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Dynamic growth and yield modeling with climate: A model for plantation Douglas-fir in the Pacific Northwest

Kevin Robert Gehringer

A dissertation submitted in partial fulfillment
of the requirements for the degree of

Doctor of Philosophy

University of Washington

2001

Program Authorized to Offer Degree: College of Forest Resources

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Abstract

Dynamic growth and yield modeling with climate: A model for plantation Douglas-fir in the Pacific Northwest

by Kevin Robert Gehringer

Chair of Supervisory Committee

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A pair of nested, dynamic, stand level growth and yield models with an annual base time step were developed to examine the potential effects of climate, represented by the Palmer Drought Severity Index (PDSI), on the growth of untreated plantation Douglas-fir (*Pseudotsuga menziesii*) stands in the Pacific Northwest. The nested models consist of a system of three autonomous, nonlinear, ordinary differential equations represented in a canonical mathematical formulation called an S-system. The models project stand density, quadratic mean diameter, and top height. The models assume only that size-density relationships exist coupling stand density and QMD, that size-size relationships exist coupling QMD and top height, and that climate modifies mortality and average growth rates.

A two step procedure was used to develop the nested models. First, a base model without climate effects was defined and calibrated. Second, annual average PDSI values were converted into a multiplicative modifier used to adjust stand mortality and average growth rates. Parameters for both models were estimated using a least squares criterion, keeping the base model parameters fixed when calibrating the climate model to isolate the climate effects.

Data from 167 untreated Douglas-fir stands containing 270 research plots, and spanning the region from southern Oregon to southern British Columbia, west of the Cascade Mountains were used to calibrate the models. The PDSI based climate modifier was then used to calibrate the extended model. The Douglas-fir measurement data spanned the growing years 1969 through 1997, which included at least one drought, with initial stand ages ranging from 4 to 113 years and measurement histories ranging from 2 to 26 years.

The base model performed well and was generally unbiased, producing stand trajectories that were indistinguishable from the data. The extended model and the base model were also indistinguishable, indicating that with these models and this data, a direct climate effect was not detectable. Several factors contributing to this result were identified, the most important being that average climate effects on mortality and growth rates are already incorporated into the model through the measurement data and the least squares parameter estimation process.

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ACKNOWLEDGMENTS

First, I would like to thank my family. Without their continued love and support this achievement would not have been possible. Their faith in me was invaluable.

Next, I would like to thank my advisor and the members of my committee. Eric Turnblom, the chair of my committee, provided me with the opportunity to be involved in the College of Forest Resources and in this research. Tom Hinckley, Jay Johnson, and Dave Peterson were always available and maintained an interest in my development and encouraged me throughout my education, research, and the writing of this dissertation.

A special thanks must go to Bruce Bare. First for helping me to enter the College of Forest Resources, and second for his continued interest in my progress. Thank you for your time and consideration through the good times and the not so good times.

I would also like to thank several of my fellow graduate students for their encouragement and willingness to listen when my research was going well and, more importantly, when it was not going so well. These comrades are: Kim Brown, Kevin Ceder, Mark Cullington, Graham MacKenzie, Dan Peplow, Edie Sonne, Larry Mason, Diana Olson, and Mike Safoutin. Your support was greatly appreciated and cannot be underestimated.

Thanks also must go the Stand Management Cooperative, at the University of Washington, and its staff for providing the data I used for my research, the semi-annual meetings as a forum in which to present my research, and funding for the final stretch. Specifically I would like to thank Randy Collier, John Haukaas, Megan O'Shea, and Dave Briggs, the director, for their interest and support of my research.

Chapter 1

INTRODUCTION

In the United States, particularly in the Pacific Northwest. much federal forest land has been removed from the harvestable pool to protect the habitat of various wildlife species and to maintain existing reserves of old growth forest. Harvest restrictions are also being placed on private land holders in the Pacific Northwest and elsewhere. This reduction in the federal forest land available for harvest, and the removal of portions of private land for habitat, have led to increased pressure on the remaining private forest holdings to supply more of the demand for wood. In order to meet the increased demand for wood, forest management practices on private forest lands will become more intensive than ever before. In order to support the more intensive management of forests and to address habitat concerns, better growth and yield models will need to be developed for forest management and decision making. A new generation of forest growth and yield models is necessary, and must provide more accurate and detailed predictions than ever before to meet the demands that will be placed upon them.

The new generation of forest growth and yield models will need to better model the growth dynamics of individual trees and the development of forest stands [2, 9, 23. 52, 54, 53, 76, 82]. These new models must eventually incorporate climate and other environmental effects on forest growth [11, 35, 46, 86, 161, 171], as do global vegetation distribution models [100, 168], and the effects of the intensive management practices used to enhance wood production, wood quality, or habitat suitability, to adequately serve the needs of forest managers [44, 60, 70]. The new generation of forest growth and yield models will need to be dynamic, process based models that do more than simply describe the data that were used to calibrate them, that is, they will need to be models developed within a mathematical-explanatory modeling framework [9, 10, 72, 79, 108, 113, 149, 150, 151]. The new generation of growth and yield models must represent the best current understanding of the processes involved in forest development [13, 12].

Growth and yield models may fail to meet the future needs of forest managers because they dependence on a statistical-empirical, regression based modeling framework [5, 8, 17, 19, 24, 27, 58, 65, 153], and have a limited scope of development and use for wood production. A *modeling framework* is the combination of a methodology for defining models and then calibrating them, for example fitting a polynomial to data using least squares regression. Many of the original reasons for the use of a statistical-empirical modeling framework, namely the large volume of data that needed to be processed and the lack of high speed computers and appropriate mathematical software, are no longer relevant, but this modeling framework is still the most common [17, 40, 41, 155, 162].

The most critical factor influencing the future development of growth and yield models is their potential for use in regional decision making processes [42, 48]. This use demands that the models represent the cause-effect relationships for forest development, tree growth, harvesting, and other forest attributes, that is, that the models contain a simplified but correct representation of the real world. Representing cause-effect relationships requires a shift away from the typical statistical-empirical models. These models simply describe the data used in their calibration, and hence cannot be used to deduce cause-effect relationships, to mathematical-explanatory or process oriented models, which attempt to mimic the actual dynamic processes of forest development and tree growth. There has been some success with incorporating cause-effect relationships in forest simulation models, for example, gap models for forest succession [13, 12, 130], dynamic biogeochemical models [113], and models of tree and forest development [10, 108, 151, 152, 161]. However, the acceptance of these alternative model formulations as being of more than academic interest is, however, slow, possibly due to the perception the their data requirements are excessive.

Three factors are critical for the development of more robust forest growth and yield models. First, at all levels of model development and use the development of forest ecosystems and tree growth must be recognized as dynamic processes. This implies that changes to different components within a forest system occur simultaneously and with feedback, that is, one component may inhibit another, such as a size-density relationship. Thus, there is a need to bring the modeling framework used for growth and yield modeling into better alignment with the dynamic system being modeled. Second, current growth and yield models use of a variety of convenience parameters, such as site index, stand age, and maximum tree sizes, as fundamental components of the models. These convenience parameters are generally unnecessary [13, 12, 10, 130, 151, 152], and their ongoing use may hamper the effectiveness of growth and yield, or forest simulation, modeling [82, 84, 83, 95]. Third, climate, represented as temperature and precipitation or some index such as the Palmer Drought Severity Index [104], and other external factors, such as nutrient cycling, nitrogen deposition, CO_2 concentrations, must eventually be incorporated into the models.

1.1 Research objectives

This research address three critical factors identified as important for the future development of forest growth and yield models, the dynamic nature of forest stand development, the use of convenience parameters, and the lack of climate effects. These issues are addressed through the development of a nested pair of dynamic, stand level growth and yield model for plantation Douglas-fir (*Pseudotsuga menziesii*) in the Pacific Northwest. The nested models consist of a base model, that does not include climate effects, and an extended model that includes climate effects. Climate effects are included in the extended model by using a simple climate index derived from the Palmer Drought Severity Index (PDSI) [104].

The base Douglas-fir growth and yield model consists of three autonomous differential equations which project stand density as trees per hectare, quadratic mean diameter (QMD), and top height, the average height of the 100 largest diameter trees per hectare. The model is formulated as an S-system, a canonical mathematical representation for systems of nonlinear ordinary differential equations [118, 119, 120, 123, 158]. The S-system growth and yield model developed does not use any of the typical convenience parameters, such as site index, stand age, or maximum tree sizes. Although not included in the model, site index values for the stand data used to calibrate the model are reproduced by the model. Further, the model appears to capture the stand dynamics and tree growth characteristics of plantation Douglas-fir, as represented by the available data.

The base model is then extended to incorporate the effects of climate on stand development and average tree growth. The model extension includes climate effects through a climate index based on PDSI, and is performed in a nested manner, creating two compatible growth and yield models, one with climate effects and one without climate effects. The nested model extension, therefore, allows the recovery of the base growth and yield model by using a nominal or average climate value of one. The development of both a base model and an extended model with climate effects was done to avoid the introduction of biases from a model calibrated for a different region or to a different data set.

Chapter 2 addresses model representation and formulation issues for the base model. The primary weaknesses in using the typical statistical-empirical modeling framework for growth and yield modeling are identified, and a dynamic modeling framework based on the S-system and a least squares criterion, called the S-system modeling framework, is proposed as an alternative to the statistical-empirical modeling framework. Finally, a dynamic, stand level growth and yield model for plantation

Douglas-fir in the Pacific Northwest is then developed within the S-system modeling framework and shown to produce results that are consistent with the available stand measurement data. Chapter 3 addresses the extension of the base Douglas-fir growth and yield model to include climate effects. Finally, Chapter 4 presents some concluding remarks and several ideas for future work, including the extension of the dynamic Douglas-fir growth and yield model to include an individual tree based submodel.

Chapter 2

A DYNAMIC, STAND LEVEL GROWTH AND YIELD MODEL FOR PLANTATION DOUGLAS-FIR IN THE PACIFIC NORTHWEST

A dynamic, stand level growth and yield model has been developed for plantation Douglas-fir (*Pseudotsuga menziesii*) in the Pacific Northwest, west of the Cascade Mountains, USA. The model may be used to project stand density, quadratic mean diameter, and top height from a specified initial stand condition into the future. The model has been formulated using a minimum number of biologically relevant assumptions to define a tightly coupled system of three nonlinear ordinary differential equations, one for each state variable, which define the family of possible stand trajectories. The coupled system of differential equations which comprise the model is represented as an S-system, a canonical mathematical formulation for nonlinear ordinary differential equations (ODEs) based on a power-law formalism. The S-system representation for the model was chosen for its flexibility, high representational power, and its overall ease of use and interpretation. The canonical S-system representation also leads naturally to the idea of model extraction from data, or system identification, through the solution of a straightforward but nontrivial parameter estimation problem. The stand level Douglas-fir growth and yield model requires a minimum number of inputs: an initial stand density, quadratic mean diameter, top height, and the length of the projection interval. The model performs quite well when compared to available stand measurement data. Key features of the model are its ability to reproduce site index without explicitly including it, and its ability to produce maximum tree sizes that are within the range of the maximum observed Douglas-fir tree sizes without *a priori* limiting the maximum tree size.

2.1 Introduction

Modern forest growth and yield modeling, as typically practiced, relies almost exclusively on a statistical-empirical modeling framework [5, 8, 17, 19, 24, 27, 36, 58, 65, 140, 153, 154, 163]. Within this framework, a one dimensional perspective dominates, despite the multidimensional nature of forest stand development and tree growth. The equations representing the processes involved in stand development and tree growth within a growth and yield model are treated independently [5, 24, 36, 58, 65, 94, 153], although in an actual forest the processes represented by these equations occur simultaneously. Further, many statistical-empirical growth and yield models also include a variety of convenience parameters, such as site index. stand age, or various measures of inter-tree competition, e.g., crown competition factor or stand density index, that are correlated with the main state variables to improve the fit of the model to the calibrating data [24, 36, 65, 153], or maximum tree sizes to limit the potential size of trees grown using a model [24, 36, 65, 153]. For these reasons, the statistical-empirical growth and yield models do not provide the most appropriate representation for the dynamics of forest stand development.

A brief overview of modern growth and yield modeling from the typical statistical-empirical modeling framework follows. The overview identifies issues believed to be hindering the effectiveness of growth and yield models. The issues fall into two broad categories relating to the model formulation and representation, or the model definition. and model use, and the statistical and biological assumptions necessary to capture stand dynamics and tree growth within a growth and yield model.

An alternative modeling framework is proposed for forest growth and yield modeling. The alternative modeling framework represents forest stand dynamics as an autonomous system of ordinary differential equations, represented in a canonical, mathematical representation called an S-system. This alternative framework will be called the S-system modeling framework. A stand level, growth and yield model for plantation Douglas-fir is developed within the S-systemmodeling framework. The model projects stand density, QMD, and top height, representing a forest stand as a tree of average size and the number of trees per hectare. The formulation of the S-system growth and yield model is justified in terms of its mathematical and biological relevance. The model was calibrated using stand measurement data from 167 Douglas-fir installations consisting of 270 research plots spanning the Pacific Northwest region west of the Cascade Mountains. The model appears to perform quite well when compared with the available stand measurement data.

2.1.1 A brief overview of modern growth and yield modeling

Modern growth and yield models come in essentially three types: (1) stand level models, which simulate diameter and height growth for an average tree and stand density; (2) distance independent individual tree models, which simulate diameter and height growth for individual trees and stand density without spatial information for each tree; and (3) distance dependent individual tree models, which simulate diameter and height growth for individual trees and stand density with spatial locations for each tree [23, 24, 153]. The degree of detail contained within a growth and yield model is greatest for distance dependent individual tree models, maintaining the spatial location, the current size and growth history, and the species of each tree. The detail is least for stand level models, maintaining the proportion of a stand in each of several species and an average tree size, or size distribution, for each species. These models are frequently used for monoculture, plantation forests. Distance independent individual tree models are hybrid models maintaining the current size and growth history and species of each tree. but not the spatial location, and are by far the most common type of growth and yield model in use, particularly in the Pacific Northwest [36, 58, 94].

Each type of modern growth and yield model contains two or three increment equations for each species, that are used to project tree size and stand density, where increments are generally negative for stand density, representing mortality. The three equations estimate increments for number of trees, or density, for each species, and, hence, the stand as a whole, ΔN, diameter, ΔD, and height, ΔH, for a particular time increment ΔT, generally one, five, or ten years [36, 58, 94]. The five and ten year time increments are typical since they tend to correspond to management time frames. Models that contain only two equations use tree diameter or height, and volume only rather than both height and diameter [24, 153]. Models that use stand density, diameter, and height increments rather than volume have become common, due to the availability of high speed, large storage capacity computers, and because tree volume is derivable from tree diameter and height.

The statistical-empirical approach to growth and yield modeling is illustrated using a typical growth and yield modeling scenario. The specific forms of the increment equations are not important in this presentation, and will be treated generically. A plethora of model formulations may be found in the literature for those interested in the specifics [24, 36, 51, 65, 153]. The data commonly used in the growth and yield models, generic model formulation, and the overall methodology are of specific interest. The intent is to set the stage for a critique of the statistical-empirical growth and yield modeling methodology, and to identify some of its weaknesses. The S-system modeling framework is then be presented and shown to overcome the primary weaknesses of the statistical-empirical modeling methodology, without additional assumptions.

A typical growth and yield model for a particular species consists of three equations. The three equations predict potential changes in stand density or tree growth as increments ΔN, ΔD, ΔH, for a fixed time interval ΔT. It is convenient to think of these increments in terms of an idealized single species stand, and either a stand level model growing an average tree or an individual tree model growing one tree.

Each of the three equations predicts its increment based upon the current conditions, N, D, H, stand age, A, and site index, S,

$$\Delta N = f_N(N, D, H, A, S; \alpha_N) \tag{2.1}$$

$$\Delta D = f_D(N, D, H, A, S; \alpha_D) \tag{2.2}$$

$$\Delta H = f_H(N, D, H, A, S; \alpha_H), \tag{2.3}$$

where f_N, f_D, and f_H, are the increment prediction functions and α_N, α_D, and α_H are vectors of unknown regression parameters estimated from data and calibrating each increment function. It is assumed that the increment functions are derived in a manner requiring them to reproduce shapes appropriate for the three components of stand development or tree growth they are intended to represent. Generally, either the equation for ΔD or ΔH is considered to be the primary size increment equation, the other being secondary [5, 24, 36, 58, 94, 153]. The values obtained from the primary size increment equation are then used in the secondary size increment equation, either

$$\Delta H = f_H(N, D, H, A, S, \Delta D; \alpha_H) \tag{2.4}$$

or

$$\Delta D = f_D(N, D, H, A, S, \Delta H; \alpha_D) \tag{2.5}$$

to obtain both of the size increments for the model. The equation for the density increment or mortality, ΔN, is generally based on a size-density relationship, uses only one size variable, usually D, and may or may not include age, A, explicitly. A typical equation may then be represented as

$$\Delta N = f_N(N, D, S; \alpha_N) \tag{2.6}$$

which includes variables for a size-density relationship and site differences through site index [24, 153]. When used as described, these three increment equations are loosely coupled, or uncoupled in the case of mortality, which implies a weak feedback mechanism.

Once values for the potential growth increments ΔN, ΔD, ΔH are obtained, a modifier m_N, m_D, and m_H is applied to each potential increment

$$\begin{aligned} \Delta N' &= \Delta N m_N \\ \Delta D' &= \Delta D m_D \\ \Delta H' &= \Delta H m_h, \end{aligned}$$

to obtain adjusted increments $\Delta N'$, $\Delta D'$, $\Delta H'$ [5, 36, 58, 94, 153]. The modifier values will generally have a value in the range of zero to two, and all three modifiers need not be used in any particular model. The modifiers are intended to bring the potential increments into better agreement with actual increments by using additional information that may be available in the model, such as a crown competition factor [36], a size-density index [5, 36, 58], local crown competition for spatially explicit models [94], tree vigor [36], or a limiting function to prevent increments from taking on biologically unreasonable values [36]. For convenience, a single modifier is used here, but a particular growth and yield model may have several modifiers applied in sequence [36, 58, 94]

Yield values are obtained for a simulation beginning with a specified initial stand condition, by computing the adjusted increment values for one time interval, ΔT, and accumulating them for the average tree, or each individual tree, and the stand density. This process is then repeated with the new values for the stand condition for the next time increment, until the cumulative time exceeds the simulation length. This process then produces simulated stand dynamics and tree growth.

As of yet, the regression parameters defined by the vectors α_N, α_D, and α_H, in the three increment functions f_N, f_D, and f_H, respectively, have not been addressed. These parameter vectors represent the unknown quantities that calibrate a particular set of increment equation formulations to a particular set of data, given appropriate formulations for each increment equation. The regression parameter vectors α_N, α_D, and α_H are typically obtained from separate, least squares regressions, one for each equation, rather than from treating the equations simultaneously [24, 65, 153]. This is necessitated by the fact that the secondary size increment equation, Equation 2.5 or Equation 2.4 requires the predicted value from the primary size increment equation, Equation 2.3 or Equation 2.2, respectively. In some cases, several regressions are performed in a sequence in an attempt to improve the overall fit of the model [85]. These separate regressions will tend to reinforce the already loose coupling, further weakening the feedback, among the three growth and yield model increment equations.

2.1.2 Typical growth and yield model assumptions

Consider a typical growth and yield model within a statistical-empirical modeling framework, as defined by the set of three increment equations Equation 2.2, Equation 2.4, and Equation 2.6, or Equation 2.3, Equation 2.5, and Equation 2.6. Assume that the specific model formulation agrees well with the available data. It is, again, useful to think of this in terms of the stand dynamics for a single species stand of trees. There are, then, two questions that are of significant importance when considering the development or use of such a growth and yield model.

- What are the primary biological and statistical assumptions, whether explicit or implicit, that are necessary in order to develop such a growth and yield model?
- Given the necessary assumptions, is the model formulation within the statistical-empirical modeling framework the most appropriate?

The first question addresses issues of model definition, that is, the features of forest stand dynamics and tree growth that a growth and yield model is intended to represent. The issues of primary interest are the biological properties to be represented by the model, the data relationships involved in obtaining their desired representation, and the requisite statistical assumptions for the data. The second question address the appropriateness of a particular mathematical or statistical representation or formulation for a growth and yield model. The issues of model interpretation and parsimony in model formulation are the primary concerns.

There are four primary biological assumptions made for growth and yield models. First, size-density relationships exist, i.e., stand density and average tree size are coupled [24, 36, 37, 47, 66, 71, 74, 102, 143, 144, 158]. Second, size-size relationships exist, i.e., tree diameter and height growth are coupled [34, 77, 78, 108, 149, 150, 151, 152, 159, 160, 167, 166]. Third, stand development and tree growth rates are based on the current stand condition or tree size, that is, there is feedback within the forest system. This is evident from the formulation of the increment equations. Fourth, stand development and tree growth are dynamic processes, nominally continuous time processes with feedback. These four biological assumptions are appropriate and pose no significant difficulties when developing a growth and yield model. Specific formulations of these four assumptions form the basis of the increment functions for all growth and yield models. The relationships among the state variables, and primary increments, within a growth and yield model are based on their statistical correlations, or generally linear interactions, constrained by a particular model formulation given by the increment equations to produce appropriately shaped curves.

Growth and yield models developed within a statistical-empirical modeling framework also generally include site index, or a surrogate for site index such as vegetation class [139], and stand age in their increment equations [5, 36, 58, 94, 153]. Site index and age are included in the models for the benefits derived from their strong correlations with size development and changes in stand density. They have only a statistical relevance in the model, and no biological function [139, 10, 150, 151, 152]. Although site index is used *a priori* to differentiate site quality among for stands in different locations, the growth and yield models may not reproduce it [26, 139]. This reinforces the purely correlational nature of site index within these growth and yield models. Stand age, if included in a model, is not explicitly relevant within the models due to the assumption that stand development and tree growth are based on the current stand density and tree size [139, 149, 150, 151, 152]. The direct inclusion of site index and stand age in a growth and yield model are hence problematic. Site index, in particular, becomes an additional parameter that is required *a priori* for each stand in order to use a growth and yield model. The use of age or site index in a growth and yield model, *a priori*, appears to add unnecessary additional assumptions [140, 139, 12, 10, 154, 151, 152], benefitting only the model developer through their strong correlations with the state variables of interest [24, 65, 153].

The three primary statistical assumptions made for growth and yield models developed within the statistical-empirical modeling framework are: the stand measurement data are independent, the stand measurement data span the range of desired values, and all factors not included in the model, e.g., climate, are constant [7, 153, 172]. The first assumptions, that the data be independent, is known not to hold *a priori* [31]: measurement data for a remeasured stand cannot be statistically independent. That is, a stand trajectory consists of *dependent* measurement data. Stand trajectories, or measurement data, from geographically different stands are statistically independent, but the separate measurements within a trajectory are not independent. The failure of the independence assumption within the statistical-empirical modeling framework may cause a significant amount of difficulty due to the autocorrelation or dependencies within each stand trajectory.

The second assumption, that the stand measurement data span the desired range of values, may not be adequately satisfied in general, though it may be for the limited range of stand densities, tree sizes, and rotations used commercially. This assumption is necessary to ensure that the statistical-empirical growth and yield model is able to reproduce the desired characteristics as represented by the stand measurement data. A statistical-empirical growth and yield model is a purely descriptive representation of a particular set, or sets, of data, and as such cannot necessarily be trusted to correctly produce behavior beyond the range of the data used in its calibration [24, 65, 153, 172]. With careful model construction, problems stemming from this issue may, however, be greatly reduced, but never eliminated.

The third assumption, that all factors not represented in a growth and yield model are constant, or randomly varying about a mean value which average out, is of fundamental importance, but rarely explicitly stated. This assumption is a necessary consequence of the static, descriptive nature of the parameter vector estimates in the increment equations, as obtained from a least squares regression, or some other parameter estimation method, because the parameter vectors are constant so is the model relative to all factors not included.

2.1.3 Necessary growth and yield model assumptions

Given the general description of the three increment equations in a typical growth and yield model, the parameter estimation procedures, and the biological and statistical assumptions just outlined, it is clear that the commonly used statistical-empirical modeling framework is *not* the most appropriate modeling framework for two reasons. First, the model formulation, parameter estimation, and model use have only superficial agreement with the actual development of a forest stand and tree growth. Second, the statistical assumptions are inappropriate, possibly difficult to meet, or are side effects of the statistical-empirical modeling framework. A set of necessary properties for a more appropriate modeling framework is now derived through a consideration of the shortcomings of the statistical-empirical modeling framework as typically applied to forest growth and yield.

Forest stand development and tree growth are dynamic processes. This implies that the fundamental characteristics of stand development and tree growth occur in continuous time, relative to an appropriate time scale, and that these processes affect each other through positive or negative feedback [15]. The fundamental stand level processes are represented as size-density relationships and size-size relationships for tree growth. The processes described by these relationships all occur *simultaneously* for an actual forest stand, that is, the stand development and tree growth are tightly coupled in reality. As described, a typical growth and yield model represents these processes as a set of three increment equations representing the simultaneous rates of change in stand density, diameter, and height for a specified time increment, and the rates are determined by the current stand density, diameter, and height values and their interactions. This general representation poses no problems *per se*, but provides the key that leads to a more appropriate modeling framework.

Difficulties with the typical approach arise due to the independent formulation and use of the three increment equations through a sequential relationship that exists for the primary and secondary size increment equations. Values for the parameter vector of each equation are also determined independently. At best, this independent formulation and parameter estimation allows a loose coupling, or weak feedback mechanism, which is a poor approximation of the simultaneous nature of the actual processes. A modeling framework that provides for tight coupling, or strong feedback, and treats the three increment equations simultaneously is necessary for the formulation of growth and yield models agreement with the actual development of forest stands and tree growth. This modeling framework should also allow the simultaneous estimation of the parameter vectors for the equations and the simultaneous generation of the rates of change.

The explicit inclusion of age or site index in the formulation of the increment equations introduces further difficulties for typical growth and yield models. Age is a convenient attribute of a forest stand or tree from a human perspective, but it is irrelevant in the context of stand development or tree growth. The physiology of tree growth is *not* directly dependent on numerical tree age but *is* dependent upon tree size [12, 10, 77, 78, 102, 154, 151, 152], which is strongly correlated with age [24, 65, 153]. Similarly for stand development mortality is *not* dependent upon stand age but *is* dependent on inter-tree competition, site conditions, and climate. The time variable of relevance for growth and yield modeling is the time *increment*, which allows the projection of the current state, through the computation of the rates of change for stand density and tree size, to a future state. A modeling framework based on time increments, preferably for an arbitrary or user defined increment value, that captures the essential characteristics of stand development and tree growth for a particular reference time scale is necessary.

The explicit use of site index in the typical statistical-empirical growth and yield models has been shown to be unnecessary, particularly where a greater emphasis is placed on the biological and physiological processes, [13, 12, 10, 139, 149, 150, 151, 152]. Site index, defined as the average height of the dominant and codominant trees for a specified reference age, was originally conceived as a simple to measure, direct way to distinguish sites based upon their potential productivity [81, 95, 96]. Site index was used in lieu of other more relevant, but difficult to obtain or use, measures of site quality based on soil properties and other local site characteristics. Site index was not originally intended to become the predominant indicator for site quality [81]. Site index has frequently been decried as an indicator of site quality [81, 95, 96, 139], and has been perceived as a stumbling block hindering the advancement of growth and yield, or forest ecosystem, modeling. In particular, site index is generally not applicable to mixed species forests or mixed age forests, and its ongoing use in these situations is problematic [81, 95, 96, 139, 169].

The statistical nature of site index, as used within a typical growth and yield model, becomes obvious when it is considered as a second, (pseudo)height measurement. Site index then provides a constraint on stand height through the regression process which calibrates the growth and yield model, standardized via the fixed reference year. So, when projecting stand development with a typical model, a forward progression of the current state is not based solely on that state, but is modulated by the additional (pseudo)height measurement given by site index. The use of site index effectively turns the projection of stand development with a growth and yield model from an initial value problem [15, 3], based on the current forest state and the interactions of the state variables, into a two point boundary value problem [15, 3], at least in height. Forest stands develop and trees grow in a forward time process that is more consistent with the initial value problem, thus making the additional objective of hitting an *a priori* site index or (pseudo)height target apparently unnecessary. Thus, a modeling framework that allows a forward time based representation for stand development and tree growth, mimicking the actual processes, without explicitly including site index is necessary.

The statistical assumptions for the typical growth and yield model also suffer from weaknesses. The statistical independence assumption is violated *a priori*. A modeling framework that automatically accounts for lack of independence for the data within a stand or stand trajectory is necessary. The data requirements to meet the necessary data coverage for a statistical-empirical growth and yield model may be excessive. A modeling framework which minimizes the data requirements necessary to achieve a good model is desirable. The assumption that all factors not included in a typical growth and yield model are constant or average out over time or space is a by product of the parameter estimation procedure in the statistical-empirical modeling framework. A modeling framework which includes this assumption as a fundamental attribute, rather than as a statistical side effect, is therefore preferred.

Finally, cause-effect relationships are *not* directly obtainable from growth and yield models developed within a statistical-empirical modeling framework but must be inferred by some external oracle [7, 64, 172]. However, obtaining an understanding of the cause-effect relationships influencing the development of a forest stand is one of the objectives of growth and yield modeling. So, a modeling framework which permits the direct inference of cause-effect relationships, or which is closer to permitting the direct representation of cause-effect relationships, is then highly desirable.

2.1.4 An alternative modeling framework for forest growth and yield

Modern growth and yield models attempt to represent the simultaneous rates of change of stand density and tree size over time via their respective increment equations and the current state variable values, Equation 2.3, Equation 2.5, and Equation 2.6, for example. The simultaneous rates of change for a dynamic phenomenon as a function of the current state is *exactly* what autonomous systems of ordinary differential equations (ODEs) represent [15, 57]. An autonomous system of ODEs inherently represents a dynamic system, and hence seems a natural choice for forest growth and yield models based on size-density and size-size relationships.

An autonomous system allows for tight coupling among the growth equations and the simultaneous estimation of the unknown model parameters, both of which mimic the tightly coupled dynamics of actual forest stand development. An autonomous system approach also allows for a minimum number of biological and statistical assumptions. For example, the future state of a stand is predicted using only the current state as an initial condition, and there is no *a priori* need for the explicit inclusion of site index, age, or other correlated variables. The dependence of the repeated measurements within a stand trajectory is automatically taken into account by the dynamic system representation, and need not be accounted for by additional adjustments or the inclusion of additional correlated variables. Further, the parameter estimation procedure that calibrates an autonomous system based growth and yield model. and the solution procedure, or integration, of such a system to obtain stand trajectories involves only time increments, not age. The forward time solution of an autonomous system based growth and yield model is consistent with the actual forward time processes of stand development. Finally, the assumption that all factors not included in the model are constant is now a fundamental part of the model formulation. because by definition autonomous systems of ODEs are independent of time, rather than a side effect of the parameter estimation procedure. This may enhance the ability to determine cause-effect relationships or biologically relevant growth laws.

A simple example addressing the data requirements for an autonomous system based growth and yield model and a typical statistical-empirical growth and yield model, demonstrates the superiority of the autonomous system approach. Given the set of three increment equations for a statistical-empirical model and the three equation autonomous system model for stand development, both of which are attempting to represent the vector field defining the simultaneous rates of change in a forest stand. Consider the problem of estimating the unknown model parameters in each case from a small set of complete stand trajectories spanning the operational range of interest. Obtaining a model that represents stand development throughout the operational range is theoretically impossible *a priori* for a statistical-empirical growth and yield model, because only minimal data coverage has been used [7, 172]. A statistical-empirical model estimated for this situation would have uses limited to conditions near the set of available stand trajectories. The behavior of the empirical-statistical model away from the supplied stand trajectories is uncertain, and cannot be guaranteed. Obtaining a model that represents stand development throughout the operational range may be possible for the autonomous system growth and yield model. An autonomous system model estimated for this situation should be usable throughout most, if not all, of the operation range. The behavior of the autonomous system model away from the supplied stand trajectories must conform to the vector field defined by the autonomous system, and this is constrained to produce stand trajectories within the envelope of the supplied stand trajectories by the parameter estimation. This has been demonstrated empirically for a size-density simulation model. In the case of the autonomous system model, the set of stand trajectories is representative of the dynamics of the system, but in the statistical-empirical model they are not necessarily representative of the system, but only of themselves.

2.1.5 A dynamic modeling framework for forest growth and yield

A mathematical-explanatory modeling framework that naturally includes the dynamic nature of forest growth and may be extended to include climate and other environmental processes that affect forest development and tree growth is necessary for the advancement of growth and yield, and forest ecosystem, modeling [9, 72].

A direct shift to dynamic, process oriented models is generally not feasible at this time, due primarily to the inherent complexity of forest ecosystems, and the lack of understanding of how processes at different spatial and temporal scales, such as photosynthesis and nutrient cycling, interact and affect stand development and tree growth [141, 103, 168], although some significant progress has been made in these areas [13, 12, 10, 100, 112. 113, 130, 131, 150, 151, 152, 161].

An intermediate dynamic modeling framework, therefore, seems necessary to help bridge the gap between the typical statistical-empirical growth and yield models and the desired mathematical-explanatory process models of forest development and tree growth. This intermediate modeling framework would provide a bridge by using the standard mensurational data available for the typical growth and yield models, stand density, tree diameter, and tree height, but in the natural dynamic model representation of an autonomous system of differential equations. In addition to directly capturing the dynamics of forest growth, this intermediate modeling framework should have the following characteristics of the statistical-empirical modeling framework. First, its model definition should be straightforward and based essentially on variable interaction or correlation. Second, the final model specification or calibration should be performed through a straightforward parameter estimation process, e.g., least squares. These two features of the statistical-empirical modeling framework are singled out because they are considered to be its most powerful attributes, and will allow for a smoother transition to a dynamic modeling framework.

A dynamic modeling framework based on a system of first order, nonlinear ordinary differential equations, and the least squares estimation of parameter values, was chosen for developing forest growth and yield models meeting the requirements of accurately representing the dynamics of forest development and tree growth. This type of representation seems quite natural for application to forest growth and yield modeling, and has actually been in use for quite some time: the von Bertallanffy equation [98, 153], the Chapman-Richards equation [24, 153], and the Lotka-Volterra predator-prey equations [57, 98], are all examples of this type of modeling approach.

A regional dynamic stand level growth and yield model for plantation Douglas-fir in the Pacific Northwest is developed. The model spans the region from southern Oregon to southern British Columbia, west of the Cascade Mountains. The dynamic growth and yield model will represent stand density as trees per hectare (TPH), average diameter as quadratic mean diameter (QMD), and average stand height as top height. The growth and yield model is developed within a dynamic modeling framework using the concept of dynamic model extraction to specify the model through a nonlinear least squares parameter estimation problem. The model is formulated as an S-system with the dynamic relationships among the three state variables defined through interactions derived from a size-density relationship coupling stand density and QMD and a size-size relationship coupling QMD and top height.

Section 2.2 defines the canonical S-system representation for dynamic systems. Section 2.3 defines the concept of dynamic model extraction and its use. Section 2.4 defines the S-system representation for the dynamic, stand level growth and yield model for plantation Douglas-fir. In particular, this section identifies the specific assumptions used to define the S-system growth and yield model, and demonstrates the compatibility of the modeling assumptions and the development of an actual forest stand. Section 2.5 describes the stand measurement data that were used to calibrate and validate the model. Also identified are potential problems that may occur due to some of the specific characteristics of this particular data set. Section 2.6 identifies the methods used to estimate the S-system model parameters, and defines the model validation procedures. Section 2.7 presents the results of the parameter estimation procedures and the model validation results. Section 2.8 presents a brief consideration of several stand management issues. First is the ability of the S-system model to recover of site index. which was not explicitly included in the model. Second is the potential of the growth and yield model to predict thinning response without the explicit inclusion of thinning effects in the model. Third is the ability of the model predictions to meet the expectations of forest managers. Finally, differences between the variation of short term stand predictions and long term stand predictions are used to hypothesize a possible climate based effect on stand development.

2.2 S-system definition, interpretation, and examples

An S-system is a canonical mathematical representation for coupled or uncoupled systems of first order, linear or nonlinear ordinary differential equations (ODEs) based on a power-law formalism [118, 119, 120, 121]. Each equation consists of two terms, a growth term and a decay term. Within each term, juxtaposition of state variables implies an interaction among those state variables. The "S" in S-system is intended to indicate that S-systems may easily represent saturable, synergistic, and oscillatory phenomena, e.g., plant growth and predator-prey relationships [122, 123, 124, 158, 98, 156, 159, 160]. For convenience, the term "S-system" is used to refer to any single ODE or set of ODEs which conforms to the definition of an equation within an S-system.

The S-system representation for systems of ordinary differential equations has its origins in chemical kinetics, the dynamics of chemical reactions, [118, 119, 120, 123, 124], and is currently finding application both inside and outside of this original domain [138]. S-systems have repeatedly been demonstrated to capture the types of behavior observed in biological systems. S-system based models are capable of exhibiting oscillatory behavior, asymptotically stable behavior. S-systems have been successfully used to model a two variable, theoretical size–density relationship for monoculture plant stands [158], and the actual size-density relationships of red pine (*Pinus resinosa*) in the lake states region [148]. The S-system model formulation has also been used to represent nutrient dynamics in a tropical forest [147], and to compress or simplify the dynamics of a complex process models of forest stand development to a few essential dynamic variables [88], to model forest growth [159, 160, 166], and to model a variety of biological systems [125, 126, 127, 156, 157].

Each equation in an S-system is represented mathematically as the difference between two terms. The first term represents all of the influx or growth influencing characteristics, and the second term represents all of the efflux or decay influencing characteristics of a particular phenomenon [123, 158]. Symbolically an S-system model is represented as the system of equations

$$\dot{X}_i = F_i^+(X_1, X_2, \ldots, X_{n+m}) - F_i^-(X_1, X_2, \ldots, X_{n+m}), \quad i = 1, 2, \ldots, n, \qquad (2.7)$$

where $\dot{X}_i$ is the time derivative, or rate of change, of state variable X_i; F_i^+ and F_i^- are both positive functions which represent the influx and efflux for state variable X_i, respectively. $X_1, X_2, \ldots, X_n$ are the n dependent, dynamic state variables and $X_{n+1}, X_{n+2}, \ldots, X_{n+m}$ are m independent state variables that are assumed to be unaffected by the dynamics of the model. The n dependent state variables represent the state of the phenomena of interest and its changes over time, which will generally include feedback effects, and the m independent variables represent factors which may affect the phenomenon of interest but are not affected by it. The m independent state variables may be thought of as representing *forcing functions* that modify the trajectory that the phenomena of interest takes through the state space.

The canonical S-system representation for ODEs further specifies that the influx, F_i^+, and efflux, F_i^-, terms are represented as products of power law functions in each state variable, e.g., functions of the form ax^b [123, 124, 158]. Taylor's theorem from the Calculus and the general theory of function approximation guarantee that the functions F_i^+ and F_i^- can be represented in this way even if their exact structure is not known *a priori* [128, 158]). A complete understanding of the theory of approximating functions is not required to use S-systems for modeling, but the fact that there is a rigorous mathematical proof permitting the S-system representation of arbitrary functions is encouraging, and lends strong support to the use of S-systems.

Each influx term and each efflux term consists of a product of power law functions, one power law function for each variable, X_j, that directly contributes to a particular influx or efflux term. Thus, each term requires a nonnegative multiplier, from the a in each individual power-law, and an exponent for each contributing variable X_j, the b in each individual power-law. For the influx term F_i^+, the nonnegative multiplier is denoted α_i and the exponents associated with the contributing variables X_j are denoted g_{ij}. For the efflux term F_i^-, the nonnegative multiplier is denoted β_i and the exponents associated with the contributing variables X_j are denoted h_{ij}. So, a generic S-system has the form

$$\dot{X}_i = \alpha_i \prod_{j=1}^{n+m} X_j^{g_{ij}} - \beta_i \prod_{j=1}^{n+m} X_j^{h_{ij}} \quad i = 1, 2, \ldots, n, \tag{2.8}$$

where $\alpha_i \geq 0$ and $\beta_i \geq 0$, $i = 1, 2, \ldots, n$.

The S-system parameters have easily understood and consistent interpretations regardless of the context in which an S-system is used. The nonnegative coefficients α_i and β_i are overall rates of influx, or growth, and efflux, or decay, respectively, for the ith rate equation. The exponents g_{ij} and h_{ij} indicate the relative rates of change of F_i^+ or F_i^- for a given small change in the variable X_j. The S-system exponents then provide a direct measure of the sensitivity of the influx and efflux terms in each equation to small changes in the values of the variables.

As presented, an S-system is an autonomous system of differential equations, that is, a system of equations in which time does not appear explicitly. Physically, this is a system in which the parameters of the system are independent of time, and are, hence constant [15]. Autonomous systems of differential equations have three characteristic properties that are useful for modeling with S-systems [15].

1. Through any point in the phase or state space, there is at most one trajectory.

2. The trajectories of autonomous systems asymptotically approach a steady state as time increases for any initial state that is not the steady state.

3. A trajectory that crosses itself must be a closed curve, providing a periodic solution.

Properties 1 and 3 imply that for a nonperiodic solution, trajectories or solutions of autonomous systems never cross, that is, there is nothing acting within the system that would cause a trajectory to change. This is consistent with the "all other things being equal" notion commonly applied when developing models. These features of autonomous systems of ODEs form the basis for performing parameter estimation, or system identification, through dynamic model extraction with S-systems.

A dynamic modeling framework based on the canonical S-system model formulation is proposed. This dynamic modeling framework easily allows the addition of climate or environmental variables to an S-system model, either directly, or as nested submodels, in a forest growth and yield model. The S-system modeling framework may therefore provide a unifying framework within which the components of forest growth and yield models may be developed. In addition, efficient numerical algorithms exist for the solution of S-system based models on digital computers [67, 68], and effective algorithms for identifying S-system model parameters also exist [89, 33, 4]. For these reasons, the S-system modeling framework should prove to be a useful tool for forest growth and yield modeling.

2.3 Dynamic model extraction with S-systems

An objective of any modeling endeavor is to obtain a concise statement or representation which captures the essence of a particular complex phenomenon. This is true for conceptual models, e.g., nutrient flow or carbon and water cycle diagrams [129]; for statistical models, e.g., regression models predicting tree taper [75, 97], or standing wood volume [18, 116]; for mathematical models, e.g., photosynthesis within a leaf [77, 78, 69], or simple exponential decay for radioactive compounds [15]. Some type of *model extraction* is involved to obtain each type of model. A primary objective of quantitative modeling is to use some form of model extraction to obtain a concise mathematical or empirical representation of some complex phenomenon.

Dynamic model extraction is the process of obtaining a concise mathematical representation of a dynamic phenomenon from a collection of appropriate time series data, with a minimum number of modeling assumptions, using a generic dynamic model formulation [133]. The concise mathematical representation obtained should capture the essential characteristics of the dynamic phenomenon. There are three requirements for performing dynamic model extraction. First, an appropriate general mathematical formulation for dynamic models must be selected to represent the dynamic phenomenon. Examples include the logistic equation [15, 98], a von Bertalanffy equation [24, 98, 153], or system of linear or nonlinear differential equations or difference equations [15, 91, 98]. A wide variety of generic dynamic model formulations are available through the literature and are in widespread use. Second, appropriate time series data which characterize the state variables of interest for the dynamic phenomenon must be obtained. The time series data implicitly define the dynamic phenomenon of interest, the dynamic characteristics of interest, state variables that represent those characteristics, and the sampling interval must all be considered to obtain appropriate time series data. Third, a reliable parameter estimation or system identification methodology must be available to transform the generic dynamic model representation into a specific representation for the particular phenomenon being modeled. The parameter estimation or system identification may be viewed as an optimization problem and posed according to a least squares based perspective [24, 21, 63, 64, 65, 153], a maximum likelihood perspective [7, 64, 21], or an information theoretic perspective [64, 6, 21].

The selection of a general mathematical representation for developing quantitative dynamic models for dynamic phenomena is of utmost importance. Dynamic phenomena are generally characterized by feedback relationships among their state variables and continuous time [15, 53, 153]. A model representation that does not support, or incompletely supports, the feedback and continuous time properties of dynamic systems makes less effective use of the information contained in the time series data than a dynamic model representation. The canonical S-system formulation for ODEs is a general mathematical representation for dynamic phenomena that has a wide applicability [123, 124, 125, 126, 127, 128, 134, 136, 156]. An S-system model has a consistent interpretation, regardless of domain of application, is extremely flexible when defining a model, has a high representational power, and makes a small number of necessary assumptions. All of these characteristics make the S-system representation an ideal candidate for dynamic model extraction, or system identification, from time series data.

Presuming the availability of appropriate time series data, a least squares minimization procedure may be used to estimate the parameters in a properly defined and constrained S-system model. The S-system representation strongly suggests a least squares problem formulation based on the differences between approximate rates of change for each equation, $\widehat{\dot{X}_i}$, derived from the time series data, and the S-system right hand side evaluated for a particular set of parameter values. Under perfect conditions, the value of this difference should be zero for each data point and each S-system equation,

$$\widehat{\dot{X}_i} - \left(\alpha_i \prod_{j=1}^{n+m} X_j^{g_{ij}} - \beta_i \prod_{j=1}^{n+m} X_j^{h_{ij}} \right) = 0 \quad i = 1, 2, \ldots, n,$$

potentially making the least squares problem a small residual problem [33]. In practice, however, this problem will likely be a medium to large residual problem [33], depending on the amount of variation in the time series data. The least squares problem defined by these equations is straightforward but must include the nonnegativity of the rate coefficients α_i and β_i as constraints. These constraints pose no difficulty, as algorithms and software for solving this type of problem are readily available [89].

The dynamic model extraction approach outlined above has been successfully applied by the author to two simulated data sets using a least squares criterion. The first example data set was derived from a two-variable S-system model of a size-density relationship [158], and the second example data set was derived from a predator-prey model [57]. The two variables in the size-density model approach asymptotes, an asymptote of zero for the density variable and a positive asymptote for the size variable [158]. The two variables in the predator-prey model exhibit periodic behavior as a limit-cycle [57]. These two models exhibit fundamentally different behavior, and in each case the S-system based model extraction procedure recovered the parameter values and, hence, the behavior of each of these dynamic systems.

The modeling framework comprised of the S-system canonical formulation for ODEs and least squares minimization for parameter estimation will be referred to as the *S-system modeling framework*, and greatly facilitates the use of dynamic model extraction to represent complex dynamic phenomena. The S-system modeling framework will provide the foundation for developing a regional stand level, dynamic growth and yield model for Douglas-fir in the Pacific Northwest.

The S-system modeling framework is used within the context of dynamic model extraction [133], using time series stand measurement data or stand trajectories to obtain a dynamic growth and yield model. Dynamic model extraction is based on the premise that the measurement data implicitly represent the processes of stand development and tree growth, and thus, with an appropriate model formulation and representative data, the essential characteristics of the stand dynamics may be extracted from the data through the solution of a straightforward parameter estimation problem [133]. This approach is appropriate for developing models where the underlying physical or physiological process are not well understood, such as stand dynamics and tree growth, given the overall complexity of forest ecosystems.

Dynamic model extraction using the S-system modeling framework provides a means for maximizing the use of the readily available, standard mensurational data to obtain dynamic growth and yield models. The models developed using this approach may be though of as process models where the lower level processes influencing the state variables are implicit, having been integrated into the measurement data, and are not directly represented. Only the direct relationships among the state variables are represented in the differential equations. This interpretation is consistent with the definition of an S-system as an autonomous system based representation for a growth and yield model.

2.4 Growth and yield model assumptions and formulation

When developing quantitative models for physical or biological phenomena it is desirable to limit the number of assumptions to the minimum that are absolutely necessary. This parsimony in assumptions can reduce or eliminate the inadvertent introduction of biases into the model formulation, as well as allowing the data used in the calibration of the model to "speak for itself" within the constraints of a particular modeling methodology. Thus, a modeling methodology with a small number of assumptions and that is capable of representing a wide variety of phenomena is highly desirable for model building. A stand level growth and yield model for monoculture Douglas-fir plantations is developed using such a methodology. In developing the growth and yield model, an attempt was made to minimize the number of assumptions made to those that were deemed essential, through consideration of both model representation and biological relevance. Each assumption is justified as being mathematically or biologically necessary before it is accepted and used as a part of the stand level Douglas-fir growth and yield model.

Forest stand development and tree growth are simultaneous dynamic processes, that is. processes that include feedback and are generally continuous in time [15]. Inter-tree competition and the well known size-density relationships, such as the $-3/2$ self-thinning rule [66, 74, 102, 173, 174], provide examples of feedback in forest stand development. The processes involved in photosynthate partitioning and distribution within a tree and respiration, provide examples of feedback within an individual tree

[59, 77, 78, 87, 166, 167]. Further, at the spatial and temporal scales of interest for growth and yield modeling, tree size growth and stand mortality are effectively continuous. Photosynthesis and many of the processes and factors influencing it occur at a much faster rate, fractions of a second to minutes or hours [37, 77, 78], and much smaller spatial scale, the leaf or needle, than growth and yield models typically represent. A typical growth and yield model has time increments on the order of years and a spatial resolution on the order of one hectare [5, 36, 58, 94, 153], and at these resolutions stand development, tree growth, and tree mortality are effectively continuous processes, that is, they occur on much smaller temporal and spatial scales, allowing a *fraction* of a tree to be considered alive without introducing any inconsistencies.

Given the dynamic nature of forest stand development and individual tree growth, a dynamic model formulation is necessary for developing a model of Douglas-fir stand dynamics and tree growth. Before defining the model, however, the assumptions required of an S-system model are shown to be consistent with the modeling of forest stand dynamics and tree growth.

The canonical S-system formulation for systems of differential equations makes only four assumptions, when used for modeling complex dynamic phenomena such as forest stand development and tree growth. Two of the four assumptions are explicit in the mathematical representation of an S-system. The remaining two assumptions are implicit and related to the particular phenomenon being modeled and the data used for model calibration or parameter estimation or model validation.

The explicit assumptions made when using S-systems for modeling are:

1. The rate(s) of change of a dynamic phenomenon may be defined by the difference(s) between aggregate, nonnegative *growth* term(s) and aggregate, nonnegative *decay* term(s);

2. Each term in an S-system rate equation may be represented as a product of

power-law functions.

These two assumptions determine the types and shapes of functions that may be represented by an S-system. Neither of these assumptions poses any difficulties for using S-system models to represent forest stand development and growth and yield. Tree size growth is a saturable phenomenon that is generally related to the difference between photosynthesis and respiration, and as such, is representable as an S-system. Changes in stand density are generally represented as a size coupled decay based phenomenon, also easily represented as an S-system. Power laws have long been in use for developing allometric relationships in forestry, e.g., the $-3/2$ self thinning rule [66, 74, 102, 173, 174], and taper equations [75], and there is a strong mathematical basis for an S-system based representation of functions [122, 123, 124]. Further, power laws may play a fundamental role within biological systems which could a theoretical basis for their use in modeling these types of systems [165].

The implicit assumptions made when using S-systems for modeling are:

1. Events affecting the phenomenon of interest, but occurring more slowly than it, are considered to be constant relative to the state variables used in the model;
2. Events affecting the phenomenon of interest, but occurring more rapidly than it, are considered to be integrated into the state variables used in the model.

These two assumptions also pose no difficulties for using S-systems to model forest stand development and growth and yield. The first implicit assumption is common and widely accepted within traditional forest stand development and growth and yield modeling [24, 36, 58, 94, 153], as well as within the broader scope of modeling biological systems [77, 161]. This assumption is generally presented as "all other things being equal, ..." within a particular modeling situation, and is intended to convey the idea that those elements not explicitly represented in the model are not relevant or are averaged out at the relevant time scale. The second implicit assumption when applied to a forest is a straightforward statement of how forest stands actually develop and trees grow when viewed on typical human oriented time scales. As a forest stand develops and individual trees grow within it, all of the factors having an influence, e.g., competition, climate, and nutrient availability, are integrated into the current state of the system as it develops.

The two implicit assumptions are also related to the remeasurement or time series data that may be used to calibrate or validate a model. If events affecting the phenomenon of interest occur on a much longer time span than the sampling interval, they are effectively constant across the sampling periods, and do not affect the values of the sampled state variables [158]. If events affecting the phenomenon of interest occur with a time span much smaller than the sampling interval, their effects are integrated or averaged into the sampled state variables [158]. Thus, the implicit assumptions for modeling complex phenomena with S-systems have a consistent interpretation both for an S-system model of the phenomena and for the sample data that are used to calibrate or validate the model.

At this point, the S-system representation and its requisite modeling assumptions have been shown to be compatible with the processes of forest development at the stand level: stand dynamics and tree size growth. Now the particular characteristics of forest stand development that are to be represented in the stand level Douglas-fir growth and yield model must be defined. Only fundamental, biologically relevant relationships and readily available state variables are desired for use in the growth and yield model.

To develop the stand level Douglas-fir growth and yield model, two spatial scales must be considered in order to select the particular forest stand characteristics to be represented in the model. The first scale is that of the stand, and the second is that of the individual tree. At the stand scale, the size-density relationship is the fundamental characteristic determining stand development. A classical size-density relationship generally specifies a limiting relationship between average plant size, usually volume or weight, and the number of plants per unit area, or density, as an allometric power law $S = aN^b$, where $a > 0$ and $b < 0$ [158]. The size-density relationship specifies that once a stand obtains the limiting relationship, or becomes near to it, stand density must decrease in order for average plant size to increase [158]. This relationship identifies a coupling of stand density and average size growth that affects their respective rates of change, so a size-density relationship should be a component in the stand level Douglas-fir growth and yield model.

The second scale is that of the individual tree, which will determine the size component, or components, of the size-density relationship. Size-density relationships typically relate average plant volume or weight to stand density. For forestry applications, individual tree volume is usually a derived variable, obtained as a function of the measured variables tree diameter and tree height [18, 24, 65, 153]. Thus, tree diameter and tree height are the fundamental tree size measurements, and average tree diameter and average tree height may be used to define the size component in a size-density relationship.

Stand density affects tree diameter growth and tree height growth differently. Dominant height growth is generally unaffected by stand density, particularly for coniferous tree species [94, 102]. Diameter growth, however, is strongly affected by stand density [5, 36, 153], and stand basal area and density are often used to create density management charts that help determine optimal stocking levels for wood production [37, 102]. Thus, a size density relationship emphasizing the relationship between stand density and average tree diameter will be used to further specify the stand level Douglas-fir growth and yield model.

Tree diameter and tree height, and their growth rates, are linked through a variety of processes, e.g., photosynthate allocation, water transmission, transpiration, and respiration. These relationships between tree diameter, tree height, and their growth rates allows stand density to affect tree height growth indirectly through tree diameter growth. Thus, it seems reasonable and appropriate to include a size-size relationship relating average tree diameter and average tree height in the stand level Douglas-fir growth and yield model.

Finally, plant growth rates must eventually decrease, and plant shape and growth habit, or life history traits, affect the rate of plant growth [13, 12, 10, 150, 153, 151, 152]. This leads to the assumption that tree growth rates depend on current tree size and morphology, as well as stand density via a size-density relationship as outlined above. The stand level Douglas-fir growth and yield model then needs to account for size effects on growth rates. Douglas-fir morphology is presumed to be accounted for through the dynamic size growth characteristics exemplified by average tree diameter and average tree height as included in the growth and yield model.

To recap, the growth and yield model will include size-density effects, by coupling stand density, measured as trees per unit area, and average tree diameter. Further, the model will also include a size-size relationship for average tree diameter and average tree height. To complete the definition of the model, all that remains is to select the specific state variables to be used for stand density, average tree diameter, and average tree height, and to specify the exact model formulation as a three equation S-system, one equation each for stand density, average tree diameter, and average tree height.

The stand level growth and yield model for plantation Douglas-fir represents a Douglas-fir stand as a vector of three state variables: the number of live trees per hectare (TPH) as the stand density state variable, quadratic mean diameter (QMD) as the average tree diameter state variable, and top height, the average height of the 100 largest diameter trees per hectare, as the average height state variable. These three state variables were selected for a variety of biological and practical reasons. First, QMD is proportional to the basal area of a tree having the average individual tree basal area for a stand [24]. Thus, the product of stand density and QMD is proportional to stand basal area. Second, the product of QMD^2 and top height is proportional to the volume of an individual dominant or codominant tree. Thus, the three term product of stand density, QMD^2, and top height is proportional to stand volume, and more importantly commercially realizable stand volume. Third, top height at a reference age, 50 years breast height age for Douglas-fir, is generally used as an indicator of site quality and potential productivity, and is commonly used to define site index values [16]. Fourth, these three state variables, TPH, QMD, and top height provide a common, standard, and readily available set of stand level measurements. Finally, if the stand level growth and yield model for Douglas-fir correctly reproduces these three state variables for individual stands, all of the derived quantities and relationships for this set of state variables are also correctly reproduced, e.g., stand volume, average tree volume, and self-thinning relationships.

The final S-system growth and yield model formulation was obtained through a trial and error process guided by the biology of forest stand development and tree growth. The starting point was the full three dimensional S-system model, which was systematically reduced to obtain the final model. The guiding biological principles were the existence of size-density interactions, and the assumptions that stand density must remain constant or decrease and tree size must increase or remain constant. The following six model selection heuristics, in order of their application, were used to judge the quality of each S-system model formulation tried and its estimated parameter values. If a lower numbered heuristic failed to be met, higher numbered criteria could not be considered.

1. The estimated S-system model must be integrable for all of the Douglas-fir stands used to estimate its parameters.

2. The integrated stand trajectories obtained from the S-system model must remain within, or near, the envelope defined by the Douglas-fir stand measurement data.

3. The integrated stand trajectories must be consistent with the biology of forest

stand development and tree growth: stand density may only remain constant or decrease and tree size may only increase or remain constant.

4. Long term simulation results for 300 and 1000 year simulations for the S-system model must produce biologically obtainable values for stand density, QMD, and top height. That is, stand density must remain positive, and QMD and top height must be within the range of observed values for maximum Douglas-fir tree sizes for these stand ages.
5. The residuals for stand density, QMD, and top height must be centered at zero and have symmetric distributions.
6. The estimated parameter values must be consistent, that is, for repeated parameter estimation attempts with different initial parameter values similar parameter values must be obtained.

The first two heuristics relate to issues of model parameterization. A model which failed to meet these two criteria was considered to contain too many parameters. A model failing to meet these two criteria would have its number of parameters reduced through a consideration of the biological principles. The next two heuristics relate to the biological consistency of a particular model formulation. A model failing to meet one of these criteria was modified in a manner intended to resolve the particular failure. Finally, the last two heuristics relate to the statistical properties or fit of a particular model formulation. No models meeting the first four heuristic criteria failed to meet these two criteria. The first S-system model formulation meeting all six of these heuristic criteria was chosen as the model formulation for the Douglas-fir growth and yield model.

A sequence of S-system model formulations were fit to the Douglas-fir growth data described in Section 2.5 in an effort to obtain a biologically consistent growth and yield model that was compatible with the data. State variable definitions for

Table 2.1: S-system state variable descriptions for the stand level, dynamic Douglas-fir growth and yield model defined by Equations 2.13.

State variable	State variable description
X_1	Stand density (TPH)
X_2	Quadratic mean diameter, QMD (cm)
X_3	Top height (m)

the S-system models may be found in Table 2.1. The first model tried was the full S-system model Equations 2.9. This model was not expected to work, but was tried to provide some insight into modeling with S-systems and the S-system parameter estimation problem. There was also the off chance that it might work. The full model formulation failed to meet the first heuristic criteria: the model could not be integrated for each Douglas-fir stand used to estimate its parameters.

$$\begin{aligned} \dot{X}_1 &= \alpha_1 X_1^{g_{11}} X_2^{g_{12}} X_3^{g_{13}} - \beta_1 X_1^{h_{11}} X_2^{h_{12}} X_3^{h_{13}} \\ \dot{X}_1 &= \alpha_2 X_1^{g_{21}} X_2^{g_{22}} X_3^{g_{23}} - \beta_2 X_1^{h_{21}} X_2^{h_{22}} X_3^{h_{23}} \\ \dot{X}_1 &= \alpha_3 X_1^{g_{31}} X_2^{g_{32}} X_3^{g_{33}} - \beta_3 X_1^{h_{31}} X_2^{h_{32}} X_3^{h_{33}} \end{aligned} \tag{2.9}$$

The full S-system model Equations 2.9 was then modified by removing the efflux terms for the QMD and top height equations, $\dot{X}_2$ and $\dot{X}_3$. This enforces the constraint that tree sizes may only increase. The stand density equation $\dot{X}_1$ was unchanged for this model formulation, allowing stand density to change or remain the same. This model, defined by Equations 2.10 satisfied the first two heuristic criteria, but failed to satisfy the third and fourth criteria. The dynamics produced by the model were generally correct within the envelope of the data except for stand density which began to increase for large trees. Further, the both QMD and top height growth rates failed to decrease over time, producing trees that were much larger than biologically possible.

$$\begin{aligned} \dot{X}_1 &= \alpha_1 X_1^{g_{11}} X_2^{g_{12}} X_3^{g_{13}} - \beta_1 X_1^{h_{11}} X_2^{h_{12}} X_3^{h_{13}} \\ \dot{X}_1 &= \alpha_2 X_1^{g_{21}} X_2^{g_{22}} X_3^{g_{23}} \\ \dot{X}_1 &= \alpha_3 X_1^{g_{31}} X_2^{g_{32}} X_3^{g_{33}} \end{aligned} \tag{2.10}$$

The model defined by Equations 2.10 was then modified to produce the model defined by Equations 2.11. Stand density and QMD are known to interact strongly [24, 37, 65, 102, 153], and stand density and dominant tree height are known to not interact [94, 102], so top height was removed from the stand density equation, $\dot{X}_1$. A Chapman-Richards decay term was added to the QMD equation [24, 65, 153], $\dot{X}_2$, to reduce the QMD growth rate for larger, forcing it to decline eventually. This approach was seen as superior to specifying *a priori* a maximum tree diameter in the model as is commonly done [13, 12, 36, 58, 94, 153]. Two modifications were made to the top height equation, $\dot{X}_3$. A Chapman-Richards decay term was added to the top height equation, $\dot{X}_3$ and stand density, X_1, was removed from the growth term. The former was done to force a reduction in the top height growth rate for larger trees, and the latter because the height growth rates of the dominant trees in a stand is generally unaffected by stand density [94, 102]. Using the Chapman-Richards decay term in this equation was also preferred to specifying an *a priori* a maximum top height.

$$\begin{aligned}
\dot{X}_1 &= \alpha_1 X_1^{g_{11}} X_2^{g_{12}} - \beta_1 X_1^{h_{11}} X_2^{h_{12}} \\
\dot{X}_1 &= \alpha_2 X_1^{g_{21}} X_2^{g_{22}} X_3^{g_{23}} - \beta_2 X_2 \qquad (2.11) \\
\dot{X}_1 &= \alpha_3 X_2^{g_{32}} X_3^{g_{33}} - \beta_3 X_3
\end{aligned}$$

The model defined by Equations 2.11 performed much better than the others, but again failed to meet the third and fourth heuristic criteria: stand density eventually increased and tree size measured at 300 and 1000 years were not biologically possible. An examination of these effects indicated that the QMD growth rate eventually began increasing, and at an accelerating rate, potentially causing both the increase in stand density and the excessively large tree dimensions. Given the strong size-density interaction for QMD, top height X_3 was removed from the equation for QMD, $\dot{X}_2$, to obtain the model formulation given by Equations 2.12. This model almost met the first four heuristic criteria, failing only for stand density which eventually began to increase. To resolve this problem, QMD X_2, was removed from the stand density equation, $\dot{X}_1$, to obtain the final S-system model formulation Equations 2.13.

$$\begin{aligned} \dot{X}_1 &= \alpha_1 X_1^{g_{11}} X_2^{g_{12}} - \beta_1 X_1^{h_{11}} X_2^{h_{12}} \\ \dot{X}_1 &= \alpha_2 X_1^{g_{21}} X_2^{g_{22}} - \beta_2 X_2 \\ \dot{X}_1 &= \alpha_3 X_2^{g_{32}} X_3^{g_{33}} - \beta_3 X_3 \end{aligned} \tag{2.12}$$

The final S-system model formulation, Equations 2.13, met all six of the heuristic criteria and was therefore chosen as the formulation for the Douglas-fir growth and yield model. Recall that in an S-system, the juxtaposition of state variables in a term indicates that those state variables are coupled in their effects on that term. Thus, the forms of the influx and efflux terms in the S-system equate the juxtaposition of state variables to the physical interaction of those state variables. This type of interaction based modeling has been shown to be appropriate, both as a conceptual tool and in applications [61, 80, 81, 91, 98, 120, 123, 125, 126, 127, 135, 134, 136, 137, 138, 156].

$$\begin{array}{lclllllll} \dot{X}_1 & = & \alpha_1 & X_1^{g_{11}} & & & - \ \beta_1 & X_1^{h_{11}} & X_2^{h_{12}} & \\ \dot{X}_2 & = & \alpha_2 & X_1^{g_{21}} & X_2^{g_{22}} & & - \ \beta_2 & & X_2 & \\ \dot{X}_3 & = & \alpha_3 & & X_2^{g_{32}} & X_3^{g_{33}} & - \ \beta_3 & & & X_3 \end{array} \tag{2.13}$$

The state variables in each term of the final S-system model are arranged in columns to permit a straightforward visual identification of those variables which contribute to a specific term. With this in mind, the state variables in each term for each equation are justified in terms of their consistency with the biological phenomena they are intended to represent.

The equation for $\dot{X}_1$ defines the rate of change of stand density over time. The decay term, $\beta_1 X_1^{h_{11}} X_2^{h_{12}}$, defines the size-density effects influencing the rate of change of stand density by including both stand density, X_1, and QMD, X_2. The growth term, $\alpha_1 X_1^{g_{11}}$, includes only stand density, X_1, and is considered to be an inertia term that offsets the stand density component in the decay term. Stand density should change only if acted upon through the size-density relationship. Given the form of this equation, $h_{11} > 0$, because stand density may promote a decline in stand density, $h_{12} > 0$, and because QMD promotes a decline in stand density, $g_{11} > 0$, to offset the decay component of stand density, and g_{11} and h_{11} should be approximately equal, they must offset the stand density components in the growth and decay terms. The magnitudes of α_1 and β_1 should both be small, given that stand density is measured in trees per hectare, a large number initially, and that mortality is generally much smaller than the number of live trees.

The equation for $\dot{X}_2$ defines the rate of change of QMD over time. The growth term $\alpha_2 X_1^{g_{21}} X_2^{g_{22}}$ defines the size-density effects influencing the QMD growth rate, and includes stand density, X_1, and QMD, X_2. The decay term, $\beta_2 X_2$, is based on the Chapman-Richards equation [24, 153], and includes only QMD, X_2. This term acts as a size dependent modifier for the QMD growth rate that will cause the growth rate to decline to an asymptote, possibly zero, as QMD increases. Given the limited range of data, this then causes QMD to approach an asymptote or maximum value. Given the form of this equation, $g_{21} < 0$, because stand density should inhibit diameter growth, $g_{22} > 0$, because QMD should promote QMD growth. The magnitudes of α_2 and β_2 are not expected to be large, but should be on the order of one.

The equation for $\dot{X}_3$ defines the rate of change of top height over time. The growth term $\alpha_3 X_2^{g_{32}} X_3^{g_{33}}$ defines the size-size effects influencing the top height growth rate, and includes QMD, X_2, and top height, X_3. The decay term $\beta_3 X_3$, is also based on the Chapman-Richards equation and includes only top height, X_3. This term acts as a size dependent modifier for the top height growth rate that will cause the growth rate to asymptotically approach zero as top height increases. As for QMD, this then causes the top height to approach an asymptote or maximum value. Given the form of this equation. g_{32} could be positive or negative, promoting or inhibiting top height growth, and $g_{33} > 0$, because top height should promote top height growth. The magnitudes of α_3 and β_3 are also not expected to be large, but should be on the order of one.

The growth and yield model defined by Equations 2.13 is not yet complete. Appropriate initial conditions for the state variables still need to be defined. The selection of live TPH, QMD, and top height as state variables imposes several additional constraints that must be incorporated into the S-system model or taken into consideration when using the model.

The definitions of QMD and top height both require diameter at breast height (DBH) measurements from the individual trees in a stand. So, the model cannot represent trees that are shorter than breast height, 1.3 m. Given this minimum top height requirement, it seems reasonable to further assume that the stand is fully stocked, that is, the number of live trees per hectare cannot increase. This is equivalent to stating that a stand is modeled as a single cohort without recruitment of new trees, which is an appropriate assumption for monoculture plantation forests. Finally, tree size is assumed to only increase with time. Only external events that cause breakage can reduce the height of a tree, and these events are not included in the model.

The constraints defining allowable initial conditions imply that stand density must be positive, $X_1 > 0$, QMD must be positive, $X_2 > 0$, and top height must be at least breast height, $X_3 \geq 1.3$ m. The stocking and size growth assumptions imply that stand density may decrease or remain constant over time, $\dot{X}_1 \leq 0$, QMD may increase or remain constant over time, $\dot{X}_2 \geq 0$, and top height may increase or remain constant over time $\dot{X}_3 \geq 0$. The constraints on stand and tree size development are presented as Equations 2.14, and are enforced in the model when it is integrated from a valid initial condition.

$$
\begin{aligned}
\dot{X}_1 &= \begin{cases} \alpha_1 X_1^{g_{11}} - \beta_1 X_1^{h_{11}} X_2^{h_{12}} & \text{if } \alpha_1 X_1^{g_{11}} - \beta_1 X_1^{h_{11}} X_2^{h_{12}} \leq 0 \\ 0 & \text{otherwise} \end{cases} \\
\dot{X}_2 &= \begin{cases} \alpha_2 X_1^{g_{21}} X_2^{g_{22}} - \beta_2 X_2 & \text{if } \alpha_2 X_1^{g_{21}} X_2^{g_{22}} - \beta_2 X_2 \geq 0 \\ 0 & \text{otherwise} \end{cases} \\
\dot{X}_3 &= \begin{cases} \alpha_3 X_2^{g_{32}} X_3^{g_{33}} - \beta_3 X_3 & \text{if } \alpha_3 X_2^{g_{32}} X_3^{g_{33}} - \beta_3 X_3 \geq 0 \\ 0 & \text{otherwise} \end{cases}
\end{aligned}
\qquad (2.14)
$$

The dynamic, stand level, growth and yield model for monoculture, plantation Douglas-fir integrates survivor growth to obtain the yield for the living trees [153], accounting for mortality and size growth in continuous time. However, the model definition is not quite complete: even though the model uses continuous time, a base time step for the model must still be selected. The selection of the base time step must take into consideration both the data available and the use of the model. A typical use of growth and yield models involves the annual projection of stand development to obtain wood volume and other stand characteristics for planning purposes [24, 36, 58, 94, 153]. Thus, an annual base time step for the growth and yield model seems quite reasonable.

The Douglas-fir growth and yield model is now completely defined. What remains is to determine values for the 13 model parameters, α_i, β_i, g_{ij}, and h_{ij} that will complete its specification. The base time step for the model is set by the parameter values. To obtain an annual base time step, some form of annualization will need to be performed on the stand remeasurement data before estimating the parameter values. see Section 2.5 for the details. Parameter values will be determined through a least squares procedure using a wide variety of Douglas-fir stand measurement data obtained from locations throughout the Pacific Northwest, west of the Cascade Mountains.

2.5 *Douglas-fir stand measurement data sources and description*

Stand measurement data used to calibrate and test the stand level Douglas-fir growth and yield model were provided by the Stand Management Cooperative (SMC), in the College of Forest Resources at the University of Washington [26]. The SMC maintains a high quality database of individual tree measurements for stands located on commercial production forest lands in the Pacific Northwest. The SMC data span the region from southern Oregon, to Southern British Columbia, west of the Cascade

Mountains. The SMC database is composed primarily of Douglas-fir and western hemlock (*Tsuga heterophylla*) measurements, some of which span approximately 30 years, with some representation of other tree species common to the Pacific Northwest.

The data in the SMC database are derived from two sources: the Regional Forest Nutrition Research Project (RFNRP) and the SMC proper. The RFNRP was a Pacific Northwest regional research project begun in 1969 and designed to investigate the effects of fertilization, on wood production and wood quality for commercial forest land west of the Cascade Mountains [111]. Most of the RFNRP installations were in stands 25 to 60 years old, and the intent of the project was to determine whether fertilization or fertilization and thinning would improve wood production by the end of rotation harvest [111]. In 1985, when the SMC was established, the data obtained from the RFNRP was absorbed by the SMC and became a part of its forest nutrition project [26]. The SMC continued to measure the RFNRP plots until 1992, when the project ended [111, 26].

The SMC has established its own research plots, again on commercial forest land spanning the Pacific Northwest region west of the Cascade Mountains, to collect growth data on very young stands, and stands which are at the stage where a pre-commercial thinning, or even a commercial thinning, are possible [26]. The stand ages for these stands are from 4 to 25 years. These data augment the older stand data from the RFNRP which generally lacked very young stands. The SMC has implemented a wide variety of treatments within its young stands, including a wide range of planting densities, fertilization regimes, thinning levels, and pruning levels to investigate their effects on the development of the young stands, both singly and in various combinations. The SMC database thus contains a sufficient range of data to be considered representative of the Pacific Northwest region west of the Cascade Mountains.

The stand density, measured as trees per unit area, QMD, and top height data used to calibrate and test the Douglas-fir growth and yield model described in Section 2.4

were obtained from the plot summary table of the SMC database. This table provides a stand level summary of the individual tree measurements organized by research plot and measurement. The SMC data are stored in its database using the Imperial units system, measuring stand area in acres, tree diameter in inches, and tree height in feet. The stand level data in the SMC plot summary table were derived from the number of live trees on a plot, the plot sizes, and the individual DBH and height measurements obtained for each tree on a plot.

The stand density values, in trees per acre (TPA), from the SMC plot summary table were computed from the number of live trees per plot and the plot size according to the formula

$$\text{TPA} = \frac{\text{number of trees per plot}}{\text{plot size (ac)}}.$$

All trees on a plot are assumed to be Douglas-fir trees for the purposes of calibrating the stand level Douglas-fir growth and yield model. This computation simply scales the number of live trees to a standard area. Given the division by plot size, small plot sizes will produce larger scale factors than larger plot sizes. Through this relationship, plot size affects the accuracy of the derived TPA values: TPA values obtained from larger plot sizes will be more accurate than TPA values obtained from smaller plot sizes. Each tree on a smaller plot represents more trees when scaled to TPA, so the plot size scaling will, in particular, inflate stand mortality values for small plot sizes. In addition, the fact that not all trees on a plot are Douglas-fir trees is exacerbated by the scaling effects of the small plots as well. These are issues that must be addressed when selecting the Douglas-fir data from the SMC database.

The SMC measures the diameter, taken at breast height, for all live trees on each research plot for each measurement. These individual tree diameters were then used to compute the QMD values in the SMC plot summary table using the formula

$$\text{QMD} = \sqrt{\frac{\sum_{i=1}^{n} d_i^2}{n}},$$

where n is the number of trees at or above breast height for that plot and measurement, and the d_i are the individual diameter measurements from the live trees [24]. The QMD value, then, is a *net* value, and does not include any trees that died during a measurement interval.

The SMC also measures individual tree heights for all trees in its younger stands but only for a representative subset of trees in its older stands, because height measurement is more difficult and time consuming than diameter measurement. Missing tree heights are filled in by using a height-diameter relationship,

$$\mathrm{H} = \mathrm{BH} + b_0 \exp\left(-\frac{b_1}{\mathrm{DBH}}\right),$$

that is estimated for each plot and measurement that has missing heights [45]. In the height-diameter relationship, BH is breast height, and b_0 and b_1 are unknown parameters for each measurement, generally estimated using least squares and the data from trees with both DBH and height measurements. When estimating a height-diameter relationship for each measurement, all tree species are assumed for convenience to follow the same height-diameter curve. The fact that many tree heights are estimated from height diameter relationships causes a reduction in the variability of tree heights: all unmeasured heights lie on a single height-diameter curve, rather than being randomly distributed around the height-diameter curve. This reduction in height variability should not affect the average stand height, but it may affect the stand top height.

Top height values for the SMC plot summary table were then obtained by computing the average height for the appropriate number of the largest diameter trees per plot, N_{th}. Top height is defined to be the average height of the 100 largest diameter trees per hectare, or equivalently, the average height of the 40 largest diameter trees per acre. The appropriate number of trees to use for a particular plot is determine using the plot size and the appropriate number of trees depending on the measurement units, e.g.,

$$N_{th} = \left\lfloor 40 \times \text{plot size (ac)} + \frac{1}{2} \right\rfloor$$

Table 2.2: Imperial to metric conversion factors used to convert the SMC plot summary data into metric units. To convert from Imperial units to metric units, multiply by the conversion factor. To convert from metric units to Imperial units, divide by the conversion factor. Conversion factors are from [65].

Conversion	Conversion factor
Acres to hectares	0.4047
Inches to centimeters	2.5400
Feet to meters	0.3048

for Imperial units, or

$$N_{th} = \left\lfloor 100 \times \text{plot size (ha)} + \frac{1}{2} \right\rfloor$$

for metric units, where $\lfloor x \rfloor$ returns the largest integer less than x. Again, plot size can be seen to affect a stand level attribute: smaller plot sizes will have greater uncertainty than larger plots sizes when computing top height due to fewer trees being used in the computation.

The stand density, QMD, and top height data were extracted from the plot summary table within the SMC database. A variety of ancillary information describing the stands was also extracted from the SMC database for each stand, including site index, elevation, Douglas-fir percentages in basal area and number of stems, and stand total age. The data were then converted into metric units, hectares, centimeters, and meters, respectively, for this modeling effort. The Imperial to metric units conversions used are found in Table 2.2 [65]. Final data selection involved filtering the stand measurement data were according to the following criteria:

- Selected stands must be untreated, pure Douglas-fir stands. A pure stand is a stand in which the Douglas-fir basal area is at least 80% of the total stand basal area.

- The plot size must be at least 0.0407 ha. This minimum plot size was chosen to avoid the potential introduction of overlarge biases caused by scaling stand density data for extremely small plots.

- The site index value for the stand must be valid, i.e., nonzero, non blank.
- Both QMD and height measurements were present and nonzero.

These criteria selected only pure Douglas-fir stands using a basal area criterion. The criteria also selected only the most reliable stand density and top height values by filtering out the smallest plot sizes. Finally, the criteria removed any obviously erroneous stand measurements from the data set. These mechanically filtered stand measurement data were then subjected to a visual inspection to discover, and remove, any remaining anomalous measurements or stands. At this stage, anomalous measurements or stands were those that when plotted appeared to be visually very different from the majority of the data set.

A straightforward approach was taken when selecting and filtering the extracted data to obtain the final stand measurement data. Only simple inequalities were used to obtain agreement of the data with the assumptions of the S-system growth and yield model. This was done for three reasons. First, the database is large, and a hand selection of the data would be an onerous and time consuming undertaking. Second, the selection criteria were kept simple to avoid selecting only stands that were ideal candidates for model calibration, for example by selecting stands which were very similar. Thus, if the data set contains less appropriate data, the model can only improve if these data are removed from the model calibration data set. Third, the data selection approach taken is fairly typical of the data selection procedures used for growth and yield model development [24, 65, 94, 153].

The data selection and filtering process yielded 167 SMC Douglas-fir installations spanning the Pacific Northwest region from southern Oregon to southern British Columbia, west of the Cascade Mountains, Figure 2.1. Douglas-fir remeasurement data from 270 research plots were obtained from these installations, providing a set of stand trajectories. The stand trajectories were comprised of 1238 sets of stand density, QMD, and top height measurements with a total of 968 measurement intervals, spanning the time period from 1969 through 1997. Stand trajectory lengths varied from 2 to 26 years. Measurement intervals were nominally two or four years. Measurement intervals actually obtained were two, four, or six years, due to the data selection criteria, and the fact that there were incomplete measurement sets for some measurements on some plots.

Because an annual base time step is desired for the Douglas-fir growth and yield model defined in Section 2.4, the remeasurement data must be annualized in some way. A typical approach is to compute the periodic annual increment (PAI) for each multiple year measurement interval [24, 153]. The PAI is defined as the difference between the final and initial measurements of a measurement interval, divided by the number of years separating the measurements,

$$\mathrm{PAI} = \frac{X_f - X_0}{t_f - t_0},$$

where X_0 is the initial measurement, X_f is the final measurement and t_0 and t_f are the initial and final times, in years, respectively. The PAI value provides the average annual rate of change for the measurement interval, or the slope of the line connecting the end points of a measurement interval. This line may then be used to interpolate between the end points

$$X_{t_y} = X_0 + (t_y - t_0)\,\mathrm{PAI}$$

to obtain approximate measurement values for the years t_y, $t_0 < t_y < t_f$.

The linear interpolation of stand measurements produced 3098 annualized measurements or 2828 annualized measurement intervals. This annualization approach was chosen, rather than simply using the actual measurement intervals and their PAI values directly, for compatibility with the climate model development described in Chapter 3. This annualization procedure also provided a natural data weighting for each measurement interval in the parameter estimation procedure, giving one annualized measurement for each year in the measurement interval. The augmented annualized data set contains no more information than is contained in the 1238 actual

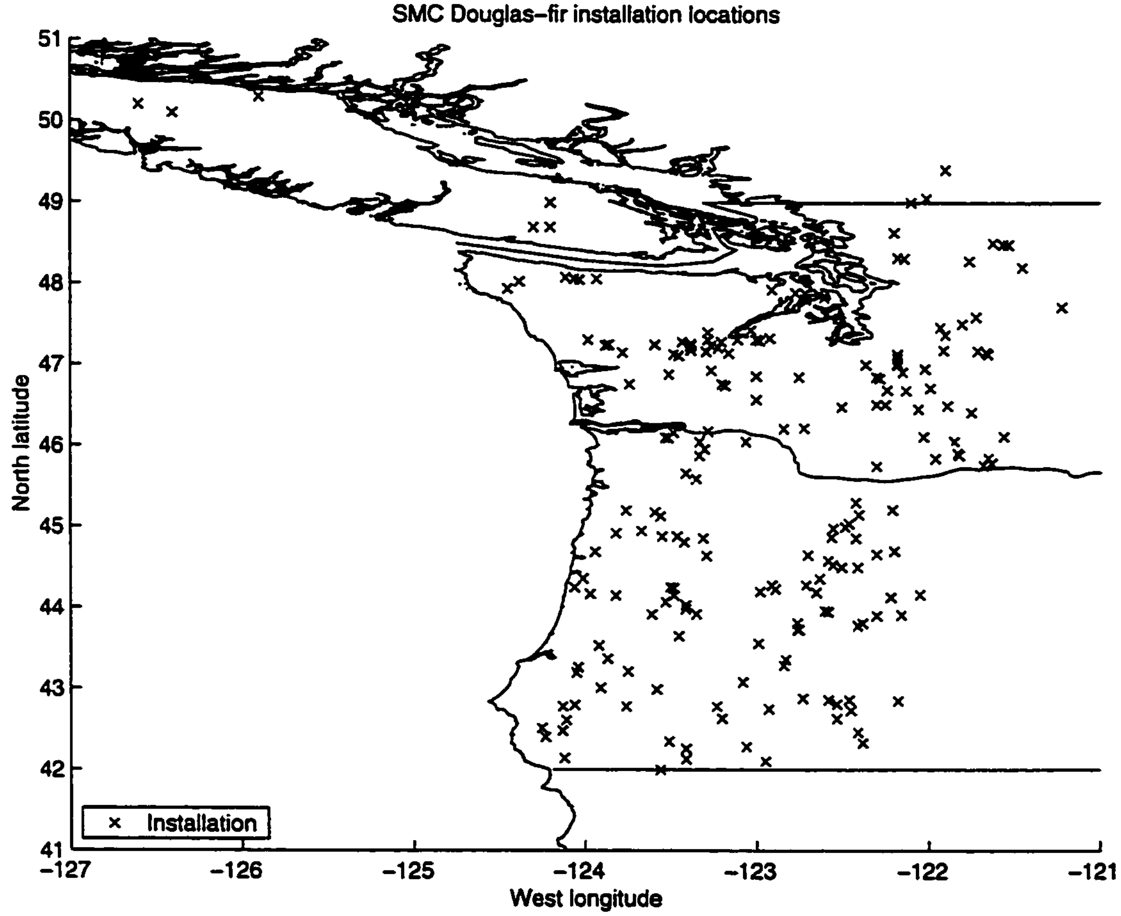

Figure 2.1: SMC installation locations for the stand measurement data. The SMC installations span the region from southern Oregon, through Washington, and into southern British Columbia, west of the Cascade Mountains. No map projection was used to correct for changes in latitude, hence the apparent elongation of Vancouver Island.

measurements, but it will be treated as though it were a real set of annual measurements for convenience and consistency of presentation. In addition to the annualization of the stand measurement data, the ancillary site index, elevation, Douglas-fir percentages in both basal area and number of stems, and stand total age, were also interpolated or replicated, as appropriate, to maintain a consistent data set.

A final filtering of the annualized measurement intervals was performed to remove any intervals which violate the assumptions of the Douglas-fir growth and yield model defined in Section 2.4. Specifically, annualized measurement intervals meeting any of the three inequalities

$$\begin{aligned} X_1^f - X_1^0 &> 0 \\ X_2^f - X_2^0 &< 0 \\ X_3^f - X_3^0 &< 0 \end{aligned} \tag{2.15}$$

were removed from the data set. The superscripts 0 and f indicate the initial and final measurements of a measurement interval, and the subscripts identify the state variables of the growth and yield model. Measurements meeting at least one of these inequalities were removed because they violate the growth and yield model assumptions that stand density may only remain constant or decrease, $\dot{X}_1 \leq 0$, and that tree size may only remain constant or increase, $\dot{X}_2 \geq 0$ and $\dot{X}_3 \geq 0$, respectively, removing 317, 0, and 10 annualized measurement intervals. A fourth inequality

$$X_1^f - X_1^0 < -150, \tag{2.16}$$

was also used to filter the data, and 70 annualized measurement intervals were removed. Measurements meeting this inequality were excluded as having mortality that was too large for an actual annual per hectare mortality for Douglas-fir stands. Mortality is considered to be the rate of change in stand density over time, and hence must be a number that is less than or equal to zero. Greater or lesser mortality, refers to a larger or smaller magnitude negative number, respectively. This interpretation of mortality was chosen for consistency with the size related variables with respect to rates of change.

The value of -150 in Equation 2.16 represents the loss of 6 trees on a 0.0407 ha plot, and was considered to be a very high, but possible, annual mortality level for Douglas-fir. The highest mortality values in these data occurred in a range of stand ages from 30 to 70 years. Mortality values ranging from -100 to -10 TPHper year are typical for Site Class II Douglas-fir stands within this age range [30, 92, 142]. Greater mortality than -150 trees per hectare per year was deemed to be highly unlikely, and possibly a result of plot sizes that were too small to adequately represent the natural variability in mortality within a stand. This final filtering of the data yielded 2431 annualized measurement intervals that will be used to calibrate and validate the Douglas-fir growth and yield model.

The 2431 annualized stand measurement intervals and their associated ancillary variables are the primary data used for the Douglas-fir growth and yield model fitting and the subsequent analysis. The data are viewed first as independent measurement intervals, the reasons for which will become obvious, and then as a contiguous set of stand measurements defining a set stand trajectories. The $N = 2431$ stand measurement intervals obtained are denoted by Y_k^0 and Y_k^f, the initial and final state variable or yield measurements, with the initial and final measurements taken at stand ages t_k^0 and t_k^f, $k = 1, 2, \ldots, N$. The time differences $t_k^f - t_k^0$ for the measurement interval view of the data are all one year, since the data were annualized prior to the final data filtering. The 2431 measurement intervals represent $S = 270$ stand measurement trajectories whose measurements will be denoted by Y_{ks} for stand ages t_{ks}, $k = 0, 1, 2, \ldots, N_s - 1$ and $s = 1, 2, \ldots, S$, where $N_s + 1$ is the number of annualized measurements for a particular stand trajectory. The time differences $t_{k+1s} - t_{ks}$ will not all be equal to one year because annualized measurements that did not meet the final filtering criteria in Equations 2.15 and Equation 2.16 were removed. Each stand measurement is a three element vector containing the stand density, QMD, and top height values, i.e, $Y_k^0 = [Y_{1k}^0, Y_{2k}^0, Y_{3k}^0]^T$, $Y_k^f = [Y_{1k}^f, Y_{2k}^f, Y_{3k}^f]^T$ and $Y_k s = [Y_{1ks}, Y_{2ks}, Y_{3ks}]^T$, for

Table 2.3: Douglas-fir numerical data summary for the annualized measurements. The annualized sample size was 2431, except for stand elevation which was 2401 due to missing elevations for a few plots. *MAD is mean absolute deviation.

Stand attribute	Mean	Standard deviation	Min.	Med.	Max.	MAD*
Initial age (years)	38.49	17.28	4.00	40.00	113.00	13.94
Site index 50 (m)	34.89	5.62	17.98	36.58	45.72	4.53
Elevation (m)	452.64	282.98	4.57	420.62	1341.12	229.51
Initial TPH	1322.17	866.01	123.55	1104.56	5337.48	618.45
Final TPH	1294.56	850.98	123.55	1082.32	5275.70	604.03
Initial QMD (cm)	21.50	9.20	1.98	21.29	68.57	7.29
Final QMD (cm)	22.16	9.06	3.49	21.84	68.83	7.20
Initial top height (m)	24.91	10.09	2.80	26.91	51.74	8.36
Final top height (m)	25.59	9.96	3.69	27.61	51.94	8.25
% Douglas-fir (BA)	97	5	80	99	100	4
% Douglas-fir (stems)	90	14	38	97	100	11
Plot size (ha)	0.07	0.05	0.04	0.04	0.20	0.04

the initial and final measurements of an interval and a stand trajectory measurement, respectively.

A brief summary highlighting some of the key features of these data follows. For these summaries, the data should be interpreted as independent measurement intervals. Table 2.3 presents a numerical summary of the annualized stand measurement data and the ancillary variables of interest. Figure 2.2 through Figure 2.4 provide histograms of the same information and characterize the distributions of the variables, most of which are not symmetric and may be highly skewed.

Figure 2.2 presents histograms for the annualized stand ages, site index values, and stand elevations. Stand ages range from 4 to 115 years. The bulk of the measurements have stand ages between 25 and 60 years, with approximately 20% of the measurements having stand ages less than 20 years, and 10% of the measurements with stand ages greater than 60 years. This age distribution is not unexpected given that the stand measurement data are from a commercially oriented cooperative in the

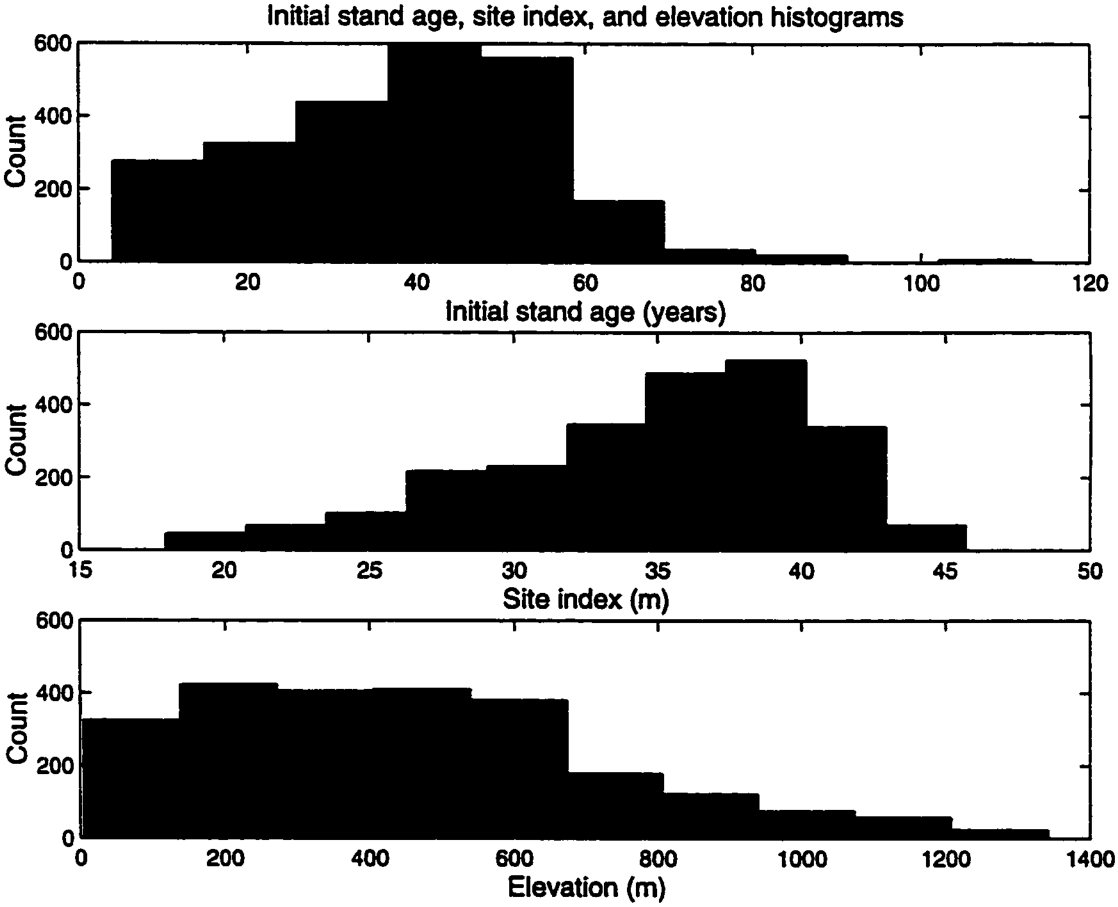

Figure 2.2: Histograms of annualized initial stand age (top), site index at age 50(middle). and elevation (bottom). The sample size is 2431 points for stand age and site index, and 2401 points for elevation; several stands did not have valid elevation values.

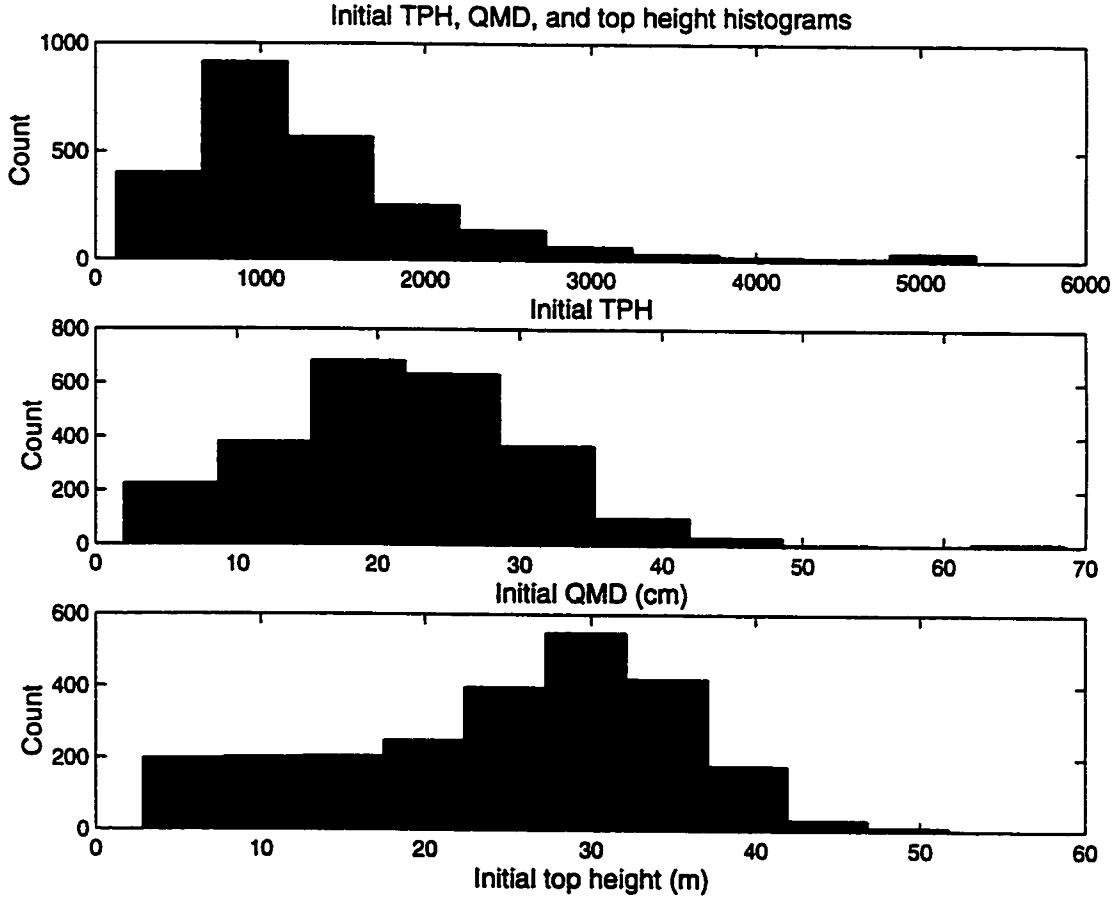

Figure 2.3: Histograms of annualized initial stand density (top), QMD(middle), and top height (bottom). The sample size is 2431 points for each of initial stand density, initial QMD, and initial height.

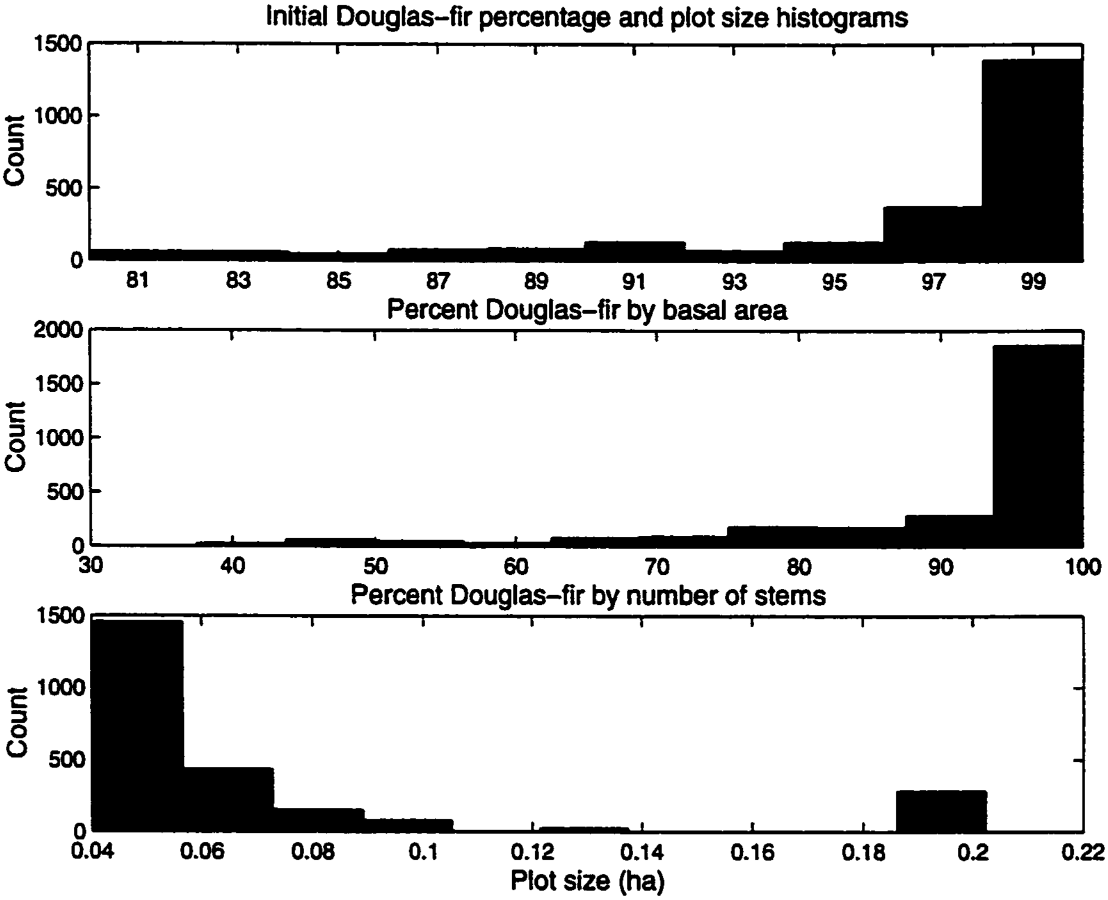

Figure 2.4: Histograms of annualized initial Douglas-fir percentages (top and middle) and plot size (bottom). The sample size is 2431 points for the Douglas-fir percentages and the plot sizes.

Pacific Northwest where Douglas-fir rotation ages are generally from 50 to 60 years. Site index values, with a reference age of 50 years for Douglas-fir, range from 18 m to 45 m. The bulk of the site index values lie between 33 m and 42 m, high site class III to low site class I [73, 16], and are indicative of highly productive Douglas-fir sites. The remaining site index values constitute less than 20% of the measurements, with most of these being less than 33 m, indicating progressively lower productivity. The distribution of Douglas-fir site index values is highly left skewed, and is generally consistent with what would be expected for a commercially oriented organization such as the SMC; it is more difficult to make money on lower quality sites. Stand elevations for the measurement data range from sea level to approximately 1350 m, but are generally below 600 m. Less than 20% or the measurements are from plots at elevations higher than 600 m. The higher elevation stands are generally located on the western slopes of the Cascade Range.

Stand age is a somewhat tricky value to pin down, and it has historically been derived from average breast height age, obtained from tree cores adjusted for differences in site quality as indicated by site index values [92, 73, 16]. Stand ages are herein defined to be the time since planting, for the recent SMC planted stands [26]. For the older RFNRP stands, which may have been naturally regenerated but are generally of unknown stand origin, stand ages will be interpreted as the average age for an initial sample of trees from a plot adjusted for differences in site index [73, 16, 26], and then incremented by the remeasurement interval for each subsequent measurement to obtain current stand age. Historical stand ages are typically not known exactly, and the newer SMC planted stands have different ages of planting stock, ranging from one to four years old at planting, so even the new stand ages are not exact, though ages from seed could be derived. The inexactness of stand age is not an issue for the growth and yield model described in Section 2.4 because the model *does not* include stand age explicitly. The S-system model when integrated and the parameter estimation procedures rely on only the time differences between measurements, and these are known, regardless of the stand origin or actual age. This is one benefit of using an autonomous system for modeling forest dynamics. Stand age is used herein as a common and convenient variable for displaying results, but is not fundamental to obtaining the results.

Figure 2.3 presents histograms of the initial stand density, initial QMD, and initial top height for each measurement interval. The initial stand densities ranged from approximately 120 TPH to over 5300 TPH, with the bulk of the measurements between 500 TPH and 1500 TPH. Less than 20% of the initial stand densities were outside this range, but the distribution is strongly right skewed. The initial QMD values ranged from 2 cm to 69 cm. The initial QMD distribution is nearly symmetric with a mean near 20 cm, but slightly right skewed, with the bulk of the data between 10 cm and 35 cm. Less than 10% of the initial QMD values are less than 10 cm and approximately 5% are greater than 35 cm. Initial top heights ranged from approximately 3 m to 52 m. The initial top height distribution is somewhat symmetric with a mean near 28 m, but is left skewed. The bulk of the measurements lie between 20 m and 38 m, with approximately 25% of the initial top heights less than 20 m, and approximately 5% of the initial top heights greater than 38 m. Histograms for the distributions of the final of stand density, QMD, and top height measurements were similar to those presented and are omitted.

Figure 2.4 presents histograms of the initial percentage of live trees on a plot that are Douglas-fir, measured by stand basal area and by number of trees, and the distribution of plot sizes. By definition, pure Douglas-fir stands are those stands with Douglas-fir representing at least 80% of the stand basal area. The SMC database, being oriented toward plantation Douglas-fir, contains many stands with very high Douglas-fir basal area percentages, and over 75% of the stand measurements have a Douglas-fir basal area percentage greater than 90%. The percentage of Douglas-fir by stems for these measurements is strongly left skewed, with a range from 38% to 100%, with approximately 80% of the measurements greater than 90% Douglas-fir.

The stands with lower percentages of Douglas-fir by stem count fall in the range of 1000 TPH to 3500 TPH. Somewhat surprisingly, the very high density stands are predominantly Douglas-fir. This variability in Douglas-fir percentage by number of stems for *pure* stands supports the restriction of stand mortality values performed, and may indicate that the value of -150 trees per hectare per year may still be too large. The distribution of sample plot sizes is strongly right skewed, indicating that smaller plot sizes are more common. Approximately 80% of the plot sizes are less than 0.07 ha, with the majority of these being 0.04 ha. This predominance of very small plot sizes necessarily introduces more uncertainty into the stand density and mortality values, because large scaling factors are necessary to convert to a per hectare basis. This, coupled with the greater variability in Douglas-fir percentages by stem count, will in turn make the Douglas-fir stand density projections derived from these data more uncertain.

From the data description there is a clear *a priori* quality assessment that may be placed on these data. The highest quality data are the QMD values obtained from a 100% tally of the live Douglas-fir tree diameters. The intermediate quality data are the top height values, for two reasons: many tree heights were estimated from height diameter relationships, and plot size affects the accuracy and precision of top height values. The lowest quality data are the stand density values; the density values are assumed to represent only Douglas-fir trees, and were scaled from small plot sizes to a per hectare basis. The per hectare stand density values may, therefore, not reflect the true stand density. The increased uncertainty in the stand density values aside, this data set should be ideal, or nearly so, for modeling monoculture Douglas-fir plantations in the Pacific Northwest.

2.6 *Model analysis and validation methods*

The procedures used to complete and validate the specification of the S-system based, stand level, Douglas-fir growth and yield model follow. The procedures must address the following issues. First, a closed form solution to the S-system model will in general not exist [15], so a numerical method for solving the S-system model must be selected. Second, the parameter estimation procedures must be appropriate for the type of data and model, without adding assumptions which are incompatible with the sample data or model. Third, the stand density, QMD, and top height stand measurement data must be treated as multidimensional data. Fourth, the nonlinear nature of the model and the complex parameter estimation process make the computation and interpretation of parameter standard errors using classical statistical approaches difficult [39, 145]. Finally, given the large sample size, traditional statistical tests for goodness-of-fit may produce spurious results [7, 172]. Each of these issues is addressed in the discussion of the procedures used, and the particular procedures are justified in terms of their applicability and their ability to address each of these concerns.

2.6.1 Solving S-systems

S-systems are first order systems of autonomous ODEs, which with a specified initial condition becomes an initial value problem, and there are a variety of methods for solving, or integrating, these types of problems [15, 57, 20, 3]. The two most common approaches for solving initial value problems are Runga-Kutta methods and Adams-Bashforth-Moulton methods [57, 20]. Both methods use Taylor series expansions of the unknown function about the current point in the state space and project the current state based on a small time increment [57, 20]. High quality and efficient implementations of these methods exist and are in common use [57, 90]. There is, however, a specialized algorithm for solving initial value problems specified as S-systems [67, 68]. This method is also based on a Taylor series expansion about the current state, but in logarithmic coordinates, and is generally more effective computationally than the traditional Runga-Kutta or Adams-Bashforth-Moulton solution methods when they are applied to S-systems [67, 68]. All of these methods perform the numerical integrations based on time increments, not actual time, so an exact knowledge of both the current state and time is not required for the integration process, only knowledge of the current state and time interval are necessary [20, 15, 57, 67, 68].

A straightforward, variable order, variable step size S-system solver was implemented in Matlab [90], and was used to integrate the S-system based Douglas-fir growth and yield model. The integration algorithm allows the solver to increase the order of the Taylor polynomial approximation, decreasing the local truncation error, or to decrease the base step size, again to decrease the local truncation error, to obtain or improve a solution to an initial value problem [67, 68, 57, 20]. All solutions of the Douglas-fir growth and yield model used a default S-system Taylor polynomial order of 7, a default time step of one year, and a default error tolerance of 10^{-3}.

2.6.2 S-system parameter estimation procedures

Estimation of the S-system parameters is performed in two steps. The first step attempts to obtain parameter estimates using the algebraic representation of the S-system and approximate rates of change for stand density, QMD, and top height obtained from a set of measurement intervals. This step presumes no *a priori* knowledge of the parameter values, although if values are known *a priori* they may be used. The second step uses the parameter values obtained in the first step and attempts to refine them by integrating each stand from its initial condition to its final condition obtaining yield values, i.e., integrating from (t_{0s}, Y_{0s}) to $(t_{N_s s}, Y_{n_s s})$.

These two parameter estimation steps treat the stand measurement data in different but compatible ways given the underlying representation for the Douglas-fir growth and yield model as an autonomous system of ODEs. In the first step, the stand measurement data are treated as a set of independent measurement intervals used to obtain estimates of *growth* for estimating the S-system parameter values. This approach is based on the algebraic representation of the S-system and will be referred to as the *algebraic* parameter estimation step. In the second step, the stand measurements are treated as dependent stand trajectories, and are used as *yield* values for refining the S-system parameter values obtained from the algebraic parameter estimation step. This procedure is based on repeatedly solving the system the S-system for each stand and will be referred to as the *shooting* parameter estimation step or the parameter refinement step. The two steps of the S-system parameter estimation procedure are presented in their general forms, and then specialized to the Douglas-fir growth and yield model defined in Section 2.4.

To define the algebraic parameter estimation step, consider an S-system with n equations and m independent state variables. Let N be the number of measurement intervals obtained from one or more samples, and let Y_{ik}^0, Y_{ik}^f, t_k^0, t_k^f, $k = 1, 2, \ldots, N$ be the initial state variable or yield measurements, the final state variable or yield measurements, the times of the initial yield measurements, and the times of the final yield measurements for each interval. The time differences $t_k^f - t_k^0$ need not all be equal. The S-system parameters are estimated by solving a simply constrained nonlinear least squares minimization problem derived from the algebraic form of the S-system representation in the following way. First, compute an approximate rate of change for each S-system equation for each measurement interval,

$$\widehat{\dot{X}_{ik}} = \frac{Y_{ik}^f - Y_{ik}^0}{t_k^f - t_k^0},$$

to approximate the left-hand-side of the S-system. Next, compute approximate state variable values or yields for these rates of change as the midpoint of each measurement interval,

$$X_{ik} = \frac{Y_{ik}^0 + Y_{ik}^f}{2}.$$

Finally, solve the nonlinear least squares problem

$$\text{minimize} \quad f_a = \sum_{k=1}^{N}\sum_{i=1}^{n}\left(\widehat{\dot{X}_{ik}} - \left(\alpha_i \prod_{j=1}^{n+m} X_{jk}^{g_{ij}} - \beta_i \prod_{j=1}^{n+m} X_{jk}^{h_{ij}}\right)\right)^2$$

subject to:

$$\alpha_i \geq 0, \quad i = 1, 2, \ldots, n$$

$$\beta_i \geq 0, \quad i = 1, 2, \ldots, n$$

to determine the estimated S-system parameter values using a least squares minimization algorithm. Notice, in particular, that this procedure does not use the time variable explicitly, only the time differences. This is due to the fact that an S-system is an autonomous system of ODEs and the rate approximations are based on time differences. The solution to this minimization problem is a parameter vector p_a.

Although the measurement interval data appear to be treated independently by this least squares parameter estimation procedure, any dependencies among the measurement intervals are *automatically* accounted for by the underlying autonomous S-system model. This becomes clear after a consideration of what an autonomous system of ODEs represents mathematically, and a consideration of the measurement interval data and its use in this least squares procedure.

An autonomous system of ODEs compactly, concisely, and completely represents the vector field of a dynamic process in the phase or state space [15, 57], providing the simultaneous rates of change for a dynamic process for any point in the state space. Stated another way, for any point in the state space, an autonomous system defines the instantaneous direction and magnitude of the motion or flow of a dynamic process. The image of a skier skiing down a bumpy mountain with many small peaks and valleys may be useful. The way the skis point indicates the direction but not the magnitude of motion, because that is gravity dependent, as the skier flows down the mountain. The surface of the mountain, thus, represents the phase or state space.

The measurement interval data are used to compute approximations to the simultaneous rates of change for a dynamic process, $\widehat{\dot{X}_{ik}}$, and appropriate state variables for those rates of change, X_{ik}. These derived values provide a direct, empirical approximation to the vector field defined by some unspecified autonomous system, by assumption since an S-system is being used to represent the process. The least squares procedure is then simply attempting to identify which specific autonomous system represents the approximate vector field, on average. The algebraic least squares parameter estimation procedure simultaneously performs the parameter estimation for all equations, and thus treats the data as vectors of linked measurements. Thus, the data used in the least squares procedure and the underlying mathematical representation are compatible, and used in a consistent manner to estimate the S-system parameters.

To define the stand trajectory or shooting parameter refinement estimation step, again consider an S-system with n equations and m independent state variables. Let S be the number of independent trajectories that have been sampled, each with $N_s + 1$ state variable or yield measurements Y_{iks} obtained at times t_{ks}, $k = 0, 1, \ldots, N_s$ and $s = 1, 2, \ldots, S$. The time differences $t_{k+1s} - t_{ks}$ need not all be equal. The S-system parameter estimates are refined by solving the simply constrained nonlinear least squares minimization problem derived as a shooting method applied to the S-system model defined by the parameters obtained in the algebraic parameter estimation step [57, 20, 3]. The nonlinear least squares problem is

$$\text{minimize} \quad f_s = \sum_{s=1}^{S} \sum_{k=1}^{N_s+1} \sum_{i=1}^{n} \left(Y_{iks} - \hat{Y}_{iks}\right)^2$$

subject to:

$$\alpha_i \geq 0, \quad i = 1, 2, \ldots, n$$

$$\beta_i \geq 0, \quad i = 1, 2, \ldots, n,$$

where $\hat{Y}_{iks}$ are predicted yield values obtained at the times t_{ks} by integrating the S-system identified in the algebraic parameter estimation step for each trajectory. The trajectories are integrated from their initial measurement time, t_{0s}, and initial stand condition $Y_{0s} = (Y_{10s}, Y_{20s}, \ldots, Y_{n0s})^T$, to their final measurement time, t_{N_s}, and the intermediate times t_{ks}, $k = 1, 2, \ldots, N_s - 1$, for each stand,$s = 1, 2, \ldots, S$, using the S-system parameter value estimates p_a. This procedure does not explicitly use the time variables t_{ks} in the integration of the S-system, but it uses the time increments $t_{k+1s} - t_{ks}$, $k = 0, 2, \ldots, N_s$ to cumulatively generate the trajectories. The solution to this minimization problem is a parameter vector p_s.

The shooting parameter refinement step recognizes the fact that a dynamic process moves *forward* in time through the integration process. It also explicitly accounts for the dependencies among the state variable or yield measurements for each particular trajectory sampled. This step attempts to modify the estimated parameters to account for the complete trajectories. The shooting least squares parameter refinement procedure also simultaneously performs the parameter estimation for all equations, and hence treats the data as vectors of linked measurements. This procedure and its use of the trajectory data are also compatible with the underlying mathematical representation of a dynamic process as an S-system, and are also used in a consistent manner to refine the S-system parameter estimates.

The two step algebraic-shooting parameter estimation procedure was applied to the stand level Douglas-fir growth and yield model defined in Section 2.4, using the $N = 2431$ annualized measurement intervals obtained from the Douglas-fir stand remeasurement data and the $S = 270$ stand density, QMD, and top height trajectories. The `lsqnonlin` procedure, in the Matlab Optimization Toolbox [90, 89], was used to solve both of the least squares minimization problems for the 13 element parameter vector

$$\begin{aligned} p &= [\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, g_{11}, g_{21}, g_{22}, g_{32}, g_{33}, h_{11}, h_{12}]^T \\ &= [p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}, p_{12}, p_{13}]^T. \end{aligned}$$

Initial estimates for the first parameter estimation step were

$$p_0 = [0.05, 1.0, 1.0, 0.05, 1.0, 1.0, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5]^T,$$

and were chosen somewhat arbitrarily based upon some initial numerical experiments with the parameter estimation procedures on a subset of the stand measurement data. The intent was to select initial estimates for the parameter values that were *a priori* not informative, but of a reasonable magnitude, and to let the least squares optimization discover appropriate parameter values by extracting them from the stand measurement data. The parameter values obtained from the algebraic parameter estimation step, p_a, were then used as the initial parameter guesses for the shooting parameter refinement step to obtain parameter values p_s. The combination of parameter estimation methods should remove any obviously incorrect model behavior, such as loops, spirals, or periodic solutions. Stopping criteria used for the least squares minimizations were 10^-6 and 10^-4 for the algebraic and shooting estimation procedures, and they were chosen because the rates used in the algebraic parameter estimation procedure were generally two orders of magnitude smaller than the yield values used in the shooting based parameter refinement. These stopping criteria cause `lsqnonlin` to halt if changes in the objective functions, f_a or f_s were less than their respective tolerances, or if the magnitudes of the respective gradients were less than the tolerances [3, 33, 89].

2.6.3 Bootstrap computation of parameter standard errors

The nonlinear nature of the S-system model and the least squares optimization process used to obtain the parameter values makes the computation and interpretation of parameter standard errors difficult [39, 145]. This is in large part due to the fact that the predominant methodology for deriving standard errors for model parameters, whether for linear or nonlinear models, assumes *a priori* that the distribution of the model residuals follows a normal or multinormal distribution [4, 14, 7]. This assumption about the as yet unknown distribution of the model residuals is made solely for the purpose of obtaining an explicit formula for the standard errors because it makes the mathematics tractable, and not out of any inherent general applicability [39, 145]. If the model residuals are *not* normally distributed, as may happen with some nonlinear models or models with poor fits, but standard errors are generally still computed as though they were, then the interpretation of the standard errors may become questionable or even misleading.

The purpose of the standard errors it to provide a quality assessment of the parameter values and an indication of the repeatability of the process used to obtain a particular set of parameter values when using similar or augmented data sets. This interpretation of parameter standard errors holds regardless of the final residual distribution. The bootstrap procedure is an alternative to the assumption of normally distributed residuals which still maintains the spirit of the standard errors, as well as being generally easy to understand and implement [39, 145].

The bootstrap procedure is a resampling based procedure that is commonly used for estimating parameter standard errors [39, 145]. The underlying premise of the bootstrap procedure is that the consistency and robustness of parameter estimates may be determined by repeatedly performing the parameter estimation procedure for randomly selected subsets of a larger data set. The mean and standard deviation may then be computed for the multiple sets of parameter estimates, and will provide an empirical estimate of the variation of the parameter estimates for a particular problem [39. 145].

The algorithm for the bootstrap procedure may be stated quite simply. Let $G(x)$ be a function or process which returns a set of parameters, $p = \{p_1, p_2, \ldots, p_m\}$, estimated from a data set $x = \{x_1, x_2, \ldots, x_n\}$, where x_k may be either a scalar or vector. The function G may be thought of as an optimization procedure without any loss in generality. Then, given a set of N data values $X_1, X_2, \ldots, X_N$, a number B of bootstrap trials, and a bootstrap sample size N_B, the bootstrap procedure as defined by the following four steps may be used to compute parameter standard errors [39].

Step 1 Randomly select with replacement N_B data values $X_b = \{X_{b1}, X_{b2}, \ldots, X_{bN_B}\}$ from the N available data values. This is called the bootstrap sample.

Step 2 Obtain estimated parameter values from the bootstrap sample, $p_b = G(X_b)$.

Step 3 Repeat **Step 1** and **Step 2** for $b = 1, 2, \ldots, B$, obtaining estimated parameter vectors $p_1, p_2, \ldots, p_B$.

Step 4 Compute the bootstrap mean parameter values, $\bar{p}_i = \frac{1}{B}\sum_{b=1}^{B} p_{ib}$, and the standard deviations of the parameter estimates, $s_{ib} = \sqrt{\frac{1}{B-1}\sum_{b=1}^{B}(p_{ib} - \bar{p}_i)^2}$, $i = 1, 2, \ldots, m$, to get the bootstrap standard errors.

The bootstrap procedure was used to compute mean parameter values and parameter standard errors for the algebraic parameter estimation procedure, and for the shooting based parameter refinement procedure. The algebraic parameter estimation bootstrap procedure for the S-system parameters in the stand level Douglas-fir growth and yield model used $B = 100$ bootstrap trials, and a bootstrap sample containing 90% of the 2431 stand measurement intervals, giving a value of $N_B = \lfloor 0.90N \rfloor + 1 = 2188$ measurement intervals. All of these trials began with the initial parameter vector p_0. The 100 bootstrap parameter estimates were used to compute the mean parameter vector $\bar{p}_a^B$ and the parameter bootstrap standard errors s_a^B for the algebraic parameter estimation procedure. The shooting based parameter refinement procedure for the S-system parameters in the model used $B = 100$ bootstrap trials, and a bootstrap sample containing 90% of the 270 stand trajectories, giving a value of $N_B = \lfloor 0.90S \rfloor + 1 = 243$ stand trajectories. All of these trials began with the initial parameter vector p_a. The 100 bootstrap parameter estimates were used to compute the bootstrap mean parameter vector $\bar{p}_s^B$ and the parameter bootstrap standard errors s_s^B for the shooting parameter estimation procedure.

The decision to use a $B = 100$ bootstrap trials for the parameter estimation procedures was made for convenience, and because it should be large enough for the task at hand [39]. A bootstrap sample size of 90% of the number of measurement intervals and stand trajectories was made because there are two distinct subsets in the stand measurement data. The first, and larger, subset consists of older stands, 20 to 60 years old at the initial measurement, with annualized trajectories containing 10 or more measurements. The second, and smaller, subset consists of very young stands, 4 to 14 years old at the initial measurement, with annualized trajectories containing less than 10 measurements, and generally only two or four. Experiments with a smaller bootstrap sample size, say 50% of the data, generally did not contain enough data from the very young stands to influence the least squares optimization, causing poorer fits for these stands. Bootstrap sample sizes containing 90% of the measurement intervals or stand trajectories gave adequate representation for both subsets of the Douglas-fir data.

2.6.4 Residual computation and interpretation

Growth and yield models, as indicated by the name, attempt to represent the growth, or change per unit time, and the yield, or the cumulative growth over time of a forest stand. The accuracy and precision of both growth and yield are therefore important for the validation of any growth and yield model. Ascertaining the accuracy and precision of the yield is, however, sufficient to demonstrate the validity of both growth *and* yield, since yield is cumulative growth. If yield is incorrect, then the cumulative growth is incorrect. If growth is incorrect, then cumulative growth or yield will be incorrect. Therefore, the validation of the stand level Douglas-fir growth and yield model will be based upon residual values determined from predicted yields. Two residual computation scenarios are examined: an annual yield based scenario and a stand trajectory based scenario. These residual computation scenarios consider annual growth and cumulative growth, respectively. Both residual computation scenarios use the parameter estimates, p_s, obtained from the shooting based parameter refinement procedure estimated with all of the data.

The first residual computation scenario computes residuals at an annual time step, by integrating the S-system model using the parameter values defined by p_s for one year, to predict the final measurement, $\hat{Y}_{ik}^f$, from the initial measurement Y_{ik}^0, for each of the N measurement intervals,

$$R_{ik}^s = Y_{ik}^f - \hat{Y}_{ik}^f, \quad i = 1,2,3, \quad k = 1,2,\ldots,N,$$

where i indexes the state variable values for stand density, QMD, and top height, respectively. The residuals computed for this scenario will be referred to as the *single step* residuals, since only a single annual time step is involved, and will be indicated by the superscript s. This residual computation scenario tests the ability of the model to predict yield at an annual time step, the model calibration base time increment, which simultaneously demonstrates the ability of the model to predict annual growth or PAI. The single step residual values do not contain any zero values for the initial conditions used to obtain the annual yield predictions, as these would be identically zero. Any zero residual values are, therefore, actual zero values for the final annualized measurements for each measurement interval.

The second residual computation scenario computes residuals by integrating the S-system model using the parameter values defined by p_s, from the initial stand state Y_{i0s} and stand age t_{0s} to the final stand age t_{N_s} to get the cumulative growth or yield, $\hat{Y}_{iks}$. for each intermediate age, t_{ks} for the S stand trajectories,

$$R_{iks}^l = Y_{iks} - \hat{Y}_{iks}, \quad i = 1,2,3, \quad k = 1,2,\ldots,N_s, \quad s = 1,2,\ldots,S.$$

where i indexes the state variable values for stand density, QMD, and top height, respectively. The residuals computed for this scenario will be referred to as the *lifetime* residuals. since the entire simulated lifetime of the stand is involved, and will be indicated by the superscript l. This residual computation scenario tests the ability of the model to predict yield for longer time spans than one year, demonstrating the ability of the model to predict stand trajectories. The lifetime residual values do not contain any zero values for the stand initial conditions used to obtain the predicted stand trajectories, as these would be identically zero. Any zero residual values are, therefore, actual zero values for the annualized stand measurements on a stand trajectory.

These two yield based residual computation scenarios are singularly appropriate for validating the Douglas-fir growth and yield model for two reasons. First, the single step and lifetime stand yield predictions are computed in a forward time manner through the integration of the S-system model [145]. This mathematical and computational process mimics the natural forward time manner in which tree growth and stand dynamics actually occur. Second, given the variable step size nature of the S-system solver, multiple time steps may be required to integrate the model for a single year, and so a direct comparison of growth rates may not be feasible. By far, the first reason is the more important from a biological perspective [145].

2.6.5 Model validation procedures

No single model can be proven to be the correct model of a particular phenomenon. Thus, the search is for a good model, as determined by a set of objective and subjective validation procedures [77, 101, 117, 153, 161, 176]. Model validation consists of building a strong circumstantial case for the correctness or reasonability of a model. Given a reasonable model specification, that is, a model definition that is consistent in both its mathematical formulation and its interpretation of a phenomenon, the quality of the model is determined quantitatively through a comparison of model predictions with actual data. This comparison of model predictions with actual data for the same, or similar, set of conditions is paramount for building a case for the quality of a particular model: if a model does not agree well with data that it was designed to reproduce or predict, then it is not a high quality model.

The stand level Douglas-fir growth and yield model validation approach taken presumes that the model definition given in Section 2.4, and further specified by the estimated S-system parameter values, is a biologically and mathematically reasonable model for the stand dynamics of plantation Douglas-fir. The model definition has been shown to be consistent with the biology of the size-density and size-size relationships of tree growth and forest stand development, and the mathematical representation of the model is also consistent with these phenomena. Given this, a quantitative definition of how to determine the quality of the model, through its agreement with actual data, is needed. A good model must meet the following set of criteria.

1. Residuals computed from model predictions and actual data must be symmetrically distributed around zero, and, hence, have approximately a zero mean.
2. The residual distributions should have small variances identifying the strength of the distribution mode.
3. Residuals computed from the model predictions and actual data should be uncorrelated with the specific variables used in the model, as well as ancillary variables that are not explicitly represented in the model but are relevant.
4. There should be a strong one-to-one linear agreement between the model predicted state variables and their actual values.
5. The model must produce reasonable results for conditions beyond those represented in the model calibration or fitting data, and the model must not behave inappropriately, that is, the model dynamics must be consistent with the biology of Douglas-fir stand development.

For a particular model, criteria 1-4 imply that the information contained in the actual data, whether used to calibrate the model or not, is well represented by the model.

These five criteria, in particular, indicate for the fitting data set that all of the information has been gleaned from the data during the model calibration and incorporated into the model, and what remains is essentially random noise. The final criterion is more heuristic in nature, but is also an important consideration, since models will frequently be used beyond the range of the data used in their calibration.

The quality of the stand level Douglas-fir growth and yield model will be assessed using these five model validation criteria, the single step and lifetime residuals, R^s_{ik} and R^l_{iks}, and the actual and predicted state variable values, $\hat{Y}_{ik}$ and $\hat{Y}_{iks}$, obtained from the two residual computation scenarios. The following methods were used to assess the quality of the Douglas fir model for each of the model validation criteria. The numbers associated with the assessment methods agree with the numbers associated with the model validation criteria, respectively.

1. Examine the distribution of the single step and lifetime residuals histograms for the stand density, QMD, and top height state variables. Also examine the distribution of residuals *vs.* stand age and initial top height to be sure that it is symmetric, or nearly so, throughout the range of these variables by plotting the residuals against the stand age and initial top height.

2. Compute a standard statistical summary of the residuals for each state variable and residual computation scenario. The residual summary will contain: the mean, standard deviation, minimum value, median, maximum value, mean absolute deviation, and sample size.

3. Simple linear regression models and residual r^2 values are computed for standardized single step and lifetime residuals to assess whether there is a simple dependence of the stand density, QMD, or top height residuals for a variety of independent variables. The lifetime and single step residuals were standardized by dividing the residuals by their standard deviation [172, 7]. This removes

problems of scale when comparing the simple linear regression coefficients.

The independent variables used for the linear models and r^2 values are: initial stand age, final stand age, initial stand density, initial QMD, initial top height, site index, and elevation. Final stand age is simply the stand age for the predicted values. The linear model fits were obtained using the Matlab function `polyfit`, which uses a least squares procedure to fit a line to the data.

In addition, the single step and lifetime residuals were plotted against final stand age, the age for the model predictions, and top height to look for trends. The figures will also include the straight line fits for these independent variables, and a 51 point residual moving average plus or minus two moving standard deviations, giving an empirical 95% confidence interval.

4. Simple linear regression models and r^2 values were computed for predicted stand density *vs.* actual stand density, predicted QMD *vs.* actual QMD, and predicted top height *vs.* actual top height, for both the single step and lifetime scenarios. The linear model fits were obtained using the Matlab function `polyfit`, which uses a least squares procedure to fit a line to the data.

   In addition, the predicted state variable values for the lifetime scenario were plotted against their respective actual values to visually assess the quality of the agreement. These figures will also include the straight line fits for these predicted *vs.* actual state variable plots, and a 51 point residual moving average plus or minus two moving standard deviations, giving a real world 95% confidence interval.

   The data, the line, and the moving average and standard deviation all fall on top of each other, so figures for the single step predicted *vs.* actual state variable results were omitted.

5. The Douglas-fir growth and yield model was used to project stands out as far as 1375 years to visually detect anomalous behavior. In addition, the tree size values predicted by the model for approximate stand ages of 150, 300, 1000 and 1375 years were compared with actual tree size values at these stand ages. Stand density values were also compared, but there is a much greater uncertainty in actual stand density values, and no actual stand density values were available for stand ages of 300, 1000, and 1375 years. Douglas-fir tree size data for ages 100 and 300 are from [92], and tree sizes for ages 1000 and 1375 are from [106] and [102], respectively.

Finally, a brief discussion of the error sources, the manner in which they were controlled, if possible, and impacts on the validation of the stand level Douglas-fir growth and yield model is in order. In order of increasing control over the error source, they are: natural variation, sampling error, discretization error, model specification error, model definition error, and measurement error. These error sources are unfortunately not mutually exclusive, and their interrelationships are also of import for understanding and interpreting the model validation results.

- Natural variation exists and cannot be controlled directly.

- Sampling error may be controlled to the extent that the sampled data are considered to be representative of a larger population. The Douglas-fir stand measurement data obtained from the SMC database are well distributed over the region of interest and should be representative of the larger population of Douglas-fir stands in the region.

- Discretization error, is linked to sampling error, and may be controlled to the extent that data sampling occurs at appropriate temporal and spatial scales. Again, the stand measurement data from the SMC database are sampled at two to four year intervals, which are short time intervals relative to the commercial and natural lifespan of a Douglas-fir stand, so the temporal discretization error should be small. Behavior at temporal scales that are shorter than the remeasurement interval length, however, may not be represented adequately by these data. The spatial scale, or plot size, of the sample plots were deemed to be of a reasonable size to be representative of the stand dynamics, but if the plot sizes are too small, this will adversely affect the ability to represent characteristics of stand dynamics that occur at larger spatial scales, e.g., stand mortality. Both the temporal and spatial scales are considered acceptable for a growth and yield model with an annual time step and a per hectare spatial scale.

- Model specification error is generally associated with the estimation of unknown model parameters from available data, and is linked to sampling error, discretization error, and model definition error. If the data are not representative, the temporal or spatial scales are not commensurate, or the model definition is not reasonable, the model specification may be inadequate. Given the representative nature of the stand measurement data, and the generally reasonable temporal and spatial scales for the measurements, the data sources should not exacerbate this type of error. To help further control this error source, the stand measurement data were annualized before they were used in a least squares parameter estimation procedure, giving a natural, annual weighting of the data. A biologically reasonable model definition was provided to give even greater control over this error source.

- Model definition error is generally associated with the form and representation of a model: the selection of what state variables to include in the model and the dependencies or relationships among those variables. This error source was controlled in the Douglas-fir growth and yield model by using a dynamic model representation for the stand dynamics, and by defining the model equations from a biological perspective, attempting to include only those relationships that were deemed biologically relevant.

- Measurement error was generally assumed to be insignificant, and hence unnecessary to control. Exceptions to this statement may occur, but they will most likely be due to the influence of sampling error or discretization error upon the measurement error.

2.7 *Results*

The parameter estimation, model behavior, and model validation analysis results for the stand level Douglas-fir growth and yield model follow. First, the parameter estimates obtained for the S-systemmodel are discussed. Second, the general behavior of the model is shown to agree quite strongly with the stand measurement data used to calibrate the model. Third, the model validation results will be presented, broken down according to the five model validation criteria outlined in Section 2.6.

Table 2.4 displays the estimated parameter values, bootstrap means, and bootstrap standard errors for both the algebraic parameter estimation method, p_{ia}, $\bar{p}^B_{ia}$, s^B_{ia}, respectively, and for the shooting based parameter refinement method, p_{is}, $\bar{p}^B_{is}$, s^B_{is}. Notice that the refined parameters, p_{is} are nearly identical to those obtained from the algebraic parameter estimation method, p_{ia}. This indicates that the algebraic parameter estimation method produced excellent initial estimates for the S-system growth and yield model parameter values. This is further evidenced by the very small bootstrap standard errors for the shooting based parameter refinement method, and the agreement to at least four decimal places between the parameter estimates obtained using all of the data and the bootstrap mean parameter values for the parameter refinement. The parameter values obtained from the algebraic parameter estimation method and the shooting based parameter refinement method are all statistically significant, having nonzero values, based on an examination of the bootstrap standard errors, that is, the intervals $p_{ia} \pm s^B_{ia}$ and $p_{is} \pm s^B_{is}$ do not contain zero.

A closer examination of the S-system parameter values estimated using the algebraic parameter estimation method, p_{ia}, indicates that there are discrepancies between some of these values and their corresponding bootstrap mean parameter values $\bar{p}_{ia}^{B}$. The greatest discrepancies are for the parameters of the QMD rate equation, $\dot{X}_2$: α_2, β_2, g_{21}, and g_{22}. The bootstrap standard errors for the algebraic parameter estimates also identify these four parameter values as being the most variable. An examination of the distributions for these four bootstrap parameter values indicated a bimodal structure with a strong mode and a weak mode. These discrepancies, therefore, are an indication that there are at least two distinct subgroups contained within the measurement interval data.

In fact, there are three subgroups that may be identified *a priori*, see Section 2.5. The first, and largest subgroup consists of somewhat older stands that are predominantly Douglas-fir, greater than 80% Douglas-fir by stand density, with stand ages ranging from 25 to 60 years. This subgroup comprises approximately 60% to 70% of the stand measurement intervals. The second subgroup consists of fast growing, young, generally 100% pure Douglas-fir stands, with stand ages in the range of 4 to 20 years. This subgroup comprises approximately 15% to 20% of the stand measurement intervals. The third subgroup consists of stands identified as pure Douglas-fir by being at least 80% Douglas-fir by basal area but which have Douglas-fir stand density percentages less than 80%. This subgroup also comprises 15% to 20% of the stand measurement intervals.

Aside from the discrepancies between the parameter values estimated using all of the data with the algebraic method and their bootstrap mean values for the QMD equation $\dot{X}_2$, the bootstrap mean values and standard errors indicate that the remaining parameter values are generally consistent in magnitude and sign. Considering that the algebraic parameter estimates were used only as initial guesses for the parameters in the shooting parameter refinement, the discrepancies in algebraic parameter values and bootstrap mean values may be ignored, particularly in light of the overall consis-

Table 2.4: Estimated S-system parameter values, bootstrap means, and bootstrap standard errors for the Douglas-fir growth and yield model defined by Equations 2.14. A subscript a indicates that the parameter estimates are from the algebraic parameter estimation procedure. A subscript s indicates that the parameter estimates are from the shooting based parameter refinement procedure. The superscript B indicates that the values were obtained using the bootstrap procedure.

p_i	p_{ia}	$\bar{p}^B_{ia}$	s^B_{ia}	p_{is}	$\bar{p}^B_{is}$	s^B_{is}
α_1	0.0449	0.0637	0.0151	0.0451	0.0451	0.0003
α_2	3.2055	1.6083	0.7976	3.2056	3.2057	0.0003
α_3	0.8371	0.9959	0.1187	0.8373	0.8372	0.0003
β_1	0.0451	0.0638	0.0151	0.0444	0.0444	0.0009
β_2	0.0952	0.3150	0.1579	0.0952	0.0952	0.0001
β_3	0.6373	0.7994	0.1239	0.6386	0.6381	0.0007
g_{11}	1.6339	1.6360	0.0649	1.6140	1.6319	0.0361
g_{21}	-0.2059	-0.1005	0.0564	-0.2062	-0.2060	0.0001
g_{22}	0.4171	0.7190	0.1597	0.4173	0.4171	0.0002
g_{32}	0.0181	0.0154	0.0047	0.0180	0.0180	0.0001
g_{33}	0.9128	0.9269	0.0211	0.9128	0.9128	0.0002
h_{11}	1.6326	1.6350	0.0653	1.6139	1.6320	0.0361
h_{12}	0.0040	0.0035	0.0024	0.0085	0.0078	0.0037

tency of the parameter values obtained by the shooting based parameter refinement. The refined parameter values, p_{is}, have been shown to be consistent with the stand measurement data, via the bootstrap procedure, and are the final parameter estimates specifying the S-system based Douglas-fir growth and yield model. The refined parameter values were used to obtain all of the subsequent results for the S-system growth and yield model validation and analyses.

Figure 2.5 through Figure 2.7 present plots containing all of the annualized stand measurement data and of 120 year model projections for 70 randomly selected stands from the 270 available. Each stand projection was begun using the first stand density, QMD, and top height measurement for each of the 70 selected stands. The same 70 stands were used for each of these three figures. For each figure, note that the model projections generally span the envelope defined by the data, remaining within the envelope, indicating the growth and yield model is capable of representing the dynamics of Douglas-fir stand development for a wide range of stand conditions.

Figure 2.5 displays the stand density *vs.* stand age, and represents the number of trees per hectare surviving. Note the generally good agreement with the data for initial stand densities less than 2000 TPH. For intermediate stand densities, 1000 TPH to 2000 TPH and stand ages between 50 and 100 years, the model appears to slightly underpredict the number of surviving trees, i.e., it overpredicts mortality. This effect is predominantly influenced by three factors. First, there are large scale factors, averaging 19.5, involved in converting the number of trees per sample plot to TPH, so the death of a single measured tree indicates, on average, the death of 20 TPH. So the loss of multiple trees on a research plot may exaggerate the actual mortality for the stand which contains it. Second, not all of the trees on the pure (by basal area) Douglas-fir plots selected are Douglas-fir trees. This may influence the overprediction of stand mortality due to a higher mortality rate for the tree species other than Douglas-fir. Third, there are few data available beyond 60 years of age. This influences the model predictions by giving more weight in the parameter estimation to younger stands which will have a higher mortality rate.

The overprediction of stand mortality should pose little if any operational difficulties for use of the S-system growth and yield model. The mortality overpredictions generally appear after typical Douglas-fir rotation ages in the Pacific Northwest, which are from 35 to 50 years [29]. The problem may be corrected by cleaning up the model calibration data set and augmenting the calibration data set with older Douglas-fir stands.

Figure 2.6 displays QMD *vs.* stand age, and represents the net QMD of the surviving trees for each prediction year. Again, note the general agreement between the model projections and the data for the entire range of QMD values. By approximately age 80, all of the stand projections appear to become parallel, indicating that they have the same, or nearly the same growth rates. This may be an artifact of the model, or it may be a legitimate representation of Douglas-fir average diameter growth. The Douglas-fir growth rates eventually approach an asymptote of approximately 0.2 cm/year. Whether these projections are consistent with old growth Douglas-fir stands remains to be determined. The existence of the QMD growth rate asymptote may, however, be considered an emergent feature of the model, that is, it was not programmed into the model form or structure, but is derived from the data and implicitly represented within the model [25]. These projections agree well for the available data, out to a stand age of approximately 80 years.

The constant rate of the projected Douglas-fir QMD growth rates over very long periods of time should also pose few if any operational difficulties for using the S-system growth and yield model in practice. The constant growth rates occur well after the typical Douglas-fir rotation ages in the Pacific Northwest, and the model agrees well with the available data in this range. The constant QMD growth rate issue may also be resolved by augmenting the model calibration data set with data from older Douglas-fir stands, and by comparison of model predicted growth rates for very old Douglas-fir trees.

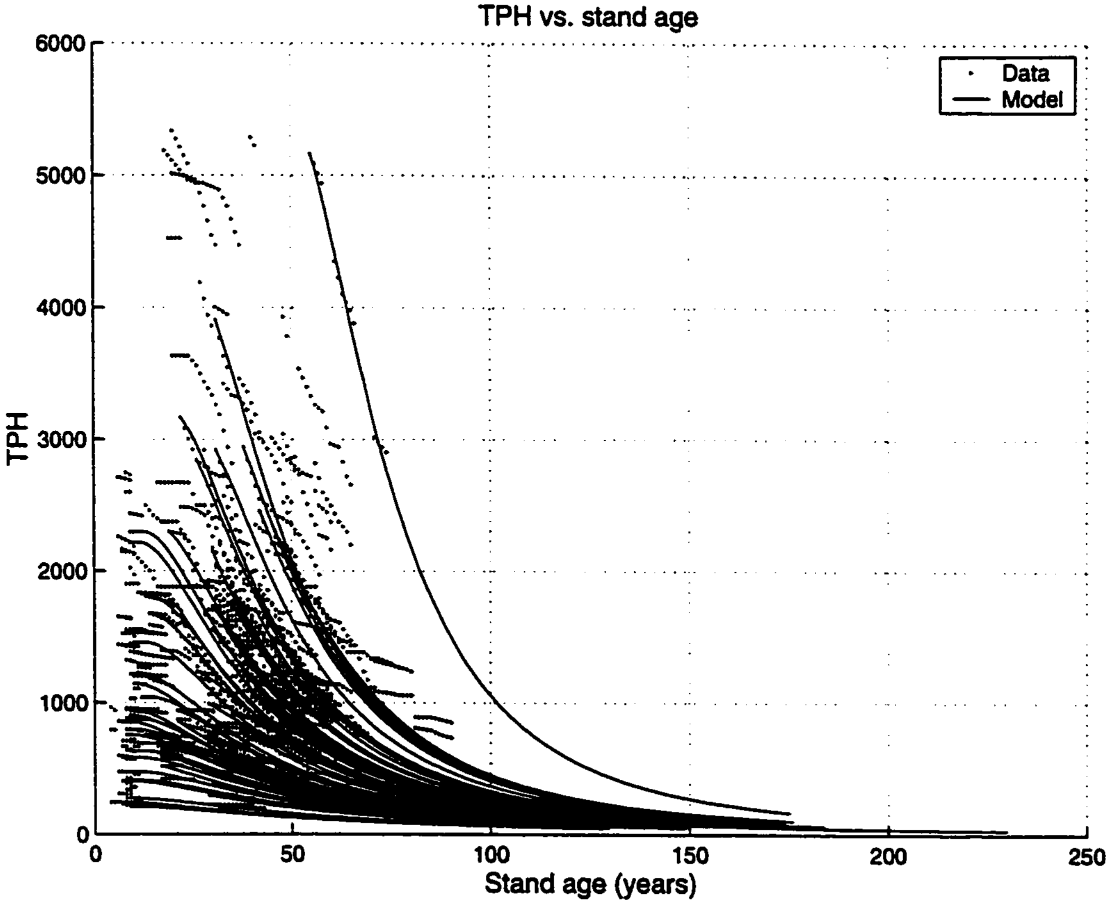

Figure 2.5: Predicted TPH *vs.* stand age for a 120 year simulation time from the initial stand ages and measurements for a random sample of 70 of the 270 available stands.

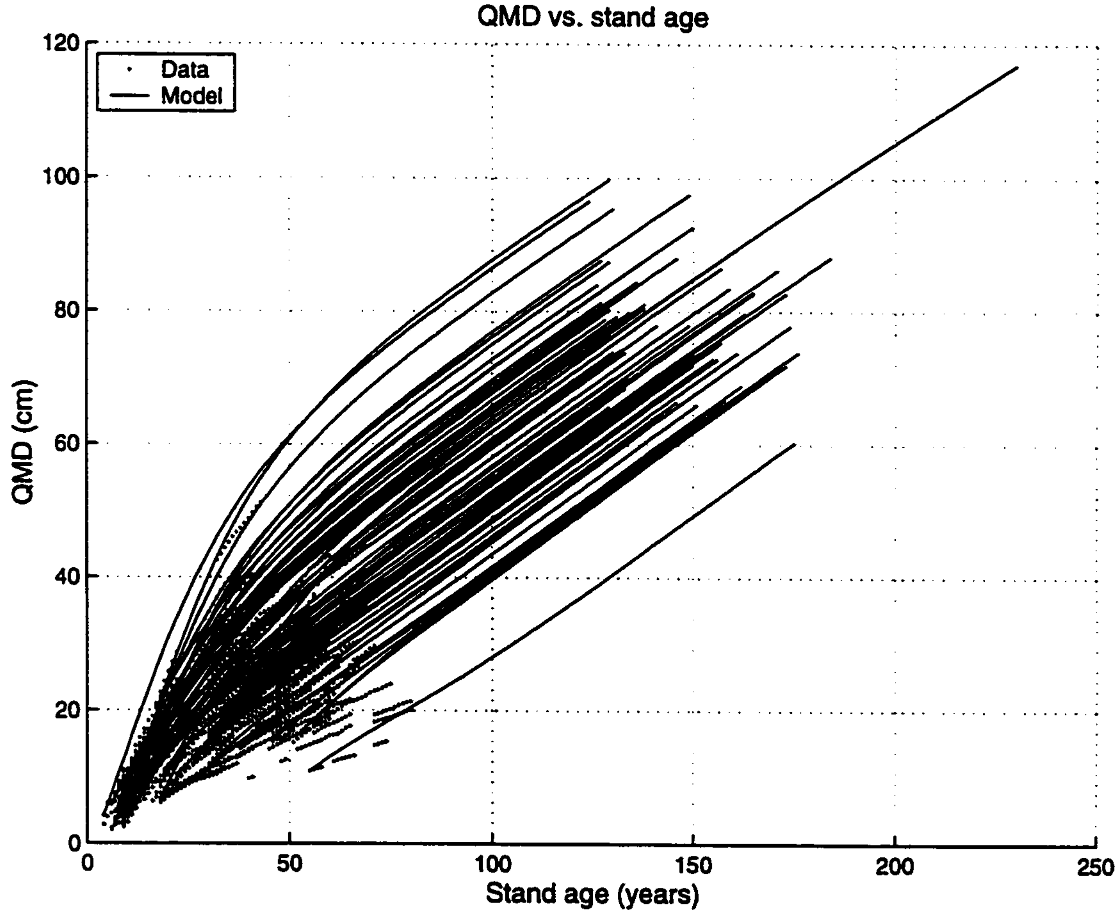

Figure 2.6: Predicted QMD *vs.* stand age for a 120 year simulation time from the initial stand ages and measurements for a random sample of 70 of the 270 available stands.

Figure 2.7 displays the top height *vs.* stand age, and represents the net dominant height of the surviving trees. Again, note the general agreement between the model projections and the data for the entire range of top height values. By approximately age 80, all of the stand projections appear to be flattening out, which is expected, indicating that their growth rates are slowing down. This model behavior is also an emergent property of the Douglas-fir growth and yield model. It was not programmed into the model form or structure, but is derived from the data and implicitly represented within the model [25].

The projected Douglas-fir top height growth rates are consistent with expectations, and agree well with the available data. The top height projections agree very well with the data, spanning the typical Douglas-fir rotation ages in the Pacific Northwest, which are from 35 to 50 years [29].

Figure 2.8 and Figure 2.9 present the phase space, or state space, view of the complete set of annualized stand measurement data and the same 120 year model projections just described for the 70 randomly selected stands. The phase space or state space view of the stand measurement data and the 70 growth and yield model projections allows a visual assessment of the quality of the S-system based model to be obtained. The S-system model is an autonomous system, and represents the direction and magnitude of flow from any point in the phase or state space. The state variable values produced by integrating the model, or those given by the annualized stand measurement data, then provide a set of stand trajectories following a phase manifold or surface defined by the S-system growth and yield model, or the biological system that the model represents. If the phase manifolds defined by the data and the model agree, then the data and model agree in all three state variables simultaneously.

In these two figures, time moves from the nearest lower right corner to the left, up, and into the figure. Key features to note are that the phase surfaces defined by the data and the model projections both narrow from right to left and up. There also appears to be a general agreement in the density of points or lines and the slopes

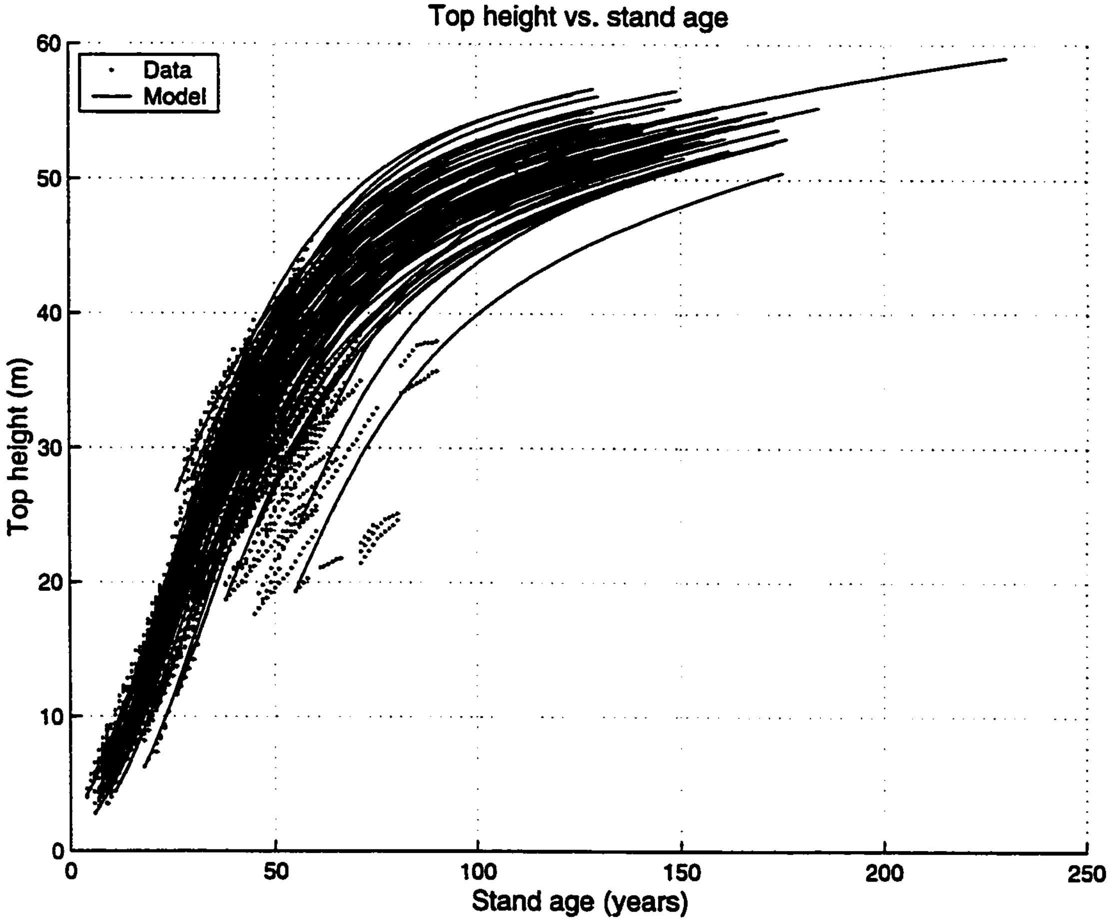

Figure 2.7: Predicted top height *vs.* stand age for a 120 year simulation time from the initial stand ages and measurements for a random sample of 70 of the 270 available stands.

for the same general region of the data and model based phase manifolds. Thus, the stand measurement data and the model projections appear to agree quite well. Notice, in particular, the twist in the phase surface that is readily apparent for the model projections, but less so for the data. This twist in the surface represents the biological phenomenon where trees planted less densely will simultaneously grow more rapidly in diameter and more slowly in height than like trees at higher stand densities [102]. This is yet another emergent feature of Douglas-fir stand dynamics that is captured by the S-system growth and yield model, yet was not explicitly programmed into the model structure [25].

Figure 2.10 through Figure 2.12 provide plots of the three two dimensional phase or state plane perspectives of the three dimensional phase manifolds. These figures simply place the stand measurement data and the model projections on the same axes to provide further evidence for the quality of their agreement. Again, notice that the model projections generally span the envelope defined by the data and stay within this envelope for each perspective. These perspective plots indicate the degree of overlap between the measurement data and the model projections, which was difficult to achieve if both the data and model projections were combined into a single three dimensional plot.

The dynamic, stand level S-system model for Douglas-fir growth and yield appears to perform qualitatively quite well. The parameter estimation procedures produced consistent results with acceptable variability in the parameter estimates, as indicated by the bootstrap standard errors. The stand projections produced from the S-system model as specified by the parameter vector p_s have the correct behavior over time, and agree very strongly with the stand measurement data used to obtain the parameter values. This initial examination of the model behavior indicates that its projections are consistent with the stand measurement data. A more thorough validation of the model using the five criteria outlined in Section 2.6 follows.

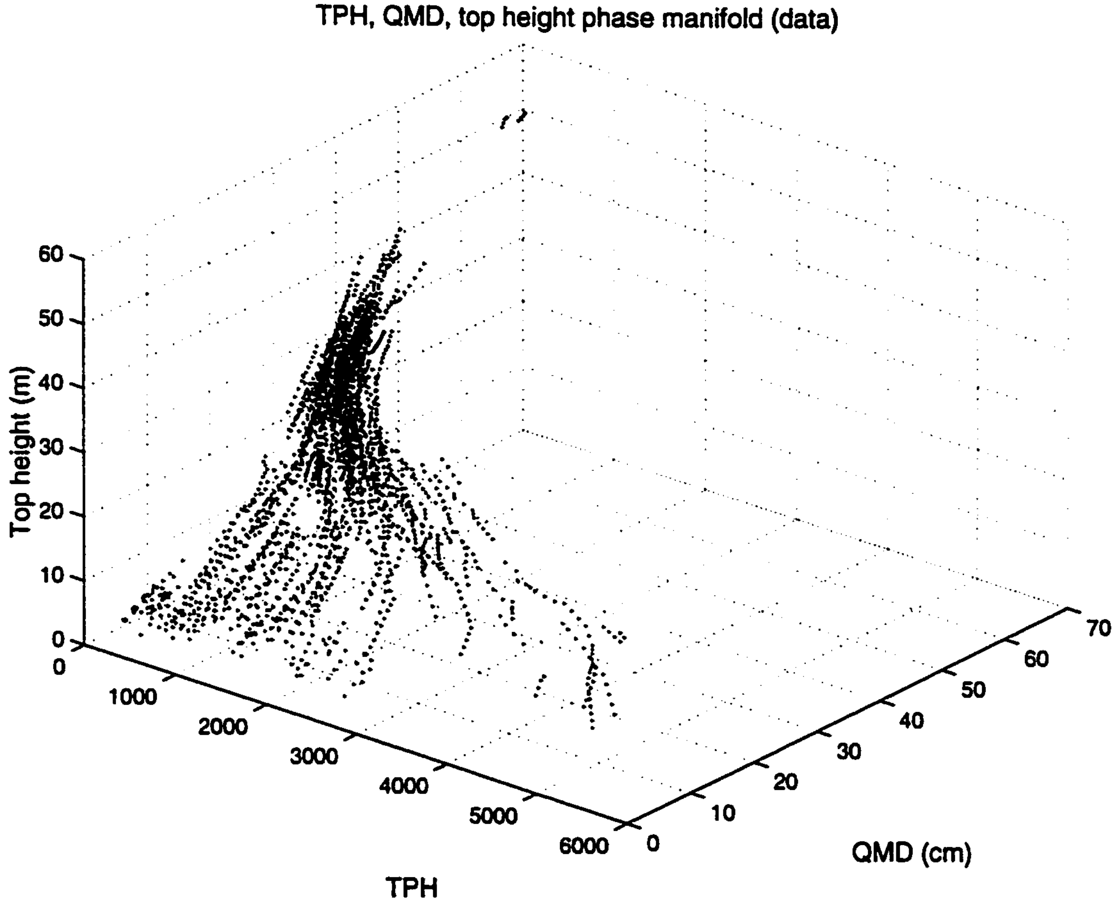

Figure 2.8: TPH, QMD, top height phase manifold derived from the annualized stand measurement data from the 270 stands. Time moves from the nearest lower right corner to the left, up, and into the figure. Note in particular the strong agreement with Figure 2.9.

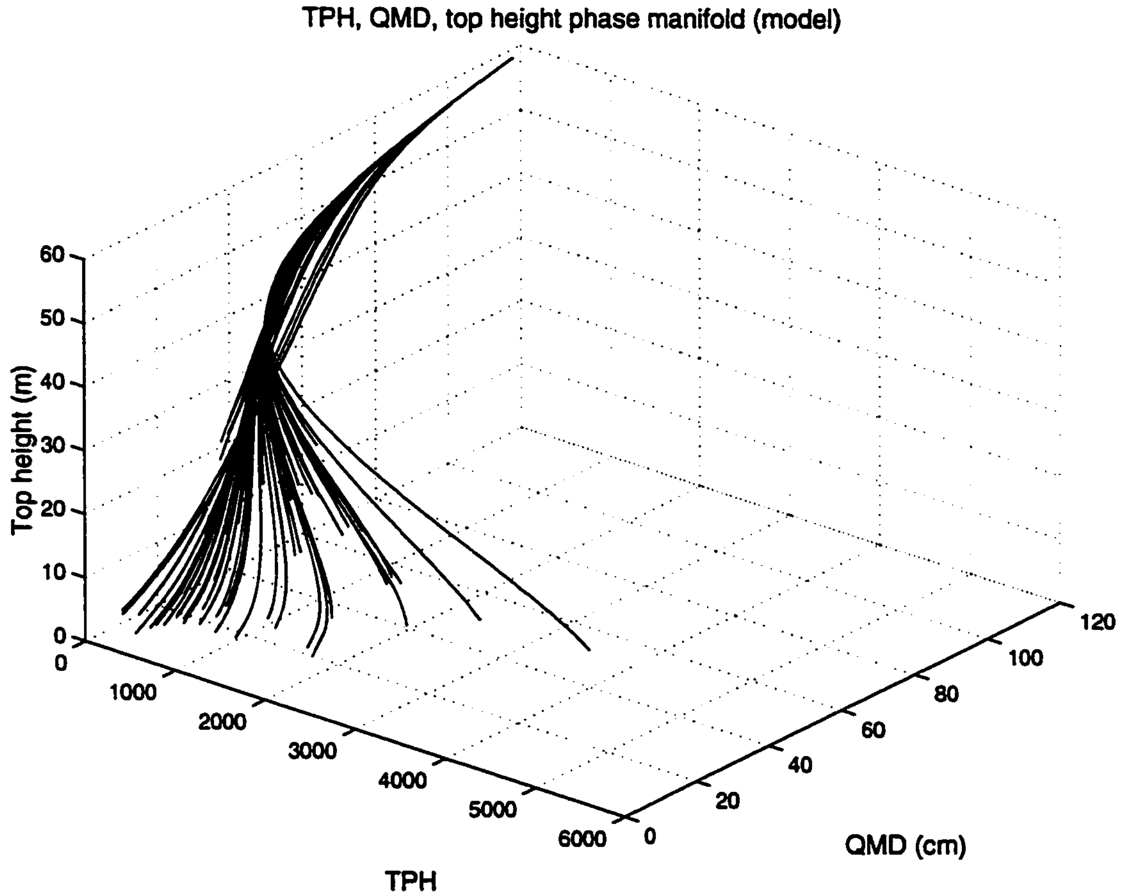

Figure 2.9: TPH, QMD, top height phase manifold obtained for a random sample of 70 of the 270 available stands for a 120 year simulation time from their initial stand ages and measurements. Time moves from the nearest lower right corner to the left, up, and into the figure. Note in particular the strong agreement with Figure 2.8.

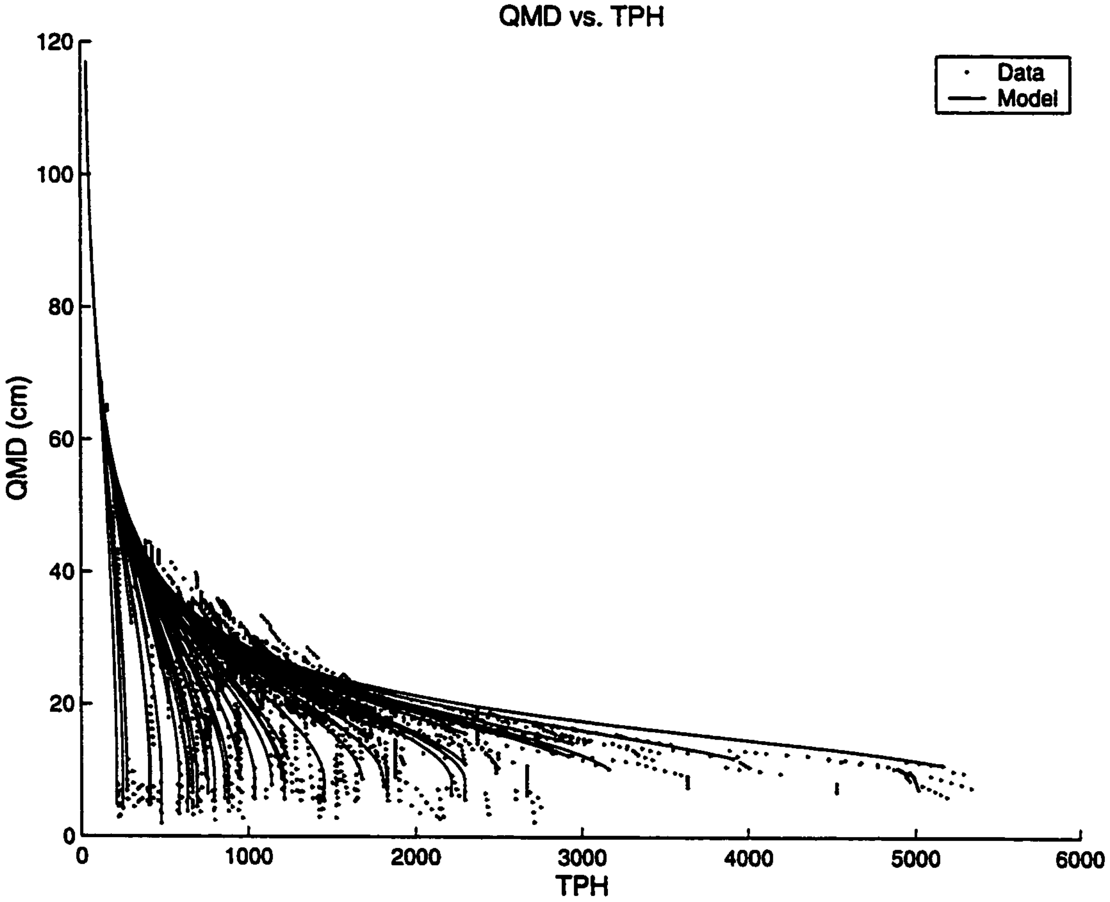

Figure 2.10: QMD *vs.* TPH size density plot obtained for a random sample of 70 of the 270 available stands for a 120 year simulation time from their initial stand ages and measurements.

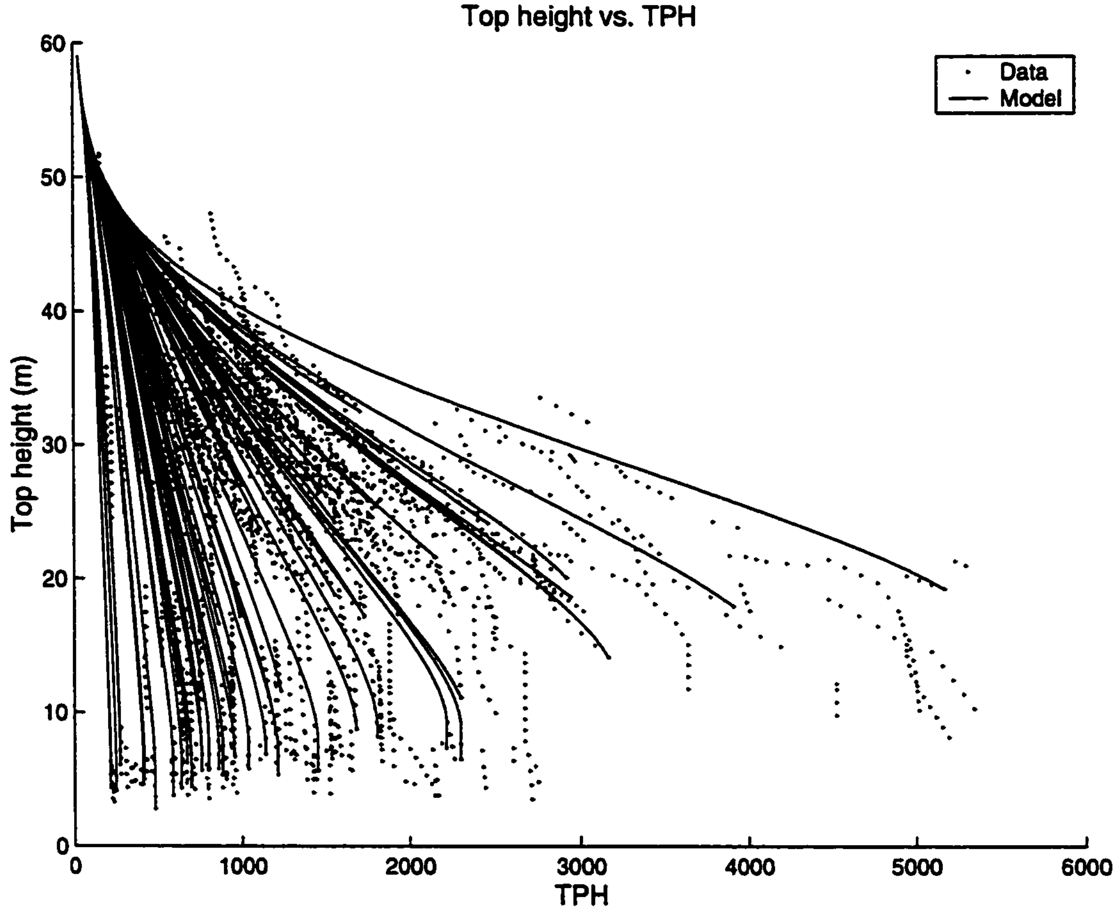

Figure 2.11: Top height *vs.* TPH size density plot obtained for a random sample of 70 of the 270 available stands for a 120 year simulation time from their initial stand ages and measurements.

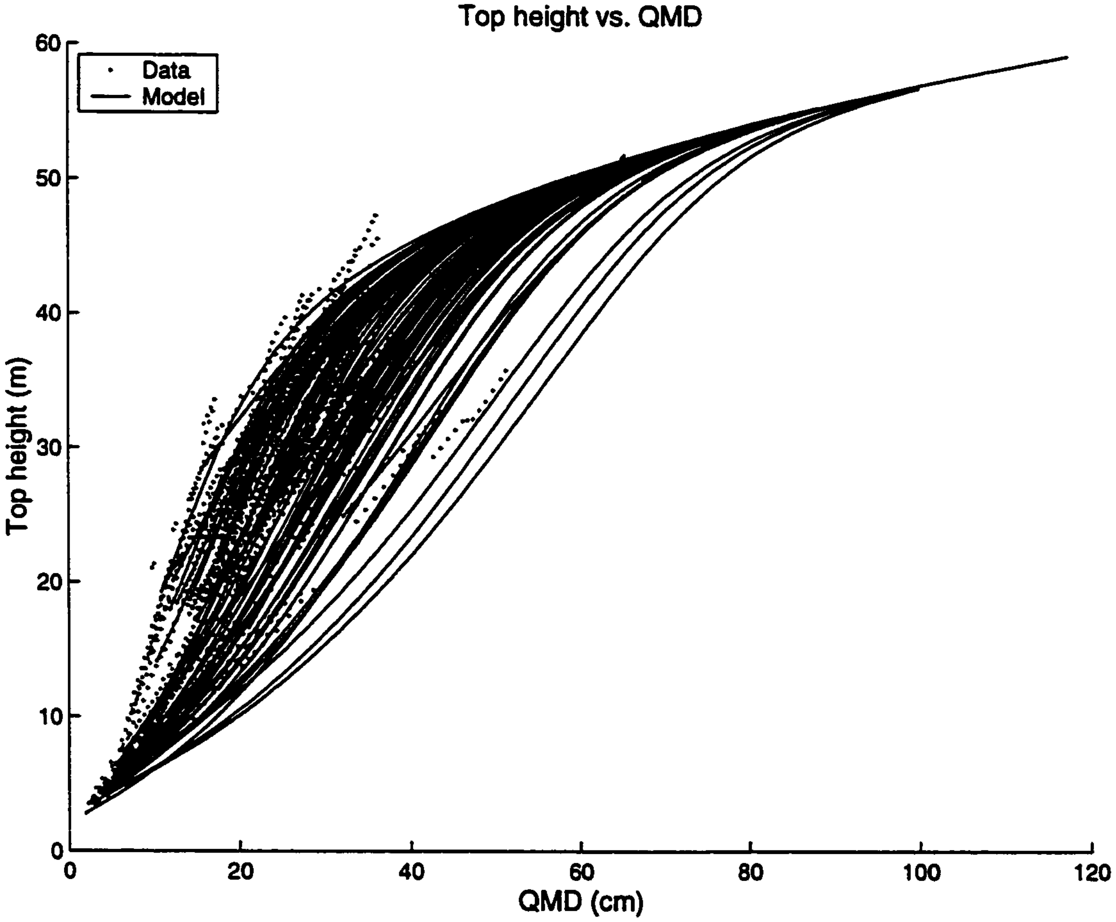

Figure 2.12: Top height *vs.* QMD plot obtained for a random sample of 70 of the 270 available stands for a 120 year simulation time from their initial stand ages and measurements.

2.7.1 Residual distribution

Figure 2.13 through Figure 2.15 present the single step or annual yield prediction based residuals, R^s_{ik}, for stand density, QMD, and top height as a set of histograms and scatterplots. The residuals are plotted against stand age and initial top height. Figure 2.16 through Figure 2.18 present the lifetime, cumulative growth or stand trajectory prediction based residuals, R^l_{iks}, for stand density, QMD, and top height as a set of histograms and scatterplots. Again, the residuals are plotted against stand age and initial top height. Scatterplots of the residuals *vs.* initial QMD and initial stand density demonstrated behavior comparable to initial top height and are omitted.

The single step residuals represent the lack of agreement between the actual and predicted annual changes in a Douglas-fir stand; if the predicted annual yields agree with the actual annual yields, the simultaneous rates of change for the state variables are correct with respect to these data. The lifetime residuals represent the lack of agreement between the actual and predicted stand trajectories, or cumulative growth, for the Douglas-fir stands; if the predicted stand trajectories agree with the actual stand trajectories, the evolution of the stand, as represented by the S-system model is correct with respect to these data. Positive residual values indicate an underprediction by the Douglas-fir growth and yield model, and negative residual values indicate an overprediction by the model for each state variable. The residuals for each state variable are displayed independently for convenience and ease of interpretation, but they were computed simultaneously: (1) by integrating the growth and yield model one year for each measurement interval to obtain the single step residuals, and (2) by integrating the growth and yield model for the available time span for each stand, to get yield values for each annualized measurement, and obtain the lifetime results.

Neither the single step residuals nor the lifetime residuals contain zero values for the initial conditions used to obtain the predicted annual yields and predicted stand trajectories. Any zero residual values are, therefore, actual zero values for the annual yields or annualized stand trajectories. Thus, there are 2431 annualized residual values for each residual computation scenario.

The three residual histograms in Figure 2.13 clearly indicate that the stand density, QMD, and top height residuals have strongly symmetric distributions. Zero is located at or near the center, or mode, of each distribution as well. The stand density residual distribution appears to be slightly left skewed, and the QMD residual distribution appears to be slightly right skewed, indicating a small bias toward overprediction and underprediction, respectively, for these state variables by the model. The top height residual distribution has zero very nearly at its center, indicating that the prediction of top height is generally unbiased. The distributions of the single step residual values for the 2431 annualized measurement intervals and for the subset of 968 actual measurement intervals were statistically indistinguishable. Separate t-tests were performed using $\alpha = 0.05$ for the annualized and actual stand density, QMD, and top height residuals, obtaining observed t-values (p-values) of -0.111 (0.456), -0.509 (0.305), and -0.111 (0.456), respectively.

Figure 2.14 and Figure 2.15 also show a generally symmetric distribution of the stand density, QMD, and top height residuals throughout the range of stand ages and initial top height values. In particular, for stand ages between 30 and 60 years and initial top heights between 22 m and 38 m, where the bulk of the stand measurement data are found, the symmetry of the residuals is very strong, with no obvious trend toward overprediction or underprediction. For young stands, or stands with small trees, both QMD and top height appear to be slightly underpredicted, and for older stands and stands with larger trees, QMD and top height also appear to be slightly underpredicted. Stand density does not appear to have any readily apparent departures from symmetry about zero. The underprediction of QMD and top height for young stands with small trees fast growing trees and for older stands with large trees is most likely an artifact of the least squares parameter estimation procedure caused by the under representation of these two stand types in the data set.

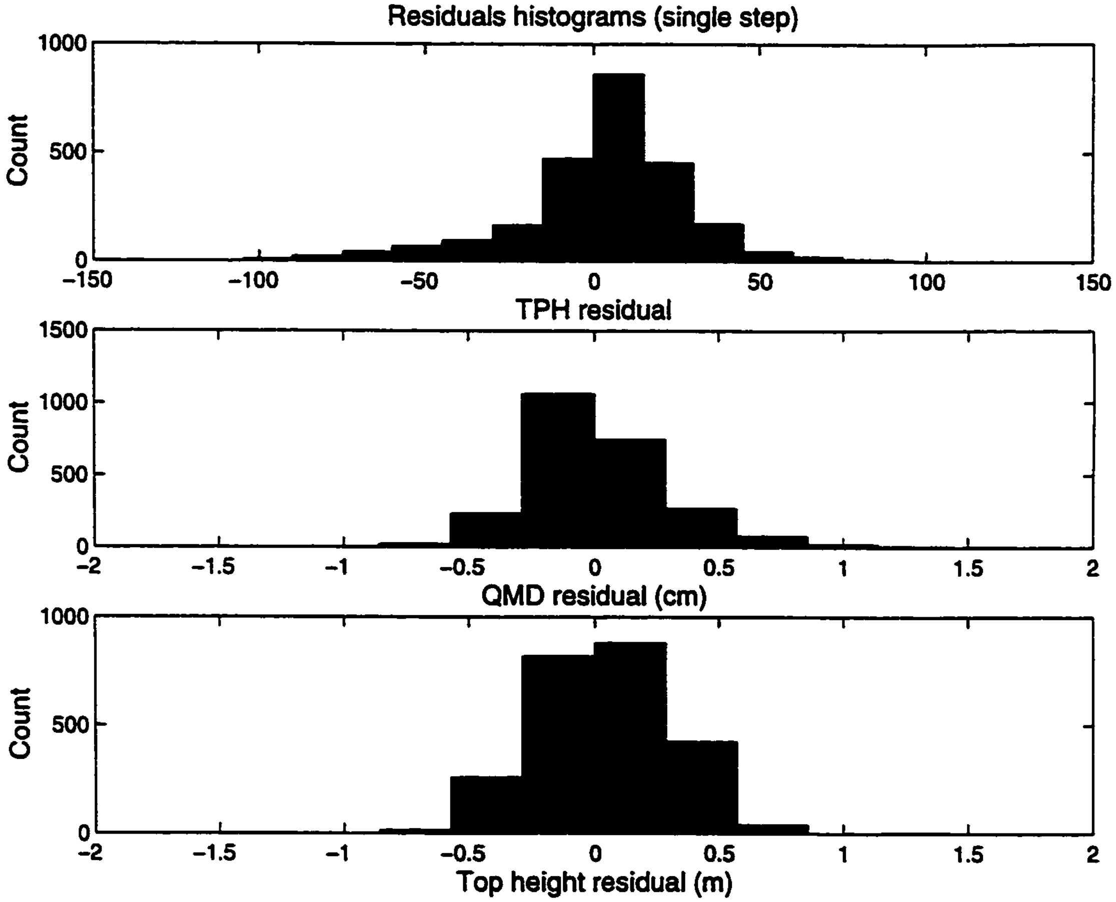

Figure 2.13: Histograms of residuals for the single step residuals computation scenario R^s_{ik}. The TPH residuals, $i = 1$, are on top, QMD residuals, $i = 2$, are in the middle, and top height residuals, $i = 3$, are on the bottom.

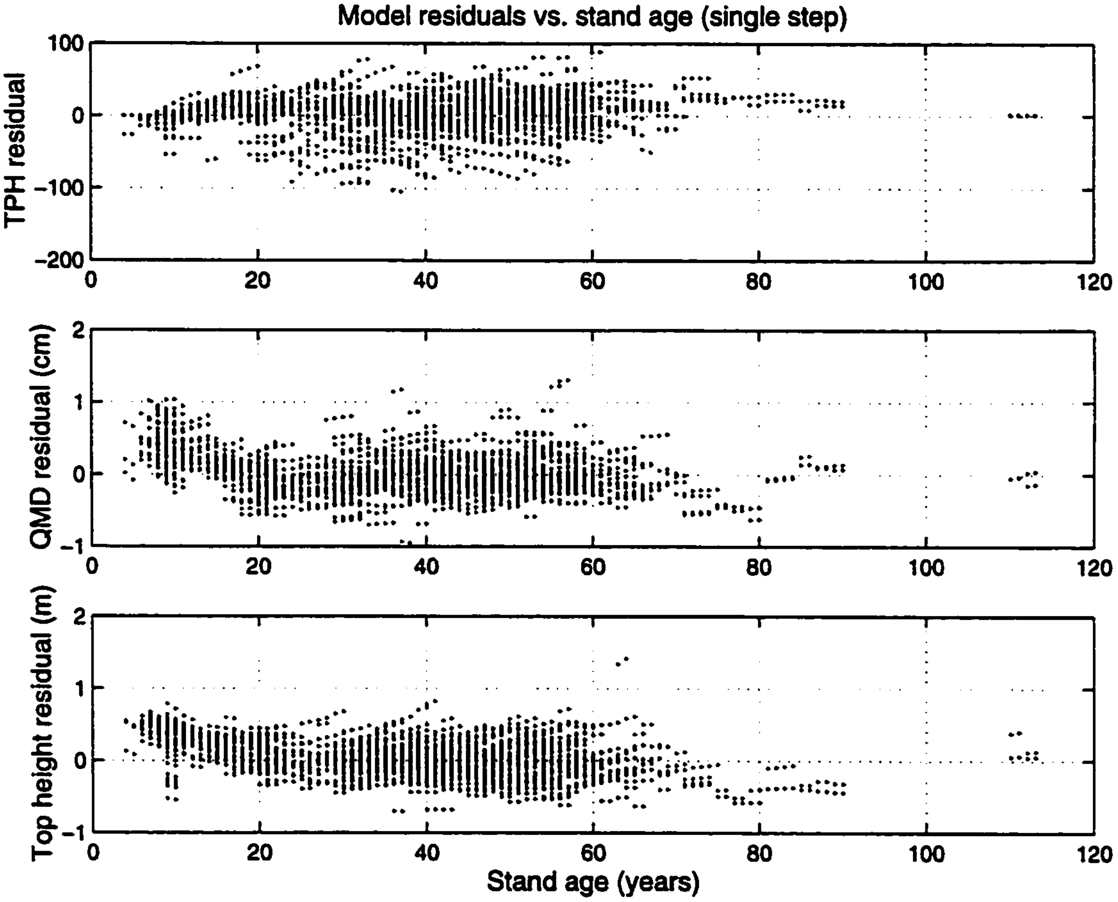

Figure 2.14: Single step residuals, R^s_{ik}, *vs.* stand age. The TPH residuals, $i = 1$, are on top, QMD residuals, $i = 2$, are in the middle, and top height residuals, $i = 3$, are on the bottom. The dot density provides an indication of the number of data points.

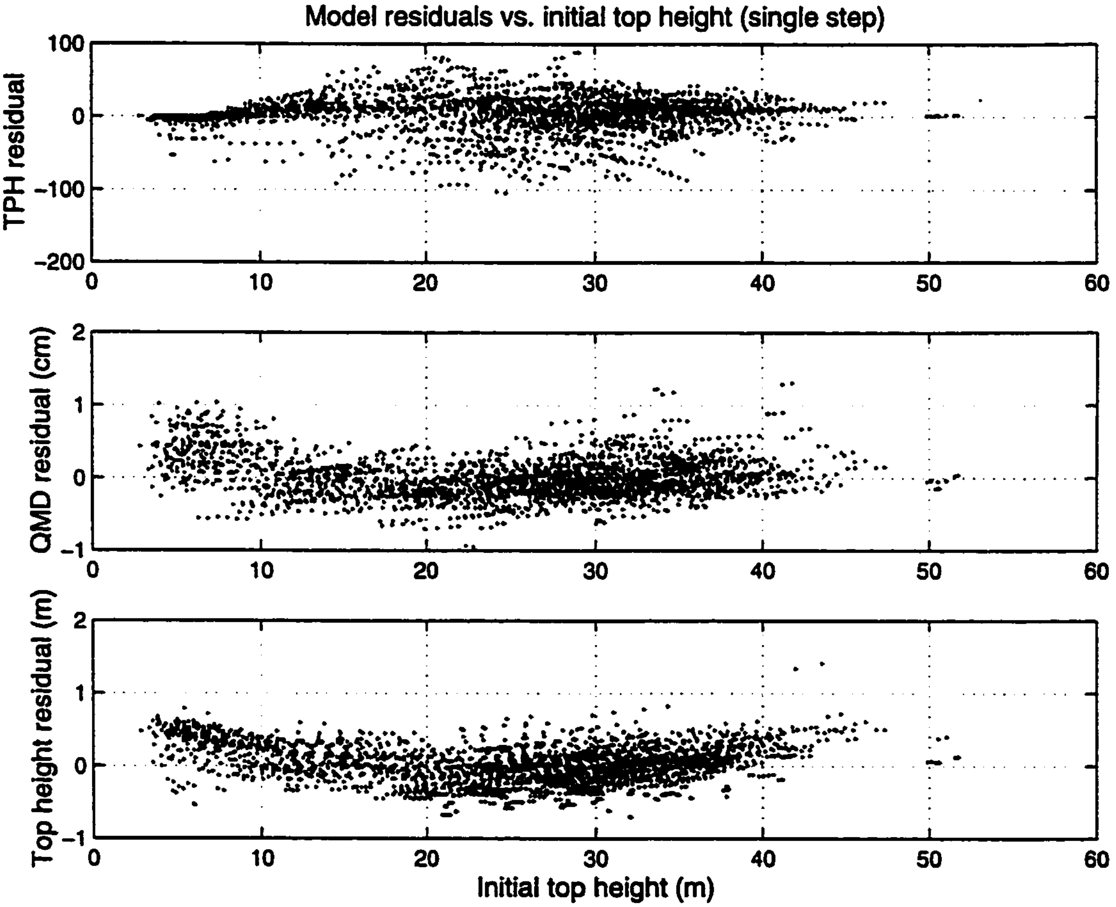

Figure 2.15: Single step residuals, R^s_{ik}, *vs.* top height. The TPH residuals, $i = 1$, are on top. QMD residuals, $i = 2$, are in the middle, and top height residuals, $i = 3$, are on the bottom. The dot density provides an indication of the number of data points.

The three residual histograms in Figure 2.16 also clearly indicate that the stand density, QMD, and top height residuals have strong symmetric distributions, though with a wider range than for the single step residuals. Zero is located at or near the center, or mode, of each distribution as well. The stand density residual distribution may be slightly right skewed, indicating a small bias toward underpredicting stand density. The top height residual distribution also appears to be slightly right skewed, indicating a small bias toward underprediction of top height by the model. The QMD residual distribution has zero very nearly at its center, indicating that the prediction of QMD is generally unbiased. The distributions of the lifetime residual values for the 2431 annualized stand measurements and for the subset of 968 actual stand measurements were statistically indistinguishable. Separate two-sided t-tests were performed using $\alpha = 0.05$ for the annualized and actual stand density, QMD, and top height residuals, obtaining observed t-values (p-values) of -1.41 (0.079), 0.965 (0.167), and 0.144 (0.443), respectively. The stand density residuals were almost significantly different at this α-level. This seems to be related to the magnitude of the residual values, small differences in the residual distributions near zero, and the large sample size which can exaggerate small differences.

Figure 2.17 and Figure 2.18 also show a generally symmetric distribution of the stand density, QMD, and top height residuals throughout the range of stand ages and initial top height values. In particular, for stand ages between 30 and 60 years and initial top heights between 22 m and 38 m, where the bulk of the stand measurement data are found, the symmetry of the residuals is very strong, with no obvious trend toward overprediction or underprediction. For young stands, both QMD and top height appear to be slightly underpredicted, but for older stands they may be overpredicted, when plotted against stand age, but may be underpredicted when plotted against initial top height. For stands with smaller trees, QMD is slightly underpredicted and top height appears symmetric about zero, whereas stands with larger trees, QMD appears to be symmetric about zero and top height appears to be underpredicted.

Stand density, again, does not appear to have any readily apparent departures from symmetry about zero. Also apparent from these two figures is that stand trajectories which disagree initially tend to continue to disagree, as evidenced by the tendrils trending away from the central bulk of the residuals. Notice, however, that some of the tendrils appear to be flattening, or curving toward zero, giving an indication that although the model and data were initially different, they may be coming into better agreement.

The single step and lifetime residuals have been shown to be essentially symmetrically distributed about zero, indicating that the S-system Douglas-fir growth and yield model is essentially unbiased, whether predicting annual growth or stand trajectories. The single step and lifetime residuals have also been shown to be generally symmetrically distributed throughout the range of stand ages and initial top height values, reinforcing the fact that the growth and yield model is generally unbiased. There were no visible differences in the residual distributions when histograms of residuals for the actual measurement data were compared to histograms of the augmented annualized measurement data set. Further, the largest value of the differences in absolute relative frequency between the residuals for the actual and annualized data sets was only 2%, indicating a good overall agreement in the residual distributions. Finally, the remaining residual scatterplots will use top height as the x-axis, because the residual distributions through the ranges of stand age and top height are generally similar.

2.7.2 Residual statistics

Standard statistical summaries for the single step and lifetime residuals are presented in Table 2.5 and Table 2.6. The single step average residuals for stand density, QMD, and top height are 3.43 TPH, 0.013 cm, and 0.043 m, with standard deviations of 25.26 TPH, 0.28 cm, and 0.26 m, respectively. The stand density mean residual and standard deviation are 0.18 and 1.30, respectively, when considered in terms of measured trees, obtained by dividing by 19.5. The lifetime average residuals for stand

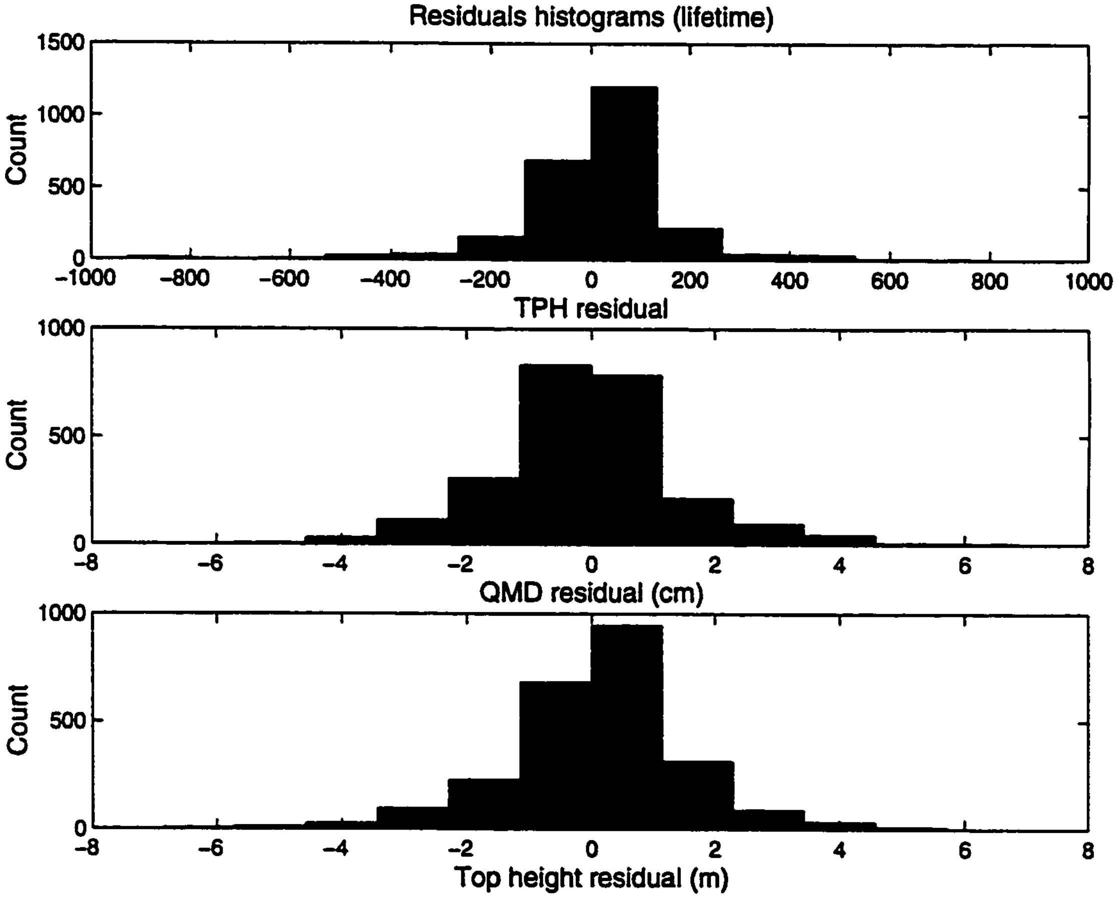

Figure 2.16: Histograms of residuals for the lifetime residuals computation scenario R^l_{iks}. The TPH residuals, $i = 1$, are on top, QMD residuals, $i = 2$, are in the middle, and top height residuals, $i = 3$, are on the bottom.

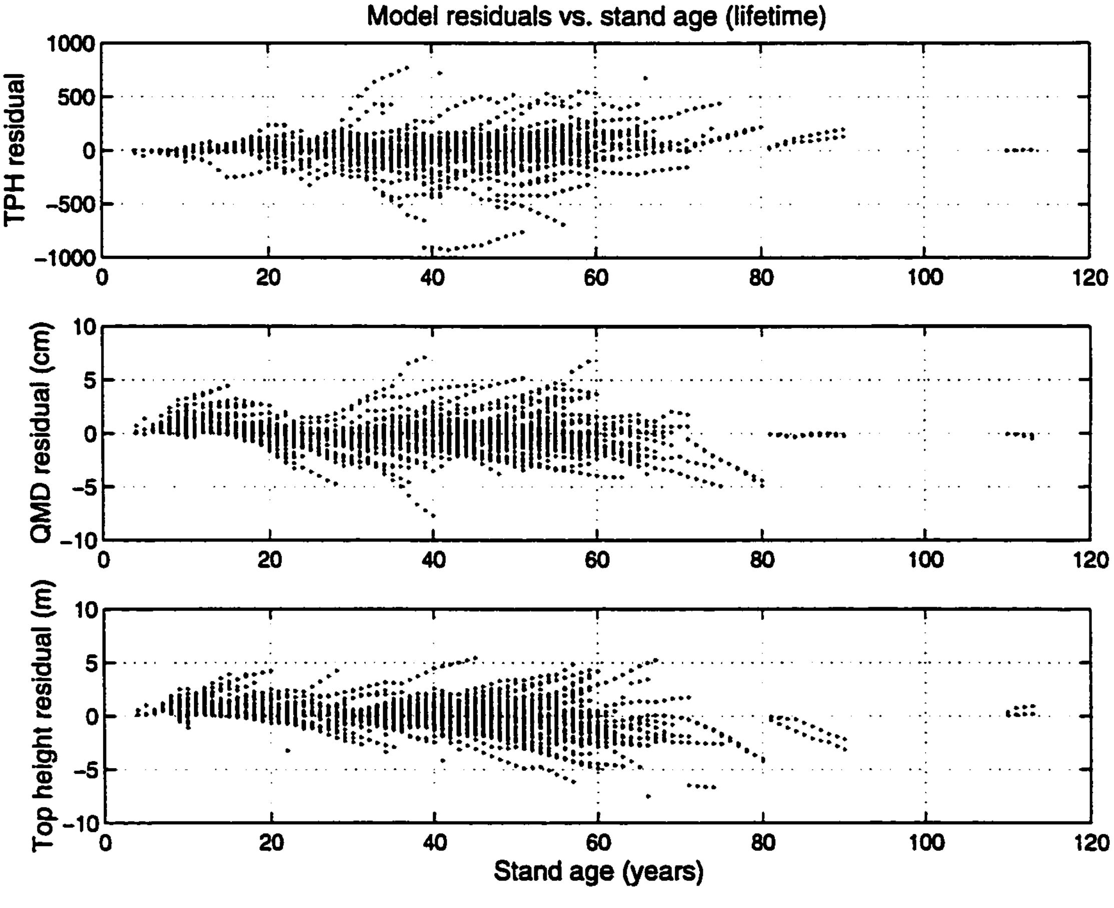

Figure 2.17: Lifetime residuals, R^l_{iks}, *vs.* stand age. The TPH residuals, $i = 1$, are on top. QMD residuals, $i = 2$, are in the middle, and top height residuals, $i = 3$, are on the bottom. The dot density provides an indication of the number of data points.

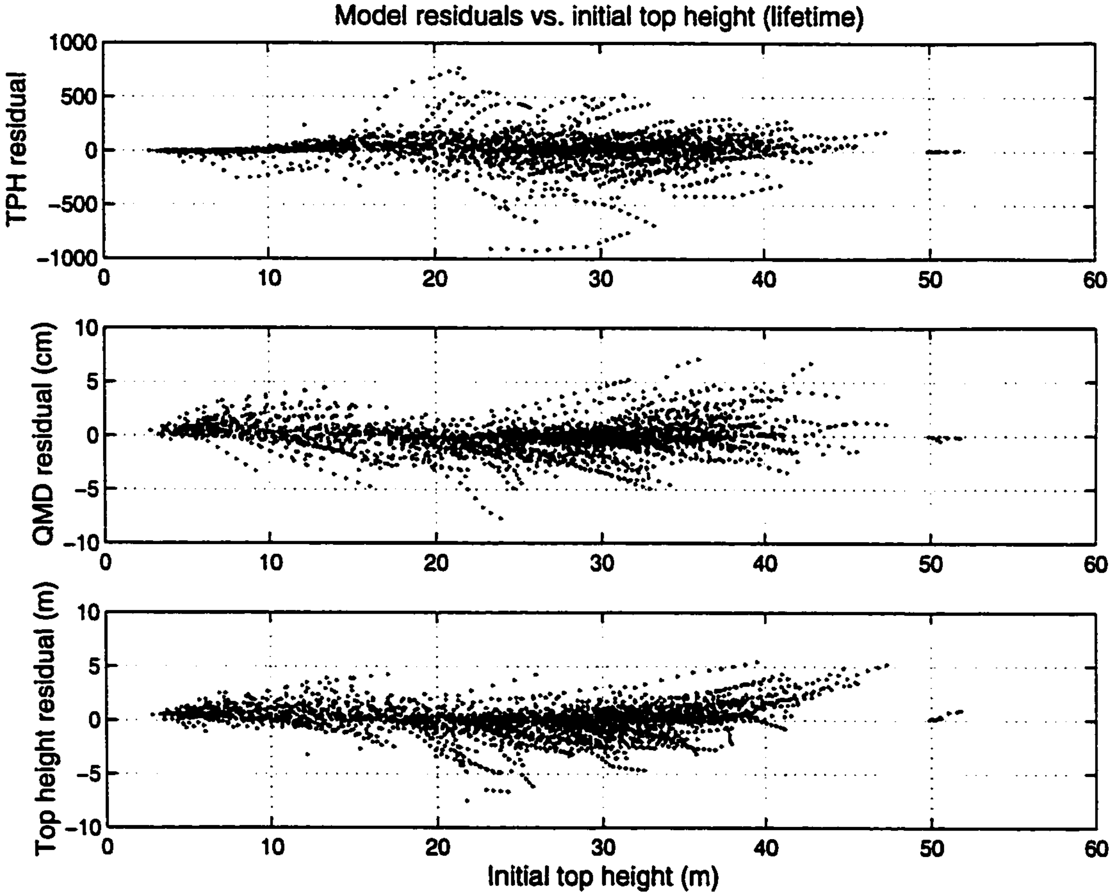

Figure 2.18: Lifetime residuals, R^l_{iks}, *vs.* top height. The TPH residuals, $i = 1$, are on top, QMD residuals, $i = 2$, are in the middle, and top height residuals, $i = 3$, are on the bottom. The dot density provides an indication of the number of data points.

density, QMD, and top height are 12.85 TPH, -0.059 cm, and 0.093 m, with standard deviations of 151.88 TPH, 1.49 cm, and 1.44 m, respectively. The stand density mean residual and standard deviation are 0.66 and 7.79, respectively, when considered in terms of measured trees, obtained by dividing by 19.5.

The mean residual values for QMD and top height are significantly less than one unit for both the single step and lifetime residuals, indicating that the Douglas-fir growth and yield model is accurately predicting the average tree sizes for both scenarios. For stand density, the per tree average residuals are within a single tree and the per hectare average residuals are on the order of the average scale factor used to convert sample plots to a hectare basis, indicating that the growth and yield model is also accurately predicting stand density for the available data. The zero centered symmetric distributions of the single step and lifetime mean residuals provides strong evidence for the quality of the model specification: it is accurate both at the calibration time step of one year and for longer time spans, though the variability increases for the longer time spans.

The precision of the Douglas-fir growth and yield model may be assessed by comparing the variation of the single step and lifetime residuals with the variation in the state variables that were predicted. The variation comparisons are performed by taking the ratios of the single step or lifetime standard deviations and the standard deviations of the state variables. These ratios may be interpreted as the percentage of variation in the data *not* accounted for by the model. The single step ratios of standard deviations are 0.030, 0.031, and 0.026 for stand density, QMD, and top height, respectively, indicating that only two to three percent of the variation in the data is not accounted for by the model. The lifetime ratios of standard deviations are 0.179, 0.165, and 0.145 for stand density, QMD, and top height, respectively, and indicate that 14% to 18% of the variation in the data is not accounted for by the model. These results indicate that the Douglas-fir growth and yield model is generally precise as well as accurate.

Table 2.5: Single step residuals summary for TPH, QMD, and top height. This summary indicates the ability of the S-system growth and yield model to predict Douglas-fir stand annual growth. *MAD is mean absolute deviation.

	TPH residual	QMD residual	Top height residual
Mean	3.434473	0.013365	0.042838
Standard deviation	25.261141	0.283599	0.260188
Minimum	-104.740336	-0.952858	-0.703518
Median	6.138521	-0.026796	0.032630
Maximum	88.993298	1.312038	1.415023
MAD*	17.956016	0.217416	0.209471
Number of points	2431	2431	2431

An interesting characteristic of the single step and lifetime residuals may be obtained by examining the ratios of the residual standard deviations for the two residual scenarios. Let the single step residual standard deviations be given by s_{i_s} and the lifetime residual standard deviations be given by s_{il}, $i = 1,2,3$. Then, the ratio values s_{il}/s_{i_s} are: 6.01, 5.32, and 5.54 for stand density, QMD, and top height respectively. These large ratios are all statistically significant for $\alpha = 0.05$ when squared, representing an F-statistic [7, 172], but the differences in variability cannot be explained in terms of the numerical integration and the time spans involved alone. A possible interpretation of these ratios is given in Section 2.8.

Comparisons of the standard normal distribution with the single step and lifetime stand density, QMD, and top height residuals are presented in Table 2.7 and Table 2.8, respectively. The tables compare the normal probabilities $P(|z| \leq m)$, with the single step residual proportions $\hat{P}(|R^s_{ik}| \leq m s_{i_s})$ and the lifetime residual proportions $\hat{P}(|R^l_{ik_s}| \leq m s_{il})$, $i = 1,2,3$, and $m = 0.5, 1.0, 1.5, 2.0, 3.0$. These comparisons permit an assessment of the amount of information potentially remaining in the residuals [21], from the perspective that a standard normal distribution is considered to be random noise, that is indicative of a totally random process with a mean of zero.

An examination of Table 2.7 indicates that the single step stand density residuals

Table 2.6: Lifetime residuals summary for TPH, QMD, and top height. This summary indicates the ability of the S-system growth and yield model to predict Douglas-fir stand yield, or cumulative growth or stand trajectories. *MAD is mean absolute deviation.

	TPH residual	QMD residual	Top height residual
Mean	12.849642	-0.059423	0.092699
Standard deviation	151.881722	1.491796	1.442787
Minimum	-928.280003	-7.683184	-7.489693
Median	15.212011	-0.057689	0.157809
Maximum	770.522952	7.138518	5.444309
MAD*	91.275177	1.052275	1.023427
Number of points	2431	2431	2431

and QMD residuals have greater than expected proportions near zero, for $m = 0.5, 1.0$, and 1.5, but slightly heavier tails, indicated by the reversal of this relationship for $m = 2.0, 3.0$. The top height residual proportions very closely resemble the normal probabilities for this table. This is likely a result of the fact that most of the tree heights used to compute the top height values were estimated from height diameter relationships. An examination of Table 2.8 indicates that the lifetime stand density residuals, QMD residuals, and top height residuals have greater than expected proportions near zero than would be expected given a normal distribution for $m = 0.5, 1.0$, and 1.5, but, again, slightly heavier tails, indicated by the reversal of this relationship for $m = 2.0, 3.0$.

The stronger central tendency of the single step and lifetime model residuals relative to the normal distribution provides additional evidence that the S-system growth and yield model captures the essential characteristics of Douglas-fir stand dynamics. Further, this comparison reinforces the conclusions that the S-system growth and yield model is both accurate as well as precise. Given this comparison, the residuals are essentially random noise, and it is, therefore, unlikely that any additional improvement in the performance of the model will be possible with this data set and

Table 2.7: Single step residuals comparison with standard normal distribution. Values represent the probability, or proportion, of residuals in the range $0 \pm m$ for the normal distribution, and $0 \pm ms_{is}$, $i = 1, 2, 3$, where s_{is} is the standard deviation of the stand density, QMD, or top height single step residuals, respectively.

m	$N(0,1)$ $P(\lvert z\rvert \le m)$	TPH $\hat{P}(\lvert R^s_{1k}\rvert \le ms_{1s})$	QMD $\hat{P}(\lvert R^s_{2k}\rvert \le ms_{2s})$	top height $\hat{P}(\lvert R^s_{3k}\rvert \le ms_{3s})$
0.5000	0.3829	0.4768	0.4192	0.3797
1.0000	0.6827	0.7491	0.7408	0.6565
1.5000	0.8664	0.8717	0.8762	0.8601
2.0000	0.9545	0.9325	0.9486	0.9609
3.0000	0.9973	0.9856	0.9881	0.9979

Table 2.8: Lifetime residuals comparison with the standard normal distribution. Values represent the probability, or proportion, of residuals in the range $0 \pm m$ for the normal distribution, and $0 \pm ms_{il}$, $i = 1, 2, 3$, where s_{il} is the standard deviation of the stand density, QMD, or top height lifetime residuals, respectively.

m	$N(0,1)$ $P(\lvert z\rvert \le m)$	TPH $\hat{P}(\lvert R^l_{1ks}\rvert \le ms_{1l})$	QMD $\hat{P}(\lvert R^l_{2ks}\rvert \le ms_{2l})$	top height $\hat{P}(\lvert R^l_{3ks}\rvert \le ms_{3l})$
0.5000	0.3829	0.6059	0.5208	0.4977
1.0000	0.6827	0.8145	0.7483	0.7470
1.5000	0.8664	0.9120	0.8717	0.8791
2.0000	0.9545	0.9457	0.9350	0.9412
3.0000	0.9973	0.9786	0.9893	0.9864

this model formulation.

2.7.3 Residual correlations

In Figure 2.19 through Figure 2.21 the single step stand density, QMD, and top height residuals are plotted against initial top height along with the simple linear regression line for each set of residuals. A 51 point moving average with plus or minus two moving standard deviations is also plotted to provide an empirical 95% confidence interval for each set of residuals. These figures are representative of the residual plots for the other independent variables appearing in Table 2.9 through Table 2.11. Note in particular that in each of these figures the value $y = 0$ is contained within the confidence interval defined by the moving average and twice the moving standard deviation, indicating an overall lack of bias in the model predictions. Also notice that the line fits are all more or less horizontal. The QMD and top height figures show a slight U-shaped structure in their respective residuals. This may be explained, in large part, by the fact that the majority of the data lie between top heights of 22 m and 38 m, and hence the agreement should be better within this interval than outside it, as is the case. The nearly horizontal moving average within this range of top height values supports this conclusion, though, on average, the model does slightly overestimate QMD and top height within this range.

Table 2.9 through Table 2.11 provide simple linear regression coefficients and r^2 values for standardized single step stand density, QMD, and top height residuals for the independent variables: initial stand age, final stand age, initial TPH, initial QMD, initial top height, site index, and elevation. The slopes and intercepts for the simple linear regressions should have nominal values of $a = 0$ and $b = 0$, indicating that there is no trend and no bias, and the r^2 values should have nominal values of zero, $r^2 = 0$, indicating that there is no correlation between the standardized residuals and each of the independent variables. Regression intercept values within the range 0 ± 1 will be considered acceptable, that is effectively zero, since they indicate that the regression intercept falls within one standard deviation of zero. Given the sample size, all of the simple linear regression coefficients are statistically significant for $\alpha = 0.05$, but what must be determined is the relevance of the values obtained [6, 64, 145, 172]. Thus, table values which are noticeably different from zero need to be explained.

Consider first the simple linear regression intercept values b. Almost all of the intercept values are within the interval $-1 \leq b \leq 1$, and most of the values are actually in the much smaller interval $-0.50 \leq b \leq 0.50$. Thus, most of the regression lines pass near the origin, $x = 0$ and $y = 0$. The notable exceptions are for the independent variable site index and the three sets of residuals, and the initial and final stand ages for top height and possibly QMD. Only the initial stand age will be considered, as the results are essentially identical for the final stand age. The regression line intercepts for site index have values of 1.2, -2.1, and -3.2 for the TPH, QMD, and top height residuals, respectively. The regression line intercepts for initial stand age have values of 0.65, and 1.01 for the QMD, and top height, residuals respectively. Thus, these intercept values may indicate that site index and stand age have relevant relationships with the single step model residuals.

Consider the simple linear regression slope values a for the single step residuals. Almost all of these values are in the interval $-0.05 \leq a \leq 0.05$, and indicate that the regression lines are almost all horizontal. The notable exceptions occur for the independent variable site index, with the QMD and top height slopes having values of 0.06 and 0.10, respectively. The TPH slope for the independent variable site index has a value of -0.03, and is the only slope for the standardized TPH residuals having a magnitude greater than 0.01. So again, site index may have a relevant relationship with one or more of the sets of single step model residuals.

Finally, consider the coefficients of determination, or r^2 values, which indicate the strength of a straight line relationship between the residual values and the independent variables. Values of r^2 near one indicate a strong linear relationship exists, and values near zero indicate that no linear relationship exists. Most of the r^2 values are in the interval $0 \leq r^2 \leq 0.05$, indicating that there is effectively no linear relationship between the residuals and the independent variables. The notable exceptions are for the independent variables site index and initial stand age and for the QMD and top height residuals. The r^2 values of interest for site index are 0.12 and 0.30, for the QMD and top height residuals, respectively. The r^2 values of interest for initial stand age are 0.07 and 0.14, again for QMD and top height, respectively. Yet again, site index and initial stand age are indicated as potentially having a relationship, though a very weak one, with the single step residuals.

Site index and initial stand age have both been identified as being potentially having linear relationships with the single step residuals from the Douglas-fir growth and yield model. There are three reasons that these variables appear to stand out in this analysis. First, the Douglas-fir stand measurement data set contains a small subset of stands, approximately 20%, that are young and growing very rapidly, and a larger data set of older stands growing more slowly. The growth on the young stands is consistently underestimated, giving their residuals a large amount of leverage in the simple regression analyses. Second, the majority of site index values, approximately 70%, fall within the rather limited range of 33 m to 40 m, but span a range from 17 m to 46 m. The residuals follow a predictable pattern relative to site index, overestimating for lower site index values and underestimating for higher site index values. Thus, the residuals for the site index values outside the range of 33 m to 40 m also have a significant amount of leverage in the simple linear regression analyses. Third, and compounding the first two reasons, the young fast growing stands are nearly all within the upper portion of 33 m to 40 m site index range, and thus exert significant leverage in the simple linear regression analyses.

Given the large amount of leverage exercised by a relatively small subset of the stand measurement data, and the generally weak linear relationships discovered, it seems reasonable to conclude that the single step residuals are generally uncorrelated with site index and stand age. The fact that the single step residuals appear to be

Table 2.9: Single step standardized TPH residual line fits and r^2 values. Regression coefficients are for the model $y = ax+b$ applied to the standardized TPH residuals for each x variable. Values of $a = 0$ and $b = 0$ imply that there is not a linear relationship or bias, and $r^2 = 0$ implies a lack of correlation with the x variable.

x	a	b	r^2
Initial stand age (years)	0.007636	-0.157956	0.017415
Final stand age (years)	0.007636	-0.165592	0.017415
Initial TPH	-0.000036	0.183172	0.000956
Initial QMD (cm)	0.009937	-0.077634	0.008364
Initial top height (m)	0.003615	0.045904	0.001332
Site index (m)	-0.031256	1.226326	0.030899
Elevation (m)	0.000655	-0.165437	0.034084

uncorrelated with the majority of the other independent variables used in the simple linear regression analyses lends credence to this conclusion as well. Thus, the Douglas-fir growth and yield model again appears to perform well, predicting annual growth, with generally small residuals, as evidenced by the single step residual analyses.

The regression coefficients obtained for the standardized residuals may be converted into the original residual units by multiplying by the appropriate residual standard deviation from Table 2.5. For example, to convert the standardized QMD residual regression coefficients $a = -0.013513$ and $b = 0.383773$ into centimeters, multiply them by $s_{2_s} = 0.283599$ obtaining $a = -0.003832$ and $b = 0.108838$, the values obtained for the least squares line in Figure 2.20.

The lifetime stand density, QMD, and top height residuals plotted against initial top height appear in Figure 2.22 through Figure 2.24, respectively, along with the simple linear regression line for each set of residuals *vs.* initial top height. A 51 point moving average plus or minus twice the moving standard deviation is also plotted to provide an empirical 95% confidence interval for the mean residuals. These figures are representative of the residual plots for the other independent variables appearing in Table 2.12 through Table 2.14. Note in particular that in each of these figures that $y = 0$ is contained within the confidence interval defined by the moving

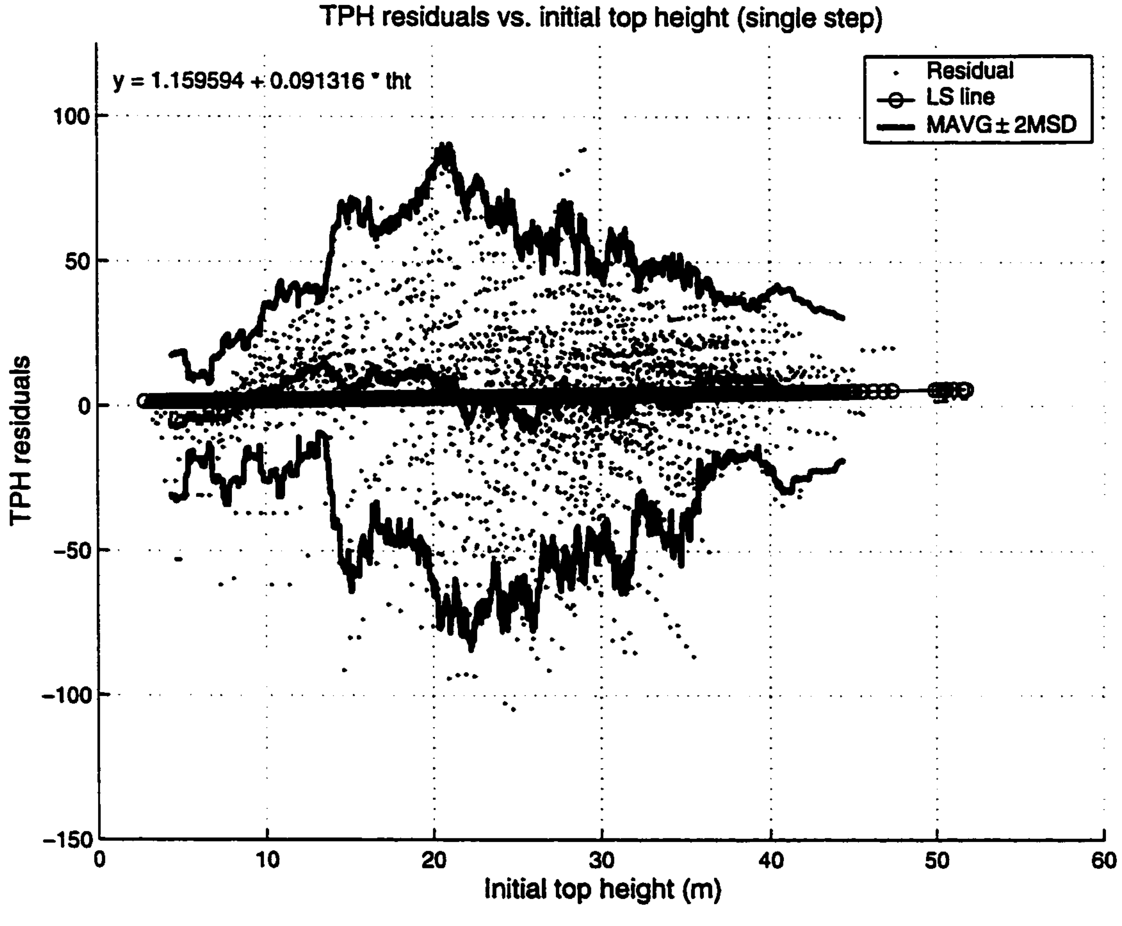

Figure 2.19: Single step TPH residuals, R^{s}_{1k}, *vs.* top height. The figure also includes the least squares line and the 51 point moving average ± twice the moving standard deviation. Regression coefficients for the model $y = ax + b$ applied to these data yields the model $y = 0.091316x + 1.159594$ with r^2= 0.001332. See Table 2.9 for the other linear regression coefficients and residual r^2 values.

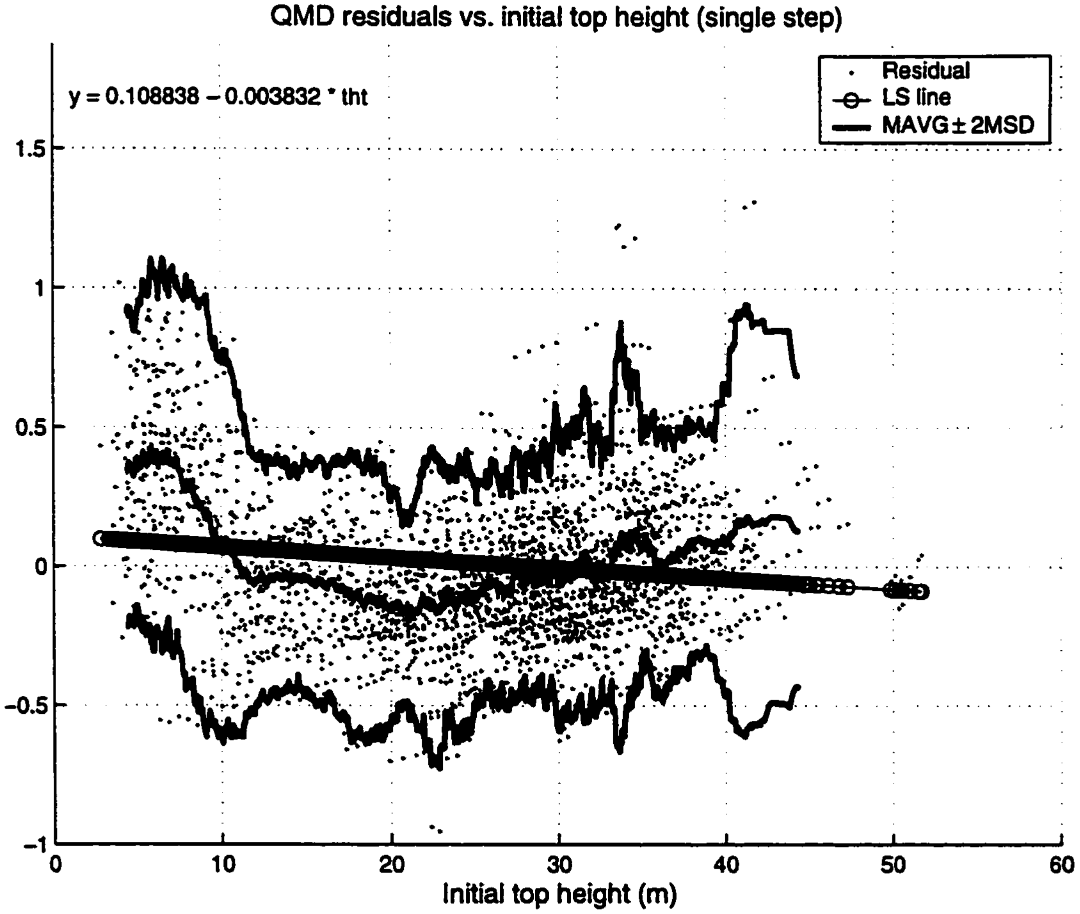

Figure 2.20: Single step QMD residuals, R^{l}_{2ks}, *vs.* top height. The figure also includes the least squares line and the 51 point moving average ± twice the moving standard deviation. Regression coefficients for the model $y = ax + b$ applied to these data yields the model $y = 0 - 0.003832x + 0.108838$ with $r^2 = 0.018608$. See Table 2.10 for the other linear regression coefficients and residual r^2 values.

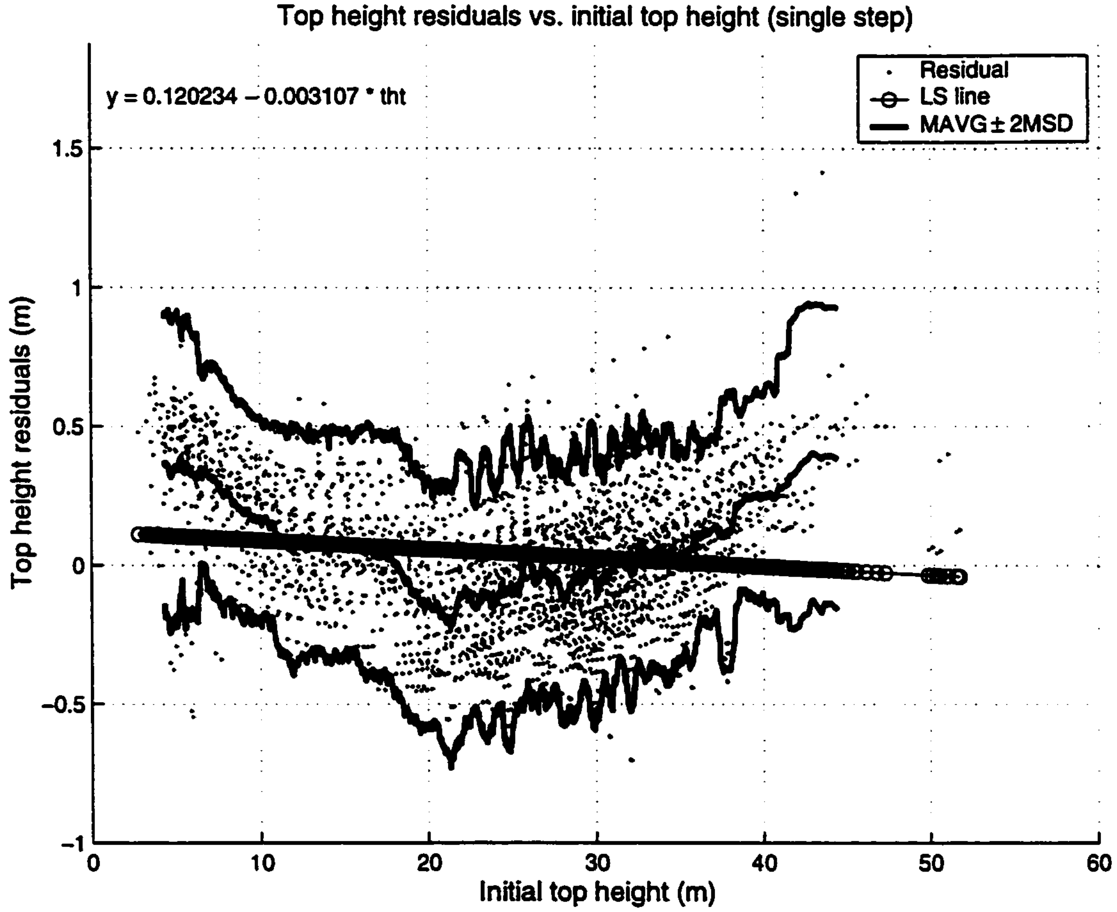

Figure 2.21: Single step top height residuals, R^{l}_{3ks}, *vs.* top height. The figure also includes the least squares line and the 51 point moving average ± twice the moving standard deviation. Regression coefficients for the model $y = ax + b$ applied to these data yields the model $y = -0.003107x + 0.120234$ with $r^2 = 0.014528$. See Table 2.11 for the other linear regression coefficients and residual r^2 values.

Table 2.10: Single step standardized QMD residual line fits and r^2 values. Regression coefficients are for the model $y = ax + b$ applied to the standardized QMD residuals for each x variable. Values of $a = 0$ and $b = 0$ imply that there is not a linear relationship or bias, and $r^2 = 0$ implies a lack of correlation with the x variable.

x	a	b	r^2
Initial stand age (years)	-0.015705	0.651607	0.073664
Final stand age (years)	-0.015705	0.667312	0.073664
Initial TPH	-0.000156	0.252920	0.018169
Initial QMD (cm)	-0.005573	0.166914	0.002631
Initial top height (m)	-0.013513	0.383773	0.018608
Site index (m)	0.061473	-2.097380	0.119523
Elevation (m)	-0.000252	0.163574	0.005180

Table 2.11: Single step standardized top height residual line fits and r^2 values. Regression coefficients are for the model $y = ax + b$ applied to the standardized top height residuals for each x variable. Values of $a = 0$ and $b = 0$ imply that there is not a linear relationship or bias, and $r^2 = 0$ implies a lack of correlation with the x variable.

x	a	b	r^2
Initial stand age (years)	-0.021868	1.006312	0.142816
Final stand age (years)	-0.021868	1.028180	0.142816
Initial TPH	-0.000164	0.381394	0.020156
Initial QMD (cm)	-0.012589	0.435242	0.013425
Initial top height (m)	-0.011940	0.462103	0.014528
Site index (m)	0.097070	-3.221717	0.298032
Elevation (m)	-0.000759	0.510776	0.046829

average and twice the moving standard deviation. This indicates an overall lack of bias in the model predictions. Also notice that the line fits are all more or less horizontal. The QMD and top height figures show a similar U-shaped structure to the single step residuals, though it is less pronounced due to the greater range in the residual values. This U-shape may also be explained in large part by the fact that the majority of the data lie between top heights of 22 m and 38 m, and hence the agreement should be better within this interval than outside it, as is the case. Further complicating the interpretation of these residuals is the fact that the calibrating data set contains both planted stands and naturally regenerating stands which may have different growth characteristics. The nearly horizontal moving average within this range of top height values supports this conclusion, though, on average, the model does slightly overestimate QMD and top height within this range.

In general, the lifetime residuals indicate that stands having strong initial agreement with the measured stand trajectories continued to have a strong agreement throughout the projection. Similarly, stands which did not have strong initial agreement with the actual trajectories tended to continue to disagree. This explains the tendrils that may be seen in the three lifetime residual figures moving away from the x-axis. The increasing trend of top height underprediction seen in Figure 2.24 is most likely a result of the lack of data for stands with larger trees to help calibrate the model. There were very few stands having a top height greater than 38 m, only about 10% of the available data. This lack of data for stands with large trees combined with the Chapman-Richards decay term for top height may cause height growth to slow somewhat prematurely.

The top height growth slowdown is then compounded by the integration process for successive measurements, increasing the top height underestimation. Other factors which may be influencing this result are the stands whose composition is not predominantly Douglas-fir by number of stems, which could have an effect through the coupling of QMD and top height. The top height underestimation may also be due to properties of the stands involved, e.g., some of them may have been thinned or fertilized at some point in their pre-measurement history. Finally, the top height underestimation may be an artifact of the height estimation process used to fill in unmeasured tree heights. Further investigation into the nature of the top height underestimation is necessary. The fact that the QMD residuals do not show a similar trend to the top height residuals for large trees lends some credence to the idea that the slowdown is not simply due to the Chapman-Richards decay term formulation. However, as will be seen shortly, long term top height projections fall within the expected range of values, and well within the maximum observed sizes for Douglas-fir trees.

Table 2.12 through Table 2.14 provide simple linear regressions and r^2 values for standardized lifetime stand density, QMD, and top height residuals for the independent variables: initial stand age, final stand age, initial TPH, initial QMD, initial top height, site index, and elevation. The slopes and intercepts for the simple linear regressions should have nominal values of $a = 0$ and $b = 0$, indicating that there is no trend and no bias, and the r^2 values should have nominal values of zero, $r^2 = 0$, indicating that there is no correlation between the standardized residuals and each of the independent variables.

There were no surprises in the simple linear regression residual analyses of the standardized lifetime residuals. Site index and stand age were again identified as potentially having relevant correlations, but were dismissed for the same reasons as in the single step analyses, a high degree of leverage.

2.7.4 Predicted vs. actual values

Predicted stand density, QMD, and top height for the lifetime, or stand trajectory, scenario, Y_{iks}, $i = 1, 2, 3$, are plotted against their actual values in Figure 2.25 through Figure 2.27, respectively. The figures also contain the simple linear regression lines and a 51 point moving average ± twice the moving standard deviation. There is

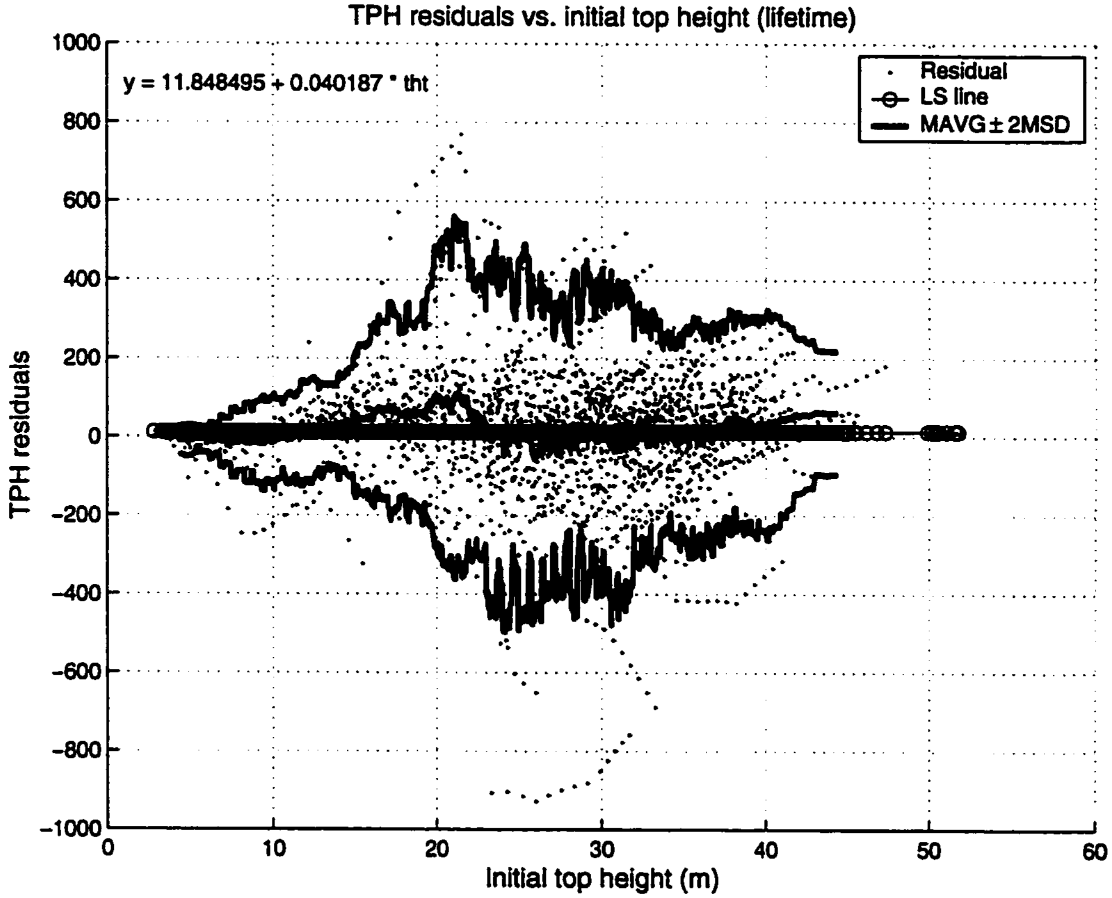

Figure 2.22: Lifetime TPH residuals, $R^l_{1k_s}$, *vs.* top height. The figure also includes the least squares line and the 51 point moving average ± twice the moving standard deviation. Regression coefficients for the model $y = ax + b$ applied to these data yields the model $y = 0.040187x + 11.848495$ with $r^2 = 0.000007$. See Table 2.12 for standardized linear regression coefficients and residual r^2 values.

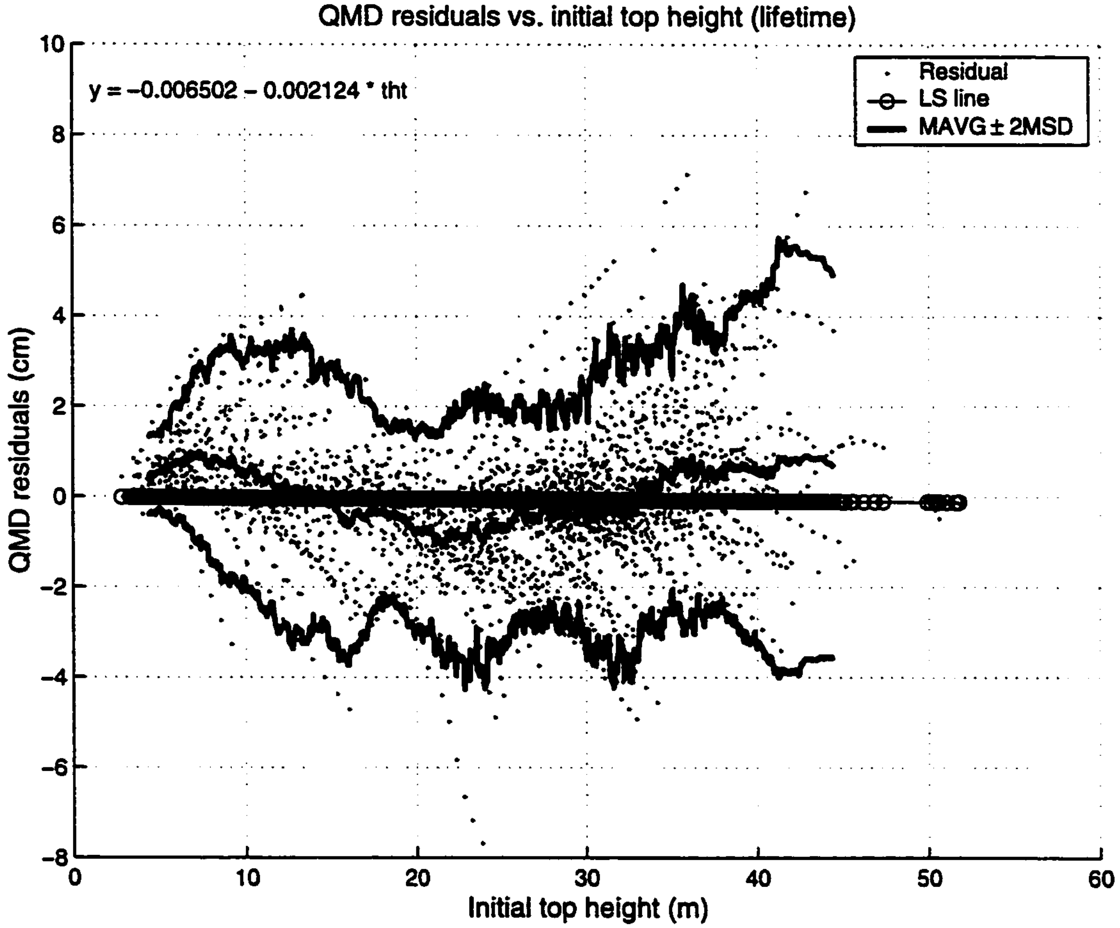

Figure 2.23: Lifetime QMD residuals, $R^l_{2k_s}$, *vs.* top height. The figure also includes the least squares line and the 51 point moving average ± twice the moving standard deviation. Regression coefficients for the model $y = ax + b$ applied to these data yields the model $y = -0.002124x + 0.006502$ with $r^2 = 0.000207$. See Table 2.13 for standardized linear regression coefficients and residual r^2 values.

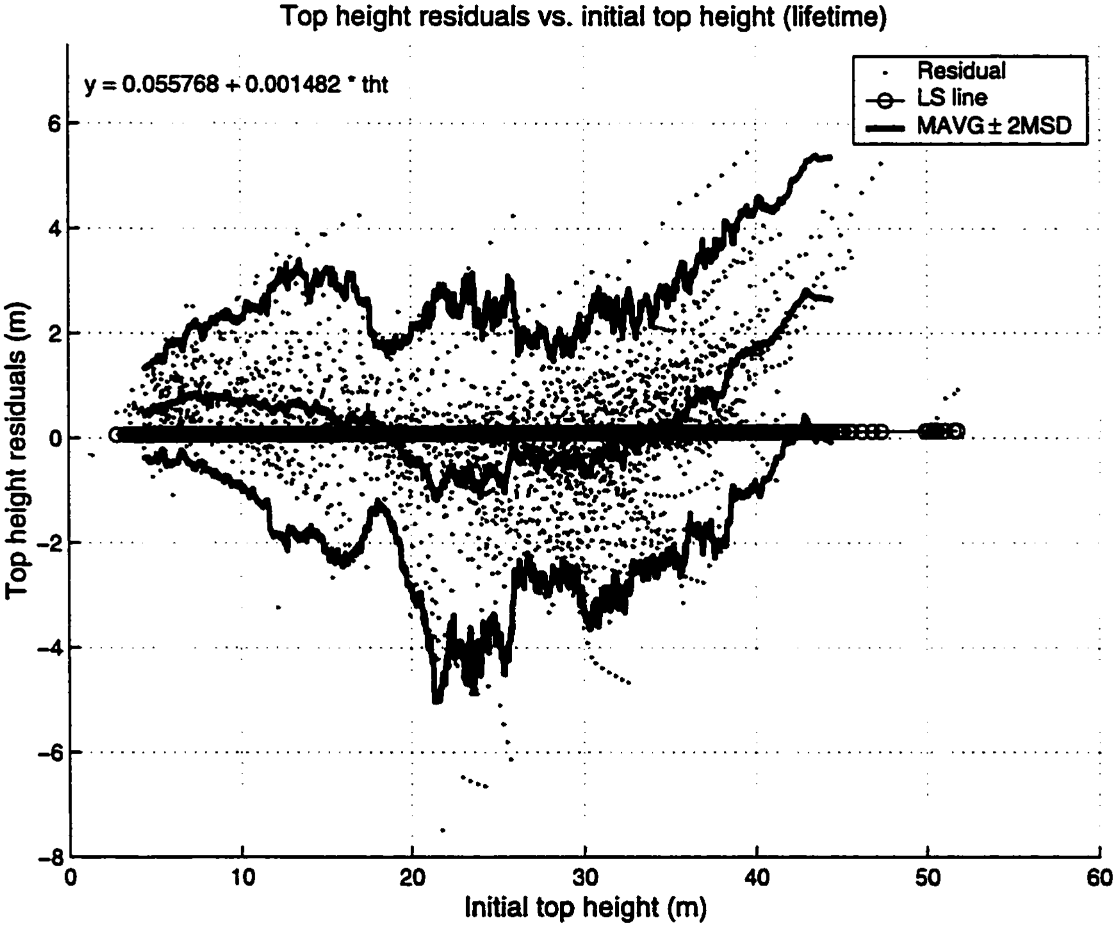

Figure 2.24: Lifetime top height residuals, R^{l}_{3ks}, *vs.* top height. The figure also includes the least squares line and the 51 point moving average ± twice the moving standard deviation. Regression coefficients for the model $y = ax + b$ applied to these data yields the model $y = 0.001482x + 0.055768$ with $r^2 = 0.000108$. See Table 2.14 for standardized linear regression coefficients and residual r^2 values.

Table 2.12: Lifetime standardized TPH residual line fits and r^2 values. Regression coefficients are for the model $y = ax + b$ applied to the standardized TPH residuals for each x variable. Values of $a = 0$ and $b = 0$ imply that there is not a linear relationship or bias, and $r^2 = 0$ implies a lack of correlation with the x variable.

x	a	b	r^2
Initial stand age (years)	0.007447	-0.202011	0.016561
Final stand age (years)	0.007447	-0.209457	0.016561
Initial TPH	0.000143	-0.104195	0.015292
Initial QMD (cm)	0.002288	0.035427	0.000443
Initial top height (m)	0.000265	0.078011	0.000007
Site index (m)	-0.037023	1.376174	0.043354
Elevation (m)	0.000676	-0.223053	0.036196

Table 2.13: Lifetime standardized QMD residual line fits and r^2 values. Regression coefficients are for the model $y = ax + b$ applied to the standardized QMD residuals for each x variable. Values of $a = 0$ and $b = 0$ imply that there is not a linear relationship or bias, and $r^2 = 0$ implies a lack of correlation with the x variable.

x	a	b	r^2
Initial stand age (years)	-0.012966	0.459230	0.050212
Final stand age (years)	-0.012966	0.472196	0.050212
Initial TPH	-0.000196	0.219186	0.028783
Initial QMD (cm)	0.008686	-0.226538	0.006391
Initial top height (m)	-0.001424	-0.004358	0.000207
Site index (m)	0.072730	-2.577058	0.167307
Elevation (m)	-0.000233	0.062391	0.004410

Table 2.14: Lifetime standardized top height residual line fits and r^2 values. Regression coefficients are for the model $y = ax + b$ applied to the top height residuals for each x variable. Values of $a = 0$ and $b = 0$ imply that there is not a linear relationship or bias, and $r^2 = 0$ implies a lack of correlation with the x variable.

x	a	b	r^2
Initial stand age (years)	-0.019098	0.799322	0.108931
Final stand age (years)	-0.019098	0.818420	0.108931
Initial TPH	-0.000166	0.283958	0.020709
Initial QMD (cm)	0.000296	0.057881	0.000007
Initial top height (m)	0.001027	0.038653	0.000108
Site index (m)	0.102415	-3.508560	0.331755
Elevation (m)	-0.000547	0.307332	0.024358

clearly a very strong one-to-one linear relationship represented by the data in these figures. From the figures it is apparent that the majority of the data points are symmetrically distributed about the regression lines and generally remain within the confidence intervals defined by the 51 point moving averages and standard deviations. This, yet again, indicates the quality of the S-system based Douglas-fir growth and yield model. No figures are presented for the single step predicted stand density, QMD, and top height values since the data, the regression lines, and the moving averages ± twice the standard deviations all overlapped, obscuring each other. No initial stand values were used in the simple linear regression analyses, nor are any plotted in the figures.

For each of the figures, note in particular that the widening of the confidence intervals occurs where there is the smallest amount of data: for very high stand densities and for stand with larger trees. The TPH and QMD moving averages track the regression line extremely well throughout the range of the data. The top height moving average tracks the regression line well until a top height of approximately 38 m, where it then begins to fall below the regression line, indicating an underprediction by the growth and yield model. This is also where the confidence interval widens due to a lack of data for stands with larger trees. This underestimation in top height for stands having a top height greater than 38 m was already identified in the lifetime residuals analysis as being largely data dependent: the lack of data for stands did not permit an adequate calibration for this range of top height values.

The coefficients for the simple linear regressions of the predicted *vs.* actual values and their corresponding r^2 values are presented in Table 2.15 and Table 2.16 for the lifetime and single step scenarios, respectively. Nominal values for the simple linear regression coefficients are $a = 1$ for the slopes and $b = 0$ for the intercepts, indicating a strong one to one, unbiased, linear relationship. Nominal values of $r^2 = 1$ further indicate a strong linear relationship with little variability.

The lifetime predicted *vs.* actual simple linear regression coefficients and r^2 values indicate that the predicted stand trajectories are in very good agreement with the actual stand trajectory data. Slope values for the lifetime predicted *vs.* actual values are 0.9756, 0.9835, and 0.9964 for stand density, QMD, and top height, respectively. All three values are very close to the nominal value $a = 1$. Intercept values for the lifetime predicted *vs.* actual values are 18.683 TPH, 0.426 cm, and -0.001 m for stand density, QMD, and top height. The top height intercept value is exceptionally good, being almost zero. The QMD intercept value is also quite good, being within half a centimeter of zero. The TPH intercept may seem large at first, but when converted into measured trees, dividing by the average scale factor 19.5, it is within one measured tree of zero, and is seen to be quite good. The lifetime predicted *vs.* actual r^2 values are all in excess of 0.960, indicating very strong one-to-one linear relationships with little variation about the line.

The single step predicted *vs.* actual simple linear regression coefficients and r^2 values indicate that the predicted annual growth is in excellent agreement with the actual stand measurement data. Slope values for the single step predicted *vs.* actual values are 1.0001, 1.0006, and 1.0003 for stand density, QMD, and top height, respectively, and are all very close to the nominal value $a = 1$. Intercept values for the single step predicted *vs.* actual values are -3.587 TPH, -0.027 cm, and -0.110 m for

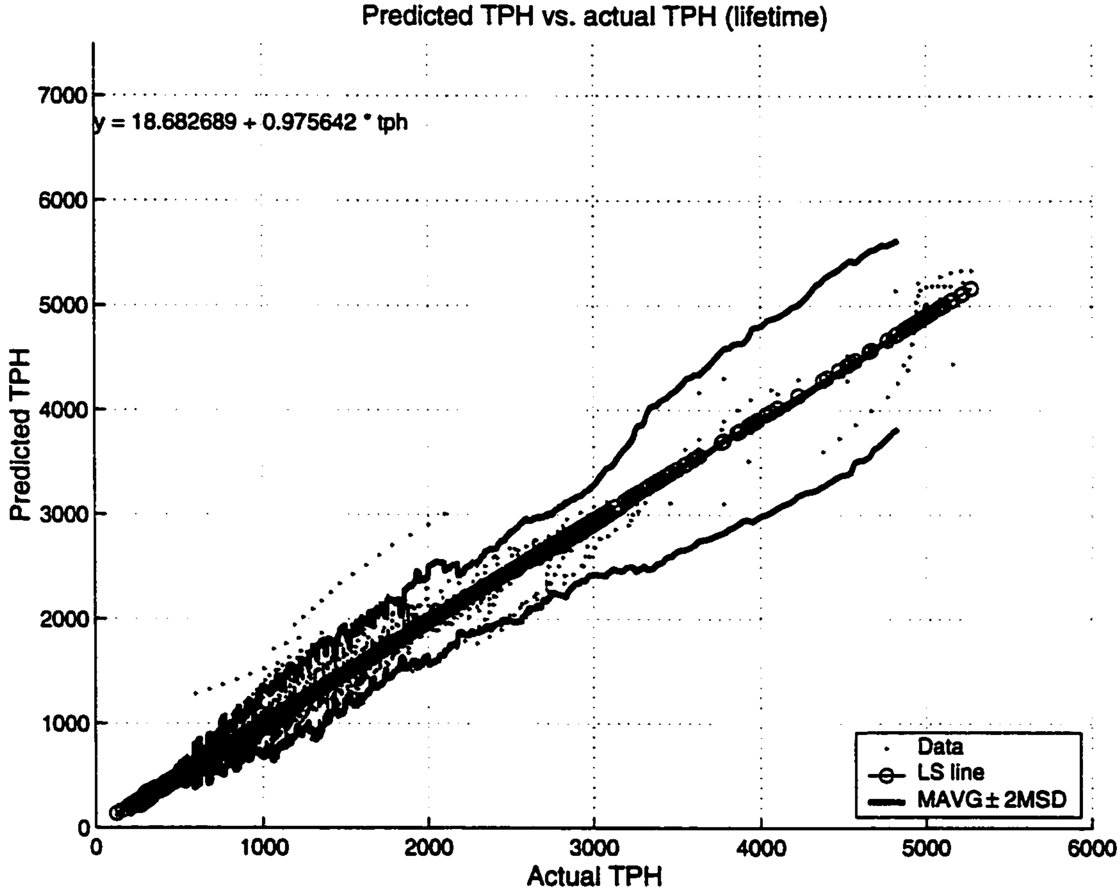

Figure 2.25: Lifetime predicted TPH, $\hat{Y}_{1ks}$, *vs.* actual TPH, Y_{1ks}. The figure also includes the least squares line and the 51 point moving average ± twice the moving standard deviation. Regression coefficients for the model $y = ax + b$ applied to these data yields the model $y = 0.975642x + 18.682689$ with $r^2 = 0.968202$. See Table 2.15 for the other lifetime linear regression coefficients and r^2 values, and Table 2.16 for the single step coefficients and r^2 values.

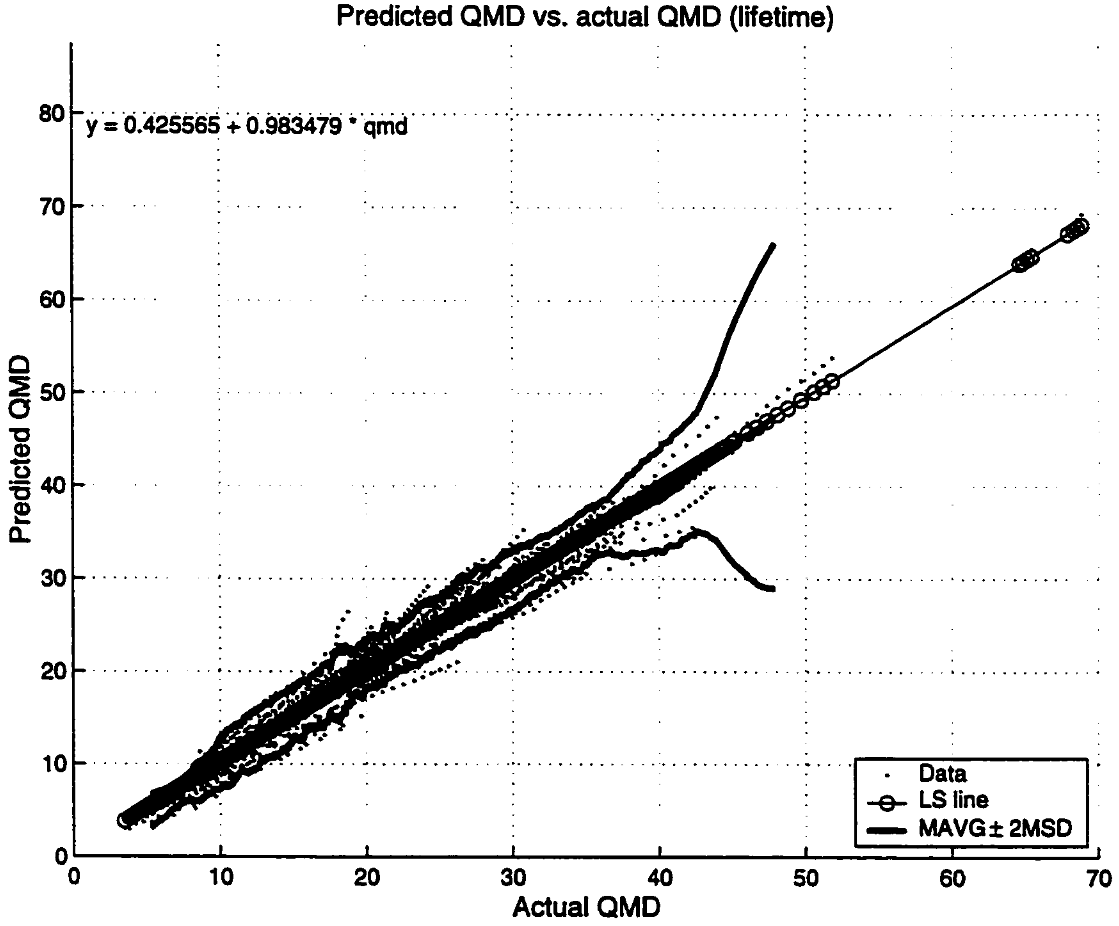

Figure 2.26: Lifetime predicted QMD, $\hat{Y}_{2ks}$, *vs.* actual QMD, Y_{2ks}. The figure also includes the least squares line and the 51 point moving average ± twice the moving standard deviation. Regression coefficients for the model $y = ax + b$ applied to these data yields the model $y = 0.983479x + 0.425565$ with r^2= 0.972990. See Table 2.15 for the other lifetime linear regression coefficients and r^2 values, and Table 2.16 for the single step coefficients and r^2 values.

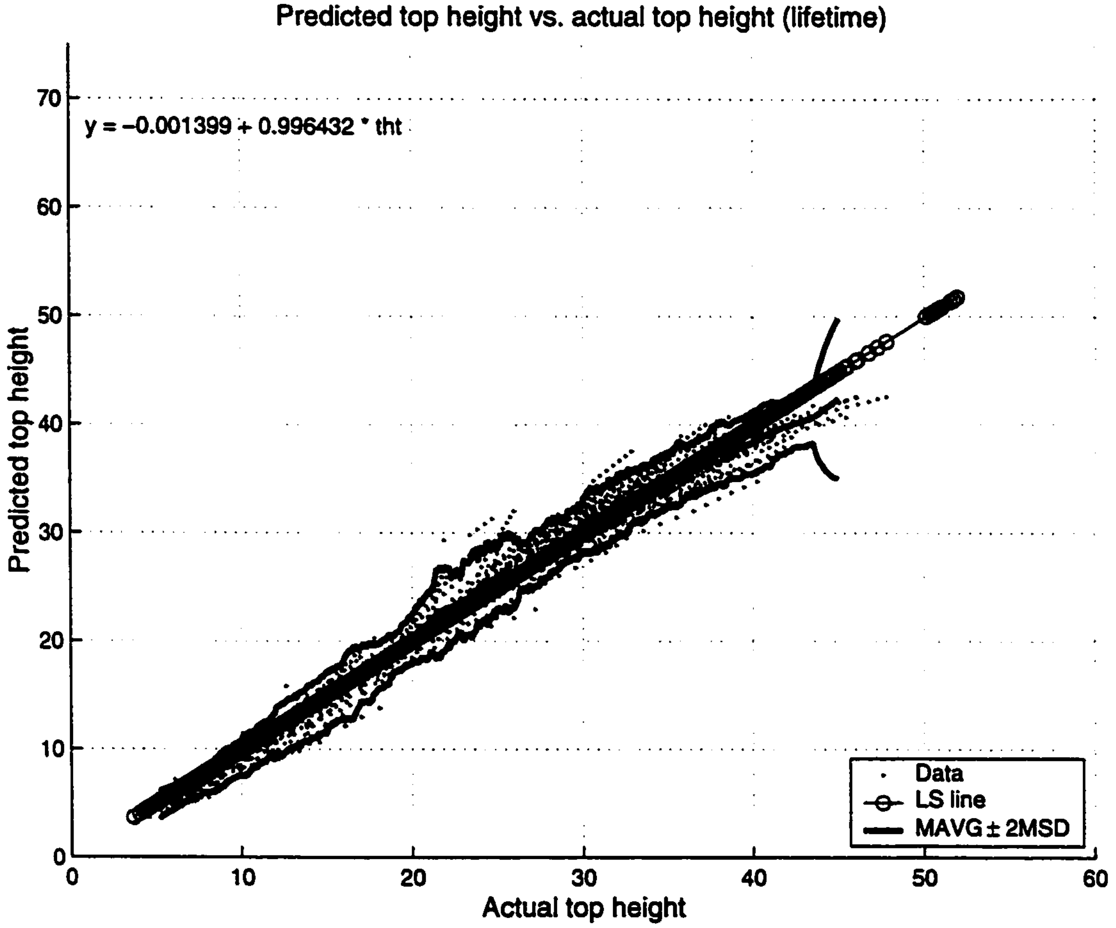

Figure 2.27: Lifetime predicted top height, $\hat{Y}_{3ks}$, *vs.* actual top height, Y_{3ks}. The figure also includes the least squares line and the 51 point moving average ± twice the moving standard deviation. Regression coefficients for the model $y = ax + b$ applied to these data yields the model $y = 0.983479x - 0.001399$ with $r^2 = 0.979312$. See Table 2.15 for the other lifetime linear regression coefficients and r^2 values, and Table 2.16 for the single step coefficients and r^2 values.

Table 2.15: Lifetime predicted *vs.* actual line fits and r^2 values. Regression coefficients are for the model $y = ax+b$ applied to the predicted state variable values, $\hat{Y}_{iks}$, *vs.* the actual state variable values, Y_{iks}. Values of $a = 1$ and $b = 0$ imply that there is strong one-to-one linear relationship and no bias, and $r^2 = 1$ implies a strong correlation between the predicted and actual values. See Table 2.16 for the single step coefficients and r^2 values.

State variable	a	b	r^2
TPH	0.975642	18.682689	0.968202
QMD (cm)	0.983479	0.425565	0.972990
Top height (m)	0.996432	-0.001399	0.979312

stand density, QMD, and top height. The QMD and top height intercept values are exceptionally good, being within one tenth of a unit of zero. The TPH intercept is also quite good, and when converted into measured trees, by dividing by the scale factor 19.5, it is well within one measured tree of zero, and also demonstrates excellent agreement with the measured data. The single step predicted *vs.* actual r^2 values are all in excess of 0.999, indicating exceptionally strong one-to-one linear relationships with almost no variation about the line.

The predicted *vs.* actual value simple regression analyses clearly demonstrate the excellent agreement between the Douglas-fir growth and yield model and the stand measurement data. The single step results demonstrate that the growth and yield model correctly predicts annual growth, and the lifetime results indicate that the model correctly predicts cumulative stand development, or stand trajectories.

2.7.5 Long term simulations

Projections for the stand level Douglas-fir growth and yield model for 150, 300, 1000, and 1375 years are presented in Table 2.17. Values for the growth and yield model were taken as the average values of projections for the 270 available stands, beginning from their initial measurements, and projecting to the desired stand age. The values used for comparison were obtained from four independent sources [93, 92, 102, 106],

Table 2.16: Single step predicted *vs.* actual line fits and r^2 values. Regression coefficients are for the model $y = ax + b$ applied to the predicted state variable values, $\hat{Y}_{ik}$, *vs.* the actual state variable values, Y_{ik}. Values of $a = 1$ and $b = 0$ imply that there is strong one-to-one linear relationship and no bias, and $r^2 = 1$ implies a strong correlation between the predicted and actual values. See Table 2.15 for the lifetime coefficients and r^2 values.

State variable	a	b	r^2
TPH	1.000118	-3.586719	0.999120
QMD (cm)	1.000612	-0.026923	0.999022
Top height (m)	1.002622	-0.109944	0.999328

and represent average values or maximum attained values for the tree sizes at the specified stand ages. For stand ages of 150 and 300 years, the comparison values represent average values for DBH and height, not QMD and top height [92]. Stand density values were generally not obtainable for stands older than 150 years, and the value for 150 years is only approximate due to the application of normal stocking to the determination of stand densities [92].

The table clearly indicates the generally good agreement between the S-system Douglas-fir growth and yield model and the observed data for QMD and top height. As already indicated, the model may slightly underestimate top height, and that may be reflected in the table as well. The long term size projections of the growth and yield model demonstrate that the model does indeed approach asymptotic values for tree diameter and dominant height that are reasonable, and in particular, values that are not too large, the projections being less than observed maximum values. The QMD and top height values predicted by the growth and yield model appear to be in good agreement with typical tree sizes for Douglas-fir in the Pacific Northwest, which generally attain maximum diameters of 250 cm to 275 cm and heights of approximately 85 m [92]. The predicted stand density at 150 years seems reasonable, though probably a bit low [92]. This is consistent with the assessment that the S-system growth and yield model over predicts stand mortality. Stand densities for stand ages

Table 2.17: Long term simulation results. Projections for 150, 300, 1000, and 1375 years for the S-system based Douglas-fir growth and yield model. Values in parentheses were obtained form McArdle and Meyer for ages 150 and 300 [92], Van Pelt for age 1000 [106], and McWirtter for age 1375 [93] as referenced in Oliver and Larson [102], and are provided for comparison. Values indicated by an asterisk (*) are maximum observed values. A question mark, '?', indicates that values were not readily available.

Age	QMD (cm)	Top height (m)	TPH
150	79 (71)	54 (61)	100 (≈150)
300	138 (165)	61.2 (65.7)	21 (?)
1000	345 (520*)	74.6 (119*)	2 (?)
1375	450 (550*)	78 (127*)	1 (?)

greater than 150 years were not available, but the values listed may be low by a factor of about five. There were few data beyond a stand age of 80 years used in the model calibration, so the long term values, particularly for stand density, are only gross estimates.

To recap, the stand level Douglas-fir growth and yield model has been shown to be in general agreement with the available stand measurement data. Model projections generally remained well within the envelope defined by the data. The TPH, QMD, and top height residuals are symmetrically distributed about zero for both the single step and lifetime residual computation scenarios. The single step and lifetime residuals are also generally symmetrically distributed throughout the range of stand ages and the range of top height values. The statistical summaries of the single step and lifetime residuals indicate that the standard deviations are small relative to the average magnitude of the state variables. The simple linear regression and correlation analysis demonstrated that the single step and lifetime residuals are not correlated with stand age, initial stand density, initial QMD, initial top height, and elevation above sea level, but may be mildly correlated with site index. A very strong one-to-one linear relationship between the predicted and actual state variable values was also shown, for both the single step and lifetime model integrations. Long term projections of the growth and yield model also appeared to produce reasonable values when compared with values from independent sources.

The stand level Douglas-fir growth and yield model appears to be generally unbiased, and it appears to be both accurate and precise, producing small residuals. The model also appears to have made very effective use of the data, as indicated by the lack of correlation with any of the variables considered in the correlation analyses, and the greater than expected proportion of residuals near zero. These results indicate that the residuals are effectively random noise, mean zero random errors, and that no further information will be extractable from them. These results present a very strong case for the validity of the stand level Douglas-fir growth and yield model, and demonstrate the power of the S-system modeling framework for dynamic model extraction.

2.8 Discussion

The stand level, dynamic Douglas-fir growth and yield model has been shown to be in very good agreement with the available data, capturing the essential characteristics of stand dynamics in Douglas-fir plantations in the Pacific Northwest, west of the Cascade Mountains. The model has some surprising characteristics when considered within the context of current, or typical, growth and yield modeling research and development. First, the model does *not* contain site index as a fundamental parameter to distinguish site quality, yet it is clearly in strong agreement with the data. Second, although not specifically an objective of developing the dynamic growth and yield model, the model produces appropriate stand level responses to thinning. Third, the relative error in the model when projecting stand development is generally quite small given the simple model representation. Finally, there is a large, and unexpected, apparent discrepancy in the variability between the single step and lifetime model residuals, s_{i_s} and s_{il}, $i = 1, 2, 3$, for stand density, QMD, and top height, respectively.

This apparent discrepancy must be accounted for. Each of these model characteristics will be addressed in turn.

2.8.1 Whither site index?

A great deal of effort was expended in an attempt to include site index in the S-system growth and yield model Equations 2.13 as an exogenous, i.e., nondynamic state variable, due to its perceived biological relevance. None of these attempts succeeded. In these attempts, either the S-system exponents became nearly zero, indicating no site index effect, or a numerical feedback would occur between the S-system exponent for site index and one or more of the other S-system parameters in each equation. In this latter situation, though the parameter estimation process would converge to a solution, the resulting dynamics of the S-system model were not appropriate, diverging wildly from the expected dynamics when the model was integrated. These results were somewhat surprising given the widespread use of site index in growth and yield models, and warrant further discussion with regard to the S-system growth and yield model.

Site index is generally defined to be the average height of the dominant and codominant trees on a particular site at a specific reference age [24, 65, 153], a breast height age of 50 years for Douglas-fir [16, 73]. Site index is also assumed to be constant for a particular location. The top height of a forest stand, i.e., the average diameter of the 100 largest diameter trees per hectare, is considered to be an objective and concise way of determining site index values [16, 153]. Site index is used as a surrogate for site quality, e.g., soil characteristics, nutrient status, and climate, to account for the differences in growth potential for forest stands in different locations [24, 65, 81, 153]. In traditional growth and yield modeling, site index is used as an index, as is stand age, to differentiate among the possible growth or yield curves that are represented by a set of regression equations [16, 24, 73, 92, 153]. The role of site index in this context is to provide a second, fixed, consistent reference point that is used *a priori* to sort the stand measurement data into different classes through a common variable, which helps the least squares regression procedures used to estimate growth and yield model parameters.

Scatter plots of the stand density, QMD, and top height residuals for the single step and lifetime residual computation scenarios are plotted against site index in Figure 2.28 and Figure 2.29, respectively. There appear to be slight trends in these residuals. The majority of the site index values, however, fall in the range of 33 m to 40 m. Within this range, the residuals are generally symmetrically distributed about zero. The simple linear regression analyses in Section 2.7 indicated only weak linear relationships for both the single step and lifetime residuals with respect to site index, with the stronger relationships being a result of the large influence of the underrepresented extremes.

Top height, as a predictor of site index, provided the largest standardized slope values and r^2 values in the residual regression and correlation analyses, see Table 2.9 through Table 2.11 for the single step results and Table 2.12 through Table 2.14 for the lifetime results. The unscaled simple linear regression slope coefficients of $a = 0.0253$ and $b = 0.1478$ were obtained for the single step and lifetime residuals, respectively. These indicate 2.53% and 14.78% change per meter of site index, and over the entire range of site index values amount to approximately 0.737 m and 4.29 m. A site class is approximately 6 m wide, so over four and one half site classes, the single step top height residuals are potentially in error by at most 12.3% and the lifetime residuals are potentially in error by at most 71% of a site class. This latter result is not as bad as it seems, as will become apparent, due to the large influence on the regression line of the underrepresented, lower site index values.

Site index is generally perceived to be a fundamental biological parameter of forest stands, and this is usually its justification for use within growth and yield models or forest simulation models [5, 24, 65, 94, 153]. This research, and other recent research, has demonstrated that it is not necessary to include site index explicitly in a forest growth and yield model [10, 108, 149, 150, 151, 152]. These models, as well as the current dynamic, S-system based Douglas-fir growth and yield model are able, generally, to reproduce the site index values for stands.

Figure 2.30 presents a scatterplot of predicted top height at stand age 55, an approximate breast height (BH) age of 50, and actual or annualized top heights for SMC stands that have surpassed a stand age of 55 years versus the historical site index values associated with each stand by the SMC. The historical site index values are for a BH reference age of 50 years [16, 73], and a 5 year adjustment was made to approximately convert the 50 year BH age to stand total age [16]. The predicted top heights are for the 257 of 270 stands which had initial stand ages less than 55 years, and were computed by integrating the Douglas-fir growth and yield model from the initial stand measurement and age to a stand age of 55 years, for an approximate BH age of 50 years. Actual or annualized top height values for a stand age of 55 years were available for 53 of the 270 Douglas-fir stands. Note the overall agreement between the S-system model predicted top heights at stand age 55 and the actual top heights at stand age 55, and their generally linear, and approximately one-to-one association with the historical site index values.

Of the 53 stands having actual or annualized top height measurements for a stand age of 55 years there were 51 stands which had initial stand ages less than 55 years. For these 51 stands r^2 values were computed to assess the strength of the linear relationship among the historical site index values, the actual or annualized top height values, and the S-system predicted top height values. The r^2 values are 0.8163 for historical site index *vs.* actual top height values, 0.7660 for historical site index *vs.* S-system predicted top height values, and 0.9241 for the actual top height values *vs.* the S-system predicted top height values. All three of these r^2 values indicate strong linear relationships, and it is encouraging that the S-system based Douglas-fir growth and yield model has a stronger agreement with reality, the actual measurements, than it does with the historical site index values.

The largest disagreements between the S-system predicted and historical site index values appears for site index values between 17 m and 30 m, a subset of the data that has already been identified as small. Further, these large site index differences, which appear in the upper left of Figure 2.30 near 35 m, are for stands in the rain shadow of the Olympic Mountains, or at the highest elevations in the data, greater than 800 m. Neither of these effects were accounted for explicitly in the model, and some very localized, large deviations are to be expected when considering such a large data set and region. Given this, the agreement between the predicted top heights and historical site index values is quite strong, as is the agreement between the predicted top heights and the actual top heights.

Figure 2.31 presents nonparametric probability density estimates for the predicted top height values at age 50, actual top height values at age 50, and the historical site index values [55, 56, 115, 114, 132, 146]. The three density estimates overlap significantly, particularly those for the S-system predicted top height values at a stand age of 55, breast height age of 50, and the historical site index values. They differ only for the lower site index values, as already stated. The density estimate for stands having actual or annualized top height values for a stand age of 55 appears to be shifted to the left, relative to the other two density estimates. This is due to the fact that actual top height values are available only for stands with lower historical site index values, and hence lower expected top heights. Mean values and standard deviations (in parentheses) are 34.8 m (5.5 m), 37.7 m (3.7 m), and 33.2 m (5.6 m), for the historical site index values, the S-system predicted top height values, and the actual top height values, respectively. The three mean site index values are all clearly similar, and are representative of the same site class.

The S-system growth and yield model appears to be capable of reproducing site index *without* explicitly including site index in the model. This result is quite surprising at first, but upon reflection it becomes clear that the model must be capable of this feat. The reason is a straightforward consequence of using an autonomous system to model the stand dynamics. The S-system model, as an autonomous system, captures the essence of the three dimensional phase manifold defined by stand density, QMD, and top height, and the simultaneous rates of change for these three state variables. Therefore, the model *implicitly* includes site index as a component of the phase manifold. In a sense, the phase manifold is *indexed* simultaneously by stand density, QMD, and top height to select the appropriate simultaneous rates of change for these state variables. Hence, the selection of an initial stand condition defines the "site curve" that the top height component of the stand trajectory will take.

This relationship becomes clear with a demonstration of how site index curves are generally used. Given the age and current top height of a stand, the site index is determined by finding the site index curve that passes through that age and top height, projected out to the reference age. A stand at the same age and having a larger top height would yield a higher site index value. Similarly, a smaller top height would yield a smaller site index value. The key concept is that from a given age and top height, the top height of a stand is *projected* into the future or the past to obtain the site index value. Thus, site index curves provide a family of curves indexed by age and top height that are used as an essentially dynamic model of top height development over time.

2.8.2 Thinning response

Most managed stand growth and yield models have an independent set of equations that are used to modify the initially predicted growth based upon treatments that may have been applied to the stand, e.g., thinning or fertilization [5, 36, 58, 94, 153]. The S-system based Douglas-fir growth and yield model does not explicitly include fertilization or thinning effects on stand dynamics. Changes in growth rates due to fertilization effects must be added to the model, if desired, since they affect the soil characteristics and hence the growth directly, and may be represented as modifications

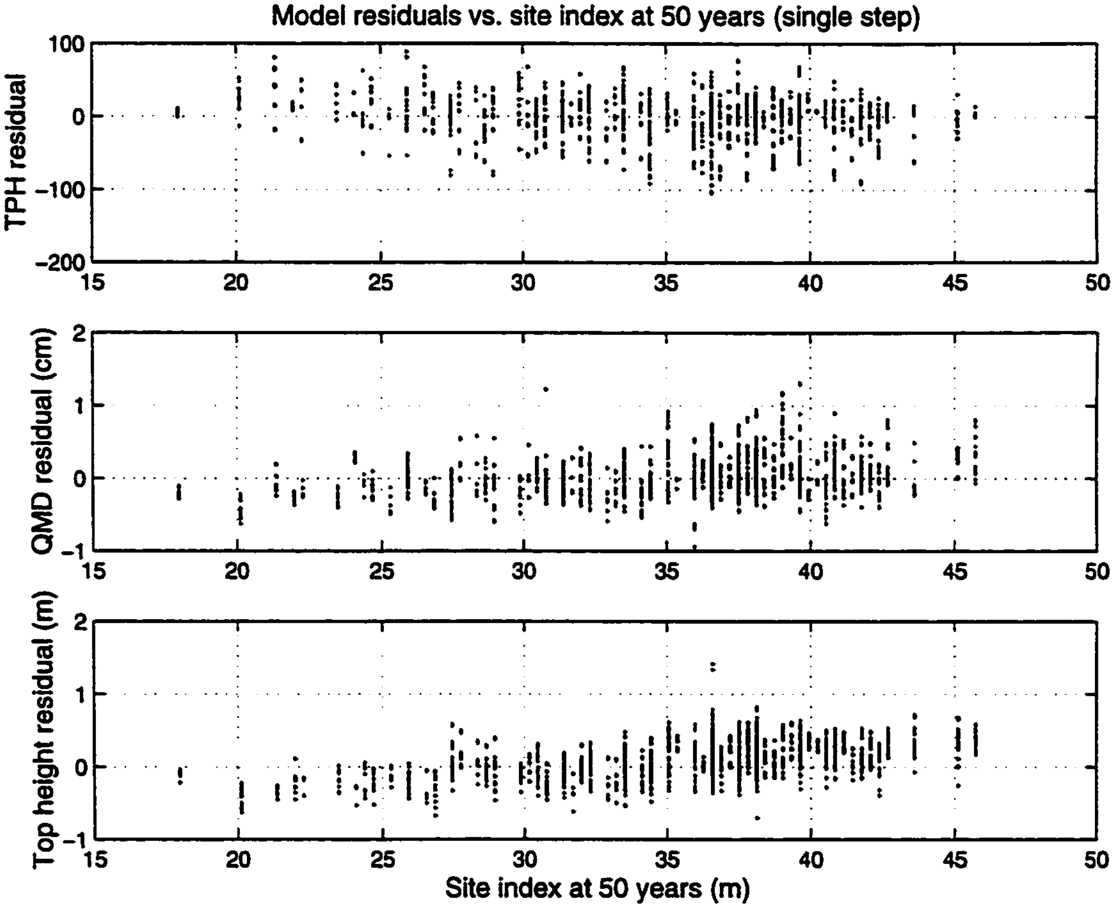

Figure 2.28: Single step residuals, R^s_{ik}, *vs.* site index. The TPH residuals, $i = 1$, are on top. QMD residuals, $i = 2$, are in the middle, and top height residuals, $i = 3$, are on the bottom.

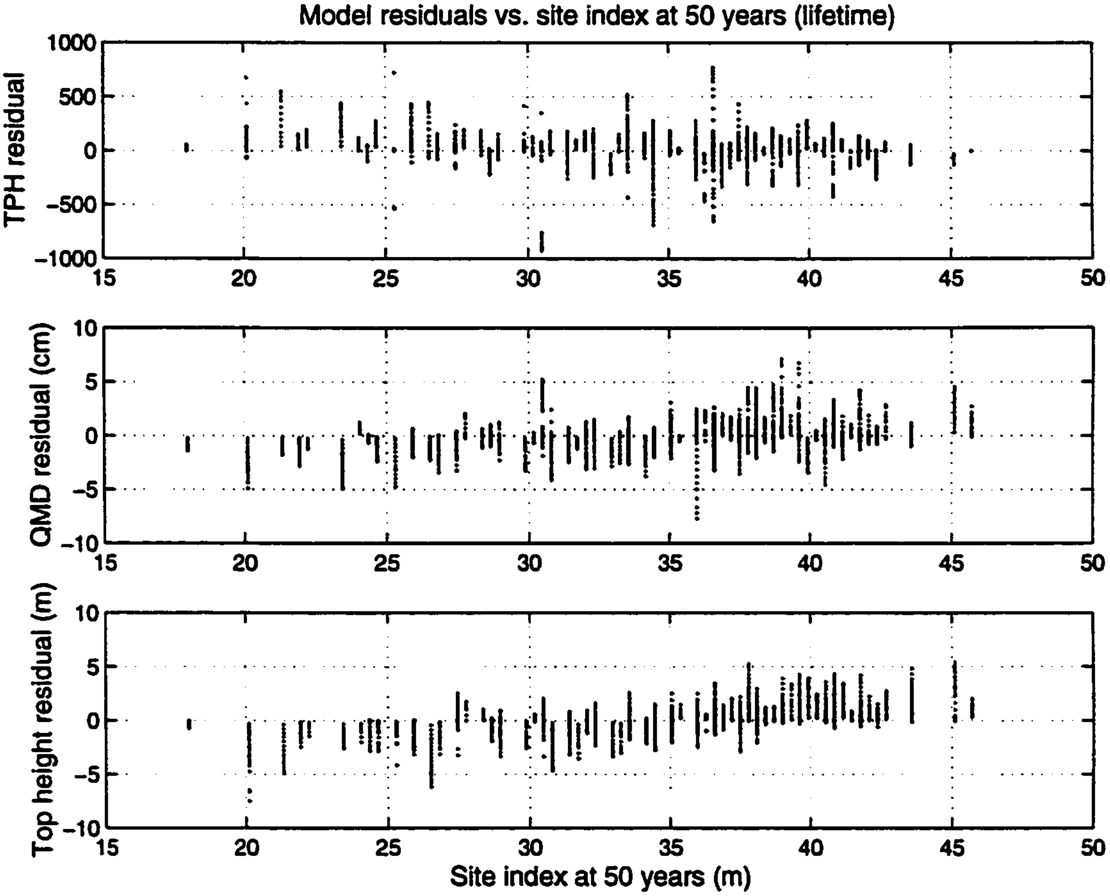

Figure 2.29: Lifetime residuals, R^l_{iks}, *vs.* site index. The TPH residuals, $i = 1$, are on top. QMD residuals, $i = 2$, are in the middle, and top height residuals, $i = 3$, are on the bottom.

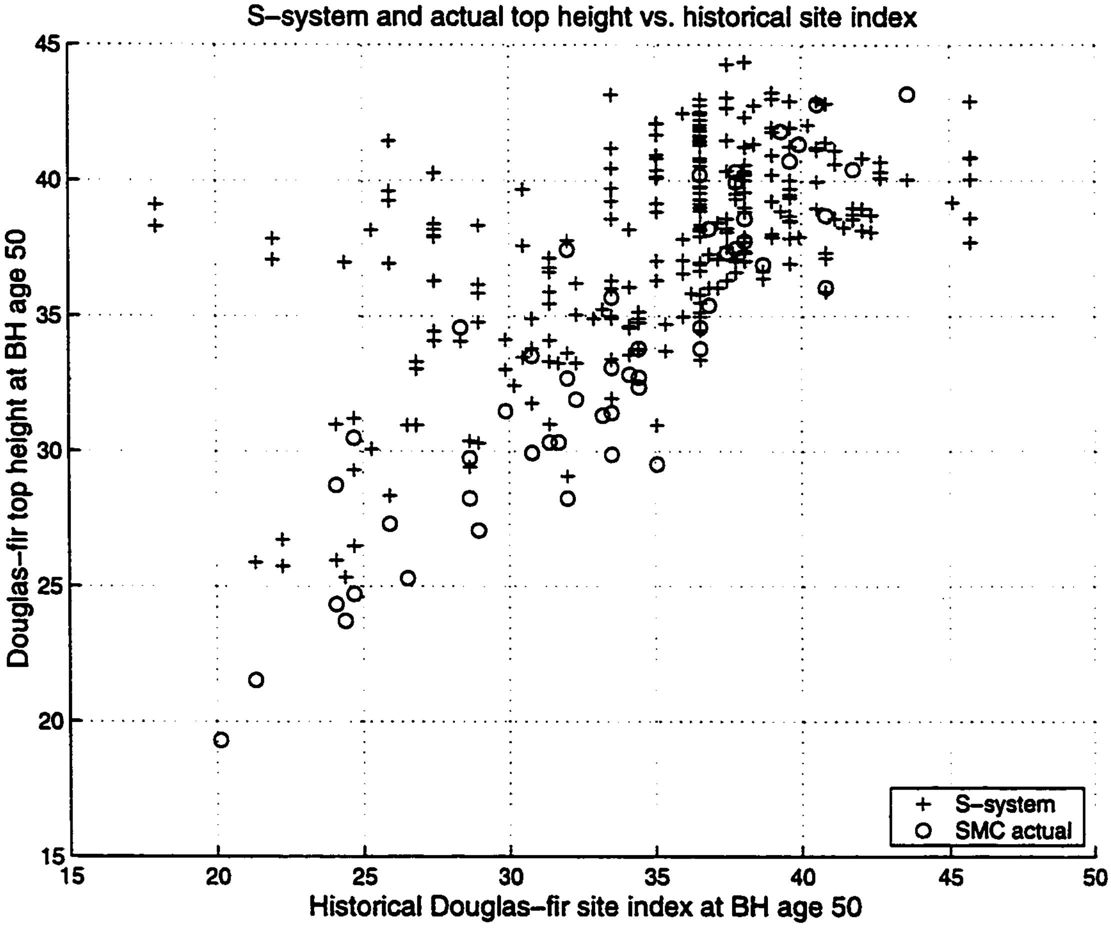

Figure 2.30: Predicted and actual top height values *vs.* historical site index for the 270 SMC plots. Note the strong agreement between the S-system predicted top height values and the actual top height values.

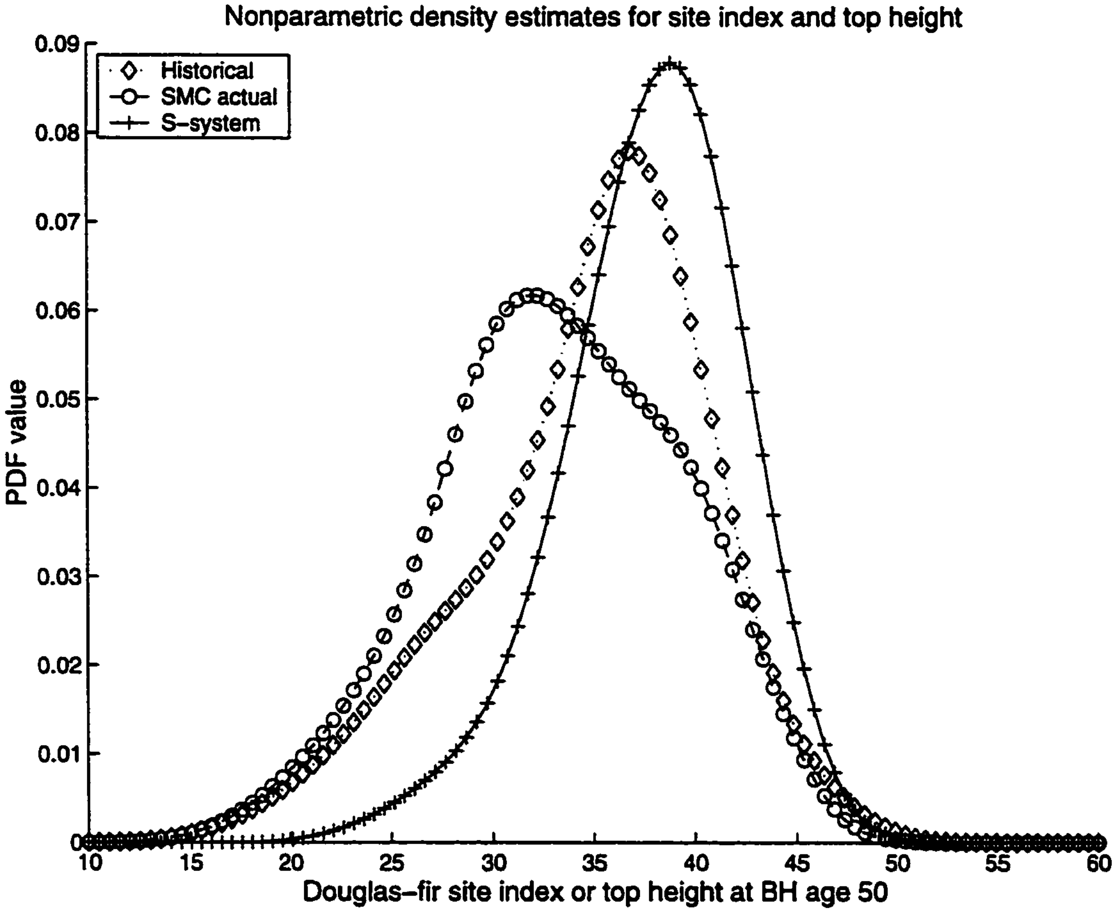

Figure 2.31: Nonparametric probability density estimates for the S-system predicted top height values, the actual top height values, and the historical site index values. There is a generally strong agreement among these three distributions, particularly the predicted top heights and the historical site index values.

of site index. Changes in growth rates due to thinning effects, however, do not appear to need explicit inclusion in the S-system model. The S-system based Douglas-fir growth and yield model appears to automatically account for changes in growth rates due to thinning. Again, as for site index, this is a result of the fact that the S-system model represents simultaneously the phase manifold for stand density, QMD, and top height.

Figure 2.32 through Figure 2.34 present model predictions for three thinned plots in a single Douglas-fir stand from the SMC database. The three plots had widely varying initial stand densities and thinning levels. The thinnings on each plot were triggered by reaching specified relative density values [28]. The plots were then thinned to lower relative density values, with the relative density changes being indicative of heavy, moderate, or light thinnings. Relative density is used to provide an index of the degree of competition for a stand [28]. Thinnings occurred at stand ages of 19 years, for plot 3 and plot 12, 21 years for plot 2, and 28 years for a second thinning on plot 3. In Figure 2.32, there is good general agreement between the stand density model predictions and the data. After each thinning the slope of the stand density trajectories becomes smaller, indicating that mortality will generally be lower after a thinning. In Figure 2.33, there is also good agreement between the QMD model predictions and the data. After each thinning the slope of the QMD trajectories increases, indicating that QMD growth is initially greater after a thinning. Finally, in Figure 2.34, there is good agreement between the top height model predictions and the data. There is no visible change in the top height trajectories after a thinning, but the model does generally track the data.

2.8.3 Management considerations

From a management perspective, growth and yield models are used to help make economic decisions about harvesting schedules, investments, and economic returns [24]. Land managers using growth and yield models are typically not experts in

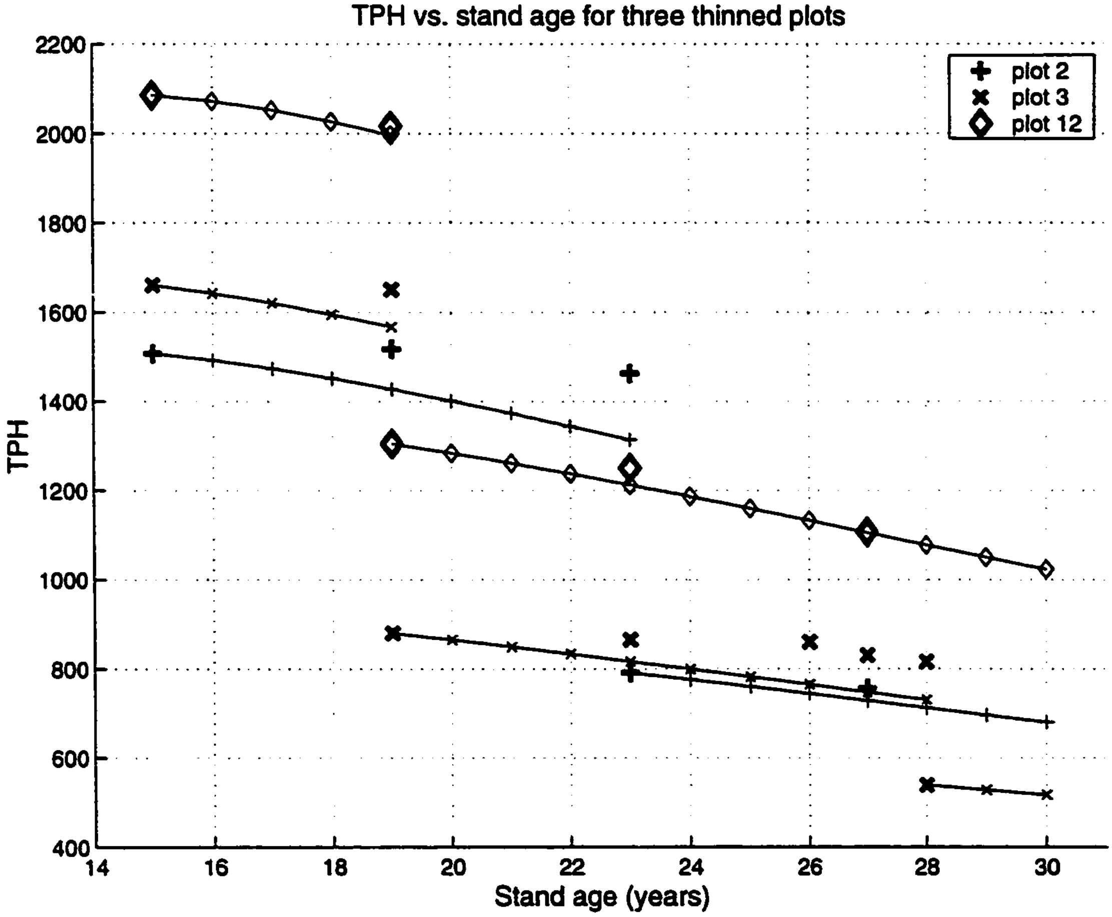

Figure 2.32: Predicted TPH response to thinning for three thinned plots in a stand. Large symbols represent the measurement data. Small symbols with lines represent the S-system growth and yield model.

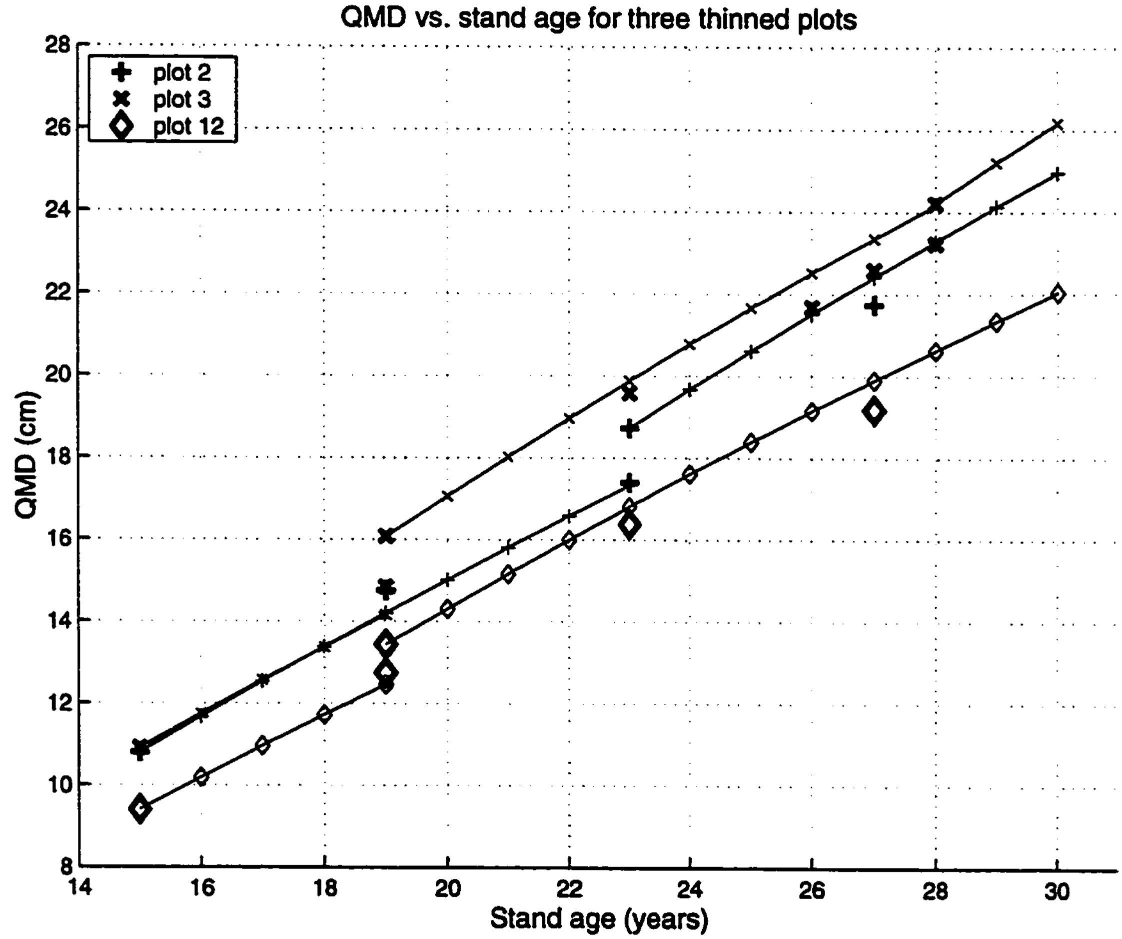

Figure 2.33: Predicted QMD response to thinning for three thinned plots in a stand. Large symbols represent the measurement data. Small symbols with lines represent the S-system growth and yield model.

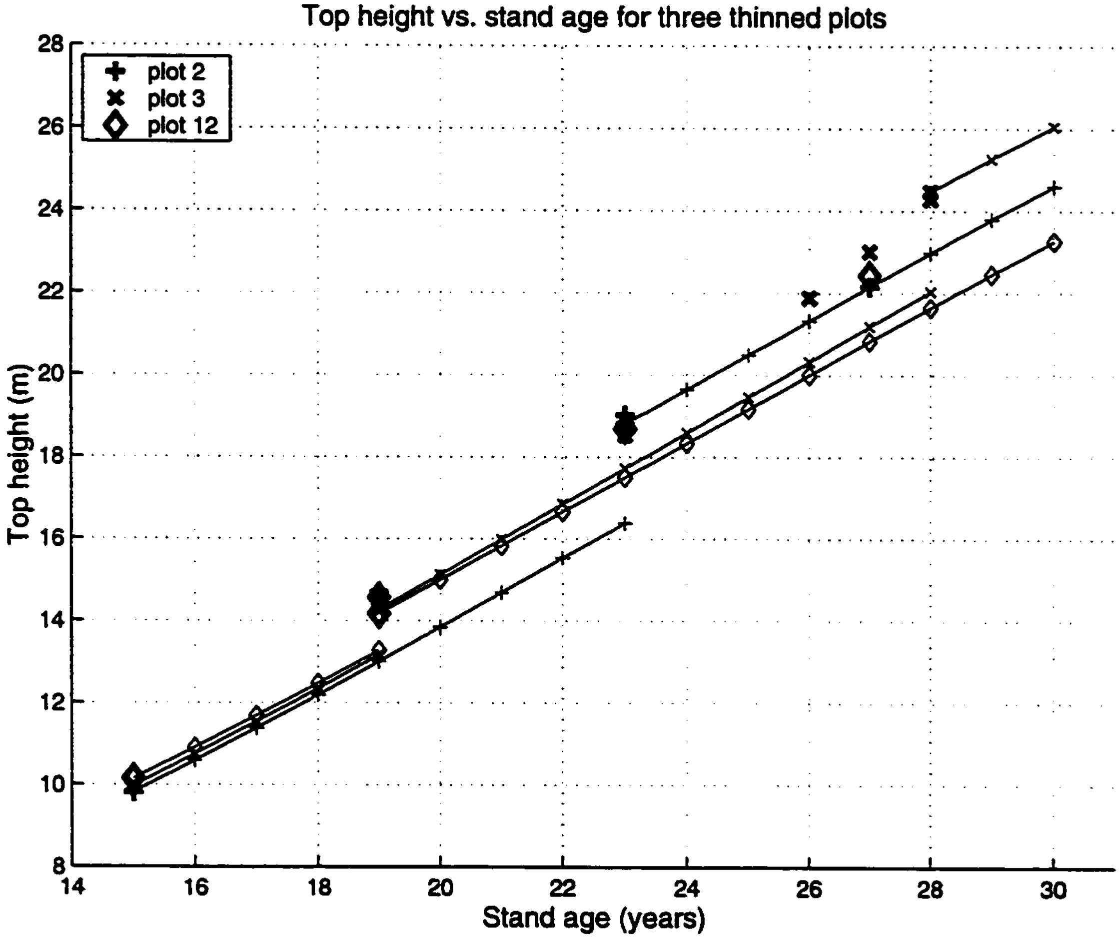

Figure 2.34: Predicted top height response to thinning for three thinned plots in a stand. Large symbols represent the measurement data. Small symbols with lines represent the S-system growth and yield model.

growth and yield modeling, and may not be aware of the variety of assumptions, traps, or pitfalls contained within a particular growth and yield model, nor should they necessarily need to be aware of them [139, 153]. Growth and yield models should produce results to the degree of accuracy and precision required by forest land managers, generally to within an error of plus or minus 5% to 10% in each state variable [139]. Currently, the error of growth and yield models is on the order of plus or minus 30% [139]. An indication of the proportion of prediction errors that should be within the desired ranges is not mentioned. It is the responsibility of the model developer to guarantee that the model meets the needs and expectation of those using it, to ensure correct use, and current models may not be meeting the expectations of the forest managers. For the sake of the following discussion, assume that an acceptable model is one having at least 80% of its prediction errors within plus or minus 10%, giving at most a 20% overall error rate. This proportion is consistent with the Douglas-fir data set used to calibrate the growth and yield model: approximately 20% of the stand measurement data could not be considered to be pure Douglas-fir by stems per hectare. A proportion of 90% to 95% would be expected for a model calibrated with a less noisy data set.

The performance of the Douglas-fir growth and yield model is evaluated from a management perspective having a maximum relative error tolerance of at most 10% for the state variables, with maximum error rate of 20%. The single step, measurement oriented, and lifetime, trajectory oriented, residuals from the S-system based Douglas-fir growth and yield model were converted into relative errors in stand density, QMD, and top height. The proportion of the individual relative errors falling within the ranges $0 \pm p$, for relative error bounds $p = 0.20, 0.15, 0.10$, and 0.05 were computed. These relative error bounds represent errors of 20% to 5%. The proportions were computed at two levels for the single step and lifetime residuals. The first level is that of the individual measurement, giving a short term assessment of the relative errors. The second level is that of the stand, computed as the average relative error for each stand trajectory, giving a long term assessment of the relative errors. A fourth proportion was also computed for each level: the proportion of measurements or stands which simultaneously had their stand density, QMD, and top height relative residuals within the ranges $0 \pm p$. This proportion provides a simultaneous estimate of the overall quality of the S-system Douglas-fir growth and yield model from this particular management perspective.

Table 2.18 and Table 2.19 present the single step, or annual growth prediction, relative error results for the measurement and stand average proportions for stand density, QMD, top height, and the three state variables considered simultaneously. Clearly, the single step S-system based growth and yield model meets the 10% error criterion for annual time steps. At least 98% of the measurement based relative errors, taken individually or simultaneously, are within the management error tolerance, and at least 95% of the stand average relative errors, again individually or simultaneously, are within the management error tolerance. For annual predictions, the stand level Douglas-fir growth and yield model appears to meet the expectations of forest land managers.

Table 2.20 and Table 2.21 present the lifetime, or stand trajectory prediction, relative error results for the measurement and stand average proportions for stand density, QMD, top height, and all three state variables considered simultaneously. Again, the S-system based growth and yield model meets the 10% error criterion at the measurement level for the stand trajectories. At least 56% of the measurement based relative errors, taken individually or simultaneously, are within the management error tolerance, with the at least 71% of the individual relative errors being within the tolerance. At least 62% of the stand average relative errors, again individually or simultaneously, are within the management error tolerance, with at least 80% of the individual relative errors being within the tolerance. These percentages increase to 78% and 83% if the management error tolerance is increased to a 15% relative error.

Given that the stand density values, and hence their residuals, have the greatest

Table 2.18: Single step relative error summary by measurement. The values are the proportion of the single step residual relative errors that are within the range $0 \pm p$ for TPH, QMD, or top height. The final column represents the proportion of the relative errors which meet the tolerances p simultaneously.

p	TPH	QMD	Top height	All
0.2000	1.0000	0.9992	1.0000	0.9992
0.1500	1.0000	0.9988	1.0000	0.9988
0.1000	0.9992	0.9918	0.9947	0.9877
0.0500	0.9823	0.9527	0.9494	0.9165

uncertainty, consider only the two size variables QMD and top height. If only QMD and top height relative errors are considered simultaneously, then at least 77% of the measurement based relative errors are within the 10% tolerance. If only QMD and top height relative errors are considered simultaneously, then at least 72% of the stand average relative errors are within the 10% tolerance. The stand trajectory predictions of the Douglas-fir growth and yield model also appear meet the expectations of forest land managers, performing better for the average tree size predictions than for stand density and size combined because of the greater measurement uncertainty in the stand density values.

2.8.4 Interpreting the lifetime - single step standard deviation ratios

The stand density, QMD, and top height residual standard deviations for the lifetime, s_{il}, and single step, s_{is}, residual computation scenarios, $i = 1, 2, 3$, respectively, have been identified as having markedly different values for each of the state variables. In fact. the ratios of the standard deviations s_{il}/s_{is} had values of 6.01, 5.32, and 5.54 for stand density, QMD, and top height, respectively. All three of these values, when squared to obtain F-statistics, are statistically significantly different for $\alpha = 0.05$.

These large relative differences in variability cannot be wholly explained by the accumulated numerical integration error from the longer simulation lengths of the lifetime residual computation scenario relative to the one year simulation lengths

Table 2.19: Single step relative error summary by stand. The values are the average proportion of the single step residual relative errors for a stand that are within the range $0 \pm p$ for TPH, QMD, or top height. The final column represents the proportion of the relative errors which meet the tolerances p simultaneously.

p	TPH	QMD	Top height	All
0.2000	1.0000	1.0000	1.0000	1.0000
0.1500	1.0000	1.0000	1.0000	1.0000
0.1000	1.0000	0.9741	0.9815	0.9556
0.0500	0.9963	0.8556	0.8259	0.7963

Table 2.20: Lifetime relative error summary by measurement. The values are the proportion of the lifetime residual relative errors that are within the range $0 \pm p$ for TPH, QMD, or top height. The final column represents the proportion of the relative errors which meet the tolerances p simultaneously.

p	TPH	QMD	Top height	All
0.2000	0.9272	0.9712	0.9815	0.8918
0.1500	0.8585	0.9288	0.9490	0.7824
0.1000	0.7158	0.8178	0.8782	0.5652
0.0500	0.4912	0.6055	0.6499	0.2583

Table 2.21: Lifetime relative error summary by stand. The values are the average proportion of the lifetime residual relative errors for a stand that are within the range $0 \pm p$ for TPH, QMD, or top height. The final column represents the proportion of the relative errors which meet the tolerances p simultaneously.

p	TPH	QMD	Top height	All
0.2000	0.9778	0.9741	0.9852	0.9481
0.1500	0.9556	0.9259	0.9333	0.8370
0.1000	0.8704	0.8000	0.8074	0.6222
0.0500	0.6185	0.5370	0.5444	0.1815

of the single step residual computation scenario. Residuals for both scenarios were computed from model projections with a base time step of 0.1 years, and there were no appreciable differences in the variability of the results from the annual time step. Further, the greater range in the lifetime residuals was not seen to contribute to the large ratios: the ratios of standard deviations did not change appreciably when the most extreme values were trimmed from each set of residuals before performing the ratio computations. This trimming of the residuals compares the variation for data where the model performed well for both the single step and lifetime integrations. Some of the differences in variation between the lifetime and single step residuals is undoubtedly due to lack of model fit for some of the data, but given the overall agreement of the model with the data lack of fit alone cannot completely account for the differences.

Given that the same data and model were used in each residual computation scenario, that the numerical integration error is not a factor, and that lack of model fit can only explain part of the differences, how can the differences in variation between the two residual computation scenarios be explained? Any explanation for this apparent discrepancy must be consistent with the biology, the mathematical representation of the model, and the stand measurement data.

To help gain an insight into the apparent discrepancy between the lifetime and single step residual variation, consider a dynamic growth and yield model that incorporates both site and climate effects. In this situation, what value would be expected for the standard deviation ratios s_{il}/s_{i_s}? The only unmodeled effects are related to the natural variation of Douglas-fir plantations, which may be composed of microsite variation and genetic variation among the stands or trees, or other natural variation. Thus, the ratios of standard deviations for the lifetime and single step scenarios should all have a nominal value of one, $s_{il}/s_{i_s} = 1$; the variation in each case should only be the natural variation. The actual ratios of standard deviations have values greater than five, so there must be a site or climate effect that is not taken into account by the Douglas-fir growth and yield model, and which becomes apparent through a comparison of the lifetime or stand trajectory integration residuals and the single step integration residuals.

The possible existence of an unmodeled site or climate effect is consistent with the biology, the mathematical representation, and the data. The stand level Douglas-fir growth and yield model represents only the dynamics of stand density, QMD, and top height over time, and, hence, does not *a priori* contain effects of differences in site characteristics or climate. Both site quality and climate are known to affect tree growth. The interpretation of site and climate effects on stand development as initiating, and possibly maintaining, deviations from a nominal stand trajectory is consistent with the biology and mathematical representation. Consistency with the data is obtained by recognizing that the stand measurement data have integrated the site and climate effects over the remeasurement intervals for each stand trajectory. The single step integrations, beginning at the initial condition for each annualized measurement interval, and projecting one year to the final measurement account, approximately, for the site or climate effects through the changes in the initial conditions for each measurement interval within a stand trajectory. Thus, the single step residuals take into account the site or climate effects as integrated into the stand measurement data for each measurement interval, through the changing initial conditions, whereas the lifetime residuals accumulate differences using only the initial stand condition for a stand trajectory and the average climate as represented by the autonomous S-system model.

Which effect site or climate is the most likely contributor to the increase in variation for the lifetime integrations? A site effect may be ruled out for two reasons. First, the correlation between the residuals, lifetime or single step, and the site index values for each stand are low. The lack of correlation is due in part to the overall uniformity of the Douglas-fir sites, as indicated by site index values predominantly in the range 33 m to 40 m, essentially a single site class. Second, for a particular location, the site based effects are assumed to be constant over time, site index does not change with time, or to vary with climate changes.

Therefore, it seems reasonable to assume that the lifetime residuals cannot account for climate influences, except possibly for the first measurement interval, since they are dependent upon *only* the initial stand condition and the autonomous S-system model. Deviations from the projected stand trajectories due to climate for the lifetime integrations will generally be larger than deviations for the single step integrations, which include changes in climate through the changing initial stand conditions. This appears to be the situation at hand, and leads to an investigation of whether the addition of climate to the dynamic, stand level Douglas-fir growth and yield model will improve its behavior when integrating stand trajectories.

In closing, the dynamic, stand level Douglas-fir growth and yield model appears to work quite well. The use of an interaction based approach to define the model proved to be straightforward, economical, and parsimonious, in terms of necessary assumptions and the number of parameters. The S-system modeling framework, and dynamic model extraction using a least squares criterion also proved to be quite effective, producing a growth and yield model that agreed well with the stand measurement data used to calibrate the model. The model correctly reproduced the observed size density relationships, as well as representing the relative differences between diameter growth and height growth for low density and high density stands, an emergent feature of the model that was not constrained to occur *a priori*. Finally, addition, differences in the variability of the residuals computed using whole stand trajectories and separate measurement intervals may indicate the existence of an unmodeled climate effect.

Chapter 3

A DYNAMIC, STAND LEVEL GROWTH AND YIELD MODEL WITH CLIMATE FOR PLANTATION DOUGLAS-FIR IN THE PACIFIC NORTHWEST

Climate is known to affect the processes of stand development and tree growth [77, 78, 107, 129, 161]. The typical forest growth and yield models developed within a statistical-empirical modeling framework do not generally include climate effects [24, 36, 58, 65, 94, 153]. Interest in extending the statistical-empirical models to include climate effects has been growing recently [105, 164, 170]. These models, being essentially descriptive models, cannot be used to obtain cause-effect relationships between climate and stand development and tree growth. The S-system modeling framework and dynamic model extraction with a least squares criterion are used to extend the dynamic, stand level Douglas-fir growth and yield model of Chapter 2 to include climate effects. Climate is included in the growth and yield model using a variable based on annual average Palmer Drought Severity Index (PDSI) values [104]. The extension of the base growth and yield model to include climate is done in a nested manner, so that the model without climate effects may be recovered by using a nominal climate value of one. This automatically produces compatible models with and without climate effects. The resulting S-system model with climate was, unfortunately, indistinguishable from the base model. A variety of factors which may have contributed to this result are discussed. A significant possibility is that the base model already includes average climate effects.

3.1 Introduction

Climate is known to affect the regional distribution of vegetation [100, 168], and the processes of stand development and tree growth [77, 78, 102, 107, 129]. The variation of temperature and precipitation patterns throughout the year and across regions also influences the growth characteristics of vegetation [77, 78, 161]. One of the best known examples of the influences of climate on vegetation and forest development is the change in forest type and growth rates moving from the equator toward the north or south poles [13, 12, 43, 130, 168]. More localized climate effects on vegetation and forest development are also well known [77, 102, 107, 161].

The need for more realistic forest growth and yield models necessitates the inclusion of the effects of climate on forest stand development and tree growth. Recently several process based forest models that include the effects of climate have appeared [10, 113, 151, 152, 161]. These models typically incorporate climate effects on the rate of photosynthesis via a photosynthesis submodel, which subsequently modifies growth rates or mortality. As early as 1972, climate effects were included in the forest succession model JABOWA as direct multiplicative modifiers of tree growth [13, 12]. The JABOWA forest succession model was the progenitor for a wide variety of forest gap models, all of which include climate effects, though the specific methods of incorporating climate vary [12, 130].

The typical, statistical-empirical forest growth and yield models however, do not include climate effects [5, 24, 36, 58, 65, 94, 153]. These models generally assume that the effects of climate average out over time, a reasonable assumption for long enough time periods. With a decrease in rotation lengths, climate effects on growth will not necessarily average out, and may have a significant impact on wood volume and value. For these reasons, and others, interest in extending the statistical-empirical growth and yield models to include climate effects has been growing recently [105, 164, 170]. Extending these models to include climate effects, however, is somewhat problematic;

being correlational models, they cannot be used to obtain cause-effect relationships between climate and stand development and tree growth [175].

The S-system modeling framework and an autonomous system approach to stand level growth and yield modeling have been shown to be appropriate and to work well. With this in mind, the S-system modeling framework and dynamic model extraction with a least squares criterion are used to extend the dynamic, stand level Douglas-fir growth and yield model of Chapter 2 to include climate effects. The growth and yield model without climate will be referred to as the base model, and the model which includes climate will be referred to the extended model or the climate model. The extension of the base growth and yield model to include climate is done in a nested manner [32, 78, 103, 164, 170], so that the base model may be recovered by using an average or nominal climate value of one. This automatically produces compatible models with and without climate effects.

Climate was included in the dynamic, stand level Douglas-fir growth and yield model using a variable based on annual average Palmer Drought Severity Index (PDSI) values [104]. Annual average PDSI values were chosen as the basis for the climate variable because PDSI combines both temperature and precipitation into a standardized climate variable, and PDSI values are readily available from the National Climatic Data Center (NCDC) [104, 22]. The PDSI values are a measure of meteorological drought, and provide an indication of the severity of dry or wet spells, based on the temperature, precipitation and local available water content of the soil [22, 99, 62, 104]. The PDSI values are generated by a model based on a simplified representation of evapotranspiration [104], which though imperfect [1, 38], produces reasonable results [99, 62].

3.2 Extending the growth and yield model to include climate

The dynamic, stand level growth and yield model for Douglas-fir defined in Section 2.4, the base model, is extended to include climate effects. Climate effects are added to the base model using an exogenous or forcing S-system state variable, that is, a state variable not represented by one of the differential equations in an S-system. The climate variable is derived from the Palmer Drought Severity Index (PDSI) [104]. This approach is consistent with the idea that at the stand level, climate affects the development of the stand and the growth of the trees, but the stand does not affect climate. Local, within stand climate variation is not considered directly, but is included indirectly through its integrated effect on tree size and mortality, as represented by the stand measurement data. The external climate is seen as modulating or modifying the within stand climate, which then affects stand development and tree growth [77, 78, 161, 168, 151]. Although technically not correct, the transpiration and respiration of forests do influence climate, but generally on a much larger spatial scale than the forest stand [77, 78, 102, 129, 161, 168]. A regional interpretation of climate-forest-landform interaction, rather than stand level interaction, is also consistent with the regional climate zone divisions of the National Climatic Data Center [22], and some recent research [100, 164, 168, 170].

Climate effects are generally added to growth and yield or forest simulation models via a multiplicative effect on the rates of change predicted by a base model [13, 12, 130, 164, 170]. More sophisticated, process based forest simulation models that directly include climate also apply the climate effects in a multiplicative manner, affecting the photosynthesis rates, which in turn affect growth rates and mortality [10, 77, 78, 150, 151, 152, 161, 168]. This interpretation is consistent with a common sense view of forest development and tree growth: there are average expected rates of change for a particular locale which are modified by climate, rates of change improving for beneficial climate changes, and rates of change worsening for detrimental climate changes. The multiplicative approach to adding climate effects is used here to extend the base S-system growth and yield model from Chapter 2 to include climate effects.

The multiplicative approach of adding climate to a base growth and yield model implies that climate effects are represented by a variable having a mean value of one, representing nominal or average climate. The variability of this climate variable is then presumed to be representative of the changes in climate, or the effects of climate on the rates of change, which are assumed to be proportional; a similar approach is used in dendrochronology [50]. This is a highly desirable feature from a modeling point of view, since it easily permits the parsimonious extension of existing models to include new information without the necessity of starting from scratch.

The S-system representation for a dynamic growth and yield model naturally accommodates the inclusion of multiplicative climate effects, since it is based on products of power-law functions. Given the directly proportional nature of a multiplicative climate modifier on the rates of change, and the form of an S-system equation which has both a growth and a decay term, four approaches for incorporating the climate multiplier present themselves. In each approach, an exponent for the climate multiplier in an equation is presumed to allow for differences in the effect of climate for each term or equation.

The first approach uses the multiplier to adjust the predicted rates of change for stand density, QMD, and top height, $\dot{X}_i$, $i = 1, 2, 3$, respectively. This approach adds three new parameters, and applies the same climate effect simultaneously to the growth and decay terms of the S-system growth and yield model. The three climate multiplier exponents for this approach must all be greater than or equal to zero, due to the direct proportionality assumption. The second approach uses the climate multiplier separately for each term of the stand density, QMD, and top height rate equations, F_i^+ and F_i^-, $i = 1, 2, 3$, respectively. This approach adds six new parameters, but allows a separate climate modification for each of the growth and decay rates. The three climate multiplier exponents for the growth terms, F_i^+, must all be

Table 3.1: S-system state variable descriptions for the stand level, dynamic Douglas-fir growth and yield model with climate as defined by Equations 3.1.

State variable	State variable description
X_1	Stand density (TPH)
X_2	Quadratic mean diameter, QMD (cm)
X_3	Top height (m)
X_4	Scaled Palmer drought severity index (PDSI)

greater than or equal to zero, due to the direct proportionality assumption, and the three climate multiplier exponents for the decay terms, F_i^-, must all be less than or equal to zero, for the same reason. The third approach uses the climate multiplier to adjust only the decay terms of the stand density, QMD, and top height rate equations, F_i^-, $i = 1, 2, 3$, respectively. The three climate multiplier exponents for this approach must all have values less than or equal to zero, due to the direct proportionality assumption. The fourth approach uses the climate multiplier for only the growth terms of the stand density, QMD, and top height rate equations, F_i^+, $i = 1, 2, 3$, respectively. This approach adds three new parameters, and allows climate to modify only the growth terms. The three climate multiplier exponents for this approach must all have values greater than or equal to zero, due to the direct proportionality assumption. The last approach is the one taken, since it adds only three new parameters, and has a plausible biological justification for each equation.

The S-system representation for the stand level Douglas-fir growth and yield model with climate effects is presented as Equations 3.1, with state variable definitions provided in Table 3.1. The state variables in each term of the S-system model are arranged in columns to permit a straightforward visual identification of those variables which contribute to a specific term in the S-system model.

$$
\begin{array}{lclllllllll}
\dot{X}_1 & = & \hat{\alpha}_1 & X_1^{\hat{g}_{11}} & & & X_4^{g_{14}} & - & \hat{\beta}_1 & X_1^{\hat{h}_{11}} & X_2^{\hat{h}_{12}} & \\
\dot{X}_2 & = & \hat{\alpha}_2 & X_1^{\hat{g}_{21}} & X_2^{\hat{g}_{22}} & & X_4^{g_{24}} & - & \hat{\beta}_2 & & X_2 & \\
\dot{X}_3 & = & \hat{\alpha}_3 & & X_2^{\hat{g}_{32}} & X_3^{\hat{g}_{33}} & X_4^{g_{34}} & - & \hat{\beta}_3 & & & X_3
\end{array}
\tag{3.1}
$$

The climate multiplier, X_4, is applied only to the growth terms. A brief biological justification for this formulation of climate effect follows for each S-system equation.

Climate effects on stand density may best be understood in terms of inertia. For a single cohort, stand density will tend to remain constant unless acted upon by an external stressor, such as climate or nutrient availability, or an internal stressor such as a size-density relationship [37, 66, 102, 158]. Beneficial climate will tend to increase the stand density inertia, decreasing the mortality rate, and detrimental climate will tend to decrease the stand density inertia, increasing the mortality rate. Thus the inclusion of the climate multiplier in the growth term of the stand density rate equation: it acts to modify the stand density inertia by increasing or decreasing the ability of the positive influence of the current stand density to offset the size-density interaction in the decay term of the equation. So, climate affects the overall mortality rate by exerting its influence to maintain the current stand density despite a size-density relationship.

Climate is generally believed to affect the QMD and top height growth rates of trees by modifying the rates of production and allocation of photosynthates to new growth and tree maintenance [10, 77, 78, 102, 151]. Within the context of the S-system growth and yield model, photosynthate allocation for new growth is controlled by the growth terms in the QMD and top height rate equations. The climate multiplier applied to these size growth terms adjusts the allocation of available photosynthate for new growth, as measured by the QMD and top height growth rates for a given tree size. The decay terms for the QMD and top height rate equations are not modified. They represent the underlying average rates for the metabolic processes of transpiration and respiration for a given tree size. These rates, though they may vary with climate, are assumed to do so only through the use of photosynthates, removing them from the pool available for new growth.

The S-system model with climate adds three new parameters, $g_{i4} \geq 0$, $i = 1, 2, 3$, and only these new parameters were estimated for the growth and yield model with climate. The base model parameters, indicated by the parameters with hats, e.g., $\hat{g}_{ij}$ or $\hat{\alpha}_i$, were held fixed, allowing the development of the nested models. In addition, the constraints $g_{i4} \geq 0$, $i = 1, 2, 3$, were imposed during the parameter estimation procedures. This was done to ensure that the effects from the climate multipliers were applied as directly proportional modifiers of the estimated growth rates. Without these constraints, it would be possible for the exponents to take on negative values, indicating an inversely proportional effect, which would be incorrect.

The additional constraints imposed by the base Douglas-fir growth and yield model are also enforced for the extended growth and yield model which includes climate. These constraints define the allowable initial conditions, and imply that stand density must be positive, $X_1 > 0$, QMD must be positive, $X_2 > 0$, and top height must be at least breast height, $X_3 \geq 1.3$ m, and the allowable signs for the rates of change, and imply that stand density may decrease or remain constant over time, $\dot{X}_1 \leq 0$, QMD may increase or remain constant over time, $\dot{X}_2 \geq 0$, and top height may increase or remain constant over time $\dot{X}_3 \geq 0$. The constraints on the allowable rates of change are enforced exactly as in Equations 2.14, with the addition of the climate modifier as state variable X_4 in the growth term of each equation, and need not be repeated here.

3.3 *Climate and stand measurement data sources and description*

The Douglas-fir stand measurement data for this modeling effort were obtained from the Stand Management Cooperative in the College of Forest Resources at the University of Washington [26]. The SMC maintains a high quality database of individual tree measurements for stands located on commercial production forest lands in the Pacific Northwest. The SMC data span the region from southern Oregon, to Southern British Columbia, west of the Cascade Mountains. The SMC database is composed primarily of Douglas-fir and western hemlock measurements, some of which span approximately 30 years, with a small component of other tree species common to the Pacific Northwest.

The climate data were obtained from the National Climatic Data Center (NCDC) via the World Wide Web. The NCDC climate data are provided on a regional basis, and are associated with the major climatic divisions or climate zones for each state [22]. The raw data obtained were monthly mean values for temperature and Palmer Drought Severity Index, and monthly precipitation for the period of record, 1895 through 1997, the last growing year for which stand measurement data were available for this research.

3.3.1 Douglas-fir stand measurement data

The Douglas-fir stand measurement data used to calibrate and validate the growth and yield model which includes climate are the same data that were used in Chapter 2 to calibrate the base growth and yield model. No modifications to the data or the data processing procedures were made for their use in extending the base growth and yield model to include climate. Figure 3.1 shows a map of the locations for the 167 SMC installation which supplied data for this modeling effort. The figure also includes approximate boundaries for the NCDC climate zones for the region of interest. The approximate climate zone boundaries were obtained by interactively tracing the boundaries for the NCDC climate zone maps for Washington and Oregon, and then scaling the resultant $x - y$ data to obtain latitude and longitude values. See Section 2.5 for a complete description of the Douglas-fir stand measurement data and its initial processing.

3.3.2 Climate data

As obtained from the NCDC, the raw climate data were monthly mean or monthly total values. Given the annual time step of the growth and yield model, climate variables with an annual basis for the time period of interest, the 1969 through 1997

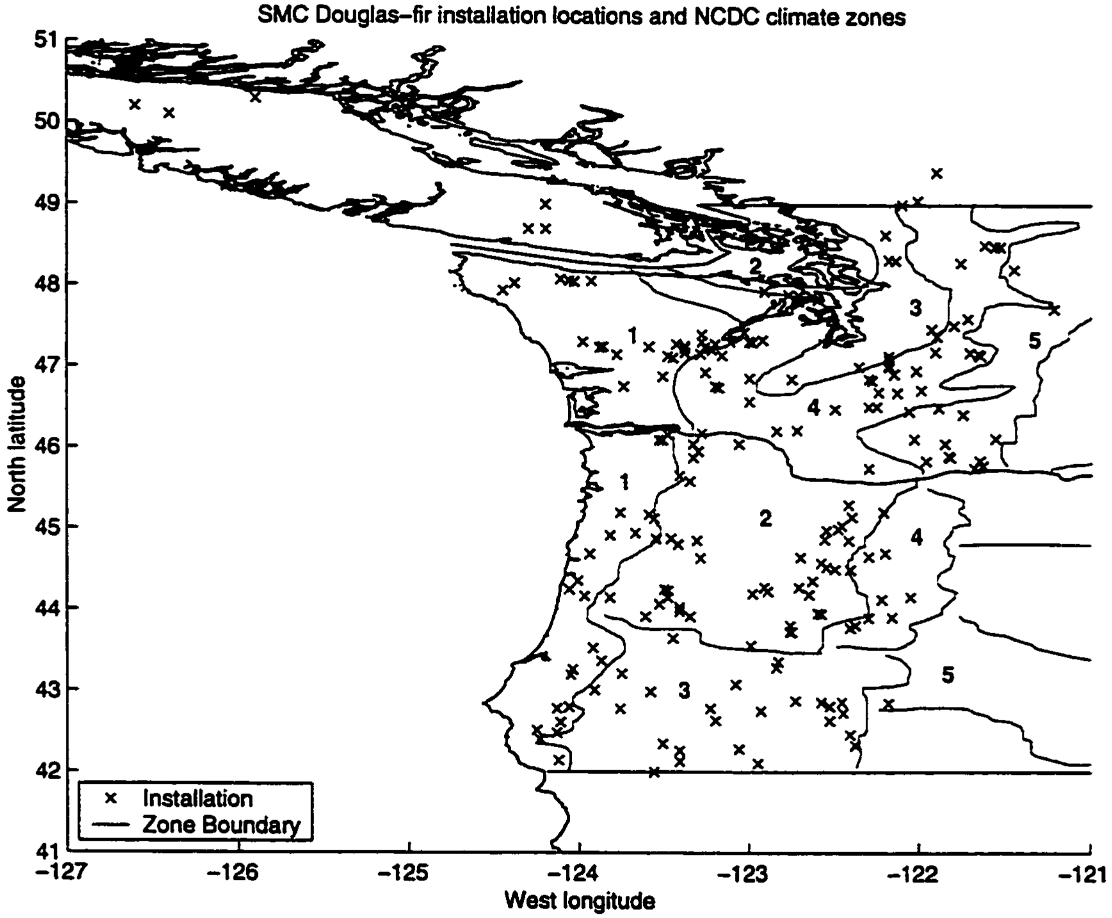

Figure 3.1: SMC installation locations for the stand measurement data with the NCDC climate zones and climate zone IDs. The SMC installations span the region from southern Oregon, through Washington, and into southern British Columbia, west of the Cascade Mountains. No map projection was used to correct for changes in latitude, hence the apparent elongation of Vancouver Island.

growing years, were desired. Although the Palmer Drought Severity Index (PDSI) is the climate variable of primary interest, the temperature and precipitation data were also kept as ancillary, related variables that could be used in the model validation procedures.

Three data processing steps were performed on the raw climate data to obtain annual climate variables for the period of interest. First, the raw climate data were converted to metric units, °C for temperature and centimeters for precipitation; PDSI is a unitless variable and no conversion was necessary. Second, the data for the period of interest were extracted from the data for the period of record. Third, annual average values for the three climate variables were computed from the monthly values to obtain the annual basis for climate desired for the growth and yield model. Two other factors affecting the data processing must be considered at this time, one that is related to the biology of Douglas-fir and the other related to the distribution of rainfall throughout the year.

Douglas-fir exhibits a preformed growth pattern, that is, the growth for the current growing year is set during the prior year [77, 102]. This means that the climate of both the prior year and the current year have an affect on tree growth in the current year. So, in the selection of the climate data used in the model, climate variable values were obtained for both of these years. The prior year and current year climate values were then averaged to obtain an estimate of the combined climate effects for the two years. This averaging of the climate variables from the prior year and current year is consistent with the manner in which forest stands, and individual trees within a stand. respond to climate changes. The response of forest stands and trees to changes in climate is attenuated, that is, it is not an instantaneous response at the scale of the stand or tree, but an average response integrated over time and space, though it may appear instantaneous at the level of leaves and stomata [77, 78, 69, 103, 112, 165, 168]. The two year average of the annual average PDSI values will be referred to as the two year average PDSI value.

The climate of the Pacific Northwest region is characterized by warm, dry summers and cool, wet winters [49, 77], with the majority of rainfall between November through March. This period of time is generally referred to as a water year, and is frequently used as the basis for climate, primarily precipitation, in forest models rather than an annual time period. Water year average PDSI was considered as a climate variable rather than annual average PDSI, but it was shown to be effectively equivalent to annual average PDSI for this situation. For the years of interest, 1968 through 1997, the annual average PDSI values and the water year average PDSI values were strongly related, having simple linear regression coefficients in the range of 0.87 to 1.04 for the slope values, and -0.25 to -0.01 for the intercept values, using annual average PDSI as the independent variable in the regressions. The r^2 values ranged from 0.55 to 0.66, indicating a moderate linear relationship with some variability. Given this relationship, the simpler climate variable, annual average PDSI was used, along with annual average temperature and precipitation as ancillary variables.

Table 3.2 presents the range of possible PDSI values and their nominal interpretation in terms of dry periods and wet periods [22, 104]. Loosely interpreted, negative PDSI values indicate that the demand for water is greater than soil water availability, positive PDSI values indicate that the demand for water is less than the soil water availability. PDSI values near zero indicate that the demand for water and the availability of water are balanced. Larger negative values, therefore, represent periods of drought, and larger positive values indicate periods of excessive moisture. The nominal range for PDSI values is -6 to +6, with occasional larger values [22, 104].

Table 3.3 presents a summary of the annual average PDSI values for the time period of interest, growing years 1968 through 1997. The summary averages the annual average PDSI values for Oregon and Washington, and the five NCDC climate zones for each state, giving a regional indication of the climate for this period. The table contains statistics for the year preceeding the current growing year, the current growing year, and the average of the two growing years. The range of annual average

PDSI values is roughly between -5 and +5, so the nominal interpretations from the monthly PDSI values from Table 3.2 are generally applicable for the annual averages as well. Notice from this table that the average climate is well within the normal range, ±0.5, for the prior and current growing years and for the two year average, though there are some extremes represented as well.

Figure 3.2 plots the two year average PDSI values for each NCDC climate zone for Washington and Oregon and the current growing years for the period of interest: 1969 through 1997. The values for Oregon and Washington are presented separately because their climates differ; Washington is generally slightly cooler and wetter than Oregon [49]. Clearly evident is the occurrence of a drought beginning approximately in 1985, and extending to approximately 1995. A smaller, shorter drought also appears to have occurred in the middle to late 1970s. Also of note is the difference in the variability of the two year average PDSI values for Oregon and Washington. The five climate zones in Oregon appear to be strongly related, following the same general course through time. The Washington climate zones appear to be more variable, not tracking together as strongly, though there are some strong trends between 1975 and 1985. The appearance of strong wet and dry periods in both Oregon and Washington, for all climate zones within the time span of interest, should enhance the ability to detect climate effects and incorporate them into a growth and yield model.

Figure 3.3 and Figure 3.4 plot the single step and lifetime residuals for the base model *vs.* growing year. A brief visual inspection of these two figures indicates that there may be some slight variability in the average base model residuals from one growing year to the next. Further, there appears to be a decrease in the average QMD residuals near 1984 to 1985, particularly for the single step residuals. This is an indication that the base model is overpredicting QMD. This is the expected behavior for the base model given the initiation of a drought at about the same time; the base model should overpredict QMD for years in which there is a drier climate. The base model residuals appear to contain trends that may be attributed to an

Table 3.2: PDSI value interpretations. Monthly PDSI values nominally fall within the interval $-6 \leq \text{PDSI} \leq 6$, with occasional values exceeding this range [22, 104]. The PDSI values provide an indication of the severity of a dry or wet period, with negative values indicating dry periods and positive values indicating wet periods [22, 104].

PDSI value range	Interpretation
$\text{PDSI} \leq -4$	extreme dry period
$-4 < \text{PDSI} \leq -3$	severe dry period
$-3 < \text{PDSI} \leq -2$	moderate dry period
$-2 < \text{PDSI} \leq -1$	mild dry period
$-1 < \text{PDSI} \leq -1/2$	incipient dry period
$-1/2 < \text{PDSI} \leq 1/2$	normal
$1/2 < \text{PDSI} \leq 1$	incipient wet period
$1 < \text{PDSI} \leq 2$	mild wet period
$2 < \text{PDSI} \leq 3$	moderate wet period
$3 < \text{PDSI} \leq 4$	severe wet period
$4 < \text{PDSI}$	extreme wet period

Table 3.3: Summary of annual average PDSI values for the growing years 1969 to 1997. The PDSI value summaries are for the year preceeding the current growing year, the current growing year, and the average value for the two growing years. The sample size was 2431, one PDSI value for each stand and annualized measurement. *MAD is mean absolute deviation.

PDSI value	Mean	Standard deviation	Min.	Med.	Max.	MAD*
Preceeding growing year	0.17	1.39	-3.68	0.12	4.78	1.09
Current growing year	0.25	1.47	-3.68	0.17	4.78	1.14
Average of growing years	0.21	1.19	-2.71	0.12	4.25	0.91

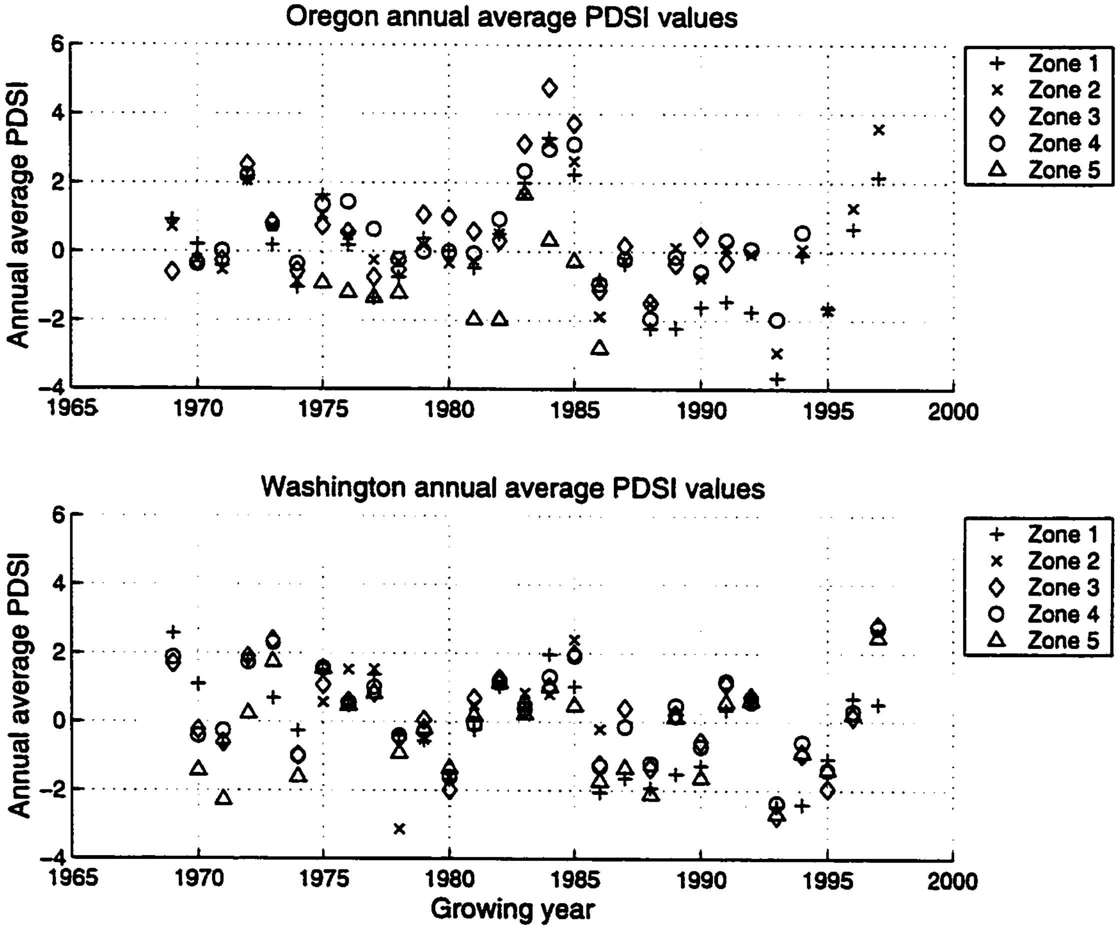

Figure 3.2: Two year average PDSI values *vs.* current growing year for Oregon and Washington. The annual average PDSI values for the five NCDC climate zones west of the Cascade Mountains for Oregon and Washington are shown. Note in particular the strong dry period which begins near 1984, indicated by the shift from positive values to negative values.

unmodeled climate effect.

To enable the association of the climate data with the Douglas-fir stand measurement data, each SMC installation was visually assigned to an NCDC climate zone. An interactive procedure was used, plotting the location, latitude and longitude, of each SMC installation, and then assigning it to a climate zone based the boundaries in Figure 3.1. Once the climate zone associations were obtained, average PDSI values for the current and preceeding growing years were associated with each stand measurement and growing year in the Douglas-fir stand measurement data. Annual average PDSI values for the SMC installations in British Columbia were obtained by extending the adjacent NCDC climate zones from Washington. These installations were a very small subset of the data, so this will have no noticeable effect on the results.

Before the two year average PDSI values may be used in the growth and yield model defined in Section 3.2 they must be converted to values which indicate a proportional increase or decrease in predicted growth rates from the base growth and yield model. The new variable should have a mean value of one, indicating normal climate, and a range which represents the proportional variability in growth expected from climate. A variability of ±20% has been reported for the influence of climate on diameter growth [12, 109, 175], and this is used as a basis for the scaling procedure.

The range of the two year average PDSI values is contained within the interval from -5 to +5. Dividing the values within this interval by the interval width of ten, produces a range of proportions spanning ±50%. Though slightly wider than the ±20% that has been reported, this is the range of proportions used to represent the effects of climate on the predicted rates of change from the base growth and yield model. Specifically, scaled PDSI values, P_s, are obtained from the formula

$$P_s = \begin{cases} 0.5, & \text{if } \frac{\bar{P}_2}{10} < -\frac{1}{2} \\ 1 + \frac{\bar{P}_2}{10}, & \text{if } -\frac{1}{2} \leq \frac{\bar{P}_2}{10} \leq \frac{1}{2} \\ 1.5, & \text{if } \frac{1}{2} < \frac{\bar{P}_2}{10}, \end{cases}$$

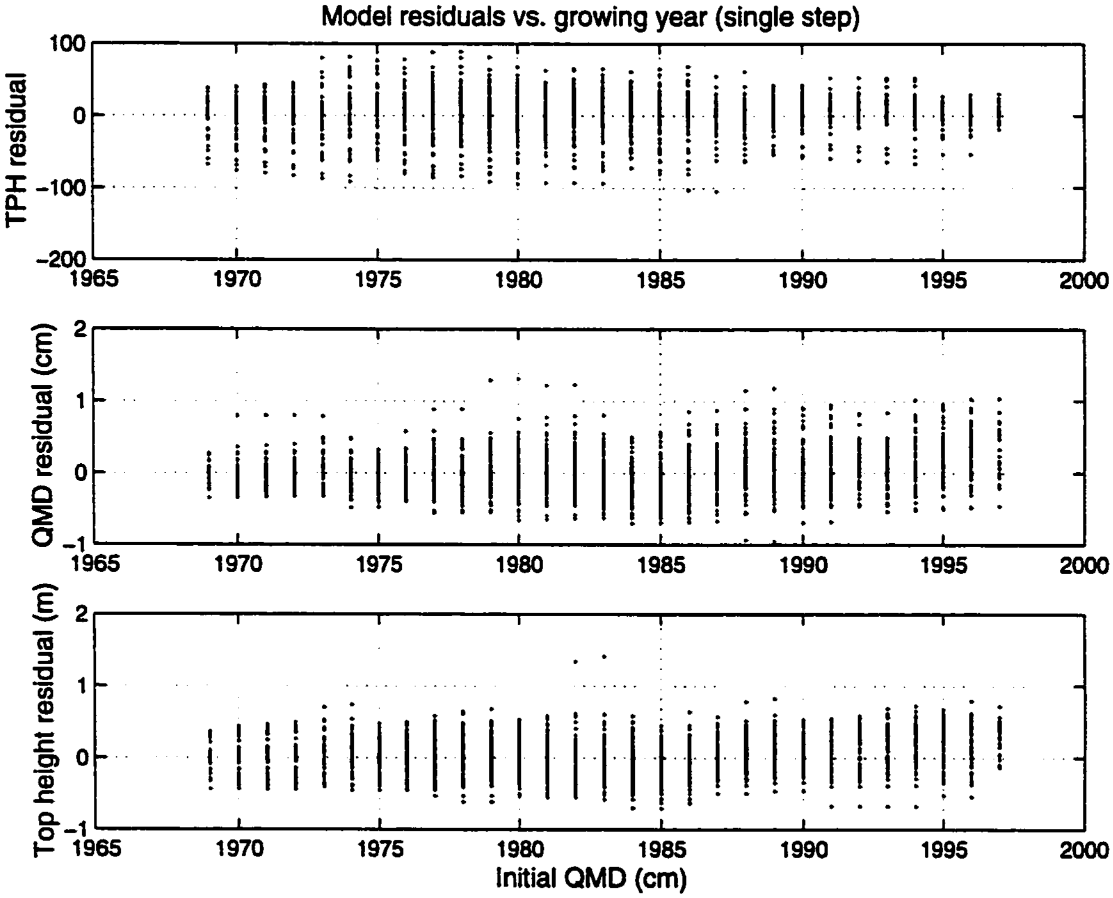

Figure 3.3: Base model single step residuals, R^s_{ik}, *vs.* growing year. The TPH residuals, $i = 1$, are on top, QMD residuals, $i = 2$, are in the middle, and top height residuals, $i = 3$, are on the bottom. The dot density provides an indication of the number of data points.

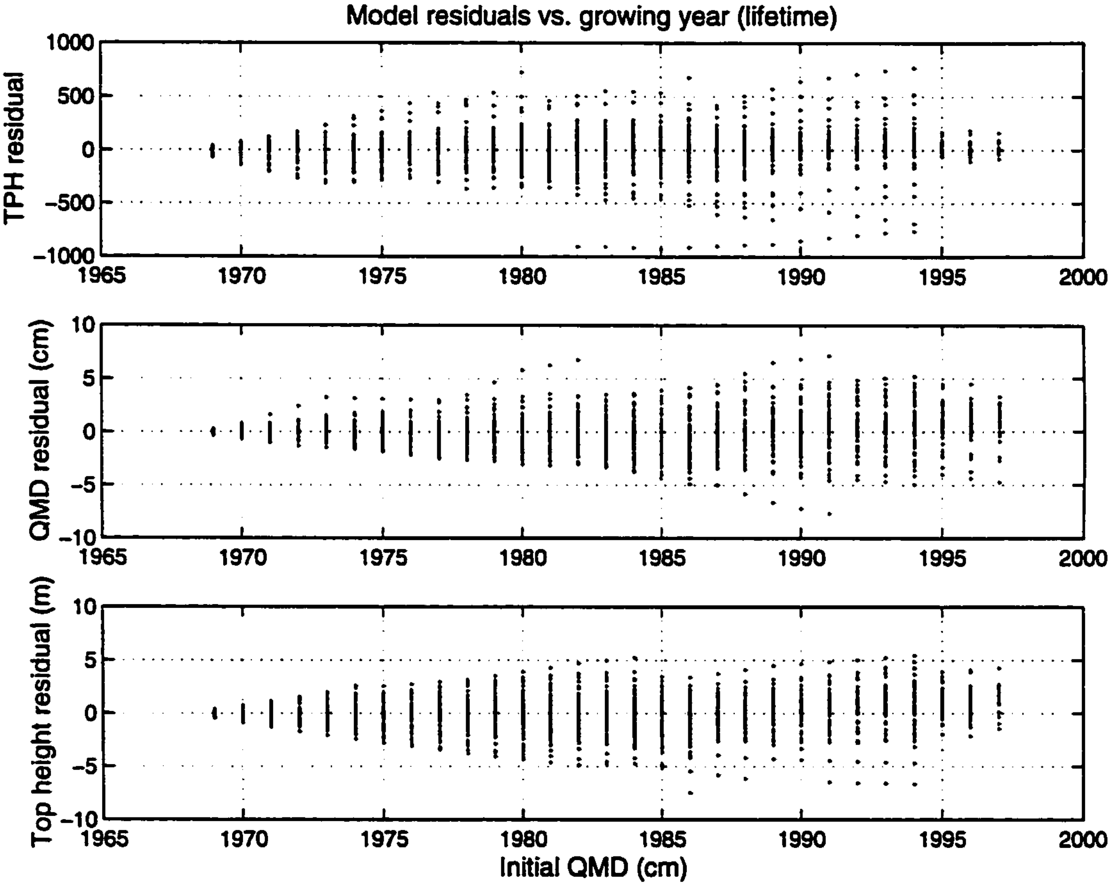

Figure 3.4: Base model lifetime residuals, R^l_{iks}, *vs.* growing year. The TPH residuals, $i = 1$, are on top, QMD residuals, $i = 2$, are in the middle, and top height residuals, $i = 3$, are on the bottom. The dot density provides an indication of the number of data points.

where $\bar{P}_2$ is the two year average PDSI value,

$$\bar{P}_2 = \frac{P_p + P_c}{2},$$

computed using the annual average PDSI values from the prior growing year, P_p, and the annual average PDSI value from the current growing year P_c. The scaled two year average PDSI values ranged from 0.52 to 1.37, with a mean of 1.01. 83% of the scaled PDSI values were within the interval $0.8 \leq P_s \leq 1.2$, which is in strong agreement with the ±20% variability due to climate that has been reported.

The formula used to compute P_s limits the direct effects of climate on growth rates to ±50% of the nominal predicted growth rates from the base model. Within each equation of the climate model, these P_s values are modified by an exponent $g_{i4} \geq 0$, $i = 1, 2, 3$, allowing the potential range of modification due to climate to be greater than, or less than, the basic ±50%, depending on the value of the exponent for each equation and the value of the scaled PDSI value used, and the effects may be different for each equation in the model. The values P_s become the climate variable X_4 in the extended S-system model.

3.4 Model analysis and validation methods

The parameter estimation procedures used to obtain values for the three S-system parameters g_{i4}, $i = 1, 2, 3$, are identical to those described in section Section 2.6. Only parameter values for the exponents of the scaled climate values were estimated, all other parameter values remained fixed to isolate the effects of climate on the base model. This permits the construction of nested models, having the base model without climate as a submodel which was extended to include the climate effects.

The bootstrap procedure, outlined in Section 2.6, was also used to obtain bootstrap means and standard errors for the three climate exponents. Again, $B = 100$ bootstrap trials were performed using 90% of the measurement intervals for the algebraic parameter estimation procedure and 90% of the stands for the shooting based parameter refinement procedure.

Finally, the model validation procedures for the Douglas-fir growth and yield model which includes climate effects were identical to those described in Section 2.6. Briefly, these procedures consisted of a visual inspection of the residuals, simple linear regression analyses and the computation of r^2 values for the stand density, QMD, and top height residuals and the annual average of PDSI, temperature, and precipitation climate variables. These procedures were applied for both the single step residuals, to determine growth related effects, and the lifetime residuals, to determine stand trajectory effects.

3.5 Results

The results of the parameter estimation procedures for the extended stand level Douglas-fir growth and yield model follow. The results indicate that there was no detectable climate effect for these data and this time period. Simple linear regression analyses, line fits and r^2 values for the single step and lifetime residuals obtained from the base model, were performed to determine if another climate variable might contribute a detectable effect for this model. The parameter estimates are presented first, followed by the single step residuals analysis, and finally by the lifetime residuals analysis.

Table 3.4 displays the estimated parameter values, bootstrap means, and bootstrap standard errors for both the algebraic parameter estimation method, p_{ia}, $\bar{p}^B_{ia}$, s^B_{ia}, respectively, and for the shooting based parameter refinement method, p_{is}, $\bar{p}^B_{is}$, s^B_{is}. Clearly the parameter estimates obtained using all of the stand measurement data are nearly zero for both the algebraic parameter estimation method and the shooting parameter refinement. The largest parameter values are on the order of 10^{-4} and 10^{-5}, respectively, for these parameter estimation approaches. These results imply that there is not a detectable, direct climate effect that is obtainable for the model

Table 3.4: Estimated S-system parameter values, bootstrap means, and bootstrap standard errors for the extended Douglas-fir growth and yield model defined by Equations 3.1 which includes climate effects. A subscript a indicates that the parameter estimates are from the algebraic parameter estimation procedure, and a subscript s indicates that the parameter estimates are from the shooting based parameter estimation procedure. The superscript B indicates that the values were obtained from the bootstrap procedure.

p_i	p_{ia}	$\bar{p}^B_{ia}$	s^B_{ia}	p_{is}	$\bar{p}^B_{is}$	s^B_{is}
g_{14}	3.25×10^{-4}	5.56×10^{-4}	6.8×10^{-4}	2.56×10^{-8}	1.62×10^{-4}	2.07×10^{-4}
g_{24}	7.53×10^{-10}	8.07×10^{-6}	8.01×10^{-5}	3.83×10^{-7}	8.01×10^{-3}	2.36×10^{-2}
g_{34}	8.54×10^{-7}	7.10×10^{-5}	1.69×10^{-4}	1.77×10^{-5}	2.65×10^{-4}	3.96×10^{-4}

and these data. The climate based effects, as determined from the scaled PDSI values and the S-system exponents, are all essentially equal to one, implying that the Douglas-fir growth and yield model which includes climate is indistinguishable from the base growth and yield model of Chapter 2.

The bootstrap parameter means and standard errors also support this conclusion. The bootstrap means are all very near zero, though not always as close to zero as the parameter estimates using all of the data. Further, the bootstrap standard errors are at least twice as large of as their respective parameter estimates derived from all of the data, and are generally at least an order of magnitude larger. The bootstrap confidence intervals for these parameter values all contain zero, so zero cannot be ruled out as a possible parameter value. Therefore, no direct climate effect is extractable from theses data.

3.5.1 Base model residual plots vs. climate variables

Figure 3.5 and Figure 3.6 present plots of the single step and lifetime residuals for the base Douglas-fir growth and yield model *vs.* their associated PDSI values. The stand density, QMD, and top height residuals in each case are clearly symmetrically distributed about zero throughout the range of PDSI values, for both the single step and lifetime residuals. This supports the conclusions derived from the estimated parameter values and bootstrap standard errors. There is no detectable, direct climate influence represented by the base model residuals and the scaled PDSI climate data. So, relative to PDSI as an indicator of climate, the base model residuals are effectively randomly distributed.

PDSI failed to have a detectable climate effect for this model and these data, but might there be a temperature or precipitation? Figure 3.7 and Figure 3.8 plot the single step and lifetime base model residuals *vs.* two year average temperature, respectively. The distribution of the residuals throughout the range of the temperature values is symmetric about zero for both sets of residuals, making it highly unlikely that temperature would contribute to a detectable climate effect. Figure 3.9 and Figure 3.10 plot the single step and lifetime base model residuals *vs.* two year average temperature and precipitation, respectively. Again, the distribution of the residuals throughout the range of the precipitation values is symmetric about zero for both sets of residuals, making it highly unlikely that precipitation would contribute to a detectable climate effect. Thus, neither the use of temperature nor the use of precipitation, rather than PDSI, would have produced a detectable climate effect.

3.5.2 Base model residual correlations with climate variables

Table 3.5 through Table 3.7 present the results for the simple linear regression analyses of the single step stand density, QMD, and top height standardized residuals for the base model *vs.* the climate variables two year average PDSI, two year average temperature, and two year average precipitation. The temperature and precipitation two year averages were derived in an identical manner to the two year average PDSI values. An examination of these tables clearly shows that the base model residuals are generally uncorrelated with the three climate variables. The regression intercepts for the standardized residuals are generally near zero, ranging from $b = -0.42$ to $b = 0.39$, and the regression slopes are also nearly zero, ranging from $a = -0.08$ to

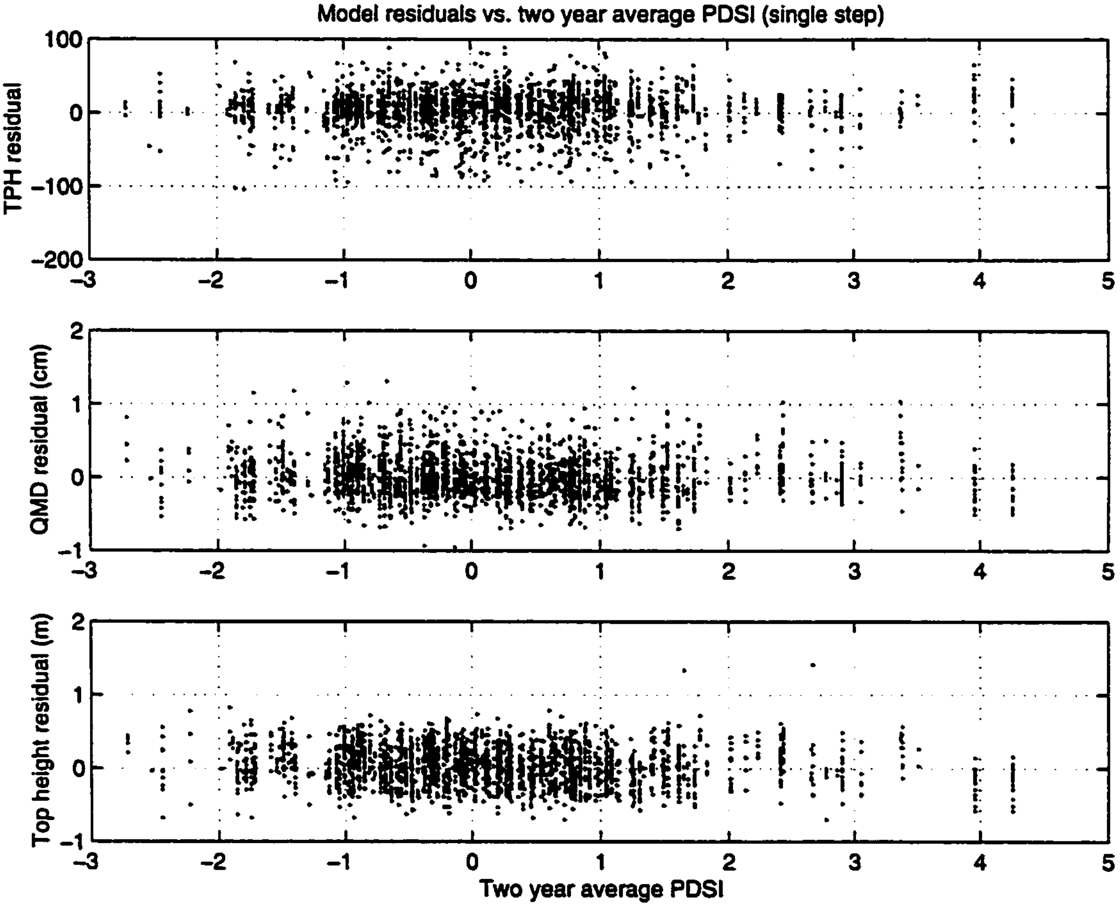

Figure 3.5: Single step residuals, R^s_{ik}, for the base model *vs.* two year average PDSI. The TPH residuals, $i = 1$, are on top, QMD residuals, $i = 2$, are in the middle, and top height residuals, $i = 3$, are on the bottom. The dot density provides an indication of the number of data points.

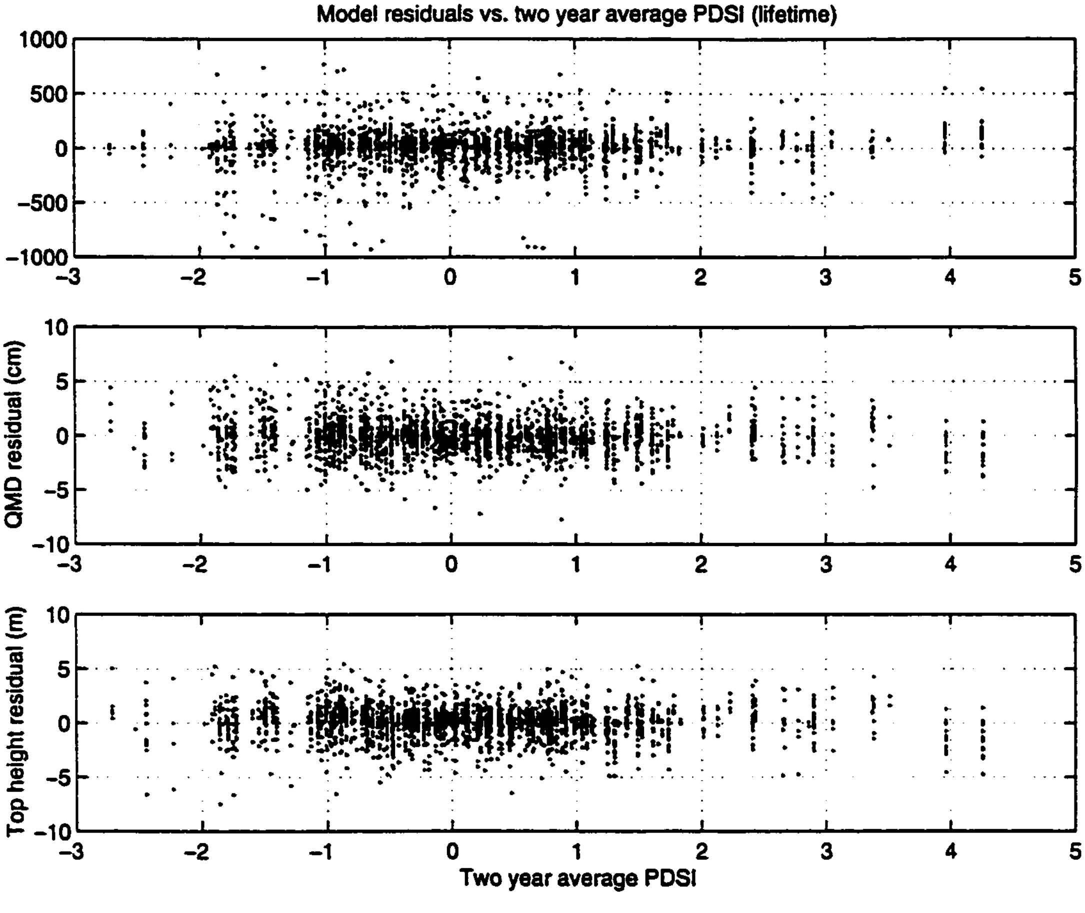

Figure 3.6: Lifetime residuals, R^l_{iks}, for the base model *vs.* two year average PDSI. The TPH residuals, $i = 1$, are on top, QMD residuals, $i = 2$, are in the middle, and top height residuals, $i = 3$, are on the bottom. The dot density provides an indication of the number of data points.

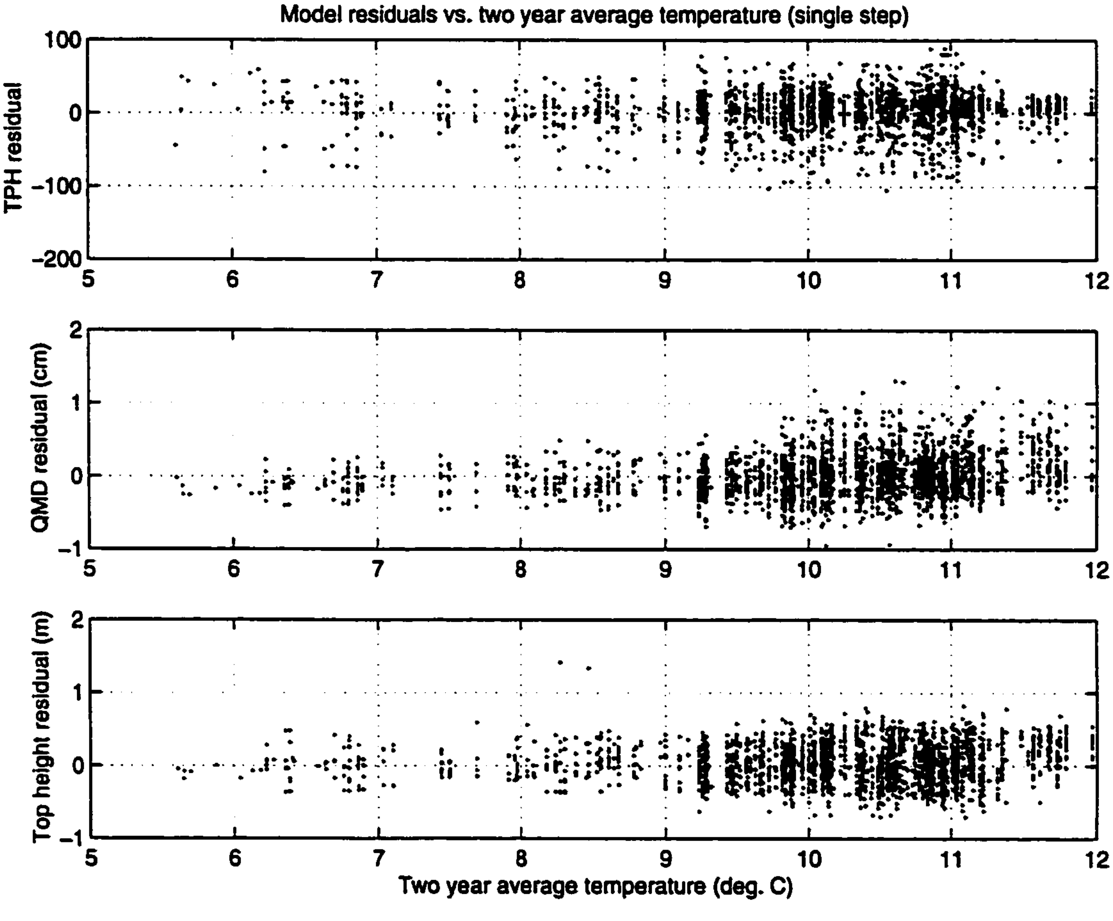

Figure 3.7: Single step residuals, R^s_{ik}, for the base model *vs.* two year average temperature. The TPH residuals, $i = 1$, are on top, QMD residuals, $i = 2$, are in the middle, and top height residuals, $i = 3$, are on the bottom. The dot density provides an indication of the number of data points.

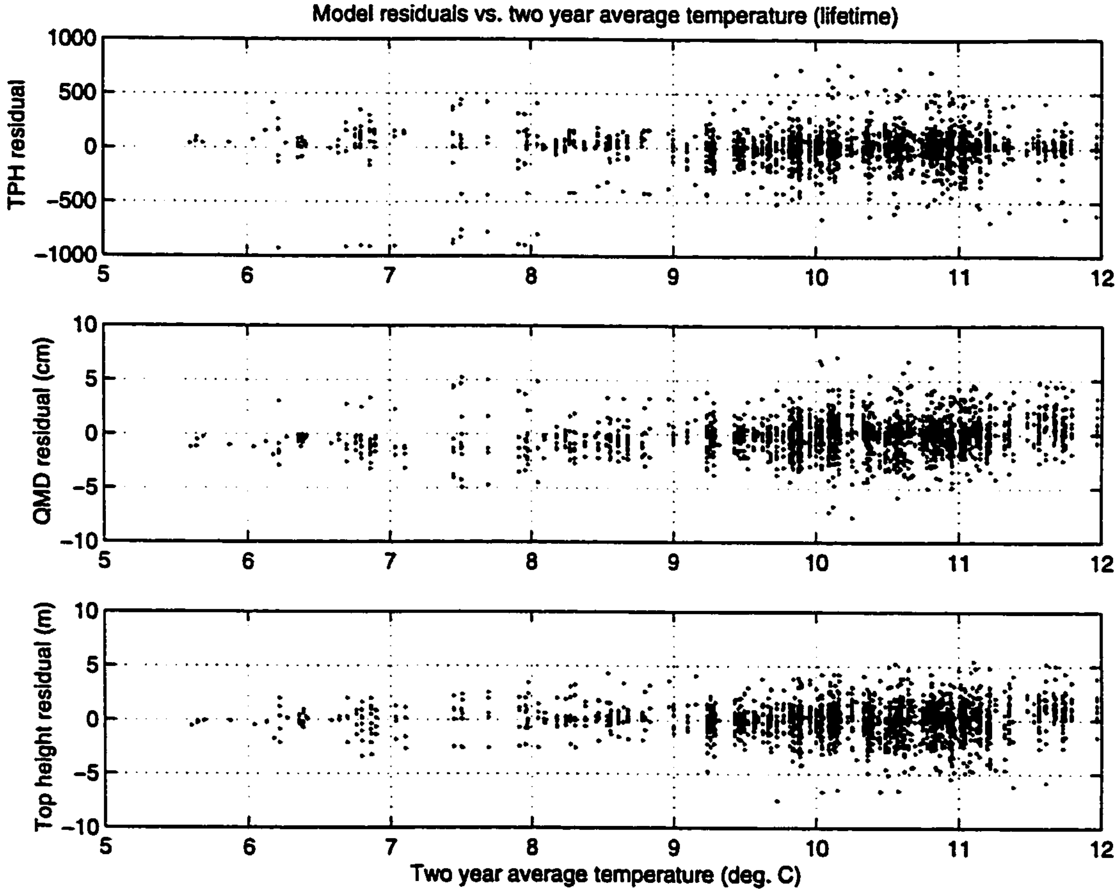

Figure 3.8: Lifetime residuals, R^l_{iks}, for the base model *vs.* two year average temperature. The TPH residuals, $i = 1$, are on top, QMD residuals, $i = 2$, are in the middle, and top height residuals, $i = 3$, are on the bottom. The dot density provides an indication of the number of data points.

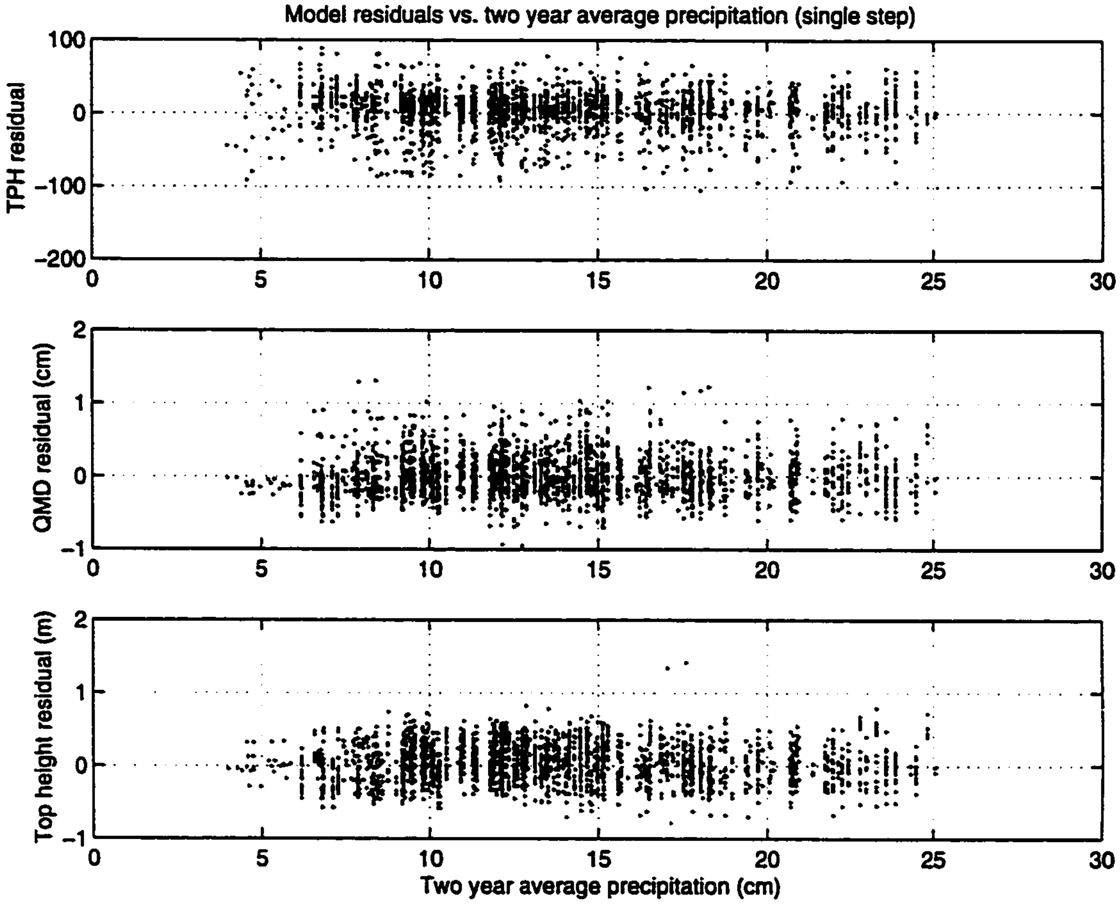

Figure 3.9: Single step residuals, R^s_{ik}, for the base model *vs.* two year average precipitation. The TPH residuals, $i = 1$, are on top, QMD residuals, $i = 2$, are in the middle. and top height residuals, $i = 3$, are on the bottom. The dot density provides an indication of the number of data points.

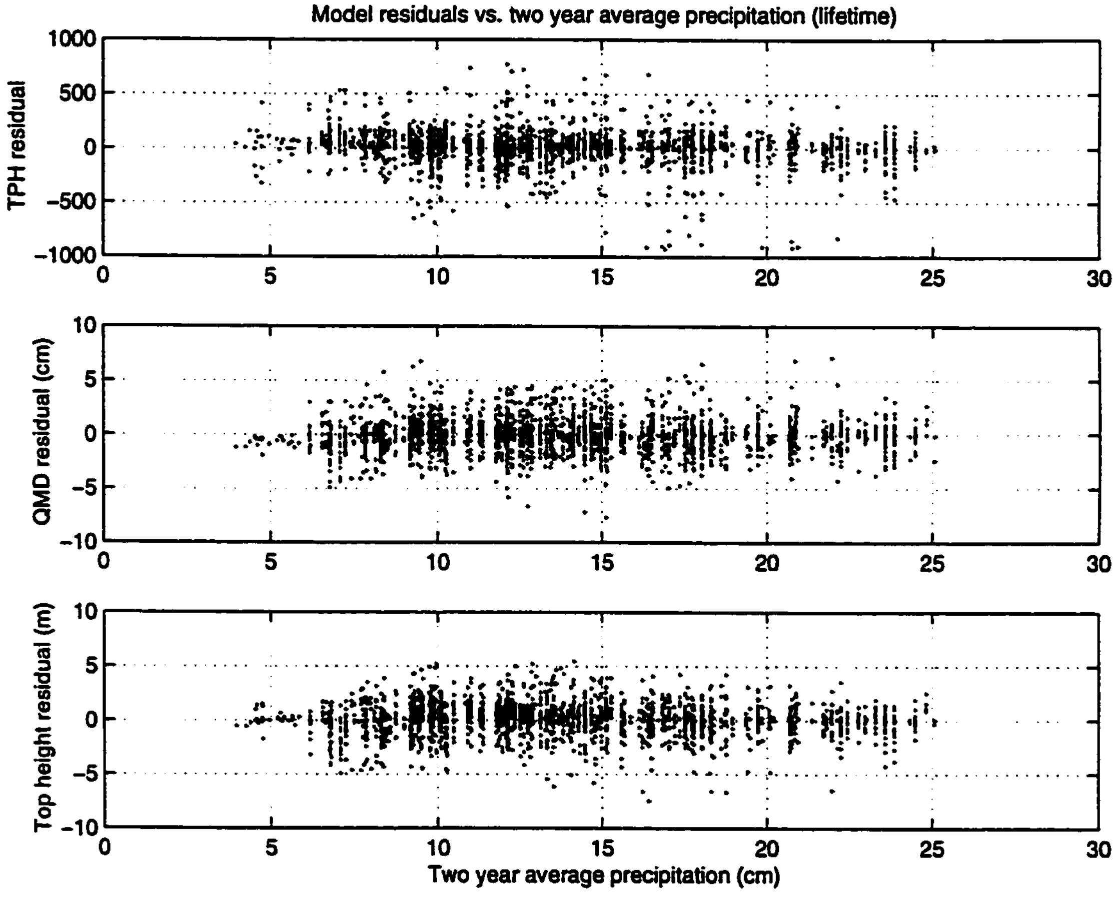

Figure 3.10: Lifetime residuals, R^l_{iks}, for the base model *vs.* two year average precipitation. The TPH residuals, $i = 1$, are on top, QMD residuals, $i = 2$, are in the middle. and top height residuals, $i = 3$, are on the bottom. The dot density provides an indication of the number of data points.

Table 3.5: Single step standardized TPH residual line fits and r^2 values *vs.* the annual average climate variables PDSI, temperature, and precipitation. Regression coefficients are for the model $y = ax + b$ applied to the standardized TPH residuals for each x variable. Values of $a = 0$ and $b = 0$ imply that there is not a linear relationship or bias, and $r^2 = 0$ implies a lack of correlation with the x variable.

x	a	b	r^2
PDSI	0.027116	0.130307	0.001036
Temperature °C	0.041178	-0.284798	0.001908
Precipitation (cm)	-0.018541	0.390654	0.007054

$a = 0.06$. The exception is for the QMD residuals and two year average temperature, for which the slope is $a = 0.18$ and the intercept is $b = -1.8$. An examination of the single step residuals plotted against the two year average temperature values indicates that there are few very low temperatures, between 5°C and 7°C, where the majority of the temperature data lie in the range 8°C and 12°C. The small number of lower temperatures, coupled with the larger residuals being associated with the higher temperature range, gives a large amount of leverage to these lower temperatures, which in turn causes the larger slope and intercept values for QMD residuals and temperature.

An examination of the r^2 values further supports the results of the simple linear regression analyses. All of the r^2 values are less than 0.04, with the majority of the values being less than 0.01. This indicates a general lack of correlation between the stand density, QMD, and top height residuals with two year average PDSI, temperature, and precipitation. The largest r^2 value, $r^2 = 0.04$, was for the QMD residuals and temperature, and is again appears to be a result of the strong leverage exerted by a small number of lower temperature values.

Table 3.8 through Table 3.10 present the results for the simple linear regression analyses of the lifetime, or stand trajectory, stand density, QMD, and top height standardized residuals for the base model *vs.* the climate variables two year average PDSI, two year average temperature, and two year average precipitation. There are

Table 3.6: Single step standardized QMD residual line fits and r^2 values *vs.* the annual average climate variables PDSI, temperature, and precipitation. Regression coefficients are for the model $y = ax+b$ applied to the standardized TPH residuals for each x variable. Values of $a = 0$ and $b = 0$ imply that there is not a linear relationship or bias, and $r^2 = 0$ implies a lack of correlation with the x variable.

x	a	b	r^2
PDSI	-0.057858	0.059760	0.004717
Temperature °C	0.179733	-1.788961	0.036346
Precipitation (cm)	-0.002349	0.079911	0.000113

Table 3.7: Single step standardized top height residual line fits and r^2 values *vs.* the annual average climate variables PDSI, temperature, and precipitation. Regression coefficients are for the model $y = ax+b$ applied to the standardized TPH residuals for each x variable. Values of $a = 0$ and $b = 0$ imply that there is not a linear relationship or bias, and $r^2 = 0$ implies a lack of correlation with the x variable.

x	a	b	r^2
PDSI	-0.081927	0.181790	0.009458
Temperature °C	0.058028	-0.428326	0.003789
Precipitation (cm)	0.002912	0.124641	0.000174

Table 3.8: Lifetime standardized TPH residual line fits and r^2 values *vs.* the annual average climate variables PDSI, temperature, and precipitation. Regression coefficients are for the model $y = ax + b$ applied to the standardized TPH residuals for each x variable. Values of $a = 0$ and $b = 0$ imply that there is not a linear relationship or bias, and $r^2 = 0$ implies a lack of correlation with the x variable.

x	a	b	r^2
PDSI	0.060023	0.072054	0.005077
Temperature °C	0.049776	-0.424016	0.002788
Precipitation (cm)	-0.026631	0.450406	0.014552

no surprises here. The regression intercepts are generally near zero, ranging from $b = -0.42$ to $b = 0.45$, and the regression slopes are also nearly zero, ranging from $a = -0.06$ to $a = 0.06$. The exception is for the QMD residuals and two year average temperature, for which the slope is $a = 0.16$ and the intercept is $b = -1.7$. Again, the discrepancy for QMD residual and temperature is due to the strong leverage exerted by the small number of lower temperature values.

Finally, all of the r^2 values for the lifetime stand density, QMD, and top height residuals are less than 0.03, with the majority of the values being less than 0.01. This indicates a general lack of correlation between the stand density, QMD, and top height residuals with PDSI, temperature, and precipitation. The largest r^2 value, $r^2 = 0.03$, was for the QMD residuals and temperature, and is again a result of the strong leverage exerted by a small number of lower temperature values.

The single step and lifetime residual analyses clearly demonstrate, counter to expectations, that a direct, detectable climate effect, based upon the inclusion of PDSI, temperature, or precipitation, in the Douglas-fir growth and yield model would be unlikely. The single step, or annual yield, residuals would be the most likely to demonstrate a climate effect, due to the annual base time step, but they show no such relationship. The lifetime, or stand trajectory, residuals, though they have some relatively large deviations for the actual stand measurements, would be expected to demonstrate a consistent bias due to the lack of climate effects within the growth and

Table 3.9: Lifetime standardized QMD residual line fits and r^2 values *vs.* the annual average climate variables PDSI, temperature, and precipitation. Regression coefficients are for the model $y = ax + b$ applied to the standardized TPH residuals for each x variable. Values of $a = 0$ and $b = 0$ imply that there is not a linear relationship or bias, and $r^2 = 0$ implies a lack of correlation with the x variable.

x	a	b	r^2
PDSI	-0.060212	-0.026804	0.005109
Temperature °C	0.157816	-1.652063	0.028022
Precipitation (cm)	0.001089	-0.054366	0.000024

Table 3.10: Lifetime standardized top height residual line fits and r^2 values *vs.* the annual average climate variables PDSI, temperature, and precipitation. Regression coefficients are for the model $y = ax + b$ applied to the standardized TPH residuals for each x variable. Values of $a = 0$ and $b = 0$ imply that there is not a linear relationship or bias, and $r^2 = 0$ implies a lack of correlation with the x variable.

x	a	b	r^2
PDSI	-0.060773	0.077001	0.005204
Temperature °C	0.034793	-0.291258	0.001362
Precipitation (cm)	-0.002699	0.101348	0.000149

yield model, but they also demonstrate no consistent climate related effects. Given these results and the fact that climate does affect stand development and tree growth, an explanation of these counterintuitive results is necessary.

3.6 Discussion

The inability to include a direct climate effect in the stand level Douglas-fir growth and yield model is somewhat surprising, given that climate has a definite effect on both stand development and tree growth [77, 78, 102, 107, 161]. There is some evidence that climate effects alone may not significantly affect the yields obtained for a particular forest stand, at least not without including the effects from increased levels of CO^2 [151, 152]. Accepting this as a possible explanation, however, is somewhat dissatisfying. There should be a more compelling interpretation of these results.

A consideration of the data, the model and its requisite assumptions, and the parameter estimation methodology provides six possible contributing factors that could potentially have confounded the ability to incorporate a direct climate effect into the Douglas-fir growth and yield model. The six factors, listed in increasing order of potential influence or relevance to these results, are given in the following list.

1. The Douglas-fir research plots are buffered.
2. Climate effects are small, on average.
3. Stand density effects may dominate climate effects.
4. The stand level attributes average out climate effects.
5. Climate effects are masked by young, fast growing trees.
6. The base model already includes average climate effects.

The confounding factors listed are not necessarily mutually exclusive, and in particular the first five are related. A brief discussion of each factor, identifying its relevance as a confounding factor, and a possible resolution, if possible, are be presented for future investigation.

3.6.1 Buffered research plots

The majority of the research plots used for the model calibration or parameter estimation were buffered, that is surrounded by a border of trees, isolating the research plot within the center of a much larger plot. Tree measurements taken from trees within the buffered research plots may not show the full effects of climate, due to an amelioration or reduction in the severity of climatic extremes which occurs from being centrally located within a larger stand [107, 102]. The effects of climate are most noticeable near the edges of stands, where effects of the buffering are minimized [102]. A resolution to this problem involves sampling trees that are near the edges of the stands containing the research plots to maximize the climate effects, at least for some trees. Using individual tree measurements from the research plots to develop and calibrate an individual tree model that is then coupled with stand density, may also provide better access to the climate information within the data.

3.6.2 Climate effects are small relative to the data variability

Climate effects on growth generally represent a small fraction of the nominal expected growth that would have occurred for the average climate. The stand level Douglas-fir growth and yield model was intended to span the region from southern Oregon to southern British Columbia, west of the Cascade Mountains. The raw variability contained within this wide ranging data set may swamp the variability due to climate, making it undetectable at this scale. This problem may be addressed by identifying smaller regions within which climate effects are investigated, possibly for each NCDC climate zone separately.

3.6.3 Stand density effects may dominate climate effects

Climate effects on tree growth are known to be small, generally ±20% or less of the actual annual growth [109, 110, 175]. It seems likely that stand density effects, particularly for younger stands as in some of the data used here, may dominate the climate effects, making them undetectable. This seems particularly reasonable, given that the largest QMD and top height residuals occurred for the youngest stands in the data set, and these stands were growing during a drought, 1984-1995. There is also evidence to support the existence of a strong stand density effect on the growth rates of young Douglas-fir stands. Further, stand density effects are generally not included in dendrochronological analyses of tree rings [50, 175]; trees selected for these studies are chosen based on whether or not they were potentially isolated trees, presumably to minimize stand density effects. This could potentially exaggerate the effects of climate on tree growth, if the stand density effect is large relative to a climate effect. Further investigation on the relative effects of stand density and climate on tree growth is necessary to sort out this issue.

3.6.4 The stand level attributes average out climate effects

The stand level Douglas-fir growth and yield model uses average tree size information, QMD and top height, derived from individual trees on a research plot. Averaging the individual tree diameters and the tree heights to obtain these average size values may effectively eliminate the effects of climate on the changes in tree size. The climate effects will generally vary for each tree based on its own local microsite conditions and growing space [102, 107]. A possible solution to this problem is to develop an individual tree based Douglas-fir growth and yield model as a submodel of the stand level growth and yield model. Climate effects may be detectable for a dynamic model based on individual trees and individual tree measurements.

The Douglas-fir measurement data are also averaged in time due to the multi-year remeasurement intervals, usually 2 or 4 years. Thus, the size increments for the individual trees, when computed, provide only an indication of average size growth over the remeasurement interval. This issue could be resolved by obtaining annual measurements on some plots, but the cost to do this would be prohibitive. The use of multi-year remeasurement intervals is most likely less important than the issue of using average tree dimensions for the development and calibration of a growth and yield model, because the number of years between measurements is small. Further, climate effects on tree growth being cumulative, it may be possible to determine appropriate annual weights to scale the growth increments for each year in a multi-year remeasurement interval.

3.6.5 Young, fast growing stands

Recall that the Douglas-fir stand measurement data set used to calibrate the S-system growth and yield model contained a significant subset consisting of very young, rapidly growing stands. These stands comprised approximately 20% of the data. The base S-system growth and yield model generally underpredicted the growth for these stands, as seen in Section 2.7. The residuals for both the single step and lifetime scenarios showed no appreciable linear relationships or correlations with any of the initial state variables, stand density, QMD, and top height, nor were there appreciable relationships with the ancillary variables, site index, elevation, and stand age. However, when the average residuals for both scenarios are plotted against the growing year, as for climate, a strong trend becomes readily apparent for both QMD and top height, though not for stand density.

Figure 3.11 and Figure 3.12 plot the average stand density, QMD, and top height model residuals for the single step and lifetime residuals scenarios, respectively. These two figures separate the average residuals for Oregon and Washington to identify any state specific trends. Notice first that there are no readily apparent differences between the average residuals for Oregon and Washington. From 1969 through 1985, the average residuals for both QMD and top height are nearly zero. But, beginning in 1985, there is a strong upward trend in the average QMD and top height residuals, indicating a consistent, and increasing though possibly slowing, underprediction of both QMD and top height for this period of time.

This dramatic underprediction of QMD and top height for this time span is being driven by a reduction in the stand ages available within the SMC database during the period 1985 to the present. Figure 3.13 plots the stand ages, with the mean and standard deviation for each year, for the SMC database *vs.* growing year. The downward trend after 1985 is clearly demonstrated. The younger stands demonstrate faster growth, and Figure 3.14 plots the QMD growth rates, again with the mean and standard deviation for each year. Notice, in particular, that the dramatic decrease in average stand age and the increase in average QMD growth rate occur simultaneously in 1985. A gradual shift to younger stands may have begun somewhat earlier, but the rate of the decrease in stand ages became significant at about 1985. The results for the rate of change of top height are similar to those for QMD, and are omitted.

Unfortunately, from the perspective of adding climate effects to a growth and yield model, this shift to younger, faster growing stands occurred during a dry period, extending from 1985 to approximately 1995. The effects on average growth due to the dry climate may have been obliterated over this time period by the rapid growth rates of the younger stands. Given the generally small average effects of climate on growth, particularly diameter growth and height growth, this long term systematic trend of underestimation swamps the least squares parameter estimation process, making the resolution of the climate effects effectively impossible, as was discovered. Further, considering the regional nature of the stand level Douglas-fir growth and yield model, even these trends are swamped by the residual variation, indicated by the fact that zero is contained within the interval defined by the mean residual value plus or minus twice the standard deviation for each growing year.

A closer examination of the two average residuals figures indicates that average

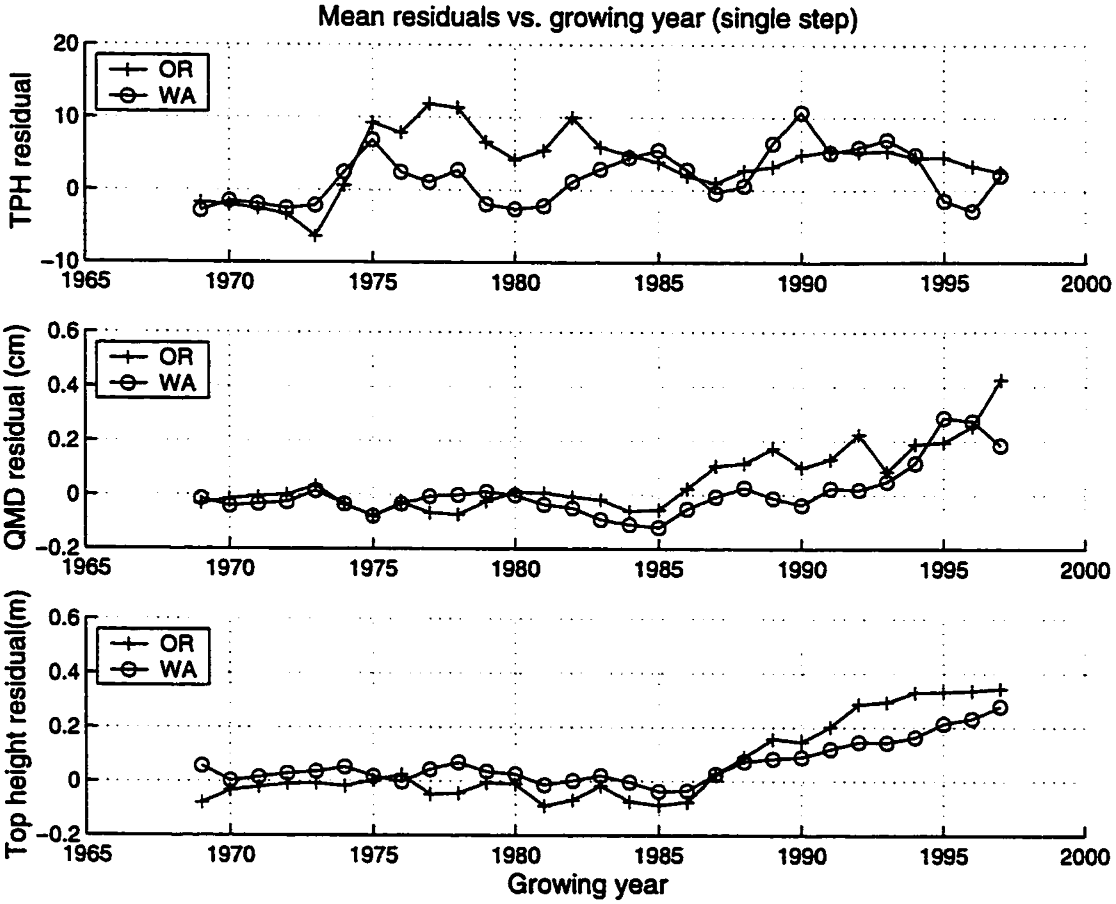

Figure 3.11: Single step average residuals, R^s_{ik}, for the base model without climate *vs.* growing year. The TPH residuals, $i = 1$, are on top, QMD residuals, $i = 2$, are in the middle, and top height residuals, $i = 3$, are on the bottom. Standard deviations are 25.26, 0.28, and 0.26. respectively for stand density, QMD, and top height. Note the increasing trend in the residuals after 1985.

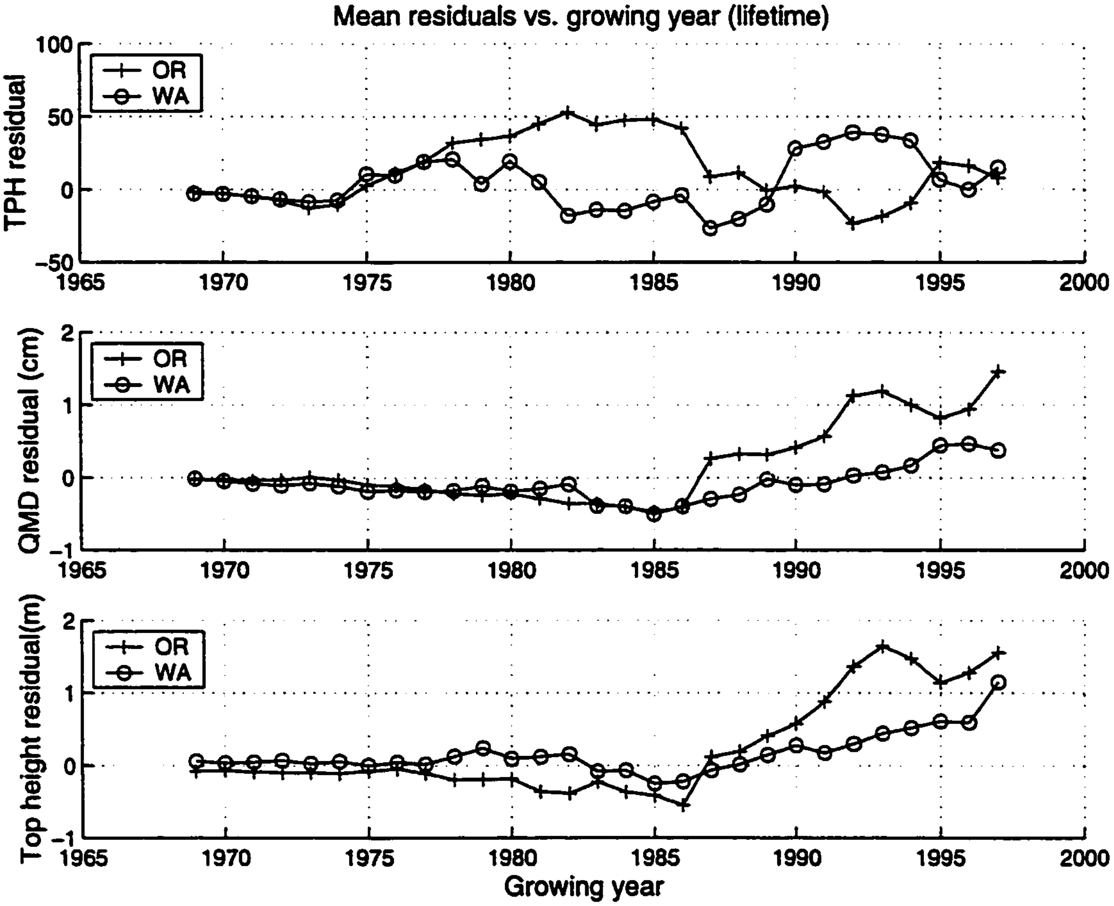

Figure 3.12: Lifetime average residuals, R^{l}_{iks}, for the base model without climate *vs.* growing year. The TPH residuals, $i = 1$, are on top, QMD residuals, $i = 2$, are in the middle, and top height residuals, $i = 3$, are on the bottom. Standard deviations are 151.98, 1.49, and 1.44. respectively for stand density, QMD, and top height. Note the increasing trend in the average residuals after 1985.

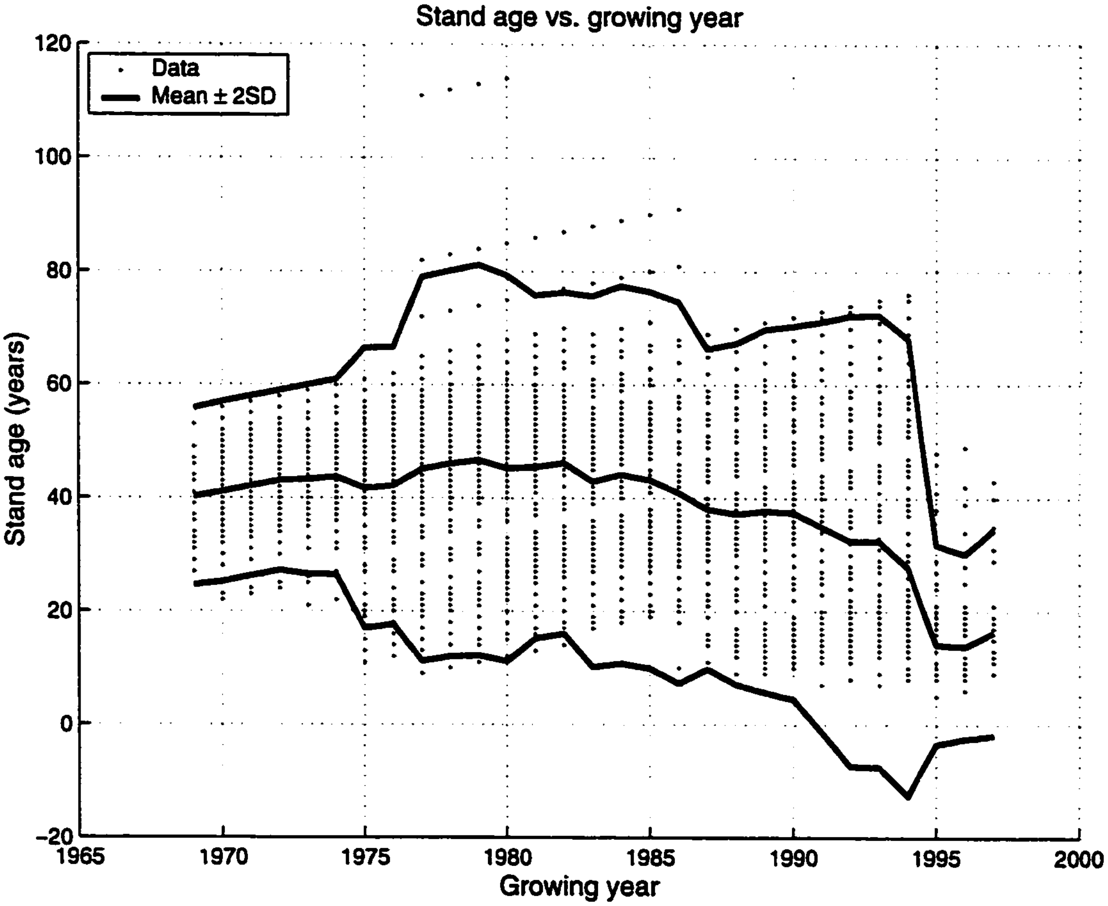

Figure 3.13: Stand age *vs.* growing year with yearly mean and standard deviation. Notice that at approximately 1984 the average stand age begins to decrease. The dot density provides an indication of the number of data points.

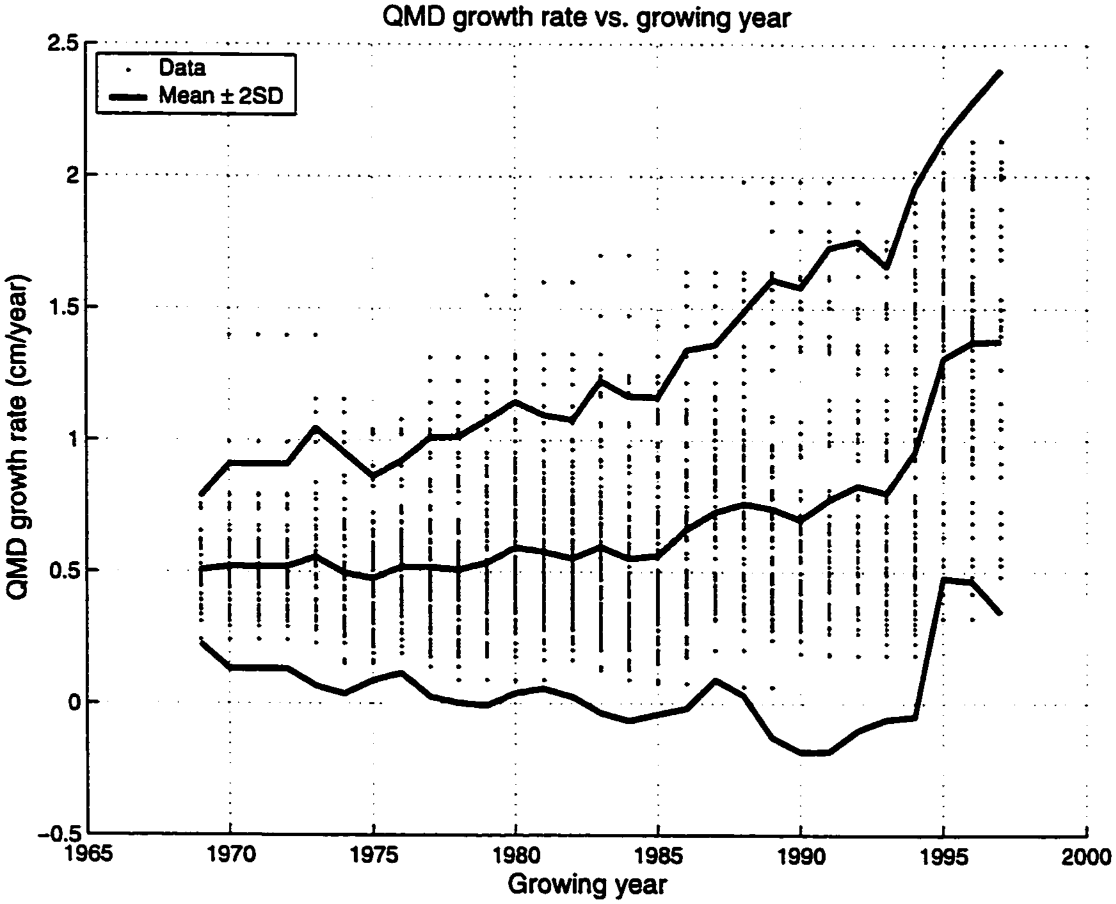

Figure 3.14: Actual QMD PAI *vs.* growing year with the annual mean and standard deviation. Notice that at approximately 1984 the average QMD growth rate begins to increase. The dot density provides an indication of the number of data points.

climate effects may be seen in the plots of the average QMD residual, even if they could not estimated and directly incorporated into the growth and yield model. The lifetime QMD residuals in Figure 3.12 appear to show the correct behavior given the minor drought which occurred in the late 1970s, consistently over estimating QMD, on average, until 1985. The single step QMD residuals in Figure 3.11 show a similar behavior, though more localized and not as pronounced, over predicting QMD near 1975, and again overpredicting just prior to 1985. These results suggest that it may be possible to eventually incorporate climate effects into the Douglas-fir growth and yield model, although with this data set, on average, the climate effects are too small to be detected with any authority.

3.6.6 The base model may already include average climate effects

Recall the discussion of the stand density, QMD, and top height residual standard deviation ratios for the lifetime and single step residual computation scenarios, the ratios s_{il}/s_{i_s}, $i = 1, 2, 3$, respectively. The larger than expected values for these three ratios were used as an argument for a potential climate effect on growth rates. The reasoning was that something must be shifting the trajectories for the individual, annual measurement intervals away from the nominal stand trajectory defined by the S-system model and the initial stand condition. This reasoning is still valid, and through an examination of the model, the least squares parameter estimation process applied to the stand measurement data, and the stand measurement data, it forms the basis for an explanation of why the base model performs so well without the direct inclusion of climate effects.

The actual stand measurement data integrate the climate changes which occurred during the measurement interval into the stand mortality and tree size growth for each stand. The annualized measurement data, then, represent the average annual yield values for the longer measurement period, for each stand. Thus, the annualized stand measurement data already contain, on average, the relevant effects on stand density, QMD, and top height from climate.

The algebraic least squares parameter estimation procedure minimizes the discrepancies among the simultaneous rates of change for stand density, QMD, and top height, and the average stand density, QMD, and top height, for the specified autonomous S-system model. The model specified through this procedure then represents an average phase surface in stand density, QMD, and top height, as in Figure 2.9. This average phase surface approximates the actual phase surface, Figure 2.8, which has a certain amount of thickness, or variability, in each coordinate direction, some of which is attributable to climate, among, possibly, other things.

So, for any particular set of stand density, QMD, and top height values within the actual phase manifold or surface, the model reproduces the average, simultaneous rates of change for the nearby stand density, QMD, and top height values, and when integrated produces the average phase surface. An averaging occurs in the parameter estimation process across all factors not explicitly included in the model, but which are reflected in the stand measurement data, such as site effects or climate. Thus, the least squares estimation procedure averages the site and climate, as integrated into the stand measurements, and, hence, the base S-system growth and yield model implicitly includes average climate effects, indexed simultaneously by stand density, QMD, and top height. This interpretation is also consistent with the mathematical representation of the growth and yield model as an S-system.

Diameter growth is known to be strongly affected by both stand density and climate [24, 77, 78, 102, 12, 153]. Figure 3.15 and Figure 3.16 present intensity contours for actual and estimated periodic annual increment (PAI) values for QMD plotted against log(stand density) and the two year average PDSI values. The actual values were computed from the annualized stand measurement data, and the estimated values were computed using the base growth and yield model given by Equations 2.14, and the average stand density, QMD, and top height values obtained as the midpoints of the annualized stand measurement intervals, to directly predict the simultaneous rates of change. The actual and estimated QMD rates were then binned and averaged, using 20 uniformly spaced bins spanning the range of both log(stand density) and the two year average PDSI values. The contour surfaces were then produced using the Matlab function `interp2`, which performs a recursive interpolation in three dimensions. Three recursion levels were used to fill in the surfaces, interpolating between the binned values. The color scale in each figure is the same, and was determined from the actual QMD rate bin averages. Red indicates the largest QMD rates, dark blue indicates the smallest QMD rates, light green indicates the average QMD rates, and black indicates an absence of data.

The apparent, implicit inclusion of average climate, as represented by two year average PDSI values, on the QMD growth rates of base Douglas-fir growth and yield model becomes clear through a comparison of these two figures. The strong effects of stand density are also readily apparent. Notice the strong correlation in the locations of the peaks and valleys in the QMD rate values, indicated by the red and blue regions in the two figures. The peaks are not as strong for the estimated QMD rates, but this is to be expected, due to the averaging of the least squares procedure. It appears that the base model represents the QMD growth rates quite well, relative to the stand density and climate variables. This final view of the base model results clearly demonstrates the quality of the dynamic, stand level Douglas-fir growth and yield model represented as an S-system; the model strongly identifies almost every feature contained in the actual data. Those features not identified strongly by the model are generally for under-represented stand conditions from the stand measurement data set.

The dynamic, stand level Douglas-fir growth and yield model from Chapter 2 was extended to include the effects of climate. Climate was represented in the model as a scaled, two year average PDSI value, having a mean value of one, indicating the nominal or average climate, and variability representative of the effects of climate on stand development and tree growth. Climate was incorporated into the growth

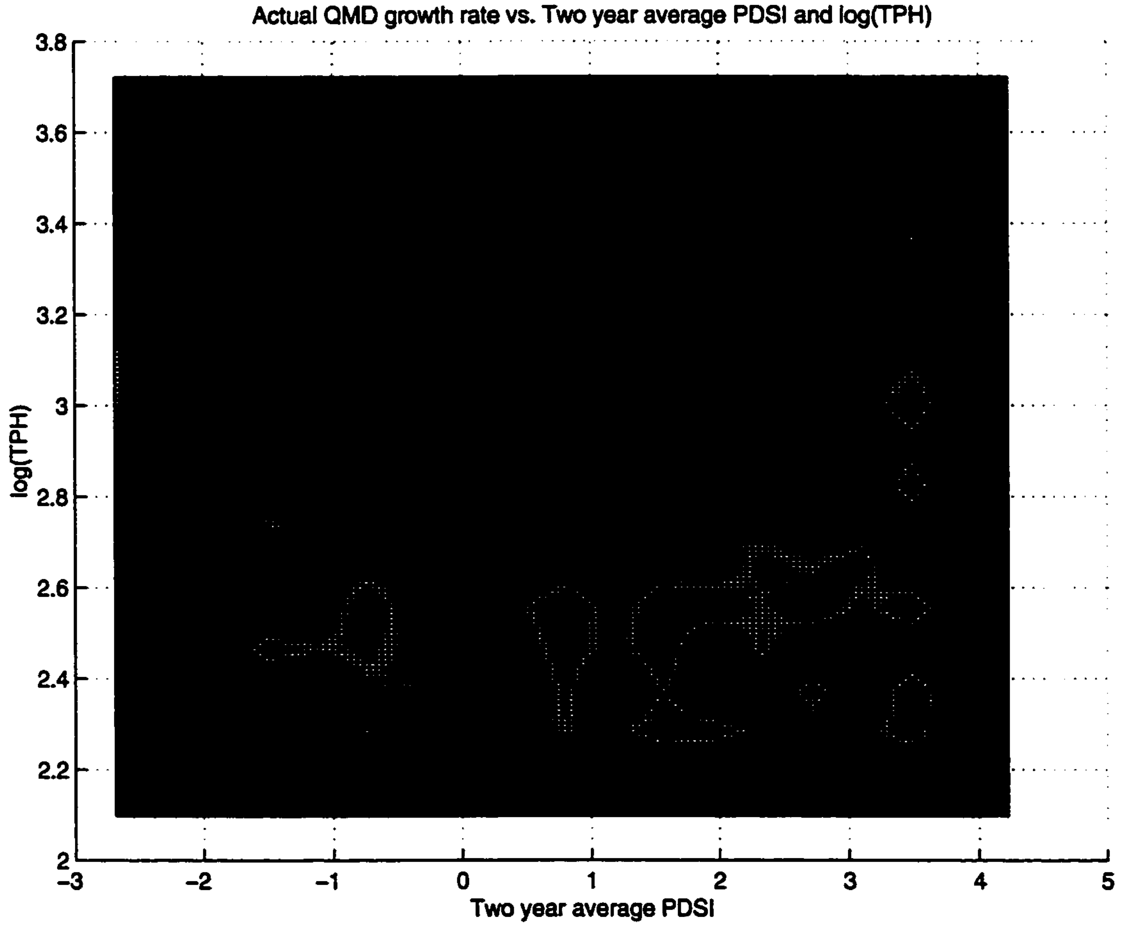

Figure 3.15: Actual QMD PAI *vs.* log(TPH) and two year average PDSI. Red indicates a peak, deep blue indicates a valley, and light green indicates zero. Regions with no data are black. The minimum and maximum actual QMD PAI values are 0.06 cm/year and 2.13 cm/year, respectively. The color scale is determined by these minimum and maximum QMD PAI values, and is identical to that in Figure 3.16.

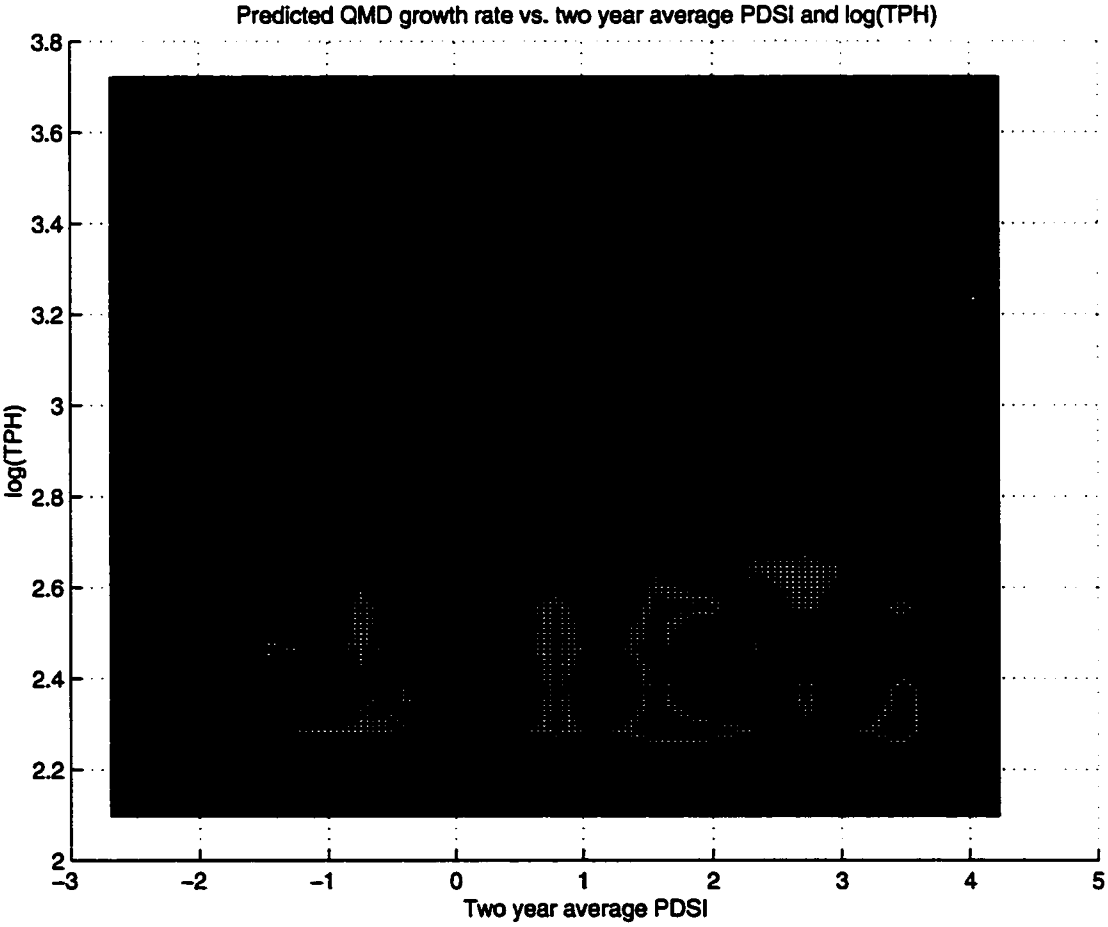

Figure 3.16: Predicted QMD PAI *vs.* log(TPH) and two year average PDSI. Red indicates a peak, deep blue indicates a valley, and light green indicates zero. Regions with no data are black. The minimum and maximum predicted QMD PAI values are 0.08 cm/year and 1.81 cm/year, respectively. The color scale is identical to that in Figure 3.15.

and yield model as a directly proportional multiplicative modifier of the individual growth terms of the S-system growth and yield model. The scaled PDSI values were allowed to be modified independently by S-system exponents for each of the three rate equations, potentially allowing the climate effect to vary among the equations. This manner of extending the base growth and yield model to include climate nests the base model, with out climate, within the broader model which includes climate. The base model may be recovered, seamlessly, by using the nominal climate value of one. This approach produces models which are guaranteed to be consistent.

The results of this modeling effort indicate that average climate effects were not detectable in a manner that would permit their direct inclusion in the dynamic, stand level Douglas-fir growth and yield model, using these data and the specific methods employed. Seven factors were identified as contributing to the inability to directly incorporate climate effects for a stand level, average tree growth and yield model.

Some of the results, however, indicate that it may be possible to extract the climate effects using individual tree measurements and the scaled PDSI values, rather than the stand averages QMD and top height. The stand level model slightly overpredicted QMD and slightly underpredicted top height, on average, but the effects were very small relative to the inherent variability of the tree size residuals. This is promising, and an individual tree model could easily be developed for Douglas-fir in the same Pacific Northwest region, again using the S-system modeling framework, and then extended to incorporate climate effects. The stand level model and the individual tree model could then be used in conjunction to project the growth of the individual trees, which would be aggregated to obtain QMD and top height, and then used to project the development of a forest stand.

The overall approach used, that of creating nested dynamic models and using a least squares model extraction criterion, is still believed to be appropriate for this situation, and although this specific application was not entirely successful, the methodology is still sound. In particular, dynamic model extraction with S-systems worked quite well, appearing to include average climate effects. Finally, an issue was raised concerning the relative importance of stand density and climate on tree growth. This issue deserves further investigation, as it could have ramifications for dendrochronological research.

Chapter 4

CONCLUDING REMARKS AND FUTURE WORK

A pair of nested, dynamic, stand level growth and yield models were developed for plantation Douglas-fir in the Pacific Northwest to assess the effects of climate on stand development and tree growth. The nested models were formulated as an autonomous system of nonlinear ordinary differential equations represented as an S-system, and consisted of a base model without climate effects and an extended model that included climate effects through a multiplicative modifier based on PDSI. The growth and yield model was defined by assuming only that size-density relationships exist for forest stands, that size-size relationships exist within a tree, and that climate modifies stand mortality and tree growth rates.

The growth and yield model projects stand density, QMD, and top height, representing a forest stand as the number of trees per hectare and an average tree size. The base model was calibrated using a wide variety of stand measurement data for Douglas-fir plantations in the Pacific Northwest and performed surprisingly well when stand projections were compared to the calibration data. The base growth and yield model did not explicitly include site index or maximum tree sizes for Douglas-fir, yet it was able to reproduce site index values, measured as top height at an approximate breast height age of 50 years, as well as plausible maximum tree dimensions for Douglas-fir in the Pacific Northwest. Further, the base model did not include any potential growth modifiers to adjust the model projections, as are commonly found in current growth and yield models.

Although climate is known to affect stand development and tree growth rates, the extended growth and yield model was indistinguishable from the base model, indicating that climate effects on growth could not be detected for these data and the stand level model. Both temperature and precipitation were examined to determine whether they could possibly have provided a detectable climate effect, but neither of these variables demonstrated identifiable trends with the base model residuals. These results may indicate that climate effects are not extractable from aggregated stand attributes such as QMD and top height.

Several data based issues were identified as factors which could have confounded the ability to detect a direct climate effect. First, changes in the SMC database composition, in particular a shift to younger fast growing stands that occurred coincidentally with a drought, may have obscured climate effects. Second, not all of the Douglas-fir stands selected were pure Douglas-fir based on the number of stems per hectare, and this would have affected the stand density and QMD rate equations, and may have also contributed to the masking of climate effects. Third, small plot sizes may have exaggerated mortality when the number of trees on a plot were scaled to trees per hectare, again affecting the stand density and QMD equations and possibly masking climate effects. Finally, tree growth patterns in Oregon and Washington are different, and there could have been some cancellation of climate effects caused by combining the data for these two states. Each of these issues may be addressed through a careful consideration of the Douglas-fir data set to select stands which may maximize the ability to detect climate effects, and this work is underway.

4.1 Implications for current growth and yield models

Possibly the most important implication of this research for current growth and yield models is that site index is not a necessary variable for growth and yield modeling. This is not to say that site effects on stand development do not exist, but only that the direct inclusion of site index provides no guarantee that site effects have actually been included in a model. In fact, the role of site index in current growth and yield models may be solely as a second pseudo-height measurement, creating a relative calibration of the increment predictions in a model. Thus, site index may not be providing the *a priori* information on differences in site quality that it was originally intended to do. Further, site index is of limited use for mixed species or mixed age forests, and its ongoing use in these areas of growth and yield modeling may be limiting the effectiveness or applicability of the models developed.

The base growth and yield model did not assume, and incorporate, maximum tree sizes for Douglas-fir, nor were shapes of the growth curves explicitly defined. Instead, the rates of change in QMD and top height were modified by a Chapman-Richards like decay term, and a least squares procedure was used to estimate the parameters of the autonomous S-system model, extracting the shapes of the growth curves from the Douglas-fir stand measurement data. Without maximum tree sizes and the *a priori* determination of growth curve shapes, the base model reproduced tree sizes that were consistent with observed maximum Douglas-fir tree sizes for ages up to and beyond 1000 years, and growth curves consistent with the Douglas-fir data, and the Pacific Northwest in general, were also reproduced by the model. Thus inclusion of maximum tree sizes and the *a priori* definition of growth curve shapes may not be necessary, and, further, may introduce biases into growth and yield models.

The base growth and yield model performs quite well without the application of modifiers, as commonly used in current growth and yield models, to adjust its growth projections in order to obtain better agreement with the stand measurement data used for the model calibration. The primary reason for this is most likely the simultaneous nature of the S-system growth and yield model. Within the S-system growth and yield model, the three state variables are always considered simultaneously. The model definition and parameter estimation or calibration were performed simultaneously for the three model equations, and the use of the model to project stand attributes into the future is also a simultaneous process. In contrast, many current growth and yield models treat their model equations independently, obtaining separate fits for each equation. The simultaneous treatment of the three state variables in the S-system growth and yield model is consistent with the actual biology of forest stand development and tree growth, and may explain the quality of the base S-system model.

Finally, the failure of the extended growth and yield model to detect, and hence incorporate climate effects, is also significant. This may imply that improvement in the current statistical-empirical growth and yield models obtained by the addition of climate variables may not be attributable to actual climate effects. The improvement may simply be a statistical artifact caused by the addition of one or more new variables that are correlated with the model errors, and not indicative of a cause-effect relationship. The development of an individual tree S-system model may help to clarify this issue.

4.2 Future work

Several factors were identified as contributing to the inability to obtain a direct climate effect on tree growth in the stand level growth and yield model. The development of an individual tree submodel that could be used in conjunction with the stand level model may allow the direct inclusion of climate effects. The growth response of individual trees to climate, even within the buffered research plots, should be more pronounced than the average climate response for a group of trees, some of which may have done better, some of which may have done worse.

The next step in applying the S-system modeling framework to growth and yield modeling is the development of an individual tree model, whether it permits the direct addition of climate or not. The individual tree model may then be used with the stand level model to project stand development through a three step procedure. First, compute the current QMD and top height for the stand and obtain the future stand density by projecting the current stand density, QMD, and top height one year. Second, grow each tree in the stand for one year. Third, kill the trees that are the least vigorous to obtain the projected stand density. The least vigorous trees are typically determined by considering both tree size and growth rate. Initial experiments with estimating an individual tree S-system model appear promising.

The S-system modeling framework, as presented for the Douglas-fir growth and yield model, is not specialized in any way to Douglas-fir. Application of the techniques and model developed here to other tree species should be performed to see how widely applicable this approach may be. In particular, the method should be applied to tree species which do not have strong pre-formed growth patterns to determine whether the allometric assumptions of the size-density and size-size relationships in the model remain valid for these types of trees.

The base growth and yield model may be extended to include effects derived from the long term nutrient status of a site in a similar manner to that used for climate effects. A new S-system state variable may be defined as a directly proportional nutrient effect using some set of soil or site characteristics, e.g., carbon-nitrogen ratio and soil type. Two approaches for including this modifier readily present themselves. First, and simplest, is to add soil nutrient status as a decay process which is based on the nutrient uptake by the trees. This involves the addition of a fourth rate equation, $\dot{X}_5$, having the form

$$\dot{X}_5 = -\beta_5 X_5^{h_{55}} \tag{4.1}$$

for the nutrient decay process. This variable may then be included in the S-system model in the same manner as the climate effect. The second approach recognizes that nutrient deposition may occur, and adds an exogenous S-system state variable representing nutrient deposition in addition to the decay process, yielding an equation of the form

$$\dot{X}_5 = \alpha_5 X_6^{g_{56}} - \beta_5 X_5^{h_{55}}. \tag{4.2}$$

Again, this variable may then be included in the growth and yield model in the same manner as the climate effect.

The S-system modeling framework allows the development of highly localized growth and yield models with minimal effort. All that is required is stand measurement data and standard least squares optimization software. The use of this technique to develop highly customized growth and yield submodels for different areas within a larger stand is conceivable, allowing for the development of customized treatment regimes for the smaller areas.

The failure to obtain a direct climate effect in the Douglas-fir growth and yield model could imply that stand density effects on tree growth, via size-density relationships, are much stronger than climate effects on tree growth. A better understanding of the relative importance of size-density effects and climate effects on tree growth is therefore necessary to correctly identify the type of growth influence.

Finally, work to extend the dynamic S-system based growth and yield modeling framework to include other treatments, such as fertilization or pruning will continue. In addition, the development of a software system for stand projection allowing the inclusion of a variety of silvicultural practices is planned.

4.3 Closing remarks

The S-system modeling framework has the two primary features which make the statistical-empirical modeling framework so popular. First, the S-system modeling framework permits the straightforward definition of models based on state variable interactions, much like the typical statistical-empirical modeling framework does through correlation among the state variables. The S-system modeling framework therefore retains the convenience and flexibility of the statistical-empirical modeling framework for model definition. Second, model calibration is achieved through the solution of a straightforward optimization problem, generally a least squares or maximum likelihood problem. The S-system modeling framework has the additional benefit of automatically accounting for temporal autocorrelation among repeated measurements for a forest stand or tree, something the typical statistical-empirical modeling framework cannot do directly. This added benefit is derived solely from the fact that the S-system modeling framework uses a mathematical representation that concisely represents dynamic systems.

The S-system modeling framework is not a panacea, as for all dynamic modeling situations, as it is not a substitute for understanding. It does, however, provide a consistent and powerful methodology for constructing dynamic models which inherit the essential characteristics of the dynamic behavior represented by the data for a particular phenomenon, here forest stand dynamics and tree growth. In situations where a model for a complex dynamic process is unavailable or a dynamic process is poorly understood, the S-system modeling framework may provide a means for capturing the essential dynamics of the process. Dynamic models produced in this manner are not substitutes for detailed process based models and the understanding necessary to produce them, but they do provide a bridge from the descriptive, statistical-empirical models to the dynamic, mathematical-explanatory models.

The path that forest growth and yield modeling must take in the future leads from a descriptive, statistical-empirical modeling framework to a dynamic, mathematical-explanatory modeling framework. This path, if followed, will lead to a better understanding of forest ecology, and to the development of more realistic forest models. This research provides but one step on the path toward this goal. As with any journey and the first steps upon it, though the goal is clear, the path ahead is not. As more is demanded of forests, the stewards of those forests, their understanding of the forests, and the tools they use must be up to the task of meeting the demands. At the present time, forest growth and yield modeling research appears to be at a fork in the path. One branch leads down a path which limits growth and yield modeling research by adhering to the classical applications of wood production and a statistical-empirical modeling framework. The other path offers limitless opportunity for growth and yield modeling research by broadening the scope of the research to include the dynamic nature of forest ecosystems in their entirety. The choice of branch is clear: a forest ecosystem model may be used as a growth and yield model if it produces tree dimensions, but a growth and yield model which only produces tree dimensions cannot be a forest ecosystem model.

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VITA

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I have excellent problem solving and analytical skills. I have a strong background in applied mathematics, statistics, and computer science, and am currently applying these skills in forestry. I am developing a regional dynamic growth and yield model for plantation Douglas-fir in the Pacific Northwest. I have also developed a customized tree list generation database for growth and yield modeling. I have a demonstrated track record in understanding complex problems, and determining appropriate solutions in a broad spectrum of areas related to mathematics and computer programming. I am highly goal oriented, with an insatiable desire to learn. I work well individually or in a group setting. I have excellent verbal and written communication skills and experience in public speaking and lecturing.

Professional Experience

September 1998 - July 2000 The Stand Management Cooperative, University of Washington Forest Resources, University of Washington, Seattle. I designed, developed, implemented, tested, and documented a set of programs and a portable subroutine library to create and manage a customized database for tree list generation. The programs were written in portable Fortran 90/95. The tree list generation database currently supports the generation of stands dominated by Douglas-fir, western hemlock, and mixtures of these two species for the Pacific

Northwest, including Oregon, Washington and southern British Columbia, west of the Cascade Mountains. The tree list generation database software may be extended to include other dominant species, and the methodology is not limited to any particular location.

February 1992 - August 1996 Jet Propulsion Laboratory, Pasadena, California. Member of the Technical Staff. Navigation and Ancillary Information Facility (NAIF), Navigation Systems Section. I was involved in all aspects of software development: design, implementation, documentation, customer support, and software maintainance. The software supported an information system developed to support the space science community by easily allowing the correlate spacecraft and planetary position and velocity information with the scientific data collected by a variety of instruments. The NAIF group develops and distributes programs and a portable subroutine library for correlating spacecraft position and orientation information with scientific data.

Summer 1990 Computer Programmer. IBM Corporation, Systems Integration Division, Houston, Texas. Developed prototype client and server applications for a remote data acquisition product using the C programming language and the socket interface of the TCP/IP communications protocol.

Summer 1989 Computer Programmer. Amoco Production Company, Research Center, Tulsa, Oklahoma. Developed a graphical user interface for use with geologic well log data and extended the capabilities of existing software to support conversion of files and automatic generation of scripts for third party geologic well log analysis software.

Summer 1988 Summer Intern. MPSI Americas, Inc., Tulsa, Oklahoma. Examined the numerical optimization methods used in a retail market analysis model.

Identified problems and proposed alternative optimization strategies. Implemented various methods to test the ideas and presented results and recommendations to the director of research and development.

Education

June 2001 Ph.D. in Forest Management, specializing in growth and yield modeling. College of Forest Resources, University of Washington, Seattle, Washington. Dissertation title: "Dynamic growth and yield modeling with climate: A model for plantation Douglas-fir in the Pacific Northwest."

December 1998 - Present University of Washington, Seattle, Washington. Ph.D. Candidate Quantitative Resources Management, College of Forest Resources. Research topics included tree list generation methodologies and growth and yield modeling.

September 1996 - December 1998 University of Washington, Seattle, Washington. Graduate student in the Doctoral Program in Quantitative Resources Management, College of Forest Resources. Cumulative G.P.A.: 3.80/4.00.

September 1990 - December 1991 Rice University, Houston, Texas. Doctoral program in Mathematical Sciences, Cumulative G.P.A. 4.04/4.33. No degree obtained.

Summer 1991 Rice University, Houston, Texas. Investigated model formulation and solution methods for large scale mixed integer programming problems with applications to minimizing flight delay times in the United States air traffic system.

September 1989 - May 1990 Rice University, Houston, Texas. Doctoral program in Statistics, Cumulative G.P.A. 4.07/4.33. No degree obtained.

September 1987 - May 1989 The University of Tulsa, Tulsa, Oklahoma. Applied Mathematics M.S. degree obtained May 1990, with an emphasis on numerical analysis and numerical computing. Graduate GPA: 4.000/4.000. Master's Thesis: "Nonparametric Probability Density Estimation Using Normalized B-Splines."

September 1983 - May 1987 The University of Tulsa, Tulsa, Oklahoma. Mathematics B.S. degree obtained May 1987, Magna Cum Laude, with a Computer Science minor. Cumulative GPA: 3.808/4.000. Senior Honors Thesis: "The Calculation of Pharmacokinetic Parameters through the use of Symbolic Computation and Rotational Discrimination Non-Linear Regression Analysis."

Computing and Software

Programming languages Fortran 95, Fortran 90, Fortran 77, the C Programming language, MATLAB (matrix-vector oriented computation and analysis), S-Plus (statistical data analysis), VAX 11/750 and 11/780 assembly language, Intel x86 assembly language (8088, 8086, 80286), MACSYMA and MAPLE (symbolic computation). Also mixed language programming in C and Fortran 77 programming on UNIX platforms: Sun, Hewlett-Packard, Silicon Graphics, NeXT.

Software systems Matlab, TeX and LaTeX for document preparation; graphical user interface development with the X-Window System and the MOTIF toolkit on UNIX platforms Dreamweaver, by Macromedia, for web site development and maintenance; and Microsoft Office products: Word, Access, Powerpoint.

Hardware and operating systems Sun workstations and Sparcstations under the UNIX operating system (Solaris and SunOS), NeXT workstations under the NeXTStep operating system, Silicon Graphics workstations under the UNIX operating system, Hewlett-Packard workstations under HP-UX, IBM style PCs under Windows, Windows 9x/NT, PC-DOS or MS-DOS, IBM mainframes under the CMS and TSO/MVS operating systems, DEC Micro VAX II, VAX 11/750, and VAX 11/780 under the VMS operating system.

Awards and Honors

Jet Propulsion Laboratory Certificate of Recognition for the NAIF Toolkit, National Aeronautics and Space Administration, September 19, 1997.

Jet Propulsion Laboratory Mention in *NASA Tech Briefs*, "The SPICE System," October 1997, Vol. 21, No. 10.

University of Washington, Seattle, Washington Full tuition Scholarship in the Quantitative Resources Management Ph.D. Program in the College of Forest Resources.

Member: Alpha Chapter, Xi Sigma Pi, Forestry Honor Society.

Rice University; Houston, Texas Full tuition scholarship in the Ph. D. program of the Department of Mathematical Sciences, now the Department of Computational and Applied Mathematics.

Teaching assistantship and full tuition scholarship in the Ph.D. program in the Department of Statistics.

The University of Tulsa; Tulsa, Oklahoma Teaching assistantship and full tuition scholarship in the Department of Mathematics and Computer Sciences. Honors Scholarship, University Scholar Scholarship. Graduated magna cum laude, G.P.A. 3.808/4.000, May 1987.

Member: University of Tulsa Honors Program; Phi Gamma Kappa honor society.

Publications

1. Gehringer, Kevin R. (1990) Master's Thesis, Department of Mathematics and Computer Science, The University of Tulsa. Nonparametric Probability Density Estimation Using Normalized B-Splines.
2. Gehringer, Kevin R and Richard A. Redner (1992) "Nonparametric Probability Density Estimation Using Normalized B-Splines." Communications in Statistics, Part B: Simulation and Computation, Vol. 21, No. 3, pp 849-878.
3. Gehringer, Kevin R., Gonyea, Bob, Turnblom, Eric C. (1997) Growth response of thinned Douglas-fir stands to fertilization at Foley Ridge. A technical report prepared for the Stand Management Cooperative, College of Forest Resources, University of Washington.
4. Gehringer, Kevin R. and Turnblom, Eric C. (1997) Pruning response in Douglas-fir, an initial analysis. Stand Management Cooperative 1997 Annual Report.
5. Gehringer, Kevin R. and Turnblom, Eric C. (1998) "An Evaluation of the Tree and Stand Simulator (TASS) Fertilization Response." A technical report prepared for the British Columbia Ministry of Forests, Research Division.

Presentations and Conferences

1. Dynamic Growth and Yield Modeling: An application of S-systems to Plantation Douglas-fir in the Pacific Northwest. Stand Management Cooperative Fall Meeting 2000, a joint meeting with the Virgina Tech Cooperative. Worthington Conference Center, St. Martin College, Lacey, Washington.

2. Tree list generation for growth and yield modeling. Project final report. Stand Management Cooperative Spring Meeting 2000. McMenamins, Troutdale, Oregon.

3. Constructing a virtual forest: A nearest neighbor tree list generation procedure. Invited presentation at the Tree List Generation Symposium, January 21, 2000, University of British Columbia, BC, Canada.

4. Tree list generation for growth and yield modeling. Project status report. Stand Management Cooperative Fall Meeting 1999. Pacific Forestry Center, Canadian Forest Service, Victoria, British Columbia, Canada.

5. Is site index necessary for growth and yield modeling? Silviculture Lab Brown Bag Lunch, November 29, 1999, College of Forest Resources, University of Washington.

6. An Evaluation of the Fertilization Response of the Tree and Stand Simulator (TASS). Western Mensurationsists Conference, 1998, Port Ludlow, Washington.