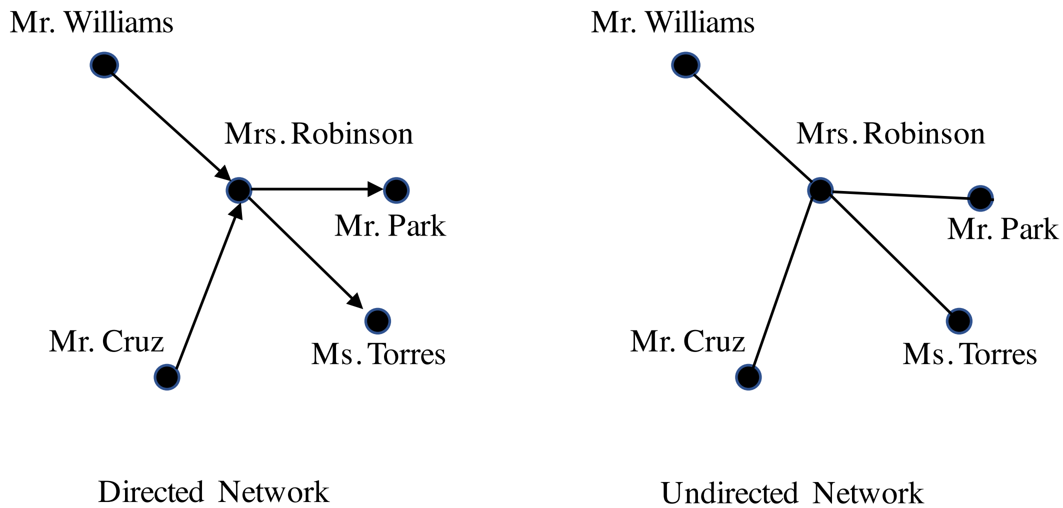
The present study contributes to the research on modeling social network change over time by comparing the various approaches using simulated datasets, as well as with real data from an NSF-funded science teacher advice network measured annually for four years. There were two sets of simulations. One set included two time points (minimum number for a longitudinal study), four node sizes (sample sizes of 20, 40, 60, and 80, which were held constant across time points), three levels of density (.06, .18, and .36), four levels of density linear growth (increases of 0%, 10%, 25%, and 50%), and four levels of density quadratic growth (deceleration of 0%, 2%, 4%, and 6%), crossed with each other for a total of 4 x 3 x 4 x 4 = 192 scenarios. The second set was a narrower version of the first, focusing only on the small sample size condition of $N = 20$ nodes (mirroring the applied dataset), as well as only the two more extreme levels of density (.06 and .36). Simulation parameters were specifically selected to ensure both representativeness of real data as well as capture a wide range of plausible situations. Given the computational intensity and length of time needed to generate a single simulated dataset, only one simulated dataset per crossed combination of conditions was generated for analysis. For each of these datasets, each of the longitudinal SNA models were conducted and each of those estimates saved. Those estimates were then analyzed using customized mixed model analyses of variance (within-subjects factor was model type, and between-subjects factors included each condition type) including tests of main effects as well as tests of two-way interactions with SNA model type. Similar to the simulated data, for the observed teacher network data, the same SNA models were conducted, including ERGMs at each time point, TERGM (estimated two ways), STERGM, and SABM. The chief aim across all analyses was to provide context for researchers when selecting the best approach for analyzing longitudinal social network data.

Chapter 2. BRIEF OVERVIEW OF SOCIAL NETWORK ANALYSIS (SNA) TERMINOLOGY

There are many ways to approach social network analyses (SNAs). Broadly, there are three kinds of SNAs that researchers may employ: qualitative, quantitative descriptive, and quantitative probabilistic or "stochastic" models. The subject of the present dissertation is the last type, stochastic models, which would be used on observed data to draw inferences about future network behavior (i.e., quantifying the likelihood of a connection, or "tie," forming between two individuals, or "nodes," given the model parameters). This said, even if researchers do not implement a stochastic SNA, they may wish to use descriptive analyses to quantify aspects of their networks. The purpose of this second chapter is therefore to provide a brief overview of common terms used for quantitatively describing and modeling a social network.

Networks, also referred to as **graphs**, express relationship patterns between individuals or entities. Included in networks are **nodes** (also referred to as actors and vertices) which may be linked together by **ties** (also referred to as edges, links, and arcs) (Robins & Lusher, 2013). In Figure 1, each node is represented with a black dot and each tie is represented by a black line linking two nodes together. First and foremost, a network may be **directed** or **undirected**, indicating the nature of the connections (see Figure 1).

Figure 1. *Directed and Undirected Networks*



A **directed network** is one in which an individual (also referred to as a node or actor, and in some disciplines, the vertex, or ego) may identify a **tie** (also referred to as edges, links, or connections) with another, but the other individual may not identify the same tie. The 'direction' of ties is indicated with an arrow. For example, Mr. Williams may identify Mrs. Robinson as someone he asks for advice, but Mrs. Robinson may not identify Mr. Williams as someone from whom she seeks advice. Conversely, the network may be **undirected** such that the only information that is known is that there is a link between the two nodes such as Mr. Williams and Mrs. Robinson spoke to each other. These data from these networks can be captured in adjacency matrices, with 1s indicating ties and 0s indicating no tie (for examples corresponding to Figure 1 see Figure 2 and Figure 3).

Figure 2. *Adjacency Matrix for Directed Network in Figure 1*

	Mr. Cruz	Mr. Park	Mrs. Robinson	Ms. Torres	Mr. Williams
Mr. Cruz	-	0	1	0	0
Mr. Park	0	-	0	0	0
Mrs. Robinson	0	1	-	1	0
Ms. Torres	0	0	0	-	0
Mr. Williams	0	0	1	0	-

Note. For undirected networks, adjacency matrices are symmetric.

Figure 3. *Adjacency Matrix for Undirected Network in Figure 1*

	Mr. Cruz	Mr. Park	Mrs. Robinson	Ms. Torres	Mr. Williams
Mr. Cruz	-	0	1	0	0
Mr. Park	0	-	1	0	0
Mrs. Robinson	1	1	-	1	1
Ms. Torres	0	0	1	-	0
Mr. Williams	0	0	1	0	-

Note. For directed networks, those in rows identify, or nominate, those in the columns. Row one shows that Mr. Cruz identified Mrs. Robinson, but did not identify anyone else.

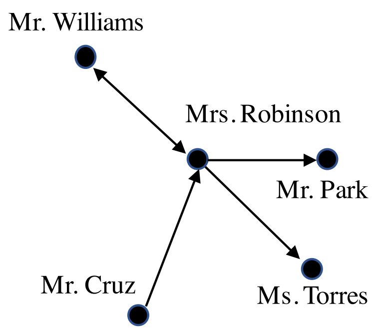
Reciprocity, also called mutuality or dyadic reciprocity, may be observed in a directed network, and indicates the nodes who identified each other (see Figure 4).

Figure 4. *Reciprocal and Non-Reciprocal Ties*

Reciprocal or Mutual Tie   Non-reciprocal or non-mutual Tie

Now let us imagine referring to the nodes as senders and receivers for a moment (see Figure 5).

Figure 5. *Directed Network with Reciprocal Tie & Corresponding Adjacency Matrix*

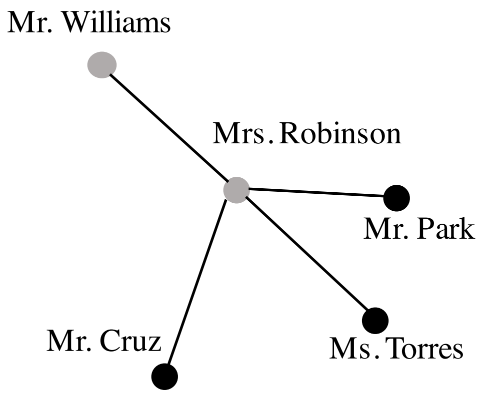


	Mr. Cruz	Mr. Park	Mrs. Robinson	Ms. Torres	Mr. Williams
Mr. Cruz	-	0	1	0	0
Mr. Park	0	-	0	0	0
Mrs. Robinson	0	1	-	1	1
Ms. Torres	0	0	0	-	0
Mr. Williams	0	0	1	0	-

Note. The adjacency matrix is interpreted as a directed network. The only reciprocal tie in the network can be seen in the matrix in rows three (Mrs. Robinson identified Mr. Williams) and five (Mr. Williams identified Mrs. Robinson).

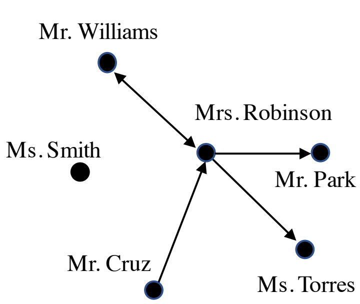
Mr. Williams may ‘send’ advice to Mrs. Robinson, and if she also ‘sends’ advice to Mr. Williams, the tie is reciprocal. Reciprocity is calculated by taking the average number of reciprocal ties out of the number of ties that exist. Of the same vein, **homophily** is a term used to describe individuals having the same attribute, such as individuals that share the same school, teaching same subject, or race (Lusher & Robins, 2013). In Figure 6, the color of the dots indicates what school the individual is from, with gray dots indicating the nodes being from one school and the black dots indicating that nodes are from a different school. Because Mr. Williams and Mrs. Robinson are both from the same school indicated by the same node color, their relationship is homophilious. This is the notion that people who are alike may be more likely to form ties with each other (Goodreau, Kitts, & Morris, 2009).

Figure 6. *Undirected Network with Homophilous Relationship*



In a given study, individuals may be surveyed on who to turn to for advice. Imagine that the group of individuals surveyed are all science teachers. Some of these science teachers may identify, or nominate, a math teacher, a principal, or another individual who was not surveyed. Information from those who were surveyed is the only information available, thus there is no way to know if the math teacher, Mr. Park, identified by the science teacher, Mrs. Robinson also seeks advice from the Mrs. Robinson. Mr. Williams and Mrs. Robinson who are both science teachers and were therefore both surveyed, each identified the other as someone who they seek advice from, thus the tie between them in reciprocal (bidirectional). Perhaps one of these science teachers, Ms. Smith, did not identify anyone as someone from whom they seek advice, no one identified her as someone they seek advice from. Ms. Smith would be an **isolate**, or a node with no connections. These concepts are illustrated in Figure 7.

Figure 7. *Directed Network with One Isolate & Corresponding Adjacency Matrix*

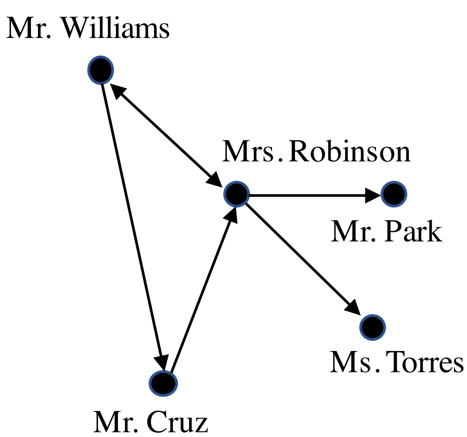


	Mr. Cruz	Mr. Park	Mrs. Robinson	Ms. Smith	Ms. Torres	Mr. Williams
Mr. Cruz	-	0	1	0	0	0
Mr. Park	0	-	0	0	0	0
Mrs. Robinson	0	1	-	0	1	1
Ms. Smith	0	0	0	-	0	0
Ms. Torres	0	0	0	0	-	0
Mr. Williams	0	0	1	0	0	-

A variety of terms indicate how well connected the network is by evaluating certain aspects as compared to the entire network. **Centralization** (also known as centrality) describes how well connected the nodes are or if nodes have the same number of ties. **Degree centrality,** one common measure of centrality, is the number of edges attached to a node. In a directed graph, this is separated into **in-degree centrality,** also called **popularity**, the number of ties directed at node (e.g., the number of times a node is nominated or identified by another node), and **out-degree centrality**, the number outward ties a node has (e.g., the number of nominations a node makes) (Lusher & Robins, 2013). In Figure 7, Mrs. Robinson's in-degree centrality is two because she was nominated by Mr. Cruz and Mr. Williams, while her out-degree centrality is three because she nominated Mr. Williams, Mr. Park, and Ms. Torres.

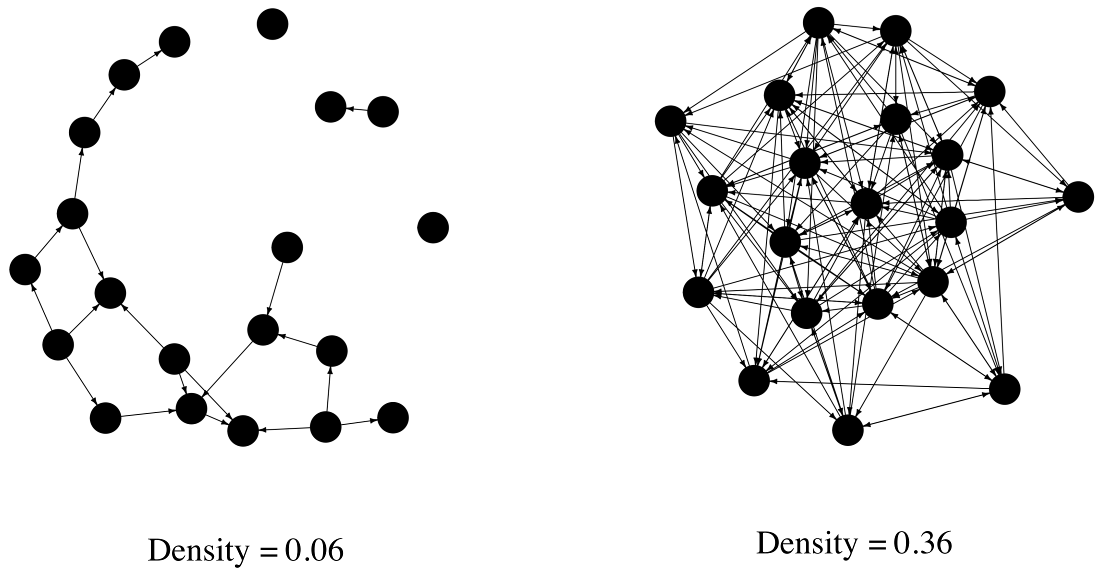
Another popular type of centrality is **betweenness centrality**, which indicates which nodes act as bridges between other nodes in a network (which nodes link other nodes together) (Grunspan, Wiggins, & Goodreau, 2014). In Figure 7, Mrs. Robinson has the highest betweenness centrality because her central position the network linking others together (e.g., Mrs. Robinson links Mr. Williams and Mr. Park, Mr. Williams and Ms. Torres, and Mr. Williams and Mr. Cruz). This can be measured a variety of ways, two of which will be briefly discussed here: distance and diameter (see Figure 8 for an illustration). **Distance** is a measure of how long a path is, on average, from one node to another. If we consider a single path, such as from Mr. Williams to Mr. Cruz, the **geodesic** (or shortest path) distance would be one. However, if we consider the path from Mr. Cruz to Mr. Williams, the distance is two because the directions of the arrows require Mr. Cruz to go through Mrs. Robinson on the path to Mr. Williams. For an entire network, this is calculated by averaging the shortest path (geodesic) between all pairs of nodes. Conversely, **diameter** is a measure of the longest geodesic (longest shortest path). For example, in Figure 8 the geodesic diameter would be two because the longest geodesic distance from one node to another is two (e.g., Mr. Williams to Mr. Park involves two ties). Taken together, we can get a sense of how connected or centralized the nodes are. If these paths are shorter, relatively, if the network is more centralized, thus information may pass more efficiently throughout the network. In other words, for information to travel from one node to any other node, it can do so without having to utilize many other nodes.

Figure 8. *Illustration of Geodesics*



Density is a measure of average connectedness calculated by dividing the observed number of connections by the number of total possible connections the network could contain. Thus, it ranges from 0 – 1, with a higher density indicated a more connected network. You may imagine a network of teachers with low density (0.06) in which teachers keep to themselves and rarely converse with other teachers, as compared to one with high density (0.36) in which teachers speak with most of the other teachers in the school, (see Figure 9).

Figure 9. *Examples of Directed Networks with 20 Nodes with Low (6%) and High (36%) Density*



Chapter 3. METHODS

This study will employ four different stochastic approaches to estimate network change over time based on the exponential random graph model (ERGM), including the temporal ERGM (known as TERGM), the separable temporal ERGM (known as STERGM), and the stochastic actor-based model (SABM). Across all of these models, it is assumed that the network *N* is being observed at a specific time *t,* even though there are several possibilities of what can happen to the network between time points, such as the formation or dissolution of a tie (connection between individuals), or both. It is also assumed that time is continuous and that one network change can influence another network change (i.e., interdependence of relationships) (Snijders & Koskinen, 2013). Each of the modeling approaches will be tested on two sets of simulations (one with varying sample sizes and one with a small sample size, where initial density levels and density change over time is varied; the former with two time points and the latter with four time points), and one observed teacher advice network dataset involving a small sample size and low density level measured at four time points.

3.1 ERGM, TERGM, STERGM, & SABM

3.1.1 *Exponential Family Random Graph Model (ERGM)*

The ERGM estimates the overall probability of the observed graph (i.e., social network) occurring given the network statistics, such that a set number of possible permutations of the graph (i.e., where relationships among the nodes/actors in the network were re-ordered at random, but upheld network structure) are evaluated for their likelihood given the parameters while the observed network characteristics (i.e., density, reciprocity, or other network statistics) are held constant. Each of these "possible" networks has a specific probability of existing, and together these probabilities comprise a probability distribution (Robins & Lusher, 2013). The ERGM as expressed in a probability density function with a mean of network (*N*) and a variance of the parameters ($\boldsymbol{\theta}$),

$$P(N, \boldsymbol{\theta}) = \frac{\exp\{\boldsymbol{\theta}^{\mathsf{T}} \mathbf{h}(N)\}}{\sum_{N^* \in \mathcal{N}} \exp\{\boldsymbol{\theta}^{\mathsf{T}} \mathbf{h}(N^*)\}}, \tag{1}$$

is equal to the exponentiated sum of weighted network statistics (($\boldsymbol{\theta}^{\mathsf{T}}$) $\mathbf{h}(N)$), where ($\boldsymbol{\theta}^{\mathsf{T}}$) is the transpose of the vector of model coefficients and $\mathbf{h}(N)$ is the vector of network statistics divided by the sum of weighted network statistics (e.g., number of ties, number of reciprocal ties, centrality measures) for all networks that could have been observed, where a specific permutation of the network (N^*) is an element of all possible network permutations with a fixed number of vertices ($\mathcal{N}$) (Leifeld & Cranmer, 2016). The denominator can alternatively be expressed as $c(\boldsymbol{\theta})$, where *c* is a normalizing constant (Leifeld, Cranmer, & Desmarais, 2017). Dependencies, endogenous or structural effects and exogenous or covariate effects, can be included in the $\mathbf{h}(N)$ vector (Leifeld & Cranmer, 2016; Snijders, Koskinen, & Schweinberger, 2010a). In sum, ERGM model includes network and node level or dyad level covariates.

The ERGM model makes assumptions outlined in (Robins & Lusher, 2013) as follows.

1) Social networks emerge locally.
2) Ties (connections among individual nodes/actors) in a network are self-organized, interdependent, and influenced by both exogenous factors and actor attributes.
3) The patterns in a network are evident of structural processes.
4) Several processes can occur simultaneously.
5) Social networks are stochastic (probabilistic) and structured.

These theoretical assumptions are based on social networks being representations of social processes. They are constructed of relational ties and therefore, are interdependent.

Researchers may decide, based on theory, which characteristics of network structures and structural effects are important and incorporate these into the ERGM model to test hypotheses. As with other statistical models, variables included should be grounded in theory. The model allows for node/actor and structural effects to be considered *simultaneously* as a representation of social processes. Unfortunately, the simultaneous modeling nature of the ERGM can lead to degeneracy issues, where the model is unable to converge because the density distribution of the networks created by the ERGM focuses on a few of the network configurations. This often happens when the model focuses on either empty networks with no ties at all, or full networks with all possible ties (Handcock, 2003b; Snijders, 2002). It is also assumed that in the ERGM the network comes from an unchanging distribution. The ERGM focuses on overall network characteristics (e.g., number of ties) rather than on actors (e.g., the SABM, discussed in section 3.1.4), such that the results of ERGM-based models are the predicted log-odds of a hypothetical next tie forming in the network. This may be specifically informative for examining certain types of ties or structural features of the network, though this is taken together across all time points so we do not know how it may change from one time point to the next.

3.1.2 *Temporal Exponential Random Graph Model (TERGM)*

The TERGM extends the ERGM by adding a temporal component. Specifically, the **network at a specific time *t***, is estimated.

$$Pr(N^t \,\middle|\, N^{t-K}, \ldots, N^{t-1}, \boldsymbol{\theta}) = \frac{\exp\left(\boldsymbol{\theta}^{\top}\mathbf{h}(N^t, N^{t-1}, \ldots, N^{t-K})\right)}{c(\boldsymbol{\theta}, N^{t-K}, \ldots, N^{t-1})} \qquad (2)$$

Here the network *N* at a specific time *t* (N^t) – which is often the last time point measured – is conditional on the network at one or more previous time points (*t-K*). In the numerator, network statistics (**h**(*N*)) for each time point (*t to t-K*) are weighted by the transpose of the parameters

($\boldsymbol{\theta}^{\intercal}$). Like the ERGM, a normalizing constant is included in the denominator (c) and the network at previous time is considered. An important specification is K, where $K \in \{0,1,\ldots,T\text{-}1\}$, is assumed to include dependencies of the specific network (N^t), so the networks before N^{t-K} are independent of N^t, and conditional upon the previous time points (Leifeld, Cranmer , & Desmarais, 2017b). Equation 3 shows the probability of the network at time t given previous networks t-K is equal to the product across all time periods:

$$Pr(N^{K+1},\ldots,N^T|N^1,\ldots,N^K,\boldsymbol{\theta}) = \prod_{t=K+1}^{T} P(N^t|N^{t-K},\ldots,N^{t-1},\boldsymbol{\theta})\,. \quad (3)$$

Equation 4 shows the vector of network statistics **h** includes "memory terms" such that the temporal dependencies between time points can be included (Leifeld & Cranmer, 2016). "Memory terms" exclude network structure, while including temporal processes; see Leifeld and colleagues (2017) for an in-depth description of "memory terms." The model should account for a temporal statistic, for example the dyadic stability term, which relates endogenously to the network structure and can capture the process over time:

$$h_m = \sum_{i\neq j} N_{ij}^t N_{ij}^{t-1} + (1 - N_{ij}^t)(1 - N_{ij}^{t-1}). \quad (4)$$

The dyadic stability term h_m increases when the status of a tie does not change between time points, such as when the lack or presence of a tie remains consistent (Leifeld & Cranmer, 2016). It creates a count of the dyads (N_{ij}) and ties (1-N_{ij}) that remain the same between t-1 and t. Notably, this method does not make assumptions about the *duration* of time between time points or about the dissolution or formation of ties, but rather includes the number of ties (Krivitsky & Handcock, 2014).

The two TERGMs that are estimable include one that uses maximum pseudolikelihood (MPLE) with bootstrapped confidence intervals, which incorporates conditional dyadic probabilities, and one that uses MCMC MLE estimation based on simulated network data. The main advantages in using MPLE is that it is much less computationally demanding than MCMC MLE since it does not require simulation (Leifeld & Cranmer, 2016). On the other hand, the MPLE method can result in downwardly biased variance estimates, though this has now been corrected for in the *R* package (**`btergm`**) (Leifeld et al., 2017b) used for analysis in the present study. This model can incorporate various node sizes and can accommodate missing data.

3.1.3 *Separable Temporal Exponential Random Graph Model (STERGM)*

The STERGM, another form of the TERGM, separates the total ties into estimates describing the formation of ties separate from the dissolution of ties. The model estimates the probability of a network that has the same structural characteristics as the observed network (this is referred to as the current network and is specific to the corresponding time point) (*N*) equal to the observed network (*n*) given that the previous network (n^{t-1}) is equal to the observed network at the previous time point, and the parameters (see Equation 5 below).

$$Pr(N = n|N^{t-1} = n^{t-1};\ \theta) = \frac{\exp\{\eta(\theta)\cdot g(n^t, n^{t-1})\}}{c_{\eta^+ . g^+}(\theta^+, n^{t-1})c_{\eta^- . g^-}(\theta^-, n^{t-1})}, \qquad (5)$$

In the model above, η is the vector of network parameters involving both the formation (η^+) and the dissolution parameters (η^-), θ is the vector of tie count network parameters, $g(n^t, n^{t-1})$, and the denominator is the formula's normalizing constant which is equivalent to the normalizing constant in the TERGM (Krivitsky & Handcock, 2014).

Across all of the ERGM-family longitudinal models (TERGM and STERGM) the probability that the observed network arises from the parameterized network is maximized. As a result, the conditional probability of a tie forming is modeled (in logits) and can be tested against a null of zero; for STERGM, the conditional probability of a tie forming *and dissolving* is modeled (again, logits). However, across each of these, it is assumed that tie formation (and dissolution in the case of STERGM) is held constant over time – in other words, that tie formation and dissolution do not interact with time (Krivitsky & Handcock, 2014). The advantage of TERGM and STERGM over ERGM is the ability to take into account information about the network from prior time point(s) in estimating the network properties; nevertheless, none of these models quantify how a network at one given time point *differs* from that same network at another time point. In order for this model to be estimated, there must be no missing data in the adjacency matrices, meaning that sample size must remain consistent across time points. This model is also computationally demanding, and has difficulty estimating small and sparse networks. If the model estimates for formation and dissolution do not converge within 20 iterations, it is considered degenerate and will not return results (Handcock et al., 2015).

3.1.4 *Stochastic Actor-Based Model (SABM)*

While TERGMs focus more on network parameters, the SABM focuses on modeling the network from the view of the actor (node). Estimates provided in this model indicate the average log-odds actor *i* having a tie with actor *j;* thus the estimates will be higher for SABM than the tie based models because the SABM considers a tie's existence from the actor perspective (each tie is considered twice once from *i* to *j* and *j* to *i*). Though TERGMs and SABMs can incorporate endogenous covariates and dyad dependencies, SABM further considers the "mini-steps" in the time in between each observed time point, such that there are infinite steps between *t* and *t*-1*,* referred to as "mini-steps" (Leifeld & Cranmer, 2016). At each mini-step there are two functions that define how actors (nodes) change their partners: one that describes the **rate-of-change** and one that is referred to as **objective**. The former describes the rate at which an actor (node) is linked with another actor, or selected, as part of a random Poisson process:

$$\forall i : \lambda_i(N^t) = \rho_t \quad (6)$$

At each mini-step every actor *i* has a probability of being selected, defined by the average rate of being selected to node *i* multiplied by the network *N* at time *t*, which is equal to the true rate of change at time *t* (Leifeld & Cranmer, 2016; Snijders, 2001). The second function at the mini-step is the objective function:

$$f_i(\theta, N) = \sum_k \theta_k h_{ik}(N). \quad (7)$$

This function considers the possible ties formed, dissolved, or persistent for the focal actor *i*. The function for every actor *i* of the parameters and the network, is equal to the sum of the outgoing dyad *k* weighted by the parameters of the outgoing dyad *k,* and the vector of network statistics for the dyad $h_{ik}(N)$. The computations are focused on the actor and the current time point. Let us compare reciprocity in the TERGM (Equation 8) and in the SABM (Equation 9) (Leifeld & Cranmer, 2016; Snijders, 2001a). Reciprocity (a bidirectional tie indicating both individuals identify the other) in TERGM is calculated by counting across all dyads *ij* as:

$$h_{reciprocity.ergm} = \sum_{i \neq j} N_{ij} N_{ji}. \quad (8)$$

In SABM, reciprocity is calculated with the specific actor *i* as:

$$h_{reciprocity.ergm}(i) = \sum_j N_{ij} N_{ji}. \quad (9)$$

The probability that actor *i* will form a tie with partner *j* in the network is expressed as (Leifeld & Cranmer, 2016):

$$Pr(N_{ij}) = \frac{\exp(f_i(\theta, N))}{\sum_{N* \in \mathcal{N}} \exp(f_i(\theta, N^*))}. \quad (10)$$

The probability of this change in the network is equal to the exponential function for actor *i* of the parameters and the network, divided by the sum of exponential function for actor *i* of the parameters and the specific permutation of the network (*N**), for all networks that could have been observed, where a specific permutation of the network (*N**) is an element of all possible network permutations with a fixed number of vertices ($\mathcal{N}$). Thus, the probability of an actor (node) making a change is equal to the proportion of the objective function of the resulting network's exponential transformation (Snijders et al., 2010b). Major limitations of this model include its difficulty in interpretation, especially since it considers actors having ties, rather than ties existing between actors, it is quite computationally demanding, and it has a hard time estimating small and sparse networks.

3.2 RESEARCH QUESTIONS

The research questions for this study focus on comparing approaches for modeling longitudinal social network change, as follows.

1. a) What are the practical considerations (such as length of time to model the estimates) for using these longitudinal social network analysis approaches? Specifically, how do TERGM with MPLE with bootstrapping, TERGM with MCMC MLE, STERGM, and SABM approaches to modeling network change over time compare?

   b) Are certain characteristics of the network data (such as node size, initial density, density change over time, and number of time points) more or less amenable to these approaches to longitudinal social network analyses?
2. a) What are the differences and similarities among the different approaches?

   b) Does the answer to 2a depend on certain characteristics of the network data (such as node size, initial density, density change over time, and number of time points)?

3.3 Data Analysis Plan

Simulated data simulation procedures are described in the next chapter. However, to analyze both the simulated data as well as applied dataset, I used *R* software. Specifically, for each dataset, I estimated the following models (and saved the results for secondary analyses).

- ERGM for the first time point, Time 0 (Model 1a)
- ERGM for the second time point, Time 1 (Model 1b)
- ERGM for the third time point, Time 2 (Model 1c) (note: this only applies to one of the simulation sets and the applied data analysis)
- ERGM for the fourth time point, Time 3 (Model 1d) (note: this only applies to one of the simulation sets and the applied data analysis)
- TERGM (Model 2a estimated using maximum pseudolikelihood with bootstrapped confidence intervals, and Model 2b estimated using MCMC MLE)
- STERGM (Model 3)
- SABM (Model 4)

For the ERGMs and the STERGM, I used the **`statnet`** suite of packages (Handcock et al., 2016; Handcock et al., 2008) in *R*. For the TERGMs, I used the **`btergm`** package (Leifeld et al., 2017a). Finally, for the SABM, I used the **`RSiena`** package (Ripley et al., 2013).

For the simulated datasets in particular, after results were saved, I conducted mixed model analyses of variance on tie formation estimates to test the main effects of model type (within-subjects factor with four levels: STERGM, TERGM-B, TERGM-M, SABM), node size (between-subjects factor for the first simulation set only), initial density (between-subjects factor), and linear/quadratic growth over time (between-subjects factors), as well as 2-way interactions among model type and each of the other factors. Finally, post-hoc *t*-tests employed the Dunn-Sidak *p*-value adjustment to control Type I error rate to 0.05. All of these secondary analyses were conducted in *SPSS* version 19.

Chapter 4. SIMULATION METHODS AND RESULTS

Data were simulated based on observed network characteristics as well as analysis model constraints. First, I surveyed the recent literature on social networks in education research to glean information about real network characteristics. Second, in order to compare longitudinal social network analysis (SNA) modeling approaches, I reviewed the model constraints to ensure that each dataset I simulated would be comparable. Finally, I constructed a single simulated dataset per condition in *R* based on node size (number of actors), density (network cohesiveness), and growth parameters. Two time points per simulation condition were considered for all conditions; for a subset of conditions, four time points were constructed so as to provide linkage with the applied dataset (which has four time points; see Chapter 5).

4.1 Real Network Properties: A Brief Review of Applied Data

I performed a brief review of recent studies published in peer-reviewed journals using the University of Washington library's *Education Source* database. I specifically searched for studies in which any quantitative probabilistic social network analysis was applied to educational network data. I then reviewed these articles to determine network characteristics to inform simulation parameters. Twelve studies were ultimately found and reviewed. Table 2 provides a breakdown of each study and corresponding network characteristics.

Across these twelve social network studies from seven sources, I found three focused on Twitter data, six focused on "advice" professional development networks, and three centered around other types of networks (one identifying negative relationships among educational leaders, one identifying who teachers form relationships within and outside of his or her school, and one following email correspondence patterns). Most of the studies only measured the network at one time point, but two measured at three or four times. Node sizes varied tremendously from 17 to 211 individuals (nodes); however, most of the advice networks were smaller, ranging from 17 to 35 nodes and one with 100 nodes). There was less variability in densities, however: most of the networks were low density, less than 23%. Importantly, values for the simulation parameters for the present study were informed by the results of this literature search with an emphasis on the studies involving professional development advice networks, as well as the real dataset's characteristics for comparability purposes (see Chapter 6 for observed data analysis).

Table 2. *Review of Studies Employing Stochastic Social Network Analyses*

Study	Network Type	*N*	Time Points	Density	Reciprocity
Daly, Moolenaar, Liou, Tuytens, & del Fresno (2105)	Educational Leaders	78	1		
Feldman (2016)	Collaboration (Advice)	100	1	0.12	0.10
Rientes & Kinchin (2014)	Academic	54	1		
Shields (2016)	Twitter (Hashtags)	211	1	0.23	
Shields (2016)	Twitter (Mentions)	211	1	0.00	>0.99
Shields (2016)	Twitter (Followers)	211	1	0.05	0.95
Sweet (2016)	Teacher Advice (Literacy)	35	1	0.07	0.36
Sweet (2016)	Teacher Advice (Literacy)	17	1	0.16	0.14
Sweet (2016)	Teacher Advice (Math)	35	1	0.05	
Sweet (2016)	Teacher Advice (Math)	17	1	0.10	
Thompson & Richards (2016)	Collaboration (Advice)	27-43	4	0.02-0.05	0.18-0.40
Uddin, Thompson, Schwendimann, & Piraveenan (2014)	Email	38	3		0.40-0.46

4.2 SIMULATION PARAMETERS

Using the findings from the literature review of real social networks, directed networks with varying characteristics were simulated using the **`network`** package in the **`statnet`** suite (Butts, 2008, 2015; Butts, 2016; Handcock et al., 2008) in *R* to mimic a network measured at multiple time points. Specifically, a simulated network was created for the first time point (Time 0), and then a simulated network for the second time point (Time 1) was sampled from the characteristics of the first time point. Each combination of parameter conditions included at least two time points (minimum number for longitudinal modeling) (one set involved two time points, and another, smaller set involved four time points). For the four time point conditions, the same re-sampling procedure was used to generate the third and fourth time points using the subsequent time point network.

For each simulation, the node size was held constant across the time points (treated as a bounded network), and the reciprocity level was held constant due to the simulation software's inability to incorporate reciprocity into the dataset generation (i.e., reciprocity was specified to be logit = 0, which is 50%). The density of the network at the second time point (Time 1) was manipulated to reflect specific linear and quadratic growth rates in logits. Notably, because reciprocity could not be controlled, the average network reciprocity for the first time point turned out to be 20% (Time 0), and 22% for the second time point (Time 1). Parameter conditions for the two time point datasets were set as follows.

- Node size (number of individuals/actors): 4 levels (20, 40, 60, and 80); note that, because the separable temporal exponential random graph model (STERGM) requires the same network size at each time point, network size was held constant over time.
- First time point (Time 0) initial network density level (percentage of ties in the network that exist out of the total possible): 3 levels (6%, 18%, and 36%); note that, because STERGM and stochastic actor-oriented model (SABM) had extreme difficulty estimating networks with sparse densities, 6% was the lowest possible condition (rather than, say, 2% as observed in the real data analysis in Chapter 6).
- Linear density growth rate: 4 levels (no growth, 10% increase, 25% increase, and 50% increase)

- Quadratic density growth rate: 4 levels (no growth, 2% decrease, 4% decrease, and 6% decrease)

Combined, these parameters include 4 * 3 * 4 * 4 = 192 crossed conditions. Note that, due to computational intensity and lengthy times for data generation and analyses, only one simulation per crossed condition was generated.

In addition to the two time point simulation set, I also generated a set of simulations with four time points, focusing on the small network case ($N = 20$) with two levels of density instead of three (just low initial density, 6%, and high initial density, 36%). Linear and quadratic growth rates in density were the same as above. This set of data was specifically simulated to mirror the applied data analysis for ease of comparability (see Chapter 5), yielding 2 * 4 * 4 = 32 crossed conditions. Again, observed reciprocity (recalling again that it is not controllable with the software in its current state) for the first time point averaged 3%, and for the second, third, and fourth time points averaged 8%.

Across all simulated networks, each of the stochastic models were conducted, with estimates from each saved for secondary data analysis. The primary interest of the analyses was in the likelihood of a tie across time points.

4.3 Model Estimation Issues & Considerations

Before examining the results of the model comparisons across simulations, it is important to note that there were several issues encountered during the simulation process. Issues included requirements for equal node sizes across time points, duration of estimation, and degeneracy issues. I describe these challenges below.

4.3.1 *Node Size*

The STERGM requires a consistent network size across time points. This model requires each node to be measured at every time point in order to assess formation and dissolution of ties, and if there are not changes in the network, the model cannot estimate that. Therefore, in some simulations, the STERGM dissolution parameters could not be estimated. In a single simulation of the four time point condition, the tie term returned was negative infinity, indicating that all ties dissolved. A similar problem occurred when estimating the dissolution parameter for reciprocal ties in 29% of the two time point simulations and 80% of the four time point simulations. This was either due to 1) reciprocal ties not existing at a given time point, or 2) all reciprocal ties dissolving from time point to time point. In these instances, the dissolution parameter for reciprocal ties returned an estimate of negative infinity; no model fit was returned, though it was statistically significant. Interestingly, when all reciprocal ties persisted from time point to time point, a very high logit value, such as 15 or larger, was returned which is a probability of greater than 99%; notably, model fit and statistical significance were inestimable. This problem occurred in the simulations with two and four time points, affecting a single simulation in each.

4.3.2 *Duration of Model Estimation, Errors, and Estimation Issues*

The average length of time for each model to be estimated varied (see Table 3). As node size, density, and number of time points increased, the models took longer to estimate. Across all network characteristics, STERGM and SABM took the most time to run; often these models required three or more times that of the length of the two TERGMs to run. As can also be seen, SABM often took more than two times as long as STERGM.

Conditions in which the networks had low initial density required extra attention not shown in the table above. While exponential random graph models (ERGMs) and temporal exponential random graph models (TERGMs) did not present issues, SABM and STERGM did present estimation problems, most often with the lowest density networks. SABM, the most computationally demanding of the models, had trouble converging without adding additional iterations. Further, STERGM often did not converge for the four time point simulations (i.e., it was experiencing degeneracy issues where the model would not converge). Without the ability to increase the number of iterations (which is the case with STERGM), problematic simulations were discarded and a new network was generated. This process was repeated for the problematic simulations, generally no more than approximately five times per simulation. Problematic simulations tended to be with for small and sparse (least dense) networks with four time points.

Table 3. *Model Estimation Time (in Seconds) for Networks with 2 Time Points*

Time Points	Node Size	Density	ERGM T0	ERGM T1	ERGM T2	ERGM T3	TERGM-B	TERGM-M	STERGM	SABM
2	20	0.06	2.09	1.99	-	-	1.68	2.29	7.57	3.10
		0.18	2.48	2.41	-	-	1.43	2.73	9.45	11.28
		0.36	2.53	2.50	-	-	1.01	2.86	9.89	9.05
	40	0.06	2.42	2.34	-	-	1.41	2.89	9.93	13.65
		0.18	2.83	2.81	-	-	1.45	3.59	10.79	22.84
		0.36	3.28	3.25	-	-	1.41	4.24	12.19	19.18
	60	0.06	2.56	2.51	-	-	1.56	3.15	10.13	28.08
		0.18	3.26	3.32	-	-	1.24	4.68	12.71	28.21
		0.36	4.28	4.34	-	-	1.28	7.29	15.85	34.23
	80	0.06	2.97	2.90	-	-	1.37	3.99	13.35	39.39
		0.18	3.98	4.13	-	-	1.43	6.41	16.05	39.05
		0.36	6.02	6.65	-	-	1.15	9.85	20.45	38.57
4	20	0.06	2.28	2.29	2.21	2.16	1.92	2.49	10.92	21.27
		0.36	2.57	2.58	2.60	2.57	1.87	3.27	13.18	29.27

Note. Model estimates were obtained using an Intel i7 2.8GHz processor with 32gb of ram.

4.4 Results of Simulations for Directed Networks with 2 Time Points

The results of the simulations are reported first for the set of two time point simulations, including both the ERGM SNA model for each time point separately, as well as the longitudinal SNA model comparisons.

4.4.1 *Results for Directed Networks with 2 Time Points, No Growth in Density Conditions*

For clarity of results, the no-growth condition was examined separate from the growth conditions. Results of the model estimation of the likelihood of a tie to form in networks with two time points are provided in Table 4. The log-odds of a tie forming in the network across time points is reflective of the network's density, with a lower log-odds indicating a tie being less likely to occur.

For reference, a log-odds of zero would indicate a probability of 50%, a negative log-odds indicates a probability of < 50%, while a positive log-odds indicates a probability of > 50%. The ERGMs show how close the model density estimate is to the observed density estimate at each network time point. Of interest is how the longitudinal models perform in estimating the log-odds of the network density; said another way, this is the log-odds of a tie forming in the network. For all densities and network sizes, the TERGM estimates of a tie forming across network time points are closest to the true average log-odds of the density across time points. In all models, higher density is indicated by the log-odds becoming nearer to zero, meaning that a tie is more likely, which is what was expected.

Table 4. *Model-Predicted Tie Estimates for Networks with 2 Time Points, No Growth in Density*

Density	Node Size	True Log-Odds Time 0	ERGM Time 0	True Log-Odds Time 1	ERGM Time 1	True Log-Odds (Avg Time 0 and Time 1)	TERGM-B	TERGM-M	STERGM (Formation)	SABM
0.06	20	-2.65	-2.67	-2.39	-2.44	-2.52	-2.55	-2.55	-2.32	-1.51
	40	-2.60	-2.56	-2.78	-2.76	-2.69	-2.66	-2.66	-2.78	-1.55
	60	-2.82	-2.86	-2.70	-2.68	-2.76	-2.76	-2.76	-2.79	-1.40
	80	-2.89	-2.90	-2.73	-2.76	-2.81	-2.83	-2.82	-3.05	-1.40
0.18	20	-1.51	-1.54	-1.96	-2.21	-1.73	-1.84	-1.85	-2.06	-1.31
	40	-1.44	-1.39	-1.56	-1.54	-1.50	-1.47	-1.46	-1.92	-0.82
	60	-1.53	-1.52	-1.49	-1.50	-1.51	-1.51	-1.51	-2.17	-0.75
	80	-1.53	-1.54	-1.53	-1.56	-1.53	-1.55	-1.55	-2.68	-0.80
0.36	20	-0.46	-0.41	-0.70	-0.74	-0.58	-0.58	-0.58	-0.46	-0.41
	40	-0.56	-0.48	-0.58	-0.57	-0.57	-0.52	-0.52	-1.31	-0.31
	60	-0.63	-0.71	-0.58	-0.58	-0.60	-0.64	-0.64	-1.90	-0.21
	80	-0.55	-0.55	-0.54	-0.53	-0.54	-0.54	-0.54	-2.42	-0.24

Note. Coefficient estimates reported are log-odds. True log-odds are averages of observed density of the networks. Sample size consistent across time points. All models take into account the previous time point when estimating the current time point; however the TERGMs and STERGM assume independence between consecutive networks, while the SABM does not (Leifeld & Cranmer, 2016). The SABM is scaled differently and should not be directly compared to the other models (Snijders et al., 2010b).

A 4 (fixed within-subjects; model type) * 4 (fixed between-subjects; node size) * 3 (fixed between-subjects; initial density level) mixed model ANOVA was used to test the main effects of each of the three factors as well as all 2-way interactions with model type on the model-predicted log-odds of a tie occurring in the network (recall that there were $N = 12$ simulations in the no-growth condition). Mauchly's test of sphericity was significant, so Greenhouse-Geisser adjusted F-tests for the within-subjects effect and interactions were used to determine significance.

These results showed that there was a significant main effect of model type, *Greenhouse Geisser Adjusted* $F(1.04, 6.22) = 110.62$, $p < 0.001$, *partial* $\omega^2 = 0.95$, on tie formation across network time points. Follow-up pairwise t-tests with a Dunn-Sidak adjustment for multiple comparisons showed significant differences between the SABM and each of the other models (collapsed across node sizes and density levels, SABM estimates a higher likelihood because of its actor-focused perspective), all adjusted *p*s < 0.002. There was also a significant main effect of initial density, $F(2, 6) = 113.871$, $p < 0.001$ (follow-up *t*-tests revealed that there is a significant difference between each of the densities as would be expected, with higher densities indicating greater likelihood of a tie across time points, all adjusted *p*s < 0.001). There was no significant main effect of node size, $F(3, 6) = 1.15$, $p = 0.404$.

More interestingly, there were two significant interactions. First, there was a significant interaction between model type and node size, *Greenhouse-Geisser Adjusted* $F(3.11, 6.22) = 6.91$, $p = 0.021$, *partial* $\omega^2 = 0.78$. Follow-up simple effects tests comparing each model type within each node size, with a Dunn-Sidak adjustment for multiple comparisons, showed significant differences between SABM and each of the other models, for all node sizes, adjusted *p*s < 0.002. For the 20-node condition, there was a significant difference between the SABM and each of the STERGMS, adjusted *p*s < 0.001. For the 40 and 60-node conditions, there was a significant difference between the SABM and each of the other models. Additionally, for the 80-node condition, there were also significant differences between each of the two TERGMs with STERGM, as well as between the SABM and each other model, adjusted *p*s < 0.001. The interaction is plotted in Figure 10.

The second significant interaction was between model type and initial density, *Greenhouse-Geisser Adjusted* $F(2.07, 6.22) = 10.20$, $p = 0.011$, *partial* $\omega^2 = 0.77$. The interaction is plotted in Figure 10.

Follow-up simple effects *t*-tests of the log-odds of a tie across network time points for each model type by initial density, with a Dunn-Sidak adjustment for multiple comparisons, again showed significantly higher likelihoods of a tie for SABM compared with each of the other models (same pattern as for densities), adjusted *p*s < 0.001. Also similar to the pattern of results for densities, follow-up simple effects tests showed that both TERGMs were significantly higher than STERGM, adjusted *p*s < 0.001.

Figure 10. *Node Size by Model for Networks with 2 Time Points, No Growth in Density*

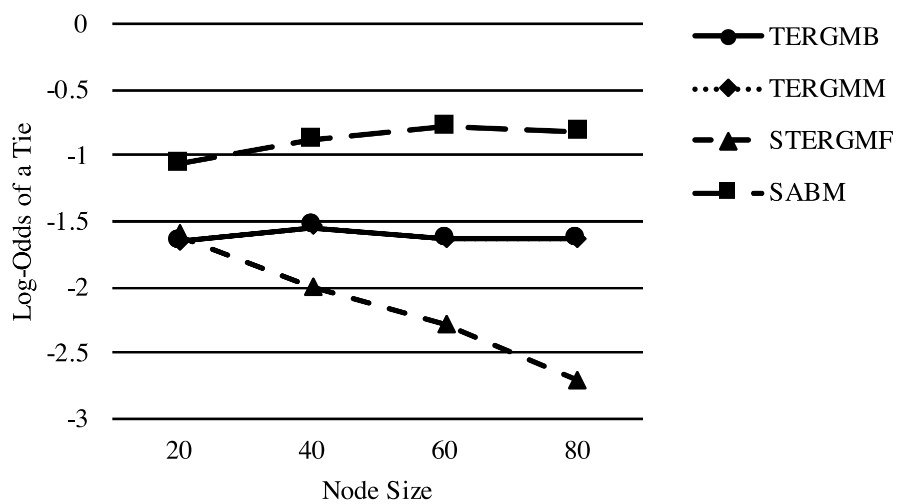
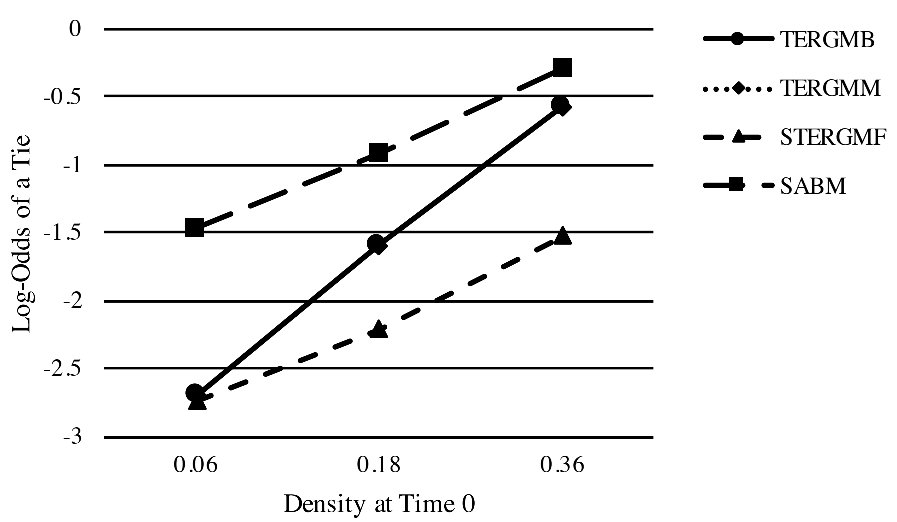


Figure 11. *Density by Model for Networks with 2 Time Points, No Growth in Density*



The SNA models also computed other statistics; for example, each model also estimated the log-odds of a reciprocal tie (see Table 5), but recall that the data generation process did not permit control over the reciprocal tie parameter settings (i.e., they were left to chance at 50%). Recall that estimates for the dissolution of ties and reciprocal ties are also estimated by STERGM (see Table 6). This said, I do not know what the true log-odds are for these estimates because the formation and dissolution estimates differ from the global estimation that occurs in the TERGMs because the TERGMs only assess change in ties over time and do not consider formation and dissolution of ties as two separate occurrences.

Table 5. *Model Estimates for Reciprocal Ties for Networks with 2 Time Points, No Growth in Density*

Density	Node Size	True Log-Odds Time 0	ERGM Time 0	True Log-Odds Time 1	ERGM Time 1	True Log-Odds (Avg Time 0 and Time 1)	TERGM-B	TERGM-M	STERGM (Formation)	SABM
0.06	20	-2.44	0.23	-1.95	0.48	-2.19	0.41	0.42	-0.56	0.37
	40	-3.26	-0.69	-3.08	-0.33	-3.17	-0.51	-0.51	-0.19	-0.24
	60	-2.31	0.55	-3.29	-0.62	-2.80	0.04	0.05	-0.58	-0.34
	80	-2.86	0.04	-2.34	0.42	-2.60	0.27	0.26	0.10	0.25
0.18	20	-1.37	0.18	-0.86	1.36	-1.11	0.70	0.71	0.48	0.78
	40	-1.65	-0.26	-1.64	-0.10	-1.65	-0.18	-0.18	0.22	-0.05
	60	-1.56	-0.03	-1.45	0.06	-1.50	0.01	0.00	0.06	0.04
	80	-1.45	0.09	-1.41	0.15	-1.43	0.12	0.12	-0.02	0.10
0.36	20	-0.54	-0.13	-0.62	0.11	-0.58	0.00	0.00	-0.52	0.06
	40	-0.70	-0.22	-0.60	-0.03	-0.65	-0.13	-0.13	0.05	0.00
	60	-0.49	0.22	-0.59	-0.01	-0.54	0.10	0.10	-0.18	-0.10
	80	-0.55	0.00	-0.55	-0.03	-0.55	-0.01	-0.01	-0.15	-0.03

Note. Coefficient estimates reported are log-odds. True log-odds are averages of observed density of the networks. Sample size consistent across time points. All models take into account the previous time point when estimating the current time point; however the TERGMs and STERGM assume independence between consecutive networks, while the SABM does not (Leifeld & Cranmer, 2016). The SABM is scaled differently and should not be directly compared to the other models (Snijders et al., 2010b).

Table 6. *STERGM Estimates for Networks with 2 Time Points, No Growth in Density*

Density	Node Size	STERGM	
		Dissolution	
		Tie	Reciprocal Tie
0.06	20	-2.44	-
	40	-2.53	-
	60	-1.45	-
	80	-0.68	0.80
0.18	20	-2.15	-
	40	-0.68	-0.34
	60	0.21	-0.06
	80	0.83	0.26
0.36	20	-0.58	-0.18
	40	0.39	0.17
	60	1.46	-0.17
	80	1.91	0.05

Note. Coefficient estimates reported are log-odds of a tie. Sample size consistent across time points. Some coefficients were inestimable due total dissolution of reciprocal ties between time points.

Though not of focal interest for the present study, model fit indices can be helpful when decided what terms to include in the model. For the present study, fit indices could only be obtained for the individual time point ERGMs and for STERGM (see Table 7). This said, note that the model fit indices for STERGM are not useful here because 1) STERGM estimates both formation and dissolution of ties and so there are model fit indices for formation and dissolution, and 2) no fit indices for the other longitudinal models are currently available.

Table 7. *Model Fit Indices for Networks with 2 Time Points, No Growth in Density*

Density	Node Size	ERGM Time 0		ERGM Time 1		STERGM Formation		STERGM Dissolution	
		AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC
0.06	20	188.30	196.20	223.20	231.10	209.00	216.80	-	-
	40	788.00	798.70	697.60	708.30	640.00	650.60	-	-
	60	1534.00	1547.00	1662.00	1674.00	1431.00	1443.00	-	-
	80	2607.00	2620.00	2913.00	2927.00	2229.00	2243.00	429.00	436.60
0.18	20	364.00	371.80	282.50	290.40	241.30	248.80	-	-
	40	1527.00	1538.00	1444.00	1455.00	1008.00	1018.00	383.50	390.90
	60	3326.00	3338.00	3383.00	3395.00	1948.00	1960.00	873.60	882.50
	80	5930.00	5943.00	5920.00	5933.00	2467.00	2480.00	1372.00	1382.00
0.36	20	511.00	518.90	486.70	494.60	292.80	299.70	194.90	200.90
	40	2048.00	2058.00	2040.00	2050.00	1044.00	1053.00	766.20	774.90
	60	4573.00	4585.00	4626.00	4639.00	1720.00	1732.00	1223.00	1233.00
	80	8307.00	8320.00	8324.00	8338.00	2185.00	2197.00	1766.00	1777.00

Note. No fit indices could be measured when all dissolution parameter estimates were negative infinity.

Finally, SABM's rate parameters, which are estimates unique to the SABM, were also estimated and are shown in Table 8. Recall that the rate parameter is the number of times an actor (node) has the opportunity to change the status of a tie between successive time points; notably this number is higher than the actual changes made by an actor (Snijders et al., 2010b). For example, for this 20 node network with 2 time points, 6% density at the initial time point, an actor has the opportunity to change the status of a tie 13.88 times from Time 0 to Time 1.

Table 8. *SABM Rate Parameter Estimates for Networks with 2 Time Points, No Growth in Density*

Density	Node Size	SABM Rate Parameter
0.06	20	13.88
	40	19.08
	60	14.59
	80	11.59
0.18	20	27.06
	40	19.05
	60	14.69
	80	10.49
0.36	20	26.39
	40	17.50
	60	10.77
	80	8.56

4.4.2 *Results for Directed Networks with 2 Time Points, Growth in Density Conditions*

Results of the model estimation (Table 9) for directed networks with two time points, with varying levels of linear and quadratic linear growth, show that for each node size and density level, as expected, the log-odds of a tie in the network is reflective of the network's density. This is evident in the lower log-odds, which corresponds to a lower probability.

Table 9. *Model-Predicted Tie Estimates for Networks with 2 Time Points, Growth in Density Conditions*

Density	Node Size	True Log-Odds Time 0	ERGM Time 0	True Log-Odds Time 1	ERGM Time 1	True Log-Odds (Avg Time 0 and Time 1)	TERGM-B	TERGM-M	STERGM (Formation)	SABM
0.06	20	-2.72	-2.73	-2.62	-2.68	-2.67	-2.28	-2.70	-2.67	-1.72
	40	-2.78	-2.78	-2.52	-2.50	-2.65	-2.63	-2.63	-2.53	-1.38
	60	-2.73	-2.73	-2.54	-2.53	-2.64	-2.62	-2.62	-2.68	-1.31
	80	-2.76	-2.76	-2.58	-2.57	-2.67	-2.66	-2.66	-2.94	-1.28
0.18	20	-1.49	-1.50	-1.22	-1.25	-1.36	-1.37	-1.37	-1.21	-0.71
	40	-1.54	-1.54	-1.33	-1.34	-1.43	-1.43	-1.43	-1.62	-0.66
	60	-1.51	-1.49	-1.38	-1.37	-1.45	-1.43	-1.43	-2.14	-0.62
	80	-1.50	-1.51	-1.42	-1.42	-1.46	-1.47	-1.47	-2.64	-0.62
0.36	20	-0.59	-0.61	-0.18	-0.18	-0.39	-0.41	-0.42	-0.18	-0.09
	40	-0.59	-0.60	-0.39	-0.38	-0.49	-0.49	-0.49	-1.19	-0.09
	60	-0.57	-0.56	-0.47	-0.48	-0.52	-0.52	-0.52	-1.99	-0.12
	80	-0.57	-0.57	-0.52	-0.51	-0.54	-0.54	-0.54	-2.53	-0.09

Note. Coefficient estimates reported are log-odds. True log-odds are averages of observed density of the networks. Sample size consistent across time points. All models take into account the previous time point when estimating the current time point; however the TERGMs and STERGM assume independence between consecutive networks, while the SABM does not (Leifeld & Cranmer, 2016). The SABM is scaled differently and should not be directly compared to the other models (Snijders et al., 2010b).

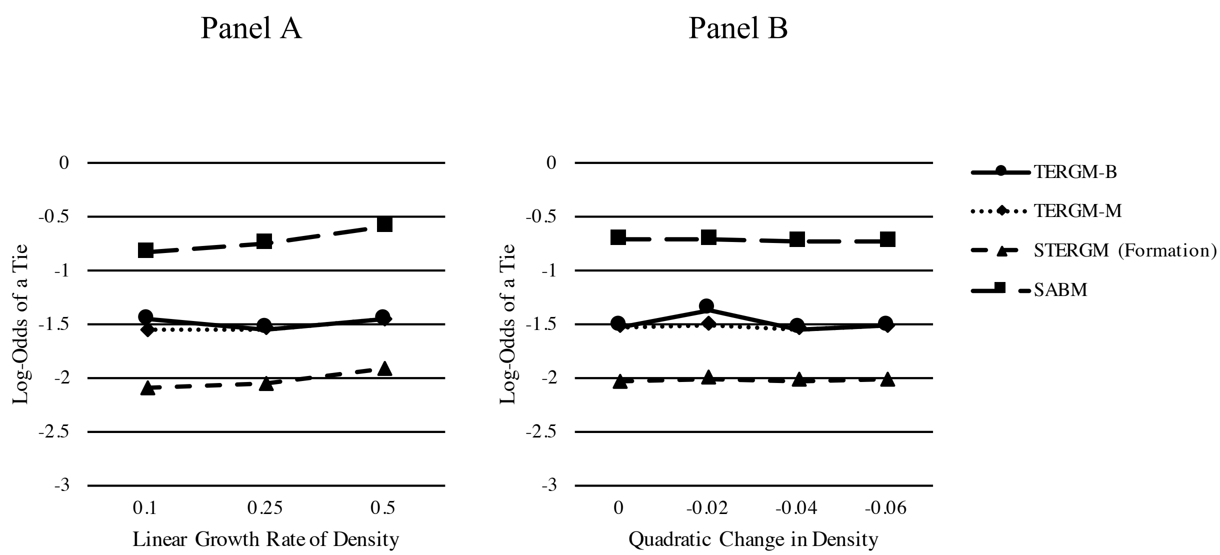
Consistently, across node sizes and densities, the TERGM estimates are closest to the true log-odds of a tie across network time points, even in cases of linear growth in density. This is consistent with findings for the no growth in density conditions as well.

Similar to the no-growth conditions, a mixed model ANOVA was used to test the effects of model type, parameter conditions (which included initial density, node size, linear change in density, and quadratic change in density), and 2-way interactions among model type and parameter conditions, as well as the 2-way interaction between linear and quadratic change in density over time and its interaction with model type on the model-implied log-odds of a tie occurring across time points. There were $N = 144$ simulations used for these analyses (no-growth conditions were excluded). Again, because Mauchly's test of sphericity was significant, adjusted F-tests for the within-subjects (model type) effect and related interaction tests were used to determine significance.

There was a significant main effect of model type, *Greenhouse Geisser Adjusted* $F(1.50, 190.85) = 523.07$, $p < 0.001$, *partial* $\omega^2 = 0.81$. Follow-up pairwise *t*-tests with a Dunn-Sidak adjustment for multiple comparisons showed significant differences between SABM and each of the other models, as well as between both TERGMs and STERGM (adjusted *p*s < 0.001). This pattern is identical to that observed in the no-growth in density conditions: SABM results estimate the highest likelihood of a tie, then TERGMs, and then STERGMs.

The ANOVA results also showed significant main effects of node size, $F(3, 127) = 28.25$, $p < 0.001$, initial density, $F(2, 127) = 1138.85$, $p < 0.001$, and linear change over time, $F(2, 127) = 8.44$, $p < 0.001$. Follow-up simple effects *t*-tests with a Dunn-Sidak adjustment showed that 20 differed from 60 and 80, and 40 nodes differed from 80 nodes, (adjusted *p*s < 0.001), that densities of 6%, 18%, and 36% each differed significantly from one another (adjusted *p*s < 0.001), and that linear growth levels of 0.10 differed significantly from 0.50 (adjusted $p = 0.001$). There was no significant main effect of the quadratic change over time, nor was there any interaction between linear and quadratic factors with each other or with model type. Nevertheless, for reader interest, plots of tie likelihood estimates by each model by the linear and quadratic factors of density are illustrated in Figure 12 (Panel A and B, respectively), collapsed across other conditions.

Figure 12. *Linear and Quadratic Change by Model for Networks with 2 Time Points, Growth in Density Conditions*



More interestingly, there were again two significant interactions. First, there was a significant interaction between model type and node size, *Greenhouse-Geisser Adjusted* $F(4.51, 190.85) = 6.27$, $p < 0.001$, *partial* $\omega^2 = 0.43$. Follow-up simple effects *t*-tests showed significant differences between the SABM and each of the other models for models with any node size, adjusted *p*s < 0.001. For networks with 40, 60, and 80 nodes, there were also significant differences between TERGM-M and STERGM, and between each model and SABM for all node sizes, adjusted *p*s < 0.001. And, for networks with 80 nodes, there was a significant difference between TERGM-B and STERGM. Figure 13 displays the mean likelihood of a tie across time points by node size, collapsed across other conditions.

Second, there was a significant interaction between model type and initial density, *Greenhouse-Geisser Adjusted* $F(3.01, 190.95) = 35.42$, $p < 0.001$, *partial* $\omega^2 = 0.34$. Follow-up simple effects *t*-tests showed that, across all density levels, SABM estimated significantly greater tie likelihoods across time points than each of the other models, and STERGM was significantly less likely than the two TERGMs (adjusted *p*s < 0.001). Figure 14 displays the mean model estimates by density levels, collapsed across other conditions.

Figure 13. *Node Size by Model for Networks with 2 Time Points, Growth in Density Conditions*

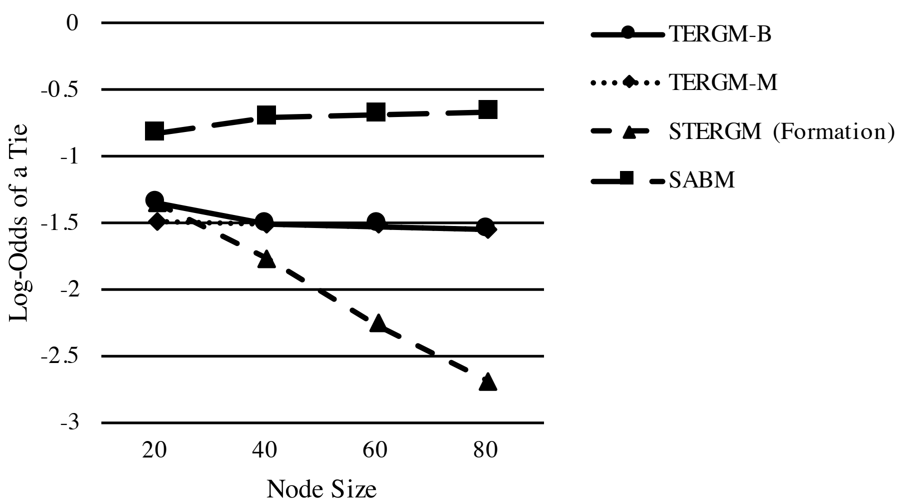
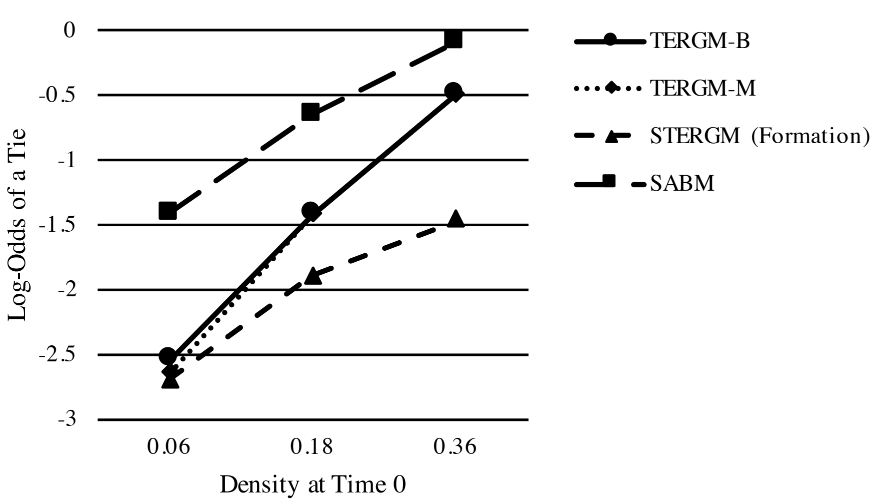


Figure 14. *Density Level by Model for Networks with 2 Time Points, Growth in Density Conditions*



Again, reciprocal ties were also estimated for each of the models (see Table 10) but were not of interest in this study. Other additional estimates that are not of interest in this study include the STERGM estimates for dissolution of ties and reciprocal ties (see Table 11), fit indices for ERGMs and STERGM (see Table 12), and SABM rate parameters (see Table 13). The rate parameters provided in SABM are an estimate of the rate-of-change function (see Chapter 3), and is the total number of opportunities an actor has to create, maintain, or dissolve a tie, this number is larger than what was actually happened (Snijders et al., 2010b). The true rate parameters are not known, but these values can indicate small or large potential for changes in the network between consecutive time points.

Table 10. *Model Estimates for Reciprocal Ties for Networks with 2 Time Points, Growth in Density Conditions*

Density	Node Size	True Log-Odds Time 0	ERGM Time 0	True Log-Odds Time 1	ERGM Time 1	True Log-Odds (Avg Time 0 and Time 1)	TERGM-B	TERGM-M	STERGM (Formation)	SABM
0.06	20	-2.30	0.34	-1.81	0.90	-2.01	-0.81	0.50	-1.17	-0.13
	40	-2.95	-0.17	-2.77	-0.26	-2.86	-0.15	-0.15	-0.06	-0.15
	60	-2.84	-0.11	-2.69	-0.16	-2.77	-0.11	-0.11	-0.01	-0.09
	80	-2.86	-0.10	-2.69	-0.12	-2.77	-0.09	-0.09	-0.11	-0.07
0.18	20	-1.45	0.06	-1.15	0.10	-1.30	0.11	0.11	-0.07	0.06
	40	-1.54	0.00	-1.32	0.02	-1.43	0.03	0.03	0.01	0.01
	60	-1.60	-0.10	-1.42	-0.05	-1.51	-0.06	-0.06	-0.05	-0.02
	80	-1.47	0.04	-1.40	0.03	-1.43	0.04	0.04	0.01	0.01
0.36	20	-0.56	0.04	-0.21	-0.03	-0.39	0.06	0.06	-0.08	-0.02
	40	-0.59	0.01	-0.41	-0.03	-0.50	0.00	0.00	-0.04	-0.02
	60	-0.58	-0.02	-0.46	0.02	-0.52	0.00	0.00	0.03	0.03
	80	-0.57	0.00	-0.53	-0.02	-0.55	-0.01	-0.01	-0.03	-0.02

Note. Coefficient estimates reported are log-odds. True log-odds are averages of observed density of the networks. Sample size consistent across time points. All models take into account the previous time point when estimating the current time point; however the TERGMs and STERGM assume independence between consecutive networks, while the SABM does not (Leifeld & Cranmer, 2016). The SABM is scaled differently and should not be directly compared to the other models (Snijders et al., 2010b).

Table 11. *STERGM Dissolution Estimates for Networks with 2 Time Points, Growth in Density Conditions*

Density	Node Size	STERGM Dissolution	
		Tie	Reciprocal Tie
0.06	20	-2.53	-
	40	-2.29	-
	60	-1.41	9.17
	80	-0.38	-0.20
0.18	20	-1.22	1.19
	40	-0.36	-0.31
	60	0.58	-0.02
	80	1.21	0.08
0.36	20	-0.13	0.11
	40	0.97	-0.04
	60	1.71	0.04
	80	2.33	-0.01

Note. Coefficient estimates reported are log-odds. Sample size consistent across time points. Not all parameters could be measured when all dissolution parameter estimates were negative infinity.

Table 12. *Model Fit Indices for Networks with 2 Time Points, Growth in Density Conditions*

Density	Node Size	ERGM Time 0		ERGM Time 1		STERGM Formation		STERGM Dissolution	
		AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC
0.06	20	193.12	201.00	199.33	207.21	178.53	186.30	-	-
	40	696.47	707.18	834.33	845.00	776.92	787.51	-	-
	60	1632.92	1645.08	1861.17	1873.67	1595.00	1607.42	319.85	326.70
	80	2846.83	2860.25	3237.08	3249.50	2357.75	2371.17	507.18	514.24
0.18	20	365.96	373.83	409.63	416.83	333.12	340.58	81.72	86.21
	40	1461.00	1471.92	1601.08	1611.67	1159.00	1169.25	372.95	380.18
	60	3348.42	3360.83	3555.92	3568.17	1947.50	1959.42	829.08	837.98
	80	6004.08	6017.42	6238.58	6252.17	2539.92	2553.00	1233.50	1243.67
0.36	20	496.78	504.01	520.29	528.16	335.41	342.39	186.18	191.98
	40	2033.17	2043.92	2101.42	2112.25	1084.00	1093.75	653.58	661.53
	60	4635.00	4647.17	4715.00	4727.33	1681.08	1692.75	1091.18	1101.53
	80	8263.50	8277.00	8356.17	8369.92	2116.67	2129.50	1381.50	1393.08

Note. No fit indices could be measured when all dissolution parameter estimates were negative infinity.

Table 13. *SABM Rate Parameter Estimates for Networks with 2 Time Points, Growth in Density Conditions*

Density	Node Size	SABM Rate Parameter
0.06	20	12.05
	40	25.86
	60	16.43
	80	11.31
0.18	20	40.31
	40	19.12
	60	12.49
	80	9.29
0.36	20	32.52
	40	14.01
	60	9.42
	80	6.97

The rate parameters indicate the number of opportunities an actor has to change tie status between Time 0 and Time 1. For the larger densities, as the number of nodes increases, the opportunity to change ties between time points decreases. There is not a clear pattern in the case of a less dense network, but overall for the more dense networks there are fewer opportunities to change ties as node size increases.

4.5 Results of Simulations for Directed Networks with 4 Time Points

In this section, results from the set of simulated directed networks with four time points are reported. For this set, I used a small sample size of 20 nodes and compared the lower density (6%) condition with the higher density (36%) condition. Linear and quadratic growth terms for density remained the same as with the two time point network data. I specifically chose the small sample size as it is most comparable to the real dataset analyzed (see Chapter 5). Also, it is notable that these network analyses are a bit more difficult to estimate as the data generation and model estimation process was not as smooth as that for networks with only two time points; not surprisingly, the time involved was also lengthier also due to the additional computing required with four time points.

4.5.1 *Results for Directed Networks with 4 Time Points*

For this section with an increased number of time points, the no growth condition and growth conditions are considered together. Of interest is how close the model estimates are to the observed log-odds of a tie across time points, which is the network density in the form of log-odds. In these instances, the true log-odds of a tie in each ERGM is the average of the observed density for the corresponding time point. The true log-odds of a tie for the longitudinal models was calculated by taking the average of the observed densities across time points.

Results of the model estimation of tie formation across time points for networks with two time points are provided Table 14. As can be seen, the log-odds of a tie in the network is reflective of the network's density, with a lower log-odds indicating a tie being less likely to occur. Recall that the log-odds of zero would indicate a probability of 50%; negative log-odds indicate a probability below 50% and positive log-odds indicate a probability above 50%.

For all small networks with high and low densities, the TERGM and STERGM estimates are closest to the true average log-odds of the density across time points. The estimates of these models are comparable. In all models, higher density is indicated by the log-odds becoming nearer to zero, meaning that a tie is more likely, which is what was expected.

A 4 (fixed within-subjects; model type) * 1 (fixed between-subjects; node size) * 2 (fixed between-subjects; initial density level) mixed model ANOVA was used to test the effects of model type, parameter conditions (which included initial density, linear change in density, and quadratic change in density), and 2-way interactions among model type and parameter conditions, as well as the 2-way interaction between linear and quadratic change in density over time and its interaction with model type on the model-implied log-odds of a tie occurring in the network. There were $N = 32$ simulations used for these analyses (no-growth conditions were included). Again, because Mauchly's test of sphericity was significant, adjusted *F*-tests for the within-subjects (model type) effect and related interaction tests were used to determine significance.

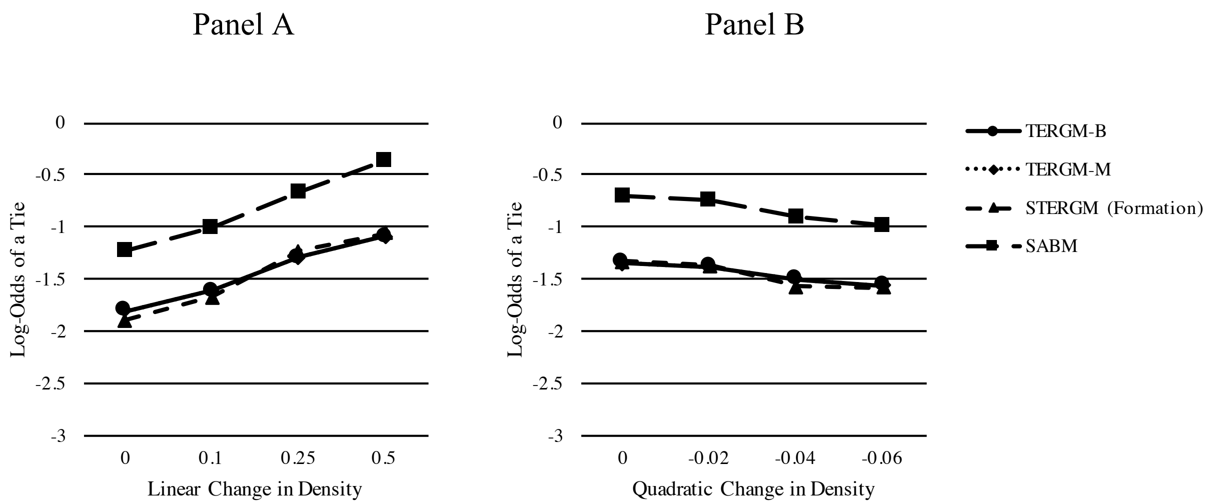
Results showed a significant main effect of density, $F(2, 24) = 2183.30$, $p < 0.001$, a significant main effect of linear change, $F(3, 24) = 63.48$, $p < 0.001$, and a significant main effect of quadratic change, $F(3, 24) = 7.34$, $p < 0.001$. Follow-up simple effects *t*-tests of the log-odds of a tie for each, with a Dunn-Sidak adjustment for multiple comparisons, showed significant differences the two density levels of 0.06 and 0.36, between a linear change in density of 0 and both 0.25 and 0.50 linear change in density, 0.10 linear change in density and both 0.25 and 0.50 linear change in density, and between a quadratic change in density of 0 and -0.06 quadratic change in density, adjusted $ps < 0.001$. Linear and Quadratic change are plotted in Figure 15.

Table 14. *Model-Predicted Tie Estimates for a Networks with 4 Time Points, Growth in Density Conditions*

Density	Node Size	True Log-Odds Time 0	ERGM Time 0	True Log-Odds Time 1	ERGM Time 1	True Log-Odds Time 2	ERGM Time 2	True Log-Odds Time 3	ERGM Time 3	True Log-Odds (Avg Times 0-3)	TERGM-B	TERGM-M	STERGM (Formation)	SABM
0.06	20	-2.84	-2.81	-2.53	-2.56	-2.45	-2.44	-2.49	-2.48	-2.58	-2.54	-2.54	-2.53	-1.57
0.36	20	-0.60	-0.64	-0.33	-0.30	-0.12	-0.12	-0.06	-0.09	-0.28	-0.35	-0.35	-0.39	-0.10

Note. Four time points used in analyses. Coefficient estimates reported are log-odds. True log-odds are averages of observed density of the networks. Sample size consistent across time points. All models take into account the previous time point when estimating the current time point; however the TERGMs and STERGM assume independence between consecutive networks, while the SABM does not (Leifeld & Cranmer, 2016). The SABM is scaled differently and should not be directly compared to the other models (Snijders et al., 2010b).

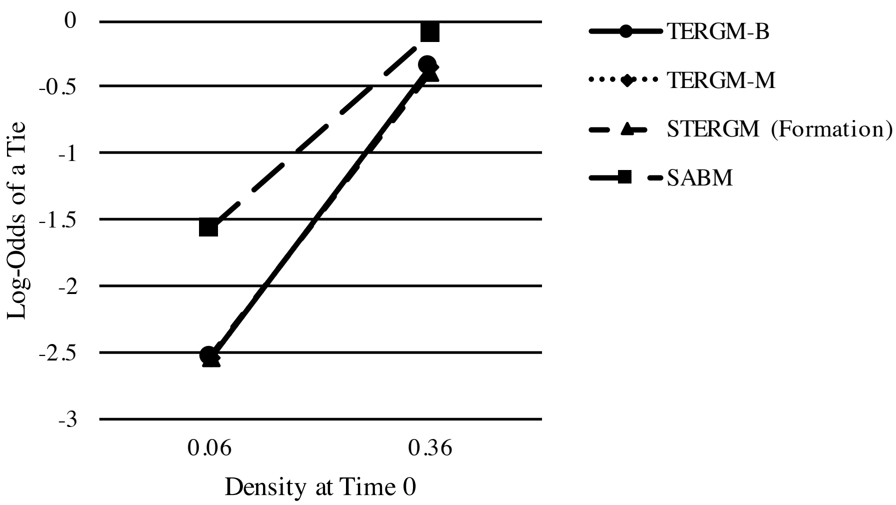
Figure 15. *Linear and Quadratic Change by Model for Networks with 4 Time Points, Growth in Density Conditions*



There was also a significant main effect of model type, *Greenhouse Geisser Adjusted* $F(1.83, 44.02) = 298.84, p < 0.001,$ *partial* $\omega^2 = 0.93$. Follow-up simple effects *t*-tests of the log-odds of a tie for each model type, with a Dunn-Sidak adjustment for multiple comparisons, showed significant differences between the SABM and each of the other models on likelihood of a tie across time points, all adjusted *p*s < 0.001.

Additionally, there was a significant interaction between model type and initial density, *Greenhouse-Geisser Adjusted* $F(1.84, 44.02) = 99.33, p < 0.001,$ *partial* $\omega^2 = 0.81$. Follow-up simple effects *t*-tests of the log-odds of a tie for each model type by initial density, with a Dunn-Sidak adjustment for multiple comparisons, showed significant differences between the SABM and each of the other models, adjusted *p*s < 0.001, which was true for all densities. The interaction can be seen in Figure 16. Inspection of the means revealed that SABM log-odds of a tie are higher than the other models for each density. Again, recall that the tie is considered from the actor perspective in SABM, thus we expect these estimates to be different.

Figure 16. *Density by Model for Networks with 4 Time Points, Growth in Density Conditions*



Estimates of reciprocal tie (Table 15), dissolution estimates from the STERGM (Table 16), model fit indices for ERGMs and STERGM (Table 17), and SABM rate parameters (Table 18) are reported. The SABM rate parameters indicate that actors have the opportunity to change tie status decreases over time, but this decreases more drastically for the low density conditions.

Table 15. *Model Estimates for Reciprocal Ties for Networks with 4 Time Points, Growth in Density Conditions*

Density	Node Size	True Log-Odds Time 0	ERGM Time 0	True Log-Odds Time 1	ERGM Time 1	True Log-Odds Time 2	ERGM Time 2	True Log-Odds Time 3	ERGM Time 3	True Log-Odds (Avg Times 0-3)	TERGM-B	TERGM-M	STERGM (Formation)	SABM
0.06	20	-0.76	0.28	-1.99	0.46	0.07	0.13	-1.27	0.16	-0.01	-2.58	-0.01	0.08	0.05
0.36	20	-0.59	0.02	-0.38	-0.08	0.48	0.01	-0.02	0.07	0.13	-0.28	0.13	0.03	0.04

Note. Four time points used in analyses. Coefficient estimates reported are log-odds. True log-odds are averages of observed reciprocity of the networks. Sample size consistent across time points. All models take into account the previous time point when estimating the current time point; however the TERGMs and STERGM assume independence between consecutive networks, while the SABM does not (Leifeld & Cranmer, 2016). The SABM is scaled differently and should not be directly compared to the other models (Snijders et al., 2010b).

Table 16. *STERGM Dissolution Estimates for Networks with 4 Time Points, Growth in Density Conditions*

Density	Node Size	STERGM Dissolution	
		Tie	Reciprocal Tie
0.06	20	-1.94	7.90
0.36	20	0.03	0.13

Note. Coefficient estimates reported are log-odds. Sample size consistent across time points. Not all parameters could be measured when all dissolution parameter estimates were negative infinity.

Table 17. *Model Fit Indices for Networks with 4 Time Points, Growth in Density Conditions*

Density	Node	ERGM Time 0		ERGM Time 1		ERGM Time 2		ERGM Time 3		STERGM Formation		STERGM Dissolution	
		AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC
0.06	20	186.54	194.40	205.71	213.07	224.06	231.95	256.53	264.41	571.12	581.04	102.68	107.76
0.36	20	496.06	503.94	515.37	523.25	507.01	514.89	468.33	476.21	880.98	889.97	609.12	617.42

Note. No fit indices could be measured when all dissolution parameter estimates were negative infinity.

Table 18. *SABM Rate Parameter Estimates for Networks with 4 Time Points, Growth in Density Conditions*

Density	Node Size	SABM Rate Parameter		
		1	2	3
0.06	20	32.58	24.29	7.77
0.36	20	32.14	26.64	18.45

Chapter 5. APPLIED DATA AND RESULTS

5.1 CONTEXT: TEACHER ADVICE & NETWORK IMPROVEMENT COMMUNITY

This chapter provides a demonstration of applying stochastic social network analyses (SNAs) in a realistic setting. Data for this chapter were collected annually as part of a longitudinal teacher advice network professional development project funded by the National Science Foundation. Specifically, University of Washington (UW) researchers and one urban, diverse public school district collaborated to 1) refine a set of Ambitious Science Teaching (AST) practices supporting the vision of next generation science standards (NGSS; Windschitl et al., 2012), and 2) develop a professional learning model in which teaching practices are the focal point of study and instructional experimentation. The professional development learning model used was 1) job-embedded, meaning that school teams iterate on teaching practices in classrooms where they can receive feedback from students, 2) inclusive of all 46 science teachers at every secondary school in the district, and 3) included multiple role actors—secondary science teachers, English learner coaches, science coaches, principals, district leaders, and university faculty and staff. This professional "advice" network (technically, this was a network *improvement* community, compared to other types of advice networks, such as network *expert* communities and network *leadership* communities) afforded the opportunity for participating members to share and empirically test teaching practices and tools such that productive variations of practices and tools can be generated.

Teacher learning in traditional professional development has been studied extensively (e.g., Ball & Cohen, 1999; Cohen & Hill, 2001; Grossman, Wineburg, & Woolworth, 2001; Odden, 1991); however, there is little research on how teachers *collaborate* (e.g., Lieberman & Pointer Mace, 2009; Moolenaar, 2012; Stein & Coburn, 2007; Stein & D'Amico, 2002). Use of

SNA is the first step toward understanding whether the teachers' *network* improved over time (i.e., using network density); additionally, secondary modeling analyses as well as qualitative follow-up analyses will be used to further examine how network change may relate to teaching practice change (but these follow-up analyses are outside the scope of the present study).

Thus, the analyses used in this chapter were aimed at quantifying network change during the first four years of the teacher professional development project. In the first year, the network was measured retrospectively (prior to professional development activities); in the second and third years, intensive professional development activities, including "studio days," were conducted by UW researchers; and finally, the fourth year of the project was a transition year in which UW researchers withdrew as focal leaders so that the teachers in the network could become teacher leaders. Although there were a number of individuals who participated in different years of the project, only data from those in the network who participated in all four years were considered for the analyses presented for the purpose of maintaining comparability across each of the longitudinal SNA models.

5.2 METHOD

Science teachers, instructional coaches, and researchers participating in the AST project ($N = 21$) in one diverse public school district were followed for four academic years: 2012-13, 2013-14, 2014-15, 2015-16. Among other things, they were asked to nominate individuals they seek professional (science teaching) advice from. There were up to seven nominees possible, and a drop-down menu of individuals within the district and research team was provided to aid recall; however, individuals could nominate anyone, not just those on the list, and they could also choose to nominate no one. The number of teachers, coaches, and researchers nominated by the 21 participants across each year of the project is shown in Table 19.

Table 19. *Number of Nominees by Time and Role*

Time Point			Network Role		
				Instructional	
	School *n*	Network *n*	Teachers	Coach	Researcher
Year 0 (2012-13)	8	27	19	4	4
Year 1 (2013-14)	8	33	24	5	4
Year 2 (2014-15)	9	38	27	5	6
Year 3 (2015-16)	9	43	31	7	5

Note. N = 21 nominators (teachers, instructional coaches, and researchers).

For each year of the study, a combined dataset of the 21 participants and all possible nominees was created as an adjacency matrix in *R* using base *R* and **`statnet`**. This was done because the STERGM approach requires the same number of nodes in the network at each time point, and missing values are not allowed. Thus, an individual who was nominated for the first time in the fourth year was added to the adjacency matrix for the three previous years, but only as being a node without being nominated as an advice-giver and without nominating anyone; similarly, an individual nominated in the first year but no other years would be considered a node in all subsequent years, but without any nominators or nominees. This method of handling "missingness" in network data over time is the best of the recommended options (see Handcock & Gile, 2010) because 1) deleting a nominee would imply the network to be smaller than it is, and 2) given the sparseness of this network, it is reasonable to assume that the individual can be treated as an isolate in years where they were not nominated. As such, the data analyses are based on a combined total of $N = 56$ nodes.

Recall that an adjacency matrix – the data analyzed in any stochastic SNA – is a people-by-people matrix populated with 1s and 0s. For a *directed* adjacency matrix (i.e., to distinguish nominators from nominees in order to understand reciprocity or "mutual ties"), the rows indicate the nominators, and columns indicate nominees, with 1 indicating a tie, and a 0 indicating no tie. If the tie is reciprocal, there will be a 1 to indicate the individual in the row nominating the other in the column, and this will be true for both individuals. (See again Figure 7 in Chapter 2 for an example of a directed adjacency matrix.) Importantly, because the survey was administered only to participating teachers, coaches, and researchers, the survey was not administered to those who participants identified as advice-givers. In other words, we cannot know whether those outside the main list of individuals in the drop-down menu who were nominated by advice-seekers would have also nominated the individual who nominated them (or anyone else in the network for that matter). Those who were nominated but did not have the opportunity to nominate others (because they were not surveyed) are included in the adjacency matrix, but would have all 0's in their corresponding row. This is a standard limitation in any network model.

5.3 Applied Data Analysis Results

5.3.1 *Descriptive Statistics for the Network across Time*

Descriptive statistics, including density, reciprocity, distance, and diameter, for the network at each time point are shown in Table 20.

Table 20. *Network Descriptive Statistics*

Time Point	Nodes	Ties	Density	Reciprocity	Distance	Diameter
Year 0 (2012-13)	27	15	0.02	0.40	1.12	2.00
Year 1 (2013-14)	33	53	0.05	0.34	2.64	6.00
Year 2 (2014-15)	38	56	0.04	0.18	2.44	6.00
Year 3 (2015-16)	43	61	0.03	0.26	2.50	5.00

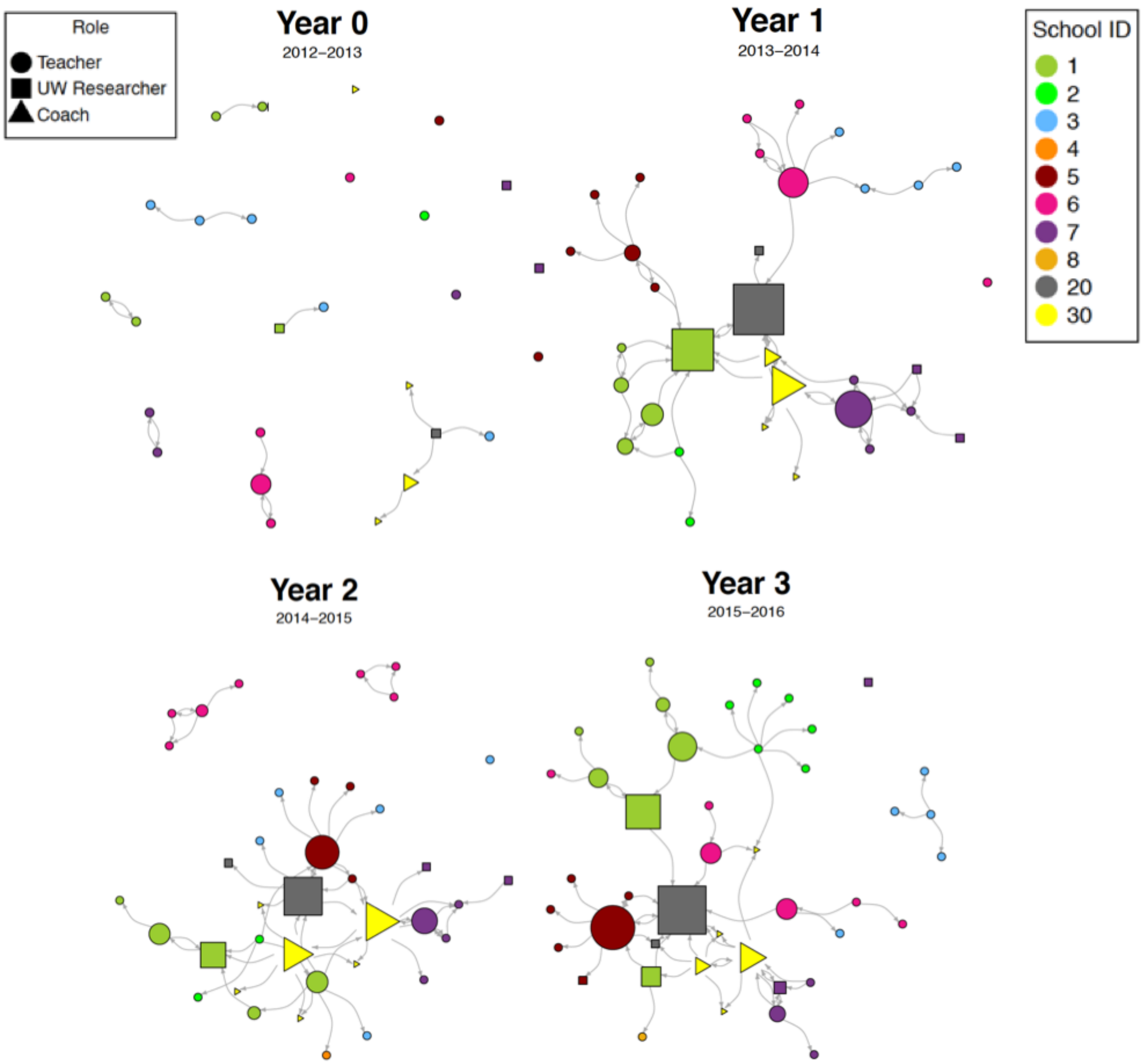
Recall that **density** is the proportion of ties observed divided by all *possible* ties in the network (and recall that a tie is a connection or "edge" between two nodes). The near-zero values in this network indicate that it was very sparse with few ties. Over time, it appears that the network had some linear and quadratic growth, first increasing from 2% in the first year (Year 0) prior to professional development to 5% in the second year (Year 1), but then decreasing to 3% in the final year when the researchers were transitioning out of the leadership role (but still ending with a higher density than prior to professional development).

Reciprocity indicates the number of reciprocal (mutually nominating) ties divided by the total number of existing ties. For the first three years we see a decline in the proportion reciprocal ties, from 40% (of the 2% density, which is approximately 0.8% of the whole network) down to 18% (of the 4% density, approximately 0.7% of the whole network), but then in the fourth (transition) year, the proportion increases up to 26% (of the 3% density, approximately 0.8% of the whole network).

Recall that **distance** is the average path length between any two nodes in the network and that **diameter** is a measure of the shortest-longest path between any two nodes in the network; the larger the diameter the less connected the network is. Taking these two measures together, we see that network appears to be less connected over time, which will make sense shortly when inspecting the sociograms showing that individuals in the network tended to nominate individuals within their own schools rather than those outside their own schools – making cross-network information sharing difficult.

Figure 17 displays the **sociograms**, which are graphics based on node- and tie-level descriptive information, for the network at each of the four time points. I used **`igraph`** to generate the graphs (Csardi & Nepusz, 2006). As can be seen, the network size (number of nodes) does become larger over time, and over time it appears that there is more connectivity *within schools*. To best interpret the graphic, the size of the nodes represents betweenness centrality, with larger nodes indicating a more central position in the network or having a larger number of geodesics (shortest paths) between any two nodes pass through that particular node.

Figure 17. *Sociograms of the Network for Each Time Point*



5.3.2 *Stochastic Social Network Analyses for the Network for Each Time Point*

Prior to analyzing the data longitudinally, an exponential random graph model (ERGM) was conducted using **`statnet`** in *R* for each time point separately as a preliminary means for assessing which parameters should be included in the longitudinal models (Handcock et al., 2008). Results (see Table 21) showed that the best fitting model included ties (a measure of density), mutuality (indicates reciprocal relationships), and ties within school (a measure of a tie within the same school).

Table 21. *ERGM Results for Each Time Point*

	Time 0 (2012-2013)				Time 1 (2013-2014)				Time 2 (2014-2015)				Time 3 (2015-2016)			
Parameter	*Coeff*	*(SE)*	*p*		*Coeff*	*(SE)*	*p*		*Coeff*	*(SE)*	*p*		*Coeff*	*(SE)*	*p*	
Tie	-4.55	(0.37)	<0.001	***	-3.98	(0.24)	<0.001	***	-3.84	(0.19)	<0.001	***	-4.20	(0.20)	<0.001	***
Reciprocal Tie	3.66	(0.91)	<0.001	***	1.19	(0.56)	0.035	*	0.83	(0.59)	0.161		1.43	(0.53)	0.007	**
Tie in Same School	1.16	(0.47)	0.014	***	2.70	(0.35)	<0.001	***	2.16	(0.30)	<0.001	***	2.35	(0.29)	<0.001	***
Fit Indices																
AIC	131.2				323.0				415.3				443.2			
BIC	144.8				337.9				431.0				459.7			

*p < 0.05, **p < 0.010, ***p < 0.001.

Model results for Times 0-3 as estimated by the ERGM, are shown comparatively in Table 21 revealed that the likelihood of tie occurring in the future for any network was significantly lower than 50% as indicated by the negative tie term in each ERGM. However, should a tie form, it is most likely to be reciprocal or within the same school. Reciprocal ties are most likely at Time 0, but above 50% at each of the time points. Ties in the same school were even more likely to form at each of the time points than reciprocal ties.

5.3.3 *Stochastic Longitudinal Social Network Analyses Across Time Points*

Recall that the longitudinal SNA models being compared for this study are largely based on the ERGM framework. In other words, the focus of the analysis remains the probability of tie formation, or for longitudinal data, the probability of a tie across network observations. Recall also that there are four modeling approaches being compared: temporal ERGM estimated two ways, separable temporal ERGM, and stochastic actor-based modeling (SABM).

As discussed in Chapters 3 and 4, temporal ERGM (TERGM) is an ERGM with a *temporal* component included into the model to take into account the network state in the previous time point(s), thus providing an "overall" probability of tie formation across all time points. In addition, there are two estimation algorithms that can be used for TERGM: TERGM-B, which uses maximum pseudolikelihood with bootstrapping, and TERGM-M, which uses MCMC MLE. Collaboration networks like the one in this study lend themselves better to the MCMC MLE estimation (Leifeld, Cranmer, & Desmarais, 2017); indeed, the TERGM-B model had trouble estimating the standard errors and the *p*-values for the present data.

Separable temporal ERGM (STERGM) estimates the probability of both the *formation* and *dissolution* of ties, taking into account the network state in the previous time point(s). Although both TERGM and STERGMs can evaluate longitudinal networks, they are limited to providing *overall* network statistics, and are unable to provide differences among specific time points. Specifically, each of the ERGM-based approaches, including ERGM for individual time points, estimates the log-likelihood of another tie forming in the network (i.e., a connection between two individuals, irrespective of direction) as well as the likelihood of a reciprocal tie occurring across the ties that do exist (all in the logit scale), based on covariates included the model. For the purpose of the present analysis, I included school membership as a covariate so that the likelihood of a tie forming within the same school could be specifically estimated (separate from the likelihood of any tie occurring in the network). It is also worth noting that, in preliminary models, I tested the significance of node role (teacher, coach, researcher) on tie formation, but inclusion of node role did not improve model fit and was subsequently dropped.

Model results for the first three longitudinal model approaches (TERGM-B, TERGM-M, and STERGM), shown comparatively in Table 22 revealed that the likelihood of any tie occurring in the future for the network was significantly lower than 50% (i.e., a logit of 0 translates to a 50% probability), and that the estimates were remarkably consistent across the modeling approaches (TERGM-B and TERGM-M estimated a coefficient of -5.06, and STERGM estimated a coefficient of -5.05; this mathematically translates to a predicted probability of 0.6% (less than 1%) that another tie would occur in the network in the future.

Table 22. *Comparison of SNA Results from TERGM-B, TERGM-M, and STERGM*

Parameter	TERGM-B				TERGM-M				STERGM					
	Coeff	*(SE)*	*p*		*Coeff*	*(SE)*	*p*			*Parameter*	*Coeff*	*(SE)*	*p*	
Tie	-5.06	(0.28)	<0.001	***	-5.06	(0.11)	<0.001	***	*Formation*	Tie	-5.05	(0.14)	<0.001	***
Reciprocal Tie	2.47	(0.35)	<0.001	***	2.49	(0.29)	<0.001	***		Reciprocal Tie	1.46	(0.30)	<0.001	***
Ties in Same School	2.20	(0.08)	<0.001	***	2.19	(0.15)	<0.001	***		Ties in Same School	2.21	(0.19)	<0.001	***
									Dissolution					
										Tie	-0.88	(0.31)	0.005	**
										Reciprocal Tie	1.32	(0.62)	0.033	*
										Ties in Same School	0.43	(0.37)	0.253	

Note. Four time points used in analyses. All models take into account the previous time point when estimating the current time point. The TERGMs and STERGM assume independence between consecutive networks (Leifeld & Cranmer, 2016). *$p < 0.05$, **$p < 0.010$, ***$p < 0.001$.

All other coefficients estimated in the models are interpreted as *conditional on the tie term*. Estimates for the likelihood of a reciprocal tie occurring in the future for the network were remarkably similar across the two TERGMs, with coefficients of 2.47 and 2.49 for TERGM-B and TERGM-M, respectively. When translated from its logit value, both estimated that the likelihood of a new reciprocal tie occurring (in the future) among ties formed in the future was 92%. STERGM's estimate was different but still significant: when translated, it estimated the likelihood of a future reciprocal tie was 81%. However, note that STERGM also estimates the likelihood of a tie *dissolving* (dissolution) in the future. Again, a mathematical translation is helpful for interpretation: STERGM's predicted probability of any tie in the network dissolving in the future was 29%, and of these dissolved ties, 79% were predicted to be reciprocal.

What is perhaps most interesting about this particular network is the within-school probability of a tie. Of the ties predicted to form in the future, all three models estimated that 90% of those ties would be within the participant's school. In addition, STERGM estimated that there was a 61% chance that a future dissolved tie would be within-school (which was not significantly different from chance, or 50%).

Finally, the SABM approach's results are reported in Table 23. Recall that SABM considers the perspective that the actor (node) is an agent in the network and forms and dissolves ties as he or she chooses. Specifically, this model estimates the conditional probability of a tie occurring *for any given node* in the network, at any given time point, rather than the probability of a single tie forming across the network of ties (Snijders, 1996). Because of this, we expect the SABM estimates to be somewhat different from the ERGM, TERGM, and STERGM estimates. In addition, SABM estimates $k - 1$ "rate" parameters (where k = number of time points), which indicate, on average, the number of time that a given node will have the *opportunity* to change their tie status, thus this number may be higher than the observed change in ties (Snijders et al., 2010b).

Table 23. *SNA Results from SABM*

Parameter	SABM			
	Coeff	*(SE)*	*p*	
Tie	-3.22	(0.16)	<0.001	***
Reciprocal Tie	1.44	(0.22)	<0.001	***
Tie in Same School	1.63	(0.16)	<0.001	***
Rate Parameters				
Period 1	3.32	(1.00)		
Period 2	3.47	(0.71)		
Period 3	4.45	(0.88)		

Note. Four time points used in analyses. The model uses the current time point to estimate the next. *$p < 0.05$, **$p < 0.010$, ***$p < 0.001$.

As can be seen, SABM's coefficient estimate of -3.22 for tie formation is significant (i.e., lower than 50% chance), and when mathematically translated, there is a 4% probability that a *given node in the network will form a new tie* in the future with any other node in the network. Recall that this quantity differs in meaning from the TERGM and STERGM tie formation estimates, as the latter models estimated the probability of *any new tie occurring in the network out of the ties that are possible* (which was predicted at 0.6%). In other words, the denominator for SABM is based on the number of nodes, whereas the denominator for TERGM and STERGM is based on the number of ties among nodes that are possible.

The other model estimates from SABM show that 1) the probability of a *node* in the network forming a future reciprocal tie (from a tie already formed) was at 81% (same estimate as STERGM, but lower than TERGM), and 2) that the likelihood a node would form a future tie within the same school as their own was 84% (slightly lower than the 90% estimated by TERGM and STERGM approaches). Finally, the rate parameter estimates showed that nodes had few opportunities for tie change at each time point: in periods 1, 2, and 3 (which translates the time between Time 0 and Time 1, between Time 1 and Time 2, and Time 2 and Time 3, respectively).

5.4 Model Comparison Discussion

The aim of analyzing these data was twofold: 1) to demonstrate how SNA can be applied to real data in context, and 2) to substantively understand whether the professional development teacher advice network changed over time.

Each of the SNA models provided slightly different information about the network, even though all models estimated the likelihood of a tie, reciprocal tie, and tie between individuals in the same school across time points. Whereas TERGM approaches network structure from a global view, examining total changes in network ties across all time points, STERGM does the same thing but separates estimates into tie formation and dissolution. SABM, which takes the perspective that each actor (node) in the network chooses to have a tie or not have a tie based on the entire network structure at that moment in time, which then impacts the probability of the same process for the next actor (node) in the network. It also is able to estimate the likelihood of an opportunity for tie change for a given actor (node) at different time points (i.e., the "rate" parameters). This last approach may indeed be preferred if one is seeking to further theory on how important individual and contextual factors are in a changing network (Ripley et al., 2017), but its drawback is that it can be difficult to estimate and interpret (Snijders et al., 2010b).

Because the network is the focal point of the current study, I believe TERGM or STERGM would actually be the recommended model selection for these particular data. Specifically, the researchers involved in the present study were interested in global networking in terms of ties (i.e., the idea that information needs to be spread amongst as many individuals as possible, even if it is limited to within-school connections), rather than node-oriented tie estimates that SABM offers. If pressed, I would actually recommend that STERGM be the model of choice since it also offers insight on tie persistence (given that it can estimate dissolution as well as formation).

This said, an advantage in using TERGM is that it can handle missing data much more easily than STERGM can, and does not require each time point to have the same number of nodes as STERGM does. With the constraint of having equal nodes at each time point, there may be systematic missingness that can require making assumptions about tie relationships that the researcher is not comfortable with.

Chapter 6. DISCUSSION AND CONCLUSIONS

This chapter will consider the comparative results of stochastic social network analysis (SNA) models across the simulations and applied analysis, limitations, and recommendations for future research. Recall that the methodological approaches examined in this study consider network characteristics from different perspectives. All of the models, however, share in common the ability to predict future network behavior based on network characteristics at prior time points. In the present study, the temporal exponential random graph model (TERGM; with two estimation approaches), separable temporal random graph model (STERGM), and stochastic actor-based model (SABM) were estimated and compared for two sets of simulation data as well as one applied dataset for a teacher advice network. Across these modeling approaches, the focal unit of analysis was the likelihood of a future tie forming given information about the network at one or more time points. Therefore, the model-derived estimate of a tie in a longitudinal model is the estimate of the total change across the time points. While TERGM and STERGM estimate the log of the likelihood of a tie forming across the network (with STERGM simultaneously estimating the probability of tie dissolution), SABM estimates the log of the likelihood of an actor (node) in the network forming a new tie, as well as the frequency of the opportunities for tie change between consecutive network observations.

All stochastic SNA methods studied employ adjacency matrices as their datasets (see again Chapter 3 for an explanation), and longitudinal SNA models require a network list of adjacency matrices. If covariates are available, they can also be added. Unlike the other models, SABM requires more computational power because it uses “ministeps” during estimation that change the network for each actor at each time point. In some instances, additional iterations will need to be used to obtain estimates. In addition, SABM estimates can be difficult to interpret (Snijders et al., 2010b).

Recall that STERGM is the model that requires the same number of nodes for each time point. Although it offers an additional set of parameter estimates (focused on tie dissolution), it is unrealistic in educational data that the network size would remain constant over time. Participating students and teachers within a network may easily come and go. Thus, in order to use STERGM, one must create combined datasets that fold in all possible nodes and then make an assumption about whether or not nodes would have had a previous tie, had it been observed (when in fact it is really missing data).

The simulation data analyses offer the ability to compare the model estimates across different conditions, including node size, network density, level of density change over time, and number of time points. The applied data analyses, on the other hand, demonstrate how these models can be applied and interpreted in practice, given a specific context in which a social network, rather than individuals, is the focal interest.

6.1 Simulation Data: Two and Four Time Point Estimates

Simulation results for the no-growth, two-time point data showed that, though model estimates of tie formation were quite similar in magnitude across node and density conditions (with SABM understandably systematically different due to the node-oriented quantity it estimates), both of the TERGMs' estimates were comparatively closest to the true density parameter compared to STERGM and SABM. Yet in the simulations using only the small network with four time points, TERGM and STERGM provided comparable tie formation estimates.

For no growth conditions for two time points, there was an interaction between model type and node size, and between model type and initial density. For the 80 node condition there was a difference between tie estimates for both TERGM-B and STERGM, as well as TERGM-M and STERGM. Both TERGM-B and TERGM-M estimated ties to be more likely than STERGM. For all levels of density, the SABM estimates systematically found a node to be more likely to form a tie with another node, than for a tie to form in the TERGMs or STERGMs. Additionally, for all levels of density, both TERGM-B and TERGM-M estimated ties to be more likely than STERGM.

Analysis of growth conditions for networks with two time points showed a main effect for model type and node size. Again, SABM was found to be significantly different from the other models, and there was a significant difference also between the TERGMs and the STERGM. The SABM estimate had the greatest likelihood of a node forming a tie with another node, while both the TERGM-B and TERGM-M estimated a higher likelihood of a tie occurring in the network than the STERGM. There was an interaction between model type and node size, and between model type and initial density. Networks of all node sizes showed a difference between SABM and all other models, while networks with 40, 60 and 80 nodes showed a significant difference between TERGM-M and STERGM. For the 80 node condition there was also a difference between tie estimates for both TERGM-B and STERGM. Both TERGM-B and TERGM-M estimated ties to be more likely than STERGM, while SABM estimates were more likely than those in any other model. For all levels of density, the SABM estimates systematically found a node to be more likely to form a tie with another node, than for a tie to form in the any of the other models. Additionally, for density levels of 0.18 and 0.36, both TERGM-B and TERGM-M estimated ties to be more likely than STERGM.

For the four time points, where no growth and growth conditions are combined, there were main effects for model type, density, linear change, and quadratic change. There were significant differences between densities of 0.06 and 0.36, as well as between linear change of 0 and 0.25, between 0 and 0.50, and between quadratic changes of 0 and -0.06. For each of these differences, a node having a tie with another node in the SABM is more likely than a tie in the other models. There were significant interactions between model type and initial density. Again, SABM estimates were significantly different from all the other models, as expected.

In addition to model estimates, the length of time it took to conduct the models was examined as a matter of practicality for researchers. The STERGM and SABM took the longest to estimate with two and four time points with SABM taking quite a bit longer.

6.2 Applied Data: Teacher Advice Network Improvement Community

Overall, models found the network to be sparse at each time point. Ties are less than 50% likely to occur, according to each of the models, yet if a tie were to occur, it is likely to be reciprocal or between individuals in the same school. However, only the tie term, and the tie in same school terms were significant. The longitudinal models differed in their perspective, the TERGM providing overall tie change, the STERGM providing tie change separately for formation and dissolution, and SABM providing the likelihood for an *actor* to form a tie with another actor in the network.

The STERGM is preferred to analyze this case study as it best addresses the research interest by estimating both formation and dissolution of ties. This is particularly helpful in assessing the effectiveness of PD on collaborations. However, STERGM requires making a decision on how to deal with missing data. In particular, the SABM model may not be the optimal model choice. This model considers the possibility that each actor could have a tie with any other actor in the network, however, this is unlikely in this circumstance because the actors probably do not often have the potential to interact with those from different schools.

6.3 STUDY LIMITATIONS

This section considers limitations of both the simulated and applied analyses.

6.3.1 *Simulation Data Analysis Limitations*

Like any study, there are a number of limitations in the simulations and the applied data analysis. The simulations were quite limited to only one random draw per condition given the computational intensity of generating the adjacency matrix data and analyzing the data with the four types of longitudinal SNA model approaches (not to mention the individual time point models). Most simulation studies employ multiple simulations per condition to account for sampling error. A second limitation of the simulations was that I was not able to directly control the reciprocity levels during data generation, and I was unable to generate data for very sparse data at the 2% level because of the longitudinal SNA model constraints (as such, I used 6%, 18%, and 36%). Third, I did not incorporate tests of covariates. Finally, I was limited to a small sample network (node size of 20) for the four time point data simulations due to the computational intensity of the data generation and analysis process. All of these limitations can be addressed of course in future simulation work.

The simulation was largely limited by the constraints of the modeling processes. For example, density and relative reciprocity can be manipulated, however specific density *and* specific reciprocity cannot both be manipulated at the same time. Reciprocity can only be manipulated relatively speaking, but this is not satisfactory for a simulation in which the conditions should be controlled closely. Notably, reciprocity only makes sense in a directed network as there is no information about directionality inherent in an undirected network. While this study did not suffer from degeneracy issue when estimating ERGMs, ERGMs can be prone to degeneracy issues (Handcock, 2003b).

Estimates with two parameters were obtained for each of the models because the TERGMs require at least two parameters to be estimated. This eliminated the possibility of simply including tie as the only parameter in the models.

Additionally, the true log-odds of a tie over time, which I compared to the longitudinal model estimates, was computed by averaging the observed networks' densities. When simulating the networks by year, this is the best approximation of the truth available.

Models differed in length of time it took to estimate parameters, and some methods seemed to be more prone to returning errors during the estimation process. Across all the models, STERGM and the SABM took the longest to estimate parameters, and experienced the most errors, particularly with smaller, sparser networks.

Model fit indices such as Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) were not available for all methods, but were available for exponential random graph models (ERGM), and STERGM. This is particularly helpful when determining what parameters should be included in the model. However, these may not be comparable across models even if available because estimation procedures differ across models.

Missing data proved to be post problematic with the STERGM, which requires the same numbers of observations at each time point. For the purposes of comparison in this study, all data were kept comparable, such that the applied data used in all the files fit the requirements of the STERGM; TERGM and SABM do not require that missing data be handled.

6.3.2 *Applied Data Analysis Limitations*

The applied analysis study's teacher advice network improvement community was a very sparse and very small network. The networking survey was only completed by a portion of the people who participated in professional development project, thus we did not have information from people who were nominated as advice-givers and cannot know if they would seek advice from those who seek their advice (i.e., there may be reciprocal ties that exist that were not captured in the scope of this study). Further, because STERGM requires the same number of nodes at each time point, only those who participated in the survey each year ($N = 21$) could be included. Additionally, due to inconsistencies in survey implementation, nominators could nominate fewer nominees in the first and second year (up to seven) and up to 10 in the third and fourth year. Thus, for the purpose of the present study, only the first seven of the 10 nominated in the third and fourth years were included in analyses. Despite these limitations, small networks may actually be typical in educational research; my brief review of the applied analyses that exist in the literature were overwhelmingly small and with low density.

6.4 CONTRIBUTIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

This study contributes to the gap in educational research methods knowledge by 1) estimating the prevalence of using social network analyses in educational research, 2) being the first to date to directly compare four viable approaches for modeling longitudinal stochastic social networks: TERGM-B, TERGM-M, STERGM, and SABM, and 3) providing a demonstration of how to implement each of these methods with a small-sample, low-density network of professional development participants who were followed for four years. There are of course several avenues of course for future research, particularly in the area of simulations.

Practically speaking, network analysis can be implemented at the school or district level to help better understand the complexity of teacher interactions within schools, as well as to explore the effectiveness of programming such as teacher retention support systems. In particular, longitudinal network analysis can evaluate whether there is change in the network over time (e.g., Do teachers maintain the same collaborative interactions/support relationships over the entire school year, or do these connections change (How often, and with whom)?). For instance, if a principal collaboration program aimed at improving teacher retention is implemented at the beginning of the school year, a TERGM could be used to see whether ties are likely to increase, or, if interested in formation and dissolution of ties, a STERGM could be employed. Perhaps of most usefulness to school leaders and policymakers would be the incorporation of sociograms in helping to better understand the networks (patterns) of communication and supports within schools and across districts. As a simple visual representation of these relationships, sociograms could be used as descriptive data (e.g., density, centrality measures) to illuminate those teachers within the network who are well connected to others in the school and identify teachers who have few connections or are isolates with no connections. Simply by knowing the structure of the network, principals can work to ensure that each teacher is a well-connected member of the network, increasing the likelihood that they will get the support they need to be effective in the classroom, and remain in the profession.

First, future research should use more than one simulation per condition, perhaps by simulating data on multiple processors to speed up analytic time (i.e., one model per dataset per machine). Second, future research could include other parameter values other than those used in the present study, such as higher density levels or decreases in density levels over time. Third, future work should incorporate covariates, like school membership (which are nearly always present in educational data).

Another important area of interest would be to test out how missing data (missing at random with 0s, missing at random with random draws of 0s and 1s, and missing at random with 1s) might bias network tie estimates, particularly for STERGM. Although STERGM seems to offer the most informative estimates about network change because it considers formation and dissolution, it requires equal node sizes across time points which is unrealistic in longitudinal educational research. Currently, the choices for handling missing data are limited, and require a convincing justification.

Recall that, across all the methods examined in the current study, the network's future characteristics (i.e., tie formation) is estimated, given prior network characteristics. None of the models tell the researcher if the network has significantly changed in density over time, or whether one time point's ties differ significantly from another time point (i.e., there's no slope for "time"). Additional work could thus examine more of a "two-step" approach that takes node-level information for use within a structural or hierarchical growth model. For example, using the number of nominations an individual receives within the network (a network centrality measure) to estimate whether in-degree centrality, also called popularity, significantly changes over time, whether it be a linear, quadratic, or piece-wise function. It may very well be the case that in questions regarding growth over time, a non-SNA model may ultimately be preferred.

Given that there are educational researchers already using SNA, particularly for professional development research, it seems important that we consider what models are available and what each may or may not offer. This study aimed to do such a comparison for the purpose of helping researchers make an informed decision. Of course, this is not to say that the quantitative approach to social network analysis will be sufficient on its own – it is likely that stochastic SNA model results would be best used in the context of complementary qualitative methods so that we can better understand how tie formation estimates translate to how information is spread and used across a network to improve individual and collective behavior.

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APPENDIX:

R Code for Generating and Analyzing Networks at 4 Time Points (ERGMs, TERGMs, STERGM, and SABM)

```
library("statnet")
library("igraph")
#Set Seeds
seed1 <- sample(1:10000, 1, replace=T)
seed1
seed2 <- sample(1:10000, 1, replace=T)
seed2
seed3 <- sample(1:10000, 1, replace=T)
seed3
#################### Time 0 #################
g0<- network(20,density=.06,directed=TRUE)
#summary(g0) #Summary of created network, includes density of created network
m0=as.matrix(g0)
GT.0=graph.adjacency(m0,mode="directed",weighted=NULL)
graph.density(GT.0) # Density OF OBSERVED GRAPH
reciprocity(GT.0) # Reciprocity of observed graph
erg.T.0<-ergm(g0~edges+mutual)
summary(erg.T.0) #ERGM Results
##################### Time 1 ERGM ###############
#RECIPROCITY (LOG-ODDS = 0 INDICATES PROBABILITY = 0.5)
g1 <- simulate(~edges+mutual, nsim=1, coef=c(-3.8918, 0),
                     basis=g0, control=control.simulate(
                       MCMC.burnin=1000,
                       MCMC.interval=100), seed=seed1)
m1=as.matrix(g1)
GT.1=graph.adjacency(m1,mode="directed",weighted=NULL)
graph.density(GT.1)
reciprocity(GT.1)

erg.T.1<-ergm(g1~edges+mutual)
summary(erg.T.1) # ERGM Results
##################### Time 2 ERGM ###############
g2 <- simulate(~edges+mutual, nsim=1, coef=c(-3.8918, 0),
                     basis=g1, control=control.simulate(
                       MCMC.burnin=1000,
                       MCMC.interval=100), seed=seed2)
m2=as.matrix(g2)
GT.2=graph.adjacency(m2,mode="directed",weighted=NULL)
graph.density(GT.2)
reciprocity(GT.2)
erg.T.2<-ergm(g2~edges+mutual)
summary(erg.T.1) # ERGM Results
##################### Time 3 ERGM ###############
g3 <- simulate(~edges+mutual, nsim=1, coef=c(-3.8918, 0),
                     basis=g2, control=control.simulate(
                       MCMC.burnin=1000,
                       MCMC.interval=100), seed=seed2)
m3=as.matrix(g3)
GT.3=graph.adjacency(m3,mode="directed",weighted=NULL)
graph.density(GT.3)
reciprocity(GT.3)
```

```
erg.T.3<-ergm(g3~edges+mutual)
summary(erg.T.3) # ERGM Results

######STERGM######
g.net <- list()
g.net[[1]] <- g0
g.net[[2]] <- g1
g.net[[3]] <- g2
g.net[[4]] <- g3

st.g.net <- stergm(
  g.net,
  formation = ~ edges + mutual,
  dissolution = ~ edges + mutual,
  estimate = "CMLE",
  control = control.stergm(
    init.form = NULL,
    init.diss = NULL,
    init.method = 0,
    force.main = FALSE,
    SA.restart.on.err = FALSE
  ),
  times = 1:4
)
summary(st.g.net) # STERGM Results

######SABM#####
library(RSiena)
library(statnet)
library(ggplot2)
betterAlgorithm <- sienaAlgorithmCreate( projname = 'Dissertation',
                                          diagonalize = 0.2, n3=4000)
X1<-as.matrix(g0)
X2<-as.matrix(g1)
X3<-AS.MATRIX(G2)
X4<-AS.MATRIX(G3)

#20 Nodes 4 Times
mynet1 <- sienaDependent(array(c(X1, X2, X3, X4), dim=c(20,20,4)))
mydata <- sienaDataCreate(mynet1)
myeff <- getEffects(mydata)
myeff
myeff <- includeEffects(myeff, recip)
myeff1 <- includeTimeDummy(myeff, balance, R, timeDummy="4")
ans3 <- siena07( betterAlgorithm,
                 data = mydata, effects = myeff, batch = T)
summary(ans3)
tt3 <- sienaTimeTest(ans3)

#####TERGMs######
library(btergm)
g.btergm.fit <- btergm(g.net ~ edges + mutual, R = 500)
summary(g.btergm.fit) # TERGM-B Results
btergm.se(g.btergm.fit, print = FALSE)
## M-TERGM
g.net.mtergm.fit <- mtergm(g.net ~ edges + mutual, R = 500)
summary(g.net.mtergm.fit) # TERGM-M Results
```