Three Essays on Foreign Trade, Offshoring and International Rivalry

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Abstract

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This dissertation consists of three essays, covering the topics of foreign trade, offshoring and international rivalry. In particular, Chapter 1 analyzes the *strategic capacity allocation* of an international oligopoly. Because a line of products shares specific inputs that are fixed in the short run, a multiproduct oligopolist faces a capacity constraint in the production. Not being able to produce the desirable quantities to meet demand, an oligopolist strategically allocates its capacity among different products against its rival. If the market were monopolistic, a firm would mainly concern the effective profitability of a product when allocating its capacity and when responding to a capacity expansion. Identical duopolists that compete in a Cournot fashion should have identical capacity allocation. However, in a sequential game, while the Stackelberg leader allocates all its scarce capacity towards the more profitable product, the follower should still allocate some capacity towards the unprofitable product. This matches the observation that Boeing, the incumbent in the large commercial aircrafts (LCA) industry, specializes in smaller planes, while Airbus allocates resources more evenly towards both superjumbo planes and smaller planes.

Chapter 2 provides an explanation to the observation that international oligopolists, which are similar in many ways (subject to the same state of technology, have equal market shares, etc.), may engage in significantly different degrees of offshoring. Different from previous studies, which considered fragmentation to be affected by global exogenous factors only, this essay sees fragmentation as an endogenous variable. A firm can invest on R&D of its own fragmentation technology to enable certain degrees of fragmentation, so that offshoring of those fragmented subparts can be achieved. An important implication of endogenous fragmentation is that the government now has a policy alternative to export subsidy. Very often, when export subsidy is prohibited under an FTA, a government has incentive to subsidize fragmentation of a firm, which can stimulate both export and offshoring.

Chapter 3 investigates Macao's and Singapore's questionable goal to diversify among two tourism services - gambling and convention. Macao has a cost advantage in gambling while Singapore has a cost advantage in convention. When a city operates as a regional monopoly, the simple multiproduct model shows that it is optimal for a city to diversify in response to an expansion in the markets of the tourism services. If the two cities operate as a Cournot duopoly instead, there will be a higher degree of product differentiation between the cities. Yet, both cities diversify more when there is a market expansion. On the other hand, Osaka is a potential entrant. The three-city model shows that if Osaka's relative cost of producing convention is even lower than Singapore’s, both Macao and Singapore will produce greater proportions of gambling compared to the two-city case. In general, Macao and Singapore respond to Osaka’s rivalry by strategizing their product mixes to avoid head-on competition with Osaka.

TABLE OF CONTENTS

Page

List of Figures 7

List of Tables 8

Chapter 1: Product Line Rivalry with Strategic Capacity Allocation 12

1.1 Introduction 12

1.2 Monopoly 17

1.3 Duopoly – Cournot Competition 27

1.4 Duopoly – Stackelberg Competition 37

1.5 Concluding Remarks 44

Chapter 2: Foreign Trade, Endogenous Fragmentation and Optimal Offshoring 55

2.1 Introduction 55

2.2 Fragmentation Function 59

2.3 Inactive Foreign Government 61

2.4 Two Active Governments 81

2.5 Concluding Remarks 87

Chapter 3: Strategic Export of Gambling and Convention Services: The Cases of Macao, Singapore and Osaka 91

3.1 Introduction 91

3.2 The Case of a Single City 94

3.3 The Case of Two Cities 100

3.4 The Case of Three Cities 107

3.5 Concluding Remarks 114

Appendix A: Proofs and Additional Explanation of Chapter 1 131

Appendix B: Mathematical Proofs of Chapter 3 133

B.1 The Case of Two Cities 133

B.2 The Case of Three Cities 135

LIST OF FIGURES

Figure Numbers Page

1.1 Optimal Capacity Allocation of Multiproduct Monopolist 47

1.2 Effect of Increase of Capacity, Z 48

1.3 Nash-Cournot-Equilibrium Capacity Allocation of Identical Firms 49

1.4 Nash-Cournot-Equilibrium Capacity Allocation of Differentiated Firms 50

1.5 Strategic Capacity Allocation Under Different Market Structures 51

1.6 Effect of Capacity Expansion 52

2.1 Fragmentation of Boeing 787 89

2.2 Production Organization of Boeing 787 and Airbus A350 90

3.1 Number of Visitors to Macao and Singapore 116

3.2 ϕ as Determinant of $\tilde{G}$ and $\tilde{V}$ 117

3.3 Movement of $\tilde{\alpha}$ in Response to Market Expansion 118

3.4 Reponses of $\tilde{G}$ and $\tilde{V}$ to Market Expansion 119

3.5 Competition Leads to Higher Degree of Product Differentiation 120

3.6 Each City Diversifies More in Response to Increase of Demand 121

3.7 Effects of Osaka’s Entry on Macao’s and Singapore’s Proportions of Gambling Depends on ϕ^O 122

3.8 Effects of Osaka’s Entry on Macao’s and Singapore’s Product Strategies Depends on Osaka’s Product Strategy 123

LIST OF TABLES

Table Numbers Page

1.1 Deliveries of Boeing 787 and Airbus A380 and A350 53

1.2 Orders of Boeing 787 and Airbus A380 and A350 54

3.1 A Numerical Example 124

3.2 A Numerical Example of $\phi^M > 1 > \phi^S > \phi^O$ 125

GLOSSARY

DOD: The US Department of Defense

DSB: Dispute Settlement Body

FTA: Free Trade Agreement

LCA: Large Commercial Aircraft

MICE: Meetings, Incentives, Conventions and Exhibitions

NASA: National Aeronautics and Space Administration

R&D: Research and Development

ROW: Rest of the World

WTO: World Trade Organization

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DEDICATION

to my beloved ones

Chapter 1

PRODUCT LINE RIVALRY WITH STRATEGIC CAPACITY ALLOCATION

1.1 Introduction

Some industries require firms to install expensive, specialized inputs before production starts. Naturally, there is a huge entry cost. These firms enjoy economies of scale by spreading the cost over cumulative production. With little trade barriers, the firms grow into an international oligopoly. For example, Apple, Huawei and Samsung divide most of the world's smartphone market. The plane-makers, Boeing and Airbus, form a duopoly in the world market of large civil aircrafts (LCA).[1] An international oligopolist does not pay a huge cost for inputs that can only be used to produce one product. Rather, it develops a line of products that utilize the same inputs. For examples, Apple hires computer-engineering expertise to design both iPhones and iPads. Airbus builds specially equipped factories for manufacturing both A380 and A350. Hence, an international oligopolist can spread the cost of inputs over multiple products. In this case, there are "economies of scope" because it is less costly to manufacture multiple products collectively in one firm than individually in different firms (Baumol, Panzar, & Willig, 1982).[2] Nevertheless, inputs so specific to a firm's product line must be limited, at least in the short run. Using more of a specific input to make one product means less of the input is available for making another

[1] Large civil aircraft (LCA) is a term dubbed by the WTO and the U.S. International Trade Commission (2001). Boeing refers to them as "commercial airplanes" and Airbus calls them "passenger aircrafts."

[2] Baumol et al. (1982) assumed firms take the price of each product as given before entering the markets. Their outcome is similar to perfect competition. This chapter focuses on industries in which entry is limited. Therefore, an oligopoly model is necessary, especially for analyzing strategic interactions.

product. Even if the world market is great enough, an international oligopolist cannot freely produce the desirable quantities to meet demand. With such *capacity constraint*, a firm needs to allocate its capacity among different products wisely and it does so strategically when there is competition. This essay provides a framework to analyze such *strategic capacity allocation* of an international oligopoly.

It turns out that the product selection of an international oligopoly can be puzzling. The LCA industry is a good example.[3] Boeing was a near monopoly until the entry of Airbus, and the two have become a duopoly since the 1980s.[4] Boeing and Airbus are equally resourceful and have virtually identical production technology. In particular, both are capable of manufacturing a family of planes. In early 1990s, Boeing and Airbus considered building "super-jumbo" planes together, but Boeing quit (The Economist, 2001). Airbus became the first to introduce the world's largest and the world's only double-decker jet airliner, the A380, in 2007.[5] Boeing has continued to devote all of its resources to the production of smaller planes, including the newest Boeing 787. Table 1.1 and Table 1.2 show that both orders and deliveries for Boeing 787 have skyrocketed since 2011. After witnessing the commercial success of Boeing 787, Airbus introduced the A350 as a head-to-head rival and started its deliveries in 2014. However, Airbus has not reallocated much resource from the A380 to the A350. As shown in Table 1.1, production of A380 has been steady since 2008.

What is puzzling here is that despite having the established advantage of an incumbent and despite having the ability to produce large planes, Boeing has given it up to Airbus. In particular,

[3] The LCA industry have always interested trade theorists because it had an oligopolistic, integrated world market where government policies were prominent (Dixit & Kyle, 1985).
[4] Baldwin and Krugman (1988) observed that Lockheed and McDonnell-Douglas were no competition to Boeing in the 1980s, and concluded that the market was not large enough to sustain more than one firm. The sizeable orders in Table 3.1 imply there is a much bigger demand today, so a duopoly is sustainable.
[5] A380 has 40% more floor space than the next largest airliner, Boeing 747-8. Boeing 747 is only a partial double-decker jet airliner.

this contradicts the common argument that an incumbent often preempts the product space to deter entry of substitutes. These previous arguments emphasized on product relation on the demand side. Two products are “related” because consumers use the products as *substitutes* or *complements*. Substitutability discourages product proliferation because sales of one product hurt sales of another product. For example, Brander and Eaton (1984) modeled firms that competed in a line of four products. They assumed there were no economies of scope and products were substitutes. They showed that in spite of the discouraging circumstances, incumbents would preempt the product space to deter entry. In particular, the incumbents produced two distant substitutes, rather than two close substitutes, when facing the threat of entry. However, this does not match the observations in the LCA industry. Boeing, the incumbent, does not deter the entry of Airbus by crowding out the product space. Rather, it yields the large-plane market to Airbus. In the model in this chapter, the incumbent may allocate all of its limited capacity towards the more profitable small planes, pushing the new firm to allocate some of its limited resources to the less profitable large planes. Also, this result is robust to different demand-side product relations – products can be substitutes, complements or unrelated.

Judd (1985) assumed there were no economies of scope and firms competed in two substitutes. He examined the credibility of product proliferation as an entry deterrent. Judd (1985) included an exit stage and found that if exit cost was not prohibitive, an incumbent might exit one of the markets to avoid head-to-head competition. The result was a differentiated duopoly – each firm produced a different product. This result does not match the observation in the LCA industry either. While Boeing specializes in small planes, Airbus makes both large planes and small planes. Boeing has never put large planes into production. There is no need to "exit." With a capacity constraint, Boeing cannot satisfy the entire demand for small planes, and has no capacity left for producing large planes.

Gilbert and Matutes (1993) also modeled competition with product line rivalry and investigated whether product preemption was credible. They considered two products. There were brand differentiation in the sense that consumers considered a product made by different firms to be different. Similar to Brander and Eaton (1984) and Judd (1985), Gilbert and Matutes (1993) assumed the products to be substitutes.[6] Different from the other two papers, Gilbert and Matutes (1993) assumed strong economies of scope. This should provide incentives for product proliferation. They concluded an incumbent's product spatial preemption was credible if brand differentiation was sufficiently large (substitutability was sufficiently weak). However, this essay assumes no brand differentiation because airlines consider Boeing and Airbus planes of the same sizes to be virtually perfect substitutes and stock up both brands in their fleets. Boeing and Airbus themselves also consider their planes (such as Boeing 787 and A350) to be head-to-head competition. In this case, Gilbert and Matutes (1993) would say that product proliferation could not be a credible entry deterrent. Expectedly, Boeing does not preempt the product space to deter entry. Rather, Boeing seems to welcome Airbus' entry into the large-plane market. Hence, this chapter focuses on a rather "entry-welcoming" product selection of the incumbent.

This essay offers a unique analysis on *strategic capacity allocation* in a model of international oligopoly, and seeks for an explanation for the puzzling, entry-welcoming product selection of an incumbent, such as what Boeing has done in the LCA industry.[7] I shall investigate

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[6] In Gilbert and Matutes (1993), the products, "basic" and "premium" were substitutes. "Basic" ("premium") products made by different firms were also imperfect substitutes.

[7] This essay is not the first to model production technology that is used by different products. For example, in Röller and Tombak's (1990) and Dixon's (1994) models, a firm had to develop a costly "flexible technology" in order to produce different products. Hence, the cost increased with scope, generating diseconomies. However, the approach here is very different. Believing that firms tend to develop new products that they can manufacture using existing

capacity allocation under different market structures. The next section presents a multiproduct monopoly that faces a capacity constraint. In this simple model, I derive basic insights of capacity allocation and assess the effect of capacity expansion. Section 1.3 presents a duopoly model, in which firms compete in a Cournot fashion. It analyses how a firm responses to the rival's capacity allocation strategies. To investigate how an incumbent interacts with an entrant, I will then look into a model of Stackelberg competition, which is closest to the type of competition between Boeing and Airbus in Section 1.4. The model provides insights into how Boeing can strategically utilize the constrained capacity to its advantage by yielding the large-plane market to Airbus. Finally, Section 1.5 provides concluding remarks.

technology, this essay assumes that the "flexible technology" is already in place and its cost is sunk. As a result, firms spread cost by manufacturing additional products, resulting in scope economies, rather than diseconomies.

1.2. Monopoly

Before the entry of Airbus, Boeing was a near monopoly in the LCA market. Monopoly is the simplest case in which important insights can be drawn. Hence, let's first analyze how a firm allocates its capacity in an environment that is free of competition.

1.2.1 Model

Consider a firm in Home, which sells planes to the rest of the world (ROW). The firm has a line of two products – "large planes" (product 1) and "small planes" (product 2). The constant marginal costs of large planes and small planes are C^1 and C^2 respectively.[8a, b]

The production technology is simple but distinctive. Suppose production of any plane from the product line requires an input that is highly specific in its use. This can be the specially equipped factory, the high-tech components, etc. Due to this nature of the input, by the time production starts, the firm cannot alter its amount. For simplicity, think of the fixed input as "factory space" in this model. Let Z acres of factory space be the production capacity that the firm has.

Developing planes is costly. Let K be the sunk cost, including the cost of building a factory of Z acres. By the time the firm decides on the scope, K is sunk. In other words, whether the firm makes no product, one product or two products from the line, the sunk cost is K. Joint production

[8a] Constant marginal costs are the simplest that can fulfill the purpose of this chapter. Some models of international oligopoly assume decreasing marginal costs. If the second partial derivatives were sufficiently small, the results would be the same as this model. Also, the focus here is the cost-side relation between products. Economies of scope are present due to the fixed input, but economies of scale of just one product are not emphasized.

[8b] If there are government subsidies, S^1 and S^2, the after-subsidy marginal costs will be $C^1 - S^1$ and $C^2 - S^2$. This chapter focuses on the multiproduct feature of international oligopolies. Chapter 2, on the other hand, will give spotlight to industrial policies and their welfare effects.

of the products is less costly than producing them separately. Hence, here exist *economies of scope*.

Because the factory space is fixed, once it is filled up, the firm cannot produce any more plane. Moreover, the factory space must be *rival* in its use. When the firm occupies certain area to produce a plane, it cannot produce another plane in the same area at the same time.[9] In other words, the firm's production is subject to a *capacity constraint*. More precisely, suppose producing a large plane requires θ^1 acres of factory space, and a small plane takes up θ^2 acres to build. Without a loss of generality, assume a large plane requires more factory space than a small plane such that $\theta^1 > \theta^2$. If the firm produces Q_1 large planes and Q_2 small planes, then

$$\theta^1 Q_1 + \theta^2 Q_2 \leq Z. \tag{1}$$

(1) says that the space used for producing large planes and small planes cannot sum up to more than the factory space the firm has. The emphasis of this chapter is on how a firm allocates capacity among different products, so throughout the chapter, there have to be two global assumptions.

(G1) *Small-capacity assumption*: Capacity, Z is sufficiently small; otherwise, there will be no constraint to the capacity.

(G2) *Large-demand assumption*: Demand for each product is sufficiently large, so that profit-maximizing Q_1 and Q_2 cannot be both equal to zero.

[9] Without this assumption, the capacity constraint is not necessarily linear in Q_1 and Q_2. A more general capacity constraint would be some function, $\Phi(Q_1, Q_2) \leq Z$.

With these assumptions, the capacity constraint in (1) is binding. Later, I shall verify this by checking the Kuhn-Tucker conditions.

Now denote the quantities demanded of large planes and small planes as X_1 and X_2 respectively. The inverse demand of each product is a function of both quantities. That is, for $i = 1,2$, $P^i = P^i(X_1, X_2)$. Also define $P^i_j \equiv \frac{\partial P^i}{\partial X_j}$ and $P^i_{ij} \equiv \frac{\partial^2 P^i}{\partial X_i \partial X_j}$, where $i = 1, 2$ and $j = 1, 2$. Assume the demand functions satisfy standard properties that $P^i_i < 0$ and $P^i_{ii} < \xi$, where ξ is a sufficiently small positive number. Note that if consumers consider large planes and small planes to be *substitutes*, then $P^1_2 < 0$ and $P^2_1 < 0$.[10] This essay rules out the case that they are perfect substitutes; otherwise, they cannot be differentiated as two products. Hence, $P^1_2 \neq P^1_1$ and $P^2_1 \neq P^2_2$. If consumers consider them to be *complements*, then $P^1_2 > 0$ and $P^2_1 > 0$. If consumers consider them to be *unrelated* goods, then $P^1_2 = 0$ and $P^2_1 = 0$.

When the market of each product is in equilibrium, $X_i = Q_i$, and there is a single world price for each product: $P^i = P^i(Q_1, Q_2)$, for $i = 1,2$. The firm chooses the output level of each plane to maximize total profit:

$$\pi = P^1 Q_1 - C^1 Q_1 + P^2 Q_2 - C^2 Q_2 - K \quad \text{subject to} \quad \theta^1 Q_1 + \theta^2 Q_2 \leq Z, Q_1 \geq 0, Q_2 \geq 0 \quad (2)$$

taking the demand functions, the marginal costs and the sunk cost as given.[11] Note that I assume output levels to be non-negative.

To highlight the role of the capacity constraint and for simplification, I express the variables in *effective terms* (denoted by lowercase letters).

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[10] If $P^1_2 < 0$ and $P^2_1 < 0$, Brander and Eaton (1984) would call the products, "*q*-substitutes." Appendix A shows that if large planes and small planes are "*q*-substitutes," they are also "*p*-substitutes."

[11] Brander and Eaton (1984) explained how the central insights remain the same whether quantity or price is the choice variable. This study considers quantity decisions because a firm can allocate its capacity by choosing output levels, which is the focus of this chapter.

Definitions: The capacity allocated to the production of product *i* is $q_i \equiv \theta^i Q_i$,

the effective marginal cost of product *i* is $c^i \equiv \frac{C^i}{\theta^i}$, and

the effective price of product *i* is $p^i \equiv \frac{P^i}{\theta^i}$,

for $i = 1,2$. In effective terms, the profit maximization problem becomes:

$$\pi = p^1 q_1 - c^1 q_1 + p^2 q_2 - c^2 q_2 - K \qquad \text{subject to} \qquad q_1 + q_2 \leq Z, q_1 \geq 0, q_2 \geq 0. \qquad (3)$$

Notice that (3) preserves the structure of the constrained profit maximization problem in (2). I solve (3) using the Lagrangean ($\mathcal{L}$) method.[12] The Kuhn-Tucker conditions are

$$p^1 + p^1_1 q_1 - c^1 + p^2_1 q_2 - \lambda \leq 0, \quad q_1 \geq 0, \quad q_1(p^1 + p^1_1 q_1 - c^1 + p^2_1 q_2 - \lambda) = 0 \qquad (4a)$$

$$p^2 + p^2_2 q_2 - c^2 + p^1_2 q_1 - \lambda \leq 0, \quad q_2 \geq 0, \quad q_2(p^2 + p^2_2 q_2 - c^2 + p^1_2 q_1 - \lambda) = 0 \qquad (4b)$$

$$q_1 + q_2 \leq Z, \quad \lambda \geq 0, \quad \lambda(q_1 + q_2 - Z) = 0. \qquad (4c)$$

Some of the conditions in (4) contradict with (G1) and (G2).[13] To be consistent with the assumptions, λ must be positive, so the capacity constraint in (1) is binding. Fig. 1.1 illustrates how the monopoly allocates its capacity to large planes and small planes. I constructed the figure with linear demands and parameters that satisfy the model assumptions. The firm achieves optimal capacity allocation at the point where the isoprofit curve is tangent to the capacity line,

[12] Alternatively, the maximization problem can be solved using the substitution method. By substituting the capacity constraint into the objective function, the multivariate constrained maximization problem becomes a univariate unconstrained maximization problem.

[13] If capacity, Z is small enough and demands are large enough (a^1 and a^2 are large enough), then it is true that $Z < \frac{a^1 - c^1}{2b^1}$, $Z < \frac{a^2 - c^2}{2b^2}$ and $Z < \frac{(2b^2 - \gamma^1 - \gamma^2)(a^1 - c^1) + (2b^1 - \gamma^1 - \gamma^2)(a^2 - c^2)}{4b^1 b^2 - (\gamma^1 + \gamma^2)^2}$.

ZZ. Point M, at which the firm allocates more capacity to small planes than to large planes, is just one possible solution. Different parameter space gives rise to different solutions along the ZZ line, which include the corner solutions of $(Z, 0)$ and $(0, Z)$.

To analyze the conditions for each solution, assume the demand functions are linear such that $P^1(X_1, X_2) = A^1 - B^1X_1 - \Gamma^1X_2$ and $P^2(X_2, X_1) = A^2 - B^2X_2 - \Gamma^2X_1$. A^1, A^2, B^1 and B^2 are positive constants. Γ^1 and Γ^2 determine how, if at all, the products are related. Once again, express the coefficients of the demand functions in effective terms.

Definitions: For $i = 1, 2$, $a^i \equiv \frac{A^i}{\theta^i} > 0$, $b^i \equiv \frac{B^i}{(\theta^i)^2} > 0$ and $\gamma^i \equiv \frac{\Gamma^i}{\theta^1\theta^2}$.

The first-order conditions with respect to q_1 and q_2 are

$$a^1 - c^1 - 2b^1q_1 - (\gamma^1 + \gamma^2)q_2 - \lambda = 0, \quad (5a)$$

$$a^2 - c^2 - 2b^2q_2 - (\gamma^1 + \gamma^2)q_1 - \lambda = 0, \quad (5b)$$

$$q_1 + q_2 = Z. \quad (5c)$$

(5) shows that a firm's capacity allocation depends on how the products are related (or unrelated) on the demand side. The airlines (the consumers) may consider large plane and small plane as *substitutes* because they function similarly in providing air transportation. In this way, $\gamma^1 > 0$ and $\gamma^2 > 0$. According to (5), marginal profit of a product will depend on the production of (and capacity devoted to) another product negatively. This is because higher sales of one product will reduce sales of the other product, which is known as "cannibalization."[14] The higher is the

[14] For example, the term "cannibalization" appeared in Lambertini (2003).

substitutability of the products, the stronger is the effect of cannibalization. On the other hand, airlines may want to buy both large planes and small planes in order to diversify the fleet and serve different routes. That means the products can *complement* each other. If $\gamma^1 < 0$ and $\gamma^2 < 0$ in (5), marginal profit of a product will depend on the production of (and capacity devoted to) another product positively. The more complementary the products are, the more likely the airlines will buy both products. Finally, if the airlines consider the products to be *unrelated* (or if the substitution effect and the complementary effect exactly offset each other), $\gamma^1 = 0$ and $\gamma^2 = 0$ in (5). Marginal profit of a product does not depend on the production of (and capacity devoted to) another product.

Define $\mathcal{L}_{ij} \equiv \frac{\partial^2 \mathcal{L}}{\partial q_i \partial q_j}$ where $i = 1, 2$ and $j = 1, 2$. The second-order conditions for profit maximization are such that $\mathcal{L}_{11} = -2b^1 < 0$, $\mathcal{L}_{22} = -2b^2 < 0$ and that the determinant of the Hessian matrix, $H \equiv 2(b^1 + b^2 - \gamma^1 - \gamma^2) > 0$. The first two conditions are consistent with the model assumptions that b^1 and b^2 are positive. As explained in the Appendix, H must be positive whether the products are complements, (imperfect) substitutes or unrelated.

Solving (5) by Cramer's rule,

$$q_1^M = \frac{(a^1 - c^1) - (a^2 - c^2) + (2b^2 - \gamma^1 - \gamma^2)Z}{H} = Z - q_2^M, \qquad (6)$$

where the "*M*" superscript denotes optimal capacity allocation of the monopoly. Recall that the capacity in (1) has to bind in this essay, so q_1^M implies $q_2^M = Z - q_1^M$. The term, $(a^1 - c^1) - (a^2 - c^2)$ in (6) is important throughout the chapter. The greater the difference between the vertical intercept of the demand curve, A, and the marginal cost, C, the greater is the marginal profit.[15] $(a - c)$ is simply $(A - C)$ adjusted for capacity requirement (θ) of the product. Hence, $(a^1 - c^1) - (a^2 - c^2)$ tells how the effective profitability of large planes compares to that of small planes. Orders for a product in Table 1.2 is a proxy of the demand for the product. The higher orders for small planes indicate that there is greater demand for small planes than for large planes. This implies that $a^1 < a^2$. Also, it is reasonable to believe that a small plane costs less to make than a large plane That is, $c^1 > c^2$. Hence, this chapter mainly focuses on the situation when

$$(a^1 - c^1) - (a^2 - c^2) < 0\,. \tag{A1}$$

That is, small planes are *effectively more profitable* than large planes.

With (6), I can compare the monopolist's capacity allocation to different products. The following assumption is useful.

$$(a^1 - c^1) - (a^2 - c^2) < (b^1 - b^2)Z\ . \tag{A2}$$

How b^1 and b^2 compare is unknown in general. It depends on the price elasticity of demand for each plane. If b^1 and b^2 are not too different, (A2) is basically an assumption about the relative effective profitability of small planes.

Lemma 1: If (A2) is true, then $q_2^M > q_1^M$. A monopolist allocates more capacity to small planes than to large planes because the effective profitability of small planes is sufficiently higher.

[15] Effective marginal profit of a large plane is $a^1 - 2b^1 q_1 - \gamma^1 q_2 - c^1$. Effective marginal profit of a small plane is $a^2 - 2b^2 q_2 - \gamma^2 q_1 - c^2$.

Note that capacity requirements also play a part. Recall that a large plane requires more capacity to make than a small plane (i.e., $\theta_1 > \theta_2$). Hence, even if $q_1 = q_2$, the firm will have greater output of small planes than large planes (i.e. $Q_2 > Q_1$). In other words, even if the products are effectively equally profitable and $b^1 \approx b^2$, the firm will produce more small planes simply because each large plane has a greater capacity requirement.

1.2.2 Capacity Expansion

(6) shows that capacity allocation depends on how big the factory is. This subsection analyzes the effect of a capacity expansion. Differentiating (6) with respect to Z:

$$\frac{\partial q_1^M}{\partial Z} = \frac{2b^2 - \gamma^1 - \gamma^2}{H}, \tag{7a}$$

$$\frac{\partial q_2^M}{\partial Z} = \frac{2b^1 - \gamma^1 - \gamma^2}{H}\,. \tag{7b}$$

If the products are complements or unrelated goods ($\gamma^1 \leq 0$ and $\gamma^2 \leq 0$), then (7a) and (7b) will be positive. If the products are substitutes ($\gamma^1 > 0$ and $\gamma^2 > 0$), then the signs of $2b^2 - \gamma^1 - \gamma^2$ and $2b^1 - \gamma^1 - \gamma^2$ are generally unknown. However, the second-order condition assumes that $b^1 + b^2 - \gamma^1 - \gamma^2 > 0$. If b^1 and b^2 are not too different, then it will as well be that $2b^2 - \gamma^1 - \gamma^2 > 0$ and $2b^1 - \gamma^1 - \gamma^2 > 0$, so (7a) and (7b) will be positive.

Lemma 2: If both (7a) and (7b) are positive, then capacity allocation to each product depends on factory space positively. Capacity expansion has a *normal* effect on capacity allocation. Otherwise, expansion has an *inferior* effect on capacity allocation.

Fig. 1.2 illustrates how capacity allocation changes when factory space, Z doubles. If capacity expansion is "normal," the new allocation is northeast to the original allocation at M. In the figure, M_2, M_3 and M_4 are some of the normal cases. If the capacity expansion is "inferior," the new allocation will not be at the northeast of M. Some of the inferior allocations are M_1 and M_5.

Now let's take a closer look at each of the five zones in Fig. 1.2. If (7a) is positive but (7b) is negative, the new capacity allocation falls into the "ultra-pro-large" zone. In other words, when the factory expands, the firm allocates more capacity to large planes, but less capacity to small planes. Small planes are "inferior." In Fig. 1.2, M_1 is an example of ultra-pro-large allocation. Conversely, (7a) can be negative while (7b) is positive, the new capacity allocation falls into the "ultra-pro-small" zone. That is, when the factory expands, the firm allocates more capacity to small planes, but less capacity to large planes. Large planes are "inferior." In Fig. 1.2, M_5 is an example of ultra-pro-small allocation.

Lemma 3: If (7a) is positive but (7b) is negative, monopolistic capacity expansion is *ultra-pro-large*. If (7b) is positive but (7a) is negative, monopolistic capacity expansion is *ultra-pro-small*.

Within the "normal" zone, there are the "pro-large," the "neutral" and the "pro-small" zones. After a factory expansion, if the proportion of capacity allocated to large planes increases, while the proportion of allocated to small planes decreases, the firm's capacity allocation is pro-large. In Fig. 1.2, M_2 is an example of pro-large allocation. Oppositely, if an expansion to the capacity decreases the proportion of capacity allocated to large planes, but increases that of small planes, the firm's capacity allocation is pro-small. In Fig. 1.2, M_4 is one pro-small allocation. Finally, if both proportions remain unchanged after a factory expansion, the firm's capacity allocation is neutral. M_3 is the neutral allocation in Fig. 1.2. The conditions on the proportions in the three normal zones are as follow:

Lemma 4: Assume both (7a) and (7b) are positive. If $(a^2 - c^2) > (a^1 - c^1)$, capacity expansion is *pro-large*. If $(a^1 - c^1) = (a^2 - c^2)$, capacity expansion is *neutral*. If $(a^1 - c^1) > (a^2 - c^2)$, capacity expansion is *pro-small*.

Lemma 4 seems to be counter-intuitive at the first glance, but it is not. Let's take the "pro-large" case as an example. Recall from Lemma 1 that if small planes are sufficiently more profitable than large planes, the firm will allocate most of its limited capacity to small planes. That is, facing a capacity constraint, the firm neglects large planes to certain extent. Following a "normal" capacity expansion, the firm allocates more capacity to both products. However, since small planes already took up a lot of capacity, the firm will assign more of the additional capacity to large planes. Therefore, by proportion, the firm allocates more of the new capacity to the effectively less profitable product.

1.3. Duopoly – Cournot Competition

The monopoly case provides basic insights to a firm's capacity allocation. What is more interesting is how a firm strategizes its capacity allocation when facing rivalry. This section and the next section explore how competition influences capacity allocation. They can provide more understanding about the competition between Boeing and Airbus.

1.3.1 Model

Suppose a firm in the domestic country and a firm in the foreign country sell planes to the ROW.[16] Hereafter, I will use asterisks (*) to distinguish variables of the foreign firm. The purpose of this study is to compare similar firms (i.e., Boeing versus Airbus), so I first simplify the model by assuming the firms to be identical. In particular, the firms have same constant marginal costs, C^1 and C^2, same sunk cost, K and same capacity, Z. Also, the factory space needed for producing a large plane is θ^1 and that for a small plane is θ^2, regardless of who produces them. In the next subsection, I shall relax this assumption and investigate how firm differentiation can affect the firms' strategic capacity allocation.

Assume the market of each product is in equilibrium. Henceforth, $X_i = Q_i + Q_i^*$ and the world price is $P^i = P^i(Q_1 + Q_1{}^*, Q_2 + Q_2{}^*)$, for $i = 1,2$.

In this section, I assume the firms make decisions *simultaneously*. In other words, the firms compete in *Cournot* fashion. The domestic firm chooses the acres of factory space to be allocated to each plane to maximize total profit subject to the capacity constraint, taking the foreign firm's capacity allocation, the demand functions, the marginal costs and the sunk cost as given:

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[16] As mentioned in the introduction, there is vertical (intra-firm) product differentiation, but no horizontal (inter-firm) product differentiation. For models that include both dimensions of differentiation, see Brander and Eaton (1984), Canoy and Peitz (1997) and Gilbert and Matutes (1993).

$$\pi = p^1 q_1 - c^1 q_1 + p^2 q_2 - c^2 q_2 - K \qquad \text{subject to} \qquad q_1 + q_2 \leq Z, q_1 \geq 0, q_2 \geq 0. \tag{8}$$

Using the Lagrangean method, the Kuhn-Tucker conditions are

$$p^1 + p_1^1 q_1 - c^1 + p_1^2 q_2 - \lambda \leq 0, \quad q_1 \geq 0, \quad q_1(p^1 + p_1^1 q_1 - c^1 + p_1^2 q_2 - \lambda) = 0 \tag{9a}$$

$$p^2 + p_2^2 q_2 - c^2 + p_2^1 q_1 - \lambda \leq 0, \quad q_2 \geq 0, \quad q_2(p^2 + p_2^2 q_2 - c^2 + p_2^1 q_1 - \lambda) = 0 \tag{9b}$$

$$q_1 + q_2 \leq Z, \quad \lambda \geq 0, \quad \lambda(q_1 + q_2 - Z) = 0. \tag{9c}$$

Similarly, the foreign firm chooses the amount of fixed input to be allocated to each plane to maximize

$$\pi^* = p^1 q_1^* - c^1 q_1^* + p^2 q_2^* - c^2 q_2^* - K \qquad \text{subject to} \qquad q_1^* + q_2^* \leq Z, q_1^* \geq 0, q_2^* \geq 0. \tag{10}$$

taking the domestic firm's capacity allocation, the demand functions, the marginal costs and the sunk cost as given. The Kuhn-Tucker conditions are

$$p^1 + p_1^1 q_1^* - c^1 + p_1^2 q_2^* - \lambda^* \leq 0, \quad q_1^* \geq 0, \quad q_1^*(p^1 + p_1^1 q_1^* - c^1 + p_1^2 q_2^* - \lambda^*) = 0 \tag{11a}$$

$$p^2 + p_2^2 q_2^* - c^2 + p_2^1 q_1^* - \lambda^* \leq 0, \quad q_2^* \geq 0, \quad q_2^*(p^2 + p_2^2 q_2^* - c^2 + p_2^1 q_1^* - \lambda^*) = 0 \tag{11b}$$

$$q_1^* + q_2^* \leq Z, \quad \lambda^* \geq 0, \quad \lambda^*(q_1^* + q_2^* - Z) = 0. \tag{11c}$$

Note that the global assumptions (G1) and (G2) must continue to hold and the conditions on the parameters are stricter than when there was only one firm.[17] Now, the demands have to be big enough for both firms to profitably produce some output. Also, because the firms share the

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[17] If capacity, Z is small enough and demands are large enough (a^1 and a^2 are large enough), then it is true that $Z < \frac{(b^1-\gamma^1+2b^2-2\gamma^2)(a^1-c^1)+(b^1-\gamma^1)(a^2-c^2)}{2b^1H+2b^1b^2-2\gamma^1\gamma^2-(b^1-\gamma^1)^2}$, $Z < \frac{(2b^1-2\gamma^1+b^2-\gamma^2)(a^2-c^2)+(b^2-\gamma^2)(a^1-c^1)}{2b^2H+2b^1b^2-2\gamma^1\gamma^2-(b^2-\gamma^2)^2}$ and $Z < \frac{(3b^2-\gamma^1-2\gamma^2)(a^1-c^1)+(3b^1-2\gamma^1-\gamma^2)(a^2-c^2)}{9b^1b^2-(2\gamma^1+\gamma^2)(\gamma^1+2\gamma^2)}$.

markets, each firm needs a smaller factory for production. With (G1) and (G2), the capacity constraint must be binding, so I only consider cases when $\lambda > 0$ among the Kuhn-Tucker conditions in (9) and (11).

Given q_1^* and q_2^*, the first-order conditions of the domestic firm are:

$$p^1 + p_1^1 q_1 - c^1 + p_1^2 q_2 - \lambda = 0, \tag{12a}$$

$$p^2 + p_2^2 q_2 - c^2 + p_2^1 q_1 - \lambda = 0, \tag{12b}$$

$$q_1 + q_2 = Z. \tag{12c}$$

The second-order conditions are the same as before: $\mathcal{L}_{11} < 0$, $\mathcal{L}_{22} < 0$ and $H > 0$.

The first-order conditions of the foreign firm are:

$$p^1 + p_1^1 q_1^* - c^1 + p_1^2 q_2^* - \lambda^* = 0, \tag{13a}$$

$$p^2 + p_2^2 q_2^* - c^2 + p_2^1 q_1^* - \lambda^* = 0, \tag{13b}$$

$$q_1^* + q_2^* \leq Z. \tag{13c}$$

Define $\mathcal{L}_{ij}^* \equiv \frac{\partial^2 \mathcal{L}^*}{\partial q_i^* \partial q_j^*}$ where $i = 1, 2$ and $j = 1, 2$.The second-order conditions are $\mathcal{L}_{11}^* = \mathcal{L}_{11} < 0$, $\mathcal{L}_{22}^* = \mathcal{L}_{22} < 0$ and $H > 0$.

In the next subsection, I will solve (12) and (13) for reaction functions and derive the Nash-Cournot equilibria.

1.3.2 Nash-Cournot Equilibrium of Identical Firms

Solving (12) gives reaction function:

$$q_1 = \varphi - \frac{q_1^*}{2}, \tag{14}$$

where $\varphi \equiv \frac{(a^1-c^1)-(a^2-c^2)+(3b^2-2\gamma^1-\gamma^2)Z}{H}$ is a constant. Note that I have simplified the expression using the binding capacity constraint: $q_2 = Z - q_1$. Similarly, solving (13) yields reaction function:

$$q_1^* = \varphi - \frac{q_1}{2}, \tag{15}$$

Fig. 1.3 plots the two reaction functions. In Panel (a), the intersection point of the reaction curves, point *e*, gives the Nash-Cournot-equilibrium capacity allocation to large planes. Denote the Nash-Cournot-equilibrium capacity allocation with superscript "*D*." With a binding capacity constraint, the capacity allocation to small planes are simply, $q_2^D = Z - q_1^D$ and $q_2^{*D} = Z - q_1^{*D}$. Since the two reaction functions are symmetric, the firms have identical capacity allocation in Nash-Cournot equilibrium. Solving the two reaction functions simultaneously yield:

$$q_1^D = q_1^{*D} = \frac{2}{3}\frac{(a^1-c^1)-(a^2-c^2)+(3b^2-\gamma^2-2\gamma^1)Z}{H}. \tag{16a}$$

which implies that

$$q_2^D = q_2^{*D} = \frac{2}{3}\frac{(a^2-c^2)-(a^1-c^1)+(3b^1-\gamma^1-2\gamma^2)Z}{H}. \tag{16b}$$

Lemma 5: Identical duopolists that compete in a Cournot fashion have identical capacity allocation in the equilibrium.

According to (16b), if

$$(a^2 - c^2) - (a^1 - c^1) \leq -(3b^1 - 2\gamma^2 - \gamma^1)Z \quad , \tag{A3}$$

each duopolist will assign no capacity to small planes: $q_2^D = q_2^{*D} = 0$.[18] That is, each duopolist will assign all the capacity to produce large planes: $q_1^D = q_1^{*D} = Z$. Fig. 1.3 Panel (b) illustrates this case. When small planes are much less profitable (i.e., large planes are much more profitable), the intersection point of the reaction curves at point *A* is above each firm's capacity of Z. Given the foreign capacity allocation, domestic firm's best response is to allocate all the capacity to produce large planes at point *B*. If the domestic firm allocates all capacity to large planes, the foreign firm's best response is also to allocate all the capacity to large planes. Hence, the equilibrium point is at point *e*.

By the same token, (16a) says that if

$$(a^1 - c^1) - (a^2 - c^2) \leq -(3b^2 - 2\gamma^1 - \gamma^2)Z \quad , \tag{A4}$$

each duopolist will assign no capacity to large planes: $q_1^D = q_1^{*D} = 0$. That means each duopolist will assign all the capacity to produce small planes: $q_2^D = q_2^{*D} = Z$. Notice that $\varphi \leq 0$. As shown in Panel (c) of Fig. 1.3, the intersection point of the reaction functions at point *A* lies below the origin, which is not possible because capacity allocation cannot be negative. Given the foreign capacity allocation, the best the domestic firm can do is to allocate no capacity to produce large planes at point *B*. If the domestic firm allocates no capacity to large planes, the foreign firm will not allocate any capacity to large planes either. Thus, point *e*, the origin, is the equilibrium point.

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[18] I assumed output levels (and so the capacity allocation) to be non-negative.

Lemma 6: If (A3) is true, then $q_1^D = q_1^{*D} = Z$. In Nash-Cournot equilibrium, both duopolists allocate all the capacity to produce large planes. If (A4) is true, then $q_1^D = q_1^{*D} = 0$. In Nash-Cournot equilibrium, both duopolists allocate all the capacity to produce small planes.

Lemma 5 and Lemma 6 have crucial meaning. If Boeing and Airbus were identical and they compete in a Cournot fashion, they would have the same capacity allocation. It would not give rise to the situation that Boeing produced small planes only, while Airbus produced both small and large planes. As will be shown next, I will adjust the assumptions in Lemma 5 and Lemma 6 in order to yield results that match real-life observations.

1.3.3 Nash-Cournot Equilibrium of Differentiated Firms

Many argue that Boeing and Airbus have similar production technology. That is why the previous subsections assumed the duopolists to be identical. Without the assumption, the firms can have different constant marginal costs, sunk cost, capacity, and even different capacity requirements for making each product. For example, suppose it is effectively more costly for the foreign firm to manufacture small planes than the domestic firm. In particular, assume the duopolists have the same firm characteristics except that $c^{2*} > c^2$. The reaction function of the foreign firm becomes:

$$q_1^* = \varphi^* - \frac{q_1}{2}, \tag{17}$$

where $\varphi^* \equiv \frac{(a^1 - c^{1*}) - (a^2 - c^{2*}) + (3b^2 - 2\gamma^1 - \gamma^2)Z^*}{H}$ is a constant. Notice that if $c^{2*} > c^2$, $c^{1*} = c^1$ and $Z^* = Z$, then $\varphi^* > \varphi$. As shown in Panel (a) of Fig. 1.4, the intersection point at point A gives a negative q_1^D, which is impossible for the domestic firm. Given the foreign firm's capacity allocation, the domestic firm's best response not to allocation any capacity to large planes. At the same time, given the domestic firm's action, the foreign firm's best response is to allocate some capacity to each product at point *e*. Hence, point *e* is the Nash-Cournot equilibrium point.

Similarly, suppose it costs less for the foreign firm to produce large planes, $c^{1*} < c^1$, while all other firm characteristics are the same. According to (17), $\varphi^* > \varphi$. As illustrated in Panel (b) of Fig. 1.4, the Nash-Cournot equilibrium point is at point *e*, just as how it was derived in Panel (a).

Lemma 7: Other things equal, if $c^{2*} > c^2$ (or $c^{1*} < c^1$), it is possible that $q_1^D = 0$ and $0 < q_1^{*D} < Z$. If it is effectively more costly for the foreign firm to produce small planes (or effectively less costly to produce large planes), the domestic firm may allocate no capacity to large planes while the foreign firm allocates capacity to both products in the Nash-Cournot equilibrium.

Therefore, if Airbus is less (more) efficient than Boeing in making small (large) planes, it is well possible that Boeing specializes in small planes, but Airbus does not.

It is also possible that the firms differ in resources. Suppose the duopolists only differ in their capacity such that the foreign firm is more resourceful, $Z^* > Z$. As indicated in (17), $\varphi^* > \varphi$. As shown in Panel (c) of Fig. 1.4, the capacity of the foreign firm, $Z^*$is higher than that of the domestic firm, Z. The intersection point at point *A* gives a negative q_1^D, so the domestic firm's best response is to allocate no capacity to large planes. The foreign firm's best response is to allocate some capacity to large planes (and small planes) because it has sufficient capacity to do so. Therefore, point e gives the Nash-Cournot equilibrium point.

Lemma 8: Other things equal, if $Z^* > Z$, it is possible that $q_1^D = 0$ and $0 < q_1^{*D} < Z$. The less resourceful domestic firm allocates no capacity to large planes, but the more resourceful foreign firm allocates capacity to both products in the Nash-Cournot equilibrium.

Hence, Airbus can allocate capacity to produce both products, but Boeing cannot maybe simply because Airbus has a larger factory in the first place.

Firm differentiation is one possible answer to the puzzling product selection of Boeing. However, this has not solved the whole mystery entirely yet. Boeing does not seem to act simultaneously as Airbus. Indeed, Boeing acts first as an incumbent. Rather than engaging in a Cournot competition, the two seem to engage in a Stackelberg competition, which shall be explored in the next section.

1.3.4 Cournot Duopoly Versus Monopoly

Before moving onto the next section, let's see how competition has affected capacity allocation by comparing the results here to the monopoly. As in Subsection 1.3.2, let's assume again that firms are identical in order to compare (16) to (6). When comparing (16) and (6), the comparison of γ^1 and γ^2 is necessary, which is unknown in general. They depend on how well each product acts as a substitute or a complement to the other product. If large planes are stronger substitutes (or weaker complements) to small planes than small planes are to large planes, then

$$\gamma^1 > \gamma^2 . \tag{A5}$$

If (A5) is true, (A6) is stricter than (A1):

$$(a^1 - c^1) - (a^2 - c^2) < (\gamma^2 - \gamma^1)Z. \tag{A6}$$

Lemma 9: If (A5) and (A6) are true, $q_1^D > q_1^M$. A firm allocates more capacity to large planes (less capacity to small planes) as a duopolist than as a monopolist.

If large planes more strongly substitute (or more weakly complement) small planes, large planes should be the unfavorable one along the product line. Together with (A6) that small planes are much more profitable than large planes, a monopolist should find small planes more attractive. However, when there is competition, any profit is shared away by the foreign firm. Hence, Lemma 9 concludes that a firm will allocate less capacity to small planes as a duopolist than it did as a monopolist. In Fig. 1.5, point *M* is the capacity allocation of the monopolist and point *D* is that of the duopolist. The figure assumes (A5) and (A6), so point *M* may lean more towards the small-plane side than point *D*.

Let's also compare the effect of capacity expansion of a duopoly to that of a monopoly. The monopoly's case is an internal expansion. As what Fig. 1.2 has illustrated, the domestic firm's own factory space has doubled to $2Z$ as a monopolist. When the market of large planes is in equilibrium, $x_1^M = q_1^M$. The duopoly's case is an external expansion. The capacity in the world increased because the foreign firm has joined the market, bringing along more factory space. In other words, $Z + Z^* = 2Z$. When the market of large planes is in equilibrium, $x_1^D = q_1^D + q_1^{*D}$.

Lemma 10: (A5) and (A6) imply $(a^1 - c^1) - (a^2 - c^2) < 2(\gamma^1 - \gamma^2)Z$, so $x_1^D < x_1^M$. An expanded monopoly allocates more capacity to large planes (less capacity to small planes) than a duopoly as a whole.

Lemma 9 and Lemma 10 show that under the same assumptions, q_1^D and q_1^M do not compare in the same way as x_1^D and x_1^M. If the duopolists are identical, duopolistic capacity expansion looks "neutral." However, according to Lemma 4, monopolistic capacity expansion will be "pro-large" if small planes are sufficiently effectively more profitable than large planes. Hence it is possible that the allocation of the expanded monopoly will lean more towards the large-plane side than the joint allocation of the duopoly.

I plot points *M* and *D* in Fig. 1.6 the same way I did in Fig. 1.5, assuming the same conditions. After capacity expansion, the monopoly's capacity allocation is at point *M'*. Point *D*+*D** refers to the combined capacity allocation of the duopoly. Fig. 1.6 illustrates the possible comparison that point *M'* leans more towards large planes than point *D*+*D**.

1.4. Duopoly - Stackelberg Competition

The previous section shows how competition influences a firm's capacity allocation. This section will show that the impact of rivalry is even more protruding when firms allocate capacity *sequentially*. History has it that Boeing was the incumbent and Airbus was the entrant. Boeing could make production decisions before Airbus. In other words, Boeing was the *Stackelberg leader* while Airbus was the *follower* in the competition. Previous section explains Boeing's product selection by assuming the firms have different production technology. In this section, I shall prove that even if the firms are assumed to be identical, it is possible that Boeing allocates no capacity to large planes at all simply because Boeing is taking advantage of its follower through strategic capacity allocation.

1.4.1 Model

Consider a situation when the domestic firm chooses its capacity allocation in the first stage and the foreign firm does so in the second stage, taking the domestic firm's actions as given. In other words, the domestic firm is a Stackelberg leader and the foreign firm is a follower. I can solve the two-stage game by backward induction. In the second stage, the foreign firm allocates capacity to each product to maximize total profit, taking the domestic firm's capacity allocation, the demand functions, the marginal costs and the sunk cost as given.

$$\pi^* = p^1 q_1^* - c^1 q_1^* + p^2 q_2^* - c^2 q_2^* - K \qquad \text{subject to} \qquad q_1^* + q_2^* \leq Z^*, q_1^* \geq 0, q_2^* \geq 0. \tag{18}$$

The first-order conditions of the foreign firm are:

$$p^1 + p_1^1 q_1^* - c^1 + p_1^2 q_2^* - \lambda^* = 0, \tag{19a}$$

$$p^2 + p_2^2 q_2^* - c^2 + p_2^1 q_1^* - \lambda^* = 0, \tag{19b}$$

$$q_1^* + q_2^* \leq Z. \tag{19c}$$

The second-order conditions are $\mathcal{L}_{11}^* < 0$, $\mathcal{L}_{22}^* < 0$ and $H > 0$. The reaction function is:

$$q_1^* = \varphi - \frac{q_1}{2}. \tag{20}$$

Again, $\varphi \equiv \frac{(a^1-c^1)-(a^2-c^2)+(3b^2-2\gamma^1-\gamma^2)Z}{H}$ is a constant, and I have simplified the expression using the binding capacity constraint: $q_2^* = Z - q_1^*$. From (20), I can derive the reaction function of capacity allocated to small planes in response to large planes, $q_2^* = q_2^*(q_1)$.

In the first stage, the domestic firm allocates capacity to each product to maximize total profit:

$$\pi = p^1 q_1 - c^1 q_1 + p^2 q_2 - c^2 q_2 - K \qquad \text{subject to} \qquad q_1 + q_2 \leq Z, q_1 \geq 0, q_2 \geq 0, \tag{21}$$

foreseeing the foreign firm's best responses in (20) and taking the demand functions, the marginal costs and the sunk cost as given. Substituting (20) into (21), the effective prices are $p^i = p^i(q_1, q_2) = p^i\big(q_1 + q_1^*(q_1), q_2 + q_2^*(q_1)\big)$ for $i = 1,2$. The first-order conditions of the domestic firm are:

$$p^1 + p_1^1 q_1 - c^1 + p_1^2 q_2 - \lambda = 0, \tag{22a}$$

$$p^2 + p_2^2 q_2 - c^2 + p_2^1 q_1 - \lambda = 0, \tag{22b}$$

$$q_1 + q_2 = Z. \tag{22c}$$

The second-order conditions are: $\mathcal{L}_{11} < 0$, $\mathcal{L}_{22} < 0$ and $H > 0$. Continue to assume (G1) and (G2), the solution to (22) is:

$$q_1^S = \frac{(a^1 - c^1) - (a^2 - c^2) + 2(b^2 - \gamma^1)Z}{H} = Z - q_2^S, \tag{23}$$

where "*S*" refers to Stackelberg leader's optimal capacity allocation.

To solve for the foreign firm's equilibrium capacity allocation, I substitute (23) back into (20):

$$q_1^{*F} = \frac{1}{2}\frac{(a^1 - c^1) - (a^2 - c^2) + (4b^2 - 2\gamma^2 - 2\gamma^1)Z}{H} = Z - q_2^{*F}, \tag{24}$$

where "*F*" refers to follower's optimal capacity allocation.

1.4.2 Stackelberg Leader Versus Follower

Let's compare (23) and (24) to see how the Stackelberg leader have a different strategy than the follower.

Lemma 11: (A5) and (A6) imply that $(a^1 - c^1) - (a^2 - c^2) < (3\gamma^1 - 2\gamma^2)Z$, so $q_1^S < q_1^{*F}$.

Lemma 11 assumes small planes to be more favorable: (i) small planes are effectively much more profitable than large planes, and (ii) small planes are stronger complements (or weaker substitutes). Under these assumptions, the Stackelberg leader will take advantage of its role and allocate more capacity to small planes than the follower does. This can explain why Boeing devotes relatively more resources into manufacturing small planes, while Airbus devotes relatively more resources to large planes. However, this does not fully answer why Boeing takes on the extreme route to not produce any large planes at all. This extreme case is possible in the present model. Setting $q_1^S = 0$ in (23),

$$(a^1 - c^1) - (a^2 - c^2) = -2(b^2 - \gamma^1)Z, \tag{25}$$

and substituting (25) into (24) yields

$$q_1^{*F} = \frac{(b^2 - \gamma^2)Z}{H} \quad . \tag{26}$$

Recall that the products are not perfect substitutes, so $b^1 > \gamma^1$ and $b^2 > \gamma^2$. Hence, $0 < q_1^{*F} < Z$. Then it must be true that $0 < q_2^{*F} < Z$. While the Stackelberg leader gives up the less appealing large planes altogether, the follower still devotes resources to produce both large planes and small planes.

Lemma 12: If $q_1^S = 0$, then $0 < q_1^{*F} < Z$. When the Stackelberg leader allocates all the capacity to produce the effectively more profitable small planes, the follower allocates capacity to produce both products.

Fig. 1.5 illustrates the possible relative positions of point *S*, the capacity allocation of the Stackelberg leader, and point F^*, that of the follower. Point *S* is at the end of the capacity line while point F^* is somewhere along the line.

In fact,

$$\pi^S - \pi^{*F} = [(p^2 - c^2) - (p^1 - c^1)]q_1^{*F} > 0\,. \tag{27}$$

In other words, the Stackelberg leader earns higher profit than the follower. The analysis shows how the Stackelberg leader can take advantage of its position to strategize its capacity allocation against the follower. The orders for Boeing 787 and Airbus A350 in Table 1.2 and the fact that small planes should be less costly to make imply that the small planes are the more profitable product. Hence, Boeing has the established advantage to allocate its limited acres of factory space into manufacturing small planes; and Airbus can only respond by satisfying the remaining demand for small planes and using its limited resources to produce large planes. Therefore, what seems to be an *entry-welcoming* move by the incumbent is indeed a profitable one.

The results can also have implications on the firms' position in the competition. Recall in Section 1.3 that identical duopolists that compete in a Cournot fashion should have identical capacity allocation. If they are not identical, it is possible that the domestic firm specializes in small planes, while the foreign firm does not. In this section, the firms are assumed to be identical, but since the domestic firm is a Stackelberg leader, it can specialize in small planes, making the foreign firm produces both products.

Lemma 13: Given $q_1 = 0$ and $0 < q_1^* < Z$. If the firms are identical, the domestic firm should be a Stackelberg leader and the foreign firm should be a follower.

If Boeing and Airbus had different costs of production and/or different capacity, it is possible that the two competed in a Cournot fashion. If Boeing and Airbus were identical, it must be that Boeing was a Stackelberg leader while Airbus was a follower.

1.4.3 Stackelberg Duopoly, Cournot Duopoly and Monopoly

To see how different types of competition affects a firm's capacity allocation, this section compares results in (23) and (24) to those in previous sections. Comparing (23) to (6) shows that

Lemma 14: If (A5) is true, then $q_1^S < q_1^M$. A Stackelberg leader allocates less capacity to large planes (more capacity to small planes) than a monopolist.

When there was no rivalry, the domestic firm might not care about how its own large planes substitute (or complement) its small planes as long as it occupies both markets. However, as a Stackelberg leader, the domestic firm cannot ignore how the rival large planes strongly substitute (or weakly complement) its small planes. Therefore, the Stackelberg leader's capacity allocation inclines more towards small planes than it did as a monopolist. Since Fig. 1.5 assumes (A5), point *S* is farther on the small-plane side compared to point *M*.

The lemma below is based on (24) and (16).

Lemma 15: (A5) and (A6) imply that $(a^1 - c^1) - (a^2 - c^2) < 2(\gamma^1 - \gamma^2)Z$, so $q_1^{*F} > q_1^D$. A follower allocates more capacity to large planes (less capacity to small planes) than a duopolist in a Cournot competition.

The follower allocates its capacity in a disadvantageous way. When small planes are more favorable, a firm can allocate less capacity toward small planes as a follower in the competition than when it could act simultaneously as the rival. Fig. 1.5 assumes (A5) and (A6), point F^* leans more towards the large-plane side than point *D*.

With assumptions (A5) and (A6), Lemma 9, Lemma 14 and Lemma 15 together yield the result that $q_1^{*F} > q_1^D > q_1^M > q_1^S$, which is illustrated in Fig. 1.5.

As in Subsection 1.3.4, let's also compare the effect of an internal capacity expansion in the monopoly and an external capacity expansion in the duopoly. Under Stackelberg competition, $x_1^{SF} = q_1^S + q_1^{*F}$ when the market of large planes is in equilibrium.

Lemma 16: (A5) and (A6) imply that $(a^1 - c^1) - (a^2 - c^2) < 2(\gamma^1 - \gamma^2)Z$, so $x_1^{SF} < x_1^D$.

Together with Lemma 10, which has the same assumptions, $x_1^{SF} < x_1^D < x_1^M$. As explained in Subsection 1.3.4, with competition, the industry as a whole allocates more capacity to the product that is more demanded. Duopolistic capacity allocation also differs under different types of competition. Stackelberg competition results in capacity allocation that inclines towards the more demanded small planes than Cournot competition. This is because the Stackelberg leader has the advantage to allocate much more capacity towards small planes, and may even give up large planes altogether. In Fig. 1.6, point *S*+*F** refers to the combined capacity allocation of the Stackelberg leader and the follower. With the assumptions that (A5) and (A6), Fig. 1.6 shows how point *S*+*F** has the greatest capacity allocated to small planes, followed by point *D*+*D** and point *M'*.

1.5. Concluding Remarks

In some industries, the huge set-up cost is a natural entry barrier. The resulting economies of scale lead to an international oligopolistic market structure. The characteristics of and the competition in these international oligopolies motivate this chapter. It is well known that the rivalry between the gigantic oligopolists is fierce. They do not compete in one product but a line of products. These products are not only related on the demand side – they can be substitutes or complements. On the cost side, products can share an input. Because the sunk cost for the input can be spread over scope, there are economies of scope. This chapter maintains that the input specific to a firm's product line must be scarce. When such capacity constraint is binding, an oligopolist needs to strategize its capacity allocation among different products because it cannot produce the desirable quantities to satisfy the large world demand. The simple model of monopoly illustrates how capacity allocation is done. It turns out that if small planes are sufficiently (effectively) more profitable, a monopolist already allocates more capacity towards small planes. Hence when there is a capacity expansion, the extra capacity will be left for producing large planes. That is, a "pro-large" allocation results. Competition has a significant effect on the capacity expansion. The duopolists' combined capacity allocation would lean more towards the more demanded small planes than the expanded monopoly. Whether the duopolists make production decisions simultaneously or sequentially is also crucial. The model shows that if the duopolists are identical, their capacity allocation should also be identical under Cournot competition. If duopolists are different, such as having different marginal cost in producing a product, it is possible that one duopolist allocates all capacity to one product, but the duopolist does not. Since many believe Boeing is more efficient, this is a possible reason why Boeing produces small planes only. However, many argue that Boeing and Airbus have similar production technology. Also, since Boeing was the incumbent, it should have acted like a Stackelberg leader in the game. The model of Stackelberg competition shows that the leader has the advantage to allocate all its capacity to the profitable product, leaving the follower to spend some precious capacity on the unprofitable product. This provides insight into why Boeing, the incumbent "yielded" the large-plane market altogether to Airbus. When indeed, Boeing was at the advantageous position to earn greater profit than Airbus by doing so. The results also have implications on the firms' position in the competition. If Boeing and Airbus had different costs of production and/or different capacity, it is possible that the two were competing in a Cournot fashion. If Boeing and Airbus were identical, to have such capacity allocation, it must be that Boeing was a Stackelberg leader while Airbus was a follower.

Even though the production choices of the international LCA oligopoly has motivated the model of this chapter, the model basically provides a framework for analyzing the strategic capacity allocation of any multiproduct oligopoly. For example, it will be interesting to see whether the capacity allocations of Apple, Huawei and Samsung can be explained by this model.

This model is also open to possible extensions. While this model sheds some light on how strategic capacity allocation works, there is a lack of dynamics. It is of my interest to extend the model to incorporate cost adjustment over time. One natural approach is to allow a firm to invest (whose cost is included in K) to obtain greater amount of the fixed specific inputs, Z in the future. We can also consider the learning effect on the marginal costs, c^1 and c^2 if "the fixed input" in the model includes not only physical capital, but also human capital. Even more intriguing will be to allow the capacity requirements, θ^1 and θ^2, to vary over time. *Learning* may cause both θ^1 and θ^2 to drop over time. θ^1 and θ^2 may drop at different rates, so the learning curves of different products may not be equally steep. The capacity constraint itself can also take a more general functional form, rather than linear.

It is also possible to allow one market to clear before the other. There can be a monopoly or a duopoly in each market. The duopolists can act simultaneously or sequentially. One interesting variation is a four-stage game, in which one market clears before the other market and the Stackelberg leader acts first in each market. In this case, products are produced in different periods. To analyze strategic capacity allocation, an intertemporal capacity constraint is necessary.

Finally, it will be interesting to test this theoretical model empirically, using detailed data of the four aircraft models. Certainly, a more complicated model will be necessary for the purpose. For example, determinants of demand should include the prices of pre-owned LCAs and fuel prices.[19]

[19] Benkard (2004) treated aircraft purchases as rentals because the market for used LCAs is efficient. There are low transaction costs.

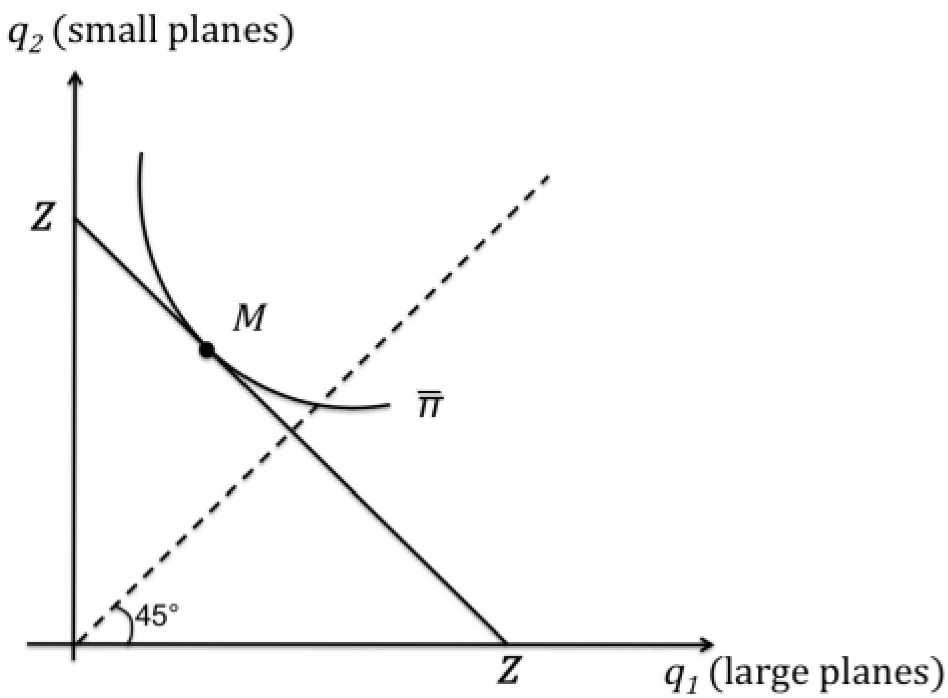

Fig. 1.1. Optimal Capacity Allocation of Multiproduct Monopolist

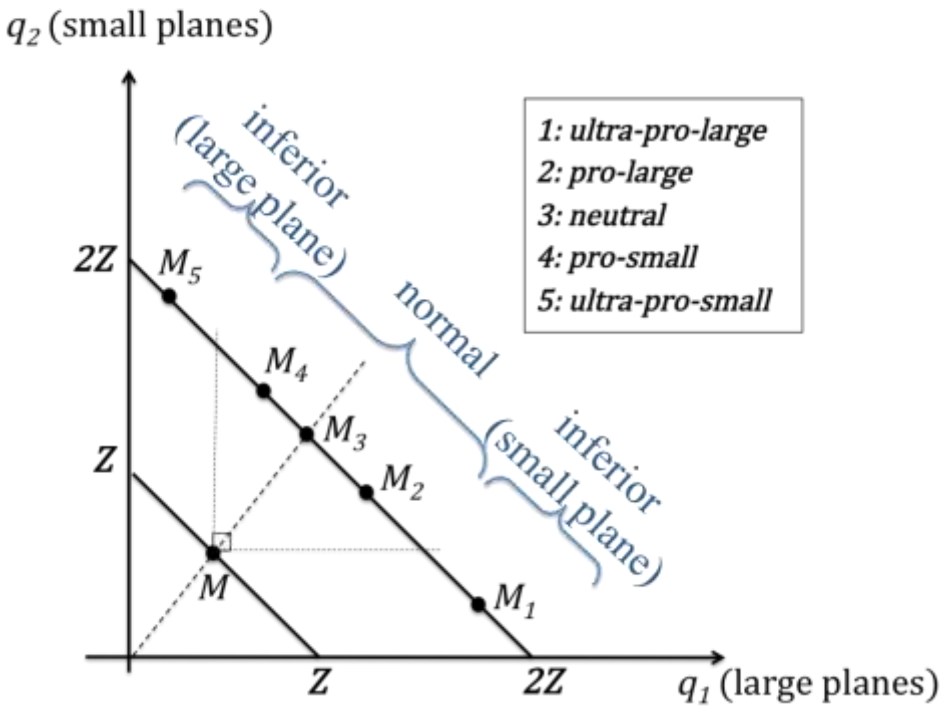

Fig. 1.2. Effect of Increase of Capacity, *Z*

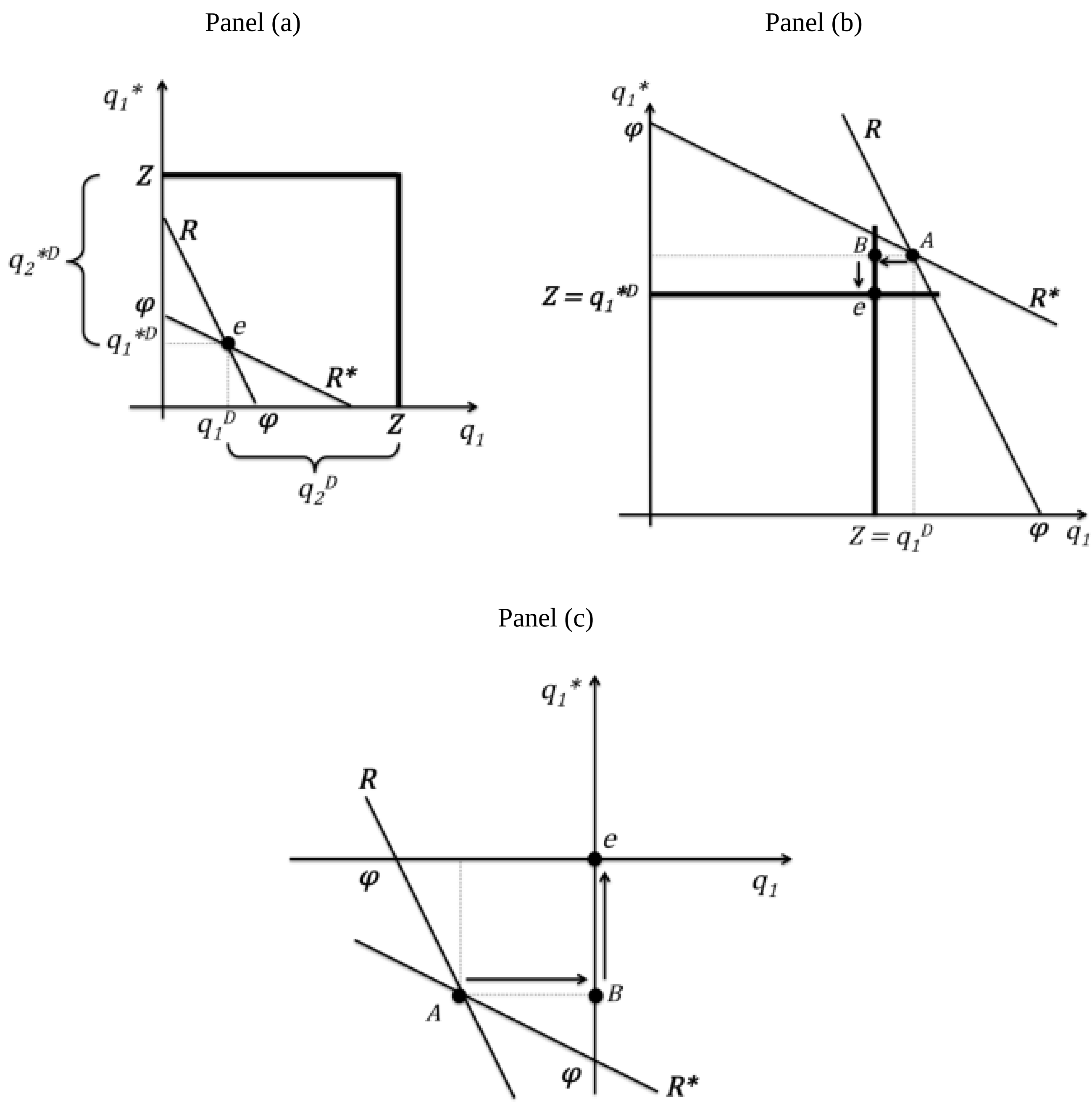

Fig. 1.3. Nash-Cournot-Equilibrium Capacity Allocation of Identical Firms

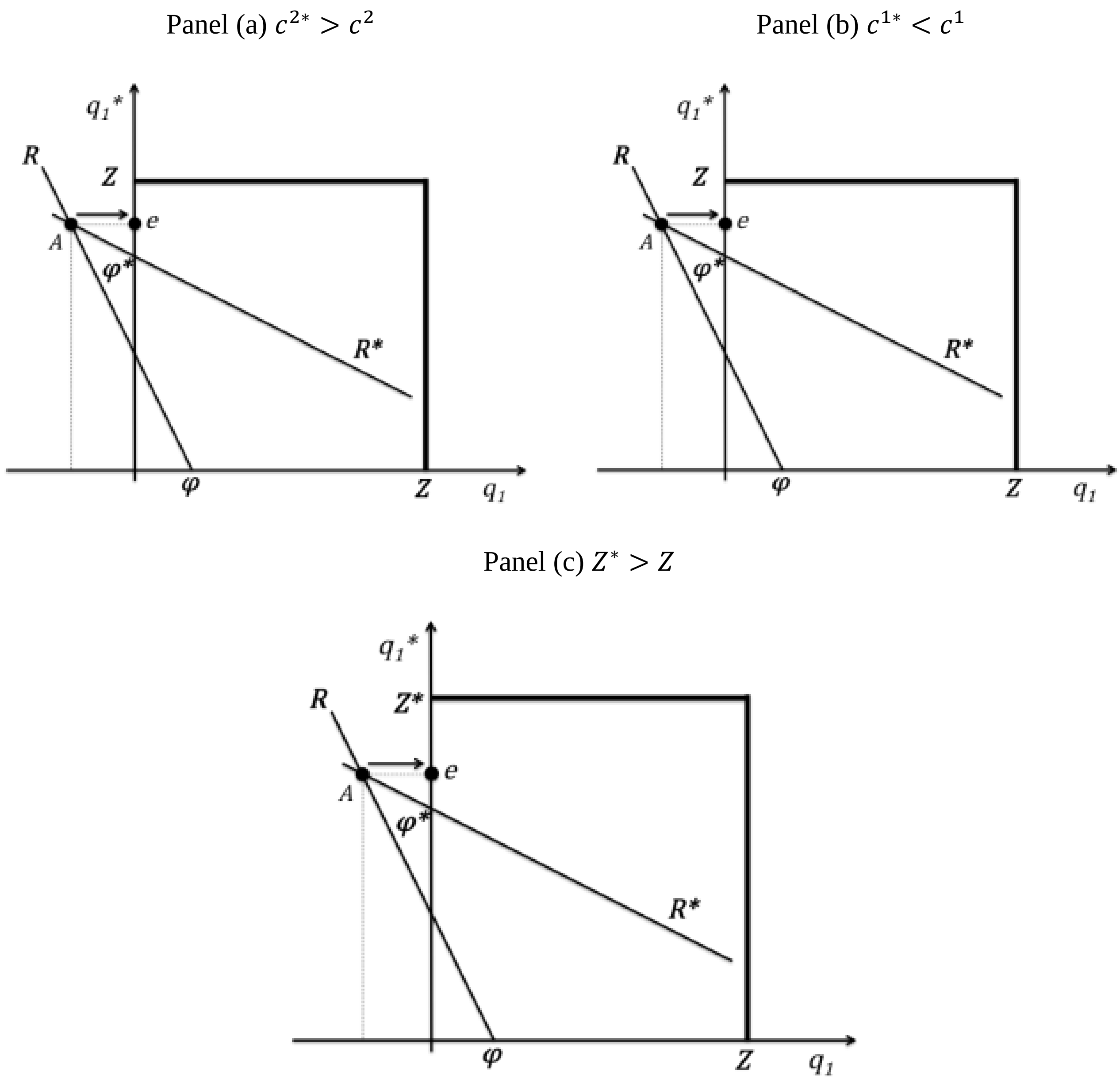

Fig. 1.4. Nash-Cournot-Equilibrium Capacity Allocation of Differentiated Firms

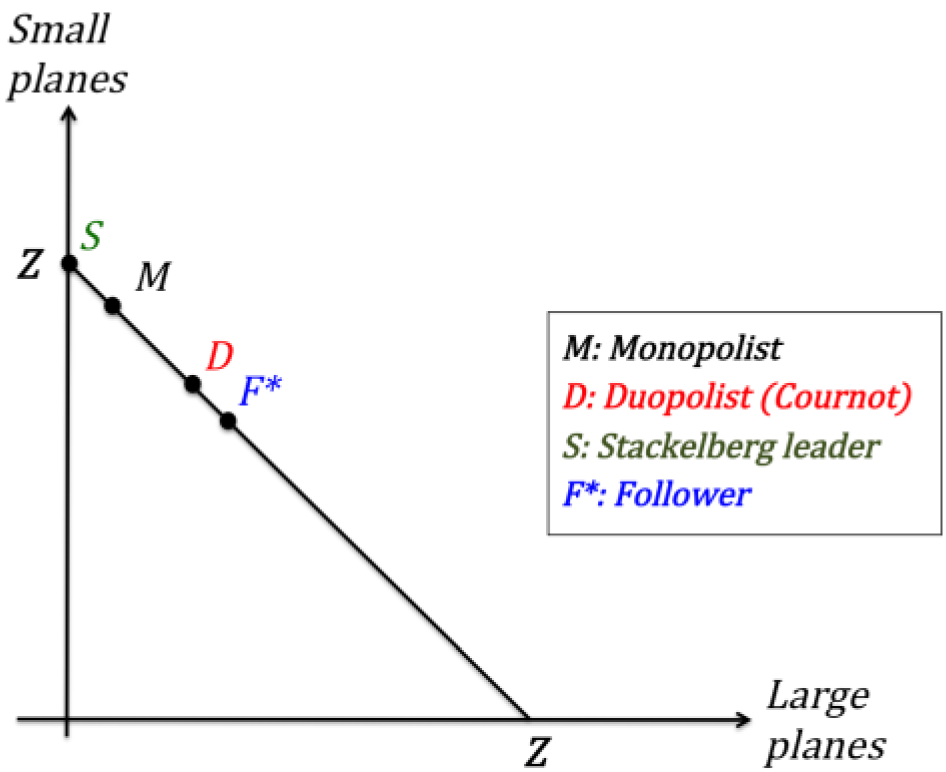

Fig. 1.5. Strategic Capacity Allocation Under Different Market Structures

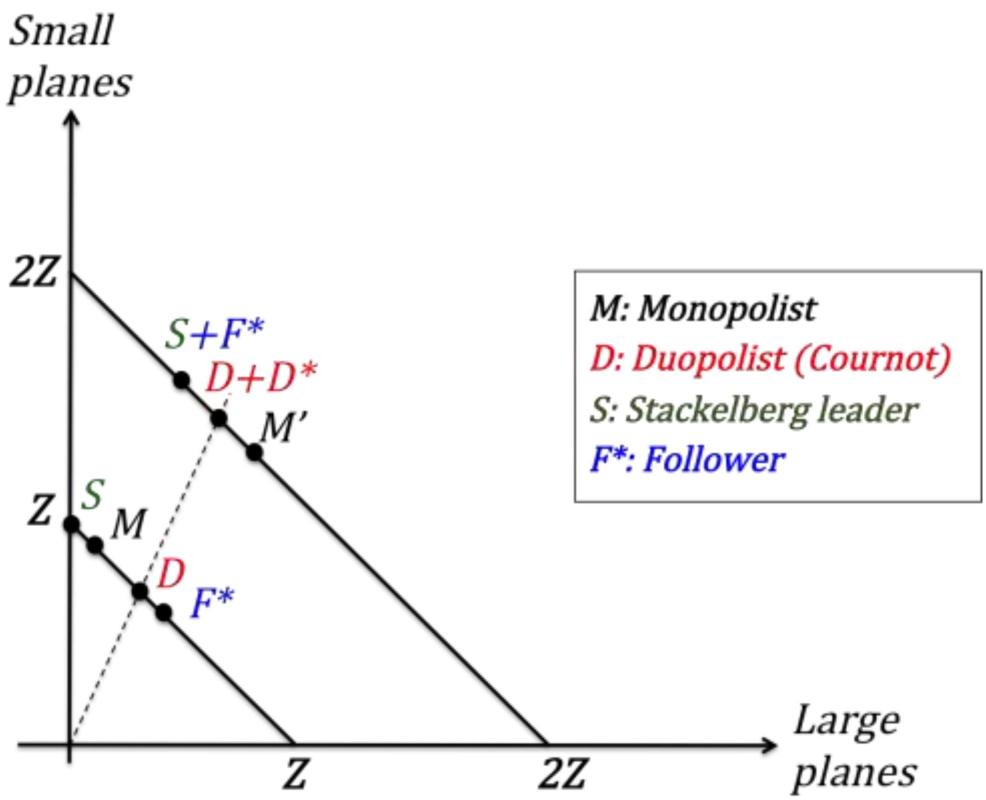

Fig. 1.6. Effect of Capacity Expansion

Table 1.1
Deliveries of Boeing 787 and Airbus A380 and A350

Product Year	Boeing 787 "small plane"	A380 "large plane"	A350 "small plane"
2007	0	1	0
2008	0	12	0
2009	0	10	0
2010	0	18	0
2011	3	26	0
2012	46	30	0
2013	65	25	0
2014	114	30	1
2015	135	27	14
2016	137	28	49
Total	363	179	15

Source: Airbus, Boeing.

Table 1.2.
Orders of Boeing 787 and Airbus A380 and A350

Product Year	Boeing 787 “small plane”	A380 “large plane”	A350 “small plane”
2001	0	85	0
2002	0	10	0
2003	0	34	0
2004	56	10	0
2005	235	20	0
2006	157	7	2
2007	369	23	292
2008	93	9	163
2009	-59	4	51
2010	-4	32	78
2011	13	19	-31
2012	-12	9	27
2013	182	42	230
2014	41	13	-32
2015	71	2	-3
2016	58	0	41
Total	1142	319	777

Source: Airbus, Boeing.

Chapter 2

FOREIGN TRADE, ENDOGENOUS FRAGMENTATION AND OPTIMAL OFFSHORING

2.1. Introduction

Fragmentation refers to “the splitting of a production process into two or more steps that can be undertaken in different locations but that lead to the same final product” (Deardorff, 2001). In this sense, fragmentation is *not* offshoring, but without fragmentation, offshoring is not possible. Offshoring occur in many industries. Large civil aircrafts (LCAs) serve as good examples because data of their product design are available to the public and are covered extensively by the media. Such firm-level data are usually inaccessible to researchers in other industries. Fig. 2.1 illustrates how Boeing’s newest LCA, the 787 Dreamliner, is built. The aircraft consists of many *subparts*. Boeing manufacture some of the subparts in the US and offshores some to other countries. Boeing then brings all the subparts together for final assembly. Based on a figure like Fig. 2.1 and data from the company’s website, I can calculate the percentage of production that is offshored as

$$\text{percentage of production offshored} = \frac{\text{value of subparts offshored}}{\text{value of all the subparts}} .$$

This percentage serves as a measure of the degree of offshoring.

Boeing has only one rival in the LCA industry − Airbus. Boeing and Airbus are similar in many ways. They have basically equal global market shares. The two appear to be equally advanced in technology. They even face the same manufacturing problems − both companies are based in countries where wages are higher than the rest of the world, and both suffer occasional labor strikes and delivery delays. Airbus A350 is designed to compete head-to-head with Boeing 787. The two aircrafts are very comparable. If Boeing engages in offshoring when making its 787, Airbus is expected to do the same for its A350. Fig. 2.2 compares how Boeing 787 and Airbus A350 are built. Boeing adopts two modes of offshoring − outsources 53% of its 787 subparts to foreign suppliers and produces 10% through foreign subsidiaries. Proxying the degree of offshoring as the percentage of subparts offshored, the above numbers indicate a total of 63% of offshoring. In contrast, Airbus outsources 16% of its A350 subparts to suppliers located outside EU countries and does not produce any A350 part through foreign subsidiaries, so these indicate only 16% of offshoring.[20] In other words, in terms of number of subparts, Boeing offshores 47% more than Airbus does.

In search for a possible explanation of how two similar firms can have significantly different degrees of offshoring, I found that the US and the EU seemed to have different policy strategies. Both the US and the EU are WTO members and the WTO strictly prohibits export subsidies. Both countries have filed numerous disputes against each other, concerning the export subsidies given on each side. On the other hand, the WTO have given benign neglect toward research and development (R&D) subsidies, presuming that they do not distort trade (Maskus, 2015). Not until recently in 2011 did the WTO find some of the NASA and US Department of Defense (DOD) aeronautics R&D subsidies constitute "specific subsides" toward Boeing ("WTO Dispute," 2013). On the other hand, there has been no complaint concerning R&D subsidies

[20] Airbus has sites located in four EU countries: France, Germany, the UK and Spain. Production of these sites is considered to be in-house production of Airbus.

towards Airbus. These motivate the essay to establish a link between R&D subsidy and offshoring of a firm.

Many studies consider offshoring to be a product of globalization. One of the earliest studies of fragmentation, Jones and Kierzkowski (1990), asserted that advancement in transportation and telecommunication technologies lowers the cost of coordination between firms and their subpart suppliers and subsidiaries. Consequently, offshoring is made easier. However, Boeing and Airbus face the same global state of technology when developing their 787 and A350 roughly in the same period. The global economic factors cannot explain why firms have different degrees of offshoring. In fact, Boeing 787 is well-known for its material evolution. The use of composite material makes it possible to produce larger subparts.[21] In other words, fragmentation depends on a firm's own product design, and a firm can research and develop its own *fragmentation technology*. Therefore, this study sees that a firm can determine its own fragmentation and its subsequent offshoring *endogenously*. This enable a government to directly subsidize the R&D of fragmentation technology and influence the degree of offshoring of a firm, especially when export subsidy is not available.

The present model is closely related to the novel model of strategic trade policy in Brander and Spencer (1983; 1985). Brander and Spencer (1983) also showed how each government could use export subsidy and R&D subsidy to support the local firm and to discourage the rival firm. However, their R&D subsidy applied to an intermediate good that entered the cost function. They did not specify any cost function and simply assumed R&D to be cost-reducing. In contrast, this essay considers a very different type of R&D – one that is specifically for

[21] According to Marsh (2005), Boeing could even make the wings and the fuselage in plastic. Boeing could make a section of the fuselage as large as 7 meters long by 6 meters as a single piece. In contrast, the manufacturer has to assemble older LCA models in-house, one small component at a time.

fragmentation technology. Hence, the R&D subsidy is different from the one in Brander and Spencer (1983). More importantly, this chapter emphasizes on how fragmentation enters the cost function. Cost-saving is through offshoring, which is not considered in Brander and Spencer's (1983) model.

Recent studies of offshoring mainly adopted models of heterogeneous firms due to Melitz (2003) and Antràs and Helpman (2004).[22] In firm heterogeneity models, more productive firms produce with lower marginal costs and are more likely to offshore. In contrast, the model in this chapter shows that even if firms start with equal marginal costs, they can still have different degrees of offshoring.[23] That is, firm heterogeneity models look into a firm's choice of *whether or not* to offshore. These models measure the degree of offshoring as the volume of "intra-industry trade" of the industry as a whole. In contrast, this chapter investigates *by how much* a firm chooses to offshore and measures the degree of offshoring as the foreign content of the final good. In other words, this essay narrows the scope to "intra-product trade."

This chapter extracts elements from three strands of literature – fragmentation, strategic trade and industrial policies and offshoring, but establishes the endogeneity of fragmentation. Hence, this essay is able to provide a framework for analyzing how the availability of a fragmentation subsidy affects a government's policy strategies and subsequently their effects on welfare, export and offshoring. Section 2.2 introduces the fragmentation function. Section 2.3 provides the main model and presents the main results of unilateral government intervention. Section 2.4 extends the model to cover bilateral government intervention. The last section concludes.

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[22] Examples are Grossman, Helpman and Szeidl (2006) and Díez (2014).

[23] Recall from the example of the LCA industry that Boeing and Airbus are similar in many ways. Firm heterogeneity is not a characteristic of the industry.

2.2. Fragmentation Function

To define fragmentation, f, consider a final good that is made up of numerous components. To build the final good, a firm can simply join all the components in-house. Alternatively, the firm can join some components into *subparts*, possibly somewhere else. Then the firm can assembled the subparts in-house into the final product. This chapter refers to this way of breaking down the production into re-locatable processes as "fragmentation." In this chapter, $f \in [0, 1]$ is the proportion of production fragmented by a firm.

Previous studies take the degree of fragmentation as given, in the sense that it is only affected by global economic factors such as the current state of technology. For example, communication technology advancement can raise the level of offshoring, and so the underlying assumption is that the degree of fragmentation is also raised for *all* firms. On the other hand, this study sees that a firm's own product design dictates which components to be joined in-house and which ones to be put together as subparts. That is, a firm can invest on R&D of *manufacturing engineering* to obtain a higher degree of fragmentation.[24] Because different firms may invest different amounts on R&D of fragmentation technology, different firms may have different degrees of fragmentation and thus different levels of offshoring.

Suppose a firm invests k on R&D of fragmentation technology. The degree of fragmentation is a function of k, $f = f(k)$. In general, the function of fragmentation can take on any form as long as it is increasing in k. That is, the first derivative of f with respect to k is $f' > 0$. This

[24] In Chapter 2 of the *Manufacturing Engineering Handbook*, Dewhurst (2004) explained the design for manufacture and assembly (DFMA) in details. Dewhurst (2004) emphasized the importance of "the design structure which is likely to minimize […] manufacturing costs, assembly cost, and other cost sources." Since I assume fragmentation to be cost-reducing in the model, it is reasonable to believe that more *capable* manufacturing engineers pursue higher degree of fragmentation.

essay assumes f to be a continuous function that is sandwiched between 0 and 1 for all positive k. Hence, the second derivative of f with respect to k must be $f'' < 0$.

The degree of fragmentation and the level of offshoring have a direct relation. Suppose a firm incurs a constant marginal cost of θ if it produces the final good in-house. If the firm produces the final good in the rest of the world (ROW) instead, the constant marginal cost will be ϕ, which is lower than θ. The cost difference, $\theta - \phi$, can be due to the comparative advantage of the ROW in producing subparts.[25] The lower cost in the ROW should be appealing to the firm, so it seeks to produce as much as possible overseas. As a result, a firm offshores all the re-locatable, fragmented proportion to the ROW. In particular, $1 - f$ is the proportion produced in-house and f is the proportion offshored. Hence, the marginal cost is

$$c = \theta(1 - f) + \phi f = \theta - (\theta - \phi)f. \tag{1}$$

Without fragmentation, the firm faces a marginal cost of θ. With fragmentation, the firm's marginal cost reduces by $(\theta - \phi)f$. In this sense, fragmentation is *cost-reducing*. In the following sections, the marginal cost, as defined in (1) will appear in the profit function of a firm.

[25] I rule out the case of $\phi \geq \theta$ that a firm has no motivation to offshore at all. This is because offshoring is already a well-observed phenomenon in the world. The central question of this chapter is not whether or not the firms offshore, but *by how much* they offshore.

2.3. Inactive Foreign Government

Consider two countries – the domestic country and the foreign country. Asterisks (*) distinguish variables of the foreign country. There is a global duopoly – one firm is located in the domestic country and the other located in the foreign country. Suppose in the beginning, neither government intervenes its firm. However, when policy options become available, it may be beneficial for a government to deviate from the "free trade" paradigm. This section explores the optimal policy of the domestic government when the foreign government is inactive. In the next section, I will explore the case when both governments are active.

The three-stage game is as follows.

i. In stage 1, the domestic government chooses export subsidy and/or fragmentation subsidy to maximize welfare.
ii. In stage 2, each firm chooses the level of R&D investment on fragmentation technology to maximize profit simultaneously.
iii. In stage 3, each firm chooses the level of output to maximize profit simultaneously.

2.3.1 Export Subsidy

One of the common industrial and trade policies governments adopt is export subsidy. Suppose the output levels of the domestic firm and the foreign firm are q and q^* respectively. Assume the ROW to be the only market for the final good.[26] Hence, q and q^* are exports to the ROW. The domestic government can subsidize its firm's export by deducting s from the firm's

[26] In the aircraft industry, it is reasonable to consider the global market as a single market, because the airlines, the buyers themselves, are multinational corporations (MNCs). Alternatively, assume the national markets to be fully integrated under free trade and zero transportation cost so that in equilibrium, the firms face the same price. These assumptions will yield the same results as the single-market assumption.

marginal cost. In this subsection, consider export subsidy to be the only policy option the domestic government has.

Suppose the quantity demanded of the final good is X and so the (inverse) demand is $P = P(X)$. The first and second derivatives of the demand function have normal properties such that $P'(X) < 0$ and $P''(X) < \xi$, where ξ is a sufficiently small positive number. Total supply is $Q = q + q^*$. When the market is in equilibrium, $X = Q$, so price is $P = P(q + q^*)$.

The three-stage game can be solved through backward induction. In stage 3, the domestic firm considers k, its investment on R&D of fragmentation technology in the previous stage, as a *sunk* cost. The domestic firm chooses the level of output, q to maximize profit:

$$\pi = Pq - cq + sq - k = P(q + q^*)q - [\theta - (\theta - \phi)f(k)]q + sq - k, \tag{2}$$

taking the foreign firm's output, q^*, local marginal cost, θ, marginal cost in the ROW, ϕ, and its government's export subsidy, s, as given. The first-order condition is

$$\frac{\partial \pi}{\partial q} = P'q + P - \theta + (\theta - \phi)f(k) + s = 0. \tag{3}$$

The second-order condition,

$$\frac{\partial^2 \pi}{\partial q^2} = P''q + 2P' < 0, \tag{4}$$

is satisfied for a normal demand function. Simultaneously, the foreign firm considers k^*, its investment on R&D of fragmentation technology as a sunk cost and chooses the level of output, q^*, and, to maximize profit:

$$\pi^* = Pq^* - c^*q^* - k^* = P(q+q^*)q^* - [\theta^* - (\theta^* - \phi)f^*(k^*)]q^* - k^*, \tag{5}$$

taking the domestic firm's output, q, local marginal cost, θ^* and marginal cost in the ROW, ϕ, as given. The first-order condition is

$$\frac{\partial \pi^*}{\partial q^*} = P'q^* + P - \theta^* + (\theta^* - \phi)f^*(k^*) = 0. \tag{6}$$

The second-order condition,

$$\frac{\partial^2 \pi^*}{\partial {q^*}^2} = P''q^* + 2P' < 0, \tag{7}$$

is satisfied for a normal demand function. In summary, the model has the following basic assumptions.

Condition A: $\theta > \phi$, $\theta^* - \phi$, $f' > 0$, $f'' < 0$, $f^{*\prime} > 0$, $f^{*\prime\prime} < 0$, $P' < 0$, $P''q + 2P' < 0$,

$P''q^* + 2P' < 0$ and $\tilde{P}''\tilde{q} + \tilde{P}''\tilde{q}^* + 3\tilde{P}' > 0$.

I solve (3) and (6) simultaneously for Nash-Cournot-equilibrium $\tilde{q}$ and $\tilde{q}^*$, where tildes denote equilibrium levels. Back-substituting the solutions into the first-order conditions and total differentiate the equations with respect to k, solving the two equations simultaneously by Cramer's rule yield

$$\frac{\partial \tilde{q}}{\partial k} = -\frac{(\theta - \phi)f'\left(\tilde{P}''\tilde{q}^* + 2\tilde{P}'\right)}{D} > 0, \tag{8a}$$

$$\frac{\partial \tilde{q}^*}{\partial k} = \frac{(\theta - \phi)f'\left(\tilde{P}''\tilde{q}^* + \tilde{P}'\right)}{D} < 0, \tag{8b}$$

where $D \equiv \tilde{P}'\left(\tilde{P}''\tilde{q} + \tilde{P}''\tilde{q}^* + 3\tilde{P}'\right) > 0$ by Condition A. Similarly, with respect to k^*,

$$\frac{\partial \tilde{q}}{\partial k^*} = \frac{(\theta^* - \phi) f^{*\prime}\left(P''\tilde{q} + \tilde{P}'\right)}{D} < 0, \tag{9a}$$

$$\frac{\partial \tilde{q}^*}{\partial k^*} = -\frac{(\theta^* - \phi) f^{*\prime}\left(P''\tilde{q} + 2\tilde{P}'\right)}{D} > 0. \tag{9b}$$

The signs of (8) and (9) are consistent with Condition A. (8) and (9) basically tell that:

Lemma 1: *A firm's production depends on its own R&D investment on fragmentation technology positively, but depends on its rival's R&D investment on fragmentation technology negatively.*

Lemma 1 holds true particularly because a firm's R&D only enters its own fragmentation function and helps only the firm itself to save cost by offshoring.[27]

Using the same technique, I derive the comparative statics with respect to domestic export subsidy, s:

$$\frac{\partial \tilde{q}}{\partial s} = -\frac{\tilde{P}''\tilde{q}^* + 2\tilde{P}'}{D} > 0, \tag{10a}$$

$$\frac{\partial \tilde{q}^*}{\partial s} = \frac{\tilde{P}''\tilde{q}^* + \tilde{P}'}{D} < 0. \tag{10b}$$

The signs are consistent with Condition A. The signs are as expected such that a domestic export subsidy encourages domestic export, but discourages foreign export.

[27] Even if I modeled *spillover effects* by assuming f depends on k*, and f* depends on k positively, Lemma 1 would still hold as long as own effect of R&D on fragmentation dominates cross effect.

In stage 2, each firm has perfect information of how its investment on R&D of fragmentation technology, which determines its level of offshoring, affects exports in the next stage. The domestic firm chooses the R&D investment on fragmentation technology, k, to maximize profit:

$$\tilde{\pi} = \tilde{P}\tilde{q} - [\theta - (\theta - \phi)f(k)]\tilde{q} + s\tilde{q} - k, \tag{11}$$

taking the foreign firm's R&D investment on fragmentation technology, k^*, local marginal cost, θ, marginal cost in the ROW, ϕ, and export subsidy, s, as given. The first-order condition is

$$\frac{\partial\tilde{\pi}}{\partial k} = \tilde{P}'\tilde{q}\frac{\partial\tilde{q}^*}{\partial k} + (\theta - \phi)f'\tilde{q} - 1 = 0. \tag{12}$$

The second-order condition is

$$\frac{\partial^2\tilde{\pi}}{\partial k^2} = [\tilde{P}''\tilde{q}\left(\frac{\partial\tilde{q}}{\partial k} + \frac{\partial\tilde{q}^*}{\partial k}\right) + \tilde{P}'\frac{\partial\tilde{q}}{\partial k}]\frac{\partial\tilde{q}^*}{\partial k} + \tilde{P}'\tilde{q}\frac{\partial^2\tilde{q}^*}{\partial k^2} + (\theta - \phi)(f''\tilde{q} + f'\frac{\partial\tilde{q}}{\partial k}) < 0. \tag{13}$$

To assure (13) to hold, I make further assumptions about the properties of the demand function (such as the magnitude of P''') and the fragmentation function. As in previous studies (e.g. Brander and Spencer (1983)), it is not an easy task to ensure the second condition to hold. It is also necessary for

$$\Delta \equiv \frac{\partial^2\tilde{\pi}}{\partial k^2}\frac{\partial^2\tilde{\pi}^*}{\partial {k^*}^2} - \frac{\partial^2\tilde{\pi}}{\partial k\partial k^*}\frac{\partial^2\tilde{\pi}^*}{\partial k^*\partial k} > 0, \tag{14}$$

to hold globally in order to ensure uniqueness and stability of the equilibrium. Since the interest of this chapter is the comparative statics of well-behaved cases, it assumes (13) and (14) to hold. One such well-behaved demand function is a linear one. That is,

$$P'' = 0. \tag{15}$$

(15) simplifies conditions (12) ad (13) a great deal:

$$\frac{\partial \tilde{\pi}}{\partial k} = \frac{4}{3}(\theta - \phi)f'q - 1 = 0, \tag{12'}$$

$$\frac{\partial^2 \tilde{\pi}}{\partial k^2} = \frac{4}{3}(\theta - \phi)[f''q - \frac{2}{3}(\theta - \phi)\frac{f'^2}{P'}] < 0. \tag{13'}$$

Hence, hereafter, this essay assumes a linear demand in order to obtain results that are easy to interpret without losing the central insights of a more general model.[28]

Simultaneously, the foreign firm chooses the R&D investment on fragmentation technology, k^*, to maximize profit:

$$\tilde{\pi}^* = \tilde{P}\tilde{q}^* - [\theta^* - (\theta^* - \phi)f^*(k^*)]\tilde{q}^* - k^*, \tag{16}$$

taking the domestic firm's R&D investment on fragmentation technology, k, local marginal cost, θ^*, marginal cost in the ROW, ϕ, and export subsidy, s, as given. The first-order condition is

$$\frac{\partial \tilde{\pi}^*}{\partial k^*} = \frac{4}{3}(\theta^* - \phi)f^{*\prime}q^* - 1 = 0. \tag{17}$$

The second-order condition is

$$\frac{\partial^2 \tilde{\pi}^*}{\partial k^{*2}} = \frac{4}{3}(\theta^* - \phi)[f^{*\prime\prime}q^* - \frac{2}{3}(\theta^* - \phi)\frac{f^{*\prime 2}}{P'}] < 0. \tag{18}$$

[28] With the linear demand assumption, $\Delta \equiv$

$$\left(\frac{4}{3}\right)^2 (\theta - \phi)(\theta^* - \phi)[\left(2f''q - (\theta - \phi)\frac{f'^2}{P'}\right)\left(2f^{*\prime\prime}q^* - (\theta^* - \phi)\frac{f^{*\prime 2}}{P'}\right) - \frac{f''f^{*\prime\prime}qq^*}{P'} > 0,$$

which hold under Condition A.

Condition B: (13), (14), (15) and (18) are satisfied.

Before proceeding to stage 1, notice that the firm does not minimize overall cost in (12'). If it did, the cost-minimizing condition would be:

$$\frac{\partial}{\partial k}[c\tilde{q}+k] = -\frac{2}{3}(\theta-\phi)f'\frac{c}{P'} - (\theta-\phi)f'\tilde{q}+1 = 0. \tag{19}$$

Hence the cost-minimizing f' is greater than the profit-maximizing f', implying that the cost-minimizing k is less than the profit-maximizing k. This is an important observation that the firm actually invests on R&D of fragmentation technology more than necessary to minimize cost.

I solve (12') and (17) simultaneously for Nash-Cournot equilibrium $\tilde{k}$ and $\tilde{k}^*$. Back-substituting the solutions into the first-order conditions, total differentiating the equations with respect to s and solving the two equations simultaneously by Cramer's rule yield

$$\frac{\partial\tilde{k}}{\partial s} = \frac{\frac{8}{9\tilde{P}'}(\theta-\phi)f'\frac{\partial^2\tilde{\pi}^*}{\partial k^{*2}} + \frac{4}{9\tilde{P}'}(\theta^*-\phi)f^{*\prime}\frac{\partial^2\tilde{\pi}}{\partial k\partial k^*}}{\Delta} > 0, \tag{20a}$$

$$\frac{\partial\tilde{k}^*}{\partial s} = -\frac{\frac{8}{9\tilde{P}'}(\theta-\phi)f'\frac{\partial^2\tilde{\pi}}{\partial k\partial k^*} + \frac{4}{9\tilde{P}'}(\theta^*-\phi)f^{*\prime}\frac{\partial^2\tilde{\pi}}{\partial k^2}}{\Delta} < 0. \tag{20b}$$

The signs are consistent with Condition A and Condition B. Since $f' > 0$ and $f^{*'} > 0$, (20) can be interpreted as the effect of domestic export subsidy on the firms' degree of offshoring.

(10a) and (10b) are as expected - the export subsidy can stimulate local export, but lower the rival firm's export. (20a) and (20b) are less obvious. When the policy encourages the domestic firm to produce more, the firm also searches for the least costly way to do so. Since offshoring is cost-reducing, the domestic firm invests more on fragmentation technology in order to achieve a higher level of fragmentation and offshoring. On the other hand, the foreign firm will want to cut its production, so it will also offshore less.

Next let's turn to stage 1. Plug the Nash-Cournot equilibrium levels of R&D investment into the Nash-Cournot equilibrium levels of production, so that $\tilde{k}$, $\tilde{k}^*$, $\tilde{\tilde{q}}$ and $\tilde{\tilde{q}}^*$ are functions of s and other parameters. Hence, the domestic government chooses export subsidy, s, to maximize welfare defined as follows: [29]

$$w = \tilde{\tilde{\pi}} - s\tilde{\tilde{q}} = \tilde{\tilde{P}}\tilde{\tilde{q}} - [\theta - (\theta - \phi)f(\tilde{k})]\tilde{\tilde{q}} + s\tilde{\tilde{q}} - \tilde{k} - s\tilde{\tilde{q}}. \quad (21)$$

The first-order condition is

$$\frac{\partial w}{\partial \mathrm{s}} = \frac{\partial \pi}{\partial q^*}\left(\frac{\partial \tilde{q}^*}{\partial k}\frac{\partial \tilde{k}}{\partial s} + \frac{\partial \tilde{q}^*}{\partial k^*}\frac{\partial \tilde{k}^*}{\partial s} + \frac{\partial \tilde{q}^*}{\partial s}\right) - s\left(\frac{\partial \tilde{q}}{\partial k}\frac{\partial \tilde{k}}{\partial s} + \frac{\partial \tilde{q}^*}{\partial k^*}\frac{\partial \tilde{k}^*}{\partial s} + \frac{\partial \tilde{q}}{\partial s}\right) = 0, \quad (22)$$

and the second-order condition, $\frac{\partial w^2}{\partial s^2} < 0$ is assumed to hold. I can rearrange (21) as

$$s^e = \frac{\tilde{\tilde{P}}'\tilde{\tilde{q}}\left(\frac{\partial \tilde{q}^*}{\partial k}\frac{\partial \tilde{k}}{\partial s} + \frac{\partial \tilde{q}^*}{\partial k^*}\frac{\partial \tilde{k}^*}{\partial s} + \frac{\partial \tilde{q}^*}{\partial s}\right)}{\left(\frac{\partial \tilde{q}}{\partial k}\frac{\partial \tilde{k}}{\partial s} + \frac{\partial \tilde{q}^*}{\partial k^*}\frac{\partial \tilde{k}^*}{\partial s} + \frac{\partial \tilde{q}}{\partial s}\right)} > 0, \quad (23)$$

in which the signs are consistent with Condition A and Condition B. (20) and (23) provide one reason why two firms can have different degrees of offshoring. Suppose the duopolists are identical (i.e. $\theta = \theta^*$ and $f(\,) = f^*(\,)$). This is a reasonable assumption for firms that have similar production technology such as Boeing and Airbus. In this case, export subsidy is positive

[29] Since only the ROW consumes, the domestic welfare function does not include consumer surplus.

and is granted unilaterally by the domestic government. Hence, by (20), the domestic firm invests more on R&D of fragmentation technology, while the foreign firm invests less. Since $\tilde{k}^e > \tilde{k}^{*e}$, by the properties of the fragmenation functions, $\tilde{f}^e > \tilde{f}^{e*}$. Since the level of fragmentation determines the degree of offshoring. The model gives the following result.

Lemma 2: *Given Condition A and Condition B are satisfied, when the foreign government does not intervene the industry, it is optimal for the domestic government to impose a positive export subsidy,* $s = s^e > 0$, *which encourages offshoring by the domestic firm, but discourages offshoring by the foreign firm. Hence, the domestic firm offshores more than the foreign firm,* $\tilde{f}^e > \tilde{f}^{e*}$.

This is a counter-argument to previous understanding of how offshoring only depends on exogenous global factors. When fragmentation is an endogenous variable, a firm's decision to offshore is affected by its government's supportive policy.

Lemma 2 implies that the domestic government has incentive to deviate from any free trade agreement (FTA) previously signed with the foreign government. This result is in line with the findings of Brander and Spencer (1983) and subsequent studies. However, under the WTO's rules, member countries have to eliminate their export subsidies. Export subsidy is no longer a realistic policy strategy. Yet, the incentive to deviate from a "free trade" paradigm remains. In the next section I will consider an alternative policy strategy.

2.3.2 Fragmentation Subsidy

While the WTO strictly prohibits export subsidy, R&D subsidies have been subject to relatively less enforcement (Maskus, 2015). In the present model, each firm can invest on R&D of fragmentation technology. Hence, there is a channel for the government to subsidize a firm's R&D. Suppose the only industrial trade policy available to the domestic government is a fragmentation subsidy, u. The subsidy is an ad valorem subsidy, so a percentage can be deducted from the domestic firm's spending on fragmentation technology.

In stage 3, the domestic firm chooses the level of output, q to maximize profit:

$$\pi = Pq - cq - (1-u)k = P(q+q^*)q - [\theta - (\theta - \phi)f(k)]q - (1-u)k, \tag{24}$$

taking k, its investment on R&D of fragmentation technology as a *sunk* cost, and taking the foreign firm's output, q^*, local marginal cost, θ, marginal cost in the ROW, ϕ, and its government's fragmentation subsidy, u, as given. The first-order condition is

$$\frac{\partial \pi}{\partial q} = P'q + P - \theta + (\theta - \phi)f(k) = 0. \tag{25}$$

Note that, u does not enter the first-order condition. The second-order condition is the same as (4) and should hold given a normal demand function.

Simultaneously, the foreign firm considers k^*, its investment on R&D of fragmentation technology as a sunk cost and chooses the level of output, q^*, to maximize profit, taking the domestic firm's output, q, local marginal cost, θ^* and marginal cost in the ROW, ϕ, as given. The profit function, the first-order condition and the second-order condition are the same as those in (5), (6) and (7).

I solve (25) and (6) simultaneously for Nash-Cournot equilibrium $\tilde{q}$ and $\tilde{q}^*$, back-substitute the solutions into the first-order conditions and total differentiate the equations with respect to k

and with respect to k^*. Solving each set of equations simultaneously by Cramer's rule yield the same equations as (8) and (9). Hence, Lemma 1 still holds here that a firm's R&D investment on fragmentation technology encourages own export, but discourages rival firm's export.

In stage 2, with perfect information of how its R&D investment on fragmentation technology, k, affects exports in stage 3, the domestic firm chooses k to maximize profit:

$$\tilde{\pi} = \tilde{P}\tilde{q} - [\theta - (\theta - \phi)f(k)]\tilde{q} - (1 - u)k, \tag{26}$$

taking the foreign firm's R&D investment on fragmentation technology, k^*, local marginal cost, θ, marginal cost in the ROW, ϕ, and fragmentation subsidy, u, as given. The first-order condition is

$$\frac{\partial \tilde{\pi}}{\partial k} = \frac{4}{3}(\theta - \phi)f'\tilde{q} - (1 - u) = 0, \tag{27}$$

and the second-order condition is the same as (13').

Simultaneously, the foreign firm chooses the R&D investment on fragmentation technology, k^*, to maximize profit, taking the domestic firm's R&D investment on fragmentation technology, k, local marginal cost, θ^*, and marginal cost in the ROW, ϕ, as given. The profit function, the first-order condition and the second-order condition are the same as those in (16), (17) and (18).

I solve (27) and (17) simultaneously for Nash-Cournot equilibrium $\tilde{k}$ and $\tilde{k}^*$. Substituting the solutions back into the first-order conditions, total differentiating the equations with respect to u and solving the two equations simultaneously by Cramer's rule yield

$$\frac{\partial \tilde{k}}{\partial u} = -\frac{\frac{\partial^2 \tilde{\pi}^*}{\partial k^{*2}}}{\Delta} > 0, \tag{28a}$$

$$\frac{\partial \tilde{k}^*}{\partial u} = \frac{\frac{\partial^2 \tilde{\pi}}{\partial k \partial k^*}}{\Delta} < 0. \tag{28b}$$

The signs are consistent with Condition B. (28) have rather expected signs. Since $f' > 0$ and $f^{*'} > 0$, it implies an impact of domestic fragmentation subsidy on the firms' degree of offshoring. I can also plug the Nash-Cournot-equilibrium levels of R&D investment into the Nash-Cournot-equilibrium levels of production, so that $\tilde{k}$, $\tilde{k}^*$, $\tilde{\tilde{q}}$ and $\tilde{\tilde{q}}^*$ are functions of u and other parameters. In other words, u does not enter (25), $\tilde{\tilde{q}}$ and $\tilde{\tilde{q}}^*$ are affected by $\tilde{k}$ according to (8), which in turn, depend on u according to (28).

In stage 1, the domestic government chooses fragmentation subsidy, u, to maximize welfare defined as:

$$w = \tilde{\tilde{\pi}} - u\tilde{k} = \tilde{\tilde{P}}\tilde{\tilde{q}} - [\theta - (\theta - \phi)f(\tilde{k})]\tilde{\tilde{q}} - (1-u)\tilde{k} - u\tilde{k}. \tag{29}$$

The first-order condition is

$$\frac{\partial w}{\partial u} = \frac{\partial \pi}{\partial q^*}\left(\frac{\partial \tilde{q}^*}{\partial k}\frac{\partial \tilde{k}}{\partial u} + \frac{\partial \tilde{q}^*}{\partial k^*}\frac{\partial \tilde{k}^*}{\partial u}\right) - u\frac{\partial \tilde{k}}{\partial u} = 0, \tag{30}$$

and the second-order condition, $\frac{\partial w^2}{\partial u^2} < 0$ is assumed to hold. Rearrange (29) to be

$$u^f = \frac{\tilde{\tilde{P}}'\tilde{\tilde{q}}\left(\frac{\partial \tilde{q}^*}{\partial k}\frac{\partial \tilde{k}}{\partial u} + \frac{\partial \tilde{q}^*}{\partial k^*}\frac{\partial \tilde{k}^*}{\partial u}\right)}{\frac{\partial \tilde{k}}{\partial u}} > 0. \tag{31}$$

The signs of the terms in (31) are consistent with Condition A and Condition B. It is optimal for the domestic government to grant a fragmentation subsidy. Hence, when export subsidy is banned by the WTO or certain FTA, a government finds it beneficial to use a fragmentation subsidy as an alterative. Such subsidy, on the other hand, is less likely to be disputed at the WTO.

(28) and (31) imply that the firms can have different degrees of offshoring. Assuming the firms have identical production technology (e.g. it is reasonable for Boeing and Airbus), $\theta = \theta^*$ and $f(\,) = f^*(\,)$. Since the fragmentation subsidy is positive and unilateral, the domestic firm invests more on R&D of fragmentation technology than the foreign firm. Since $\tilde{k}^f > \tilde{k}^{*f}$, $\tilde{f}^f > \tilde{f}^{f*}$.

Lemma 3: *Given Condition A and Condition B are satisfied, when the foreign government is inactive, the domestic government has incentive to impose* $u = u^f > 0$, *which encourages domestic export and offshoring, but discourages foreign export and offshoring. Hence, the domestic firm offshores more than the foreign firm.* $\tilde{f}^f > \tilde{f}^{f*}$.

2.3.3 Export Subsidy and Fragmentation Subsidy

Subsections 2.3.1 and 2.3.2 show how the domestic government can improve welfare and encourage export and offshoring when either export subsidy or fragmentation subsidy is an available policy strategy. This subsection analyses the case when both policy strategies are available and finds out whether it is optimal for a government to adopt both policies at the same time.

As in the previous subsections, in stage 3, the domestic firm considers its investment on R&D of fragmentation technology, k as a sunk cost and chooses the level of output, q, to maximize profit:

$$\pi = Pq - cq + sq - (1-u)k = P(q+q^*)q - [\theta - (\theta - \phi)f(k)]q + sq - (1-u)k, \quad (32)$$

taking the foreign firm's output, q^*, local marginal cost, θ, marginal cost in the ROW, ϕ, its government's export subsidy, s, and fragmentation subsidy, u, as given. The first-order and second-order conditions are the same as (3) and (4).

Simultaneously, the foreign firm considers k^*, its investment on R&D of fragmentation technology as a sunk cost. It chooses the level of output, q^*, to maximize profit, taking the domestic firm's output, q, local marginal cost, θ^* and marginal cost in the ROW, ϕ, as given. The profit function, the first-order condition and the second-order condition are the same as those in (5), (6) and (7).

I solve (3) and (6) simultaneously for Nash-Cournot equilibrium $\tilde{q}$ and $\tilde{q}^*$ and back-substitute the solutions into the first-order conditions. I total differentiate the equations with respect to k and with respect to k^*. Solving each set of equations simultaneously by Cramer's rule yields the same equations as (8) and (9). In other words, as before, Lemma 1 holds that a firm's R&D investment on fragmentation technology encourages own export, but discourages rival firm's export.

Similarly, I derive the comparative statics with respect to domestic export subsidy, s as in (10). Hence, it still holds here that the domestic export subsidy is favorable to domestic export but unfavorable to foreign export. In stage 2, the domestic firm has perfect information of how its R&D investment on fragmentation technology, k, affects exports in stage 3 and chooses k to maximize profit:

$$\tilde{\pi} = \tilde{P}\tilde{q} - [\theta - (\theta - \phi)f(k)]\tilde{q} + s\tilde{q} - (1-u)k, \tag{33}$$

taking the foreign firm's R&D investment on fragmentation technology, k^*, local marginal cost, θ, marginal cost in the ROW, ϕ, export subsidy, s, and fragmentation subsidy, u, as given. The first-order and the second-order conditions are the same as (27) and (13').

The foreign firm simultaneously chooses the R&D investment on fragmentation technology, k^*, to maximize profit, taking the domestic firm's R&D investment on fragmentation technology, k, local marginal cost, θ^*, marginal cost in the ROW, ϕ, and export subsidy, s, as given. The profit function, the first-order condition and the second-order condition are the same as those in (16), (17) and (18).

Simultaneously solving (27) and (17) for Nash-Cournot equilibrium $\tilde{k}$ and $\tilde{k}^*$, substituting the solutions back into the first-order conditions, total differentiating the equations with respect to s, and solving the two equations simultaneously by Cramer's rule yield same comparative statics as (20). It is still true here that the export subsidy raises domestic offshoring, but reduces foreign offshoring. The comparative statics with respect to u are as those in (28). Hence, the fragmentation subsidy increases domestic offshoring, but decreases foreign offshoring.

By plugging the Nash-Cournot-equilibrium levels of R&D investment into the Nash-Cournot equilibrium levels of production, $\tilde{k}$, $\tilde{k}^*$, $\tilde{\tilde{q}}$ and $\tilde{\tilde{q}}^*$ are functions of s, u and other parameters.

In stage 1, the domestic government chooses export subsidy, s and fragmentation subsidy, u, to maximize welfare defined as:

$$w = \tilde{\pi} - s\tilde{\tilde{q}} - u\tilde{k} = \tilde{\tilde{P}}\tilde{\tilde{q}} - \left[\theta - (\theta - \phi)f\left(\tilde{k}\right)\right]\tilde{\tilde{q}} + s\tilde{\tilde{q}} - (1-u)\tilde{k} - s\tilde{\tilde{q}} - u\tilde{k}. \tag{34}$$

The first-order conditions are

$$\frac{\partial w}{\partial s} = \frac{\partial \pi}{\partial q^*}\left(\frac{\partial \tilde{q}^*}{\partial k}\frac{\partial \tilde{k}}{\partial s} + \frac{\partial \tilde{q}^*}{\partial k^*}\frac{\partial \tilde{k}^*}{\partial s} + \frac{\partial \tilde{q}^*}{\partial s}\right) - s\left(\frac{\partial \tilde{q}}{\partial k}\frac{\partial \tilde{k}}{\partial s} + \frac{\partial \tilde{q}^*}{\partial k^*}\frac{\partial \tilde{k}^*}{\partial s} + \frac{\partial \tilde{q}}{\partial s}\right) - u\frac{\partial \tilde{k}}{\partial s} = 0, \tag{35}$$

$$\frac{\partial w}{\partial u} = \frac{\partial \pi}{\partial q^*}\left(\frac{\partial \tilde{q}^*}{\partial k}\frac{\partial \tilde{k}}{\partial u} + \frac{\partial \tilde{q}^*}{\partial k^*}\frac{\partial \tilde{k}^*}{\partial u}\right) - s\left(\frac{\partial \tilde{q}}{\partial k}\frac{\partial \tilde{k}}{\partial u} + \frac{\partial \tilde{q}^*}{\partial k^*}\frac{\partial \tilde{k}^*}{\partial u}\right) - u\frac{\partial \tilde{k}}{\partial u} = 0. \tag{36}$$

The second-order conditions, $\frac{\partial w^2}{\partial s^2} < 0$, $\frac{\partial w^2}{\partial u^2} < 0$ and $\frac{\partial w^2}{\partial s^2}\frac{\partial w^2}{\partial u^2} - \frac{\partial w^2}{\partial s \partial u}\frac{\partial w^2}{\partial u \partial s} > 0$ are assumed to hold. Through substitution, (35) and (36) become:

$$s^{ef} = \frac{\tilde{\tilde{P}}'\tilde{\tilde{q}}[\frac{2}{3}\frac{\Delta}{\frac{\partial^2 \tilde{\pi}^*}{\partial {k^*}^2}}\left(\frac{\partial \tilde{q}^*}{\partial k^*}\right)^2 + \frac{\partial \tilde{q}^*}{\partial s}]}{\frac{2}{3}\frac{\Delta}{\frac{\partial^2 \tilde{\pi}^*}{\partial {k^*}^2}}\frac{\partial \tilde{q}^*}{\partial k^*}\frac{\partial \tilde{q}}{\partial k^*} + \frac{\partial \tilde{q}}{\partial s}} > 0, \tag{37}$$

$$u^{ef} = -\frac{4}{3}(\theta^* - \phi){f^*}'q\frac{\frac{(\theta^* - \phi){f^*}'}{(\theta - \phi)f'}\frac{\partial^2 \tilde{\pi}}{\partial k^2}\frac{\partial^2 \tilde{\pi}}{\partial k \partial k^*} + \frac{(\theta - \phi)f'}{(\theta^* - \phi){f^*}'}\frac{\partial^2 \tilde{\pi}^*}{\partial {k^*}^2}\frac{\partial^2 \tilde{\pi}}{\partial k \partial k^*} + 2\left(\frac{\partial^2 \tilde{\pi}}{\partial k \partial k^*}\right)^2 - \frac{1}{2}\frac{\partial^2 \tilde{\pi}}{\partial k^2}\frac{\partial^2 \tilde{\pi}^*}{\partial {k^*}^2}}{-\frac{(\theta^* - \phi){f^*}'}{(\theta - \phi)f'}\left(\frac{\partial^2 \tilde{\pi}}{\partial k \partial k^*}\right)^2 + 2\frac{(\theta - \phi)f'}{(\theta^* - \phi){f^*}'}\frac{\partial^2 \tilde{\pi}^*}{\partial {k^*}^2}\frac{\partial^2 \tilde{\pi}}{\partial k \partial k^*} + \frac{\partial^2 \tilde{\pi}}{\partial k^2}\frac{\partial^2 \tilde{\pi}^*}{\partial {k^*}^2} - 2\frac{\partial^2 \tilde{\pi}^*}{\partial {k^*}^2}\frac{\partial^2 \tilde{\pi}}{\partial k \partial k^*}} < 0, \tag{38}$$

in which the signs are consistent with Condition A and Condition B. According to (37) and (38), if both policies can be adopted, the domestic firm will impose a positive export subsidy and a negative fragmentation subsidy (positive fragmentation tax). Comparing the results here and those in the previous subsection shows that when the government does not have to rely on the stimulation of a fragmentation subsidy alone, it chooses to subsidize exports and tax fragmentation instead. This tax is indeed used to induce cost minimization. Different from (19), when the domestic government can intervene fragmentation through u,

$$\frac{\partial}{\partial k}[c\tilde{q} + (1-u)k] = (\theta - \phi)f'\tilde{q} - (1-u) = \frac{\partial \tilde{\pi}}{\partial k} + \frac{1}{3}(\theta - \phi)f'\tilde{q} + u = 0. \qquad (39)$$

(27) and (39) imply that the profit-maximizing k is greater than the cost-minimizing k. Hence a negative u can mitigate the situation that the firm invests on R&D of fragmentation technology more than necessary to minimize cost.

Lemma 4: *Given Condition A and Condition B are satisfied, when the foreign government does not intervene the industry, it is optimal for the domestic government to subsidize export,* $s = s^{ef} > 0$ *and tax fragmentation,* $u = u^{ef} < 0$.

Because the export subsidy encourages offshoring, while the fragmentation tax discourages offshoring, in general, the overall effect may not be positive. The domestic firm may not offshore more than the foreign firm when both policies are available to the domestic government. A numerical method is necessary to determine the exact level of offshoring of each firm. The next section provides a numerical example. The numerical example is also capable of providing a comparison of the levels of export, offshoring and welfare when two policy options versus one policy option are available. The comparison reflects the advantages when a government has choices of different kinds of subsidies.

2.3.4 A Numerical Example

In order to derive explicit solutions for export, offshoring and welfare, I have to make assumptions about the demand function and the fragmentation function. Without losing generality, one of the simplest functional forms are linear demand in the form of $P = a - b(q + q^*)$ where a and b are positive constants, and asymptotic fragmentation function such as $f(k) = 1 - \frac{1}{1+k}$ and $f^*(k^*) = 1 - \frac{1}{1+k^*}$. Even with these simple functional forms, solving for the Nash-Cournot-equilibrium levels of k and k^* involve solving two cubic equations. In the process, I reject solutions that do not satisfy the model assumptions (e.g. negative k and k^*).[30] A handy numerical example is to set $a = 100$, $b = 1$, $\theta = \theta^* = 10$ and $\phi = 1$. With such numerical example, I can confirm the model results in the previous subsections as well as present some meaningful comparisons. For easy comparison, define:

Case 1: Let $\tilde{\tilde{q}}^e$, $\tilde{f}^e$ and w^e be the respective levels of export, offshoring and welfare of the domestic country when the domestic government imposes the optimal export subsidy. Also let $\tilde{f}^{*e}$ to be the level of offshoring in the foreign country in this case.

Case 2: Let $\tilde{\tilde{q}}^f$, $\tilde{f}^f$ and w^f be the respective levels of export, offshoring and welfare of the domestic country when the domestic government imposes the optimal fragmentation tax-cum-subsidy. Also let $\tilde{f}^{*f}$ to be the level of offshoring in the foreign country in this case.

Case 3: Let $\tilde{\tilde{q}}^{ef}$, $\tilde{f}^{ef}$ and w^{ef} be the respective levels of export, offshoring and welfare of the domestic country when the domestic government imposes the optimal export subsidy

[30] Assuming more complex functional forms (such as quadratic demand functions and logarithmic fragmentation functions) would require solving for polynomials of higher degrees, but the central insights derived here would remain.

and the optimal fragmentation tax-cum-subsidy. Also let $\tilde{f}^{*ef}$ to be the level of offshoring in the foreign country in this case.

As discussed in Lemma 2 and Lemma 3, the domestic firm offshores more than the foreign firm when the domestic government can impose either export subsidy or fragmentation tax-cum-subsidy while the foreign government is inactive. The present numerical example shows the same results. In the case when the domestic government both subsidizes export and taxes fragmentation, the present model shows that the favorable effect of the export subsidy more than offsets the unfavorable effect of the fragmentation tax, so the domestic firm offshores more compared to the foreign firm.

Lemma 5: *In the present model, when only the domestic government is active, the domestic firm offshores more than the foreign firm when one of export subsidy and fragmentation tax-cum-subsidy is a policy option or when both policies are available. That is,* $\tilde{f}^{e} > \tilde{f}^{*e}$, $\tilde{f}^{f} > \tilde{f}^{*f}$ and $\tilde{f}^{ef} > \tilde{f}^{*ef}$.

As shown in Subsection 2.3.1, the export subsidy stimulates export directly in stage 3. In Subsection 2.3.2, the fragmentation subsidy has positive impact on export only through its impact on fragmentation in stage 2. In this present model, the firm both exports and offshores the most when the domestic firm only focuses on export subsidy. This is followed by the case when the domestic government also taxes fragmentation at the same time, offsetting some of the positive effect of the export subsidy. When both policies are available, the domestic government will tax fragmentation to induce the firm to minimize cost. The government subsidizes fragmentation only when export subsidy is not an option. In the present model, fragmentation subsidy alone has the least effect on export and offshoring among the three cases.

Lemma 6: *In the present model, the domestic firm exports the most when there is only an export subsidy. It exports and offshores more when the government both subsidizes export and taxes fragmentation than when the government only subsidizes fragmentation. That is,* $\tilde{\tilde{q}}^{e} > \tilde{\tilde{q}}^{ef} > \tilde{\tilde{q}}^{f}$ and $\tilde{f}^{e} > \tilde{f}^{ef} > \tilde{f}^{f}$.

Even though the export subsidy alone appears to have the greatest positive impact on export and offshoring, when given the options to intervene both export and fragmentation, the domestic government will choose to tax fragmentation at the same time as it subsidizes export. In this present numerical example, the case when the domestic government has the flexibility to use both policy tools generates the highest welfare, followed by export subsidy alone and fragmentation subsidy alone.

Lemma 7: *In the present model, the welfare of the domestic country is the highest when the government is able to intervene both export and fragmentation. Welfare is higher the domestic government can subsidize export only than when it can subsidizes fragmentation only.* $w^{ef} > w^{e} > w^{f}$.

Section 2.3 already presented the main results of the study. For the sake of completeness, the next section briefly explains how the actions of the foreign government come into the picture. However, the central insights of the main model remain.

2.4. Two Active Governments

Noticing that the other party of an FTA uses policy strategies against itself, a government may adopt certain trade and industrial policy as a retaliation. For example, if the WTO Dispute Settlement Body (DSB) rules that one government has imposed illegal subsidies, it may allow the trading partner to retaliate. Hence, this section considers the actions of both governments. When both the domestic and the foreign governments can intervene their export and fragmentation, the three-stage game is as follows.

i. In stage 1, each government chooses export subsidy and/or fragmentation tax-cum-subsidy to maximize welfare simultaneously.

ii. In stage 2, each firm chooses the level of R&D investment on fragmentation technology to maximize profit simultaneously.

iii. In stage 3, each firm chooses the level of output to maximize profit simultaneously.

As in the previous subsections, in stage 3, the domestic firm considers its investment on R&D of fragmentation technology, k as a sunk cost and chooses the level of output, q, to maximize profit, taking the foreign firm's output, q^*, local marginal cost, θ, marginal cost in the ROW, ϕ, its government's export subsidy, s, and fragmentation subsidy, u, as given. The profit function, the first-order condition and second-order condition are the same as (32), (3) and (4).

Simultaneously, the foreign firm considers k^*, its investment on R&D of fragmentation technology as a sunk cost. It chooses the level of output, q^*, and, to maximize profit:

$$\pi^* = P(q + q^*)q^* - [\theta^* - (\theta^* - \phi)f^*(k^*)]q^* + s^*q^* - (1 - u^*)k^*, \tag{40}$$

taking the domestic firm's output, q, local marginal cost, θ^*, marginal cost in the ROW, ϕ, its government's export subsidy, s^*, and fragmentation subsidy, u^*, as given. The first-order condition is:

$$\frac{\partial \pi^*}{\partial q^*} = P'q^* + P - \theta^* + (\theta^* - \phi)f^*(k^*) + s^* = 0. \tag{41}$$

The second-order condition is the same as (7).

I solve (3) and (41) simultaneously for Nash-Cournot equilibrium $\tilde{q}$ and $\tilde{q}^*$. I back-substitute the solutions into the first-order conditions and total differentiate the equations with respect to k and with respect to k^*. Solving each set of equations simultaneously by Cramer's rule yields the same equations as (8) and (9). Thus, Lemma 1 holds for both firms that a firm's R&D investment on fragmentation technology encourages its own export, but discourages the rival firm's export.

Similarly, I derive the comparative statics with respect to domestic export subsidy, s as in (10). The comparative statics with respect to foreign export subsidy, s^* are

$$\frac{\partial \tilde{q}}{\partial s^*} = \frac{\tilde{P}''\tilde{q} + \tilde{P}'}{D} < 0. \tag{42a}$$

$$\frac{\partial \tilde{q}^*}{\partial s^*} = -\frac{\tilde{P}''\tilde{q} + 2\tilde{P}'}{D} > 0. \tag{42b}$$

Hence, an export subsidy is favorable to the country's own export but unfavorable to the other country's export.

In stage 2, both firms have perfect information of how their R&D investment on fragmentation technology affects exports in stage 3. The domestic firm chooses R&D investment on fragmentation technology, k to maximize profit, taking the foreign firm's R&D investment on fragmentation technology, k^*, local marginal cost, θ, marginal cost in the ROW, ϕ, domestic export subsidy, s, domestic fragmentation subsidy, u, and foreign export subsidy, s^* as given. The profit function, the first-order condition and the second-order condition are the same as (33), (27) and (13').

The foreign firm simultaneously chooses the R&D investment on fragmentation technology, k^*, to maximize profit,

$$\tilde{\pi}^* = \tilde{P}\tilde{q}^* - [\theta^* - (\theta^* - \phi)f^*(k^*)]\tilde{q}^* + s^*\tilde{q}^* - (1 - u^*)k^*, \tag{43}$$

taking the domestic firm's R&D investment on fragmentation technology, k, local marginal cost, θ^*, marginal cost in the ROW, ϕ, foreign export subsidy, s^*, foreign fragmentation subsidy, u^*, and domestic export subsidy, s, as given. The first-order condition is

$$\frac{\partial\tilde{\pi}^*}{\partial k^*} = \frac{4}{3}(\theta^* - \phi)f^{*\prime}\tilde{q}^* - (1 - u^*) = 0. \tag{44}$$

The second-order condition is the same as (18).

I simultaneously solve (27) and (43) for Nash-Cournot equilibrium $\tilde{k}$ and $\tilde{k}^*$, substitute the solutions back into the first-order conditions and total differentiate the equations with respect to s. Solving the two equations simultaneously by Cramer's rule yields same comparative statics as (20). With respect to s^*:

$$\frac{\partial\tilde{k}}{\partial s^*} = -\frac{\frac{8}{9\tilde{P}'}(\theta^* - \phi)f^{*\prime}\frac{\partial^2\tilde{\pi}}{\partial k\partial k^*} + \frac{4}{9\tilde{P}'}(\theta - \phi)f'\frac{\partial^2\tilde{\pi}^*}{\partial k^{*2}}}{\Delta} < 0. \tag{45a}$$

$$\frac{\partial\tilde{k}^*}{\partial s^*} = \frac{\frac{8}{9\tilde{P}'}(\theta^* - \phi)f^{*\prime}\frac{\partial^2\tilde{\pi}}{\partial k^2} + \frac{4}{9\tilde{P}'}(\theta - \phi)f'\frac{\partial^2\tilde{\pi}}{\partial k\partial k^*}}{\Delta} > 0. \tag{45b}$$

Lemma 3 holds true for both firms that an export subsidy raises a firm's offshoring, but reduces its rival firm's offshoring. The comparative statics with respect to u are as those in (29). The comparative statics with respect to u^* are

$$\frac{\partial \tilde{k}}{\partial u^*} = \frac{\frac{\partial^2 \tilde{\pi}^*}{\partial k^* \partial k}}{\Delta} < 0, \tag{46a}$$

$$\frac{\partial \tilde{k}^*}{\partial u^*} = -\frac{\frac{\partial^2 \tilde{\pi}}{\partial k^2}}{\Delta} > 0. \tag{46b}$$

The fragmentation subsidy increases a firm's offshoring, but decreases the rival firm's offshoring.

By plugging the Nash-Cournot equilibrium levels of R&D investment into the Nash-Cournot-equilibrium levels of production, $\tilde{k}$, $\tilde{k}^*$, $\tilde{\tilde{q}}$ and $\tilde{\tilde{q}}^*$ are functions of s, u, s^*, u^* and other parameters.

In stage 1, the domestic government chooses export subsidy, s and fragmentation subsidy, u, to maximize welfare defined in (34), taking foreign export subsidy, s^*, and foreign fragmentation subsidy, u^*, as given. The first-order and second-order conditions are the same as those in (35) and (36). The second-order conditions, $\frac{\partial w^2}{\partial s^2} < 0$, $\frac{\partial w^2}{\partial u^2} < 0$ and $\frac{\partial w^2}{\partial s^2}\frac{\partial w^2}{\partial u^2} - \frac{\partial w^2}{\partial s \partial u}\frac{\partial w^2}{\partial u \partial s} > 0$ are assumed to hold. By substitution, (35) and (36) become (37) and (38). Notice that $s > 0$ and $u < 0$ for all s^* and u^*.

The foreign government chooses export subsidy, s^* and fragmentation subsidy, u^*, to maximize welfare:

$$w^* = \tilde{\tilde{\pi}}^* - s^*\tilde{\tilde{q}}^* - u^*\tilde{k}^*$$

$$= \tilde{\tilde{P}}\tilde{\tilde{q}}^* - \left[\theta^* - (\theta^* - \phi)f^*\left(\tilde{k}^*\right)\right]\tilde{\tilde{q}}^* + s^*\tilde{\tilde{q}}^* - (1-u^*)\tilde{k}^* - s^*\tilde{\tilde{q}}^* - u^*\tilde{k}^*, \qquad (47)$$

taking domestic export subsidy, s and domestic fragmentation, u, as given. The first-order conditions are

$$\frac{\partial w^*}{\partial s^*} = \frac{\partial \pi^*}{\partial q}\left(\frac{\partial \tilde{q}}{\partial k}\frac{\partial \tilde{k}}{\partial s^*} + \frac{\partial \tilde{q}}{\partial k^*}\frac{\partial \tilde{k}^*}{\partial s^*} + \frac{\partial \tilde{q}}{\partial s^*}\right) - s^*\left(\frac{\partial \tilde{q}^*}{\partial k}\frac{\partial \tilde{k}}{\partial s^*} + \frac{\partial \tilde{q}^*}{\partial k^*}\frac{\partial \tilde{k}^*}{\partial s^*} + \frac{\partial \tilde{q}^*}{\partial s^*}\right) - u^*\frac{\partial \tilde{k}^*}{\partial s^*} = 0, \quad (48)$$

$$\frac{\partial w^*}{\partial u^*} = \frac{\partial \pi^*}{\partial q}\left(\frac{\partial \tilde{q}}{\partial k}\frac{\partial \tilde{k}}{\partial u^*} + \frac{\partial \tilde{q}}{\partial k^*}\frac{\partial \tilde{k}^*}{\partial u^*}\right) - s^*\left(\frac{\partial \tilde{q}^*}{\partial k}\frac{\partial \tilde{k}}{\partial u^*} + \frac{\partial \tilde{q}^*}{\partial k^*}\frac{\partial \tilde{k}^*}{\partial u^*}\right) - u^*\frac{\partial \tilde{k}^*}{\partial u^*} = 0. \qquad (49)$$

The second-order conditions, $\frac{\partial w^{*2}}{\partial s^{*2}} < 0$, $\frac{\partial w^{*2}}{\partial u^{*2}} < 0$ and $\frac{\partial w^{*2}}{\partial s^{*2}}\frac{\partial w^{*2}}{\partial u^{*2}} - \frac{\partial w^{*2}}{\partial s^*\partial u^*}\frac{\partial w^{*2}}{\partial s^*\partial u^*} > 0$ are assumed to hold. Solving (48) and (49) simultaneously yields

$$s^* = \frac{\tilde{\tilde{P}}'\tilde{\tilde{q}}^*[\frac{2}{3}\frac{\Delta}{\frac{\partial^2\tilde{\pi}}{\partial k^2}}\left(\frac{\partial \tilde{q}}{\partial k}\right)^2 + \frac{\partial \tilde{q}}{\partial s^*}]}{\frac{2}{3}\frac{\Delta}{\frac{\partial^2\tilde{\pi}}{\partial k^2}}\frac{\partial \tilde{q}}{\partial k}\frac{\partial \tilde{q}^*}{\partial k} + \frac{\partial \tilde{q}^*}{\partial s^*}} > 0, \qquad (50)$$

$$u^* = -\frac{4}{3}(\theta-\phi)f'q^*\frac{\frac{(\theta-\phi)f'}{(\theta^*-\phi)f^{*'}}\frac{\partial^2\tilde{\pi}^*}{\partial k^{*2}}\frac{\partial^2\tilde{\pi}}{\partial k\partial k^*} + \frac{(\theta^*-\phi)f^{*'}}{(\theta-\phi)f'}\frac{\partial^2\tilde{\pi}}{\partial k^2}\frac{\partial^2\tilde{\pi}}{\partial k\partial k^*} + 2\left(\frac{\partial^2\tilde{\pi}}{\partial k\partial k^*}\right)^2 - \frac{1}{2}\frac{\partial^2\tilde{\pi}}{\partial k^2}\frac{\partial^2\tilde{\pi}^*}{\partial k^{*2}}}{-\frac{(\theta-\phi)f'}{(\theta^*-\phi)f^{*'}}\left(\frac{\partial^2\tilde{\pi}}{\partial k\partial k^*}\right)^2 + 2\frac{(\theta^*-\phi)f^{*'}}{(\theta-\phi)f'}\frac{\partial^2\tilde{\pi}}{\partial k^2}\frac{\partial^2\tilde{\pi}}{\partial k\partial k^*} + \frac{\partial^2\tilde{\pi}}{\partial k^2}\frac{\partial^2\tilde{\pi}^*}{\partial k^{*2}} - 2\frac{\partial^2\tilde{\pi}}{\partial k^2}\frac{\partial^2\tilde{\pi}}{\partial k\partial k^*}}$$

$$< 0, \qquad (51)$$

Hence, $s^* > 0$ and $u^* < 0$ for all s and u.

Lemma 8: *When both governments can intervene both export and offshoring, both subsidize export and tax fragmentation. That is,* $s > 0$, $u < 0$, $s^* > 0$ *and* $u^* < 0$.

To derive the Nash-Cournot equilibrium levels of $\tilde{s}$, $\tilde{u}$, $\tilde{s}^*$ and $\tilde{u}^*$, I have to solve the four equations, (37), (38), (50) and (51), simultaneously for the four unknowns. Since it is not a linear system of equations, there can be multiple equilibria. However, for the interest of this chapter, Lemma 8 has already provided the important insights that each government subsidize export and tax fragmentation in the equilibrium. Hence, if the WTO DSB allows the foreign government to retaliate, it will.

2.5. Concluding Remarks

Due to the observations in the LCA industry, this essay raises a question: How can two similar firms have significantly different degrees of offshoring in the production of their competing products? This essay builds a function of fragmentation and incorporates it into a standard strategic trade model. In the model, firms can choose their levels of R&D investment of fragmentation technology before engaging in Cournot competition. Once a firm has chosen its optimal R&D investment of fragmentation technology, the corresponding fragmentation and offshoring are also determined. With such a model, I can analyze the policy strategies of each government. Same as previous studies, it is optimal for a government to subsidize export if it is the only available policy option. This stimulates both exports and offshoring. In the present model, even if export subsidy is prohibited, a government can subsidize fragmentation to achieve the same purpose. The model also shows when both policies are available, the government will rely on export subsidy to stimulate export and offshoring, but will tax fragmentation to induce the firm to minimize cost. By doing so, the government achieves higher welfare than when only one policy is available. Finally, to complete the analysis, the model introduces intervention by the foreign government. It shows that when allowed to do so, the foreign government will retaliate by imposing both export subsidy and fragmentation tax.

This study provides a possible reason why Boeing offshores a much higher portion of its 787 subparts than Airbus does to its A350. The WTO ruling that the US government has subsidized some R&D programs associated with Boeing supports this hypothesis. It will be interesting to find out if the same is true in other international oligopolies such as the automobile industry. The challenge will be to find data of fragmentation and offshoring such as those presented in Fig. 2.1 and Fig. 2.2.

There are some possible extensions to the present model. For example, by allowing the rival firm's R&D investment of fragmentation technology to enter a firm's fragmentation function makes it possible to analyze the spillover of fragmentation technology. Such model is especially applicable to industries where firms outsource to the same subpart suppliers. In such case, it is likely that the subpart suppliers quickly learn the fragmentation technology developed by a firm and provides such skills to another firm.

The fragmentation function can also easily adapt to different cost functions. For example, in a model of firm heterogeneity, firms may not only differ in productivity, but also in their endowment of fragmentation technology. An additional stage can be introduced, so that each firm can choose how much to invest on its R&D of fragmentation technology. As a result, more productive firms may not always be more likely to offshore because they may not be endowed with higher fragmentation technology in the first place.

There can also be further examination on the timing that a government chooses optimal policies. As in the present framework, fragmentation tax-cum-subsidy is granted before the firm invests on R&D of fragmentation technology, but the timing of export subsidies is adjustable. Export subsidy may be granted before, after or at the same time as fragmentation tax-cum-subsidy. As discussed in Brander and Spencer (1983), there is a fairly large taxonomy of different cases.

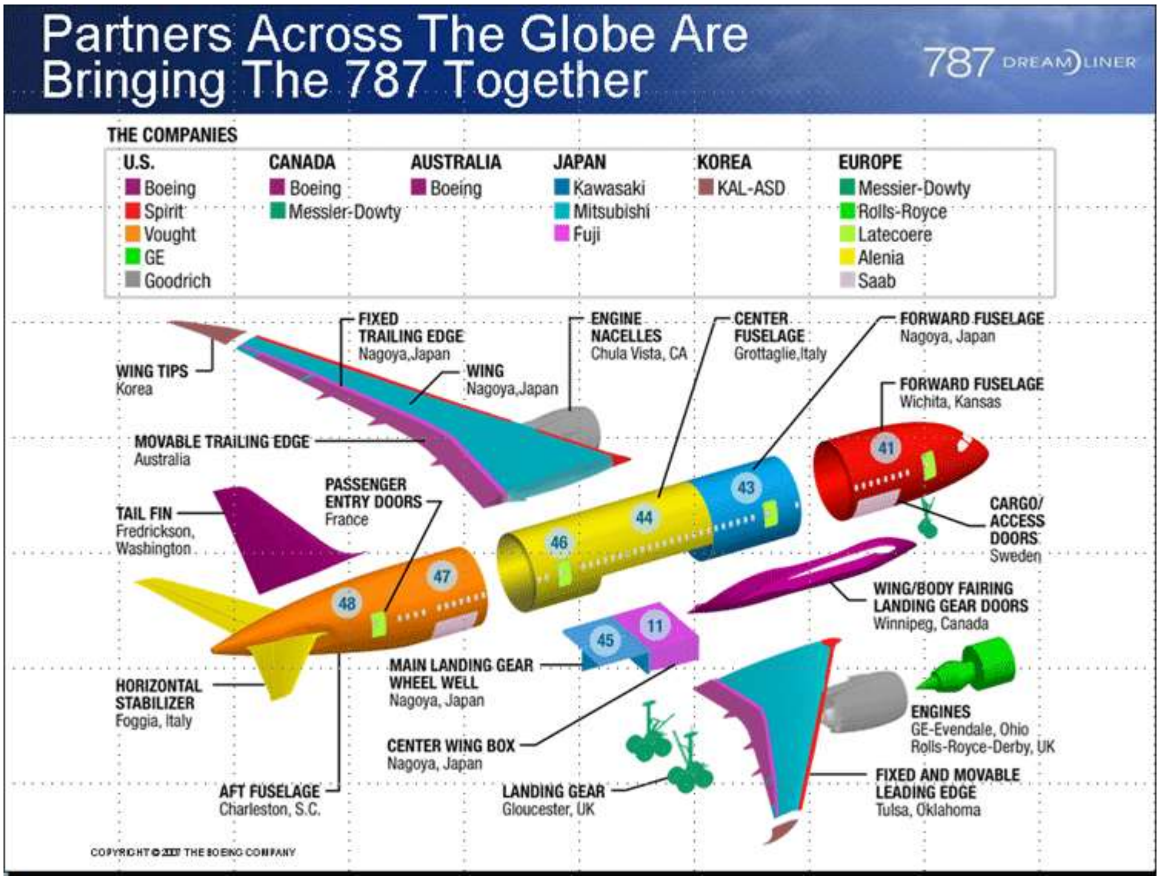


Fig. 2.1. Fragmentation of Boeing 787. Source: Boeing.

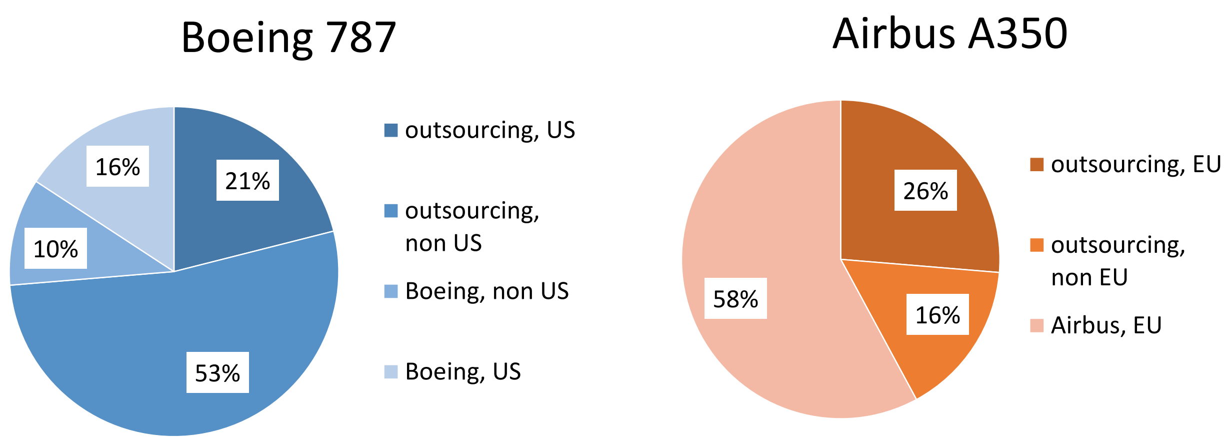


Fig. 2.2. Production Organization of Boeing 787 and Airbus A350. Sources: Boeing, Airbus and their suppliers, author's calculations.
Note: The value of the subparts is not available, so a proxy is calculated by dividing the number of subparts of interest by the total number of subparts.

Chapter 3

STRATEGIC EXPORT OF GAMBLING AND CONVENTION SERVICES: THE CASES OF MACAO, SINGAPORE AND OSAKA

3.1. Introduction

Having an area of merely 11.8 square miles, Macao has very scarce natural resources. Inevitably, Macao relies on exports of services as the main income source – namely tourism. Yet, Macao's economy has attracted little international attention until 2008 when it finally replaced Las Vegas to receive the highest gambling revenue in the world (McCartney, 2008). With a population of merely 648,500, Macao has a very specialized labor force that produces gambling service (DSEC). In fact, the gambling industry has a long history in Macao. The city has served as the only legal casino hub in the region, serving mainly Hong Kong gamblers since the 20th century. While Macao's gambling industry flourishes, the Macao government addresses a policy goal to diversify its economy, particularly towards developing the convention industry (Macao government, 2016). Given the fact that Macao already has a competitive edge in producing gambling, it is not obvious why the Macao government pursuits such a new challenge. When accessing the motive behind Macao government's objective, many cite social factors. For instance, there are concerns of gambling addictions of the local people, related crimes such as money laundering and conflicts with religious values (Lewis, 2016). There are political factors as well. During China's high official, Zhang Dejiang's visit to Macao, he emphasized the need of the city to diversify its economy (Wu & Master, 2017). However, as discussed in McCartney (2008), solid economic explanations are lacking.

On the other hand, Singapore seems to be in an opposite situation. Though larger than Macao, Singapore is a small city with an area of 278 square miles and a population of 5,612,300. Singapore also relies on service export. Its labor force is one of the most educated and competitive in the world (Education First). Singapore has long established itself as one the world's biggest business centers. Singapore is one of the main destinations for companies to hold meetings, incentives, conventions and exhibitions (MICE) (Your Singapore, 2016). It is among the world's top 10 and is ranked Asia's number 1 convention city (ICCA, 2017; Singapore Tourism Board, 2013). According to Singapore Tourism Board (2014), business travelers spent 1.7 time more than leisure visitors in 2013. As Singapore's convention businesses thrive, however, Singapore chooses to diversify among convention and gambling - two casinos were opened in Singapore in 2010 (Jia, 2015). The Singaporean government's move seems to be counter-intuitive. Many have complained about how the business-friendly image of Singapore might be affected (Pierson, 2011). To tackle gambling addiction, the Singaporean government even imposes an entry fee of 100 Singaporean dollars on citizens and permanent residents (Barnard, 2017).

In search for an explanation to the two cities' motivation to diversify among gambling and convention, I observe that an important economic event has influenced both cities in the past decade – the surge of tourists from Mainland China. Due to the rise of the Chinese economy and lower visa requirements on Chinese visitors, Chinese tourists have become the major source of tourism revenue in the recent decade. As two of the main tourist destinations, both Macao and Singapore have experienced an increase of visitors (see Fig. 3.1). This chapter shall show that the growing markets of gambling and convention lead to diversification in both cities. In Section 3.2, the model analyzes the case when a single city provides gambling and convention in a regional market and investigates what determines the degree of diversification among the two tourism services. In Section 3.3, I consider competition between Macao and Singapore and find out if rivalry has an effect on the degree of diversification chosen by each city.

On the other hand, Japan legalized casino gaming in 2016 (Shaffer, 2016). The Osaka Prefecture proposed to open Japan's first legal gambling complex in 2002 (Johnson, 2017). Seeing the potential of Osaka's entry, I present a model with Osaka as an additional competitor in Section 3.4. Depending on Osaka's relative costs of convention to gambling, its entry may have different impacts on Macao's and Singapore's degrees of diversification.

3.2. The Case of a Single City

Consider a city that produces two tourism services - gambling (G) and convention (V). Tourists consider gambling and convention to be related. Hence, the (inverse) demand for G and the (inverse) demand for V are $P_G = P_G(G, V)$ and $P_V = P_V(V, G)$ respectively. For simplicity, assume the demand functions to be linear and symmetric such that $P_G = a - bG - \theta V$ and $P_V = a - bV - \theta G$, where a and b are positive constants. These standard demand functions are normal in the sense that a is assumed to be sufficiently positive to justify positive consumption in the markets, and $\frac{\partial P_G}{\partial G} = -b < 0$, $\frac{\partial P_V}{\partial V} = -b < 0$. Tourists may consider gambling and convention to be substitutes or complements. $\frac{\partial P_G}{\partial V} = -\theta$ and $\frac{\partial P_V}{\partial G} = -\theta$ have unspecified signs because θ can be positive or negative. If gambling and convention are substitutes (complements), $\theta > 0$ ($\theta < 0$).[31] It is standard to assume $b > \theta$. If the two tourism services are complements, $b > \theta$. If the two tourism services are substitutes, $b > \theta$ assumes that $|\frac{\partial P_G}{\partial G}| > |\frac{\partial P_G}{\partial V}|$ and $|\frac{\partial P_V}{\partial V}| > |\frac{\partial P_V}{\partial G}|$. In other words, own-price effect is greater than cross-price effect on the quantity demanded of each tourism service.

Assume it costs $C_G = c_0 + c_1 G + c_2 G^2$ to produce gambling, and $C_V = \phi c_0 + \phi c_1 V + \phi c_2 V^2$ to produce convention, where c_0, c_1, c_2 and ϕ are positive constants. This is the simplest cost structure that satisfies the purpose of this study – C_G and C_V exhibit increasing marginal costs and the parameter ϕ tells which tourism service is more costly to produce.[32] More

[31] This definition of product relations is called the "q-definition." As explained in Chapter 2, if G and V are "q-substitutes" ("q-complements"), they are also "p-substitutes" ("p-complements") according to the conventional "p-definition."
[32] It is common to assume increasing marginal costs in simple models like this one. The second-order condition for stable solutions can still be satisfied as long as c_2 is not be too negative. Even if c_1 were negative, the main results would not be affected as long as a was sufficiently large – the demands were large enough.

precisely, suppose $G = V$ so that $C_G = \phi C_V$. In this case if $\phi > 1$ ($\phi < 1$), then convention is more (less) costly than gambling; and if $\phi = 1$, then the two tourism services are equally costly.

Suppose a city acts as a regional monopoly. Its objective is to choose the quantities of G and V to be produced in order to maximize profit. When the markets are in equilibrium, the profit function is defined as follows:

$$\max_{\{G,V\}} \pi = P_G G + P_V V - C_G - C_V$$

$$= (a - bG - \theta V)G + (a - bV - \theta G)V - (c_0 + c_1 G + c_2 G^2)$$

$$-(\phi c_0 + \phi c_1 V + \phi c_2 V^2). \quad (1)$$

The first-order conditions are

$$\frac{\partial \pi}{\partial G} = a - c_1 - 2(b + c_2)G - 2\theta V = 0, \quad (2a)$$

$$\frac{\partial \pi}{\partial V} = a - \phi c_1 - 2(b + \phi c_2)V - 2\theta G = 0. \quad (2b)$$

The second-order conditions are $\frac{\partial^2 \pi}{\partial G^2} = -2(b + c_2) < 0$, $\frac{\partial^2 \pi}{\partial V^2} = -2(b + \phi c_2) < 0$ and the determinant of the Hessian matrix $H = 4[(b + c_2)(b + \phi c_2) - \theta^2] > 0$, which are satisfied in this model. Solving the first-order conditions simultaneously yields the profit-maximizing production levels,

$$\tilde{G} = \frac{2}{H}[(b - \theta + \phi c_2)a - (b - \phi\theta + \phi c_2)c_1], \quad (3a)$$

$$\tilde{V} = \frac{2}{H}[(b - \theta + c_2)a - (\phi b - \theta + \phi c_2)c_1], \quad (3b)$$

using "~" to denote optimal levels of G and V. (3) show that ϕ is an important determinant of the production levels a city chooses. (3) indicate that

$$\tilde{G} \overset{>}{\underset{<}{=}} \tilde{V} \quad if \quad \phi \overset{>}{\underset{<}{=}} 1. \tag{4}$$

(4) shows that it is optimal for the city to produce more (less) gambling than convention if $\phi > 1$ ($\phi < 1$). Fig. 3.2 plots $\tilde{G}$ and $\tilde{V}$ against ϕ, holding other parameters constant. The $\tilde{G}$ line and the $\tilde{V}$ line intersect at $\phi = 1$. For $\phi < 1$, the $\tilde{G}$ line is below the $\tilde{V}$ line. For $\phi > 1$, the $\tilde{G}$ line is above the $\tilde{V}$ line.

For the comparison of the levels of $\tilde{G}$ and $\tilde{V}$, define the production of gambling as a proportion of the total production as $\alpha \equiv G/(G+V)$. In other words, if the city produces more (less) gambling than convention, α will be greater (less) than 0.5. Denote $\tilde{\alpha}$ to be α evaluated at the optimal levels, $\tilde{G}$ and $\tilde{V}$. Hence, if $\phi > 1$ ($\phi < 1$), it is optimal for the city to choose an $\tilde{\alpha} >$ 0.5 ($\tilde{\alpha} < 0.5$). If $\phi = 1$, the city choses a proportion of exactly 0.5.

More precisely, differentiating $\tilde{\alpha}$ with respect to ϕ yields:

$$\frac{\partial \tilde{\alpha}}{\partial \phi} = 4\frac{(b-\theta+c_2)[c_2 a+(b+\theta)c_1](a-c_1)}{[(\tilde{G}+\tilde{V})H]^2} > 0. \tag{5}$$

Hence, as convention becomes relatively more costly, the city will produce a higher proportion of gambling instead.

Proposition 1: (i) If it is more (less) costly to produce convention than gambling, a city produces a larger (smaller) proportion of gambling, and if it is equally costly to produce the two tourism services, a city produces half of each; i.e., if $\phi \gtreqless 1$, then $\tilde{\alpha} \gtreqless 0.5$.

(ii) The more costly it is to produce convention relative to gambling, the greater is the proportion of gambling produced, and vice versa; i.e., $\frac{\partial \tilde{\alpha}}{\partial \phi} > 0$.

Fig. 3.3 illustrates some possible positions of optimal $\tilde{\alpha}$ along the number line. The closer $\tilde{\alpha}$ is to 1, the larger is the proportion of gambling produced. The closer $\tilde{\alpha}$ is to 0, the larger is the proportion of convention produced. In Panel (a), $\tilde{\alpha} > 0.5$ is a possible position when $\phi > 1$. In Panel (b), $\tilde{\alpha} < 0.5$ is a possible position when $\phi < 1$. The closer $\tilde{\alpha}$ is to one of these extremes indicates that the city opts for greater specialization in one kind of tourism services. The closer $\tilde{\alpha}$ is to the mid-point of 0.5 indicates greater degree of *diversification* among gambling and convention.

The interest of this chapter is to find out what determines the degree of diversification of a city. Whether it is optimal for a city to diversify more depends on changes in demand. Because the model assumes gambling and convention to have identical market size, a, the tourism services experience the same market expansion. Differentiating (3) with respect to a yields

$$\frac{\partial \tilde{G}}{\partial a} = \frac{b - \theta + \phi c_2}{H} > 0, \tag{6a}$$

$$\frac{\partial \tilde{V}}{\partial a} = \frac{b - \theta + c_2}{H} > 0. \tag{6b}$$

Hence, both $\tilde{G}$ and $\tilde{V}$ increase with a. Fig. 3.4 illustrates how both the $\tilde{G}$ line and the $\tilde{V}$ line shift up in response to an increase in a.

Notice that (6a) and (6b) can be different. Hence, the increase in $\tilde{G}$ and the increase in $\tilde{V}$ can be of different extents. As a result, the effect of expansion in the markets can move the position of $\tilde{\alpha}$. Differentiate $\tilde{\alpha}$ with respect to a:

$$\frac{\partial\tilde{\alpha}}{\partial a} = \frac{(1-\phi)c_1}{\left(\tilde{G}+\tilde{V}\right)^2 H}. \tag{7}$$

The model has previously specified the parameter values in (7) except for ϕ. Therefore, the sign of (7) depends on the value of ϕ. In particular, $\tilde{\alpha}$ decreases in response to a rise of a if $\phi > 1$. On the other hand, if $\phi < 1$, $\tilde{\alpha}$ increases when a rises. Fig. 3.3 shows the direction of how $\tilde{\alpha}$ moves in response to an expansion in the markets. In the example of $\phi > 1$, $\tilde{\alpha}$ is greater than 0.5. Before market expansion, $\tilde{\alpha}$ is relatively closer to 1; after market expansion, it moves closer to 0.5. In the example of $\phi < 1$, $\tilde{\alpha}$ is less than 0.5. Before market expansion, $\tilde{\alpha}$ is relatively closer to 0; after market expansion, it moves closer to 0.5. In the example of $\phi = 1$, $\tilde{\alpha}$ remains at 0.5 before and after market expansion.

Proposition 2: Expansion in the markets of gambling and convention leads higher degree of diversification; i.e., if $\phi \gtreqless 1$, then $\tilde{\alpha} \gtreqless 0.5$ and $\dfrac{\partial\tilde{\alpha}}{\partial a} \lesseqgtr 0$.

The results from this simple model have important implications toward real-life cases such as Macao and Singapore. Scarce in natural resources, cities like Macao and Singapore rely heavily on export of services, especially tourism services. Macao is known for its casinos. Macao’s long history of gambling industry and its well-trained labor force clearly generate a cost advantage in providing gambling service. Using the language of this present model, Macao's ϕ

should be greater than 1. The model finds that Macao's $\tilde{\alpha}$ should be greater than 0.5, leaning towards gambling. However, in Macao's recent annual policy addresses, the government expressed strong will to diversify, especially towards convention (Macao Government, 2016). That is, its objective is to move Macao's $\tilde{\alpha}$ closer to 0.5.

On the other hand, Singapore should be on the other side of the spectrum. It is one of the biggest convention centers in the world. Singapore's well-developed business environment is particularly appealing to commercial customers. Its highly-educated, multilingual labor force is known for providing high-quality business services. According to the analysis of the present model, Singapore's ϕ should be less than 1, so its optimal $\tilde{\alpha}$ should be lower than 0.5. Yet, the Singaporean government also seeks to diversify, launching two casinos 2010 (Jia, 2015). In other words, the goal is to move Singapore's $\tilde{\alpha}$ closer to 0.5.

The present model can provide an explanation to a city's objective to diversify even when it has cost advantage in one particular tourism service. According to Proposition 2, a rise of a causes $\tilde{\alpha}$ to approach 0.5 regardless of the value of ϕ. In other words, when experiencing an expansion in both markets, a city chooses to diversify. In the recent decades, the number of Mainland Chinese tourists has skyrocketed due to the rise of the Chinese economy and the liberalization of visa requirements towards Chinese travelers (China Contact). Macao and Singapore are two of their main destinations, so there has been a significant increase in the market size. Hence, even though Macao specializes in gambling while Singapore specializes in convention, each city chooses to diversify more in response to such market expansion.

3.3. The Case of Two Cities

The basic model in the previous section describes a situation when a city operates rather like a local monopoly in its own region. The city acts individually without considering competition. However, as the industry grows, the city may cover not just the regional market, but also the international market, in which international competitors await. In this section, suppose there are two cities Macao and Singapore, denoted with the "M" superscript and the "S" superscript respectively. Their cost structures are the same as the one described in Section 3.2. Macao incurs $C_G^M = c_0 + c_1 G^M + c_2 G^{M2}$ and $C_V^M = \phi^M c_0 + \phi^M c_1 V^M + \phi^M c_2 V^{M2}$ in its production and Singapore incurs $C_G^S = c_0 + c_1 G^S + c_2 G^{S2}$ and $C_V^S = \phi^S c_0 + \phi^S c_1 V^S + \phi^S c_2 V^{S2}$ in its production. The costs of the cities differ only in the parameter ϕ. Without a loss of generality, assume $\phi^M > 1 > \phi^S$. As discussed previously, it is reasonable to believe that Macao has cost advantage in gambling relative to convention, while Singapore has cost advantage in convention relative to gambling.

The total production of the two cities are $G^M + G^S$ and $V^M + V^S$. With linear and symmetric inverse demand functions, in equilibrium, prices are $P_G = a - b(G^M + G^S) - \theta(V^M + V^S)$ and $P_V = a - b(V^M + V^S) - \theta(G^M + G^S)$.

The cities compete in a Nash-Cournot fashion. Macao maximizes profit by choosing G^M and V^M, taking G^S and V^S as given:

$$\max_{\{G^M, V^M\}} \pi^M = P_G G^M + P_V V^M - C_G^M - C_V^M. \tag{8}$$

The first-order conditions are

$$\frac{\partial \pi^M}{\partial G^M} = a - c_1 - 2(b + c_2)G^M - 2\theta V^M - bG^S - \theta V^S = 0, \tag{9a}$$

$$\frac{\partial \pi^M}{\partial V^M} = a - \phi^M c_1 - 2(b + \phi^M c_2)V^M - 2\theta G^M - bV^S - \theta G^S = 0. \tag{9b}$$

The second-order conditions are the same as those in Section 3.2 and are satisfied in the model: $\frac{\partial^2 \pi^M}{\partial {G^M}^2} = -2(b + c_2) < 0$, $\frac{\partial^2 \pi^M}{\partial {V^M}^2} = -2(b + \phi^M c_2) < 0$ and $H^M = 4[(b + c_2)(b + \phi^M c_2) - \theta^2] > 0$. Solving (9) simultaneously would yield best-response functions.

Singapore maximizes profit by choosing G^S and V^S, taking G^M and V^M as given:

$$\max_{\{G^S, V^S\}} \pi^S = P_G G^S + P_V V^S - C_G^S - C_V^S. \tag{10}$$

The first-order conditions are

$$\frac{\partial \pi^S}{\partial G^S} = a - c_1 - 2(b + c_2)G^S - 2\theta V^S - bG^M - \theta V^M = 0, \tag{11a}$$

$$\frac{\partial \pi^S}{\partial V^S} = a - \phi^S c_1 - 2(b + \phi^S c_2)V^S - 2\theta G^S - bV^M - \theta G^M = 0. \tag{11b}$$

The second-order conditions are the same as those in Section 3.2 and are satisfied in the model: $\frac{\partial^2 \pi^S}{\partial {G^S}^2} = -2(b + c_2) < 0$, $\frac{\partial^2 \pi^S}{\partial {V^S}^2} = -2(b + \phi^S c_2) < 0$ and $H^S = 4[(b + c_2)(b + \phi^S c_2) - \theta^2] > 0$. Solving (11) would yield best-response functions. Solving (9) and (11) simultaneously yield the Nash-Cournot equilibrium production levels denoted with "^":

$$\hat{G}^M = \frac{\Omega(a - c_1) + \Gamma^M a + \gamma^M c_1}{\Delta}, \tag{12a}$$

$$\hat{V}^M = \frac{\Omega(a - c_1) + \mathrm{N}^M a + \nu^M c_1}{\Delta}, \tag{12b}$$

$$\hat{G}^S = \frac{\Omega(a - c_1) + \Gamma^S a + \gamma^S c_1}{\Delta}, \tag{12c}$$

$$\hat{V}^S = \frac{\Omega(a - c_1) + \mathrm{N}^S a + \nu^S c_1}{\Delta}, \tag{12d}$$

where $\Delta \equiv \frac{9}{16} H^M H^S + \{3(b^2 - \theta^2)[(\phi^M + \phi^S + 2)b + (\phi^M + \phi^S)c_2] + b^2(11\phi^M + 11\phi^S + 3\phi^M\phi^S + 3)c_2 + 7(\phi^M + \phi^S + 2)bc_2^2 + 7\phi^M\phi^S c_2^3\}c_2 > 0$; and Ω, Γ^i, γ^i, N^i and ν^i for $i = M, S$ are also expressions that contain parameters, b, θ, c_2, ϕ^M and ϕ^S, which are defined in Appendix B.1. (12) show that

$$\hat{G}^M - \hat{V}^M = \frac{(\Gamma^M - N^M)a + (\gamma^M - \nu^M)c_1}{\Delta} > 0, \tag{13a}$$

$$\hat{G}^S - \hat{V}^S = \frac{(\Gamma^S - N^S)a + (\gamma^S - \nu^S)c_1}{\Delta} < 0, \tag{13b}$$

where the signs of $(\Gamma^i - N^i)$, $(\gamma^i - \nu^i)$ for $i = M, S$ are proved in Appendix B.1. (13) imply that $\hat{\alpha}^M \equiv \hat{G}^M/(\hat{G}^M + \hat{V}^M) > 0.5$ while $\hat{\alpha}^S \equiv \hat{G}^S/(\hat{G}^S + \hat{V}^S) < 0.5$. Fig. 3.5 illustrates the positioning of $\hat{\alpha}^M$ and $\hat{\alpha}^S$.

Proposition 3: In Nash-Cournot equilibrium, Macao produces a greater proportion of gambling, while Singapore produces are a greater proportion of convention; i.e., $\hat{\alpha}^M > 0.5$ and $\hat{\alpha}^S < 0.5$.

Furthermore, I can analyze the effect of rivalry by plugging ϕ^M in place of ϕ in the equations of Section 3.2. I can then compare $\hat{\alpha}^M$ of Section 3.3 to $\tilde{\alpha}^M$ of Section 3.2. The difference of the two is

$$\hat{\alpha}^M - \tilde{\alpha}^M = 2\frac{(\beta^M a + \delta^M c_1)(a - c_1) + \varepsilon^M a c_1}{\left(\tilde{G}^M + \tilde{V}^M\right)\left(\hat{G}^M + \hat{V}^M\right) H^M \Delta} > 0, \tag{14}$$

where β^M, δ^M and ε^M are positive expressions of parameters b, θ, c_2, ϕ^M and ϕ^S. They are presented in Appendix B.1. Thus Macao produces a bigger proportion of gambling under competition than it would as a regional monopoly. That is, it has a greater degree of product differentiation if it considers Singapore as a rival.

By plugging ϕ^S in place of ϕ in the equations derived in Section 3.2, I can obtain $\tilde{\alpha}^S$. The difference of $\hat{\alpha}^S$and $\tilde{\alpha}^S$ is

$$\hat{\alpha}^S - \tilde{\alpha}^S = 2\frac{(\beta^S a + \delta^S c_1)(a - c_1) + \varepsilon^S a c_1}{\left(\tilde{G}^S + \tilde{V}^S\right)\left(\hat{G}^S + \hat{V}^S\right) H^S \Delta} < 0, \tag{15}$$

where β^S, δ^S and ε^S are negative expressions of parameters b, θ, c_2, ϕ^M and ϕ^S. They are presented in Appendix B.1. If it considers the competition of Macao, Singapore produces a larger proportion of convention than in a regional monopoly. That is, it also has a greater degree of product differentiation when reacting with a rival.

Proposition 4: Both Macao and Singapore adopt greater degrees of product differentiation when competition is considered; i.e., $\hat{\alpha}^M - \tilde{\alpha}^M > 0$ and $\hat{\alpha}^S - \tilde{\alpha}^S < 0$, implying $\hat{\alpha}^M - \hat{\alpha}^S > \tilde{\alpha}^M - \tilde{\alpha}^S$.

Differentiating $\hat{\alpha}^M$ with respect to ϕ^M and $\hat{\alpha}^S$ with respect to ϕ^S:

$$\frac{\partial\hat{\alpha}^M}{\partial\phi^M} = \frac{2(c_1 + 2c_2\hat{V}^M)}{\left(\hat{G}^M + \hat{V}^M\right)^2\Delta^2}\{\left(\frac{3}{4}H^S + b\phi^S c_2^2\right)\hat{V}^M$$

$$+ [4(b + c_2)H^S + b(b^2 - \theta^2) + b^2\phi^S c_2 + \theta^2 c_2]\hat{G}^M\} > 0, \qquad (16a)$$

$$\frac{\partial\hat{\alpha}^S}{\partial\,\phi^S} = \frac{2(c_1 + 2c_2\hat{V}^S)}{\left(\hat{G}^S + \hat{V}^S\right)^2\Delta^2}\{\left(\frac{3}{4}H^M + b\phi^M c_2^2\right)\hat{V}^S$$

$$+ [4(b + c_2)H^M + b(b^2 - \theta^2) + b^2\phi^M c_2 + \theta^2 c_2]\hat{G}^S\} > 0. \qquad (16b)$$

These show that a city's proportion of $\hat{\alpha}$ depends on its own ϕ positively. As expected, a city produces a bigger proportion of gambling when convention becomes more costly. While a city produces relatively more gambling when ϕ increases, its rival city will react by producing a smaller proportion of gambling. This can be proved by differentiating $\hat{\alpha}^M$ with respect to ϕ^S and $\hat{\alpha}^S$ with respect to ϕ^M:

$$\frac{\partial\hat{\alpha}^M}{\partial\phi^S} = -\frac{2\left(c_1 + 2c_2\hat{V}^S\right)}{\left(\hat{G}^M + \hat{V}^M\right)^2\Delta^2}\left\{(3(b^2 - \theta^2)\theta + 2(2b - \theta)(b + \phi^M c_2) + 4b\theta c_2)\hat{V}^M\right.$$

$$\left. + [3(b^2 - \theta^2)b + 4b(2b + c_2)c_2]\hat{G}^M\right\} < 0, \qquad (17a)$$

$$\frac{\partial\hat{\alpha}^S}{\partial\,\phi^M} = -\frac{2\left(c_1 + 2c_2\hat{V}^M\right)}{\left(\hat{G}^S + \hat{V}^S\right)^2\Delta^2}\left\{(3(b^2 - \theta^2)\theta + 2(2b - \theta)(b + \phi^S c_2) + 4b\theta c_2)\hat{V}^S\right.$$

$$\left. + [3(b^2 - \theta^2)b + 4b(2b + c_2)c_2]\hat{G}^S\right\} < 0. \qquad (17b)$$

Proposition 5: When convention becomes more costly, a city produces more gambling while the rival city produces less gambling, and vice versa; i.e., $\frac{\partial\hat{\alpha}^M}{\partial\phi^M} > 0$, $\frac{\partial\hat{\alpha}^S}{\partial\phi^S} > 0$, $\frac{\partial\hat{\alpha}^M}{\partial\phi^S} < 0$ and $\frac{\partial\hat{\alpha}^S}{\partial\phi^M} < 0$.

As in the previous section, an increase in demand size, a in the markets of the tourism services should move the positions of α. Differentiate $\hat{\alpha}^M$ and $\hat{\alpha}^S$ with respect to a:

$$\frac{\partial\hat{\alpha}^M}{\partial a} = -\frac{1}{\Delta}[(b^2-\theta^2)(2\phi^M-\phi^S-1)c_1+2(b-\theta)(\phi^M-\phi^S)c_1c_2 + 2[(\phi^S+1)b+2\phi^S c_2](\phi^M-1)c_1c_2] < 0, \quad (18a)$$

$$\frac{\partial\hat{\alpha}^S}{\partial a} = \frac{1}{\Delta}[(b^2-\theta^2)(\phi^M-2\phi^S+1)c_1+2(b-\theta)(\phi^M-\phi^S)c_1c_2+2[(\phi^M+1)b + 2\phi^M c_2](1-\phi^S)c_1c_2] > 0. \quad (18b)$$

According to Proposition 3, $\hat{\alpha}^M > 0.5$, so it decreases toward 0.5 when a rises. On the other hand, $\hat{\alpha}^S < 0.5$, so it increases towards 0.5 when a rises. Hence, same as the single-city case in Section 3.2, a market expansion moves both $\hat{\alpha}^M$ and $\hat{\alpha}^S$ closer to 0.5. Fig. 3.6 shows how both $\hat{\alpha}^M$ and $\hat{\alpha}^S$converges to 0.5 in response to the market expansion.

Proposition 6: When there is a market expansion of the two tourism services, each city adopts a greater degree of diversification; i.e., $\hat{\alpha}^M > 0.5$, $\hat{\alpha}^S < 0.5$, $\frac{\partial\hat{\alpha}^M}{\partial a} < 0$ and $\frac{\partial\hat{\alpha}^S}{\partial a} > 0$.

The analysis above provides intuition to a possible real-life situation that Macao and Singapore face. As explained in the previous section, it is reasonable to believe that Macao can produce gambling relatively less costly while Singapore can produce convention relatively less costly. Therefore, this section assumes $\phi^M > 1 > \phi^S$. When the two cities compete as a duopoly, Proposition 3 asserts that it is optimal for Macao to produce more gambling than convention: $\hat{\alpha}^M$ should be higher than 0.5, while Singapore should produce a larger proportion of convention: $\hat{\alpha}^S$ should be lower than 0.5. Whether the cities consider each other as a rival makes a difference. According to Proposition 4, both cities specialize more if they compete as a duopoly than if they operate as a regional monopoly.

When Macao has a competitive edge in gambling and it is advantageous for Singapore to specialize more on convention, it is not apparent why both governments aim at a higher degree of diversification. The recent surge of Mainland Chinese tourists is a concrete piece of evidence of market expansion of the tourism services. According to Proposition 6, the larger markets give both Macao and Singapore incentives to move away from their specialties. That is, $\hat{\alpha}^M$ responds to a negatively, but $\hat{\alpha}^S$ reacts to a positively. Recall that the previous section considers each city individually and shows that a city, regardless of its relative cost ϕ, has incentive to diversify in response to a market expansion. The same logic still applies in this section. It is not easy measure the rivalry, if any, between Macao and Singapore. However, both the single-city model and the two-city model identify market expansion as a plausible economic reason behind the governments' objectives to diversify among gambling and convention.

3.4. The Case of Three Cities

The previous section shows how each of Macao and Singapore specializes more in the tourism service that it is good at providing when they consider each other as a rival; i.e. each duopolist chooses its equilibrium product mix strategy. This section extends the model to cover more competitors. One potential competitor is Osaka. In this model, variables of Osaka will be denoted with the "O" superscript. Osaka's cost structure is the same as the other cities': $C_G^O = c_0 + c_1 G^O + c_2 G^{O2}$ and $C_V^O = \phi^O c_0 + \phi^O c_1 V^O + \phi^O c_2 V^{O2}$. I assume the three cities have different ϕ's. The assumption that $\phi^M > 1 > \phi^S$ remains, but how ϕ^O compares to ϕ^M and ϕ^S is yet to be analyzed.

The total production of the three cities are $G^M + G^S + G^O$ and $V^M + V^S + V^O$. With the linear and symmetric inverse demand functions, in equilibrium, the prices are $P_G = a - b(G^M + G^S + G^O) - \theta(V^M + V^S + V^O)$ and $P_V = a - b(V^M + V^S + V^O) - \theta(G^M + G^S + G^O)$.

The cities compete in a Nash-Cournot fashion. (8) and (10) already described the profit maximization problems faced by Macao and Singapore. Note, however, that Macao and Singapore need to take G^O and V^O as given when choosing their production levels. Hence, the first-order conditions of Macao's profit maximization are

$$\frac{\partial \pi^M}{\partial G^M} = a - c_1 - 2(b + c_2)G^M - 2\theta V^M - b(G^S + G^O) - \theta(V^S + V^O) = 0, \tag{19a}$$

$$\frac{\partial \pi^M}{\partial V^M} = a - \phi^M c_1 - 2(b + \phi^M c_2)V^M - 2\theta G^M - b(V^S + V^O) - \theta(G^S + G^O) = 0. \tag{19b}$$

Solving (19) simultaneously yield best-response functions. The second-order conditions are the same as those in Section 3.3 and are satisfied in the model. Singapore's first-order conditions of the profit maximization problem are

$$\frac{\partial \pi^S}{\partial G^S} = a - c_1 - 2(b + c_2)G^S - 2\theta V^S - b(G^M + G^O) - \theta(V^M + V^O) = 0, \tag{20a}$$

$$\frac{\partial \pi^S}{\partial V^S} = a - \phi^S c_1 - 2(b + \phi^S c_2)V^S - 2\theta G^S - b(V^M + V^O) - \theta(G^M + G^O) = 0. \tag{20b}$$

Solving (20) simultaneously yields best-response functions. The second-order conditions are the same as those in Section 3.3 and are satisfied in the model. Similarly, Osaka maximizes profit by choosing G^O and V^O, taking G^M, V^M, G^S and V^S as given:

$$\max_{\{G^O, V^O\}} \pi^O = P_G G^O + P_V V^O - C_G^O - C_V^O. \tag{21}$$

The first-order conditions are

$$\frac{\partial \pi^O}{\partial G^O} = a - c_1 - 2(b + c_2)G^O - 2\theta V^O - b(G^M + G^S) - \theta(V^M + V^S) = 0, \tag{22a}$$

$$\frac{\partial \pi^O}{\partial V^O} = a - \phi^O c_1 - 2(b + \phi^O c_2)V^O - 2\theta G^O - b(V^M + V^S) - \theta(G^M + G^S) = 0. \tag{22b}$$

Solving (22) simultaneously yields best-response functions. The second-order conditions are the same as those in Section 3.2 and are satisfied in the model: $\frac{\partial^2 \pi^O}{\partial {G^O}^2} = -2(b + c_2) < 0$, $\frac{\partial^2 \pi^O}{\partial {V^O}^2} = -2(b + \phi^O c_2) < 0$ and $H^O = 4[(b + c_2)(b + \phi^O c_2) - \theta^2] > 0$.

Solving (19), (20) and (22) simultaneously yields the Nash-Cournot equilibrium production levels (denoted with an upper bar):

$$\bar{G}^M = 2\frac{\mathrm{X}^M(a - c_1) + \chi^M c_1}{\Sigma}, \tag{23a}$$

$$\bar{V}^M = 2\frac{\mathrm{T}^M(a - c_1) + \tau^M c_1}{\Sigma}, \tag{23b}$$

$$\bar{G}^S = 2\frac{\mathrm{X}^S(a-c_1)+\chi^S c_1}{\Sigma}, \tag{23c}$$

$$\bar{V}^S = 2\frac{\mathrm{T}^S(a-c_1)+\tau^S c_1}{\Sigma}, \tag{23d}$$

$$\bar{G}^O = 2\frac{\mathrm{X}^O(a-c_1)+\chi^O c_1}{\Sigma}, \tag{23e}$$

$$\bar{V}^O = 2\frac{\mathrm{T}^O(a-c_1)+\tau^O c_1}{\Sigma}, \tag{23f}$$

where Σ, X^i, χ^i, T^i and τ^i for $i = M, S, O$ are expressions that contain parameters, b, θ, c_2, ϕ^M, ϕ^S and ϕ^O, which are defined in Appendix B.2. (23) show that

$$\bar{G}^M - \bar{V}^M = 2\frac{(X^M - T^M)(a-c_1)+(\chi^M-\tau^M)c_1}{\Sigma} > 0, \tag{24a}$$

$$\bar{G}^S - \bar{V}^S = 2\frac{(X^S - T^S)(a-c_1)+(\chi^S-\tau^S)c_1}{\Sigma} < 0, \tag{24b}$$

$$\bar{G}^O - \bar{V}^O = 2\frac{(X^O - T^O)(a-c_1)+(\chi^O-\tau^O)c_1}{\Sigma} \begin{matrix} > \\ = 0 \\ < \end{matrix} \quad if\ \phi^O \begin{matrix} > \\ = 1, \\ < \end{matrix} \tag{24c}$$

where the signs of $\left(X^i - T^i\right)$ and $\left(\chi^i - \tau^i\right)$ for $i = M, S, O$ are proved in Appendix B.2. (24) imply that $\bar{\alpha}^M \equiv \bar{G}^M/(\bar{G}^M + \bar{V}^M) > 0.5$, $\bar{\alpha}^S \equiv \bar{G}^S/(\bar{G}^S + \bar{V}^S) < 0.5$. Fig. 3.8 illustrates the positioning of $\bar{\alpha}^M$ and $\bar{\alpha}^S$. As in Sections 3.2 and 3.3, because $\phi^M > 1 > \phi^S$, it is optimal for Macao to produce more gambling while Singapore should produce more convention. On the other hand, how $\bar{\alpha}^O \equiv \bar{G}^O/(\bar{G}^O + \bar{V}^O)$ compare to 0.5 is unknown without specifying how ϕ^O compares to 1. More importantly is how ϕ^O compares to ϕ^M and ϕ^S. In general, for $i = M, S, O$, $j = M, S, O$,$k = M, S, O$ and $i \neq j \neq k$,

$$\frac{\partial \bar{\alpha}^i}{\partial \phi^i} = \frac{2}{(\bar{G}^i + \bar{V}^i)^2\Sigma^2}\left\{\left[\frac{\partial X^i}{\partial \phi^i}(a-c1)+\frac{\partial \chi^i}{\partial \phi^i}c1\right]\bar{V}^i - \left[\frac{\partial T^i}{\partial \phi^i}(a-c1)+\frac{\partial \tau^i}{\partial \phi^i}c1\right]\bar{G}^i\right\} > 0, \tag{25}$$

where $\frac{\partial X^i}{\partial \phi^i} > 0$, $\frac{\partial \chi^i}{\partial \phi^i} > 0$, $\frac{\partial T^i}{\partial \phi^i} = 0$ and $\frac{\partial \tau^i}{\partial \phi^i} < 0$ are proved in Appendix B.2. Hence, the greater ϕ^i is, the higher $\bar{\alpha}$ is. Suppose $\phi^M > \phi^S > \phi^O$, then $\bar{\alpha}^M > \bar{\alpha}^S > \bar{\alpha}^O$. If $\phi^M > \phi^O > \phi^S$, then $\bar{\alpha}^M > \bar{\alpha}^O > \bar{\alpha}^S$. Finally if $\phi^O > \phi^M > \phi^S$, then $\bar{\alpha}^O > \bar{\alpha}^M > \bar{\alpha}^S$. The analyses of the three cases are as follow.

Case 1:

As illustrated in Case 1 of Fig. 3.8, when $\phi^M > 1 > \phi^S > \phi^O$, $\bar{\alpha}^M > 0.5 > \bar{\alpha}^S > \bar{\alpha}^O$. To see the effect of Osaka's entry into competition, compute the following differences:

$$\bar{\alpha}^M - \hat{\alpha}^M = \frac{(\eta^M a + \kappa^M c_1)(a - c_1) + \mu^M a c_1}{\left(\hat{G}^M + \hat{V}^M\right)(\bar{G}^M + \bar{V}^M)\Delta\Sigma} > 0, \tag{26a}$$

$$\bar{\alpha}^S - \hat{\alpha}^S = \frac{(\eta^S a + \kappa^S c_1)(a - c_1) + \mu^S a c_1}{\left(\hat{G}^S + \hat{V}^S\right)(\bar{G}^S + \bar{V}^S)\Delta\Sigma} > 0. \tag{26b}$$

where η^i, κ^i and μ^i for $i = M, S$ are expressions of parameters b, θ, c_2, ϕ^M and ϕ^S, presented in Appendix B.2. In general, it is very difficult to derive the signs of (26) in this model of 6 first-order conditions and 6 unknowns. However, as shown in Appendix B.2, $\eta^M > 0$ and $\eta^S > 0$ in this case. Together with an a that is large enough, these are sufficient conditions for (26) to be positive. On the other hand, $\kappa^i > 0$ and $\mu^i > 0$ for $i = M, S$ can also be sufficient conditions for (26) to be positive. Thus while Osaka has the greatest degree of specialization in convention, Macao and Singapore prevent head-on competition by producing more gambling instead. Fig. 3.8 illustrates how both Macao and Singapore adopt greater α's in Case 1.

Proposition 7: Given (i) a is sufficiently large, or (ii) $\kappa^i > 0$ and $\mu^i > 0$ for $i = M, S$, when considering Osaka as a competitor, Macao and Singapore produce greater proportions of gambling; i.e., $\bar{\alpha}^M > 0.5 > \bar{\alpha}^S > \bar{\alpha}^O$, $\bar{\alpha}^M - \hat{\alpha}^M > 0$ and $\bar{\alpha}^S - \hat{\alpha}^S > 0$.

Case 2(a):

When $\phi^M > \phi^O > \phi^S$, $\bar{\alpha}^M > \bar{\alpha}^O > \bar{\alpha}^S$. As shown in Appendix B.2, the signs of (26) are not obvious in these cases. In general, each of (26) can be positive or negative depending on how close ϕ^O is to Macao's ϕ^M or Singapore's ϕ^S. Fig. 3.7 shows how (26a) and (26b) can be positive or negative depending on the value of ϕ^O. Such conditions on the ϕ's are complicated. However, it is fairly easy to derive numerical results. I construct Fig. 3.8 using numerical method. In Case 2(a), ϕ^O is greater than, but closed to ϕ^S. In this case, the effect of Osaka's rivalry on Singapore's convention is so strong that Singapore will react by adopting a greater proportion of gambling instead. As in Case 1, Macao tries to distinguish itself from the other competitors by producing an even bigger proportion of gambling. Case 2(a) of Fig. 3.8 describe such movements.

Case 2(b):

In Case 2(b), ϕ^O is somewhere in the middle of and maintain quite a distance between ϕ^S and ϕ^M. In this case, Osaka's presence pushes both Singapore and Macao to the gambling side. It is as though each city occupies its own territory on the number line. See Case 2(b) of Fig. 3.8 for the illustration.

Case 2(c):

Case 2(c) is the symmetric case of Case 2(a). When ϕ^O is still less than, but closed enough to ϕ^M, Macao's gambling experience sufficiently strong rivalry, so it adopts a greater proportion of convention instead. In this case, Singapore tries to spread out from the other competitors by producing an even bigger proportion of convention. Case 2(c) of Fig. 3.8 shows their adjustments.

Case 3:

By symmetry, the case of $\phi^O > \phi^M > 1 > \phi^S$ is the opposite results of Case 1. Hence, it is not discussed in details here to avoid redundancy. As illustrated in Case 3 of Fig. 3.8, when Osaka's relative cost of convention is even greater than that of Macao, or in other words, it is the least relatively costly for Osaka to produce gambling, both Macao and Singapore react to Osaka's competition by spreading out toward the convention side.

Osaka proposed to open Japan's first legal casino in 2002. The potential rivalry of Osaka is definitely in the air. It is not known whether Macao, Singapore and Osaka will see themselves as an international oligopoly. If they do, the analysis above shows that how Macao and Singapore adjust their product mix strategies in response to Osaka's entry depends on Osaka's relative cost of convention. The history provides some clues. Among the three cities, Osaka (and the whole country of Japan) is the only one with no experience in operating casinos. Macao's economy was largely based on casino revenue throughout its history. During the colonial era of Singapore, casinos had been popular (Remember Singapore, 2011). On the other hand, Osaka is one of the most competitive business centers in Asia and should have a cost advantage in producing convention service. If Osaka's ϕ^O is even lower than Singapore's ϕ^S, Proposition 7 asserts that Osaka's entry should push both Macao and Singapore to the gambling side. Each city needs to distinguish itself from the competitors by choosing a unique product mix, spreading their $\bar{\alpha}$'s apart from each other and avoiding head-to-head competition with the new rival.

3.5. Concluding Remarks

The goal of Macao and Singapore to diversify the two tourism services - gambling and convention – appears to be unjustifiable by any economic reason. Macao has a cost advantage in producing gambling while Singapore can produce convention with relatively lower cost. The model shows that when each city operates as a regional monopoly, Macao chooses a product mix consisted of more gambling than convention, while Singapore's is consisted of more convention than gambling. Due to the surge of Mainland Chinese tourists, each city experiences growing markets of gambling and convention. In response to the market expansion, each city chooses to produce a more even split between gambling and convention. In other words, this simple model can provide a plausible reason why a city seeks to diversify and produces more of a product that it cannot produce with a cost advantage. Macao and Singapore seem to pursue opposite policy goals, but in fact, the two cities have the same underlying objective to diversify.

I also consider the situation when Macao and Singapore compete consider themselves as a international duopoly. In this model, because Macao's relative cost of convention is higher compared to Singapore, Macao strategically chooses a Nash-Cournot-equilibrium product mix that inclines towards gambling while Singapore inclines towards convention. When comparing their product mixes to those when they consider themselves a regional monopoly, it is clear that competition pushes the cities to the farther ends of the spectrum. Each city attempts to distinguish its product mix from the competitor's. When the markets expand, the equilibrium product mixes consist of more even distribution of gambling and convention. Hence, the surge of Mainland Chinese tourists is the key reason behind the cities' diversification objective whether competition is considered or not.

Even though Osaka's plan to launch an integrated casino resort is still up in the air, it is insightful to analyze the effect of this potential entry. The three-city model suggests that if Osaka's relative cost of producing convention is lower than both Macao's and Singapore's, both Macao and Singapore will react by producing greater proportions of gambling compared to the duopoly scenario. In general, when choosing an optimal product mix, cities try to avoid direct competition with its rivals, so their product strategies depend on their relative costs of the tourism services.

The model results in this chapter match the real-life observations of the development of gambling and convention in Macao and in Singapore. The study provides economic insights into a city’s pursuit to diversify multiple tourism services even if it has clear cost advantage in one particular kind of tourism service. As Osaka’s plan to open casinos is expected to be realized in the near future, further investigation into the reactions of Macao and Singapore to the new rivalry will soon be possible.

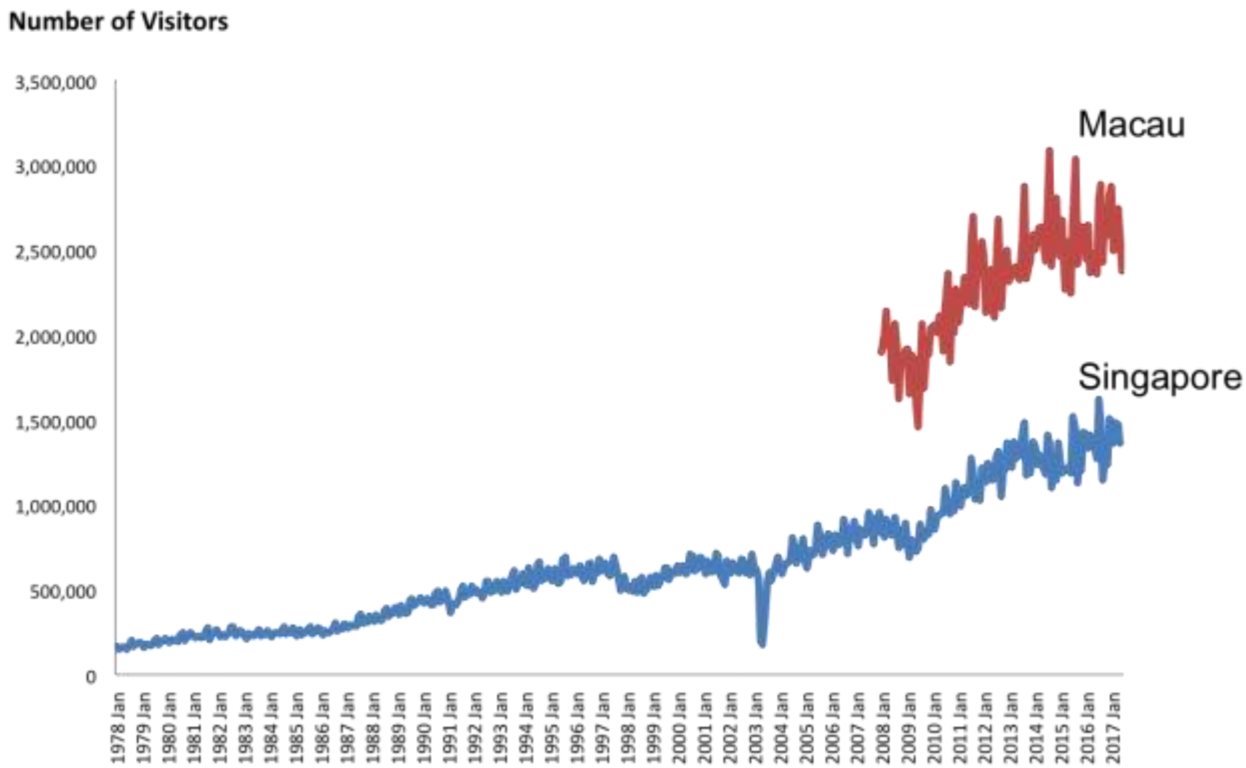


Fig. 3.1. Number of Visitors to Macao and Singapore. Source: DSEC and SingStat.

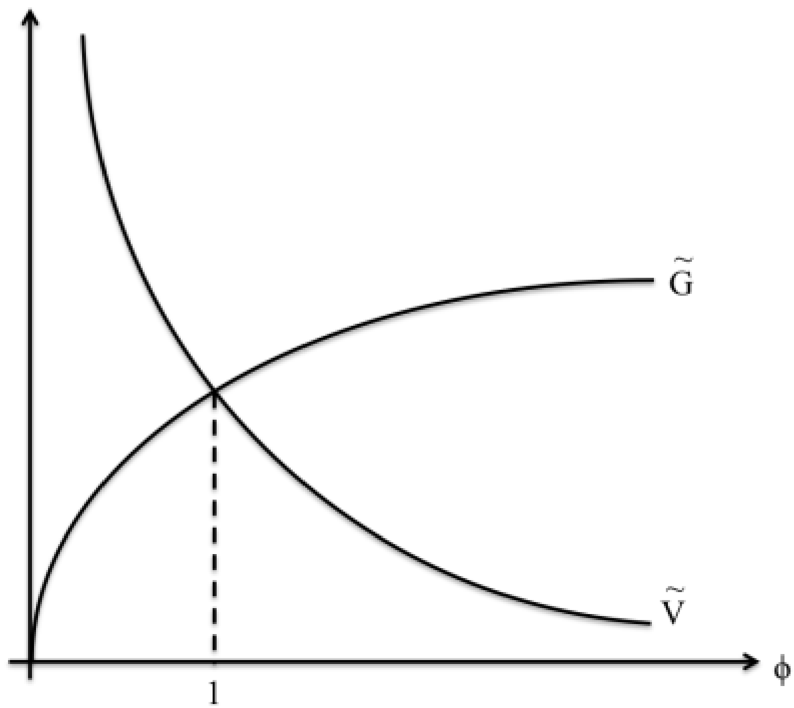
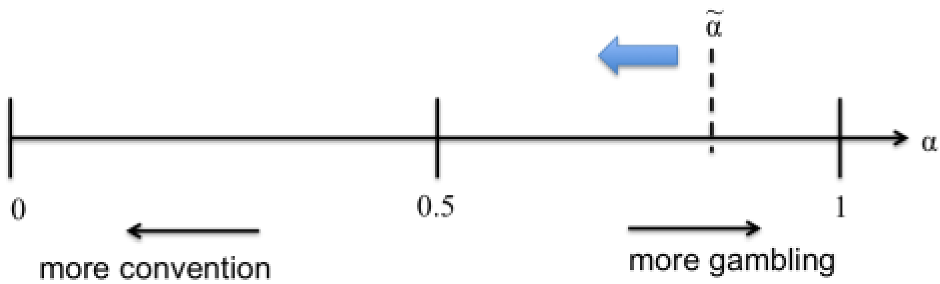
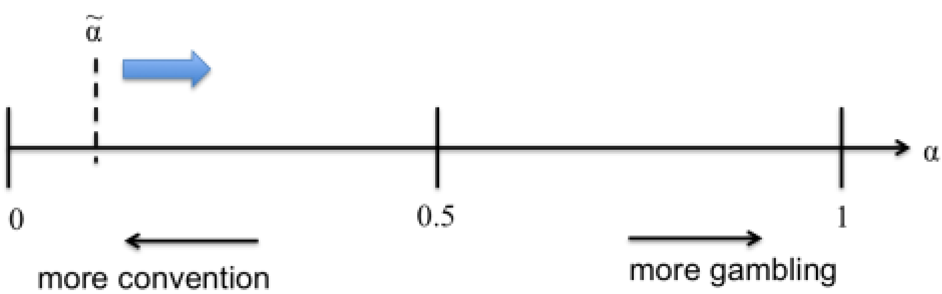


Fig. 3.2. ϕ as Determinant of $\tilde{G}$ and $\tilde{V}$

Panel (a) An example of $\phi > 1$



Panel (b) An example of $\phi < 1$



Panel (c) $\phi = 1$

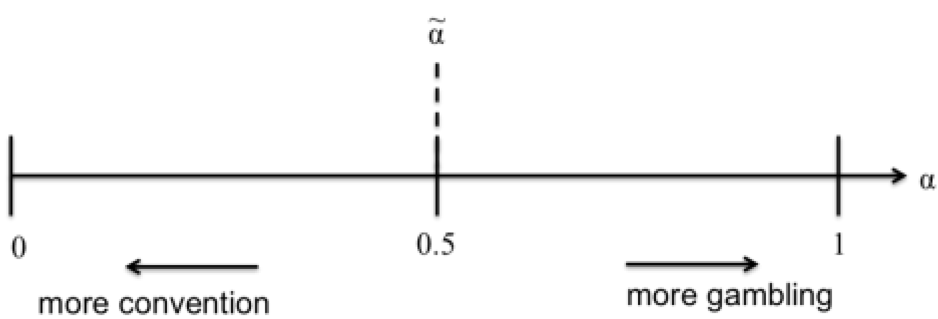


Fig. 3.3. Movement of $\tilde{\alpha}$ in Response to Market Expansion

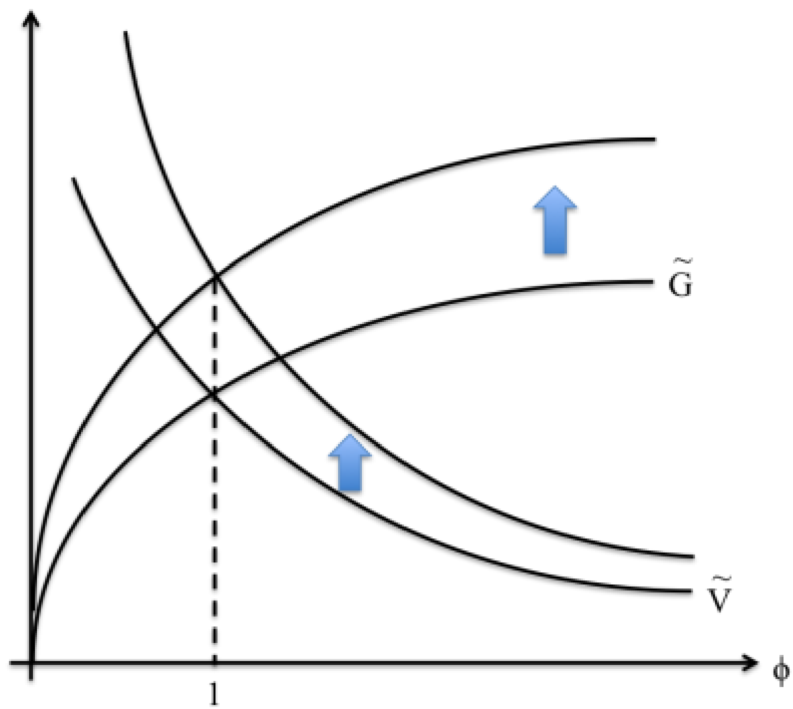


Fig. 3.4. Responses of $\tilde{G}$ and $\tilde{V}$ to Market Expansion

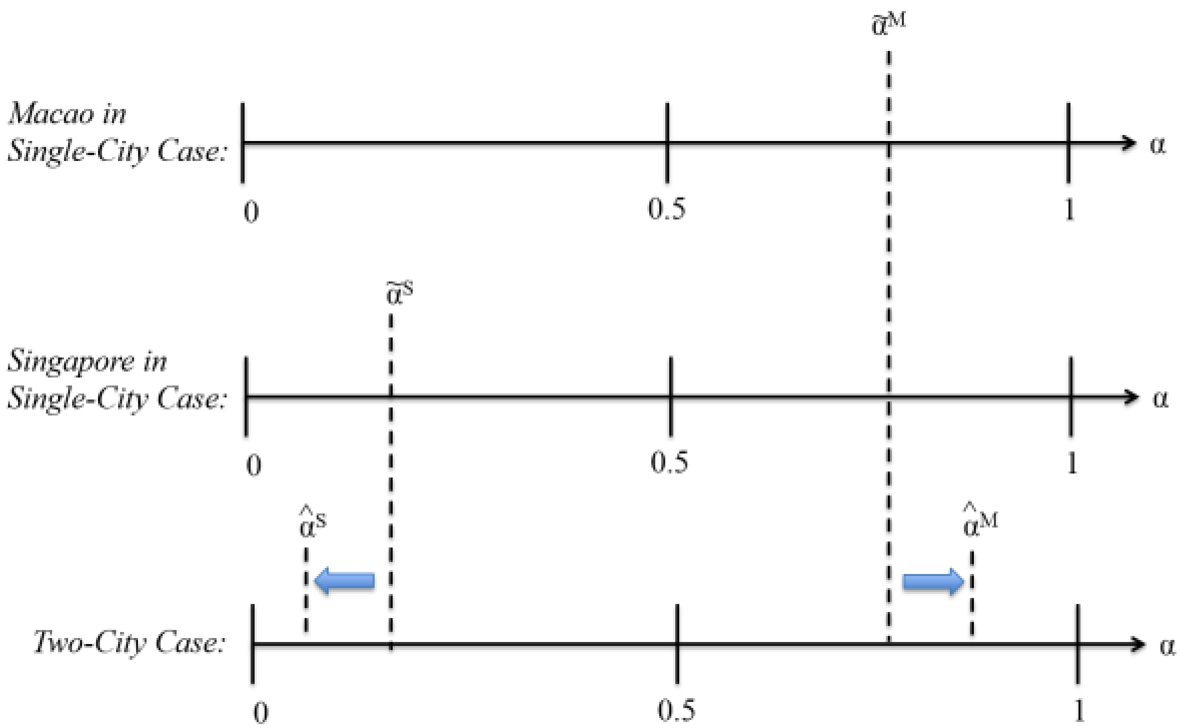


Fig. 3.5. Competition Leads to Higher Degree of Product Differentiation

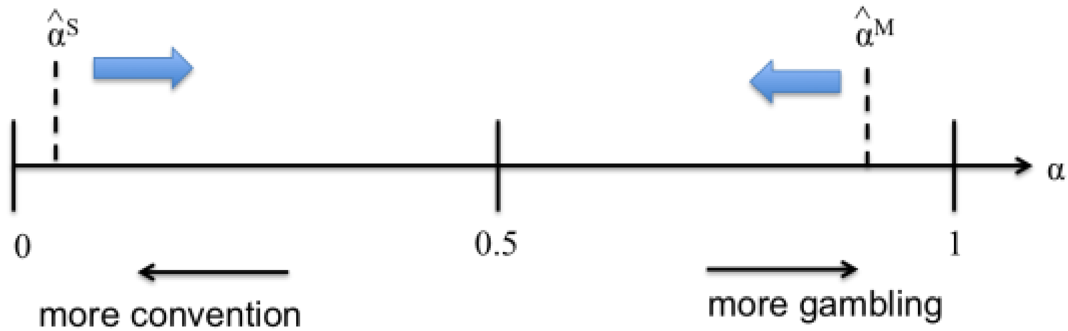


Fig. 3.6. Each City Diversifies More in Response to Increase of Demand

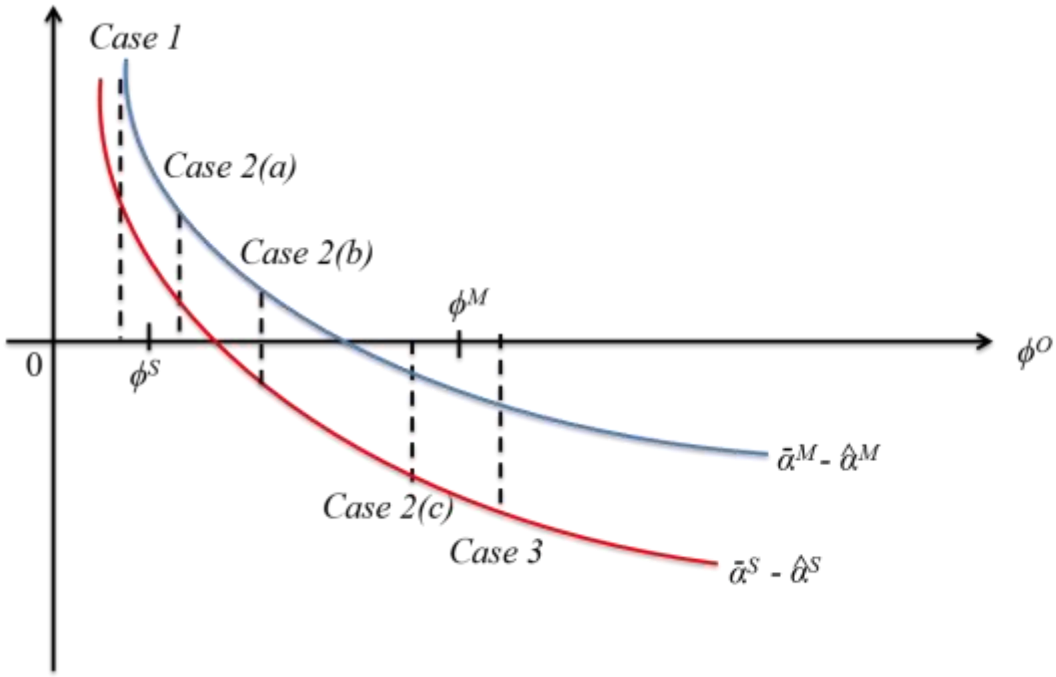


Fig. 3.7. Effects of Osaka's Entry on Macao's and Singapore's Proportions of Gambling Depends on ϕ^O

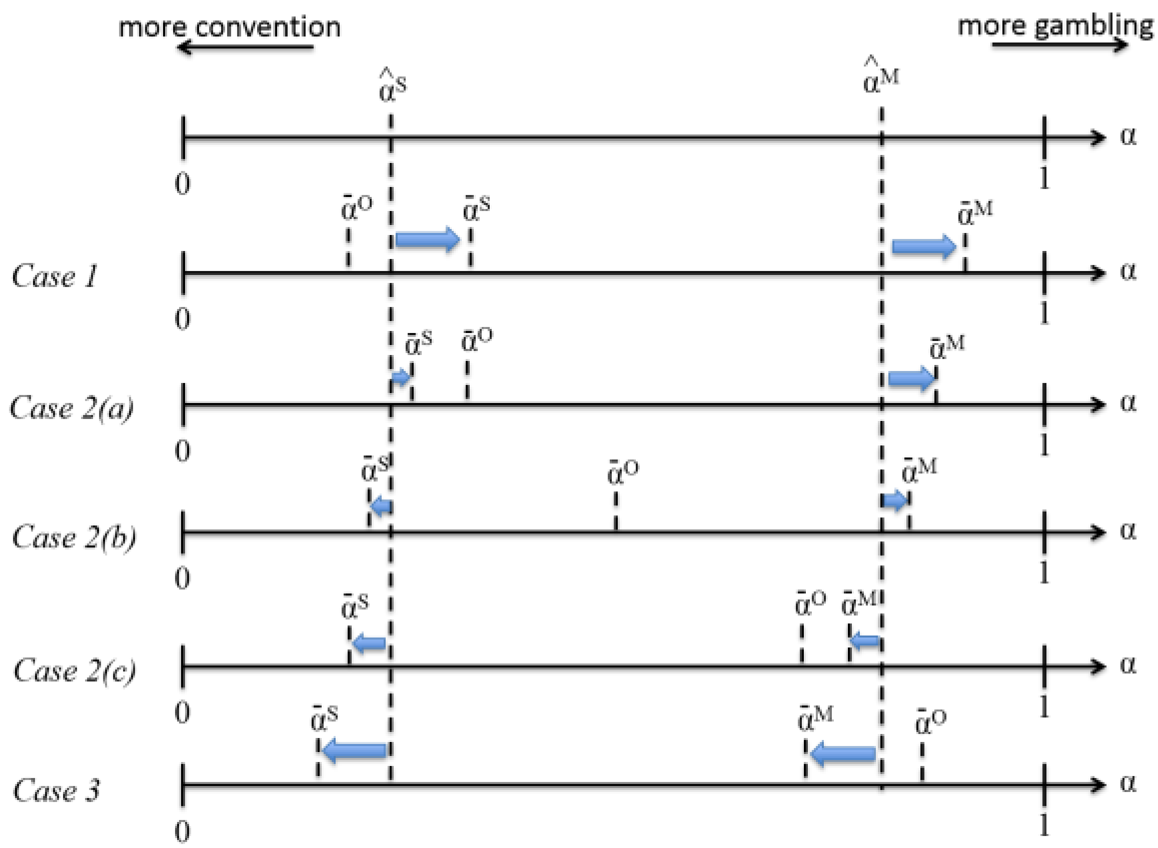


Fig. 3.8. Effects of Osaka's Entry on Macao's and Singapore's Product Strategies Depend on Osaka's Product Strategy

Table 3.1.
A Numerical Example

	Macao ($\phi^M = 2$)		Singapore ($\phi^S = 0.5$)	
	Single City	Two Cities	Single City	Two Cities
G	21.39	16.12	17.72	13.68
V	12.43	8.34	27.09	22.97
α	0.63	0.66	0.40	0.37
P_G	72.39	54.54	68.72	54.54
P_V	76.87	53.78	64.05	53.78
π	1642.04	860.22	2208.14	1478.79

Note: $a = 100$, $b = 1$, $\theta = 0.5$, $c_0 = 1$, $c_1 = 2$ and $c_2 = 1$. Corrected to 2 decimal places.

Table 3.2.
A Numerical Example of $\phi^M > 1 > \phi^S > \phi^O$

	Macao ($\phi^M = 2$)		Singapore ($\phi^S = 0.5$)		Osaka ($\phi^O = 0.25$)
	Two Cities	Three Cities	Two Cities	Three Cities	Three Cities
G	16.12	12.94	13.68	11.20	10.20
V	8.34	5.68	22.97	16.13	22.17
α	0.66	0.70	0.37	0.41	0.31
P_G	54.54	43.67	54.54	43.67	43.67
P_V	53.78	38.85	53.78	38.85	38.85
π	860.22	502.25	1478.79	820.26	1047.13

Note: $a = 100$, $b = 1$, $\theta = 0.5$, $c_0 = 1$, $c_1 = 2$ and $c_2 = 1$. Corrected to 2 decimal places.

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Appendix A

PROOFS AND ADDITIONAL EXPLANATION OF CHAPTER 1

The conventional definition of product relationship uses the demand functions instead of the inverse demand functions. This yields the "*p*-definitions." Differentiate the demand functions:

$$\frac{\partial X^1}{\partial P^1} = -\frac{B^2}{B^1B^2 - \Gamma^1\Gamma^2}, \tag{i}$$

$$\frac{\partial X^2}{\partial P^2} = -\frac{B^1}{B^1B^2 - \Gamma^1\Gamma^2}, \tag{ii}$$

which are negative only if $B^1B^2 - \Gamma^1\Gamma^2 \geq 0$. Hence the condition, $B^1B^2 - \Gamma^1\Gamma^2 \geq 0$, must be satisfied in order to be consistent with the law of demand. Also,

$$\frac{\partial X^1}{\partial P^2} = \frac{\Gamma^1}{B^1B^2 - \Gamma^1\Gamma^2}, \tag{iii}$$

$$\frac{\partial X^2}{\partial P^1} = \frac{\Gamma^2}{B^1B^2 - \Gamma^1\Gamma^2}. \tag{iv}$$

If the products are *p*-substitutes, $\frac{\partial X^1}{\partial P^2}$ and $\frac{\partial X^2}{\partial P^1}$ will be positive, requiring $\Gamma^1 > 0$ and $\Gamma^2 > 0$. If the products are *p*-complements, $\frac{\partial X^1}{\partial P^2}$ and $\frac{\partial X^2}{\partial P^1}$ will be negative, so $\Gamma^1 < 0$ and $\Gamma^2 < 0$. If the products are unrelated, $\frac{\partial X^1}{\partial P^2} = \frac{\partial X^2}{\partial P^1} = 0$, so it requires $\Gamma^1 = \Gamma^2 = 0$.

Also, if the products were perfect substitutes, $\frac{\partial X^1}{\partial P^2}$ and $\frac{\partial X^2}{\partial P^1}$ would approach infinity. This would happen when $B^1B^2 - \Gamma^1\Gamma^2 = 0$, but this study rules out perfect substitutability. Together with the requirement to be consistent with the law of demand, this study assumes $B^1B^2 -$

$\Gamma^1\Gamma^2 > 0$. It is useful to note that, in effective terms, $b^1b^2 - \gamma^1\gamma^2 = \frac{B^1}{{\theta^1}^2}\frac{B^2}{{\theta^2}^2} - \frac{\Gamma^1}{\theta^1\theta^2}\frac{\Gamma^2}{\theta^1\theta^2} = \frac{B^1B^2 - \Gamma^1\Gamma^2}{(\theta^1\theta^2)^2} > 0$.

This model uses the less conventional "*q*-definitions" instead of "*p*-definitions," but the two have same requirements. In effective terms, $\frac{\partial p^1}{\partial x^1} = b^1$, $\frac{\partial p^2}{\partial x^2} = b^2$, $\frac{\partial p^1}{\partial x^2} = \gamma^1$ and $\frac{\partial p^2}{\partial x^1} = \gamma^2$. If the products are *q*-substitutes, γ^1 and γ^2 will be positive. If the products are *q*-complements, γ^1 and γ^2 will be negative. If the products are unrelated, γ^1 and γ^2 will equal zero. Also, if the products were perfect *q*-substitutes, $\frac{\partial p^1}{\partial x^1} = \frac{\partial p^1}{\partial x^2}$ and $\frac{\partial p^2}{\partial x^2} = \frac{\partial p^2}{\partial x^1}$. This would happen when $b^1 - \gamma^1 = 0$ and $b^2 - \gamma^2 = 0$. However, since this study rules out perfect substitutability, it assumes $b^1 - \gamma^1 > 0$ and $b^2 - \gamma^2 > 0$. These are slightly stricter than the requirement of the "*p*-definitions" that $b^1b^2 - \gamma^1\gamma^2 > 0$.

Appendix B

MATHEMATICAL PROOFS OF CHAPTER 3

B.1 The Case of Two Cities

$\Omega \equiv (b-\theta+2\phi^M c_2)(b-\theta+2\phi^S c_2)(3b+3\theta+2c_2)+2(b^2-\theta^2)(1-\phi^M-\phi^S)c_2 > 0$

For $i = M, S$, $j = M, S$ and $i \neq j$,

$$\Gamma^i \equiv \left[2(b-\theta)\left(b-3\phi^j\theta+2\phi^j\left(1-2\phi^i\right)c_2\right)+4\left(b-5\phi^j\theta\right)\phi^i c_2\right]c_2,$$

$$\gamma^i \equiv -\left\{2(b+\theta)\left[b+2\left(\phi^i+\phi^j-2\phi^i\phi^j\right)c_2\right]-6\theta\left(\phi^i b-\phi^j\theta\right)-2\theta\phi^i\phi^j\,(3b+8c_2)\right.$$
$$\left.+2b\theta\right\}c_2+3\theta(b^2-\theta^2)\left(1-2\phi^i+\phi^j\right),$$

$$N^i \equiv \left[-4(b^2-\theta^2)\left(\phi^i+\phi^j-1\right)+2(b-\theta)\left(3\phi^j b-\theta\right)\right.$$
$$+4(b+c_2-\theta)\left(2\phi^j-\phi^i-3\phi^i\phi^j\right)c_2+4\left(b+\phi^i c_2\right)\left(1+\phi^j\right)c_2$$
$$\left.-24\theta\phi^i\phi^j c_2\right]c_2,$$

$$\nu^i \equiv \left\{6(b^2-\theta^2)\left(2\phi^j-\phi^i\right)-4(b+\theta)\left[\phi^i\left(1+\phi^j\right)-2\phi^j\right]c_2-b\left(6\phi^i b-2\theta\right)\left(1+\phi^j\right)\right.$$
$$\left.+4\left[b^2+\phi^j\theta\left(b+4\phi^i c2\right)\right]\right\}c_2+3b(b^2-\theta^2)\left(1-2\phi^i+\phi^j\right).$$

Note that $\Omega + N^i = (3b-3\theta+2c_2)[b^2-\theta^2+2b\left(1+\phi^j\right)c_2-4\phi^j c_2^2]$. Hence,

$$\Gamma^i - N^i = \left\{2(b^2-\theta^2)\left(2\phi^i-\phi^j-1\right)+4(b-\theta)\left(\phi^i-\phi^j\right)c_2\right.$$
$$\left.+4\left[\left(\phi^j+1\right)b+2\phi^j c_2\right]\left(\phi^i-1\right)c_2\right\}c_2 \gtrless 0 \qquad if \quad \phi^i \gtrless 1 \gtrless \phi^j,$$

and

$$\gamma^i - \nu^i = 3[b^3-\theta^3+(b-\theta)b\theta]\left(2\phi^i-\phi^j-1\right)+\{6(b^2-\theta^2)\left(\phi^i-\phi^j\right)+(b+\theta)[6(\phi^j$$
$$+1)b+12\phi^j c_2](\phi^i-1)\}c_2 \gtrless 0 \qquad if \quad \phi^i \gtrless 1 \gtrless \phi^j.$$

$$\beta^i \equiv \{(b-\theta)^2[(\phi^i - 2\phi^j + 1)(b+\theta) + 2(\phi^i - \phi^j)c_2]$$

$$+ 2(b-\theta)[(1-\phi^i)b + 2\phi^i c_2](1-\phi^j)c_2\}c_2 \gtrless 0 \qquad if \quad \phi^i \gtrless 1 \gtrless \phi^j,$$

$$\delta^i \equiv (\phi^i - 1)\theta\{(b^2-\theta^2)(4\phi^i - 5\phi^j + 1) + 4[(1-\phi^j)\phi^i c_2 + b(\phi^i - \phi^j)]c_2$$

$$+ 2b(1+\phi^i)(1-\phi^j)c_2\}c_2 \gtrless 0 \qquad if \quad \phi^i \gtrless 1 \gtrless \phi^j,$$

and

$$\varepsilon^i \equiv 3(\phi^i - \phi^j)((b^2-\theta^2)^2 + \{(b+\theta)(b^2-\theta^2+2bc_2)(\phi^i - 2\phi^j + \phi^i\phi^j)$$

$$+ 2(b+\theta)[(2\phi^i\phi^j(\phi^i - 1)c_2 - \phi^{i2}(1-\phi^j)b]c_2$$

$$+ 2(b^2-\theta^2)(\phi^i - \phi^j)(3b + \phi^i b - 2\phi^i\theta + 3\phi^i c_2)\}c_2 \gtrless 0 \qquad if \quad \phi^i \gtrless 1 \gtrless \phi^j.$$

B.2 The Case of Three Cities

$$\Sigma \equiv \begin{vmatrix} -2(b+c_2) & -2\theta & -b & -\theta & -b & -\theta \\ -2\theta & -2(b+\phi^M c_2) & -\theta & -b & -\theta & -b \\ -b & -\theta & -2(b+c_2) & -2\theta & -b & -\theta \\ -\theta & -b & -2\theta & -2(b+\phi^S c_2) & -\theta & -b \\ -b & -\theta & -b & -\theta & -2(b+c_2) & -2\theta \\ -\theta & -b & -\theta & -b & -2\theta & -2(b+\phi^O c_2) \end{vmatrix} > 0$$

For $i = M, S, O$, $j = M, S, O$, $k = M, S, O$ and $i \neq j \neq k$,

$$\begin{aligned} X^i \equiv\ & 8\big[2(b-\theta)\big(\phi^i\phi^j + \phi^i\phi^k + \phi^j\phi^k\big) + \theta\phi^i\big(\phi^i + \phi^k\big) + 2\phi^i\phi^j\phi^k(b+c_2)\big]c_2^4 \\ & + 4\big\{\big[\phi^i(\phi^j\phi^k b^2 + b\theta + \big(\phi^j + \phi^k\big)\theta^2\big] \\ & + b(b-\theta)\big[3\big(\phi^i + \phi^j + \phi^k\big) + 4\big(\phi^i\phi^j + \phi^i\phi^k + \phi^j\phi^k\big)\big] \\ & + \theta(b-\theta)\big(3\phi^i\phi^j + 3\phi^i\phi^k - 2\phi^j\phi^k\big)\big\}c_2^3 \\ & + 2(b-\theta)\big\{(b^2-\theta^2)\big[6\big(\phi^i + \phi^k + \phi^j\big) + \phi^j\phi^k\big] + 2b(b+\theta)\phi^i\big(\phi^j + \phi^k\big) \\ & + (b-\theta)^2\phi^j\phi^k + (b-\theta)\big[4b + \big(4\phi^i - \phi^j - \phi^k\big)\theta\big] + \theta[4b \\ & + 2\big(4\phi^i + \phi^j + \phi^k\big)\theta]\big\}c_2^2 \\ & + (b^2-\theta^2)\big\{3(b-\theta)^2\big(\phi^j + \phi^k\big) + 3(b^2-\theta^2)\phi^i \\ & + 2(b-\theta)\big[4b + \big(\phi^j + \phi^k\big)\theta\big]\big\}c_2 + 2(b^2-\theta^2)^2(b-\theta), \end{aligned}$$

$$\begin{aligned} \chi^i \equiv\ & 4\theta\big(4\phi^i\phi^j\phi^k - \phi^i\phi^j - \phi^i\phi^k - 2\phi^j\phi^k\big)c_2^4 \\ & + 2b\theta\big(8\phi^i\phi^j\phi^k - 2\phi^i - 3\phi^j - 3\phi^k + 3\phi^i\phi^j + 3\phi^i\phi^k - 6\phi^j\phi^k\big)c_2^3 \\ & + \big[(b^2-\theta^2)\theta\big(2\phi^i - 7\phi^j - 7\phi^k + 12\phi^i\phi^j + 12\phi^i\phi^k - 12\phi^j\phi^k\big) \\ & + 4\theta^3\big(\phi^i - \phi^j - \phi^k + \phi^i\phi^j + \phi^i\phi^k - \phi^j\phi^k\big) + 4b^2\theta(\phi^i\phi^j\phi^k - 1)\big]c_2^2 \\ & + 4(b^2-\theta^2)b\theta\big(2\phi^i - \phi^j - \phi^k + \phi^i\phi^j + \phi^i\phi^k - \phi^j\phi^k - 1\big)c_2 \\ & + (b^2-\theta^2)^2\theta(3\phi^i - \phi^j - \phi^k - 1), \end{aligned}$$

$$T^i \equiv (2b - 2\theta - c_2)[b^2 - \theta^2 + 2bc_2\big(1+\phi^j\big) + 4\phi^j c_2^2][b^2 - \theta^2 + 2bc_2(1+\phi^k) + 4\phi^k c_2^2],$$

$$\tau^i \equiv -\{16(\phi^i - 1)\phi^j\phi^k c_2^5 + 8b(2\phi^i\phi^j + 2\phi^i\phi^k - 8\phi^j\phi^k - \phi^j - \phi^k + 6\phi^i\phi^j\phi^k)c_2^4$$
$$+ 4\{(b^2 - \theta^2)(12\phi^i\phi^j + 12\phi^i\phi^k - 12\phi^j\phi^k - 7\phi^j - 7\phi^k)$$
$$+ b^2[3\phi^i - 1 + 9(\phi^i - 1)\phi^j\phi^k] + 2\theta^2(3\phi^i\phi^j + 3\phi^i\phi^k - 3\phi^j - 3\phi^k$$
$$- \phi^j\phi^k)\}c_2^3$$
$$+ 2b[(b^2 - \theta^2)(18\phi^i - 15\phi^j - 15\phi^k + 18\phi^i\phi^j + 18\phi^i\phi^k - 22\phi^j\phi^k - 6)$$
$$+ 4b^2\phi^i\phi^j\phi^k + 4\theta^2(\phi^i - \phi^j - \phi^k + \phi^i\phi^j + \phi^i\phi^k - \phi^j\phi^k - 1)]c_2^2$$
$$+ (b^2 - \theta^2)[(b^2 - \theta^2)(27\phi^i - 13\phi^j - 13\phi^k - 9)$$
$$+ b^2(\phi^i\phi^j + \phi^i\phi^k - \phi^j\phi^k) + 8\theta^2(2\phi^i - \phi^j - \phi^k - 1)]c_2$$
$$+ 2b(b^2 - \theta^2)^2(3\phi^i - \phi^j - \phi^k - 1)\}.$$

Hence,

$$X^i - T^i = 16(\phi^i - 1)\phi^j\phi^k c_2^5$$
$$+ 8[(b - \theta)(2\phi^i\phi^j + 2\phi^i\phi^k - 4\phi^j\phi^k) + b(2\phi^i\phi^j\phi^k - \phi^j - \phi^k) + \theta(\phi^i\phi^j$$
$$+ \phi^i\phi^k - 2\phi^j\phi^k)]c_2^4$$
$$+ 4[3(b^2 - \theta^2)(\phi^i\phi^j + \phi^i\phi^k - \phi^j\phi^k)$$
$$+ (b - \theta)^2(3\phi^i - 3\phi^j + \phi^i\phi^j + \phi^i\phi^k - 2\phi^j\phi^k)$$
$$+ \theta(b - \theta)(4\phi^i - 5\phi^j + \phi^k + \phi^i\phi^j + \phi^i\phi^k - 2\phi^j\phi^k) + b^2(\phi^i\phi^j\phi^k - 1)$$
$$+ \theta^2(\phi^i - \phi^j - \phi^k + \phi^i\phi^j + \phi^i\phi^k - \phi^j\phi^k)]c_2^3$$
$$+ 2(b^2 - \theta^2)[3(b - \theta)(2\phi^i - \phi^j - \phi^k) + 2b(\phi^i\phi^j + \phi^i\phi^k - \phi^j\phi^k - 1)$$
$$+ 2\theta(2\phi^i - \phi^j - \phi^k)]c_2^2 + b(b^2 - \theta^2)^2(3\phi^i - \phi^j - \phi^k - 1)c_2,$$

such that $X^M - T^M > 0$, $X^S - T^S < 0$ and $X^O - T^O \gtreqless 0$ if $\phi^O \gtreqless 1$.

$$\chi^i - \tau^i = 16(\phi^i - 1)\phi^j\phi^k c_2^5$$
$$+ 8[2(b-\theta)(\phi^i\phi^k - 4\phi^j\phi^k + 3\phi^i\phi^j\phi^k) + b(2\phi^i\phi^j - \phi^j - \phi^k)$$
$$+ \theta(\phi^i\phi^k - \phi^i\phi^j + 10\phi^i\phi^j\phi^k - 10\phi^j\phi^k)]c_2^4$$
$$+ 4[(b^2-\theta^2)(12\phi^i\phi^j + 12\phi^i\phi^k - 10\phi^j\phi^k - 7\phi^j - 7\phi^k)$$
$$+ b(b-\theta)(3\phi^i - 11\phi^j\phi^k + 9\phi^i\phi^j\phi^k - 1)$$
$$+ (b-2\theta)\theta(\phi^i\phi^j + \phi^i\phi^k - \phi^j - \phi^k) + b\theta(\phi^i - 1)(17\phi^j\phi^k + 1)]c_2^3$$
$$+ 2\{b(b^2-\theta^2)(6\phi^i + 6\phi^i\phi^j + 6\phi^i\phi^k - 6\phi^j\phi^k - 5\phi^j - 5\phi^k - 2)$$
$$+ \theta(b^2-\theta^2)(2\phi^i - 7\phi^j - 7\phi^k + 12\phi^i\phi^j + 12\phi^i\phi^k - 12\phi^j\phi^k)$$
$$+ 4\theta^2(b-\theta)(\phi^i\phi^j + \phi^i\phi^k - \phi^j - \phi^k) + 4b[(\phi^i - 1)(\phi^j\phi^k b^2 + \theta^2)$$
$$+ (\phi^i\phi^j\phi^k - 1)b\theta] + 4\theta^3(\phi^i - 2\phi^j - 2\phi^k + 2\phi^i\phi^j + 2\phi^i\phi^k - \phi^j\phi^k)\}c_2^2$$
$$+ [(b^2-\theta^2)^2(11\phi^i - 5\phi^j - 5\phi^k - 1) + 8b(b+\theta)(b^2-\theta^2)(2\phi^i - \phi^j - \phi^k$$
$$+ \phi^i\phi^j + \phi^i\phi^k - \phi^j\phi^k - 1)]c_2 + 2(b+\theta)(b^2-\theta^2)^2(3\phi^i - \phi^j - \phi^k - 1).$$

such that $\chi^M - \tau^M > 0$, $\chi^S - \tau^S < 0$ and$\chi^O - \tau^O \gtrless 0$ if $\phi^O \gtrless 1$.

$$\frac{\partial X^i}{\partial \phi^i} = 16\phi^j\phi^k c_2^5 + 8[2(b-\theta)(\phi^j + \phi^k) + 2b\phi^j\phi^k + \theta(\phi^j + \phi^k)]c_2^4$$
$$+ 4[3(b^2-\theta^2)(\phi^j + \phi^k) + b(b-\theta)(\phi^j + \phi^k + 3) + b^2\phi^j\phi^k + b\theta + \theta^2(\phi^j$$
$$+ \phi^k)]c_2^3$$
$$+ 4[3(b^2-\theta^2)(b-\theta) + b(b^2-\theta^2)(\phi^j + \phi^k) + 2\theta(b-\theta)^2 + 4\theta^2(b$$
$$- \theta)]c_2^2 + 3(b^2-\theta^2)^2 c_2 > 0$$

$$\frac{\partial \chi^i}{\partial \phi^i} = 8\theta(4\phi^j\phi^k - \phi^j - \phi^k)c_2^4 + 4b\theta(3\phi^j + 3\phi^k + 8\phi^j\phi^k - 2)c_2^3$$
$$+ 4[2\theta^3(\phi^j + \phi^k + 1) + \theta(b^2-\theta^2)(6\phi^j + 6\phi^k + 1) + 2b^2\theta\phi^j\phi^k]c_2^2$$
$$+ 8b\theta(b^2-\theta^2)(\phi^j + \phi^k + 2)c_2 + 6\theta(b^2-\theta^2)^2 > 0$$

$$\frac{\partial T^i}{\partial \phi^i} = 0$$

$$\frac{\partial\tau^i}{\partial\phi^i} = -16\phi^j\phi^k c_2^5 - 16b\left(\phi^j + \phi^k + 3\phi^j\phi^k\right)c_2^4$$
$$- 4\left[6(2b^2 - \theta^2)\left(\phi^j + \phi^k\right) + 3b^2\left(3\phi^j\phi^k + 1\right)\right]c_2^3$$
$$- 4b\left[(9b^2 - 7\theta^2)\left(\phi^j + \phi^k + 1\right) + 2b^2\phi^j\phi^k\right]c_2^2$$
$$- (b^2 - \theta^2)\left[27b^2 - 11\theta^2 + 8b^2\left(\phi^j + \phi^k\right)\right]c_2 - 6b(b^2 - \theta^2)^2 < 0$$

$$\eta^i \equiv 2(b-\theta)\left[(b + 2c_2)\left(b + 2\phi^j c_2\right) - \theta^2\right]\Big\{16\phi^i\phi^j(1 - \phi^k)c_2^4$$
$$+ 8\left[(b-\theta)\left[\phi^i\left(\phi^j - \phi^k\right) + \left(\phi^i - \phi^k\right)\phi^j\right] + b(1 - \phi^k)(\phi^i + \phi^j + 2\phi^i\phi^j\right]c_2^3$$
$$+ 4\left[(b^2 - \theta^2)(1 - \phi^k)\left(\phi^i + \phi^j\right)\right.$$
$$+ (b-\theta)(2b+\theta)\left[\left(\phi^j - \phi^k\right)\phi^i + \left(\phi^i - \phi^k\right)\phi^j\right] + b(b-\theta)\left(\phi^i + \phi^j - 2\phi^k\right)$$
$$\left.+ b^2(1 - \phi^k)\left(1 + \phi^i + \phi^j + \phi^i\phi^j\right)\right]c_2^2$$
$$+ 2(b^2 - \theta^2)\Big[(b-\theta)\left(\phi^i + \phi^j - 2\phi^k\right)$$
$$- 2b\left[\left(1 + \phi^i\right)(1 - \phi^k) + \phi^j - \phi^k + \left(\phi^i - \phi^k\right)\phi^j\right]\Big]c_2$$
$$+ (b^2 - \theta^2)^2\left(1 + \phi^i + \phi^j - 3\phi^k\right)\Big\}c_2 > 0 \qquad if \quad \phi^i > 1 > \phi^j > \phi^k$$

$$
\begin{aligned}
\kappa^i \equiv {} & 64\phi^i{\phi^j}^2(1-\phi^k)(\phi^i-1)c_2^6 + 32b\phi^j(\phi^i-1)\{2\phi^i - 2\phi^j\phi^k \\
& + \phi^j(1+3\phi^i)(1-\phi^k) + 4\phi^i(\phi^j-\phi^k)\}c_2^5 \\
& + 16\Big\{\Big[\phi^i(\phi^j-\phi^k) + \big(9\phi^i{\phi^j}^2 + {\phi^i}^2\phi^j\big)(\phi^i-\phi^k) + \phi^i\phi^j\phi^k - {\phi^j}^2\Big](b^2 \\
& - \theta^2) \\
& + \Big[[\phi^j(\phi^i-1) - 6 - 8\phi^j](\phi^i-\phi^k) \\
& + \Big[\phi^i(3+5\phi^j)(\phi^i-1) + 4{\phi^i}^2\phi^j + 2\big({\phi^i}^2-1\big)\phi^j\Big](\phi^j-\phi^k) \\
& + \phi^i\phi^j(\phi^i-\phi^j) + \big(\phi^i - 2{\phi^j}^2\phi^k - {\phi^i}^2\phi^j\big)(\phi^i-1) \\
& - \phi^j(\phi^i+2\phi^j)(1-\phi^k) + 2\phi^j\big({\phi^i}^2-1\big) + 3{\phi^i}^2\phi^j(1-\phi^j\phi^k)\Big]b^2 \\
& + \Big[7{\phi^j}^2(\phi^i-\phi^k) + \phi^i\phi^j(\phi^j-\phi^k) + {\phi^i}^2\phi^j(\phi^i-\phi^j) \\
& + 3\phi^i\phi^j\phi^k(\phi^i-1) + 2\phi^i\phi^j(1-\phi^k)\Big]\theta^2\Big\}c_2^4 \\
& + 8b\Big\{\Big[\phi^i\phi^j(11+12\phi^j)(\phi^i-\phi^k) + {\phi^i}^2(8\phi^j+9)(\phi^j-\phi^k) \\
& + 4\phi^j\big({\phi^i}^2-1\big) + 5\phi^i\phi^k\big(1-{\phi^j}^2\big) + \phi^i\phi^j(\phi^i-\phi^j\phi^k)\Big](b^2-\theta^2) \\
& + \Big[\Big[(4+\phi^j)(\phi^i-1)\phi^j - 21\phi^j - 12{\phi^j}^2 - 4\Big](\phi^i-\phi^k) \\
& + \Big[3{\phi^i}^2 - 1 - 2\phi^j + 4\big({\phi^i}^2-1\big)\phi^j - 4\phi^j\Big](\phi^j-\phi^k) - {\phi^j}^2\phi^k(\phi^i-1) \\
& + [\phi^j(1-\phi^j\phi^k)+1]\big({\phi^i}^2-1\big) + 2({\phi^i}^2-\phi^j)\Big]b^2 \\
& + \Big[\phi^j(21+11\phi^j)(\phi^i-\phi^k) + 6\phi^j(\phi^j-\phi^k) + (2\phi^i+{\phi^j}^2)(1 \\
& - \phi^k)\Big]\theta^2\Big\}c_2^3 \\
& + 4(b^2 \\
& - \theta^2)\Big\{\Big[\big(4{\phi^i}^2 + \phi^i\phi^j\big)(\phi^j-\phi^k) + (2{\phi^j}^2 + 13\phi^i\phi^j)(\phi^i-\phi^k)\Big](b^2 \\
& - \theta^2) \\
& + \Big[\Big[(9+16\phi^j)(\phi^i-1) + 3\phi^i{\phi^j}^2 - 4\phi^j - 3\Big](\phi^i-\phi^k) \\
& + \big(8{\phi^i}^2 - 8 + 4{\phi^i}^2\phi^j - 4\phi^j - 6\phi^j - 1\big)(\phi^j-\phi^k) - 5\phi^j\phi^k(\phi^i-\phi^j) \\
& - 3\phi^j(1-\phi^k) + 3\phi^j(\phi^i\phi^j-1)(\phi^i-2\phi^k) - 3\phi^j(\phi^i\phi^j-\phi^k) - 3(1 \\
& - \phi^j\phi^k)\Big]b^2 \\
& + \Big[(\phi^i+2\phi^j)(1-\phi^k) + 10\phi^j(\phi^i-\phi^k) + 3(\phi^i+\phi^j)(\phi^j-\phi^k) \\
& + 3\phi^j\phi^k(\phi^i-1) + {\phi^i}^2 - \phi^j\phi^k\Big]\theta^2\Big\}c_2^2 \\
& + 2b(b^2-\theta^2)^2\Big\{\big(13\phi^i - 13 + \phi^i\phi^j - \phi^j + 2{\phi^j}^2 + 12\phi^i\phi^j - 3\big)(\phi^i \\
& - \phi^k) + \big(4{\phi^i}^2 + \phi^i\phi^j - 5\phi^j - 8\big)(\phi^j-\phi^k) - 8\phi^k(\phi^i-\phi^j) \\
& - 3(1-\phi^k) - 3\phi^j\phi^k(\phi^i-1)\Big\}c_2 \\
& + (b^2-\theta^2)^3\Big\{(\phi^i-1)\big[5\phi^i - 4\phi^k - 1 + 5(\phi^j-\phi^k)\big] + (\phi^i+\phi^j-2)(1 \\
& + \phi^i - \phi^j - \phi^k)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
\mu^i \equiv\ & 64\phi^i\phi^{j^2}\phi^k(\phi^i-1)c_2^7 \\
& + 32\phi^j\{\phi^i(4\phi^i\phi^j+\phi^i\phi^k-5\phi^j\phi^k)(b-\theta) \\
& + (\phi^i-1)[3(1+\phi^j)\phi^i\phi^k-2(\phi^i-\phi^k)\phi^j]b\}c_2^6 \\
& + 16\{\phi^j[2(\phi^i-\phi^k)\phi^j+3\phi^i\phi^k(\phi^i-1)](b^2-\theta^2) \\
& + 2[\phi^i\phi^k(1+\phi^j)(\phi^i-\phi^j)+5\phi^j(\phi^i+\phi^j+2\phi^i\phi^j)(\phi^i-\phi^k)]b(b-\theta) \\
& + [\phi^j(3+2\phi^i+3\phi^j-10\phi^i\phi^j)(\phi^i-\phi^k)+(\phi^j-4\phi^{i^2}\phi^j-5\phi^{i^2})(\phi^j-\phi^k) \\
& + \phi^j\phi^k(\phi^i-1)+\phi^j[2\phi^i(\phi^i\phi^{j^2}-1)+(\phi^{i^2}-1)\phi^{j^2}-3(\phi^{i^2}(1-\phi^j))]]b^2 \\
& + \phi^j[9\phi^i\phi^j(\phi^i-\phi^k)+\phi^{i^2}(\phi^j-\phi^k)]\theta^2\}c_2^5 \\
& + 8\{[10\phi^j(\phi^i+\phi^j)(\phi^i-\phi^k)+2\phi^i\phi^k(\phi^i-\phi^j)](b^3-\theta^3) \\
& + [4\phi^i(1-2\phi^i)(\phi^j-\phi^k)+2\phi^j(1+4\phi^j-9\phi^i\phi^j)(\phi^i-\phi^k) \\
& + 2\phi^j(2\phi^k-\phi^i)(\phi^i-\phi^j)+2\phi^j(4\phi^k-\phi^j)(\phi^{i^2}-1)]b(b^2-\theta^2) \\
& + [(5\phi^i+20\phi^j+32\phi^{j^2}+44\phi^i\phi^j+28\phi^i\phi^{j^2})(\phi^i-\phi^k)+\phi^j(\phi^j-\phi^k) \\
& + 2(\phi^{i^2}-\phi^j\phi^k)+\phi^i\phi^k(4+\phi^j)(\phi^i-\phi^j)]b^2(b-\theta) \\
& + [[1+2(1-\phi^j)\phi^j-26\phi^i\phi^j-16\phi^{j^2})(\phi^i-\phi^k) \\
& + (1+3\phi^j+2\phi^{i^2}(1-\phi^j)-\phi^{i^2}\phi^j)(\phi^j-\phi^k)+(\phi^k+\phi^i\phi^{j^2})(\phi^i-1) \\
& + \phi^{j^2}(\phi^i\phi^k(\phi^i+1)-2)-\phi^{i^2}(1-\phi^j)]b^3\}c_2^4 \\
& + 4(b-\theta)\{[3\phi^j(\phi^i-\phi^k)+2\phi^k(\phi^{i^2}-\phi^j)+2\phi^i\phi^k(\phi^i-1)](b^3-\theta^3) \\
& + [(43\phi^j+12\phi^i)(\phi^i-\phi^k)+19\phi^i\phi^j(\phi^j-\phi^k)+6\phi^k(\phi^{i^2}-\phi^j) \\
& + (2+17\phi^j)(\phi^{i^2}-\phi^j\phi^k)+3\phi^{j^2}(1-\phi^k)]b(b^2-\theta^2) \\
& + [\phi^j(2\phi^j+27\phi^i+9\phi^i\phi^j)(\phi^i-\phi^k)+\phi^{i^2}(10+\phi^j)(\phi^j-\phi^k)+3\phi^{j^2}(\phi^i-1) \\
& + \phi^{i^2}-\phi^{j^2}+7\phi^i(\phi^i-\phi^j\phi^k)]b^2(b-\theta) \\
& + [(15+20\phi^j+10\phi^{j^2}-2\phi^i-10\phi^i\phi^j)(\phi^i-\phi^k) \\
& + (3+7\phi^j-8\phi^{i^2}\phi^j-19\phi^i\phi^j-12\phi^{i^2})(\phi^j-\phi^k)-\phi^i\phi^j(11+3\phi^k)(\phi^i-\phi^j) \\
& - 2\phi^i(\phi^i-\phi^{j^2})+\phi^j(\phi^i\phi^j-\phi^k)+3\phi^k(\phi^i\phi^{j^2}-1)]b^3 \\
& + [(2\phi^{j^2}+17\phi^i\phi^j)(\phi^i-\phi^j)+\phi^{j^2}(\phi^i-1)]\theta^3 \\
& + [(3+2\phi^j)(\phi^i-\phi^k)+3(\phi^j-\phi^k)+7\phi^k(\phi^i-\phi^j)+\phi^k(3+2\phi^j)(\phi^i-1) \\
& + 3\phi^j\phi^k(\phi^{i^2}-1)]b^2\theta\}c_2^3 \\
& + 2(b^2-\theta^2)(b \\
& - \theta)\{[5(\phi^i+4\phi^j)(\phi^i-\phi^k)+2(\phi^{i^2}-\phi^j\phi^k)+\phi^j(\phi^j-\phi^k)](b^2-\theta^2) \\
& + [(17\phi^i\phi^j+9\phi^i+2\phi^{j^2})(\phi^i-\phi^k)+6\phi^i(\phi^j-\phi^k)+3\phi^{j^2}(\phi^i-1) \\
& + 4\phi^{i^2}(1-\phi^k)+2\phi^i\phi^j(1-\phi^j)]b(b-\theta) \\
& + [[39+32\phi^j+6\phi^{j^2}-2\phi^i(1+\phi^j)](\phi^i-\phi^k)+(3+6\phi^j-8\phi^{i^2})(\phi^j-\phi^k) \\
& + \phi^i[(8\phi^j+3)(\phi^i-1)+2(\phi^i-\phi^{j^2})+5(\phi^i-\phi^j)]]b^2 \\
& + [3(\phi^i-\phi^k)+3(\phi^j-\phi^k)+3\phi^k(\phi^i-1)+3\phi^k(\phi^i-\phi^j)]b\theta\}c_2^2 \\
& + (b \\
& - \theta)(b^2-\theta^2)^2\{[(5\phi^i-1)(\phi^i+\phi^j-2\phi^k)+\phi^k(1-\phi^j)+(\phi^{i^2}-\phi^{j^2})](b-\theta) \\
& + [(38+14\phi^j)(\phi^i-\phi^k)+4\phi^k(2\phi^i-\phi^j-1)+2\phi^j(1+\phi^j-2\phi^k)]b\}c_2 \\
& + 6(b-\theta)(b^2-\theta^2)^3(\phi^i-\phi^k)
\end{aligned}
$$