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Aeroelastic Modeling, Modal Testing, and Wind Tunnel System Identification of the MARGE II Aeroservoelastic Model

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Abstract

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Wind tunnel testbeds and their mathematical models are crucial for the development of aeroelastic control laws. Yet, developing accurate aeroelastic mathematical models adequate for control law synthesis remains a challenge. Aeroelastic models often require fine tuning of the unsteady aerodynamic matrices and the finite element (FE) structural model through ground vibration testing to predict the wind tunnel frequency response well. In this thesis, an overview of the aeroelastic mathematical modeling, modal testing, and wind tunnel system identification process of MARGE II—the University of Washington’s new aeroservoelastic wind tunnel model—is presented. An alternative to FE modeling for obtaining structural mode shapes is explored: digital image correlation (DIC) modal testing, making it possible to create aeroelastic models without an FE model. Light is then shed on the challenges of wind tunnel frequency response system identification. Finally, the accuracy of two aeroelastic mathematical models (created with FE and DIC-based structural modes) is evaluated by comparing the resulting frequency responses to wind tunnel data.

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Nomenclature

Acronyms

AE	Aeroelastic
AFS	Active Flutter Suppression
AIC	Aerodynamic Influence Coefficients
ASE	Aeroservoelastic
DIC	Digital Image Correlation
DLM	Doublet Lattice Method
FEA	Finite Element Analysis
FRF	Frequency Response Function
GAF	Generalized Aerodynamic Forces
GLA	Gust Loads Alleviation
GVT	Ground Vibration Testing
IPS	Infinite Plate Spline
LES	Large Eddy Simulation

MARGE Models for Aeroelastic Response to Gust Excitation

MDOF Multiple Degrees of Freedom

ODS Operating Deflection Shape

RBF Radial Basis Functions

RFA Rational Function Approximation

SDOF Single Degree of Freedom

URANS Unsteady Reynolds-Averaged Navier–Stokes

Greek Symbols

α Angle between camera view plane and modal deflection (deg)

$\bar{\eta}$ y-coordinate on the DLM aerodynamic surface

$\bar{\xi}$ x-coordinate on the DLM aerodynamic surface

β Aerodynamic lag term

Δx Pixel length

δ Control surface deflection angle (deg)

ϵ_k Strain at point k

$\epsilon_k^{(i)}$ Contribution to strain at point k by mode i

η Modal loss factor

γ_{xy} Coherence

$\hat{\gamma}_{xy}$ Coherence estimate

κ	Plate curvature (m^{-1})
λ	Eigenvalue
ω	Angular frequency (rad/s)
Φ	Frequency response function phase (rad)
ϕ_i	Mode shape vector i
ϕ_{ik}	Deflection at point k due to mode i
ρ_∞	Freestream density (kg/m^3)
τ	Iteration index
θ	Pitch angle (deg)
$\theta^{(i)}$	Contribution to pitch angle by mode i (deg)
$\tilde{\phi}$	Velocity perturbation potential (m^2/s)
ξ_i	Component i of the modal coordinate vector
ζ	Modal damping coefficient
$\{\xi\}$	Modal coordinate vector

Subscripts and Indices

a	Aileron
b	Aerodynamic lag root index
c	Control surface
e	Elevator

g	Gust vane
h	Structural mode
i, j, r	Mode indices
k, l	Point indices
n	Natural frequency
o	Index of pixel in the subset

Latin Symbols

$\bar{g}$	Gravitational acceleration (m/s ²)
$\bar{K}$	Kernel function
$\bar{n}$	Total number of DLM panels
$\bar{P}_i$	Point force applied to the infinite plate (N)
$\bar{q}$	Dynamic pressure (Pa)
$\bar{R}, \bar{S}$	V-g-f Generalized eigenvalue matrices
$\hat{S}_{xx}, \hat{S}_{xy}, \hat{S}_{yy}$	Power spectral density estimates
$\hat{X}$	Chirp-z DFT of input signal
$\hat{Y}$	Chirp-z DFT of output signal
a	Speed of sound (m/s)
a_0, a_1, a_2	Spline boundary conditions
a_k	Acceleration at point k (m/s ²)

A_{ij}, A_{hc}, A_{hg}	Generalized aerodynamic forces (m ³)
C	Structural damping matrix (kg/s)
c_i	Normalization constant of mode i
C_{hh}	Modal damping matrix (N/s)
C_N	DIC cost function value
d	Speckle diameter (pixels)
f_i	Frequency of mode i (Hz)
F_k	Force applied at point k (N)
FRF_k	Frequency response function of deflection at point k
g	Dummy damping in V-g analysis
gst_i	Hysteretic damping corresponding to structural mode i
$H(f)$	Frequency response function
H_1	H_1 estimate
I	Brightness
j	Imaginary unit
K	Structural stiffness matrix (N/m)
k	Reduced frequency
K_{hh}	Modal stiffness matrix (N)
M	Structural mass matrix (kg)

m	Mass (kg)
M_∞	Freestream Mach number
M_{hh}	Modal mass matrix (kg)
N	Length of a subset in terms of number of pixels
n	Total number of lag terms
n_f	Number of frequency points
p	Pressure (Pa)
p'	Pressure potential (Pa)
P_i	Roger approximation matrix i
R_i	Residue of mode i (m/N s ²)
r_{ij}	Distance between deflecting and applied-force points in the IPS (m)
S	Surface area of DIC panel (m ²)
s	Laplace variable
S_{uu}, S_{vv}	Input/Output noise power spectral densities
S_{xx}, S_{xy}, S_{yy}	Power spectral densities
T	Period (s)
U	Airspeed (m/s)
u	x-component of translation vector
v	y-component of translation vector

V_x	Pixel velocity in the x-direction
V_y	Pixel velocity in the y-direction
W_j	Deflection of the infinite plate (m)
x, x'	x-coordinate
X_l	Structural deflection at point l (m)
y, y'	y-coordinate/Spanwise coordinate (m)

Chapter 1

Introduction

This thesis presents the aeroeservoelastic modeling and wind tunnel validation of the University of Washington's all-new aeroservoelastic wind tunnel model, MARGE II. In particular, the focus is on different ways to obtain structural dynamic information (Finite Element Analysis vs. Digital Image Correlation Ground Vibration tests) for the purpose of aeroelastic modeling and matching aeroelastic models to wind tunnel frequency response data. In doing so, the aim is to shine light on methods of creating aeroelastic models that avoid the time-intensive finite-element model correction process and highlight reasons for similarities and differences between aeroelastic models and wind tunnel frequency response measurements. It is the hope that the discussion of aeroelastic modeling and system identification in this thesis will help speed up the process of aeroelastic modeling and wind tunnel model validation for the reader.

1.1 Motivation

Given the increasing flexibility of modern transport aircraft, aircraft control systems must be designed based on dynamic models that not only include the rigid body but also the flexible dynamics

of the aircraft. Including the flexible dynamics is crucial to prevent unplanned control structure interaction (CSI) with potentially catastrophic results. Including flexible dynamics in aircraft dynamic models is also crucial for the design of controllers for aeroelastic behavior objectives. Such controllers include Gust Load Alleviation (GLA) [1–8] controllers that reduce structural dynamic internal load overshoots as well as fatigue, ride comfort controllers, modal suppression controllers that increase structural damping (as on Boeing 787-10) [9] and Active Flutter Suppression (AFS) controllers [10–18], which may be certified in the future.

Aeroservoelastic (ASE) models couple structural dynamic (such as Finite Element) and aerodynamic models together. Although Finite Element (FE) models capture the low-frequency modes well, they require substantial fine-tuning to capture higher-order modes. Finite element models also show sensitivity to boundary conditions and the way components of a structure are joined. Aeroelastic predictions after fine tuning of FE models by ground vibrations tests can lead to significant changes in aeroelastic predictions such as flutter speed [19] compared to predictions based on the untuned models, and experience has shown that correction of FE models has played an important role in making accurate aeroelastic predictions [20].

Traditionally, GVTs used to fine-tune and validate structural models were based on covering the aircraft structure with accelerometers, which sometimes introduced non-negligible added mass and damping [21], especially in the case of wind tunnel models or very small aircraft. Developments in optics based Ground Vibration Testing (GVT) and in particular in Digital Image Correlation (DIC) based Ground Vibration Testing have allowed modal testing to be conducted without the need for accelerometers [22] making it possible to obtain the structural information needed to create ASE models directly without Finite Element models that need corrections or accelerometer-covered GVTs. Once ASE models are created, they are validated through flight or wind tunnel system identification.

It should be noted, however, that DIC GVT based modes can be obtained only after an airframe is ready for testing. Aeroelastic analyses carried out during the design optimization of an airplane

must be FE-based.

1.2 Prior Work

There is a limited number of publicly available resources that present detailed correlations between ASE mathematical models and test data. Some representative ASE math/test correlations can be found in [20, 23–26]. In these references, Finite Element model structure is coupled with Doublet Lattice Method (DLM) unsteady aerodynamics. To the best of the author’s knowledge, no prior work was published to date on the coupling of DIC-based GVT modes with unsteady aerodynamic models to create aeroelastic math models. DIC has been used, however, in aeroelastic applications such as hypersonic and wind turbine aeroelasticity, mainly to track the structure as it undergoes aeroelastic interactions [27, 28].

Once an aeroelastic mathematical model is created for a given airframe (whether that of a wind tunnel model, a small UAV, or a full size flight vehicle), ASE system identification can be carried out in the wind tunnel or in flight. There is extensive literature on the frequency-domain system identification of aeroelastic vehicles that is used to validate the ASE models. But major challenges exist in this area, one of which is the difficulty in obtaining consistently high coherence during identification. Efforts to increase coherence through input design and data processing techniques are discussed in [29–31]. Past work revealed that the coherence is highly dependent on the modes of the structure, as well as modal damping, and the presence of aeroelastic nonlinearities [32–34].

1.3 Contribution

In this thesis, the possibility of using DIC-based mode shapes for ASE modeling to bypass the process of having to correct finite element models with ground vibration tests is investigated. By comparing the resulting ASE mathematical models to wind tunnel tests to assess the accuracy of

the mathematical models, the thesis adds to the existing limited literature on the development of accurate ASE mathematical models in general and the MARGE-II system in particular.

1.4 Thesis Organization

The thesis is organized as follows: Chapter 2 reviews the mathematical background for digital image correlation (DIC), frequency-domain system identification and aeroelastic modeling. Chapter 3 introduces the MARGE II wind tunnel model and the experimental set-ups used for DIC-GVT and wind tunnel system identification. Chapter 4 provides the details of the aeroelastic models created using DIC and FEA mode shapes. Chapter 5 explains the wind tunnel identification process of MARGE II. Chapter 6 presents the comparison of the aeroservoelastic (ASE) mathematical model against wind tunnel frequency response data. Chapter 7 then presents the concluding remarks.

Chapter 2

Theoretical Background

This chapter begins with the discussion of the methods of extracting structural mode shapes using DIC ground vibration testing. Then, methods of using the DIC or FEA mode shapes to create ASE models are explained. The chapter is concluded with a discussion of frequency-domain system identification and the associated input design that are required to validate these ASE models.

The theory discussed in this chapter is later used to develop and validate the ASE model of the Model for Aeroelastic Response to Gust Excitation (MARGE) II system. MARGE II is a flexible ASE wind tunnel model designed to be fitted in University of Washington's 3x3 wind tunnel. MARGE II model contains four servo-actuated control surfaces, potentiometers that measure the control surface positions, accelerometers and strain gages that measure the structural deformation as well as a pitch angle sensor. The 3x3 wind tunnel also houses a separate gust vane system used to excite MARGE II with gust inputs.

2.1 Modal Analysis Using Digital Image Correlation

Many methods for structural modal testing exist. The most common among them is to cover a structure with an array of accelerometers and measure their motions. But for a structure as small

and lightweight as the MARGE II wind tunnel model, the damping and inertia introduced by an accelerometer array may skew the result.

DIC is a broad term describing the set of algorithms used to calculate object motion and deformations from a series of images of the object undergoing deformation. Other than being non-contact, DIC also has the advantage that it may require less reliance on interpolation. Accelerometers can only provide information about the point at which they are attached. However with DIC, vibration of any point in the structure satisfying certain color gradients can be tracked allowing for a dense mesh of points (up to equal to the number of pixels) and minimal reliance on interpolation.

2.1.1 Calculation of Image Displacements

DIC does not directly calculate the deformation of the structure but computes the motion of patterns in the image of a structure in pixel coordinates (“image displacement”) resulting from the deformation of the object. Firstly, one must specify the desired points in the image where the image displacement will be computed. Then, the image displacement at those points is calculated based on the motion of a grid of N -by- N pixels surrounding the points, called the subsets. N is an integer. Each point has an associated subset and one displacement vector is calculated per subset. Using a subset of pixels when calculating the displacement of a point helps avoid potential ambiguities and provides robustness [22].

The displacement of a subset is computed with respect to a reference frame and the frame’s corresponding light intensity distribution. The reference image (frame) usually represents the undeformed state of the structure. When the structure deforms, the subset associated with a point translates and deforms as shown in Figure 2.1.

The aim is to find the translation of the subset in images representing the deformed state of the structure. In other words, it is sought to find the deformed version of the subset in a frame that captures the deformed state of the structure. The location of the deformed subset corresponds to the translation of the subset. The location of the subset is identified based on its light intensity

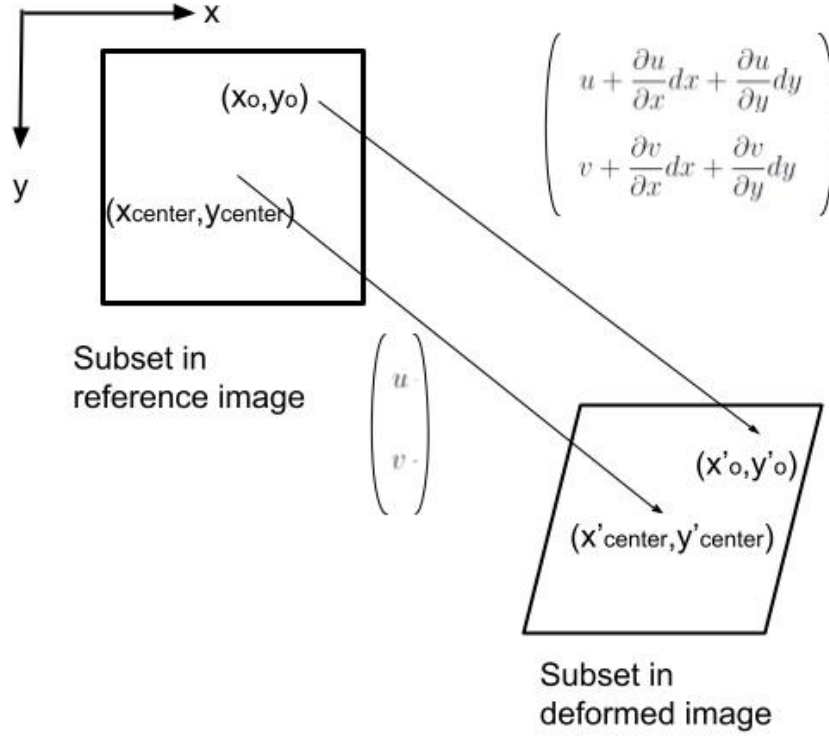


Figure 2.1: Shearing and displacement of the subset and its associated center point

pattern $I(x, y)$ where the arguments x and y denote location in image pixel coordinates. However, because the translation of the subset usually corresponds to a non-integer value, and $I(x, y)$ is only known for every pixel, the light intensity across the subset is interpolated to determine the non-integer pixel translation of the subset in the reference image [35].

Suppose there is an initial subset containing the point o at image coordinates (x_o, y_o) and this point translates to the image location (x'_o, y'_o) due to structural deformation and rigid body motion. It is assumed that its brightness stays the same (called brightness constancy assumption) and any change in brightness is considered noise I_n [35]. The brightness constancy assumption is expressed as Equation 2.1:

$$I_{translated}(x'_o, y'_o) = I_{initial}(x_o, y_o) + I_n. \quad (2.1)$$

The motion of point o from its location (x_o, y_o) in the reference subset to (x'_o, y'_o) in the deformed

subset is due to both translation (u, v) but also deformations. When the deformations are small, the motion of all points in the subset is described as shown in Equation 2.2 [35]:

$$x' = x + u + \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy, \quad y' = y + v + \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy. \quad (2.2)$$

The solution for the six unknowns per subset: $u, v, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ is obtained by minimizing cost function Equation 2.3 [35]:

$$C_N = \sum_{o=1}^{N^2} [I_{initial}(x_o, y_o) - I_{translated}(x'_o, y'_o)]^2. \quad (2.3)$$

Minimizing this cost function with respect to the six parameters corresponds to looking for a displacement and deformation of the subset image that lead to the smallest change in brightness I compared to the reference frame. The difference in brightness I of the translated and deformed subset and the original subset is evaluated for every pixel of the N -by- N subset in the reference image.

There are many implementations of DIC. One such implementation is to assume that the image only translates but does not deform. This assumption is known to be accurate for small (sub-pixel) motions. This implementation of DIC simply becomes the Lucas-Kanade Algorithm [36]. This assumption reduces the unknowns to only displacements (u, v) . The Lucas-Kanade algorithm is explained below.

The equation relating the light intensity of a pixel in the initial (reference) image to its intensity in the deformed image is given in Equation 2.4:

$$I_{translated}(x + u, y + v, t + \Delta t) = I_{initial}(x, y, t) + I_n + \frac{dI}{dt}\Delta t + H.O.T. \quad (2.4)$$

Neglecting the higher order terms and substituting Equation 2.1 (the brightness constancy equation) into Equation 2.4, it is found that $\frac{dI}{dt} = 0$. The rate of change of brightness with respect to

time is further expanded in Equation 2.5:

$$\frac{dI}{dt} = \frac{\partial I}{\partial t} + \frac{\partial I}{\partial x}V_x + \frac{\partial I}{\partial y}V_y = 0 \quad (2.5)$$

where $V_x = \frac{dx}{dt}$ and $V_y = \frac{dy}{dt}$. But if every time increment is taken to be the change between one frame to the next, $\frac{\partial I}{\partial t}$ becomes the change in brightness of a pixel between one frame to the next, and V_x, V_y simply become the image displacements (u, v) . Now, if Equation 2.5 is enforced for all pixels in the subset, the system of Equations 2.6 is obtained [37]:

$$\begin{bmatrix} I_x^{(1)} & I_y^{(1)} \\ I_x^{(2)} & I_y^{(2)} \\ \vdots & \vdots \\ I_x^{(N^2)} & I_y^{(N^2)} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -I_t^{(1)} \\ -I_t^{(2)} \\ \vdots \\ -I_t^{(N^2)} \end{bmatrix}. \quad (2.6)$$

The terms I_x, I_y and I_t denote the partial derivatives of the light intensity distribution with respect to the pixel coordinates and time. System of Equations 2.6 are overconstrained and can be solved with the standard least squares approach [37] resulting in the closed-form solution provided in Equation 2.7:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}^{-1} \begin{bmatrix} -\sum I_x I_t \\ -\sum I_y I_t \end{bmatrix} \quad (2.7)$$

Although a closed form solution to minimize the cost function Equation 2.3 can be obtained, the closed-form approach was not used in the work described here due to the approach being first order. Instead, in Algorithm 1 [22] was used.

By calculating the displacement of the subset incrementally $(\delta u, \delta v)$ over iterations (τ) until the increments are less than a small value (ϵ) , the accuracy of the determined displacements is increased. This approach was adopted for the DIC experiments described here, based on the implementation in the *pyIDI* Python package written by Masmeijer et al [22].

Algorithm 1 Iterative Lucas–Kanade Algorithm

1: Initialize $u_0 \leftarrow 0, v_0 \leftarrow 0$

2: $\tau \leftarrow 0$

3: **loop**

4: Solve

$$(\delta u_\tau, \delta v_\tau) = \arg \min_{\Delta u, \Delta v} \sum_{o=1}^{N^2} [I_{\tau+1}(x_o, y_o) - I_{initial}(x_o + u_\tau + \Delta u, y_o + v_\tau + \Delta v)]^2$$

5: $u_{\tau+1} \leftarrow u_\tau + \delta u_\tau$

6: $v_{\tau+1} \leftarrow v_\tau + \delta v_\tau$

7: **if** $\sqrt{\delta u_\tau^2 + \delta v_\tau^2} < \varepsilon$ **then**

8: **return** $(u_{\tau+1}, v_{\tau+1})$

9: **end if**

10: $\tau \leftarrow \tau + 1$

11: **end loop**

2.1.2 Cautions and Best Practices for Image Displacement Calculation

In order for DIC to be successful, several conditions must hold. Firstly, the motion of points in the image must only be a result of the motion of the structure. Secondly, because of the brightness constancy assumption, any dark spots and glares and sudden lighting changes on the structure will dramatically increase the noise term in Equation 2.1 and result in poor accuracy. Thirdly, because the displacement at a specified point is calculated based on the time-change of light intensity of a small subset of pixel around the desired point, deformation continuity must hold on the part of the structure that is projected onto this subset of pixels [38]. For example, any holes on the structure that fall within the pixel subset will invalidate the calculation. Finally, the light intensity at different x, y pixels in the image $I(x, y)$ (resulting from the object and its lighting conditions) must contain sufficient spatial variation in light intensity in both the image x and y direction. Else, the DIC algorithms will suffer from the aperture problem or low signal-to-noise ratio leading to inaccurate displacement computation. The need for light intensity gradients in both the image x and y direction can be addressed by applying a speckle pattern onto the object.

Firstly, the speckle patterns should be high contrast (such as black speckles on a white back-

ground) to increase signal-to-noise ratio [39]. The size of the speckles is also important. Very large speckled patterns will lead to most of the subset being covered by one speckle and will reduce the gradients in the subset making it harder to identify the motion of the subset. However, the speckles also cannot be too small to satisfy the spatial Nyquist condition given in Equation 2.8 [35]:

$$2\Delta x < d \quad (2.8)$$

where Δx is the pixel size and d is the pixel diameter. It is for this reason that the optimal speckle diameter is between 3-7 pixels [38]. The speckles should cover about 50% of the area, with the rest being background [40]. The distribution and sparsity of these speckles control the light intensity distribution, which is crucial for accurate gradient calculation.

The final consideration that is of importance is camera distortion. For this purpose, models such as a camera pin hole model can be used. Or, one can directly specify the physical distance (at the location of the measured object) corresponding to each of the pixels. This approach is only valid if the object-to-camera angle and distance do not significantly change such as in a vibration experiment. Regardless of which method is used, it is necessary to convert structural motion in pixels to structural deformation in physical units, for which lens distortion must be taken into account.

2.1.3 Extraction of Mode Shapes

Once the image displacement is related to the deformation and motion of the structure, the displacement of the structure, $X_l(t)$, for every point observed on the structure can be obtained. Because modal displacement is assumed to be in the out of surface normal direction, $X_l(t)$ is a scalar value representing the deformation of the structure along the surface normal vector. Suppose that the excitation force applied at point k during the GVT, $F_k(t)$ is recorded as well. The frequency response function can then be obtained between F_k and the displacement of every point of interest on the

structure X_l . Let this frequency response function be denoted FRF_{lk} . The value of $FRF_{lk}(f)$ at every point can be used to obtain the mode shapes.

The mode shape at natural frequency f_i is obtained by using the frequency response at frequency f_i ($FRF_{lk}(f_i)$). $FRF_{lk}(f_i)$ represents the operating deflection shape (ODS) of the structure at that natural frequency as excited by the applied force. In order for the ODS $FRF_{lk}(f_i)$ to approximate the mode shape ϕ_i at frequency f_i , one must choose the point of force excitation on the structure such that the mode shape is well excited. If the natural frequencies of the structure are well separated, the separation ensures that the ODS of the structure is mainly a response due to the mode shape ϕ_i . Even so, some of the points on the structure may lie close to the nodes of mode ϕ_i and the antinodes of the surrounding modes. For points that are close to the antinodes of surrounding modes, it is difficult to say that $FRF_{lk}(f_i) \approx \phi_i$. To partially correct the presence of the effects of surrounding modes on $FRF_{lk}(f_i)$, it is possible to use a consistent phase reference for all of the points on the structure Φ_{ref} , representing the phase of mode ϕ_i relative to the excitation force to filter out response due to other modes. Using a consistent phase reference, the DIC approximation for mode shape i at every point l ($\phi_{il_{DIC}}$) is obtained as shown in Equation 2.9:

$$\phi_{il_{DIC}} = \text{Real} \left(|FRF_l(f_i)| \angle (\Phi_{FRF_l(f_i)} - \Phi_{ref}) \right). \quad (2.9)$$

Now that mode shape $\phi_{i_{DIC}}$ is available, the mode must be normalized. One common way of normalization is mass-normalization. Mass normalization means that every mode satisfies Equation 2.10:

$$\phi_i^T M \phi_i = 1. \quad (2.10)$$

Since the FRFs of all points near f_i are dominated by the response of the i -th mode (provided that the modes are well separated in frequency or, in the case of two modes that are very close in natural frequency, excitation force locations that minimize response of one mode and maximize the response of the other), the FRF can be approximated to be a result of only the i -th mode in the

vicinity of the modal frequency f_i as shown in Equation 2.11 [41]:

$$FRF_{lk}(f_i) = \frac{X_l}{F_k}(f_i) = \sum_{r=1}^m \frac{R_{r,kl}}{(1 + j\eta_r)(2\pi f_r)^2 - (2\pi f_i)^2} \approx \frac{R_{i,kl}}{j\eta_i(2\pi f_i)^2} \quad (2.11)$$

where R_i is the modal residual, η_i is the modal loss factor of the i -th mode and j is the complex unit. It is important to note, again, that the contributions to $FRF_{lk}(f_i)$ from other modes ($r \neq i$) can be neglected only when there is sufficient frequency separation between the modes. The residual of mode i is defined as the product of the normalized modal deflection at point k and l as shown in Equation 2.12:

$$R_{i,kl} = \phi_{il_{normalized}} \cdot \phi_{ik_{normalized}}. \quad (2.12)$$

The mass-normalized modal deflections are obtained by dividing the DIC-measured mode shapes by the normalization constant ' c_i ':

$$\phi_{i_{normalized}} = \frac{\phi_{i_{DIC}}}{c_i}. \quad (2.13)$$

By assuming single degree of freedom system behavior near frequency f_i , the modal loss factor η_i is approximated using the modal damping coefficient ζ_i [41]. The simplified relationship between the loss factor and damping coefficient near frequency f_i is provided in Equation 2.14:

$$2\zeta_i = \eta_i. \quad (2.14)$$

The modal damping coefficients can be determined by frequency sweeps or by log-decrement methods (see more about determining the modal damping coefficient in a subsequent section). Substituting Equations 2.12, 2.13 and 2.14 into Equation 2.11 and taking the absolute value leads to Equation 2.15:

$$|FRF_{kl}(f_i)| = \frac{\phi_{il_{DIC}}\phi_{ik_{DIC}}}{2\zeta_i(2\pi f_i)^2 c_i^2}. \quad (2.15)$$

Equation 2.15 can be rearranged to solve for the normalization c_i , allowing the DIC mode shapes to be converted to mass-normalized mode shapes.

To make the process of mass-normalization clearer, a single degree of freedom system may be considered. Equation 2.16 provides the governing equation of motion of a point mass M , attached to a spring of stiffness K and damper with magnitude C excited by a sinusoidal force F at frequency ω :

$$(-\omega^2 M + j\omega C + K) X(j\omega) = F(j\omega) \rightarrow (-\omega^2 + 2j\zeta_n \omega \omega_n + \omega_n^2) X(j\omega) = \frac{F(j\omega)}{m}. \quad (2.16)$$

Excitation at the natural frequency $\omega = \omega_n$ simplifies Equation 2.16 into Equation 2.17:

$$2j\zeta_n \omega_n^2 X(j\omega_n) = \frac{F(j\omega_n)}{m}. \quad (2.17)$$

Since both motion $X(j\omega_n)$ and excitation force $F(j\omega_n)$ are known, the mass M can be calculated directly from the motion and excitation force using Equation 2.18:

$$M = \frac{F(j\omega_n)}{2j\zeta_n \omega_n^2 X(j\omega_n)} = \frac{1}{2\zeta_n \omega_n^2 \left(j \cdot X(j\omega_n)/F(j\omega_n) \right)}. \quad (2.18)$$

In other words, the mass of the system can be evaluated based on its natural frequency, damping ratio, and frequency response at the natural frequency.

In the multi degree of freedom (MDOF) case, the single degree of freedom (SDOF) Equation 2.18 remains applicable to the system when the system oscillates at each natural mode i . The mass M determined by Equation 2.18 is, then, the generalized mass. Finally, to obtain mass-normalized mode shapes, the measured modes are divided by the square root of the generalized mass $c_i = \sqrt{M}$.

2.2 Aeroservoelastic Modeling

Aeroelastic (AE) models couple aerodynamic and structural models to describe the dynamics of flexible vehicles in flight or in the wind tunnel. These models can have varying levels of complexity depending on the fidelity of the aerodynamic and structural models. Structural nonlinearities can include geometric, material, and local nonlinearities such as control surface freeplay. Aerodynamic nonlinearities can include flow separation and shocks.

Due to the moderate flexibility of MARGE II, an overall linear modeling approach can be used without a significant loss of accuracy. The linear structural model results in mode shapes that can be used in the aerodynamic solver to obtain unsteady generalized loads without having to iteratively solve coupled nonlinear structural / aerodynamic equations. When Finite Element based structural mathematical models are used for aeroelastic purposes, it is common to develop lower fidelity FEA models (such as stick-beam models in the case of high aspect ratio wings) [42] with lumped concentrated masses. More detailed and complex FE models are often reduced in order (using Guyan static-reduction, for example) [43] [44] to decrease the computational effort needed to compute the mode shapes.

For vehicles operating in the low-speed flow regime, applicable aerodynamic models in decreasing order of complexity are Large Eddy Simulations (LES), Unsteady Reynolds-Averaged Navier-Stokes (URANS), Euler equations methods, and inviscid panel methods. Common inviscid potential flow methods include the Vortex Lattice Method (VLM), the Doublet Lattice Method (DLM), ZAERO panel aerodynamics models, and several other panel methods for wing / body combinations. Recent books on the unsteady aerodynamic panel modeling of aircraft configurations include [45] and [46].

The Doublet Lattice Method (DLM), conceived in the 1960s and developed over many years, has proven to be very successful for modeling the linear unsteady aerodynamics of flight vehicles at low angles of attack and subsonic high speeds. DLM unsteady aerodynamic modeling was used

in the work described here for the AE modeling of the MARGE II wind tunnel model.

2.2.1 The Doublet Lattice Aerodynamic Math Model

The Doublet lattice aerodynamic model is based on a solution of the linear small disturbance velocity potential PDEs describing the system. The perturbation velocity potential and pressure are related through Equations 2.19 and 2.20 [47]:

$$(1 - M_\infty^2) \tilde{\phi}_{xx} + \tilde{\phi}_{yy} + \tilde{\phi}_{zz} - \frac{1}{a_\infty^2} \tilde{\phi}_{tt} - \frac{2U_\infty}{a_\infty^2} \tilde{\phi}_{xt} = 0, \quad (2.19)$$

$$p = -\rho_\infty \left(\phi_t + U \tilde{\phi}_x \right). \quad (2.20)$$

However, the DLM does not solve for a velocity potential, then a pressure perturbation. Instead, the DLM directly solves the pressure potential (p') equation (Equation 2.21) created by combining the pressure and velocity potential PDEs [47]:

$$\beta^2 p'_{xx} + p'_{yy} + p'_{zz} - \frac{2U}{a^2} p'_{xt} - \frac{1}{a^2} p'_{tt} = 0. \quad (2.21)$$

The pressure potential equation can be solved for any arbitrarily time-varying pressure source or doublet. However, DLM restricts its solutions to sinusoidal time-variations of the motions of the body in the flow and the strengths of the pressure doublets. Based on the fundamental pressure doublet solution, at a given air density ρ speed U , Mach number M_∞ and reduced frequency k , the complex pressure jump amplitude Δp on an aerodynamic surface can be related to the complex downwash velocity amplitude w induced at any point (x, y, z) by Equation 2.22 that integrates the Kernel function $\bar{K}$ on the aerodynamic surface with surface coordinates $\bar{\xi}$ and $\bar{\eta}$ [47]:

$$\frac{w(x, y, z)}{U} = \frac{-1}{8\pi} \int \int_S \frac{2\Delta p(\bar{\eta}, \bar{\xi})}{\rho U^2} \bar{K}(x, y, z, \bar{\xi}, \bar{\eta}, k, M_\infty) d\bar{\xi} d\bar{\eta}. \quad (2.22)$$

The solution of this integral equation is challenging because of the singularities in its kernel function $\bar{K}$.

Approximating and correcting the integral has been the topic of research for a number of decades resulting in many versions of the DLM code. In this research, the 1972 Rodden parabolic approximation of the non-singular part of the Kernel function [48] is used in the spanwise integration of the equation. The 1972 implementation includes coplanar surface corrections which were not included in the first 1969 DLM formulation [49]). The coplanar surface corrections are important for MARGE II which includes a coplanar wing and tail. The 1998 quartic approximation ([50]) for the spanwise integration was omitted, as the 1972 and 1998 approximations of the Kernel function are known to yield similar results when using sufficiently low aspect ratio (<3) aerodynamic panels in aerodynamic surface meshing [51]. The more modern approximation Desmarais [52] approximation for Landahl's I1 and I2 terms needed to calculate the Kernel function [53] instead of the Laschka approximation was used to improve accuracy. Evaluating equation 2.22 results in the final relationship between the downwash velocity w and the pressure jump Δp related through the Aerodynamic Influence Coefficient matrix (AIC) defined in Equation 2.23:

$$\frac{w}{U} = [AIC(k, M_\infty)] \frac{\Delta p}{\bar{q}}. \quad (2.23)$$

As is the industry standard, at every reduced frequency k , the AIC matrix produced by the DLM is corrected using the VLM to adjust the quasi-steady part of the aerodynamic forces.

Now, to solve for the pressure jump over the aerodynamic surfaces, the appropriate boundary conditions are required. Due to the no penetration boundary condition on the surface of the aeroelastic model, the downwash due to motion in a structural mode shape ϕ_i is given by Equation 2.24 [47]:

$$\frac{w}{U} = \frac{\partial \phi_i}{\partial x} + \frac{jk}{b} \cdot \phi_i. \quad (2.24)$$

Other boundary conditions are automatically taken care of. The far field conditions are met due

to the diminishing perturbation caused by the pressure doublet at infinite distance. The wake conditions need not be explicitly modeled to obtain the correct forces on the lifting surface. Noting that the wake is modeled to have no pressure jump ($\Delta p = 0$), Equation 2.22 shows that modeling the wake surface explicitly would not lead to a change in the aero panel downwash velocities. Hence, the force on $\bar{n}$ aero panels (F_{aero}) with surface areas S due to any (structural or control) mode shape ϕ_i can be expressed as in Equation 2.25 [43]:

$$\{F_{i_{aero}}\} = \begin{bmatrix} S_1 & 0 & \cdots & 0 \\ 0 & S_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & S_n \end{bmatrix} [AIC(k, M_\infty)]^{-1} \left(\frac{\partial \vec{\phi}_i}{\partial x} + \frac{jk}{b} \cdot \vec{\phi}_i \right). \quad (2.25)$$

As a result, the work done by mode i on mode j (the generalized self-induced force) is given by Equation 2.26:

$$A_{ij} = \{\phi_i\}^T \{F_{j_{aero}}\}. \quad (2.26)$$

Generalized forces due to gust velocity inputs can be calculated from the known distribution of the local angles of attack due to the gust field. In the work presented here, gusts are generated by oscillating two vanes at the entrance to the wind tunnel test section. Those vanes are modeled as additional control surfaces (in addition to the wing and tail control surfaces). The DLM model of MARGE II is, therefore, just a model of lifting surfaces and control surfaces in the flow that move in a combination of self-induced motions and externally-commanded motions.

2.2.2 Mode Shape Splining

To enforce the boundary conditions in Equation 2.24, mode shape motion information at each of the aerodynamic panels is required. A mode shape and its derivative ($\phi_i, \frac{\partial \phi_i}{\partial x}$), obtained from ground vibration testing or finite element modeling, have to be interpolated onto the aerodynamic mesh.

For interpolation of the modes ϕ_i , in a way similar to NASTRAN, an in-house infinite plane spline method (IPS) was used due to its superior interpolation capability compared to polynomial-based splining [54].

In essence, the finite element/ground vibration mode shape is treated as the deflection of an infinite plate. Then, the forces, $\bar{P}_i$, and boundary conditions, a_0 , a_1 and a_2 , that would need to be applied at each of the ground vibration test/finite element mesh points to cause the mode shape deflection at these points, $\phi(x_{FE/GVT}, y_{FE/GVT})$, are computed. The computed forces are then used to calculate the displacement at the points that correspond to the aerodynamic mesh $\phi(x_{aero}, y_{aero})$. To do so, the following force $\bar{P}_i$ -deflection W_j relationship in Equations 2.27, 2.28, 2.29 and 2.30 of an infinite plate [54]:

$$\sum \bar{P}_i = 0 \quad (2.27)$$

$$\sum x_i \bar{P}_i = 0 \quad (2.28)$$

$$\sum y_i \bar{P}_i = 0 \quad (2.29)$$

$$W_j = a_0 + a_1 x_j + a_2 y_j + \sum_i \bar{P}_i \cdot r_{ij}^2 \cdot \ln(r_{ij}). \quad (2.30)$$

are used twice. Although physical arguments were used to derive the IPS method, it can be viewed, in essence, as a radial base function (RBF) method with the kernel function $r^2 \ln(r)$.

For interpolating the derivative $(\phi_i, \frac{\partial \phi_i}{\partial x})$, the IPS method can be problematic and lead to singularities when points on the GVT/FE and aerodynamic mesh coincide. Simple linear interpolation is used in such cases.

2.2.3 Aeroservoelastic State-Space Formulation

The governing equation of an AE system excited by self-induced aeroelastic forces, control surface, and gust vane inputs is provided in Equation 2.31 [55]:

$$[s^2 M_{hh} + s C_{hh} + K_{hh}] \{\xi\} - \bar{q} [A_{hh}(s) \{\xi\} - A_{hc}(s) \{\delta_c\} - A_{hg}(s) \{\delta_g\}] = 0 \quad (2.31)$$

where Equation 2.31 describes a structural system with modal coordinates $\{\xi\}$ forced by aerodynamic terms that correspond to self-induced ($A_{hh}\{\xi\}$), control surface input ($A_{hc}\{\delta_c\}$) and gust vane input ($A_{hg}\{\delta_g\}$) aerodynamics. By assuming that the control surface positions are the input rather than the commands to the actuators (and actual control surface motions are measured in addition to the commands to the actuators), an irreversible control surface model can be used in the AE system identification process. In other words, the input (which is the control surface position) is fully known during the AE system identification process. It is also assumed, as an approximation, that the control surfaces are mass balanced or, alternatively, that the mass unbalance of the control surfaces is negligible, especially at higher wind tunnel speeds.

The modal mass, damping and stiffness matrices are normalized as in Equation 2.32:

$$M_{hh} = I, \quad C_{hh} = \begin{bmatrix} 2\zeta_1\omega_1 & & \\ & \ddots & \\ & & 2\zeta_m\omega_m \end{bmatrix}, \quad K_{hh} = \begin{bmatrix} \omega_1^2 & & \\ & \ddots & \\ & & \omega_m^2 \end{bmatrix}. \quad (2.32)$$

The generalized aerodynamic forces A_{hh} , A_{hc} and A_{hg} calculated in the frequency domain by DLM are approximated in the time-domain using the Roger Rational Function Approximation

(RFA) leading to the time-domain version of the governing AE equation (Equation 2.33):

$$[s^2 M_{hh} + s C_{hh} + K_{hh}]\{\xi\} - \bar{q} \left[P_0 + s P_1 + s^2 P_2 + \sum_{b=1}^n \frac{s}{s + \beta_b} \cdot P_{b+2} \right] \begin{bmatrix} \{\xi\} \\ \{\delta_c\} \\ \{\delta_g\} \end{bmatrix} = \{0\}. \quad (2.33)$$

The measurements of strain and pitch angle are given by the sum of contributions from different modes as expressed in Equation 2.34:

$$\theta = [\theta^{(1)} \quad \theta^{(2)} \quad \dots \quad \theta^{(m)}]\{\xi\} \quad \text{and} \quad \epsilon_k = [\epsilon_k^{(1)} \quad \epsilon_k^{(2)} \quad \dots \quad \epsilon_k^{(m)}]\{\xi\}. \quad (2.34)$$

The measurement by an accelerometer placed on the structure is given by the contributions of accelerations from the individual modes and the effect of gravity, which depends on the local surface normal, hence the modes. For a vertically hanging wing with spanwise coordinate y' , the acceleration at a point ' k ' is given by Equation 2.35:

$$a_k = [\phi_{1k} \quad \phi_{2k} \quad \dots \quad \phi_{mk}]s^2\{\xi\} - \bar{g} \cdot \left[\frac{d\phi_{1k}}{dy'} \quad \frac{d\phi_{2k}}{dy'} \quad \dots \quad \frac{d\phi_{mk}}{dy'} \right]\{\xi\}. \quad (2.35)$$

2.2.4 Flutter Determination Using the V-g Method

The V-g method is used to determine the flutter speed and frequency. The control surface GAFs are removed from the equation. For the flutter analysis of MARGE II the control surfaces are considered locked and irreversible. Removing the control surface and gust vane aerodynamic contributions and assuming hysteretic damping g_{st_i} leads to Equation 2.36:

$$[-\omega^2 M_{ss} + (1 + jg) \begin{bmatrix} 1 + jg_{st_1} & & \\ & \ddots & \\ & & 1 + jg_{st_n} \end{bmatrix} K_{ss} - \frac{1}{2}\rho U^2 A_{hh}(jk)]\{\xi\} = 0 \quad (2.36)$$

where g is an artificial “dummy” structural damping added mathematically to force the system into simple harmonic motion. Dividing Equation 2.36 by ω and substituting $\frac{U^2}{\omega^2} = \frac{b^2}{k^2}$, the generalized eigenvalue problem described in Equations 2.37, 2.38, 2.39 and 2.40 is obtained:

$$0 = [\bar{R} + \lambda \bar{S}] \{\xi\} \quad (2.37)$$

$$\bar{R} = -M - \frac{1}{2} \frac{b^2}{k^2} \rho U^2 A_{hh}(jk) \quad (2.38)$$

$$\lambda = \frac{1 + jg}{\omega^2} \quad (2.39)$$

$$\bar{S} = \begin{bmatrix} 1 + jg_{st_1} & & \\ & \ddots & \\ & & 1 + jg_{st_n} \end{bmatrix} K \quad (2.40)$$

where $\bar{R}$ and $\bar{S}$ are the generalized eigenvalue problem matrices and λ is the generalized eigenvalue. Solving the eigenvalue λ for a range of reduced frequencies (that cover the flight envelope), the corresponding natural frequency ω , structural damping g and speed U are obtained through Equation 2.41:

$$\omega = \sqrt{Re(\lambda)}, \quad g = \frac{Im(\lambda)}{Re(\lambda)}, \quad U = \frac{\omega b}{k}. \quad (2.41)$$

In the V-g analysis, branches of dummy damping versus speed and frequencies versus speed are tracked to determine the lowest speed at which any “dummy” damping value would cross from negative to positive. The dummy damping cross-over speed is the predicted flutter speed of the system (see a detailed discussion in Ref. [45]).

2.3 Frequency-Domain System Identification

Treating the MARGE II aeroelastic system as a quasi-linear system under small control surface and gust vane inputs and small angles of attack, the system’s frequency response in the wind tunnel and during the modal test $H(f)$ can be estimated. The theory of frequency response function

estimation and input design below is summarized from Ref. [30].

2.3.1 Frequency Response Estimation

To calculate the frequency response of the system $H(f)$, the inputs and outputs of the system are recorded and are Fourier-transformed using the Discrete Fourier Transform (DFT) technique. The DFT of the input and output are denoted $\hat{X}$ and $\hat{Y}$ respectively.

Although multiple methods for evaluating the DFT exist (standard FFT, Zoom FFT, etc.), the Chirp-z transform FFT is preferred in this work to evaluate $\hat{X}$ and $\hat{Y}$. The Chirp z-transform is generally advantageous compared to the regular FFT for aeroelastic system identification. Firstly, when using a window of time T_{win} and n_f samples per window, the standard FFT creates a frequency grid with intervals $\Delta f = \frac{1}{T_{win}}$ in the interval $f = \{\frac{1}{T_{win}}, \frac{n_f}{T_{win}}\}$. So, the standard FFT gives frequency information over a large frequency interval but with coarse frequency grid Δf . For aeroelastic systems, the frequency ranges of interest during identification are low frequencies. However, low frequency ranges must be identified with high frequency resolution near the modal peaks, especially for modes that are not well damped. The Chirp z-transform allows one to select a smaller frequency range by distributing the n_f -point frequency grid over this smaller range resulting in a higher frequency resolution without having to increase the number of samples in the window.

Once $\hat{X}$ and $\hat{Y}$ are calculated, the auto and cross power spectral density (PSD) estimates of the input and output are computed as shown in Equation 2.42:

$$\hat{S}_{xx} = \hat{X}^* \hat{X} \quad \hat{S}_{xy} = \hat{X}^* \hat{Y} \quad \hat{S}_{yy} = \hat{Y}^* \hat{Y}. \quad (2.42)$$

Having computed the PSD estimates using the recorded input and output signals, the frequency response of the system $H(f)$ can finally be estimated. The frequency response $H(f)$ of a linear system can be calculated with either the H_1 or H_2 estimator [30]. H_1 and H_2 estimators both use

the auto and cross PSD of the input and the output signals to approximate true frequency response of the system $H(f)$. Under no noise, both estimators will result in the same $H(f)$. Depending on whether the system process or measurement noise is greater, H_1 or H_2 will result in a more accurate estimate of $H(f)$. It is common in aircraft system identification to assume that the control surface command input is noise-free. Hence, the H_1 estimator was used for the work described here. H_1 is defined in Equation 2.43 as the ratio between the cross PSD estimate $\hat{S}_{xy}$ and the auto PSD estimate of the input signal $\hat{S}_{xx}$:

$$H_1(f) = \frac{\hat{S}_{xy}}{\hat{S}_{xx}}. \quad (2.43)$$

When there is no input (process) noise, the auto and cross PSD estimates equal the true PSDs. In other words, $\hat{S}_{xy} = S_{xy}$ and $\hat{S}_{xx} = S_{xx}$. Hence, under no process noise, the H_1 estimator becomes equal to the true frequency response of the system, $H(f)$ as shown in Equation 2.44 [30]:

$$H_1(f) = \frac{\hat{S}_{xy}}{\hat{S}_{xx}} = \frac{S_{xy}}{S_{xx}} = H(f). \quad (2.44)$$

However, if there is process noise $U(f)$ with process noise auto PSD S_{uu} so that $\hat{S}_{xx} = S_{xx} + S_{uu}$, the H_1 estimate no longer equals the $H(f)$ as shown in Equation 2.45:

$$H_1(f) = \frac{\hat{S}_{xy}}{\hat{S}_{xx}} = \frac{S_{xy}}{S_{xx} + S_{uu}} < \frac{S_{xy}}{S_{xx}} = H(f). \quad (2.45)$$

Consequently, the H_1 estimator underestimates the true frequency response function $H(f)$ when process noise is non-zero.

The quality of the H_1 estimate can be evaluated using the coherence metric γ_{xy}^2 based on the auto PSD of the output signal $S_{yy}(f)$ and output noise $S_{vv}(f)$. The definition of the coherence

metric is provided in Equation 2.46 [56]:

$$\gamma_{xy}^2(f) = \frac{1}{1 + \frac{S_{vv}(f)}{S_{yy}(f)}}. \quad (2.46)$$

The coherence is a measure of the extent to which the output noise degrades the measured output quality. Other factors that contaminate the frequency response function estimate such as nonlinearities and process noise (filtered through system dynamics) are represented as effective measurement noise in Equation 2.46. In practice, the quality of the frequency response functions is evaluated using the coherence function estimate $\hat{\gamma}_{xy}^2$ based on the PSD estimates as in Equation 2.47:

$$\hat{\gamma}_{xy}^2 = \frac{|\hat{S}_{xy}|^2}{|\hat{S}_{xx}||\hat{S}_{yy}|}. \quad (2.47)$$

It is possible to verify that when process noise is set to 0 (as the effect of process noise filtered through dynamics is viewed as measurement noise) and substitute $\hat{S}_{xy} = S_{xy} = (HS_{xx})^2$ and $\hat{S}_{yy} = S_{yy} = H^2S_{xx}$, the coherence and its estimate in Equations 2.46 and 2.47 are equivalent.

2.3.2 Input Design for System Identification

The goal of the control surface and gust vane input design for system ID tests is to obtain a high quality and high coherence frequency response function estimate. For frequency response identification, it is common to use frequency-varying sweeps (chirps). The sweep input and data collection must meet the following criteria [30]:

- The sweep must cover the range of frequencies of interest. Hence, one or multiple sweeps must cover the full range of frequencies in which the system is to be identified.
- The frequency increase during the sweep must be smooth.
- Data collected must include the response of the system at trim condition (at least 3 seconds

before and after the sweep). The trim data allows the response to be “demeaned” cleanly and the trim condition to be removed.

- The sweep input must be repeated at least twice for noise averaging.
- The amplitude of the sweep should be high to increase the response signal-to-noise ratio but not overstress the structure or lead to a response that is large enough to introduce significant nonlinear aerodynamic effects.
- Although a good guideline for sweep time is $T_{rec} > \frac{5}{f_{min}}$ (sweep time must be larger than 5 times the period corresponding to the lowest frequency contained in the sweep), this time must be further extended for identification of structural dynamics to have a sufficiently small frequency grid and high resolution to capture modal peaks that are not well-damped.

Chapter 3

MARGE II Model and Experimental Set-ups

In this chapter, the experimental set up used for the wind tunnel system identification of MARGE II and modal testing of MARGE II is presented. The wind tunnel test set-up is presented first, in Section 3.1. The DIC-based modal testing set-up is presented in Section 3.2.

3.1 MARGE II Wind Tunnel System Identification Set-up

In this section, the hardware and design of MARGE II is discussed. For the purpose of brevity, the discussion of the experimental hardware is kept short. Readers who are interested in learning more about the design, sizing, and hardware of MARGE II are referred to [57].

MARGE II (shown in Fig. 3.1) was built based on the design of MARGE I [58]. Similar to MARGE I, MARGE II is a flexible half span aircraft model that is designed to be mounted in the University of Washington's 3x3 low speed wind tunnel. MARGE II is free to rotate in pitch and can, therefore, emulate the short-period mode of real aircraft.

The wing spar, tail spar, and the fuselage of the model are made of Aluminium and Low

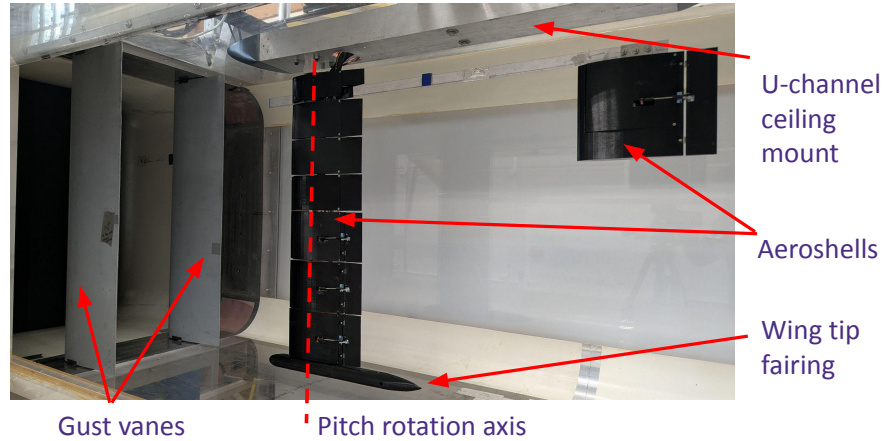


Figure 3.1: MARGE II in the 3x3 wind tunnel

Carbon Steel plates, respectively. The aerodynamic shapes of the wing and tail are given by 3-D printed ABS aeroshells that form a NACA 0012 profile around the wing and tail spar. There are six aeroshells along the span of the wing. The aeroshells are hollow and are attached to the wing and tail spar at only two points as to introduce minimal stiffness. The low stiffness of the aeroshells allows for a simplified FE model that does not need to include the aeroshells for its modeling of the stiffness of the structure. The wing also includes a cylindrical brass tip mass, used to lower the structural frequencies and include more modes within the bandwidth of the servo actuators. The tip mass is housed in the 3-D printed wing tip fairing.

MARGE II features complex actuation capabilities including three servo-actuated ailerons and an elevator, with a -3dB bandwidth of 18 Hz, range of ± 20 deg and freeplay of 0.5 deg. In addition to the control surfaces on MARGE II, there are gust vanes that sit upstream of MARGE II in the wind tunnel, capable of exciting the MARGE II model with gusts of any desired shape, including random gusts, up to 15 Hz. MARGE II also houses a plethora of sensors: three accelerometers, five strain gages, a hall effect sensor that measures the model root pitch angle, and four potentiometers that measure the position of the control surfaces. The locations of the control surfaces and sensors are shown in Figure 3.2.

The locations of the accelerometers and strain gages were finalized based on the FE analysis

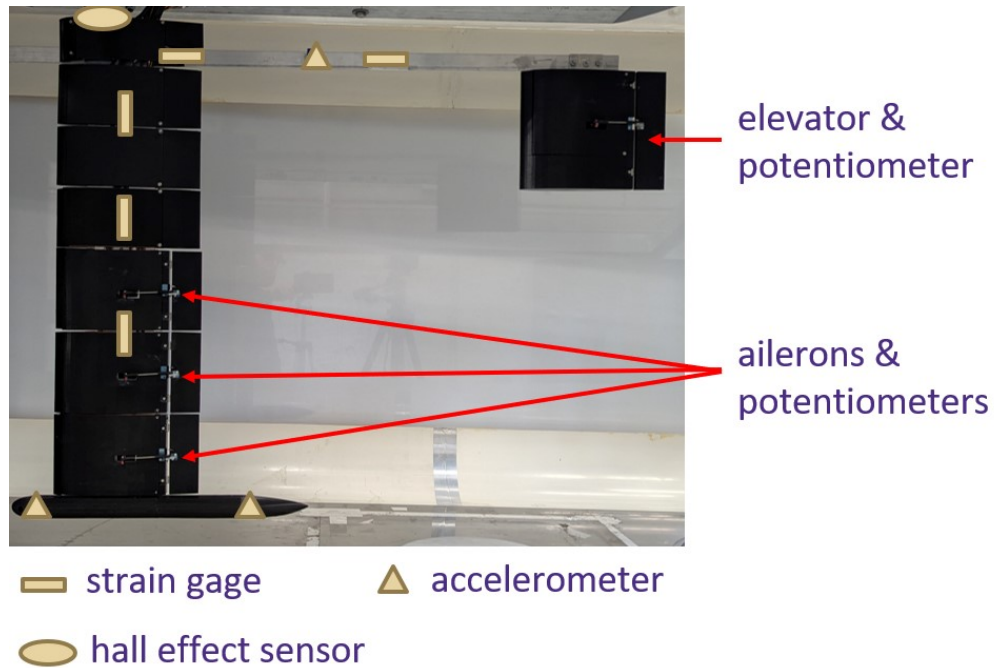


Figure 3.2: MARGE II sensors and control surfaces

of the model. In particular, the fuselage accelerometer was placed close to the antinode of the fuselage bending mode, and strain gauges were distributed throughout the wing to increase the observability of not only the first wing bending, but higher order wing modes that fall within the control bandwidth. The strain gauges were positioned away from locations of intense stress concentrations (such as cutouts, wing root lap joint, and spar-aeroshell connections) that would require a very detailed FE mesh to model. The measurements of the control surface position by the potentiometers allow for bare-airframe system identification of the model. That is, the potentiometer measurement capability allows identification of the “pure” aeroelastic model (AE) without its actuators.

All signals from the sensors were filtered and amplified through a signal conditioning board that includes a 2nd order RC filter with a 40 Hz cut-off. The signals are then sampled at a rate of 500 Hz.

Simulink was used as the common user interface to record and observe signals from all of the sensors in real time, and send commands to the gust vanes and control surfaces. The command and

sensing architecture is shown in Figure 3.3.

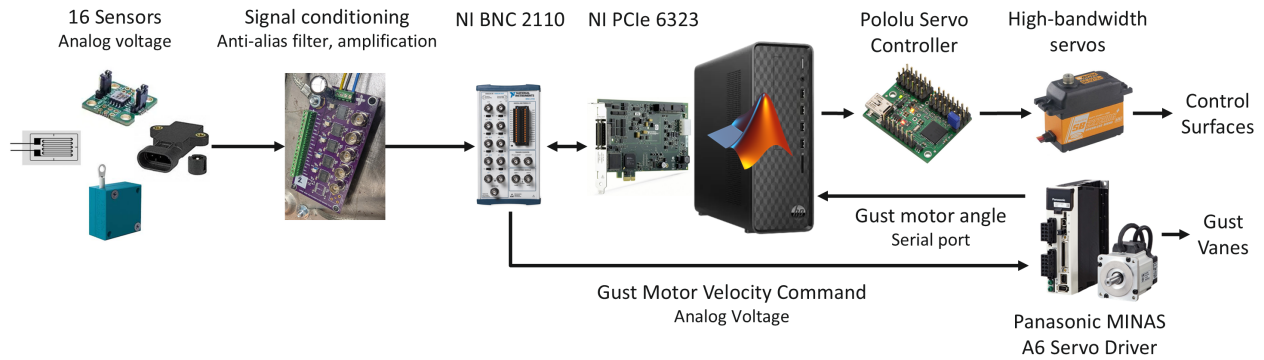


Figure 3.3: Real time sensing and actuation architecture, using a desktop PC with Simulink Desktop Real Time software (credit: Alexander F. LaFranchi)

3.2 Digital Image Correlation Ground Vibration Testing

Digital image correlation (DIC) is used to identify the mode shapes during ground modal testing. To do conduct the GVT, MARGE II is placed on a ground vibration test fixture in order to replicate the wind tunnel boundary conditions. The model was covered with speckle pattern stickers to enable performing DIC (see Figure 3.4).

The motion of the structure was recorded with both 100 and 200 fps rates by a 1052x800 pixel camera with a 12-bit depth. Because the images cannot be filtered in an analog way, higher frequency light noise sources may alias down to low frequencies, close to the modes of the structure. Comparing the mode shapes from 100 and 200 fps is used to reveal any aliasing artifacts. Studio lights were used to illuminate MARGE II's surface to make sure that the surface pattern is illuminated in an even way. Significant spatial variations in lighting could break the brightness-constancy assumption. The lighting and the speckle pattern was chosen to avoid both white and black camera saturation. White and black saturation in DIC can quintuple the noise in the measured displacements [59]. A speckle pattern with less contrast helped avoid saturation and push more of the observed pixels into the linear grayscale range of the camera (see Figure 3.5).

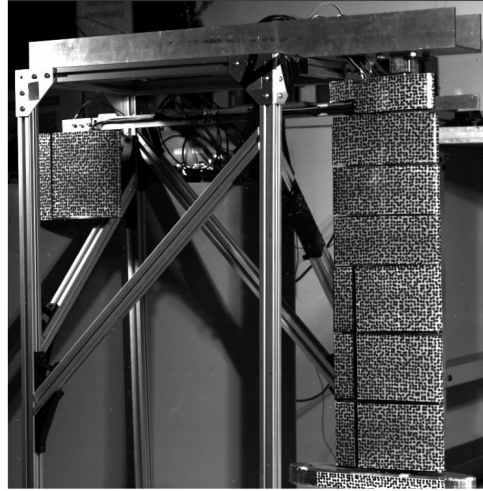


Figure 3.4: MARGE II covered with speckle patterns for Digital Image Correlation-based GVT

Figure 3.5 shows that the high contrast black speckles on black background led to white saturation while the pattern with less contrast resulted in a bimodal distribution (with each peak representing either the background or the speckles) within the linear grayscale range of the camera.

The Marge II structure was excited with a shaker from multiple points including the tail, fuselage and wing. Multiple points were used to excite the modes near their antinodes (that is, to put maximum energy into the structure in a way that would excite particular modes). Some of the points of excitation are shown in Figure 3.6.

While exciting the structure, the force sensor on the stinger tip (between shaker and structure) and the sensors on board MARGE II were recorded. Recording the force allows the creation of force-displacement frequency response functions. Some modal testing methods (such as Operational Modal Analysis) can recover the mode shapes without measuring the input force. However, as discussed earlier, the input force must be known in order to mass-normalize the modes.

The motion of the structure was recorded with one camera. Because only one camera was used, the 3D motion of the points on the structure could not be fully recovered. But with a small angle

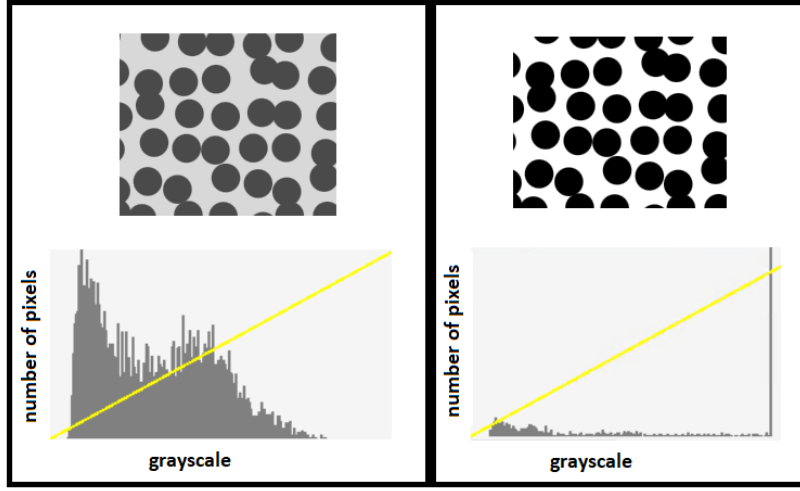


Figure 3.5: Different speckle patterns on MARGE II and the related grayscale intensity of a subset of pixels on MARGE II speckle pattern

assumption, it could be assumed that the deflection of the structure is normal to the plate surface. Figure 3.7 shows the angle between the assumed direction of the modes and the camera view plane.

Hence, the relationship between the motion of any point l viewed by the camera in the horizontal direction u_l and the deflection X_l is defined in Equation 3.1:

$$u_l = X_l \cdot \cos(\alpha) \quad (3.1)$$

where α is the angle shown in Figure 3.7. It is important to note that the camber curvature may distort the measured structural deflections. But such a distortion can easily be checked. If the same plunge and pitch angle are obtained at different chordwise locations at a given wing spanwise location, it could be said that the distortion due to camber is minimal. In other words, mode shapes should show no chordwise variation. Any chordwise variation is an indicator of distortion due to camber effects. In the case of high aspect ratio wings with relatively stiff chord, assuming no

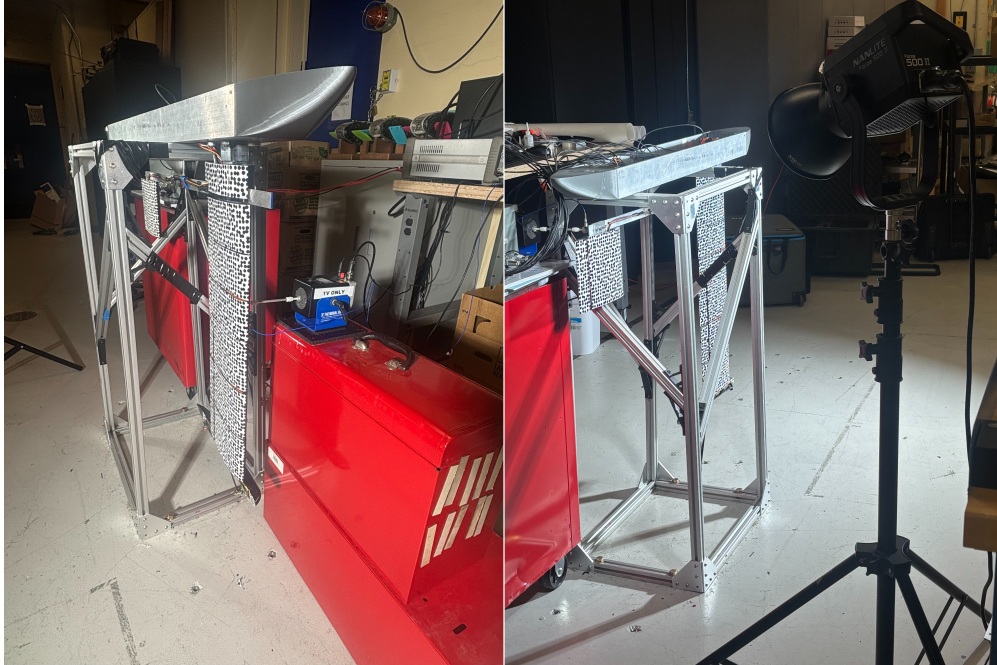


Figure 3.6: MARGE II excited from rear fuselage and wing

variation of the pitch angle along the chord is a valid and useful approach.



Figure 3.7: Top view of MARGE II and camera during ground vibration testing

Chapter 4

Aeroelastic Mathematical Model

This chapter focuses on the aeroelastic mathematical model. First, the mode shapes and frequencies created by FEA are shown in Section 4.1. The thesis then proceeds to comparing these frequencies and mode shapes to those obtained during DIC ground vibration testing in Section 4.2. Finally, details are provided on the aerodynamic modeling, required to obtain the aeroelastic state-space model. The details of the aeroelastic model are also summarized in Ref. [60].

4.1 Finite Element Structural Model

A finite element model of MARGE II was constructed in NASTRAN. Plate elements were used to model the wing spar, tail spar, and the fuselage. The aeroshells, servos, and electrical wires were modeled as concentrated mass, CONM2, elements with mass and rotational inertias. Since the servos “sit” in the cut-outs of the wing plate, each servo mass had to be connected to the structure so that it would move with the wing plate. RBE3 elements were used to for this purpose. Using the RBE3 elements helped avoid introducing artificial stiffness that would be created by the RBE2 element. Assuming no freeplay, the control surfaces were rigidly attached to the wing spar with RBE2 elements. The modeling assumptions resulted in a medium-fidelity finite element

model that is lower in detail than what would have been obtained by modeling the aeroshells in detail and using solid elements but higher in fidelity than typical stick-plate aeroelastic models because multiple plate elements were used in the chordwise direction. The FE simulation of the load carrying plates using plate elements made it possible to include in the FE model details of the cutouts in the plate structure into which actuators were inserted. The out-of-plane mode shapes resulting from the NASTRAN model are shown in Figure 4.1 with mode shape frequencies given in Table 4.1.

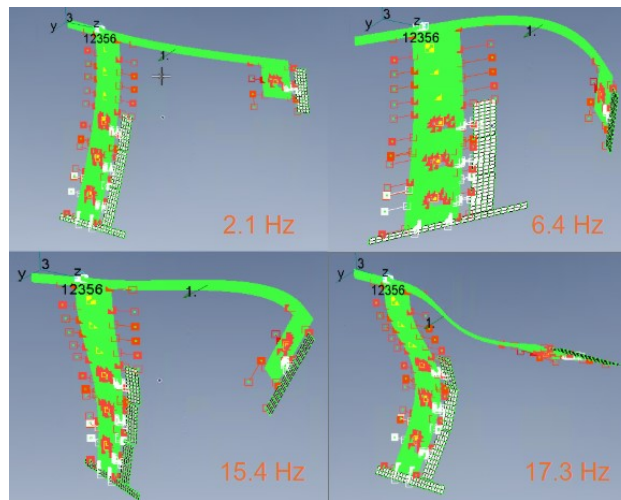


Figure 4.1: Nastran MARGE II mode shapes

Mode Shape	Frequency (Hz)
Rigid body pitch mode	0
First wing bending mode	2.1
Fuselage bending mode	6.4
Wing torsion mode	15.4
Second wing bending mode	17.3

Table 4.1: NASTRAN mode shapes and frequencies

Although the rigid body pitch mode was modeled as having 0 Hz natural frequency, in reality, due to the sensor cables, the pitch mode is not a perfect rigid body mode. Adding a torsional stiffness to the NASTRAN model to emulate the cable stiffness, however, did not lead to significant improvement in aeroelastic frequency response predictions.

Finally, strains corresponding to mode shape i (ϵ_i) at the locations of the strain gages were simply extracted using the modal curvature of the plates (κ_i) and their thickness (t) using the classic thin-plate theory Equation 4.1:

$$\epsilon_i = \frac{t\kappa_i}{2}. \quad (4.1)$$

4.2 DIC Structural Model

The DIC-based structural modes were extracted and mass-normalized as explained in Section 2.1. In order to do obtain the modes, good quality FRFs between the force input and the structure deflections had to be obtained. The objective was to identify all the modes up to 20 Hz. To satisfy this objective, 85 second linear chirps covering the 0.005-25Hz range were commanded to the shaker. Because of the limited RAM of the camera, there was an inherent trade-off between the number of chirp repeats, chirp duration and sampling rate. Sweeps shorter than 85 seconds resulted in poor coherence and identification quality in lower frequencies (<1.5 Hz). Such poor coherence in the <1.5 Hz frequency region is shown in Figure 4.2 where a 45 second sweep was used for identification.

A window of 5000 samples led to good coherence throughout the range of frequencies of interest. Lower window size led to issues near the modal peak of the 2.1 Hz low-damped wing bending mode (with damping ratio $\zeta = 0.013$) to capture the modal peak magnitude and damping accurately. Choosing a large enough window size to frequency response near modal frequencies accurately was of major importance for mass-normalizing the mode accurately. The frequency response functions for window sizes $N=2000$ and $N=5000$ are shown in Figure 4.3.

The chirp amplitude also had to be selected carefully. The voltage amplitude of the chirps was kept between 0.03-0.05 V. This range of amplitudes turned out to be the sweet-spot for modal testing. Although increasing the voltage amplitude increased signal-to-noise-ratio, further increases would have resulted in displacement amplitudes of more than 2 pixels for the structure. Large dis-

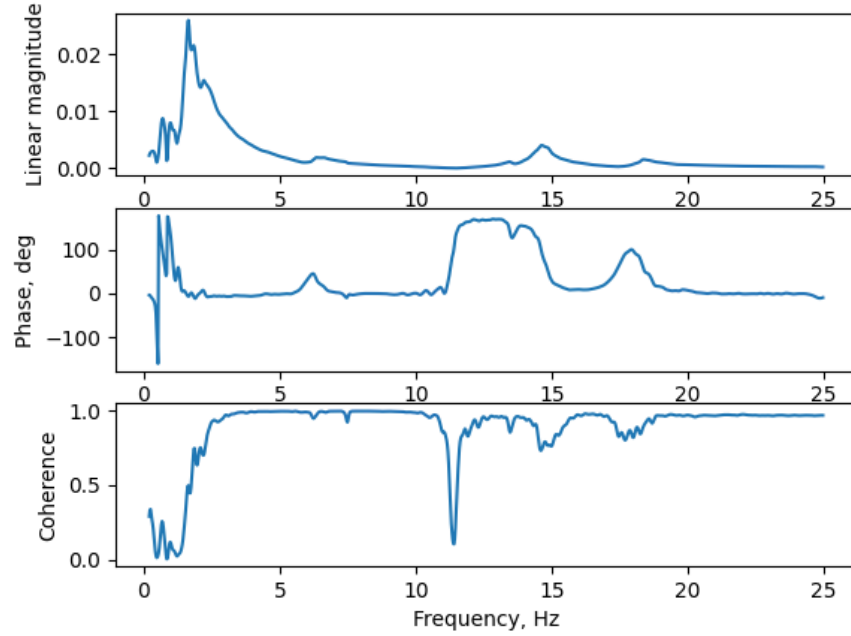


Figure 4.2: FRF obtained by placing shaker at the tail when excited by 45 second sweeps

placements are not calculated accurately by DIC due to the zeroth displacement calculation (that neglects the first order shearing of the pixel subsets and higher order effects).

The sweep amplitude, time and windowing considerations led to consistently high quality modal identification throughout the whole range of frequencies for most points. An example FRF (between mid-fuselage force excitation and an imaged point) is shown in Figure 4.4.

Figures 4.5 and 4.6 show side-by-side comparisons of the FEA and DIC-extracted wing bending and the fuselage modes. The figures show a good match between the two. In addition to the match in mode shapes, the difference in frequencies was less than 5% for both modes.

The mass normalization of the DIC modes was sensitive to the coherence of the measurements. The sensitivity is not surprising since while natural frequencies could be obtained quite accurately from frequency sweep FRF figures, the measurement of modal structural damping accurately is more challenging. But once good coherence was obtained, the mass-normaliation method resulted in mode shapes of similar amplitudes to FEA (both sets mass normalized). No further adjustments to these amplitudes was deemed necessary or led to better match of the aeroelastic model with

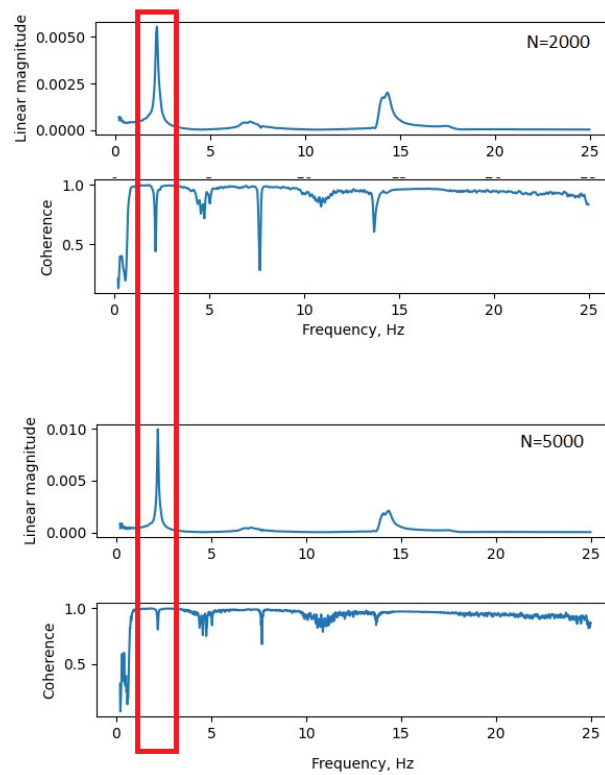


Figure 4.3: FRF with shaker placed at the wing, analyzed with window sizes $N=2000$ (upper plot) and 5000 (lower plot). The 2.1 Hz modal peak is encircled for emphasis.

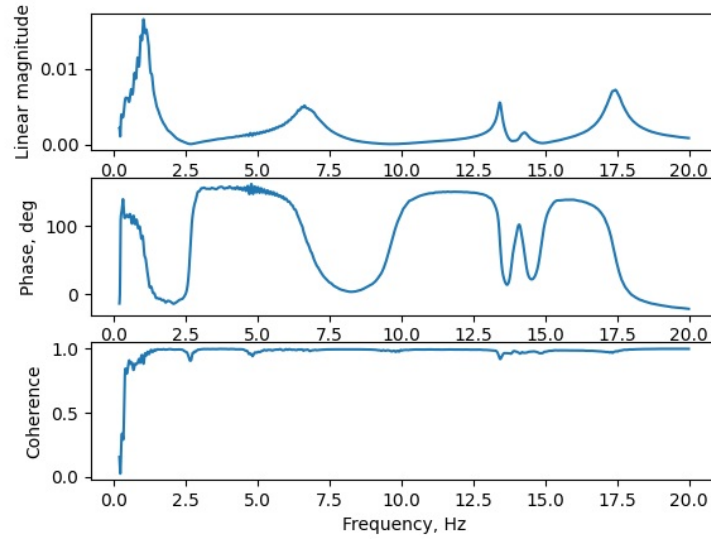


Figure 4.4: Receptance FRF of a tracked-point under mid-fuselage force excitation

wind tunnel system ID data. Because of the good coherence (high signal-to-noise ratio), except for the pitch mode shape, no smoothing had to be applied to the mode shapes. All mode shapes (other than the pitch mode) shown are unsmoothed. Smoothing of the flexible mode shapes did not improve the aeroelastic frequency response predictions.

One key aspect of this methodology is the importance of paying attention to the placement of the actuation force. When the excitation force is applied close to the node or nodal line of a mode, the mode would not be excited well. For example, the first wing bending mode shape shown in Figure 4.5 was obtained by applying the force excitation at the wing. The same mode shape (with a correctly mass-scaled amplitude) was still obtained when the structure was excited at the tail. However, when the structure was excited at middle of the fuselage (near the node of the wing bending mode), the mode was not excited sufficiently. In fact, the modal peak cannot even be seen in Figure 4.4. The excitation near the modal node resulted in the erroneous identification of the mode shape as shown in Figure 4.7.

The higher frequency modes (at and above 13 Hz) did not show a good match with the FE mode shapes in either frequency or shape. A possible reason for the mismatch at higher frequencies is

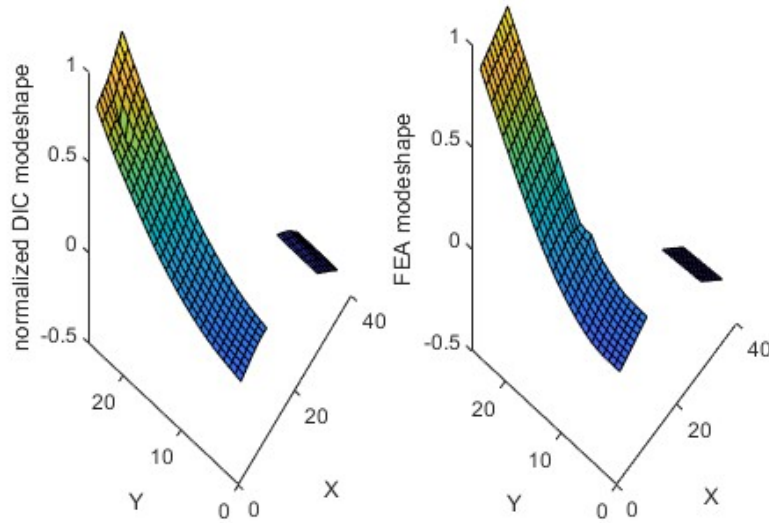


Figure 4.5: Wing bending normalized DIC vs. FEA modeshape

the lack of explicit aeroshells in the FEA model. The aeroshells were only included as lumped mass elements, without modeling their stiffness. However, in our case, errors in higher-frequency FE modes is not a major issue because MARGE II's current servos are not capable of controlling higher order modes due to their phase delay. In addition, the first three modes (that were modeled accurately by FEA) were sufficient for the flutter analysis to predict the flutter characteristics well. The higher order mode shapes and frequencies as determined by DIC are shown in Figure 4.8 with modal frequencies given in Table 4.2.

Mode Shape	Frequency (Hz)
Fuselage torsion	13.5
2nd Wing bending	14.3
Wing torsion	17.4

Table 4.2: DIC higher order mode shapes and frequencies

Although the operating deflection shape at the modal frequency was assumed to be a good

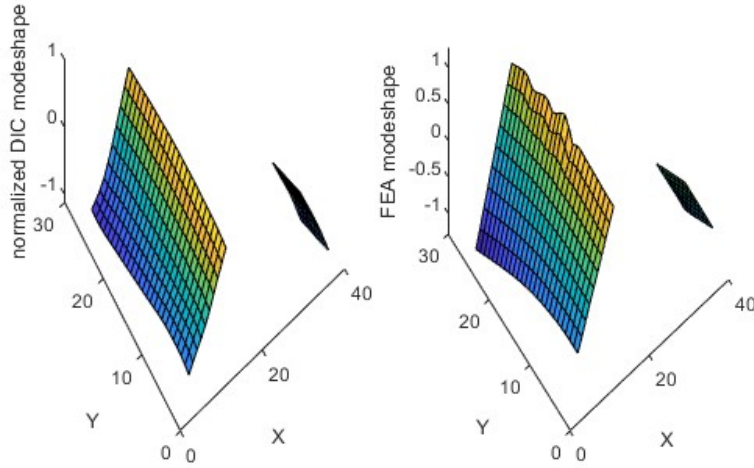


Figure 4.6: Fuselage bending normalized DIC vs. FEA modeshape

approximation of the mode shape, this assumption had to be validated, especially in the case of the out-of-plane fuselage bending mode (at 6.65 Hz) where a nearby in-plane fuselage-bending mode at 7.5 Hz exists. With two modes close in frequency, the fact that the 7.5 Hz mode has minimal impact on the ODS at 6.65 Hz had to be validated in order ensure that the 6.65 Hz mode could be accurately obtained from the frequency response at 6.65 Hz. The shape of the in-plane mode determined by FEA is shown in Figure 4.9:

The in-plane mode caused a projection onto the x-axis of the camera lens and was picked up by the DIC method. To ensure that this in-plane mode did not lead to a large inaccuracy in the identification of the 6.65 Hz out-of-plane (hence aeroelastically important) fuselage bending mode, the mode shapes were verified by fitting polynomial functions to each of the FRFs (using the pyEMA Python package [61]) to recover the mode shapes as shown in Figure 4.10.

The out-of-plane fuselage bending determined by using the pyEMA tool, that takes into account the coalescence effect from the nearby 7.5 Hz mode, did not show significant differences compared to the ODS at the 6.65 Hz modal peak.

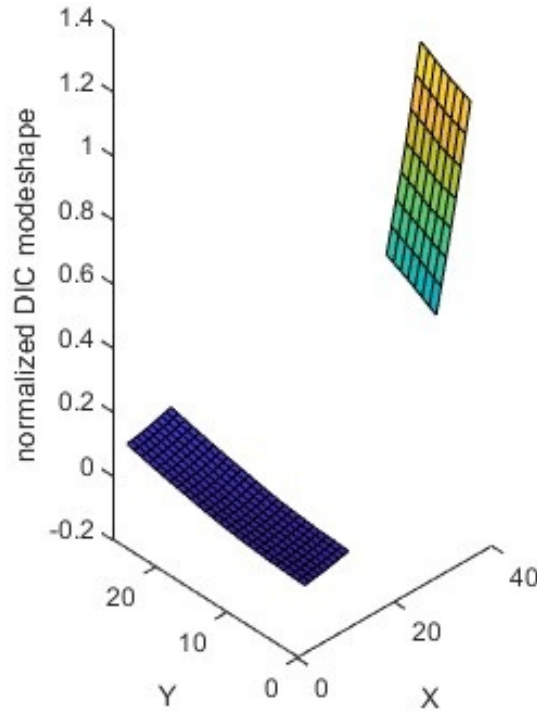


Figure 4.7: Receptance FRF of a tracked-point under mid-fuselage force excitation

In general, when using one shaker or a small number of shakers, it can be difficult to bring the structure to oscillate in a pure natural mode shape even when the excitation is at exactly the natural frequency. In the one-shaker DIC based technique presented here, the structure could be brought to shake at natural frequencies in shapes that were close to the corresponding natural mode shape but not perfectly so. The excited motion at the natural frequencies was, then, complex in nature. Still, the modes determined by the pyEMA tool revealed that more than 94% of the magnitude of all of the points in every mode was in-phase with each other. In other words, all of modes were nearly real (only less than 6% of magnitude of each the mode shape was imaginary) indicating that mode shapes were identified with good accuracy.

With more shakers distributed and coordinated properly, the identification of the ‘pure’ modes

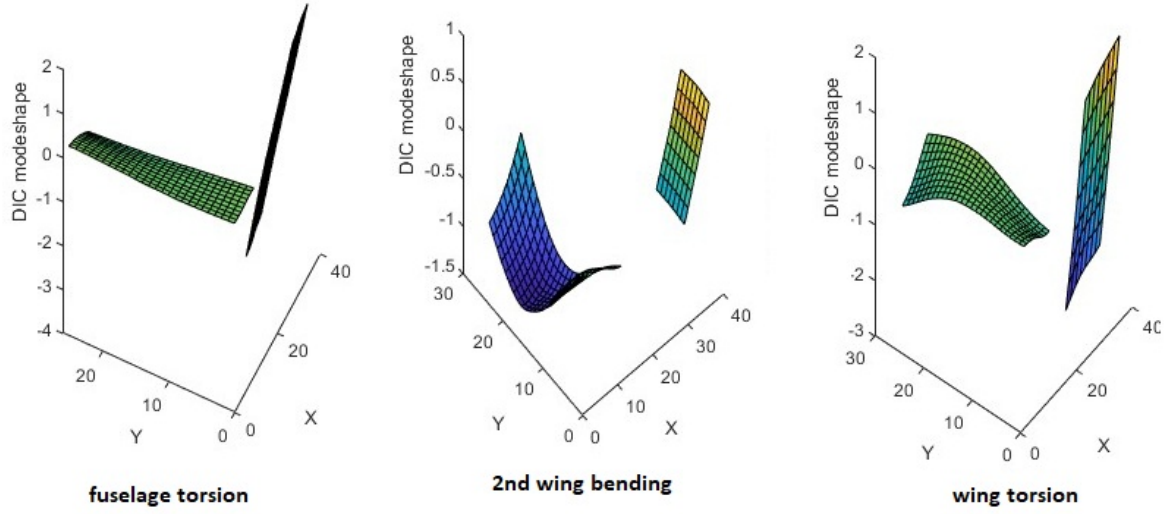


Figure 4.8: DIC-measured higher order modes

can be improved. As a matter of fact, it was quite common in ground vibration tests of the 1960s and 1970s to use arrays of shakers to excite the structure so that it would shake in pure modes. Consider the structural dynamic equations of motion for a structure with mass M , damping C and stiffness K excited at a natural frequency (ω_n) excited by force $F(j\omega_n)$ as shown in Equation 4.2:

$$[-\omega_n^2 M + j\omega_n C + K] \{X(j\omega_n)\} e^{j\omega_n t} = \{F(j\omega_n)\} e^{j\omega_n t}. \quad (4.2)$$

Then, the excitation forces can be distributed as described in Equation 4.3 such that the excitation force cancels out the damping term:

$$\{F(j\omega_n)\} = [j\omega_n C] \{X(j\omega_n)\}. \quad (4.3)$$

As a result of the cancellation of damping by the force input, the structure is left to move according to Equation 4.4:

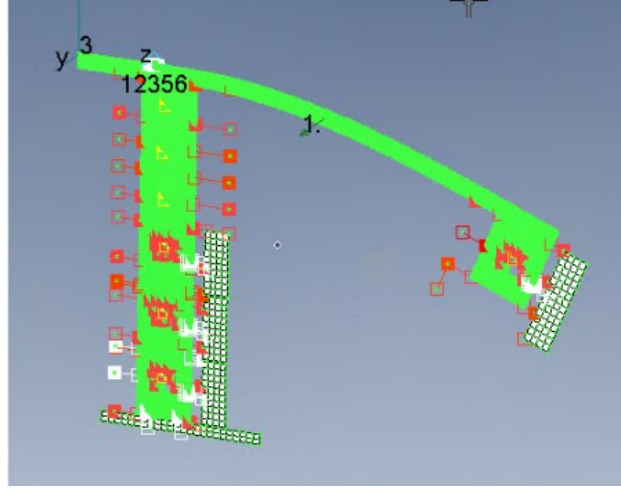


Figure 4.9: In-plane fuselage bending mode at 7.5 Hz

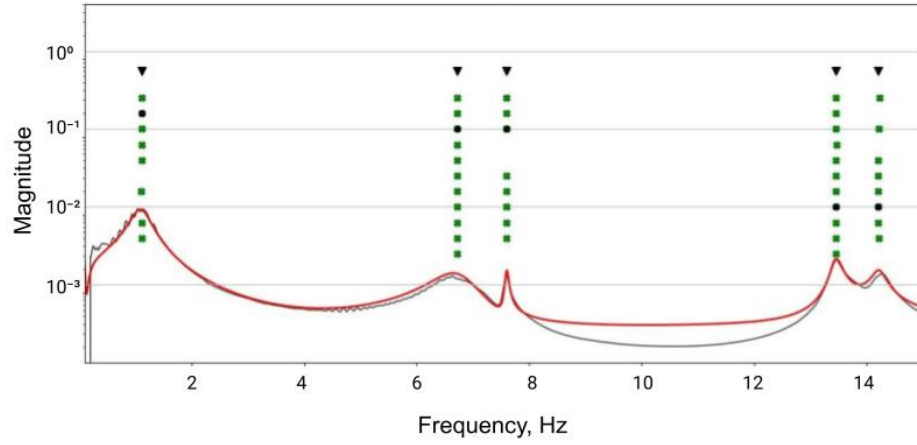


Figure 4.10: Polynomial fitting of frequency response functions to determine the mode shapes

$$[-\omega_n^2 M + K] \{X(j\omega_n)\} = \{0\}. \quad (4.4)$$

Thus, the vector of displacements would be real, all points would move in phase, and the structure would move in the ‘pure’ mode shape corresponding to ω_n . This method of identifying the ‘pure’ modes applies to damping forces of any type, where canceling them by the correct distribution of external forces would lead to the oscillation of the structure in pure natural mode shapes.

4.3 Aerodynamic Model

The DLM mesh of the wind tunnel walls, gust vanes, model's wing, tail and wing tip fairing is shown in Figure 4.11.

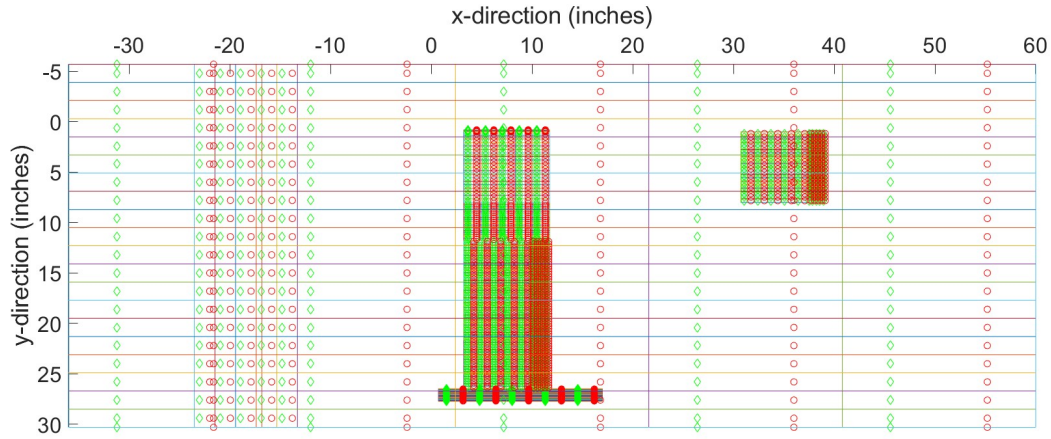


Figure 4.11: Side View of the DLM mesh showing gust vanes, wind tunnel walls, wing, tip fairing, and tail

As shown in Figure 4.11, all DLM boxes were kept under an aspect ratio of three as required to achieve good accuracy using the 1971 DLM implementation [51]. The chordwise edges of all boxes at the spanwise locations were aligned to avoid singularities [62]. The effect of mesh refinement was also studied. Firstly, Figure 4.11 shows a rather coarse meshing of the wind tunnel walls but further refinement was not necessary because DLM modeling with and without the walls showed a very small difference in the GAFs. Secondly, the effect of mesh refinement was studied on the ailerons. As shown in Figure 4.11, each aileron was nominally modeled with five chordwise and 20 spanwise boxes. Figure 4.12 shows the comparison between GAFs obtained by five versus ten chordwise aileron boxes. No significant difference in the GAFs was observed, showing that the DLM-scheme has converged, even at high reduced frequencies that often require more boxes for

correct modeling [51].

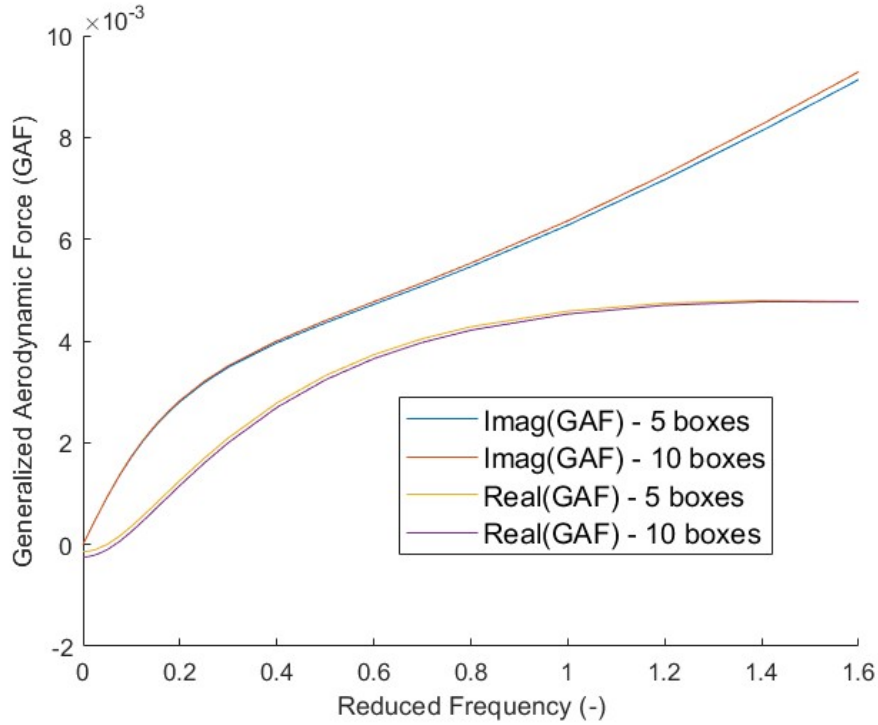


Figure 4.12: Aerodynamic work done by outboard aileron on fuselage bending mode under different number of chordwise aileron DLM boxes

The different parts of the wing, tail, and control surfaces were splined separately to ensure accuracy during FEA/DLM mesh mode-shape splining. Excellent splining accuracy was achieved for all mode shapes as shown in Figure 4.13.

Finally, the GAFs calculated by the DLM were brought to the time domain with the Roger rational function approximation (RFA) of four lag roots ($\beta = 0.2, 0.4, 0.6, 0.9$). Four lag roots led to a good match between all frequency-domain GAFs and time-domain RFAs for the range of reduced of frequencies of interest ($k \leq 1.2$). An example of the match between the frequency-domain gust GAFs the time-domain gust RFA is shown in Figure 4.14).

Because the gusts in the 3x3 tunnel are created by the test section entrance gust vanes which are modeled as additional control surfaces, one may expect the resulting generalized aerodynamic forces (GAFs) to be, as functions of reduced frequency, similar to the GAFS of the other control

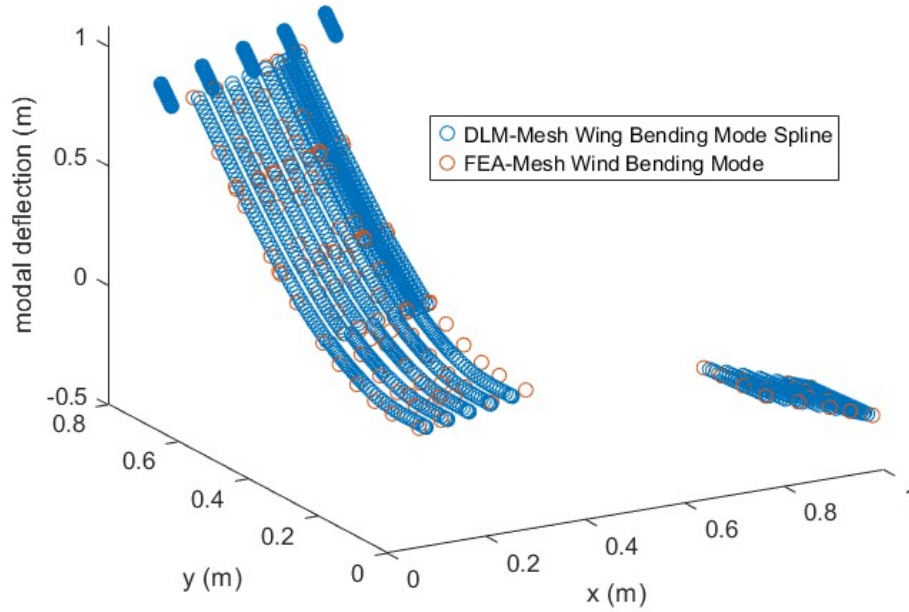


Figure 4.13: Splined (on DLM mesh) vs. FEA-mesh wing bending mode shape

surfaces. But that is not the case. The gust vane created GAFs are found to behave as functions of reduced frequency similar to the classic behavior of gust GAFs that are captured mathematically by Sears Function type behavior. The gust field is convected by the tunnel and is hitting the different parts of the model with delay. This delay is what gives the characteristic spiral shape and is what makes it difficult to approximate the gust response with RFAs [43]. This complexity has driven engineers to create separate RFAs for gust (with more lag roots) and control surfaces or adopt identification/reduction based methods such Loewner's framework [43] to model gust response in the time domain. In our case, four lag roots made it possible to capture this reduced frequency complex dependence well.

It is also important to note that the Sears-like shape of the gust vane GAFs is a testament to the fact that gust vanes in wind tunnels lead to aerodynamics that simulate atmospheric turbulence well, making gust vanes a good choice as a test bed, even if they do not exactly replicate

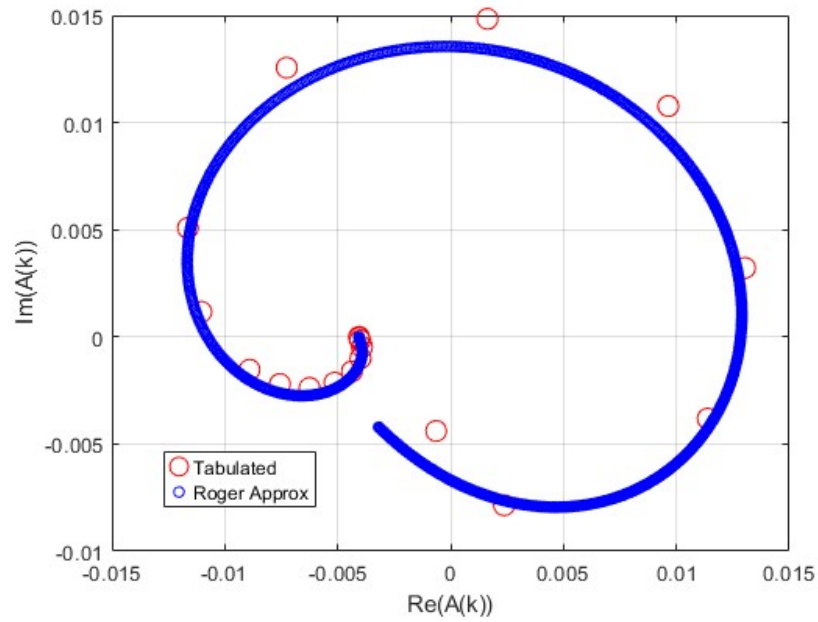


Figure 4.14: Comparison of RFA and DLM reduced frequency dependent predictions of gust vane aerodynamics in root locus form

atmospheric gust fields.

Chapter 5

Wind Tunnel System Identification

The objective of the chapter is to present the wind tunnel frequency response and flutter identification techniques and results. Section 5.1 describes the methods used to obtain good coherence and the inhibitors that lead to lower coherence. Section 5.2 presents the flutter testing process of MARGE II. Most of the results presented in this chapter are also presented Ref. [57]. It is also important to note that Alexander F. LaFranchi has been an equal contributor to the wind tunnel system identification of the model. Please see the Acknowledgements section for further specifics.

5.1 Gust Vane & Control Surface Frequency Response

MARGE II's frequency response was identified under both gust vane and control surface excitation in the wind tunnel. The gust vane system identification was conducted up to 15 Hz to abide by the Torque-Speed envelope of the gust motor. Control surface system identification was conducted to at least 20 Hz to determine the aeroelastic behavior of MARGE II at frequencies up to the control surface -3dB bandwidth. The identification was conducted at 200, 300, 400 and 500 Pa dynamic pressures. Identification below 200 Pa was not conducted due to the significant impact of the bearing friction and cable stiffness compared to aerodynamic forces at such low dynamic

pressures. Identification above 500 Pa was not conducted to avoid the 520 Pa flutter onset.

Identification was of the Single Input/Multiple Output (SIMO) type. Frequency sweeps were executed by each of the control surfaces and gust vanes individually, while data from all of the sensors were collected simultaneously. An example of a sweep (commanded versus measured by control surface potentiometer) is shown in Figure 5.1.

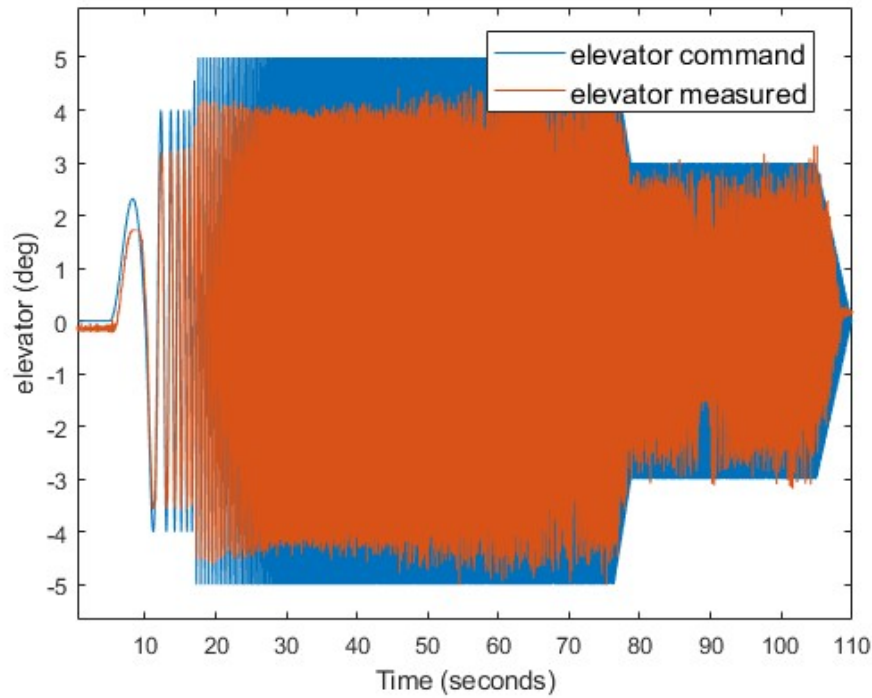


Figure 5.1: Elevator sweep 0.1-20 Hz at 400 Pa

As is the challenge with much of aeroelastic system identification, sufficiently exciting the system to obtain a good signal-to-noise ratio without introducing significant aerodynamic nonlinearities and overexciting the system was challenging which made the amplitude of the sweep an important design parameter.

The amplitude of sweeps used for identification was limited to 6 degrees as applying larger amplitude sweeps resulted in changes in frequency response data, indicating presence of significant aerodynamic nonlinearities. An example of a frequency response identified using sweeps of

different amplitudes is shown in Figure 5.2 where changing the sweep amplitude from 4 deg to 10 deg resulted in increase in coherence of the identification (due to the higher signal-to-noise ratio) but also resulted in significant phase shifting at frequencies greater than 15 Hz as a result of the aerodynamic nonlinearities.

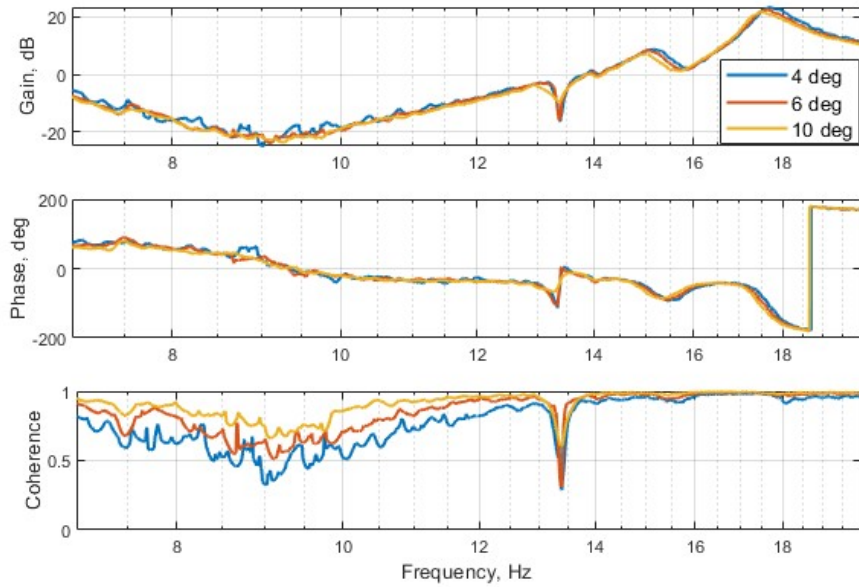


Figure 5.2: 200 Pa Elevator to Fuselage Aft Strain Gauge Frequency Response Functions at 4, 6, and 10 degree amplitudes

Up to 300 Pa, aerodynamic nonlinearities were the limiting constraint for the control surface sweep amplitude. At higher dynamic pressures, the elevator sweep amplitudes had to be further reduced down to 4 degrees to avoid overstressing the structure.

In addition to controlling the sweep amplitude as a whole, amplitude notching was used to control the excitation around certain frequencies. Notching was applied around the 2.1 Hz first wing bending mode to avoid overstressing the structure. The elevator input was also notched for frequencies below 1 Hz to avoid excessive pitch angles that could introduce large aerodynamic nonlinearities. The notching is shown in Figure 5.1 where the elevator sweep amplitude amplitude was lowered for the first 15 seconds while exciting the rigid body and first wing bending modes.

Sweeps applied for 95 seconds covering the full 0.1-20 Hz range led to frequency regions of poor coherence. To remedy this problem, “targeted” frequency sweeps covering smaller frequency ranges were conducted. These targeted sweeps were also 95 seconds long but covered smaller frequency ranges (0.1-5 Hz, 4.8-12.7 Hz and 12.5 - 20 Hz). Identification using targeted sweeps resulted in a significant increase in coherence. The small overlap in the frequency range of the targeted sweeps prevented coherence drops. The improvement in coherence is shown in Figure 5.3.

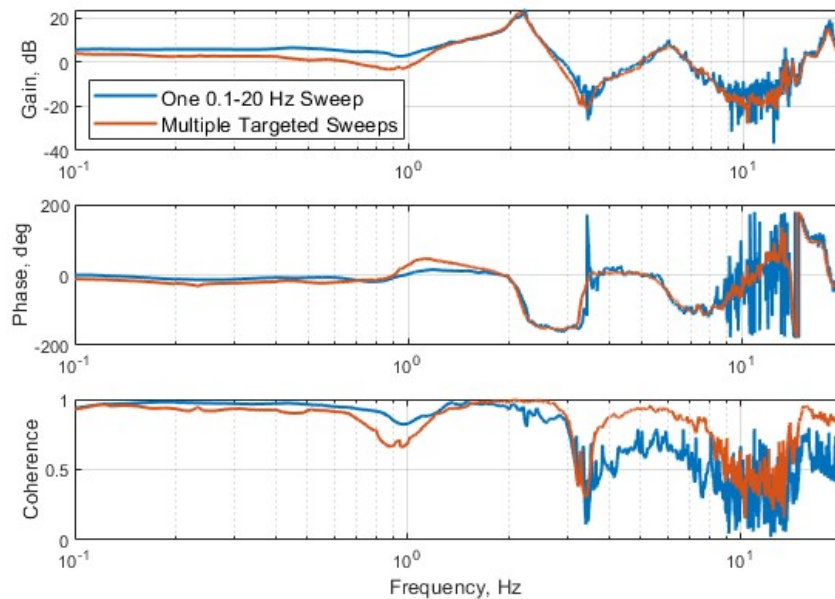


Figure 5.3: Midboard aileron to mid-fuselage strain gage FRFs at 300 Pa

Low coherence was encountered due to two reasons. Firstly, extreme-low gain regions in the input-output relationships placed the system dynamics close the noise floor. Extreme-low gain regions were common in the 7-12 Hz regions of different frequency responses because there are no modes with natural frequencies in the 7-12 Hz range. However, the exact nature and frequencies of the coherence drop manifested differently in different input-output relationships due to the changing nature of the controllability and observability when excited by different control/gust vanes and observed by different sensors. A second reason for low coherence was the low damping, observed

near certain modal peaks. Low damping near modal peaks led to both increased response to excitation by tunnel turbulence and a persistent (slow-decaying) transient response. Both the persistent transient response and noise excitation contaminated the frequency response data.

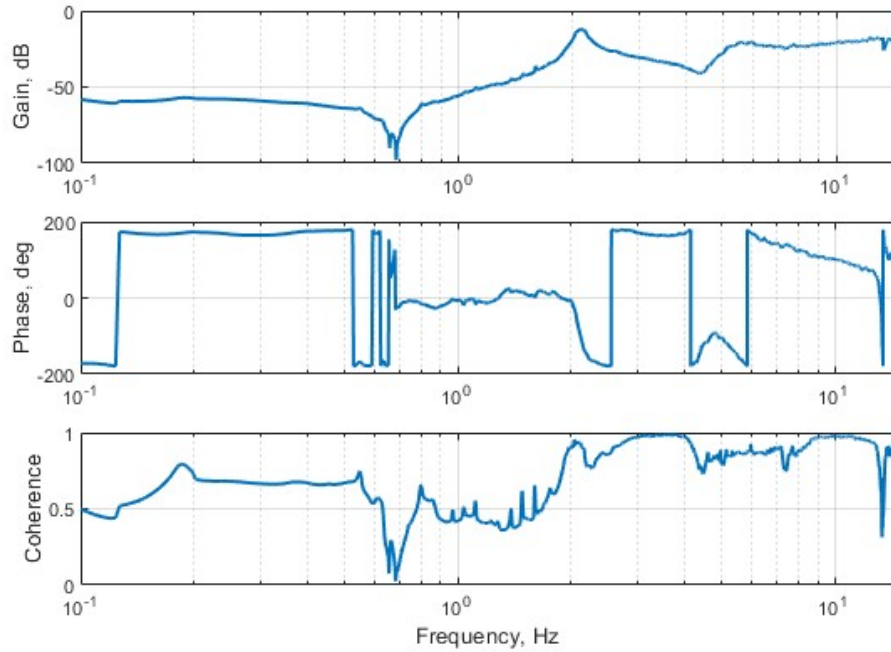


Figure 5.4: Gust vane to wing tip leading edge accelerometer FRF at 500 Pa

The reduction of coherence due to both low gain and transience effects can be seen in Fig. 5.4. A small drop in coherence just above the wing bending frequency (2.1-2.7Hz) can be seen due to the transience created by the sweep at the 2.1 Hz wing bending mode. In other words, as the sweep frequency was increased beyond 2.1 Hz, the frequency response until 2.7 Hz was contaminated with the transient response of the 2.1 Hz wing bending mode. The low damping ratio and attenuation rate of the 2.1 Hz wing bending mode's response reduced the coherence up to 2.7 Hz. On the other hand, the low coherence below the wing bending frequency was a direct result of the low gain, caused by the gust vanes not exciting the rigid body mode well.

Varying the sweep amplitude, notching schedule, duration, repeats, and using frequency-targeted sweeps generally resulted in high coherence identification of the frequency response data, with

some input-output relationships showing higher coherence than others. The coherence of identification was mostly consistent across the 200-500 Pa region. This consistency is shown in Figure 5.5.

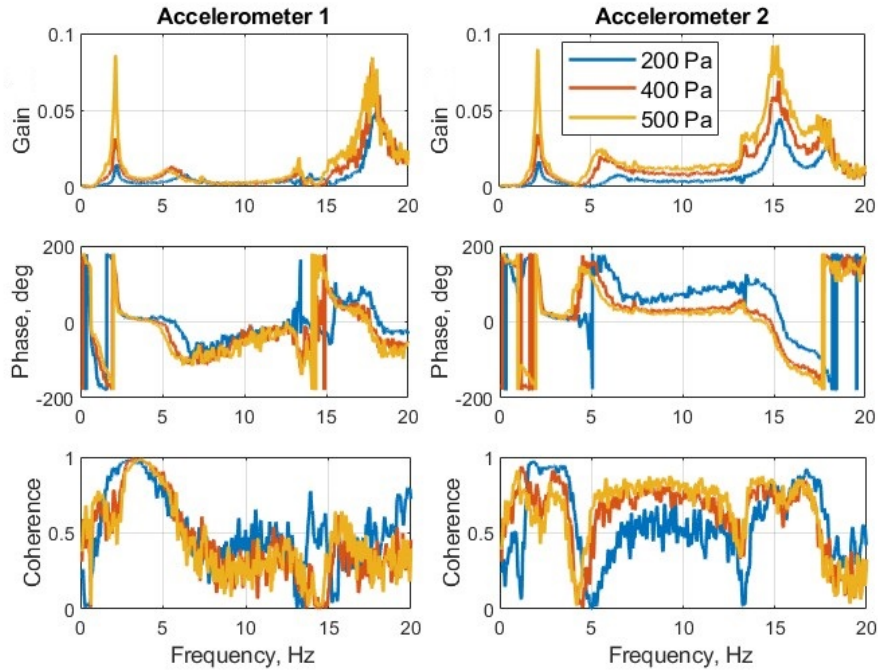


Figure 5.5: Frequency response estimates resulting from the same inboard aileron sweep input at different dynamic pressures

Despite this consistency, the dominant issues that reduced signal-to-noise ratio in the various values of tunnel dynamic pressure were different. Near 200 Pa, the control surfaces (especially ailerons, and outboard aileron in particular) were not very effective. Near 500 Pa, both the aerodynamic damping of the model was reduced (towards flutter) and the tunnel turbulence levels were increased leading to more test noise. The combined effect of these two contributors was quantified by looking at the noise measured by the sensors at different dynamic pressures when no input was provided. Studying the sensor output was a more effective way to understand the noise characteristics than quantifying the wind tunnel turbulence length and time scales because the reduction in coherence depends not only on the turbulence magnitude but also on its interaction with the model.

For example, the noise as sensed by the wing root strain gage is shown in Figure 5.6. The significant increase in sensor noise (compared to the wind-off condition) reveals that most of the noise is due to turbulence excitation. Placing or removing the gust vanes upstream of MARGE II also resulted in a minimal change in the perceived noise, indicating no significant downstream wake effects on the MARGE II model.

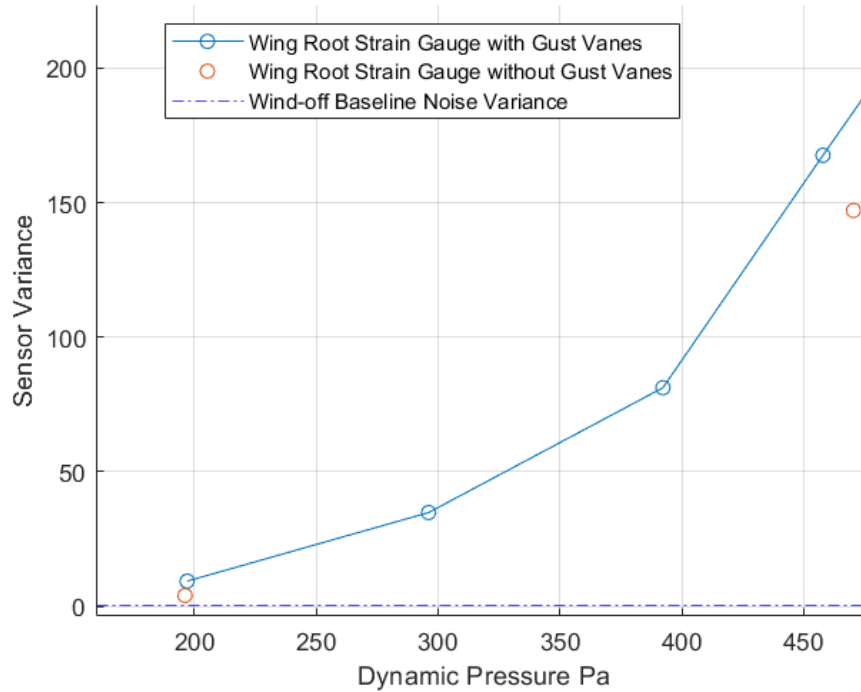


Figure 5.6: Wind On Wing Root Strain Noise Across Dynamic Pressures

5.2 Determination of the Flutter Onset

The initial technique used for determining the flutter speed was to track variation of the damping ratio and frequencies of the critical flutter mode as a function of dynamic pressure. An extrapolation to find the flutter speed was planned. A method commonly used for such extrapolation is Zimmerman and Weissenburger's flutter margin method [63].

The initial method used to estimate the damping ratio of the critical flutter mode (2.1 Hz wing bending mode) was using the log decrement method. The system was excited using 2.1 Hz gust vane input, and the excitation was terminated with the system's oscillations left to decay. The damping coefficients were extracted using the decay rate of the subsequent peaks. However, due to increased tunnel turbulence near 500 Pa, an accurate estimate of the damping ratio as a function of speed could not be obtained with the log-decrement method as shown in Figure 5.7. Figure 5.7 shows a very large uncertainty in the determined damping ratios, even after using multiple peaks to estimate decay rate.

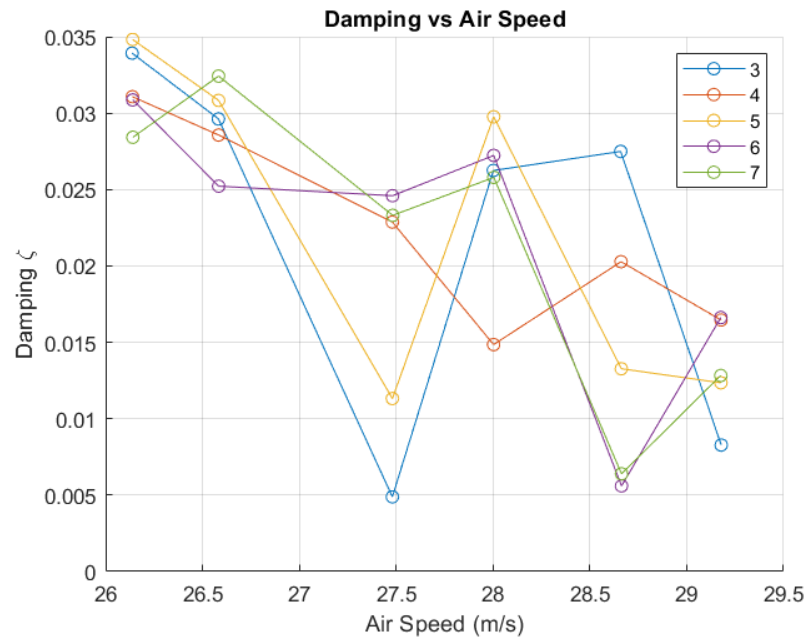


Figure 5.7: Log Decrement with different numbers of peaks for damping ratio calculation

A more successful approach for determining damping ratios was using the half-power method. The system was excited with a gust vane sweep covering a small range of frequencies around the wind bending mode. Despite the high tunnel turbulence and low damping of the system, a high coherence frequency response estimate that could be used to calculate the damping could be obtained as shown in Figure 5.8.

Windowing size had to be carefully chosen in order to capture the modal peaks accurately. To

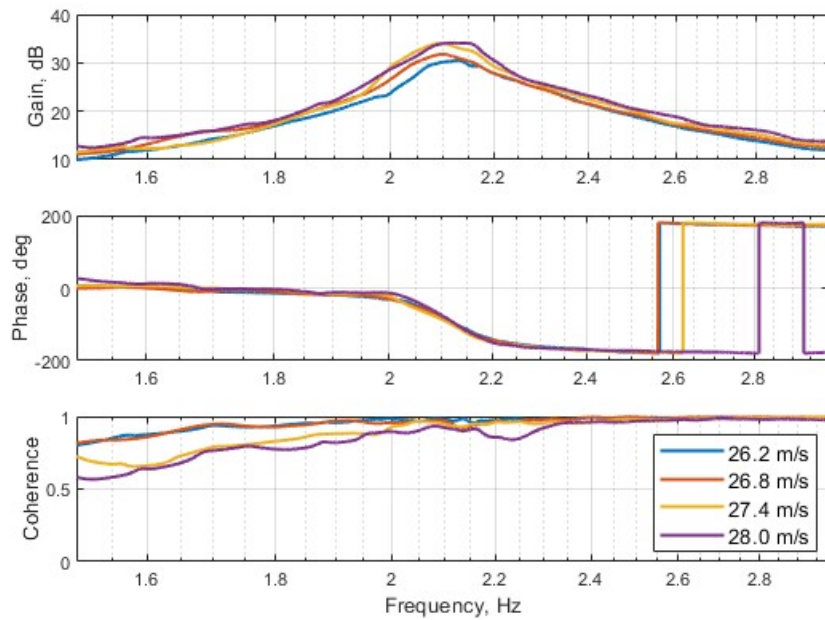


Figure 5.8: Mid-wing strain gage FRF under gust vane input at speeds and frequencies near flutter

ensure accuracy in capturing the modal peaks, the window size had to be increased until the peak height and width converged resulting in a large window size of 10000 (hence a frequency grid containing 10000 points over the 1.5-3 Hz range).

The damping ratios obtained using the half-power method were in good agreement with the results of the V-g-f analysis as shown in Figure 5.9:

Although an extrapolation to determine the flutter speed was planned based on subcritical results, flutter was unexpectedly encountered at 520 Pa. One reason for this unexpected encounter was that the MARGE II model configuration evolved throughout testing while the mathematical model during did not evolve at the same rate, leading to the unexpected flutter encounter. Specifically, the wing tip fairing surface had not yet been included in the mathematical model when the unexpected flutter encounter occurred. Secondly, the wind tunnel speed drifted after being set. With a flutter mechanism that displays large changes in minimum damping with small changes in speed, a drift of approximately 1 Pa in wind tunnel dynamic pressure was sufficient to produce flutter that

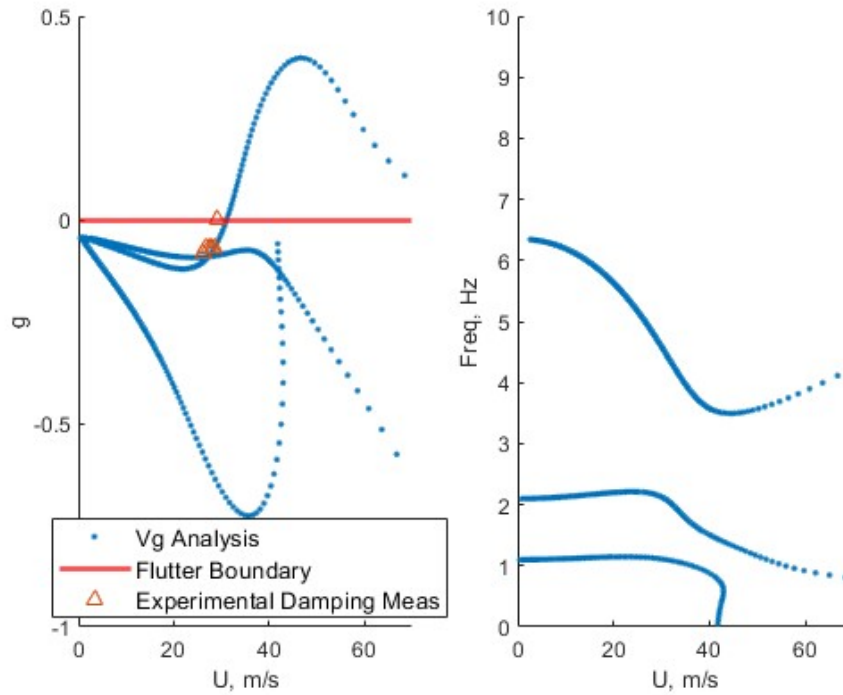


Figure 5.9: V-g flutter analysis of MARGE II

was, by analysis, was expected a bit later.

The final mathematical model (shown in the V-g-f plot) predicted a flutter speed of 31 m/s while flutter was encountered at 29 m/s. The 2 m/s difference in prediction is mostly due to the wing tip fairing not being modeled well by DLM. In wind tunnel tests conducted without the wing fairing, flutter had not been encountered even at 600 Pa (31 m/s). The effect of the fairing was under predicted by DLM (the wingtip fairing was modeled as a thin plate) which led to the 2 m/s gap between the predicted and actual flutter speed.

The critical flutter mode (wing bending mode) was also correctly identified by analysis. Hence, the aeroelastic mathematical model predicted the flutter mechanism accurately.

In the V-f plot of Figure 5.9, it is possible to see that the 2.1 Hz wing bending mode and the rigid body mode are close in frequency and interact. Hence, MARGE II encounters body-freedom-flutter (BFF). The wing bending and pitch mode do not fully coalesce but the importance of the

rigid body mode in the flutter mechanism is seen once the rigid body mode is removed from the flutter analysis. Figure 5.10 clearly shows that the removal of the rigid body mode from the flutter analysis drastically changes the flutter behavior (leading to an incorrect fuselage and wing bending mode coalescence behavior).

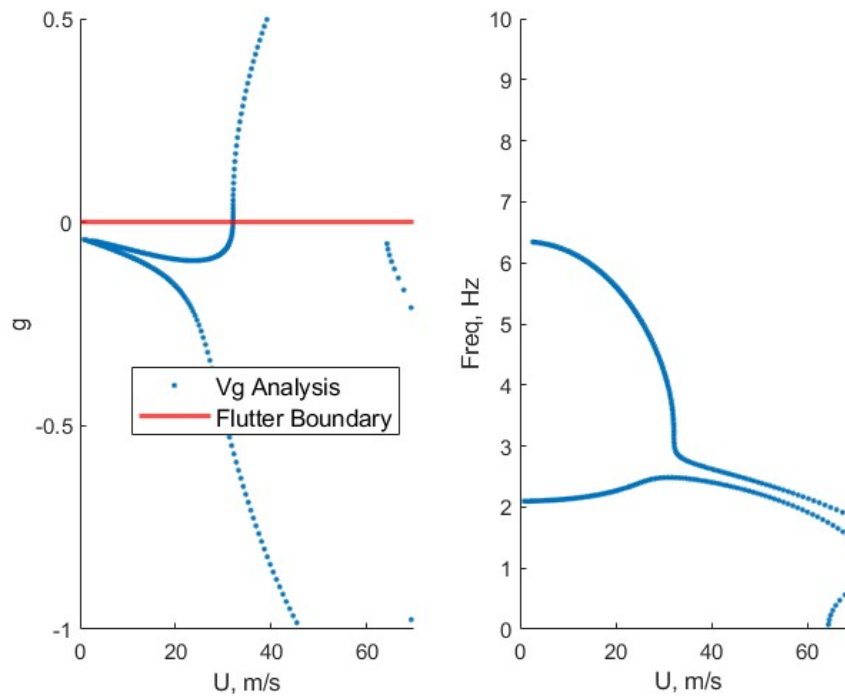


Figure 5.10: V-g flutter prediction of MARGE II without the rigid body pitch mode

Chapter 6

Validation of the Aeroelastic Model

In this chapter, the frequency response predictions of both of the Finite Element and the DIC-based mathematical models are compared to the wind tunnel data. The chapter is organized as follows: Firstly, the fine tuning process of the mathematical models are described. Secondly, physical reasons for the mismatches between the early versions of the mathematical model and the MARGE II's wind tunnel frequency response is discussed, including the methods and the extent to which the mismatches were mitigated. Finally, the conclusions about the accuracy of the mathematical models under different control surface and gust vane inputs are presented.

Both the DIC-based and FE-based aeroelastic mathematical models were first fine tuned using the wind tunnel frequency response data. The fine tuning followed a step-by-step process. First, the frequencies in the structural stiffness matrix, K_{ss} , were adjusted to align the peaks of the numerical frequency response functions with the peaks of the wind tunnel frequency response functions, though significant changes to the K_{ss} matrix were not necessary. Secondly, the structural damping matrix, C_{ss} , was fine-tuned to get a good match between the wind tunnel frequency response functions and aeroelastic models. Then, the control surface and gust vane GAFs were scaled by a factor in the range 0.35-0.42 to match the gains of the predicted and wind tunnel-measured frequency response. Finally, the state-space output matrix entries corresponding to strain gages

were scaled to obtain match the gain against wind tunnel frequency response data. All frequency response predictions later presented in this chapter have undergone this fine tuning process.

The fine tuning procedure of the mathematical models and the modifications on the MARGE II testbed were driven by the need to improve the match between the mathematical models and the wind tunnel frequency response. Firstly, the initial frequency responses predicted by the mathematical models seemed to have significantly higher gain than recorded in the wind tunnel. The control surface and gust vane GAFs were scaled by a factor of 0.35-0.42 to alleviate the gain mismatch. This range of unsteady aerodynamic oscillatory flap effectiveness factors is much lower than the commonly used range of 0.6-0.8 needed to correct DLM predictions. The need for lower correction factors in the work presented here were likely due to many factors including control surface/lifting surface gaps, wing root and tip effects due to nearby non-lifting bodies, as well as some flow separation on the control surfaces. Indeed, reducing the gaps between the control surfaces and the main lifting surface with tape led to significant increases in gain and improvements in math model/physical model phase match above 4 Hz. The change in gain and phase after reducing the gap is shown in Figure 6.1. Secondly, in addition to the gain, significant differences between the phase and modal frequencies were initially observed. The reason for this mismatch was attributed to the boundary conditions. The boundary conditions play an important role in determining the aeroelastic response. In the case of MARGE II, rotary shaft bolt of MARGE II (pitch axis) was replaced with a new shoulder bolt that led to a lower level of freeplay. The difference between the aeroelastic responses is seen in Figure 6.2. The figure shows that the reduction in freeplay (hence correction of the boundary condition) led to a much better match between the math model and the wind tunnel frequency response. Although the boundary condition of the MARGE II system was improved by reducing the free play, the boundary condition of the mathematical model and the physical system still continue to differ due to the presence of pitch stiffness and damping in the real system induced by the cables. The effect of the boundary condition difference induced by the cables is seen in the strain gage frequency response plots. Although most of the strain gage re-

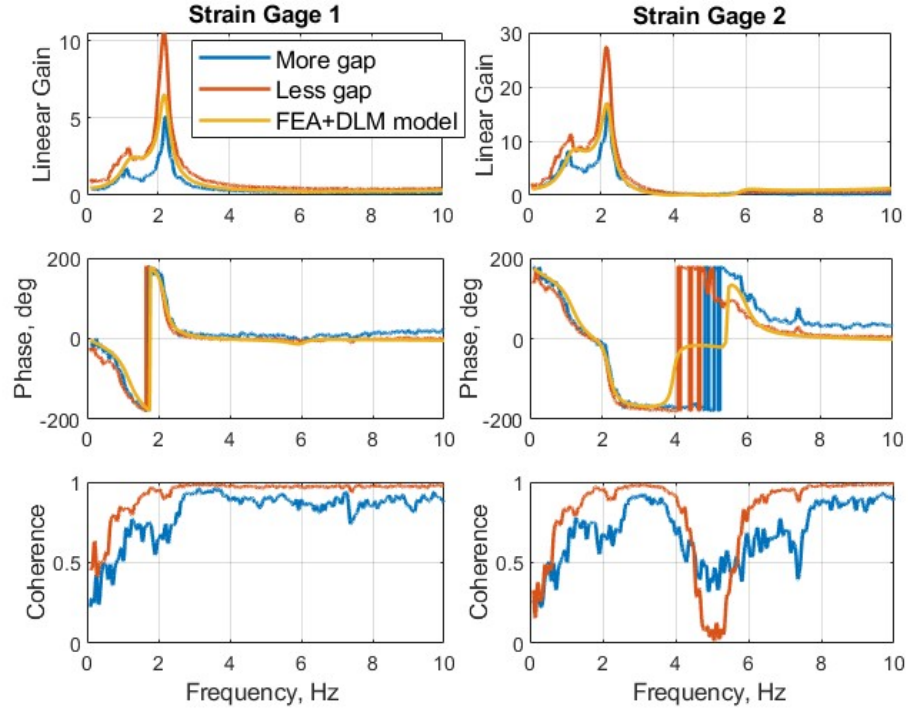


Figure 6.1: 300 Pa inboard aileron FRFs with and without tape to reduce control surface gap

sponses were modeled well under gust vane and elevator inputs, some of the strain gage frequency responses showed significant mismatches, especially below 1.5 Hz where the frequency response is dominated by the pitch mode. An example of such a strain gage frequency response is seen in Figure 6.3. The lack of cable stiffness and damping model in the FE analysis resulted in a rigid body pitch mode, hence zero contribution from the pitch mode to the strain output. In reality, the pitch mode is not exactly a rigid body mode due to the pitch stiffness created by the sensor and power cables, contributing to the strain output in way that is not modeled by the FE analysis. This being said, significant improvements to the precited frequency response below 1.5 Hz could not be obtained by trying to model the cable stiffness and damping in the FE model, likely due to the complexity of the physics involved.

Having discussed the fine tuning and mitigation of the mathematical and physical model's frequency response mismatches, the final comparisons between the two mathematical models and

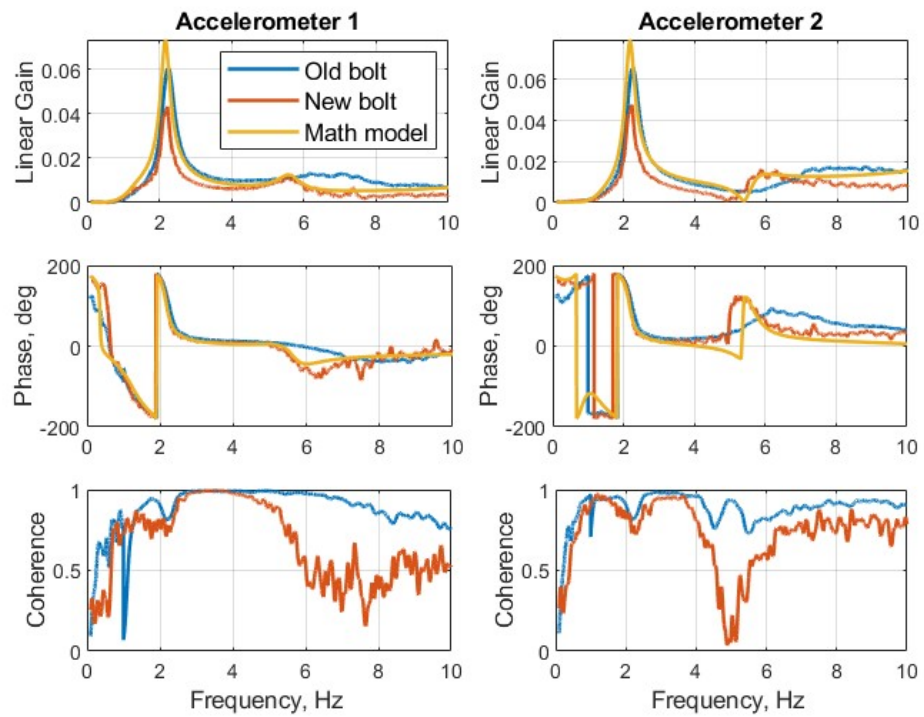


Figure 6.2: Difference in aeroelastic response after using a new bolt with less freeplay under midboard aileron input at 400 Pa

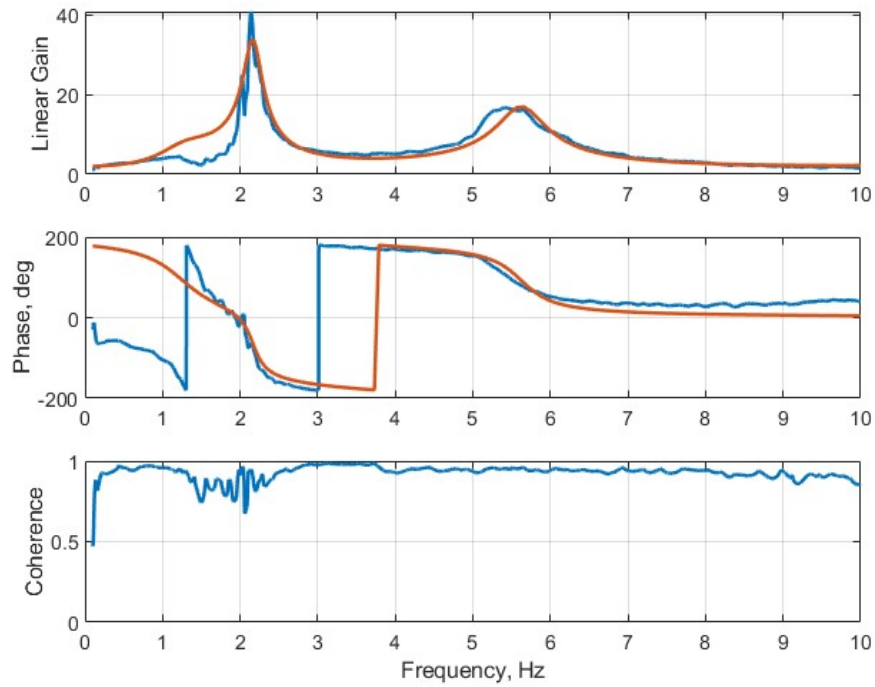


Figure 6.3: 400 Pa elevator input to mid-fuselage strain gage

the frequency response data are presented next. Both the DIC and FE-based math models predicted well the accelerometer and pitch angle response of the system when excited by gust vanes and the elevator as shown in Figures 6.4, 6.5, 6.6, and 6.7. The DIC model struggled to predict the response to aileron excitation beyond 3.5 Hz as shown in Figures 6.8 and 6.9. In Figures 6.8 and 6.9, all ailerons were taped together, and the gaps between the ailerons and the wing lifting were reduced while collecting the wind tunnel data.

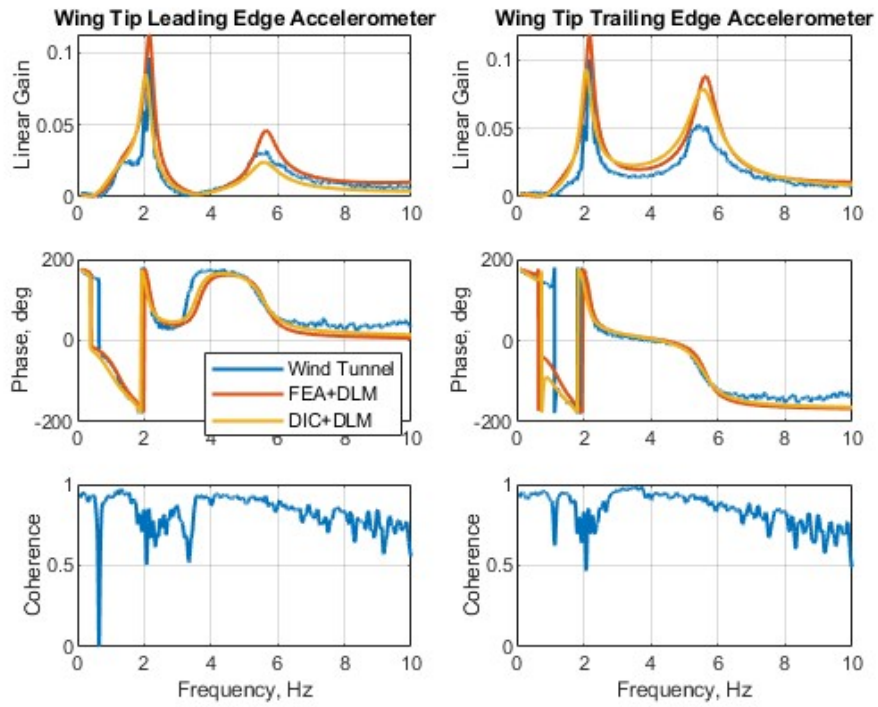


Figure 6.4: 400 Pa elevator input to wing accelerometers

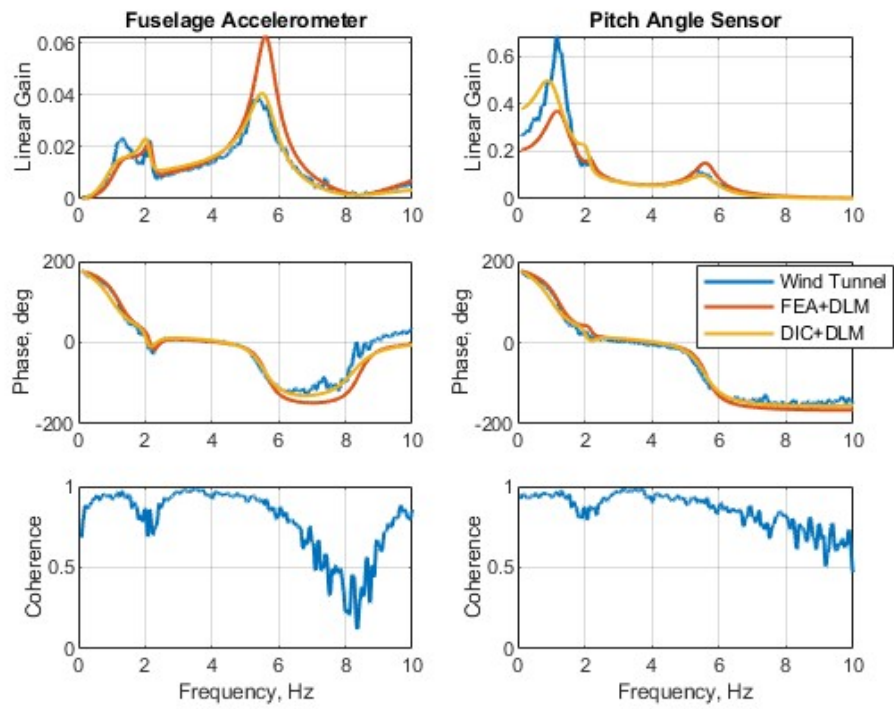


Figure 6.5: 400 Pa elevator input to fuselage sensors

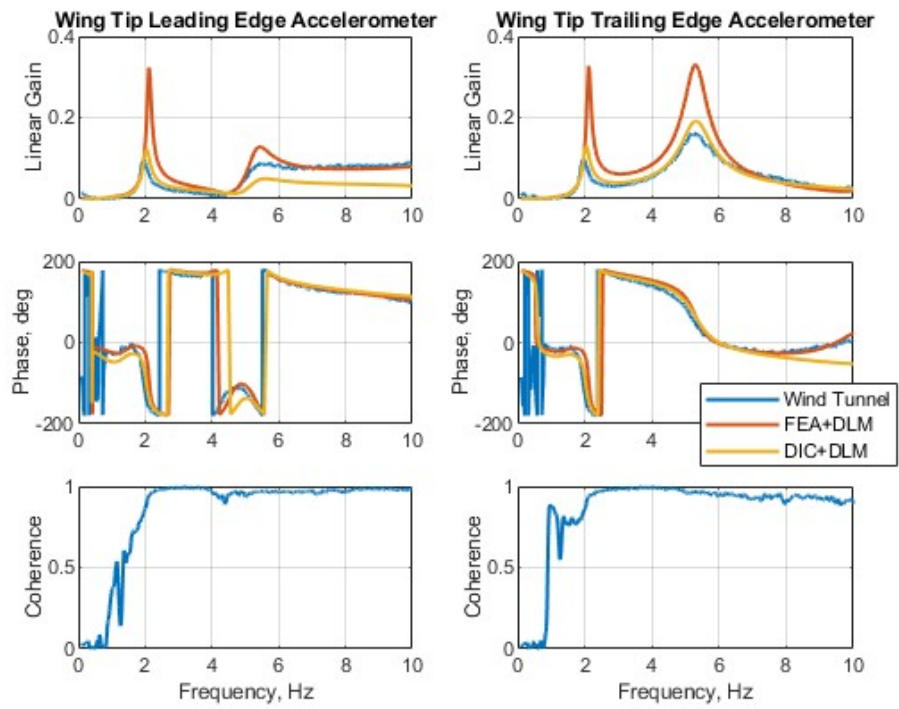


Figure 6.6: 500 Pa gust vane input to wing accelerometers

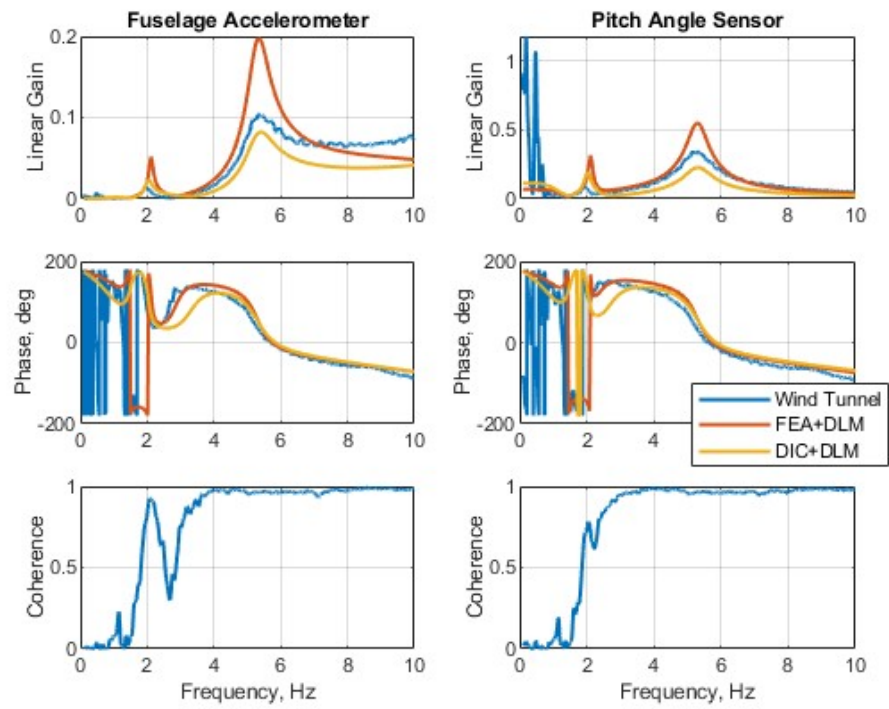


Figure 6.7: 500 Pa gust vane input to fuselage sensors

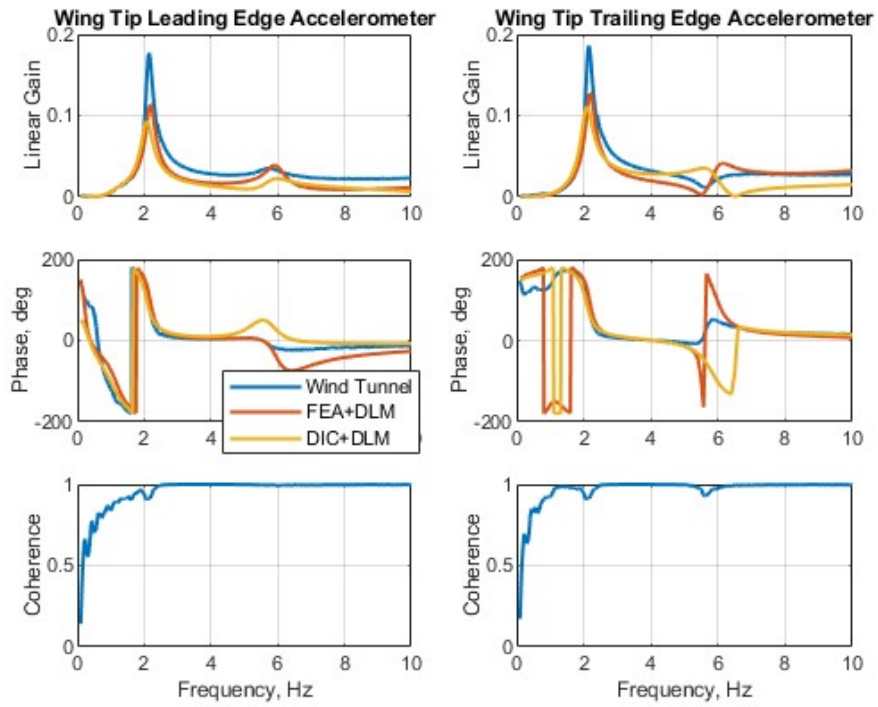


Figure 6.8: 300 Pa ailerons input to wing accelerometers

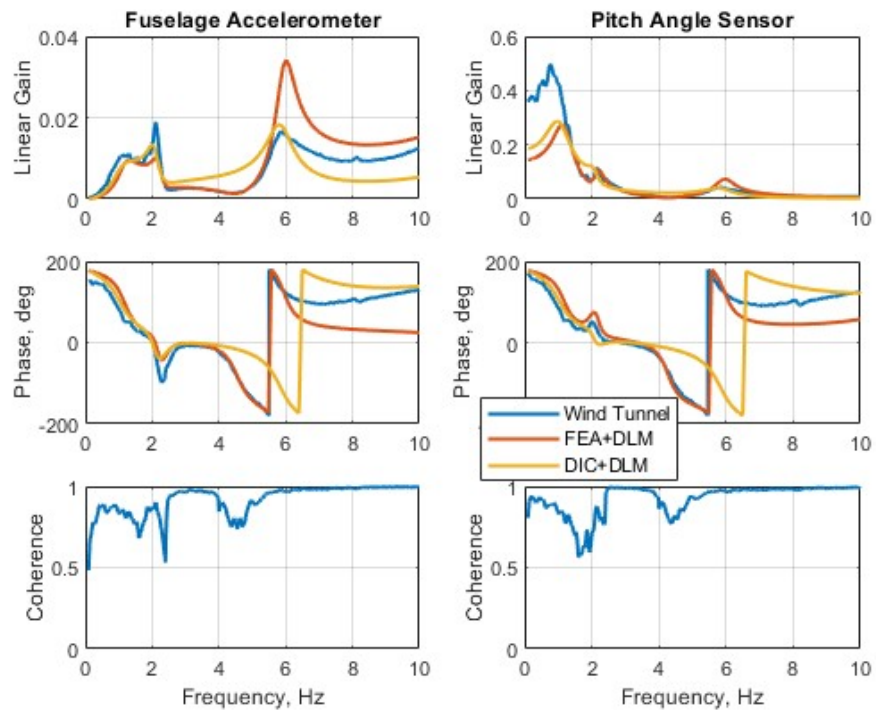


Figure 6.9: 300 Pa ailerons input to fuselage sensors

Chapter 7

Conclusion

Both the aeroelastic mathematical modeling and validation of the MARGE II aeroservoelastic wind tunnel model were studied via testing in the work described here.

To build the structural dynamic mathematical model, two techniques were used and compared; an FEA-based technique and a DIC-based technique. In the corresponding GVT, the structure was excited at multiple points in order to determine its natural modes of vibration, with excitation force locations that were designed to improve the quality of the capture of the mode shapes. A good match between the FEA and DIC modes was obtained for the low frequency wing and fuselage bending modes. The mode shapes, as well as natural frequency and damping information, were coupled with a DLM aerodynamic model to create aeroelastic models of the system.

To identify the frequency response of the system under control surface and gust vane excitation sweeps, frequency domain SIMO system identification methods were used. Exciting the model sufficiently without overstressing the system and creating aerodynamic nonlinearities to obtain good coherence in the 0.1-20 Hz range presented a challenge.

Flutter speed and the damping of the critical flutter mode were well predicted by the aeroelastic mathematical models.

It proved more challenging to predict the frequency response accurately. The frequency re-

sponse under elevator and gust vane inputs were predicted better than those excited by the ailerons. The FEA-based model predicted the model's frequency response better under aileron input than the DIC-based model. The accuracy in predicting responses to elevator and gust vane inputs were similar for both models.

Future work on aeroelastic modeling should include supplementing of the DLM aerodynamics with URANS or LES-based CFD simulations to study the effects of higher-fidelity CFD simulations on the AIC matrices. In addition, inclusion of the the aeroshells in the finite element model should improve FEA predictions of the higher order modes. DIC modal shape identification may be improved when multiple actuators are used. Multiple actuator identification is an area that calls for more work. Research aimed at improving coherence in AE system ID tests also continue to be of major importance, to which more work should be dedicated.

Overall, this work will add to the literature on aeroelastic modeling and testing. While MARGE II is a relatively simple system, it shares much of the physical behavior and mathematical modeling challenges that developers of full size flight vehicles encounter. Lessons learned from research done with MARGE II will benefit the fields of aeroelasticity and aeroservoelasticity in general. Beyond the focus on mathematical modeling and system identification, there are the challenges of designing and implementing controllers that would shape dynamic aeroelastic response in desirable ways, including gust load alleviation, ride comfort, handling qualities, and active flutter suppression. MARGE II will serve as a testbed for the development of such technologies.

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