Distributed Method to Optimal Profile Descent

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Abstract

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Current ground automation tools for Optimal Profile Descent (OPD) procedures utilize path stretching and speed profile change to maintain proper merging and spacing requirements at high traffic terminal area. However, low predictability of aircraft's vertical profile and path deviation during decent add uncertainty to computing estimated time of arrival, a key information that enables the ground control center to manage airspace traffic effectively. This paper uses an OPD procedure that is based on a constant flight path angle to increase the predictability of the vertical profile and defines an OPD optimization problem that uses both path stretching and speed profile change while largely maintaining the original OPD procedure. This problem minimizes the cumulative cost of performing OPD procedures for a group of aircraft by assigning a time cost function to each aircraft and a separation cost function to a pair of aircraft. The OPD optimization problem is then solved in a decentralized manner using dual decomposition techniques under inter-aircraft ADS-B mechanism. This method divides the optimization problem into more manageable sub-problems which are then distributed to the group of aircraft. Each aircraft solves its assigned sub-problem and communicate the solutions to other aircraft in an iterative process until an optimal solution is achieved thus decentralizing the computation of the optimization problem.

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Chapter 1

INTRODUCTION

Optimal Profile Descent (OPD) has been a subject of extensive flight simulations and demonstrations due to its potential benefits that include reductions in fuel consumption, CO_2 emission, flight time and airport community noise level during descent [1, 2]. Since Flight Management Systems (FMS) installed on most modern commercial aircraft is already capable of programming OPD procedures into flight plan and flying them using LNAV (Lateral Navigation) and VNAV (Vertical Navigation) functions for guidance control, many airports around the world have already implemented or are considering implementing it in near future. Tong *et al.* describe the development of OPD procedure based on a 2° Flight Path Angle (FPA) descent path with different Instrument Landing System (ILS) glideslope intercept altitudes at George Bush Intercontinental Airport in Houston, Texas [3]. At Louisville International Airport, a major hub of operations for UPS, Clarke *et al.* use OPD procedures to demonstrate significant fuel savings as well as noise reductions for night time flights [4, 5, 6]. Another case study conducted by Shresta *et al.* at Denver International Airport illustrate similar benefits of using OPD procedures [7]. These benefits are realized when an aircraft is allowed to fly a continuous descent from cruise altitude to end of de-

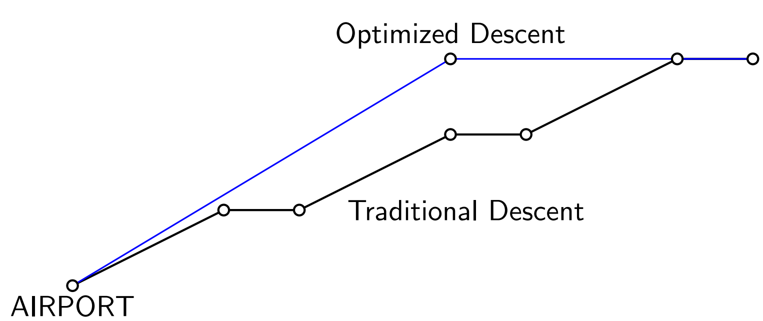

Figure 1.1: Traditional descent vs. OPD

scent thus eliminating inefficient constant altitudes called level segments that are typically associated with traditional step-down descents.

1.1 OPD Implementation

In general, a continuous descent is separated into two sub-descents. The first sub-descent is from cruise altitude to Metering Fix (MF) which serves as an entry point to Terminal Radar Approach Control (TRACON). The second sub-descent is from MF to Final Approach Fix (FAF) which typically indicates a starting point of final approach course. Airlines prefer to fly both segments of the descent at idle thrust to minimize fuel consumption and noise level. However, this can be challenging operationally under medium to high traffic conditions where each aircraft must meet the level of separation assurance at MF or FAF required by airport airspace restriction.

Low Predictability of Vertical Profile

The FPA of the descent path is derived from the following steady state energy equation for a point mass aircraft defined in [6]:

$$\tan\gamma = \frac{(T-D)}{W} - \frac{1}{g}\frac{d(V_g)}{dt} \tag{1.1}$$

Since each aircraft has a different level of idle thrust, it affects the FPA of the descent path which, in return, determines the location of Top of Descent (TOD) points. This makes it difficult for Air Traffic Controller (ATC) to reliably predict aircraft's vertical profile and compute estimated time of arrival (ETA), a crucial data required for accurate airspace traffic management [8]. Other aircraft specific parameters such as gross weight, descent speed profile, cruise altitude and predicted deicing band entry can further affect the TOD location. To increase the predictability of vertical profile during descent, a method of flying constant FPA has been suggested [3, 9, 10]. In order to maintain a constant FPA during descent, thrust needs to be set a little higher than idle. This extra thrust available during descent allows aircraft to control speed and to maintain the constant FPA path while maintaining most of the benefits that come from flying idle thrust descents. Since the constant FPA method increases the predictability of each aircraft's vertical profile, it allows the ATC to better manage separation assurance among aircraft using a ground automation tool such as Traffic Management Adviser (TMA) that manages time-based planning.

Path Deviation

ATC uses another ground tool such as En-route Descent Adviser (EDA) that recommends path stretching (path options) as a means to maintaining separation assurance when speed control alone is not enough [10, 11, 12]. During descent, when an aircraft receives a speed change request, it uses various control surfaces such as flaps and speed brake to manage the speed. However, when there is not enough control over speed, VNAV commands the aircraft to deviate from its idle vertical profile to achieve necessary acceleration/deceleration. In this situation, the aircraft no longer follows its predicted trajectory introducing further uncertainty to the predicted aircraft's vertical profile. To avoid OPD trajectory change, Cao *et al.* give a method of delaying an initiation of OPD procedure until the required airspace clearance is issued [13].

Centralized Method to OPD Optimization

For a group of aircraft trying to land at the same airport within a small time window, each aircraft has two main goals. First is to maintain individual OPD procedure which reflects the optimized descent path based on the aircraft's parameters such as thrust capability and gross weight. Second is to maintain separation assurance with other aircraft. Currently, Air Traffic Controller (ATC) has a responsibility of maintaining separation assurance among aircraft and issuing clearance for OPD procedure and path deviations to each aircraft. This centralized method can become inefficient and ineffective in near future when air traffic is expected to grow rapidly especially with the inclusion of unmanned aerial vehicle to the airspace. This problem is addressed by setting up cost functions and minimizing the total cost. Bertsimas *et al.* describe a method of assigning a time delay cost function to each aircraft using different cost weights [14]. Hoekstra *et al.* discusses a concept of Free Flight where the task of maintaining separation assurance is moved from ATC to each aircraft through a system called Airborne Dependent Surveillance Broadcast (ADS-B) [15].

1.2 OPD Optimization

In this paper, we define an OPD optimization problem for a group of aircraft by assigning time cost function to each aircraft and separation cost between a pair of aircraft. This optimization problem utilizes both path options and speed profile change on a constant FPA descent path. Since a small FPA change during descent is shown to exhibit a small penalty in fuel burn and flight time [10], we also allow the OPD trajectory to change in a systematic way to provide more delay capability and to account for unknown variables such as unexpected wind changes. . This method provides necessary speed and time control to maintain OPD procedure and highly predictable vertical profile. Such 3D path operational concept can be considered as a near-term enabler of Next Generation Air Transportation System (NextGen) for Federal Aviation Administration (FAA) [16]. However, we will not discuss how the path options for a group of aircraft will be formulated in a collision-free manner since the subject of conflict resolution for ATC has been well documented in other papers [17, 18, 19]. Next, we solve the optimization problem in a distributed manner by breaking the global problem into a set of more manageable sub-problems using dual decomposition techniques. This involves an iterative process during which each aircraft solves sub-problems using local variables for other aircraft. At each step of the process, the local variables are communicated to other aircraft and the process repeats until an agreeable convergence is achieved. Twu *et al.* use the dual decomposition method to solve the optimization problem of autonomous merging and spacing between two aircraft [20]. We take a similar approach but expand the problem to include any number of aircraft and consider each aircraft's OPD procedure.

Figure 1.2 gives a high level description of the paper. Chapter 2 gives an overview of the concept of Lagrangian Duality and dual decomposition method. In Chapter 3, we set up the OPD optimization problem and decompose it into sub-problems using the dual de-

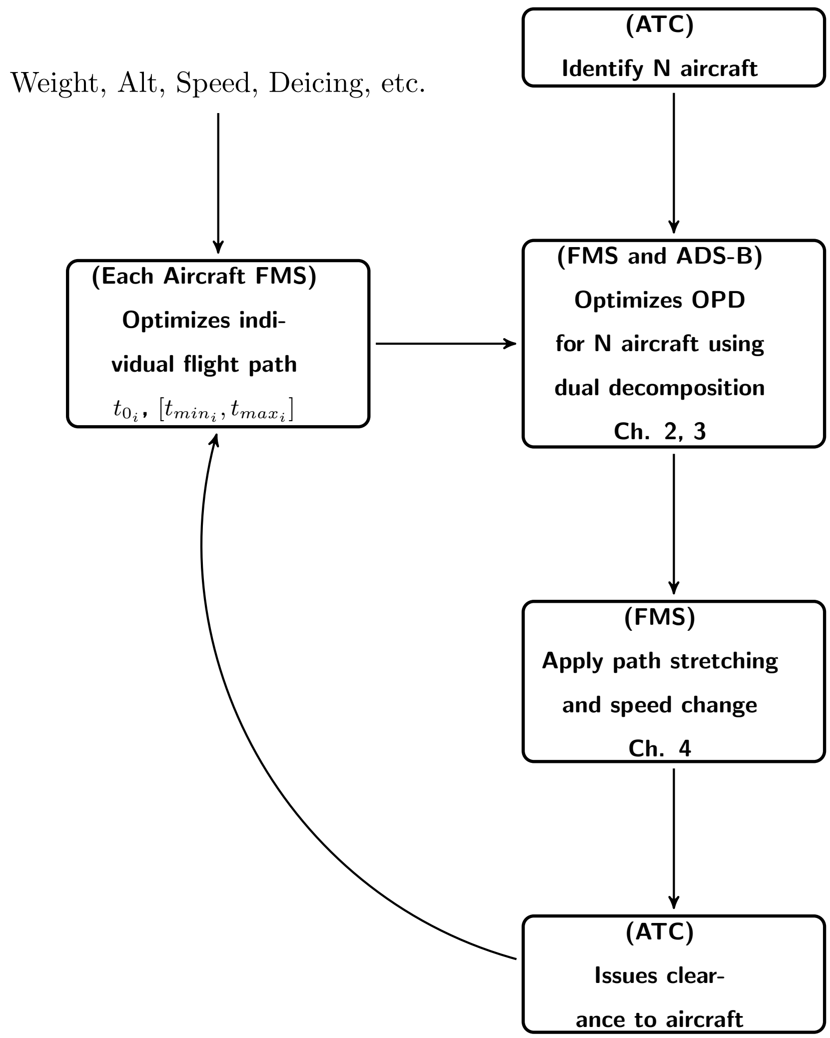

Figure 1.2: Process Overview

composition method. We also show that the problem is convex thus it is possible to achieve a global optimal value. In Chapter 4, we explain how the path options and speed change can be utilized to provide required time delay for separation assurance between a pair of aircraft. In Chapter 5, we present various simulations to demonstrate the validity of the algorithm. Then we conclude in Chapter 6 with future work.

Chapter 2

DUAL DECOMPOSITION

The main goal of this paper is 1) define an optimization problem for a group of aircraft flying different OPDs and 2) solve the optimization problem in a distributed manner using dual decomposition techniques. Once the cost functions for aircraft's OPDs are defined, the optimization problem is formulated and the concept of Lagrangian duality is used to give the best lower bound of the optimization solution. Moreover, the minimum solution of the optimization problem can be achieved under the condition of strong duality.

The theory of dual decomposition is an old but useful concept that allows a complex problem to be decomposed into more manageable sub-problems. These sub-problems can be solved concurrent by each aircraft without any intervention from ATC. There are also many papers and books that do an excellent job of explaining the concept of dual decomposition [21, 22, 23]. Since it is an indispensable tool for what this paper attempts to do, its basic concept will be described in this section along with the concept of Lagrangian Duality. This section borrows heavily from applicable sections of Chapter 4 and 5 of Boyd's book [21].

2.1 Optimization problem

Any optimization problem can simply be summarized as minimizing a cost function among all its variables. Consider a cost function $f_0(x) : R^n \rightarrow R$ and a variable $x \in R^n$. Its optimization problem can be stated as:

$$\min f_0(x) \quad \text{where } f_i(x) \leq 0,\ i = 1, ..., m \text{ and } h_i(x) = 0,\ i = 1..., p \tag{2.1}$$

Here, $f_i(x) : R^n \rightarrow R$ is a set of inequality constraints and $h_i(x) : R^n \rightarrow R$ is a set of equality constraints. When a x satisfies both constraints, x is said to be *feasible*. Among the feasible set, the optimal value is as follows:

$$p^* = \inf\{f_0(x) \mid f_i(x) \leq 0,\ i = 1, ..., m,\ h_i(x) = 0,\ i = 1, ..., p\} \tag{2.2}$$

The standard optimization problem (2.5) becomes a *convex* problem when $f_0, ..., f_m$ are convex and the equality constraint is restricted to an affine function $a_i^T x = b_i, i = 1, ..., p$. This convex problem definition will become useful later when strong duality is discussed.

2.2 Lagrange dual function

For the optimization problem (2.1), its *Lagrangian* is defined by adding the inequality and equality constraints to the optimization problem.

$$L(x, \lambda, \nu) = f_0(x) + \sum_{i=1}^{m} \lambda_i f_i(x) + \sum_{i=1}^{p} \nu_i h_i(x), \quad \lambda_i \geq 0 \tag{2.3}$$

λ_i and ν_i are called the *Lagrange multipliers* for the inequality and equality constraints respectively. The minimum value of the Lagrangian is called Lagrange dual function and is defined as:

$$g(\lambda, \nu) = \inf L(x, \lambda, \nu) = \inf \left(f_0(x) + \sum_{i=1}^{m} \lambda_i f_i(x) + \sum_{i=1}^{p} \nu_i h_i(x) \right) \tag{2.4}$$

If $\widetilde{x}$ is feasible for the optimization problem (2.1), then the summation of the inequality constraints $\sum_{i=1}^{m} \lambda_i f_i(\widetilde{x})$ must be non-positive since λ is non-negative, and the summation of the equality constraints $\sum_{i=0}^{p} \nu_i h_i(\widetilde{x})$ is zero. Then (2.3) becomes:

$$L(\widetilde{x}, \lambda, \nu) = f_0(\widetilde{x}) + \sum_{i=1}^{m} \lambda_i f_i(\widetilde{x}) + \sum_{i=0}^{p} \nu_i h_i(\widetilde{x}) \leq f_0(\widetilde{x}) \tag{2.5}$$

Now the following important relationship is established:

$$g(\lambda, \nu) = \inf L(x, \lambda, \nu) \leq L(\widetilde{x}, \lambda, \nu) \leq f_0(\widetilde{x}) \tag{2.6}$$

Using the definition from (2.5), (2.6) becomes:

$$g(\lambda, \nu) \leq p^* \tag{2.7}$$

Since (2.5) is true for any feasible point, (2.7) states that the Lagrangian dual function gives a lower bound on p^*.

2.3 Lagrange dual problem

Having computed the lower bound of the optimization problem, it naturally follows that $g(\lambda, \nu)$ should be maximized to get the best lower bound. This leads to another optimization problem called the Lagrange dual problem:

$$\max g(\lambda, \nu) \text{ where } \lambda \geq 0 \tag{2.8}$$

If the optimal value for (2.8) is denoted as d^* then (2.7) results in:

$$d^* \leq p^* \tag{2.9}$$

The next section describes the relationship between d^* and p^*.

2.4 Strong duality

The difference between p^* and d^* describes how close the best lower bound given by the Lagrange dual function is to the optimal value of the optimization problem. When the difference is zero, it is described as *strong duality* and it means that the optimal value can be obtained by solving the Lagrange dual function. On the other hand, when the difference is non-zero, it is described as *weak duality*. Among various conditions under which strong duality is guaranteed to exist, Slater's condition requires the following two:

1. $f_0(x), ..., f_m(x)$ are convex.

2. x is strictly feasible meaning $f_i(x) < 0,\ i = 1, ..., m$.

When these two conditions are met, Slater's condition guarantees strong duality.

2.5 Dual decomposition

Dual decomposition is a method of dividing the optimization problem (2.1) into multiple sub-problems and the optimal value of the optimizaton problem can be obtained by solving the sub-problems. Consider the following optimization problem:

$$\min f(x_1, x_2) = f_1(x_1, x_2) + f_2(x_1, x_2) \tag{2.10}$$

To decompose this problem, we introduce local variables for x_1 and x_2. Then (2.10) can be re-written as:

$$\min f(x_1, x_2) = f_1(x_1, y_2) + f_2(y_1, x_2) \quad \text{where } x_1 = y_1 \text{ and } x_2 = y_2 \tag{2.11}$$

f_1 uses y_2 which is a local variable of x_2. Likewise, f_2 uses y_1 which is a local variable of x_1. This is the optimization problem (2.1) without the inequality constraints.
The Lagrangian (2.3) can be reformulated to:

$$L(x_1, x_2, y_1, y_2, \nu_1, \nu_2) = f_1(x_1, y_2) + f_2(y_1, x_2) + \nu_1^T(x_1 - y_1) + \nu_2^T(x_2 - y_2) \tag{2.12}$$

Now the Lagrangian dual function (2.8) becomes:

$$g(\nu_1, \nu_2) = \inf \big(f_1(x_1, y_2) + f_2(y_1, x_2) + \nu_1^T(x_1 - y_1) + \nu_2^T(x_2 - y_2)\big) \tag{2.13}$$

This function can now be divided into two independent sub-problems:

$$\begin{cases} g_1(\nu_1, \nu_2) = \inf \big(f_1(x_1, y_2) + \nu_1^T x_1 - \nu_2^T y_2\big) \\ g_2(\nu_1, \nu_2) = \inf \big(f_2(y_1, x_2) - \nu_1^T y_1 + \nu_2^T x_2\big) \end{cases} \tag{2.14}$$

Finally, the Lagrange dual problem (2.8) can be reformulated:

$$\begin{cases} \max g_1(\nu_1, \nu_2) \\ \max g_2(\nu_1, \nu_2) \end{cases} \tag{2.15}$$

If both f_1 and f_2 are convex, then since there are no inequality constraints, Slater's condition states that the minimum value of f can be achieved by solving the Lagrange dual problems concurrently.

In the next section, an OPD optimization problem is defined for a group of aircraft and it will be shown how its Langrage dual problem has a property of strong duality. Then finally the optimal value will be solved in a distributed manner using dual decomposition.

Chapter 3

DISTRIBUTED METHOD TO OPTIMAL PROFILE DESCENT

Modern FMS is already capable of computing an economic vertical profile that optimizes flight mission based on numerous variables such as gross weight and speed profile. Under today's environment, the main goal of any commercial passenger aircraft would probably be saving fuel. This saving is realized when the aircraft does not deviate from its original estimated time of arrival (ETA) since the flight is already based on a trajectory that optimizes fuel saving. For a freighter airplane carrying time-sensitive shipment, its mission is to arrive on time. Twu *et al.* demonstrate how a quadratic cost function can be defined for an aircraft deviating from its original ETA [20]. We define a similar cost function as follows:

$$J_T(t) = k_T(t - t_0)^2 \tag{3.1}$$

t_0 is the aircraft's original ETA and k_T represents a cost weight for the cost function. Sometimes, a specified time of arrival (STA) may be issued to aircraft. For example, an airplane that makes a temporary stop at a connecting airport must arrive at the airport by its STA or it risks not meeting the schedule for the connecting flights. For this airplane, there should be a penalty for deviating from its STA. In this situation, t_0 becomes the aircraft's STA and k_T should a cost weight associated with the STA.

When each aircraft is considered individually, the optimal solution results in each aircraft meeting its original ETA or STA. As soon as the aircraft is forced to deviate from its ETA or STA, the penalty increases. Thus conflicts arise when there are multiple aircraft trying to land at the same airport within a small time window. For example, when aircraft i has an ETA which is equal to the STA of aircraft j, then there is a conflict which must be resolved in a way that results in the lowest overall cost for both aircraft. In addition, most airports require that aircraft maintain a minimum spacing requirement. A metering fix is usually considered as a point where the time separation is required. Thus, anther cost function must be introduced. This separation cost function becomes non-zero when there is not enough spacing between two aircraft. However, when they are sufficiently far apart then there should be no penalty. Between aircraft i and j, the cost function is defined as:

$$J_M(t_i, t_j) = \begin{cases} 0 & \text{for} \quad \mid t_i - t_j \mid \geq T_S \\ k_M(\mid t_i - t_j \mid - T_S)^2 & \text{for} \quad \mid t_i - t_j \mid < T_S \end{cases} \tag{3.2}$$

k_M is a cost weight for the separation cost function. Since the minimum spacing requirement is directly related to the safety of aircraft operation, it is usually set higher than k_T though it will be up to the aircraft operators and regulators to come up with this value.

3.1 OPD Optimization for two aircraft

Consider a situation near an airport where aircraft i is trying to meet its ETA and aircraft j is trying to meet its STA. If the notation J^i is used to indicate the cost function for airplane i, then the total cost functions are:

$$\begin{cases} J^i(t_i, t_j) = J_T(t_i) + \frac{1}{2} J_M(t_i, t_j) \\ J^j(t_j, t_i) = J_T(t_j) + \frac{1}{2} J_M(t_j, t_i) \end{cases} \tag{3.3}$$

Observe that the separation cost is shared between two aircraft and it is the same for both aircraft. Then, the total cost that arises when aircraft i and j are trying to land at the same airport is defined as:

$$J(t_i, t_j) = J^i(t_i, t_j) + J^j(t_j, t_i) = J_T(t_i) + J_T(t_j) + J_M(t_i, t_j) \tag{3.4}$$

The OPD optimization problem for two aircraft is defined as:

$$\min J(t_i, t_j) \tag{3.5}$$

To decompose the optimization problem (3.5), let's introduce local variables for t_i and t_j and their equality constraints. If the notation t_j^i denotes aircraft i's local variable for the arrival time t of aircraft j, then (3.3) can be reformulated to:

$$\begin{cases} J^i(t_i^i, t_j^i) = \frac{1}{2} J_T(t_i^i) + \frac{1}{2} J_T(t_j^i) + \frac{1}{2} J_M(t_i^i, t_j^i) \\ J^j(t_j^j, t_i^j) = \frac{1}{2} J_T(t_i^j) + \frac{1}{2} J_T(t_j^j) + \frac{1}{2} J_M(t_j^j, t_i^j) \end{cases} \tag{3.6}$$

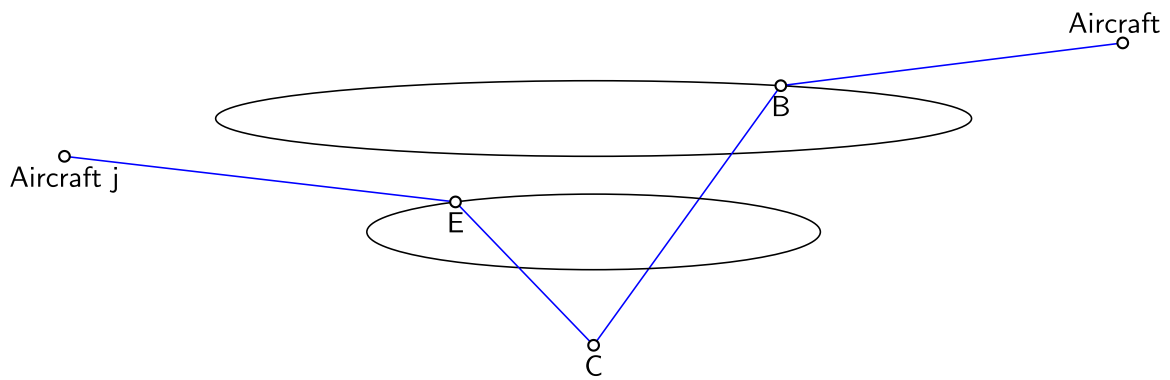

Figure 3.1: OPD for two aircraft

Since each aircraft is now able to calculate the other aircraft's time cost function, the time cost can also be shared at this time. The optimization problem (3.5) can be reformulated to:

$$\min J(t_i^i, t_j^i, t_i^j, t_j^j) = \min J^i(t_i^i, t_j^i) + J^j(t_j^j, t_i^j) \quad \text{where } t_i^i = t_i^j \text{ and } t_j^j = t_j^i \tag{3.7}$$

This is in the same form as (2.11) and its Lagrangian becomes:

$$L(t_i^i, t_j^i, t_j^j, t_j^j, \nu_{ij}, \nu_{ji}) = J^i(t_i^i, t_j^i) + J^j(t_j^j, t_i^j) + \nu_{ji}(t_i^i - t_i^j) + \nu_{ij}(t_j^j - t_j^i) \tag{3.8}$$

Now the Lagrangian dual function is defined as:

$$g(v_{ij}, v_{ji}) = \inf L(t_i^i, t_j^i, t_j^i, t_j^j, \nu_{ij}, \nu_{ji}) \tag{3.9}$$

And this can be divided into two sub-problems each aircraft can solve concurrently:

$$\begin{cases} g_i(v_{ij}, v_{ji}) = \inf \left(J^i(t_i^i, t_j^i) + \nu_{ji} t_i^i - \nu_{ij} t_j^i \right) \\ g_j(v_{ij}, v_{ji}) = \inf \left(J^j(t_j^j, t_i^j) - \nu_{ji} t_i^j + \nu_{ij} t_j^j \right) \end{cases} \tag{3.10}$$

Finally, the Lagrange dual problem can be defined as:

$$\begin{cases} \max g_i(v_{ij}, v_{ji}) \\ \max g_j(v_{ij}, v_{ji}) \end{cases} \tag{3.11}$$

Each Lagrange dual problem can also be solved concurrently. Since the cost functions are convex and satisfies Slater's constraint qualification [21], this dual decomposition method results in an optimal solution.

3.2 OPD Optimization for N airplanes

Now we can generalize the cost functions to accommodate N aircrafts. Once again, to use dual decomposition, each aircraft needs to have local values of other aircraft's arrival times. We use the notation t^i_j to indicate aircraft i's local variable t for aircraft j for $i, j = 1...N$. The time cost function for airplane i can be defined as:

$$J^i_T(t^i) = \frac{1}{N}\sum_{j=1}^{N} k_T(t^i_j - t_{0_j})^2 \tag{3.12}$$

For the separation cost, aircraft i needs to maintain a required separation from all other aircraft j where $j \neq i$. This introduces the following terms for the separation cost:

$$J^i_{M_1}(t^i) = \frac{1}{N}\sum_{\substack{j=1\\j\neq i}}^{N} \begin{cases} 0 & \text{for} \quad |\ t^i_i - t^i_j\ | \geq T_S \\ k_M(|\ t^i_i - t^i_j\ | - T_S)^2 & \text{for} \quad |\ t^i_i - t^i_j\ | < T_S \end{cases} \tag{3.13}$$

In addition, since aircraft i controls the local values of t for other aircraft, it needs to also make sure that they also maintain the required separation. This introduces the following extra terms for the separation cost:

$$J^i_{M_2}(t^i) = \frac{1}{N}\sum_{\substack{j=1\\j\neq i}}^{N}\sum_{\substack{k=j+1\\k\neq i}}^{N} \begin{cases} 0 & \text{for} \quad |\ t^i_j - t^i_k\ | \geq T_S \\ k_M(|\ t^i_j - t^i_k\ | - T_S)^2 & \text{for} \quad |\ t^i_j - t^i_k\ | < T_S \end{cases} \tag{3.14}$$

Putting the cost functions together, the total cost for aircraft i is defined as:

$$J^i(t^i) = J^i_T(t^i) + J^i_{M_1}(t^i) + J^i_{M_2}(t^i) \tag{3.15}$$

Then, the OPD minimization problem becomes:

$$\min \sum_{i=1}^{N} J^i(t^i) \quad \text{where } t^i_i = t^j_i \text{ and } i \neq j \tag{3.16}$$

The equality constraints explain that aircraft j's local value of t_i should be equal to the true value of t_i which is controlled by aircraft i. The Lagrangian is defined as:

$$L(t, \nu) = \sum_{i=1}^{N} J^i(t^i) + \sum_{i=1}^{N}\sum_{\substack{j=1\\i\neq j}}^{N} \nu_{ji}(t^i_i - t^j_i) \tag{3.17}$$

Observe that $J^i_M(t^i_j, t^i_k)$ is equal to $J^i_M(t^i_k, t^i_j)$ and (3.14) guarantees that the separation cost between aircraft j and k is not double counted. The Lagrangian dual function is defined as:

$$g(\nu) = \inf \left(\sum_{i=1}^{N} J^i(t^i) + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} \nu_{ji}(t^i_i - t^j_i) \right) \tag{3.18}$$

This function can now be divided into multiple independent sub-problems each aircraft can solve:

$$g_i(\nu) = \inf \left(J^i(t^i) + \sum_{j=1, i \neq j}^{N} (\nu_{ji} t^i_i - \nu_{ij} t^i_j) \right) \tag{3.19}$$

This can be solved using the gradient descent method. The Lagrangian dual problem is then:

$$\max g_i(\nu) \tag{3.20}$$

For the N aircraft problem, each Lagrange dual problem can also be solved concurrently. Since the cost functions are convex and satisfies Slater's constraint qualification, this dual decomposition method results in an optimal solution to the optimization problem (3.16).

Chapter 4

PATH OPTIONS

Using path options as a means of delaying aircraft has been discussed in [9, 10]. It usually occurs when speed change alone is not sufficient to to maintain separation assurance among aircraft at MF or FAF. Three separate points are required to specify a triangular path option in a FMS flight plan: maneuver start point, turn back point and maneuver end point. To

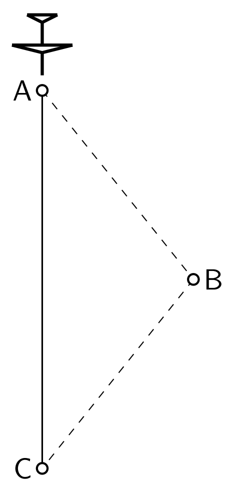

Figure 4.1: Triangular Path Option

illustrate the need for the path options, we consider the following scenario. While flying at a cruise altitude, an aircraft receives a new specified time of arrival (STA) at MF from the ground control center. Unfortunately, this STA is much later than the current estimated time of arrival (ETA) and the aircraft cannot meet the STA using only the speed control. Since the aircraft lacks necessary speed control to achieve its STA, it needs to use a path option to provide extra time delay using path options. At the maneuver start point, the aircraft starts deviating from its original path at a specified angle. Then it continues to deviate until it hits the turn back point where the aircraft starts flying towards the MF.

Since there are infinite ways of inserting extra time delay, the next two sections describe a systematic way of inserting path options in the FMS flight plan based on the location of the maneuver start point.

4.1 Maneuver Start Point before Top of Descent

FMS computes optimized descent path based on numerous variables such gross weight, cost index, wind/temperature forecasts, cruise altitude, speed and etc. In Figure 4.2, the arc represents a set of Top of Descent (TOD) locations where the FPA of the initial descent path can be maintained. For this method, the TOD of the initial descent path is located

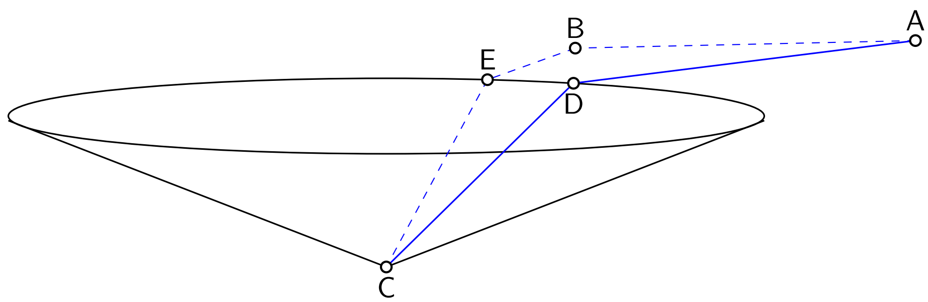

Figure 4.2: Maneuver Start Point before Top of Descent

at D and the distance between the turn back point and the maneuver end point is greater than the descent path distance. The turn back point can be inserted into FMS flight plan using the method of Place, Bearing, Distance (PDB) as described in [10]. The place would be a descent waypoint that is associated with C. The bearing (θ_{CRZ}) is the lateral deviation angle at C relative to the original path. And the distance is simply BC. The stretched path from A to B is defined by the law of cosine:

$$\begin{aligned} (AB)^2 =& (ADC)^2 + (BC)^2 - 2(ADC)(BC)\cos(\theta_{CRZ}) \\ & \text{where } (BC) \geq (CD) \end{aligned} \tag{4.1}$$

If V_{CRZ} is the aircraft ground speed during cruise, then the arrival time at TOD is given by:

$$\begin{aligned} T_{CRZ} =& T_{current} + \frac{(AB) + (BC) - (CD)}{V_A} \\ &\text{where } V_A \in [V_{min}, V_{max}] \end{aligned} \tag{4.2}$$

4.2 Maneuver Start Point After Top of Descent

When the aircraft follows its vertical profile, it starts descending at the TOD point. Figure 4.3 shows the descent path. Once the descent has started, any lateral or vertical deviation from the path will result in a FPA change. However, a small FPA change is shown to have a small penalty in fuel burn and flight time thus we will consider inserting a path option in descent using the same PDB method discussed above. The maneuver start point can be positioned anywhere in the descent but it is usually placed at an waypoint that already exists in the flight plan. Given the lateral deviation angle (θ_{DES}) relative to the original path at A, the new FPA after the lateral deviation is defined as:

$$\tan(\gamma_{new}) = \tan(\gamma_0)\cos(\theta_{DES}) \quad \text{where } \gamma_{new} \in (\gamma_{min}, \gamma_{max}) \tag{4.3}$$

The range of the FPA γ is specified by the individual characteristics of aircraft such as thrust and drag as defined (1.1). If V_{DES} is the aircraft ground speed during descent, then

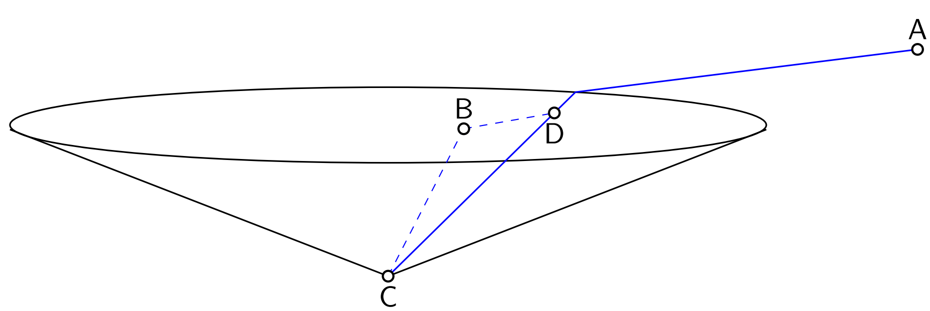

Figure 4.3: Maneuver Start Point after Top of Descent

the arrival time at C is given by:

$$
\begin{aligned}
T_{DES} =& T_A + \frac{2(DBC)}{V_{DES} + \sqrt{V_{DES}^2 + 2(DBC)\dot{V}_{DES}}} \\
&\text{where } (DBC) = \frac{(DC)}{\cos(\theta_{DES})}
\end{aligned}
\tag{4.4}
$$

4.3 Multiple Path Options

It is possible to utilize both path options discussed above during the flight. Since the first method does not affect the FPA of the original OPD procedure, both path options can be inserted before the aircraft reaches point A in Figure 4.2. In this case, the time cost function (3.1) changes to:

$$
J_T(t) = k_{T_{CRZ}}(t_{CRZ} - t_{CRZ_0})^2 + k_{T_{DES}}(t_{DES} - t_{DES_0})^2 \tag{4.5}
$$

In fact, it is possible to add any number of path options and the cost for each path option can be controlled by the cost weight k_T.

Chapter 5

SIMULATIONS

To illustrate how the OPD optimization problem can be used to maintain a separation assurance, let's assume that there are eight aircraft of the same type entering an airspace that imposes a separation assurance T_S of two time unit. Since all aircraft are of the same type, the ground command center gives each aircraft a time cost weight k_T of one. It also assigns an separation cost weight k_M of 100. The ratio of cost weights is calculated as:

$$k = \frac{k_M}{k_T} = 100 \tag{5.1}$$

If the aircraft's ETAs are more than two time units apart, then the problem becomes uninteresting. The aircraft just need to follow their original OPD procedures. However, consider a situation where their ETAs are exactly one time unit apart. Also assume that each aircraft is scheduled to arrive as early as possible meaning that they cannot get to the destination earlier than the current ETAs. While the time window is constrained from below, each aircraft has necessary time control to meet any time delay requirements via path options and speed change.

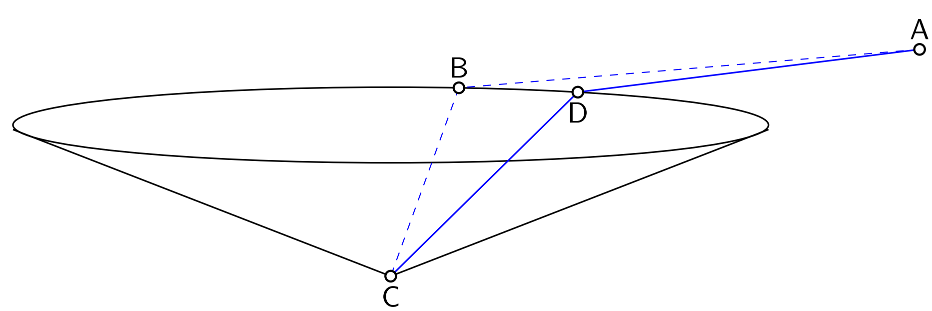

Figure 5.1: Path Option for Simulation

	Aircraft							
	1	2	3	4	5	6	7	8
Original ETA	20	21	22	23	24	25	26	27
Earliest Time	20	21	22	23	24	25	26	27
Latest Time	31.42	32.42	33.42	34.42	35.42	36.42	37.42	38.42

Table 5.1: OPD Problem for Eight Aircraft with $T_S = 2$ and $k = 100$

5.1 Separation Assurance with Fixed k

In this simulation, we utilize a path option where each aircraft turns towards the destination right at the arc as displayed in Figure 5.1. Also the maximum lateral deviation at C relative to the original path θ_{CRZ} is set at 60°. As mentioned above, we will assume that all aircraft will deviate from the original fight path in a collision-free fashion. Table 5.1 shows the original ETA and time window for each aircraft. The nominal solution is expected to be a two time unit separation between each aircraft starting with the first aircraft arriving at 20. This solution makes sense when we are unable to perform cost comparisons for the OPD procedures. When the cost functions are available, we can begin to optimize the OPD procedures. In Table 5.2, the optimal solution shows that the total cost is minimized when the separation is slightly less than two time units between each aircraft. This means that,

ETA	Aircraft							
	1	2	3	4	5	6	7	8
Original	20	21	22	23	24	25	26	27
Nominal	20	22	24	26	28	30	32	34
Optimal	20	21.77	23.55	25.35	27.17	29.02	30.91	32.85

Table 5.2: Optimal OPD Solution for Eight Aircraft with $T_S = 2$ and $k = 100$

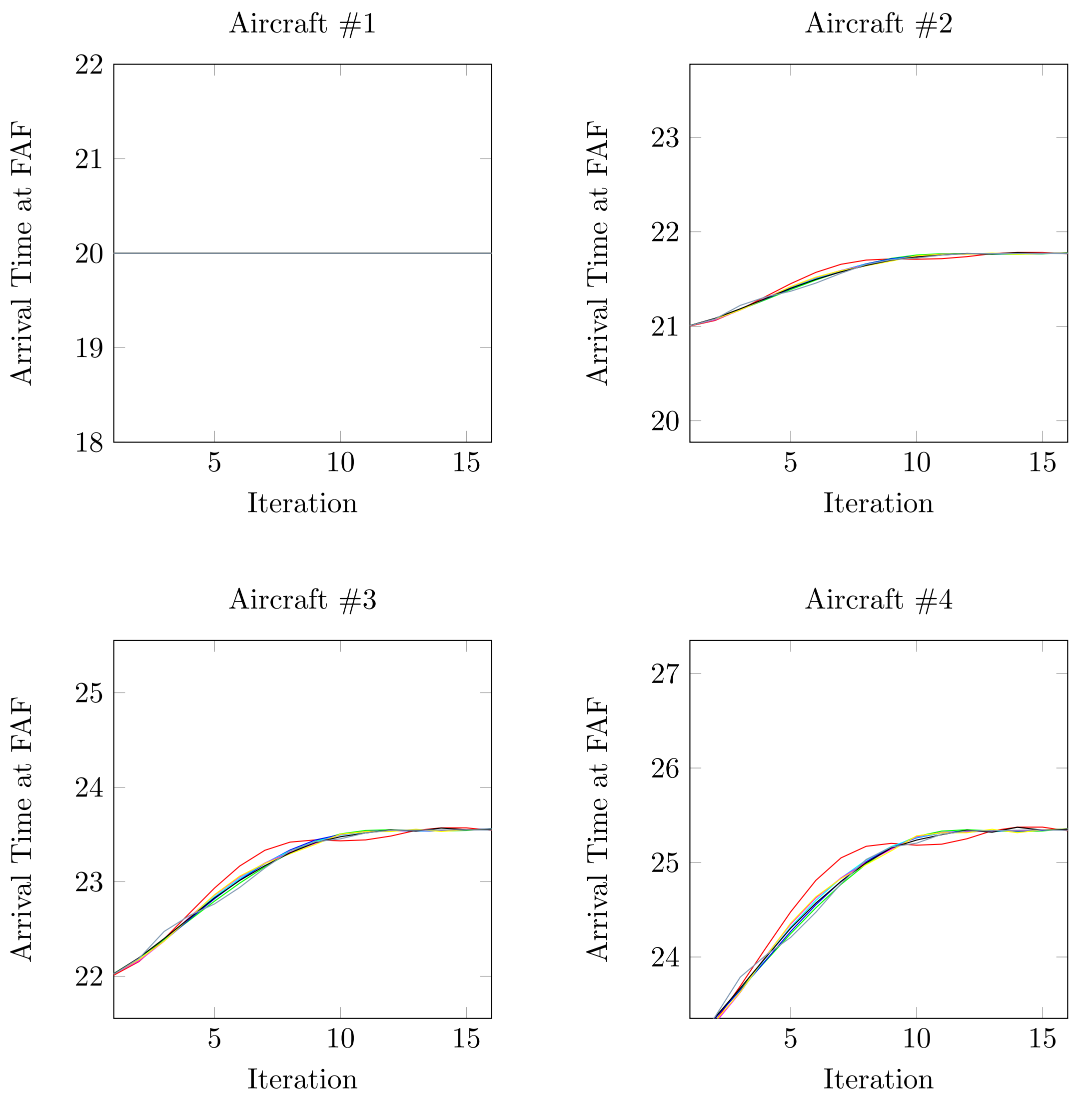

Figure 5.2: Sim 1 ETAs for Airplane 1, 2, 3 and 4

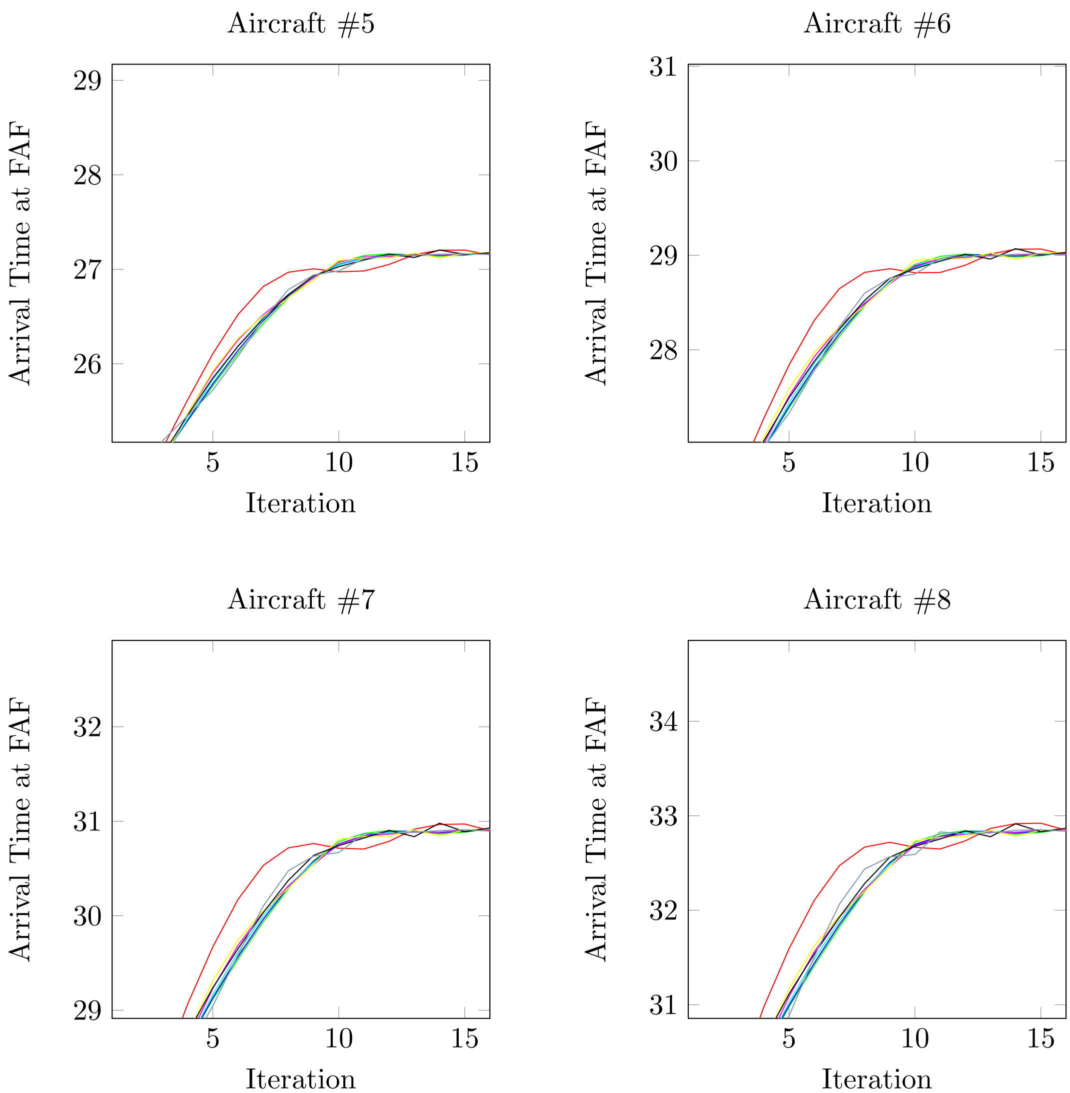

Figure 5.3: Sim 1 ETAs for Airplane 5, 6, 7 and 8

beyond this separation, the cost to deviate from the original ETAs becomes larger than the cost to maintain the separation assurance. The previous two pages display plots that show the computed optimal solutions at each iteration. On the figure, individual curve represents the solution computed by each aircraft. After a certain number of iterations, the solutions converge to an optimal value. The number of iterations is dependent on the level of accuracy for the convergence. If the resources for broadcasting aircraft parameters are expected to be expensive, then the level accuracy of the convergence will need to decrease to accommodate a smaller number of iterations. Recall that Lagrangian dual problem maximizes $g(\nu)$. Since the OPD optimization problem is convex and satisfies the conditions for strong duality, the final value of $g(\nu)$ should be equal to the minimum value of the optimization problem. Figure 5.4 shows the value of $g(\nu)$ at each iteration. When computed in a centralized

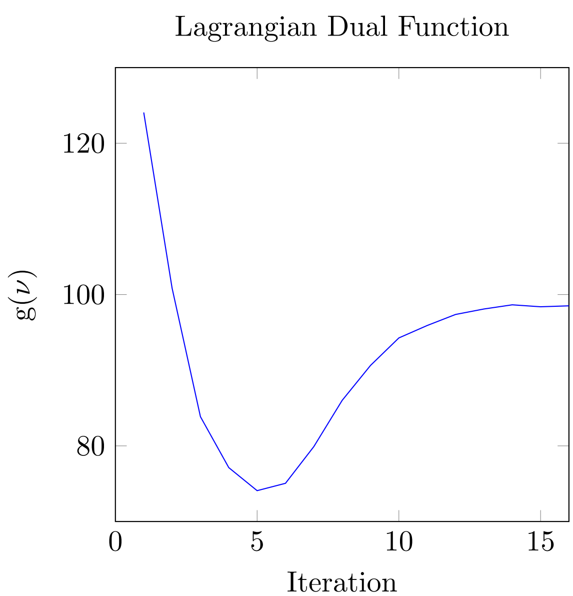

Figure 5.4: Lagrangian Dual Problem for Eight Aircraft

manner, the minimum value of the optimization problem becomes 93.52. The final value of $g(\nu)$ computed using the dual decomposition techniques is 93.51. The small difference can be explained by different exit criteria utilized in the subgradient method of each algorithm. Interestingly, the time separation of Aircraft 1 and Aircraft 2 is smaller than that of Aircraft

ETA	Aircraft							
	1	2	3	4	5	6	7	8
$k = 5$	20	21.07	22.16	23.28	24.64	25.73	27.14	28.79
$k = 50$	20	21.62	23.25	24.91	26.61	28.35	30.17	32.07
$k = 500$	20	21.95	23.90	25.85	27.81	29.77	31.75	33.73
$k = 5000$	20	21.99	23.99	25.98	27.98	29.98	31.97	33.97

Table 5.3: Optimal OPD Solution for Ten Aircraft with $T_S = 2$ and Various k

9 and Aircraft 10. Any time delay that incurs to Aircraft 2 also has an cost impact on all other aircraft that come after it. This explains why the difference between a pair of aircraft gets progressively larger. While this simulation result demonstrates variable time separations, there may be a situation in the current air traffic management where the separation assurance must be maintained at all cost. The algorithm is able to handle this by increasing the ratio of cost weights as the separation assurance becomes more important. Table 5.1 shows what happens to the optimal solution when k increases from 50 to 1000. As expected, the higher k is, the closer the optimal solution gets to the nominal solution. Consider now that the time window is no longer constrained from below allowing the aircraft

	Aircraft							
	1	2	3	4	5	6	7	8
Original ETA	28	29	30	31	32	33	34	35
Earliest Time	19	20	21	22	23	24	25	26
Latest Time	33.87	34.87	35.87	36.87	37.87	38.87	39.87	40.87
Optimal ETA	24.70	26.66	28.61	30.54	32.46	34.39	36.34	38.30

Table 5.4: Optimal OPD Solution for Eight Aircraft without Time Constraint

to slow down. In this situation, we lengthen the cruise portion of the flight so that aircraft would have more room to maneuver. This also moves up each original ETA by 8 time units. Table 5.4 shows the original ETAs, time windows and optimal solutions. The optimal solutions suggest that the aircraft ETAs will need to move out to both directions from the center point 30.5 resulting Aircraft 1 4 to speed up and Aircraft 5 8 to slow down.

5.2 *Aircraft with Time-Sensitive Cargo*

Using the scenario described in Table 5.4, assume that Aircraft 3 is now carrying time-sensitive cargo meaning that it now has a high priority of meeting its original ETA. The nominal solution would be to retain Aircraft 3's original ETA and push other aircraft out to meet the separation assurance. For this OPD optimization problem, we increase the time cost weight of Aircraft 3 to be equal to the separation cost weight. Table 5.5 shows that the optimal solution for Aircraft 3 is 29.90 which is a little lower than the original ETA of 30. More aircraft are arriving after Aircraft 3 thus it is better to speed up Aircraft 3 rather than moving Aircraft 4 8 out further. The minimum value of the OPD optimization problem is 49.01 which matches well with the Lagrangian dual problem, 49.04. The next two pages displays the convergence of optimal solutions for each aircraft.

Another interesting situation arises when there are more than one aircraft with time-sensitive cargo. Assume that Aircraft 4 is also carrying time-sensitive cargo and its time

ETA	Aircraft							
	1	2	3	4	5	6	7	8
Original	28	29	30	31	32	33	34	35
Nominal	26	28	30	32	34	36	38	40
Optimal	25.95	27.93	29.90	31.77	33.65	35.55	37.47	39.43

Table 5.5: Optimal OPD Solution for One Aircraft with Time-Sensitive Cargo

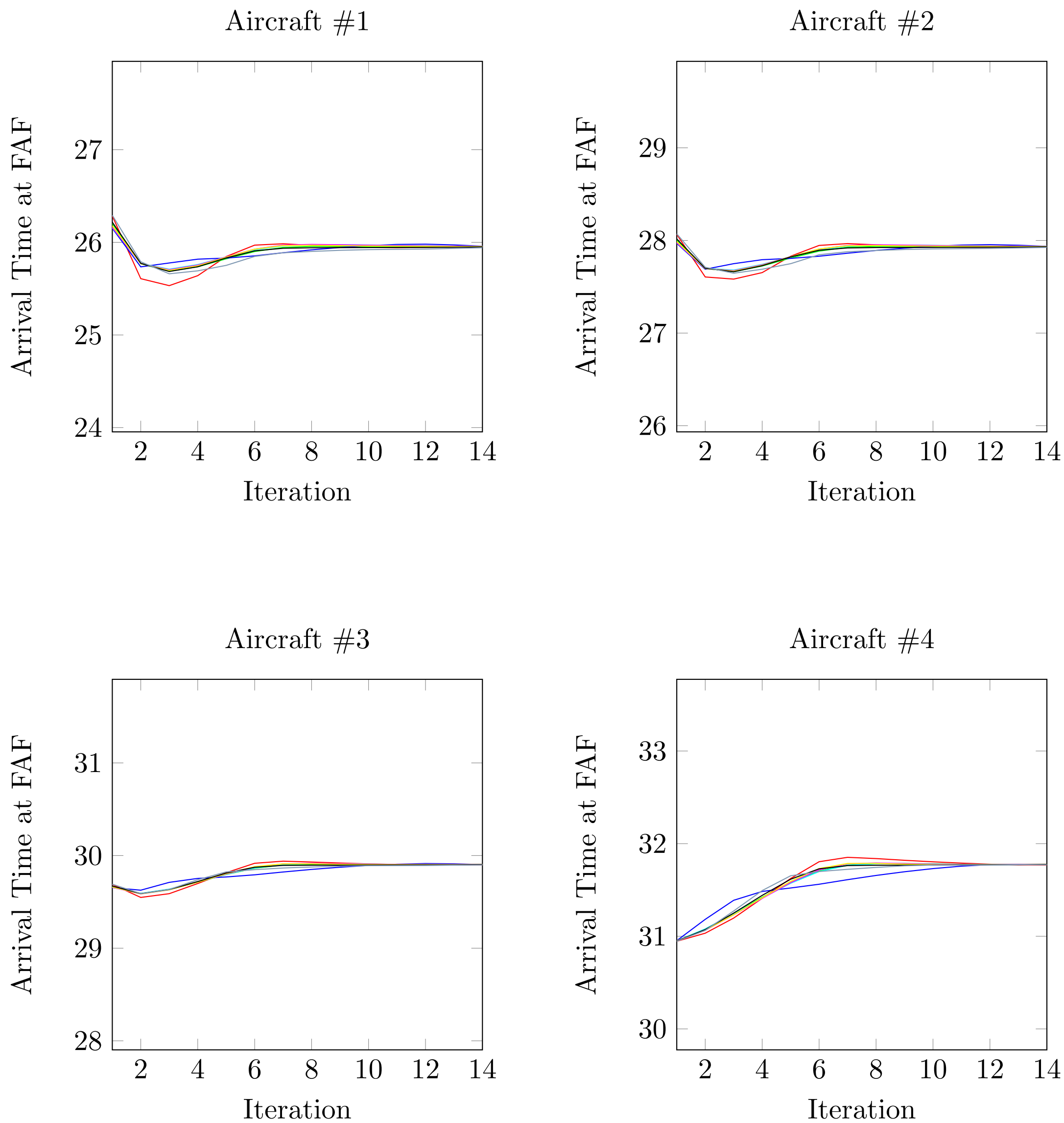

Figure 5.5: Sim 2 ETAs for Airplane 1, 2, 3 and 4

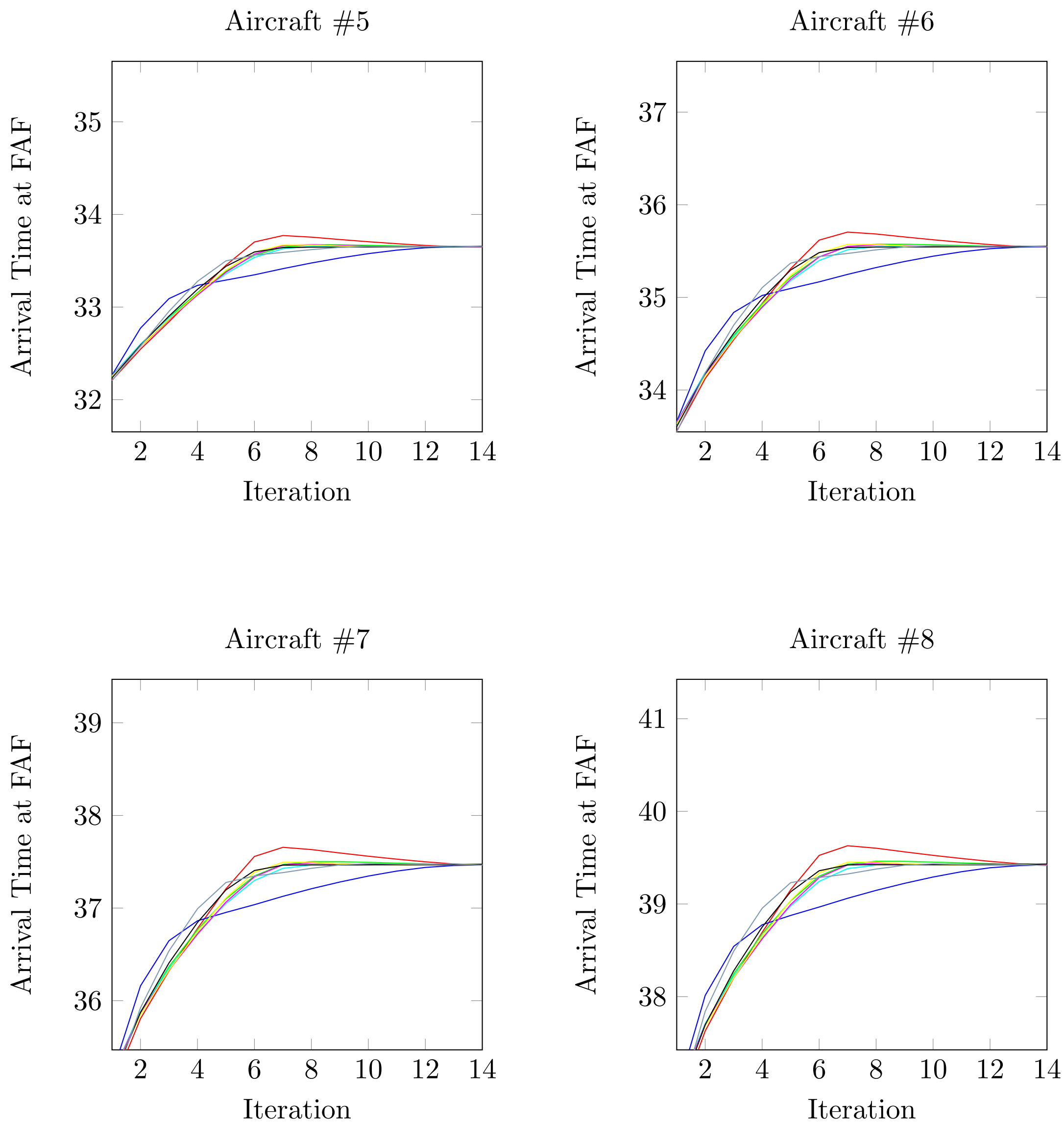

Figure 5.6: Sim 2 ETAs for Airplane 5, 6, 7 and 8

ETA	Aircraft							
	1	2	3	4	5	6	7	8
$k = 1$	25.71	27.69	29.66	31.28	33.17	35.08	37.01	38.97
$k = 0.1$	25.97	27.95	29.92	31.07	32.98	34.89	36.82	38.78
$k = 0.01$	26.04	28.02	29.99	31.01	32.91	34.83	36.76	38.73

Table 5.6: Optimal OPD Solution for Various k for Aircraft 3 and 4

cost weight becomes 100. In this situation, the nominal solution would be to move Aircraft 3 earlier and Aircraft 4 later but it is not very obvious how much they should move. Also considering that the ratio of cost weights k is 1 for these aircraft, the separation between them should not be as big as the separation for other pairs of aircraft. In fact, as k decreases, the separation should approach the difference of their original ETAs. Table 5.6 shows the optimal solutions for when k is 1, .1 and .01 for Aircraft 3 and 4. Figure 5.7 shows the $g(\nu)$ for the Lagrangian dual problem when k is .01.

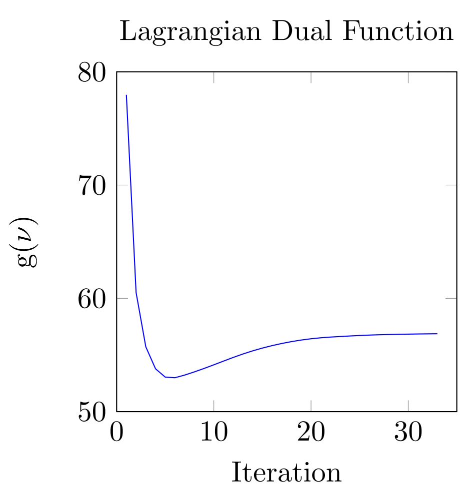

Figure 5.7: Lagrangian Dual Problem $g(\nu)$: $k = .01$ for Aircraft 3 and 4

Chapter 6

CONCLUSION

We have proposed an optimization problem that utilizes both path options and speed change to minimize the cost of flying an OPD procedure that uses a constant FPA descent path. This optimization problem assigns a time cost function to each aircraft and a separation cost function to a pair of aircraft and minimizes the cumulative cost of performing OPD procedures for a group of aircraft. It also provides high predictability of aircraft's vertical profile and necessary path deviation during decent to maintain merging and spacing requirements under high traffic condition. In addition, to decentralize the computation of an optimal solution, the problem is divided into multiple sub-problems and they are assigned to each aircraft. Then, using the dual decomposition method, an optimal solution to the OPD optimization problem is computed.

While this paper is concerned about flying OPD procedures near the terminal area, a similar method can be applied to different phases of flight for both manned and unmanned aircraft. For a group of aircraft trying to takeoff from the same airport within a small time window, an optimization problem can be defined to solve the takeoff time of each aircraft such that the total time delay could be minimized while controlling minimum takeoff separation time among aircraft. For the cruise flight phase, the problem can address the lateral separation assurance as well as vertical separation assurance. Since there are multiple choices for the cruise altitude, each aircraft needs to choose one that will result in an optimal result. In these different flight phases, the problem can also consider different cost functions that involve changes in trip distance, fuel usage and speed change to better reflect the needs of aircraft operators.

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