

Doctoral Students Learning to Become Equity-Oriented Mathematics Teacher Educators

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Abstract

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This study sought to understand how doctoral students in mathematics education learn to become equity-oriented mathematics teacher educations (EOMTEs) by analyzing the experiences of current doctoral students in mathematics education at three universities across the US. I focus on doctoral students learning in mathematics education because doctoral students often go on to become mathematics teacher educators, therefore providing doctoral students in mathematics education with the potential to influence the practices and dispositions of future mathematics teachers. I used case study tradition to investigate the communities of practice (CoP) doctoral students participate in that support their learning and the ways doctoral student identities as EOMTEs develop related to engagement, alignment, and imagination (Wenger, 1998).

I found two kinds of CoPs that support doctoral student learning to become EOMTES: institution-based mixed member and peer created CoPs. Institution-based mixed member CoPs provided opportunities for doctoral students to learn to apply theories into practice and expand their conceptions of what it means to be an EOMTE. Key features of productive institution-based mixed member groups are (1) there are shared interests in collaboration by all members,

regardless of their status in the group, (2) participants perceive that all members of the CoP have shared understandings and commitments to equity, (3) new members learn the norms of participation in these groups by observing the interactions between more veteran members, and (4) these groups have knowledgeable and experienced members who guide the learning of novice members. Peer created CoPs are established when doctoral students feel a necessary part of their learning is missing from the institution-based mixed member CoPs they participate in. Key features of productive peer created CoPs are (1) the learning focuses on a genuine interests of the doctoral students in the group, (2) members can opt-in or opt-out with no social or academic consequences, and (3) members perceived they have similar status.

Additionally, findings indicate there are specific ways an identity as an EOMTE is either encouraged or discouraged in doctoral student experiences. I found evidence that when doctoral students have both meaningful opportunities to engage in practice as EOMTEs and feel their beliefs and stances are aligned with other members of the CoP, then there are opportunities for doctoral students to imagine themselves in increasingly sophisticated roles as EOMTEs. Conversely, if there is a lack of either meaningful opportunities of engagement or alignment between the doctoral student and other members of a CoP, imagination is limited.

Dedication

Dedicated to all the Guys in my life. To my father, who always encouraged me to strive to be my best; and to my son who has brought more joy to my life than I could have ever hoped.

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Chapter One: Context and Problem Statement

Doctoral students in teacher education often go on to take positions as teacher educators in university settings, thereby providing this group of people meaningful opportunities to influence the approaches, dispositions, and stances preservice teachers take with them into their careers as teachers. Attending to doctoral student learning as they become teacher educators is critical given the significant role they will play in supporting future generations of teachers. While there is much to learn from and study about doctoral student learning in teacher education broadly, I focus in this dissertation on how doctoral students learn to become equity-oriented mathematics teacher educators (EOMTEs). Mathematics education is the focus content area of this dissertation because mathematics has long been positioned as a culture-less and apolitical subject that has led to further cement racist, sexist, and ableist beliefs about who is and is not capable of doing mathematics. This underlying context makes mathematics education a particularly compelling subject area to consider issues of equity.

In what follows, I briefly describe how I draw on Critical Race Theory (CRT) as a lens for understanding mathematics as a political subject. I also provide a definition of equity in mathematics education that does not directly come from CRT work but adheres to many of the goals and principles of CRT. I then give a summary of situated learning as a framework for how I make sense of learning as it applies to becoming an EOMTE.

Critical Race Theory

I draw on Critical Race Theory (CRT) as a lens for understanding why and how mathematics is a political subject for which attention to issues of equity is necessary. In this section, I introduce CRT, explain the connection between CRT and equity in mathematics, and describe a definition of equity in mathematics education.

CRT was born out of critical legal studies whose work “questions the very foundation of liberal order, including equality theory, legal reasoning, Enlightenment rationalism, and neutral principles of constitutional law” (Delgado & Stefancic, 2017, p. 3). CRT interprets differences in outcomes for groups based on race as symptoms of broader hierarchies of power and privilege. In this way, CRT allows us see interactions and outcomes as products of biased laws that have led to the widespread acceptance of discrimination that occurs at the intersection of identities such as race, class, gender, ability, etc. While there is no set of agreed upon tenets that define CRT (Delgado & Stefancic, 2017), I draw on five common themes from CRT literature that are applicable to education. I briefly describe these principles here; a more thorough explanation of CRT is presented in Chapter Two.

1. *Permanence of Pervasiveness of Racism* establishes that “racism is ordinary” (Delgado & Stefancic, 2017, p. 8). That is, that racism is an inherent part of US society that is present in all aspects of life in America, including education (Tate, 1997).

2. *Interest Convergence* is the reason why the permanence and pervasiveness of racism exists. Real social change, CRT scholars argue, is not likely because those in power (namely elite Whites) have little incentive to change a system that has benefitted them. Ladson-Billings (2013) writes, “White people will seek racial justice only to the extent that there is something in it for them” (p. 38), thus validating that change is only possible when there is a convergence of interests that also benefits Whiteness.

3. *Race as a Social Construct* challenges notions that race is an “objective, inherent, or fixed” quality. This tenet proposes instead that race is a construct created by society as “a mechanism for creating hierarchy and an ideology of White supremacy” (Ladson-Billings, 2013, p. 39).

4. *Unique Voice of Color* acknowledges that people of color have had unique experiences in American society that can be used as counter narratives to challenge the normativity of dominant narratives that have normalized racism (Bell, 1987; Delgado, 1993).

5. *Intersectionality* orients people to consider the intersectional identities people bring with them into interactions and the way those multiple, overlapping identities influence how they are perceived and treated (Crenshaw, 1991).

These tenets help us question the so-called “universal” nature of mathematics and the influence this assumption has had on the ways people experience mathematics learning environments. For example, the claim of neutrality and objectivity has led to an interpretation of achievement discrepancies in mathematics related to race as indicators of innate ability. The logic goes, if mathematics is free from structures of society and civilization, then those who are successful in mathematics must be naturally gifted at it because societal injustices are below mathematics. The result of this has been stereotypes about who is and is not capable of doing mathematics, marking White and Asian students as highly capable and other groups of color as much less capable. Stereotypes also play a role in the ways schools treat Black and Brown presence in mathematics spaces (McGee, 2013) and in how Black and Brown people come to see themselves as less capable of doing mathematics (Steele, 1997).

This stereotype, CRT scholars may argue, is according to design. CRT helps us see the gap between racial group achievement in mathematics as flaws in the quality of educational opportunities students receive and high-stakes standardized assessment. These assessments privilege biased forms of life experiences and opportunities that are more likely available to White and middle-class families rather than as the inherent “failures” of non-White and middle class students, their families, and communities (Flores, 2007). For example, we may

acknowledge that segregated educational opportunities for students based on race through “high” and “low” level tracks (Oakes, 1985) have led to more robust learning opportunities for White and middle-class students or how property taxes were historically the main funding source for public education, thereby affording schools serving affluent student demographics access to more financial resources to support student learning. We know that while teachers certainly do the day-to-day work of perpetuating or dismantling systems of oppression in their classrooms, it seems unlikely that an entire system will change on the good intentions and dedication to change of teachers alone. This is due to the pervasiveness and permanence of Whiteness (Bell, 1992) that is baked into all the institutions we live in and the drive of the system to preserve itself. Given university-based teacher education is the dominant mechanism for certifying new teachers, a systems level analysis would suggest changes in teacher education focused on teacher learning. For change to occur, however, we must consider the knowledge and learning of those who support mathematics teacher learning, namely mathematics teacher educators.

A Definition of Equity

I am interested in the learning of novice mathematics teacher educators who aim to be equity oriented in their work. Equity, however, is a term that is often used without clear definition. In what follows, I briefly describe a definition of equity in mathematics education.

CRT helps us see that while attending to systems are crucial for social change, the systems that currently exist are not likely to change unless there is some level of interest convergence. Thus, even though we may want to focus on systemic change, we must also acknowledge this change is not likely. Although bleak, Bell (1992) helps us make sense of this “hard-to-accept fact,” saying “we must acknowledge it, not as a sign of submission, but as an act of ultimate defiance...because, while no one escapes death, those who conquer their dread of it

are freed to live more fully” (p. 12). That is, through the recognition and acceptance of this truth that allows those who seek equity to “confront and conquer” the limitations of what progress is possible and thereby allows movements to have a clearer “reason and the means for further struggle” (p.12).

In mathematics education this means equity needs to attend to what Gutierrez (2002) calls the “critical” and “dominant’ axes of equity. Gutierrez’s conception of equity in mathematics education stems from an understanding of the permanence and pervasiveness of Whiteness and of how race has operated as a social construct in ways that have led to further marginalization and stereotyping of mathematics ability. She calls us to attend to the critical by challenging current systems through integrating identity and power in mathematics learning experiences. A critical approach to addressing equity in mathematics “squarely acknowledges students are members of a society rife with issues of power and domination. It takes students’ cultural identities and builds mathematics around them in such ways that doing mathematics necessarily takes up social and political issues in society, especially highlighting the perspectives of marginalized groups” (Gutierrez, 2002, p. 151). The dominant axis attends to supporting students to learn currently privileged mathematics knowledge. Privileged mathematics knowledge, Gutierrez (2002) contends, is mathematics content most closely aligned with the experiences of “Whites, men, and middle-class or wealthy individuals” (p. 151) who have occupied positions of power and have normalized their beliefs and interests so well “that most people are not consciously aware of them” (p. 151). This stems from an acknowledgement of the pervasiveness of Whiteness.

In her work, Gutierrez explicitly names equity as a process rather than an end. If equity was conceptualized as finite and attainable in the relative future, there is a risk of limiting what

can be done to forward equity. That is, arguments could be made that implementing a set of initiatives will create equity, as if it would be attained by a few changes (Gutierrez, 2002). Instead, Gutierrez promotes a vision of equity that takes time to develop because, as CRT helps us see, equity in mathematics education is an issue that extends beyond single classrooms or schools. Equity issues in mathematics are a symptom of broader social structures that are designed to protect Whiteness at the expense of others. As such, doctoral student learning to become EOMTEs can be conceptualized as a process that may begin before doctoral programs and is ongoing through their career.

Situated Learning

CRT is used in this study as a frame for seeing the problem context in which this study is situated. At its core, my research questions are interested in issues of learning. I use situated learning theory to understand the process of learning as a sociocultural process. Sociocultural theories of learning understand that learning occurs in sociocultural contexts that include artifacts and interactions with others. Sociocultural theory has been used widely since the 1990's to understand learning (Jonassen & Land, 2012) and suggests that learning is an inherently social process (Vygotsky, 1978). That is, that learning occurs through social and cultural interactions that are frequently mediated by cultural tools (Bruner, 1997; Wertsch, 1988). Situated theory adds that all learning is situated in social, cultural, historic, and political contexts (Nasir, Rosebery, Warren, & Lee, 2006) For example, doctoral students learn in university contexts with faculty advisors, other faculty mentors, doctoral peers, in research projects, and coursework. Many of these relationships and spaces forward doctoral student learning through their participation in particular kinds of practice (Lave & Wenger, 1991). Membership on a research team, for example, can support a doctoral student's understanding of how to conduct research,

knowledge of the literature on mathematics education, and develop ways of interacting with other researchers and participants of studies. In this dissertation, I draw on situated learning, and Communities of Practice (CoPs) in particular, to understand how learning occurs for doctoral students working to become EOMTEs. In brief, CoPs are communities of people engaged in practice together (Wenger, 1999). Members of a CoP work together to forward common goals and typically have a shared repertoire of practices and language. Members develop identities with a practice through their participation in CoPs based on their opportunities to participate in the CoPs practices and the quality of those opportunities. Cultivating an identity as an EOMTE, then, requires that doctoral students have opportunities to advance their participation in EOMTE related CoPs.

It is with this backdrop that I investigated how doctoral students learn to become equity-oriented MTEs who are committed to and capable of addressing issues of equity in their work as teacher educators. In particular, I ask

- What CoPs support doctoral student learning to become EOMTEs?
- How do doctoral students develop identities as EOMTEs?

Before outlining the claims and contributions I offer through this dissertation, I provide a summary of the literature on doctoral student learning in mathematics education and provide an explanation of how this study builds off and is a departure from this literature base.

Literature on Doctoral Student Learning in Mathematics Education

The literature on doctoral learning in teacher education in the US is limited. The few scholars who conduct research on teacher educator learning argue that while teaching experience is necessary for becoming a teacher educator, it is insufficient because supporting preservice teacher learning requires knowledge and practices that go beyond the scope of what is typically

part of K-12 teaching (Abell, Rogers, Hanuscin, Lee, & Gagnon, 2009; Dunn, 2016; Goodwin et al., 2014; Hollins, Luna, & Lopez, 2014; Jacobs, Yendol-Hoppey, & Dana, 2015; Zeichner, 2005). What this necessary knowledge is, however, is not completely defined. A small, but growing body of literature identifies the knowledges, skills, and dispositions necessary for future mathematics teacher educators (Reys, 2018; Reys, 2017; Reys & Dossey, 2008; Shih, Reys, & Engledowl, 2016). This current literature base identifies the following knowledges and skills for doctoral students in mathematics education to obtain: Mathematics content knowledge, research skills, experience with mathematics curriculum, teaching and learning in mathematics, equity, policy, and to a much lesser degree, the process of this learning.

Mathematics Content Knowledge

Mathematics content knowledge, unsurprisingly, is consistently identified as necessary for doctoral learning in mathematics education. The “plus six” content requirement was first introduced at a conference on doctoral programs in mathematics education in a session led by Dossey and Lappan (2001). This requirement states doctoral students should have expertise in school mathematics content six years beyond the highest level of mathematics education they study (e.g. a high school mathematics teacher educator should have mastery of mathematics content through a two-year master degree in mathematics) The scholars describe this level of mathematical knowledge as necessary for PhD candidates to “engage in conversations with teachers over the full K-12 range” at the elementary level and “can discuss articulation issues across the boundaries of elementary, secondary, and post-secondary education” at the secondary level (p. 68). From this perspective, some scholars argue graduate level mathematics courses should be a requirement in doctoral programs in mathematics education.

While there seems to be no controversy in the idea that doctoral students in mathematics education must have a firm understanding of mathematics content, not all scholars agree to the “plus six” requirement. The conversation about what the necessary amount of mathematics content to study has been taken up and left unanswered over two decades. Despite this lack of consensus, a study of 500 PhD graduates in mathematics education from 1997 to 2014 found that while 23% of doctoral students took no mathematics courses as part of their doctoral studies, 95% of doctoral students in this same study also reported feeling well prepared for their future professions post-graduation (Shih, Reys, Reys, & Engledowl, 2019). These statistics suggest that many doctoral students in mathematics teacher education feel prepared to do their professional work regardless of the amount of mathematics coursework they completed in their doctoral programs.

Some scholars support the idea of the “plus six” criterion while simultaneously suggesting what is more important is to cultivate a learner’s stance. For example, Chazan and Lewis (2008) contend that doctoral students should develop a stance as a “life-long learner of mathematics” (p. 81). Given the unresolved nature of these conversations, the Association of Mathematics Teacher Educators (AMTE), the leading collective of mathematics teacher educators, describes the mathematics content necessary for doctoral students in mathematics education as “broad and deep mathematical knowledge both to identify the big ideas in the pre-K-14 mathematics curriculum and to examine how those ideas develop throughout the curriculum” (AMTE, 2002, p. 4). This is a slightly softer stance on the “plus six” requirement that emphasizes understanding mathematics content in relation to student learning and curriculum.

Research Skills

The most cited set of knowledges and skills described in this literature is the importance of cultivating scholars who can produce meaningful and rigorous research. Scholars seem to unanimously believe that doctoral students are best prepared when they are exposed to learning opportunities related to multiple traditions of research (Thornton, Hunting, Shaughnessy, Sowder, & Wolff, 2001). This category of knowledges and skills is cited as one of the areas that many doctoral students have the least amount of previous experience given many doctoral students have histories as K-12 educators or in mathematics (Ferrini-Mundy, 2008). Given these perspectives, it is interesting then that Shih, Reys, Reys, and Engledown (2019) report that in their survey of 500 recent graduates in mathematics education from 1997 to 2014, 90% of doctoral graduates reported their preparation in qualitative methods as “adequate” or “very well addressed” and only 70% rated their preparation in quantitative methods as “adequate” or “very well addressed” in their programs. These numbers suggest that while most doctoral graduates in mathematics teacher education feel prepared to engage both qualitative and quantitative research work, fewer doctoral graduates feel prepared to enact quantitative research.

While there is little consensus on how much research related experience or coursework doctoral students in mathematics education should receive, there is consensus that doctoral students must be able to implement multiple methodological traditions in educational research and should learn it through a combination of coursework and apprenticeship (Reys, Glasgow, Ragan, Simms, 2001; Lester & Carpenter, 2001; Lambdin & Wilson, 2001; and Wolff, 2001). Little, however, is suggested about the quality of research related learning experience doctoral students are exposed to during these apprenticeships.

Experience with Mathematics Curricula

Among the various knowledges and skills described in this literature base is mathematics curriculum. AMTE (2002) suggests doctoral students should be able to design effective mathematics curricula which requires knowledge “of current theories and research about human learning, how to connect different areas of mathematics, and how students come to appreciate mathematics as a discipline” (p 7). In addition, Long (2001) and Silver and Walker (2008) describe the importance of understanding the past and current political context around mathematics curriculum at the local (school/district), state, and national levels.

Although the emphasis on understanding the political and historical contexts that surround curriculum development and selection might suggest there are institutional structures involved in the development and adoption of curriculum, few scholars dig deeply into the equity implications of this. Taylor and Kitchens (2008) advocate for increased attention to the ways equity is attended to in doctoral programs in mathematics education. In particular, they point to the importance of doctoral learning around effective curricula for diverse learners and the potential challenges that surround this issue. In addition to supporting diverse learners through curriculum, Taylor and Kitchens (2008) suggest doctoral students also need to learn more about mathematics curriculum in relation to how it can address issues of social justice.

Teaching and Learning Mathematics

Given 40% of the work mathematics teacher educators do involves teaching (mathematics content or methods) courses, it is no surprise that a large focus of the core knowledge and skills doctoral students need to learn focuses on teaching and learning mathematics at various levels (K-12, undergraduate, of mathematics teachers). Wilson and Franke (2008), suggest “teaching mathematics to teachers involves an additional layer of

complexity [than teaching mathematics to K-12 students]. Preparing teachers requires an enhanced set of knowledge and skills-the learner is a teacher of mathematics as well as a student of mathematics” (p 103). This involves an understanding of the “fundamental theories of learning mathematics” (AMTE, 2002, p. 8), also known as specialized knowledge of mathematics needed for teaching (Wilson & Franke, 2008). Other scholars name the importance of fluency in using technology to promote mathematics learning (Heid & Smith, 2008), knowledge of learning theories (Reys, Glasgow, Teuscher, & Nevels, 2008), and the ability to communicate effectively with teachers, administrations, and teacher candidates (Fennell, Briars, Crites, Gay, & Tunis, 2001). For many institutions this requires doctoral students have previous K-12 teaching experiences (Reys, Reys, Shih, 2019; Reys, 2017; Fennell, Briars, Crites, Gay, & Tunis, 2001; Lambdin & Wilson, 2001; Ragan, 2001; Thornton, Hunting, Shaughnessy, Sowder, & Wolff, 2001) and experiences teaching at the university level (AMTE, 2002; Blume, 2001; Fennell, Briars, Crites, Gay, & Tunis, 2001; Lambdin & Wilson, 2001; Ragan, 2001). In addition to the experience of teaching at the K-12 and university levels, many scholars agree that doctoral students must learn to be “critically reflective” about their own practice and develop dispositions towards improving their teaching craft (AMTE, 2002).

Equity

Shih, Reys, Reys, and Engledowl (2019) describe a profile of mathematics education doctoral graduates’ backgrounds and preparation in the US by analyzing survey results from 500 mathematics education doctorates from 1997 to 2014 who attended 23 different institutions. As stated earlier, approximately 95% of respondents reported feeling satisfied with their preparation as doctoral students. In addition, one of the most under addressed topics doctoral graduates identified in Shih, Reys, Reys and Engledowl’s study was knowledge related to topics of

diversity and equity. While conversations about the preparation of doctoral students in mathematics education have been ongoing for the last few decades, equity has been a topic that is underdeveloped. Nondescript language that alludes to equity as important but does not clarify what is meant by diversity and equity are often used. For example, terms such as “political awareness” (Presmeg & Wagner, 2001, p. 75) and “historical, social, political, and economic contexts of education” (AMTE, 2002, p. 5) are used to signal attention to inequities as important without any additional description of what it meant by these phrases. Taylor and Kitchens (2008) are the only scholars who give equity more than a single sentence or section in their writing on doctoral student learning. While they do not define a specific vision of what equity in mathematics education looks like, they name topics and experiences doctoral students should have related to equity and diversity. Topics include: how diverse learners learn, strategies for supporting struggling learners; the role of parents in mathematics education; and strategies for preparing teacher candidate to work with diverse learners. In addition to these necessary knowledges, Taylor and Kitchens (2008) also suggest doctoral students require experiences working in a diversity of settings that are unfamiliar to them and for doctoral students to cultivate an “appreciation of diversity/equity issues” and theories regardless of whether issues of diversity and equity are at the core of their research work. These topics and experiences, according to Taylor and Kitchens (2008) are important because they support us to consider the need for doctoral programs to evolve to address the needs of the diversifying student population.

Policy

As alluded to in the curriculum section, policy is a less described area of expertise that doctoral students in mathematics education need exposure. Middleton and Dougherty (2008) suggest this is important for doctoral students to understand the context in which their research is

situated and will allow them to produce more meaningful and impactful research (Fey, 2001). While many of the issues related to policy in mathematics education can be learned about through coursework, scholars who write about the importance of policy in doctoral education suggest doctoral students should engage in active or observational experiences in real policy work (e.g. school board meetings) (Long, 2001; Middleton & Dougherty, 2001).

Processes of Learning

While the previous categories described above identify the major content and skills doctoral students in mathematics education should learn, a few scholars write about the importance of considering the processes for learning. Golde (2008), for example offers the idea of *intellectual communities* that are broadly inclusive of different status and expertise, purposeful and knowledge-centered, flexible, respectful and generous, and deliberately tended to meet the learning needs of doctoral students. Teuscher, Marshall, Newton, and Ulrich (2008) who were doctoral students at the time of their publication, note the shared authority and high level of interaction available to them in *intellectual communities* they are members of were meaningful to their learning. In addition, Blume (2001) describes the importance of attending to what he terms *non-course experiences* and the contributions to student learning around mathematics, teaching, research, and joining scholarly communities.

Final Thoughts on Doctoral Student Learning in Mathematics Teacher Education

It is important to note here that while these major categories of doctoral student learning in mathematics education come from the literature, they are not equally represented or explored across these texts. Mathematics content knowledge, for example, is described in the majority of articles, whereas experiences and knowledge about equity was only brought up in three articles or reports, two of which simply name it as important to learn about. Table 1 shows the major

focal categories identified and described in articles on doctoral student learning in mathematics education. The aim of this study is to elaborate on the equity category by describing what it means to be an equity-oriented mathematics teacher educator and how doctoral students are supported to become EOMTEs.

Table 1

Number of Articles that Pay Serious Attention Paid to of Each Category of Learning

Topic	Mathematics Content Knowledge	Research Skills	Mathematics Curricula	Teaching and Learning Mathematics	Equity	Policy
Number of Articles	17	18	5	16	1	2

It is important to note that this literature base includes the perspectives and experiences of faculty members in mathematics education and mathematics teacher education who currently support doctoral students in these fields. While we can certainly learn from the perspective of those who have been successful in completing doctorate degrees in mathematics teacher education and currently mentor doctoral students into this field, there is much more that can be learned about the process of learning to become EOMTEs, and perhaps other, skills, knowledges, and practices from the learner’s perspective.

While current mathematics teacher educators provide a robust set of knowledge and experience to learn about doctoral student learning, most mathematics teacher educators in the US share a similar history of being doctoral students in mathematics education who have been successful in academia. This means that in some ways, this group of people have been socialized into a realm of mathematics education that privileges the status quo. This is evident, for example, in the disproportionate attention given to the amount of school mathematics doctoral students

need to learn. The emphasis on cultivating K-16 mathematics knowledge suggests what most scholars are concerned about is the preservation of the field of mathematics as it currently exists. In addition, issues of equity, while listed as important in a few articles, has been the central focus of only one article on doctoral student learning in mathematics education. While consensus is largely lacking in terms of what exactly doctoral students need to learn in all of the categories described above, the fact remains that issues of equity in society at large, and in mathematics in particular, should be considered more carefully given that many teacher education programs (of which many doctoral students in MTE go on to work for) suggest they value equity. I argue that a commitment to cultivating EOMTEs is crucial because if we are not actively seeking to develop an MTE force who are dedicated and committed to forwarding equity agendas, systemic change is not likely to occur (Bell, 1992). Systems of oppression and power are ubiquitous, meaning they are all around us and so infused in dominant ways of being that go unnoticed. Thus, if we are not supporting the development of MTEs who are prepared and committed to forwarding equity in their work we are, at best, unintentionally allowing systems of oppression to continue operating in ways that privilege some while oppressing others.

In this study, I build on the work of Taylor and Kitchens (2008) in its focus on the importance of attending to the equity orientations doctoral students bring with them and take on in their mathematics teacher education preparation. This study is a departure from this literature base in two major ways. First, I focus on the process of learning rather a set of knowledges and skills to obtain. A focus on the process of learning allows us to consider the structures that support learning to become an EOMTE. The second way this study departs from the current literature on doctoral student learning in mathematics teacher education is that it draws on the

experiences and perspectives of the learners themselves. This perspective allows us to attend to the developing identity of doctoral students as they learn to become EOMTEs.

In what follows, I provide a brief description of the study context.

Study Context

As a reminder, the research questions I ask are: What CoPs support doctoral student learning to become EOMTEs? And how do doctoral students develop identities as EOMTEs? In this dissertation, I present my analysis of the narratives and observed experiences of eight doctoral students who are learning to become EOMTEs. In particular, I focused on the quality of experiences doctoral students had in CoPs and their opportunities of engagement, alignment, and imagination related to an EOMTE identity. Participants in this study attend three universities across the US that are distinct from one another in size, student demographic, and private/public status.

Summary of Claims

I make two major claims in this dissertation. The first claim attends to the quality of CoPs doctoral students find are helpful for their development as EOMTEs. Chapter Four offers an overview of the ways doctoral students make sense of what it means to be an EOMTE as it is informed by both their experiences prior to their doctoral program and experiences in CoPs during their doctoral program. The analyses I conducted points to two kinds of CoPs: institution-based-mixed membership CoPs and peer-created CoPs. Institution-based-mixed-membership CoPs are groups of mathematics teacher educators comprised of doctoral students and faculty who work collaboratively on endeavors associated with university work. This might be an advising group, a research group, or a teaching group. In these groups, doctoral student and faculty work together to meet goals specific to the university (e.g. teaching a mathematics

methods course). In these groups, doctoral students learned about what it means to be an EOMTE by implementing theories of equity ideas in academic work. For example, doctoral students may expand their knowledge of the literature on equity in mathematics education by engaging with how more seasoned scholars discuss and make sense of ideas across texts in research. These experiences can support and shift doctoral student perspectives about what it means to be an EOMTE. Institution-based CoPs that were reported as being helpful to doctoral student learning related to their EOMTE development had four notable features. First, there are shared interests in collaboration by all members, regardless of their status in the group. Second, participants perceive that all members of the CoP have shared understandings and commitments to equity. Third, new members learn the norms of participation in these groups by observing the interactions between more veteran members. Finally, these groups have knowledgeable and experienced members who guide the learning of novice members. Peer-created CoPs differ from institution-based CoPs in two ways. First, all members are doctoral students and second, the goals of the community are to forward doctoral student learning rather than a practice related to the university. One reason peer-created CoPs were productive for doctoral student learning was that the aims of the CoP were focused on doctoral student learning that was prompted by doctoral student interest, and the lack of mixed-membership allowed doctoral students to participate without status consciousness.

The second claim comes from an analysis of the identity development doctoral students have as they learn to become EOMTEs. Situated learning theory highlights the importance of three aspects of identity: engagement, alignment, and imagination. That is, that meaningful opportunities to engage in the practices of a COP alongside alignment of vision and values of members and opportunities to imagine oneself in increasingly sophisticated roles in the CoP are

crucial to a person developing an identity as an EOMTE. An analysis of doctoral student experiences related to their identity development as EOMTEs suggests that when there are meaningful opportunities to engage in the practices of a CoP and there is alignment in the beliefs and values of the members of the CoP, doctoral students have opportunities to imagine themselves in increasingly sophisticated roles and therefore, develop strong identities as EOMTEs. In cases when there is a lack of either opportunities for meaningful engagement or alignment of values and beliefs, opportunities to imagine oneself in increasingly sophisticated roles diminishes, thus resulting in doctoral students questioning their place in academia.

Summary of Contributions

The current literature on doctoral student learning in mathematics education identifies the knowledge doctoral students in mathematics education should learn. This study contributes to this literature in four ways. First, this study illuminates the process of learning rather than identifying knowledge outcomes. Second, this study's focus on the learner provided a pathway to understand how doctoral students develop identities as EOMTEs. These first two contributions may support those who support doctoral student learning to be EOMTEs in making decisions about how they structure learning opportunities for doctoral students. Third, this study's methodology draws uniquely on the perspectives and experiences of doctoral students themselves rather than of the perspectives and experiences of those who are established mathematics teacher educators. This added perspective is crucial for understanding the current landscape in which doctoral students are learning to become EOMTEs. Last, given the lack of research on doctoral preparation with respect to equity, this study's focus on equity may prime those who support doctoral student learning in mathematics teacher education to consider how equity should be central to the learning of mathematics teacher educators.

This study also makes a conceptual contribution to our understanding of identity development. In *Communities of Practice*, Wenger (1999) describes the three aspects of identity (engagement, alignment, and imagination) as “complementary,” suggesting they are separate entities that build upon one another. My findings suggest there is a particular relationship between engagement and alignment that must be coordinated for a learner to begin to imagine themselves in more sophisticated practices within a CoP.

Organization of Remaining Chapters

In Chapter Two, I elaborate on the theoretical and conceptual constructs that drive these research questions and study. I provide a more thorough description of concepts from Critical Race Theory, provide a definition for understanding equity, and describe a situated approach to understanding learning. Chapter Three offers a description of the methodological process used in this study. Chapter Four contains a description of doctoral students’ equity orientations in mathematics teacher education. Chapter Five addresses the first research question, attending to the kinds of learning opportunities doctoral students describe as meaningful to their EOMTE development. Chapter Six investigates the ways doctoral students develop identities as EOMTES. Finally, Chapter Seven offers a discussion of the implications of this research along with future directions for research.

Chapter Two: Conceptual Framework

In this dissertation, I investigate the CoPs that support doctoral student learning to become EOMTEs and how doctoral students develop identities as EOMTEs. Both these things are, at their core, questions about equity and learning. As such, this chapter provides a foundation for what is meant by “equity oriented” by starting with a lens for understanding the disparity in opportunities and access that are often categorized as an “achievement gap” in education by broader news and social media outlets. Next, I describe a framework for understanding equity in mathematics education. Last, I describe the theory of learning I use throughout this study to make sense of how doctoral students become EOMTEs.

Critical Race Theory as a Tool for Understanding the World

Critical Race Theory (CRT) is a theoretical perspective born out of legal studies that offers a way of understanding the history of racialized structures in society that were established in legal system and how they exist in our lives today (Delgado, 1995). For example, rights to land ownership, to vote, to speak your non-English native language in schools, to gain US citizenship, and to enter particular establishments have all been US laws that have normalized the marginalization of people of color (Freeman, 2013; Harris, 1993; Smith, 2006). This provides White people with privileges that are protected by law, thus making Whiteness property (Harris, 1993). By providing privileges to White people, the law and its application have necessarily and systemically kept large numbers of people of color in positions of poverty despite the belief held by some that we live in a post-racial, meritocratic society (Delgado & Stefancic, 2017). For example, laws around property rights have historically prevented non-White people from owning land, thereby restricting their families from building generational wealth that have preserved the privilege and economic advantages of White people. We might

look at Seattle as an example. Although this region has been occupied and cared for by the Duwamish people since time immemorial, White settlers forcibly removed the Duwamish people from the region, burning down Long Houses and making it illegal for the Duwamish to live in Seattle. Later in Seattle's history, neighborhoods developed covenants that restricted non-White people from living in neighborhoods unless they worked as servants for White families. This history of exclusion of people of color from Seattle has allowed White families to have financial opportunities to build generational wealth through the exclusion of non-White people. Despite changes to these laws that have ended the exclusion of non-White people from neighborhoods, Black presence in these places continues to be challenged. More broadly, the narrative that we live in a post racial society feeds off common beliefs that we live in a meritocratic society where the hardworking get what they earn. This view, that America was founded on and has continued to be a society driven by the merit of individuals makes it challenging for those who have benefitted from histories of institutional racism to see their opportunities as anything except the result of their hard work. This mentality works to prioritize White privilege despite changing laws.

In addition to a commitment to understand "how a regime of White supremacy and its subordination of people of color have been created and maintained in America" (Crenshaw, Gotanda, Peller, & Thomas, 1995, p. xiii), CRT scholarship is also committed to change the "vexed bond between law and racial power" (Crenshaw, et al., 1995, p. xiii) despite the commonly held conviction that "Black people will never gain....White dominance" (Bell, 1992, p.12). While Bell (1992) offers an existential reality check about the permanence of White dominance, he does so because while "hard to accept...we must acknowledge it, not as a sign of submission, but as an act of ultimate defiance" (p.12). By this, Bell means that to truly forward

any movement that challenges White supremacy, we must confront the reality of the possible outcomes such that we are (1) more fully able to live as people who are not defined by oppression and (2) to make realistic plans for progress that can further the cause.

A CRT lens helps us see issues in mathematics education as systemic. Many scholars have taken up CRT to unpack and understand how racist policies operate in math classrooms (e.g. Martin, 2013; Gutierrez, 2013; Nasir & Hand, 2006). These scholars critically analyze the ways common assumptions about mathematics, including how it should be taught and learned, promote *Whiteness as Property* (Harris, 1993). For example, narratives about the *neutral* and *universal* nature of mathematics situates mathematics as absent of any political criticism. These assumptions stoke stereotypes about people based on race, which in mathematics has positioned Asian people as inherently gifted in mathematics. Thus, when White and East Asian students disproportionately occupy advanced mathematics courses and mathematics related occupations, merit and innate ability are the only ways to explain this discrepancy which further validates common notions that non-East Asian folks of color are less capable of doing mathematics. This also suggests that the lack of Black, Latinx, and Indigenous representation in these spaces occurs because these groups are incapable of doing mathematics rather than questioning the learning opportunities these groups of students had in schools. This notion extends beyond classrooms and into mathematics related fields where Black presence is constantly questioned (McGee, 2013). The widespread adoption of these beliefs preserves mathematics competence as the innate property of Whites and Asians (Battey & Leyva, 2016). While some may contend White privilege is not a factor in mathematics education because Asians are seen as capable mathematicians, Tuan (1997) provides us with a compelling argument about the limitations of *honorary White* status. *Honorary Whites* are groups of historically marginalized groups who are

given access to a limited amount of White privilege in bounded contexts. Asian Americans, for example, are given *honorary White* status in mathematics spaces, but constantly have their sense of belonging in the US questioned (e.g. where are you really from?) (Tuan, 1997). In addition, Shah (2019) echoes this sentiment with analysis of how the “Asians are good at math” stereotype threatens Asian personhood through the dehumanization of Asians as a monolithic group of emotionless, uncreative calculators.

Other scholars also work to attend to the ways Whiteness are preserved in mathematics education by integrating Ethnomathematics, a field of mathematics that draws on ways of engaging in mathematics that acknowledge the deeply cultural nature of doing mathematics (Powell, 2002). Ethnomathematics requires that we question our assumptions about what counts as mathematics and pushes us to see the dominant ways of engaging with mathematics as just one cultural way, rather than the only way. This serves to unveil assumptions about the universality of mathematics content, its application, and ways of engaging with mathematics that have long contributed to White supremacy.

CRT allows us to see issues in mathematics at a systems level that can be address through multiple avenues. I see teachers as a powerful lever for changing the experiences and opportunities students have in mathematics. Teachers, however, are not the end or beginning of the system that created the inequities we see today. Teachers are trained in university-based teacher education programs by faculty members, and faculty members often entered this work as students in doctoral programs. As such, I sought to understand how doctoral students learn to become equity-oriented mathematics teacher educators precisely because, as CRT helps us see, if we do not intentionally focus on equity, we are not likely to address the normative structures in

mathematics education that have led to such different expectations and opportunities for students of mathematics based on race.

To make sense of the experiences and narratives of the doctoral students in this study, I begin by describing a conception of equity that helped me make sense of how doctoral students experiences in relation to equity that is informed by CRT. Next, I clarify the definition of equity I use as a framework for understanding how doctoral students in this study make sense of what it means to be EOMTEs.

Equity

A common concern in work that aims to create sustained social change is the weakening of the meaning behind terms. *Equity* and *social justice*, for example, have come to represent a large range of commitments and ideologies in literature on teacher education (McDonald & Zeichner, 2009). As new, powerful terms come to the fore (e.g. culturally responsive teaching, abolitionist and antiracist teaching), the co-opting of these ideas have the potential to compromise the meaning of these terms in ways that preserve and protect the status quo under the guise of progressive language. As such, work that attempts to address issues of power, privilege, and systemic change must describe the terms related to issues of power that are used (Kendi, 2019). I draw on Gutierrez' (2007; 2012; 2013) definition of equity as it aligns with the goals of CRT. Equity, according to Gutierrez (and others) relies on multiple planes. Gutierrez names these planes the dominant and critical axes of equity.

According to Gutierrez' description of equity, one way equity is often approached is through a dominant lens. This lens aims to support students to develop the mathematical skills and knowledge they need to engage in the mathematics pipeline. That is, participation and access to high levels of mathematics in school, majoring in mathematics or mathematics related fields,

and pursuing professions in mathematics related fields. Supporting students to participate in the mathematics pipeline means students need to *achieve* in mathematics-this means performing well in courses and on assessments that have historically been used to measure mathematical proficiency. This includes standardized tests at the school, district, state, and national levels. Additionally crucial in this approach to equity is the *access* students have to the resources necessary for achieving in mathematics. These resources include but are not limited to: opportunities students have to a wide range of courses (e.g. does the school offer an AP calculus class?), after school tutoring, high quality teachers, up to date textbooks, technology, etc. Central to this approach to equity is that participation in mathematics is helpful for the students because it provides them with access to better futures through better paying jobs. Implicit in this approach is that participating in the mathematics pipeline makes students' lives better. In many ways, this is aligned with Bell's (1992) acknowledgement that while "peaks-of-progress" (p. 12) are possible, White dominance, and therefore the norms of White supremacy, namely a valuing of one's upward mobility in a capitalist society that values the advancement of mathematics and technology, will persist. Thus, we must prepare students who have been historically marginalized with access to the tools, knowledges, and ways of being that they need to "play the game" (Gutierrez, 2013, p. 52) according to the structure that currently exist.

A second approach for understanding equity in mathematics education is what Gutierrez terms the critical axis. This approach aims to support students to develop identities as mathematicians that is harmonious with the identities students bring into the classroom. That is, that students are able to engage in mathematics in ways that does not feel separate from who they are as people outside of the mathematics classroom. This means that as students engage with mathematics, the mathematics content and focus are relevant to student lives and can be used as

a tool to interrogate systems of power. This could mean providing students with opportunities to see the mathematics in schools as a particular form of mathematics that is privileged by dominant society and that there are other ways of engaging in mathematics that are used in other cultures (e.g. ethnomathematics). This can help students understand that mathematics can be more relevant to their lived and cultural experiences, especially for students who do not typically see themselves represented in school curricula (François, 2009). In addition to complicating what counts as mathematics, this approach to understanding equity takes the stance that increasingly diverse student participation in mathematics improves the field of mathematics itself. A diversity of experiences and perspectives allows mathematics to become a better field. That is, that while narratives about equity that take a dominant stance begins with the assumption that mathematics will improve the lives of marginalized students, the critical axis begins with the assumption that the experiences, perspectives, and histories marginalized students bring with them will improve mathematics. For example, modeling is a meaningful practice in mathematics. A person's approach to modeling problems is often limited by, in part, the assumptions we developed through the collection of our life experiences. Homogenous groups of people working on a problem may not have access to other ways of thinking or being that could allow for different approaches to solving or modeling a problem. The experience of Mary Jackson, popularized in the recent film *Hidden Figures* paints Mary as a genius Black woman who was able to solve mathematical problems that allows human beings to travel into space. While Jackson certainly was a mathematical visionary, her positionality as a Black woman provided her with life experiences that were distinctly different than those of the White men she worked with. Jackson brought more than diversity of personhood, but also a diversity of experience, thought, and problem solving that lived outside of the paradigms of those around her. Thus, diversity adds to

what we can accomplish in mathematics. This perspective challenges the status quo (and Whiteness) because it takes the assumption that having more diverse perspectives, life experiences, axiologies, and epistemologies improves any endeavor (Center for Community Organizations, 2019). The underlying assumptions within the critical dimension of equity is that mathematics can and should be used as a tool for social change and mathematics education should uplift student identities in ways that challenge the status quo, including a shift to understand that diversity is an asset.

While seemingly at odds with one another, Gutierrez proposes that these two approaches are equally important. That is, that students need to be supported to both perform well in school mathematics as well as learn to use mathematics as a tool for social change. Given the deep socialization of the assumptions about mathematics (e.g. the neutrality and universality of mathematics; ideas about quickness as the same as smartness; the existence of “math people”) teachers are often more readily able to implement teaching that attends to the dominant axis than the critical (Gregson, 2013; Rubel, 2017). This challenge for teacher learning suggests teacher educators likely also find it more challenging to support teacher learning around the critical axis.

These two seemingly oppositional goals are seen in the writings of other prominent scholars. Gloria Ladson-Billings (1995) describes a Culturally Relevant Pedagogy in which effective teachers of Black youth attend to three aspects of student learning: academic achievement, cultural competence, and sociopolitical consciousness. We can see clear connections between the dominant (academic achievement) and critical (cultural competence and sociopolitical consciousness) axes of Gutierrez’ framework. Gutstein (2007) names four aspects of social justice mathematics education: academic success, sociopolitical consciousness, a sense of social agency, and positive cultural and social identities. We can see academic success fits

under a dominant approach to equity and sociopolitical consciousness, a sense of social agency, and positive cultural and social identities align with a critical approach to equity. Like Ladson-Billings and Gutstein, Gutierrez's work equally forefronts both the dominant and critical axes of equity, acknowledging the challenge of holding these two seemingly opposite objectives simultaneously and, in another example of how diversity of cultures, experiences, and ideas support improvement, offers a different paradigm for engaging in both the dominant and critical that reframes this opposition.

Gutierrez draws on the work of Gloria Anzaldua to offer a rationale for holding two seemingly conflicting agendas. This work, Anzaldua suggests, is both possible and necessary for the advancement of a more just society. Anzaldua (2002) offers the term *Nepantla*, a Spanish word that embodies "the overlapping spaces between different perceptions and belief systems" (p. 541). Said another way, *Nepantla* is "the uncomfortable space where there is no solid ground, that has no official recognition. That contributed to the expansion of new ways of asking questions, new theories, more interdisciplinary approaches to understanding the world around us" (Gutiérrez, 2012, p. 35). In a society that encourages binary categorization of ideas, people, and things, *Nepantla* is a state in which we sit with uncertainty between ideas, places, people, etc. in an attempt to better understand those ideas, people, and places and the relationships between them. Through this process, of choosing to sit with uncertainty rather than mark things as the same as opposite, we are able to imagine new ways of being, doing, and seeing. This process is called *conocimiento* (Anzaldua, 2002). *Conocimiento* "requires that you...confront what you've programmed yourself (and have been programmed by your cultures) to avoid...to confront the traits and habits distorting how you see reality and inhibiting the full use of your facultades" (Anzaldua, 2002, p. 541). That is, "conocimiento questions conventional

knowledge's current categories, classifications, and contents" (Anzaldua, 2002, p. 541) and thereby enables one to see things that were previously in opposition in new ways. The critical and dominant then, do not have to be diametrically opposed. This way of processing binaries can help people envision new ways of being and new ways of moving equity agendas forward.

Given this lens, the paradox one might feel about undertaking a commitment to develop both dominant and critical aspects of Gutierrez's axes of equity can be seen as an act of *conocimiento*, something that can lead to new ways of existing in the world and new ways of shaping student experiences in mathematics classes. This is particularly key because as we have seen in past civil right progress, strides towards equity have primarily been in service of those who were already in power. Critical Race scholars term this phenomenon interest convergence (Bell, 1992). Integration in schools, for example, was only made a requirement when there was pushback on the US government's war on communism, citing that the US had no grounds to dictate how other countries should operate when their own Black citizens had extremely different educational experiences than White citizens (Bell, 1973). Another example of this phenomenon can be seen in statistics that despite widespread narratives that affirmative action benefitted African Americans most, White women have obtained the majority of affirmative action positions (Guy-Sheftall, 1993). Thus indicating, yet again, that seemingly progressive changes to address racism in policy occur when there is interest convergence for the benefit of White people. Given past attempts for progress have tended to continue to protect the status quo, a new way forward is necessary. *Conocimiento* proports that by holding things in tension, we can arrive at new ways of existing that previously were not conceptualized. This is necessary in justice and equity work as older and current ways of approaching equity have not been able to sustain social change.

I was interested in learning about how doctoral students learn to become not just mathematics teacher educators, but ones who are committed to equity within their mathematics teacher educator roles. Gutierrez's work on equity in mathematics supported me in two ways in this dissertation. First, I relied on this definition of equity as I did initial investigations of mathematics teacher education programs and PhD programs, looking for programs that suggested an interest in cultivating mathematics teachers committed to both dominant and critical axes of equity Gutierrez described. Second, I used the critical and dominant definitions of equity in a descriptive analysis of doctoral student interviews to make sense of the ways participants were understanding what it meant to be an EOMTE. More details about this process and its use are outlined in the chapters three and four.

Situated Learning

My interest in the processes and experiences of doctoral students becoming EOMTEs is, at its core, an inquiry about learning. In this work, I conceptualize learning using a situated learning framework. Situated learning takes the stance that learning is intimately linked to participation that occurs within particular practices (Lave & Wenger, 1991). That is that when we are learning we are seeking to adopt increasingly more sophisticated ways of participating relative to a collective of do-ers of a particular activity. For example, learning to become a mathematician typically begins with learners engaging in enterprises like solving problems with known solutions. As a mathematics learner continues to develop into a full-fledged mathematician, they engage in mathematical practices that become more advanced (e.g. solving more complex problems, attending conferences to learn about advances in those solutions, presenting their own solutions to other mathematicians, etc.). This positions learning as more than an intellectual process alone. It suggests learning includes one's advancing participation and

sense of identity as a mathematician or EOMTE with others who are also mathematicians or EOMTEs. This theory of learning attends to participation, membership, and identity in ways that allow us to make sense of how doctoral students learn to become EOMTEs.

Lave and Wenger (1991) use the term *communities of practice* (CoPs) to describe “a set of relations among person, activity, and the world over time and in relation with other tangential and overlapping communities of practice” (p. 98). Said another way, CoPs exist in the interactions and relationships between people who engage in cultural practices together. These cultural practices can be formal or informal, but must meet three criteria: mutual engagement, joint enterprise, and shared repertoire (Wenger, 1999). Mutual engagement suggests members engage in practice together. These might be formal meetings in an academic department or informal interactions like family celebrations of birthdays. Joint enterprise refers to the common goals and practices that the group engages in. Though these goals can range in nature, the joint enterprise is what member of a CoP identify, either explicitly or implicitly through their actions, as the goals of the group. Last, shared repertoire refers to the ways of engaging that are established as normative for the group. This can include “routines, words, tools, ways of doing things, stories, gestures, symbols, genres, actions, or concepts that the community has produced or adopted in the course of its existence, and which have become part of its practice” (Wenger, 1999, p. 83). That is, what defines a CoP is a collective of members working together towards a shared purpose who use common tools and language. CoPs are defined by the shared practices its members engage in and can range in formality but have high levels of interaction between its members (Wenger, 1999; Hoadley, 2012). CoPs are “persistent, sustained social network of individuals who share and develop an overlapping knowledge base, set of beliefs, values, and experiences focused on a common practice” (Barab & Duffy, 2012, p. 41). A mathematics

methods team at a university, for example, can operate as a CoP when its various members work collectively to plan and implement preservice teacher (PST) learning around how to teach mathematics to students. This collective would likely share goals related to PST learning and have developed ways of interacting, structuring learning, and use particular jargon in these interactions.

A situated lens is particularly well suited for this study as the transition from classroom teacher to doctoral student training to become a mathematics teacher educator is one that requires a shift in practices. Doctoral students learn to become EOMTEs in universities through their participation in multiple CoPs where collectives of faculty, staff, and other doctoral students support teacher learning and forward research about teacher learning. I was is interested in the various CoPs doctoral students are members of and how those CoPs support their learning to become EOMTEs.

Constellations of Practice

Literature that takes a situated lens for understanding learning often describes ways new members develop identities within a particular group (e.g. basketball identity development while on a particular basketball team (Nasir & Hand, 2008)). Many identities, EOMTEs included, are developed across a person's participation in various related CoPs and have implications for the ways individuals participate in these CoPs. As such, it becomes important to consider not only the one or two mathematics teacher educator CoPs doctoral students join, but also the more tangential CoPs they are a part of that also inform how they see themselves as EOMTEs. This idea might be made clearer with an example. A doctoral student engages in various experiences and various communities of practice in their doctoral program. Some of these experiences are more tightly tied to MTE, for example TA-ing or being the instructor of record for a mathematics

methods course. Others might be more loosely tied to the actual work of mathematics teacher education, but may contribute to one's thinking about teacher education and therefore be connected to the decision making and thinking one does when in their methods course. For example, a doctoral student might take a class on Critical Race Theory. While the content and conversation of this class may not address the day-to-day work of a mathematics teacher educator, or even education at all, ideas from the course may provide a doctoral student with a lens that they use as an instructor of PSTs. Other examples of these spaces include informal relationships with peers, advising relationships, grading work, ongoing relationships with others in academia, friendships with teachers, or independent readings a doctoral student chooses to do. All these experiences and relationships contribute to the identities doctoral students develop and influence the ways they participate as they move in and across different teacher education spaces. Doctoral student identities as EOMTEs are intersectional and extend beyond the boundaries of their participation in any one community of practice. Intersectionality (Crenshaw, 1991) as it is described in CRT acknowledges that the multiple identities a person has are simultaneously important for understanding that person's experiences. Crenshaw (1991) writes about the experiences of Black females and emphasizes that these women's experiences are unique to the intersection of their race and gender by juxtaposing their experiences to that of women of different races. A core point of Crenshaw's work is that our intersectional identities are influenced by hierarchies of power (e.g. race and gender). While Crenshaw (1991) advocates for an intersectional approach to understanding how a person experiences the world, she also addresses the importance of *anti-essentialism*, noting that while people of the same intersecting identities may experience things similarly, they also are unique and individual people who experience the world in their own unique ways. Situated learning helps us see that we have

identities related to the various practices we participate in. The various and related identities we have that are related to the various and related communities of practice we participate in are called *constellations of practice* (Wenger, 1999). Acknowledging the contributions of experience in multiple CoPs on the identity development of doctoral students is important for understanding the messy process of learning to become something as complex as an EOMTE.

I conceptualizes the various learning experiences these doctoral students have (both inside and outside of the university context) while enrolled in the PhD as a constellation of practice (Lave & Wenger, 1991). These constellations of practice are comprised of the doctoral student experiences across multiple CoPs that are related and connected by and through the experiences of each doctoral students. As such, this research considered the constellations of practice that doctoral students are connected to that relate to their equity orientations in mathematics teacher education to understand how their identity as EOMTEs are cultivated and negotiated across these spaces.

Legitimate Peripheral Participation and Identity

While this paper uses situated learning as way to understand doctoral student learning to become EOMTEs, it is also important to consider how learning occurs within particular CoPs. Legitimate peripheral participation (LPP) is a way of understanding how new members learn to become a part of a CoP (Lave & Wenger, 1991). LPP is not a best practice nor a pedagogy, but rather, is a framework for making sense of the ways new members learn to participate in a CoP. When new members join a CoP, their participation often begins with peripheral roles. Engaging in practices in this way supports new members to learn more about the practices of the CoP which include the shared language, routines, practices, values, and goals of the group. As new members continue developing proficiency in peripheral practices and come to understand the

common practices and mutual enterprise of the CoP, they begin to take on increasingly advanced practices, thereby advancing their own membership. What it means to participate in advancing ways is largely determined by the social and cultural norms of the CoP. As such, the goal of LPP is to support members to move from more peripheral (or novice) forms of participation towards full participation in a community of practice. Through the advancement of participation, new members also develop stronger identities with the CoP.

LPP can take many forms and is particular to social and cultural contexts. Rogoff, Paradise, Arauz, Correa-Chavez, and Angelillo (2003) provide an example of LPP, noting the social and cultural nature of this learning process. In this mode, new members “observe and listen with intent concentrations and initiative” (p. 176) in order that they might learn about the nuances of cultural practices they are to be embedded in (Rogoff et al., 2003). Through this process, novice members become “increasingly deeply involved” (p. 183). This form of learning to participate was observed in children in some cultural groups, suggesting that intent participation is a cultural process of LPP.

Successfully joining a CoP is indicated by what Wenger (1999) terms *competent participation*. Competent participation is participation that includes mutuality of engagement, accountability to the enterprise, and negotiability of the repertoire. That is, the ability to engage with other members in meaningful cultural practice, to understand the goals and purpose of the CoP in ways that allow for a member to “take some responsibility for it and contribute to its pursuit” (p. 137) including the reification of the CoP, and the status to propose changes to the practices of the community. While competent participation is demonstrative of the development of learning, also central to full participation is “an increasing sense of identity” (Lave & Wenger,

1991, p. 111). Conversely, when a new member is not able to advance their membership, aspects of their identity as a member in the CoP is likely limited.

It is important to continue to emphasize that LPP is not a best approach to joining a community of practice, but rather is a lens for understanding how a person joins that can either be productive or unproductive. For example, Lave and Wenger (1991) describe a CoP of butchers in grocery stores and the limitation of the induction of new members when job responsibilities were siloed in ways that new members are unable to advance their participation. Additionally, the absence of interactions between more advanced and novice members, lack of ability to negotiate practice, and the lack of reification of the practice prevented new members from developing identities within the community of practice.

Identity

Developing identity with a CoP is related to one's ability to competently engage in practice. As new members advance their participation, they also advance their identity. In this way, identity and learning are linked. Barab and Duffy (2012) summarize this idea, "there are no clear boundaries between the development of knowledgeable skills and the development of identities; both co-arise as individuals participate and become central to the CoP" (p. 33). This positions identity as an ongoing process that is "constantly renegotiated" (Wenger, 1999, p. 154) through participation. The identity members form in CoPs largely depends on opportunities for engagement, alignment, and imagination.

Engagement, Alignment, and Imagination. Descriptions about membership frequently include narratives about participation and identity. While members have a variety of reasons for joining a CoP, a member's ability to develop a sense of identity with a CoP is important for advancing their learning and participation (Engeström, 2009; Gau, 2014; Greeno & Engeström,

2006). Wenger (1999) offers three analytic tools for making sense of the identity formation of members in CoPs: engagement, alignment, and imagination that can aid an analysis of doctoral student learning to become EOMTEs. We might consider how doctoral students are developing a sense of self that crosses boundaries of communities of practice into a larger constellation of practice.

Engagement suggests that identity formation depends on the extent to which members have opportunities to have “active involvement in mutual process of negotiation of meanings” (Wenger, 1999, p. 173). That is, that members can contribute to the evolution of a CoP’s practice. In engaging in this negotiation, members might identify themselves along different trajectories based on their opportunities to engage in meaningful practice (e.g. if not able to contribute to the group’s practice meaningfully, the member might only be able to access the CoP along a peripheral trajectory) (Nasir & Hand, 2006). Through experiences in engagement, these members develop a shared history of learning and evolution of practice that contributes to a member’s forming sense of identity with the CoP.

Alignment refers to the coherence across the individual’s beliefs and practices and the practices of other members of the community. That is, that the extent to which a member can coordinate “their energies, actions, and practices” (Wenger, 1999, p. 179) with other members has implications for the alignment a member feels to the CoP.

Imagination describes a member’s ability to conjure “images of the self and images of the world that transcend engagement” (Wenger, 1999, p. 177). That is, that imagination is about being able to see oneself in a range of advancing practices and roles within a CoP. In part, having access to other members with a diverse range of memberships and along different trajectories is

important because it allows new members to observe a full sense of the culture and practices of the CoP and imagine themselves in different roles.

While these three constructs are separate, Wenger describes them as “complementary” (Wenger, 1999, p. 187) because together, they support identity development. This treatment, of engagement, alignment, and imagination as separate entities, suggests those who support learning consider the opportunities learners have along these three constructs separately. That is, one should consider how to provide meaningful opportunities along these three constructs in way that do not compromise the others. This is evident in studies that take up this framework to understand learning and identity development that describe engagement, alignment, and imagination as separate constructs (Nasir, 2002; Au, 1997; Adema, et al., 2019). Looking at the multiple opportunities doctoral students have related to the engagement, alignment and imagination as an EOMTE across multiple CoPs provides us a way of seeing the complex process of learning.

Conclusion

In this chapter, I described CRT and used it as a lens for making sense of how mathematics education is a political field. I provided a definition of equity that informed the participant recruitment, data collection, and analysis processes that will be further unpacked in the next chapter. I then described a theoretical framework for understanding learning that relies on understanding learning as mediated through participation in communities of practice. This understanding of learning is used to explore the kinds of learning opportunities and experiences doctoral students have that facilitate their becoming EOMTEs. Next, I highlighted how identity development is cultivated through opportunities for engagement, alignment, and imagination. These constructs are used to make sense of the experiences doctoral students have in becoming

EOMTEs in the various communities of practice they participate in. The next chapter describes the methodological approach used for this study's design including data collection and analysis.

Chapter Three: Methodology

I take a qualitative approach to understanding how doctoral students learn to become EOMTEs. A qualitative methodology is appropriate for this study because it is centrally concerned with “how people interpret their experiences, how they construct their worlds, and what meaning they attribute to their experiences” (Merriam & Tisdell, 2016, p. 6). The questions I am asking are concerned with the ways doctoral students learn to become EOMTEs by analyzing doctoral student narratives about their experiences and when possible, observations of doctoral students engaging in learning opportunities they describe as meaningful.

More specifically, I draw on case study tradition. This method provides an “in-depth description and analysis of a bounded system” (Merriam & Tisdell, 2016, p. 37) that allows for a thorough description of a small number of cases while also allowing for each case to be complex. Each case is bound by the experiences of the doctoral student as each doctoral student interacts with people and the institution social, cultural, historical, and political ways. While no two doctoral students have the same experiences, there are aspects of doctoral experiences that are shared. In order to be able to speak to what is particular to a context and what is more general, I provide multiple cases to identify how successful induction is experienced in specific and across various settings (Stake, 1995). As a reminder, the two research questions I ask in this dissertation are:

- What CoPs support doctoral student learning to become EOMTEs?
- How do doctoral students develop identities as EOMTEs?

In what follows, I describe the setting and participants in this study, data set, and analytic process or addressing these two questions.

Research Setting and Participants

Site Selection

I identified three focal institutions with the assistance of my committee. After discussing my research interests, my committee produced a list of 25 R1 institutions they believed could be reasonable sites to study based on their knowledge of the research interests of faculty at those institutions. It is important to note R1 institutions were the only sites considered because doctoral programs are not available at other types of academic institutions. Purposeful sampling was then used to select three focal sites. This method for sampling was most appropriate given I draw on a specific definition of equity in mathematics education and a desire to learn about doctoral student learning than was (1) productive and (2) had an equity-orientation within mathematics teacher education. The identification and recruitment of participants to this study occurred in two phases, first focused on university sites and second, on individual doctoral students. To begin the process of identifying possible sites where there was (1) development of mathematics teacher educators and (2) a focus on equity in mathematics teacher education, I compiled information about each of these 25 sites including information about their teacher education programs, mission and vision related to the kind of teachers they aspire to support the development of, the various roles doctoral students have related to the teacher education programs (TEPs), and the role faculty play in TEPs and in doctoral student learning. From this initial list of 25 sites, I narrowed my list down to seven institutions of interest based on (1) explicit commitments to equity and (2) doctoral opportunities to work in TEPs. The intentions behind these criteria are that if a program, and the mathematics education part of a TEP, is committed to forwarding equity agendas, then doctoral students involved in those programs are more likely to have learning opportunities around becoming EOMTEs.

Once the seven sites were identified, I reached out to either faculty or directors of programs at each of the institutions to learn more about doctoral student learning opportunities to become EOMTEs. This set of seven institutions was further narrowed to four based on program comfort with being studied on the experiences of doctoral students learning to become EOMTEs and returned communication. Of these four sites, three agreed this study would be able to learn from the experiences of the doctoral students and allowed me to recruit individual students (described in the following section). The fourth institution suggested they were not confident in their ability to support the development of EOMTEs and was removed from the potential study sites. In what follows, I provide a brief description of each of the focal institutions including information about the racial demographic of the undergraduate student population (often demographic information of the graduate school or the college of education specifically were not available), number of faculty in mathematics education, the number of doctoral students in mathematics education, approximate size of the university, and the private/public status of the school. Specific details that might compromise the anonymity of these institutions (e.g. geography) were intentionally omitted from these descriptions.

Table 2

Key Features of Focal Institutions

	Colos University	Rickard University	Seneca University
Population of White students (Undergraduate except when noted)	47%	62%	70% (undergraduate); 87% (all graduate students across university)
Number of faculty in mathematics education	3	10	2
Approximate number of doctoral students in mathematics education	10	100	2
Size	Medium	Large	Large
Private/Public	Private	Public	Public

Rickard University. Rickard University is a large public predominantly White institution (PWI) whose student population is 62% White. As such, the undergraduate and masters level teacher education programs Rickard serve a mostly White student population and most students in the graduate school are also White. Rickard University's COE has a relatively large number of faculty (>10) whose work focuses on Mathematics Education, although these faculty belong to multiple departments across the College of Education (STEM education and teacher education). Each year approximately 25 new mathematics education doctoral students join the program. Due to its large size, mathematics education doctoral students at Rickard have little interaction with other mathematics education doctoral students above or below their starting year except for in a few informal spaces (e.g. Black graduate student union).

Colos University. Colos University is a medium sized private university that offers teacher education programs for masters students that are approximately 47% White. There is no undergraduate teacher education program available at Colos. There are three faculty who support doctoral learning in mathematics teacher education at Colos. As such there are relatively low numbers of doctoral students in mathematics education. Given these low numbers, in addition to institutional structures that provide time and space for doctoral students in mathematics education to regularly meet, doctoral students in mathematics teacher education at Colos have many opportunities to interact with one another.

Seneca University. Seneca University is a large public PWI whose college of education serves a White undergraduate and graduate population of 70% and 87%, respectively. The mathematics education department at Seneca is extremely small, comprising of two faculty.

Given there are only two doctoral students in mathematics education at Seneca and they share an advisor, these doctoral students worked closely in coursework, research, and advising.

Participant Selection

Once the focal sites were established, TEP directors and faculty at each of the three focal sites provided guidance on how to best recruit students at their institutions. These directors and faculty assisted in this process by sending introductory emails between myself and doctoral students they knew were interested in issues of equity in mathematics teacher education. I then set up one-on-one meetings with each doctoral student to explain the project and the data collection process. During these meetings, I explicitly shared that I was interested in learning about their experiences becoming EOMTEs. In some cases, doctoral students nominated peers who they thought might also be a good fit for the study. In the end, I was able to recruit a total of eight doctoral students across three institutions.

Participants by Institution

I provide brief descriptions of each doctoral student and their path to enrolling in mathematics education PhD program. Identifying details that would compromise the anonymity of participants were intentionally omitted in these descriptions (see Table 3 below provides a snapshot of the participants' demographic

Table 3*Summary of Participants*

Participant	Institution	Years in PhD Program	Gender	Race	Other Identities Participants Shared	Prior Career/ Experiences in Education
Maya	Rickard	4	F	Black	First generation college student	High School Mathematics Teacher Community College Mathematics Teacher
Esther	Rickard	4	F	Black		High School Mathematics Teacher
Marvin	Rickard	2	M	Black	Family of academics	High School Mathematics Teacher Mathematics Instructional Coach
Anna	Colos	4	F	White	First generation college student	High School Mathematics Teacher
Naomi	Colos	3	F	White	Jewish	Middle School Mathematics Teacher
Grace	Colos	2	F	Asian		Elementary Teacher
Kemé	Seneca	2	Non-Binary	Latinx/Indigenous	Queer	High School Mathematics Teacher
Manny	Seneca	3	M	Asian		High School Mathematics Teacher

Rickard University.

Maya. Maya is a Black woman who is also a first-generation college student. Maya attended an HBCU for both her undergraduate and master's degrees. In those settings, Maya had multiple mentors encourage her to pursue a PhD. At the time of data collection Maya was in her fourth and fifth years of her doctoral program. In a career teaching mathematics at both the high school and community college levels, Maya became concerned with the type of instruction her students received when in other teachers' classrooms. These teachers largely drew on a lecture-

based model of instruction in which students listened to 75-minute lectures then went on to work on similar problems independently. In these classrooms, students had “no interaction” with the instructor or other students and received little to no feedback on their learning of mathematical concepts. As such, mathematics was taught in ways that were devoid of any “relationship or...connections.” This approach, Maya knew, was likely causing many students to struggle through mathematics courses.

Maya decided to pursue a PhD in mathematics education to have a wider impact on the kinds of experiences students were having in mathematics classes and to do research that elevated the voices of “underserved populations,” focusing particularly on the mathematics learning experiences of Black girls. As a doctoral student, Maya has been able to teach multiple courses in Rickard University’s TEPs.

Esther. Esther is a Black woman who attended an HBCU for her undergraduate degree before going on to earn her masters, obtain a teaching credential, teach mathematics at the high school level, and enter a PhD program. At the time of data collection, Esther was in her fourth and fifth years as a doctoral student. Esther’s motivation for joining a PhD program was to be part of a larger system of change. In her role as a classroom teacher, Esther was confident in her ability to meet the needs of every one of her students. Despite this confidence, Esther wondered if there was more for her to learn and if a teacher educator role might allow her to have a wider impact on the quality of experiences students have in mathematics classrooms by supporting new teacher learning. As a doctoral student, Esther took on numerous roles that allowed her to develop her identity as an EOMTE. These experiences have spanned her own research and work as a teacher educator in multiple roles throughout the college of education.

Esther's commitment to caring for all students is driven by what Esther calls the "mantra of Black women," that is, "if we don't do it, who will?" Here, Esther is suggesting that the most marginalized in society will not be cared for except by Black women. This has been both Esther's lived experiences as well as a common theme in the documented experiences of Black women throughout history (Collins, 2002). Given this perspective, Esther shared that she is sometimes at the margins of the types of discourse and stances that are normative at Rickard University. People outside of Rickard University are often surprised that Esther is a doctoral student at Rickard University because, as Esther says, "I talk the way I talk and...I'm very, very pro-Black," meaning that this stance of being *pro-Black* and speaking in ways that do not "fall in line" with the status quo, something Esther says is typical of "the type" of people of color at Rickard University. While sometimes at the margins in a predominantly White setting, Esther has found a community of Black folks she describes as a "lifeline." This is important, Esther shared, because without the community of other Black folks with whom she can openly share her ideas and frustrations, she questions whether she would still be in this PhD program.

Marvin. Marvin is a Black man who attended Rickard University for two undergraduate and one master's degrees. Through his experiences at Rickard University, Marvin found Black mentors in education who provided ongoing encouragement to continue to "apply himself" and pursue advanced degrees. At the time of data collection, Marvin was in his first and second year of the PhD program.

In Marvin's K-16 education his intelligence and presence in advanced mathematics courses was constantly questioned. From his earliest days in elementary to the mathematics courses he took as an undergraduate, his status as a competent and capable being were challenged despite his ability to perform well on the assessments given to him. As an

undergraduate, Marvin noticed that other Black and Brown students, who were in the top of their classes in High school were struggling in remedial mathematics courses at Rickard University. This, Marvin concluded was because of a lack of a rigorous mathematics education offered to many Black and Brown students in K-12 schooling. And so, Marvin decided to become a high school mathematics teacher. Beyond supporting students to learn mathematics in ways that would prepare them for college level mathematics, Marvin also saw that as a mathematics teacher he could (1) provide an example for students that a Black man can be a mathematician, something he never had as a student, and (2) be an advocate for students who, like him, were harassed by a system that refused to acknowledge Black brilliance in mathematics.

After years of teaching at the high school level, Marvin went on to become an instructional coach for mathematics teachers. This role supported Marvin to see teacher education as an important path towards improving the learning experiences of Black and Brown students. This, paired with the ongoing encouragement he received from both family and mentors to pursue a PhD led Marvin to Rickard University for a fourth degree. As a doctoral student, Marvin has had opportunities to work with both in-service and preservice teachers in mathematics in professional learning spaces.

Colos University.

Anna. Anna is a White woman and first-generation college student who, at the time of data collection was entering her fourth year in the PhD program. Anna attended a large public institution for her undergraduate degree before pursuing a master's degree and credential from a private university. Anna shared that she became a high school mathematics teacher because when she studied mathematics as an undergraduate student she was exposed to the “transformative” power of mathematics. For the first time in mathematics courses, Anna was

pressed to consider the question: “What counts as doing mathematics?” For Anna mathematics became less about “procedures to memorize and regurgitate” to a field that involved “critical thinking and logic and making connections.” In addition to this understanding of what mathematics is, Anna worked as a tutor for low-income first-generation college students. Students in this program were enrolled in pre-baccalaureate classes which meant “they [would not] get college credit for taking these classes” regardless of the grades they earned. Anna’s noticed the majority of students in these situations were people of color and came to see mathematics education as deeply racialized. This motivated her to pursue a career as a mathematics teacher.

After years of teaching high school mathematics, Anna became interested in pursuing a PhD in mathematics education because she saw that while she could have an impact on students in a year, those students would have other teachers that would put her students through what she called “math trauma.” Anna decided to pursue teacher education “to be a part of the movement that gets more like-minded teachers in the classroom. Especially in schools where they can have the greatest impact which are typically places where there’s not a whole lot of privilege.” That is, Anna saw a PhD program as a way to positively impact mathematics education for students who have been historically marginalized at a larger scale. In her PhD program, Anna has been able to participate in both teacher education work at the undergraduate and master’s level as well as engage in research that focuses on mathematical identity development in students who are typically marginalized in mathematics related fields.

Grace. Grace is a Korean-American woman who previously taught at the elementary level. At the time of data collection, Grace was in her first and second years of the PhD program. The school Grace worked in partnered with mathematics education researchers who supported

teachers to “engage in a shift in practice” to include more collaborative approaches to mathematics. Through these interactions, a few researchers broached the possibility of Grace enrolling in a PhD program, sharing that they knew Grace “would be a really great candidate.” Grace shares that after having a conversation with doctoral student at Colos University, she applied for two reasons. First, to work with her current advisor because Grace knew they had shared values and commitments related to students and supporting learning and, second, because the administration changes at Grace’s school left her feeling burnt out.

When asked what she hoped to get out of a PhD program, Grace shared that she wanted to learn more about teacher education and how to be “healthy while being a teacher” dedicated to equity and justice. As Grace entered the PhD program she found it helpful to be in a place where she was “supporting and mentoring other women of color in the profession.” For Grace, this passion was a way for her to share her growing knowledge about what being “healthy” while also being dedicated to equity and justice as an educator with other women of color. In part, this was motivated by Grace’s own reflection about her experiences as a woman of color who “felt unsupported, experienced microaggressions, [and] racism” in teacher education spaces. As such, she wanted to be “a supportive figure for teachers of color who are really egregiously underrepresented...in preservice space and in-service spaces.” In her doctoral program, Grace has been able to work in pre-service teacher education and Research Practice Partnerships with a local district working on improving mathematics instruction.

Naomi. Naomi is a White woman who at the time of data collection was in her second and third years of her doctoral program. Naomi began her path to becoming a mathematics teacher through an alternative, fast-tracked credential program. During her time in this program, Naomi struggled to find meaningful support and encountered personal health issues that led her

to leave the fast-tracked credential program. Naomi then spent the next few years working with students through a non-profit organization that affirmed her desire to become a teacher. This led Naomi to seek out a “justice-oriented” teacher education program. Her interest in finding a program that centered issues of justice in education stemmed from her training as an undergraduate major in African American Studies and her experiences working in an educational program that partnered with middle school students in under resourced schools. It was in these settings Naomi learned to be critical of institutional structures and traditional approaches to learning mathematics that focused on rote memorization and developed a desire to learn more about a constructivist approach to learning, although many of the examples she saw related to this were in literacy and social studies. In her new teacher education program, Naomi was inspired to become a mathematics educator who could integrate the ideas she valued from the constructivist approach she was exposed to in her undergraduate years.

As a teacher, Naomi worked primarily with students of color as a middle school teacher and used an instructional approach that prioritized collaborative sense making of mathematical ideas. Naomi’s experience in her second teacher education program, alongside her experiences engaging students in mathematics under this pedagogy allowed her to see how a discourse-based approach to instruction could impact the quality of student learning opportunities.

Naomi was drawn to a PhD program after feeling burnt out because of the bureaucracy that surrounded her teaching work. Naomi knew from her research experience as a master’s student that she enjoyed the process of classroom based-action research. She was also, however, frustrated by her understanding of academic research as removed from the realities of the classroom. Naomi has since been inspired by the work of female scholars who have integrated research into meaningful improvements at the classroom level. This motivated Naomi to seek out

opportunities to be able to do “school-based work” along-side research. This path, however, was not one that Naomi always felt confident in following. Naomi shared she felt “intimidated” to apply to a PhD program, saying she felt like she did not have enough “expertise” but was encouraged to apply for a doctoral program by a personal mentor. In the doctoral program, Naomi has been able to work for a teacher education program in a methods sequence for a secondary mathematics teacher education program.

Seneca University.

Manny. Manny is a Filipino-American man who, at the time of data collection was in his third year of the doctoral program. Prior to entering a doctoral program, Manny worked as a high school mathematics teacher. As a teacher, he saw the systems in schools preparing Black and Brown student populations for prison. In looking back on his experiences, Manny describes schooling systems as “essential [about] behavior, labor, control, and management.” That is, that the ways Black and Brown students were treated in schools with regard to behavioral issues, rigid forms of demonstrating knowledge, and the prioritizations of very particular knowledges could be described as a system of control and management.

Manny was drawn to a PhD program because he saw it as an opportunity to learn more about what he was noticing in schools and be a part of eliminating these experiences for students of color. As a doctoral student, Manny was able to draw connections between what he was learning in coursework (e.g. about histories of colonization and oppression) with the practices he had seen in schools (e.g. standardized testing and behavior management). For example, in learning about the Catholic church’s influence in countries like the Philippines to assimilate young Filipinos into Catholicism and English language, including the values and taken for granted assumptions that underpin the English language and Christianity that prioritize, for

example, rugged individualism or guilt, oftentimes telling non-western people their culture, practice, and ways of being are inherently flawed.

As a doctoral student, Manny has had opportunities to engage in research that has helped him analyze, in community with other scholars, the impact of systems of oppression in schools.

Kemé. Kemé is a gender non-binary indigenous person who at the time of data collection was in their first and second years in the doctoral program. Kemé entered academia as a master's student wanting to know more about what they could do to better the experiences of their high school mathematics students of color. As a teacher, Kemé noticed the unfair labeling of students as not-math-abled that they saw deeply linked to unjust and biased perceptions of who their students of color are. After completing a master's program, Kemé advisor recommended they pursue a PhD at Seneca University because of compelling work being done there. Kemé researched the work various faculty were doing at Seneca in mathematics education and found an advisor whose work closely aligned with their interests. Kemé was hoping a doctoral program would offer the time and space they needed to learn more about the systems that were creating the negative experiences they and many of their students had in schools.

Kemé has been guided through their life and academic experiences by family and elder in their own and other nations as well as by particular faculty at Seneca. Through these relationships and Kemé's own life experiences, they have come to understand what it means to be an EOMTE in relation to what it means to be a person committed to justice. That is, that who we are as people in academia must be in relation to who we are as people.

Data Sources

This study focuses on the experiences and perspectives of doctoral students aiming to become EOMTEs. Data was collected from April 2020 through January 2021. During this time, I

conducted initial interviews with each participant, an initial analysis of those interviews, and developed individualized protocols for follow up interviews. In addition to interviews, when it was appropriate and possible, I conducted observations of interactions and activities doctoral students described as meaningful for their learning to become EOMTEs.

Initial Interviews

I conducted and audio-recorded semi-structured virtual interviews with each of the eight focal doctoral students (see protocol in Appendix A). Interviews lasted, on average, 60 minutes. I chose a semi-structured interview because each participants' experiences, while similar in some ways, were unique given various aspects of their context, identity, and path to the PhD. As the interviews occurred, I took handwritten notes about what participants were saying to refer to specific statements, people, or interactions they described in follow up questions. I also audio-recorded the interviews so I could be fully present with the conversation.

While semi-structured, the initial interviews focused on gathering information about three main topics. The participants' personal history, including what motivated them to enroll in PhD programs; current conceptions about what it means to be an EOMTE, including what it means to be an equity-oriented mathematics educator; and naming moments, aspects, or interactions of their experience that they found supportive of their learning. I also asked, as a follow up, which (if any) of these interactions or activities I might be able to observe.

I did not ask specifically about any of the demographic information except for the number of years participants were in the doctoral program and their work prior to entering the doctoral program. All other information, including their gender, race, and first-generation college student status were aspects of identity that doctoral students openly shared when describing themselves, their perspectives, and experiences. I did not explicitly ask about doctoral student

identities that are salient to their becoming EOMTEs because I did not want to impose my own vision of identity on participants during the interviews. Instead, I asked follow-up questions when identities were raised organically by participants. For example, Esther described the importance of her experiences attending a HBCU and the relationships she built with other Black doctoral students at Rickard University. I asked a follow up question about whether she saw it as important to have a Black community within Rickard University and she went on to further describe the importance of this identity to her experience.

Initial Member Check

Following the initial interview, I transcribed and analyzed each interview and created an interview summary form that attended to five categories. The categories are:

1. Prior work, motivation, and expectations: what were the participants doing prior to joining the doctoral program, why did they apply, and what are they hoping to learn and do after getting their degree
2. Definitions of equity with examples
3. Challenges in their experiences learning to become EOMTEs
4. Forms of support: what are the places/people/interactions that support their learning.
5. Types of work related to EOMTE development

Once these were created, I wrote up brief summaries to send back to participants that included their definitions of equity and forms of support. Seven of the eight focal doctoral students responded to this email affirming my summary. The purpose of these member checks were to ensure my interpretations of their interviews were accurately portraying their thinking. The last participant did not respond. Here is an example of an email that was sent to doctoral students after the initial interview:

I hope this email finds you well! I have been listening to your interview and wanted to share back a short summary of your thinking around equity and the places/spaces/relationships you find critical to your becoming an equity oriented math teacher educator. I also wanted to see if you might have time to schedule the 2nd 1 hour interview to talk more about these places/spaces/relationships.

For your thinking about equity-you focus on the structural issues that underpin systems of oppression. In particular, you named things like standardized testing and the school-to-prison pipeline. You see teachers as integral in disrupting these systems and see important work in "turning as many of them against the system" so that change might begin to happen. You shared that in graduate school you've come to see the complexity of this systemic oppression as beyond and bigger than any one aspect of it (e.g. standardized testing). You have also come to understand that as we tear down the system, we need to rebuild and replace those systems in a more just way.

The places/spaces/relationships that have sustained and supported you in this work are:

- 1. Relationship with advisor (and doctoral peer)*
- 2. Your own research*
- 3. Attending mathematics methods courses taught by your advisor*
- 4. Course work and your relationship with [faculty name]*
- 5. Equity and Justice Fellowship*

Observations

It became clear in the initial interviews that many of the spaces, relationships, and activities doctoral students felt supported their learning would be inaccessible to me. There are multiple reasons for this. First, some of the relationships doctoral students described were so personal that it would be inappropriate for me to join. For example, Kemé shared that their interactions with their advisor were so personal that it would be uncomfortable and inauthentic to have an observer join. Second, some of the spaces and activities doctoral students described as sustaining to their beings were racially specific. For example, Maya described the significance of the Black graduate student union to her experience, however, my being a non-Black person makes it inappropriate for me to join. Third, my presence in some activities (e.g. courses) would require the participant to share with their colleagues that they were in a study and anonymity was an important status for many of the doctoral students in this study. Fourth, given data collection occurred during the COVID pandemic, some of the supportive CoPs and interactions doctoral

students described were paused during the data collection period. In very few cases, all members of the interaction agreed to be recorded or observed. As such, I was able to collect observational data from three interactions (see Table 4). Given this data was collected during the Covid-19 Pandemic, all meetings were held via zoom and participants recorded them for me to watch later. The observed events ranged from 60-150 minutes.

Table 4

Summary of Observations

Participant	Observation	Number of People Present	Length of Observation Event
Naomi	Planning session for Whiteness Awareness group with fellow doctoral student	2	60 minutes
Grace	Planning session for mathematics methods course	3	90 minutes
Anna	Instruction for undergraduate course on mathematics education	>20	150 minutes

While few in number, these observations allowed me to observe the nature of the interaction and prepare more specific follow up questions for the second interview.

Interview Two

The second interview was set up with doctoral participants after member checking and any observations. Given that seven of the eight participants responded, I was able to attain second interviews from seven of the focal participants. These interviews lasted between 60-90 minutes and focused on clarifying noticings from initial analyses and learning more about the particular structures, interactions, and relationships that doctoral students describe as helpful for their becoming EOMTEs. Each of these interviews were tailored to the unique experiences each

doctoral student brought up during interview 1. As such, there was no one interview protocol; instead, I created tailored interviews topics for each participant (see Appendix B for an example).

I began these interviews by sharing back (1) the equity descriptions doctoral students described in the first interview and (2) the specific examples about their vision of equity as it related to their work as mathematics teachers and mathematics teacher educators. This was presented to doctoral students in a chart that included notes about where these values, beliefs, and stances were developed (See Table 5 for an example)

Table 5

Example of Participant Descriptions of Equity

Description of Equity at the Classroom Level	As it Relates to Teacher Education	Origin of Thinking
<p>A teacher’s work is to support students to develop a positive math identity. Teachers need to:</p> <ul style="list-style-type: none"> - focus on student ideas, experiences, and interests - make tasks more culturally sustaining, more inclusive, more student centered, ambitious, and inquiry based - be an advocate for students who are marginalized 	<p>A teacher educator’s work is to help PSTs learn to do the left things by:</p> <ul style="list-style-type: none"> - Integrating equity coherently across courses <p>Practicing teacher educators should:</p> <ul style="list-style-type: none"> -Share resources/readings/language that can help move forward abolitionist agendas <p>Research should:</p> <ul style="list-style-type: none"> - focus on the multiply marginalized and their voices - be in service of teachers and students; prioritize their needs 	<ul style="list-style-type: none"> -Prior experiences as POC -teaching in a school that was committed to reforming teaching practice -being a participant in research

Participants were asked to add, remove, or amend any aspect of these tables. In all seven second interviews, doctoral students conveyed the tables created for them accurately represented their thinking and experiences and did not add, remove, or amend any of the text I shared with them. Next, I shared back the salient challenges doctoral students described experiencing in their

becoming EOMTEs in the institution. Doctoral students were similarly asked to add, remove, or change any aspect of the list (see Table 6 for an example).

Table 6

Example of Challenges Doctoral Students Faced

Location	Challenge
All university related work (work with teachers; course work; supervisory work)	Being the lone voice of equity; working with White folks who are <i>uncomfortable</i> speaking about issues related to race. Therefore, issues related to race become participant’s responsibility
Undergraduate courses in institution (TEP)	There are no courses dedicated to equity and diversity
Faculty are underprepared to address issues of equity	Faculty have to integrate equity but do not have expertise in it, and therefore are not well suited to facilitate discussions
Doctoral Student Colleagues	Many of whom are supporting teachers to do complex work without experience in the classroom; but have backgrounds in math. Cannot provide PSTs with a holistic support

Next each doctoral student was asked to describe the pre-identified relationship, interactions, and activities they found to be helpful. The second interview largely forced on member checking to ensure my interpretations of the first interview and observations were accurate, gaining additional information about their experiences as doctoral students related to learning to become an EOMTE, and having doctoral students identify faculty who supported their EOMTE development.

Methods of Analysis

Given the purpose of this study is to understand how doctoral students learn to become EOMTEs, I engaged in multiple phases of iterate data analysis (Merriam, 2009). The first round of analysis was largely descriptive in nature, seeking to understand how each doctoral student made sense of what it means to be an EOMTE. This was important because understanding what doctoral students were hoping to become would have implications for how I would be able to

make sense of their learning opportunities (Stake, 1995). Once I completed descriptions of each participant's conception of equity, I looked across doctoral narratives to identify common themes. The second phase of analysis involved coding transcripts and observations for opportunities for meaningful learning moments. During this analysis, I identified aspects of doctoral students common positive and productive experiences that supported their EOMTE learning. The third phase of analysis looked at the ways engagement, alignment, and imagination were at play in the various experiences doctoral students had with learning to become an EOMTE. Each of the phases are described more thoroughly below.

Phase One

Phase One of analysis began creating interview summary forms (ISFs) for each participant. As described earlier, the ISFs logged analytic notes and quotes about the following five categories:

1. Prior work, motivation, and expectations: what were the participants doing prior to joining the doc program, why did they apply, and what are they hoping to learn and do after getting their degree
2. Definitions of equity with examples
3. Tensions in their work/learning
4. Forms of support: what are the places/people/interactions that support their learning.
5. Types of work related to becoming an EOMTE

Table 7 provides an example of one row of the ISF for category 4: Forms of support. Bold font was used to highlight parts of a quote that stood out to me at the moment the ISF was created.

Table 7

Example of Forms of Support in ISF

	Quote	AN
<p>Work with advisor (and other doc student)</p>	<p>we just decided to treat it as a course where we use those milestone things as assignments for the course. And then also give us a chance to talk about the literature in a more like organized way so that we're kind of you know, keeping keeping a good pace, I guess. So it's worked out good because we've, it's forced us to kind of meet more often and talk, especially now, when it's so isolating</p> <p>we're starting up like a joint advising thing now where we're just going to have the three of us like we're just going to work together, like we both have a paper we're trying to push out. So we're going to share with it's going to be a three way, like, comment on each other's plus advisor. So we're doing that kind of thing. Now, which is the kind of thing that's that was the whole, kind of into this for us signing up for the course with her. So it's worked out good.</p> <p>because I think if [my advisor] wasn't there because she does you know her work explicitly focused on social justice mathematics...But without a math ed. scholar who does equity work in the program. I don't think I would have, I don't think I would still be here. That's a huge because to me it was like, I realized like, once I realized that nobody knew anything about the work that I wanted to do. I was like, this is not going to be helpful. I'm not going to make it through this</p> <p>And like I think to like just knowing that these people all have the same kind of passion as I do about this kind of stuff like that. It helps kind of validate like when I, you know sometimes I feel like I'm overreacting. Right. You know, and then I talk, you know, will mean talking. It's like, yeah, no, no. You're, you should feel that way. And I'm like, Okay. So that you know just having that having that that group to, because it is like you go like we're doing a paper on micro aggressions from this project on navigating,</p>	<p>AN</p> <p>Space to learn</p> <p>Feedback relationship</p> <p>Forwarding work in both academic and personal ways</p>

Once ISFs were completed for each participant, I created a descriptions of equity analysis for each participant and began to look for themes across the participants. During this process, I distilled each participant’s descriptions of equity into themes (see Table 8 for an example).

Table 8

Example of Description of Equity for a Participant

Description of Equity
<p>In TE, be sure to position teachers as having expertise <i>And one of the kind of commitments, I feel like I came into my program with that I'm now kind of realizing is like trying to understand and support the work of expert teachers because I think there's a lot of like deficit view of teachers, especially in teacher ed programs and so, yeah. Did I answer your question that kind of?</i></p> <p>Being explicit about WHO is the focus of equity (naming Black folks) <i>I think one piece is just being explicit about which groups are struggling and why and what are the systems that are that are keeping them behind, so to speak, and like unpacking achievement gap and unpacking that language.</i></p> <p>Equity as connected to pedagogy <i>help students make those connections and like understand kind of have us all be on the same page with pushing people to think in those ways. So that, that's like one piece, but then I also think really, one thing I would love to see and work on because I don't know fully how to do this. But to really support students to see that like all these things we're talking about with teaching math conceptually and listening to students thinking and centering certain thinking, that's equitable pedagogy. Equitable pedagogy, it's not a separate thing. Like our identities are always happening in the classroom, all the time and the decisions we make about which work we sequence when that affects academic and social status. But those things are also related to students multi layered identity.</i></p> <p>Bringing your whole self to the classroom <i>can I ground it specifically in like an equitable learning environment. Because I think there's different ways to think about it, right, like if we achieve equity, you know, then, to me, that means like achievement patterns are not predictable based on specific characteristics like race or gender or sexuality or a lot of different things. So we shouldn't be able to predict based on people's demographics. But that's just like the outcome of equity right so then it's like, well, how do you make an environment equitable so that that can be the outcome. And so I guess to me that I would describe like an equitable learning environment as a place where people, where all people feel like they can bring their full selves and contribute and grow and learn and be uncomfortable and learning process, but still feel some element of like safety. Mm hmm. And can really like. I mean, my main, yeah, I think, to me, the two main things are you can bring your full self and you are pushed to grow and improve in ways that that you're kind of on board with</i></p>

I think I came in thinking more about the learning and the growth part. Oh, yeah. Everyone should be able should have access to learning. Everyone should be supported to learn, everyone- but I think that like being able to be your full self piece, while I thought about it as a teacher, especially with like the age group I had, I was so concerned about, definitely concerned about people feeling, kids feeling comfortable participating and I thought a lot about that. But I think honestly in my, noticing my own discomfort in certain Colos classes made me and feeling like I couldn't bring my full self made me be like oh shit that's I forgot how shitty that feels like I haven't felt this way in a while. I've felt that I'm sure at some point. But like so that actually kind of ironically, like crystallized that for me in feeling like I couldn't be my full self, kind of the other way.

All eight participants' equity summaries were transferred into an excel chart that allowed me to see where there was an overlap in the ways doctoral students were describing what it means to be an EOMTE (See Table 9).

Table 9

Snap Shot of Excel Chart with Summaries of Equity Descriptions by Participant

D	E	F	G	H	I	J
Big ideas around equity	Naomi	Grace	Anna	Maya	Esther	Marvin
-concrete school based work (not just elistic "bullshit, academia")						
Focusing on students' ideas, experiences, and interests		X	X			
Coherence in messaging for PSTs -needs to be throughout, not just one day	X	X			X	X
Making tasks more culturally sustaining, more inclusive, more student centered, ambitious, and inquiry based		X				
Focusing on those who are underserved/marginalized/	X	X	X	X		X
Elevating the voices of the multiply marginalized - be an advocate for students who are marginalized	X	X				
Distributing academic power by sharing resources		X			X	
Math should be something useful; should change the world			X	X	X	
Math is a gatekeeper for non dominant groups			X			
Student identity-need to help students cultivate positive (mathematician) identities		X	X	X		
Debunking: students learn at the same pace			X			
Debunking: who is capable of doing math and therefore, positioning students as competent	X		X			X
Debunking: who math is for			X			

The descriptors of equity were then categorized further according to larger categories such as research, self-awareness, the purpose of mathematics education, and addressing equity

which were then uploaded into a Dedoose code book. These codes were informed by a CRT lens. For example, the code “knowing self, knowing bias” was assigned when doctoral students acknowledged a system of inequity that required teachers to unpack their taken-for-granted and normative ways of understanding the world that attempts to mask the political nature of mathematics education. This code, in particular, is informed by doctoral student awareness of the prevalence and pervasiveness of Whiteness. The code “the individual” was assigned when doctoral students spoke about the importance of seeing students for the unique people they are based on their range of experiences, this, I argue is related to intersectionality. These codes were used to recode all collected data (see Table 10).

As I coded the interviews, previously unidentified codes related to equity emerged and were subsequently added to the initial code lists. Once all interviews were coded multiple times, I pulled excerpts from Dedoose that related to each code and identified themes that were shared across multiple participants by addressing the question: how do doctoral students consider equity in relation to their work as EOMTE? I used the excerpts related to the codes to describe, in better and clearer detail, the frequently stated stances related to equity doctoral students shared. Next, I looked across the excerpts and themes and considered, what is common across these in terms of the experiences that surrounded them, the beliefs held, and in particular, how they relate to Gutierrez’s framework for understanding equity in mathematics education. To do this, I looked at each of the doctoral student narratives and identified whether the sentiment behind participant statements related to Gutierrez’s description of equity and the CRT tenets.

Table 10

Codes Related to Equity Descriptions

Code	Descriptor
Equity in MTE	Any sentiment related to how participants see equity in MTE
Research	Sentiments about equity that are particular to research
The (multiply) marginalized	Attending to the “who” of research-and learning more about the experiences of a particular group of folks who have been marginalized in society (and in math ed)
Purpose of research-equity	The purpose of research is to create more equitable educational experiences for students
Self-Awareness and ways of being	Narratives about equity are, at their root, about self-awareness and cultivating a particular way of being as an EOMTE
Compassion	Narratives about ways of being that involve having compassion for students as human beings
The individual	Narratives about the importance of seeing students as unique individuals who have unique experiences, needs, and hopes. Includes anti-essentializing narratives
Knowing yourself, knowing your bias	Narratives that emphasize the importance of knowing yourself, your identities, and unpacking assumptions about social structure, mathematics math education, etc.
The purpose of mathematics education	Narratives about what the purpose of education is
Math as a tool for SJ	Narratives that suggest math should be a tool for social justice; useful and relevant to students’ lives
Other	
Ways of addressing equity issues	Narratives about what doctoral students do/hope to do to support the development of teachers with commitments to equity in math ed
Playing the game while changing it	Narratives that describe PSTs need to learn how to engage in creative insubordination, to be creative in how they teach within the parameter of “the game”
Preparing PSTs for working in “diverse” settings	Narratives that describe PSTs need to learn about what it means to work in a “diverse” setting-typically these narratives focus on things that PSTs have lacked experience in
Sharing power through sharing resources	Narratives that describe participants desire/commitment to share the power/knowledge they gain with others who are currently working in schools

Green-highlighted rows are parent codes, white colored rows are child codes

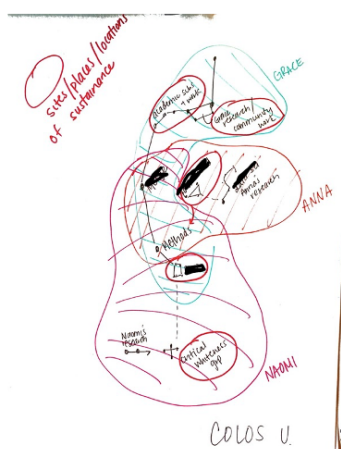
A summary of these noticings is described in Chapter Four.

Phase Two

To learn about the CoPs that supported doctoral students learning to become an EOMTE, I reread each interview and marked where there were participants described participating in a CoP related to becoming an EMOTE. Next, I created maps of the various CoPs within an institution that were present across the narratives of all the doctoral students in each university and added layers to represent how each participant engaged with the various CoPs in that institution in order to map the constellation of practices each doctoral student was a part of (see Figure 1 below). This process allowed me to see the overlap between the CoPs participants brought up in their interviews and where there were commonalities in which CoPs were supporting multiple participants. For example, Figure 1 helps us see that the CoP that supported Naomi's development as an EOMTE lived outside of her interactions with Anna and Grace. Noticing this pattern helped me see that the only space that Naomi found support was in the Critical Whiteness Group, a collective that was peer-created. Names of some of the CoPs are blacked out to protect the anonymity of the site and the doctoral students in the study.

Figure 1

Example of Mapping of Constellations of Practice at Participants at Colos Identified



This mapping enabled me to see the various categories of CoPs doctoral students were members of. The categories that emerged were: teacher education CoPs, Mentor relationships, peer relationships, university at large, and outside of the university. Then, given multiple doctoral students may experience a particular CoP differently, I created + and – codes for each of these learning spaces to mark whether the experience doctoral students shared related to a CoP were helpful or harmful to their identity development as an EOMTE. These codes were uploaded to Dedoose and all interviews were coded. To clarify, positive codes were applied to experiences in which doctoral students described an interaction as helpful to their becoming an EOMTE. For example, mathematics methods planning sessions in which doctoral students were asked to share their ideas and asked questions were coded as positive. Negative codes were applied to experiences in which doctoral students described an interaction as a barrier or distraction to their becoming an EOMTE. For example, teacher education meetings where doctoral students felt they needed to reorient and explain why particular approaches to mathematics education (e.g. tracking) perpetuate the status quo to their supervisors.

I then pulled the positive codes associated with the various interactions and experiences doctoral students had within their institutions and engaged in an iterative writing process that allowed me to identify two distinct kinds of CoPs doctoral students are part of that support their EOMTE development. These two distinct CoPs, institution-based-mixed-member and peer-created CoPs provided unique learning opportunities for doctoral students as they engaged in EOMTE work. Claims from this analysis are presented in Chapter Five.

Given the first research question is about the qualities of CoPs that are supportive to doctoral student learning to become EOMTEs, I did not look at the negatively coded excerpts for the analysis of question 1. I did, however, return to the negative codes for each participant to

make sense of why some CoPs were helpful while others were not. In particular, this analysis helped me see that peer-created CoPs were initiated because of an accumulation of negative experiences in institution based CoPs. This claim is elaborated in Chapter Five.

Phase Three

To address the second research question: How do doctoral students develop identities as EOMTEs?, I added codes engagement, alignment, and imagination to the code book established in the Phase Two analysis. These codes were added based on the descriptions of equity offered by Lave and Wenger (1991). An analysis of how these constructs were at play in interactions and participation within particular CoPs allowed me to make sense of the ways doctoral student identity along the construct engagement, alignment, and imagination was developed (or hindered). In addition to coding excerpts for engagement, alignment, or imagination, I also applied positive or negative codes. An excerpt was labeled positive if doctoral student descriptions indicated this aspect of identity development forwarded their identity as an EOMTE. For example, Marvin described his participation in a mathematics teacher education CoP in which he was able to voice his beliefs and stances about issues of equity in mathematics education and enact those beliefs in his interactions with teachers in professional development settings. This instance was marked “engagement positive” because it was a meaningful opportunity for Marvin to engage in the practices of an EOMTE.

Once I mapped these experiences for all participants, I looked across these opportunities for engagement, alignment, and imagination and identified similarities across the ways engagement, alignment, and imagination operated for doctoral students in both instances that were positive and those that were negative. In this analysis, I drew on counter-narrative approaches developed by CRT scholars that take seriously the perspectives and interpretations of

experiences from the learner's perspective (Delgado, 1995). This was important because doctoral students were reporting on their interpretation of their experiences learning in situations they described as both affirming and harmful to their development as EOMTEs. My analysis took seriously the reported experiences of the doctoral students, and in particular their negative experiences of the doctoral students of color and those who have been marginalized along other intersecting planes of their identity, as perspectives that support us to take a non-dominant perspective of these interactions. An analysis of this process is described in Chapter Six.

Researcher Identity and Positionality

I share a similar trajectory for joining a PhD program in mathematics teacher education and who hopes to become an EOMTE a many of the participants in this study. Three identities with the participants in this study: I was previously a teacher, I am currently a doctoral student, and I aspire to bring equity into my work as a mathematics teacher educator. These shared identities allowed me to relate, in many ways, to the stories and experiences doctoral students shared in their interviews. In addition, these shared identities supported me to build rapport with the participants of this study in very genuine ways that in some cases, led to ongoing email correspondence about their experiences in the doctoral program and life.

While I share multiple identities with many of the doctoral students in this study, one distinct difference is our institutional contexts. Each institution has a unique culture that create a specific context that has implications for the ways doctoral students experience the place. My outsider perspective, as a person from a different institution, allowed me to see aspects of their experiences that may have been assumed to be universal (e.g. the structure of a doctoral program, requirements for engaging in certain teacher education work, norms around collaboration with

faculty). This outsider perspective gave me the opportunity to ask questions of what I was seeing as meaningful opportunities for learning.

Many scholars have identified that it is impossible to create a study that does not use a particular lens (Miles & Huberman, 1984; Patton, 2002; Wolcott, 1994). That is, that all data are collected and analyzed based on decisions about what to include or not to include. As such, all data has already taken on a particular lens that has the “residue” of cultural norms that contain histories of power (Esmonde, 2017). Given that I draw on CRT, I draw on the assumption that all of our experiences occur at the intersection of our race, gender, class, etc, As such, I made sense of the narratives doctoral students shared with me in relation to the intersectional identities they shared with me. For example, Marvin shared that he constantly had his intellect questioned in his K-16 mathematics experiences. I relate this narrative to the work of McGee and Martin (2011) who studied how Black presence in mathematics spaces are often challenged because of stereotypes about Black people being able to do mathematics because I see Marvin’s experience as racialized.

Limitations of Design

This study is limited in the scope of data I was able to collect. For example, I was unable to observe all doctoral students engaging in the learning opportunities they described because of (1) Covid-19 limitations, (2) the deeply personal and private relationships involved in the interactions doctoral students describe, and (3) the membership required to join a particular group (e.g. based on race). While depending on doctoral student narratives provides a lens on how the participants make sense of their own experiences, I was not able to conduct a triangulated analysis of the ways they engaged in these spaces. As such, iterative member checking and detailed descriptions of the participants and the universities they are enrolled in are

provided for triangulation and to aid the trustworthiness of the claims I make in this dissertation.

I am also deeply grateful for the time and vulnerability these doctoral students shared with me. Being open about their experiences required vulnerability; doctoral students had to share details about things that happened to them that often touched on relationships where there was often a more powerful other involved. I have the responsibility to protect the participants' identities, and therefore felt a commitment to be extremely careful in the descriptions of the individuals and their university contexts. As such, there are places in this dissertation where the reader may be left wanting to know more about the context; these places are intentionally ambiguous to protect the participants.

Chapter Four: Doctoral Student Equity Descriptions

To make sense of the experiences doctoral students have related to becoming EOMTEs, it is important to understand what equity-oriented means to them. While no two doctoral students are the same there are a few common aspects of their understandings about what it means to be an EOMTE. The beliefs described in this chapter were collected in the interviews I conducted with each doctoral student. The foundational belief they all shared is that issues of equity in mathematics education exist because of larger systemic and institutional structures that work to preserve the privilege of some over others. This understanding of how inequities operate influenced the current understandings doctoral students have about what it means to work towards equity as teachers of mathematics that can be mapped on to both the Gutierrez's dominant and critical axes. While they shared this common foundational understanding of equity, doctoral student beliefs differed when they described how the work of equity-oriented mathematics teaching is enacted, these differences are highlighted in the section below. Many of the beliefs these doctoral students held about what it means to be an equity-oriented teacher stemmed from doctoral students' own experiences as both students and mathematics educators in schools. This chapter will first elaborate on the commonly held sentiment that issues of equity in mathematics education exists because of larger systemic and institutional structures. Doctoral student understandings of equity more broadly influenced how they understood what it means to be an EOMTE. In this text, I describe the ways doctoral students made sense of what it means to be an equity-oriented teacher and highlight what it is they hoped to instill in the teachers they work with.

Equity as a Systemic and Institutional Issue Applied in Schooling

I found that all eight of the participants in this study understood issues of equity as related to broader systems and structures that have been imbedded in our society. For clarity's sake, what makes something a systemic injustice rather than simply a coincidental difference in experiences is the pattern of behavior and experiences particular groups of people encounter. It is hard to call a single instance or interaction that is not explicitly racist, for example, racist. We might not be able to ascertain that a White man speaking over a Latinx female in one meeting is indicative of the entitlement that White man has developed because of Whiteness as property. We can, however, notice patterns of interactions, for example a White man speaking over women of color more generally or acknowledging ideas that were originally shared by women of color only when the idea is restated by a White woman or another male. Scholars have long analyzed these patterns of behavior and noted that Black women and transfolk have been abandoned when the rights of Black men and of White women were lobbied around (hooks, 1981). In what follows, I provide two examples of how doctoral students spoke about equity in education and describe how I interpreted their narratives to be indicative of the principle of CRT related to the prevalence and pervasiveness of Whiteness. These two examples were selected because they represent a span of beliefs shared by the participants in this study, more examples of how doctoral students thought about equity as a systemic issue are presented later in this chapter.

One perspective doctoral students used to describe the systemic nature of issues of equity in school related to the capitalism embedded in schooling experiences. This approach to understanding equity issues begins with the assumption of the permanence and pervasiveness of Whiteness. Kemé shared that they believe “everything we learn about what to do in school is all

about production.... everything is so much about production, production, production.” Given Kemé used language like capitalism and colonialism in their description of educational issues, I interpreted Kemé’s narratives to suggest that what is valued in schools as learning is a process of production that is akin to what is valued in a capitalist society. Through these experiences in schools, students are socialized to buy into this system of learning which has the effect of perpetuating capitalism, which has, historically maintained “conditions of ideological hegemony” (Burbules & Burke, 1999) that are intertwined with race (Mayorga & Picower, 2018), thus suggesting the emphasis on production and the preparation of students to participate in capitalist systems occurs because Whiteness has created structures to protect reify inequities based on race.

Esther described what equity issues in mathematics education are that focused on both unjust systems and anti-essentialism (Crenshaw, 1991). In brief, anti-essentialism is related to the CRT tenet intersectionality (Crenshaw, 1991); where intersectionality helps us see how there are multiple intersecting identities that impact the way we experience the world, we must also recognize that not all people with the same demographic markers will have the same experience. Esther said that in her high school mathematics classrooms “your race is not a factor. Now, this particular person may need more [help] than another. And the person who needs more might be a minority, while the one that doesn't need as much isn't. But that's not because of me. That's because of society and our system.” While the sentiment “your race is not a factor” may be initially interpreted as indicative of a color-blind approach to understanding equity, Esther was not suggesting that race plays no role in the opportunities and experiences students have in schools. Esther was suggesting that in her interactions with students, she focuses on what students each need individually while also acknowledging systems in our society have caused

people from minoritized backgrounds to need more support in schools. Here we see Esther simultaneously recognizing that not every person of color will have the same experience while also pointing to common occurrences (a minoritized person needing more help than a non-minoritized one) and attributing that to “society” and a broader “system.” This understanding of issues in mathematics education as disproportionately impacting minorities alongside a commitment to seeing students as individuals suggests an understanding of both the prevalence and pervasiveness of Whiteness, as indicated by Esther’s comment about differences in experiences and preparation are due to “society and our system,” as well as anti-essentialism, as evidenced by Esther’s stance that her job is to help each student learn, regardless of their race.

While not all doctoral students used the same language, they all pointed to issues in education stemming from a long-established history of unjust and unequal opportunities for people based on identities such as race, sexuality, gender, socioeconomic status, immigration history, and other social markers that have privileged the experiences of White folks, and White men in particular. As Grace called it, “a fraught history of causing harm and not being in service of Black, and Indigenous, and other students of color.” That is, each participant used a lens that assumed historical and institutional structures exist to explain the patterns they noticed in mathematics education related to inequity. While I did not include the voices of all doctoral students in this section, the aim of the examples I shared was to establish that doctoral students understood equity issues were systemic in nature. In what follows, I describe how this lens is used to make sense of what it means to doctoral students to be an equity-oriented mathematics teacher and how these beliefs are connected to an institutional understanding of inequity that often relates back to doctoral students’ lived experiences. These descriptions highlight more doctoral student voices.

Equity-Orientations in Mathematics Education

While I focused on understanding how doctoral students become equity-oriented mathematics teacher educators, doctoral students in this study often began their descriptions of what it means to be an EOMTE by describing what it means to be an equity-oriented teacher. There were two common themes about what it means to be an equity-oriented mathematics teacher that were evident across the doctoral student narratives. First, that teachers must work to understand who they are and how they may have been complicit in the normalizing of the prevalence and pervasiveness of Whiteness. Second, teachers considered what they might change in their work to work against the prevalence and perverseness of Whiteness.

One quality of equity-oriented mathematics teachers is they unpack their own privilege, biases, and understand that issues of equity are systemic in nature. This is important because if a teacher has not done the tough work of unraveling the political underpinnings of education and mathematics education, they are likely to default to the normative and problematic assumptions society has socialized them to have about students who have been historically marginalized. Marvin gave an example of how this played out when he worked with a majority White teacher group. He explained, when teachers learn about issues of equity, they begin to ask themselves: “what can we do to kind of help and save the students?” Marvin had multiple conversations with in-service and preservice teachers where he clarified for teachers who hold this stance that:

“Nobody needs you to save them. We just need you to advocate and be allies...It's more like if you're going to be in that environment, understand the environment. But you're not going to come in here, some kind of superhero and just enact all these things, it's going to save all the all the Black and Brown kids. I mean, it's not some kind of mission trip to the low socioeconomic status community where you can just, you know, post pictures on Facebook

about how you're saving--that's not what we're doing here. And if we have that kind of thought process you need to have a really deep reflection about who you are and how you view students and how you view students of color.” (Marvin)

In the quote above, Marvin described the White savior complex (DiAngelo, 2018) that many teachers take up when they come to realize there are issues of equity at play in their own classrooms. The common way to address the issue is to see yourself as the solution provider without considering how that mentality, the belief that you can save the *less fortunate*, in and of itself is positioning yourself as better and more capable than these *other* people without educating yourself about the powerful work folks from historically marginalized backgrounds have done and are able to do.

Maya had a similar observation that many of her White peers and colleagues thought equity meant that we should treat every person the exact same way. Maya clarified that what is important in understanding your privilege and biases is that you understand the larger landscape that the inequity exists within. Maya said this means more than “treating everybody fairly” and considering student identities to understand “what makes them who they are and could [this identity] be impacting their math experiences?” Maya went on to name socioeconomic class and race and potential identities that might influence how a student is experiencing a classroom. Anna similarly shared this might look like being open to seeing patterns in things like “student achievement” to identify who the system is and is not serving. An example Anna gave is of students she tutored in pre-baccalaureate mathematics classes in her undergraduate years. She noticed the students who were put on academic watch were first generation students of color. Becoming comfortable and adept at seeing patterns in the ways historically marginalized

populations continue to be marginalized support teachers to identify when their own practices and ways of being are also perpetuating the status quo.

Esther provided another example of the importance of noticing when neutral language is used as a proxy for issues of race. Esther noticed that in the teacher education program she worked for, schools being described as “high performing [were] majority White and Asian schools” and those described as “low performing” served “Hispanic and African American” students. The use of high and low performing labels without consideration of how those terms are used in racialized ways are important for teachers to learn to identify. By unpacking our bias, these doctoral students were suggesting that teachers interrogate the ways patterns they notice in and around mathematics education serve as tools to hide the racial disparities that exist across our educational system.

This quality, being reflective of their own biases, assumptions, and privilege, requires that teachers are able to unpack commonly held notions about mathematics that have served to promote the status quo (e.g. the universality of mathematics). All doctoral students suggested this foundational quality must be in place for teachers to enact any meaningful equity work in their mathematics classrooms

The second quality of an equity-oriented mathematics teacher is they reform what it means to do mathematics. So often students come to believe that mathematics is an individualistic activity where one performs rote procedures and memorizes facts. Student experiences with this kind of mathematics leads students to believe they either are or are not capable of engaging in rigorous mathematics. This is largely due to the “traditional” approach to mathematics education in which learning follows a pattern where teachers show students how to solve problems and students are then asked to solve similar problems using the same steps their

teacher showed them (Silver & Stein, 1996). This, alongside an emphasis on quickness over developing conceptual mathematical understanding, the lack of relevant or cultural connection many students feel about the content of mathematics classes, and stereotypes about who is good at mathematics (Aguirre, Mayfield-Ingram, & Martin, 2013) can lead students to question themselves and their intellect, thus causing what Anna called “math trauma.”

Participants highlighted that one approach to teaching mathematics that combats the “math trauma” many students have experienced involves engaging students in problem solving and collaboration to support student understanding of mathematical ideas. This approach to teaching should, in Naomi’s opinion, attend to student identities and support them to develop conceptual understandings of mathematical ideas. Naomi described how a classroom that provides opportunities for students to engage in collaborative sense making can support students to develop strong identities as mathematicians in ways that elevate their various identities. Naomi said, “the decisions we [as teachers] make about which work we sequence...that affects academic and social status. But those things are also related to students multi-layered identity.” By multi-layered identity, Naomi was describing the intersectionality of student identities and the ways those overlapping and layered identities inform a student’s experience in the world. Naomi’s approach to reforming mathematics education is for teachers to consider student identities and how those identities are supported (or denied) in mathematics classrooms. One of a teacher’s goals, then, should be to identify ways to elevate student identities and status in their instruction. This requires, as Marvin made clear, a belief that all students are capable of learning rigorous mathematics. As a Black student who excelled in mathematics, Marvin experienced a questioning of his mathematics ability and place in advanced mathematics courses at every phase of his K-16 mathematics education. He described going from being “labeled as academically

gifted” to being labeled “retarded” because of a few behavioral issues he was having in school. This questioning of Marvin’s ability to learn mathematics repeated in his experience in high school when he expressed an interest in pursuing engineering and was told “I don’t think you have innate math ability” by a teacher and again in his undergraduate experience when a professor “accused [Marvin] of cheating” because he was performing well in the class. In order to value student identity and status, teachers must believe in the capabilities of all their students.

Anna shared that mathematics is often seen as sterile and removed from the social world. She countered that if teachers rethought what it means to do mathematics to see “the point of the work is to help other people, is to answer actual pressing questions and then to be able to communicate those answers like in a story or in a way that like an audience is going to understand them” students would start to build an identity as mathematicians. As such, Anna believes a teacher should reform what it means to engage in mathematics by providing students with more relevant and meaningful applications of mathematics that allow students to expand what it means to do mathematics.

Other doctoral students emphasized that this expansion of what mathematics learning involves includes a major shift in curriculum. Grace, for example, described the need for a “culturally sustaining” approach to teaching that engages students in tasks that are inclusive to the various identities students bring with them. Maya and Esther both mentioned that the tasks teachers select are central to supporting student identities and supporting students to see mathematics as more than the rote memorization, but as a tool for social change. In particular, Maya and Esther pointed to the work of Eric Gutstein (2005) and using social justice issues as contexts for tasks students are engaged in to support students to see mathematics can be used as a tool for social change. Esther described this saying she wants students to know “math isn’t just a

subject, it's a tool we can use to do almost anything...and if we're not helping our students realize how math is a tool, we're doing them a disservice." Similarly, Marvin shared that including Ethnomathematics (D'Ambrosio, 1997) in the curriculum is also a way to support students from historically marginalized backgrounds, for whom mathematics has largely been culture free, to see themselves and their cultural practices as relevant in the mathematics classroom.

Along those lines, Kemé described the reformation of what it means to do mathematics to include an expansion of what it means to connect with mathematics. We often consider mathematics as something that is performed or completed. Kemé wondered how might "our relationship with mathematics...be similar to the ways in which we are in relation to other people?" That is, that what we would be most concerned about is student relationships with mathematics rather than students' ability to solve problems. Taking this approach to the goals of mathematics would shift the kinds of curricular and pedagogical decisions teachers make in the classroom. Thus, mathematics can become a place for healing rather than the perpetuation of "math trauma."

In sum, while not all doctoral students in this study had the exact same vision of what it means to reform mathematics, they all described a shift away from a traditional understanding and enactment of mathematics. The vision of mathematics they described wanting to work towards includes a focus on student identity as mathematicians through more relevant and meaningful curriculum and instruction.

Equity-Orientations in Mathematics Teacher Education

Given these beliefs about what it means to be an equity-oriented mathematics teacher, doctoral students saw the work of an EOMTE as two-fold. First, supporting teachers to confront

their biases, assumptions, and privileges and unpack how those biases, assumptions, and privileges impact their interactions with students in harmful ways. This includes supporting teachers to develop an arsenal of resources to engage students in mathematics in new ways. The second aspect of being an EOMTE involves engaging in research that can support the field, more broadly, to do this work. We saw this, for example, in Marvin's work supporting White educators to reframe their savior complex by considering why they believe they can "save" Black and Brown children.

Interestingly, doctoral student themes about equity largely revolved around the critical axes of Gutierrez's framework. It feels important to note that doctoral student unanimously suggested that students need to be supported to engage in rigorous mathematics as it currently exists in addition to how they see mathematics education needing to be reformed. Despite doctoral student beliefs that the dominant is important, they spent the majority of the time in our interviews describing their commitments related to the critical. This may be because as found by Gregson (2013) and Rubel (2017), teachers are much more prepared to enact dominant agendas in mathematics education than to enact aspects of the critical axis. Thus, doctoral students may have attended to the critical aspects of equity because they saw critical work as more challenging to enact.

In this chapter, I provided a description of commonly held beliefs doctoral students have about issues of equity in society at large and in mathematics education more specifically. As shared in Chapter Three, doctoral students in this study enrolled in the doctoral program to have a greater impact on the learning experiences students have through their work with preservice teachers. The next two chapters delve into how this learning occurs. Chapter Five looks at the

ways learning occurs across the CoPs doctoral students participate in. Chapter Six analyses the kinds of identity development doctoral students experiences through these learning opportunities.

Chapter Five: CoPs that Support Doctoral Student EOMTE Learning

Doctoral experiences span a multitude of kinds of learning opportunities. Common among these are coursework, research assistantships, teaching assistantships, and instructional coaching. In addition to these more formal spaces where there are structured interactions with pre-established goals of the community of practice, doctoral students also join informal spaces that differ in the kinds of members who are involved in those CoPs (e.g. student groups). These various experiences collectively add to doctoral student learning at large and can support EOMTE learning. This chapter seeks to understand and unpack the learning experiences mathematics education doctoral students have in the various CoPs they join during their doctoral experience that contribute to their development as EOMTEs. This analysis draws on interviews with doctoral students where they described their learning experiences and, when possible, observations of their interactions in these spaces. Table 11 shows the interviews and observations brought into the analysis in this chapter.

Table 11

Data Used in This Analysis

	Interviews	Observations
Maya	2	
Esther	2	
Marvin	2	
Anna	2	1
Naomi	2	1
Grace	2	1
Kemé	2	
Manny	1	

I found there are two categories of CoPs doctoral students join that support the development of their learning as EOMTEs. The first of these two categories of CoPs are those that are institutionally created and whose purpose is to forward some aspect of the university

structure (e.g. a course or a research project) and are typically made up of members with different status within a university (e.g. novice graduate student, advanced graduate student, faculty, staff, etc.). The second of these categories of CoPs are those that are created and maintained by doctoral students and whose purpose is determined by the needs of the group. Doctoral students who join these peer groups often participate in these spaces because it supports their future institutional work as university-based MTEs in ways their more formal experiences are unable to. These groups can be formally created or more informally developed relationships. In what follows, I describe both categories of CoPs, the kinds of learning doctoral students engage are afforded in each of these spaces, and highlight key qualities of productive institution-based mixed membership groups and peer groups.

Institution-Based Mixed Membership CoPs

Institution-Based Mixed Membership CoPs are groups of people who work together to forward a goal related to the institution. Institutional work includes but is not limited to, the preparation of preservice teachers or research. Often when doctoral students enter PhD programs they join several institution-based mixed membership CoPs through roles such as teaching assistants for methods courses or research assistants on faculty research projects. One defining feature of the category of CoPs described in this section is the range of status members hold. For example, in many mathematics methods CoPs there may be faculty (of different statuses), doctoral students (of different statuses), and classroom-based teachers (with different levels of experience working in teacher education). As such, when new doctoral students join these CoPs they often have opportunities to interact with and learn from a variety of members in these CoPs about how to engage in MTE practice that doctoral students may take with them into their future work as EOMTEs. In what follows, I describe examples from Grace, Manny, and Marvin's

experiences of institution-based mixed membership CoPs that supported their learning to become EOMTEs. These three cases were selected because they represent the range of types of learning doctoral students reported experiencing in institution-based mixed membership CoPs. Next, I look across the three examples to name qualities of these kinds of CoPs that allow for those learning opportunities.

Grace

As a reminder to the reader, Grace is a Korean-American woman with an elementary teaching background who was in the first two years of her doctoral program at Colos University. At the time of data collection, Grace was a new member of an elementary mathematics methods CoP. This CoP was comprised of one tenure line faculty and a handful of rotating doctoral students of various experiences teaching elementary mathematics methods. This mathematics teacher educator CoP was responsible for a cohort-based mathematics methods experience for masters' students seeking an elementary teaching credential. The mathematics methods sequences spanned three consecutive quarters and each quarter a different pair of mathematics teacher educators taught the course. When new members joined this mathematics teacher educator CoP, they were paired with the faculty member during their first quarter teaching any course in the three-quarter sequence. Because a new set of members taught the course each quarter, all members of the group interacted regularly to maintain coherence and share information about the teacher cohort across their mathematics methods experience. Additionally, as a pair plans for their particular methods course, it was not uncommon to invite those who taught the prior course in the sequence to shed light on their experience working with this cohort of teachers or to invite those who will teach the next course in the sequence to plan for a coherent learning experience for PSTs. In addition, Grace perceived that this CoP treated all

members as instructors regardless of the titles assigned to each of them through the university (e.g. TA vs instructor). As a new member of the CoP, Grace's official role was Teaching Assistant. However, Grace shared she was positioned by the faculty and other doctoral students as an equal instructor. That meant Grace was encouraged to share her ideas and ask questions freely by all members of this group. In what follows, I provide an account of Grace's experience in the CoP based on a planning meeting observation involving Grace, a faculty member, and an advanced doctoral student who had taught this course multiple times.

In one planning meeting, Grace and the faculty member were preparing for an upcoming course section by reviewing the syllabus and basic course structure. During the first half of the meeting, Grace and the faculty member shared ideas about what should be included or altered in the syllabus. When proposing ideas, both Grace and the faculty used language like, "do we want to..." to bring up new ideas. This use of languages that offers ideas through questions allowed this pair to talk through decisions in ways that invited all members to wrestle with the purpose of assignments and the purpose of the course more generally.

Halfway through the planning session, another doctoral student joined the meeting. This doctoral student taught the course preceding the course Grace and the faculty member were currently planning for. While the focus of their discussion was on the structure and assignments of the course, there were moments when the group discussed other social issues that are meaningful to them. For example, they discussed White passing folks of color and the complexity of allowing people to self-identify into racial groups for particular activities in the TEP. Their conversation moved toward White scholars who pose as Black and Indigenous people. While the doctoral students described these scholars with a tone that indicated disdain, the faculty member mused about the mental health issues these people may have. As the

conversation moved back to the organization of Canvas, Colos' online learning platform, the other doctoral student shared a belief that PSTs should be fluent in Canvas given they have had multiple courses and quarters using it. This doctoral student then shared frustration about one course section that recently offered prompts for the PSTs to talk about the Black Lives Matter movement. In this class session, a White student interrupted to ask logistical questions about Canvas. The other doctoral student went on to explain the lack of awareness this PST displayed in the timing of their question and how this behavior was representative of a common practice of ignoring or making unimportant issues related to racial injustice. The faculty member acknowledged this frustration by validating the other doctoral student's thinking, and put the PSTs actions in context with the challenging nature of the current political, economic, social, and educational uncertainty PSTs were currently experiencing. The group continued selecting core readings and thinking through issues of the course's organization. After a few minutes, the faculty member paused this planning and returned to the issue the other doctoral student had previously brought up. The faculty again validated the doctoral student's frustration, calling it "a hard tension" in which conversations that privilege the experiences of the marginalized are often interrupted and displaced. The faculty then posed a question to the group: "What would it mean to hold both these conversations?" The "both" this faculty member was referring to are (1) conversations about social issues related to race and (2) the questions students have about the logistics of the program. The faculty member wondered aloud how to have these very important social justice conversations while also validating the needs of students. At this point the other doctoral student agreed that the question the PST interrupted with was a valid question and expressed frustration that at this point in the program the student was still having a hard time navigating the online platform and that the student did not understand there were other, more

appropriate venues to get support with this question. The faculty then paused and offered a solution to address this issue in the future, suggesting there are university supports for logistical questions about the online learning platform and if a user question arose in class during a conversation that aimed to address a larger social issue, the group refer PSTs to these resources. While the doctoral students continued to share the frustration they felt at the ignorance embodying this PSTs actions, the faculty looked up university resources for supporting students with technology related questions and added this information to the syllabi. The faculty then summarized what learning the doctoral students might take away from this by asking the question: How do we [have a class where] there is space for young people to be whatever [they want to be], but at the same time, you can't use that as an excuse to be continually displacing and disrupting?" The faculty went on to say that one thing this group could do is anticipate when these breaches might occur and be prepared to reorient the conversation while also quickly pointing off topic questions to available resources to support them.

Throughout the conversation, the faculty member repeatedly affirmed the perspectives of the more advanced doctoral student while also reframing the conversation to support the doctoral students to see the humanity in the PSTs with whom the doctoral students were frustrated. In essence, the faculty member was supporting the doctoral students to see their role as EOMTEs as to remember the humanity of all their PSTs while holding them accountable for the important conversations PSTs need to have to become good and just teachers. I argue this exchange allowed for two meaningful learnings. First, Grace and the other doctoral student were able to see a model for what it looks like to be committed to issues of equity without being embittered by the inevitability of distractions to the work by PSTs. Much like Anzaldua's *conocimiento*, the faculty member took these agendas that were conflicting in a moment to reimagine what it might

look like to hold both a commitment to holding PSTs accountable to engage with issues of equity without positioning this student as deficit. This was made clearer in the questions the faculty member posed to the group, to wonder about possible ways to engage as a teacher educator in this way.

Second, I argue that Grace learned more about acceptable ways of participating in this CoP. While Grace listened more than she verbally contributed to the conversation, she certainly did agree with the other doctoral student that the PSTs interruption was inappropriate and derailed a conversation meant to open space for the voices and experiences of those who have been historically marginalized. As a new member, Grace was still learning what it meant to participate in this CoP. I argue that Grace's opportunity to see how another, more advanced, doctoral student brought up an issue and how the faculty member responded was important for Grace to witness because it allowed her to see that raising your frustrations with a faculty member is acceptable practice in this group. A common refrain among doctoral students was that status differences between doctoral students and faculty can made them feel uneasy about raising challenges because of a fear of making a mistake, being wrong, or being positioned as a novice. In this case, Grace was able to observe how the faculty member generously validated the concerns and perspectives of doctoral students and reframed them in ways that supported the doctoral students to engage in their work as EOMTEs in different, more sustainable ways.

Manny

As a reminder to the reader, Manny is a Filipino-American man who was in the third year of a doctoral program at Seneca University. Manny was a member of an intellectual community comprised of his advisor and academic peer. These three met regularly to share their written work and offer one another feedback. Manny shared he believed all members of this COP have

shared ideas about the way structural and institutional racism, classism, etc. have been established and perpetuated in schools, and so they often came to share writing that had similar perspectives. As part of these feedback cycles, this group spent time describing their ideas and pushing one another to clarify their thinking to improve their scholarship. In addition to serving as a space to improve their scholarship, this collective also dedicated time to make sense of the experiences doctoral students have in the mathematics methods course taught by their advisor. Often these conversations about research, writing, and methods courses overlapped and influenced how doctoral students make sense of what it means to be an EOMTE. In what follows, I draw an interview with Manny where he described his shifting ideas about what it means to be an EOMTE.

Prior to joining the doctoral program, Manny taught in both public and parochial schools. These experiences led him to see the unjust experiences many low-income students of color experience in schools as the result of unfair and biased stereotypes held by teachers and school administrations. As such, Manny entered the doctoral program viewing institutional issues, such as racism, classism, and colonization as central components of the US education system. In particular, Manny described how the histories of colonization and capitalism have historically led to the socialization of youth from colonized countries to be assimilated into colonial and capitalist values through “behavior labor control and management” through forced assimilation into religion and language. We can clearly see this playing out in schools, Manny said, when we look at “all the structures that are in place to reinforce the system.” He pointed to structures such as standardized testing, discipline, and a reliance on “objective” statistics related to testing and discipline to “measure” a child’s worth. These measures, while touted as objective and bias-free, are seen as necessary in schooling systems that rely on record like these for high stakes decisions

like college admissions. These experiences and perspectives led Manny to enter the doctoral program convinced that real change required we “tear the system down...and start over.” This sentiment was a central assumption in Manny’s scholarship when he joined this CoP.

Through ongoing conversations within his advising CoP Manny began to expand his conception of what real change required. He shared that in conversations about scholarship, debriefing his experiences in the mathematics methods course, and reflecting on where his own beliefs originated led him to expand his belief that the system required a complete “tear down.” In these conversations, Manny shared that the perspectives of his fellow CoP members in tandem with the literature this group exposed him to, challenged the tear down approach to educational justice he came to Seneca with. In particular, Manny pointed to the ways abolitionist teaching challenged him to see justice work as requiring more than tearing down. Through their work in the mathematics methods course, Manny observed there were things you could do as an EOMTE to “fight from the inside.” For example, he saw that in mathematics methods courses, you could support teachers to take on abolitionist teaching stances, thereby using an existing and dominant system (e.g. schooling) that would take a long time to tear down, and focus on “disrupting” the status quo in schools through teacher practice. Thus, Manny began to see that he needed to take on a “dual role.” He elaborated saying, “you can’t just be tearing things down without a plan to move forward,” instead this work also includes “building it up the right way.” Manny began to see his role as an EOMTE as supporting teachers to work against unjust systems they encounter at both the school and classroom levels.

Manny acknowledged that the context surrounding his evolving ideas about what it means to generate real systemic change in education has shifted, in large part, because of the nature of interactions he has in his advising CoP. Manny’s description of his experience in this

group highlighted particular ways of interacting that afforded opportunities for shifts like this to occur. I argue that because Manny felt all members were welcome to share their authentic ideas and thinking and because of the mutual respect members had for one another, Manny was able to take on ideas that were slightly different than his own. This was possible, in part, because of the common assumptions about equity this group had and the level of trust its members shared with one another.

Marvin

As a reminder to the reader, Marvin is a Black man who was in his second year in the doctoral program at Rickard University. After feeling like he was not getting the learning opportunities he hoped for in his first year, Marvin asked a faculty member whose research focuses on equity in mathematics education if he could join their research team. This research team serves as a CoP whose aim is to advance research about culturally responsive teaching practices in mathematics classrooms and was comprised of one faculty member, one advanced doctoral student, and Marvin. In what follows, I describe Marvin's experiences in this CoP as he shared them with me in interviews.

At the time of Marvin's second interview, this group was in the process of writing and Marvin had the opportunity to watch this process unfold. In this CoP, Marvin worked with two women of color whose research focused on forwarding equity issues in mathematics education. This, Marvin felt, was important because he was able to fully be a learner. This sat in contrast to his other experiences working in institutional based CoPs in which he felt like the token equity person whose responsibility was to bring up any and all issues of equity as they became relevant. Being with more knowledgeable others in this new CoP allowed Marvin to be part of a conversation that went beyond Marvin's current understanding of theories of equity in

mathematics education. This was the first time Marvin was part of a university-based mix member CoP where he was certain his values, stances, and experiences were shared with other members and felt like the other members had knowledge that would help him develop into the kind of EOMTE he hoped to become. As such, Marvin participated intently (Rogoff, Paradise, Arauz, Correa-Chaves, & Angelillo, 2003), that is, he focused on soaking in all the knowledge and practices of those who are more experienced than him in this work. Given this group was working on writing, much of their time together was dedicated to talking through aspects of the paper, including the theoretical and conceptual frameworks that drove their work. During these conversations, Marvin shared he listened intently to learn from his more knowledgeable colleagues about the theories he had long been interested in. For example, the group talked about Culturally Responsive Teaching and the various language and theories related to it such as, “culturally sustaining pedagogy, culturally compatible pedagogy, ...and social justice mathematics.” This helped Marvin understand the academic terrain he aimed to contribute to. He shared that listening to these conversations sometimes felt overwhelming and made him worry about his lack of knowledge and ability to add value to this group. He also noted that while he sometimes felt like he was not contributing to the brain trust of the group, that feeling was overrun by the excitement he felt as he learned about these concepts from the way his colleagues described them.

I argue that Marvin came into this group with a thirst to learn more about culturally responsive teaching and was investigating research and works that would have provided him with information about these terms and the nuances between them, being part of this group allowed Marvin to learn about the connections and evolution of these ideas in ways that were difficult to do alone. In addition to learning about these concepts, Marvin was also learning about

these concepts in a research context. This opportunity for praxis of theory and research allowed Marvin to further internalize the conceptual learning he had related to theories of equity doing while developing skills related to research.

Learning Across these Cases

When I looked across Grace, Manny, and Marvin's cases I noticed two kinds of learning that were available to doctoral students. In Marvin's case, the new ideas he gained were a more formalized version of the ideas he already had. Expanding one's knowledge of the current research and literature of your discipline is a necessary part of being in academia. In addition to better preparing Marvin for academia, this experience validated the ideas about equity in mathematics education Marvin brought with him, thus affirming his sense of belonging in the doctoral program. It is important to note that this sense of alignment with an institutionally based CoP and its goals is the first experience in graduate school Marvin had where he felt validated and supported at Rickard University. Marvin's case is an example of a learning opportunity that both forwarded his knowledge and skills related to academic research and support Marvin to see himself in increasingly advanced roles in EOMTE practice.

The second kind of learning I noticed involved a shift in the beliefs a doctoral student held. Grace and Manny's learning required a reframing of their perspectives and beliefs about what it meant to be an EOMTE. In this process, they challenged their previous notions about what it meant to engage as an EOMTE through ongoing conversations with their supervisors and colleagues. For Grace, it was learning to anticipate and productively address challenges that may arise when supporting PST learning around equity while also reframing what Grace would have previously deemed ignorance as a legitimate concern. This reframing also put the onus of work on the teacher educator to be more strategic about how to support equity-orientations for teacher

learning. For Manny, it was learning a modified approach for working towards equity more broadly. These learnings were essential for doctoral students to persevere in the academy because experiences like these were likely to occur again for Grace as she continued working in teacher education in university settings because these interactions may lead to burn out because of the resentment and bitterness that can result from not understanding the ways systemic hierarchies one can have when they recognize will preserve themselves at all costs (Bell, 1990).

Qualities of Productive Institution-Based Mixed Membership CoPs

My analysis showed that for both of these learnings, there were four common qualities of these CoPs that allowed for these learnings to occur. First, participants perceived there was a genuine interest on the part of all members of the group to collaborate. This meant that every member was committed to debunking hierarchies of knowledge and expertise. Doctoral students in each of these settings described this being done explicitly by the faculty members who made moves to invite doctoral student voices and ideas and authentically take up those ideas. This did not mean that those with more experience and expertise were unable to share their ideas, it was that those who had less experience of expertise were not made to feel like they do not have anything to contribute. The second quality of these groups was that all the members in the groups described by Grace, Manny, and Marvin had similar views and commitments to equity that they talked about explicitly. While these views were similar, it is important to note they were not always the same. Shared foundational beliefs, alongside a genuine interest in collaboration across all members of the group allowed for ideas that were in conflict to be tried on by other members in the group without fear of being perceived as unknowledgeable or intellectually inferior. This process, of being able to try on new ideas that are not the same as your own is an important part of a productive collaborative learning environment (Grossman, Wineburg, & Woolworth, 2001).

The third characteristic of these groups was that doctoral students who were new to these CoPs had opportunities to learn how to participate in these groups by observing the ways other members interacted, and in particular, how more advanced doctoral students interacted with other members. Establishing a CoP in which typical hierarchies of power and expertise are challenged can be hard to do as those who are typically assigned the most power need to consciously position themselves and others in ways that work against the norms of the hierarchy. Also, those who typically have less power may be unsure of what these interactions could look like given their past experiences with hierarchies play out in the interactions of academic settings. Being able to observe how more seasoned doctoral students interact with faculty provide newer doctoral students with an image of what their participation might look like in this group. Finally, a fourth feature of these learning experiences was the presence of a more knowledgeable other. These more knowledgeable others shared their experiential and content expertise in ways that pushed doctoral students learning in ways that ultimately supported doctoral students to have more sustained futures as EOMTEs in academia. For example, should Grace not learn how to find compassion and humanity in frustrating moments when breaches of her vision of equity occur, existing in an academic space may become soul sucking.

It is important to note that while Grace, Manny, and Marvin's cases demonstrate examples where institution-based mixed membership groups contributed to their understanding and learning to become EOMTEs, not every doctoral student reported having an experience like this. As described in the features of these groups that allow for these learnings, a genuine interest to collaborate that break down traditional power related interactions, shared values and commitments to equity, models for what participation as a doctoral student look like, and a more knowledgeable other are centrally important for these learnings to occur. While the institution-

based mixed membership groups highlighted in this paper focused on the positive learning experiences doctoral students felt, not all doctoral students had opportunities to learn in this way. Maya, Esther, and Naomi reported never finding an institution-based mixed membership group that felt productive for their EOMTE development. These participants cited challenges related to the four qualities of mixed membership groups listed above. That is, frustration at the lack of shared vision about what it means to be an EOMTE and therefore differing views on the work they undertook, or different views about the hierarchy of a mix membership group which sometimes suggested that doctoral students should participate in the CoP in limited ways that were determined by the senior members and faculty of the group, or not having confidence in a faculty members knowledge about equity related to the teacher education work they were engaged with. When this was the case, doctoral students tended to create new avenues for their learning in the form of peer created CoPs. The next section of this paper describes what a peer created CoP is, how it functions, and the qualities of these groups that allowed for learning to occur.

Peer Created CoPs

Peer created CoPs are groups of doctoral students who choose to work together for a particular aim. These aims often span multiple purposes and support doctoral students to advance their understandings of what it means to be an EOMTE in ways that were not available to them in their institution-based mixed membership CoPs. Interestingly, these groups can be set up to be formal working or informal support groups. Two defining features of these groups are that they are comprised entirely of doctoral students and they aim to support doctoral student learning around becoming an EOMTE. In what follows, I describe two cases of peer created CoPs and the learning doctoral students can experience as a result of their participation in these groups. Next, I

look across the two examples to name qualities of these CoPs that afford those learning opportunities. Data for these cases draw on interviews, and when possible (in Naomi's case), observations. As stated above, Maya, Esther, and Naomi did not identify an institution-based mixed membership CoP they were members of that supported their becoming EOMTEs. As such, the experiences Maya, Esther, and Naomi are highlighted in this section. Maya and Esther's case are combined as they co-created the peer-created CoP described below.

Maya and Esther

As a reminder to the reader, Maya and Esther are both Black women in their fourth year of the doctoral program at Rickard University. When Maya and Esther entered their doctoral program, they quickly realized they were in the racial minority. This was a stark change from their previous experiences in higher learning because they both attended HBCUs in their undergraduate experiences. In addition to the visually present racial differences between Maya, Esther and their peers, they also noticed other students and the faculty they worked with often had different perspectives about issues of equity in mathematics. While Maya and Esther were both members of institution-based mixed membership CoPs related to the teacher education courses they were assigned to teach or be a Teaching Assistant for, they reported these spaces did little to expand or support their equity-orientations in mathematics teacher education. For example, Esther was a teaching assistant for a faculty member who, when there was a misalignment between what Esther and the faculty thought PSTs might benefit from learning, would immediately report Esther to her supervisor instead of talking with Esther. These interactions left Esther feeling disrespected because "it felt like she was tattling" on Esther instead of having a conversation where Esther could "listen to [the] concerns and address them." This kind of interaction, Esther expressed, led her to realize she had to go outside of this

relationship to learn to be a “good teacher educator” who is one of “the type of teachers we expect teachers” to be. Maya’s experience differed from Esther’s as Maya reported having positive rapport with her co-instructor, she also shared that the course she was teaching was not necessarily supporting her development as an EOMTE because of the limitations of what she was able to teach in the assessment course she was assigned. Maya, Esther, and the one other Black doctoral student in their cohort noted they often felt solace in one another’s presence in class. Their ability to have side conversations to make sense of the statements their peers were making solidified their bond and helped to stave off feelings of their own lack of belonging in these spaces. Through their ongoing debrief conversations about their life, work, and experiences in the doctoral program, these students discussed problems of practice they were noticing in their work with PSTs. In particular, they noted that the homogeneity of the PSTs they worked with often meant these PSTs had very little knowledge or experience working with non-White and non-middle-class youth. Maya said they often talked about “the lack of awareness [their students] had about [issues of diversity] and how that’s going to...really hurt them when they actually get to the classroom.” This CoP of doctoral students shared feelings of uncertainty about PST preparation and the possible harm these future teachers could cause when their assumptions about what learning, communication, and the experiences of students should look like are brought into their interactions with students from diverse backgrounds. Maya, Esther, and their colleague turned then, to consider what they might be able to do to support their PSTs learning to understand their experiences are not every students’ reality. Thus, the informal support group Maya, Esther, and their colleague created developed into a working relationship that allowed them to advance their practice as EOMTEs in a mathematics methods course and publish an article about integrating issues of equity into mathematics methods courses for PSTs.

Two of the three members of this CoP were assigned to be instructors of the mathematics methods course. Despite their separate sections and one person not receiving any funding for their involvement in the courses, Maya, Esther, and their colleague worked together to plan and co-teach learning activities for PSTs in mathematics methods that supported PSTs to understand their own experiences are not the experiences all people have. Although Maya, Esther, and their colleague did not feel like they had a faculty member to seek out assistance for how to engage in this work, they were pointed to Eric Gutstein's *Rethinking Mathematics* in a course they took. Gutstein's work shaped the decisions they made about which global and social issues to highlight in their methods course. As a result of this work, PSTs engaged in mathematical tasks that also supported them to challenge their conceptions of what is *normative* in terms of life experiences and what it means to engage in mathematics in schools.

Expanding one's conception of what is *normative*, however, is not easy work as it requires a paradigm shift that can shake one's sense of their goodness or merit. While the main purpose of these sessions was to support PSTs to understand that their experience was not the experience of all people or students, a second and unintended learning was an expansion of what it means to learn mathematics. The approach Maya, Esther, and their colleague took to instruction was one that asked PSTs to engage in mathematics differently than they had previously. It was a major shift from the lecture, notes, and practice of procedures many of their PSTs were accustomed to. As such, the changes to their methods courses brought up conflict in many of their PST students in terms of their understanding of themselves in the world and as doers of mathematics. This conflict showed up in classes as PSTs pushed back against the legitimacy of what their instructors were exposing them to. Esther reflected on one particular student who was "upset because he felt...everybody wouldn't be receptive [to this kind of

instruction.] And he [was] not going to lose his job over this.” Despite Esther’s reassurance that all the lessons they worked on were aligned with appropriate mathematics content standards, some students felt deeply uncomfortable integrating mathematics in contexts that could be interpreted as political.

As part of their learning in how to enact a mathematics methods course that forefronts issues of equity and reframes what it means to engage in mathematics for PSTs, Maya, Esther, and their colleague had to learn about how to address the issues of discomfort their largely White middle-class students felt. The experiences Maya, Esther, and their colleague navigated together provided them with an opportunity to enact their vision of what it meant to be an EOMTE in ways they were unable to do in their institution-based CoPs. In their peer created mathematics methods CoP, they collaboratively tried on new ideas, refined their practice, and learned about potential challenges in doing this work in the future.

Naomi

As a reminder to the reader, Naomi is a White woman and was in her third year of the doctoral program at Colos University. Naomi began her doctoral program eager to learn about how to enact her equity ideas into mathematics teacher education. She began by observing the secondary mathematics methods sequence and moved on to a TA role for a faculty member. In her role as a TA, Naomi reported feeling like she was invited to share her ideas about the readings assigned or the structure of interactions PSTs would experience (e.g. jigsaw), but decisions about the class and its content were ultimately determined by the faculty member. Naomi’s felt her ability to participate fully was limited. While Naomi shared that she learned about the structure of a mathematics methods course, she did not expand her understanding or knowledge of how to enact an equity focused mathematics methods sequence that addressed

issues in mathematics beyond those related to gender. In this, and other institution-based CoPs, Naomi shared she felt her “opinions about [issues of equity] weren’t welcomed. [She] couldn’t figure out where they were welcomed and how to nurture those ideas and not get shut down.” This happened for Naomi along two planes. First, she encountered these experiences as oppositional. That is, that she had different epistemological approaches to understanding issues of equity from some of her peers and professors. Naomi’s background in institutions where others shared her beliefs and commitments about equity, she reported feeling unprepared to do the “emotional and intellectual work of pushing against the norm” that these peers and faculty were presenting. In these cases, Naomi felt shut down because her ideas and beliefs were different. A second way Naomi encountered feeling shut down was when sharing ideas with colleagues who shared her beliefs about issues of equity. This stemmed from, in Naomi’s understanding, Colos’ attempt to be “apolitical.” For example, in coursework, some professors engaged students in conversations about deeply personal and political topics without doing work to establish trust or norms around engaging in these discussions, partly because of beliefs about the apolitical nature of these topics. As a result, Naomi described her peers of color feeling pushed to the margins and frustrated by the lack of attention paid to issues of equity. This also has caused the development of a common practice of calling-out people to respond to comments or interactions that were not critical. This culture of interacting, Naomi shared, led her to feel guarded about how she participated with her peers. Naomi’s desire to learn and grow to become an EOMTE led her to seek out other learning opportunities for her to expand her knowledge and skills in community with others.

Naomi found and joined a smaller specialization within her college that focused on issues of systemic justice and equity in education. While she shared learning more about systemic

justice and equity in this group, she wanted to respect this space as an alternative space for supporting the learning of her doctoral student peers of color. Naomi said, “[White students] can work together on [issues of equity in our work] without students of color, having to have the burden placed on them to educate their White peers.” Naomi, conscious of her positionality in this space as a White woman for whom the academy had largely been accepting of, did not want to shift the space to prioritize her own learning at the expense of her peers of color.

Naomi found one other White doctoral student who felt similarly and the two of them met with faculty of color to clarify their goals and ask questions about how a group that attended to White student learning might function. Through ongoing conversations with faculty of color about how to institutionalize this group, Naomi and her colleague decided a peer only group might be the best way forward. While the group itself was made up of doctoral students only, Naomi and her colleague received guidance from an administrator in the GSE who supported Naomi and her colleague to be strategic in their language to convey who this group was for, being clear this group of open to all but that they “especially encourage White folks” to participate. This was necessary as Colos had a strict policy about exclusion of any group of people. The institutional fear of exclusion based on identity, while grounded in beliefs about equity opportunity and access, takes a color-blind approach to understanding that issues of equity require different engagement of people based on their positionality (another example how Colos attempted to be apolitical). This hard stance by the institution was representative of Naomi’s experience in the various institution-based CoPs she was a part of that did not address the core questions she was asking.

In the end, Naomi and her peer reached out to other members of their doctoral cohort to invite them to join an informal student-only group focused on investigating their relationship

with White supremacy, making connections between their commitments and the work they did as teacher educators and researchers. Naomi described this collective of White doctoral students as a response to the “burden of “cultural taxation” (Padilla, 1994) that is too often placed on students of color in GSE spaces, we hope to build a space for students to educate themselves about anti-Blackness and racism.” She went on to explain that educating oneself about anti-Blackness and racism is important socially and academically because the research one engages in is shaped by our beliefs, stances, and understandings about issues of systemic injustice. In essence, Naomi was looking for an “intellectual home” where she could continue to investigate race and racism in education in relation to her own work (and therefore her own positionality) without putting the burden of that learning on her BIPOC peers and faculty.

At the time of data collection, Naomi was a leader and facilitator of a group of White doctoral students in education committed to learning how to bring equity orientations into their work, many of whom were interested in teacher education and research. Through this group, Naomi had opportunities to learn more about how to facilitate conversations about Whiteness and White supremacy that she previously did not have opportunities to do. In a planning meeting Naomi and a colleague considered what learning and thinking they hoped to engage members in during their upcoming group meeting. Naomi and her colleague threw out a rapid succession of questions that were quickly refined through their conversation. The questions began with “how do we understand these connections?” referring to the connections participants might see in Bettina Love’s *We Want to do More Than Survive* and their work in education. The question turned to “what are these connections and beyond?” which then turned to “what could we do about those connections” and finally “how can we use our roles as people in educational institutions? What in our roles could work to disrupt these patterns?” What was clear from the

evolution of these prompts was an emphasis on moving beyond a purely intellectual understanding of Whiteness, anti-Blackness, and systemic injustice and towards an embodied understanding of these ideas that became enacted in practice. This was seen in the shift in questions that focused on ‘understanding’ towards a focus on actions that might ‘disrupt’ the status quo. During this planning session, Naomi and her co-facilitator also anticipated challenges that might arise in the group, for example, the fact that as TAs many members of this group felt like they had limited power to enact some of the changes they wanted to make in the courses they supported. To address this while also not leaving room for the conversation to stagnate in frustrations about their lack of power, Naomi and her co-facilitator developed the question “how do you enact this work when you don’t have full agency?” In a follow-up interview about this planning meeting and the enactment of the planned session, Naomi shared that this space felt markedly different from the other institutional-based mixed membership groups she was a part of, largely because she felt able to try out ideas without fear of retribution or condemnation.

Naomi also shared that being the facilitator of this group provided her with opportunities to approximate facilitating discussions about Whiteness with PSTs. She named skills she developed like reorienting conversations that begin to draw on deficit thinking and explicitly naming what was happening in a conversation to support discourse that brought a conversation back to its intended purpose. These are skills Naomi hoped to use when she can work, with more agency, in mathematics methods courses with PSTs.

Part of the opt-in nature of the group meant that people who joined had a shared set of commitments. In this group it is first that they were working to unpack their Whiteness, and second, were willing and eager to engage in this vulnerable process with others. This sense of common ground allowed Naomi to be vulnerable about how she made sense of her experiences

in ways she did not feel comfortable doing in other spaces at Colos. This is due, in part to the level of trust members of this group shared. This trust was possible because of the shared commitment its members had.

Learning Across these Cases

Looking across Maya, Esther, and Naomi's experience in peer-created CoPs, I argue they created these CoPs to address learning they felt was missing in their experiences in institution-based mixed membership CoPs. All three of these participants were able to learn about how to enact their visions of what it means to be an EOMTE in the context of their MTE work in collaboration with other doctoral students who had similar interests. For Maya and Esther, it focused on enacting their vision of equity in a PST mathematics methods course; for Naomi, it was developing ideas for actions she could take as a methods instructor and researcher to enact her beliefs about equity. In all these cases, doctoral students were learning to enact their visions in their MTE work.

Qualities of Productive Peer Created CoPs

As stated earlier, the learning opportunities doctoral students had when they engaged in these peer-created CoPs were guided by their own interests and self-defined needs. This was due, in part, because these groups were created with the purpose of supporting doctoral student learning to become EOMTEs and all members were onboard with this purpose. While doctoral student learning might be a subgoal of the institution-based mixed member groups, it rarely was the sole or primary focus of the group. As such, the activities and amount of time that focused on doctoral student learning and advancement in these peer-created groups were dramatically different than many other institution-based CoPs.

In addition to the ability to focus on doctoral student learning, members of these groups opted in and therefore also had the capacity to opt out. Thus, issues of alignment across members of the group were not an issue in the same way it might be in an institution-based CoP (an example of this will be described in Chapter Six). If a member did not feel like they shared values with other members of the group or the purpose of the group, they were free to disengage without any repercussions. This also suggests one aspect of why these groups were so important for doctoral student learning was that there were shared visions of equity, that, while not exactly the same are shared enough to establish trust and therefore an ability to listen to and press one another in ways that did not shut learning down.

Additionally, the lack of mixed membership in these groups allowed doctoral students to participate without being concerned with status differences between themselves and faculty members. That is, that doctoral students felt free to participate openly in these groups without fear of retribution, being looked down on, or perceived as uninformed or feeling the pressure to educate someone with a higher status about issues related to equity.

It is crucial to note not every doctoral student in this study found or needed to find these communities. Some doctoral students who had strong alignment of visions of equity with their faculty mentors and whose interactions with their faculty mentors position them as equal participants did not express a need for these peer-created CoPs.

Chapter Six: EOMTE Identity Development

In the previous chapter, I described the kinds of learning doctoral students can have in both institutionally based mixed-membership and peer created CoPs and the qualities of those CoPs that allow for that learning. Where Chapter Five focused on the kinds of learning doctoral students had and the aspects of the structure of the CoPs that allowed for that learning, this chapter looks at the ways doctoral students develop EOMTE identities, including the challenges they face during this identity development. To investigate how doctoral students develop identities as EOMTEs in various CoPs, I analyzed doctoral student interviews for opportunities for engagement, alignment, and imagination in the various CoPs they were a part of that related to mathematics teacher education (see Table 12). This analysis looks at the experiences of two doctoral students, Kemé and Anna. These doctoral students were selected as focal cases for this chapter because their experiences represent a range of experiences that are representative of the other doctoral students' experiences. That is, while Kemé and Anna share identities as first-generation college students, other aspects of their identity such as race and gender, are different. As a reminder to the reader, Kemé is an Indigenous non-binary doctoral student at Seneca University and Anna is a White female doctoral student at Colos University. Kemé and Anna's were selected as focal cases because their experiences with engagement, alignment, and imagination represent the range of conceptual relationships between the constructs that are representative of all the participants. I chose to highlight both helpful and harmful experiences Kemé and Anna had related to developing identities as EOMTEs for two reasons. First, to accurately portray their experiences as neither completely affirming or discouraging. Second, because the work of an EOMTE and therefore our identities as EOMTEs transcend our participation in any one CoP. Sharing multiple experiences Kemé and Anna had is important to

conveying their identities as EOMTEs span multiple CoPs. In sum, this paper attends to the ways engagement, alignment, and imagination were experienced as doctoral students developed identities as EOMTEs.

Table 12

Data Used in Identity Development Analysis

	Interviews
Kemé	2
Anna	2

Lave & Wenger (1991) describe identity as “the way a person understands and views himself, and is viewed by others, a perception of self which is fairly constant” (p. 81). They go on to suggest that a person’s identity is developed through their participation with a particular practice. In the context of this study, doctoral student learning to become an EOMTE occurred in conjunction with their developing identities as EOMTEs. Wenger (1999) later provides more specific constructs to make sense of the ways identity develops. The three constructs (engagement, alignment, and imagination) suggest that one requirement of developing a meaningful identity as an EOMTE is a sense of belonging with a community of practice. That is, to imagine “our sense of place” as an EOMTE (Wenger, 1999, p. 176), there must be meaningful opportunities for engagement and a shared vision of the purpose and goals of the work doctoral students are doing with others that enables learners to see themselves as members of this practice. Although Wenger (1999) treats these constructs as separate, he calls them “complementary” as they all contribute to identity development. As described in the conceptual framework chapter, opportunities for meaningful engagement, alignment, and imagination are crucial for doctoral students to develop identities as EOMTEs. As such, this analysis looks at the

ways doctoral students had meaningful and conflicting opportunities with engagement, alignment, and imagination in the various CoPs related to developing an EOMTE identity.

The first set of cases highlights experiences Kemé and Anna had that were meaningful opportunities for doctoral students to experience engagement, alignment, and imagination. When doctoral students have meaningful opportunities to engage in the work of MTE in ways that are aligned with their vision of what it means to be an EOMTE, their ability to imagine themselves continuing to work and be an EOMTE is enhanced. Looking at examples where there are opportunities for engagement, alignment, and imagination that are coherent allow us to see the intricate ways in which engagement, alignment, and imagination are related. The second set of cases feature interactions Kemé and Anna had in CoPs that challenged their identities as EOMTEs. In these cases, engagement, imagination, and alignment were linked, but in ways that highlight how engagement and alignment can challenge opportunities for imagination.

Engagement, Alignment, and Imagination Coordinated

The cases described below are examples of experiences that support the development of doctoral student identities as EOMTEs. These cases highlight the ways coordinated and coherent experiences with engagement, alignment, and imagination support doctoral students to develop identities as EOMTEs despite some initial hesitancy. In each case, I describe that hesitancy and an experience that helped the doctoral students move past it through an analysis of their opportunities for engagement, alignment, and imagination. Kemé and Anna's cases draw on interviews about their experiences as doctoral students.

Kemé

Kemé entered the work of mathematics teacher education with some skepticism. While Kemé already believed preservice teacher education was an important place to work on

educational justice at scale, they were hesitant to work in teacher education because, as Kemé described “What I do is not for White students.” By this, Kemé meant that their work in education has been driven by a commitment to promoting educational justice with and for BIPOC students. Given Kemé’s work always involved BIPOC students, the move teacher education would require (to work with PSTs who are predominantly White) was unnerving. This was in part because Kemé is a person of color who had to learn how to navigate the world as “somebody who hasn’t been racialized as White” and so Kemé’s commitment and what they felt prepared to share about how they navigated racialized spaces is not for White folks. Rather, Kemé hoped their experiences might serve as a tool for other BIPOC folk to make sense of make decisions about their own experiences. Additionally, Kemé reported their prior experiences supporting White people learning about issues of systemic racism were intellectually and emotionally draining. As such, the switch to working in mathematics teacher education, a field that while diversifying racially is still largely White, was daunting to Kemé. In addition to this sense of who Kemé teaches related to race, Kemé also shared that they were still unpacking their own understanding of themselves as a teacher of adults. Kemé’s background as a high school teacher is a core part of Kemé’s identity, as they consider themselves a teacher of adolescent aged students, not adults. Given much of the work of an EOMTE involves the work of teacher education, these barriers to participation seem significant to address to support Kemé to become an EOMTE. These reasons for Kemé’s uncertainty about their future position as a teacher educator are potentially problematic for Kemé’s development as an EOMTE as mathematics teacher education is a significant part of that work and involves working with adults, many of whom will be White and all of whom will be adults.

Kemé's work with the mathematics methods at Seneca began with their participation in the mathematics methods course led by their advisor. This mathematics methods course focused on supporting a cohort of all White preservice teachers to address the question "What does it take to be an abolitionist, anti-racist teacher?" In order to do this, preservice teachers were asked to make sense of systems of oppression in schools and how teachers can be complicit in perpetuating the status quo in the classroom in ways that are damaging or harmful to all students and to students who have been historically marginalized, in particular. This work, Kemé acknowledged, takes time, in large part because unveiling and unpacking first, that Whiteness exists; and second, how it has operated in the lives of White folks and folks of color, especially when confronting these ideas for the first time, is a process.

In their observations of the course, Kemé was able to see how their advisor navigated these "difficult conversations" and supported White preservice teachers to "change and challenge themselves about the ways they have been taught, the ways they have not been taught, and the limitations of their knowledge because of their lack of exposure to other people." I argue that observing how a more experienced EOMTE facilitated this learning for White preservice teachers was important for Kemé because it served as a proof case that allowed Kemé to see that mathematics methods courses in PWI university-based teacher education programs can support White preservice teachers to meaningfully learn about issues related to systemic racism in ways that did not leave Kemé feeling intellectually and emotionally drained. My analysis suggests this experience was significant for Kemé because it supported them to imagine the possibility of being in this role, something they were and continue to be hesitant about given the age and racial demographics of PST students.

In addition to their observations of their mentor leading conversations with White preservice teachers, Kemé also acted as an active participant and was often positioned by their advisor as an expert classroom teacher during the mathematics methods course. As such, Kemé shared stories from their experiences as a student and as a teacher to support the PSTs developing ideas about what it means to be an abolitionist and antiracist educator. Given the focus of much of this discussion was about experiences with systemic marginalization, many of the stories Kemé shared were deeply personal and laid bare their past in ways that required them to exhibit vulnerability with these PSTs. While the process of laying bare Kemé's experiences with racial injustice in schools was something they described as being taxing to their intellectual and emotional being, Kemé felt differently about engaging in this way in mathematics methods setting versus in their own coursework with White peers. A crucial difference in these experiences, I argue, was Kemé had the ability to choose what to share and was prepared to do this sharing given the framing and foundations of the methods course that were largely unavailable to Kemé when responding to comments by classmates in course conversations. This preparation included, for example, opportunities to write about and reflect on their experience with racism and policing as a student before sharing the story with the PSTs.

In addition to the prework Kemé did to prepare themselves for the intellectual and emotional work they would need to do to share their story, they also engaged in ongoing conversation with their mentor to unpack the ways Kemé was experiencing being in the mathematics methods class. In these check-ins, Kemé was able to speak freely and truthfully with their advisor about how they experienced the class. I argue that the combination of being positioned as a veteran teacher in a mathematics methods course, being provided with time and a process to prepare for supporting White PST learning about issues related to inequality, and the

ongoing check-ins with their advisor allowed Kemé to feel optimistic about the ways they engaged their EOMTE identity, beliefs, and experiences in the mathematics methods space.

The support Kemé received prior to and after methods sessions helped them see methods work as doable. This was enhanced by the ways Kemé was provided the agency to enact their vision of equity in mathematics education in the course. Kemé shared that when they told their stories in the methods classes, they noticed the looks on the PSTs' faces. In particular, Kemé noted the guilt they saw. Guilt, and in particular White Guilt, is well known for being a point of stagnation for many well-intentioned privileged people while learning about the ways their privilege has caused harm to others. Instead of allowing PSTs to wallow in their guilt, Kemé described that they offered PSTs a process to move beyond it by inviting them to interrogate where their guilt was coming from and the ways their institutionally sanctioned power and role as a mathematics educator may be reinforcing biases, stereotypes, and the status quo. Kemé challenged the students to “step back” and deeply consider the power they will have to help or harm students. In that moment, Kemé also positioned themselves as someone who has to continue to “step back...as a non-Black person” to consider the ways they might be perpetuating antiBlackness. I interpreted these moments as place where Kemé had agency to ask White PSTs to reflect on their own positionality and the effect their privilege may have on the experiences of their future BIPOC students thereby supporting future action instead of just feeling guilt. This agency to ask PSTs to hear their story and reflect on their own biases and practices was markedly different than the education of White folks Kemé often felt they need to do with their White peers in their own coursework. I argue interactions like the ones Kemé had in the methods course contribute to Kemé's ability to see themselves engaging in a deeply personal, yet non-taxing way, with White people. Taken together, we can see this sense of optimism, about the impact

engaging in this way can have on White PST's learning, and how it contributed to Kemé's ability to see themselves as a future EOMTE.

In Kemé's experience working in and with the mathematics methods course, they reported were able to engage in ways that felt right for them. That is, Kemé had some control over what they shared, how they engaged, when they choose to press, or step back. This is important for Kemé's development as an EOMTE, particularly because of their learned hesitation about being an educator of White folks. Using Wenger's conception of identity formation, my data analysis showed that this kind of engagement also allowed Kemé to imagine themselves in a university-based teacher educator role in ways they were previously skeptical about. As described in earlier chapters, this university-based CoP allowed Kemé to feel comfortable while also expanding their identity as an EOMTE. This is possible, in large part, because of the alignment shared between Kemé and their advisor. This alignment goes beyond the basic beliefs they shared about the purpose and problems in society and education but also extended to the ways of being and interacting these two individuals brought into their relationship with one another. The ongoing checking in, the validation and welcoming of emotional processing in academic settings, and the appreciation for vulnerability Kemé and their advisor shared in their interactions opened opportunities for Kemé to engage in the ways described above, which in turn, led them to expand their view of what is possible for them as an EOMTE in a university-based setting.

Anna

Equity in mathematics education, for Anna, is about making mathematics more inclusive for the groups of students who have been historically left out of mathematics. While stating her ideals and beliefs about mathematics with conviction, Anna also conveyed that she herself still

had to work through her own identity and positionality as a White female when engaging in work that prioritizes the experiences of students who are racially different than her. I argue Anna's way of engaging in the work was indicative of her uncertainty about what it meant for her to be an EOMTE. This uncertainty was two pronged. First, Anna felt extreme imposter syndrome about being at Colos University which included her sense of being an academic and teacher educator in a university setting. Second, Anna expressed she was constantly cautious about the ways her privilege as a White person was surfacing in her ideas and actions. While an important process to engage in as a part of what Anna believed is the work of a White EOMTE, this reflection sometimes stymied her participation in MTE CoPs at Colos. Anna shared she was constantly questioning, "Am I doing the wrong thing? Am I learning the wrong approach? Am I going to be ill prepared for the profession?" These questions challenged Anna's identity as an EOMTE.

One space that assuaged Anna's sense of imposter syndrome and encouraged her to reflect on who was benefitting from her work was a collective of MTEs comprised of doctoral students, faculty, and staff, all of whom had experiences working as K-12 mathematics educators. This group's goal was to understand how a curricular reform could provide students with more robust learning opportunities by drawing on real world issues that were relevant to students' lives. In addition to using relevant contexts, Anna described an approach to curricular reform as a process of learning that emphasized problem solving and communicating solutions. Anna's interest in curriculum reform was in the potential to reform what it means to learn mathematics. Part of Anna's work with this collective of mathematics teacher educators was to learn more about how curricular reform might be used in schools to provide more meaningful and inclusive learning experiences for students who have been historically left out of

mathematics. Curricular reform, Anna said, has the capacity to “help other people...answer actual pressing questions and...be able to communicate those answers...in a way that an audience is going to understand them.” This process of engaging in real world problem solving is important because it can improve peoples’ lives. Anna sees this approach to curriculum reform as a “place for women and girls to feel...like they belong in [mathematics] spaces” because of its commitment to explore topics that are relevant to students and the shift away from traditional models of learning. While Anna’s research focused on the experiences of women and girls in classrooms that embrace this kind of curriculum reform and the impact of those experiences on affirming their identities as mathematicians she also problematized her work saying she wanted to (1) incorporate gender beyond a binary in her study and (2) attend to student intersectional identities, and in particular, considering the intersection between gender and race. While this work investigated how this kind of curriculum reform was enacted in school settings, Anna saw implications for this work in her research as a teacher educator working with practicing mathematics teachers.

Given all members of this group had K-12 mathematics, but not this particular approach to curriculum reform, teaching experience, this group operated as a collective of educators who sought to learn more about how to enact and engage in curriculum reform with students in schools. The group chose to approach this learning by welcoming rough draft and in process ideas. Anna described it as a space where “the whole organization is trying to learn together.” Anna shared that the norm of engaging as learners supported her to be comfortable with not being an expert which combatted the imposter syndrome she often felt in other spaces at Colos where being an expert and speaking with authority was valued. In her experiences with the teacher educator CoP, Anna felt like her ideas, concerns, and questions were elicited and heard. This

sense of feeling like she and her ideas mattered supported Anna to imagine herself as a competent and capable mathematics teacher educator and helped her develop a stronger sense of being an EOMTE.

In addition to having a shared vision of how curriculum reform might provide students who have been historically marginalized in the field of mathematics with opportunities to meaningfully engage with mathematics, this mathematics teacher educator group also had concerns about how this approach to curriculum would be implemented. Anna shared the group constantly discussed the implications of “how [a course that draws on this kind of curriculum reform] gets positioned and who gets recruited into this class; and that when we research how this class gets implemented, what are the outcomes for students and which students?” which included “being very mindful of sort of the pathway and tracking that could happen in integrating a new topic and a new course into high school.” That is, this group was concerned not only with developing curriculum that would be potentially meaningful and impactful to students, but also with the necessary guidance for curriculum implementation for future users to ensure that it does not continue to perpetuate the inequities Anna and her colleagues sought to disrupt. For Anna, being a part of a group that felt a sense of responsibility in how their work was taken up validated the way of engaging as an EOMTE Anna hoped to have. This was reassuring for Anna because it allowed her to engage in this group in ways that were in sync with her beliefs about equity in mathematics education. This alongside the shared beliefs this group had about mathematics as a space for those who have been historically marginalized in the field, helped Anna see she has a place and a future as an EOMTE. This case highlights a second example of a CoP that supported doctoral student development to be an EOMTE.

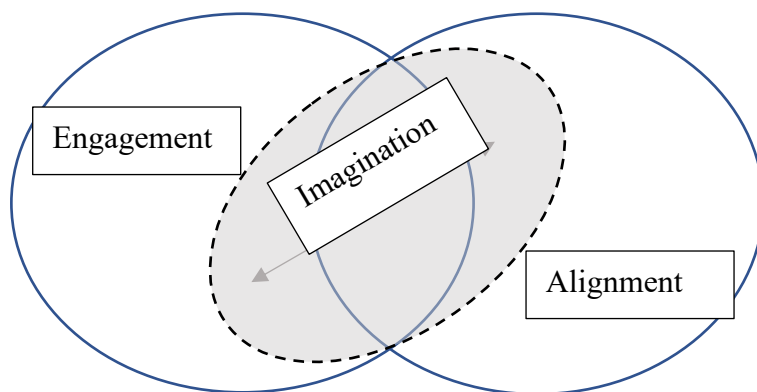
Cross Case Analysis

When I looked across these two cases, I saw that both Kemé and Anna had strong alignment as a foundation for their relationship and interactions with other members of these CoPs that directly addressed issues of equity in mathematics education. This foundational alignment was important because the shared visions, beliefs, and dispositions about mathematics as an issue of equity members of these groups brought with them allowed for particular ways of engaging and interacting that supported Kemé and Anna to engage in competent participation (Wenger, 1999). In Kemé's case, it was the opportunity to engage with White PSTs in a way that was on their own terms, they could share their stories when and how they felt comfortable doing it. In Anna's case, it was the ability to feel like being curious about new ideas (curriculum reform in particular) was encouraged by members with various status and feeling like issues about the application of their work related to the perpetuation of the status quo were taken seriously by the group. For both Kemé and Anna, these ways of participating supported them to develop an imagined way of being they previously were hesitant about because they were both able to feel like they had some ownership over the pursuits of the group. This kind of experience allowed Kemé to see working with White PSTs as a possible and sustainable activity to engage in; it supported Anna to see EOMTEs do not need to be rigid experts and dampened her ever present imposter syndrome. These cases affirm Wenger's (1999) assertion that engagement, alignment, and imagination are important for the development of an identity as an EOMTE. This analysis looked not only at the opportunities for engagement, alignment, and imagination, but also at the interplay between these three constructs. For example, the ways of engaging that were available to Kemé and Anna in these interactions was largely in line with the values and beliefs about equity and mathematics education that was shared between group members. That is, that the

alignment in beliefs and purpose that Kemé and Anna felt with other members in these CoPs, in part, dictated the various ways they were invited to engage in the work of the CoP. In addition, Kemé and Anna’s ability to imagine themselves engaging in EOMTE work was due to their comfort in the ways they engaged. Thus, engagement and alignment were complementary in ways that supported Kemé and Anna to imagine themselves in more advancing EOMTE roles. Figure 2 represents the intertwined nature of engagement and alignment that Kemé and Anna experienced in the examples described above. Key features of this model of Kemé and Anna’s experiences were that engagement and alignment were impacting one another in synergistic ways across the various experiences they had in these CoPs and this synergy between engagement and alignment supported their ability to imagine themselves as future university-based EOMTEs. In this model imagination existed in a plane at the intersection of engagement and alignment. A large, shared region between engagement and alignment resulted in a relatively large cross-section for imagination.

Figure 2

Imagination at the intersection of Engagement and Alignment



The meaningfulness of engagement and alignment in participant's ability to imagine themselves as an EOMTE, particularly when there was hesitancy on the part of the participant was the same for all doctoral students in this study. Manny and Marvin, for example described finding institution-based CoPs related to EOMTE work that allowed them to actualize and envision their place in the institution when they previously were feeling tokenized. Grace shared that her relationship with her advisor paired with her advisor's openness about what it meant to be an academic of color supported her to believe that if she wanted to stay in academia, she could. The participants' developing imagination as EOMTEs typically came after they had a less affirming experience in which they questioned the sustainability of university based EOMTE roles. Relatedly, my analysis found all the doctoral students of color described the emotional and intellectual labor they felt obligated to provide to their White peers or students. It is interesting to note that Kemé is the only person of color in this study who has found institutional space to protect them from that burden.

In the next section of this chapter, I present cases where there is less congruence between the ways Kemé and Anna experienced engagement, alignment, and imagination and present an altered model for these experiences.

Engagement or Alignment with No Opportunities for Imagination

All doctoral students in this study pursued a PhD because they believed the doctoral program would offer them coursework, experiences, and mentors that would be able to support their development as EOMTEs. While they believed the doctoral program would provide them with opportunities to learn to be EOMTEs, doctoral students in this study found supportive learning environments to be rare. Much more frequently doctoral students in this study described feeling frustrated at the lack of opportunities they had to develop into the kind of mathematics

teacher educator they had hoped the program was already prepared to shape them into. The cases described below are examples of experiences that challenged doctoral student identities as EOMTEs. These cases highlight the influence of and challenges to engagement and alignment on opportunities for imagination. Kemé's case is about an experience in a course that made Kemé question their place in a university setting. Anna's case highlights the challenging experiences Anna had when interacting with doctoral student peers. In both cases, we see a disparity in either engagement or alignment that challenged doctoral student imagination of themselves as EOMTEs in a university setting. These cases drew on interviews with Kemé and Anna.

Kemé

Kemé pointed to a particular kind of interaction they repeatedly experienced as a doctoral student that challenged their EOMTE identity development in ways that caused them to question whether they belonged in academia. Kemé described multiple experiences where their desire to challenge the status quo was shut down in coursework by faculty. Below is a vignette of an exchange Kemé described in an interview:

Kemé entered the classroom and was greeted by the faculty. Kemé was already excited about this course because the faculty leading it were scholars whose work was known for being critical. At the start of the class, students were given note cards and asked to write what they hoped this course would be able to do for them. Kemé, optimistic about the opportunity to share this with faculty who seemed eager to adapt to students' interests wrote, "a safe space to ask questions and to challenge theory." After gathering and reading the cards aloud, the faculty paused at Kemé's card and emphasized the sentiment Kemé wrote, suggesting this was exactly what the faculty intended the class would be for students. Kemé, having had their card read aloud and validated publicly by the faculty felt even more encouraged about the learning they

would be able to engage in because of the invitation to think critically. Later in the class Kemé shared their interest in thinking more about how to fuse Critical Race Theory (CRT) and Educational Psychology together, hoping to spur a conversation about how to bring a critical lens to educational psychology. Kemé's wondering was met with a strong message that they needed to use "the language of [their] discipline" and CRT would not fit into an educational psychology tradition. Kemé, thinking this was an opening for further conversations shared they felt the language of Educational Psychology is "inherently racist" and therefore disrespectful to people like Kemé. Kemé was told, at this point, that tradition is something that must be adhered to in academia. Kemé thus came to realize this course would not be a safe space to explore ideas and push back on theory, particularly if the challenge went against the norms of academia seen by these faculty. Kemé reflected back on this and other similar experiences that left them "questioning why I was in the doc program because I felt like, I felt like I was not in a position where I could service my community."

Educational psychology is a field that captures learning theories that span from behaviorist to sociocultural. Many scholars have, in fact, integrated critical theories into understandings of learning (Esmonde & Booker, 2017). The examples in Esmonde and Booker (2017) help us see how scholars have pressed the boundaries of what is taken for granted in sociocultural learning theory in particular. While faculty can use their positions in universities to challenge the status quo as it shows up in theories, this is not always the case. I argue that in Kemé's experience in this course, the faculty disingenuously framed the course as a place to press boundaries. This instance may seem like a potentially bad day for course instructors and a student, but Kemé reported this kind of interaction happened multiple times in their two years as

a doctoral student. The repeated experience of feeling like their question and ideas were not welcomed resulted in Kemé considering leaving the program.

In relation to engagement and imagination, the experience Kemé had in this class shows an initial coherence between opportunities for meaningful ways of participating (engagement) and shared commitments with the course instructors (alignment). The opportunity for meaningful engagement was initiated in the opening task of sharing your hopes for the class followed by the public validation of Kemé's ideas. Kemé's initial belief that they had shared commitment with the course instructors was something they brought with them from their prior knowledge of the kind and quality of research these instructors produced. In many ways, the way Kemé described this experience fits in with that of Figure 2, where there were coordinated and coherent experiences related to engagement and alignment. Where Kemé's case differs however is when they participated in ways they thought were accepted (e.g. asking questions Kemé perceived to challenge some of the traditions of educational psychology) and received a response from the course instructors that indicated that kind of participation was not allowed. This resulted in a shift to both Kemé's opportunities for meaningful engagement and alignment with the course instructors that resulted in increasingly less overlap between engagement and imagination. I argue that as this happened, Kemé's identity as an EOMTE was increasingly challenged.

At the end of the exchange described above, Kemé shared they opted out of continuing the conversation and changed the way they participated in that course for the duration of the course, in large part because Kemé's past experiences with these kinds conversations would demand a lot of emotional and intellectual labor and likely not support their development as an equity-oriented scholar and teacher educator. Where this space held promise as a place to

cultivate their EOMTE identity, the rigidity of what the kind of participation this space allowed left Kemé unmotivated to continue to develop their identity with this group of people.

While not the only kind of experience Kemé had in courses, Kemé reported this kind of interaction was not rare. This and other experiences like this led Kemé to feel, in many ways, like becoming a scholar and teacher educator would mean being “limited to a particular place,” a particular way of being, and therefore a particular way of engaging in the teacher education and research work that was not aligned with Kemé’s hopes for their work as an EOMTE. The collection of these experiences over time led Kemé to question why they were in the doctoral program and contemplate dropping out of the program.

Anna

Anna described feeling like an imposter through her various experiences as a graduate student. This sense of being an imposter in academic settings began for Anna in her undergraduate years. As Anna described it, she took a single math class as an undergraduate that was a prerequisite for a business major. While intrigued by mathematics, she was afraid of declaring a major in mathematics because she feared she was not capable. After finding success in one mathematics course that positioned students as competent sense makers of mathematics, Anna was motivated to take more mathematics courses and eventually switched her degree from business to mathematics with a new sense of being capable of engaging in rigorous mathematics. Anna reported this imposter syndrome continued as Anna went on to pursue a master’s degree in teaching secondary mathematics. Her sense of being an imposter was so great that Anna immediately wondered if the admissions team made a mistake in admitting her. Anna called the teacher education program asking a made-up question as a guise to confirm she was, in fact, admitted into the program. The same sense of feeling like an imposter followed Anna into the

doctoral program. This imposter syndrome, paired with a culture of competition that spanned nearly every peer interaction Anna had in her first few years as a doctoral student served to make Anna question whether she belonged at Colos.

Anna shared many of the beliefs about equity in mathematics education that her peers in this study held. For example, when Anna worked as a tutor in her undergraduate program she noticed the vast majority of students who were underprepared for the rigor of their programs were folks of color. Given her belief that mathematics could be transformative for people if taught in ways that allowed students to meaningfully engage with it, Anna interpreted the disproportionate amount of “first generation, low income...students of color” in these classes as structural issues. This sentiment, that students of color are underserved by institutions like schools alongside Anna’s belief that more equitable and inclusive approaches in PST’s learning was a way to improve the educational experiences historically marginalized students, was also a commonly held belief of other students at Colos. Therefore, Anna had relatively strong alignment in her beliefs with her peers regarding the harm schools currently inflict on students from groups that have been historically marginalized and that changes in the structure of mathematics education holds great promise for working towards greater educational justice. Despite this alignment, Anna found it challenging to participate in equity-oriented spaces at Colos in ways that were meaningful to her ideas about what it means to be an EOMTE.

What hindered Anna’s ability to develop her EOMTE identity with many of her peers were the norms of engagement that were established for participating in equity-oriented work more generally at Colos. All three doctoral students at Colos in this study describe Colos as “competitive” or “performative.” Grace, in particular said students frequently felt “worried about confidentiality...to a point where [people think] about things like, *this is my intellectual*

property” and are unwilling to share ideas or collaborate more generally. Grace believed this culture was perpetuated by faculty who “actively foster that kind of mindset” in part because of the “unforgiving tenure approval rate” Colos faculty experienced. Naomi echoed the institutional trickle-down of this culture suggesting that there was evident discord between some faculty that played out in the way their doctoral students interacted. In addition to this, Naomi shared that the culture of interacting with peers at Colos was such that people felt the need to position themselves with more status. Naomi shared that seeing peers get shut down in their interactions with other peers led her to disengage in participating in some conversations or participate in very cautious ways. This competitive culture, as Anna experienced it, played out in a type of *Woke Olympics* where students were frequently most concerned with positioning themselves as more socially conscious than others. For example, Anna reflected on how many of her peers would share information about the kinds of neighborhoods and students they worked in as teachers to gain status as a person who was dedicated to equity. This is not to suggest that sharing this kind of information was not meaningful to people’s identities or relevant to understanding the context of a particular situation, but rather Anna reported it felt like people shared this information solely to elevate their position as a person who was more equity oriented than those around them.

One common practice of this culture of wokeness is that people are often called-out, rather than a more educative approach of calling-in. Calling people out often leads the person being called-out to feel backed into a corner and behave defensively. This also has the effect of positioning the person who was called-out as less woke, thereby increasing the status of those doing the calling out. While Anna largely did not disagree with what she saw being called-out, her aversion to participate in this culture of calling out and competing for advanced status as

equity or justice oriented stemmed from her prior experiences attempting to educate her White family about Whiteness and White privilege. Anna shared she first began interrogating her own Whiteness and White privilege in her master's in teaching program. During this program Anna fell into the practice of calling-out and shaming people into changing their language. She learned, however, that when she engaged in this kind of interaction with people, they were completely unable to hear any explanation for why particular language or sentiments were indicative of larger social and institutional isms. Through these interactions, Anna learned "when you're overly aggressive, they can't hear you. They just go automatic defenses up." This led Anna to conclude that if she wanted someone to really hear her, she had to take a different approach. Instead of being concerned with calling people out, Anna became concerned with having a conversation that had the potential to educate the other person. Anna described her current approach by asking the question, "How do you challenge a person in a way that allows them to actually hear you instead of...defending the f----- up thing they just said." And so, Anna entered the Colos PhD program with a reframing strategy for discussing issues of equity that was less direct than her previous approach. In the interview, Anna used tentative language rather than declarative language where she provided counterevidence to statements she heard that held biased assumptions. For example, my analysis showed Anna was careful to use "I" statement that offer perspectives and ideas that she framed as belonging to her and not universal as a tool to challenge broad assumptions she heard people make about groups of students. Anna's inclination to engage in conversations about systemic injustice in ways that were at odds with much of the culture of interactions at Colos led her to "retract" herself from these interactions when the conversations became harmful to advancing progress for her and the other people involved. The

ways some of her peer colleagues responded to Anna's work suggested to her that she was seen as *less woke* and less prepared to engage in rigorous equity related research.

Despite a shared vision of the driving forces that cause equity issues in education, the clash in understandings about what it meant to engage in this work between Anna and her peers made Anna's imposter syndrome "flare up". Imposter syndrome leaves us in a state of questioning our place and ability to be successful in a space. As such, I argue Anna's ability to imagine cultivating her EOMTE with her peers was limited. While it is fortunate Anna had other venues and CoPs to engage in that did support her EOMTE identity development, given that we know peer created groups can play a valuable role for doctoral students learning to become EOMTEs, it feels important to consider the ways doctoral students are able to cultivate their EOMTE identity with their peers.

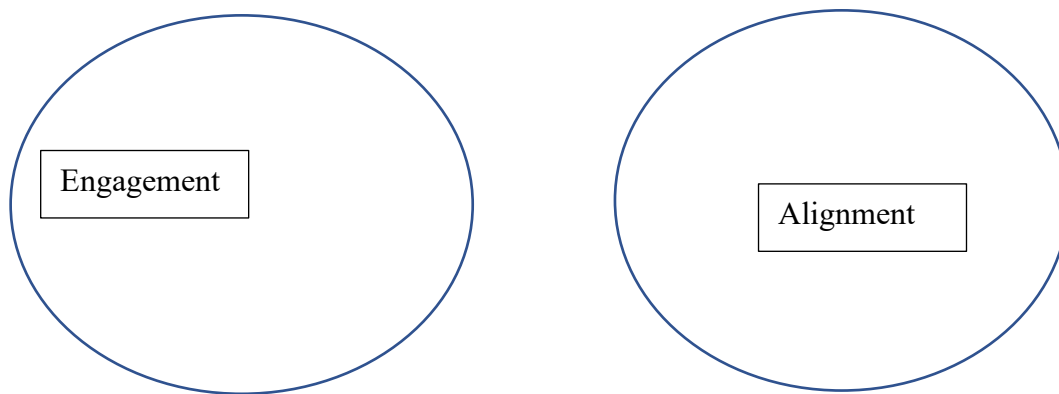
Cross Case Analysis

When I looked across these two cases, Kemé and Anna both had experiences that were different in nature but had the effect of causing them to question how they identified as an EOMTE because they did not feel like they were able to engage in competent participation (Wenger, 1999). In Kemé's case, they were invited to participate in a course in ways that Kemé thought would advance their learning of what it means to be an EOMTE. Yet, there was some underlying discrepancies between Kemé's vision of equity and those held by course instructors led Kemé to feel frustrated with the institution to the point of questioning whether they should continue. Anna found alignment with the larger vision of equity shared with most of her peers, yet the modes of engagement that were available to her that are acceptable with her peers were at odds with Anna's learned ways of engaging in this work. As such, she perceived she was positioned by some of her peers as less equity oriented which often led Anna to wonder if she

belonged at Colos or in a PhD program. Figure 2 presents a representation featuring imagination as a plane that exists at the intersection of engagement and alignment. Building on this, Figure 3 attempts to represent Kemé's and Anna's experience, where a lack of overlap between engagement and alignment makes a plane for imagination impossible. Wenger (1999) suggests that when advancing membership within a CoP is hindered, so too is one's identity as a member in the CoP. In both Kemé and Anna's cases, their opportunities for meaningful engagement and alignment with others resulted in a loss of opportunities for imagination that made them both seriously question their sense of belonging in the institution.

Figure 3

Engagement and Alignment as Unrelated



These two cases highlighted that in relationships or spaces where doctoral students had a lack of coherence with meaningful opportunities for engagement or alignment, they were likely to also have a lack of opportunities for imagination. These findings continue to affirm Wenger's (1999) assertion that complementary experiences with engagement, alignment, and imagination are crucial for one's identity development. This analysis adds a more specific relationship about these constructs; that a person's ability to imagine themselves was tied to both engagement and alignment. That is, that when either opportunities for meaningful engagement or alignment were

compromised, one's ability to imagine themselves as an EOMTE was also compromised. In Kemé's case we saw a lack of meaningful opportunities to engage and alignment in understandings about the purpose and intent behind those forms of engagement that led to hindered opportunities for imagination. Similarly, Anna's case highlighted how one can have shared beliefs with those they work with, but if the culture of a CoP is such that the forms of engagement that are seen as aligned with that vision are at odds with a person's understanding of what that engagement should look like, imagination is also hindered.

This finding, that when opportunities for meaningful engagement or alignment between a person and other members of the CoP are compromised result in a challenge in one's ability to imagine themselves as an EOMTE occurred for all doctoral students at some point in their doctoral experience.

Summary

In this chapter, I provided cases to highlight the ways engagement, alignment, and imagination operated for doctoral students attempting to develop an EOMTE identity. In this analysis we affirmed that engagement, alignment, and imagination are all crucial aspects of EOMTE identity development and that the way one of these constructs manifest themselves often rely on the other constructs. That is, often what is determined to be acceptable ways of engaging are determined by the vision of equity one has. And when that vision of equity is aligned between members of a CoP, members are more likely to be able to envision themselves being able to continue engaging in the practices of the CoP.

While these constructs certainly do influence the learning experiences and opportunities members have, Kemé and Anna's less affirming experiences help us see that while their ability to imagine futures as EOMTEs was tied to both engagement and alignment, it was possible for

engagement and alignment to be at odds with one another. If either meaningful opportunities for engagement or alignment in vision and beliefs were disparate, then doctoral students were less likely to be able to develop a vision of themselves continuing in this work. In fact, 6 of the 8 doctoral students shared questioning, at some point in their programs, whether they should continue in their PhD programs because of issues related to engagement or alignment within the various CoPs they participated in.

This is not to say that to develop an identity as an EOMTE one must only have opportunities for engagement that are aligned with their own beliefs. Rather, conflict is an unavoidable part of the process of identity development, especially as doctoral students develop identities as EOMTEs across a multiple CoPs in their constellation of practices. Lave and Wenger (1991) say “conflict is experienced and worked out through everyday practice in which differing viewpoints and common stakes are in interplay. Learners can be overwhelmed, overawed, and overworked. Yet even when submissive imitation is the result, learning is never simply a matter of the ‘transmission’ of knowledge or the ‘acquisition’ of skill; identity in relation with practice, and hence knowledge and skill and their significance to the subject and the community, are never unproblematic” (p. 116). Lave and Wenger (1991) remind us that it is through the making sense of these less coherent spaces that support doctoral students to develop a transcendent identity as an EOMTE that extends beyond any one moment or even the collection of experiences in one particular CoP. Said another way, engagement, alignment, and therefore imagination are all contextual aspects of identity. A doctoral student may have meaningful opportunities for engagement, alignment, and imagination related to being an EOMTE with some groups they work with and not with others. Their identity as an EOMTE is not determined by any one of these, but rather, as the sum of the collection of their experiences

related to EOMTE work. It is crucial that conflicting experiences are not the only ones available to doctoral students as coherent experiences with engagement, alignment, and imagination advance doctoral student identities as EOMTEs in meaningful ways.

Chapter Seven takes these ideas and elaborates on the implications of these findings to both the field of doctoral student learning in MTE and conceptually to understanding how identity development occurs related to Wenger's three constructs.

Chapter Seven: Discussion and Conclusion

It is clear from the literature on doctoral student learning in mathematics education, and in particular the lack of consensus in the literature, that doctoral preparation should be a key focus of our field's development. In addition, increasing calls in education at large, and mathematics education in particular, push us to reconsider the taken for granted assumptions that have driven mathematics education (Gutierrez, 2013; Martin, Gholson, & Leonard, 2010). These two bodies of literature, brought together, necessitate a focus not only on how doctoral students in mathematics education are prepared to be mathematics teacher educators, but how they are prepared to meaningfully address issues of equity in their work as future university-based mathematics teacher educators who will have a wide-spread impact on the approaches to teaching and learning that in-service and preservice teachers will take up with their students.

Without attention to this preparation, those who support doctoral student learning are not likely to change their approach or stances in doctoral student preparation and therefore will continue to perpetuate a cycle of production of teachers and teacher educators who unconsciously uphold status quo beliefs about mathematics and students that have been documented as harmful (Martin, Gholson, & Leonard, 2010; Martin, 2003). I looked at the experiences of eight doctoral students across three institutions who are committed to becoming EOMTEs.

Review of Findings and Implications

Chapter Five described two structurally different CoPs doctoral students reported as being meaningful EOMTE learning opportunities. One category of the CoPs are institutionally-based mixed member collectives that work to advance some aspect of the institution. These might be mathematics teacher educator groups whose work aims to prepare PSTs in teacher

education programs, research collectives, or advising groups that focus on providing feedback on academic writing. In these kinds of CoPs afforded two kinds of learnings. First, doctoral students were able to learn to “play the game” (Gutierrez, 2013), that is they learned skills necessary for survival in academia that also validated and encouraged doctoral student commitments to equity in mathematics education. Second, doctoral students learning involved shifting beliefs about what it means to be an EOMTE from a more knowledgeable other. In both of these cases, there were four common qualities: 1) members perceived that all members of the CoP are committed to challenging traditional hierarchies of power and invite those who may be perceived as having less expertise to bring their ideas, experiences, and beliefs to the group; 2) There is some shared commitment to equity among members of the group, although this commitment does not need to be exactly the same; 3) doctoral students have opportunities to learn how to engage and participate in this group by observing how other members of the group interact with one another; and 4) there are more knowledgeable others who provide doctoral students with examples of what it looks like to engage in EOMTE work.

The second category of CoPs described in Chapter Five are Peer-Created CoPs. These CoPs are often developed by doctoral students when they perceive the learning opportunities provided to them by the institution are not sufficient in preparing them to become the kind of EOMTE they hope to be. These peer-created CoPs are designed to address a particular need that doctoral students have identified. This could mean the CoP focuses on issues of content in a methods course or debrief what it means to interrogate Whiteness in their work as teacher educators or as researchers. These learning opportunities are possible because of two characteristics of these groups. First, the opt-in nature of these CoPs suggests there is cohesion among group members’ sense of purpose and drive for learning. Second, the lack of mixed

membership in the group allows doctoral students to participate in ways they might feel hesitant to in CoPs with faculty members who serve as evaluators and mentors to these doctoral students.

Chapter Six then offered an analysis of the ways doctoral students develop identities as EOMTEs in institution based CoPs. This analysis looked at doctoral student opportunities for meaningful engagement, alignment, and imagination and the interplay between these constructs. First, this chapter explored instances when there are meaningful opportunities for engagement and strong alignment between group members that support doctoral students to imagine sustainable and long-term success working as EOMTEs. These cases highlight the interplay between engagement and alignment for productive opportunities for imagination, and therefore one's ability to develop an identity as an EOMTE. The second set of cases analyzed instances when there was either a lack of opportunities for meaningful and engagement or a lack of alignment between group members that lead to doctoral students to question their place and potential success as EOMTEs in an institutional setting. That is, that when experiences are challenged in terms of engagement or alignment, opportunities for imagination are compromised.

Contributions to the Literature on Doctoral Student Learning in MTE

Looking back to the current literature on doctoral student learning in mathematics teacher education, we see a focus on the kinds of knowledge and skills doctoral students should develop. This study adds to the current literature on doctoral student learning in the following ways:

First, the current landscape of doctoral student in learning in mathematics education mainly focuses on the knowledge and skills doctoral students need to learn. Very few articles focus, instead, on the structure of the processes that make for fertile learning environments. Golde (2008) and Mewborn (2008) focus on the importance of developing *intellectual communities* in which doctoral students have meaningful learning opportunities in contexts that

are broadly inclusive, purposeful and knowledge-centered, encourages risk taking, respectful and generous, and deliberately tended (Golde, 2008). Since Golde introduced the idea of intellectual communities to the mathematics education community focused on doctoral student learning, many scholars have taken up this idea as important for doctoral preparation. Little, however, has been added to the literature about how these intellectual communities are established or function. I argue the CoPs described in Chapter Five are examples of intellectual communities that support doctoral student learning that contain all the qualities of productive intellectual communities Golde (2008) described. In this way, this study adds concrete examples that elaborate on the structures of productive intellectual communities. Implications of these findings might prompt those who support doctoral students in mathematics education to consider questions such as: What opportunities do doctoral students have to engage in these ways? How do I learn about how doctoral students experience institution-based CoPs? What other spaces do doctoral students have to experience intellectual communities related to becoming EOMTEs?

Second, as stated multiple times through this dissertation, the literature on doctoral student learning in mathematics education focuses on what is to be learned rather than how these knowledges and skills are learned. Given this focus, the literature offers very little on issues of the identity development of doctoral students as MTEs more generally or EOMTEs, more specifically. This study contributes to the field as the first study that conducted an analysis that attended to issues of identity as part of doctoral learning in mathematics education. In particular, this study provides insights into the ways doctoral students identities as EOMTEs are developed by unpacking their opportunities for engagement, alignment, imagination, and the interplay between these three constructs. The findings related to Chapter Six may prompt those who support doctoral student learning to attend to the ways doctoral students are experiencing

engagement, alignment, and imagination across the various CoPs they participate within to consider what additional experiences and support doctoral students might need. This might include, as Mewborn (2008) points out, opportunities for cross-institution collaboration. Using engagement, alignment, and imagination as constructs to assess the kinds of learning opportunities students need might better help faculty who support doctoral student learning to identify the different kinds of experiences doctoral students would benefit from. This would address some of the limitations doctoral programs have based on issues such as number of faculty, expertise of faculty, and budgets that many doctoral programs in mathematics education face (Reys, 2018; Taylor & Kitchens, 2008; Fey, 2001). One note of caution, however, is how the burden of these *interinstitutional collaborations* (Fey, 2001) that provide doctoral students with opportunities to develop their EOMTE identities are placed on faculty whose research and identities are already pushed to the margins of academia, thereby placing an additional weight and responsibility disproportionately on faculty of color.

Third, methodologically, the literature on doctoral student learning in mathematics education has largely drawn on the perspectives and experiences of current faculty members and in a few cases, surveys of recent doctorates in mathematics education or of self-reported narratives of current doctoral students. While much can be learned in the reflections of those who once were students and currently support doctoral student learning, much can be learned from taking the perspectives of the learner (Stake, 1995). The design of this research inquiry relies on the experiences of doctoral students in the forms of data collected and the analysis to understanding the complexity around learning to become an EOMTE.

Last, this work prioritizes equity in ways that have not previously been done. A small handful of studies name equity or some proxy for the term (e.g. political awareness or diversity)

as an after-thought to the “core” knowledges doctoral students in mathematics education should learn. Often these terms, and what is meant by them, go completely undefined. This paper provides a definition equity in mathematics education to provide context for understanding the perspectives and analysis I offer in this dissertation.

Conceptual Contribution Related to Understanding Identity Development

In this study, I drew heavily on situated learning theory as a tool for understanding learning. The foundational literature on situated theory suggests identity development is an important aspect of learning that is cultivated through participation within a CoP. The three constructs Wenger (1999) describes: engagement, alignment, and imagination, are used as tools to understand how identity is developed. While described as “complementary,” these constructs are treated as separate entities that learners experience. This separate but complementary approach to understanding engagement, alignment, and imagination is taken up in research that uses these constructs to understand identity development, often being described in isolation from one another in its use understanding the identity development of students in various contexts (Nasir, 2002), PSTs (Au, 1997), and professional identities more broadly (Adema, et al., 2019). My analysis of engagement, alignment, and imagination highlight two contributions related to these constructs.

First, my findings indicate a particular kind of relationship between engagement, alignment, and imagination. In Chapter Six, I showed when there was no overlap between opportunities for meaningful engagement or alignment, participant’s ability to imagine themselves in university-based positions and engaging in the work of an EOMTE were compromised. That is, if either engagement and alignment are not affirming of the kind of

practices and beliefs an individual hopes to develop, the individual's ability to imagine themselves in increasing more sophisticated practice in a CoP are not likely.

A second conceptual contribution of this analysis is opportunities for engagement and alignment are often related. More specifically, the ways of engaging that a member finds meaningful to their advancing membership with a CoP is reliant on the individual's sense of alignment with the purpose and beliefs of the group. As such, engagement and alignment should be treated as more than "complementary," but rather as intertwined constructs that can have a range of relationships that span from synergistic (see Figure 2) to dissonant (See Figures 3 and 4).

Limitations

The conceptual contributions I make in this study were generated based on the analysis of eight doctoral students. While the relationship between engagement, alignment and imagination outlined are evident in participant's experiences, it is unclear whether these relationships are unique to these individuals, EOMTE identity development, or identity development more generally. As such, more research and conceptual work is needed to confidently suggest the generalizability of the conceptual contributions laid out in this chapter.

Second, some of the experiences doctoral students shared required they reflect on experiences that occurred one to four years prior to the data collection for this study. As time and experiences continually influence the way we see past events, it is possible that over time doctoral students come to understand their experiences differently and begin to interpret new experiences in ways that are different from how they might have interpreted those same experiences previously. Thus, there is a complex web of learning and growth that all people experience that make these findings unique to the settings they occurred in.

Future Research

Reys, a scholar whose work has focused on doctoral student learning in mathematics education for over two decades repeatedly publishes work that calls for additional research on doctoral student learning in mathematics education. In his more recent work, he calls for research that identifies doctoral programs in mathematics education that are doing good work and study their program requirements and the experiences doctoral students have within those programs (Reys, Reys, & Shih, 2019). The current criteria offered for how to identify which programs are considered “good” at preparing doctoral students looks at the research production in high-impact journals (Reys & Reys, 2017) or asking mathematics education faculty which institutions they perceive are producing high-quality doctorates in mathematics education (Reys, Reys, Shih, & Safi, 2019). In addition to these points, Shih et al.’s (2019) report on a survey of recent graduates suggests what was missing most in their programs was attention to issues of equity. I argue that attention to issues of equity should be a central criterion for understanding how doctoral students in mathematics teacher education are prepared because if we do not center equity, we are likely not attending to it meaningfully and continuing to center that which is status quo. One way to identify which institutions are “good” at preparing doctoral students in mathematics education in ways that promote equity is to create a scholarship map that outline the academic lineage of scholars whose work in both research and teacher education focus on issues of equity meaningfully. Studying the programs structures and the experiences of doctoral students in those programs may better shed light on what we should hope doctorates in mathematics education learn and are able to do. Along those same lines, doctoral programs might also study what it means to have equity orientations given approaches and the focus of equity work change over time.

In addition to the structures of doctoral programs and the experiences of doctoral students, an interesting future line of research may investigate the perspectives of faculty who support doctoral student learning. While beyond the scope of this dissertation, I also collected interview data from faculty members who doctoral students in this study reported as having a positive impact on their development as EOMTEs. In these interviews, I asked faculty members what roles they saw themselves playing in supporting doctoral students to take on and take up equity orientations in their doctoral work. Interestingly, some faculty believed it was their part of their responsibility to cultivate equity orientations in their doctoral students; others believed their responsibility was to support their doctoral students in whatever interests they already had, therefore not bringing up equity if not already an interest of their students. Given institutional structures and systems are sometimes at odds with the individuals within the system, a survey of faculty may provide an opportunity to identify particular faculty who intentionally support the development of EOMTEs and learn from their perspectives, beliefs, and approaches.

In addition to these study contexts, studies that use Gutierrez' equity framework to make sense of how doctoral students learn to navigate the tension between the dominant and critical aspects of equity in their research work and in their practice with teacher education could be compelling. Few scholars (Gregson, 2013; Rubel, 2017) use this framework to make sense of teacher learning, although they treat the dominant and critical as two separate planes during their analysis. Research on doctoral student learning that focuses on how doctoral students learn to support teachers to take on stances and practices that attend to both axes may support the field to develop (1) better practices for doctoral learning and (2) teacher learning.

Other research might attend to how institutional norms, practices, and resources shape how doctoral students develop as EOMTEs. While this study did not do cross-institution

analyses of the experiences of doctoral students at Colos, Rickard, and Seneca Universities, there were differences in the institutional contexts that may have led to differences in student experiences and learning opportunities.

Last, we might consider the similarities and differences in the experiences of elementary and secondary focused mathematics education doctoral students. Grace was the only doctoral student in elementary mathematics in this study, and a variation of participates in relation to the grade level focus in mathematics education may shed light on interesting learning experiences and opportunities.

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Appendix A: Interview Protocol 1

Question	What I am trying to learn & why	How I know I've got it
Personal History		
<p>Tell me about what motivated you to start a doctoral program in math teacher education?</p> <p>→ If it hasn't come up, ask: what were you doing before enrolling in the doc program?</p>	<p>History of participant Build rapport</p>	<p>They've talked about their path AND what motivated them to join a doc program</p>
Context		
<p>Can you tell me about the types of work you have done or currently do in which you are a math teacher educator?</p> <ul style="list-style-type: none"> • How did you learn to do this work? <p>Are there other kinds of relationships/places you learned about math teacher education?</p> <p>Is there anything else you think is important for me to know to understand what it's like to be a doctoral student in math teacher education here?</p>	<p>What counts as the work of TE-what range of work they do that is MTE work.</p> <p>Is it mentoring? Debriefs? How are you being prepared: engagement --specific examples</p> <p>Begin to identify learning opportunities</p>	<p>They identify specific examples of how they've been supported to become equity-oriented mathematics teacher educators.</p> <p>Equity is defined.</p>
Context-Equity specific		

<p>Would you say the TEP / methods group you work with have a commitment to equity or social justice? -How, if at all, does it show up in your practice?</p> <p>I would love for you to imagine you are teaching a math methods course. What would you want your teachers to take away from your course in relation to your vision of equity in math education?</p> <ul style="list-style-type: none"> • How do you support teachers to learn this? <p>To what extent do you think that is shared with folks you work with in math teacher education? In what ways?</p> <p>I'm curious if there are places in the college of ed or in your doc student experiences that have pushed your beliefs about equity in math ed. → Can you describe the relationships you have with the folks/places that are grounding your work (around equity)?</p> <p>How, if at all, has your thinking around equity changed or evolved as a result of being at this institution? –probe to understand who and what supported these changes</p>	<p>context</p> <p>Try to get at what things they feel they can already do...and what things they feel like they want more support in</p> <p>What tensions are there? Alignment?</p>	<p>Understanding of the definition of equity in mathematics education the participant has and the tensions they see in enacting this definition.</p> <p>Participants have identified the alignment between their own ideas of equity with the institution/people they work with</p>
<p>Aspirational + support in this setting</p>		
<p>What are your goals for post graduation? What kind of what does that entail?</p> <ul style="list-style-type: none"> • Do you feel like you are being prepared to do that work? In what ways? 	<p>Their long-term goals, may provide insight into the trajectories they are on...their identity in terms of imagination</p>	<p>They identify what they might want to do after graduating and have articulated their interest in this work.</p>

<p>**If not mentioned, do you feel like you are (or are becoming) a math teacher educator?</p> <p>IF yes, - Can you tell me about how you are being prepared to be a math teacher educator?</p> <p>-probe for experiences that may be outside of the scope of teaching courses (e.g. conferences, introductions, etc.)</p> <p>If no, can you tell me more about why you think that is?</p>		<p>They've describe whether they feel like they are/are not being prepared to do that future work.</p>
<p>Setting up future observations & interviews</p>		
<p>I'm trying to understand doctoral student learning around becoming equity-oriented teacher educators. What activities/interactions/etc. do you think I should plan on observing?</p>	<p>Plan for future data collection & begin to learn more about the structure of the doctoral student learning experiences in this CoP</p>	

Appendix B: Example of Interview Protocol 2 for Grace

Note to self: you are trying to understand what about these various experiences are supporting the doc student's learning...and in particular, how that learning is connected to equity and MTE work

You described that equity work should always be in service of teachers and students, sharing resources and power, and elevating the voices of the multiply marginalized.

Description of equity at the classroom level	How the left column is worked on the TE	Came in with/developed in PhD
<p>A teacher's work is to support students to develop a positive math identity. Teachers need to:</p> <ul style="list-style-type: none"> - <i>focus on student ideas, experiences, and interests</i> - <i>make tasks more culturally sustaining, more inclusive, more student centered, ambitious, and inquiry based</i> - <i>be an advocate for students who are marginalized</i> 	<p>A teacher educator's work is to help PSTs learn to do the left things by:</p> <ul style="list-style-type: none"> - <i>Integrating equity coherently across courses</i> <p>Practicing educators</p> <ul style="list-style-type: none"> - <i>Share resources/readings/language that can help move forward abolitionist agendas</i> <p>Research should:</p> <ul style="list-style-type: none"> - <i>focus on the multiply marginalized and their voices</i> - <i>be in service of teachers and students; prioritize their needs</i> 	<p>-Prior experiences</p> <ul style="list-style-type: none"> - <i>teaching in a school that was committed to reforming teaching practice</i> - <i>being a participant in research</i> <p style="text-align: center;">-</p>

Here is a summary of the tension I noticed in your narrative about being an equity-oriented MTE.

Does this look right? What would you add/subtract/change?

Place	Tension
Academia	The milestones as standing in the way of work being in service to teachers and students

Frame learning as something that can be about developing practices, stances, understandings, etc.
 –I would love for us to describe the interactions you have in these spaces and what you were learning in that instance

And in particular, you talked about your commitments stemming from your experiences prior to entering Colos...but that your time at Colos has helped you ‘Intellectualize your commitments’ how did this happen?

Interaction to ask about	Additional questions to clarify what this interaction entails	- Describe what that space looked/felt/sounded like -What were you learning in that instance <i>Probe to learn: what about this interaction was helpful for their learning</i>	-How do you see this as related to your equity-orientation in MTE?
Co-Planning with Advisor			
RA work with colleague	What is the goal/purpose of this work?		
Advisee group	What is the purpose of this space? -choose coursework, Raships, research -navigating the hidden curriculum of doc programs?		
Program emphasis	How did you get synced up with [this emphasis]?		

Your own research	What forms of support were helpful for you in doing your own independent research?		
Math Ed Group	Is it important that there are no faculty members in [this group]?		