

String-stable constant-spacing platoons without communication

Henry Chang, Yudong Lin, Santosh Devasia

Abstract—This work considers string instability in constant-spacing-policy (CSP) platoons without communication. It is well known that platoons will have string instability without communication with CSP if the vehicle dynamics has at least two poles at the origin. This work considers a modification of the vehicle dynamics, using input-output feedback linearization along with stable-pole zero cancellations, to retain only one pole at the origin. As a result, it is possible to design CSP that is string stable without communication. A disadvantage, with the reduction to only one pole at the origin, is that there is non-zero steady-state error for a constant platoon-speed. However, it is shown that a Taylor-approximation-based, delayed-self-reinforcement (Taylor-approximated DSR) approach can substantially reduce the overall (transient and steady-state) spacing errors while avoiding string instability. Comparative simulations using a benchmark vehicle model show that the proposed Taylor-approximated DSR can reduce the overall error substantially (by 95%) when compared to the standard predecessor-following CSP platoons. Moreover, the resulting steady-state spacing deviation is more than 2 orders of magnitude less when compared to other string-stable approaches such as the constant-time-headway (CTH) policy.

I. INTRODUCTION

Essentially, previous work in [1] has shown that no linear controller can achieve string stability for stable constant-spacing policy (CSP), without communication (e.g., with the lead vehicle), if the vehicle dynamics has more than one pole at the origin. Currently, the main approach to achieve string stability for CSP is to use communication, e.g., [2]. However, inter-vehicle communication is not always feasible when vehicles form ad-hoc platoons without the necessary communication infrastructure or if there are security or jamming concerns, e.g., [3], [4]. In the absence of communication, since string-stability cannot be achieved, the approach has been to relax the constant-spacing requirement by allowing the spacing to increase proportional to the steady-state speed using a constant time headway (CTH) approach, typically, requiring about 1.5 – 2s of time headway resulting in 30 – 40m steady-state spacing deviation at 20m/s speed. Although CTH facilitates string stability, the large spacing can substantially decrease the overall flow of vehicles.

An alternative to CTH for string-stability without communication is work in Ref. [5] that uses delayed-self-reinforcement (DSR) [6] – the work in [5] has shown that CSP platoons can be made string-stable even in the presence of communication loss. However, for small spacing errors, CSP platoons with the DSR approach in [5] still require some communication of leader information. Recent work [7]

has developed a modified DSR approach for platoons that does not require communication for reducing the spacing errors and has been experimentally demonstrated in human-guided robotic platoons. Thus, the use of DSR can enable string-stable CSP with small spacing errors.

The current article shows that a Taylor-approximation of the DSR (Taylor-approximated DSR) also can achieve string-stable CSP and yield small spacing errors. An advantage of the proposed Taylor-approximated DSR is that it does not require the use of delay terms as in standard DSR. The proposed Taylor-approximated DSR approach uses input-output feedback linearization followed by stable pole-zero cancellation to reduce the number of poles at the origin to one for the vehicle dynamics. This allows the design of string-stable CSP. Effectively, it is shown (i) that the Taylor-approximation of DSR results in a high-gain proportional-derivative (PD) controller, (ii) that the combined Taylor-approximated DSR controller (high-gain PD and pole-zero cancellation after feedback linearization) can be implemented using a feedforward/feedback scheme without the high-gain. Additionally, the resulting Taylor-approximated DSR controller is shown to be string stable with small steady-state spacing errors when compared to other standard string-stable approaches such as the CTH policy. The improvement in performance with the proposed Taylor-approximated DSR approach is illustrated using comparative simulations.

II. PROBLEM FORMULATION

A. CSP: System description

The goal of each follower vehicle i ($i \geq 1$) is to control its position $x_i(t)$ for maintaining a desired distance $\delta_i(t)$ from the position $x_{i-1}(t)$ of preceding vehicle $i - 1$, as shown in Fig. 1(top). Therefore, the reference trajectory $r_i(t)$ for vehicle i is $r_i(t) = x_{i-1}(t) - \delta_i(t)$ with the feedback control scheme shown in Fig. 1(bottom).

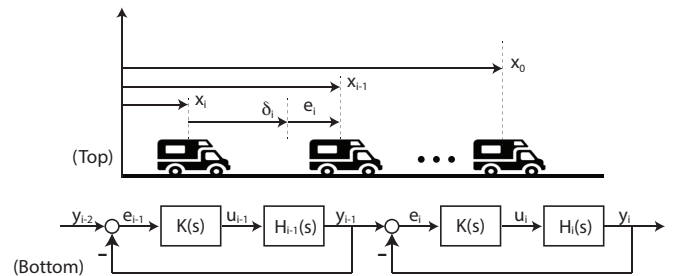


Fig. 1: (Top) schematic platoon. (Bottom) control scheme.

The platoon is assumed to satisfy the standard, constant-spacing-analysis assumptions, as in [1], that all vehicles:

- i. have the same linear single-input-single-output (SISO) dynamics $H_i(s) = H(s)$,
- ii. have the same linear feedback controller $K(s)$,
- iii. have the same constant desired spacing $\delta_i(t) = \delta$,
- iv. start from rest, and
- v. have zero initial spacing errors, with

$$x_0(0) = 0, \quad x_i(0) = -i\delta, \quad \forall i \geq 1. \quad (1)$$

With constant desired offset δ between vehicles, the positions x can be redefined as y , which are deviations from the initial positions,

$$y_i(t) = x_i(t) - x_i(0) = x_i(t) + i\delta, \quad \forall i \geq 0, \quad (2)$$

with zero initial conditions

$$y_i(0) = 0, \quad \forall i \geq 0. \quad (3)$$

B. String instability with any stabilizing controller [1]

Since the vehicles start from rest, the position $y_i(t)$ can be related to the vehicle input $u_i(t)$, in Laplace domain, as

$$Y_i(s) = H(s)U_i(s) = K(s)H(s)E_i(s) \quad (4)$$

where the error is $e_i(t) = y_{i-1}(t) - y_i(t)$ as in Fig. 1(bottom) and the capitalization, e.g., $Y_i(s)$ indicates the Laplace Transform of the corresponding time-domain signal $y_i(t)$. Then, the spacing error $e_i(t) = y_{i-1}(t) - y_i(t)$ for each vehicle $i \geq 1$ is given by

$$E_i(s) = Y_{i-1}(s) - Y_i(s) \quad (5)$$

$$= Y_{i-1}(s) - K(s)H(s)E_i(s) \quad (6)$$

resulting in

$$E_i(s) = \frac{1}{1 + K(s)H(s)} Y_{i-1}(s) \quad (7)$$

and using Eq. (4)

$$E_1(s) = \frac{1}{1 + K(s)H(s)} Y_0(s) \quad (8)$$

$$E_i(s) = \frac{1}{1 + K(s)H(s)} Y_{i-1}(s) = \frac{K(s)H(s)}{1 + K(s)H(s)} E_{i-1}(s). \quad (9)$$

Thus, the changes in the spacing error $E_i(s)$ as it propagates along the platoon depends on the complementary sensitivity function $T(s)$,

$$E_i(s) = \frac{K(s)H(s)}{1 + K(s)H(s)} E_{i-1}(s) \quad (10)$$

$$= T(s)E_{i-1}(s) \quad (11)$$

$$= [T(s)]^{i-1} E_1(s). \quad (12)$$

For string stability, i.e., for the error to not grow while it propagates down the platoon (increasing i), the magnitude $|T(j\omega)|$ of the complementary sensitivity function $T(s)$ should be less than one at all frequencies ω ,

$$\|T(s)\|_\infty = \sup_{\omega \in \mathbb{R}} |T(j\omega)| \leq 1. \quad (13)$$

However, the problem is that there are classical limits on reducing the complementary sensitivity function $T(j\omega)$ at all frequencies ω . In particular, as shown in Theorem 1 of [1], if vehicle dynamics $H(s)$ is a rational transfer function with at least two poles at the origin and if $K(s)$ is any stabilizing controller, then

$$\int_0^\infty \ln|T(j\omega)| \frac{d\omega}{\omega^2} \geq 0. \quad (14)$$

As a result, there has to be some frequency ω^* such that

$$\ln|T(j\omega^*)| > 0 \quad (15)$$

or

$$|T(j\omega^*)| > 1, \quad (16)$$

which violates the string stability condition in Eq. (13) as error in frequency ω^* will grow as it propagates along the platoon, e.g., as in Eq. (12). Moreover, for zero steady-state error at constant speed V , the open loop function $L(s) = K(s)H(s)$ should have at least two poles at the origin.

C. Research problem

The strong negative result in [1] showed that there exists no stabilizing controller $K(s)$ with the decentralized control scheme in Fig. 1(bottom) which is string stable (i.e., no amplification of error at any frequency) if the vehicle dynamics $H(s)$ has at least two poles (for zero-steady-state error with CSP) at the the origin because of the requirement of Eq. (16). The research question is to see if there are alternate decentralized control schemes to achieve string-stability with a stabilizing controller $K(s)$ that seeks to maintain constant spacing between the vehicles. Moreover, the stabilizing controller should also result in small spacing error when the lead vehicle has constant speed V , with position, i.e., when

$$y_0(t) = Vt. \quad (17)$$

III. PROPOSED APPROACH

The negative result on lack of stabilizing string-stable linear controllers in [1] relies on the open loop system $L(s)$

$$L(s) = K(s)H(s) \quad (18)$$

having two poles at the origin, as shown below. Therefore, the proposed approach removes this condition using a modified control scheme, which then enables potential string-stable, linear stabilizing controllers.

A. Motivation for the proposed control scheme

The lack of stabilizing string-stable linear controllers follows from [8] (see Corollary 13.4.1), which shows that provided the open-loop function L is has at least one pole at the origin and provided the closed-loop system is stable,

$$\int_0^\infty \ln \left| T \left(\frac{1}{-j\vartheta} \right) \right| d\vartheta = -\frac{\pi}{2} L_0^{-1} + \pi \sum_{i \in \mathbb{Z}} \frac{1}{\zeta_i} + \frac{\pi}{2} \tau, \quad (19)$$

$$\bar{u}_i(t) = -ke_i(t) = k[y_{i-1}(t) - y_i(t)] \quad (31)$$

with open-loop function $\bar{L}(s) = k/s$. Since the open-loop function only has one pole at the origin, it does not violate the conditions for the negative result that any linear stabilizing controller cannot be a string stable. Note that the closed-loop complementary sensitivity function is

$$\bar{T}(s) = \frac{k}{s+k}, \quad (32)$$

which implies that each system is stable provided the controller gain is positive, i.e.,

$$k > 0. \quad (33)$$

Moreover, the gain of complementary sensitivity function is less than one since

$$\|\bar{T}(s)\|_\infty = \sup_{\omega \in \mathbb{R}} \frac{k}{\sqrt{\omega^2 + k^2}} = 1. \quad (34)$$

Finally, the spacing error in follower vehicle $i \geq 1$ can be found using Eqs. (8) and (12), for a constant-speed lead-vehicle trajectory in Eq. (17), as

$$\begin{aligned} E_i(s) &= [\bar{T}(s)]^{i-1} \frac{1}{1 + \bar{L}(s)} Y_0(s) \\ &= [\bar{T}(s)]^{i-1} \frac{1}{1 + \bar{L}(s)} \frac{V}{s^2}. \end{aligned} \quad (35)$$

Therefore, the steady-state spacing error for a constant speed V lead-vehicle trajectory is, since $\lim_{s \rightarrow 0} \bar{T}(s) = 1$,

$$\begin{aligned} \lim_{t \rightarrow \infty} e_i(t) &= \lim_{s \rightarrow 0} E_i(s) \\ &= \lim_{s \rightarrow 0} s [\bar{T}(s)]^{i-1} \frac{1}{1 + \bar{L}(s)} \frac{V}{s^2} \\ &= \lim_{s \rightarrow 0} \frac{1}{1 + \bar{L}(s)} \frac{V}{s} = \lim_{s \rightarrow 0} \frac{1}{1 + \frac{k}{s}} \frac{V}{s} \\ &= \lim_{s \rightarrow 0} \frac{V}{s+k} = \frac{V}{k}. \end{aligned} \quad (36)$$

The spacing error is proportional to speed V and can be reduced by choosing the controller gain k to be sufficiently large in the operating bandwidth ($\omega_{BW} \approx k$ rads/s) of the vehicle.

Remark 4 (Steady-state error). *Reducing the vehicle dynamics to only have one pole at the origin (rather than two poles at the origin) reduces the type of the open-loop function. Therefore, the steady-state error in Eq. (36) for a constant platoon speed V is to be expected.*

D. Proposed approach to reduce the spacing error

The goal of this section is to reduce the the steady-state error in Eq. (36) for a constant platoon speed V . Note that the reduction of the vehicle dynamics to a single integrator can be leveraged to design of vehicle control with small spacing error, e.g., using delayed-self-reinforcement (DSR) [6] for reducing spacing errors: (i) even with loss of communication as in [5] and (ii) with communication-free controllers as in [7]. Such DSR controllers are simplified using the Taylor approximation and this Taylor-approximated DSR control is

analyzed to show that it maintains string-stability and leads to small tracking errors.

1) *DSR control*: Consider the following DSR controller from [6], [7] that is a modification of the string-stable proportional control in (31)

$$\begin{aligned} \bar{u}_i(t) &= k e_i(t) + \frac{\beta_1}{\tau_d} (e_i(t) - e_i(t - \tau_d)) \\ &\quad + \frac{\beta_2}{\tau_d} (y_i(t) - y_i(t - \tau_d)), \end{aligned} \quad (37)$$

where the controller gain k is positive $k > 0$, τ_d is the sampling time, or in the Laplace domain

$$\begin{aligned} \bar{U}_i(s) &= k E_i(s) + \frac{\beta_1}{\tau_d} E_i(s) (1 - e^{-s\tau_d}) \\ &\quad + \frac{\beta_2}{\tau_d} Y_i(s) (1 - e^{-s\tau_d}). \end{aligned} \quad (38)$$

2) *Taylor-approximated DSR*: The Taylor approximation

$$e^{-s\tau_d} \approx 1 - s\tau_d,$$

modifies Eq. (38) to

$$\bar{U}_i(s) = k E_i(s) + \beta_1 s E_i(s) + \beta_2 s Y_i(s). \quad (39)$$

Applying the control law in Eq. (39) to the modified vehicle dynamics in Eq. (29) yields, in the Laplace domain,

$$s Y_i(s) = k E_i(s) + \beta_1 s E_i(s) + \beta_2 s Y_i(s). \quad (40)$$

Collecting similar terms and rearranging yields (provided $\beta_2 < 1$)

$$Y_i(s) = \left[\frac{k + \beta_1 s}{s(1 - \beta_2)} \right] E_i(s) = L_{DSR}(s) E_i(s). \quad (41)$$

This results in a high-gain PD controller for input $\bar{U}(s)$

$$\bar{U}_i(s) = \left[\frac{k + \beta_1 s}{(1 - \beta_2)} \right] E_i(s) \quad (42)$$

$$= K_{DSR}(s) E_i(s), \quad \text{provided } \beta_2 < 1, \quad (43)$$

where $L_{DSR}(s)$ is the new open-loop function of the DSR control scheme illustrated in Fig. 3 with the Taylor-approximated controller $K_{DSR}(s)$

$$K_{DSR}(s) = \left[\frac{k + \beta_1 s}{(1 - \beta_2)} \right] \quad \text{where } \beta_2 < 1. \quad (44)$$

Since the open-loop function $L_{DSR}(s)$ only has one pole at the origin, it does not violate the conditions for the negative result that any linear stabilizing controller cannot be a string stable.

Remark 5 (Avoids delay terms in DSR). *An advantage of the Taylor-approximated DSR controller $K_{DSR}(s)$ in Eq. 44 is that it does not require any delay terms as in standard DSR. Nevertheless, since the controller is developed by approximating the DSR, the controller $K_{DSR}(s)$ is referred to as the Taylor-approximated DSR.*

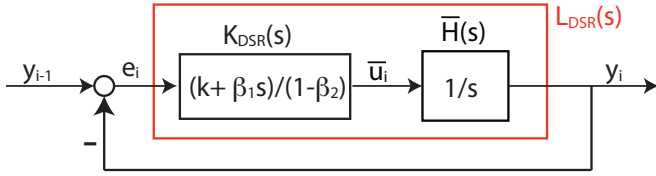


Fig. 3: The Taylor-approximated DSR control scheme with open-loop function $L_{DSR}(s)$ given in Eq. (41) with $\beta_2 < 1$.

Remark 6 (Connection of Taylor-approximated DSR control to feedforward). *Note that if the control constants β_1 and β_2 in the DSR control in Eq. (39) are both one $\beta_1 = \beta_2 = 1$, then the dynamics in Eq. (40) reduces to*

$$\dot{y}_i(t) = k e_i(t) + \dot{y}_{i-1}(t), \quad (45)$$

which has a feedforward component of the predecessor's velocity $\dot{y}_{i-1}(t)$, and represents the ideal, exponentially-stable, tracking controller,

$$\dot{e}_i(t) = -k e_i(t). \quad (46)$$

3) *Stability of Taylor-approximated DSR control:* The complementary sensitivity function with the Taylor-approximated controller in Eq. (44), is

$$\begin{aligned} T_{DSR}(s) &= \frac{L_{DSR}(s)}{1 + L_{DSR}(s)} \\ &= \frac{(k + \beta_1 s)}{s(1 - \beta_2) + (k + \beta_1 s)} \\ &= \frac{\beta_1 s + k}{(1 + \beta_1 - \beta_2)s + k}. \end{aligned} \quad (47)$$

Therefore, with $k > 0$, the closed-loop system with the approximate DSR controller in Eq. (39) is stable if

$$\beta_2 < 1 + \beta_1. \quad (48)$$

4) *Steady-state error with Taylor-approximated DSR:* Again, the error in follower vehicle $i \geq 1$ can be found using Eqs. (8) and (12), for a constant-speed lead-vehicle trajectory in Eq. (17), as [7]

$$\begin{aligned} E_i(s) &= [T(s)]^{i-1} \frac{1}{1 + L_{DSR}(s)} Y_0(s) \\ &= [T(s)]^{i-1} \frac{1}{1 + L_{DSR}(s)} \frac{V}{s^2}. \end{aligned} \quad (49)$$

The steady-state error for any constant speed V lead-vehicle trajectory in Eq. (17) is, since $\lim_{s \rightarrow 0} T_{DSR}(s) = 1$,

$$\begin{aligned} \lim_{t \rightarrow \infty} e_i(t) &= \lim_{s \rightarrow 0} s E_i(s) \\ &= \lim_{s \rightarrow 0} s [T(s)]^{i-1} \left(\frac{1}{1 + L_{DSR}(s)} \right) \frac{V}{s^2} \\ &= \lim_{s \rightarrow 0} \left(\frac{1}{1 + L_{DSR}(s)} \right) \frac{V}{s} \\ &= \lim_{s \rightarrow 0} \left(\frac{s(1 - \beta_2)}{s(1 - \beta_2) + k + \beta_1 s} \right) \frac{V}{s} \end{aligned} \quad (50)$$

$$= \left(\frac{1 - \beta_2}{k} \right) V, \quad (51)$$

which can be made small by selecting $|1 - \beta_2|$ to be a small or if the bandwidth of the vehicle dynamics is high (controller gain k is large).

Remark 7 (Reduced steady-state error). *Note that the Taylor-approximated DSR has smaller steady-state error when compared to the error in Eq. (36) for a constant platoon speed V with the proportional controller.*

5) *String stability of Taylor-approximated DSR control:* The main condition for string stability is shown in the following lemma.

Lemma 1 (String-stability condition). *Let the proportional gain be positive $k > 0$. Then, for string stability with DSR control,*

$$\beta_2 < 1 \quad \text{if } \beta_1 \geq 0 \quad (52)$$

$$\beta_2 < 1 + 2\beta_1 \quad \text{if } \beta_1 < 0. \quad (53)$$

Proof: If $\beta_1 \geq 0$, then there are no closed right half plane zeros in $L_{DSR}(s)$ from Eq. (41). Moreover, the velocity constant with DSR is from Eq. (41)

$$\lim_{s \rightarrow 0} s L_{DSR}(s) = \lim_{s \rightarrow 0} \frac{k + \beta_1 s}{(1 - \beta_2)} = \frac{k}{(1 - \beta_2)}. \quad (54)$$

Then, from Eq. (21),

$$\int_0^\infty \ln |T_{DSR}(j\omega)| \frac{d\omega}{\omega^2} = -\frac{\pi}{2} \frac{1 - \beta_2}{k}. \quad (55)$$

Therefore, for string stability, i.e., for $\|T_{DSR}(s)\|_\infty \leq 1$, the integral in Eq. (55) cannot be non-negative, which requires the condition $\beta_2 < 1$ in Eq. (52).

If $\beta_1 < 0$ then there is a closed right half plane zero at $-k/\beta_1$ in $L_{DSR}(s)$ from Eq. (41). Then, from Eq. (21),

$$\begin{aligned} \int_0^\infty \ln |T_{DSR}(j\omega)| \frac{d\omega}{\omega^2} &= -\frac{\pi}{2} \frac{1 - \beta_2}{k} - \pi \frac{\beta_1}{k} \\ &= -\frac{\pi}{2} \frac{1 - \beta_2 + 2\beta_1}{k}. \end{aligned} \quad (56)$$

Again, for string stability, i.e., for $\|T_{DSR}(s)\|_\infty \leq 1$, the integral in Eq. (56) cannot be non-negative, which requires the condition $1 - \beta_2 + 2\beta_1 > 0$ in Eq. (53).

Moreover, the norm of the complementary sensitivity function $T_{DSR}(s)$ can be found from Eq. (47) as

$$\begin{aligned} \|T_{DSR}(s)\|_\infty &= \sup_{\omega \in \mathbb{R}} \left| \frac{\beta_1 j\omega + k}{(1 + \beta_1 - \beta_2)j\omega + k} \right| \\ &= \sup_{\omega \in \mathbb{R}} \frac{\sqrt{k^2 + \beta_1^2 \omega^2}}{\sqrt{k^2 + (1 + \beta_1 - \beta_2)^2 \omega^2}}. \end{aligned} \quad (57)$$

If $\beta_1 \geq 0$, then $1 - \beta_2 > 0$ from Eq. (52), results in

$$\beta_1 < \beta_1 + (1 - \beta_2) \quad (58)$$

or

$$\beta_1^2 < (\beta_1 + (1 - \beta_2))^2. \quad (59)$$

Similarly, if $\beta_1 < 0$ (i.e., $-\beta_1 > 0$), then

$$0 < -\beta_1 < \beta_1 + (1 - \beta_2) \quad (60)$$

from Eq. (53), which again leads to Eq. (59), i.e.,

$$\beta_1^2 < (\beta_1 + (1 - \beta_2))^2. \quad (61)$$

Therefore, for all β_1 ,

$$\sqrt{k^2 + \beta_1^2 \omega^2} \leq \sqrt{k^2 + (1 + \beta_1 - \beta_2)^2 \omega^2}. \quad (62)$$

resulting in $\|T_{DSR}(s)\|_\infty \leq 1$ from Eq. (57). ■

6) Implementation without high-gain ($m > 1$ case):

When the relative degree is more than one, $m > 1$ requiring the stable-pole-zero-canceling block-diagram in Fig. 3, the input $\tilde{u}_i(t)$ to the vehicle can be found from Eq. (23), and definition of $K_{DSR}(s)$ in Eq. (44), as

$$\begin{aligned} \tilde{U}_i(s) &= C_{ff}(s)\bar{U}_i(s) - C_{fb}(s)Y_i(s) \\ &= C_{ff}(s)K_{DSR}(s)\bar{E}_i(s) - C_{fb}(s)Y_i(s) \\ &= C_{ff}(s) \left[\tilde{k} + \tilde{\beta}_1 s \right] \bar{E}_i(s) - C_{fb}(s)Y_i(s) \\ &= C_{ff}(s)\tilde{k}\bar{E}_i(s) - C_{fb}(s)Y_i(s) + \tilde{\beta}_1 s C_{ff}(s)\bar{E}_i(s), \end{aligned} \quad (63)$$

where the high-gain terms $\tilde{k}, \tilde{\beta}_1$ are

$$\tilde{k} = \frac{k}{1 - \beta_2}, \quad \tilde{\beta}_1 = \frac{\beta_1}{1 - \beta_2}. \quad (64)$$

Note that from Eqs. (24) and Eq. (25)

$$sC_{ff}(s) = C_{fb}(s) + s^m, \quad (65)$$

which can be substituted in Eq. (63) to obtain

$$\begin{aligned} \tilde{U}_i(s) &= C_{ff}(s)\tilde{k}\bar{E}_i(s) - C_{fb}(s)Y_i(s) \\ &\quad + \tilde{\beta}_1 (C_{fb}(s) + s^m) \bar{E}_i(s) \\ &= C_{ff}(s)\tilde{k}\bar{E}_i(s) - C_{fb}(s)Y_i(s) \\ &\quad + \tilde{\beta}_1 [C_{fb}(s) + s^m] (Y_{i-1}(s) - Y_i(s)) \\ &= C_{ff}(s)\tilde{k}\bar{E}_i(s) - (1 + \tilde{\beta}_1) C_{fb}(s)Y_i(s) \\ &\quad + \tilde{\beta}_1 [C_{fb}(s) + s^m] Y_{i-1}(s) \\ &\quad - \tilde{\beta}_1 s^m Y_i(s) \end{aligned} \quad (66)$$

Substituting for $\tilde{U}_i(s) = s^m Y_i(s)$ from Eq. (22) into Eq. (66) results in

$$\begin{aligned} \tilde{U}_i(s) &= C_{ff}(s)\tilde{k}\bar{E}_i(s) - (1 + \tilde{\beta}_1) C_{fb}(s)Y_i(s) \\ &\quad + \tilde{\beta}_1 [C_{fb}(s) + s^m] Y_{i-1}(s) - \tilde{\beta}_1 \tilde{U}_i(s) \end{aligned} \quad (67)$$

or

$$\begin{aligned} (1 + \tilde{\beta}_1) \tilde{U}_i(s) &= C_{ff}(s)\tilde{k}\bar{E}_i(s) - (1 + \tilde{\beta}_1) C_{fb}(s)Y_i(s) \\ &\quad + \tilde{\beta}_1 [C_{fb}(s) + s^m] Y_{i-1}(s) \end{aligned} \quad (68)$$

leading to removal of the high-gain term $1/(1 - \beta_1)$, using Eq. (64),

$$\begin{aligned} \tilde{U}_i(s) &= C_{ff}(s) \frac{\tilde{k}}{(1 + \tilde{\beta}_1)} \bar{E}_i(s) - C_{fb}(s)Y_i(s) \\ &\quad + \frac{\tilde{\beta}_1}{(1 + \tilde{\beta}_1)} [C_{fb}(s) + s^m] Y_{i-1}(s) \\ &= C_{ff}(s) \frac{k}{(\beta_1 + (1 - \beta_2))} \bar{E}_i(s) - C_{fb}(s)Y_i(s) \\ &\quad + \frac{\beta_1 [C_{fb}(s) + s^m]}{(\beta_1 + (1 - \beta_2))} Y_{i-1}(s) \\ &= C_{ff}(s)K_{fb}\bar{E}_i(s) - C_{fb}(s)Y_i(s) + K_{ff}(s)Y_{i-1}(s), \end{aligned} \quad (69)$$

where

$$K_{fb} = \frac{k}{(\beta_1 + (1 - \beta_2))}, \quad K_{ff}(s) = \frac{\beta_1 [C_{fb}(s) + s^m]}{(\beta_1 + (1 - \beta_2))} \quad (71)$$

with a feedforward term

$$\tilde{Y}_{i-1}(s) = K_{ff}(s)Y_{i-1}(s) \quad (72)$$

of the predecessor vehicle and a tracking-error-feedback term $C_{ff}(s)K_{fb}\bar{E}_i(s)$, as shown in Fig. 4. The resulting feedback-feedforward implementation of the non-high-gain, Taylor-approximated DSR from Eq. (70) is shown in Fig. 4.

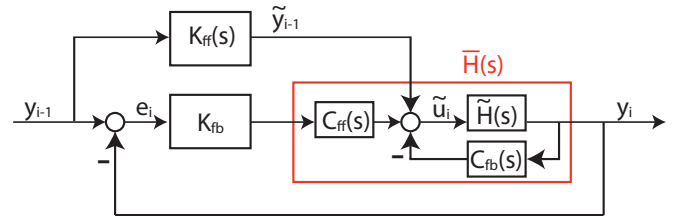


Fig. 4: Equivalent feedforward $K_{ff}(s)$ and feedback K_{fb} implementation in Eq. (70) of the Taylor-approximated DSR in Fig. 3, without high-gain terms, by using controllers $K_{ff}(s)$, K_{fb} in Eq. (71).

Remark 8 (Properties of feedforward/feedback controller). *The equivalent feedforward/feedback implementation of the Taylor-approximated DSR in Fig. (4) is stable provided the gain $K_{fb} = \frac{k}{(\beta_1 + (1 - \beta_2))}$ is positive, which is similar to stability condition for the proportional controller in Eq. (33). The relationship between the vehicle response $y_i(t)$ and the predecessor position $y_{i-1}(t)$ has not changed, and therefore string-stability and steady-state properties of the Taylor-approximated DSR are not affected by the equivalent feedforward/feedback implementation.*

Remark 9 (Sensing and implementation issues). *The feedforward controller $K_{ff}(s)Y_{i-1}(s)$ in Eq. (70) requires sensing of the predecessor vehicle's output $y_{i-1}(t)$ and its time derivatives. In implementation perspective, one approach is to sense only the predecessor vehicle's output $y_{i-1}(t)$*

and then apply differentiation to obtain derivatives of the predecessor vehicle's output $y_{i-1}(t)$ using a low-pass filter $F(s)$ to avoid differentiating high-frequency noise in the sensed predecessor position [11], [12], i.e.,

$$F(s) = \left[\frac{\omega_f}{s + \omega_f} \right]^m. \quad (73)$$

The modified feedforward term in Eq. (72) $\tilde{y}_{i-1}(t)$ becomes

$$\tilde{Y}_{i-1}(s) = K_{ff}(s) \left(\frac{\omega_f}{s + \omega_f} \right)^m Y_{i-1}(s). \quad (74)$$

Note that these filters in the feedforward path avoids tracking unwanted high-frequency noise in the sensed predecessor position, and does not impact stability of the system.

IV. RESULTS AND DISCUSSION

Simulation results are presented for the example system in [1], with and without the proposed approach (feedback-linearization and reduction of the vehicle dynamics to a first-order integrator) that facilitates the design of string-stable, constant-spacing platoons.

A. String-instability in example system

The benchmark vehicle dynamics and controller in [1] is

$$H(s) = \frac{1}{s^2(\tau_H s + 1)}, \quad K(s) = \frac{\gamma s + 1}{\tau_K s + 1} \quad (75)$$

where $\tau_H = 0.1$, $\tau_K = 0.05$ and $\gamma = 2$. The vehicle dynamics $H(s)$ corresponds to the state space model

$$\frac{d}{dt} \begin{bmatrix} y_i(t) \\ \dot{y}_i(t) \\ \ddot{y}_i(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau_H} \end{bmatrix} \begin{bmatrix} y_i(t) \\ \dot{y}_i(t) \\ \ddot{y}_i(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau_H} \end{bmatrix} u_i(t) \quad (76)$$

Note that the controller $K(s)$ can be rewritten as

$$\begin{aligned} K(s) &= \frac{\gamma s + 1}{\tau_K s + 1} = \frac{\frac{\gamma}{\tau_K} s + \frac{1}{\tau_K}}{s + \frac{1}{\tau_K}} \\ &= \frac{\frac{\gamma}{\tau_K} s + \frac{\gamma}{\tau_K^2} + \frac{1}{\tau_K} - \frac{\gamma}{\tau_K^2}}{s + \frac{1}{\tau_K}} \\ &= \frac{\gamma}{\tau_K} + \frac{\frac{1}{\tau_K} - \frac{\gamma}{\tau_K^2}}{s + \frac{1}{\tau_K}} \end{aligned} \quad (77)$$

Therefore, the controller $K(s)$ corresponds to a state space model (with controller state $z_i(t)$ acting on error $e_i(t)$)

$$\frac{d}{dt} z_i(t) = -\frac{1}{\tau_K} z_i(t) + e_i(t) \quad (78)$$

$$u_i(t) = \left(\frac{\tau_K - \gamma}{\tau_K^2} \right) z_i(t) + \frac{\gamma}{\tau_K} e_i(t). \quad (79)$$

The resulting closed-loop dynamics is

$$\begin{aligned} \frac{d}{dt} \begin{bmatrix} y_i(t) \\ \dot{y}_i(t) \\ \ddot{y}_i(t) \\ z_i(t) \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{\gamma}{\tau_H \tau_K} & 0 & -\frac{1}{\tau_H} & \left(\frac{\tau_K - \gamma}{\tau_K^2} \right) \\ -1 & 0 & 0 & -\frac{1}{\tau_K} \end{bmatrix} \begin{bmatrix} y_i(t) \\ \dot{y}_i(t) \\ \ddot{y}_i(t) \\ z_i(t) \end{bmatrix} \\ &\quad + [0 \ 0 \ \frac{\gamma}{\tau_H \tau_K} \ 1]^T y_{i-1}(t). \end{aligned} \quad (80)$$

The motion of the lead vehicle is defined by a trapezoidal input trajectory $U_0(s)$ (which seeks to accelerate the lead vehicle from rest to 20m/s over roughly 12 seconds)

$$U_0(s) = \frac{1}{s^2} (e^{-s} - e^{-3s} - e^{-11s} + e^{-13s}) \quad (81)$$

with the desired spacing $\delta = 5m$ for a 6-vehicle platoon with 5 followers $N = 5$. The resulting motion of the lead vehicle is given by

$$Y_0(s) = X_0(s) = H(s)U_0(s), \quad (82)$$

leading to the error $E_1(s)$ for the first follower from Eq. (8). The errors for the followers can be found by simulating the state-space model in Eq. (80), and are shown in Fig. 5. As seen in the plots, the error increases as it propagates along the platoon, which illustrates the string-instability issue.

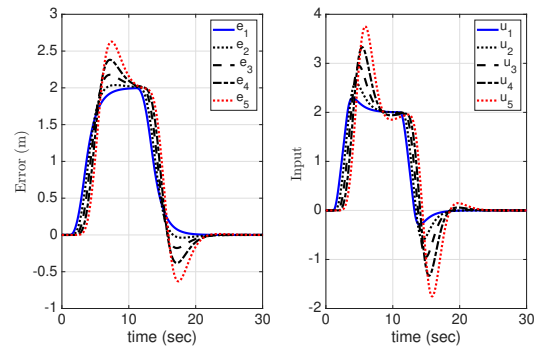


Fig. 5: Constant-spacing-policy platoon from [1] is string-unstable. (Left) Time domain plots of the errors $\{e_i(t)\}_{i=1}^5$ for the follower vehicles with the predecessor-follower strategy, showing growth in the error as it propagates along the platoon. (Right) Time domain plots of the inputs $\{u_i(t)\}_{i=1}^5$ for the follower vehicles also show a corresponding increase in magnitude.

B. Constant-time-headway policy

One approach to overcome the string-instability to use the constant-time-headway (CTH) policy where an additional velocity-dependent offset $\delta_i(t) = T_\delta \dot{y}_i(t)$ is added to the desired spacing from the predecessor, i.e., to modify the error that is controlled to

$$e_{m,i}(t) = e_i(t) - T_\delta \dot{y}_i(t),$$

which is then acted upon by the controller $K(s)$ in Fig. 1(bottom) to yield $U_i(s) = K(s)E_{m,i}(s)$. The resulting closed-loop dynamics becomes

$$\begin{aligned} \frac{d}{dt} \begin{bmatrix} y_i(t) \\ \dot{y}_i(t) \\ \ddot{y}_i(t) \\ z_i(t) \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{\gamma}{\tau_H \tau_K} & -\frac{\gamma T_\delta}{\tau_H \tau_K} & -\frac{1}{\tau_H} & \left(\frac{\tau_K - \gamma}{\tau_K^2} \right) \\ -1 & -T_\delta & 0 & -\frac{1}{\tau_K} \end{bmatrix} \begin{bmatrix} y_i(t) \\ \dot{y}_i(t) \\ \ddot{y}_i(t) \\ z_i(t) \end{bmatrix} \\ &\quad + [0 \ 0 \ \frac{\gamma}{\tau_H \tau_K} \ 1]^T y_{i-1}(t). \end{aligned} \quad (83)$$

The use of CTH enables string-stability, however, the time headway T_δ needs to be sufficiently large at around $T_\delta = 2s$ – string instability is seen with time headway of $T_\delta = 1s$

as seen in Fig. 6(left). However, a drawback with the CTH-policy is that it also leads to larger spacing error of around 40m, which is to be expected with the time headway of 2s and speed of about 20m/s. Moreover, the platoon is also sluggish in the sense that it takes about 30 seconds (compared to the 12 seconds for the lead vehicle) for all the vehicles to reach the steady-state velocity.

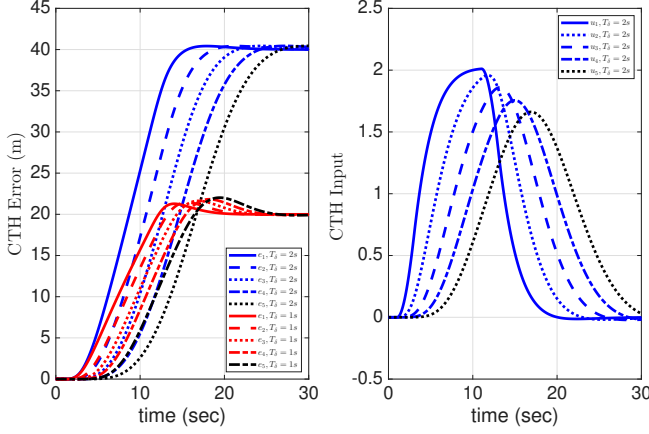


Fig. 6: Results from constant-time-headway (CTH) policy. (Left) Errors $\{e_i(t)\}_{i=1}^5$ do not show string instability when compared to the predecessor-follower strategy in Fig. 5 when the time headway T_δ is sufficiently large, $T_\delta \approx 2s$ – however, the spacing errors in the follower vehicles are substantially large. (Right) The associated inputs $\{u_i(t)\}_{i=1}^5$ to the vehicle do not increase along the platoon when compared to the predecessor follower approach in Fig. 5.

C. Application of proposed approach

The dynamics is converted into a chain of pure integrators, by choosing the input $u_i(t)$ so that the right-hand side of third line of the vehicle dynamics in Eq. (76) becomes

$$-\frac{1}{\tau_H} \ddot{y}_i(t) + \frac{1}{\tau_H} u_i(t) = \tilde{u}_i(t) \quad (84)$$

i.e.,

$$u_i(t) = \tau_H \tilde{u}_i(t) + \ddot{y}_i(t). \quad (85)$$

As a result, the vehicle dynamics in Eq. (76) becomes

$$\frac{d}{dt} \begin{bmatrix} y_i(t) \\ \dot{y}_i(t) \\ \ddot{y}_i(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_i(t) \\ \dot{y}_i(t) \\ \ddot{y}_i(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tilde{u}_i(t), \quad (86)$$

which corresponds to, as in Eq. (22),

$$\tilde{H}(s) = \frac{Y_i(s)}{\tilde{U}_i(s)} = \frac{1}{s^3}. \quad (87)$$

1) *High-gain controller:* The controller for pole-zero cancellation is, from Eqs. (23), (27), and (28)

$$\tilde{u}_i(t) = \ddot{\tilde{u}}_i(t) + 2c_1 \dot{\tilde{u}}_i(t) + c_1^2 \tilde{u}_i(t) - 2c_1 \ddot{y}_i(t) + c_1^2 \dot{y}_i(t). \quad (88)$$

and the DSR control is, from Eq. (43),

$$\bar{u}_i(t) = \frac{\beta_1(\dot{y}_{i-1}(t) - \dot{y}_i(t))}{(1 - \beta_2)} + \frac{k(y_{i-1}(t) - y_i(t))}{(1 - \beta_2)}. \quad (89)$$

Substituting for $\bar{u}_i(t)$ from Eq. (89) into Eq. (88), results in

$$\begin{aligned} \tilde{U}_i(s) &= (s^2 + 2c_1s + c_1^2)\bar{U}_i(s) - (2c_1s^2 + c_1^2s)Y_i(s), \\ &= (s^2 + 2c_1s + c_1^2) \left(\frac{k + \beta_1s}{1 - \beta_2} \right) \bar{E}_i(s) \\ &\quad - (2c_1s^2 + c_1^2s)Y_i(s), \end{aligned} \quad (90)$$

in the Laplace domain. Note the scaling in the input $\tilde{U}_i(s)$ by the high-gain term $1/(1 - \beta_2)$.

2) *Feedforward/Feedback implementation without high gains:* The input to the vehicle dynamics $\tilde{u}_i(t)$ can be implemented, without the scaling by the high-gain term $1/(1 - \beta_2)$, using the feedforward/feedback scheme in Fig. 4. The vehicle dynamics equation used for simulations is derived from Eq. (70), using $\bar{E}_i(s) = Y_{i-1}(s) - Y_i(s)$,

$$\begin{aligned} \tilde{U}_i(s) &= C_{ff}(s)K_{fb}\bar{E}_i(s) - C_{fb}(s)Y_i(s) + K_{ff}(s)Y_{i-1}(s), \\ &= -[C_{ff}(s)K_{fb} + C_{fb}(s)]Y_i(s) \\ &\quad + [C_{ff}(s)K_{fb} + K_{ff}(s)]Y_{i-1}(s). \end{aligned} \quad (91)$$

Using Eqs.(27), and (28) for $C_{ff}(s)$, $C_{fb}(s)$ and Eq.(71) for $K_{ff}(s)$, K_{fb} , in the above Eq. (91), i.e.,

$$\begin{aligned} C_{ff}(s) &= (s + c_1)^2 = s^2 + 2c_1s + c_1^2, \\ C_{fb}(s) &= 2c_1s^2 + c_1^2s \\ K_{fb} &= \frac{k}{(\beta_1 + (1 - \beta_2))} \\ K_{ff}(s) &= \frac{\beta_1 [C_{fb}(s) + s^m]}{(\beta_1 + (1 - \beta_2))} = \frac{\beta_1 [s^3 + 2c_1s^2 + c_1^2s]}{(\beta_1 + (1 - \beta_2))} \end{aligned} \quad (92)$$

as shown in Fig. 4 results in

$$\begin{aligned} \tilde{U}_i(s) &= - \left[\frac{k(s^2 + 2c_1s + c_1^2)}{(\beta_1 + (1 - \beta_2))} + (2c_1s^2 + c_1^2s) \right] Y_i(s) \\ &\quad + \left[\frac{k(s^2 + 2c_1s + c_1^2)}{(\beta_1 + (1 - \beta_2))} + \frac{\beta_1 [s^3 + 2c_1s^2 + c_1^2s]}{(\beta_1 + (1 - \beta_2))} \right] Y_{i-1}(s) \\ &= - \left[\hat{k}(s^2 + 2c_1s + c_1^2) + (2c_1s^2 + c_1^2s) \right] Y_i(s) \\ &\quad + \left[\hat{k}(s^2 + 2c_1s + c_1^2) + \hat{\beta}_1(s^3 + 2c_1s^2 + c_1^2s) \right] Y_{i-1}(s) \\ &= - \left[(2c_1 + \hat{k})s^2 + (c_1^2 + 2c_1\hat{k})s + c_1^2\hat{k} \right] Y_i(s) \\ &\quad + \left[\hat{\beta}_1s^3 + (2c_1\hat{\beta}_1 + \hat{k})s^2 + (c_1^2\hat{\beta}_1 + 2c_1\hat{k})s + c_1^2\hat{k} \right] Y_{i-1}(s) \end{aligned} \quad (93)$$

where

$$\hat{k} = \frac{k}{1 - \beta_2 + \beta_1}, \quad \hat{\beta}_1 = \frac{\beta_1}{1 - \beta_2 + \beta_1}. \quad (94)$$

The corresponding input in the time domain, is

$$\begin{aligned} \tilde{u}_i(t) &= \hat{\beta}_1 \ddot{y}_{i-1}(t) + (2c_1\hat{\beta}_1 + \hat{k})\dot{y}_{i-1}(t) \\ &\quad + (c_1^2\hat{\beta}_1 + 2c_1\hat{k})y_{i-1}(t) + (c_1^2\hat{k})y_{i-1}(t) \\ &\quad - (2c_1 + \hat{k})\ddot{y}_i(t) - (c_1^2 + 2c_1\hat{k})\dot{y}_i(t) - (c_1^2\hat{k})y_i(t) \end{aligned} \quad (95)$$

$$= \tilde{y}_{i-1}(t) - (2c_1 + \hat{k})\ddot{y}_i(t) - (c_1^2 + 2c_1\hat{k})\dot{y}_i(t) - (c_1^2\hat{k})y_i(t), \quad (96)$$

where the filtered predecessor position $y_{f,i-1}(t)$ given by, as in Eq. (73),

$$Y_{f,i-1}(s) = \left(\frac{\omega_f}{s + \omega_f} \right)^3 Y_{i-1}(s), \quad (97)$$

is used to avoid noise effects in the effective feedforward input, as in Eq. (74), modifying $\tilde{y}_{i-1}$ in Eq. (96) to

$$\begin{aligned} \tilde{y}_{i-1}(t) \approx & \hat{\beta}_1 \ddot{y}_{f,i-1}(t) + (2c_1\hat{\beta}_1 + \hat{k})\dot{y}_{f,i-1}(t) \\ & + (c_1^2\hat{\beta}_1 + 2c_1\hat{k})\dot{y}_{f,i-1}(t) + (c_1^2\hat{k})y_{f,i-1}(t). \end{aligned} \quad (98)$$

with $\tilde{y}_{i-1}(t)$ representing the effective feedforward input terms from the predecessor in Eq. (96) obtained using a low-pass filter to remove high-frequency noise. Note that there is no division by $(1-\beta_1)$ (as in Eq. (89)) in the final control law in Eq. (95). The resulting state-space equation from Eq. (86)

$$\begin{aligned} \frac{d}{dt} \begin{bmatrix} y_i(t) \\ \dot{y}_i(t) \\ \ddot{y}_i(t) \end{bmatrix} = & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -(c_1^2\hat{k}) & -(c_1^2 + 2c_1\hat{k}) & -(2c_1 + \hat{k}) \end{bmatrix} \begin{bmatrix} y_i(t) \\ \dot{y}_i(t) \\ \ddot{y}_i(t) \end{bmatrix} \\ & + [0 \ 0 \ 0 \ 1]^T \tilde{y}_{i-1}(t), \end{aligned} \quad (99)$$

with $\tilde{y}_{i-1}$ in Eq. (98) is used in simulations.

Remark 10 (Removal of high gain in vehicle input $u_i(t)$). While the Taylor-approximation of DSR in Eq. (43) appears like a high-gain proportional-derivative controller for $\tilde{u}_i(t)$ due to the division by $(1-\beta_2)$, the actual vehicle input $u_i(t)$ in Eq. (85) does not contain this division by $(1-\beta_2)$ since this dividing term is removed from $\tilde{u}_i(t)$ in Eq. (95).

The DSR controller parameter $\beta_2 = 0.99$ was chosen close to one with $\beta_1 = 1$ to achieve exponential tracking as discussed in Remark 6 and to achieve a small, steady-state error as in Eq. (51). The gain $k = 1/\tau_H$ was chosen to match the actuator bandwidth of the vehicle dynamics. The pole-zero cancellations were selected to be at $-c_1 = -1/\gamma$ to match the zeros in the standard controller $K(s)$ and the filter frequencies were at $\omega_f = 2\pi 100 \text{ rad/s}$ to remove noise as discussed in Remark 9 while avoiding modification of lower-frequency signals of interest. The norm of the complementary sensitivity function $T_{DSR}(s)$ for these parameters is, from Eq. (57),

$$\|T_{DSR}(s)\|_\infty = \sup_{\omega \in \mathbb{R}} \frac{\sqrt{k^2 + \beta_1^2 \omega^2}}{\sqrt{k^2 + (1 + \beta_1 - \beta_2)^2 \omega^2}} \quad (100)$$

$$= \sup_{\omega \in \mathbb{R}} \frac{\sqrt{\frac{1}{\tau_H^2} + \omega^2}}{\sqrt{\frac{1}{\tau_H^2} + (1.01)^2 \omega^2}} = 1. \quad (101)$$

Thus, the complementary sensitivity function $T_{DSR}(s)$ has magnitude one when frequency is zero and the magnitude decreases to $1/1.01$ as frequency increases, which satisfies the string stability condition in Eq. (13).

The proposed Taylor-approximated DSR controller for CSP does not exhibit string instability and substantially reduces the spacing errors without substantially increasing

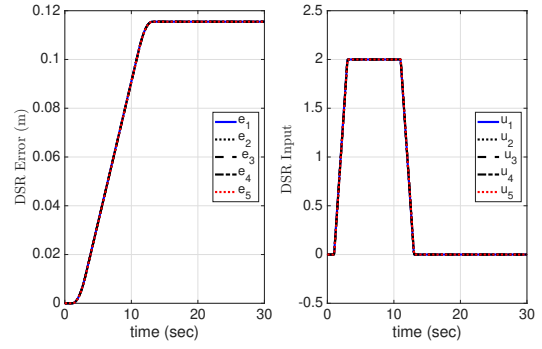


Fig. 7: Results from proposed Taylor-approximated DSR approach. (Left) Errors $\{e_i(t)\}_{i=1}^5$ for the follower vehicles are substantially reduced and does not show string instability when compared to the predecessor-follower strategy in Fig. 5. Moreover, errors are substantially lower than the constant-time-headway (CTH) policy in Fig. 6. (Right) The associated inputs $\{u_i(t)\}_{i=1}^5$ to the vehicle do not increase along the platoon and are smaller when compared to the predecessor follower approach in Fig. 5.

the required inputs — when compared to the standard predecessor follower strategy. The tracking error and input using the Taylor-approximated DSR is shown in Fig. 7. Note that the maximum spacing error magnitude is reduced by 95.6%, from with the predecessor-follower strategy 2.62m to 0.115m, with the proposed approach. Moreover, the maximum input magnitude required is reduced to the smallest possible value, i.e., the maximum desired acceleration of 2 m/s^2 — the reduction is by 46.7%, from 3.75 with the predecessor-follower strategy to 2.0 with the proposed approach. However, a drawback of keeping only one pole at the origin in the modified vehicle dynamics is that the steady-state error with the proposed approach does not go to zero (max steady-state error is 0.115m) when compared to zero steady-state error with the predecessor-follower strategy. Nevertheless, the presence of the steady-state error is similar to the CTH policy that also has a steady-state error that is proportional to speed, although the CTH steady-state error is substantially larger at 40m. Moreover, the overall error with the Taylor-approximated DSR (when including transient errors) is substantially reduced (from 40m to 0.115m) when compared to the CTH predecessor follower strategy, by more than two orders of magnitude, as seen by comparing the results in Figs. 6 and 7.

V. CONCLUSIONS

It is shown that string instability is not unavoidable in constant-spacing-policy (CSP) platoons even if the vehicle dynamics has more than one pure integrator (pole at the origin). While the string-instability limitation is true for a class of linear controllers and vehicle dynamics with more than one pole at the origin, this article showed that input-output feedback-linearization followed by stable-pole zero cancellation can remove the additional poles at the origin enabling string-stable, CSP platoons. The removal of

more than one pole at the origin implies that the steady-state error cannot be zero, when platoon speed is constant, with the proposed constant-spacing tracking. However, it is shown that a Taylor-approximation-based delayed-self-reinforcement (Taylor-approximated DSR) controller can substantially reduce the steady-state tracking error with the proposed CSP approach, without requiring larger inputs.

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