

Essays on the Issue of Piracy in the Market of
Information Goods

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Abstract

Essays on the Issue of Piracy in the Market of Information Goods

by

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Copyright infringement in markets of information goods—commonly known as *piracy*—has remained amongst the top concerns for the manufacturers as well as for governments around the world. There exists an extensive body of literature that discusses the issue of piracy along with possible remedies and responses for the manufacturers and policymakers. In my dissertation, I intend to contribute to that stream.

First, I study the economic impact of piracy on the supply chain of information goods. When information goods are sold to consumers via a retailer, in certain situations, a moderate dose of piracy seems to have a surprising positive impact on the profits of the manufacturer and the retailer, while, at the same time, enhancing consumer welfare. Clearly, such a “win-win-win” situation is not only good for the overall supply chain, but is also beneficial for the overall economy. I argue that the economic rationale for this surprising result is rooted

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in how piracy interacts with the problem of double marginalization. I explain this rationale and develop useful insights for management and policy.

Moving on to the supply-side of piracy—which has largely been abstracted away in prior literature—I compare the effects of different types of anti-piracy policies. I propose a clear distinction between efforts that restrict supply of pirated goods (*supply-side* enforcement) and those that penalize illegal consumption (*demand-side* enforcement). In my effort to compare these two different types of enforcement, I construct a parsimonious model that endogenizes the supply of pirated goods. Specifically, I capture the reality that download activities generate ad revenues for the uploaders in the piracy ecosystem, and it is that mechanism that incentivizes the suppliers of pirated goods to upload illegal content.

In my analyses, perhaps expectedly, I find the two types of anti-piracy measures to have similar impacts on the profit and welfares in the short-run, where the cost of development is sunk, and the decision of the firm is only about pricing the product. However, surprisingly, in the long-run—where the manufacturer can respond to piracy by changing its product quality as well—the two different enforcement measures may have contrasting socioeconomic implications. All in all, supply-side enforcement turns out to have a much more desirable impact in the long run. I provide explanations for my findings and discuss the implications from the perspectives of the manufacturer, consumers, and policymakers.

Keywords: Online piracy, supply chain, retailer, double marginalization, digital goods, anti-piracy measure, supply-side enforcement, demand-side enforcement, innovation, welfare.

To my dearest wife, my best friend, my greatest blessing, *Jenny Wu*

Curriculum Vitae

The author is an engineer by training, and he happily identifies himself as one. He received his Bachelor of Science from UC Davis and Master of Science from University of Michigan, Ann Arbor, in Computer Science and Engineering (CSE). Prior to joining the Foster School of Business at the University of Washington for the PhD program in Information Systems, the author was pursuing a PhD degree in CSE at the University of Michigan. He specialized in real-time embedded systems, and he was a member of Real-Time Computing Laboratory led by Professor Kang G. Shin. After learning about how technologies were “born,” the author quickly grew curious about their later stages of life, their uses in the real world, and their socioeconomic implications. Following—what later turned out to be—the logical next step, the author decided to pursue a degree in Information Systems, the interface between business and technology. During his PhD program in Information Systems, the author researched on the issues of copyright infringements and economic implications of anti-piracy efforts under the direction of professors Debabrata Dey and Atanu Lahiri. He is currently a lecturer of Master of Science in Information Systems program at the University of Washington, and the topics of his expertise are Information Security and Cloud Computing. As of July 2016, he will be joining the Kelley School of Business at Indiana University as an Assistant Professor.

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Frankly, I would not have even dreamed of going to grad school had it not been for my parents, Sangook and Junghee Kim. I am truly fortunate to have my parents as role models both in academia and in life. They have supported me wholly while I was going through the process of soul-searching, switching from one graduate program to another. In times when I feel that everything is lying on my shoulders, it is soothing to be reminded that I am

their son. I would also like to thank the other son of my parents, Michael Kim. I enjoyed my brother's company whenever he came to visit, and I tell everyone that he is getting his masters in Commercial Music Production and Sound Engineering at NYU. I actually have no idea what he does—or if that title is even remotely correct—but I get to showoff that there is someone cool in my gene pool, and I thank him for the bragging right.

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Contents

| | | |
|----------|---|-----------|
| 1 | Introduction | 1 |
| 2 | The Effect of Piracy on the Information-Goods Supply Chain | 5 |
| 2.1 | Introduction | 5 |
| 2.2 | Literature Review | 7 |
| 2.3 | Model | 12 |
| 2.3.1 | Consumer Behavior | 13 |
| 2.3.2 | Retailer's Problem | 14 |
| 2.3.3 | Manufacturer's Problem and the Equilibrium | 15 |
| 2.4 | Impacts of Piracy | 18 |
| 2.4.1 | Manufacturer's and Retailer's Profits | 18 |
| 2.4.2 | Price and Consumer Welfare | 22 |
| 2.5 | Piracy and Double Marginalization | 25 |
| 2.5.1 | Channel Profit and Social Welfare | 28 |
| 2.5.2 | Channel Efficiency | 30 |
| 2.6 | Conclusion | 37 |
| 3 | Combating Online Piracy: Short-Run Analysis | 40 |
| 3.1 | Introduction | 40 |
| 3.2 | Literature Review | 44 |
| 3.3 | Model Preliminaries | 45 |

| | | |
|----------|--|------------|
| 3.3.1 | Enforcement Environment | 46 |
| 3.3.2 | Demand-side Enforcement and Consumer Behavior | 47 |
| 3.3.3 | Supply-Side Enforcement and Behavior of Pirate Suppliers | 49 |
| 3.3.4 | Consumers' Access to Pirated Content | 50 |
| 3.3.5 | Consumer Demand | 53 |
| 3.3.6 | Supply Level at Subgame Equilibrium | 54 |
| 3.3.7 | Manufacturer's Decision Problem | 55 |
| 3.4 | Equilibrium Analysis | 58 |
| 3.5 | Results | 60 |
| 3.5.1 | Metrics | 61 |
| 3.5.2 | Comparative Statics | 63 |
| 3.6 | Discussion | 65 |
| 3.7 | Conclusion | 67 |
| 4 | Combating Online Piracy: Long-Run Analysis | 70 |
| 4.1 | Introduction | 70 |
| 4.2 | Equilibrium Analysis | 72 |
| 4.3 | Results | 76 |
| 4.3.1 | Summary and Comparison | 82 |
| 4.4 | Enforcement and Market Competition | 83 |
| 4.5 | Costs of Enforcement | 87 |
| 4.6 | Conclusion | 92 |
| 5 | Closing | 95 |
| | Bibliography | 98 |
| | Appendices | 103 |

| | |
|--|------------|
| A Appendix To Chapter 2 | 103 |
| A.1 Proofs | 103 |
| A.2 Examples of Retailing Configurations | 109 |
| B Appendix To Chapter 3 | 114 |
| B.1 Complete Analysis of the Equilibrium | 114 |
| B.2 Technical Results and Proofs | 123 |
| B.2.1 A Few Useful Observations | 123 |
| B.2.2 Proofs | 124 |
| C Appendix To Chapter 4 | 129 |
| C.1 Complete Analysis of the Equilibrium | 129 |
| C.2 Technical Results and Proofs | 136 |
| C.2.1 A Few Useful Observations | 136 |
| C.2.2 Technical Lemmas | 136 |
| C.2.3 Proofs | 137 |

List of Tables

| | | |
|-----|--|-----|
| 3.1 | Manufacturer's Optimization Problem under Different Strategies | 57 |
| 3.2 | Economic Impacts of Demand- and Supply-Side Enforcement (Short Run) | 66 |
| 4.1 | Economic Impacts of Demand- and Supply-Side Enforcement | 83 |
| A.1 | Structure of Retailing Configurations and Revenue Structures | 109 |
| A.2 | Summary of Results from Different Retailing Configurations With Nash Bargaining on p and w | 112 |
| C.1 | Boundaries Obtained from Profit Comparisons for $\beta < 1$ | 131 |
| C.2 | Boundaries Obtained from Profit Comparisons for $\beta \geq 1$ | 134 |

List of Figures

| | | |
|------|---|----|
| 1.1 | Thesis Overview | 3 |
| 2.1 | Regions of Piracy | 16 |
| 2.2 | Piracy Rate as a Function of Enforcement, for $\beta = 0.75$ | 19 |
| 2.3 | Manufacturer's and Retailer's Profits as Functions of Enforcement; $\beta=0.75$. | 20 |
| 2.4 | Equilibrium Retail Price as a Function of Enforcement; $\beta = 0.75$ | 22 |
| 2.5 | Consumer Surplus as a Function of Enforcement; $\beta=0.75$ | 24 |
| 2.6 | Channel Profit (CP) and Social Welfare (SW) as Functions of Enforcement; $\beta=0.75$ | 29 |
| 2.7 | Regions of Piracy—Manufacturer-Retailer Chain (Dashed) vs. Integrated Firm (Solid) | 32 |
| 2.8 | Manufacturer-Retailer Setup vs. Vertically-Integrated Firm, Retail Prices for $\beta = 0.75$ | 33 |
| 2.9 | Wholesale Price, Retail Price, and Margin Ratios as Functions of Enforcement, for $\beta = 0.75$ | 34 |
| 2.10 | Channel Profit—Manufacturer-Retailer Chain vs. Integrated Firm; $\beta=0.75$. | 36 |
| 3.1 | Consumers Self-Select Based on Their Relative Benefits | 54 |
| 3.2 | Relevant Partitions of the (r, e) Space in the Short Run; $\xi = \frac{2}{3}$, $\theta = 10$ | 59 |
| 4.1 | Relevant Partitions of the (r, e) Space in the Long Run; $\xi = \frac{2}{3}$, $c=0.01$ | 73 |

| | | |
|-----|--|----|
| 4.2 | Equilibrium Price-to-Quality Ratio as a Function of e and r ($\xi = \frac{2}{3}$, $c = 0.01$) | 75 |
| 4.3 | Relevant Partitions of the (K_r, K_e) Space; $\xi = \frac{2}{3}$, $c = 0.01$ | 90 |

Chapter 1

Introduction

Copyright infringements are particularly conspicuous in markets for information goods, mainly because it is nearly costless to make and distribute illegal copies. Such infringements, commonly known as *piracy*, seem to be getting only more and more serious as a result of wider dispersion of broadband networks and the ongoing shift toward digital formats. Take the sound recording industry, for example. According to the International Federation of the Phonographic Industry (IFPI), the industry's revenue from digital formats continues to grow; it has reached US \$5.9 billion in 2013, nearly two-fifth of the total (IFPI 2014). At the same time, though, the damage incurred by piracy has also become increasingly severe. Siwek (2007) estimates that, as a consequence of widespread piracy plaguing this industry, the US economy loses about US \$12.5 billion in total output and more than 70,000 jobs annually. The software manufacturers groan in similar desperation. According to the Business Software Alliance (BSA), the lost revenue due to piracy has doubled since 2004 and has crossed the US \$60 billion mark in 2011 (BSA 2011). With the rapidly growing spread of broadband and mobile networks, essentially anything that can be digitally stored and logically rendered can be pirated and delivered quickly to anywhere with internet connection. In fact, today, digital piracy is a significant component of the global counterfeiting industry that is estimated at a whopping US \$600 billion annually, accounting for 5–7% of all global trade (Bitton 2012). The presence of rampant piracy is also evident from the abundance of

pirated contents online; Price (2011), for example, reports that over 99% of the files transferred through P2P networks and over 90% of the files available for download on cyberlocker sites—cloud-based file storage and distribution services—are copyrighted material.

To counter this growing problem of piracy, a lot of efforts has been expended globally, by many governments, by manufacturers of information goods, and by different business alliances and industry lobbies. Over the years, such efforts—I refer to them as anti-piracy measures or enforcement efforts—have mostly centered around bringing pirates to justice within the existing legal framework of a country. In 2003, for example, one of the biggest news items concerning online piracy was the thousands of lawsuits brought by the Recording Industry Association of America (RIAA) against illegal music downloaders (McBride and Smith 2008). Even though, after less than six years, the RIAA announced that it would stop suing for illegal downloading, some manufacturers are still trying to discourage piracy by prosecuting illegal users (Liebelson 2014). In addition, there has been a marked increase in the number of lawsuits brought directly by many governments around the globe (de Beer and Clemmer 2009).

Piracy enforcement approaches have also evolved. Intelligent monitoring of online activities and appropriate deterrent steps against offenders have been employed all over the globe (de Beer and Clemmer 2009). In more recent times, governments have also started scanning for sites that distribute, or aid in the distribution of, pirated content. Often, governments have gone after these sites forcefully, shutting them down and prosecuting them. The most prominent example, of course, is the recent shutdown of MegaUpload.com. Even when they have been spared from being brought down completely, such pirate sites have faced significant downtime and uphill legal battles (Masnick 2012).

In this rather dynamic reality of piracy and anti-piracy efforts, there have been many studies analyzing the impact of piracy, discussing its intricate socioeconomic implications, and identifying actions that manufacturers—and governments alike—could take to combat

it. In my dissertation, I intend to contribute to that stream.

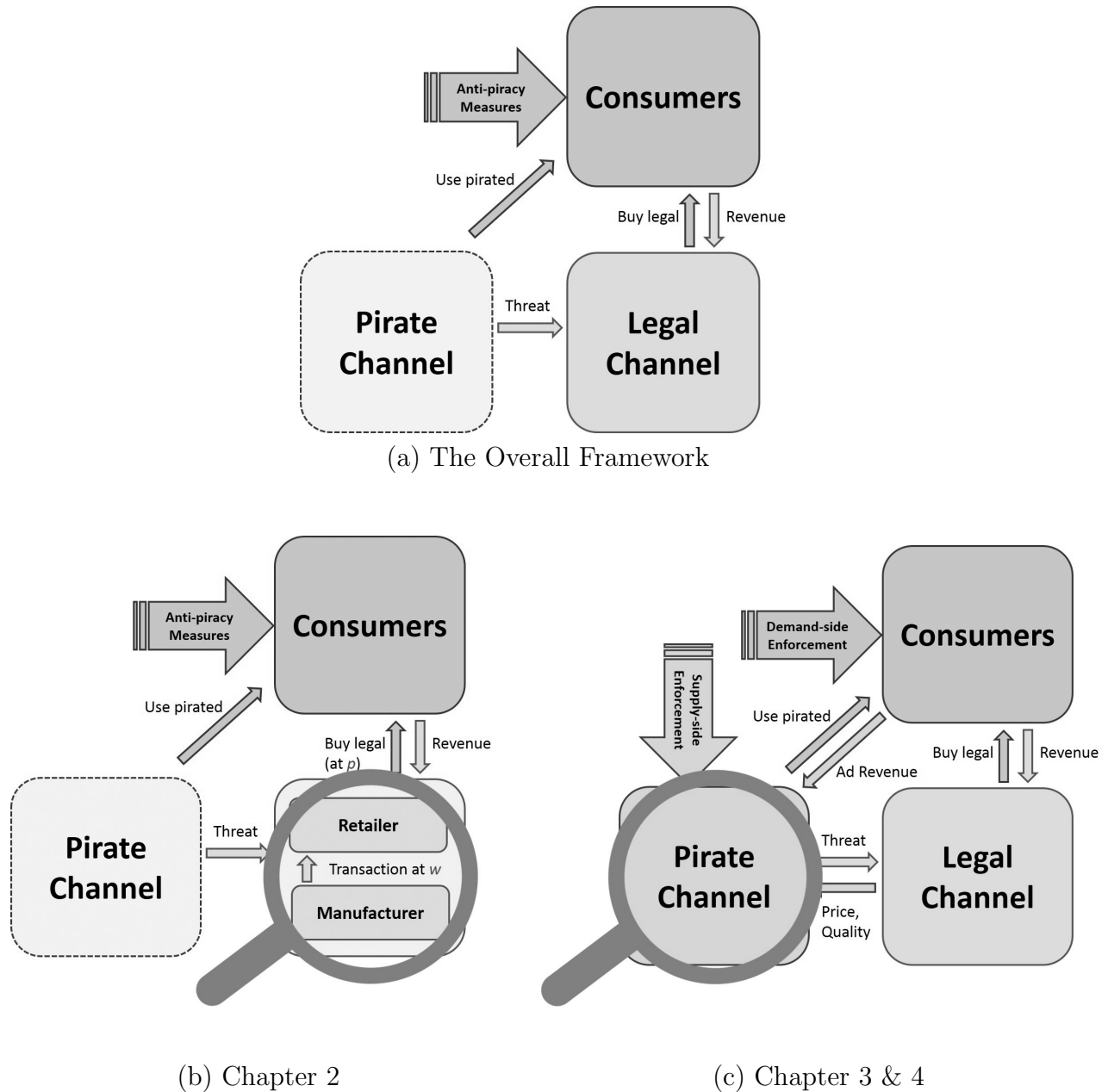


Figure 1.1: Thesis Overview

In my first study—Chapter 2—I examine the economic impact of piracy on the supply chain of information goods; See Figure 1.1(b). Specifically, I develop an economic model to describe a market where the manufacturer first sells to a retailer at a wholesale price, and the

retailer then resells to consumers. In such a setup, the legal channel has two menaces to deal with: the internal issue of channel coordination and the external issue of piracy. My interest is on the interaction of these two. The results show that both the manufacturer and retailer may be better off with a moderate dose of piracy than without. More surprisingly, perhaps, when piracy leads to a “win-win” situation for the manufacturer and the retailer, even the consumers come out ahead, indicating that piracy could further lead to a “win-win-win” situation.

Moving on to the supply-side of piracy—which has largely been abstracted away in prior literature—I look into the incentive behind providing pirated contents online “free of charge”; See Figure 1.1(c). Opening up that abstraction allows a comparison between legislative efforts that restrict the supply of pirated goods (*supply-side* enforcement) and those that penalize illegal consumption (*demand-side* enforcement). Chapter 3 is heavily dedicated to framing the problem, constructing the model, and solving for the short-run case where the product quality is fixed and exogenous to the model. I find that, in the short-run, the two types of enforcement appear to have similar effects on the metrics of interest such as piracy rate, profit, and welfares. So, perhaps, the two anti-piracy approaches are not so different in their eventual outcomes, and there is no reason to discriminate the two—or is there?

In Chapter 4, leveraging the framework developed in the previous chapter, I investigate to see what happens in the long run where the manufacturer can actually decide on—and change—the level of product quality in response to piracy. It turns out that, in the long run, the two anti-piracy approaches have a very different impact on the manufacturer’s incentive to innovate, resulting in disparate welfare implications. More specifically, supply-side enforcement turns out to have a much more desirable impact in the long run.

Chapter 2

The Effect of Piracy on the Information-Goods Supply Chain

2.1. Introduction

Drawn by the gravity of the issue of piracy, there have been many studies analyzing the toll of piracy and identifying actions a manufacturer could take to combat it. In a somewhat expected manner, most of the prior literature suggests that piracy limits the manufacturer's pricing power and, naturally, the manufacturer prefers stricter enforcement (Chen and Png 2003, Lahiri and Dey 2013). A notable exception in which piracy is thought to be a good news for the manufacturer is when the positive network effect is strong and illegal use contributes significantly to the value of the legal product (Conner and Rumelt 1991). The implications of piracy for consumer and social welfare have also been analyzed; in most cases, the findings are mixed. Often, stricter enforcement, although consistently found to be better for the manufacturer, is not necessarily so for its consumers and the society (Bae and Choi 2006, Lahiri and Dey 2013).

Notwithstanding its richness, the existing literature routinely makes one critical assumption—that the manufacturer sells *directly* to consumers. As we know, however, in many cases, information goods are actually sold through retailers, Amazon, Apple, and Barnes and Noble being some of the most prominent ones. Since a retailer may not have the same incentive to

respond to piracy as the manufacturer, it is not clear whether the results from prior research would automatically extend to these cases. This is precisely where I focus in this chapter.

Specifically, I consider a supply chain—the manufacturer first sells to the retailer at a wholesale price, and the retailer then resells to consumers at a retail price. Such a setup is indeed common in prior literature on supply chain and is also well studied in economics (cf. Tirole 1992, p. 175), though much less so in the literature on piracy. More importantly, it has also been the most widely used among online retailers and is still quite dominant in mature industries such as music (Abhishek et al. 2013). This retailing arrangement has been identified as one of the most prominent distribution models for books, both printed and electronic (Filloux 2012). The popularity and longevity of this arrangement is rooted in the fact that it is both cheaper to implement and easier to manage compared to other contractual alternatives (Cachon and Lariviere 2005).

Although easy to implement, the retail setting above can also cause the incentive of the manufacturer to diverge from that of the retailer so that the two entities would no longer act as one. In other words, the legal channel has to deal with channel coordination on top of piracy. This is indeed an important issue, because the manufacturer no longer has its hands on an important lever in the battle against piracy, the final retail price, and neither can it expect the retailer to cooperate in that regard. Naturally, the following research questions emerge:

- How does a lack in channel coordination compound a manufacturer's response to piracy? Does it affect the manufacturer and retailer the same way?
- Is it still true that the manufacturer always prefers stricter enforcement? And, what should a retailer do?
- To what extent do the results of prior research remain applicable? What are the implications for consumer and social welfare?

Answering these questions are indeed important as they not only have important implications for manufacturers and retailers, but also impact public policy regarding piracy of digital goods.

I construct a parsimonious model to describe this market setting and seek answers to the questions above. My analyses lead to several interesting results. I find that piracy can have a markedly different impact on the manufacturer from what has been suggested previously—the manufacturer, surprisingly, can be better off when enforcement is weaker. At the same time, the retailer can gain, too, from piracy or a threat of piracy. Put differently, a lack of enforcement can lead to an unexpected “win-win” situation where both the manufacturer and retailer make higher profits. What is even more surprising, though, is the result that this gain in the channel profit need not come at the expense of consumers, and piracy may, in fact, lead to a “win-win-win” situation where consumers too enjoy a greater surplus.

To get to the bottom of these findings, I conduct an extensive comparison of my model with the setup commonly assumed in the literature on piracy. This comparison reveals a curious interplay between the twin problems in my setup, piracy and double marginalization. What I find quite fascinating is that piracy can actually reduce—or completely eliminate at times—double-marginalization. The lesson in short is that, once again, the issue of piracy is not all black-and-white as one might imagine it to be—when the legal channel has a retailer between the manufacturer and its consumers, piracy may become an unlikely device of channel coordination.

2.2. Literature Review

My work is at the intersection of two distinct, but broad, streams of literature: double marginalization and digital piracy. Interestingly, the issue of double marginalization has long been recognized by economists (Spengler 1950)—when a monopolistic manufacturer

sells through a monopolistic retailer, the *vertical externality* manifests itself as a higher retail price, a lower demand, and a reduced channel profit. In recent times, though, this issue has primarily been examined by a large number of scholars in the area of supply chain management. The resulting literature is so vast that it is impossible to do justice in the short literature review section of this work, and the interested reader is referred to the excellent surveys by Cachon (2003) and Höhn (2010). This literature typically considers a manufacturer-retailer chain similar to the one I consider here. Often, the retail price is assumed to be fixed and the demand, random. The primary decision variable is the order quantity that maximizes the retailer's expected profit. The general finding is that, when the manufacturer charges the retailer a per-unit wholesale price, the retailer orders a quantity that is inefficient when compared to what a manufacturer selling directly would do. Fortunately, the literature also shows that such inefficiencies can be mitigated, and completely eliminated in some cases, using mechanisms such as *buy backs*, *quantity flexibility contracts*, or *revenue sharing agreements* (Cachon 2003). The case of a price-setting retailer has also been considered, and it has been shown that contracts that coordinate the supply chain for a price-taking retailer may no longer do so for a price-setting one (Cachon 2003). The primary overlap between my work and this literature is simply that I too deal with inefficiencies that arise from decentralized decision-making in a supply chain of an information good.

Despite this facile similarity, however, my work is quite different from the traditional literature on the supply chain of physical goods. First and foremost, this work is concerned with information goods, making the issue of inventory and associated costs moot in my case. Naturally, we need not worry about the stochastic nature of the demand¹ or the optimal order quantity, essentially rendering coordinating mechanisms such as buy backs and quantity flexibility contracts irrelevant. Second, these coordinating mechanisms are not really the

¹Given the current level of anti-piracy enforcement, the demand faced by the retailer is fully determined by the price it charges.

focus of my work and, accordingly, I do not propose any contracts or devise any mechanisms for coordinating the supply chain. I simply show that an illegal activity, namely, piracy, which is supposed to be harmful, can surprisingly mitigate certain inefficiencies that arise from decentralized decision-making. I find that, when enforcement is low, piracy cannibalizes the legal demand acutely and takes a terrible toll on the legal channel's pricing power. On the other hand, when enforcement is high, piracy is too weak to meaningfully restrain the channel and mitigate its inefficiencies. This way, I explain why a supply chain, which is inefficient to begin with, prefers only a moderate dose of enforcement, a dose potent enough to reduce the legal channel's pricing power and mitigate double-marginalization, but modest enough at the same time to not cannibalize the channel excessively. Clearly, these insights are new, and they enrich the broader literature on channel inefficiencies.

Since the inefficiencies in a supply-chain with deterministic demand arise primarily out of the lack of competition (Tirole 1992, p. 175)—and since piracy can indeed be viewed as a *shadow* competition (Lahiri and Dey 2013)—it may be tempting to think that piracy simply means more competition for the retailer, making my setting a special case of downstream competition. This impression is also not true. Even though the problem of double marginalization is most pronounced when a monopolistic manufacturer sells through a monopolistic retailer, as I explain later in this work, piracy works in a way that is fundamentally different from downstream competition, or any competition for that matter. Downstream competition makes the manufacturer better off by limiting the retailer's pricing power. Piracy, however, not only limits the retailer's power but also ties the same hand that feeds the manufacturer. This is precisely why greater competition among retailers is always better for the channel, whereas more piracy (or less enforcement) is so only up to a point. In summary, despite a vast body of scholarly work in the area of supply chain, my work identifies and addresses an important niche issue that has not been addressed before.

Let us now turn our attention to the literature on digital piracy. This literature is also

quite vast and, given the importance of this issue in today's digital world, has been growing quite rapidly. One stream of this literature challenges the common wisdom that piracy is always detrimental to the manufacturer. In fact, it identifies several situations in which a manufacturer may find it profitable to tolerate or support piracy. One such situation occurs in the presence of a positive network effect that translates illegal usage into a higher willingness-to-pay for the legal product (Conner and Rumelt 1991). Just as positive network effects alter the incentive to tolerate piracy, negative network effects do, too. For example, denying pirates critical security patches can be counterproductive when doing so makes legal users vulnerable to security attacks, lowering their willingness to pay (August and Tunca 2008). Finally, even when there are no network effects, anti-piracy measures that directly diminish the utility of the legal product can also create incentives to tolerate piracy (Vernik et al. 2011). This last paper is particularly relevant, as it also examines a setting in which retailers are present. Much like these works, I too discover positive effects of piracy on a manufacturer. However, my context is substantially different. First, in my case, there is no impact of piracy whatsoever on the utility of the legal product—neither do I model network effects that indirectly impact the value of the legal product nor do I consider anti-piracy measures that directly lower the usage value. Interestingly, despite omitting such impacts, I find that piracy—or, equivalently, lower enforcement level—can favorably impact the legal channel's profit. Second, and perhaps more importantly, none of these papers mention double marginalization, an issue central to this work, let alone discuss the impact of piracy on it.

Just as a manufacturer may have incentives to tolerate piracy, a social planner may also have his own reasons to do the same. Bae and Choi (2006) and Novos and Waldman (1984) show that social welfare can actually increase as a result of piracy. Lahiri and Dey (2013) explain why piracy may unexpectedly lead to more innovation and better quality products, again leading to higher welfare. Chen and Png (2003) highlight the tension between private profits and social welfare—in the battle against piracy, the manufacturer prefers stricter

enforcement and higher prices, whereas the society, just the opposite. That the manufacturer prefers more enforcement in the absence of network effects should not come as a surprise, since piracy offers consumers an alternative and, in that sense, works as a “competitor” to the legal product. What is rather interesting in these papers is that the society as a whole wins when piracy restrains the monopolist manufacturer’s pricing power. Intriguingly, I show that, in the case of a information-goods supply chain, piracy may even lead to a “win-win-win” situation in which the manufacturer, retailer, and consumers all gain simultaneously. This, to the best of my knowledge, is a new finding, and it does augment the broader literature on welfare implications of piracy.

There are other related interesting works, as well. They all shed light on a manufacturer’s strategy in the presence of piracy. Chellappa and Shivendu (2005) show how the pirated version of a digital good may serve as its product sample, with important implications for its pricing and versioning decisions. Wu and Chen (2008) explain why a manufacturer may offer a lower quality version of its product to combat piracy. Sundararajan (2004) discusses a monopolist’s optimal nonlinear price schedule in the presence of piracy. Gopal and Gupta (2010) explains situations in which product bundling may serve as an antidote to piracy. Johar et al. (2012) show that the manufacturer can use the content-delivery speed as a strategic lever in its battle against piracy. Tunca and Wu (2013) show why various forms of piracy may work against one another, surprisingly helping the manufacturer. Kannan et al. (2015) point out why piracy can be a motivation to build buggy products. Jain (2008) finds that piracy reduces price competition in an oligopoly setting, which can lead to an increase in the profits. In a sense, piracy induces implicit collusion between the manufacturers. In this work, I find that piracy works similarly for manufacturer-retailer as well. Put differently, Jain (2008) looks at how piracy coordinates multiple legal channels, whereas I look at how piracy coordinates a single vertical manufacturer-retailer channel. As the reader may be starting to recognize, the stream of literature on piracy is truly vast, and it is simply beyond

my means to review all works. Nevertheless, my main contribution to this vast stream is that I study piracy in the backdrop of double-marginalization and explain why piracy and its threat can serve as an unlikely method of channel coordination in a manufacturer-retailer chain.

Finally, I would like to point out that there is a growing branch of information systems literature, which looks at the role of retailers of information goods. For example, in a recent work Abhishek et al. (2013) study whether online retailers (e-tailers) should use the agency-selling format versus the more conventional wholesale format. The authors show that the decision depends on the level of competition among e-tailers and how the electronic channel affects the traditional brick-and-mortar channels. Among earlier works, Chellappa and Shivendu (2003) compare the impact of piracy on digital supply chains under different contracts (fixed-fee vs. per-copy). They find that, because of a high fixed cost, a zero marginal cost, and an uncertainty in the market size, retailers prefer a fixed-fee contract where they pay a one-time licensing fee. Khouja and Park (2007) study the effect of piracy on a creator-manufacturer supply chain for digital experience goods. They show that the royalty system does not solve the double-marginalization problem and is thus suboptimal from a supply-chain perspective. Jeong et al. (2012) explore the impact of piracy on the profitability of a digital music supply chain under different contracting arrangements between a record label and a retailer. The authors find that piracy can only reduce the channel profit, contrary to key findings of this work.

2.3. Model

I consider a traditional wholesale model where the manufacturer first decides on the wholesale price, $w > 0$. Then, the retailer chooses the retail price, $p > 0$. Finally, consumers decide whether to buy, pirate, or forgo use. As is customary, while solving the game, I will traverse this time line backwards, starting with the consumers' decision.

I hasten to add that this traditional retail model is conceptually no different from a form of agency selling (Abhishek et al. 2013). In one form of agency selling, the manufacturer gets to decide the final price p , but it must pay the retailer a fixed fee of w for every unit sold through the retailer’s platform. Note that the decisions about w and p are still made in the same order and the resulting game is identical—albeit the manufacturer decides p now, and the retailer, w . As a result, all my findings extend to that setting as well, after switching the roles of the two parties involved.²

2.3.1 Consumer Behavior

I assume that consumers are heterogeneous in terms of their valuations for the product:

Assumption 2.1. *Consumers are indexed by their valuation, v , which is uniformly distributed over $[0, 1]$.*

Further, following prior literature (Sundararajan 2004), I assume that the pirated copy is of lesser quality compared to the legal version:

Assumption 2.2. *Consumer v gets a value of $v\beta$ from using the pirated version, where $\beta \in (0, 1)$ is the degradation factor.*

A consumer using an illegal copy potentially faces a legal penalty. The expected penalty, denoted r , is exogenous in my model—it essentially serves as a proxy for the level of enforcement against consumption of pirated goods (August and Tunca 2008, Lahiri and Dey 2013). Thus, a consumer can enjoy a utility of $(v - p)$ from purchasing the legal version, or $(v\beta - r)$ from a pirated copy.

A consumer buys the legal product if the following individual rationality (IR) and incentive

²As a thought exercise, I have considered many other different possible setups during my preliminary analysis. Though not central to this work, I have summarized my conceptualizations of different retail settings in Appendix A.2 should they be of interest to the readers.

compatibility (IC) constraints are satisfied:

$$v - p \geq 0 \Rightarrow v \geq p, \quad \text{and} \quad (\text{IR-Legal})$$

$$v - p \geq v\beta - r \Rightarrow v \geq \frac{p - r}{1 - \beta}. \quad (\text{IC-Legal})$$

Similarly, a consumer would choose to procure a pirated copy if the following IR and IC constraints are satisfied:

$$v\beta - r \geq 0 \Rightarrow v \geq \frac{r}{\beta}, \quad \text{and} \quad (\text{IR-Pirated})$$

$$v\beta - r > v - p \Rightarrow v < \frac{p - r}{1 - \beta}. \quad (\text{IC-Pirated})$$

Since $\frac{p-r}{1-\beta} > p$ holds only when $p > \frac{r}{\beta}$, the legal demand, $q(p)$, for $p \in (0, 1)$, can be written as:

$$q(p) = \begin{cases} 1 - \frac{p-r}{1-\beta}, & \text{if } p > \frac{r}{\beta}, \\ 1 - p, & \text{otherwise.} \end{cases} \quad (2.1)$$

2.3.2 Retailer's Problem

As apparent from (2.1), the legal demand depends on the retail price, p . Of course, the retailer would choose p in order to maximize its profit, $\pi_r(p) = (p - w)q(p)$. In a typical manufacturer-retailer setup, where the issue of piracy is absent, the price set by the retailer is expected to be strictly increasing in the wholesale price charged by the manufacturer. However, as it turns out, even this basic intuition does not hold any longer. The threat of piracy might be compelling enough for the retailer to hold the retail price at a fixed value of $\frac{r}{\beta}$ even as the wholesale price changes. I state this curious finding as my first result.

Lemma 2.1. *The optimal retail price for a given w , $p^*(w)$, is:*

$$p^*(w) = \begin{cases} \frac{1}{2}(1 - \beta + r + w), & \text{if } w > \frac{2r}{\beta} - 1 + \beta \left(1 - \frac{r}{\beta}\right), \\ \frac{r}{\beta}, & \text{if } \frac{2r}{\beta} - 1 \leq w \leq \frac{2r}{\beta} - 1 + \beta \left(1 - \frac{r}{\beta}\right), \\ \frac{w+1}{2}, & \text{otherwise.} \end{cases}$$

Lemma 2.1 fully characterizes how the retailer would react to a given wholesale price. Its reaction can be explained as follows. When w is high, the retailer must set p high accordingly to remain profitable. Specifically, the retailer finds it preferable to tolerate some piracy and set p above $\frac{r}{\beta}$. So, the optimal retail price is obtained by maximizing $(p - w) \left(1 - \frac{p-r}{1-\beta}\right)$; see (2.1). The first order condition immediately leads to the optimal price of $p^*(w) = \frac{1}{2}(1 - \beta + r + w)$. Exactly the opposite happens when w is small and the retailer is content with a price below $\frac{r}{\beta}$ —the optimal price of $\left(\frac{w+1}{2}\right)$ is then obtained from maximizing $(p - w)(1 - p)$. However, neither of the two solutions above is meaningful when w is moderate. In such a situation, $(p - w) \left(1 - \frac{p-r}{1-\beta}\right)$ is decreasing for all $p > \frac{r}{\beta}$, whereas $(p - w)(1 - p)$ is increasing for all $p < \frac{r}{\beta}$. Hence, it becomes optimal for the retailer to set the price to $\frac{r}{\beta}$, irrespective of the value of w .

2.3.3 Manufacturer's Problem and the Equilibrium

Since Lemma 2.1 provides the retailer's reaction function to the manufacturer's choice of w , the manufacturer, as the first mover in this game, anticipates this reaction and names the optimal wholesale price, w^* , accordingly, to maximize $\pi_m(w) = wq(p^*(w))$. Note that, irrespective of the values of r and β , the manufacturer can always force the retailer to a retail price of $\frac{r}{\beta}$ by picking a wholesale price in the range $\left[\frac{2r}{\beta} - 1, \frac{2r}{\beta} - 1 + \beta \left(1 - \frac{r}{\beta}\right)\right]$. Clearly, if the manufacturer wishes to do so, it will end up choosing the corner solution of $w^* = \frac{2r}{\beta} - 1 + \beta \left(1 - \frac{r}{\beta}\right)$.

Alternatively, the manufacturer may want to confine the retailer to one of the other two regions by choosing an interior solution obtained from the first order condition. Of course, the manufacturer would do so if and only if: (i) an appropriate interior solution exists, and (ii) the manufacturer's profit from that interior solution is more than the profit from the corner solution. This way, we can determine w^* and obtain the equilibrium retail price, $p^* = p^*(w^*)$:

Proposition 2.1 (Equilibrium). Let $\rho_1 = \frac{3\beta(1-\beta)}{4-3\beta}$ and $\rho_2 = \frac{\beta(6-4\beta+\sqrt{2\beta})}{4(2-\beta)}$. When $r < \rho_1$, both the manufacturer and retailer find it optimal to tolerate some level of piracy (“Piracy Region”). In this case, $w^* = \frac{1}{2}(1 - \beta + r)$ and $p^* = \frac{3}{4}(1 - \beta + r)$. On the other hand, when $\rho_1 \leq r \leq \rho_2$, there is no piracy, but the threat of piracy affects the pricing decisions (“Threat Region”). Here, $w^* = \frac{2r}{\beta} - 1 + \beta \left(1 - \frac{r}{\beta}\right)$ and $p^* = \frac{r}{\beta}$. Finally, when $r > \rho_2$, even the threat of piracy disappears, and the manufacturer and retailer both behave as if they are in a market not affected by piracy (“Benchmark Region”). In this case, $w^* = \frac{1}{2}$ and $p^* = \frac{3}{4}$.

Figure 2.1 illustrates the three regions delineated by Proposition 2.1; the boundaries of these regions, ρ_1 and ρ_2 are also clearly identified as functions of β . Evidently, when

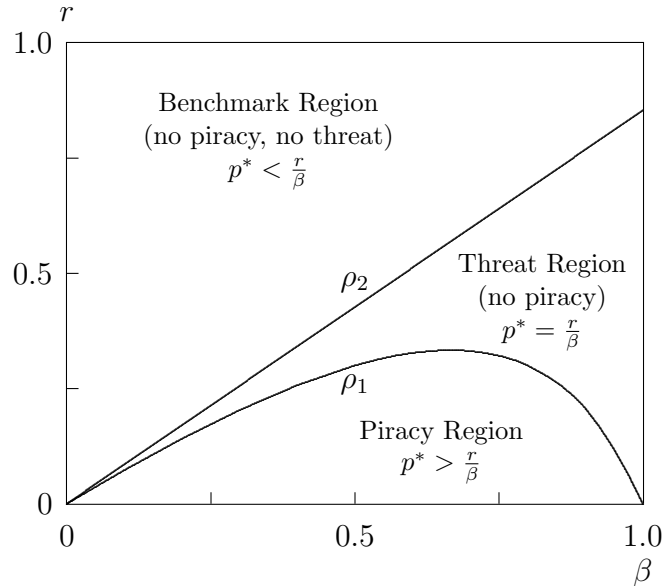


Figure 2.1: Regions of Piracy

enforcement is very high, above ρ_2 to be precise, we end up with $p^* < \frac{r}{\beta}$ in optimality, and neither piracy nor its threat has any effect on the equilibrium. As a result, in the region above ρ_2 , the equilibrium prices do not depend on r . In essence, this region is equivalent to a market that is not fraught with piracy in any manner. For this reason, I choose it as the *benchmark* for studying the impact of piracy on the supply chain.

As enforcement falls below ρ_2 , the issue of piracy becomes relevant. Specifically, in the region between ρ_1 and ρ_2 in Figure 2.1, the retail price is always $\frac{r}{\beta}$. This limit price barely keeps piracy away. Although piracy is not present in this region, the threat of piracy is evident from the fact that both p^* and w^* are now directly linked to r and β . Hence, this part of the parameter space is best termed as the *threat region*. The outcome in this region, $p^* = \frac{r}{\beta}$, is conceptually similar to the limit price in classical economics (Milgrom and Roberts 1982), where a limit price is used to discourage an entry of a potential competitor. Here, the retailer tries to stave off the “shadow” competition from piracy by naming a price at the limit level.

Finally, when enforcement becomes even weaker and r falls below ρ_1 , piracy surfaces— it is now impossible for the manufacturer, or the retailer, to weed out piracy completely. Accordingly, this region is named the *piracy region*.

Another interesting observation can be made from Figure 2.1. The relationship between ρ_1 and β is not monotonic, indicating that a higher-quality pirated product does not necessarily make piracy more likely. As expected, when β is very low, the pirated product is not quite attractive to consumers, and it takes only a little enforcement to wipe it out. As β increases, however, the pirated product becomes more appealing, requiring a higher level of enforcement. Finally, when β becomes much higher, the pirated product morphs into a near-perfect substitute for the legal version, inviting both the manufacturer and retailer to forcefully confront it through low prices. This strategic response from the legal channel causes the piracy region to narrow again.

A word is now in order on the real-world meaning of Proposition 2.1. The equilibrium regions identified by it are not just mathematical possibilities; they do provide important practical insights. For example, high prices have often been blamed for high piracy rates in many emerging economies (Karaganis 2011), and it has been suggested that the legal channel should cut prices to combat piracy effectively. What Proposition 2.1 shows is that

this argument is not completely accurate. If enforcement is extremely weak, below ρ_1 in particular, the strategy of eradicating piracy through lower prices is simply futile. There is no way a manufacturer or retailer can out-compete in price an alternative that is nearly free. However, doing so becomes not only viable but also optimal when the expected legal cost of the pirated alternative increases beyond a point; this is actually the essence of the threat region, where the pirated good, though forsaken by consumers for all practical purposes, still remains in the hands of a few and continues to pose a credible threat of shadow competition to the legal channel.

2.4. Impacts of Piracy

I now carefully analyze the equilibrium outcome in the previous section to investigate the impacts of piracy on the incentives of the manufacturer, the retailer, and consumers. Before I delve into my main findings, as a backdrop, I present what happens to the piracy rate—a metric commonly used in various industry reports (e.g., BSA 2011)—as the level of enforcement changes. This rate, denoted μ , is simply the fraction of illegal demand over the total: $\mu = \frac{p-r}{1-\beta} \frac{r}{\beta}$. As we can see in Figure 2.2, μ decreases in r and flat-lines as r goes beyond ρ_1 , exiting the piracy region. I formally state this intuitive result as the next lemma:

Lemma 2.2. *The level of piracy decreases in the level of enforcement, that is, $\frac{\partial \mu}{\partial r} < 0, \forall r < \rho_1$.*

2.4.1 Manufacturer’s and Retailer’s Profits

Now, I examine whether a reduced level of piracy increases the profits for the manufacturer and retailer. According to prior research, lower enforcement and higher piracy should lead to lower profits (Bae and Choi 2006, Lahiri and Dey 2013). The only prominent case, in which piracy is considered favorable from a profit perspective, is when the positive network effect is sufficiently strong (Conner and Rumelt 1991). In the presence of a strong positive network

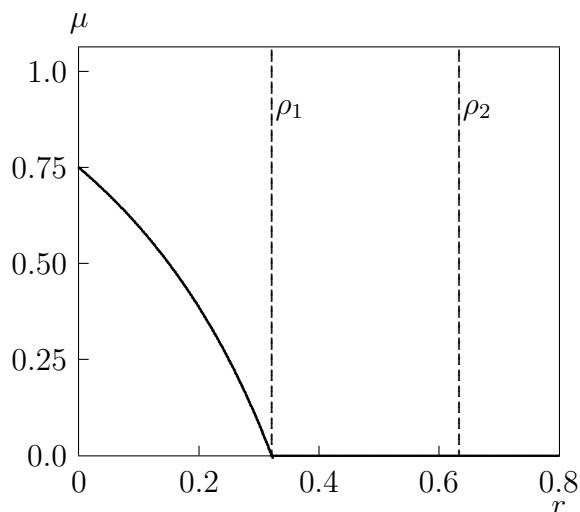


Figure 2.2: Piracy Rate as a Function of Enforcement, for $\beta = 0.75$

effect, piracy means a larger user base and, consequently, a greater willingness-to-pay for the legal product. Absent any network effect, however, there is little to be gained from piracy in this regard, and higher enforcement can only mean weaker competition from the pirated product, yielding a higher profit. Does this simple insight from prior research still hold for the supply chain I consider here?

Recall that, when $r > \rho_2$, we are in the benchmark region where piracy is irrelevant and plays no role in the equilibrium. For convenience, I will henceforth denote the benchmark profits of the manufacturer and retailer by $\pi_{m0} = \frac{1}{8}$ and $\pi_{r0} = \frac{1}{16}$, respectively. The question of interest, therefore, is how the manufacturer's equilibrium profit in the piracy region, or in the threat region, compares to π_{m0} . Likewise, one may ask a similar question about the retailer's profit. Answering these questions is critical to understanding how piracy impacts the entire supply chain.

Proposition 2.2. *Let ρ_1 and ρ_2 be as above. In equilibrium, the manufacturer's and retailer's*

profits are respectively given by:

$$\pi_m^* = \begin{cases} \frac{(1-\beta+r)^2}{8(1-\beta)}, & \text{if } r < \rho_1, \\ \frac{(\beta-r)(r-(1-\beta)(\beta-r))}{\beta^2}, & \text{if } \rho_1 \leq r \leq \rho_2, \\ \pi_{m0} = \frac{1}{8}, & \text{otherwise,} \end{cases} \quad \text{and} \quad \pi_r^* = \begin{cases} \frac{(1-\beta+r)^2}{16(1-\beta)}, & \text{if } r < \rho_1, \\ \frac{(1-\beta)(\beta-r)^2}{\beta^2}, & \text{if } \rho_1 \leq r \leq \rho_2, \\ \pi_{r0} = \frac{1}{16}, & \text{otherwise.} \end{cases}$$

Figure 2.3 shows how the equilibrium profits change with enforcement, r . Clearly, as expected, these profits are not affected by r when $r > \rho_2$. A closer look at these profits, as

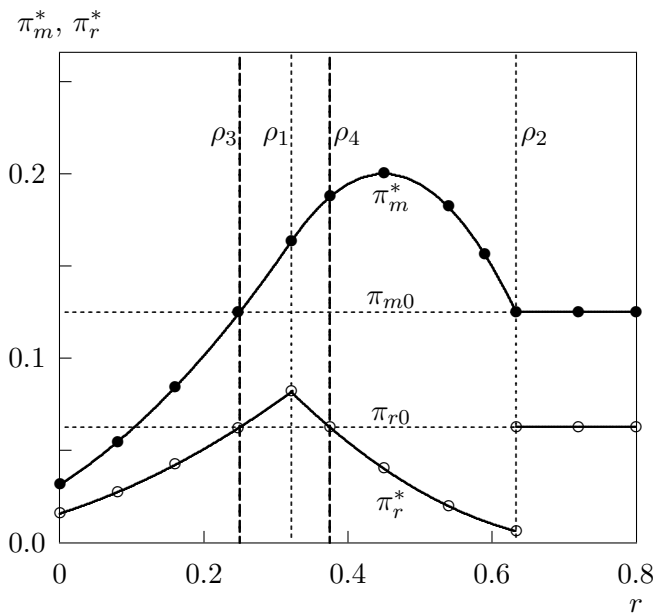


Figure 2.3: Manufacturer's and Retailer's Profits as Functions of Enforcement; $\beta=0.75$

well as a quick comparison with π_{m0} and π_{r0} , reveals several interesting insights. Somewhat counterintuitively, I find that the impacts of r on these profits are not monotonic—a higher level of enforcement does not necessarily add to the profits of the manufacturer and retailer. When the enforcement level is low and piracy is rampant, the conventional wisdom that the manufacturer's profit increases with r holds, but it need not at moderate or high levels of enforcement. In fact, two new thresholds, ρ_3 and ρ_4 emerge, and the parameter space separates into three new regions. When $r < \rho_3$, the manufacturer and retailer are both worse off in the presence of piracy. However, when $\rho_3 \leq r \leq \rho_4$, both are better off compared to

the benchmark case; note that $\rho_3 < \rho_4$ as long as $\beta < \frac{8}{9}$. Finally, when $\rho_4 < r < \rho_2$, only the manufacturer gains. I formalize these observations in the next result:

Theorem 2.1. *Let ρ_1 and ρ_2 be as above. Further, let $\rho_3 = \sqrt{1-\beta} - (1-\beta)$ and $\rho_4 = \beta - \frac{\beta}{4\sqrt{1-\beta}}$. Then, as long as $\beta < \frac{8}{9}$, $\rho_3 < \rho_4$. When $r < \rho_3$, the manufacturer and retailer are both worse off in the presence of piracy than without (“Lose-Lose Region”). When $\rho_3 \leq r \leq \rho_4$, the manufacturer and retailer are both better off (“Win-Win Region”). When $\rho_4 < r < \rho_2$, the manufacturer is better off, but not the retailer (“Win-Lose Region”).*

In Theorem 2.1, there is no “lose-win” region, where the retailer is better off but the manufacturer is not. This is expected since the manufacturer has a first-mover advantage in the sequential game. Perhaps, the real surprising part of Theorem 2.1 is the emergence of the “win-win” region, where the manufacturer and retailer both enjoy higher profits and, therefore, prefer the presence of piracy or its threat. Viewed differently, a moderate level of enforcement is preferable to both than a low—or, intriguingly, even a high—level of enforcement. A material implication is that, when situated in this region of moderate piracy, we cannot expect either the manufacturer or the retailer to complain too much about ill effects of piracy, nor should we anticipate them to lobby governments and other law-enforcing agencies to step up enforcement efforts. This result is in stark contrast with prior literature, which finds that piracy or its threat can only hurt the manufacturer (Bae and Choi 2006, Lahiri and Dey 2013). The manufacturer does benefit from piracy. And, this benefit is not a consequence of any network effect, neither is it at the expense of the retailer, who benefits as well.

Now, when the manufacturer or the retailer, or both, make a higher profit, does it come at the expense of consumer welfare? How should consumers react to higher levels of enforcement? I explore that next.

2.4.2 Price and Consumer Welfare

It turns out that, in my setup, the retail price paid by a consumer, p^* , can actually be lower in the presence of piracy (or its threat) for a significant part of the parameter space. Recall, from Proposition 2.1 that p^* can be expressed as:

$$p^* = \begin{cases} \frac{3}{4}(1 - \beta + r), & \text{if } r < \rho_1, \\ \frac{r}{\beta}, & \text{if } \rho_1 \leq r \leq \rho_2, \\ p_0 = \frac{3}{4}, & \text{otherwise.} \end{cases} \quad (2.2)$$

In order to see how this price changes with r , I plot it in Figure 2.4, and compare it with the benchmark price p_0 , as well as $\bar{p}_0 = \frac{1}{2}$, the pure monopoly price that would have been charged if the manufacturer sold the product directly to consumers. I find that, as long as

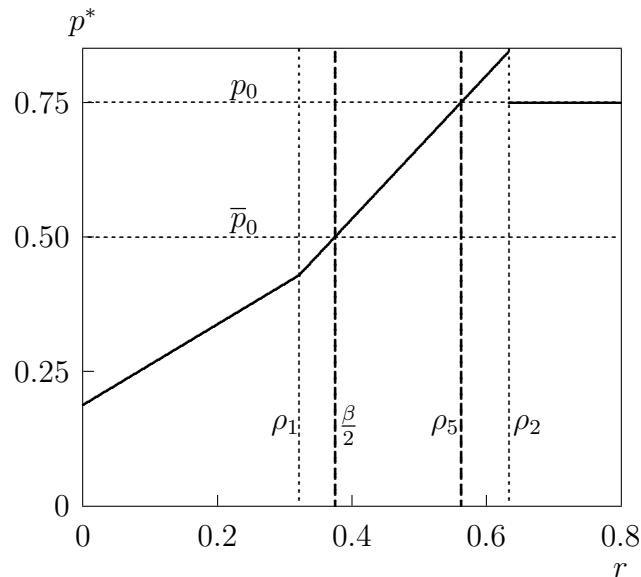


Figure 2.4: Equilibrium Retail Price as a Function of Enforcement; $\beta = 0.75$

the enforcement level is not very high—specifically, if $r < \rho_5 = \frac{3\beta}{4}$ —the retail price in the presence of piracy or its threat is lower than that without. This seems quite intuitive. After all, in the presence of piracy or its threat, the manufacturer and retailer are weakened by the competition from their own shadow. What is surprising is that there is a region—specifically,

$\rho_5 < r < \rho_2$ —where the retail price is, in fact, larger than p_0 , despite the threat of piracy; see Figure 2.4. The dynamics of the manufacturer-retailer relationship is quite interesting in this region. The manufacturer, being the first mover, squeezes the retailer by forcing it to hold p^* fixed at the limit price of $\frac{r}{\beta}$. The manufacturer relents only after the demand becomes heavily depressed as a result of the rapidly rising p^* , to an extent that it now starts to take its toll not just on the retailer but also on the manufacturer itself.

In order to understand, in more formal terms, how consumers may react to piracy, I now consider the surplus of the legal consumers. When consumer v purchases the legal product at a price p^* , he enjoys a net surplus of $(v - p^*)$. Therefore, the legal consumer surplus (CS) can be obtained by integrating $(v - p^*)$ over the set of all consumers buying the legal product:

$$CS = \begin{cases} \int_{\frac{p^*-r}{1-\beta}}^1 (v - p^*)dv, & \text{if } p^* > \frac{r}{\beta}, \\ \int_{p^*}^1 (v - p^*)dv, & \text{otherwise.} \end{cases}$$

Substituting p^* from (2.2), the above can be easily simplified. I obtain the following result.

Proposition 2.3. *The consumer surplus of legal consumers, CS , is given by:*

$$CS = \begin{cases} \frac{(1-\beta+r)((1-\beta)(6(\beta-r)+1)-r)}{32(1-\beta)^2}, & \text{if } r < \rho_1, \\ \frac{(\beta-r)^2}{2\beta^2}, & \text{if } \rho_1 \leq r \leq \rho_2, \\ CS_0 = \frac{1}{32}, & \text{otherwise.} \end{cases}$$

I plot this consumer surplus in Figure 2.5, for $\beta = 0.75$; for completeness, I also show the one including the illegal surplus as a dashed line. As expected, consumers are better off in the presence of piracy or its threat, as long as the enforcement level is not too high, that is, $r < \rho_5$. What is interesting here is that, even though the retail price is monotonic in r in the piracy and threat regions (see Figure 2.4), the consumer surplus of the legal users is not necessarily so. More specifically, the consumer surplus is increasing in r when piracy is rampant. This may seem counterintuitive at first—a higher level of piracy results in a

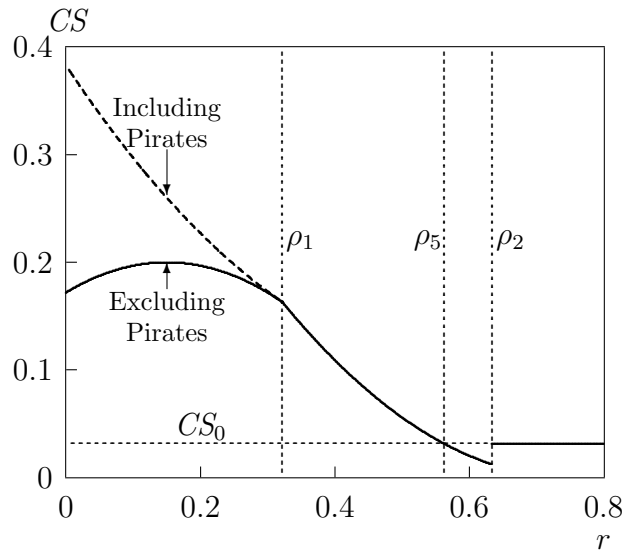


Figure 2.5: Consumer Surplus as a Function of Enforcement; $\beta=0.75$

lower price which actually should have augmented the consumer surplus. However, when the enforcement level is really low and the piracy rate high, the number of legal consumers is quite small. As r increases, the resulting increase in their number enhances the total consumer welfare, even though the per-consumer surplus shrinks somewhat. At even higher values of r , this increase in the legal consumer base is no longer sufficient to fully offset the shrinkage in the per-consumer surplus, so the total consumer surplus now starts decreasing.

Finally, it is easy to see that ρ_4 in Theorem 2.1 is always less than ρ_5 , implying that the region where consumers prefer piracy is much wider than where the manufacturer or the retailer prefers it. This leads us to the next important result.

Theorem 2.2. *Let ρ_3 and ρ_4 be as above. Then, the manufacturer, retailer, and consumers are all better off in the presence of piracy or its threat, provided $\rho_3 \leq r \leq \rho_4$.*

I conclude this section by recalling that, in calculating the consumer surplus in Proposition 2.3, the surplus generated through piracy and illegal use has been excluded. If we include this surplus, the consumer surplus would seem even higher in the piracy region—as shown by the dashed line in Figure 2.5—further strengthening the findings in Theorem 2.2.

Either way—with or without the illegal surplus—the results so far, taken together, seem to suggest that there exists an *invisible hand* of piracy; even when every player is acting in his or her own narrow self-interest—the manufacturer and retailer maximizing their profits, and consumers their own utility—somehow, the presence of piracy or its threat is making every selfish actor better off. This invisible hand surely begs for an economic explanation, which I present next.

2.5. Piracy and Double Marginalization

Before we can understand how piracy lends an invisible hand, we first need to understand the inefficiencies that exist in this supply chain in the absence of piracy and its threat. More importantly, we need to recognize the inefficiencies that are fundamentally rooted in the well-known issue of *double marginalization* or *vertical externality* (Tirole 1992, p.175).

When a monopolistic manufacturer sells directly to consumers, the issue of monopolistic inefficiency remains, but the issue of double marginalization is not relevant. However, when the supply chain has a retailer between the manufacturer and consumers, the incentives of the manufacturer and retailer—both monopolies in what they do—can, and do, diverge. It is this dissonance between the manufacturer and retailer that leads to the problem of double marginalization—in naming a wholesale price, the monopolistic manufacturer first decides on its *margin*, and then the monopolistic retailer adds its own *margin* to set a retail price. The net effect of this double marginalization is a retail price that is higher than the direct monopoly price; see p_0 and \bar{p}_0 in Figure 2.4. This higher price invariably shrinks the consumer base, resulting in reduced profits for both the manufacturer and retailer (Figure 2.3) as well as a slimmer consumer surplus (Figure 2.5). Absent any external forces or incentives, there is no way to coordinate the supply chain, since even a good-faith effort by one party would be immediately thwarted by the profit-maximizing action of the other, as neither party would want to leave a free penny on the table.

In a market for information goods, the issue of double marginalization is particularly important. For, each manufacturer acts like a local monopoly here. There is only one Microsoft Windows as a software, only one *Life is Beautiful* as a movie, and only one *Thriller* from Michael Jackson as a song. There may be other software products and other great movies or music, but they cannot be considered as substitutes for various reasons, the primary one being consumers' preferences for these specific goods. When these monopolistic manufacturers try to sell their zero-marginal cost information goods through retail channels, such as large online stores that are likely near-monopolies themselves, double marginalization surfaces, injecting inefficiencies into the market.

At the same time, information goods also suffer from the menace of piracy. Since these goods can often be shared costlessly, it is easy for consumers to locate pirated copies. Therefore, two major problems plague markets for information goods—piracy and double marginalization. What is fascinating here is that piracy or its threat can, to a degree, diminish the undesirable impact of double marginalization. At the core of this notion is the fact that double marginalization actually stems from the monopolistic nature of the market and a lack of competition at every stage of the supply chain (Tirole 1992, p.175). In contrast, when the market is competitive, its invisible hand would ensure that such inefficiencies are mitigated, resulting in socially desirable outcomes, to the liking of every constituency. What piracy provides is really a close proxy—it introduces a shadow competition for the manufacturer and the retailer, providing an alternative, a closely related version of the good to the consumer. It is in the existence of this shadow competition or the threat of its entry that, much like a competitive market, piracy helps in the alignment of the incentives of the two parties involved, reducing, or even eliminating altogether, the problem of double marginalization.

Is piracy then equivalent to competition, and is it just yet another tool to combat double marginalization? A closer look reveals that the underlying process through which piracy

curbs double marginalization is quite different from, for example, how downstream competition works. When retailers compete against one another, such competition predictably squeezes them by cutting their pricing power and, thus, always enhances the manufacturer's market position as a monopoly. The gain in channel efficiency is totally captured by the manufacturer and not shared with the retailers at all. Piracy, on the other hand, creates a shadow competition for the entire legal channel, and not just the retailers—every time a retailer loses a consumer to piracy, the manufacturer also suffers. The first order effect of this shadow competition is a simultaneous reduction in the pricing power of both parties. The second order effect is also simultaneous but, by nature's justice, works in the opposite direction—when the manufacturer responds to the competition by lowering its markup, inadvertently, it ends up helping the retailer too; likewise, when the retailer responds, the manufacturer also benefits. Interestingly, it turns out that, at moderate levels of enforcement, this second order effect can more than compensate for the first order effect, thereby increasing the profits of either one of them or of both. The resulting gain in channel efficiency can thus be beneficial not just to the manufacturer but to the retailer as well. At low levels of enforcement, however, the first order effect dominates, leading to lower profits. Indeed, as stated in Theorem 2.1, piracy, depending on its prevailing level, can result in a win-win, win-lose, or lose-lose situation.

As a word of caution, the results do not imply that the legal channel should, all of a sudden, start actively encouraging piracy. The implication is simply that, situated in a real-world context, the manufacturer and retailer should recognize that a certain level of piracy or its threat might actually be beneficial and should, therefore, exercise some moderation in their anti-piracy efforts. Finally, how should a central planner or a policymaker react in this situation? The invisible hand of a competitive market, after all, is supposed to bring about a socially desirable outcome (Smith 1776, p.477). Does piracy have a similar impact? In order to answer this and obtain a more complete picture, let us now investigate the impacts

on channel profit and social welfare.

2.5.1 Channel Profit and Social Welfare

The channel profit (CP) is simply the total profit generated by the manufacturer and retailer together, and social welfare (SW) is obtained by adding the consumer surplus to the channel profit. Since the individual profits and consumer surplus have already been estimated, we can find the channel profit and social welfare quite easily:

Corollary 2.1. *Let ρ_1 and ρ_2 be as above. In equilibrium, the channel profit (CP) and social welfare (SW) are respectively given by:*

$$CP = \begin{cases} \frac{3(1-\beta+r)^2}{16(1-\beta)}, & \text{if } r < \rho_1, \\ \frac{r(\beta-r)}{\beta^2}, & \text{if } \rho_1 \leq r \leq \rho_2, \\ CP_0 = \frac{3}{16}, & \text{otherwise,} \end{cases} \quad \text{and} \quad SW = \begin{cases} \frac{(1-\beta+r)(7(1-\beta)-r)}{32(1-\beta)^2}, & \text{if } r < \rho_1, \\ \frac{1}{2} \left(1 - \frac{r^2}{\beta^2}\right), & \text{if } \rho_1 \leq r \leq \rho_2, \\ SW_0 = \frac{7}{32}, & \text{otherwise.} \end{cases}$$

Comparing them with their benchmark values, I obtain the following important result:

Theorem 2.3. *Let ρ_3 and ρ_5 be as above, and let $\rho_6 = \max\{\rho_3, \frac{\beta}{4}\}$. Then, in the presence of piracy or its threat, the channel profit is higher for $\rho_6 \leq r \leq \rho_5$, and the social welfare is higher for $0 < r \leq \rho_5$, when compared to their respective benchmark values.*

Theorem 2.3 is better visualized in Figure 2.6, where I plot channel profit and social welfare as functions of the enforcement level, for $\beta = 0.75$. Theorem 2.3 and Figure 2.6 clearly show that, over a significant portion of the parameters space, the supply chain and the entire society perform better in the presence of piracy or its threat than without, irrespective of whether or not the pirates' surplus is included in the analysis. Such results could give a policymaker a reason for a momentary pause, perhaps to ponder whether to tolerate some piracy and exercise moderation when stepping up enforcement.

At the same time, we must also recognize that, although piracy injects a proxy competition in the market, it has its own obvious downsides. After all, it is an illegal activity and

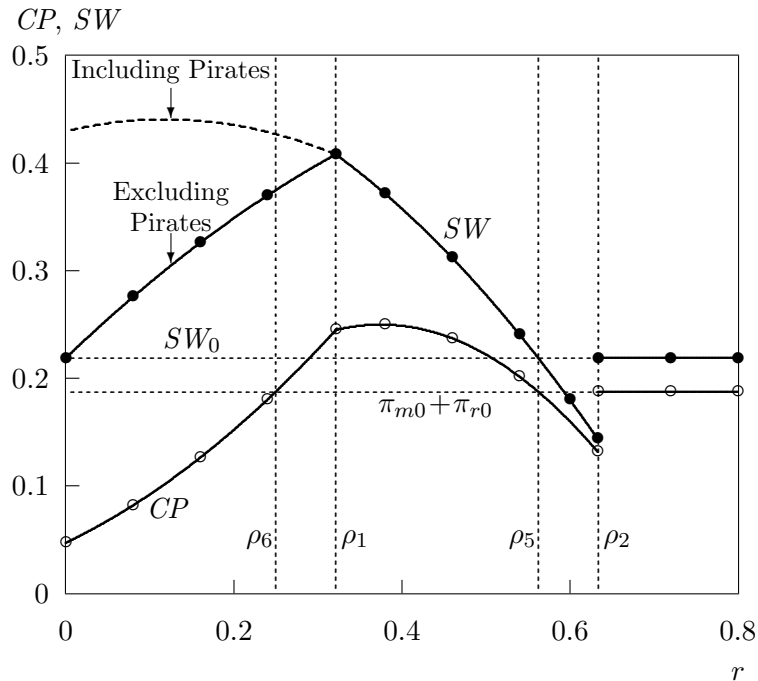


Figure 2.6: Channel Profit (CP) and Social Welfare (SW) as Functions of Enforcement; $\beta=0.75$

cannot be tolerated unabated. Moreover, when the enforcement level is too low and piracy is rampant, it eats deep into the surplus of the legal channel, which may be a cause for concern for a policymaker interested in the overall health of the industry. At the other extreme, though, when the enforcement level is very high, the diminished threat of piracy may not mitigate the problem of double marginalization at all; in fact, Figure 2.6 clearly shows that it exacerbates the problem when $\rho_5 < r < \rho_2$. It is only when the enforcement level is moderate, and the piracy rate low, that the shadow competition from piracy can truly mimic the invisible hand of a competitive market.

Before concluding this section, I wish to make a quick note. It is true that, unlike prior literature, piracy in my setting can positively impact both private profit and public welfare at the same time. However, it is still the case that the manufacturer prefers a higher level of enforcement than what may be considered socially optimal. This is because certain benefits of piracy, which accrue to consumers, or to the retailer, may not be of much value to the

manufacturer. Corollary 2.2 states this formally.

Corollary 2.2. *The level of enforcement which maximizes the social surplus is less than the level at which the manufacturer's profit is maximized.*

Note that Corollary 2.2 ignores the cost of piracy enforcement. However, in reality, governments do incur significant costs in implementing various anti-piracy laws, implying that policymakers might actually prefer an even lower level of enforcement. Moreover, if a social planner decides to also include the benefits from illegal consumption in its yardstick, it would seek a further reduction. Consequently, in practice, a serious gap might exist between what the manufacturer wants and how governments respond, quite reminiscent of the tension between private profits and public welfare discussed in prior research (e.g., Chen and Png 2003).

2.5.2 Channel Efficiency

Since we now know that a moderate dose of piracy can lend its invisible hand in abating the problem of double marginalization, a natural question arises. Is there any other way—perhaps, legally a more acceptable one—to mitigate this problem? To be sure, there are. If the manufacturer and retailer are vertically integrated, this vertical externality is obviously eliminated. However, in the context of information goods, this might be a daunting task, as it may involve merger of retailers such as Amazon with manufacturers such as Microsoft, Electronic Arts, and Paramount Pictures, to name a few. In the end, in fact, we may be left with only one giant integrated firm that has a complete, global monopoly over the entire spectrum of information goods. Clearly, such usurpation will not be palatable to anyone who believes in a market-based economy.

Interestingly, alternative contractual arrangements, such as revenue sharing or two-part tariffs, when properly designed and implemented, can also align incentives and coordinate the supply chain in a way that it virtually behaves like an integrated firm (Tirole 1992,

p.176–177). Indeed, Apple’s *iTunes Store* is based on a revenue-sharing contract. However, examples abound where such contractual forms are not feasible or easily implementable in reality. Consider the music industry where, for example, the traditional retailing setup is actually the dominant one (Abhishek et al. 2013). Such a setup is also quite common for books, both printed and electronic (Filloux 2012). And, the case of Amazon or Best Buy selling video games from large publishers such as Electronic Arts also belongs to the same setup. As we have already seen in §2.1, there are many more cases where my model is applicable. Clearly, the issue of double marginalization is a concern in all these situations, and online piracy can indeed play a mitigating role.

Recognizing the inevitability of such situations, one may now ask: How *efficient* is piracy in mitigating the vertical externality? To address this issue in a rigorous manner, it is imperative that I provide an analysis of the vertically-integrated setting. Doing so would also allow us to draw clear distinctions between the impacts of piracy discussed in prior research and those here.

Consider a supply chain where the manufacturer and retailer operate together as one entity. The profit-maximization problem faced by the integrated firm, conveniently referred to as the manufacturer henceforth, is $\max_{\bar{p}} \bar{\pi}(\bar{p}) = \bar{p}q(\bar{p})$. Similar to what we have seen in the previous section, the equilibrium outcomes can again be characterized in terms of three separate regions.

Proposition 2.4. *Let $\bar{\rho}_1 = \frac{\beta(1-\beta)}{2-\beta}$, and $\bar{\rho}_2 = \frac{\beta}{2}$. Then, $\bar{\rho}_1 < \bar{\rho}_2$, and the following three cases emerge:*

- **Piracy Region:** *When $r < \bar{\rho}_1$, the manufacturer finds it optimal to tolerate some level of piracy. In this case, $\bar{p}^* = \frac{1}{2}(1 - \beta + r)$ and $\bar{\pi}^* = \frac{(1-\beta+r)^2}{4(1-\beta)}$.*
- **Threat Region:** *When $\bar{\rho}_1 \leq r \leq \bar{\rho}_2$, there is no piracy, but the threat of piracy affects the pricing decision of the manufacturer. Here, $\bar{p}^* = \frac{r}{\beta}$ and $\bar{\pi}^* = \frac{r(\beta-r)}{\beta^2}$.*

- **Benchmark Region:** For $r > \bar{\rho}_2$, even the threat of piracy disappears, resulting in $\bar{p}^* = \frac{1}{2}$ and $\bar{\pi}^* = \frac{1}{4}$.

There are some important implications of Proposition 2.4. First, I note that piracy impacts the vertically-integrated case quite differently. This is clearly outlined by the fact that the three regions described in Proposition 2.4 are quite different from the ones in Proposition 2.1. This is illustrated in Figure 2.7, which, for $i = 1, 2$, plots $\bar{\rho}_i$, alongside the original ρ_i thresholds for comparison. Recall that ρ_1 and $\bar{\rho}_1$ represent the minimum levels of enforcement

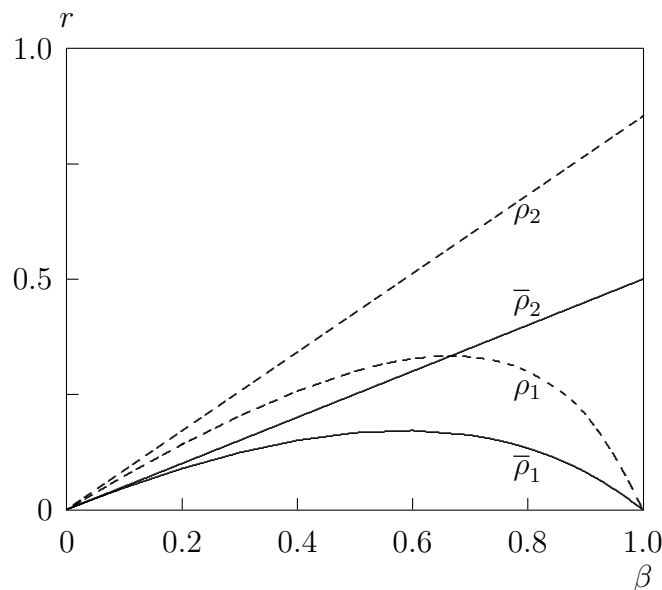


Figure 2.7: Regions of Piracy—Manufacturer-Retailer Chain (Dashed) vs. Integrated Firm (Solid)

needed to eradicate piracy in their respective setups. Similarly, ρ_2 and $\bar{\rho}_2$ represent those needed to make piracy completely irrelevant by removing even its threat. Simple algebra shows that $\bar{\rho}_i < \rho_i$, $i = 1, 2$, implying that the piracy and threat regions extend farther for the supply chain than for the vertically-integrated manufacturer. Viewed alternatively, enforcement is simply not as effective in the presence of a retailer as it is otherwise, and having the retailer in the middle makes piracy a more likely phenomenon. This is intuitive.

The higher retail price resulting from double marginalization makes piracy more appealing and, consequently, harder to eradicate.

To see the mechanics of my results—that is, how piracy interacts with “margins” in the problem of double marginalization—I take a closer look at the prices charged by the manufacturer and retailer. Superimposing the price from the vertically-integrated case (\bar{p}^*) onto the retail price from the manufacturer-retailer chain (p^*) depicted in Figure 2.4, we get Figure 2.8. When $r > \rho_2$, the equilibria for both setups fall in the benchmark region

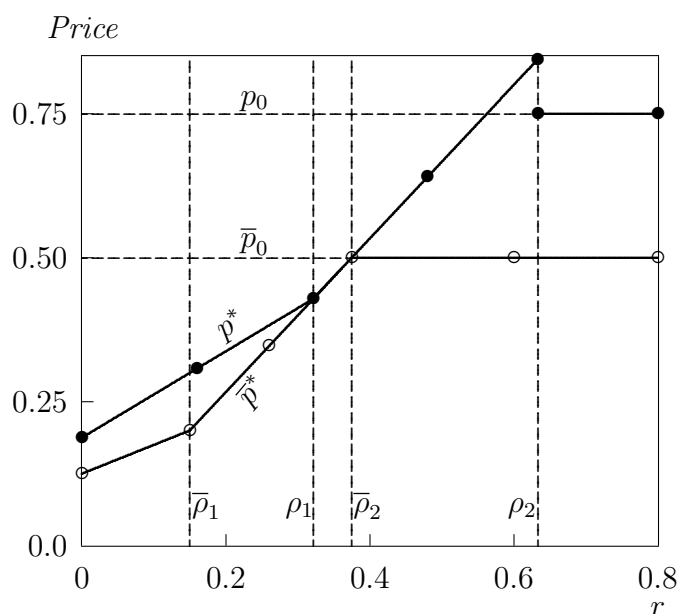


Figure 2.8: Manufacturer-Retailer Setup vs. Vertically-Integrated Firm, Retail Prices for $\beta = 0.75$

where piracy is a non-issue. Thus, I expect prior studies on double marginalization, which do not consider piracy, to apply here. That is precisely what we witness in Figure 2.8; when $r > \rho_2$, p_0 is indeed higher than \bar{p}_0 , which confirms the presence of double marginalization in the manufacturer-retailer chain I consider in this work. As we have already seen in §2.4.2, to the left of the ρ_2 boundary, p^* decreases as r decreases. This is because, as r decreases, the expected legal penalty from procuring the pirated product also decreases. In the fight against piracy, which is now more attractive to consumers, the legal channel responds with

a lower retail price. What is more interesting is that, as r keeps decreasing, p^* can decrease all the way to—and even below— \bar{p}_0 , fully offsetting the hike in the retail price that comes from double marginalization:

Lemma 2.3. *In the piracy and threat regions, weaker enforcement leads to a lower retail price. This decrease can completely offset double marginalization when $\beta > \frac{1}{3}$.*

Although Lemma 2.3 shows how piracy can put a downward pressure on the retail price—and hence, reduce the effect of double-marginalization on the retail price—it is not entirely clear whose margin piracy actually suppresses. Does piracy eat into the retailer’s margin, or is it the manufacturer’s margin that gets squeezed? Figure 2.9 presents answers to these questions. In this figure, we observe how the margins of the manufacturer and retailer are

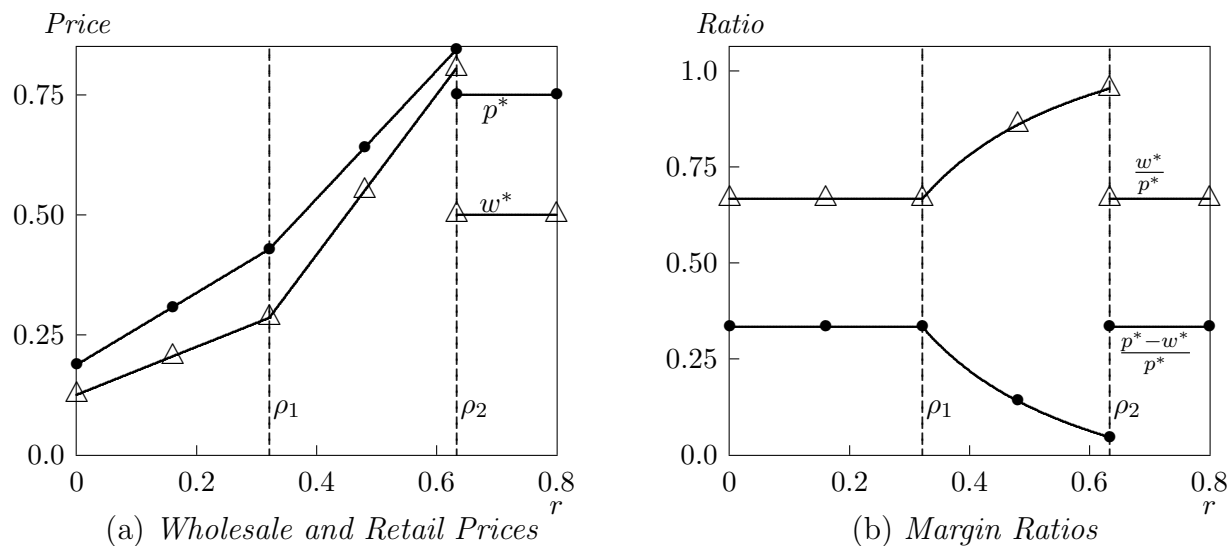


Figure 2.9: Wholesale Price, Retail Price, and Margin Ratios as Functions of Enforcement, for $\beta = 0.75$

affected by piracy. Since the marginal cost of production is zero for information goods, w^* is effectively the manufacturer’s margin. Then, the retailer’s margin can be represented by $(p^* - w^*)$. Interestingly, in both the piracy region and the benchmark region, two-thirds of

the revenue from each unit sold ends up in the manufacturer's pocket, and the retailer enjoys the other one-third. This constant ratio in the two regions shows that, although the retail price is lower in the piracy region compared to the benchmark region, the dynamics of the manufacturer-retailer relationship stays more or less the same. In other words, the existence of piracy in equilibrium suppresses the margins of the manufacturer and retailer evenly. The situation is quite different, however, in the threat region. The manufacturer's margin quickly increases in r , while the situation is quite the opposite for the retailer's margin. As seen in Proposition 2.1, in the threat region, the retailer's optimal price is $\frac{r}{\beta}$ regardless of the manufacturer's decision of w . This prompts the manufacturer to set w aggressively and to bite into the retailer's margin.

Lemma 2.4. *The ratio of the manufacturer and retailer's margins stays constant in the piracy region and benchmark region. Thus, the existence of piracy suppresses the margins of the manufacturer and retailer evenly. In the threat-region, however, the manufacturer's margin quickly increases in r , while the story is just the opposite for the retailer.*

The story that emerges from a comparison of the channel profits is also quite interesting. In Figure 2.10, when the legal channel is vertically integrated—and the issue of double marginalization absent—the channel profit increases in the level of enforcement until the point where the issue of piracy completely disappears. In other words, $\bar{\pi}^*$ increases in r , just as prior research suggests. Also, beyond the $\bar{\rho}_2$ boundary, $\bar{\pi}^*$ stays unchanged at $\frac{1}{4}$, and this profit is the ideal outcome for the legal channel, as neither double marginalization nor piracy is present any longer.

In contrast, for the manufacturer-retailer chain, the channel profit is not monotonic in r . The channel profit, $(\pi_m^* + \pi_r^*)$, initially increases with r , just as the conventional wisdom suggests. This is because the pirated good becomes less attractive to consumers, allowing the legal channel to partly reclaim its pricing power. Beyond a certain threshold of r , however,

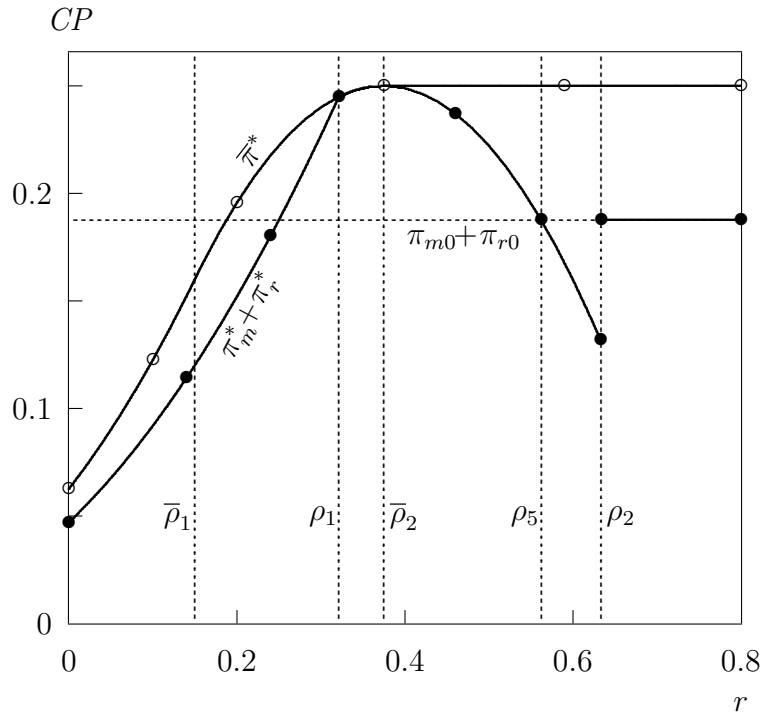


Figure 2.10: Channel Profit—Manufacturer-Retailer Chain vs. Integrated Firm; $\beta=0.75$

the pricing power has been reclaimed to an extent that the problem of double marginalization starts to dominate; now, $(\pi_m^* + \pi_r^*)$ decreases in r . Eventually, when r approaches the ρ_2 boundary, the channel profit plunges because of severe double marginalization. The net result, in short, is the lack of monotonicity in the profit plot to the left of ρ_2 . Finally, when $r > \rho_2$, the issue of piracy disappears, but double marginalization remains, presenting itself as the gap between $(\pi_{m0} + \pi_{r0})$ and $\frac{1}{4}$ in Figure 2.10.

In order to see the extent to which piracy mitigates double marginalization, I can also compare the channel profit with the profit of the vertically integrated manufacturer. This comparison can be summarized nicely using the notion of channel efficiency, η , which is essentially the ratio of the two, that is, $\eta = \frac{\pi_m^* + \pi_r^*}{\pi^*}$.

Theorem 2.4. *Let $\bar{\rho}_1$, ρ_1 , $\bar{\rho}_2$, and ρ_5 be as above. Then, piracy or its threat improves the channel efficiency by suppressing the impact of double-marginalization as long as $\bar{\rho}_1 < r < \rho_5$.*

In fact, for $\rho_1 \leq r \leq \bar{\rho}_2$, the channel efficiency reaches 100%.

Theorem 2.4 allows us to further narrow down the region where piracy is most efficient in coordinating the supply chain. The bound placed on r in Theorem 2.4 simply suggests that the enforcement level must be moderate—it cannot be too low or too high. It must also be noted that the interval $[\rho_1, \bar{\rho}_2]$ is non-empty only if $\beta \geq \frac{2}{3}$. This lower bound on β simply means that, for the pirated version to be able to completely eliminate the effect of double marginalization, quality-wise, it must be somewhat competitive against the legal product.

2.6. Conclusion

Piracy of information goods is a growing concern in today’s global economy and has received widespread attention from manufacturers, business alliances, and governments, as well as from researchers in various disciplines. Despite the large body of literature on piracy and its economic impacts, work on how piracy affects the supply chain of information goods has been scant. In this work, I seek to address this issue. Prior research examining the economic impacts of piracy has typically examined the tension between private profit and public welfare, and has found that more enforcement usually translates to more pricing power for the manufacturer and higher profits, but only so at the expense of consumers and the society. I find that this intuition does not hold for a vertical structure, where the above tension could disappear at moderate levels of enforcement.

More specifically, I ask how piracy impacts channel coordination within an information-good supply chain that faces two problems: one internal, double marginalization, and the other external, namely, piracy. In such a setting, it is indeed interesting how the supply chain reacts to piracy and its threat. When the incentives of the manufacturer and retailer do not align, one may imagine the situation to be even grimmer for the manufacturer as well as for the retailer. After all, the legal channel is now dealing with two concerns. I find that

such an intuition, however, does not have much merit. In fact, both the manufacturer and retailer may be able to make higher profits in the presence of piracy than they can when piracy is completely absent and has no influence on the market. Intriguingly, at the same time, the surplus of legal consumers can also be higher. In short, much like the *invisible hand* of a market, piracy makes everyone—every selfish actor engaged in maximizing his or her self-utility—better off.

Consider the economics behind this invisible hand of piracy. “What is worse than a monopoly? A chain of monopolies” (Tirole 1992, p.175). In other words, it is the lack of external competition that results in serious channel inefficiencies, commonly known as double marginalization. What actually happens is that piracy introduces a shadow competition, thereby limiting the pricing power of the legal channel and, in turn, reducing double marginalization.

How the two problems, double marginalization and piracy, interact is also very interesting. When anti-piracy enforcement is weak and piracy is rampant, the effect of piracy dominates, and the legal channel gets hurt, just as the conventional wisdom suggests. On the other hand, when strict enforcement wipes out piracy and all its threat completely, the issue of double marginalization dominates, and the legal channel suffers from channel inefficiencies. Thus, neither extreme is good for the legal channel—actually, the sweet spot is somewhere in the middle. When enforcement is moderate, it leads to an appropriate “prescription” of piracy—piracy is not too strong to hurt profits, but still strong enough to contain double marginalization. Not recognizing this trend only leads to overstating the benefits of increasing enforcement. In fact, it is useful not only from a policymaker’s perspective but also so from those of manufacturers and retailers who currently have little tolerance for piracy.

Do the results mean that manufacturers should immediately stop investing in anti-piracy efforts? Should, all of a sudden, governments disengage from all enforcement activities, or better still, start encouraging more piracy, perhaps by legalizing it? The answer to all such

questions is beyond the scope of a simple modeling exercise and would obviously depend on the ground realities. For example, given the prevailing high rate of piracy today,³ it may be worthwhile to invest more in enforcement activities and anti-piracy measures. At the same time, however, the results suggest that, in doing so, we must not get carried away and lose the overall perspective. We must realize that, once digital piracy is abated or controlled to an extent, further anti-piracy investment might not bear the desired fruit and can actually end up hurting both private profit and public welfare.

As my objective has been to analyze the effect of piracy on a manufacturer-retailer chain, I use a simple model that helps us accomplish exactly that. Obviously, there are factors not captured in my model. For example, I do not consider any network effect. If prior literature is of any indication, piracy should benefit the legal channel even more when network effects are incorporated. However, since interactions between different economic factors can sometimes be counterintuitive—much like my findings in this work—I can only speculate with caution, leaving necessary formal analyses to future work.

A Summary of Threshold Values

| Threshold | Definition |
|----------------|--|
| ρ_1 | $\frac{3\beta(1-\beta)}{4-3\beta}$ |
| ρ_2 | $\frac{\beta(6-4\beta+\sqrt{2\beta})}{4(2-\beta)}$ |
| ρ_3 | $\sqrt{1-\beta} - (1-\beta)$ |
| ρ_4 | $\beta - \frac{\beta}{4\sqrt{1-\beta}}$ |
| ρ_5 | $\frac{3\beta}{4}$ |
| $\bar{\rho}_1$ | $\frac{\beta(1-\beta)}{2-\beta}$ |
| $\bar{\rho}_2$ | $\frac{\beta}{2}$ |

³In a recent statement to Games Industry International, Ubisoft CEO, Yves Guillemont, claimed that the piracy rate of PC Games stands at an astounding level of 93–95% (Thier 2012). Even though Guillemont’s claim sounds a little too tall—and, perhaps, it is as well (Ployhar 2012)—claims of a high piracy rate is consistent across all types of information goods. For example, Business Software Alliance reports that the global piracy rate for PC software hovers at 42% and, among the countries at the top of software piracy charts, this number exceeds 70% (BSA 2011). Similar claims are common for music and movies as well (Verrier 2013).

Chapter 3

Combating Online Piracy: Short-Run Analysis

3.1. Introduction

To counter online piracy, significant efforts—I call them anti-piracy measures or enforcement efforts—have been expended globally. Traditionally, such efforts have mostly centered around the consumers of pirated products and have typically included monitoring and auditing for pirated use and penalties against illegal use (Danaher and Smith 2014, de Beer and Clemmer 2009, Farivar 2013, Mills 2012). In recent times, however, governments have also started scanning for sites that distribute, or aid in the distribution of, pirated content. Often, governments have gone after these sites forcefully, shutting them down and prosecuting the site operators (Danaher et al. 2014, Epstein 2012, Horwitz 2013). In capturing the essence of enforcement efforts, prior theoretical works on piracy have typically associated anti-piracy measures with a *piracy cost*—the expected loss resulting from potential legal liabilities, that is, the probability that piracy gets detected times the expected penalty assessed on detection (cf. August and Tunca 2008, Lahiri and Dey 2013). Such a conceptualization is certainly applicable to instances where penalties are imposed on illegal consumption. A higher level of enforcement increases either the probability of detection or the expected penalty on detection, or both, and consumers find piracy to be a more costly option.

However, I recognize that the above conceptualization fails to capture anti-piracy measures that are not directed at pirate consumers but affect the distribution or supply of pirated content. For instance, when a user attempts to locate a pirated content on Google, instead of the links to the pirated content—or the hosting pirate sites—the user may be faced with the following notice:

In response to multiple complaints we received under the US Digital Millennium Copyright Act, we have removed X results from this page. If you wish, you may read the DMCA complaints that caused the removals at *LumenDatabase.org*:
{Links to the complaints}.

This notice shows the Digital Millennium Copyright Act (DMCA) in effect, and the links that would have otherwise lead users to pirated contents have been filtered out. Although such an enforcement activity does abate piracy, it does not translate to an increase in the piracy cost for consumers—it neither increases the probability of detection nor does it impact the expected penalty. Evidently, in addition to the class of anti-piracy measures that attempts to curb the *demand* for piracy by making pirated goods less attractive to consumers, there is also a class that impacts the *supply* side and diminishes the visibility of pirated content. Following Danaher et al. (2014), I term the former type “demand-side” enforcement, and the latter, “supply-side” enforcement. Recently, Danaher and Smith (2014) and Danaher et al. (2014) have empirically investigated the impacts of demand- and supply-side enforcement on legal sales and found both types to have favorable impacts. Thus, although there are useful empirical findings about private profit, the impact on public welfare—consumer and social—has not been explored either empirically or analytically and remains an important open issue.

Rooted in this unexplored issue is my desire to address a set of questions, which have become increasingly prominent in the backdrop of online piracy fueled by the likes of

PirateBay.com and MegaUpload.com. For example, should anti-piracy measures be directed at illegal downloaders, or should they target sites that distribute pirated content? How do these measures impact consumer and social welfare? And, if indeed a shift in enforcement efforts were to take place, what would be the economic implications? In this work, I bring both enforcement types into one consolidated framework and investigate whether there are reasons to suspect the two approaches to have different economic implications, not only from the manufacturer's perspective, but also from consumers' and policymaker's points of view. In doing so, I theoretically scrutinize previous empirical findings on legal sales, and also examine, if the two approaches indeed have different economic consequences, which direction has a more desirable impact within a socioeconomic system.

Intuitively, either type of enforcement can curb piracy and may thus be beneficial to a manufacturer. However, it is not obvious exactly how the manufacturer will, or even should, react to these two types of enforcement, and whether that will eventually translate to gains or losses in welfare. In this chapter and the next, I investigate to see whether the two anti-piracy approaches are similar in their economic impacts and, if not, why, when, and how they differ.

In developing my model, I inherit much of the setup from prior literature. Specifically, I consider a profit-maximizing monopolist serving consumers heterogeneous in their taste for quality (e.g., Moorthy 1984, Mussa and Rosen 1978), and assume that there exists a quality difference between the pirated and legal versions (e.g., Lahiri and Dey 2013, Sundararajan 2004). Furthermore, I assume that higher levels of demand-side enforcement make piracy more costly—and hence less attractive—to consumers (e.g., August and Tunca 2008). My model, in essence, is an extension of the typical setup used in prior literature, the new elements being the ad-supported pirated content suppliers and the presence of supply-side enforcement.

My research questions and modeling approach put the spotlight squarely on how suppliers of pirated content actually operate, unraveling the mystery surrounding their “business” models. The underlying ecosystem shows a curious interdependence among some of the key entities: aggregator sites, online ad agencies, and pirate suppliers (Seidler 2011). In practice, many aggregator sites providing online access to pirated content operate on revenues generated through online advertisements. These sites include, among others, content-hosting and streaming sites known as *cyberlockers* (e.g., MegaUpload.com, FileServe.com, and RapidShare.com), *torrent indexers* such as BitSnoop.com, as well as individual blogs with clickable links to pirated content. Since the ad revenue increases with the traffic, such a site incentivizes individual pirate suppliers to share content that will be in high demand, as measured by the number of downloads. Pirate suppliers leveraging such a site typically offer free pirated content to lure hoards of downloaders to their pages. The traffic attracted by a supplier, in turn, has a direct impact on its revenue funneled through the aggregator site. It is the explicit modeling of this ecosystem that sets my work apart from prior literature.

In my analysis, I compare the two types of anti-piracy measures in their impacts on various economic metrics such as social welfare, piracy rate, manufacturer’s profit, and consumer surplus. Interestingly, or perhaps, expectedly, despite the stark difference in the approaches—one going after the illegal demand and the other going after the supply—my findings show that both types of measures have similar impacts across all the metrics I consider. Do my findings then imply that the difference between the two approaches is merely superficial? Is there no reason to discriminate the two? Maybe, the reality of scattered efforts across the camps of demand- and supply-side reflects my findings. There is a catch, though. These findings pertain to the short-run case where the product has already been developed and the only decision the manufacturer makes is the pricing. Hence, the logical question to ask is, will the findings in the short run extend to the long-run case where the manufacturer can decide on the product quality as well? This question provides a natural

segue to my next chapter.

3.2. Literature Review

The main point of this work is comparing the two forms of enforcement in terms of their impacts on a monopolist's pricing decision and the resulting impacts on consumer and social welfare. In this regard, existing research provides some interesting cues. For example, Danaher et al. (2014) scrutinize the effects of *HADOPI*—a demand-side anti-piracy law used in France to punish repeat offenders more severely—and find that it resulted in a 25% increase in legal sales. On the other hand, Danaher and Smith (2014) study the effects of shutting down *MegaUpload.com*—clearly a case of supply-side enforcement—and estimate that this shutdown resulted in a 6.5–8.5% boost in sales of digital movies. In another related work, Danaher et al. (2015) find that the blocking of piracy sites, also a supply-side measure, can be effective in boosting legal sales only when several such sites are blocked simultaneously, but the impact on legal sales is insignificant when only one site is blocked. Although empirical studies can spot such immediate impact on sales, the welfare implications of enforcement are far more elusive. The focus in this work, therefore, is complementing this line of research by qualitatively distinguishing various anti-piracy efforts and comparing their welfare implications.

There is also a vast literature on the economics of piracy using quantitative models. A branch of this literature argues that piracy—or the act of supporting it—may surprisingly benefit the manufacturers of digital goods in the presence of network effects (August and Tunca 2008, Conner and Rumelt 1991). Another branch examines when and how certain tools—digital rights management (DRM), nonlinear pricing, versioning, bundling, content delivery technology, or free trials—may help a manufacturer combat piracy (e.g., Chellappa and Shivendu 2005, Cho and Ahn 2010, Gopal and Gupta 2010, Johar et al. 2012, Sundararajan 2004, Vernik et al. 2011, Wu and Chen 2008). The closest to my work in this

chapter is the branch that examines the economic impacts of anti-piracy efforts on a manufacturer's strategy and resulting welfare (Bae and Choi 2006, Chen and Png 2003, Lahiri and Dey 2013). However, this branch considers only demand-side enforcement. For instance, Chen and Png (2003) examine and compare three ways to curb piracy, all directed at changing the relative appeal of the pirated product to a consumer, vis-à-vis the legal one. Bae and Choi (2006) and Lahiri and Dey (2013) also assume the supply of pirated goods to be exogenous and unlimited. My main contribution to this stream of literature is that: (i) I *model the ecosystem* of online piracy including pirate suppliers as strategic players, and (ii) I *endogenize the supply* of pirated content. In doing so, I am able to obtain a more complete picture and answer my research questions by comparing the economic impacts of supply- and demand-side anti-piracy measures.

Finally, it is worth mentioning that, although not explicitly recognized as such, the supply side of piracy has started garnering some attention. In particular, the role of commercial pirates, who price illegal versions to maximize profit, has been examined. For example, Jain Singh (2009) shows that the existence of commercial pirates can confound a manufacturer's response to piracy in unpredictable ways. Tunca and Wu (2013) find that increasing enforcement against individual pirates in P2P networks might make commercial pirates more competitive, harming the manufacturer in the process. Neither work, however, explicitly models the ecosystem that sustains online piracy, nor do they analyze the economic implications of disrupting this ecosystem.

3.3. Model Preliminaries

I develop an economic model with three strategic players: (i) a profit-maximizing monopolist, (ii) pirate suppliers supported by advertisements, and (iii) utility-maximizing consumers. The supplier and consumer bases are both normalized to a mass of one. The monopolist, situated within a market with certain levels of demand- and supply-side enforcement, chooses

the price and quality of its product.

The timeline is as follows: First, the manufacturer offers a product of quality $\theta > 0$ at a price $p > 0$. In the short run, the manufacturer can only set the price and the quality level is fixed at a constant. Though we can simply normalize this θ to one without any loss of generality, I carry this notation for the sake of convenience for my long-run analysis in the following Chapter 4. After the manufacturer's decision, the suppliers of pirated content decide whether to provide an illegal version or not—only when potential revenues from piracy can fully offset the cost imposed by supply-side enforcement, a supplier provides a pirated copy. This determines the supply level and availability of pirated content. Finally, each consumer decides whether to buy, pirate, or forgo use; this decision depends on the piracy cost resulting from demand-side enforcement as well as on the availability of a pirated copy.

Before I proceed, I note here that I am using the terms “short-run” and “long-run” in their traditional microeconomic sense, in which the distinction between the two terms is that (i) in the short run, firms have already incurred the (sunk) fixed costs, while there are no (sunk) fixed costs in the long run, and (ii) irrespective of the sunk cost, the firm produces in the short run as long as the market price covers the variable costs (Mankiw 2008, p.280). An information good involves an up-front development cost, which depends on the quality level chosen by the firm as a part of its long-run production decision. This up-front cost can be rather large; consider, for example, the development of a star-studded movie, an enterprise-level software, or a rich and complicated video game. Once the product is developed and introduced to the market, however, this cost is sunk and the short-run decision of the firm is only about pricing the product.

3.3.1 Enforcement Environment

Before describing the players and their behavior, it is important to first discuss the enforcement environment in which they operate. I assume that this environment is characterized

by two parameters, r and e , respectively representing the levels of demand- and supply-side enforcement activities. When demand-side enforcement is stepped up, that is, when r increases, it results in either a higher probability of getting detected when using a pirated copy or a higher penalty on detection, or both. In other words, similar to prior literature (August and Tunca 2008, Lahiri and Dey 2013), a higher r simply increases the expected legal penalty a consumer faces and is a proxy for the piracy cost in my model.

Similarly, supply-side enforcement, e , increases the “entry cost” faced by each pirate supplier; this entry cost includes the *risk* of prosecution and penalty if convicted of distributing illegal copies, and, naturally, depends, in an aggregate sense, on all actions that amplify this risk by making it difficult to supply pirated content. Such actions may include, among other things, prosecuting pirate suppliers, shutting down cyberlockers, or requiring search engines to filter out links to illegal content.

I argue that both r and e largely depend on the political and legal environment in which a business operates. For example, the cost of piracy in certain developing nations, both for consumption and supply, is quite low because, in those countries, either governments are remiss in enforcing intellectual property laws and international treaties, or the penalty on detection is low under their judicial systems (BSA 2011). In contrast, there are hundreds of piracy-related criminal prosecutions in the United States every year. Given an enforcement environment, I wish to characterize the market equilibrium and, through comparative statics, examine how this equilibrium behaves when one or the other type of enforcement is increased.

3.3.2 Demand-side Enforcement and Consumer Behavior

The consumers are heterogeneous along two orthogonal dimensions and are indexed by the pair: $\langle v, k \rangle$. Consumers’ preference for quality is represented by v ; consumer $\langle v, \cdot \rangle$ gets a value of $v\theta$ from using a product of quality θ . On the other hand, as discussed in §3.3.4, k stands for their technical ability or their affinity towards piracy. I make the following

assumption about v :

Assumption 3.1. *A consumer's preference for quality, v , is uniformly distributed over $[0, 1]$. A consumer knows his v , whereas the manufacturer only knows its distribution.*

I now explain what quality means for a digital good and what impact piracy may have on this quality. The term *quality* is being used in the classical sense—it essentially captures those characteristics of a product that are desired by all consumers (Moorthy 1984). Quality of a digital good—also a proxy for innovation in my Chapter 4—may comprise all or some such appropriate and desirable characteristics as the ease of use, speed, functionality, flexibility, portability, resolution, fidelity and encoding bit rate, among others.

Prior research has often considered the pirated product to be of lesser quality (e.g., Sundararajan 2004). Indeed, examples abound where the physical quality of the pirated copy is less than that of the original, as is usually the case with pirated movies (Karaganis 2011). Similarly, pirated software products do not often receive certain updates and patches (August and Tunca 2008), and may be missing important functionalities or contain embedded malicious codes (Lahiri and Dey 2013), implying that the quality of the pirated content, ϕ , is likely less than θ . However, it is possible to argue to the contrary as well, that is $\phi > \theta$. For example, a pirated media file may not have pesky restrictions in terms of the number of devices where it can be played (e.g., Vernik et al. 2011). Likewise, in the case of software products, such as Mathematica or SAS, a pirated copy may be viewed by some as more convenient as it may not require a periodic renewal of the license. Irrespective of whether piracy enhances or impairs quality, the pirated product is highly unlikely to have a quality level that is totally independent of the quality of the legal product. Logically, we would expect ϕ to be an increasing function of θ , satisfying $\phi|_{\theta=0} = 0$. For simplicity, I choose a linear form for ϕ , but do so without any precept about whether the pirated content is superior or inferior¹:

¹My own experience suggests that, in most real world situations, ϕ is likely to be less than θ , that is,

Assumption 3.2. For a legal product of quality θ , the quality of its pirated version is $\phi = \beta\theta$, $\beta > 0$.

A consumer faces an expected legal penalty of r if he ends up using an illegal copy. As explained earlier, this penalty is exogenous in my model and simply depends on the level of enforcement against the consumption of pirated goods. Clearly, a consumer can enjoy a utility of $(v\theta - p)$ from purchasing the legal version, and $(v\beta\theta - r)$ from a pirated copy; which one he chooses in the end depends not just on these utilities, but also on the availability of pirated content, as determined by supply-side enforcement activities. I discuss that next.

3.3.3 Supply-Side Enforcement and Behavior of Pirate Suppliers

I now consider how the behavior of suppliers of pirated content impacts availability. Because anyone who can make use of cyberlockers or other file-sharing sites can easily distribute illegal content, I assume that there is a large number of identical potential pirate suppliers. This assumption of a large number of suppliers is essentially equivalent to saying that the suppliers are *atomistic*, that is, the decision of a single supplier does not impact the supply level. The atomistic suppliers are similar to atomistic traders in financial markets, where the decision of one individual cannot impact the performance of the entire market, although the decisions of many individuals collectively can make a difference.

Despite there being a large number of pirate suppliers, not all of them will end up entering the piracy “market.” For, similar to Tunca and Wu (2013), I also take into account that each pirate supplier faces an “entry cost;” as explained earlier, this entry cost is also denoted by e , the level of *supply-side* enforcement, which determines how many pirate suppliers will enter the market—when e is very high, no one would dare to enter the market, and, when e is very small, there will be abundant supply as almost every pirate supplier will become active. Since the mass of all potential pirate suppliers has been normalized to one, I denote $\beta < 1$. The case of $\beta > 1$ is included in my analysis for the sake of completeness.

the supply level of pirated content—the number of pirate suppliers active in the market in distributing illegal copies—by $\eta(e) \in (0, 1)$. If η were zero, there would be no supply, whatsoever, of pirated copies. On the other hand, if $\eta(e)$ were one, pirated copies would be abundant and, therefore, readily available.² I do consider these two extreme cases in my overall analysis, but focus primarily on the case where $\eta(e) \in (0, 1)$.

3.3.4 Consumers' Access to Pirated Content

Let us now examine how the above supply level (η) impacts consumers' access to pirated content. Whether or not a consumer can eventually access an illegal copy depends not only on the supply level of pirated content, but also on the consumer's technical ability, which covers a wide spectrum of know-hows and skills—knowledge of appropriate keywords, familiarity with different types of web sites (such as aggregator sites, torrent indexing sites, blogs, cyberlockers, and search engines), and working knowledge of torrents, protocols, and file converters (Khantwal 2016, Veneziani 2007). The more technically savvy a consumer is, the higher would be the number of choices he ought to have in terms of pirate sites he can readily access. When enforcement, e , is stepped up and η declines as a result, the usual pirate sites a consumer is familiar with may no longer be available. As a result, a consumer with limited technical abilities may not be able to locate the pirated product anymore. In contrast, if the consumer is technically savvy, he can adopt the protocols of a new site to open up other avenues at little or no additional cost.

This conceptualization of consumer heterogeneity in terms of their technical savviness is also supported by empirical research. In particular, Danaher et al. (2015) found that, although blocking only one website may not dissuade consumers from pirating, blocking several of them can indeed force consumers to abandon piracy and switch to legal channels.

²The attentive reader will recognize here that, when e approaches zero, $\eta(e) \rightarrow 1$ and my model reduces to that in (Lahiri and Dey 2013), which assumes that there is no supply-side enforcement and the supply level is exogenous and pirated content, abundant.

This very fact—the fact that a certain minimum number of sites must be blocked before the effects of blocking can take hold—indicates that it must be a consumer’s technical savviness that ultimately determines the number of piracy site options he is likely to have at his disposal. When only one or two sites are blocked, the consumer may still have other options left for pirating a copy. In contrast, when several sites are blocked at once, he may run out of all such options and may fail to locate a pirated copy.

When faced with higher supply-side enforcement, why do some consumers run out of options, while others can still access them? Can consumers not learn and improvise, thereby increasing their piracy options? The empirical results in (Danaher et al. 2015) give us some important clues in terms of how consumers may behave in this regard. First, when 19 piracy sites in the UK were blocked at once, legal sales from the heaviest users of those sites increased by 23.6%, indicating that some consumers indeed found the learning cost to be too steep to locate and use newer sites; hence, some of them decided to go legal. Second, and more importantly, although 19 major sites were blocked, other pirate sites still remained accessible, meaning that some consumers, conceivably the more tech-savvy ones, were able to pirate—perhaps, their superior technical knowledge opened other options for them at little or no additional cost. This indicates that consumers are likely heterogeneous in terms of their savviness and, hence, in terms of the piracy options they have or could open up easily.

Because a consumer’s technical savviness determines the piracy options he has, I capture the heterogeneity in consumers’ technical abilities by indexing them with a random variable $k \geq 0$ —consumer $\langle \cdot, k \rangle$ has access to k potential pirate sites. Interestingly, k can also be viewed as consumers’ affinity towards piracy, because the more a consumer likes piracy, the more time he must have spent learning the skills and gathering the necessary technical know-how. In that sense, consumers with $k = 0$ have no inclination whatsoever to violate copyrights and can be viewed as inherently *ethical* (August and Tunca 2008). I assume that k follows a geometric distribution with parameter $g \in (0, 1)$:

Assumption 3.3. *Consumer $\langle \cdot, k \rangle$ has access to only k potential pirate sites, $k = 0, 1, 2, \dots$; the probability mass associated with consumer $\langle \cdot, k \rangle$ is $g(1 - g)^k$.*

The assumption of a geometric distribution for k simply ensures analytical tractability and is not critical for the results. Since not all potential pirate sites may be active, the probability that a consumer finds a given site useful for pirating is not necessarily one. Specifically, this probability ought to increase with the supply level, η ; hence, I simply set it to η . Now, consumer $\langle \cdot, k \rangle$ will no longer be able to locate a pirated copy only when all his k options become inactive; given an η , this happens with a probability of $(1 - \eta)^k$. Taking an expectation over k , we can find the probability of a randomly chosen consumer not having access to a pirated copy as:

$$\lambda(\eta) = \sum_{k=0}^{\infty} g(1 - g)^k (1 - \eta)^k = \frac{g}{1 - (1 - g)(1 - \eta)} = \frac{g}{g + (1 - g)\eta}.$$

Note that $\lambda(\eta)$ satisfies the following set of intuitive conditions: (i) $\lambda(0) = 1$, (ii) $\lambda(1) = g$, (iii) $\frac{\partial \lambda(\eta)}{\partial \eta} = -\frac{g(1-g)}{(g+(1-g)\eta)^2} < 0$, and (iv) $\frac{\partial^2 \lambda(\eta)}{\partial \eta^2} = \frac{2g(1-g)^2}{(g+\eta(1-g))^3} > 0$. They respectively show that: (i) no one will be able to find a pirated version when there is no supply, (ii) all consumers—except for the g -fraction of ethical consumers who would never even consider pirated contents—will be able to find an illegal copy when the supply is abundant, and (iii) the probability $\lambda(\eta)$ gets smaller as the supply increases. The last condition, which implies that the marginal impact of additional supply on $\lambda(\eta)$ is diminishing, is logical as well. For, when the market is saturated with pirate suppliers, adding one more supplier should have little impact on the availability of pirated content.

By setting $\xi = \frac{g}{1-g}$, we end up with a Tullock-type specification (Tullock 1980, pp. 99–101):

$$\lambda(\eta) = \frac{\xi}{\xi + \eta}. \tag{3.1}$$

The above transformation (from g to ξ) not only makes the analyses notationally more convenient but also demonstrates that my specification is largely consistent with the prior

literature on online content distribution, which has also employed a Tullock contest function to model the probability of attracting traffic to a content site (Dellarocas et al. 2013). Tullock contest function is used to relate the probability of winning a contest to the resources contestants devote to it (Tullock 1980, pp. 99–101). In (Dellarocas et al. 2013), the authors adopt this function to connect the probability of attracting traffic to content sites, the level of effort exerted by those sites, and consumers’ propensity towards outside alternatives (e.g., other channels). And, in the context of online *pirated* content distribution, we can draw an interesting parallel; the probability of a consumer having access to a pirated copy—that is, the probability of attracting traffic to *pirated* content sites ($1 - \lambda(\eta)$)—is determined by the level of supply η (which is determined by the aggregate level of effort exerted by pirate sites), and consumers’ propensity towards alternatives outside piracy (i.e., staying ethical). Clearly, ξ , which is a fraction of ethical consumers over unethical, represents the consumer population’s general propensity towards options outside piracy, i.e., staying ethical.

3.3.5 Consumer Demand

For a consumer to use pirated content, it must be available and attractive at the same time. The pirated version is more attractive over the legal one if the following incentive compatibility (IC) constraint is satisfied: $v\theta - p \leq v\beta\theta - r$. However, a restricted level of supply, $\eta(e) < 1$, simply means that the availability of the pirated content would be less than perfect and not all consumers will be able to locate the pirated version, irrespective of whether it is attractive or not. A consumer who cannot locate a pirated copy would buy the legal product if and only if his individual rationality (IR) constraint is met: $v\theta - p \geq 0$, that is, $v \geq \frac{p}{\theta}$; see Figure 3.1. On the other hand, when a consumer can locate a copy, he would choose to use the legal product if and only if, in addition to the above IR constraint, the following incentive compatibility (IC) constraint is also satisfied: $v\theta - p \geq v\beta\theta - r$. Figure 3.1 graphically captures these IR and IC constraints and identifies the legal and illegal demands

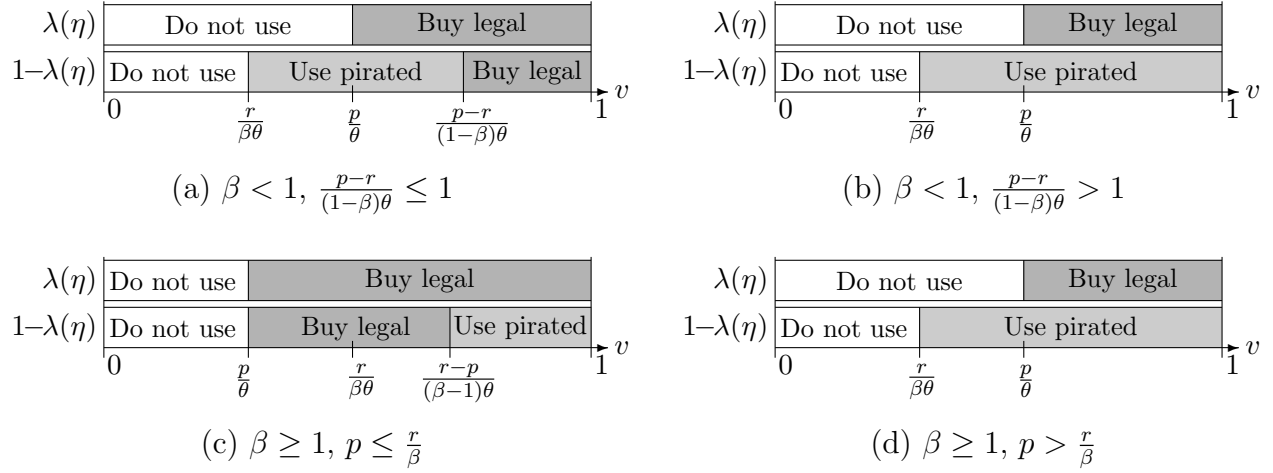


Figure 3.1: Consumers Self-Select Based on Their Relative Benefits

for various market configurations in which piracy exists. Using Figure 3.1, the legal and illegal demands, denoted q and \hat{q} , respectively, can now be expressed as:

$$q = \begin{cases} \lambda(\eta) \left(1 - \frac{p}{\theta}\right) + (1 - \lambda(\eta)) \left(1 - \min\left\{1, \frac{p-r}{(1-\beta)\theta}\right\}\right), & \text{if } \beta < 1, \\ \lambda(\eta) \left(1 - \frac{p}{\theta}\right) + (1 - \lambda(\eta)) \left(\min\left\{1, \frac{r-p}{(\beta-1)\theta}\right\} - \min\left\{\frac{p}{\theta}, \frac{r-p}{(\beta-1)\theta}\right\}\right), & \text{otherwise;} \end{cases} \quad (3.2)$$

and

$$\hat{q} = \begin{cases} (1 - \lambda(\eta)) \left(\min\left\{1, \frac{p-r}{(1-\beta)\theta}\right\} - \frac{r}{\beta\theta}\right), & \text{if } \beta < 1, \\ (1 - \lambda(\eta)) \left(1 - \min\left\{1, \max\left\{\frac{r}{\beta\theta}, \frac{r-p}{(\beta-1)\theta}\right\}\right\}\right), & \text{otherwise.} \end{cases} \quad (3.3)$$

3.3.6 Supply Level at Subgame Equilibrium

Pirate suppliers are typically paid based on the number of downloads of pirated content uploaded by them. The total advertisement revenue earned by all suppliers should then be proportional to \hat{q} , the total number of illegal downloads; henceforth, without loss of generality, I assume this constant of proportionality to be one. In the piracy “market” with identical suppliers, the revenue for each supplier is then $\frac{\hat{q}}{\eta}$, the total revenue divided by the number of suppliers participating in distributing an illegal copy. A supplier compares this revenue with his entry cost, e , and enters the market as long as the revenue is more than

the cost. Thus, in a *subgame equilibrium*, a supplier's utility is $\frac{\hat{q}}{\eta} - e = 0$, implying

$$\hat{q} = \eta e. \quad (3.4)$$

Substituting (3.1) into (3.3) and equating the resulting expression to (3.4), I solve for \hat{q} , η , and λ in the subgame equilibrium. I obtain:

$$\lambda = \begin{cases} \frac{e\theta\xi(1-\beta)}{p-\frac{r}{\beta}}, & \text{if } \frac{p-r}{(1-\beta)\theta} \leq 1 \text{ and } \beta < 1, \\ \frac{e\theta\xi}{\theta-\frac{r-p}{\beta-1}}, & \text{if } p \leq \frac{r}{\beta} \text{ and } \beta \geq 1, \\ \frac{e\theta\xi}{\theta-\frac{r}{\beta}}, & \text{otherwise,} \end{cases} \quad (3.5)$$

which can now be substituted into (3.2) to estimate the subgame-perfect legal demand.

3.3.7 Manufacturer's Decision Problem

Equation (3.5) allows us to estimate the legal demand, q , in terms of the enforcement levels, e and r . The overall equilibrium is then found by simply maximizing the manufacturer's profit. I assume:

Assumption 3.4. *The manufacturer's marginal cost of producing an additional copy is zero, and its cost of developing a product of quality level θ is $\frac{c\theta^2}{2}$, $c > 0$.*

Now, in the short run, the manufacturer's problem is to solve: $\max_p \pi = pq - \frac{c\theta^2}{2}$. As a reminder, θ is fixed at a constant in the short run, and so is the development cost. To see how the equilibrium looks like in the long run where the manufacturer can change the quality level—by investing in R&D, for example—I endogenize θ in the following Chapter 4.

Although conceptually straightforward, solving this problem and analytically characterizing its solution are not simple. This is because the manufacturer's strategy may shift as we move from one point in the parameter space to another, resulting in singularities with respect to the decision variables at the boundaries of these strategies. Depending on the context parameters, the manufacturer finds it optimal to be in exactly one of the seven cases.

Cases 1A, 1B—Limited Supply ($0 < \eta < 1$): Here, a pirated copy has limited availability. As indicated in Figure 3.1 (a) and (c), the manufacturer can name a price such that it ends up selling to both types of consumers, those who have access to a pirated copy and those who do not (Case 1A). Alternatively, as shown in Figure 3.1 (b) and (d), it can set p so high that (Case 1B), effectively shutting out, from the legal product, all consumers who have access to a pirated version.

Cases 2A, 2B—Ample Supply ($\eta = 1$): The supply of pirated content is abundant in this case, and $\eta = 1$ implies $\lambda = \frac{\xi}{\xi+1}$. Similar to Case 1, we again face two possibilities: the manufacturer may serve both groups, those with piracy as an option and those without (Case 2A), or consider just the latter (Case 2B).

Case 3A, 3B, 3C—No Piracy ($\eta = 0$): Piracy ceases to exist when no consumer has the option to use a pirated version, or if the manufacturer chooses the price and/or quality in a way that the pirated product is rendered completely uncompetitive. In equilibrium, they are equivalent, and $\lambda = 1$ in both cases. There are three ways piracy may disappear: In Case 3A, the manufacturer chooses a “limit” price such that the illegal copy barely becomes unattractive compared to the legal one.³ In Case 3B, the manufacturer chooses a “limit” quality to the same effect. Keep in mind that this case does not arise when the quality level is fixed at a constant in the short run; only when the manufacturer can change the quality level (i.e., in the long-run), does Case 3B become possible. In 3A and 3B, piracy ceases to exist, even though the threat of piracy still remains—unless the manufacturer holds the price or quality at the “limit” level, piracy can resurface. Finally, Case 3C happens when enforcement on either demand- or supply-side, or both, is high enough to suppress all threats completely, resulting in a pure monopoly.

³The concept of limit pricing here is the same as that in classical economics (Milgrom and Roberts 1982), where the limit price is used to discourage an entry of a potential competitor. Similarly, the monopolist tries

Table 3.1: Manufacturer's Optimization Problem under Different Strategies

| | Case | Objective Function | Constraints |
|----------------|---------|--|---|
| $\beta < 1$ | Case 1A | $p \left(\lambda \left(1 - \frac{p}{\theta} \right) + (1 - \lambda) \left(1 - \frac{p-r}{(1-\beta)\theta} \right) \right) - \frac{c\theta^2}{2}$ | $\lambda = \frac{e\theta\xi(1-\beta)}{p-\frac{r}{\beta}}, \frac{\xi}{\xi+1} < \lambda < 1, \text{ and } \frac{p-r}{(1-\beta)\theta} \leq 1$ |
| | Case 1B | $p\lambda \left(1 - \frac{p}{\theta} \right) - \frac{c\theta^2}{2}$ | $\lambda = \frac{e\theta\xi}{\theta-\frac{r}{\beta}}, \frac{\xi}{\xi+1} < \lambda < 1, \text{ and } \frac{p-r}{(1-\beta)\theta} > 1$ |
| | Case 2A | $p \left(\frac{\xi}{\xi+1} \left(1 - \frac{p}{\theta} \right) + \frac{1}{\xi+1} \left(1 - \frac{p-r}{(1-\beta)\theta} \right) \right) - \frac{c\theta^2}{2}$ | $\frac{e\theta\xi(1-\beta)}{p-\frac{r}{\beta}} \leq \frac{\xi}{\xi+1}, \text{ and } \frac{p-r}{(1-\beta)\theta} \leq 1$ |
| | Case 2B | $\frac{p\xi}{\xi+1} \left(1 - \frac{p}{\theta} \right) - \frac{c\theta^2}{2}$ | $\frac{e\theta\xi}{\theta-\frac{r}{\beta}} \leq \frac{\xi}{\xi+1}, \text{ and } \frac{p-r}{(1-\beta)\theta} > 1$ |
| | Case 3A | $p \left(1 - \frac{p}{\theta} \right) - \frac{c\theta^2}{2}$ | $\lambda = \frac{e\theta\xi(1-\beta)}{p-\frac{r}{\beta}} = 1, \text{ and } \frac{p-r}{(1-\beta)\theta} \leq 1$ |
| | Case 3B | $p \left(1 - \frac{p}{\theta} \right) - \frac{c\theta^2}{2}$ | $\lambda = \frac{e\theta\xi}{\theta-\frac{r}{\beta}} = 1, \text{ and } \frac{p-r}{(1-\beta)\theta} > 1$ |
| | Case 3C | $p \left(1 - \frac{p}{\theta} \right) - \frac{c\theta^2}{2}$ | $\frac{e\theta\xi(1-\beta)}{p-\frac{r}{\beta}} > 1 \text{ or } \frac{e\theta\xi}{\theta-\frac{r}{\beta}} > 1$ |
| $\beta \geq 1$ | Case 1A | $p \left(\lambda \left(1 - \frac{p}{\theta} \right) + (1 - \lambda) \left(\frac{r-p}{(\beta-1)\theta} - \frac{p}{\theta} \right) \right) - \frac{c\theta^2}{2}$ | $\lambda = \frac{e\theta\xi}{\theta-\frac{r-p}{\beta-1}}, \frac{\xi}{\xi+1} < \lambda < 1, \text{ and } p \leq \frac{r}{\beta}$ |
| | Case 1B | $p\lambda \left(1 - \frac{p}{\theta} \right) - \frac{c\theta^2}{2}$ | $\lambda = \frac{e\theta\xi}{\theta-\frac{r}{\beta}}, \frac{\xi}{\xi+1} < \lambda < 1, \text{ and } p > \frac{r}{\beta}$ |
| | Case 2A | $p \left(\frac{\xi}{\xi+1} \left(1 - \frac{p}{\theta} \right) + \frac{1}{\xi+1} \left(\frac{r-p}{(\beta-1)\theta} - \frac{p}{\theta} \right) \right) - \frac{c\theta^2}{2}$ | $\frac{e\theta\xi}{\theta-\frac{r-p}{\beta-1}} \leq \frac{\xi}{\xi+1}, \text{ and } p \leq \frac{r}{\beta}$ |
| | Case 2B | $\frac{p\xi}{\xi+1} \left(1 - \frac{p}{\theta} \right) - \frac{c\theta^2}{2}$ | $\frac{e\theta\xi}{\theta-\frac{r}{\beta}} \leq \frac{\xi}{\xi+1}, \text{ and } p > \frac{r}{\beta}$ |
| | Case 3A | $p \left(1 - \frac{p}{\theta} \right) - \frac{c\theta^2}{2}$ | $\lambda = \frac{e\theta\xi}{\theta-\frac{r-p}{\beta-1}} = 1, \text{ and } p \leq \frac{r}{\beta}$ |
| | Case 3B | $p \left(1 - \frac{p}{\theta} \right) - \frac{c\theta^2}{2}$ | $\lambda = \frac{e\theta\xi}{\theta-\frac{r}{\beta}} = 1, \text{ and } p > \frac{r}{\beta}$ |
| | Case 3C | $p \left(1 - \frac{p}{\theta} \right) - \frac{c\theta^2}{2}$ | $\frac{e\theta\xi}{\theta-\frac{r-p}{\beta-1}} > 1 \text{ or } \frac{e\theta\xi}{\theta-\frac{r}{\beta}} > 1$ |

In each of the above seven cases, the manufacturer's objective function as well as the associated constraints (or, validity conditions) can be derived easily. Consider, for example, Case 1A for $\beta < 1$. For this case, $\frac{p-r}{(1-\beta)\theta} \leq 1$ by definition, so (3.5) gives us $\lambda = \frac{e\theta\xi(1-\beta)}{p-\frac{r}{\beta}}$, and $0 < \eta < 1$ translates to $\frac{\xi}{\xi+1} < \lambda < 1$. Finally, the objective function can be obtained directly from the demand given by (3.2). The other cases are similar, and I summarize them all in Table 3.1.

to stave off the “shadow” competition from piracy by naming a price at the limit level.

3.4. Equilibrium Analysis

I now discuss the equilibrium for this model. In addition to satisfying the constraints in Table 3.1, for a particular case to occur in equilibrium, it must dominate, from the perspective of the manufacturer's profit, all other cases that also provide a valid solution satisfying relevant constraints. Below, I ensure that these requirements are met while partitioning the parameter space into different regions, each representing a different case. A complete description of these regions along with their boundaries is provided in Appendix B.1.

Let us consider the manufacturer's decision in the short-run situation, where the quality of the information good is fixed and the manufacturer can only set the price. Here, the manufacturer's decision problem becomes: $\max_p R = pq(\theta, p)$, for a fixed, exogenous θ . Figure 3.2 illustrates the partitions of the (r, e) space for $\beta = 0.75$, $\xi = \frac{2}{3}$, and $\theta = 10$, with Region i representing the part of the parameter space where Case i occurs in equilibrium; also see Appendix B.1.⁴ This figure is intuitive. At low values of e , I expect the supply to be abundant ($\eta = 1$), implying an equilibrium in Region 2A or 2B. As e increases, the supply of the pirated content becomes restricted ($0 < \eta < 1$), causing the equilibrium to move to either Region 1A or 1B. At an even higher e , there is no supply whatsoever ($\eta = 0$), and the equilibrium is found in Region 3C. Indeed, at very high values of e or r , both piracy and its threat should completely disappear, allowing the manufacturer to enjoy its full monopoly power. Prior research also identifies a similar possibility (Bae and Choi 2006, Lahiri and Dey 2013). However, unlike prior work, such a situation can surface in the equilibrium even when $r = 0$. This is because, in my model, a large e alone can stamp out all piracy completely.

The occurrence of the equilibrium in Region 3A is somewhat more curious. For example, when $r = 1$, as e increases, the equilibrium moves from Region 1A to 3A, and then to 1B. The move from 1A to 3A is again intuitive—a higher e , resulting in a lower level of supply,

⁴Since quality is fixed in the short run, Case 3B (limit quality) cannot be a part of the manufacturer's strategy space.

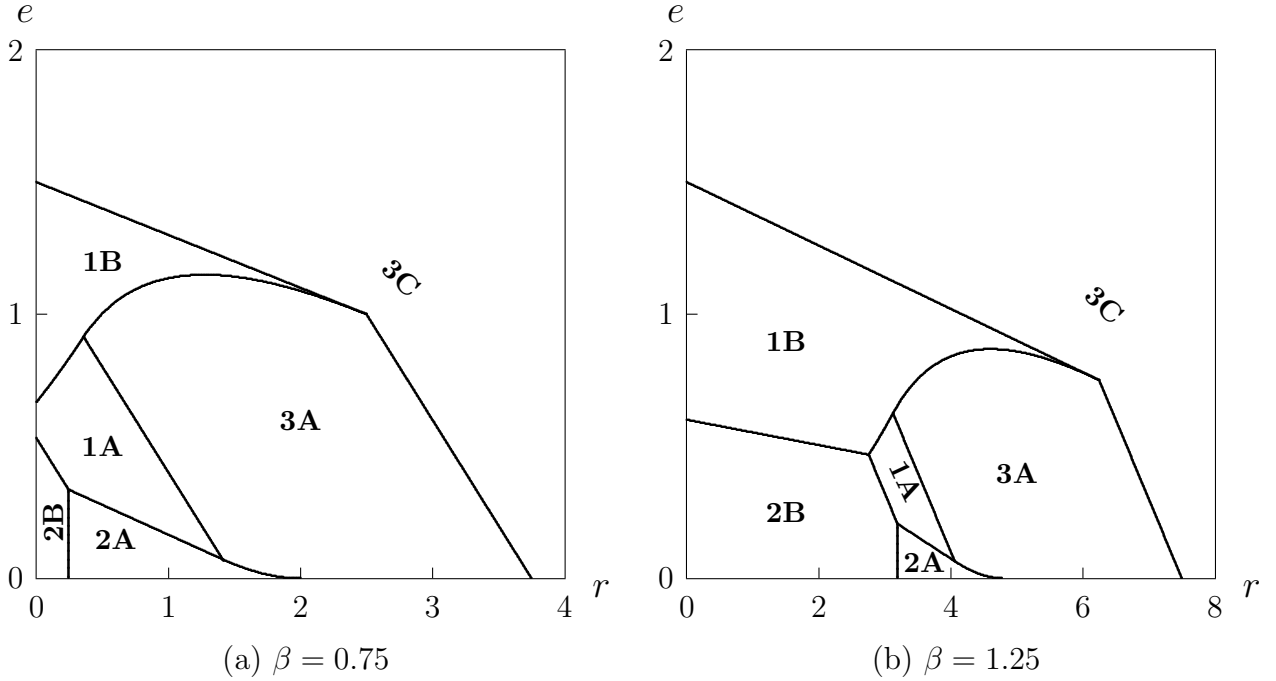


Figure 3.2: Relevant Partitions of the (r, e) Space in the Short Run; $\xi = \frac{2}{3}$, $\theta = 10$

allows the manufacturer to eliminate piracy by using the limit price. However, as e increases further, only a few consumers are able to locate a pirated copy, so the manufacturer finds it profitable to ignore them instead of luring them to the legal product through a low price. This moves the equilibrium from 3A to 1B.

Lemma 3.1. *Cases 1A and 2A are not possible in equilibrium when $\beta = 1$.*

Proof for this, as well as all other results, are available in Appendix B.2.

Proposition 3.1. *For a given θ , the equilibrium price is given by:*

$$p^*(\theta) = \begin{cases} \frac{r}{2} + \frac{\theta(1-\beta)(1+e\beta\xi)}{2}, & \text{Case 1A } (\beta < 1), \\ \frac{r}{2\beta} + \frac{e\theta\xi(\beta-1)}{2\beta}, & \text{Case 1A } (\beta > 1), \\ \frac{r+\theta(1-\beta)(1+\xi)}{2(1+\xi(1-\beta))}, & \text{Case 2A } (\beta < 1), \\ \frac{r+\theta\xi(\beta-1)}{2(\beta+\xi(\beta-1))}, & \text{Case 2A } (\beta > 1), \\ \frac{r}{\beta} + e\theta\xi(1-\beta), & \text{Case 3A } (\beta < 1), \\ r - \theta(\beta-1)(1-e\xi), & \text{Case 3A } (\beta \geq 1), \\ \frac{\theta}{2}, & \text{Cases 1B, 2B, 3B, and 3C.} \end{cases} \quad (3.6)$$

From Proposition 3.1, we can see that $p^*(\theta)$ increases with both r and e in Case 1A—as either type of enforcement increases, piracy declines, leading to a greater pricing power for the manufacturer. In Case 1B, however, the manufacturer disregards the pirates completely while setting the price, which is why $p^*(\theta)$ no longer depends on the enforcement levels.

3.5. Results

In this section, to study the effects of changing the enforcement levels, I perform comparative statics on a set of relevant metrics, with respect to e and r . As briefly mentioned earlier in §3.3.3, I restrict this analysis to the case where $\eta \in (0, 1)$, that is, only to Regions 1A and 1B, which collectively I call the *primary piracy region*:

Definition 3.1. *Cases 1A and 1B, where a limited amount of supply exists—that is, $0 < \eta < 1$, $\eta \neq 0, 1$ —is henceforth called the primary piracy region.*

Even though I am able to analytically characterize all the seven regions in the parameter space (see Appendix B.1), the only interesting region for my investigation is the primary piracy region where η is fractional. The extreme cases $\eta = 1$ (Regions 2A and 2B) and $\eta = 0$ (Regions 3A, 3B, and 3C) are uninteresting for the following reasons. First, in these regions, η is fixed at a corner value and does not respond to the changes in supply-side enforcement, e . Since I am interested in comparing the impacts of e and r , these regions are naturally of little relevance here. Second, and perhaps more importantly, there is empirical evidence that piracy is indeed impacted by a change in supply-side enforcement (Danaher et al. 2014), making the primary piracy region the only practically relevant one. Third, mathematically, the case of $\eta = 0$ is rather trivial as, then, there would be no piracy, and the other extreme case, $\eta = 1$, has already been studied (cf. Lahiri and Dey 2013).

3.5.1 Metrics

Although my primary metric for comparing anti-piracy choices is *social welfare*—and arguably should be the only one—I consider several secondary metrics as well, mainly because of their practical relevance. *Piracy rate* is often touted as a measure of significance—a mention of this rate invariably shows up in claims made by manufacturers and their alliances to shore up support for higher levels of enforcement on both sides, and, at the same time, piracy rate is much used as a valuable yardstick in policy debates and in the legal parlance (Karaganis 2011). The manufacturer’s *profit* is also an important yardstick as it usually serves as an indicator of the health of the industry. Finally, *consumer surplus* is a useful metric to understand the basic economic rationale behind consumer activism and lobbying efforts. I now define all these metrics for the short-run case, that is, as a function of θ .

Piracy Rate The *piracy rate*, μ , is defined as the number of pirated copies in use as a fraction of the total user base: $\mu = \frac{\hat{q}}{q+\hat{q}}$. Substituting the equilibrium price from (3.6) into (3.2) and (3.3), we get:

$$\mu(\theta) = \begin{cases} \frac{r(2-\beta)-\beta\theta(1-\beta)(1-e\xi(2-\beta))}{2(1-\beta)(r-\beta\theta(1-e\xi(1-\beta)))}, & \text{Case 1A } (\beta < 1), \\ \frac{r(2\beta-1)-\theta(\beta-1)(e\xi+2\beta(1-e\xi))}{(\beta-1)(r+\theta(e\xi(\beta-1)-2\beta))}, & \text{Case 1A } (\beta > 1), \\ \frac{2(\beta\theta-r)(r-\beta\theta(1-e\xi))}{2r\beta\theta(2-e\xi)-\beta^2\theta^2(2-e\xi)-2r^2}, & \text{Case 1B.} \end{cases} \quad (3.7)$$

Manufacturer’s Profit The manufacturer’s revenue—or profit since the marginal cost is zero—in equilibrium is $\pi^*(\theta) = p^*(\theta)q(\theta, p^*(\theta)) - \frac{c\theta^2}{2}$. As before, substituting (3.6) into (3.2), we get:

$$\pi^*(\theta) = -\frac{c\theta^2}{2} + \begin{cases} \frac{(r+\theta(1-\beta)(1+e\beta\xi))^2}{4\theta(1-\beta)}, & \text{Case 1A } (\beta < 1), \\ \frac{(r+e\theta\xi(\beta-1))^2}{4\beta\theta(\beta-1)}, & \text{Case 1A } (\beta > 1), \\ \frac{e\xi\beta\theta^2}{4(\beta\theta-r)}, & \text{Case 1B.} \end{cases} \quad (3.8)$$

Consumer Surplus I consider the total surplus of *all* consumers, legal and illegal. Using Figure 3.1, this surplus can be estimated from:

$$CS(\theta) = \begin{cases} \lambda \int_{\frac{p}{\theta}}^1 (v\theta - p)dv + (1-\lambda) \int_{\frac{p}{\theta}}^1 (v\theta - p)dv + (1-\lambda) \int_{\frac{r}{\beta\theta}}^{\min\{1, \frac{p-r}{(1-\beta)\theta}\}} (v\beta\theta - r)dv, & \text{if } \beta < 1, \\ \lambda \int_{\frac{p}{\theta}}^1 (v\theta - p)dv + (1-\lambda) \int_{\frac{r-p}{(\beta-1)\theta}}^{\min\{1, \frac{r-p}{(\beta-1)\theta}\}} (v\theta - p)dv + (1-\lambda) \int_{\frac{r}{\beta\theta}}^{\min\{1, \max\{\frac{r}{\beta\theta}, \frac{r-p}{(\beta-1)\theta}\}} (v\beta\theta - r)dv, & \text{otherwise.} \end{cases}$$

Substituting the equilibrium price from (3.6), we get:

$$CS(\theta) = \begin{cases} \frac{r^2(4-3\beta)}{8\beta\theta(1-\beta)} - r \left(\frac{3}{4} - \frac{e\xi(1-\beta)}{2} \right) - \frac{e^2\beta^2\theta\xi^2(1-\beta)}{8} - \frac{e\beta\theta\xi(1-\beta)}{2} + \frac{\theta(1+3\beta)}{8}, & \text{Case 1A } (\beta < 1), \\ \frac{r^2(4\beta-3)}{8\beta\theta(\beta-1)} + r \left(\frac{e\xi(\beta-1)}{2\beta} - 1 \right) - \frac{e^2\theta\xi^2(\beta-1)}{8\beta} - \frac{e\theta\xi(\beta-1)}{2} + \frac{\beta\theta}{2}, & \text{Case 1A } (\beta > 1), \\ \frac{e\beta\theta^2\xi}{8(\beta\theta-r)} + \frac{(\beta\theta-r)(\beta\theta(1-e\xi)-r)}{2\beta\theta}, & \text{Case 1B.} \end{cases} \quad (3.9)$$

Social Welfare For a zero marginal cost digital good, the total social welfare is estimated from the consumption benefits of *all* consumers, legal and illegal, taken together, minus the cost of development:⁵

$$SW(\theta) = -\frac{c\theta^2}{2} + \begin{cases} \lambda \int_{\frac{p}{\theta}}^1 v\theta dv + (1-\lambda) \int_{\frac{p}{\theta}}^1 v\theta dv + (1-\lambda) \int_{\frac{r}{\beta\theta}}^{\min\{1, \frac{p-r}{(1-\beta)\theta}\}} v\beta\theta dv, & \text{if } \beta < 1, \\ \lambda \int_{\frac{p}{\theta}}^1 v\theta dv + (1-\lambda) \int_{\frac{r-p}{(\beta-1)\theta}}^{\min\{1, \frac{r-p}{(\beta-1)\theta}\}} v\theta dv + (1-\lambda) \int_{\frac{r}{\beta\theta}}^{\min\{1, \max\{\frac{r}{\beta\theta}, \frac{r-p}{(\beta-1)\theta}\}} v\beta\theta dv, & \text{otherwise.} \end{cases}$$

⁵In estimating the social surplus, I do not consider the cost of either type of piracy enforcement. This is because both r and e are exogenous parameters in the model. Thus, my modeling experiment does not aim to provide justification for additional investment in either type of enforcement activities. My purpose is simply to find an answer to this question: if the government has already decided to invest in piracy enforcement, where is that marginal dollar better spent, on the supply or the demand side?

Once again, substituting the equilibrium price from (3.6), after some algebra, we obtain:

$$SW(\theta) = -\frac{c\theta^2}{2} + \begin{cases} -\frac{r^2(4-3\beta)}{8\beta\theta(1-\beta)} + r\left(\frac{1}{4} - \frac{e\xi(1-\beta)}{2}\right) + \frac{e^2\beta^2\theta\xi^2(1-\beta)}{8} + \frac{\theta(3+\beta)}{8}, & \text{Case 1A } (\beta < 1), \\ -\frac{r^2(4\beta-3)}{8\beta\theta(\beta-1)} - \frac{er\xi(\beta-1)}{2\beta} + \frac{e^2\theta\xi^2(\beta-1)}{8\beta} - \frac{e\theta\xi(\beta-1)}{2} + \frac{\beta\theta}{2}, & \text{Case 1A } (\beta > 1), \\ \frac{3e\beta\theta^2\xi}{8(\beta\theta-r)} + \frac{\beta^2\theta^2-r^2}{2\beta\theta} - \frac{e\xi(\beta\theta+r)}{2}, & \text{Case 1B.} \end{cases} \quad (3.10)$$

For completeness, I also perform comparative statics on the *legal social welfare* (SW_L), which excludes the surplus generated by illegal use, mostly because certain governments or policymakers may be interested in this surplus as a secondary metric. Considering the illegal surplus when choosing policy directions may, at times, be politically inconvenient. Furthermore, just as manufacturers do not like lost sales due to piracy, governments may also not like losses in tax revenues. Therefore, it is only practical that anti-piracy efforts are often moderated by political and pecuniary calculations.

3.5.2 Comparative Statics

Now, I proceed to answer my research questions regarding the economic impacts of the two potential anti-piracy instruments, namely, r and e . In general, I am interested in identifying whether the impact of changing one is more desirable than that of changing the other from the perspectives of the manufacturer, consumers, and policymakers. The manufacturer would, of course, want a higher profit; likewise, consumers would prefer a higher consumer surplus, and policymakers, a higher social surplus. Further, it is likely that the manufacturer, and sometimes the policymakers, would also prefer to see a decrease in the piracy rate. To see if these expectations can be met by an anti-piracy instrument, I am particularly interested in whether the impact of changing r or e is uniformly favorable throughout the primary piracy region. Why do I seek such uniformity? After all, every product or its market is represented by a single point in the parameter space and, to understand how its online piracy is impacted by changes in enforcement, a simple point-specific comparative statics should have sufficed. However, the purpose of this research is to identify possible unintended consequences of

different piracy enforcement activities and to seek guidance accordingly. A recommendation can be useful only if the nature of its impact is not tied to a few specific products. Put differently, there is little comfort in knowing that a prescribed approach would work only for some products facing piracy, and not all, as having different anti-piracy policies in place at the same time could be quite impractical.

Theorem 3.1. *In the short run, if $\beta < 1$, social welfare is not monotonic in the primary piracy region with respect to either r or e . On the other hand, if $\beta \geq 1$, social welfare is monotonically decreasing in both r and e throughout the primary piracy region.*

As either type of enforcement increases, piracy declines and, with it declines the consumer surplus. When $\beta \geq 1$, the negative impact of enforcement on the consumer surplus dominates, and social welfare decreases with both r and e . Interestingly, though, this relationship need not hold when $\beta < 1$. This may seem counterintuitive at first. After all, for a product with zero marginal cost, the social surplus should increase with the overall consumption level, but stronger enforcement means only a reduction in consumption. Then, why should higher enforcement not always result in a smaller social surplus, just as it does for $\beta \geq 1$? Actually, when β is small, the value generated by one consumer converting from illegal to legal generates a large upsurge (specifically, by a factor of $\frac{1}{\beta}$) in the legal surplus, which can offset the reduction in consumption.

Theorem 3.1 also shows that, for $\beta < 1$, neither enforcement policy uniformly dominates the other one. Each instrument has some positive impact, but only in parts of the primary piracy region. As mentioned above, the situation with $\beta \geq 1$ is even bleaker. There, both types of enforcement, supply- and demand-side, have a uniformly negative impact on social welfare. The implication is clear. When it is difficult to change the quality level of the product—for example, if the current state of the technology imposes a quality ceiling—very strict piracy restrictions may have an unintended, and perhaps even undesirable, impact on

social welfare. Enforcement authorities must consider such possibilities in formulating their directions. Now, what impact does piracy enforcement have on the secondary metrics? The next set of results answers that question:

Proposition 3.2. *In the short run, over the entire primary piracy region, the following relationships are observed:*

- i) The piracy rate is monotonically decreasing in both r and e .*
- ii) The manufacturer's profit is monotonically increasing in both r and e .*
- iii) The consumer surplus is monotonically decreasing in both r and e .*
- iv) The legal social welfare is monotonically increasing in both r and e .*

Proposition 3.2 has several important insights. First, the two types of enforcement are still indistinguishable in terms of their impacts. In other words, the secondary metrics, too, do not provide any clear anti-piracy directions for products with exogenous quality. Second, this indistinguishability aside, piracy enforcement seems to have a more desirable impact on the secondary metrics, with the only exception of consumer surplus. Third, Proposition 3.2 states that the consumer surplus decreases with enforcement even as the manufacturer's profit increases, suggesting that the extra legal surplus generated by additional enforcement activities accrues mostly to the monopolist manufacturer and is not duly shared with consumers. Finally, all the findings in Proposition 3.2 consistently support the basic message in Theorem 3.1 that, in the short run, anti-piracy efforts, despite the best of intentions, can sometimes have undesirable consequences. Moderation and careful considerations are needed in formulating enforcement strategies towards online piracy.

3.6. Discussion

For the sake of clarity, I now summarize my results in Table 3.2. Each cell in this table shows the direction of the impact of an anti-piracy measure for each of the economic metrics

Table 3.2: Economic Impacts of Demand- and Supply-Side Enforcement (Short Run)

| Enforcement → Metric ↓ | Demand-Side ($r \uparrow$) | | Supply-Side ($e \uparrow$) | |
|-----------------------------|------------------------------|----------------|------------------------------|----------------|
| | $\beta < 1$ | $\beta \geq 1$ | $\beta < 1$ | $\beta \geq 1$ |
| <i>Social Welfare</i> | ↑↓ | ↓** | ↑↓ | ↓** |
| <i>Piracy Rate</i> | ↓* | ↓* | ↓* | ↓* |
| <i>Manufacturer Profit</i> | ↑* | ↑* | ↑* | ↑* |
| <i>Consumer Surplus</i> | ↓** | ↓** | ↓** | ↓** |
| <i>Legal Social Welfare</i> | ↑* | ↑* | ↑* | ↑* |
| Overall Score | 2 | 1 | 2 | 1 |

NOTE: *direction desirable; **direction undesirable

I study. Whether such a direction is socially desirable or not is also indicated in the table. These can be further consolidated into *overall scores* counted as follows: Each cell with a desirable impact contributes a +1 to the overall score, but -1 if the impact is undesirable. When the directionality is unclear, no contribution is made.

However we slice it—whether we look at the individual metrics or the overall score—Table 3.2 seems to convey one clear message: despite the obvious difference between demand- and supply-side approaches, their eventual outcomes may not be so different. In other words, the difference between the two approaches may actually be only superficial, and any anti-piracy measure would move the metric of interest in the same direction. Perhaps, this is the impression that people already have, and the reality of scattered efforts across both camps of anti-piracy measures simply reflects that attitude.

However, that is not to say that all parties feel the same about anti-piracy measures. Often, the most important metric for policy debates is social welfare. In my short-run analysis, both types of enforcement seem to do well in this regard, especially when we look at the legal social welfare. The actual comparison, however, is a bit more subtle. Here, an increase in social welfare is primarily driven by a larger profit. In other words, it is possible

to argue that most of the surplus generated through enforcement activities of either type would mainly go to the manufacturer, with very little coming the consumers' way. This imbalance, sometimes coupled with additional inconveniences faced by consumers, perhaps explains why there have been so many instances of consumer uproar against enforcement legislation and activities (Bachman 2011, BBC 2008, Guarini 2013, Krebs 2005).

Finally, when it comes to choosing an anti-piracy measure, in addition to the general direction of impact that I discuss here, the cost of enforcement is also an important factor to consider. Related to the costs of enforcement are the issues of scale and effectiveness of anti-piracy measures. Given that both types of enforcement move the profit and welfares in the same direction, the natural next aspect to consider in choosing anti-piracy measures would be their relative effectiveness: If we were to invest our next dollar in curbing piracy, which approach would yield a greater return? It is a question that could only be answered empirically, and I leave that for future work. Already, some researchers are looking into such a problem (Danaher et al. 2015).

3.7. Conclusion

Much like any other markets, the “market” for pirated goods has both consumers and suppliers, and intuitively, anti-piracy efforts can be directed toward suppressing either the demand or the supply. The objective of demand-side enforcement, which has long been in use, is to reduce piracy by depressing the demand for pirated content. Such enforcement primarily involves making the pirated product less attractive by imposing penalties for illegal use. On the other hand, supply-side enforcement, which has lately started gaining popularity, does not aim to shift the demand curve—it simply pushes the supply curve down, making pirated content less available. It involves combating piracy essentially at its source, for example, by limiting the reach of pirate suppliers through shutting down their websites, filtering them out from search-engine results, and by prosecuting illegal content distribution. In this work,

I construct a framework to compare these two types of enforcement in terms of their impacts on economic metrics such as profit and welfare.

After witnessing abundantly available pirated products online—which are freely available for anyone to download—one may wonder: “What incentives do people have to upload and make all the pirated products available for free?” While some have proposed fame and ideologies—to freely share information for the greater good of the society—to be the propellants, actually, there might be a simpler reason; money. In piracy ecosystem, pirated products lure consumers to pirate sites, and the traffic generates ad revenues. In some cases, to promote suppliers to upload pirated contents, aggregator sites—such as cyberlockers—directly pay individual uploaders based on the number of downloads. It is this reality that I capture in my model.

Endogenizing the supply side of piracy ecosystem allows us to compare demand-side and supply-side anti-piracy measures. It turns out that, in the short run—where the manufacturer cannot change the quality of the product—the two types of enforcement seem quite indistinguishable in their effect on various economic metrics that I consider here. So, despite the obvious difference between the two types, perhaps, there is no strong reason to discriminate them after all. Keep in mind, though; that message is derived from the short-run analysis. Given the close relationship between piracy and innovation—how the industry generally argues that piracy kills innovation, and how the recent paper by Lahiri and Dey (2013) argues the opposite—it is natural to wonder if the findings from the short-run analysis would automatically extend to the long-run case. That is what I investigate in the following Chapter 4.

My modeling work in this chapter is limited by its simplicity, which is actually a principle that I strive to follow. In reality, the ecosystem that supports online piracy is vastly more complex. This ecosystem often involves many legitimate business entities, such as online payment systems, search engine providers, and advertisement services; they all benefit from

piracy (Seidler 2011). Thus, it is difficult to attribute legal liabilities to any specific entity. It is this difficulty that essentially makes the supply side of piracy a rather nebulous concept around which a definitive boundary is often difficult to draw. Moreover, many blog owners involved in piracy only provide links to pirated content but do not actually host any. Such instances create tensions between stopping online piracy and protecting freedom of speech. Clearly, all these issues need to be examined carefully in order to better understand the supply side of piracy and related policy implications. Furthermore, the manufacturer—either individually on its own or collectively through an industry group—could attempt to influence the public policy debate by actively engaging in political lobbying. The extent of such an action, its direct impact on the enforcement levels, and the consequent spillover effects on the price and quality decisions are also among the issues that I do not address. Despite such limitations, this work would have achieved its goal if it has succeeded in providing a new economic framework through which other researchers can conceptualize different anti-piracy measures.

Chapter 4

Combating Online Piracy: Long-Run Analysis

4.1. Introduction

In the previous chapter, I construct a framework that allows for a clear comparison between anti-piracy efforts that are geared towards punishing the illegal users (*demand-side* enforcement) and the ones that focus on taking down pirate sites and discouraging pirate uploaders (*supply-side* enforcement). I find that, when the product quality is static and the manufacturer cannot change the level of quality in response to piracy, the two anti-piracy approaches—despite the apparent difference—may not be so different in their eventual economic outcomes. For certain digital goods, their manufacturers may not be in a position to respond to piracy by altering quality, and in such cases, analysis of the short-run equilibrium would not only be appropriate but also sufficient. However, in many other real-world situations, a manufacturer may be able to adjust the quality level in response to piracy (Jain 2008, Lahiri and Dey 2013). There, it is important to consider the long-run decision, in which the manufacturer has control over θ as well. Recent research shows that a manufacturer may, in fact, respond to a higher level of demand-side enforcement by decreasing the quality of its product and lowering welfare in the process (Lahiri and Dey 2013), contradicting the common argument that stronger enforcement is necessary to foster innovation.

In this chapter, extending the previous one, I investigate whether demand- and supply-side enforcements have different impacts on the manufacturer's incentive to innovate, and if so, how those impacts propagate to other metrics of interest (e.g., piracy rate, profit, and welfare). Leveraging much of the model setup and the literature review in Chapter 3, I swiftly delve into the equilibrium analysis and results in this chapter.

In my effort, I find that supply-side enforcement, when compared to its demand-side counterpart, has a more favorable impact on innovation and welfare in the long run, for all types of digital goods afflicted with piracy. This superiority of supply-side enforcement may seem puzzling a bit—after all, either type of enforcement essentially makes piracy more difficult. How could their impacts then be so different? A closer examination reveals a subtle interplay between the enforcement type and the piracy ecosystem. Consistent with prior literature (Lahiri and Dey 2013), a higher level of demand-side enforcement makes the pirated version less appealing to consumers, and the weakening “shadow” competition from the pirated product, in turn, makes the manufacturer respond with a lower quality. Stronger supply-side enforcement, on the other hand, has no impact on the relative appeal of piracy; it only makes pirated content less available. This reduction in supply effectively expands the legal consumer base, yielding better returns on investments in product development, thereby inducing the manufacturer to invest more in quality. It is this positive impact on quality that eventually reflects itself more favorably on social welfare.

My findings relate well to the current literature and extend them logically. Results in prior research are often mixed. Some show that a lack of enforcement and higher piracy can decrease the manufacturer's revenue, killing incentives to innovate and leading to lower quality products (Bae and Choi 2006, Jain 2008). Lahiri and Dey (2013), however, tell a different story. They argue that, in certain circumstances, less (demand-side) enforcement may surprisingly lead to higher quality products, and eventually to higher consumer and social welfare. At the core of these results lies an argument that the manufacturer can

leverage a higher quality to “compete” against piracy; so, when (demand-side) enforcement is weak, the manufacturer simply responds with a lower quality, which, in turn, adversely impacts the legal consumers. I find that this argument continues to apply to my wider setting—the model in Lahiri and Dey (2013) is essentially a special case of my model—but only so far as demand-side enforcement is concerned. Interestingly, it does not extend to the supply side—the impact of supply-side enforcement on innovation is often exactly the opposite, and its long-run impacts on consumer and social welfare, strikingly different.

4.2. Equilibrium Analysis

In the long run, in addition to the price, the manufacturer has an extra lever it can use in response to piracy: The product quality. The manufacturer can decide on the level of investments in product development and associated R&D activities, which determines the quality of the resulting product. Hence, the product quality I consider here is directly connected to the manufacturer’s incentive to innovate and the overall level of innovation in the industry. We can write the manufacturer’s long-run decision problem as: $\max_{\theta} \pi(\theta) = p^*(\theta) q(\theta, p^*(\theta)) - \frac{c\theta^2}{2}$, where all the expressions are the same as in Chapter 3; see §3.3 for details.

Figure 4.1 illustrates the partitions of the (r, e) space for the same values of β and ξ as in Figure 3.2 in Chapter 3; a formal characterization is available in Appendix C.1. It is not surprising that the two figures—Figures 3.2 and 4.1—are qualitatively similar. There are, however, two main differences. First, since the quality level is now a function of e and r , the boundaries in Figure 4.1 are different from the ones in Figure 3.2. This shifting of boundaries results in a substantial expansion of Region 3A in the long-run case. Second, since the manufacturer now has discretion over quality, Case 3B becomes a possibility, resulting in the emergence of Region 3B, where the manufacturer chooses quality as a tool to eliminate

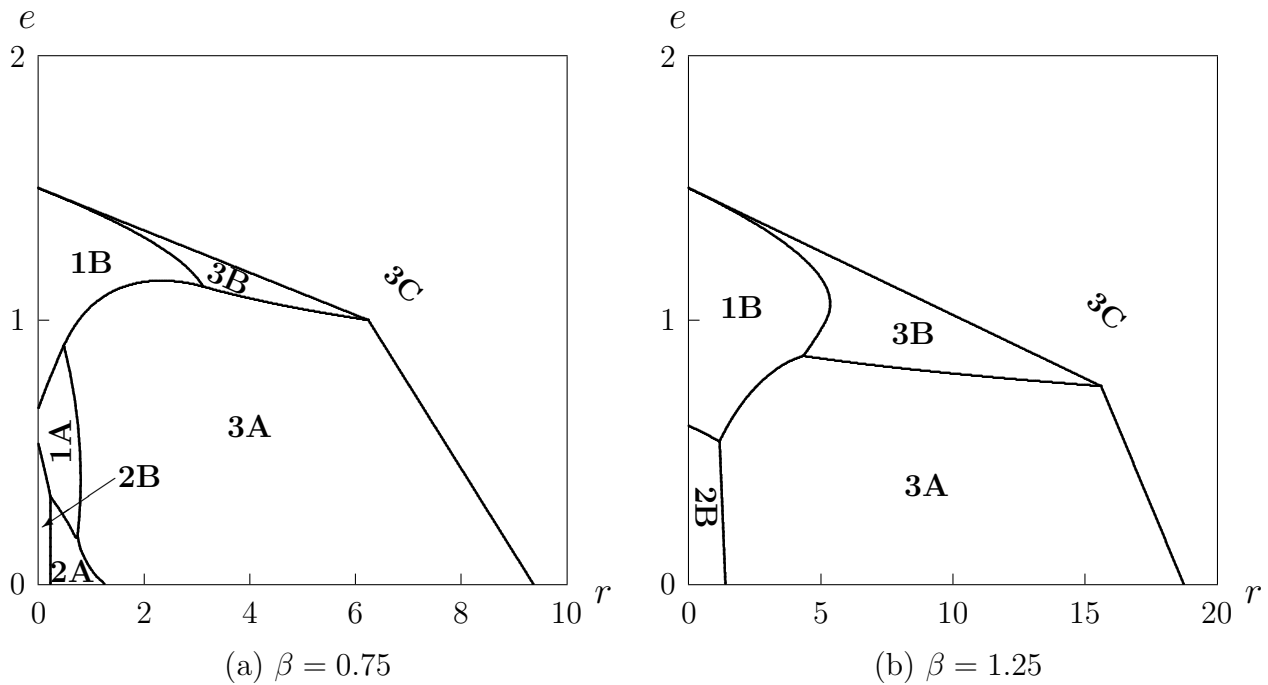


Figure 4.1: Relevant Partitions of the (r, e) Space in the Long Run; $\xi = \frac{2}{3}$, $c = 0.01$

piracy, just as it uses price to do so in Case 3A. Unlike the limit price, however, the idea that the manufacturer can use a limit quality as a means to squeeze out piracy is has not been identified in prior literature.

Lemma 4.1. *In the long-run equilibrium, Cases 1A and 2A are not possible when $\beta \geq 1$.*

Lemma 4.1 simply suggests that competing with the pirated product is futile when β is large. Rather, the manufacturer is better off just concentrating on the λ portion of the market with no access to pirated content.

Proposition 4.1. *Let θ_{1A} and θ_{2A} be the largest positive roots of the following two cubic equations, respectively:*

$$\frac{(1 - \beta)(1 + e\beta\xi)^2}{4} - \frac{r^2}{4\theta^2(1 - \beta)} - c\theta = 0 \text{ and}$$

$$\frac{(1 - \beta)(1 + \xi)}{4(1 + \xi(1 - \beta))} - \frac{r^2}{4\theta^2(1 - \beta)(1 + \xi)(1 + \xi(1 - \beta))} - c\theta = 0.$$

Further, let θ_{3AH} and θ_{3AK} be the unique positive roots of the following two cubic equations, respectively:

$$e\xi(1-\beta)(1-e\xi(1-\beta)) + \frac{r^2}{\beta^2\theta^2} - c\theta = 0, \text{ and}$$

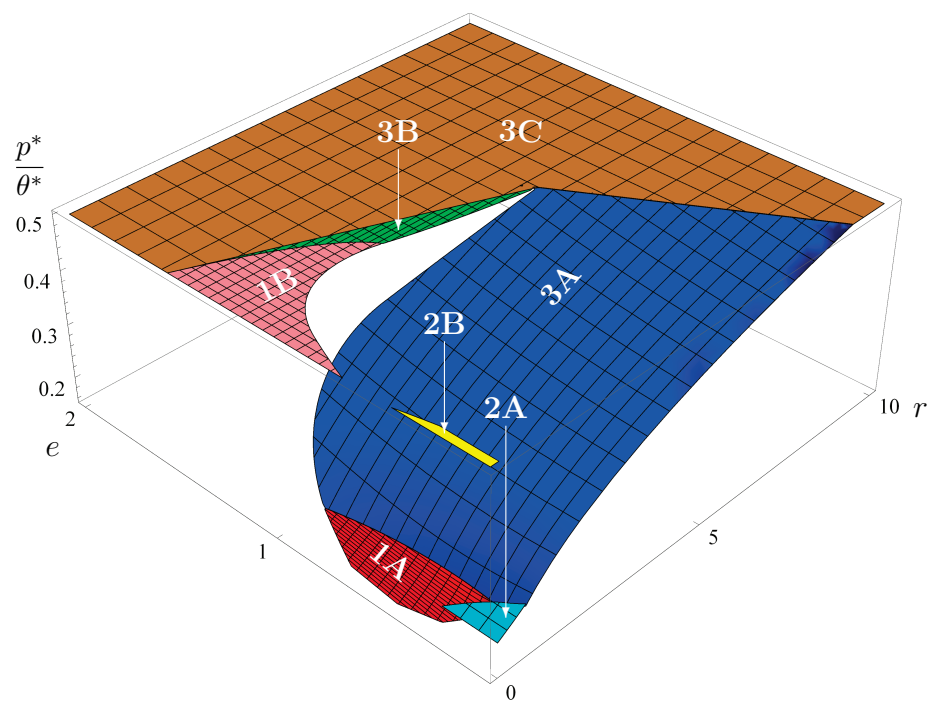
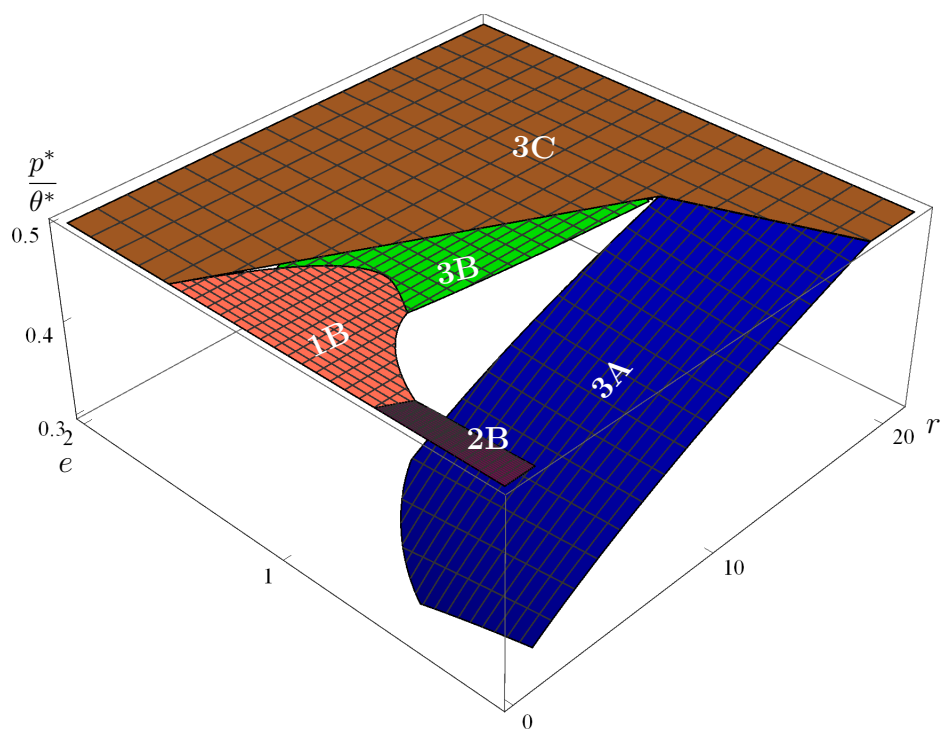
$$(\beta-1)(1-e\xi)(e\xi(\beta-1)-\beta) + \frac{r^2}{\theta^2} - c\theta = 0.$$

The equilibrium quality is then given by:

$$\theta^* = \begin{cases} \theta_{1A}, & \text{Case 1A } (\beta < 1), \\ \theta_{1B} = \frac{r}{\beta} + \frac{e\beta\xi + \sqrt{e\beta\xi(e\beta\xi - 16cr)}}{8c\beta} & \text{Case 1B}, \\ \theta_{2A}, & \text{Case 2A } (\beta < 1), \\ \theta_{2B} = \frac{\xi}{4c(1+\xi)}, & \text{Case 2B}, \\ \theta_{3AH}, & \text{Case 3A } (\beta < 1), \\ \theta_{3AK}, & \text{Case 3A } (\beta \geq 1), \\ \theta_{3B} = \frac{r}{\beta(1-e\xi)}, & \text{Case 3B}, \\ \theta_{3C} = \frac{1}{4c}, & \text{Case 3C}. \end{cases} \quad (4.1)$$

The equilibrium price in the long run can also be obtained by substituting θ^* from (4.1), on a case-by-case basis, into the optimal price expression (3.6) from Chapter 3; see §3.4 for details. It can be verified that the equilibrium price has a trend that is quite similar to that of the equilibrium quality. Put differently, the manufacturer decides to invest in quality only when it can recover the additional investment through a higher price. In order to see this relationship more clearly, I plot the equilibrium price-to-quality ratio in Figure 4.2; this ratio is essentially a metric for the relative competitiveness of the legal product against its pirated version—the higher the ratio, the lower is the competitiveness.

There are several interesting observations that can be made from this figure. First, in equilibrium, the price-to-quality ratio is constant in Regions 1B, 2B, 3B, and 3C. In Cases 1B and 2B, the manufacturer has given up on consumers who have access to pirated goods, and it essentially operates as a monopolist over the λ segment of consumers.

(a) $\beta = 0.75$ (b) $\beta = 1.25$ Figure 4.2: Equilibrium Price-to-Quality Ratio as a Function of e and r ($\xi = \frac{2}{3}$, $c = 0.01$)

Thus, in Cases 1B, 2B, and 3C, the manufacturer cares little about piracy, and the ratio—the relative competitiveness—is not impacted by either demand- or supply-side enforcement. What is perhaps unexpected is that the situation is also the same in Case 3B, where the threat of piracy is neither absent nor has the manufacturer decided to ignore this threat in its quality decision.

The behavior of the price-to-quality ratio in the other three regions (1A, 2A, and 3A) is also curious. In these regions, the ratio increases with r , but remains quite flat with the change in e . As r increases, pirated content becomes less attractive, easing the pressure on the manufacturer and allowing it to command a relatively higher price. On the other hand, e has little impact on the relative competitiveness, since the attractiveness of pirated content remains the same. An increase in e only restricts the supply of pirated content, but the manufacturer has to stay competitive for the fraction of consumers that still have access to the pirated version. I elaborate on this point in the following section.

4.3. Results

Quality is often an important strategic decision for many manufacturers of digital products, and endogenizing it is necessary for obtaining insights applicable to these products. There is another motivation for endogenizing quality—some policymakers may be specifically interested in learning the overall long-term impact of an enforcement choice, and, often times, long-term decisions center around the notion of innovation, which plays a crucial role in the development and design of a large majority of information goods. Quality of a digital good is frequently used as an indicator of innovation, and a higher quality is usually viewed as socially desirable (Brynjolfsson and Zhang 2007). Economic history of nations provides sufficient testimony that the concept of innovation is important for public policy, as the former is often linked with an economy’s rapid growth (cf. Grossman and Helpman 1993). Therefore, I start the analysis with the impact on innovation:

Theorem 4.1. *In the primary piracy region, the long-run equilibrium quality is always increasing in e but decreasing in r .*

In other words, I find that supply-side enforcement provides, to manufacturers engaged in the research, development, and design of a large variety of information goods, added incentives to innovate and invest in quality. This result is quite surprising, especially in light of exactly the opposite impact from demand-side enforcement. After all, any enforcement effort, supply- or demand-side, essentially makes it more difficult to pirate, one way or the other. Why then should they impact the equilibrium quality so differently?

The answer lies in how different enforcement types reveal themselves in the market and the piracy ecosystem. When there is an increase in demand-side enforcement, the pirated version appears less attractive to a potential consumer. A lessening “shadow” competition from the pirated copy is met with diminishing aggression from the manufacturer, in terms of a lower quality, at either the same or a higher price-to-quality ratio. On the other hand, when supply-side enforcement is stepped up, the relative attractiveness of pirated content does not suffer; only its supply reduces, making it less available to a potential copyright violator. A portion of the consumer base, however, can still access the pirated copy. If the manufacturer wishes to remain competitive for this consumer segment, it can ill afford to drastically reduce the quality or jack up the price-to-quality ratio, and this hesitance from the manufacturer is illustrated in Figure 4.2. In fact, irrespective of whether the manufacturer wants to cater to this segment or not, the presence of a bigger λ segment now provides the manufacturer with a marginally better return from investing in quality and allows the manufacturer to leverage their “loyalty” by increasing the quality level. This is essentially at the crux of the model and its results.

Although the result in Theorem 4.1 is quite edifying in itself, we must still consider social welfare to be the primary metric. All the metrics have already been defined in §3.5 for the short-run case, and the corresponding long-run versions are easily obtained by substituting

θ with its case-appropriate value from (4.1).¹

Theorem 4.2. *In the long run, if $\beta < 1$, social welfare is increasing in e over the entire primary piracy region but is not monotonic in r . For $\beta \geq 1$, social welfare is decreasing in r throughout the primary piracy region but is not monotonic in e .*

Evidently, in the primary piracy region, the two enforcement approaches have contrasting effects on social welfare. Theorem 4.2 states that, when $\beta < 1$, increasing supply-side enforcement has an overall positive impact on social welfare, but the impact from the demand side is not uniformly positive. In fact, my extensive numerical experiments suggest that, unless r or β is very small, this impact is largely negative and, as β increases beyond a threshold, social welfare actually becomes monotonically decreasing in r . Theorem 4.2 also states that, for $\beta \geq 1$, the impact of demand-side enforcement becomes uniformly negative. In contrast, for $\beta \geq 1$, the impact of e on social welfare is not monotonic—it is mostly positive, except when e is excessively large. Taken together, it appears that supply-side enforcement, overall, has a more desirable impact on social welfare when compared to its demand-side counterpart.

A point is in order. Although, for the sake of completeness, I provide analyses for both cases, $\beta < 1$ and $\beta \geq 1$, prior research has mostly considered the former (e.g., August and Tunca 2008, Bae and Choi 2006, Chellappa and Shivendu 2005, Chen and Png 2003, Jaisingh 2009, Lahiri and Dey 2013, Sundararajan 2004). The literature correctly recognizes that pirated products are usually inferior. Indeed, as I have mentioned earlier in §3.3.2, it is quite common for stolen product keys to not work, illegal copies of software to lack manufacturers' support, or illegally downloaded movies or music to not have the same resolution. Given this reality, anti-piracy discussions are expected to focus primarily on this class of information

¹Recall from my earlier discussion in §3.3.1 that r and e represent the given enforcement environment, and my analysis does not include the enforcement costs. Results in Theorem 4.2 and Proposition 4.2 should, therefore, be interpreted accordingly.

goods for which $\beta < 1$. What I find here is that, for this predominant class, supply-side enforcement is not only better than its demand-side counterpart but is also uniformly desirable across the entire primary piracy region. Moreover, as I will show shortly, whether or not $\beta < 1$, supply-side enforcement increases the social welfare generated from legal sales (that is, welfare excluding pirates) throughout the primary piracy region, whereas demand-side enforcement again falls short, further strengthening the appeal of supply-side measures from a long-run perspective.

The reason supply-side enforcement outperforms demand-side is also telling. As we have seen already, from a short-run perspective, they are similar. In other words, if we hold quality fixed, they would have similar impacts on both consumption and welfare. However, in the long run, quality is not fixed. In particular, as Theorem 4.1 establishes, θ is increasing in e but not in r . Thus, every dollar spent on raising e instead of r actually generates more innovation. It is this innovation that enhances the value from consumption, eventually translating to a higher social welfare, legal or otherwise, making the two types of enforcement fundamentally different from a long-run perspective.

In order to complete the comparison between the two approaches, I also look at the impacts on the secondary metrics:

Proposition 4.2. *In the long run, the following relationships are observed across the entire primary piracy region:*

- i) The piracy rate is monotonically decreasing in r , but it is not monotonic in e .*
- ii) The manufacturer's profit is monotonically increasing in both r and e .*
- iii) The consumer surplus is monotonically decreasing in r , but not in e .*
- iv) The legal social welfare is monotonically increasing in e , but not in r .*

There are several interesting observations that can be made from Proposition 4.2. First, even though the piracy rate decreases as r increases, the impact of e is not uniform, that

is, in certain portions of the piracy region, the piracy rate could actually increase with e . This may seem surprising. After all, the first order effect of a higher level of enforcement, regardless of its type, ought to be a reduction in the piracy rate. At least, that is what is widely believed. A quick Google search, for example, on “how to reduce the piracy rate” returns links to a large set of articles and news items, all of which point to higher levels of enforcement—a higher r or e , or both. To see the intuition behind this result, however, we must also consider the second order effect—the increase in quality resulting from a higher e has a positive impact on the illegal demand, which works in a direction contrary to the first order effect. As mentioned earlier, piracy rate is often used as a valuable yardstick in policy debates and in the legal parlance (Karaganis 2011). I find that this conventional wisdom is not always correct. It makes economic sense only in the short run, or only for demand-side enforcement, but a higher supply-side enforcement, in certain real-world contexts, may actually result in a higher piracy rate in the long run. From another angle, if one is interested in merely curbing the long-run piracy rate, according to Proposition 4.2, enforcement efforts are perhaps better directed at the demand side. This is an important point because, often, “you get what you measure.” An enforcement activity geared towards reducing the long-run piracy rate may end up doing exactly that, but it might not be a socially desirable outcome, after all.

Second, Proposition 4.2 echoes the sentiment shared by most manufacturers of digital goods that ramping either type of enforcement up is good for the industry and is corroborated by empirical findings in prior research (Danaher and Smith 2014, Danaher et al. 2014). For consumer advocates, however, it provides a brand new perspective. The long-held belief has been that piracy is beneficial to consumers in several ways. Piracy creates an opportunity to use the product free of charge, which, in turn, puts a pressure on the manufacturer to decrease the price (Lahiri and Dey 2013). Besides, certain restrictions (e.g., DRM) designed to deter piracy also reduce the utility of the legal product (Guarini 2013, Krebs 2005, Vernik et al.

2011). Naturally, consumers have often rallied against overzealous anti-piracy measures.² The result shows that such an attitude can be economically justified only in the short run, or only so far as demand-side enforcement is concerned. The story may change, though, when we turn to supply-side enforcement—an increase in e results in better quality and, in certain cases (specifically, if the current enforcement level is not excessively high), could actually improve consumer welfare in the long run. Viewed differently, consumer groups should try to understand that not all enforcement activities have the same economic impact and, if properly designed and executed, a supply-side action can, in fact, enhance overall consumer welfare. Opposition to anti-piracy measures should be moderated.

Third, Proposition 4.2 also tells us that, if a policymaker is concerned about the legal social welfare in the context of piracy, supply-side enforcement is likely to provide a uniformly positive impact, whereas such impact from the demand side is often ambiguous. Thus, the results, taken together, find supply-side enforcement in a comparatively better light.

Finally, it is worth noting that, for my exploration, enforcement cost is immaterial. To understand this further, let us consider an example. Suppose there exists a central anti-piracy agency which is in charge of enforcing anti-piracy laws, and the head of the agency can decide whether to reassign an agent from enforcing one side of anti-piracy laws to another. Clearly, here, there is no cost involved as it would simply be a reallocation of available human resources. What I am interested in knowing is whether such a directional shift in anti-piracy effort could actually be beneficial. Of course, prior to seeing the results, one cannot say whether analyses purely based only on directionality would provide the necessary insight as to what class of anti-piracy measures is the better one to invest in. It is reasonable to expect both demand- and supply-side measures to move the economic metrics in the same direction—as we have seen to be the case in the short run. If that were the case, one would

²A case in point is the DRM code used by music giant Sony-BMG on its music CDs; a massive consumer uproar was triggered by this action and, within days, Sony caved in by providing a patch to remove the DRM code (Krebs 2005).

need to consider the effectiveness and cost of enforcement efforts to see which anti-piracy measures have a greater return on the “dollar” invested. Fortunately, my results do reveal some fundamental differences between the two classes of anti-piracy measures in the long run, offering us some key insights as to what might be the smarter path to take.

4.3.1 Summary and Comparison

The comparative statics presented above have important practical implications for manufacturers and consumers, along with some broader connotations for public policy. In trying to answer my research questions, I find that supply- and demand-side enforcement activities not only manifest themselves distinctly in the piracy ecosystem but also end up impacting different economic metrics very differently in the long run. A side-by-side comparison of the results from Theorem 3.1 and Proposition 3.2 to those from Theorem 4.2 and Proposition 4.2 allows us to see the stark contrast between the short- and long-run impacts. To see this contrast more clearly, I pull together the results from the short- and long-run analyses, and summarize the results in the following Table 4.1. As in §3.6, each cell in this table shows the direction of the impact of an anti-piracy measure, either in the short or long run, for each of the economic metrics I study. Also, to be able to get a quick glance of the main insight, I aggregate the overall score at the bottom. Just looking at this score board, it becomes apparent that supply-side enforcement may indeed be a superior alternative in the long run.

Several broad insights can be gleaned from the results. First, although there is little to choose between the two enforcement types in terms of their impacts in the short run, their long-run impacts are widely different, with supply-side enforcement emerging as perhaps the desirable one. The implication is clear from a perspective of anti-piracy strategy. Unless the long-run consequences are properly taken into considerations, one may end up with a wrong policy choice.

Table 4.1: Economic Impacts of Demand- and Supply-Side Enforcement

| Enforcement \longrightarrow Metric \downarrow | Demand-Side ($r \uparrow$) | | | | Supply-Side ($e \uparrow$) | | | |
|---|------------------------------|-------------------|----------------------|----------------------|------------------------------|-------------------|----------------------|----------------------|
| | Short-Run | | Long-Run | | Short-Run | | Long-Run | |
| | $\beta < 1$ | $\beta \geq 1$ | $\beta < 1$ | $\beta \geq 1$ | $\beta < 1$ | $\beta \geq 1$ | $\beta < 1$ | $\beta \geq 1$ |
| <i>Social Welfare</i> | $\uparrow\downarrow$ | \downarrow^{**} | $\uparrow\downarrow$ | \downarrow^{**} | $\uparrow\downarrow$ | \downarrow^{**} | \uparrow^* | $\uparrow\downarrow$ |
| <i>Piracy Rate</i> | \downarrow^* | \downarrow^* | \downarrow^* | \downarrow^* | \downarrow^* | \downarrow^* | $\uparrow\downarrow$ | $\uparrow\downarrow$ |
| <i>Product Quality</i> | — | — | \downarrow^{**} | \downarrow^{**} | — | — | \uparrow^* | \uparrow^* |
| <i>Manufacturer Profit</i> | \uparrow^* | \uparrow^* | \uparrow^* | \uparrow^* | \uparrow^* | \uparrow^* | \uparrow^* | \uparrow^* |
| <i>Consumer Surplus</i> | \downarrow^{**} | \downarrow^{**} | \downarrow^{**} | \downarrow^{**} | \downarrow^{**} | \downarrow^{**} | $\uparrow\downarrow$ | $\uparrow\downarrow$ |
| <i>Legal Social Welfare</i> | \uparrow^* | \uparrow^* | $\uparrow\downarrow$ | $\uparrow\downarrow$ | \uparrow^* | \uparrow^* | \uparrow^* | \uparrow^* |
| Overall Score | 2 | 1 | 0 | -1 | 2 | 1 | 4 | 3 |

NOTE: *direction desirable; **direction undesirable

As before, each cell with a desirable impact contributes a +1 to the overall score, but -1 if the impact is undesirable. When the directionality is unclear, no contribution is made.

Second, even though supply-side enforcement seems better in the long run, its short-term consequence might not be all that great. When adopting such a policy, a policymaker should be ready to see some temporary opposition due to welfare losses in the short run. A policymaker should stay the course till the long-run benefits start showing up. At the same time, consumer protection agencies and other lobbies should be aware of this distinction between the two types of enforcement and their long- and short-run impacts. They should recognize that supply-side enforcement may foster better innovation, lead to better quality products, and may actually enhance consumer welfare in the long run, despite some short-term pains. Such insights may help them bring moderation to their opposition to anti-piracy measures.

4.4. Enforcement and Market Competition

So far, I have assumed that the manufacturer operates within a monopolistic setting. This assumption is, of course, in line with the prior literature that has typically considered manufacturers of information goods to be monopolies (cf. August and Tunca 2008, Bae and Choi

2006, Lahiri and Dey 2013, among many others). Indeed, unlike their physical counterparts, information goods intrinsically lack close substitutes; see page 25. The rationale for a monopolistic setting—the lack of competition—is clearly evidenced in the price commanded by information goods, which would surely have dropped to zero under perfect competition, since information goods have negligible marginal cost (Shapiro and Varian 1999). However, since consumers have no incentive to pirate a product that is essentially free, perfect competition would also lead to a complete eradication of piracy.

I do recognize that the very existence of digital piracy in today’s marketplace actually hints at a lack of perfect competition. At the same time, it is not true that the market is fully monopolistic in reality. For, manufacturers of information products do not have the ability to name any price that they want, without losing some demand to an imperfect substitute. This point is perhaps best illustrated by the video games industry. Although a specific game does not have a real substitute, a hike in price is likely to be met by consumers moving to similar games from another publisher (Plafke 2010). Therefore, I believe that the reality is somewhere in the *middle*—between the two extremes of monopoly and perfect competition—and, though not a monopoly, the manufacturer does retain *some* pricing power. That pricing power, however, is capped because the manufacturer faces some (imperfect) competition. In this section, I wish to examine if the bulk of my findings extend to this more realistic setting.

To capture the case of imperfect competition, I assume that the manufacturer still has a unique product but now faces a cap on the price it can charge, normalized for the quality of the product. In other words, I now assume that the manufacturer’s profit maximization problem, as outlined in §3.3.7, now faces an additional constraint: $\frac{p}{\theta} \leq \delta$, where $0 < \delta < \frac{1}{2}$. Not only does such an abstraction have the advantage of faithfully capturing the reality of the *middle*, but it also easily reduces to one of the two extreme cases—when $\delta = \frac{1}{2}$, there is no real cap on the price, and we are back to the monopoly case, whereas when $\delta = 0$, the pricing power disappears completely, as should be the case in a perfect competition. That

such caps exist in markets where consumers are informed about competing alternatives is also acknowledged in the marketing literature (e.g., Steiner 1973, 1980), where it is recognized that sellers “hesitate to raise prices above that perceived cap out of fear that consumers” may consider them “overpriced on everything they sell” (Hollifield 2014, p.15).

As before, I only consider the primary piracy region (Cases 1A and 1B) in my analysis. In that region, whenever the interior solution automatically satisfies $\frac{p}{\theta} \leq \delta$ —as is the case when δ is sufficiently large—all my earlier results continue to apply without any modification, and there is nothing more to do. Therefore, the only additional case we must examine now is when the interior solution actually violates $\frac{p}{\theta} \leq \delta$, in which case we would end up with a boundary solution of $\frac{p}{\theta} = \delta$.

Lemma 4.2. *If the price cap is binding, that is, if $\frac{p}{\theta} = \delta$, then Case 1B is not possible in the long-run equilibrium.*

Lemma 4.2 is intuitive. Recall that, in Case 1B, the monopolist manufacturer completely ignores the $(1 - \lambda)$ segment with access to pirated content and charges a monopoly price to the λ segment. However, when there is a real cap on the price the manufacturer can charge, the profit from just the λ segment decreases. The manufacturer can no longer ignore the $(1 - \lambda)$ segment and starts pricing the product to make it attractive to that segment as well, thereby moving the outcome from Case 1B to Case 1A. In other words, the shadow competition from piracy can be ignored by a monopolist in certain situations, but, in a competitive market, this shadow competition becomes a relevant issue in the manufacturer’s pricing strategy.

Lemma 4.2 indicates that, faced with a binding price cap due to competition, it is sufficient to consider just Case 1A,³ the solution for which is my next result.

³Although Case 1A was not possible in the long-run equilibrium for $\beta \geq 1$ under monopoly (Lemma 4.1), curiously, it emerges as a viable equilibrium outcome under imperfect competition.

Proposition 4.3. *If the price cap is binding, that is, if $\frac{p}{\theta} = \delta$, then the long-run equilibrium could occur under Case 1A with the following solution:*

$$\theta^* = \begin{cases} \frac{e\beta\delta\xi}{c} + \frac{\delta(1-\beta-\delta)}{c(1-\beta)}, & \text{if } \beta < 1 \\ \frac{e\delta\xi}{c} - \frac{\beta\delta^2}{c(\beta-1)} & \text{otherwise,} \end{cases} \quad \text{and} \quad p^* = \begin{cases} \frac{e\beta\delta^2\xi}{c} + \frac{\delta^2(1-\beta-\delta)}{c(1-\beta)}, & \text{if } \beta < 1 \\ \frac{e\delta^2\xi}{c} - \frac{\beta\delta^3}{c(\beta-1)} & \text{otherwise.} \end{cases}$$

It is easy to see from Proposition 4.3 that $\lim_{\delta \rightarrow 0} \theta^* = 0$. In other words, as expected, when the market is fiercely competitive, the manufacturer decides against participating in the market. Given the fixed costs of development, the manufacturer stays away from developmental activities unless it can recover those costs by charging a healthy price premium.

Examining the expression for θ^* above, we can see that it is independent of r , but is linearly increasing in e . The following result is immediate:

Corollary 4.1. *If the price cap is binding, that is, if $\frac{p}{\theta} = \delta$, then the long-run equilibrium quality does not change with r but is increasing in e .*

Thus, we find that Corollary 4.1 matches with the finding in Theorem 4.1 that supply-side enforcement may indeed result in added incentives for innovation and lead to better products. In other words, even when the product faces competition in the marketplace, supply-side enforcement seems to be a better choice as an anti-piracy instrument, from the perspective of innovation.

I conclude this section with comparative statics for private profit and public welfare. I find:

Theorem 4.3. *If the price cap is binding, that is, if $\frac{p}{\theta} = \delta$, then, in the long run equilibrium, the manufacturer's profit is always increasing in both r and e . The consumer surplus is always decreasing in r ; however, it is increasing in e for $\beta < 1$ but not monotonic for $\beta \geq 1$. Social welfare is increasing in r and e for $\beta < 1$; for $\beta \geq 1$, it is not monotonic in e but is monotonically decreasing in r .*

In conclusion, I find that Theorem 4.3 practically echoes the findings in Theorem 4.2 and Proposition 4.2. Therefore, my main results derived under a monopoly assumption extend naturally to the competitive setting. When the competition is not very fierce and δ is large, the price cap is not binding. In that case, my earlier results hold verbatim. However, as the competition increases resulting in a smaller δ , the price cap may become binding. Even in that situation, the basic message that supply-side enforcement is generally a better choice seems to hold. In practice, I expect a market to have many different types of information products, with varying degrees of competition faced by their manufacturers. Since a unique enforcement strategy for every product is not practically viable, in an overall sense, supply-side enforcement is perhaps the path to adopt, till we learn more about the actual consequences of these efforts.

4.5. Costs of Enforcement

As mentioned in §3.3.1, throughout my analyses, I have assumed that the players in my game-theoretic setting—manufacturer, pirate suppliers, and consumers—operate within a given enforcement environment, implying that both r and e are exogenous and their costs, sunk. This assumption allowed us to ignore enforcement costs and maintain analytical tractability of my modeling experiment.

In reality, however, enforcement is not free. Designing and legislating copyright laws, monitoring network traffic, scanning potential sites for pirated content, finding, blocking, or shutting down illegal sites, apprehending and prosecuting offenders—all enforcement activities, supply- or demand-side, require substantial investments for a government. Earlier, I ignored these enforcement costs and were able to conclude that supply-side enforcement has a more positive impact in an overall sense; such a conclusion was entirely based on the results from my comparative statics. In this section, I would like to study how robust that conclusion would be if these costs were included in my analysis and the enforcement levels

were endogenized. In order to do so in a rigorous manner, we must include the policymaker as a fourth player who, at the very beginning of the game, takes the enforcement costs into consideration and chooses r and e in such a way that maximizes the total social welfare net of the enforcement costs. In other words, conceptually, the policymaker would solve the following problem:

$$\max_{r,e} SW(\theta^*) - (K_r r^2 + K_e e^2), \quad (4.2)$$

where, θ^* and $SW(\cdot)$ are as given in (4.1) and (3.10), respectively. In this formulation, I assume quadratic cost functions for both demand- and supply-side enforcement, with $K_r > 0$ and $K_e > 0$ being the respective cost parameters. Solving this optimization problem would essentially transform the equilibrium regions from the (r, e) space to the (K_r, K_e) space.

Before I proceed, a word of caution is in order. I note that the purpose of the optimization problem in (4.2) is not *normative*—I am not seeking to derive the optimal enforcement level (r^*, e^*) that a policymaker might want to actually implement in a real-world situation. The reason is simple. Both r and e are abstract concepts in my model, with unspecified scales or units. Furthermore, it is quite likely that the optimal values themselves are different for different information goods. It seems impractical to have widely varying enforcement strategies for different products in a market. Viewed this way, my model should not be used to derive how much investment should be made in enforcement activities; rather, it should only be used to see the impact of shifting resources from one side to the other. The purpose of this part of the analysis is, therefore, still *positive*—I would like to examine if the desirability of supply-side enforcement as an anti-piracy instrument continues to hold in a qualitative sense.

Although conceptually straightforward, the solution to the optimization problem in (4.2) is not fully tractable. I say “not fully” because, in fact, it is partially tractable. In three of the equilibrium regions, namely Regions 1A, 2A, and 3A—see Appendix C.1 for details about

these regions—the optimal quality, θ^* , is obtained by solving appropriate cubic equations. In each case, I can and do identify the correct root and theoretically prove that it indeed exists. However, because of the sheer size of the expressions for those roots (in Mathematica), it is not appropriate to include here. That kind of size also makes it impossible to dissect it any further to identify useful analytical properties—a natural eventuality in any modeling experiment, a point is reached where it simply becomes impossible to go forward. To be sure, certain limited-impact analytical observations can still be made,⁴ but I do not consider them significant.

It turns out, however, that not all is lost, because (4.2) can still be solved numerically and, although a tedious undertaking, the solutions for different combinations of parameter values can be carefully checked and compared to test the veracity of my earlier results. To that end, I perform extensive numerical analyses with a wide variety of parameter value combinations and find that equilibrium regions themselves depend heavily on K_e . When K_e is small, the policymaker finds it optimal to completely eradicate piracy, leading to $\eta = 0$; the resulting equilibrium happens in Regions 3A and 3C. However, as K_e increases, the policymaker can no longer eliminate piracy fully and is forced to tolerate it to an extent. The equilibrium now shifts to the primary piracy region—Regions 1A or 1B—where $\eta > 0$. As K_e increases further and supply-side enforcement becomes prohibitively costly, the policymaker is forced to stop supply-side enforcement altogether, resulting in an abundant supply ($\eta = 1$) of pirated content in Regions 2A and 2B. Figure 4.3 illustrates this behavior in the (K_r, K_e) space for $\beta = 0.75$, $\xi = \frac{2}{3}$, and $c = 0.01$.

Interestingly, the influence of K_r on the equilibrium regions is a lot more subdued,⁵

⁴For example, if $\beta > \frac{3}{4}$, throughout Region 1B—and if $\beta \geq 1$, throughout the entire primary piracy region— r^* is zero, implying a complete substitution of demand-side enforcement by its supply-side counterpart in those cases. Additionally, for $\beta < 1$, throughout Region 1A, r^* is capped at a theoretical ceiling, given by σ_{1A} in Technical Lemma A1 in Appendix C.2.

⁵In contrast, K_e plays a much bigger role in defining the boundary of the primary piracy region. Region 1B is primarily defined by two thresholds of K_e and seems independent of K_r , except for very low values of K_r . The boundaries of Region 1A are also determined by K_e thresholds alone beyond low values of K_r .

although the intuitive trend of η increasing with enforcement cost is observed for K_r as well.

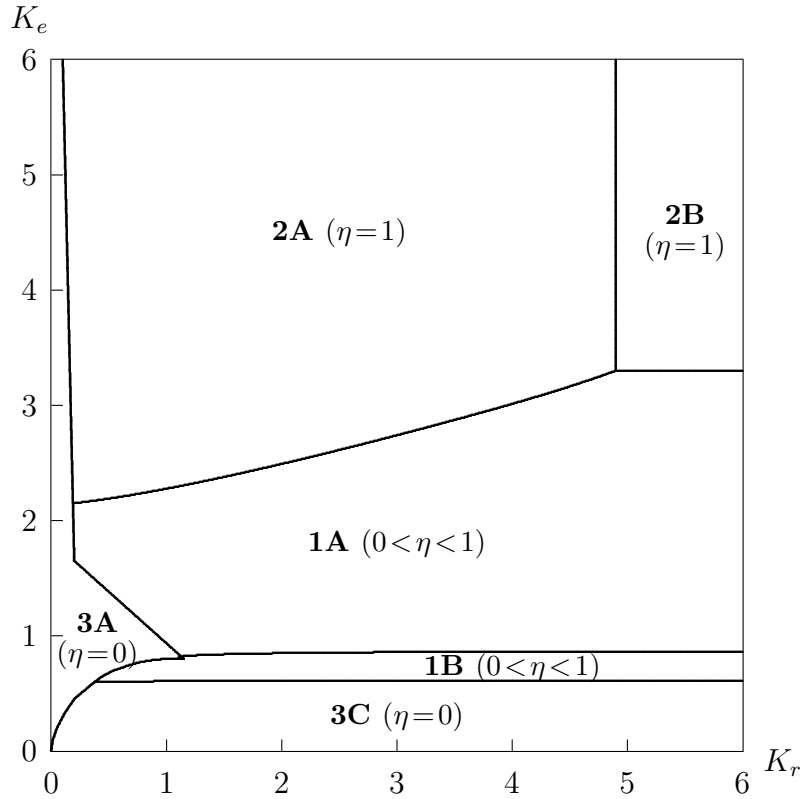


Figure 4.3: Relevant Partitions of the (K_r, K_e) Space; $\xi = \frac{2}{3}$, $c = 0.01$

By carefully comparing the results from the solutions to (4.2) obtained with a wide set of parameter values, I am able to make the following observations:

- Although it now has the option to completely eradicate piracy, in a significant portion of the relevant parameter space, the government is still willing to tolerate piracy to an extent $(0 < \eta < 1)$, that is, the primary piracy region can still occur. This is consistent with the realities of today.
- Unless supply-side enforcement becomes prohibitively costly, the government always employs it. More specifically, throughout the entire primary piracy region, e^* is strictly

greater than zero. This is intuitive; when the cost of enforcement is zero, since social welfare increases in e , it is clear that the policymaker will be inclined to increase e as much as possible. When there is a cost associated with increasing e , the optimal value would depend on the rate at which the cost increases with the enforcement level (i.e., the convexity of cost curve). The policymaker would increase e to the point at which the marginal cost equals the marginal benefit, which is, again, positive. Clearly, when operating in the primary piracy region, supply-side enforcement is an indispensable instrument for the policymaker, even after accounting for its cost.

- In contrast, a policymaker may choose to ignore demand-side enforcement altogether in several parts of the parameter space, even when K_r is fairly low. More importantly, in a significant portion of the region of interest—the primary piracy region—I find that $r^* = 0$, suggesting that it is optimal for the policymaker to fully shift resources from the demand to the supply side of enforcement activities. This is not only true for all of Region 1B, but also for all of Region 1A except a small portion. This is because, in a significant portion of the primary piracy region, the social welfare is strictly decreasing in r , leaving little reason for the policymaker to invest in it at all.

I construe that supply-side enforcement is more likely to be preferred by a policymaker, further corroborating my earlier findings.

I conclude this section by noting that, in addition to the investments made by the government, manufacturers and their industry alliances could invest in additional enforcement activities as well. As we have seen from Proposition 4.2, the profit monotonically increases in both r and e . Hence, regardless of the enforcement type, the manufacturer would always prefer to increase the enforcement level as much as it could, that is, up to the point where the marginal cost of enforcement equals the marginal profit. Perhaps, the more interesting question to ask is, how would the manufacturer's decision to invest in quality be affected

when the manufacturer has the ability to change the levels of enforcement? An interesting observation can be made here. When the manufacturer finds it optimal to increase the level of demand-side enforcement, it finds it less necessary to differentiate its product from the pirated counterpart and invests less in quality. In other words, the manufacturer deems quality and demand-side enforcement as substitutes, a finding similar to the one presented by Lahiri and Dey (2013). Therefore, the manufacturer is essentially trading off between the cost of demand-side enforcement and that of increasing quality. If enforcement is costlier, the manufacturer may focus on developing a better quality product. If, on the other hand, improving quality is more expensive, the manufacturer may instead resort to stepping up demand-side enforcement. The message is quite different for supply-side enforcement, however. There, whenever the manufacturer increases supply-side enforcement, it also finds itself in a better position to invest in quality and profit from its investment. Hence, the manufacturer considers quality and supply-side enforcement as complements in the fight against piracy.

4.6. Conclusion

In recent times, anti-piracy efforts aimed at curbing online piracy has seen a gradual shift in their focus from the demand side to the supply side. Leveraging the model constructed in Chapter 3, to answer whether a shift to the supply side indeed has merits, I compare demand- and supply-side anti-piracy measures in terms of their impacts on innovation and welfares in the long run.

I find that, although the two types of enforcement are somewhat indistinguishable with respect to their short-run impacts, there are some fundamental differences between their economic impacts in the long run. In situations where piracy exists, making the pirated product less available leads to an increase in the equilibrium quality of the legal product,

whereas making piracy less attractive decreases this quality. This contrast is indeed fascinating, because both types of enforcement actually have similar effects of protecting the manufacturer's profit. More interestingly, in terms of social welfare as well, the effect of supply-side enforcement turns out to be more desirable in the long run, when compared to its demand-side counterpart.

In order to test the robustness of my results, I consider an extension where the market imposes some competition on the manufacturer. Although the very existence of online piracy tells us that such a competition cannot be perfect, imperfect substitutes for digital goods do exist in the marketplace. In my extension, the presence of such substitutes manifests as a cap on the price premium a manufacturer can command. I find that, my results are indeed robust to such an extension—as long as piracy exists, supply-side measures seem to have a more desirable economic impact in the long run, both in terms of innovation and welfare.

My basic model does not explicitly consider the costs incurred by a policymaker in implementing the two types of enforcement. For, in this research, I am only interested in exploring whether the policymaker, situated in a real-world context, might find it favorable to shift resources from the demand to the supply side. Comparative statics from the basic model indicate that, given a total budget for enforcement activities, the relative allocation of that budget should perhaps favor the supply side. In order to further test the veracity of this result, I later consider enforcement costs and endogenize the enforcement levels. The results from my original model are further corroborated by this exercise.

Similar to the limitations described in §3.7, my approach in this chapter is also limited by some of my simplifying assumptions. Here, I assume that consumers' technical savviness remains static in the time window of the equilibrium. It is, however, conceivable that, as enforcement activities make it harder to locate pirated content, consumers may devote more time towards learning how to circumvent them and finding other avenues for piracy. Also, over time, more and more younger, and apparently more tech-savvy, consumers are likely to

enter the fray. Taken together, the overall distribution of technical abilities might shift to the right. Obviously, in future, if all consumers can swiftly learn how to expand piracy options, the results would greatly diminish in value, as would the concept of supply-side enforcement itself. At the same time, though, I also feel that it is too early to pronounce supply-side enforcement dead, mainly because of its increasing prominence in current practice as well as in recent empirical literature (e.g., Danaher and Smith 2014, Danaher et al. 2015).

I end with a word of caution. The findings should not be automatically extrapolated to justify all supply-side legislation efforts, such as SOPA, PIPA, the PIRATE Act, or any other specific piece of legislation. When it comes to the letter of the law, the devil always ought to be in the detail. It is important to remember that, indeed, the original intent of SOPA/PIPA was to limit the supply of pirated content. The massive campaign that followed against them, however, was not because they intended to limit the supply of pirated content; instead, that opposition was rooted in the vagueness and unbounded scope of these bills. I do not consider such details. My work simply provides a distinction between policies that diminish attractiveness of pirated content versus those that limit its supply, a much-needed lens through which researchers and practitioners can view various policy proposals.

Chapter 5

Closing

It is absolutely fascinating when we take a moment and think about how much the method and the overall scheme of piracy has evolved over the past several decades. The issue of piracy—or, more accurately, copyright infringement—is as old as the inception of the concept of intellectual property. Copyright infringement is actually a more broad concept under which piracy falls. When someone, without a proper permission, copies the design of a physical good or imitates the interior theme of a restaurant, that person may be infringing on someone else's copyright. The term piracy is usually used only in the context of information goods, and information goods have several noteworthy characteristics: They are usually quite expensive to develop, but their marginal cost of production is negligible. Also, it is rather easy to create a copy of the original version without the manufacturer's consent. In the old days, people commonly made copies of cassette and VHS tapes, and creating and giving a mixed tape as a gift was more or less part of the cultural norms. Students made photocopies of textbooks, and street vendors sold bootleg tapes. All these activities were illegal back in the days as well, but their impacts were limited by their very material nature.

Then came the digital world, the world of the internet. All of a sudden, many barriers of the physical world got lifted. The manufacturers—and retailers—of information goods can now reach consumers all over the world at an instant. The problem is, so can pirate suppliers. Seeing an opportunity to make a profit, many individuals started uploading pirated contents,

and platforms to connect illegal downloaders and uploaders have come to existence. Pirate suppliers also started using existing tools and services such as blogs, payment services, and ad publishing services. While it is not clear in what order all these events transpired—as it is a chicken-egg kind of situation—one thing is clear; there is now a giant ecosystem of online piracy where anything that can be digitally stored and logically rendered can be subject to trade. As amply demonstrated in the introduction, the issue of piracy is no longer a “cute” problem of mixed tapes and poorly-bound photocopied textbooks. Many reports on piracy losses throw massive figures around in the magnitude of billions of US dollars. I would say piracy has certainly outgrown its cute phase, and manufacturers, business alliances, and governments around the world are all struggling to figure out what to do with this problem that has suddenly gotten so massive.

Such a challenge in the real world presents us academicians with an interesting phenomenon to study. And, it appears that researchers have not missed a beat in providing streams of insights from their studies on the impact of piracy—and anti-piracy measures—from the perspectives of the manufacturers, consumers, and the overall society; my review of the literature can be found in §2.2 and §3.2. To summarize without repeating too much of what has already been said in the conclusion sections—§2.6, §3.7, and §4.6—in my dissertation, I intend to contribute to the literature on the issue of piracy by taking a closer look at what has largely been abstracted away previously; see Figure 1.1. Specifically, in my first piece—Chapter 2—I open up the supply-chain of the legal channel in order to study the interaction between the internal issue of channel coordination and the external threat of piracy. And, in the subsequent chapters, I shed light on the ecosystem of the supply-side of piracy, and make comparison between demand- and supply-side enforcement efforts. From my exploration, I find several interesting counter-intuitive results, and there appears to be one permeating theme across all my findings: *Things are not always as they seem on the surface!* At a glance, many would expect that when a supply-chain is faced with two

problems—the problem of piracy and double marginalization—the situation surely cannot be better than having just one. Also, given that both demand- and supply-side measures are geared toward suppressing piracy, there is no obvious reason to believe that the two approaches would have any contrasting economic implications. My surprising discoveries stem from the fact that I consider strategic players—consumers, manufacturer, retailer, and pirate suppliers—in my game theoretic settings, and the players act in their own self-interest based on other players' strategies. It is from this dynamics of action-reaction that unexpected—for better or worse—consequences can arise. The results in my dissertation throw a bit of caution in the wind by pointing out that, while deploying anti-piracy efforts, failing to consider the intricate interactions among the parties involved could lead to undesirable consequences.

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Appendix A

Appendix To Chapter 2

A.1. Proofs

Proof of Lemma 2.1

If $q(p) = 1 - \frac{p-r}{1-\beta}$, then $\pi_r(p) = (p-w) \left(1 - \frac{p-r}{1-\beta}\right)$, implying:

$$\frac{\partial \pi_r}{\partial p} = 1 - \frac{2p-r-w}{1-\beta}. \quad (\text{A1})$$

Since $\frac{\partial^2 \pi_r}{\partial p^2} = -\frac{2}{1-\beta} < 0$, the first-order condition results in $p^*(w) = \frac{1}{2}(1 - \beta + r + w)$, which, according to (2.1), must be greater than $\frac{r}{\beta}$, or $w > \frac{2r}{\beta} - 1 + \beta \left(1 - \frac{r}{\beta}\right)$, for this solution to be valid.

If, on the other hand, $q(p) = 1 - p$, then $\pi_r(p) = (p-w)(1-p)$, resulting in:

$$\frac{\partial \pi_r}{\partial p} = 1 - 2p + w. \quad (\text{A2})$$

Furthermore, since $\frac{\partial^2 \pi_r}{\partial p^2} = -2 < 0$, $\frac{\partial \pi_r}{\partial p} = 0$ results in $p^*(w) = \frac{w+1}{2}$, which must be smaller than $\frac{r}{\beta}$, or $w < \frac{2r}{\beta} - 1$, for this solution to be valid.

Recall that $r < \beta$ must hold for the problem to be interesting. However, $r < \beta$ also implies that $\frac{2r}{\beta} - 1 < \frac{2r}{\beta} - 1 + \beta \left(1 - \frac{r}{\beta}\right)$. Now, if $\frac{2r}{\beta} - 1 \leq w \leq \frac{2r}{\beta} - 1 + \beta \left(1 - \frac{r}{\beta}\right)$, $\frac{\partial \pi_r}{\partial p}$ given by (A1) is negative whereas that given by (A2) is positive. Naturally, the optimal p is simply $\frac{r}{\beta}$. ■

Proof of Proposition 2.1

From Lemma 2.1, it is evident that we have three cases to consider: (i) $w > \frac{2r}{\beta} - 1 + \beta \left(1 - \frac{r}{\beta}\right)$, (ii) $\frac{2r}{\beta} - 1 \leq w \leq \frac{2r}{\beta} - 1 + \beta \left(1 - \frac{r}{\beta}\right)$, and (iii) $w < \frac{2r}{\beta} - 1$.

For case (i), I substitute $p^*(w) = \frac{1}{2}(1 - \beta + r + w)$ into (2.1) to obtain the manufacturer's profit:

$$\pi_m = \frac{w(1 - \beta + r - w)}{2(1 - \beta)}. \quad (\text{A3})$$

Since $\frac{\partial^2 \pi_m}{\partial w^2} = -\frac{1}{1-\beta} < 0$, the first order condition, $\frac{\partial \pi_m}{\partial w} = \frac{1-\beta+r-2w}{2(1-\beta)} = 0$, results in $w^* = \frac{1}{2}(1-\beta+r)$, which, according to Lemma 2.1, must be greater than $\frac{2r}{\beta} - 1 + \beta \left(1 - \frac{r}{\beta}\right)$, or $r < \frac{3\beta(1-\beta)}{4-3\beta} = \rho_1$, for this equilibrium to be valid.

For case (ii), $p^* = \frac{r}{\beta}$. The manufacturer, unwilling to leave money on the table, always chooses the highest value from the range $\frac{2r}{\beta} - 1 \leq w \leq \frac{2r}{\beta} - 1 + \beta \left(1 - \frac{r}{\beta}\right)$, resulting in $w^* = \frac{2r}{\beta} - 1 + \beta \left(1 - \frac{r}{\beta}\right)$. This equilibrium is valid across all r values.

Finally, for (iii), $p^*(w) = \frac{w+1}{2}$, and the manufacturer's profit is:

$$\pi_m = \frac{1}{2}w(1-w). \quad (\text{A4})$$

implying $w^* = \frac{1}{2}$. According to Lemma 2.1, this w^* must be less than $\frac{2r}{\beta} - 1$, or $r > \frac{3\beta}{4} = \rho_5$.

Since $\rho_1 < \rho_5$, (ii) is the only valid equilibrium if $\rho_1 \leq r \leq \rho_5$, and $w^* = \frac{2r}{\beta} - 1 + \beta \left(1 - \frac{r}{\beta}\right)$.

If $r < \rho_1$, the manufacturer can either set $w = \frac{1}{2}(1-\beta+r)$, or set $w = \frac{2r}{\beta} - 1 + \beta \left(1 - \frac{r}{\beta}\right)$. If the manufacturer chooses $w = \frac{1}{2}(1-\beta+r)$, its profit is $\frac{(1-\beta+r)^2}{8(1-\beta)}$ from (A3). On the other hand, if it chooses $w = \frac{2r}{\beta} - 1 + \beta \left(1 - \frac{r}{\beta}\right)$, its profit becomes $w(1 - \frac{r}{\beta}) = \frac{(\beta-r)(r-(1-\beta)(\beta-r))}{\beta^2}$. Between the two choices, the manufacturer chooses the one that yields a higher profit. It is easy to verify that, at $r = \rho_1$, both options yield the same profit, and for $r < \rho_1$, the first option is always better. Thus, if $r < \rho_1$, (i) is the equilibrium outcome and $w^* = \frac{1}{2}(1-\beta+r)$.

If $r > \rho_5$, the manufacturer can either set $w = \frac{1}{2}$, or set $w = \frac{2r}{\beta} - 1 + \beta \left(1 - \frac{r}{\beta}\right)$. If $w = \frac{1}{2}$, the manufacturer's profit is $\frac{1}{8}$ from (A4), and, if $w = \frac{2r}{\beta} - 1 + \beta \left(1 - \frac{r}{\beta}\right)$, the profit becomes, as before, $\frac{(\beta-r)(r-(1-\beta)(\beta-r))}{\beta^2}$. Comparing these two profits, it is easy to verify that the manufacturer would choose the first option if $r > \frac{\beta(6-4\beta+\sqrt{2\beta})}{4(2-\beta)} = \rho_2$. Since $\rho_2 > \rho_5$ holds trivially, (iii) is the equilibrium outcome with $w^* = \frac{1}{2}$ if $r > \rho_2$.

With the closed-form solution for w^* , we can derive p^* from Lemma 2.1. ■

Proof of Lemma 2.2

Piracy rate, μ , is 0 in the threat region and benchmark region. Hence, we only need to consider the piracy region. In this region, μ can be written as $(\frac{p-r}{1-\beta} - \frac{r}{\beta})/(1 - \frac{r}{\beta})$. Substituting for p with p^* from Proposition 2.1, μ becomes $\frac{3(1-\beta)\beta-(4-3\beta)r}{4(1-\beta)(\beta-r)}$. Thus, $\frac{\partial \mu}{\partial r} = -\frac{\beta}{4(1-\beta)(\beta-r)^2} < 0$. ■

Proof of Proposition 2.2

Using p^* and w^* from Proposition 2.1, we can find the equilibrium profits for the manufacturer and retailer as $\pi_m^* = w^*q(p^*)$ and $\pi_r^* = (p^* - w^*)q(p^*)$, respectively. ■

Proof of Theorem 2.1

First, I consider the manufacturer. Since $\rho_1 = \frac{3\beta(1-\beta)}{4-3\beta}$, $\rho_3 = \sqrt{1-\beta} - (1-\beta)$, and $\beta < \frac{8}{9}$, it is easy to show that $\rho_3 < \rho_1$. Since in the piracy region ($r < \rho_1$), the manufacturer's profit, $\pi_m = \frac{(1-\beta+r)^2}{8(1-\beta)}$, is increasing in r , equating this profit to the benchmark profit of $\pi_{m0} = \frac{1}{8}$ and solving for r , I find that the manufacturer would be better off if $r \geq \rho_3$. Next, in the threat region ($\rho_1 \leq r \leq \rho_2$), the manufacturer's profit, $\pi_m = \frac{(\beta-r)(r-(1-\beta)(\beta-r))}{\beta^2}$, can never be less than π_{m0} . In other words, the manufacturer prefers piracy or its threat if $\rho_3 \leq r \leq \rho_2$.

Next, I consider the retailer. The retailer's profit, $\pi_r = \frac{(1-\beta+r)^2}{16(1-\beta)}$, is also increasing in r in the piracy region ($r < \rho_1$). Therefore, as before, equating this profit to the benchmark profit of $\pi_{r0} = \frac{1}{16}$ and solving for r , I find that the retailer would also be better off if $r \geq \rho_3$. In the threat region ($\rho_1 \leq r \leq \rho_2$), the retailer's profit, $\pi_r = \frac{(1-\beta)(\beta-r)^2}{\beta^2}$, is decreasing in r . This profit is greater than or equal to $\pi_{r0} = \frac{1}{16}$ if and only if $r < \rho_4 = \beta - \frac{\beta}{4\sqrt{1-\beta}}$. Since $\beta < \frac{8}{9}$ ensures that $\rho_4 > \rho_1$, I find that the retailer is better off in the presence of piracy or its threat if $\rho_3 \leq r \leq \rho_4$.

Since ρ_4 is always less than ρ_2 , the three regions in the theorem emerge by combining the above. ■

Proof of Proposition 2.3

CS is given by:

$$CS = \begin{cases} \int_{\frac{p^*-r}{1-\beta}}^1 (v - p^*)dv, & \text{if } p^* > \frac{r}{\beta}, \\ \int_{p^*}^1 (v - p^*)dv, & \text{otherwise.} \end{cases}$$

The desired result can now be proved by substituting p^* from (2.2) into the expression above. ■

Proof of Theorem 2.2

In the piracy region, $CS = \frac{(1-\beta+r)((1-\beta)(6(\beta-r)+1)-r)}{32(1-\beta)^2}$. Since $\frac{\partial^2(CS)}{\partial r^2} = -\frac{7-6\beta}{16(1-\beta)^2} < 0$, CS is concave in r , and is minimized at one of the two boundaries, $r = 0$ or $r = \rho_1$. Now, $CS|_{r=0} = \frac{1+6\beta}{32} > \frac{1}{32}$ and $CS|_{r=\rho_1} = \frac{1}{2(4-3\beta)^2} > \frac{1}{32}$. Clearly then, CS in the piracy region is always above the benchmark value of $CS_0 = \frac{1}{32}$.

Furthermore, the consumer surplus in the threat region, $CS = \frac{(\beta-r)^2}{2\beta^2}$ is decreasing in r . Therefore, by equating it to CS_0 , I find that consumers are better off if $r \leq \rho_5 = \frac{3\beta}{4}$.

Since $\rho_4 < \rho_5$ for all $\beta > 0$, the result follows from Theorem 2.1. ■

Proof of Corollary 2.1

Since the channel profit is given by $CP = \pi_m^* + \pi_r^*$, it can be easily calculated from Proposition 2.2. And, social welfare can be calculated from:

$$SW = \begin{cases} \int_{\frac{p^*-r}{1-\beta}}^1 v dv, & \text{if } p^* > \frac{r}{\beta}, \\ \int_{p^*}^1 v dv, & \text{otherwise.} \end{cases}$$

Substituting p^* from (2.2) into the above expression, I get the desired result. ■

Proof of Theorem 2.3

In the piracy region, $CP = \frac{3(1-\beta+r)^2}{16(1-\beta)}$ is increasing in r . Equating it to CP_0 , I get, $r = \rho_3 = \sqrt{1-\beta} - (1-\beta)$. Clearly, channel profit is higher in the presence of piracy if $r \geq \rho_3$. In the threat region, $CP = \frac{r(\beta-r)}{\beta^2}$ is concave in r . Equating it to CP_0 , and noting that the valid solution for r must be greater than ρ_1 , I find that the threat region does better in terms of channel profit if $r \leq \rho_5 = \frac{3\beta}{4}$.

As far as social welfare is concerned, in the piracy region, $SW = \frac{(1-\beta+r)(7(1-\beta)-r)}{32(1-\beta)^2}$ is increasing in r , because the derivative $\frac{\partial(SW)}{\partial r} = \frac{3(1-\beta)-r}{16(1-\beta)^2}$ is positive for $r < \rho_1$. Therefore, the minimum value of SW occurs at $r = 0$, and this minimum value happens to be the same as SW_0 , implying that piracy always leads to a higher social surplus. I now move to the threat region, where $SW = \frac{1}{2} \left(1 - \frac{r^2}{\beta^2}\right)$ is clearly decreasing in r . Equating it to SW_0 , I find that the threat region does better in terms of social welfare, again, if $r \leq \rho_5 = \frac{3\beta}{4}$. This completes the proof. ■

Proof of Corollary 2.2

An immediate implication of Corollary 2.1 is that SW is increasing for $r < \rho_1$. For, its derivative is $\frac{3(1-\beta)-r}{16(1-\beta)^2}$ there, which lies between two positive numbers $\frac{3}{16-16\beta}$ and $\frac{3}{16-12\beta}$. Between ρ_1 and ρ_2 , however, its derivative is simply $-\frac{r}{\beta^2}$, implying that SW decreases when r increases from ρ_1 to ρ_2 . Now, by noting that the value of SW at ρ_1 , $\frac{7-6\beta}{2(4-3\beta)^2}$, exceeds the benchmark surplus of $\frac{7}{32}$, we can infer that SW is indeed maximized at $r = \rho_1$.

Now, as shown in the proof of Theorem 2.1, the manufacturer's profit peaks somewhere between ρ_1 and ρ_2 . Further, using the first order condition, it is easy to verify that this peak is at $r = \frac{\beta(3-2\beta)}{2(2-\beta)}$, and simple algebra confirms that this r is indeed greater than ρ_1 . ■

Proof of Proposition 2.4

When $\bar{p} > \frac{r}{\beta}$, $\bar{\pi} = \bar{p}q(\bar{p}) = \bar{p}\left(1 - \frac{\bar{p}-r}{1-\beta}\right)$, implying $\frac{\partial \bar{\pi}}{\partial \bar{p}} = 1 - \frac{2\bar{p}-r}{1-\beta}$.

Since $\frac{\partial^2 \bar{\pi}}{\partial \bar{p}^2} = -\frac{2}{1-\beta} < 0$, the first order condition, $\frac{\partial \bar{\pi}}{\partial \bar{p}} = 0$, results in $\bar{p}^* = \frac{1}{2}(1 - \beta + r)$. Clearly, this solution must be greater than $\frac{r}{\beta}$, or $r < \frac{(1-\beta)\beta}{2-\beta} = \bar{\rho}_1$.

If, on the other hand, $\bar{p} < \frac{r}{\beta}$, then $\bar{\pi} = \bar{p}q(\bar{p}) = \bar{p}(1 - \bar{p})$, resulting in $\bar{p}^* = \frac{1}{2}$. This p^* should be less than $\frac{r}{\beta}$, or $r > \frac{\beta}{2} = \bar{\rho}_2$.

Finally, since $\bar{\rho}_1 < \bar{\rho}_2$, I must also consider the situation where $\bar{\rho}_1 \leq r \leq \bar{\rho}_2$. In that situation, the profit above is decreasing for $\bar{p} > \frac{r}{\beta}$ but increasing for $\bar{p} < \frac{r}{\beta}$. So, \bar{p}^* becomes $\frac{r}{\beta}$.

The optimal profit in each region can be found easily from $\bar{p}^*q(\bar{p}^*)$. ■

Proof of Lemma 2.3

From Proposition 2.1, we know that the equilibrium retail price, p^* , in the threat region is $\frac{r}{\beta}$, which comes out to be $p_{\rho_2}^* = \frac{6+\sqrt{2}\sqrt{\beta}-4\beta}{8-4\beta}$ at $r = \rho_2$. $p_{\rho_2}^*$ is $\frac{3}{4}$ when $\beta = 0$, and is an increasing function in β , since $\frac{\partial p_{\rho_2}^*}{\partial \beta} = \frac{1}{4\sqrt{2\beta}(\sqrt{\beta}+\sqrt{2})^2} > 0$. Since $0 < \beta < 1$, $p_{\rho_2}^*$ is always greater than p^0 , the equilibrium retail price in the benchmark region.

Also, from Proposition 2.1, we know that p^* increases in r in both the piracy region and the threat region. We also know that p^* is continuous across these two regions since it is piecewise continuous and is continuous at $r = \rho_1$. Hence, the lowest point for p^* across the piracy region and the threat region is at $r = 0$, where $p^* = \frac{3(1-\beta)}{4}$. This value is lower than $\bar{p}^0 = \frac{1}{2}$ if $\beta > \frac{1}{3}$. Hence, if $\beta > \frac{1}{3}$, it is possible to have p^* lower than \bar{p}^0 . ■

Proof of Lemma 2.4

The manufacturer and retailer's margins can be represented by w^* and $p^* - w^*$, respectively. From Proposition 2.1, we know that (1) $w^* = \frac{1}{2}(1-\beta+r)$ and $p^* - w^* = \frac{1}{4}(1-\beta+r)$ when $r < \rho_1$, (2) $w^* = \frac{2r}{\beta} - 1 + \beta\left(1 - \frac{r}{\beta}\right)$ and $p^* - w^* = \frac{(1-\beta)(\beta-r)}{\beta}$ when $\rho_1 \leq r \leq \rho_2$, and (3) $w^* = \frac{1}{2}$ and $p^* - w^* = \frac{1}{4}$ when $\rho_2 < r$.

In the piracy region and benchmark region, the ratio of the manufacturer and retailer's margins stay constant at 2 : 1.

On the other hand, in the threat region, the manufacturer's margin increases in r , whereas the retailer's decreases. More specifically, $\frac{\partial w^*}{\partial r} = \frac{2-\beta}{\beta} > 0$ and $\frac{\partial(p^*-w^*)}{\partial r} = -\frac{1-\beta}{\beta} < 0$ when $\rho_1 \leq r \leq \rho_2$. ■

Proof of Theorem 2.4

I start by noting that, when neither piracy nor its threat is present, $\eta = \frac{3/16}{1/4} = \frac{3}{4}$.

I will now show that, for $\bar{\rho}_1 < r < \rho_5$, η is larger than $\frac{3}{4}$. To do so, I make use of Corollary 2.1 and Proposition 2.4. This allows us to divide the interval $(\bar{\rho}_1, \rho_5)$ into several parts:

- When $\bar{\rho}_1 < r < \rho_1$, $CP = \frac{3(1-\beta+r)^2}{16(1-\beta)}$ and $\bar{\pi}^* = \frac{r(\beta-r)}{\beta^2}$. Therefore, $\eta = \frac{3\beta^2(1-\beta+r)^2}{16r(1-\beta)(\beta-r)}$, and $\eta - \frac{3}{4} = \frac{3(\beta(1-\beta)-r(2-\beta))^2}{16r(1-\beta)(\beta-r)} > 0$.
- If $\rho_1 \leq r \leq \bar{\rho}_2$, $CP = \bar{\pi}^* = \frac{r(\beta-r)}{\beta^2}$. Therefore, $\eta = 1$, and the channel is fully coordinated.
- Finally, when $\bar{\rho}_2 < r < \rho_5$, $CP = \frac{r(\beta-r)}{\beta^2}$ and $\bar{\pi}^* = \frac{1}{4}$. Therefore, $\eta = \frac{4r(\beta-r)}{\beta^2}$, which is greater than $\frac{3}{4}$ because $r < \rho_5 = \frac{3\beta}{4}$.

Finally, I show that the interval of 100% channel efficiency, $[\rho_1, \bar{\rho}_2]$, is not empty, that is, $\rho_1 \leq \bar{\rho}_2$. It is easy to verify that this is indeed the case as long as $\beta \geq \frac{2}{3}$. ■

A.2. Examples of Retailing Configurations

There exist many different forms of retail configurations in the online digital goods market, and I summarize as follows:

- *Agency referral (AR)*: The manufacturer sells through the retailer’s platform, and the retailer charges a referral fee. The manufacturer sets the price in this case.
- *Agency selling (AS)*: The retailer sets the price and sells to consumers directly. The manufacturer gets a certain proportion of the revenue.
- *Reselling (RS)*: The manufacturer sets the wholesale price, and the retailer sets the retail price and sells to consumers.

Further, depending on whether the transfer between the manufacturer and the retailer is a fixed fee per sale or a percentage of the total revenue, we can divide AR into *AR-fixed (ARF)* and *AR-percentage (ARP)*, and AS into *AS-fixed (ASF)* and *AS-percentage (ASP)*. From all these different types of retailing, I recognize some structural similarities between them: One party sets the price, p , and the other gets a cut of the revenue, w , where w can be a fixed fee or a percentage. Hence, from the construction of my model so far, I can classify all the retail configurations I consider as one of the following forms in Table A.1. For instance,

Table A.1: Structure of Retailing Configurations and Revenue Structures

| | A | B |
|------------------|------------------------|------------------|
| w : percentage | $\pi_A = (1 - w)pq(p)$ | $\pi_B = wpq(p)$ |
| w : fixed fee | $\pi_A = (p - w)q(p)$ | $\pi_B = wq(p)$ |

AR is the case where A is the manufacturer, and B is the retailer. Similarly, AS fits the case where A is the retailer, and B is the manufacturer. As a matter of fact, AR and AS have identical structures with the labeling of manufacturer and retailer reversed. Hence, the results of subsequent analyses for AR will be mirror-images of those from AS.

Lemma A.1. *AR and AS are mirror-cases of each other.*

By setting w as a fixed fee or a percentage, ARF, ARP, ASF, and ASP can all be modeled. Also, note that RS has the same structure as ARF, where w would be interpreted as the wholesale price. Hence, the

conceptual framework presented in Table A.1 appears to be quite comprehensive in capturing some of the most common retailing configurations available in the online digital goods market.

Deciding on p and w : Now that we have the revenue structure of each player in all retail configurations, the next question is then the timing: Is the retail price set before the transfer between the manufacturer and retailer, or is it the other way around? A number of possible sequences arise, and I summarize them below.

Among all possible sequences, some are uninteresting because they are either obvious or unlikely. For instance, when both p and w are determined by one party, it trivially leads to the case of vertical integration (VI), and whichever party sets both p and w gets the entire revenue, leaving nothing for the other party (i.e., $w = 0$). Also, in general, p cannot be determined before w ; the party setting w tries to get as much as possible from the total revenue, and anticipating that, the other party knows its revenue would always be 0 regardless of the retail price it sets. Hence, such markets cannot exist. Based on such a set of logic, I prune all obvious and unlikely cases, which leads us to the following remaining cases for evaluation.

- *Agency referral - fixed fee (ARF)*
 - *Retailer decides on $w \rightarrow$ Manufacturer decides on p :* This case is identical to the traditional reselling structure (see ASF below), with the role of retailer and manufacturer reversed.
- *Agency referral - percentage (ARP)*
 - *Retailer decides on $w \rightarrow$ Manufacturer decides on p :* This case is quite dissimilar from the reselling case. Here, the retailer gets 100 percent of the revenue, leaving nothing to the manufacturer. The retailer and the manufacturer acts as if they were vertically integrated, and all the results—including retail price and channel profit—are exactly the same as in VI. In practice, this situation translates to the case where the retailer takes large fraction of the revenue, giving the manufacturer just about enough to satisfy its participation constraint. As a note, the issue of retailer competition would play a big role in such a setting since the manufacturer will switch to a retailer with a bigger w . I speculate that w will be driven down to 0 in the case of retailer competition where identical retailers compete against one another.
- *Agency selling - fixed fee (ASF)*

- *Manufacturer decides on $w \rightarrow$ Retailer decides on p* : This is what has been commonly referred to as reselling (or wholesale).
- *Agency selling - percentage (ASP)*
 - *Manufacturer decides on $w \rightarrow$ Retailer decides on p* : This is a mirror-case of ARP; the market behaves as in VI, and the manufacturer gets all of the profit, leaving nothing for the retailer.

There are a couple of takeaways from the summary of the cases above. First, many of the retailing configurations reduce to the case of VI. VI is a good representation of the market when (i) both p and w are set by one party, or when (ii) w is a fraction of the total revenue (as in ARP and ASP). The only case where VI is not a good model of the market is when p and w are determined by separate parties and w is a fixed fee—and that is precisely the case I focus on in Chapter 2. Second, the approach above may be too simplistic in considering the cases where the revenue is shared between the two parties as a fraction of the total. In both ARP and ASP, one party gets all the revenue whereas the other is left with nothing. These cases illustrate situations where one party dominates in the revenue sharing scheme (i.e., the other party has nowhere else to go), and the results are sensitive to my monopoly assumption. Perhaps, a better way to think about revenue-sharing is as a bargaining process between the two parties, each with different negotiation power. I consider that next.

Bargaining for p and w : While p and w can be determined by separate entities without any negotiation, p and w can also be a result of the negotiation between the two parties. Nash bargaining game (Nash 1950) is an approach that has been widely used in two-party bargaining situations, and I model the negotiation between the manufacturer and retailer as an asymmetric Nash bargaining game (Roth 1979). In this setting, p and w are derived by maximizing $\Pi = \pi_m^\kappa \pi_r^{(1-\kappa)}$, where $\kappa \in (0, 1)$ is the manufacturer’s relative bargaining power, and π_m and π_r are the profits of the manufacturer and retailer, respectively. In this setup, several interesting findings surface, which I summarize below and in Table A.2:

- When w is a percentage of the total revenue, w becomes the same as one’s relative bargaining power (κ or $1 - \kappa$, depending on the party).
- When w is a fixed fee per sale, w becomes a fraction of the retail price, where the fraction is the same as one’s relative bargaining power (κ or $1 - \kappa$, depending on the party).

Table A.2: Summary of Results from Different Retailing Configurations With Nash Bargaining on p and w

| | ARF | | | ARP | | |
|---------------------------|------------------------------------|------------------------------|-------------------|------------------------------------|------------------------------|-------------------|
| ρ_1 | $\frac{(1-\beta)\beta}{2-\beta}$ | | | $\frac{(1-\beta)\beta}{2-\beta}$ | | |
| ρ_2 | $\frac{\beta}{2}$ | | | $\frac{\beta}{2}$ | | |
| | $r < \rho_1$ | $\rho_1 \leq r \leq \rho_2$ | $r > \rho_2$ | $r < \rho_1$ | $\rho_1 \leq r \leq \rho_2$ | $r > \rho_2$ |
| p^* | $\frac{1}{2}(1-\beta+r)$ | $\frac{r}{\beta}$ | $\frac{1}{2}$ | $\frac{1}{2}(1-\beta+r)$ | $\frac{r}{\beta}$ | $\frac{1}{2}$ |
| w^* | $1-\kappa$ | $1-\kappa$ | $1-\kappa$ | $(1-\kappa)p^*$ | $(1-\kappa)p^*$ | $(1-\kappa)p^*$ |
| $\pi_c (= \pi_r + \pi_m)$ | $\frac{(1-\beta+r)^2}{4(1-\beta)}$ | $\frac{r(\beta-r)}{\beta^2}$ | $\frac{1}{4}$ | $\frac{(1-\beta+r)^2}{4(1-\beta)}$ | $\frac{r(\beta-r)}{\beta^2}$ | $\frac{1}{4}$ |
| π_r | $(1-\kappa)\pi_c$ | $(1-\kappa)\pi_c$ | $(1-\kappa)\pi_c$ | $(1-\kappa)\pi_c$ | $(1-\kappa)\pi_c$ | $(1-\kappa)\pi_c$ |
| π_m | $\kappa\pi_c$ | $\kappa\pi_c$ | $\kappa\pi_c$ | $\kappa\pi_c$ | $\kappa\pi_c$ | $\kappa\pi_c$ |
| | ASF | | | ASP | | |
| ρ_1 | $\frac{(1-\beta)\beta}{2-\beta}$ | | | $\frac{(1-\beta)\beta}{2-\beta}$ | | |
| ρ_2 | $\frac{\beta}{2}$ | | | $\frac{\beta}{2}$ | | |
| | $r < \rho_1$ | $\rho_1 \leq r \leq \rho_2$ | $r > \rho_2$ | $r < \rho_1$ | $\rho_1 \leq r \leq \rho_2$ | $r > \rho_2$ |
| p^* | $\frac{1}{2}(1-\beta+r)$ | $\frac{r}{\beta}$ | $\frac{1}{2}$ | $\frac{1}{2}(1-\beta+r)$ | $\frac{r}{\beta}$ | $\frac{1}{2}$ |
| w^* | κ | κ | κ | κp^* | κp^* | κp^* |
| $\pi_c (= \pi_r + \pi_m)$ | $\frac{(1-\beta+r)^2}{4(1-\beta)}$ | $\frac{r(\beta-r)}{\beta^2}$ | $\frac{1}{4}$ | $\frac{(1-\beta+r)^2}{4(1-\beta)}$ | $\frac{r(\beta-r)}{\beta^2}$ | $\frac{1}{4}$ |
| π_r | $(1-\kappa)\pi_c$ | $(1-\kappa)\pi_c$ | $(1-\kappa)\pi_c$ | $(1-\kappa)\pi_c$ | $(1-\kappa)\pi_c$ | $(1-\kappa)\pi_c$ |
| π_m | $\kappa\pi_c$ | $\kappa\pi_c$ | $\kappa\pi_c$ | $\kappa\pi_c$ | $\kappa\pi_c$ | $\kappa\pi_c$ |

- In all cases of AR and AS, Nash bargaining leads to incentive alignment between the manufacturer and the retailer, and the two parties act as if they were vertically integrated.

The summary highlights two important facts. First, when both p and w are determined through Nash bargaining, most of the retailing configurations are essentially equivalent. For instance, it is trivial to transform AR into AS, and vice versa. Second, Nash bargaining leads to a perfect incentive alignment between the manufacturer and retailer. Prior literature in retailing shows that vertical relationship between the manufacturer and retailer leads to divergence in their incentives, leading to the issue known as *vertical*

externality (Tirole 1992, p. 174). Vertical externality manifests itself as a higher retail price and reduced channel profit. When the price decision is made jointly through bargaining, however, the issue of vertical externality disappears, and the manufacturer and retailer act in perfect coordination as if they were vertically integrated. This is intuitive. Once the manufacturer and retailer are through deciding how to split the revenue—whether via a fixed fee or as a percentage of the total revenue—their incentives align to maximize the total channel revenue.

Appendix B

Appendix To Chapter 3

B.1. Complete Analysis of the Equilibrium

The purpose of this appendix is to provide all the details related to the overall analysis of the equilibrium and characterize it rigorously. Solving the manufacturer's optimization problem outlined in Table 3.1, we can easily find the equilibrium outcome in each region. In the short run, θ is exogenous, and the manufacturer only chooses the price for a given θ .

$\beta < 1$, Case 1A: From the legal demand in (3.2), the revenue in this case is:

$$R = p \left(\lambda \left(1 - \frac{p}{\theta} \right) + (1 - \lambda) \left(1 - \frac{p-r}{(1-\beta)\theta} \right) \right).$$

I substitute $\lambda = \frac{e\theta\xi(1-\beta)}{p - \frac{r}{\beta}}$ into this to obtain $R = p + ep\beta\xi - \frac{p(p-r)}{(1-\beta)\theta}$. The first order condition is then given by $\frac{\partial R}{\partial p} = 1 + e\beta\xi - \frac{2p-r}{(1-\beta)\theta} = 0$, which can be easily solved to obtain $p^*(\theta)$. It is easy to see that the second order condition is also satisfied.

$\beta \geq 1$, Case 1A: From (3.2), the revenue is:

$$R = p \left(\lambda \left(1 - \frac{p}{\theta} \right) + (1 - \lambda) \left(\frac{r-p}{(\beta-1)\theta} - \frac{p}{\theta} \right) \right).$$

As before, I substitute $\lambda = \frac{e\theta\xi}{\theta - \frac{r-p}{\beta-1}}$ into this to obtain $R = ep\xi + \frac{p(r-p\beta)}{(\beta-1)\theta}$, differentiating which, I obtain the following first order condition $\frac{\partial R}{\partial p} = e\xi + \frac{r-2p\beta}{(\beta-1)\theta} = 0$. Since the second order condition is also satisfied, the first order condition yields the desired price.

Case 1B: In Case 1B, λ is independent of p : $\lambda = \frac{e\theta\xi}{\theta - \frac{r}{\beta}}$. Therefore, the revenue, $R = p\lambda \left(1 - \frac{p}{\theta} \right)$, is clearly maximized at $p^*(\theta) = \frac{\theta}{2}$.

The derivations for all other cases are analogous. In fact, in all cases, except Case 3A, the optimal price can be obtained by solving the appropriate first order condition. In Case 3A, the limit price is obtained by

setting $\lambda = 1$ as indicated in Table 3.1. Accordingly, I get the prices in (3.6):

$$p^*(\theta) = \begin{cases} \frac{r}{2} + \frac{\theta(1-\beta)(1+e\beta\xi)}{2}, & \text{Case 1A } (\beta < 1), \\ \frac{r}{2\beta} + \frac{e\theta\xi(\beta-1)}{2\beta}, & \text{Case 1A } (\beta > 1), \\ \frac{r+\theta(1-\beta)(1+\xi)}{2(1+\xi(1-\beta))}, & \text{Case 2A } (\beta < 1), \\ \frac{r+\theta\xi(\beta-1)}{2(\beta+\xi(\beta-1))}, & \text{Case 2A } (\beta > 1), \\ \frac{r}{\beta} + e\theta\xi(1-\beta), & \text{Case 3A } (\beta < 1), \\ r - \theta(\beta-1)(1-e\xi), & \text{Case 3A } (\beta \geq 1), \\ \frac{\theta}{2}, & \text{Cases 1B, 2B, 3B, and 3C.} \end{cases}$$

In addition to satisfying the constraints in Table 3.1, for the equilibrium to occur within a specific region, its corresponding case must dominate, from the perspective of the manufacturer's profit, all other cases that provide a valid solution. Below, I ensure that all these requirements are met in determining the boundary for each region.

Characterization of Different Equilibrium Regions for $\beta < 1$

First, I determine the boundaries based on profit comparisons between pairs of equilibrium regions. I will subsequently show that the validity conditions in Table 3.1 are automatically satisfied within each region obtained from such profit comparisons.

Profit Comparisons

Region 1A:

Equating the profits for Cases 1A and 1B, I get:

$$e = h_1(r; \theta) = \frac{(r + (1 - \beta)\theta)^2}{\beta\theta\xi(1 - \beta)(\beta\theta - r)}.$$

I find that 1A dominates if $e < h_1(r; \theta)$, 1B dominates if $e > h_1(r; \theta)$, and both provide the same profit at $e = h_1(r; \theta)$. Similarly, equating the profits from 1A and 3A, I get the following boundary:

$$e = h_2(r; \theta) = \frac{1}{\xi} \left(\frac{1}{2 - \beta} - \frac{r}{\beta\theta(1 - \beta)} \right),$$

and 1A is valid only when $e < h_2(r; \theta)$. Now, I compare the profit in 1A with that from 2A and 2B to obtain:

$$e = h_3(r; \theta) = \frac{\sqrt{1 + \xi(1 - \beta)}(r + \theta(1 - \beta)(1 + \xi)) - \sqrt{1 + \xi}(r + (1 - \beta)\theta)(1 + \xi(1 - \beta))}{\beta\theta\xi\sqrt{1 + \xi}(1 - \beta)(1 + \xi(1 - \beta))}, \text{ and}$$

$$e = h_4(r; \theta) = \frac{\theta \left(\sqrt{\xi(1 - \beta)} - (1 - \beta)\sqrt{1 + \xi} \right) - r\sqrt{1 + \xi}}{\beta\theta\xi\sqrt{1 + \xi}(1 - \beta)}.$$

1A dominates to the right of these boundaries, that is, when e is greater than both $h_3(r; \theta)$ and $h_4(r; \theta)$. Region 1A can now be fully characterized as follows:

$$\{(r, e) | \max\{h_3(r; \theta), h_4(r; \theta)\} < e \leq \min\{h_1(r; \theta), h_2(r; \theta)\}; e, r \geq 0\}. \quad (\text{RGN1Ah})$$

Point to note here is that, in obtaining the boundaries of Region 1A, its profit need not be compared with the profit from either 3B or 3C. This is because, considering the validity conditions, I see that 1A can never have a valid solution when 3B or 3C does.

Region 1B:

The result of profit comparison with 1A is already captured as $e = h_1(r; \theta)$. Next, I compare its profit with that of 3A to obtain the following boundary:

$$e = h_5(r; \theta) = \frac{4r(2r - 3\beta\theta)(1 - \beta) + \beta^2\theta^2(3 - 4\beta) + \beta\theta\sqrt{\beta(\beta\theta^2(3 - 4\beta)^2 - 16r^2(1 - \beta) - 8r\theta(1 - \beta)(1 - 4\beta))}}{8\beta\theta\xi(1 - \beta)^2(\beta\theta - r)},$$

such that Region 1B occurs only above this boundary. Define:

$$h_6(r; \theta) = \begin{cases} h_1(r; \theta), & \text{if } 0 \leq r < \rho_{1h} = \frac{\theta(1 - \beta)(3\beta - 2)}{4 - 3\beta}, \\ h_5(r; \theta), & \text{otherwise,} \end{cases}$$

where ρ_{1h} is the solution of $h_1(r; \theta) = h_5(r; \theta)$. Clearly, $e = h_6(r; \theta)$ provides a combined lower boundary for 1B. To find the only other possible lower boundary for this region, I now compare its profit with that of Case 2B to obtain:

$$e = h_7(r; \theta) = \frac{\beta\theta - r}{\beta\theta(1 + \xi)}.$$

To find the upper boundary for Region 1B, I equate its profit with that from 3C to obtain:

$$e = \frac{\beta\theta - r}{\beta\theta\xi} = \frac{(1 + \xi)h_7(r; \theta)}{\xi}.$$

Therefore, Region 1B can now be fully characterized as follows:

$$\{(r, e) | \max\{h_6(r; \theta), h_7(r; \theta)\} \leq e < \frac{(1 + \xi)h_7(r; \theta)}{\xi}; r \leq \rho_{2h} = \frac{\theta(2\beta - 1)}{2}; e, r \geq 0\}, \quad (\text{RGN1Bh})$$

where ρ_{2h} is the solution of $h_5(r; \theta) = \frac{(1 + \xi)h_7(r; \theta)}{\xi}$.

Region 2A:

Comparing the profits from Cases 2A and 2B, I get:

$$r = \rho_{3h} = \theta(1 + \xi) \left(\frac{\sqrt{\xi(1 - \beta)(1 + \xi(1 - \beta))}}{1 + \xi} - (1 - \beta) \right).$$

Region 2A occurs to the right of this boundary and 2B, to the left. The boundary with Region 1A is already derived as $e = h_3(r; \theta)$. I now find the boundary between 2A and 3A by comparing their profits:

$$e = h_8(r; \theta) = \frac{\beta\theta - 2r}{2\beta\theta\xi(1-\beta)} + \frac{1}{2\theta\xi(1-\beta)^2} \sqrt{\frac{(1-\beta)(\theta(1-\beta)(1+\xi)(\beta\theta - 2r) - r^2)}{(1+\xi)(1+\xi(1-\beta))}}.$$

Then, the combined upper boundary can be found as:

$$e = h_9(r; \theta) = \begin{cases} h_3(r; \theta), & \text{if } 0 \leq r < \rho_{4h} = \frac{\theta(1-\beta)(1+\xi)}{2-\beta} \left(2\sqrt{1-\frac{\beta\xi}{1+\xi}} - (2-\beta) \right), \\ h_8(r; \theta), & \text{if } \rho_{4h} \leq r < \rho_{5h} = \frac{\beta\theta(1-\beta)(1+\xi)}{2-\beta+2\xi(1-\beta)}, \\ 0, & \text{otherwise,} \end{cases}$$

where ρ_{4h} and ρ_{5h} solve $h_3(r; \theta) = h_8(r; \theta)$ and $h_8(r; \theta) = 0$, respectively. Region 2A can now be expressed as:

$$\{(r, e) | e \leq h_9(r; \theta); \rho_{3h} \leq r < \rho_{5h}; e, r \geq 0\}. \quad (\text{RGN2Ah})$$

Region 2B:

The possible boundaries with Regions 1A, 1B and 2A have already been found as $e = h_4(r; \theta)$, $e = h_7(r; \theta)$, and $r = \rho_{3h}$, respectively. Of these, $e = h_4(r; \theta)$ and $r = \rho_{3h}$ are valid boundaries for Region 2B only if Case 1A and Case 2A occur for the given set of parameter values. Hence, I modify them in the following manner:

$$e = h_{10}(r; \theta) = \begin{cases} h_4(r; \theta), & \text{if } h_4(r; \theta) < h_2(r; \theta), \\ \infty, & \text{otherwise,} \end{cases} \quad \text{and} \quad r = \rho_{6h} = \begin{cases} \rho_{3h}, & \text{if } \rho_{3h} < \rho_{5h}, \\ \infty, & \text{otherwise.} \end{cases}$$

I now find the possible boundary with Region 3A by comparing the profits:

$$e = h_{11}(r; \theta) = \frac{\beta\theta \left(1 - \frac{1}{\sqrt{1+\xi}} \right) - 2r}{2\beta\theta\xi(1-\beta)}.$$

The complete characterization of Region 2B is, therefore, given by:

$$\{(r, e) | e \leq \min\{h_7(r; \theta), h_{10}(r; \theta), h_{11}(r; \theta)\}; r < \rho_{6h}; e, r \geq 0\}. \quad (\text{RGN2Bh})$$

Region 3A:

The only remaining profit comparison is between Regions 3A and 3C. I do so now to obtain the boundary between them:

$$e = h_{12}(r; \theta) = \frac{\beta\theta - 2r}{2\beta\theta\xi(1-\beta)}.$$

It is now possible to express this region as:

$$\{(r, e) | \max\{h_2(r; \theta), h_9(r; \theta), h_{11}(r; \theta)\} \leq e \leq \min\{h_5(r; \theta), h_{12}(r; \theta)\}; e, r \geq 0\}. \quad (\text{RGN3Ah})$$

Region 3B: Since quality θ , is exogenous in the short run, Region 3B (limit quality) cannot occur in the short-run equilibrium.

Region 3C:

When enforcement is very high, on either side, the threat of piracy disappears completely, and we enter this region of pure monopoly. Therefore, there is no upper boundary for this region. It only has a lower boundary, shared with Regions 1B and 3A; these boundaries have been found above as $e = \frac{(1+\xi)h_7(r;\theta)}{\xi}$ and $e = h_{12}(r;\theta)$, respectively. Therefore, we can characterize this region as:

$$\left\{ (r, e) \mid e > \min \left\{ \frac{(1+\xi)h_7(r;\theta)}{\xi}, h_{12}(r;\theta) \right\}; e, r \geq 0 \right\}. \quad (\text{RGN3Ch})$$

Verifying the Validity Conditions in Table 3.1

I will first show that the condition $\frac{p-r}{(1-\beta)\theta} \leq 1$ is automatically satisfied in Cases 1A and 2A, which are characterized by (RGN1Ah) and (RGN2Ah), respectively. I prove this by contradiction. Let p^* be the equilibrium solution for a specific (r, e) point satisfying (RGN1Ah), but suppose that $\frac{p^*-r}{(1-\beta)\theta} > 1$. Then, from (3.5), we should have $\lambda = \frac{e\theta\xi}{\theta - \frac{r}{\beta}}$, the same as the expression for λ in Case 1B. This makes p^* a feasible solution in Case 1B. Furthermore, since $\left(1 - \frac{p^*-r}{(1-\beta)\theta}\right)$ is now negative, comparing the objective functions for Cases 1A and 1B in Table 3.1, we can immediately infer that the profit from Case 1B is higher. Since the profit from a feasible solution is higher, the optimal profit from Case 1B must also be higher, which is the desired contradiction. A similar argument comparing the profits from Cases 2A and 2B would show that $\frac{p^*-r}{(1-\beta)\theta} \leq 1$ is also satisfied when the equilibrium is obtained from Case 2A.

Let us now show that the condition $\frac{p-r}{(1-\beta)\theta} > 1$ is met in Cases 1B and 2B, which are characterized by (RGN1Bh) and (RGN2Bh), respectively. Let p^* now be the equilibrium solution for a specific (r, e) point satisfying (RGN1Bh), but suppose that $\frac{p^*-r}{(1-\beta)\theta} \leq 1$. Again, (3.5) tells us that we should have $\lambda = \frac{e\theta\xi(1-\beta)}{p^* - \frac{r}{\beta}}$, which is the same as the expression for λ in Case 1A, making p^* a feasible solution in Case 1A. Now, comparing the objective functions for Cases 1A and 1B, it becomes clear that the profit from Case 1A is higher, which, once again, leads to a contradiction. A similar argument applies to Case 2B as well.

Turning our attention to Case 3A, which must abide by $\frac{p-r}{(1-\beta)\theta} \leq 1$, it is easy to verify that this constraint is also automatically met, because, if not, the solution would become feasible in Case 3B, which is not possible in the short run.

I also observe that, if the equilibrium obtained from Case 1A leads to $\lambda = \frac{e\theta\xi(1-\beta)}{p^* - \frac{r}{\beta}} \leq \frac{\xi}{\xi+1}$, the solution would immediately become a feasible one in Case 2A, and, at the same time, the difference between the

Case 1A and Case 2A profits, which is simply $p^* \left(\lambda - \frac{\xi}{\xi+1} \right) \left(\frac{p^*-r}{(1-\beta)\theta} - \frac{v^*}{\theta} \right)$, would be non-positive. Since a feasible solution in Case 2A is now at least as good, Case 2A must dominate Case 1A, again a contradiction to the claim that Case 1A dominates. A similar argument can be used to prove that Case 1B can dominate Case 2B only if $\lambda > \frac{\xi}{\xi+1}$. Clearly, the converse is true as well; unless $\lambda \leq \frac{\xi}{\xi+1}$, Cases 2A and 2B cannot dominate.

Moving on to the $\lambda < 1$ constraint in Case 1A, if this constraint is relaxed to $\lambda \leq 1$, Case 1A would subsume Case 3A, and the optimal solution obtained from either case would be exactly the same whenever $\lambda = 1$. In other words, the point at which the constraint $\lambda \leq 1$ becomes binding would be precisely the one where Case 3A takes over, implying that the constraint $\lambda < 1$ is algebraically equivalent to $e < h_2(r; \theta)$. A similar argument works for Case 1B as well, since violating this constraint would put the equilibrium in Case 3B, which is impossible.

Finally, simple algebra can show that the two constraints associated with Case 3C, namely, $\frac{e\theta\xi(1-\beta)}{p^* - \frac{r}{\beta}} > 1$ and $\frac{e\theta\xi}{\theta - \frac{r}{\beta}} > 1$, are equivalent to $e > h_{12}(r; \theta)$ and $e > \frac{(1+\xi)h_7(r; \theta)}{\xi}$, respectively. So, they are also satisfied when Case 3C dominates.

Characterization of Different Equilibrium Regions for $\beta \geq 1$

Once again, as before, all the regions can be characterized from profit comparisons. As shown later, the validity conditions in Table 3.1 are automatically satisfied within each region.

Profit Comparisons

Region 1A:

Equating the profits for Cases 1A and 1B, I get:

$$e = k_1(r; \theta) = \frac{r^2}{\theta\xi(\beta-1)(\beta\theta-r)}.$$

Case 1A dominates if $e < k_1(r; \theta)$, 1B dominates if $e > k_1(r; \theta)$, and both provide the same profit at $e = k_1(r; \theta)$. Now, equating the profits from 1A and 3A, I get:

$$e = k_2(r; \theta) = \frac{1}{\xi} \left(\frac{2\beta}{2\beta-1} - \frac{r}{(\beta-1)\theta} \right),$$

and 1A is valid only when $e < k_2(r; \theta)$. Next, I compare the profit in 1A with that from 2A and 2B to obtain:

$$e = k_3(r; \theta) = \frac{\sqrt{\beta}\theta\xi(1-\beta) - r\left(\sqrt{\beta} - \sqrt{(1+\xi)(\beta-\xi(1-\beta))}\right)}{\theta\xi(1-\beta)\sqrt{(1+\xi)(\beta-\xi(1-\beta))}}, \text{ and}$$

$$e = k_4(r; \theta) = \frac{\theta\sqrt{\beta\xi(\beta-1)} - r\sqrt{1+\xi}}{\theta\xi\sqrt{1+\xi}(\beta-1)}.$$

1A dominates to the right of these boundaries, that is, when e is greater than both $k_3(r; \theta)$ and $k_4(r; \theta)$. Region 1A can now be fully characterized as follows:

$$\{(r, e) | \max\{k_3(r; \theta), k_4(r; \theta)\} < e \leq \min\{k_1(r; \theta), k_2(r; \theta)\}; e, r \geq 0\}. \quad (\text{RGN1Ak})$$

As before, Region 1A can never have a valid solution when 3B or 3C does and, in obtaining the boundaries of Region 1A, its profit need not be compared with the profit from either 3B or 3C.

Region 1B:

The result of profit comparison with 1A has already been captured as $e = k_1(r; \theta)$. Next, I compare its profit with that of 3A to obtain the following boundary:

$$e = k_5(r; \theta) = \frac{\theta\sqrt{\beta\theta(24r(\beta-1) + \beta\theta(9-8\beta)) - 16r^2(\beta-1)} - (4r\theta(\beta-1)(4\beta-1) + \beta\theta^2(4\beta(3-2\beta)-3) - 8r^2(\beta-1))}{8\theta\xi(\beta-1)^2(\beta-r)},$$

such that Region 1B occurs only above this boundary. Combining this with $e = k_1(r; \theta)$, I get:

$$e = k_6(r; \theta) = \begin{cases} k_1(r; \theta), & \text{if } 0 \leq r < \rho_{1k} = \frac{2(\beta-1)\beta\theta}{4\beta-3}, \\ k_5(r; \theta), & \text{otherwise,} \end{cases}$$

where ρ_{1k} is the solution of $k_1(r; \theta) = k_5(r; \theta) = k_2(r; \theta)$. Clearly, $e = k_6(r; \theta)$ provides a combined lower boundary for 1B. To find the only other possible lower boundary for this region, I now compare its profit with that of Case 2B to obtain:

$$e = k_7(r; \theta) = \frac{\beta\theta - r}{\beta\theta(1+\xi)}.$$

Finally, to find the upper boundary for Region 1B, I equate its profit with that from 3C to obtain:

$$e = \frac{\beta\theta - r}{\beta\theta\xi} = \frac{(1+\xi)k_7(r; \theta)}{\xi}.$$

Therefore, the full characterization of Region 1B is as follows:

$$\left\{ (r, e) | \max\{k_6(r; \theta), k_7(r; \theta)\} \leq e < \frac{(1+\xi)k_7(r; \theta)}{\xi}; r \leq \rho_{2k} = \frac{\beta\theta}{2}; e, r \geq 0 \right\}, \quad (\text{RGN1Bk})$$

where ρ_{2k} is the solution of $k_5(r; \theta) = \frac{(1+\xi)k_7(r; \theta)}{\xi}$.

Region 2A:

Comparing the profits from Cases 2A and 2B, I get:

$$r = \rho_{3k} = \theta \left(\sqrt{\xi(\beta-1)(\beta+\xi(\beta-1))} - \xi(\beta-1) \right).$$

Region 2A occurs to the right of this boundary and 2B, to the left. The boundary with Region 1A is already derived as $e = k_3(r; \theta)$. I now find the boundary between 2A and 3A by comparing their profits:

$$e = k_8(r; \theta) = \frac{(2\beta-1)\theta - 2r - \sqrt{\frac{\theta^2(\beta-1)(\beta+\xi(2\beta-1)) - r^2 - 2r\theta\xi(\beta-1)}{(\beta-1)(1+\xi)(\beta+\xi(\beta-1))}}}{2\theta\xi(\beta-1)}.$$

Then, the combined upper boundary can be found as:

$$e = k_9(r; \theta) = \begin{cases} k_3(r; \theta), & \text{if } 0 \leq r < \rho_{4k} = (\beta-1)(1+\xi) \left(\frac{2\theta\sqrt{\beta(\beta-\frac{\xi}{1+\xi})}}{2\beta-1} - \frac{\theta\xi}{1+\xi} \right), \\ k_8(r; \theta), & \text{if } \rho_{4k} \leq r < \rho_{5k} = \frac{\theta(\beta-1)(2\beta(1+\xi)-\xi)}{2\beta(1+\xi)-1-2\xi}, \\ 0, & \text{otherwise,} \end{cases}$$

where ρ_{4k} and ρ_{5k} solve $k_3(r; \theta) = k_8(r; \theta)$ and $k_8(r; \theta) = 0$, respectively. Region 2A can now be expressed as:

$$\{(r, e) | e \leq k_9(r; \theta); \rho_{3k} \leq r < \rho_{5k}; e, r \geq 0\}. \quad (\text{RGN2Ak})$$

Region 2B:

The possible boundaries with Regions 1A, 1B, and 2A have already been found as $e = k_4(r; \theta)$, $e = k_7(r; \theta)$, and $r = \rho_{3k}$, respectively. Of these, $e = k_4(r; \theta)$ is a valid boundary for Region 2B only if Case 1A and Case 2A occur for the given set of parameter values. Hence, I modify them:

$$e = k_{10}(r; \theta) = \begin{cases} k_4(r; \theta), & \text{if } k_4(r; \theta) < k_2(r; \theta), \\ \infty, & \text{otherwise,} \end{cases} \quad \text{and} \quad r = \rho_{6k} = \begin{cases} \rho_{3k}, & \text{if } \rho_{3k} < \rho_{5k}, \\ \infty, & \text{otherwise.} \end{cases}$$

I now find the possible boundary with Region 3A by comparing profits:

$$e = k_{11}(r; \theta) = \frac{\theta(2\beta-1) - 2r - \frac{\theta}{\sqrt{1+\xi}}}{2\theta\xi(\beta-1)}.$$

The complete characterization of Region 2B is, therefore, given by:

$$\{(r, e) | e \leq \min\{k_7(r; \theta), k_{10}(r; \theta), k_{11}(r; \theta)\}; r < \rho_{6k}; e, r \geq 0\}. \quad (\text{RGN2Bk})$$

Region 3A:

The only remaining profit comparison is between Regions 3A and 3C. I do so now to obtain the boundary between them:

$$e = k_{12}(r; \theta) = \frac{\theta(2\beta-1) - 2r}{2\theta\xi(\beta-1)}.$$

It is now possible to express this region as:

$$\{(r, e) | \max\{k_2(r; \theta), k_9(r; \theta), k_{11}(r; \theta)\} \leq e \leq \min\{k_5(r; \theta), k_{12}(r; \theta)\}; e, r \geq 0\}. \quad (\text{RGN3Ak})$$

Region 3B: As before, Region 3B cannot occur in the short-run equilibrium.

Region 3C:

As before, there is no upper boundary for this region. It only has a lower boundary, shared with Regions 1B and 3A. Both these boundaries have been found above. Therefore, I can characterize this region as:

$$\{(r, e) | e > \min\left\{\frac{(1+\xi)k_7(r; \theta)}{\xi}, k_{12}(r; \theta)\right\}; e, r \geq 0\}. \quad (\text{RGN3Ck})$$

Verifying the Validity Conditions in Table 3.1

The line of argument here is quite similar to the case of $\beta < 1$. I first show that the condition $p \leq \frac{r}{\beta}$ is automatically satisfied in Cases 1A and 2A, which are as characterized by (RGN1Ak) and (RGN2Ak), respectively. I prove this by contradiction. Let p^* be the equilibrium solution for a specific (r, e) point satisfying (RGN1Ak), but suppose that $p^* > \frac{r}{\beta}$. Then, from (3.5), we must have $\lambda = \frac{e\theta\xi}{\theta - \frac{r}{\beta}}$, as in Case 1B. So, p^* is a feasible solution in Case 1B. Furthermore, since $\left(\frac{r-p^*}{(\beta-1)\theta} - \frac{p^*}{\theta}\right)$ is now negative, comparing the objective functions for Cases 1A and 1B in Table 3.1, we can immediately infer that the profit from Case 1B is higher, which is impossible. A similar argument with Cases 2A and 2B shows that $p \leq \frac{r}{\beta}$ is also satisfied when the equilibrium is obtained from Case 2A.

Let us now show that the condition $p > \frac{r}{\beta}$ is met in Cases 1B and 2B, which are characterized by (RGN1Bk) and (RGN2Bk), respectively. Let p^* now be the equilibrium solution for a specific (r, e) point satisfying (RGN1Bk), but suppose that $p^* \leq \frac{r}{\beta}$. Again, (3.5) tells us that we should have $\lambda = \frac{e\theta\xi}{\theta - \frac{r-p^*}{\beta-1}}$, which is the same as the expression for λ in Case 1A, making p^* a feasible solution in Case 1A. Now, comparing the objective functions for Cases 1A and 1B, it becomes clear that the profit from Case 1A is higher, which again leads to a contradiction. A similar argument applies to Case 2B, too.

I now guide our attention to Case 3A, where $p \leq \frac{r}{\beta}$ is automatically met, because, if not, the solution would become feasible in Case 3B, which, as before, is not possible in the short run.

Furthermore, if the equilibrium obtained from Case 1A leads to $\lambda \leq \frac{\xi}{\xi+1}$, the solution would become a feasible one in Case 2A, and the difference between the Case 1A and Case 2A profits, which is $p^* \left(\lambda - \frac{\xi}{\xi+1}\right) \left(1 - \frac{r-p^*}{(\beta-1)\theta}\right)$, would be non-positive, as the existence of piracy in either case ensures that $\frac{r-p^*}{(\beta-1)\theta} < 1$. Since a feasible solution in Case 2A is now at least as good, Case 2A must dominate Case 1A,

again a contradiction. A similar argument can be used to prove that Case 1B can dominate Case 2B only if $\lambda > \frac{\xi}{\xi+1}$. Clearly, the converse is true as well; unless $\lambda \leq \frac{\xi}{\xi+1}$, Cases 2A and 2B cannot dominate.

Next, I consider the constraint $\lambda < 1$ in Case 1A; if this constraint is relaxed to $\lambda \leq 1$, Case 1A would subsume Case 3A, and the point at which the constraint $\lambda \leq 1$ becomes binding would be precisely the one where Case 3A takes over, implying that the constraint $\lambda < 1$ is now algebraically equivalent to $e < k_2(r; \theta)$. A similar argument works for Case 1B as well, as violating this constraint would place the equilibrium in Case 3B, which is impossible.

Finally, simple algebra can show that the two constraints associated with Case 3C, $\frac{e\theta\xi}{\theta - \frac{r-p^*}{\beta-1}} > 1$ and $\frac{e\theta\xi}{\theta - \frac{r}{\beta}} > 1$, are equivalent to $e > k_{12}(r; \theta)$ and $e > \frac{(1+\xi)k_7(r; \theta)}{\xi}$, respectively. So, they are also satisfied when Case 3C dominates.

B.2. Technical Results and Proofs

B.2.1 A Few Useful Observations

Before I provide all the proofs, I will state a few results that would be useful later for several of the proofs:

- In Case 1A, if $\beta < 1$, then $\frac{r}{\beta\theta} < \frac{1-\beta}{2-\beta}$. This is because in Case 1A, $e < h_2(r; \theta)$, which can be satisfied only if $h_2(r) > 0$ leading to this result. Of course, this also means that $\frac{r}{\beta\theta} < \frac{1}{2}$.
- In Case 1A, if $\beta \geq 1$, then $\frac{r}{\beta\theta} < \frac{2(\beta-1)}{2\beta-1}$. This is because in Case 1A, $e < k_2(r; \theta)$, which can be satisfied only if $k_2(r) > 0$ leading to this result.
- In Case 1B, if $\beta < 1$, then $\frac{r}{\beta\theta} < \frac{2\beta-1}{2\beta}$. This is because in Case 1B, $r < \rho_{2h} = \frac{\theta(2\beta-1)}{2}$. Since this condition can only be satisfied if $\rho_{2h} > 0$, Case 1B can occur only if $\beta > \frac{1}{2}$.
- In Case 1B, if $\beta \geq 1$, then $\frac{r}{\beta\theta} < \frac{1}{2}$. This is because in Case 1B, $r < \rho_{2k} = \frac{\beta\theta}{2}$.
- In both Cases 1A and 1B, irrespective of the value of β , $e\xi < 1 - \frac{r}{\beta\theta} < 1$. In Case 1A, when $\beta < 1$, $\lambda < 1$ and $\frac{p-r}{(1-\beta)\theta} < 1$ can be combined to obtain the result, and when $\beta \geq 1$, I can get the result from $\lambda < 1$ and $p < \frac{r}{\beta}$. Finally, in case 1B, the result follows directly from $\lambda < 1$.
- In Case 1A, for $\beta < 1$, $(1 - e\xi(1 - \beta)) > (1 - 2e\xi(1 - \beta)) > 0$. The first inequality is trivially true. To prove the second one, I note that, in this case, $e < h_2(r; \theta)$, which implies that $e\xi < \frac{1}{2-\beta} < \frac{1}{2(1-\beta)}$. The inequality follows.

B.2.2 Proofs

Proof of Lemma 3.1

When $\beta = 1$, the pirated version has exactly the same quality as the legal version. Therefore, if $p \leq r$, no consumer will consider the pirated product. That is, however, not allowed in Cases 1A and 2A, where some level of piracy must exist, by definition. On the other hand, if $p > r$, only the inherently ethical consumer segment will purchase the legal product, making the equilibrium outcome fall under either Case 1B or 2B. ■

Proof of Proposition 3.1

See Appendix B.1. ■

Proof of Theorem 3.1

In order to prove this, I will show that:

- i) If $\beta < 1$, in Case 1A, social welfare is increasing in e if $r < \frac{e\beta^2\theta\xi}{2}$; it is decreasing otherwise. In this case, social welfare is increasing in r if $e < \frac{\beta\theta(1-\beta)-r(4-3\beta)}{2\beta\theta\xi(1-\beta)^2}$, and decreasing otherwise.
- ii) If $\beta < 1$, in Case 1B, social welfare is increasing in e if $\beta < \frac{3}{4}$ or if $r > \frac{\theta\sqrt{\beta(4\beta-3)}}{2}$; it is decreasing if neither condition holds. Furthermore, social welfare is increasing in r if $r > \frac{\theta(2\beta-\sqrt{3\beta})}{2}$ and $e > \frac{8r(\beta\theta-r)^2}{\beta\theta\xi(4r(2\beta\theta-r)+\beta\theta^2(3-4\beta))}$; otherwise, it is decreasing.
- iii) If $\beta \geq 1$, over the entire primary piracy region (Cases 1A and 1B), social welfare is monotonically decreasing in both e and r .

I now make use of (3.10) to obtain the appropriate expression for social welfare in different cases.

- i) $\beta < 1$, Case 1A: Differentiating short-run social welfare with respect to e , I get:

$$\frac{\partial(SW)}{\partial e} = \frac{\xi(1-\beta)(e\beta^2\theta\xi - 2r)}{4},$$

which would be positive if $r < \frac{e\beta^2\theta\xi}{2}$ and negative otherwise. Similarly, from (3.10), I can get:

$$\frac{\partial(SW)}{\partial r} = \frac{1}{4} \left(1 - 2e\xi(1-\beta) - \frac{r(4-3\beta)}{\beta\theta(1-\beta)} \right),$$

which would be positive if $e < \frac{\beta\theta(1-\beta)-r(4-3\beta)}{2\beta\theta\xi(1-\beta)^2}$ and negative otherwise.

- ii) $\beta < 1$, Case 1B: Again, differentiating the appropriate expression for social welfare with respect to e , I get:

$$\frac{\partial(SW)}{\partial e} = \frac{\xi(4r^2 + \beta\theta^2(3-4\beta))}{8(\beta\theta - r)},$$

which is positive if either $\beta \leq \frac{3}{4}$ or $r > \frac{\theta}{2}\sqrt{\beta(4\beta-3)}$, and negative when neither condition holds. Differentiating (3.10) with respect to r yields:

$$\frac{\partial(SW)}{\partial r} = -\frac{r}{\beta\theta} - \frac{e\xi}{2} + \frac{3e\beta\theta^2\xi}{8(\beta\theta-r)^2},$$

which can be shown to be positive if and only if $r > \frac{\theta(2\beta-\sqrt{3\beta})}{2}$ and $e > \frac{8r(\beta\theta-r)^2}{\beta\theta\xi(4r(2\beta\theta-r)+\beta\theta^2(3-4\beta))}$.

iii) $\beta \geq 1$, Case 1A: In this case:

$$\frac{\partial(SW)}{\partial e} = \frac{\theta\xi(\beta-1)\left(\frac{e\xi}{2} - \frac{\beta\theta+r}{\theta}\right)}{2\beta},$$

which is always negative since $\frac{e\xi}{2} < \frac{1}{2} < 1 < \beta < \frac{\beta\theta+r}{\theta}$. Also, I find:

$$\frac{\partial(SW)}{\partial r} = -\frac{2e\xi(\beta-1)^2 + \frac{r(4\beta-3)}{\theta}}{4\beta(\beta-1)},$$

which is clearly negative.

$\beta \geq 1$, Case 1B: Here, I have:

$$\frac{\partial(SW)}{\partial e} = \frac{\xi(4r^2 - \beta\theta^2(4\beta-3))}{8(\beta\theta-r)}.$$

Now, since $r < \frac{\beta\theta}{2}$, I find that $4r^2 - \beta\theta^2(4\beta-3) < (\beta\theta)^2 - \beta\theta^2(4\beta-3) = -3\beta\theta^2(\beta-1) < 0$, implying that the numerator is negative. Since the denominator is clearly positive, the partial derivative must be negative overall. Finally, from (3.10), I get:

$$\frac{\partial(SW)}{\partial r} = -\frac{r}{\beta\theta} - \frac{e\xi}{2} + \frac{3e\beta\theta^2\xi}{8(\beta\theta-r)^2}.$$

Clearly, $\frac{\partial(SW)}{\partial r}$ is a linear function of e and is maximized at an extreme value of e , that is, either at $e = 0$ or at $e = \frac{\beta\theta-r}{\beta\theta\xi}$, the second extreme value being derived from the constraint that $\lambda < 1$ in this case. Now, since $\left.\frac{\partial(SW)}{\partial r}\right|_{e=0} = -\frac{r}{\beta\theta} < 0$, I only need to consider the other extreme value. I find that:

$$\left.\frac{\partial(SW)}{\partial r}\right|_{e=\frac{\beta\theta-r}{\beta\theta\xi}} = \frac{4r^2 - \beta\theta^2(4\beta-3)}{8\beta\theta(\beta\theta-r)},$$

the denominator of which is positive, and the numerator, as shown above, is negative, making the right hand side negative overall.

Since the maximum of $\frac{\partial(SW)}{\partial r}$, taken over the valid parameter range, is negative, $\frac{\partial(SW)}{\partial r}$ must also be negative in this entire range. ■

Proof of Proposition 3.2

- i) I start by showing that, in the primary piracy region, the piracy rate, as given in (3.7), is monotonically decreasing in both e and r .

$\beta < 1$, Case 1A: From (3.7), the piracy rate in this case is given by:

$$\mu = \frac{r(2 - \beta) - \beta\theta(1 - \beta)(1 - e\xi(2 - \beta))}{2(1 - \beta)(r - \beta\theta(1 - e\xi(1 - \beta)))}.$$

Taking partial derivative with respect to e and r , I get:

$$\frac{\partial \mu}{\partial e} = -\frac{\beta^2\theta^2\xi}{2(r - \beta\theta(1 - e\xi(1 - \beta)))^2} < 0, \quad \text{and} \quad \frac{\partial \mu}{\partial r} = -\frac{\beta\theta}{2(1 - \beta)(r - \beta\theta(1 - e\xi(1 - \beta)))^2} < 0.$$

$\beta \geq 1$, Case 1A: From (3.7):

$$\mu = \frac{r(2\beta - 1) - \theta(\beta - 1)(e\xi + 2\beta(1 - e\xi))}{(\beta - 1)(r + \theta(e\xi(\beta - 1) - 2\beta))}.$$

Again, taking partial derivative with respect to e and r , I obtain:

$$\frac{\partial \mu}{\partial e} = -\frac{2\beta^2\theta^2\xi}{(r - 2\beta\theta + e\theta\xi(\beta - 1))^2} < 0, \quad \text{and} \quad \frac{\partial \mu}{\partial r} = -\frac{2\beta^2\theta}{(\beta - 1)(r - 2\beta\theta + e\theta\xi(\beta - 1))^2} < 0.$$

Case 1B: From (3.7), the piracy rate in this case is given by:

$$\mu = \frac{2(\beta\theta - r)(r - \beta\theta(1 - e\xi))}{2r\beta\theta(2 - e\xi) - \beta^2\theta^2(2 - e\xi) - 2r^2}.$$

Taking partial derivative with respect to e and r , I get:

$$\frac{\partial \mu}{\partial e} = -\frac{2\beta^2\theta^2\xi(\beta\theta - r)^2}{(2(\beta\theta - r)^2 - e\beta\theta\xi(\beta\theta - 2r))^2} < 0, \quad \text{and} \quad \frac{\partial \mu}{\partial r} = -\frac{2e\beta^2\theta^2\xi(\beta\theta(2 - e\xi) - 2r)}{(2(\beta\theta - r)^2 - e\beta\theta\xi(\beta\theta - 2r))^2}.$$

Now, to show that $\frac{\partial \mu}{\partial r}$ is also negative, I need to prove that $\beta\theta(2 - e\xi) > 2r$, which follows directly by multiplying the last inequality below by $\beta\theta > 0$:

$$e\xi < 1 - \frac{r}{\beta\theta} \quad \Leftrightarrow \quad 2 - e\xi > 2 - \left(1 - \frac{r}{\beta\theta}\right) = 1 + \frac{r}{\beta\theta} > \frac{2r}{\beta\theta}.$$

- ii) I now show that the manufacturer's profit, as given in (3.8), is monotonically increasing in both e and r .

$\beta < 1$, Case 1A: In this case, $R^* = \frac{(r + \theta(1 - \beta)(1 + e\beta\xi))^2}{4\theta(1 - \beta)}$. I simply differentiate R^* with respect to e and r to get:

$$\frac{\partial R^*}{\partial e} = \frac{\beta\xi(r + \theta(1 - \beta)(1 + e\beta\xi))}{2} > 0 \quad \text{and} \quad \frac{\partial R^*}{\partial r} = \frac{1 + e\beta\xi}{2} + \frac{r}{2\theta(1 - \beta)} > 0.$$

$\beta \geq 1$, Case 1A: Now, $R^* = \frac{(r+e\theta\xi(\beta-1))^2}{4\beta\theta(\beta-1)}$. Differentiate R^* with respect to e and r to get:

$$\frac{\partial R^*}{\partial e} = \frac{\xi(r+e\theta\xi(\beta-1))}{2\beta} > 0 \quad \text{and} \quad \frac{\partial R^*}{\partial r} = \frac{e\xi}{2\beta} + \frac{r}{2\beta\theta(\beta-1)} > 0.$$

Case 1B: $R^* = \frac{e\xi\beta\theta^2}{4(\beta\theta-r)}$. As before, I differentiate R^* with respect to e and r :

$$\frac{\partial R^*}{\partial e} = \frac{\beta\theta^2\xi}{4(\beta\theta-r)} > 0 \quad \text{and} \quad \frac{\partial R^*}{\partial r} = \frac{e\beta\theta^2\xi}{4(\beta\theta-r)^2} > 0.$$

iii) I now show that consumer surplus, as in (3.9), is monotonically decreasing in both e and r .

$\beta < 1$, Case 1A: In this case:

$$\frac{\partial(CS)}{\partial e} = -\frac{\xi(1-\beta)(2(\beta\theta-r)+e\beta^2\theta\xi)}{4} < 0 \quad \text{and} \quad \frac{\partial(CS)}{\partial r} = \frac{r(4-3\beta)}{4\beta\theta(1-\beta)} + \frac{e\xi(1-\beta)}{2} - \frac{3}{4}.$$

The first inequality follows from $\frac{r}{\beta\theta} < 1$. To show that $\frac{\partial(CS)}{\partial r}$ is negative, I note that it is a linear increasing function of e and r , and its maximum is obtained at the extreme point (\bar{r}, \bar{e}) , where $\frac{\bar{r}}{\beta\theta} = \frac{1-\beta}{2-\beta}$ and $\bar{e} = h_2(r)$. However, even this maximum value is negative because:

$$\left. \frac{\partial(CS)}{\partial r} \right|_{\substack{e=\bar{e} \\ r=\bar{r}}} = -\frac{1}{2(2-\beta)} < 0.$$

Therefore, $\frac{\partial(CS)}{\partial r}$ must be negative over the entire range.

$\beta \geq 1$, Case 1A: Here:

$$\frac{\partial(CS)}{\partial e} = -\frac{\xi(\beta-1)(2(\beta\theta-r)+e\theta\xi)}{4\beta} < 0 \quad \text{and} \quad \frac{\partial(CS)}{\partial r} = -1 + \frac{e\xi(\beta-1)}{2\beta} + \frac{r(4\beta-3)}{4\beta\theta(\beta-1)}.$$

The first one is clearly negative as $\frac{r}{\beta\theta} < 1$. To prove that the second one is also negative, I note that it is a linearly increasing function of e and r , and, once again, its maximum is obtained at the extreme point (\bar{r}, \bar{e}) , where $\frac{\bar{r}}{\beta\theta} = \frac{2(\beta-1)}{2\beta-1}$ and $\bar{e} = k_2(r)$. It turns out that this maximum value is negative because:

$$\left. \frac{\partial(CS)}{\partial r} \right|_{\substack{e=\bar{e} \\ r=\bar{r}}} = -\frac{1}{2(2\beta-1)} < 0.$$

Therefore, $\frac{\partial(CS)}{\partial r}$ must also be negative over the entire range.

Case 1B: In this case:

$$\frac{\partial(CS)}{\partial e} = \frac{\xi(8r\beta\theta + \beta\theta^2(1-4\beta) - 4r^2)}{8(\beta\theta-r)} \quad \text{and} \quad \frac{\partial(CS)}{\partial r} = -1 + \frac{r}{\beta\theta} + e\xi \left(\frac{1}{2} + \frac{\beta\theta^2}{8(\beta\theta-r)^2} \right).$$

To show that the first one is negative, I must prove that $X(r) = 8r\beta\theta + \beta\theta^2(1-4\beta) - 4r^2 < 0$. Now $X(r)$ is clearly a concave function of r and its maximum, $r = \beta\theta$, is not a valid interior solution, as

$r < \beta\theta$. Therefore, the maximum of $X(r)$ occurs at the upper bound of r . First, for θ_{1B} to be a valid solution, r must always satisfy $\frac{r}{\beta\theta} < \frac{1}{3}$. In addition, if $\beta < 1$, r must also satisfy $\frac{r}{\beta\theta} < \frac{2\beta-1}{2\beta}$. I consider two cases. When $\beta < 1$, I set $\bar{r} = \frac{\beta\theta(2\beta-1)}{2\beta}$ and find that $X(\bar{r}) < -(1-\beta)\theta^2 < 0$. On the other hand, when $\beta \geq 1$, I set $\bar{r} = \frac{\beta\theta}{3}$ and find that $X(\bar{r}) < -\frac{\beta\theta^2(16\beta-9)}{9} < 0$.

To show that the second expression is also negative, I note that it is a linearly increasing function of both r and e , and must have its maximum at the largest possible values of r and e . Now, since $e\xi < 1 - \frac{r}{\beta\theta}$, the upper bound for e is simply $\bar{e} < \frac{\beta\theta-r}{\beta\theta\xi}$. Once again, I consider two cases. When $\beta \leq \frac{3}{4}$, I set $\bar{r} = \frac{\beta\theta(2\beta-1)}{2\beta}$ and get:

$$\left. \frac{\partial(CS)}{\partial r} \right|_{\substack{e=\bar{e} \\ r=\bar{r}}} = -\frac{1-\beta}{4\beta} < 0.$$

Now, when $\beta > \frac{3}{4}$, I set $\bar{r} = \frac{\beta\theta}{3}$ and get:

$$\left. \frac{\partial(CS)}{\partial r} \right|_{\substack{e=\bar{e} \\ r=\bar{r}}} = \frac{9-16\beta}{48\beta} < 0.$$

iv) I now show that the legal social welfare, SW_L , is monotonically increasing in both e and r .

$\beta < 1$, Case 1A: By differentiating the legal social welfare in this case, it can be easily shown that:

$$\frac{\partial(SW_L)}{\partial e} = \frac{\beta\xi(r + \theta(1-\beta + e\beta\xi(3-2\beta)))}{4} > 0 \quad \text{and} \quad \frac{\partial(SW_L)}{\partial r} = \frac{e\beta\xi}{4} + \frac{\theta(1-\beta) - r}{4\theta(1-\beta)^2}.$$

The second derivative would also be positive if $(1-\beta)\theta - r > 0$, which is indeed true because:

$$\frac{r}{\beta\theta} < \frac{1-\beta}{2-\beta} \Rightarrow (1-\beta)\theta - r > \frac{r(2-\beta)}{\beta} - r = \frac{2r(1-\beta)}{\beta} > 0.$$

$\beta \geq 1$, Case 1A: Here:

$$\frac{\partial(SW_L)}{\partial e} = \frac{\xi(r + \beta\theta(2 - e\xi))}{4\beta} \quad \text{and} \quad \frac{\partial(SW_L)}{\partial r} = \frac{1}{8} \left(\frac{2e\xi}{\beta} + \frac{2r(3\beta-2)}{\beta\theta(\beta-1)^2} \right).$$

Since $e\xi < 1$ and $\beta > 1$, the above derivatives are both positive.

Case 1B: In this case, I have:

$$\frac{\partial(SW_L)}{\partial e} = \frac{3\beta\theta^2\xi}{8(\beta\theta - r)} > 0 \quad \text{and} \quad \frac{\partial(SW_L)}{\partial r} = \frac{3e\beta\theta^2\xi}{8(\beta\theta - r)^2} > 0,$$

which completes the proof. ■

Appendix C

Appendix To Chapter 4

C.1. Complete Analysis of the Equilibrium

The long-run equilibrium can be obtained by taking the short-run outcome and simply endogenizing θ . In other words, in each case, I maximize the total profit, $\pi(\theta) = p^*(\theta)q - \frac{c\theta^2}{2}$, to find the optimal quality from the first order condition for that case. As shown in Lemma 4.1, for $\beta \geq 1$, Cases 1A and 2A cannot occur in the long run.

Case 1A: As per Lemma 4.1, I only need to consider $\beta < 1$, in which case, the manufacturer's profit can be obtained from (3.2):

$$\pi = p^*(\theta) \left(\lambda \left(1 - \frac{p^*(\theta)}{\theta} \right) + (1 - \lambda) \left(1 - \frac{p^*(\theta) - r}{(1 - \beta)\theta} \right) \right) - \frac{c\theta^2}{2},$$

where $\lambda = \frac{e\theta\xi(1-\beta)}{p^*(\theta) - \frac{r}{\beta}}$ and $p^*(\theta) = \frac{r}{2} + \frac{\theta(1-\beta)(1+e\beta\xi)}{2}$. Substituting these, I get:

$$\pi = \frac{(r + \theta(1 - \beta)(1 + e\beta\xi))^2}{4\theta(1 - \beta)} - \frac{c\theta^2}{2}$$

The first order condition can then be derived as:

$$\frac{\partial \pi}{\partial \theta} = \frac{(1 - \beta)(1 + e\beta\xi)^2}{4} - \frac{r^2}{4\theta^2(1 - \beta)} - c\theta = 0. \quad (\text{FOC1A})$$

I now show that (FOC1A) has a unique real positive root that maximizes the profit. First, I observe that (FOC1A) is essentially a cubic equation, and the signs of its coefficients indicate that the product of the three roots is negative but their sum is positive. This immediately implies that there are exactly one negative and at most two positive roots. Of these two positive roots, one is a minimum, and the other a maximum, of the underlying profit. Since $\frac{\partial \pi}{\partial \theta}$ approaches $-\infty$ as θ becomes large, the higher of the two positive roots must be the maximum.

Case 1B: In this case, the profit is: $\pi = p^*(\theta)\lambda\left(1 - \frac{p^*(\theta)}{\theta}\right) - \frac{c\theta^2}{2}$, where $p^*(\theta) = \frac{\theta}{2}$ and $\lambda = \frac{e\theta\xi}{\theta - \frac{r}{\beta}}$. Substituting these, I get :

$$\pi = \frac{e\beta\theta^2\xi}{4(\beta\theta - r)} - \frac{c\theta^2}{2}.$$

The first order condition can then be derived as:

$$\frac{\partial\pi}{\partial\theta} = \frac{e\beta\theta\xi(\beta\theta - 2r)}{4(\beta\theta - r)^2} - c\theta = 0. \quad (\text{FOC1B})$$

Since $\theta \neq 0$, (FOC1B) reduces to a quadratic equation, and the sign of the coefficients indicate that both of its roots are positive. Again, $\frac{\partial\pi}{\partial\theta}$ approaches $-\infty$ as θ becomes very large, implying that the larger of the two roots, $\theta_{1B} = \frac{r}{\beta} + \frac{e\beta\xi + \sqrt{e\beta\xi(e\beta\xi - 16cr)}}{8c\beta}$, is the unique maximum.

Along similar lines, the first order condition in Case 2A (for $\beta < 1$) can be derived as:

$$\frac{(1 - \beta)(1 + \xi)}{4(1 + \xi(1 - \beta))} - \frac{r^2}{4\theta^2(1 - \beta)(1 + \xi)(1 + \xi(1 - \beta))} - c\theta = 0, \quad (\text{FOC2A})$$

which has one negative root and at most two positive roots, and that the maximum profit is achieved at the largest positive root, θ_{2A} . In Case 2B, the profit, $\frac{\theta\xi}{4(1 + \xi)} - \frac{c\theta^2}{2}$, is convex and is maximized at $\theta_{2B} = \frac{\xi}{4c(1 + \xi)}$.

Further, I can show that, for Case 3A, the first order conditions for $\beta < 1$ and $\beta \geq 1$ are respectively:

$$e\xi(1 - \beta)(1 - e\xi(1 - \beta)) + \frac{r^2}{\beta^2\theta^2} - c\theta = 0, \quad \text{and} \quad (\text{FOC3AH})$$

$$(\beta - 1)(1 - e\xi)(e\xi(\beta - 1) - \beta) + \frac{r^2}{\theta^2} - c\theta = 0. \quad (\text{FOC3AK})$$

As the underlying profit is concave, the unique positive roots of these equations, which I name θ_{3AH} and θ_{3AK} , provide the optimal quality levels. For Case 3B, the limit quality can be obtained by setting $\lambda = 1$ as indicated in Table 3.1. In Case 3C, the profit, $\frac{\theta}{4} - \frac{c\theta^2}{2}$, is convex and is maximized at $\theta_{3C} = \frac{1}{4c}$.

Taken together, we get the optimal quality as:

$$\theta^* = \begin{cases} \theta_{1A}, & \text{Case 1A } (\beta < 1), \\ \theta_{1B} = \frac{r}{\beta} + \frac{e\beta\xi + \sqrt{e\beta\xi(e\beta\xi - 16cr)}}{8c\beta}, & \text{Case 1B}, \\ \theta_{2A}, & \text{Case 2A } (\beta < 1), \\ \theta_{2B} = \frac{\xi}{4c(1 + \xi)}, & \text{Case 2B}, \\ \theta_{3AH}, & \text{Case 3A } (\beta < 1), \\ \theta_{3AK}, & \text{Case 3A } (\beta \geq 1), \\ \theta_{3B} = \frac{r}{\beta(1 - e\xi)}, & \text{Case 3B}, \\ \theta_{3C} = \frac{1}{4c}, & \text{Case 3C}. \end{cases}$$

From this optimal quality, I can also find the manufacturer's profit in each region. Let π_i be the profit

in Region i . I can write the equilibrium profit as:

$$\pi^*(\theta^*) = \begin{cases} \pi_{1A} = \frac{(r+\theta_{1A}(1-\beta)(1+e\beta\xi))^2}{4\theta_{1A}(1-\beta)} - \frac{c\theta_{1A}^2}{2}, & \text{Case 1A } (\beta < 1), \\ \pi_{1B} = \frac{e\beta\theta_{1B}^2\xi}{4(\beta\theta_{1B}-r)} - \frac{c\theta_{1B}^2}{2}, & \text{Case 1B}, \\ \pi_{2A} = \frac{(r+\theta_{2A}(1-\beta)(1+\xi))^2}{4\theta_{2A}(1-\beta)(1+\xi)(1+\xi(1-\beta))} - \frac{c\theta_{2A}^2}{2}, & \text{Case 2A } (\beta < 1), \\ \pi_{2B} = \frac{\xi^2}{32c(1+\xi)^2}, & \text{Case 2B}, \\ \pi_{3AH} = \frac{(r+e\beta\theta_{3AH}\xi(1-\beta))(\beta\theta_{3AH}(1-e\xi(1-\beta))-r)}{\beta^2\theta_{3AH}} - \frac{c\theta_{3AH}^2}{2}, & \text{Case 3A } (\beta < 1), \\ \pi_{3AK} = \frac{(\beta\theta_{3AK}-r-e\theta\xi(\beta-1))(r-\theta_{3AK}(\beta-1)(1-e\xi))}{\theta_{3AK}} - \frac{c\theta_{3AK}^2}{2}, & \text{Case 3A } (\beta \geq 1), \\ \pi_{3B} = \frac{r(\beta(1-e\xi)-2cr)}{4\beta^2(1-e\xi)^2}, & \text{Case 3B}, \\ \pi_{3C} = \frac{1}{32c}, & \text{Case 3C}. \end{cases}$$

Characterization of Different Equilibrium Regions for $\beta < 1$

Finding the boundaries of different regions for the long-run equilibrium is now a conceptually straightforward extension of the short-run exercise. To characterize the boundaries, I again compare the profits in these regions, the solutions to which are denoted in Table C.1.

Table C.1: Boundaries Obtained from Profit Comparisons for $\beta < 1$

| Profit Comparison | Boundary Solution | Profit Comparison | Boundary Solution |
|------------------------|--------------------|------------------------|--------------------|
| $\pi_{1A} = \pi_{1B}$ | $e = H_1(r)$ | $\pi_{2A} = \pi_{2B}$ | $r = \rho_{3H}$ |
| $\pi_{1A} = \pi_{2A}$ | $e = H_2(r)$ | $\pi_{2A} = \pi_{3AH}$ | $e = H_6(r)$ |
| $\pi_{1A} = \pi_{2B}$ | $e = H_3(r)$ | $\pi_{2B} = \pi_{3AH}$ | $r = \rho_{4H}(e)$ |
| $\pi_{1A} = \pi_{3AH}$ | $r = \rho_{1H}(e)$ | $\pi_{3AH} = \pi_{3B}$ | $e = H_7(r)$ |
| $\pi_{1B} = \pi_{2B}$ | $e = H_4(r)$ | $\pi_{3AH} = \pi_{3C}$ | $e = H_8(r)$ |
| $\pi_{1B} = \pi_{3AH}$ | $e = H_5(r)$ | $\pi_{3B} = \pi_{3C}$ | $e = H_9(r)$ |
| $\pi_{1B} = \pi_{3B}$ | $r = \rho_{2H}(e)$ | | |

Region 1A:

Region 1A can be characterized as follows:

$$\{(r, e) | \max\{H_2(r), H_3(r)\} < e \leq H_1(r); r < \rho_{1H}(e); e, r \geq 0\}. \quad (\text{RGN1AH})$$

Region 1B:

Define:

$$H_{10}(r) = \begin{cases} H_1(r), & \text{if } 0 \leq r < \rho_{5H}, \\ H_5(r), & \text{otherwise,} \end{cases}$$

where ρ_{5H} is the solution of $H_1(r) = H_5(r)$. Clearly, $e = H_{10}(r)$ provides a combined lower boundary for 1B. The only other possible lower boundary for this region is $e = H_4(r)$. The upper/right boundary for Region 1B is obtained by comparing the profits from 1B and 3B. It turns out that $\pi_{1B} = \pi_{3B}$ has two roots, only one of which is valid depending on the parameter values:

$$r = \rho_{2H}(e) = \begin{cases} \frac{\beta(1-e\xi)(2e\xi-1)}{4ce\xi}, & \text{if } e \geq \frac{\beta}{24c}, \\ \frac{e^2\beta\xi^2(1-e\xi)}{2c(2-e\xi)^2}, & \text{otherwise.} \end{cases}$$

Region 1B can now be fully characterized as follows:

$$\{(r, e) | e > \max\{H_4(r), H_{10}(r)\}; r < \min\{\rho_{2H}(e), \rho_{6H}\}; e, r \geq 0\}, \quad (\text{RGN1BH})$$

where $r = \rho_{6H} = \frac{2\beta-1}{8c}$ solves $H_8(r) = H_9(r)$.

Region 2A:

The combined upper boundary for this region can be found as:

$$e = H_{11}(r) = \begin{cases} H_2(r), & \text{if } 0 \leq r < \rho_{7H}, \\ H_6(r), & \text{if } \rho_{7H} \leq r < \rho_{8H}, \\ 0, & \text{otherwise,} \end{cases}$$

where ρ_{7H} and ρ_{8H} solve $H_2(r) = H_6(r)$ and $H_6(r) = 0$, respectively. Region 2A can now be expressed as:

$$\{(r, e) | e \leq H_{11}(r); \rho_{3H} \leq r < \rho_{8H}; e, r \geq 0\}. \quad (\text{RGN2AH})$$

Region 2B:

The boundary between 2A and 2B is relevant only when Case 2A occurs, so I define:

$$\rho_{9H} = \begin{cases} \rho_{3H}, & \text{if } \rho_{3H} < \rho_{8H}, \\ \infty, & \text{otherwise.} \end{cases}$$

The complete characterization of Region 2B is then given by:

$$\{(r, e) | e \leq \min\{H_3(r), H_4(r)\}; r < \min\{\rho_{4H}(e), \rho_{9H}\}; e, r \geq 0\}. \quad (\text{RGN2BH})$$

Region 3A:

The boundary with 3C is given by $e = H_8(r) = \frac{\beta-8cr}{2\beta\xi(1-\beta)}$. It is now possible to express this region as:

$$\{(r, e) | H_{11}(r) \leq e \leq \min\{H_5(r), H_7(r), H_8(r)\}; r \geq \max\{\rho_{1H}(e), \rho_{4H}(e)\}; e, r \geq 0\}. \quad (\text{RGN3AH})$$

Region 3B:

Solving $\pi_{3B} = \pi_{3C}$, I get $e = H_9(r) = \frac{\beta-4cr}{\beta\xi}$. Then, Region 3B can be characterized as:

$$\{(r, e) | H_7(r) \leq e \leq H_9(r); \rho_{2H}(e) \leq r < \rho_{6H}; e, r \geq 0\}. \quad (\text{RGN3BH})$$

Region 3C:

When enforcement is very high, on either side, the threat of piracy disappears completely, and I enter this region of pure monopoly. Therefore, there is no upper boundary for this region. It only has a lower boundary, shared with Regions 3A and 3B. Therefore, I can characterize this region as:

$$\{(r, e) | e > \min\{H_8(r), H_9(r)\}; e, r \geq 0\}. \quad (\text{RGN3CH})$$

Verifying the Validity Conditions in Table 3.1

Verifying these conditions is quite similar to the short run case. I start by showing that the condition $\frac{p-r}{(1-\beta)\theta} \leq 1$ is automatically satisfied in Cases 1A and 2A, which are characterized by (RGN1AH) and (RGN2AH), respectively. I prove this by contradiction. Let (p^*, θ^*) be the equilibrium solution for a specific (r, e) point satisfying (RGN1AH), but suppose that $\frac{p^*-r}{(1-\beta)\theta^*} > 1$. Then, from (3.5), we should have $\lambda = \frac{e\theta^*\xi}{\theta^*-r}$, the same as the one in Case 1B. This makes (p^*, θ^*) a feasible solution in Case 1B. Furthermore, since $\left(1 - \frac{p^*-r}{(1-\beta)\theta^*}\right)$ is now negative, comparing the objective functions for Cases 1A and 1B in Table 3.1, we can immediately infer that the profit from Case 1B is higher, which is a contradiction. A similar argument comparing the profits from Cases 2A and 2B would show that $\frac{p^*-r}{(1-\beta)\theta^*} \leq 1$ is also satisfied when the equilibrium is obtained from Case 2A.

Let us now show that the condition $\frac{p-r}{(1-\beta)\theta} > 1$ is met in Cases 1B and 2B, which are characterized by (RGN1BH) and (RGN2BH), respectively. Let (p^*, θ^*) now be the equilibrium solution for a specific (r, e) point satisfying (RGN1BH), but suppose that $\frac{p^*-r}{(1-\beta)\theta^*} \leq 1$. Again, (3.5) tells us that we should have $\lambda = \frac{e\theta^*\xi(1-\beta)}{p^*-r}$, which is the same as the one for λ in Case 1A, making (p^*, θ^*) a feasible solution in Case 1A. Now, comparing the objective functions for Cases 1A and 1B, it becomes clear that the profit from Case 1A is higher, which leads to a contradiction. A similar argument applies to Case 2B as well.

I also observe that, if the equilibrium obtained from Case 1A leads to $\lambda = \frac{e\theta^*\xi(1-\beta)}{p^*-r} \leq \frac{\xi}{\xi+1}$, the solution would immediately become a feasible one in Case 2A, and, at the same time, the difference between the

Case 1A and Case 2A profits, which is simply $p^* \left(\lambda - \frac{\xi}{\xi+1} \right) \left(\frac{p^*-r}{(1-\beta)\theta^*} - \frac{p^*}{\theta^*} \right)$, would be non-positive. Since a feasible solution in Case 2A is now at least as good, Case 2A must dominate Case 1A, again a contradiction to the claim that Case 1A dominates. A similar argument can be used to prove that Case 1B can dominate Case 2B only if $\lambda > \frac{\xi}{\xi+1}$. Clearly, the converse is true as well; unless $\lambda \leq \frac{\xi}{\xi+1}$, Cases 2A and 2B cannot dominate.

Moving on to the $\lambda < 1$ constraint in Case 1A, if this constraint is relaxed to $\lambda \leq 1$, Case 1A would subsume Case 3A, in the sense that the optimal solution obtained from either case would be exactly the same whenever the first order conditions for 1A leads to $\lambda = 1$. In other words, the point at which the constraint $\lambda \leq 1$ becomes binding would be precisely the one where Case 3A takes over, implying that the constraint $\lambda < 1$ is algebraically equivalent to $r < \rho_{1H}(e)$. A similar argument works for Case 1B as well.

Between Cases 3A and 3B, the equilibrium must also satisfy: (i) $\frac{p-r}{(1-\beta)\theta} < 1$ in 3A, and (ii) $\frac{p-r}{(1-\beta)\theta} \geq 1$ in 3B. These must hold since violating one would simply move the equilibrium to the other region.

Finally, simple algebra can show that the two constraints associated with Case 3C, namely, $\frac{e\theta^*\xi(1-\beta)}{p^*-\frac{r}{\beta}} > 1$ and $\frac{e\theta^*\xi}{\theta^*-\frac{r}{\beta}} > 1$, are equivalent to $e > H_8(r)$ and $e > H_9(r)$, respectively.

Characterization of Different Equilibrium Regions for $\beta \geq 1$

When $\beta \geq 1$, Cases 1A and 2A cannot occur. I now compare the profits in the remaining five regions to characterize the boundaries, as shown in Table C.2.

Table C.2: Boundaries Obtained from Profit Comparisons for $\beta \geq 1$

| Profit Comparison | Boundary Solution | Profit Comparison | Boundary Solution |
|------------------------|--------------------|------------------------|-------------------|
| $\pi_{1B} = \pi_{2B}$ | $e = K_1(r)$ | $\pi_{3AK} = \pi_{3B}$ | $e = K_3(r)$ |
| $\pi_{1B} = \pi_{3AK}$ | $e = K_2(r)$ | $\pi_{3AK} = \pi_{3C}$ | $e = K_4(r)$ |
| $\pi_{1B} = \pi_{3B}$ | $r = \rho_{1K}(e)$ | $\pi_{3B} = \pi_{3C}$ | $e = K_5(r)$ |
| $\pi_{2B} = \pi_{3AK}$ | $r = \rho_{2K}(e)$ | | |

Region 1B:

As before, it turns out that $\pi_{1B} = \pi_{3B}$ has two roots, only one of which is valid:

$$r = \rho_{1K}(e) = \begin{cases} \frac{\beta(1-e\xi)(2e\xi-1)}{4ce\xi}, & \text{if } e \geq \frac{\beta}{24c}, \\ \frac{e^2\beta\xi^2(1-e\xi)}{2c(2-e\xi)^2}, & \text{otherwise.} \end{cases}$$

Region 1B can be characterized as follows:

$$\{(r, e) | e > \max\{K_1(r), K_2(r)\}; r < \rho_{1K}(e); e, r \geq 0\}. \quad (\text{RGN1BK})$$

Region 2B:

The complete characterization of Region 2B is given by:

$$\{(r, e) | e \leq K_1(r); r < \rho_{2K}(e); e, r \geq 0\}. \quad (\text{RGN2BK})$$

Region 3A:

The boundary with 3C is given by $e = K_4(r) = \frac{2\beta-1-8cr}{2\xi(\beta-1)}$. It is now possible to express this region as:

$$\{(r, e) | e \leq \min\{K_2(r), K_3(r), K_4(r)\}; r \geq \rho_{2K}(e); e, r \geq 0\}. \quad (\text{RGN3AK})$$

Region 3B:

Solving $\pi_{3B} = \pi_{3C}$, I get $e = K_5(r) = \frac{\beta-4cr}{\beta\xi}$. Then, Region 3B can be characterized as:

$$\{(r, e) | K_3(r) \leq e \leq K_5(r); r \geq \rho_{1K}(e); e, r \geq 0\}. \quad (\text{RGN3BK})$$

Region 3C:

When enforcement is very high, on either side, the threat of piracy disappears completely, and we enter this region of pure monopoly. Therefore, there is no upper boundary for this region. It only has a lower boundary, shared with Regions 3A and 3B. Therefore, we can characterize this region as:

$$\{(r, e) | e > \min\{K_4(r), K_5(r)\}; e, r \geq 0\}. \quad (\text{RGN3CK})$$

Verifying the Validity Conditions in Table 3.1

I first show that the condition $p > \frac{r}{\beta}$ is automatically satisfied in Case 1B and 2B, which are characterized by (RGN1BK) and (RGN2BK), respectively. I prove this by contradiction. Let (p^*, θ^*) now be the equilibrium solution for a specific (r, e) point satisfying (RGN1BK), but suppose that $p^* \leq \frac{r}{\beta}$. However, that would make (p^*, θ^*) a feasible solution in Case 1A, which is not possible in the long run. A similar argument applies to Case 2B as well.

Moving on to the $\lambda < 1$ constraint in Case 1B, I note that this constraint is subsumed by the constraint $r < \rho_{1K}(e)$. Also, between Cases 3A and 3B, the equilibrium must also satisfy: (i) $p \leq \frac{r}{\beta}$ in 3A, and (ii) $p > \frac{r}{\beta}$ in 3B. These must also hold since violating one would simply move the equilibrium to the other region.

Finally, simple algebra can show that the two constraints associated with Case 3C, namely, $\frac{e\theta^*\xi}{\theta^* - \frac{r-p^*}{\beta-1}} > 1$ and $\frac{e\theta^*\xi}{\theta^* - \frac{r}{\beta}} > 1$, are equivalent to $e > K_4(r)$ and $e > K_5(r)$, respectively.

C.2. Technical Results and Proofs

C.2.1 A Few Useful Observations

Before I provide all the proofs, I will state a few results that would be useful later for several of the proofs:

- All the conditions provided in Appendix B.2.1 for the short-run analysis must remain valid in the long run as well—a violation would simply mean that the price chosen cannot be the optimal one. Viewed differently, since all these conditions must be satisfied by $p^*(\theta)$ for any given θ , they should clearly be satisfied by $p^*(\theta^*)$.
- In Case 1B, irrespective of the value of β , $\frac{r}{\beta\theta^*} < \frac{1}{3}$. The first order condition with respect to θ leads to $c = \frac{e\beta\xi(\beta\theta^* - 2r)}{4(\beta\theta^* - r)^2}$, the second derivative of the profit with respect to θ is simply $\left(-\frac{e\beta^2\theta\xi(\beta\theta - 3r)}{4(\beta\theta - r)^3}\right)$. Since this second derivative is negative at an interior maximum, we must have $\beta\theta^* > 3r$.

C.2.2 Technical Lemmas

I only state the lemmas here; their proofs are available, along with all the other proofs, in Appendix C.2.3.

Lemma A1. *Let $X(r; \theta)$ be given by:*

$$X(r; \theta) = 8r^3 - 3r^2\beta\theta(1 - 2e\xi(1 - \beta)) - 2r\theta^2(2 - 3\beta(1 - \beta) + e\beta\xi(4 - 5\beta)(1 - \beta) + 2e^2\beta^2\xi^2(1 - \beta)^2) \\ + \beta\theta^3(1 - 2e\xi(1 - \beta))(1 - \beta)^2(1 + e\beta\xi)^2.$$

Then, the equation $X(r; \theta) = 0$ has two positive roots for $0 < \beta < 1$, the smaller of which is denoted by $r = \sigma_{1A}(\theta)$. Furthermore, for $r \in \left[0, \frac{\beta\theta(1 - \beta)}{2 - \beta}\right]$, $X(r; \theta) > 0$ if and only if $r < \sigma_{1A}(\theta)$.

Lemma A2. *Let $X_1(e; \theta)$ and $X_2(r; \theta)$ be given by:*

$$X_1(e; \theta) = -6r^3 - r^2\theta(4 - 9\beta + 2e\beta\xi(2 - 3\beta)) + 2r\theta^2(1 - \beta)^2(1 + e\beta\xi)^2 \\ - \beta\theta^3(1 - \beta)(1 + e\beta\xi)(1 - 5\beta + 7e\beta\xi(1 - \beta) + 2e^2\beta^2\xi^2(1 - \beta)), \text{ and} \\ X_2(r; \theta) = 2r^3\beta - 3r^2\theta(1 - \beta)(1 + e\beta\xi(1 - \beta)) - 2r\theta^2(1 - \beta)^2(1 + \beta + e\beta^2\xi(1 + e\xi)) \\ + \theta^3(1 - \beta)^3(1 + e\beta\xi)^2(1 + e\beta\xi(1 - \beta)).$$

- If $r \in \left[0, \frac{\beta\theta(1 - \beta)}{2 - \beta}\right]$ and $0 < \beta < 1$, the equation $X_1(e; \theta) = 0$ has three real roots, the largest of which is denoted by $e = \gamma_{1A}(\theta)$. Then, $X_1(e; \theta) > 0$ at a positive e if and only if $e < \gamma_{1A}(\theta)$.*
- The equation $X_2(r; \theta) = 0$ has two positive roots, the smaller of which is denoted by $r = \tau_{1A}(\theta)$. Then, for $r \in \left[0, \frac{\beta\theta(1 - \beta)}{2 - \beta}\right]$ and $0 < \beta < 1$, $X_2(r; \theta) > 0$ if and only if $r < \tau_{1A}(\theta)$.*

C.2.3 Proofs

Proof of Lemma 4.1

For $\beta \geq 1$, in Case 1A, the manufacturer's profit is given by $\pi = ep\xi + \frac{p(r-p\beta)}{(\beta-1)\theta} - \frac{c\theta^2}{2}$; please see Appendix B.1. Clearly, this profit can be made arbitrarily large by reducing θ arbitrarily close to 0 (as $p < \frac{r}{\beta}$ in this case). Therefore, an interior solution for Case 1A cannot exist.

For Case 2A, the profit at $p^*(\theta)$ specified in (3.6) is:

$$\frac{(r + (\beta - 1)\theta)^2 \xi}{4(\beta - 1)(2\beta - 1)\theta(1 + \xi)} - \frac{c\theta^2}{2},$$

which can again be made arbitrarily large by choosing θ arbitrarily small, and an interior solution thus cannot exist. ■

Proof of Proposition 4.1

See Appendix C.1. ■

Proof of Theorem 4.1

Case 1A: From (FOC1A), I get:

$$\pi' = \frac{(1 - \beta)(1 + e\beta\xi)^2}{4} - \frac{r^2}{4\theta^2(1 - \beta)} - c\theta.$$

Clearly,

$$\frac{\partial \pi'}{\partial e} = \frac{\beta\xi(1 - \beta)(1 + e\beta\xi)}{2} > 0, \quad \text{and} \quad \frac{\partial \pi'}{\partial r} = -\frac{r}{2(1 - \beta)\theta^2} < 0.$$

Finally, the second order condition must be satisfied in optimality, that is, $\frac{\partial \pi'}{\partial \theta} \Big|_{\theta=\theta^*} < 0$. It now follows from the Implicit Function Theorem that:

$$\frac{d\theta^*}{de} = -\frac{\frac{\partial \pi'}{\partial e} \Big|_{\theta=\theta^*}}{\frac{\partial \pi'}{\partial \theta} \Big|_{\theta=\theta^*}} > 0 \quad \text{and} \quad \frac{d\theta^*}{dr} = -\frac{\frac{\partial \pi'}{\partial r} \Big|_{\theta=\theta^*}}{\frac{\partial \pi'}{\partial \theta} \Big|_{\theta=\theta^*}} < 0.$$

Case 1B: In this case, (FOC1B) gives:

$$\pi' = \frac{e\beta\theta\xi(\beta\theta - 2r)}{4(\beta\theta - r)^2} - c\theta,$$

differentiating which, I get:

$$\frac{\partial \pi'}{\partial e} = \frac{\beta\theta\xi(\beta\theta - 2r)}{4(\beta\theta - r)^2} > 0 \quad \text{and} \quad \frac{\partial \pi'}{\partial r} = -\frac{er\beta\theta\xi}{2(\beta\theta - r)^3} < 0.$$

Once again, the second order condition must be satisfied in optimality, implying $\frac{\partial \pi'}{\partial \theta} \Big|_{\theta=\theta^*} < 0$. The rest follows from the Implicit Function Theorem, in a manner similar to the proof for 1A above. ■

Proof of Theorem 4.2

To prove this theorem, I will show that:

- i) If $\beta < 1$, social welfare is increasing in e over the entire primary piracy region. However, social welfare is not monotonic in r and is increasing either only at a low r or for a low β and a high e . In particular, in Case 1A, social welfare is increasing in r if and only if $r < \sigma_{1A}(\theta^*)$, where $\sigma_{1A}(\cdot)$ is as defined in Lemma A1. Similarly, in Case 1B, social welfare is increasing in r if $\beta < \frac{3}{4}$ and $e > \frac{8r(2\beta\theta^*-r)(\beta\theta^*-r)^3}{\beta^2\theta^{*2}\xi(20r^3+4r^2\theta^*-44r^2\beta\theta^*-11r\beta\theta^{*2}+28r\beta^2\theta^{*2}+3\beta^2\theta^{*3}-4\beta^3\theta^{*3})}$; otherwise, it is decreasing.
- ii) If $\beta \geq 1$, social welfare is increasing in e except when e is large. Specifically, it is increasing if $e < \frac{4(\beta\theta^*-r)^2(\beta\theta^*-2r)(\beta^2\theta^{*2}+r^2)}{\beta^2\theta^{*2}\xi(4r^3+4r^2\theta^*(4\beta-1)-r\beta\theta^{*2}(28\beta-13)+4\beta^2\theta^{*3}(2\beta-1))}$; it is decreasing otherwise. However, social welfare is decreasing in r over the entire primary piracy region.

I now proceed to prove each case separately.

- i) I start with $\beta < 1$. When $\beta < 1$, Cases 1A and 1B can both occur, and I need to consider them separately.

$\beta < 1$, Case 1A: I first consider the long-run impact of e . Using the chain rule, I get:

$$\left. \frac{d(SW)}{de} \right|_{\theta=\theta^*} = \left. \frac{\partial(SW)}{\partial e} \right|_{\theta=\theta^*} + \left. \frac{\partial(SW)}{\partial \theta} \right|_{\theta=\theta^*} \frac{d\theta^*}{de}.$$

The expression for $\left. \frac{\partial(SW)}{\partial e} \right|_{\theta=\theta^*}$ is already available from the proof of Theorem 3.1. Also, I use the Implicit Function Theorem and the intermediate steps from the proof of Theorem 4.1 to find:

$$\frac{d\theta^*}{de} = - \frac{\left. \frac{\partial \pi'}{\partial e} \right|_{\theta=\theta^*}}{\left. \frac{\partial \pi'}{\partial \theta} \right|_{\theta=\theta^*}} = \frac{2\beta\theta^{*3}\xi(1-\beta)^2(1+e\beta\xi)}{(1-\beta)^2\theta^{*2}(1+e\beta\xi)^2 - 3r^2}.$$

Finally, I can also find that:

$$\left. \frac{\partial(SW)}{\partial \theta} \right|_{\theta=\theta^*} = \frac{r^2(4-\beta)}{8\beta\theta^2(1-\beta)} + \frac{1+3\beta}{8} - \frac{e\beta\xi(1-\beta)}{2} - \frac{e^2\beta^2\xi^2(1-\beta)}{8}.$$

Combining all of these, I get:

$$\left. \frac{d(SW)}{de} \right|_{\theta=\theta^*} = \frac{\xi(1-\beta)X}{4Y}, \text{ where}$$

$$X = 6r^3 + 4r^2\beta\theta^* \left(\frac{4-\beta}{4\beta} + e\xi(1-\beta) \right) - 2r\theta^{*2}(1-\beta)^2(1+e\beta\xi)^2 + 3\beta^2\theta^{*3}(1-\beta)(1+e\beta\xi) \left(\frac{1+3\beta}{3\beta} - e\xi(1-\beta) \right),$$

and $Y = \theta^{*2}(1-\beta)^2(1+e\beta\xi)^2 - 3r^2$.

Now, I show that $Y > 0$. First, (FOC1A) implies that $Y = 4c(1-\beta)\theta^{*3} - 2r^2$. Then, from the second order condition, I find that $\frac{1}{4} \left(\frac{2r^2}{(1-\beta)\theta^{*3}} - 4c \right) < 0$ must hold, immediately implying that $Y > 0$. Therefore, to complete the proof, I only need to show that X is positive. I do so now.

Since $r < \frac{\beta\theta^*}{2}$, I can write:

$$\begin{aligned} X &> 6r^3 + 4r^2\beta\theta^* \left(\frac{4-\beta}{4\beta} + e\xi(1-\beta) \right) - \beta\theta^{*3}(1-\beta)^2(1+e\beta\xi)^2 + 3\beta^2\theta^{*3}(1-\beta)(1+e\beta\xi) \left(\frac{1+3\beta}{3\beta} - e\xi(1-\beta) \right) \\ &= 6r^3 + 4r^2\beta\theta^* \left(\frac{4-\beta}{4\beta} + e\xi(1-\beta) \right) + 4\beta^2\theta^{*3}(1-\beta)(1+e\beta\xi)(1-e\xi(1-\beta)) > 0, \end{aligned}$$

because $(1 - e\xi(1 - \beta)) > 0$.

I now consider the long-run impact of r . As before, I employ the chain rule:

$$\frac{d(SW)}{dr} \Big|_{\theta=\theta^*} = \frac{\partial(SW)}{\partial r} \Big|_{\theta=\theta^*} + \frac{\partial(SW)}{\partial \theta} \Big|_{\theta=\theta^*} \frac{d\theta^*}{dr}.$$

From the proof of Theorem 3.1, I can find the expression for $\frac{\partial(SW)}{\partial r}$. Also, the Implicit Function Theorem and the intermediate steps in the proof of Theorem 4.1 give us:

$$\frac{d\theta^*}{dr} = -\frac{2r\theta^*}{Y}, \text{ where, as before, } Y = \theta^{*2}(1-\beta)^2(1+e\beta\xi)^2 - 3r^2 > 0.$$

Since I already know $\frac{\partial(SW)}{\partial \theta}$ from above, I combine all of these to obtain:

$$\frac{d(SW)}{dr} \Big|_{\theta=\theta^*} = \frac{X(r; \theta^*)}{4\beta\theta^*Y}, \text{ where}$$

$$\begin{aligned} X(r; \theta) &= 8r^3 - 3r^2\beta\theta(1 - 2e\xi(1-\beta)) - 2r\theta^2(2 - 3\beta(1-\beta) + e\beta\xi(4-5\beta)(1-\beta) + 2e^2\beta^2\xi^2(1-\beta)^2) \\ &\quad + \beta\theta^3(1 - 2e\xi(1-\beta))(1-\beta)^2(1+e\beta\xi)^2. \end{aligned}$$

Since $Y > 0$, the sign of the derivative above depends on the sign of $X(r; \theta^*)$ alone. The proof then follows directly from Lemma A1.

$\beta < 1$, Case 1B: In this case, I have:

$$\frac{\partial(SW)}{\partial e} = \frac{\xi(4r^2 + \beta\theta^2(3 - 4\beta))}{8(\beta\theta - r)}, \quad [\text{from the proof of Theorem 3.1}]$$

$$\frac{\partial(SW)}{\partial \theta} = \frac{1}{8} \left(4\beta + \frac{4r^2}{\beta\theta^2} + e\xi \left(1 - 4\beta - \frac{r^2}{(\beta\theta - r)^2} \right) \right), \text{ and}$$

$$\frac{d\theta^*}{de} = -\frac{\frac{\partial \pi'}{\partial e} \Big|_{\theta=\theta^*}}{\frac{\partial \pi'}{\partial \theta} \Big|_{\theta=\theta^*}} = \frac{\theta^*}{e} + \frac{2r^2}{e\beta(\beta\theta^* - 3r)}. \quad [\text{from the Implicit Function Theorem}]$$

Combining all these with the help of the chain rule, I get:

$$\left. \frac{d(SW)}{de} \right|_{\theta=\theta^*} = \frac{X}{8e\beta^2\theta^{*2}(\beta\theta^* - r)(\beta\theta^* - 3r)}, \text{ where}$$

$$X = -e\beta^2\theta^{*2}\xi \left(4r^3 + 4r^2\theta^*(4\beta - 1) - r\beta\theta^{*2}(28\beta - 13) + 4\beta^2\theta^{*3}(2\beta - 1) \right) + 4(\beta\theta^* - r)^2(\beta\theta^* - 2r) (\beta^2\theta^{*2} + r^2).$$

Since its denominator is clearly positive, $\frac{d(SW)}{de}$ is positive at $\theta = \theta^*$ if and only if $X > 0$. Now, it is clear that X is a linear function of e and attains its minimum at one of the extreme values of e .

Because $0 \leq e \leq \frac{\beta\theta - r}{\beta\theta\xi}$ in Case 1B, I now find:

$$\begin{aligned} X|_{e=0} &= 4(\beta\theta^* - r)^2(\beta\theta^* - 2r) (\beta^2\theta^{*2} + r^2) > 0, \text{ and} \\ X|_{e=\frac{\beta\theta^* - r}{\beta\theta^*\xi}} &= (\beta\theta^* - r) \left(8r^4 + \frac{16r\beta\theta^*(\beta\theta^* + 3r)(\beta\theta^* - 3r)}{9} + \frac{11r\beta^3\theta^{*3}}{9} \right. \\ &\quad \left. + \frac{\beta\theta^{*2}(1 - \beta) (\beta\theta^*(\beta\theta^* + 31(\beta\theta^* - 3r)) + 4(\beta\theta^* - 3r)^2)}{9} \right) > 0. \end{aligned}$$

X is, therefore, always positive in the region of interest, which means $\frac{d(SW)}{de}$ is positive at $\theta = \theta^*$.

I now consider the long-run impact of r on social welfare. We know:

$$\begin{aligned} \frac{\partial(SW)}{\partial r} &= -\frac{r}{\beta\theta} - \frac{e\xi}{2} + \frac{3e\beta\theta^2\xi}{8(\beta\theta - r)^2}, && \text{[from the proof of Theorem 3.1]} \\ \frac{\partial(SW)}{\partial \theta} &= \frac{1}{8} \left(4\beta + \frac{4r^2}{\beta\theta^2} + e\xi \left(1 - 4\beta - \frac{r^2}{(\beta\theta - r)^2} \right) \right), \text{ and} \\ \frac{d\theta^*}{dr} &= -\frac{\left. \frac{\partial \pi'}{\partial r} \right|_{\theta=\theta^*}}{\left. \frac{\partial \pi'}{\partial \theta} \right|_{\theta=\theta^*}} = -\frac{2r}{\beta(\beta\theta^* - 3r)}. && \text{[from the Implicit Function Theorem]} \end{aligned}$$

Once again, combining these using the chain rule, I get:

$$\left. \frac{d(SW)}{dr} \right|_{\theta=\theta^*} = \frac{X}{8\beta^2\theta^{*2}(\beta\theta^* - r)^2(\beta\theta^* - 3r)}, \text{ where}$$

$$X = e\beta^2\theta^{*2}\xi Y - Z,$$

$$Y = 20r^3 + 4r^2\theta^* - 44r^2\beta\theta^* - 11r\beta\theta^{*2} + 28r\beta^2\theta^{*2} + 3\beta^2\theta^{*3} - 4\beta^3\theta^{*3}, \text{ and}$$

$$Z = 8r(2\beta\theta^* - r)(\beta\theta^* - r)^3 > 0.$$

Now, for $\frac{d(SW)}{dr}$ to be positive at $\theta = \theta^*$, both its numerator and denominator must have the same sign. Since the denominator is clearly positive, this implies that X must be greater than zero to make the derivative positive. To complete the proof then, I will show that: (a) if $\beta \geq \frac{3}{4}$, then $X < 0$, implying that the derivative can never be positive, and (b) if $\beta < \frac{3}{4}$, then $Y > 0$, implying $X > 0$ iff $e > \frac{Z}{\beta^2\theta^2\xi Y}$.

(a) I first consider the situation where $\frac{3}{4} \leq \beta < 1$. I note that X is a linear function of e , and the maximum value of X must be attained at one of the extreme values of e . Of course, $X|_{e=0} = -Z < 0$, so I consider the other extreme, $e = \frac{1}{\xi}$. After some algebra, I find:

$$X|_{e=\frac{1}{\xi}} = -r^3 \left(8r^2 - 40r\beta\theta^* + 13\beta^2\theta^{*2} \right) - \beta^2\theta^{*2} \left(39r^3 - 4r^2\theta^*(1+3\beta) - r\beta\theta^{*2}(12\beta-11) + \beta^2\theta^{*3}(4\beta-3) \right).$$

Since both roots of $(8r^2 - 40r\beta\theta^* + 13\beta^2\theta^{*2}) = 0$ are larger than $\frac{\beta\theta^*}{3}$, $(8r^2 - 40r\beta\theta^{*2} + 13\beta^2\theta^{*2}) > 0$ in the region of interest. On the other hand, $(39r^3 - 4r^2\theta^*(1+3\beta) - r\beta\theta^{*2}(12\beta-11) + \beta^2\theta^{*3}(4\beta-3)) = 0$ is a cubic equation in r , whose discriminant, $\beta^2\theta^{*6}(\beta(9\beta(24\beta(302\beta-1445)+38461)-113468)+1168)$, is negative at all $\beta \in [\frac{3}{4}, 1)$. So, there is only one real root. Furthermore, since the cubic polynomial approaches $-\infty$ as $r \rightarrow -\infty$, and becomes equal to the non-negative number $\beta^2\theta^{*3}(4\beta-3)$ at $r = 0$, its only real root cannot be positive. Therefore, the polynomial is non-negative at all $r \geq 0$. Taken together, I have just shown that $X|_{e=\frac{1}{\xi}} < 0$ whenever $\beta \geq \frac{3}{4}$, implying that social welfare is decreasing in r for $\beta \geq \frac{3}{4}$.

(b) Now, I move to the case of $\beta < \frac{3}{4}$. Here, I first solve $\frac{\partial Y}{\partial r} = 0$, and observe that the unique minimum and maximum of Y happen at:

$$r_{\min} = \frac{\theta^*}{30} \left(2(11\beta-1) + \sqrt{\beta(64\beta+77)+4} \right), \text{ and } r_{\max} = \frac{\theta^*}{30} \left(2(11\beta-1) - \sqrt{\beta(64\beta+77)+4} \right).$$

Since $\beta > \frac{1}{2}$ in Case 1B, it is immediate that r_{\min} is larger than $\frac{\beta\theta^*}{3}$ and that $Y|_{r=\frac{\beta\theta^*}{3}} > 0$. These, taken together with the fact that $Y|_{r=0} > 0$ for $\beta < \frac{3}{4}$, imply that Y must be positive in $\left[0, \frac{\beta\theta^*}{3}\right)$. Viewed differently, if Y actually became negative in this region, the minimum would have occurred prior to the function becoming positive again at $r = \frac{\beta\theta^*}{3}$. Thus, $Y > 0$ for all $\beta < \frac{3}{4}$ in Case 1B, implying that social welfare is increasing in r if and only if $e > \frac{Z}{\beta^2\theta^{*2}\xi Y}$.

ii) Now, I consider $\beta \geq 1$. Recall that, in the long run, Case 1A is not possible here. So, I need to consider only Case 1B.

$\beta \geq 1$, Case 1B: The expression for social welfare in this case is basically the same as the previous case (i.e., $\beta < 1$, Case 1B).

Therefore, I have:

$$\frac{d(SW)}{de} \Big|_{\theta=\theta^*} = \frac{-e\beta^2\theta^{*2}\xi Y + Z}{8e\beta^2\theta^{*2}(\beta\theta^* - r)(\beta\theta^* - 3r)}, \text{ where}$$

$$Y = (4r^3 + 4r^2\theta^*(4\beta-1) - r\beta\theta^{*2}(28\beta-13) + 4\beta^2\theta^{*3}(2\beta-1)) \text{ and } Z = 4(\beta\theta^*-r)^2(\beta\theta^*-2r)(\beta^2\theta^{*2}+r^2).$$

Clearly, $Z > 0$. Furthermore, since the denominator is positive, $\frac{d(SW)}{de}$ is positive at $\theta = \theta^*$ iff

$(-e\beta^2\theta^{*2}\xi Y + Z) > 0$. Assuming $Y > 0$, this condition is equivalent to:

$$e < \frac{Z}{\beta^2\theta^{*2}\xi Y} = \frac{4(\beta\theta^* - r)^2(\beta\theta^* - 2r)(\beta^2\theta^{*2} + r^2)}{\beta^2\theta^{*2}\xi(4r^3 + 4r^2\theta^*(4\beta - 1) - r\beta\theta^{*2}(28\beta - 13) + 4\beta^2\theta^{*3}(2\beta - 1))},$$

and the desired result would follow directly. Therefore, to complete the proof, I only need to show that $Y > 0$. Since $\frac{\partial^2 Y}{\partial r^2} = 8(3r + \theta^*(4\beta - 1)) > 0$, the minimum of Y can happen either at an interior point or at one of the extreme points of $r \in \left[0, \frac{\beta\theta^*}{3}\right)$. Solving the first order condition, I get two roots, of which I only consider the positive one: $\frac{r}{\beta\theta^*} = \frac{2-8\beta+\sqrt{4+\beta(148\beta-71)}}{6\beta}$, which is larger than the maximum possible value of $\frac{1}{3}$. Clearly, the interior minimum is beyond the admissible range of r . Therefore, the minimum must occur at one of the extreme points. I find that $Y > 0$ at both these points:

$$Y|_{r=0} = 4\beta^2\theta^{*3}(2\beta - 1) > 0, \quad \text{and} \quad Y|_{r=\frac{\beta\theta^*}{3}} = \frac{\beta^2\theta^{*3}(16\beta - 3)}{27} > 0.$$

Since this implies that $Y > 0$ over the entire valid range, the result, as stated in the theorem, has been proved.

I now consider the long-run impact of r on social welfare. I know from Theorem 3.1 that $\frac{\partial(SW)}{\partial r} < 0$, and from Theorem 4.1 that $\frac{d\theta^*}{dr} < 0$. Furthermore:

$$\frac{\partial(SW)}{\partial \theta} = \frac{1}{8} \left(4\beta + \frac{4r^2}{\beta\theta^2} + e\xi \left(1 - 4\beta - \frac{r^2}{(\beta\theta - r)^2} \right) \right),$$

which is a linear function of $e \in \left[0, \frac{\beta\theta - r}{\beta\theta\xi}\right]$ and is clearly positive at $e = 0$. At the other extreme, $e = \frac{\beta\theta - r}{\beta\theta\xi}$, I get:

$$\left. \frac{\partial(SW)}{\partial \theta} \right|_{e=\frac{\beta\theta-r}{\beta\theta\xi}} = \frac{4r(\beta\theta + r)(\beta\theta - r) + \beta\theta^2(\beta\theta - 2r)}{8\beta\theta^2(\beta\theta - r)} > 0.$$

Therefore, $\frac{\partial(SW)}{\partial \theta}$ is always positive in this case. Using the chain rule now, I get:

$$\left. \frac{d(SW)}{dr} \right|_{\theta=\theta^*} = \underbrace{\left. \frac{\partial(SW)}{\partial r} \right|_{\theta=\theta^*}}_{<0} + \underbrace{\left. \frac{\partial(SW)}{\partial \theta} \right|_{\theta=\theta^*}}_{>0} \underbrace{\left. \frac{d\theta^*}{dr} \right|_{\theta=\theta^*}}_{<0} < 0.$$

Proof of Proposition 4.2

To prove this proposition, I will show that:

- i) The piracy rate is monotonically decreasing in r , but it is not monotonic in e . In Case 1A, the piracy rate is increasing in e if $e < \frac{3r - \theta^*(1-\beta)}{\beta\theta^*\xi(1-\beta)}$, and decreasing otherwise. In Case 1B, however, the piracy rate is increasing in e if and only if $e < \frac{4r^3 - \beta\theta^*(\beta\theta^* - 3r)^2}{r\beta\theta^*\xi(\beta\theta^* - 2r)}$.
- ii) The manufacturer's profit is monotonically increasing in both e and r .

- iii) The consumer surplus is monotonically decreasing in r , but not in e . It is increasing in e for moderate or low values of e . Specifically, in Case 1A, it is increasing if and only if $e < \gamma_{1A}(\theta^*)$, where $\gamma_{1A}(\cdot)$ is as defined in Lemma A2. In Case 1B, the consumer surplus is increasing in e if and only if $e < \frac{4(\beta\theta^* - r)^3(\beta\theta^* - 2r)(\beta\theta^* + r)}{\beta^2\theta^{*2}\xi(8\beta^3\theta^{*3} - 2\beta^2\theta^{*3} - 36r\beta^2\theta^{*2} + 7r\beta\theta^{*2} + 48r^2\beta\theta^* - 4r^2\theta^* - 20r^3)}$.
- iv) The legal social welfare is monotonically increasing in e , but not in r unless r is small. More specifically, it is increasing in r if and only if $r < \tau_{1A}(\theta^*)$ in Case 1A or $r < \frac{\beta\theta^*(11 - \sqrt{73})}{8}$ in Case 1B, where $\tau_{1A}(\cdot)$ is as defined in Lemma A2.

I now prove each result one by one:

- i) I again start with the piracy rate and show that it is monotonically decreasing in r .

Case 1A: I know from Proposition 3.2 that $\frac{\partial\mu}{\partial r}\Big|_{\theta=\theta^*} < 0$. Also, from Theorem 4.1, I know that $\frac{d\theta^*}{dr} < 0$.

Finally,

$$\frac{\partial\mu}{\partial\theta}\Big|_{\theta=\theta^*} = \frac{r\beta}{2(1-\beta)(r - \beta\theta^*(1 - e\xi(1-\beta)))^2} > 0.$$

Combining everything using the chain rule, I get:

$$\frac{d\mu}{dr}\Big|_{\theta=\theta^*} = \underbrace{\frac{\partial\mu}{\partial r}\Big|_{\theta=\theta^*}}_{<0} + \underbrace{\frac{\partial\mu}{\partial\theta}\Big|_{\theta=\theta^*}}_{>0} \underbrace{\frac{d\theta^*}{dr}}_{<0} < 0.$$

Case 1B: This is quite similar to Case 1A in the long run. In this case, I get:

$$\frac{\partial\mu}{\partial\theta}\Big|_{\theta=\theta^*} = \frac{2er\beta^2\theta\xi(\beta\theta^*(1 - e\xi) + (\beta\theta^* - 2r))}{(2(\beta\theta^* - r)^2 + e\beta\theta^*\xi(\beta\theta^* - 2r))^2} > 0.$$

As before, using the chain rule, I can easily show that μ is monotonic in r in the long run.

I now consider the long-run impact of e on piracy rate. Again, I use the chain rule:

$$\frac{d\mu}{de}\Big|_{\theta=\theta^*} = \frac{\partial\mu}{\partial e}\Big|_{\theta=\theta^*} + \frac{\partial\mu}{\partial\theta}\Big|_{\theta=\theta^*} \frac{d\theta^*}{de}.$$

Case 1A: In this case:

$$\frac{\partial\mu}{\partial e} = -\frac{\beta^2\theta^2\xi}{2(r - \beta\theta(1 - e\xi(1-\beta)))^2}, \quad \frac{d\theta^*}{de} = \frac{2\beta\theta^{*3}\xi(1-\beta)^2(1 + e\beta\xi)}{(1-\beta)^2\theta^{*2}(1 + e\beta\xi)^2 - 3r^2},$$

and $\frac{\partial\mu}{\partial\theta}$ is as given above. Combining, I get:

$$\frac{d\mu}{de}\Big|_{\theta=\theta^*} = \frac{\beta^2\theta^{*2}\xi(r + \theta^*(1-\beta)(1 + e\beta\xi))(3r - \theta^*(1-\beta)(1 + e\beta\xi))}{2(\beta\theta^*(1 - e\xi(1-\beta)) - r)^2(\theta^{*2}(1-\beta)^2(1 + e\beta\xi)^2 - 3r^2)}.$$

Since, as before, $\theta^{*2}(1-\beta)^2(1 + e\beta\xi)^2 - 3r^2 > 0$, the denominator is clearly positive, implying that the above derivative will be positive iff $3r - \theta^*(1-\beta)(1 + e\beta\xi) > 0$ or $e < \frac{3r - \theta^*(1-\beta)}{\beta\theta^*\xi(1-\beta)}$.

Case 1B: In this case:

$$\frac{\partial \mu}{\partial e} = -\frac{2\beta^2\theta^2\xi(\beta\theta - r)^2}{(2(\beta\theta - r)^2 - e\beta\theta\xi(\beta\theta - 2r))^2}, \quad \frac{d\theta^*}{de} = \frac{\theta^*}{e} + \frac{2r^2}{e\beta(\beta\theta^* - 3r)},$$

and $\frac{\partial \mu}{\partial \theta}$ is as given above. Combining, I get:

$$\left. \frac{d\mu}{de} \right|_{\theta=\theta^*} = \frac{2\beta\theta^*\xi(\beta\theta^* - r)(4r^3 - \beta\theta^*(\beta\theta^* - 3r)^2 - er\beta\theta^*\xi(\beta\theta^* - 2r))}{(\beta\theta^* - 3r)(2(\beta\theta^* - r)^2 - e\beta\theta^*\xi(\beta\theta^* - 2r))^2}.$$

Since the denominator is clearly positive, the above derivative will be positive iff $4r^3 - \beta\theta^*(\beta\theta^* - 3r)^2 - er\beta\theta^*\xi(\beta\theta^* - 2r) > 0$ or $e < \frac{4r^3 - \beta\theta^*(\beta\theta^* - 3r)^2}{r\beta\theta^*\xi(\beta\theta^* - 2r)}$.

ii) Next, I show that the manufacturer's profit is monotonically increasing in both e and r . This is straightforward. Since $\pi^*(\theta) = R^*(\theta) - \frac{c\theta^2}{2}$, from the Envelope Theorem, I get:

$$\left. \frac{d\pi^*}{de} \right|_{\theta=\theta^*} = \left. \frac{\partial \pi^*}{\partial e} \right|_{\theta=\theta^*} = \left. \frac{\partial R^*}{\partial e} \right|_{\theta=\theta^*} > 0 \quad \text{and} \quad \left. \frac{d\pi^*}{dr} \right|_{\theta=\theta^*} = \left. \frac{\partial \pi^*}{\partial r} \right|_{\theta=\theta^*} = \left. \frac{\partial R^*}{\partial r} \right|_{\theta=\theta^*} > 0.$$

iii) I now show that the consumer surplus is monotonically decreasing in r . I first use the chain rule:

$$\left. \frac{d(CS)}{dr} \right|_{\theta=\theta^*} = \left. \frac{\partial(CS)}{\partial r} \right|_{\theta=\theta^*} + \left. \frac{\partial(CS)}{\partial \theta} \right|_{\theta=\theta^*} \frac{d\theta^*}{dr}.$$

I know from Proposition 3.2 that $\left. \frac{\partial(CS)}{\partial r} \right|_{\theta=\theta^*} < 0$. Furthermore, from Theorem 4.1, I know that $\frac{d\theta^*}{dr} < 0$. Therefore, to complete the proof, I need to show that $\left. \frac{\partial(CS)}{\partial \theta} \right|_{\theta=\theta^*} > 0$. I now do so for each case.

Case 1A: In this case, I already know that $e < \frac{1}{\xi}$ and $\frac{r}{\beta\theta} < \frac{1-\beta}{2-\beta}$. Furthermore, since $\beta < 1$, I must have $\beta(1-\beta) < \frac{1}{4}$. Therefore, I get:

$$\begin{aligned} \left. \frac{\partial(CS)}{\partial \theta} \right|_{\theta=\theta^*} &= \frac{1}{8} \left(1 + 3\beta - 4e\beta\xi(1-\beta) - e^2\beta^2\xi^2(1-\beta) - \frac{r^2(4-3\beta)}{\beta\theta^{*2}(1-\beta)} \right) \\ &> \frac{1}{8} \left(1 + 3\beta - e\xi - \frac{e^2\beta\xi^2}{4} - \frac{r^2(4-3\beta)}{\beta\theta^2(1-\beta)} \right) \\ &> \frac{1}{8} \left(1 + 3\beta - 1 - \frac{\beta}{4} - \frac{\beta(1-\beta)^2(4-3\beta)}{(2-\beta)^2(1-\beta)} \right) > \frac{1}{8} \left(\frac{3\beta}{4} + \frac{\beta(4-\beta-\beta^2)}{(2-\beta)^2} \right) > 0. \end{aligned}$$

Case 1B: In this case, $e < \frac{\beta\theta-r}{\beta\theta\xi}$ and $\frac{r}{\beta\theta} < \frac{1}{3}$. Then:

$$\begin{aligned} \left. \frac{\partial(CS)}{\partial \theta} \right|_{\theta=\theta^*} &= \frac{1}{8} \left(4\beta - \frac{4r^2}{\beta\theta^{*2}} - e\xi \left(4\beta + \frac{r^2}{(\beta\theta^*-r)^2} - 1 \right) \right) \\ &> \frac{1}{8} \left(4\beta - \frac{4r^2}{\beta\theta^{*2}} - \frac{\beta\theta^*-r}{\beta\theta^*} \left(4\beta + \frac{r^2}{(\beta\theta^*-r)^2} - 1 \right) \right) = \frac{1}{8} \left(\frac{4r}{\theta^*} \left(1 - \frac{r}{\beta\theta^*} \right) + \frac{\beta\theta^*-2r}{\beta\theta^*-r} \right) > 0. \end{aligned}$$

I now consider the long-run impact of e . Again, I make use of the chain rule:

$$\left. \frac{d(CS)}{de} \right|_{\theta=\theta^*} = \left. \frac{\partial(CS)}{\partial e} \right|_{\theta=\theta^*} + \left. \frac{\partial(CS)}{\partial \theta} \right|_{\theta=\theta^*} \frac{d\theta^*}{de}.$$

Case 1A: In this case:

$$\frac{\partial(CS)}{\partial e} = \frac{\xi(1-\beta)(2r - \beta\theta(2 + e\beta\xi))}{4}, \quad \frac{d\theta^*}{de} = \frac{2\beta\theta^3\xi(1-\beta)^2(1 + e\beta\xi)}{\theta^2(1-\beta)^2(1 + e\beta\xi)^2 - 3r^2},$$

and $\frac{\partial(CS)}{\partial \theta}$ is as given above. Combining, I get:

$$\left. \frac{d(CS)}{de} \right|_{\theta=\theta^*} = \frac{\xi(1-\beta)X(e; \theta^*)}{4Y}, \text{ where}$$

$$\begin{aligned} X(e; \theta) = & -6r^3 - r^2\theta(4 - 9\beta + 2e\beta\xi(2 - 3\beta)) + 2r\theta^2(1 - \beta)^2(1 + e\beta\xi)^2 \\ & - \beta\theta^3(1 - \beta)(1 + e\beta\xi)(1 - 5\beta + 7e\beta\xi(1 - \beta) + 2e^2\beta^2\xi^2(1 - \beta)) \end{aligned}$$

and, as before, $Y = \theta^2(1 - \beta)^2(1 + e\beta\xi)^2 - 3r^2 > 0$. Therefore, the above derivative would be positive iff $X(e; \theta^*) > 0$ which, according to Lemma A2, is true if $e < \gamma_{1A}(\theta)$.

Case 1B: Here,

$$\frac{\partial(CS)}{\partial e} = \frac{\xi(8r\beta\theta - \beta\theta^2(4\beta - 1) - 4r^2)}{8(\beta\theta - r)}, \quad \frac{d\theta^*}{de} = \frac{\theta^*}{e} + \frac{2r^2}{e\beta(\beta\theta^* - 3r)},$$

and $\frac{\partial(CS)}{\partial \theta}$ is as given above. Combining, I get:

$$\left. \frac{d(CS)}{de} \right|_{\theta=\theta^*} = \frac{-e\beta^2\theta^{*2}\xi X + Y}{8e\beta^2\theta^{*2}(\beta\theta^* - 3r)(\beta\theta^* - r)}, \text{ where}$$

$$X = 8\beta^3\theta^{*3} - 2\beta^2\theta^{*3} - 36r\beta^2\theta^{*2} + 7r\beta\theta^{*2} + 48r^2\beta\theta^* - 4r^2\theta^* - 20r^3, \quad \text{and} \quad Y = 4(\beta\theta^* - r)^3(\beta\theta^* - 2r)(\beta\theta^* + r) > 0.$$

Now, assuming $X > 0$, it is clear that the above derivative would be positive if and only if $e < \frac{Y}{\beta^2\theta^{*2}\xi X}$.

Therefore, the proof can be completed by simply showing that $X > 0$; I do so now.

I first solve $\frac{\partial X}{\partial r} = 0$ and find two roots. From the second order condition, I find that the first one is a minimum and the second, a maximum:

$$r_{\min} = \frac{\theta^*}{30} \left(2(12\beta - 1) - \sqrt{4 + 9\beta(1 + 4\beta)} \right), \quad \text{and} \quad r_{\max} = \frac{\theta^*}{30} \left(2(12\beta - 1) + \sqrt{4 + 9\beta(1 + 4\beta)} \right).$$

Since $\beta > \frac{1}{2}$ in Case 1B, it is immediate that r_{\min} and r_{\max} are both larger than $\frac{\beta\theta^*}{3}$. These, taken together with the facts that $X|_{r=0} = 2\beta^2\theta^3(4\beta - 1) > 0$ and $X|_{r=\frac{\beta\theta^*}{3}} = \frac{\beta^2\theta^3}{27}(16\beta - 3) > 0$, imply that $X > 0$ in $\left[0, \frac{\beta\theta^*}{3}\right)$.

iv) Finally, I show that the legal social welfare is monotonically increasing in e . Again, I make use of the chain rule:

$$\frac{d(SW_L)}{de} \Big|_{\theta=\theta^*} = \frac{\partial(SW_L)}{\partial e} \Big|_{\theta=\theta^*} + \frac{\partial(SW_L)}{\partial \theta} \Big|_{\theta=\theta^*} \frac{d\theta^*}{de}.$$

I know from Proposition 3.2 that $\frac{\partial(SW_L)}{\partial e} \Big|_{\theta=\theta^*} > 0$. Furthermore, from Theorem 4.1, I know that $\frac{d\theta^*}{de} > 0$. Therefore, to complete the proof, I only need to show that $\frac{\partial(SW_L)}{\partial \theta} \Big|_{\theta=\theta^*} > 0$.

Case 1A: In this case, $\beta < 1$. Thus, I get:

$$\frac{\partial(SW_L)}{\partial \theta} \Big|_{\theta=\theta^*} = \frac{3}{8} + \frac{r^2}{8\theta^{*2}(1-\beta)^2} + \frac{e\beta\xi}{8} (2(1-\beta) + e\beta\xi(3-2\beta)) > 0.$$

Case 1B: In this case, $\frac{r}{\beta\theta} < \frac{1}{3}$. Therefore:

$$\frac{\partial(SW_L)}{\partial \theta} \Big|_{\theta=\theta^*} = \frac{3e\beta\theta^*\xi(\beta\theta^* - 2r)}{8(\beta\theta^* - r)^2} > 0.$$

I now consider the long-run impact of r . Once again, I make use of the chain rule:

$$\frac{d(SW_L)}{dr} \Big|_{\theta=\theta^*} = \frac{\partial(SW_L)}{\partial r} \Big|_{\theta=\theta^*} + \frac{\partial(SW_L)}{\partial \theta} \Big|_{\theta=\theta^*} \frac{d\theta^*}{dr}.$$

Case 1A: In this case:

$$\frac{\partial(SW_L)}{\partial r} = \frac{e\beta\xi}{4} + \frac{\theta(1-\beta) - r}{4\theta(1-\beta)^2}, \quad \frac{d\theta^*}{dr} = -\frac{2r\theta}{\theta^2(1-\beta)^2(1+e\beta\xi)^2 - 3r^2},$$

and $\frac{\partial(SW_L)}{\partial \theta}$ is as above. Combining, I get:

$$\frac{d(SW_L)}{dr} \Big|_{\theta=\theta^*} = \frac{X(r; \theta^*)}{4\theta^*Y(1-\beta)^2}, \text{ where}$$

$$\begin{aligned} X(r; \theta) &= 2r^3\beta - 3r^2\theta(1-\beta)(1+e\beta\xi(1-\beta)) - 2r\theta^2(1-\beta)^2(1+\beta+e\beta^2\xi(1+e\xi)) \\ &\quad + \theta^3(1-\beta)^3(1+e\beta\xi)^2(1+e\beta\xi(1-\beta)), \end{aligned}$$

and, as before, $Y = \theta^{*2}(1-\beta)^2(1+e\beta\xi)^2 - 3r^2 > 0$. Therefore, the derivative in question would be positive if and only if $X(r; \theta^*) > 0$, which, by Lemma A2, is true if $r < \tau_{1A}(\theta^*)$.

Case 1B: In this case:

$$\frac{\partial(SW_L)}{\partial r} = \frac{3e\beta\theta^2\xi}{8(\beta\theta - r)^2}, \quad \frac{\partial(SW_L)}{\partial \theta} = \frac{3e\beta\theta\xi(\beta\theta - 2r)}{8(\beta\theta - r)^2}, \text{ and } \frac{d\theta^*}{dr} = -\frac{2r}{\beta(\beta\theta^* - 3r)}.$$

Combining, I get:

$$\frac{d(SW_L)}{dr} \Big|_{\theta=\theta^*} = \frac{e\theta^*\xi(4r^2 - 11r\beta\theta^* + 3\beta^2\theta^{*2})}{8(\beta\theta^* - r)^2(\beta\theta^* - 3r)},$$

which would be positive if and only if $4r^2 - 11r\beta\theta^* + 3\beta^2\theta^{*2} > 0$, or $r < \frac{\beta\theta^*(11-\sqrt{73})}{8} \approx 0.307\beta\theta^*$. ■

Proof of Lemma 4.2

In Case 1B, the manufacturer's profit is given by:

$$\pi = p\lambda \left(1 - \frac{p}{\theta}\right) - \frac{c\theta^2}{2},$$

where $\lambda = \frac{e\theta\xi}{\theta - \frac{r}{\beta}}$. When $\frac{p}{\theta} = \delta$, I can substitute $p = \delta\theta$ in the above expression and differentiate w.r.t. θ to obtain the following first order condition:

$$\frac{d\pi}{d\theta} = \frac{\theta^2 (e\beta^2\delta\xi(1-\delta)) - \theta(2er\beta\delta\xi(1-\delta))}{(\beta\theta - r)^2} - c\theta = 0.$$

Since $\theta > 0$ and $\frac{r}{\beta\theta} < 1$, I can multiply both sides by $\frac{(\beta\theta - r)^2}{\theta}$ and solve the resulting quadratic equation to obtain the following two roots:

$$\theta^* = \left\{ \frac{2cr - e\beta\delta\xi(\delta-1) + \sqrt{e\beta\delta\xi(\delta-1)}\sqrt{4cr + e\beta\delta\xi(\delta-1)}}{2c\beta}, \frac{2cr - e\beta\delta\xi(\delta-1) - \sqrt{e\beta\delta\xi(\delta-1)}\sqrt{4cr + e\beta\delta\xi(\delta-1)}}{2c\beta} \right\}.$$

However, for either root to be real, δ must be greater than 1, which contradicts the fact that $\delta < \frac{1}{2}$. Thus, Case 1B cannot be valid when the competition constraint binds. ■

Proof of Proposition 4.3

In Case 1A, the manufacturer's profit is given by:

$$\pi = p \left(\lambda \left(1 - \frac{p}{\theta}\right) + (1-\lambda) \left(1 - \frac{p-r}{(1-\beta)\theta}\right) \right) - \frac{c\theta^2}{2}, \quad \text{where } \lambda = \begin{cases} \frac{e\theta\xi(1-\beta)}{p - \frac{r}{\beta}} & \text{if } \beta < 1, \\ \frac{e\theta\xi}{\theta - \frac{r-p}{\beta-1}} & \text{otherwise.} \end{cases}$$

When $\frac{p}{\theta} = \delta$, I can substitute $p = \delta\theta$ in the above expressions, differentiate w.r.t. θ , and solve the appropriate first order conditions to obtain the desired result. It is easy to verify that the second order conditions hold as well. ■

Proof of Corollary 4.1

It is easy to see that θ^* is independent of r . Also, differentiating θ^* w.r.t. e , I get:

$$\frac{d\theta^*}{de} = \begin{cases} \frac{\beta\delta\xi}{c} & \text{if } \beta < 1, \\ \frac{\delta\xi}{c} & \text{otherwise.} \end{cases}$$

Since both terms are positive, the result follows. ■

Proof of Theorem 4.3

According to Lemma 4.2, Case 1B is no longer possible. Hence, I simply limit our attention to Case 1A.

i) I start by showing the manufacturer's profit monotonically increases in both e and r . The manufacturer's profit is given by:

$$\pi = p \left(\lambda \left(1 - \frac{p}{\theta} \right) + (1-\lambda) \left(1 - \frac{p-r}{(1-\beta)\theta} \right) \right) - \frac{c\theta^2}{2}, \quad \text{where } \lambda = \begin{cases} \frac{e\theta\xi(1-\beta)}{p-\frac{r}{\beta}} & \text{if } \beta < 1, \\ \frac{e\theta\xi}{\theta-\frac{r-p}{\beta-1}} & \text{otherwise.} \end{cases}$$

$\beta < 1$: From the Envelope Theorem, I get:

$$\frac{d\pi}{de} \Big|_{\theta=\theta^*} = \frac{\partial\pi}{\partial e} \Big|_{\theta=\theta^*} = \beta\delta\xi\theta^* > 0 \quad \text{and} \quad \frac{d\pi}{dr} \Big|_{\theta=\theta^*} = \frac{\partial\pi}{\partial r} \Big|_{\theta=\theta^*} = \frac{\delta}{1-\beta} > 0.$$

$\beta \geq 1$: Again, from the Envelope Theorem, I get:

$$\frac{d\pi}{de} \Big|_{\theta=\theta^*} = \frac{\partial\pi}{\partial e} \Big|_{\theta=\theta^*} = \delta\xi\theta^* > 0 \quad \text{and} \quad \frac{d\pi}{dr} \Big|_{\theta=\theta^*} = \frac{\partial\pi}{\partial r} \Big|_{\theta=\theta^*} = \frac{\delta}{\beta-1} > 0.$$

ii) I now show how the consumer surplus changes with e and r .

$\beta < 1$: The consumer surplus (CS) is given by:

$$CS = \frac{r(r + \beta\theta(e\xi(1-\beta) - 2\delta)) + \beta\theta^2(\delta^2 - (1-\beta)(\delta(2 + e\beta\xi) - 1))}{2\beta\theta(1-\beta)}.$$

I first investigate the effect of e on CS . By differentiating CS with respect to e and θ , I get:

$$\frac{\partial(CS)}{\partial e} = -\frac{1}{2}\xi(\beta\theta\delta - r) \quad \text{and} \quad \frac{\partial(CS)}{\partial\theta} = \frac{\beta\theta^2(\delta^2 - (1-\beta)(\delta(2 + e\beta\xi) - 1)) - r^2}{2\beta\theta^2(1-\beta)}.$$

Using the chain rule, I get:

$$\begin{aligned} \frac{d(CS)}{de} \Big|_{\theta=\theta^*} &= \frac{\partial(CS)}{\partial e} \Big|_{\theta=\theta^*} + \frac{\partial(CS)}{\partial\theta} \Big|_{\theta=\theta^*} \frac{d\theta^*}{de} \\ &= \frac{\xi(cr\theta^{*2}(1-\beta) + \delta X)}{2(c\theta^{*2}(1-\beta))}, \end{aligned}$$

where $X = \beta\theta^{*2}(2\delta^2 + (1-\delta)(3+2e\beta\xi)(1-\beta)) - r^2$. From the condition $\lambda = \frac{e\theta\xi(1-\beta)}{p-\frac{r}{\beta}} < 1$, I get $r < \beta\theta(\delta - e\xi(1-\beta))$. Hence, $X > \beta\theta^{*2}(2\delta^2 + (1-(3+2e\beta\xi)\delta)(1-\beta)) - (\beta\theta^*(\delta - e\xi(1-\beta)))^2 = \beta\theta^{*2}Y$, where $Y = (1-3\delta)(1-\beta) + 2\delta^2 - \beta\delta^2 - e^2\beta\xi^2(1-\beta)^2$. Since $r > 0$, $\beta\theta(\delta - e\xi(1-\beta)) > 0$, which shows that $e^2\beta\xi^2(1-\beta)^2 < \beta\delta^2$. Thus, $Y > (1-3\delta)(1-\beta) + 2\delta^2 - \beta\delta^2 - \beta\delta^2 = (1-\beta)(1-\delta)(1-2\delta) > 0$. Finally, since $Y > 0$, $X > 0$, and thus, $\frac{d(CS)}{de} \Big|_{\theta=\theta^*} > 0$.

Now, I show that CS decreases with r . By differentiating CS with respect to r , I get:

$$\frac{\partial(CS)}{\partial r} = \frac{r - \beta\theta\delta}{\beta\theta(1-\beta)} + \frac{e\xi}{2}.$$

From the condition $\lambda = \frac{e\theta\xi(1-\beta)}{p-\frac{r}{\beta}} < 1$, I get $\frac{e\xi}{2} < -\frac{r-\beta\theta\delta}{2\beta\theta(1-\beta)}$, and from the condition $\frac{r}{\beta\theta} < \frac{p}{\theta} = \delta$, $r < \beta\theta\delta$. Thus, $\frac{\partial(CS)}{\partial r} = \frac{r-\beta\theta\delta}{\beta\theta(1-\beta)} + \frac{e\xi}{2} < \frac{r-\beta\theta\delta}{\beta\theta(1-\beta)} - \frac{r-\beta\theta\delta}{2\beta\theta(1-\beta)} = \frac{r-\beta\theta\delta}{2\beta\theta(1-\beta)} < 0$. Since $\frac{d\theta^*}{dr} = 0$, $\frac{d(CS)}{dr} \Big|_{\theta=\theta^*} = \frac{\partial(CS)}{\partial r} \Big|_{\theta=\theta^*} < 0$.

$\beta \geq 1$: In this case, the consumer surplus is given by:

$$CS = \frac{r^2 + r\theta(e\xi(\beta - 1) - 2(\beta + \delta - 1)) + \theta^2(\beta(\beta + \delta^2 - 1) - e\xi(\beta - 1)(\beta + \delta - 1))}{2\theta(\beta - 1)}.$$

Hence, at $\theta = \theta^*$, $\frac{d(CS)}{dr} = \frac{\partial(CS)}{\partial r} = \frac{r - (\beta + \delta - 1)\theta}{(\beta - 1)\theta} + \frac{e\xi}{2}$. Since $p = \delta\theta$ and $\lambda = \frac{e\theta\xi}{\theta - \frac{r-p}{\beta-1}} < 1$, I can easily show that $\frac{r - (\beta + \delta - 1)\theta}{(\beta - 1)\theta} < -e\xi$, which implies that $\frac{d(CS)}{dr} < -e\xi + \frac{e\xi}{2} = -\frac{e\xi}{2} < 0$, at $\theta = \theta^*$.

On the other hand, CS is not a monotonic function of e . First, I write using the chain rule:

$$\begin{aligned} \left. \frac{d(CS)}{de} \right|_{\theta=\theta^*} &= \left. \frac{\partial(CS)}{\partial e} \right|_{\theta=\theta^*} + \left. \frac{\partial(CS)}{\partial \theta} \right|_{\theta=\theta^*} \frac{d\theta^*}{de} \\ &= \frac{c\xi(\beta - 1) \left(\frac{r\delta(e\xi(\beta - 1) - \beta\delta)^2}{c(\beta - 1)} + \frac{\delta^2(\beta((\beta - 1)(1 + \delta) + 2\delta^2) - 2e\xi(\beta - 1)(\beta + \delta - 1))(e\xi(\beta - 1) - \beta\delta)^2}{c^2(\beta - 1)^2} - r^2 \right)}{2\delta(e\xi(\beta - 1) - \beta\delta)^2}. \end{aligned}$$

Next, I solve $\frac{d(CS)}{de} = 0$, a quadratic equation in r , to obtain the only valid root:

$$\hat{r} = \frac{\delta(e\xi(\beta - 1) - \beta\delta)(e\xi(\beta - 1) - \beta\delta - \sqrt{\beta(\beta(2 + \delta)^2 - 4(1 - \delta)(1 + 2\delta)) - 2e\xi(\beta - 1)(\beta(4 + \delta) - 4(1 - \delta)) + e^2\xi^2(\beta - 1)^2})}{2c(\beta - 1)}.$$

Now, it can be shown that, within the valid region for Case 1A, $\frac{d(CS)}{de}$ crosses zero only once, and it is a decreasing function of r at that point. Therefore, $\frac{d(CS)}{de}$ is positive if and only if $r < \hat{r}$. In other words, CS is increasing in e as long as $r < \hat{r}$ and is decreasing otherwise.

iii) Finally, I show how the social welfare changes with e and r .

$\beta < 1$: The social welfare (SW) is given by:

$$SW = \frac{r(\beta\theta(2\delta - e\xi(1 - \beta)) - r) + \beta\theta^2((1 - \beta)(1 - c\theta) + e\beta\xi\delta(1 - \beta) - \delta^2)}{2\beta\theta(1 - \beta)}.$$

I first investigate the effect of e on SW . Using the chain rule, I get:

$$\left. \frac{d(SW)}{de} \right|_{\theta=\theta^*} = \left. \frac{\partial(SW)}{\partial e} \right|_{\theta=\theta^*} + \left. \frac{\partial(SW)}{\partial \theta} \right|_{\theta=\theta^*} \frac{d\theta^*}{de}.$$

By differentiating SW with respect to e and θ , I get:

$$\frac{\partial(SW)}{\partial e} = \frac{\xi}{2}(\beta\theta\delta - r) \quad \text{and} \quad \frac{\partial(SW)}{\partial \theta} = \frac{r^2 + \beta\theta^2((1 - \beta)(1 - 2c\theta) + \delta(e\beta\xi(1 - \beta) - \delta))}{2\beta\theta^2(1 - \beta)}.$$

Since $\frac{r}{\beta\theta} < \frac{p}{\theta} = \delta$, $r < \beta\theta\delta$. Thus, $\frac{\partial(SW)}{\partial e} > 0$.

Now, $\left. \frac{\partial(SW)}{\partial \theta} \right|_{\theta=\theta^*} = \frac{X}{2\beta\theta^{*2}(1 - \beta)}$, where $X = r^2 + \beta\theta^{*2}(1 - \beta)(1 - c\theta^* - \delta)$. Substituting θ^* in X with the expression from Proposition 4.3, I get:

$$X = \frac{(rc(1 - \beta))^2 + Y\beta\delta^2(1 - \beta)(1 + e\beta\xi) - \delta^2}{c^2(1 - \beta)^2},$$

where $Y = (1-\beta)(1-\delta)^2 + \beta\delta^2 - e\beta\xi\delta(1-\beta)$. From the condition $\lambda = \frac{e\theta\xi(1-\beta)}{p-\frac{r}{\beta}} < 1$, I get $r < \beta\theta(\delta - e\xi(1-\beta))$, and since $r > 0$, $\delta - e\xi(1-\beta) > 0$. Multiplying both sides by $\beta\delta$, I get $\beta\delta^2 - e\beta\xi\delta(1-\beta) > 0$, implying that $Y > 0$. Finally, since $Y > 0$, X must be positive, implying $\frac{\partial(SW)}{\partial\theta}\Big|_{\theta=\theta^*} > 0$. Since, $\frac{d\theta^*}{de} > 0$ as well, each term in the above chain rule is positive, so $\frac{d(SW)}{de}\Big|_{\theta=\theta^*} > 0$.

Now, I show that SW increases with r . By differentiating SW with respect to r , I get:

$$\frac{\partial(SW)}{\partial r} = \frac{\beta\delta\theta - r}{\beta\theta(1-\beta)} - \frac{e\xi}{2}.$$

From the condition $\lambda = \frac{e\theta\xi(1-\beta)}{p-\frac{r}{\beta}} < 1$, I get $\frac{\beta\delta\theta - r}{\beta\theta(1-\beta)} > e\xi$. Thus, $\frac{\partial(SW)}{\partial r} = \frac{\beta\delta\theta - r}{\beta\theta(1-\beta)} - \frac{e\xi}{2} > e\xi - \frac{e\xi}{2} = \frac{e\xi}{2} > 0$. Since $\frac{d\theta^*}{dr} = 0$, $\frac{d(SW)}{dr}\Big|_{\theta=\theta^*} = \frac{\partial(SW)}{\partial r}\Big|_{\theta=\theta^*} > 0$.

$\beta \geq 1$: In this case, social welfare is given by:

$$SW = \frac{r\theta(2\delta - e\xi(\beta - 1)) - r^2 + \theta^2(\beta(\beta - 1 - \delta^2) - e\xi(\beta - 1)(\beta - 1 - \delta))}{2\theta(\beta - 1)}.$$

Therefore, at $\theta = \theta^*$, $\frac{d(SW)}{dr} = \frac{\partial(SW)}{\partial r} = -\frac{r - \delta\theta}{\theta(\beta - 1)} - \frac{e\xi}{2}$, which is clearly negative, because $r > \beta\delta\theta > \delta\theta$.

To show that SW is not a monotonic function of e , I write using the chain rule:

$$\begin{aligned} \frac{d(SW)}{de}\Big|_{\theta=\theta^*} &= \frac{\partial(SW)}{\partial e}\Big|_{\theta=\theta^*} + \frac{\partial(SW)}{\partial\theta}\Big|_{\theta=\theta^*} \frac{d\theta^*}{de} \\ &= \frac{c\xi(\beta-1)\left(r^2 - \frac{r\delta(e\xi(\beta-1) - \beta\delta)^2}{c(\beta-1)} - \frac{\delta^2(\beta(2\delta^2 - (\beta-1)(1+\delta)) + 2e\xi(\beta-1)(\beta-\delta-1))(e\xi(\beta-1) - \beta\delta)^2}{c^2(\beta-1)^2}\right)}{2\delta(e\xi(\beta-1) - \beta\delta)^2}. \end{aligned}$$

Solving $\frac{d(SW)}{de} = 0$, a quadratic equation in r , I obtain two roots of which only one is valid:

$$\hat{r} = \frac{\delta(e\xi(\beta-1) - \beta\delta)(e\xi(\beta-1) - \beta\delta - \sqrt{\beta(\delta^2(8+\beta) - 4(\beta-1)(1+\delta)) - 2e\xi(\beta-1)(4(1-\beta+\delta) + \beta\delta) + e^2\xi^2(\beta-1)^2})}{2c(\beta-1)}.$$

Since $\frac{d(SW)}{de}$ can be shown to be a decreasing function of r within the valid region of Case 1A, $\frac{d(SW)}{de}$ is positive if and only if $r < \hat{r}$. Therefore, SW is increasing in e if $r < \hat{r}$ and decreasing otherwise. ■

Proof of Lemma A1

To prove this result, let us first recognize that $X(r; \theta)$ is a cubic polynomial in r satisfying:

$$\begin{aligned} X(-\infty; \theta) &= -\infty, \quad X(+\infty; \theta) = +\infty, \quad X(0; \theta) = (1 - 2e\xi(1 - \beta))(1 - \beta)^2\beta(1 + e\beta\xi)^2\theta^3, \text{ and} \\ X\left(\frac{\beta\theta}{2}; \theta\right) &= -\frac{\beta\theta^3}{4}(4(1-\beta) + 7\beta^2 + 2e\xi(1-\beta)(4-\beta(4+5\beta)) + 4e^2\beta\xi^2(1-\beta)^2(4-3\beta) + 8e^3\beta^2\xi^3(1-\beta)^3). \end{aligned}$$

Now, recall that $2e\xi(1-\beta) < 1$, which immediately implies that $X(0; \theta) > 0$. Moving on to $X\left(\frac{\beta\theta}{2}; \theta\right)$, note that $(4e^2\beta\xi^2(1-\beta)^2(4-3\beta) + 8e^3\beta^2\xi^3(1-\beta)^3) > 0$ holds trivially. Further, $(4(1-\beta) + 7\beta^2 + 2e\xi(1-\beta)(4-\beta(4+5\beta)))$

is linear in e , its value at $e = 0$ is positive trivially, and its value at $e = \frac{1}{\xi(2-\beta)}$ is $\frac{16(1+\beta)}{2-\beta} - \beta(22 + 3\beta)$, which can be easily shown to be positive for all $\beta \in [0, 1]$. Therefore, $(4(1-\beta) + 7\beta^2 + 2e\xi(1-\beta)(4-\beta(4+5\beta))) > 0$ always, implying that $X\left(\frac{\beta\theta}{2}; \theta\right) < 0$.

The above signs, taken together, imply that the polynomial has three real roots, one negative and two positive, and also that exactly one of the two positive roots is bigger than $\frac{\beta\theta}{2}$ while the other one is smaller. Note that $r \in \left[0, \frac{\beta\theta(1-\beta)}{2-\beta}\right]$ implies that $\beta\theta > 2r$ (as $\frac{1-\beta}{2-\beta} < \frac{1}{2}$ for $0 < \beta < 1$). Therefore, we are at most interested in $r \in \left[0, \frac{\beta\theta}{2}\right]$, which, in turn, means that the only root of interest is the smaller of the two positive roots. Let us denote it by $\sigma_{1A}(\theta)$. Then, $X(r; \theta)$ is positive if $0 \leq r < \sigma_{1A}(\theta)$. ■

Proof of Lemma A2

i) It is easy to see that $X_1(e; \theta)$, given by:

$$\begin{aligned} X_1(e; \theta) = & -6r^3 - r^2\theta(4 - 9\beta + 2e\beta\xi(2 - 3\beta)) + 2r\theta^2(1 - \beta)^2(1 + e\beta\xi)^2 \\ & - \beta\theta^3(1 - \beta)(1 + e\beta\xi) \left(1 - 5\beta + 7e\beta\xi(1 - \beta) + 2e^2\beta^2\xi^2(1 - \beta)\right), \end{aligned}$$

is a cubic expression in e . Its discriminant can be written as $4\beta^{10}\theta^{12}\xi^6(1 - \beta)^2Y\left(\frac{r}{\beta\theta}, \beta\right)$, where the function $Y(\cdot)$ is given by:

$$\begin{aligned} Y(z, \beta) = & 4(1 - \beta)^3(41 - 9\beta) - 48z(1 - \beta)^4 + z^2(1 - \beta)^2(16 - \beta(488 - \beta(1177 - 27\beta(22 - 3\beta)))) \\ & + 12z^3(1 - \beta)^3\beta(8 - 3\beta(37 - 9\beta)) - 4z^4(1 - \beta)\beta(8 - \beta(349 - \beta(977 - 3\beta(283 - 63\beta)))) \\ & - 48z^5(2 - \beta)(1 - \beta)^2\beta^2(3 - 14\beta) + 16z^6(2 - \beta)^2\beta^2(1 - 12(1 - \beta)\beta). \end{aligned}$$

Since $r \in \left[0, \frac{\beta\theta(1-\beta)}{2-\beta}\right]$, $\beta\theta > 2r$ as well (as $\frac{1-\beta}{2-\beta} < \frac{1}{2}$ for $0 < \beta < 1$). Therefore, we are only interested in values of z below $\frac{1}{2}$. I will first show that $Y > 0$ for $0 < \beta < 1$ and $0 < z < \frac{1}{2}$. To do so, I differentiate Y with respect to β multiple times to find that $\frac{\partial^6 Y}{\partial \beta^6} > 0$. Therefore, $\frac{\partial^5 Y}{\partial \beta^5}$ is an increasing function of β and is maximized at $\beta = 1$. Since even this maximum value is negative, I conclude that $\frac{\partial^5 Y}{\partial \beta^5} < 0$, implying that $\frac{\partial^4 Y}{\partial \beta^4}$ is a decreasing function of β . It is, therefore, minimized at $\beta = 1$, and this minimum value is found to be positive. Therefore, $\frac{\partial^4 Y}{\partial \beta^4} > 0$, and $\frac{\partial^3 Y}{\partial \beta^3}$ is an increasing function of β . Its maximum value at $\beta = 1$ turns out to be negative, indicating that $\frac{\partial^3 Y}{\partial \beta^3} < 0$ and $\frac{\partial^2 Y}{\partial \beta^2}$ is a decreasing function of β . Continuing this alternating pattern, the minimum of $\frac{\partial^2 Y}{\partial \beta^2}$ occurring at $\beta = 1$ is positive, making $\frac{\partial^2 Y}{\partial \beta^2} > 0$ and $\frac{\partial Y}{\partial \beta}$ an increasing function of β . Therefore, $\frac{\partial Y}{\partial \beta}$ is maximized at $\beta = 1$, and this maximum value happens to be negative. This implies that the original function,

Y , is decreasing in β . Since its minimum value, occurring at $\beta = 1$, is positive, Y must be positive over the entire valid range.

This, of course, means that the discriminant itself is positive and $X_1 = 0$ must have three real roots. I now investigate the nature of these three roots. Solving $\frac{\partial X_1}{\partial e} = 0$, and checking the second order condition, I get the two critical points of X_1 :

$$e_{\min} = \frac{-T - Q}{6\beta^2\theta\xi(1 - \beta)}, \quad \text{and} \quad e_{\max} = \frac{-T + Q}{6\beta^2\theta\xi(1 - \beta)},$$

where $Q = \sqrt{4r^2(1 - 2\beta(4 - 5\beta)) - 12r\beta\theta(1 - \beta)^2 + 3\beta^2\theta^2(1 - \beta)(11 - 3\beta)} > 0$ and $T = (1 - \beta)(9\beta\theta - 2r) > 0$. Clearly, e_{\min} is negative, making the first (smallest) of the three real roots negative and of little interest. It turns out that the second root is negative as well. To see this, I consider two separate cases. First, if $0 < \beta \leq \frac{1}{5}$, it can be shown that e_{\max} is negative as well, implying that the second root is negative. This is because $T > Q$:

$$T^2 - Q^2 = 12\beta \left(r^2(2 - 3\beta) + 2r\beta\theta(1 - \beta) + \theta(1 - \beta)(\beta\theta - 2r) + \frac{9\beta\theta^2}{5}(1 - \beta) + 6\beta\theta^2(1 - \beta) \left(\frac{1}{5} - \beta \right) \right) > 0.$$

If, on the other hand, $\frac{1}{5} \leq \beta < 1$, then I find:

$$\begin{aligned} X_1(0; \theta) &= -6r^3 - r^2\theta(4 - 9\beta) + 2r\theta^2(1 - \beta)^2 + \beta\theta^3(5\beta - 1)(1 - \beta) \\ &> -3r^2\beta\theta - r^2\theta(4 - 9\beta) + 2r\theta^2(1 - \beta)^2 + \beta\theta^3(5\beta - 1)(1 - \beta) \quad [\text{since } \beta\theta > 2r] \\ &= -4r^2\theta(1 - \beta) + 2r^2\beta\theta + 2r\theta^2(1 - \beta)^2 + \beta\theta^3(5\beta - 1)(1 - \beta) \\ &> -2r\beta\theta^2(1 - \beta) + 2r^2\beta\theta + 2r\theta^2(1 - \beta)^2 + \beta\theta^3(5\beta - 1)(1 - \beta) \quad [\text{since } \beta\theta > 2r] \\ &= 2r\theta^2(1 - \beta)(1 - 2\beta) + 2r^2\beta\theta + \beta\theta^3(5\beta - 1)(1 - \beta) \\ &> 2r\theta^2(1 - \beta)(1 - 2\beta) + 2r^2\beta\theta + 2r\theta^2(5\beta - 1)(1 - \beta) \quad [\text{since } \beta\theta > 2r] \\ &= 2r\theta^2(1 - \beta)(3\beta) + 2r^2\beta\theta > 0. \end{aligned}$$

Therefore, $X_1(0; \theta) > 0$, implying, once again, that the second root is negative. Hence, only the largest root, denoted $e = \gamma_{1A}(\theta)$, is of interest to us, and it is obvious that X_1 can be positive only to the left of this root, as X_1 approaches $-\infty$ when e is very large.

ii) Note that:

$$\begin{aligned} X_2(r; \theta) &= 2r^3\beta - 3r^2\theta(1 - \beta)(1 + e\beta\xi(1 - \beta)) - 2r\theta^2(1 - \beta)^2(1 + \beta + e\beta^2\xi(1 + e\xi)) \\ &\quad + \theta^3(1 - \beta)^3(1 + e\beta\xi)^2(1 + e\beta\xi(1 - \beta)) \end{aligned}$$

is a cubic polynomial in r . Further, its discriminant is:

$$16\theta^6(1-\beta)^6((9+22\beta+21\beta^2+12\beta^3+4\beta^4)+e\xi(45\beta+36\beta^2+12\beta^3+6\beta^4+12\beta^5)+e^2\xi^2(108\beta^2-15\beta^3+15\beta^4-12\beta^5+21\beta^6)+e^3\xi^3(144\beta^3-153\beta^4+69\beta^5-12\beta^6+22\beta^7)+e^4\xi^4(108\beta^4-159\beta^5+102\beta^6-15\beta^7+9\beta^8)+e^5\xi^5(45\beta^5-81\beta^6+57\beta^7-9\beta^8)+e^6\xi^6(9\beta^6-14\beta^7+9\beta^8)).$$

When the discriminant is expressed this way as a polynomial of e , all the coefficients of that polynomial turn out to be trivially positive if $\beta \in (0, 1)$. Since $e > 0$, the discriminant must be positive, implying that $X_2(r; \theta)$ has three real roots. And, since (i) $X_2(-\infty; \theta) \rightarrow -\infty$, (ii) $X_2(0; \theta) > 0$, and (iii) the coefficient of r^2 is negative, $X_2(r; \theta)$ must have exactly one negative and two positive roots. Between these two positive roots occurs the minimum of the function, at $r_{\min} = \frac{\theta(1-\beta)(3(1+e\beta\xi(1-\beta))+Q)}{6\beta}$, where $Q = \sqrt{9+3\beta(4(1+\beta)+e\xi(6-\beta(6-4\beta-e\xi(3-\beta(2-3\beta))))} > 0$. Now, I check whether r_{\min} is larger than $\frac{\beta\theta(1-\beta)}{2-\beta}$, the maximum permissible value of r in Case 1A. Indeed, it would be larger iff $\frac{3(1+e\beta\xi(1-\beta))+Q}{6\beta} > \frac{\beta}{2-\beta}$ or $(2-\beta)Q > T$, where $T = 6\beta^2 - 3(1+e\beta\xi(1-\beta))(2-\beta)$. If $T \leq 0$, $(2-\beta)Q > T$ holds trivially. On the other hand, if $T > 0$, this is equivalent to:

$$(2-\beta)^2Q^2 > T^2 \quad \Leftrightarrow \quad 4(1-\beta) + \beta(10-2\beta(3+\beta)) + e\beta^2\xi(2-\beta)(5-4\beta+e\xi(2-\beta)) > 0.$$

Now, the last inequality obviously holds since $0 < \beta < 1$, immediately implying that r_{\min} and the largest root of $X_2(r; \theta) = 0$ are both larger than $\frac{\beta\theta(1-\beta)}{2-\beta}$. The largest root, therefore, is of no interest to us. So, the only root of interest is the smaller of the two positive roots, which I denote by $\tau_{1A}(\theta)$. Thus, $X_2(r; \theta)$ is positive if $r < \tau_{1A}(\theta)$, and it is negative otherwise. ■