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Investigation of a Multi-Component Intervention Addressing Mathematical Reasoning and Self-Regulation of Behavior for Students with Emotional/Behavioral Disabilities

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A dissertation submitted in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

University of Washington

2013

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Program Authorized to Offer Degree:

College of Education

University of Washington

Abstract

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For students with Emotional/Behavioral Disabilities (EBD), negative student outcomes are the poorest across disability categories, including high rates of school dropouts, unemployment and incarcerations. Mathematically, students with EBD receiving instruction in special education settings experience practices not consistent with recommendations for high quality mathematics instruction, resulting in 73% of students performing below the 50th percentile on standardized mathematics achievement tests (Wagner, Kutash, Duchnowski, Epstein, & Sumi, 2005). This study focused on the development and implementation of a multi-component mathematics/behavior intervention addressing mathematical reasoning and meaningful mathematics participation for elementary students with EBD. Participants for this study included 3 upper elementary students who were identified with EBD and low performance in mathematics. Each student participated in one-on-one instruction approximately 4 days per week for 20 minute over 4 weeks (n=16 sessions). A single-subject multiple baseline research design was implemented and intervention sessions were video recorded for further analysis. The intervention was effective in improving student outcomes in developing a deeper level of place value understanding, more efficient strategy use and an increase in productive participation.

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Chapter 1: Introduction and Statement of Problem

For student with Emotional/Behavioral Disabilities (EBD), behavioral and academic deficits are interactive and highly related (Kauffman, 2001; Lane, 2004; Reid, Gonzalez, Nordness, Trout, & Epstein, 2004). Intuitively, the relationship between academics and behavior for students with EBD is logical given the identification criteria for this disability category specifies that the emotional/behavioral problems experienced by youth must adversely affect their educational performance (IDEIA 2004; Reid et al., 2004). This academic and behavioral relationship may help explain the compilation of negative outcomes exhibited within the social/behavioral and academic domains. Behaviorally, students with EBD fall victim to substance abuse, high unemployment rates and incarcerations with approximately 70 percent estimated to be arrested at least once within their lifetime (Greenbaum et al., 1996; U.S. DOE, 2005; Wagner, Kutash, Duchnowski, Epstein, & Sumi, 2005). Further, poor academic outcomes include below grade level functioning and high levels of retention and dropout rates resulting in about 50 percent of students with EBD dropping out of school, the highest dropout rate across disability categories (Greenbaum et al., 1996; Wagner et al., 2005).

Complicating matters, the school context in which services are provided are also reported to have their own set of challenges related to teacher preparedness, programing and instruction. Teachers and administrators report that students with EBD create some of the most challenging situations to handle, resulting in less instructional engagement time and high rates of disruptiveness to the classroom environments. While 75% of students with EBD spend part of their school day in general education settings, the challenges presented by this population may explain why of these students at least 56% of their school day is spent in special education settings. However, despite spending more time in these specialized settings, Kauffman (2001)

reports that the instructional and classroom management programs that serve students with EBD are limited in meeting the academic, social, and behavioral needs of the youth serviced. From an academic standpoint, special education classrooms serving students with EBD are described as consisting of a dependency on paper-pencil tasks (i.e. workbooks & worksheets), limited access to high quality instruction, a lack of curricular adaptations and attention to academic needs (Davis, et al., 2003; Jackson & Neel, 2006; Nelson, 1996; Shores et al., 1993; Wehby, Symons, & Shores, 1995). In addition, students with EBD have been observed to demonstrate a lack of academic engagement, specifically “pervasive boredom and apathy” (Nelson, 1996, p. 146-147) towards learning.

A closer examination of the academic performance of students with EBD suggests their performance remains at best stable over time (reading and writing), but often results in declining performance (mathematics; Anderson, Kutash, & Duchnowski, 2001; Lane, Barton-Arwood, Nelson & Wehby, 2008, Nelson, Benner, Lane, & Smith, 2004). When comparing students with EBD to students with Learning Disabilities (LD), longitudinal studies suggest students with EBD perform worse academically across time than do students with LD (Anderson et al., 2001). While poor academic performance is exhibited across core subject areas, students with EBD seem to struggle the most with mathematics. For example, 97% of students with EBD ages 12-14 were reported as performing below grade level in mathematics (Greenbaum et al., 1996) and more than a decade later Wagner et al. (2005) reported similar low achievement rates with 43% of students with EBD scoring in the bottom 25th percentile, and a combined 73% of the population scoring below the 50th percentile on the *Woodcock-Johnson III Tests of Achievement* – Mathematics Calculation subtest (WJ III; Woodcock, McGrew, & Mather, 2001). When comparing mathematics results by age, Nelson and colleagues (2004) found a statistically

significant decline in mathematics performance of children (elementary) compared to adolescence (middle and high school). Further supporting these findings, Lane and colleagues (2008) found that students with EBD performed well below the 25th percentile for elementary (22nd percentile) and secondary students (13th percentile), while Wagner et al. (2006) found a similar decline in mathematical calculations from elementary to high school, dropping from the 34th percentile to the 28th percentile.

Mathematics Education and Special Education

Addressing mathematics instruction for students with EBD is further complicated by the deeply rooted and often opposing traditions of mathematics education and special education (Baroody, 2011; Maccini & Gagnon, 2002; Woodward & Montague, 2002). Tension between the two communities circulates around disagreement in pedagogical practices, instructional emphasis on particular mathematical knowledge domains and philosophical debates on individual learning deficits (Woodward & Montague, 2002).

Continuum of Teacher-Centered to Student-Centered Learning. At the center of this dispute is how children learn, ranging across theories of behaviorism, cognitive psychology, and constructivism (Woodward, 2004). The special education community consistently advocates for instructional practices which are teacher-directed (grounded in behaviorism) and empirically supported by evidence-based research (Carnine, 1997; Mercer & Mercer, 2001). For example, in the recent *Response to Intervention (RtI) Math Practice Guide*, Gersten et al. (2009) advocate for explicit and systematic instruction as described in *Recommendation 3* to include “providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review” (p. 21). However, there exists a great disparity in interpretations of explicit instruction such as Baroody (2011) who defines special

education practices as “a traditional direct-instruction-and-drill approach...based on memorizing basic skills by rote” and “typically consists of verbal directions or rules regarding written symbols or procedures...unrelated to real, meaningful, and purposeful activities or experiences” (p.23). Critics of rote learning and procedural instruction believe that mathematical learning must involve sense-making and without this step, instruction will result in answers that do not make sense as they disconnect knowledge from its context (Gustein, Lipman, Hernandez, & de los Reyes, 1997). Further, explicit instruction has yet to “transfer to higher order cognitive skills such as reasoning and problem solving” (Woodward & Montague, 2002, p.91).

To address some of the contention, mathematics educators Karp & Voltz (2000) propose looking at such learning theories along a continuum of teacher-directed to student-centered learning. For instance, cognitive psychology moves beyond behaviorism to explain the mental processes that take place within the mind, and help explain how knowledge comes to be. Unlike behaviorism, cognitive psychology explores the mental activities of the mind which are not observable behaviors (Amirault & Branson, 2006). Cognitive frameworks have demonstrated their potential to shift towards the middle of the continuum in which teacher-direction is gradually released and involves more of an apprenticeship relationship between the teacher and student (Karp & Voltz, 2000). A central shift in becoming more student-centered offers a promising direction for collaboration and agreement between the two communities (mathematics education and special education), and offers further insight into the processes of learning (Woodward & Montague, 2002). For example, Gersten & Baker (1998) propose reconceptualizing instruction for students with LD using an integrated approach to explicit instruction and situated cognition. Similarly, Bottge and colleagues (2007) built on *Anchored Instruction* (Cognition and Technology Group at Vanderbilt University, 1990) developing

Enhanced Anchored Instruction, a situated problem-solving approach incorporating aspects of explicit instruction and scaffolds providing further supports and practice. An important dimension to cognitive learning is the individual's construction of mathematical knowledge without much discussion of the social construction of mathematical knowledge encompassing active, social engagement (Ball & Bass, 2000).

On the opposite side of the continuum, are constructivist theories of learning with a fully student-centered learning approach in which the learner constructs knowledge using discovery-learning pedagogy. This approach has been met with scrutiny in which an over simplified or incorrect interpretation of discovery learning is often viewed as providing learners with limited teacher interaction such as placing manipulatives in front of a learner with the hope that the intended mathematical concepts will appear salient to the learner, thus the teacher is in a passive role (Stein, Kinder, Silbert, Carnine, 2006). This ambiguous pedagogical interpretation is not, however, one that is advocated for within the field of mathematics education. Instead, a preferred approach to student-centered learning is the adoption of sociocultural learning theory viewing the learner as an active participant within a community of learners. With this constructivist approach to learning, knowledge is socially constructed, connecting children's informal mathematical knowledge to a more formal understanding of mathematics (Ball & Bass, 2000). This inquiry-oriented or guided-discovery approach to learning is further explained by Vygotsky's Zone of Proximal Development (ZPD), where the learner engages in a mediated process of learning both with external tools (i.e. concrete manipulatives, visual representations) and with more experienced peers (Vygotsky, 1978). A dominant, pedagogical practice associated with sociocultural learning theory is classroom discourse generally defined as a learning community which co-constructs knowledge through the mutual interactions of the

teacher, students, and content (Cazden, 2001). Strategic teacher questioning and talk moves supported through deep content knowledge allows the teacher to make explicit connections across student thinking and to move discussions towards new or purposeful ideas within the content discussed (Ball & Bass, 2000; Kazemi & Stipek, 2001; Lampert, 1990; O'Connor & Michaels, 1993; Sherin, 2002; Yackel & Cobb, 1996).

The special education community has largely disputed inquiry-oriented learning as not offering appropriate structure, scaffolding, or making salient intended mathematical concepts (Geary, 2003; Karp & Voltz, 2000; Woodward, 2004). Difficulties identified, particularly for students with LD, include challenges with cognitive load demands, working memory deficits, difficulty resonating with key mathematical concepts discussed, and a general passive role when working with peers not identified as having a disability (Baxter, Woodward, Voorhies, & Wong, 2002; Miller & Hudson, 2007). However given the widespread movement towards reform-based mathematics or inquiry-based learning, the above challenges must be recognized and further researched within the context of reform-based mathematics to best meet the instructional needs of students with disabilities (Woodward, 2004).

NCTM Standards and Basic Skills. Closely related to the debate on how students with disabilities learn, is special educators' tendency to focus on basic skills more closely aligned with traditional mathematics, thus limiting students access to general education curriculum as required by IDEIA (Maccini & Gagnon, 2002). In mathematics, access to general education curriculum means addressing the *National Council of Teachers of Mathematics (NCTM) Standards* emphasizing problem-solving, mathematical reasoning and communication of mathematical thinking. As an example, mathematics educators address the *Standards* by actively engaging students in making conjectures, justifying and questioning each other's ideas and

operating as mathematicians leading to deep levels of mathematical understanding (Kazemi & Stipek, 2001; Lampert, 1990; Martino & Maher, 1999; Yackel, 2002). This disconnect in addressing the *NCTM Standards* across general and special education settings is evidenced in the common practice of most special educators to rely on less rigorous curriculum, namely computation (Jackson & Neel, 2006; Maccini & Gagnon, 2002; Montague & Jitendra, 2006). However, as currently written, the *Standards* require special educators to possess the requisite skills in making appropriate curricular and pedagogical changes in terms of individualizing instruction. Rivera (1992) cautions special educators that reform-based practices have “not been researched sufficiently in the special education literature” (p.3) and that access to curricula is dependent on empirically validated and efficacious pedagogy for teaching students with disabilities. Here lies the dilemma for the field of special education in which advocating for access to general education (*Standards*-based) curriculum, but not supporting the pedagogy, is an unresolved contention requiring “a research-based model that articulates how skills and concepts are taught to students with LD [and EBD (Maccini & Gagnon, 2002)] in the context of reform-based mathematics” (Woodward, 2004, p.25).

Deficits Model. A final contention, related to this debate is special education’s focus on student needs or deficits. Research conducted on mathematics learning disabilities (MLD) has identified a variety of subtypes such as memory, language processing, cognitive development (Bryant, Kim, Hartman, & Bryant, 2006), procedural, semantic and visuospatial (Geary, 2004) all of which require adaptations to the learning environment when students access general education curriculum. Adaptations are thought of as individualized based on student strengths and needs, relevant to the intended objectives taught and effective so that learning can occur (Bryant et al., 2006). In contrast, adapting instruction based on deficits of an individual is argued

by mathematics educators as placing the deficit within the individual, operating under a deficit model, and further perpetuating the historical trend that not all students are capable of learning (Gutierrez, 2007). To be clear, while the field of mathematics education recognizes that some students do indeed have MLD, it is viewed as problematic when students identified as such are victims of ineffective or inappropriate instruction (Baroody, 2011) and therefore situating deficits within individuals and families further exasperates the perception of their inadequate mathematical ability (Gutierrez, 2007).

As a result, researchers are beginning to define populations as having mathematical difficulties (MD) or MLD with the former resulting from a host of potential causes, but not biologically based as is MLD (Mazzocco, 2007). MD is often associated with low mathematics achievement and students with MD are typically identified using a cutoff score from a standardized mathematics achievement assessment. Comparatively, students with MLD are believed to have a biologically based mathematics disorder, however consensus on definitions and criteria have yet to be determined (Mazzocco, 2005). Researcher studying students who struggle in mathematics should therefore be mindful of their intended population and generalizations made across the two categories (Mazzocco, 2007).

Moving Forward. Despite the friction between the fields of special education and mathematics education, collaborative efforts have evolved within the past 5-10 years and offer guidance for educators and researchers looking to identify commonalities for which to move forward (Boyd & Bargerhuff, 2009). In particular, Boyd & Bargerhuff (2009) share insight on some of the consistencies between the two fields including: a) the central role of building on students' background knowledge, b) relating content to meaningful and real-life context and c) the critical role highly effective general education instruction plays in limiting the

misidentification of students with disabilities. Another recommendation to facilitate collaboration is an interwoven approach in which various instructional approaches are drawn on based upon student needs (Karp & Voltz, 2000; Woodward & Montague, 2002). With this approach, teacher-centered learning may be a starting point for instruction, but the goal is to move students towards student-centered learning aligning more similarly to reform-based instruction. Such collaborative visions require practitioners and researchers from both fields to build a community focused on addressing the lack of collaboration and research gap which impact mathematics instruction for all students. In pursuit of this goal, a movement was generated in May 2011 to bring together forty special educators and mathematics educators hosted by Council for Exceptional Children (CEC) and NCTM to begin substantive collaboration and research to support students struggling in mathematics. Given the state of mathematics performance for students with EBD and the current collaborative movement, this study intends to look at how research informs mathematics instruction for students with EBD.

Purpose of Study

Based on the current status of research within the field of EBD, there are strong implications that students with EBD would benefit from further research addressing mathematics and behavior using a comprehensive instructional approach. The development of a mathematics/behavior intervention addressing mathematical reasoning skills for elementary students identified for EBD is imperative. This population of students requires a concerted effort toward the investigation of a multi-component intervention that extends beyond procedural knowledge related to basic computation, and instead targets mathematical reasoning skills and challenging behaviors/decreased motivation, which disrupts the learning environment and impedes students' ability to learn. Specifically, the purpose of this study was to investigate the

effects of a multi-component intervention targeting the development of place value concepts related to number composition and operations using multiple modes of representation (manipulatives and visual images) and increased opportunities to communicate reasoning and connections made across mathematical concepts taught. In addition, to address anticipated disruptive behaviors and increase active participation, self-regulation strategies targeting productive behaviors in mathematics were embedded resulting in a multi-component intervention tailored to the needs of students with EBD.

Research Questions

The questions for the proposed research study include:

Research Question 1: *What effect does the multi-component mathematics/behavior intervention have on student understanding of place value, number composition and efficient strategy use for problem-solving tasks?*

Research Question 2: *What effect does the multi-component mathematics/behavior intervention have on increasing on-task behaviors and decreasing disruptive behaviors?*

Research Question 3: *What effect does the multi-component mathematics/behavior intervention have on the frequency of statements students make articulating mathematical reasoning and appropriate use of mathematical materials?*

Chapter 2: Literature Review

Developing high quality mathematics instruction for students with EBD must consider that educational needs and behavioral issues of these students. Careful consideration is given not only to the emotional/behavioral needs, but also the development of mathematics instruction more aligned with best practices advocated for within the field of mathematics education.

Several aspects of the literature were reviewed in pursuit of this goal including: a) the intersection of academics and behavior, b) current practices of mathematics instruction and c) promising intervention components.

Mathematics and Behavior

Relationship between Academics and Behavior. Research within the field of EBD has sought to understand the etiology of comorbid academic and behavioral deficits which typically manifest as academic underachievement and delinquency (Hinshaw, 1992; Lane, Wehby, Little, & Cooley, 2005; Sutherland, Lewis-Palmer, Stichter, & Morgan, 2008). The preschool years are a crucial time in the development of this comorbid relationship between academics and behavior where associations are found to exist before formal schooling begins (Hinshaw, 1992). Further, the relationship is not a unidirectional relationship starting as one deficit and leading to the other. Rather, the overlap of academics and behavior are defined as “complex, bidirectional” and “reciprocal in nature” (Sutherland et al., 2008, p. 225, 229), ranging between 10-50% (depending on definitions; Hinshaw, 1992). The reciprocal connection exists between students who display externalizing behavior patterns and have co-occurring academic deficits, as well as students with academic deficits who display challenging behaviors (Lane, 2004; U.S. Dept of Ed, 2001). The Twenty-Third Annual Report to Congress summarizes the overlap stating that “academic and social failures are reciprocally and inextricably related” (U.S. Dept of Ed, 2001, p. I-34).

The most common correlates of students with EBD is attention-deficit/hyperactivity disorder (ADHD) estimated to affect two-thirds of the population or approximately 65 percent of elementary students with EBD (Wagner et al., 2005). Since limited studies have investigated the relationship between mathematics and behavior for student with EBD, the literature investigating mathematics and ADHD is drawn up. Specific to students with ADHD, approximately 85 percent are identified with the inattention subtype impacting their mathematics performance (Marshall, Hynd, Handwerk, & Hall, 1997). Zental and Smith (1993) conducted a comparison study investigating math fact fluency (i.e. accuracy & rate) of 92 elementary boys (grades 2 to 5) grouped as having hyperactivity, hyperactivity and aggression or not having either hyperactivity or aggression. Results from this study found that students with hyperactivity and aggression had the most severe difficulty, followed by students with hyperactivity only and finally students without hyperactivity (Zentall & Smith, 1993). Unsurprisingly, inattention is more pronounced for students with ADHD than students with LD and further, is thought to relate to difficulty with auditory processing in which visual formatting or cues are processed more efficiently assuming a visual cue is not embedded within a more complex visual (Zentall, 2007). In addition, difficulty focusing on salient mathematical concepts is another prominent characteristic impeding mathematical achievement for student with ADHD (Zentall, 2007) and is further evidenced by findings from Baxter et al. (2002). In Baxter et al. (2002), observational data on three target students was collected from a larger study population of 28 students. Data collected during mathematical discourse resulted in instructional dilemmas that occur such as one student attending to irrelevant patterns. The difficulty becomes paying attention during instruction and ignoring irrelevant information within complex situations (e.g. group discussions, multi-step problem situations), and instead focusing attention on key mathematical ideas. Thus,

implications for developing appropriate mathematics instruction for students with EBD include awareness of the behaviors that make learning a challenge (e.g. inattention and focusing on irrelevant information) and identifying key pedagogical practices which can be incorporated into mathematics instruction to best support student needs.

Unilateral Interventions. To address the problematic outcomes of students with EBD, historical trends within the field show an emphasis on interventions addressing the reduction of externalizing behaviors. For example, Dunlap and Childs (1996) conducted a literature review yielding 113 articles focused on interventions for students with EBD. Of the 113 studies, 53% were intended to decrease problem behaviors and increase desirable behaviors, and 17.5% of the studies measured academic outcomes and almost none of the included studies identified academic interventions as independent variables. The focus placed on externalizing behaviors assumes that positive impacts will extend beyond disruptive behavior to improve academic and social outcomes as well (Sutherland et al., 2008).

More recently, a change occurred from simply controlling inappropriate behavior to examining the effects of teaching academics and social behavior within schools (Kauffman & Landrum, 2006). However, this redirected focus has noted consistently the trend of researchers within the field of EBD to address either academics or behavioral performance. The notion that ameliorating one deficit will extend beyond the scope of the targeted skill, resulting in collateral effects on other student deficits does not appear to be efficacious. Unfortunately, a unilateral focus (either academics or behavior) continues, despite the call from those vested within the field of EBD for a combined emphasis of programming on academics *and* behavior to address the overlapping needs of students with EBD (Sutherland et al., 2008). It is surprising that two decades ago Hinshaw (1992) stated the following:

Overall, reducing problem behavior is not a sufficient intervention for youngsters with overlapping achievement and behavior problems; the promotion of academic success is critical for these children. I should make explicit that instruction in academic skills will not, in all likelihood, succeed for youngsters with comorbid underachievement and externalizing behavior unless motivation is enhanced through the use of incentives such as reinforcers and response cost procedures (p.899).

Hinshaw (1992) makes clear that for student with academic and behavioral difficulty a combined approach is best; nevertheless, a unilateral focus continues as further demonstrated by reviews of the literature which have explored either academic or social/behavioral programming. For example, common themes for literature reviews include social skills training (Kavale, Mathur, & Mostert, 2004; Maag, 2006), self-management interventions (Mooney, Epstein, Reid, & Nelson, 2003) and academic instruction (Lane, 2004; Trout, Nordness, Pierce, & Epstein, 2003). While the findings from such reviews are invaluable within the field, it continues this notion of targeting one area, academic or social/behavioral deficits for a population displaying consistent underachievement in both domains (academics and social/behavioral). To extend this literature base, another body of literature was reviewed to examine the current status of research on academic and behavioral interventions targeting outcomes related to both domains (academic and behavior) for students with or at-risk of EBD.

Trends within the literature again emphasized that when the focus was placed on a sole intervention, positive outcomes were achieved within the targeted domain. Table 1 summarizes studies in which academic deficits were targeted and resulted in positive academic outcomes (Lingo, Slaton, & Jolivette, 2006; Penno, Frank, & Wacker, 2000; Regan, Mastropieri, &

Scruggs, 2005; Scott & Shearer-Lingo, 2002; Wehby, Falk, Barton-Arwood, Lane, & Cooley, 2003).

Table 1
Academic Intervention Outcomes

Author	Participants	Intervention	Outcomes
Lingo, Slaton, & Jolivette (2006)	n=7, Grade 6-7, EBD	Reading: Corrective Reading Program	Reading: Gains in CWPM & ORF; statistically sign. Pre/post test results Behavior: No improvements observed
Penno, Frank, & Wacker (2000)	N=3, Grade 8-9, EBD	Math & Science: Instructional modifications based on FBAs	Math & Science: productivity and accuracy improved for all students Behavior: inappropriate behavior was reduced during math & science
Regan, Mastropieri, & Scruggs (2005)	N=5, Grade 6, identified EBD	Writing: Dialogue Journal Description	Writing: increase length of writing, quality of writing for 4 of 5 Behavior: increase time on-task
Scott & Lingo-Shearer (2002)	N=3, Grade 7, EBD	Reading: Teach your Child to Read in 100 Easy Lessons and Great Leaps Reading	Reading: ORF increasing trend Behavior: increase time on-task
Wehby, Falk, Barton-Arwood, Lane, et. al. (2003)	N=8, Grade 2-4, EBD	Reading: Open Court Reading & Peer Assisted Learning Strategy (PALS)	Reading: Moderate gains in reading achievement (NWF, sound naming, blending, segmenting); no improvement in standardized score Behavior: no impact on problem behavior

Note. EBD – Emotional/Behavioral Disability; FBA – Functional Behavioral Assessment; CWPM – correct words per minute; NWF – Nonsense Word Fluency; ORF – Oral Reading Fluency

Likewise, when behavior was the targeted skill, studies resulted in positive social and/or behavioral outcomes summarized in Table 2 (Alber, Anderson, Martin, & Moore, 2005; Cheney et al., 2009; Jolivette, Wehby, Canale, & Massey, 2001; Rafferty & Raimondi, 2009; Walker et al., 2009). However, inconsistency is found when measuring collateral effects on an area not

directly addressed by the intervention (Cheney et al., 2009; Lingo et al., 2006; Walker et al., 2009; Wehby et al., 2003).

Table 2
Behavioral Intervention Outcomes

Author	Participants	Intervention	Outcomes
Alber, Anderson, Martin & Moore (2005)	N=4, grade 4-6, EBD	Behavior: Recruitment training	Behavior: all increased appropriate recruitment responses per session Math: accuracy and completion increased
Cheney, Stage, Hawkens, et al. (2009)	N=121, grade 1-5, at-risk for EBD	Behavior: Check, Connect, Expect	Behavior: Reduction of externalizing and internalizing behavior Academic: no statistical increase across time
Jolivette, Wehby, Canale, & Massey (2001)	N=3, grade 1-2, EBD	Behavior: Choice-making opportunities	Behavior: 2 of 3 improved on-task engagement Math: 2 of 3 increased number of problems attempted
Rafferty & Raimondi (2009)	N=5, grade 2-3, EBD	Behavior: self-monitoring attention and performance	Self-monitoring of Performance more effective than self-monitoring attention for increasing on-task behaviors and problems correct
Walker, Seeley, Small et al. (2009)	N=200, grade 1-3, at-risk EBD	Behavior: First Steps to Success	Behavior: moderate to strong effects Reading: Letter-word identification and ORF not responsive

Note. EBD – Emotional/Behavioral Disability; ORF – Oral Reading Fluency

Multi-component Interventions. Within the past decade, a growing body of research supports the effectiveness of multi-component interventions targeting academic and behavioral deficits. Specifically, studies addressing the academic areas of reading and writing for students at-risk or identified with EBD, show promise for ameliorating deficits in both arenas (academics and behavior) (Lane, Harris et al., 2008; Lane, O'Shaughnessy, Lambros, Gresham, & Beebe-Frankenberger, 2001; Harris, Oakes, Lane, & Rutherford, 2009; Nelson, Martella, & Marchand-Martella, 2002). For example, some of the studies addressed academic deficits through Self-

Regulated Strategy Development (SRSD; Lane, Harris et al., 2008), Phonological Awareness Training for Reading (PATR; Lane et al. 2001) and Great Leaps Reading & Sonday System (Harris et al. 2009), as well as behavioral deficits through positive behavior support using tangibles (Lane, Harris et al., 2008), group-contingencies (Lane et al. 2001) and response-cost (Harris et al., 2001). A sample of study outcomes implementing multi-component interventions are for students at-risk for EBD are described in Table 3. The positive academic and behavioral outcomes demonstrated in these studies suggest that a dual approach to intervening with this population is effective with the exception of Lane & Menzies (2003). The authors explain the lack of positive impact on behaviors as a result of primarily addressing reading instruction, with limited or a secondary focus on social skills instruction (Lane & Menzies, 2003). This finding implies that future studies incorporating a dual focus should be cognizant of intervention dosage (intensity and duration).

Table 3
Multi-component Intervention Outcomes

Author	Participants	Intervention	Outcomes
Lane, O'Shaughnessy, Lambros, Gresham, & Beebe-Frankenberger (2001)	Grade 1; at-risk for EBD and reading deficits	Reading: PATR; Behavior: Group-contingency reinforcement of participation	Reading: substantial gains in NWF & ORF improvement from baseline to intervention Behavior: TDB decreased and NSI decrease for 6 of 7
Nelson, Martella, & Marchand-Martella (2002)	Grades 1-5; at-risk for EBD	Behavior: SWPBIS-Think Time, Talk it out, SOS Help for Parents, FBA/BIP; Reading: Sound Partners (tutoring)	Academics: Substantially improved academic competence for target students on the WJ-R; Social: Substantially improved social competence
Harris, Oakes, Lane, & Rutherford (2009)	Grade 1; at-risk for EBD and reading deficits	Reading: Sonday System & Great Leaps Reading; Behavior: Reinforcement System	Reading: improvements in ORF & NWF Behaviors: improved externalizing and internalizing behaviors

Lane & Menzies (2003)	Grade 1-6; at-risk for reading and behavior	Reading: John Shefelbine's Phonics Chapter Books; Social Skills: Social skills training	Reading: statistically significant improvement in reading skills for primary grades; Behavior: no statistically significant decrease of negative comments
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Note. EBD – Emotional/Behavioral Disability; SWPBS – School-wide Positive Behavioral Interventions and Support; FBA – Functional Behavioral Assessment; BIP – Behavior Intervention Plan; WJ-R – Woodcock-Johnson-Revised Tests of Achievement; PATR - Phonological Awareness Training for Reading; NWF – Nonsense Word Fluency; ORF – Oral Reading Fluency; TDB – total disruptive behavior; NSI – negative social interactions

In addition, multi-component interventions are also showing promise with another population of students. That is, students with LD and ADHD also benefit from the use of combined strategy instruction-direct instruction approach and embedded self-determination instruction (Konrad, Fowler, & Walker, 2007). For example, Konrad et al. (2007) reviewed literature on 34 studies examining academic and self-determination interventions for students with LD and/or ADHD. Findings demonstrated that strong effects were the result of interventions that combined self-management with goal setting. To date, research in the field of EBD has neglected to investigate interventions targeting underachievement in mathematics and challenging behaviors. Upon further analysis of the mathematics/behavior research gap, only one study was identified in which mathematics instruction and behavioral/social deficits were addressed through a multi-component intervention (Mulcahy & Krezmien, 2009). Unfortunately, this study conducted with middle school students only included mathematics measures and little is known about the effectiveness of the behavioral component on improving student behavior.

Mathematics Instruction for Students with EBD

Classroom context. Understanding the context of the mathematics classrooms serving students with EBD is central to conducting research on mathematics instruction for this population. However, limited research has been conducted on the mathematics environment for

students with EBD; what has been conducted offers some insight into this particular setting. Jackson and Neel (2006) conducted 60 observations in a variety of settings (general education, resource room and self-contained EBD) across 4 schools. Several findings are highlighted from Jackson & Neel (2006) including an absence of: a) evidence that students with EBD have access to standards-based content or reform-based mathematics instruction, b) small group activities, c) complex math problems and d) opportunities to communicate their mathematical thinking such as justifying & questioning mathematical content. With limited studies to draw from, and because students with LD and EBD are often compared, further insight into special education settings comes from reviewing studies investigating the context of resource settings typically serving students with LD.

Griffin, Jitendra, & League (2009) studied communication patterns, instructional practices, and teacher content knowledge across five pre-service teachers working with special education populations including one student with EBD. 20 observations and one follow-up interview per teacher were conducted along with student pre- and post-test assessments. Two noteworthy findings related to communication patterns are highlighted. First, teachers dominated the classroom discourse with student responses being limited to answering questions posed by the teacher (more than three times the rate of their students). This communication pattern follows an Initiation-Response-Evaluation (IRE) pattern typical for a traditional classroom participation structure in which the teacher initiates, students respond, & teacher evaluates (Cazden, 2001). IRE has the purpose of controlling student behavior and prescribing the content of the lesson (Zebebergen, 2000). As pointed out by Griffin et al. (2009), and further evidenced by Jackson & Neel (2006), IRE patterns limit students opportunities to expand on their mathematical thinking or to justify their reasoning.

A related second finding from Griffin et al. (2009) is the frequency of teacher press occurring during lessons. The authors define teacher press for conceptual understanding as “teachers’ concerted efforts to keep working with students until they develop understanding” (p.321), “to elaborate their ideas or to make their reasoning explicit” and “the teacher follows students’ answers with a request for deeper thinking” (p.325). On average, 4 of the 5 pre-service teachers pressed for student thinking 0 to 2 times per lesson with one teacher averaging 11 occurrences of teacher press. Such limited incidences of teacher press can have a negative impact on student achievement since pressing students to explain and justify their thinking has been demonstrated to increase students’ conceptual understanding and achievement in problem-solving (Kazemi & Stipek, 2001).

In sum, students receiving mathematics instruction in EBD settings experience: a) limited access to general education curriculum (over reliance on traditional, procedural knowledge), b) instruction from teachers with limited mathematical content knowledge and c) instruction depended on paper-pencil tasks (Jackson & Neel, 2006).

Mathematics Interventions for Students with EBD. As the disability category suggests, behavioral concerns have been at the foreground of instructing students with EBD with interventions primarily emphasizing student-directed interventions targeting on-task behavior (Lane, 2004). Several reviews of the literature have reported minimal findings of interventions that target mathematics instruction beyond automatizing basic facts and mastering procedural knowledge for students with EBD thus, nearly neglecting higher-level thinking related to problem-solving, conceptual understanding, and mathematical reasoning (Lane, 2004; Hodge, Riccomini, Buford, & Herbst, 2006; Templeton, Neel, & Blood, 2008).

While some research has begun to address the subject of mathematics for students with EBD, a pattern emerges where academic behaviors (i.e. productivity, on-task) are the focus. Of the total 13 studies reviewed by Hodge and colleagues (2006), 77 percent (n=10) included student-directed interventions (i.e. self-monitoring, self-managing, self-regulating strategies) and 92 percent (n=12) addressed basic computation skills. Further, for the studies addressing basic computation (n=12), the only measure of mathematics performance was a measure of basic facts or calculation (worksheets) which is a very limited aspect of mathematics competence and narrowly aligns with the *NCTM Standards*. For example, strategy instruction includes teaching students to memorize facts by using the acronym CCC for cover, copy, compare.

Templeton, Neel, and Blood (2008) conducted a meta-analysis of 16 studies addressing mathematics for students with EBD and several findings are highlighted. First, the mathematical domain targeted by all 16 studies was math facts or computation similar to findings from Hodge et al. (2006). Second, 87.5 percent of the studies (n=14) addressed mathematics performance as a primary focus and were found effective (percentage nonoverlapping data (PND) of 87.30) compared to the other two studies which were interested mostly in student behavior (e.g. on-task), thus mathematics performance was a secondary concern and resulted in questionable effectiveness (IPND of 62.5). This finding suggests that focusing primarily on mathematics to improve student mathematical performance is more effective than primarily focusing on behavior. However, the rate of effectiveness is still below the very effective threshold set at 90 percent or above suggesting greater improvement is possible. Third, despite the perceived effectiveness of interventions, the maintenance, as measured for only half of the studies (n=8), was extremely low at PND of 27.5. Therefore while the initial effectiveness of interventions produced a PND of 87.3 the maintenance drops to 27.5, further suggesting that addressing

performance of basic calculations as the end goal has a limited impact on long-term mathematical achievement.

The lack of studies addressing mathematics instruction beyond mathematical performance on basic computation for students with EBD necessitates researchers to look elsewhere including instruction within other disability categories namely LD to “determine whether effective strategies for students with LD are equally effective for students with EBD” (Hodge et al., 2006). Further, based on the current status of mathematics instruction for students with EBD, it is important to focus on instructional practices that promote problem-solving and higher-order mathematics skills (Jackson & Neel, 2006). Since access to general education curriculum is essential and reform-movements within the field of mathematics currently shape the direction of mathematics instruction, the context of mathematics in elementary schools includes a greater emphasis on conceptual understanding and mathematical reasoning skills. Thus, instruction necessitates more than student-directed interventions targeting math fact performance in order to produce positive outcomes for students with EBD mathematics achievement (Hodge et al., 2006; Jackson & Neel, 2006). Evidence to support access to *Standards* based mathematics instruction for students with EBD has yet to be found, resulting in more traditional mathematics instructions (Jackson & Neel, 2006).

Promising Intervention Components

Concrete-Representational-Abstract Instruction. Concrete-Representational-Abstract (CRA) instruction is a three stage instructional sequence used to teach mathematical concepts (conceptual knowledge) and procedures (procedural knowledge). CRA follows a constructivist approach building off of the work of Piaget’s stages of cognitive development and Bruner’s modes of representation: enactive, iconic, and symbolic (Witzel, Riccomini, & Schneider, 2008).

Since mathematics is highly symbolic and abstract, students need instruction that enables them to rationalize abstract mathematics (Gersten et al., 2009; Goldin & Shteingold, 2001). CRA addresses the representation of symbols using a sequenced approach which has proven successful for students with LD at all levels of school (Witzel et al., 2008). CRA instructional sequences begin with a series of lessons to teach a math concept with only concrete manipulatives, then moves to the next series of lessons using visual images, and finally the math concept is taught with abstract algorithms. Specifically, elementary students with LD have benefited from CRA over traditional mathematics instruction in both higher-level mathematics problems and mathematical computations (Jordan, Miller, & Mercer, 1998; Miller & Mercer 1993).

However, limitations for CRA are noted by the field of mathematics education. Concerns have been raised about the over simplified teaching sequence, suggesting that students mathematical thinking develops in a less rigid or sequenced manner (Russell, 1999). The belief here is that students are constantly thinking in the abstract, regardless of how the solutions are represented (i.e. concretely or visually). That is, using concrete manipulatives alone does not promote conceptual understanding and further dictating how students use materials limits their ability to make sense of the mathematical ideas in which they engage in (Moscardini, 2009; Puchner et al., 2008). Unfortunately, research conducted on CRA implementation has been limited to measuring student performance of procedural knowledge without incorporating measures of student thinking or how they make sense of the mathematical concepts taught (i.e. conceptual knowledge). For example, Flores (2010) taught at-risk elementary students (e.g. students not identified with a disability) subtraction with regrouping. While success is noted in achieving increased performance on digits correct, no measure was given to determine if CRA instruction had an impact on student understanding of mathematical thinking related to base-ten

concepts, an underlying concept of regrouping procedures. In fact, the multiple baseline study demonstrated that after students received instruction in subtraction with regrouping in the tens place, they were unable to generalize this knowledge to higher place values (hundreds place) without explicit instruction on procedural steps. Therefore it appears that CRA was successful in teaching conceptual understanding of procedural steps involved in regrouping, but not conceptual understanding of subtraction concepts where making connections to other mathematical ideas such as number composition is necessary. This supports Baroody (2011) claim that “children may not even understand the concrete model conceptually and may simply memorize the manipulative-based procedure by rote, as they would an incomprehensible written procedure” (p.39). Without measures of student thinking, research on CRA has yet to demonstrate student growth in conceptually understanding key mathematical concepts. Further, it is hypothesized that CRA as currently structured, limits students interaction with meaningful mathematical ideas by inhibiting opportunities to communicate of their reasoning and connections across mathematical concepts.

Cognitive Learning Theory-Cognitive Representation. Related to CRA, Bruner’s extension of Piaget’s theory of cognitive development is of particular interest for this study. The development of cognitive representation is focused on how knowledge comes to be not what knowledge exists. Specifically the psychological process of acquiring knowledge involving how information is represented, organized and transformed to direct actions in which developmental levels of children are identified into three modes of representation: enactive, iconic and symbolic (Bruner & Kenney, 1965). Upon abstraction, the learner “continues to rely upon the stock of imagery he has built en route to abstract mastery. It is this stock of imagery that permits him to work at the level of heuristic, through convenient and non-rigorous means of exploring problems

and relating them to problems already mastered” (Bruner & Kenney, 1965, p. 59). As students perform external representations (manipulation of concrete devices or visual images to represent mathematical ideas), the goal is for internal or psychological representations (meaning given to mathematical ideas) to develop in conjunction (Goldin & Shteingold, 2001). A theoretical perspective on how two often opposing theories, behaviorism and constructivism, complement each other is offered by Goldin & Shteingold (2001). Here the external or objective behaviors follow a more behavioral approach to learning whereas the internal or cognitive processes follow a more constructivist approach to learning. So as students represent abstract problems internally, they are externally using enactive or iconic representations, with a shared goal of developing conceptual understanding of mathematical ideas.

Representation of Mathematical Ideas. Bruner’s three modes of representation (enactive, iconic, and symbolic) relate to mathematical instruction in which manipulative objects and visual images are often used to begin teaching mathematical concepts (Gersten et al., 2009). Since mathematics is highly symbolic and abstract, this initial or concrete operations stage supports the learner in developing an understanding or rationalization of abstract mathematics (Puchner, Taylor, O’Donnell, & Fick, 2008; Shih, Speer, & Babbitt, 2011). This goes along with Piaget’s explanation of knowledge “to know is therefore to assimilate reality into structures of transformation, and these are the structures that intelligence constructs as a direct extension of our actions” (Piaget, 1969, p. 29). This construction of knowledge develops through an interaction of external and internal systems of representation (Goldin & Shteingold, 2001).

However, researchers have questioned the rigidity of Piaget’s stages, finding that omissions of language, culture, and responsiveness to external influences required Piaget’s theory to be modified or abandoned (Case & Okamoto, 1996). Instead, building on Piaget’s

theory of cognitive development, *Central Conceptual Theory* also emphasizes the central role of structures however instead of a central structure this theory postulates domain specific structures (i.e. quantities, space) and also includes the content of semantic construction, cultural influence and structural change related to maturation (Case & Okamoto., 1996).

For modes of representation (manipulatives, visual imagery) to aid in the development students mathematical thinking, diligent attention to the learner's development of internal representations is imperative; the learner must embody "personal symbolization constructs and assignments of meaning to mathematical notation...spatial representation, problem-solving strategies and heuristics, and (very important) affect in relation to mathematics" (Goldin & Shteingold, 2001, p. 2). In other words, simply using external representations is not enough to foster conceptual understanding; rather the learner must internally understand the mathematical idea presented (Puchner et al., 2008). The use of materials in the instructional sequence and the strategies used should be dictated by the students' understanding of the problem situation (Moscardini, 2009). Clements & Sarama (2007) raise concern that mathematical knowledge should not be dichotomized into concrete and abstract understanding. *Sensory-Concrete* and *Integrated-Concrete* are helpful terms for distinguishing two levels of concrete knowledge.

Sensory-Concrete refers to the sensory-motor actions involved in manipulating objects in which children rely on when solving mathematical tasks. A higher-level of concrete knowledge, *Integrated-Concrete*, describes learners who connect concrete representations to other mathematical ideas including abstract knowledge (Clements, 2000). *Integrated-Concrete* knowledge development occurs "when students connect manipulative models to their intuitive, informal understanding of concepts and to abstract symbols, when they learn to translate between representations, and when they reflect on the constraints of the manipulatives that embody the

principles of a mathematics system (Clements, 2000, p.56). Therefore, CRA as an instructional sequence has merit, but only if the sequence is used to address students making sense of the problems posed rather than memorize procedures across modes of representation and symbols. Engaging in sense-making discourse, necessary for developing *Integrated-Concrete* knowledge, for studies implementing CRA has yet to be incorporated and therefore limitations of CRA include a lack of focus on student's mathematical reasoning and sense-making, key components in Cognitively Guided Instruction (CGI) classrooms.

Cognitively Guided Instruction. To address this limitation, a well-researched teaching and professional development approach in mathematics education is CGI. CGI focuses on the development of students' mathematical thinking and instruction that fosters this development guided by teachers understanding, knowledge and beliefs of children's thinking. CGI is grounded in developing teachers understanding of the developmental trajectory of students' mathematical learning across the four operations with whole numbers and fractions (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Carpenter, Franke, Levi, 2003; Empson & Levi, 2011). Specific to the interest of this study, CGI classrooms provide an environment in which students develop informal solutions to problem situations, thus developing place value concepts, connections across mathematical ideas, computation skills, and mathematical reasoning skills. Longitudinal research investigated students grouped by their strategy use (invented algorithms, standard algorithms, or combination) and found that students' understanding of base-ten number concepts and transfer of skills to extended problems was at a higher level than their peers who relied only on standard algorithms (Carpenter, Franke, Jacobs, Fennema, & Empson, 1997). Further, when comparing CGI classrooms to traditional classrooms, students in CGI classrooms significantly outperform control students in basic skills and problem solving questions in both

standardized assessments and in clinical interviews with students (Carpenter et al., 1996; Carpenter et al., 1999; Carpenter, Fennema, Peterson, Chi-Pang, Loef, 1989; Villasenor & Kepner, 1993). A central feature of CGI classrooms is the focus on problem-solving and research has demonstrated significant achievement in this area with no loss of skills as a result of less time spent on basic skill drills (Carpenter et al., 1999). CGI frameworks and principles have not been widely drawn upon for teaching mathematics to students with disabilities. There is very limited documentation or experimentation in using CGI research with students with disabilities (Behrend, 1994; Moscardini, 2009), but the core ideas about children's thinking provide promising direction for coordination and adaptation of the CRA teaching sequence.

Children's Construction of Mathematics. Mathematical cognition emphasizes development of mathematical thinking or conceptual understanding. Such emphasis moves beyond memorizing procedural steps or basic facts and instead addresses the underlying meaning of concepts taught (Reid & Hresko, 1981). Fundamental to this approach, children's informal knowledge of mathematics is central to engaging students to develop mathematical reasoning. To accomplish this, active engagement in meaningful mathematics is an essential feature of cognitive approaches to learning, thus limiting worksheets and paper-pencil assignments.

Piaget's theory of Cognitive Development centers on equilibrium in which children assimilate information within their schemata. Upon presenting children with problem-situations (or formal knowledge), cognitive disequilibrium results in which new information does not fit within children's current mental structure resulting in mental conflict. Disequilibrium leads children to rethink and adjust their ideas or strategies called accommodation (Piaget, 1972). Connecting ideas through assimilation leads to more holistic understanding of ideas or conceptual understanding (Baroody, 2011). From this learning approach, building on informal

knowledge benefits students with weaknesses in cognitive load or working memory because rather than memorizing meaningless facts, children build on knowledge already in their repertoire (Ginsburg & Baron, 1993). Informal knowledge is then connected to known or unknown (new) knowledge, encouraging children to think about ideas through discussion or think alouds. The goal of eliciting student understanding regarding ideas which they would view as problematic or surprising “is for learners to understand mathematical relationships and structures and, more generally, for them to construct increasingly sophisticated mathematical concepts” (Wood, Cobb & Yackel, 1990, p. 509).

While some success with cognitive approaches to learning for students with behavioral challenges is evidenced in (Baroody, 1996), Baroody (2011) suggests “applying cognitive principles to teaching children with learning or behavioral difficulties may require creative adaptations or accommodations” (p.37). This is consistent with research investigating cognitive approaches for students with disabilities (Baxter, Woodward, Voorhies, & Wong, 2002).

Children’s Sense-making. Success in teaching meaningful mathematics requires teachers to engage students in discussing their mathematical reasoning thus supporting their development of mathematical concepts. For teachers to be successful in such roles, they must understand of how students make sense of mathematical concepts which requires teachers to possess mathematical content knowledge and pedagogical content knowledge which is widely recognized as essential for teachers across the fields of mathematics education and special education (Carpenter, Fennema & Franke 1996; Griffin, Jitendra, & League, 2009; Yackel & Cobb, 1996).

CGI engages teachers in research-based findings on children’s strategy use across the four operations, following the developmental trajectory of children’s mathematical learning

(Carpenter, Fennema, Franke, 1996). Learning trajectories consist of “a goal, a learning path or trajectory through which children move through levels of thinking, and instruction that helps them move along that path” (Clements & Sarama, 2007, p.463). Teachers learn to make sense of children’s thinking by engaging in pedagogical practices such as talk moves which encourage children to reason, justify, and ultimately make sense of the mathematics they interact with, thus moving them through levels understanding (Clements & Sarama, 2007). Research has demonstrated that children who make sense of mathematics through their use of invented algorithms have a more developed sense of number (higher levels of understanding) and are more likely to transfer/generalize skills to extended problems at a higher level than their peers who relied only on standard algorithms (Carpenter, Franke, Jacobs, Fennema, & Empson, 1997). Follow-up studies on teachers found that four years after receiving training in CGI practices, teachers in general were still implementing concepts discussed from the professional development, showing the lasting impact of this approach in working with teachers (Knapp & Peterson, 1995).

In relation to student performance, Bryant, D., Bryant, B., & Hammill (2000) identified key behavioral characteristics of students with weakness in mathematics as defined by their classroom teachers with some of the top ranked behaviors as follows: difficulty with the language of math, fails to verify answers and settles for first answer, and makes borrowing errors. Clearly, sense-making is a central component to all of these characteristics. That is, if students understand the problem situation (language of mathematics), they are more equipped to select a strategy that makes sense for solving the problem; then, through teacher-facilitated dialogue, students share their thinking by reasoning/justifying steps taken, thereby limiting the existence of the behaviors associated with weakness in mathematics. In their words

“mathematics is conceptually dense...students must understand the meaning of mathematical symbols and words” (Bryant et al., 2000).

Cognitive Approach to Developing Number and Operations. Achieving computational fluency has received considerable attention from both the fields of special education and mathematics education (Baroody, Bajwa, & Eiland, 2009; Geary, 2004), and align with the *National Council of Teachers of Mathematics (NCTM) Principal and Standards for School Mathematics*’ Number and Operations Standard specified as “instructional programs from prekindergarten through grade 12 should enable all students to understand numbers, ways of representing numbers, relationships among numbers, and number systems; understand meanings of operations and how they relate to another; and compute fluently and make reasonable estimates” (NCTM, 2000). Three phases for achieving basic fact fluency are: Phase 1: Counting Strategies; Phase 2: Reasoning Strategies; and Phase 3: Mastery (Baroody et al., 2009). Understanding these phases begin with the well-researched field of early childhood mathematics which informs researchers and practitioners understanding of the number and operations domain (Clements & Sarama, 2007). Children achieving mathematical success within this domain demonstrate an interconnected knowledge of numbers or number sense (Baroody et al., 2009). Number sense includes “composing and decomposing numbers, recognizing the relative magnitude of numbers, dealing with the absolute magnitude of numbers, using benchmarks, linking representations, understanding the effects of arithmetic operations, inventing strategies, estimating, and possessing a disposition toward making sense of numbers” (Clements & Sarama, 2007, p. 467).

Children with low mathematical achievement typically get stuck with counting strategies (Phase 1- counting all or counting on) and do not progress through the other two phases

(Reasoning and Mastery; Baroody et al., 2009; Jordan & Levine, 2009). From a number sense perspective, children struggling to make gains beyond immature counting strategies often lack adequate informal knowledge and/or effective instruction (Baroody, 2011; Mazzocco, 2007). Further, several risk factors identified for developing characteristics of students with EBD (Kauffman, 2001) overlap with risk factors for developing poor mathematical achievement including: low socio-economic status, single parent homes, limited parental education, and minority status (Baroody et al., 2009; Jordan & Levine, 2009). In addition, just like characteristics of students with EBD begin to manifest early on during or before the preschool years, development of number sense is critical during this same period (ages 3 to 5; Clements & Sarama, 2007). Mazzocco (2009) reminds us that limited learning opportunities are one pathway to mathematics difficulty; “children who have the prerequisite cognitive skills for successful mathematics” however “environmental influences play a significant role in the development of mathematical thinking and achievement” (p. 2). I hypothesize that there is a relationship between mathematical and emotional/behavioral development for students with EBD who struggle in mathematics and thus support adopting a number sense perspective over a cognitive processing deficit or learning disability. Currently, interventions targeting number sense for students with disabilities have addressed early mathematics concepts aligning with the developmental trajectory of students in the primary grades (kindergarten through second grade) and focus instruction on counting principles, place value and making tens (Bryant et al., 2008). However, few studies have addressed the need to develop conceptual understanding of number sense in the upper elementary grades (3rd-5th; Cawley, Parmar, Lucas-Fusco, Kilian, Foley, 2007). For example, understanding place value as a higher order concept (beyond naming the place value of digits) has implications for exploring the composition and decomposition of

numbers which extends into inventing algorithms thus building fluency across the four operations (Baroody et al., 2009; Cawley et al., 2007). Therefore addressing the needs of students with EBD during mathematics instruction will emphasize development of place value as it relates to number composition or part-whole relations, identified as a more advanced type of number sense (National Mathematics Advisory Panel 2008).

Composition and Decomposition of Numbers. Developing understanding of part-whole relationships is a foundational concept of arithmetic, and understanding composing and decomposing of numbers contribute to the development of part-whole concepts (Clements & Sarama, 2007). Further, number composition and decomposition are highly generalizable concepts reaching across all domains of mathematics and “serve as conceptual glue to connect a multitude of important elementary school mathematical concepts” (Schwartz, 2010, p. 169). Composition in its most basic understanding means the forming of units or composing a whole out of parts. A well-developed understanding of composition extends across concepts of counting, the base-ten number system, operations, geometry and area measurement (Schwartz, 2010). Naturally, the counterpart to composition is decomposition or the breaking up of a whole into its parts which has implications most notably in subtraction with regrouping and understanding fractions (Ma, 2010; Schwartz, 2010). Mathematical reasoning of number composition entails students making connections across operations such as connecting concepts of addition and subtraction.

Implications for CRA & CGI. While CRA provides a framework for designing intervention sessions, CRA is restrictive in developing students’ mathematical reasoning (Clements, 2000), problem-solving skills (Russell, 1999) and communication of mathematical thinking; all emphasized in the *NCTM Standards*. CRA interventions are designed to increase

student achievement in performing procedural steps that are conceptually understood. For example, Witzel (2005) recommends teachers to “explore hands-on and pictorial approaches that effectively represent math procedures that lead to productive and efficient math skills connected to conceptual understanding” (p. 59). The concern is the meaningfulness of the conceptual knowledge targeted, since conceptually understanding mathematical procedures is quite different from conceptually understanding mathematical ideas (Clements, 2000). As aforementioned, students should engage in making sense of concrete modeling in which CRA studies limit through the use of teacher-directed scripts. It is well acknowledged that CRA affords students opportunities to interact across modes of representation in which positive mathematical success is achieved (Witzel, 2005; Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Maccini & Hughes, 2000); however without engaging students in meaningful communication of their mathematical thinking (i.e. explanations and justifications), the teacher has no measure of what students understand beyond their ability to demonstrate a prescribed procedure. Instead, students’ use of concrete manipulatives can build conceptual understanding provided students have, “teachers who can reflect on their students’ representations for mathematical ideas and help them develop increasing sophisticated and mathematical representations” (Clements, 2000, p.47). Calls for students to increase verbal explanations of their thinking is limited in special education literature, but is increasingly discussed as a salient feature in mathematics instruction for students with disabilities (Jackson & Neel, 2006; Griffin et al., 2009).

Further, prescribing one way to solve problems and dictating the order in which representation is used negates students making sense of the mathematics. Instead, the role of representation (physical and pictorial) is as an aid in developing understanding where students should be encouraged to choose between materials available that make sense to them (e.g.

drawing a picture over using blocks; Clements, 2000; Moscardini, 2009). Butler and colleagues (2003) started this line of inquiry when they compared CRA to RA (representational-abstract) and found CRA to be more beneficial. This study will further build on this finding by investing the flexibility of moving between concrete & representational materials, without restricting the order in which they are used or the mode of representation available. Further, the mathematics intervention will be complimented by CGI practices where instruction will encourage students to reason, justify, and ultimately make sense of the mathematics they interact with.

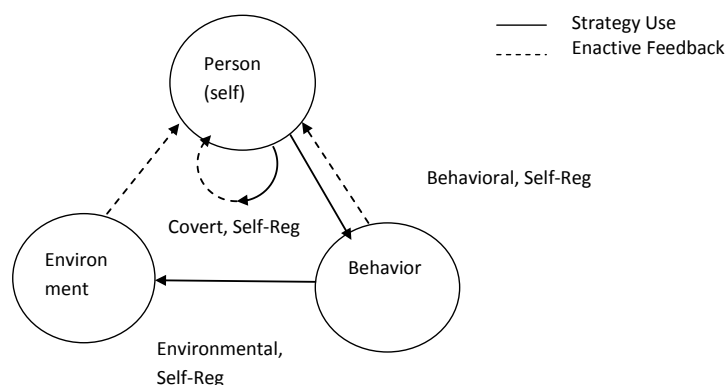
Further investigation of CRA instruction, complimented with a CGI framework for developing children's thinking across learning trajectories is warranted to evaluate the teaching structure on improving mathematics performance for students with EBD. However, additional teaching resources are also required to address the social/behavioral needs of students with EBD to increase participation during mathematics instruction.

Student-directed Instruction. A promising approach for ameliorating challenging behaviors is student-directed instruction (i.e. self-management, self-determination, and self-regulation) involving numerous interventions such as self-monitoring, self-evaluation, self-instruction, goal setting, strategy instruction, and choice making (Carter, Lane, Crnabori, Bruhn, & Oakes, 2011). Student-directed instruction has the ultimate goal of building student levels of autonomy and responsibility for their academic performance and social behaviors (Nelson et al., 2008).

Social-Cognitive Theory of Self-Regulation. In social cognitive theory, human behavior is "regulated by an interplay of self-generated and external sources of influence" (Bandura, 1991, p. 249). In applying this to the classroom, Zimmerman (2000) discussed the capabilities of students to self-regulate components of their learning behaviors, environment and internal

cognitive and affective processes. In other words, self-regulated learners are described as “metacognitively, motivationally, and behaviorally active participants in their own learning process” (Zimmerman, p. 5, 2001). An immense number of theories are drawn on to explain these principles including Information Processing, Constructivism, Social Cognition, Vygotskian and Behaviorism to name a few (Zimmerman, 2001). This study builds off of social-cognitive theory of self-regulation which analyzes the triadic relationship of processes seen in Figure 1 (Zimmerman, 1989). Social cognitive theorists assume reciprocal causation between the determinants of self-regulation (personal, environmental and behavioral) and each has strategies for increasing regulation or control of behavior, environment and covert processes (Zimmerman, 2001). Behavioral determinants inform this research study in which three principle sub-functions: self-observing, self-judgment and self-reaction will be further studied (Zimmerman, 2001). Here a student would: (a) observe their behavior/performed task, (b) evaluate their performance level with a standard/goal in mind, and (c) react to their performance with personal affirmations or tangible reinforcement contingent upon desired outcomes. In addition, self-efficacy as it translates to personal influences and social modeling related to the environment will also serve to inform the development of the mathematics/behavior intervention.

Figure 1



Self-Regulation Enhancing Mathematic Engagement. While Applied Behavior Analysis (ABA) is powerful in ameliorating problem behaviors, Polsgrove and Smith (2004) address some of the limitations of ABA strategies, including the advancement of program goals rather than the individual's, natural environments lacking consistent reinforcement, and the lack of skills necessary for maintaining motivation or emotional regulation. In other words, special education teachers may be involved in doing for their students too often thus providing them with little opportunity to do for themselves, thereby neglecting to draw from research on internal motivation and self-regulation (Deci, Hodge, Pierson, & Tomassone, 1992). As previously discussed, social-cognitive theory puts forth the belief that "reciprocity between environmental variables and individual behavior" exists (Polsgrove & Smith, 2004, p. 400). Further, in order for students with EBD to obtain a sense of freedom from teacher-directed controls over their problematic behavior, students must be taught alternative responses such as self-control (Polsgrove & Smith, 2004). One of the many strategies for teaching self-control of behavior is self-management, demonstrating effectiveness for addressing deficits around externalizing and internalizing problem behaviors (Polsgrove & Smith, 2004; Gresham & Kern, 2004).

By definition, students with EBD exhibit externalizing and internalizing behaviors that adversely impede their educational progress (Cullinan, 2004). Ignoring the emotional and behavioral factors that impact this population's ability to learn may be a significant barrier, and therefore behavioral interventions are apt to become more effective if they are embedded within the instructional curriculum. Particularly related to mathematics instruction, students with disabilities have been noted as poor self-regulators and therefore strategies involving self-regulation of behaviors and academic engagement show great promise for improving mathematic outcomes when combined with teacher-directed instruction (Sayeski & Paulsen, 2010; Miller,

Butler, & Lee, 1998). Fostering autonomy and competence in students has evidenced “enhanced conceptual understanding, greater creativity, higher self-esteem, and lower anxiety” all of which impact student success in mathematics (Deci et al., 1992, p. 459). For student with EBD, motivational self-perceptions related to autonomy and support-of-autonomy was predictive of students’ mathematics achievement reinforcing the belief that settings serving students with EBD place too much emphasis on controlling behaviors (Deci et al., 1992). In promoting student autonomy, teachers must relinquish some control over student behavior striving for a balance that still maintains the structure necessary for serving students with EBD.

Investigation of self-regulation strategies was conducted to determine the feasibility and effectiveness of approaches within this mathematics/behavior intervention. While there is disagreement in the literature regarding teacher-directed and student-directed instruction (Woodward, 2004), the intention of this study is to determine how these two approaches can inform the other and result in a viable solution to develop an innovative mathematics/behavior intervention. For example, students should be encouraged to choose from built-in strategies leading to self-efficacy, as well as selected performance outcomes they are motivated to self-monitor/observe. The goal of this study is to address Zimmerman’s qualification of self-regulation that “students’ learning must involve the use of specified strategies to achieve academic goals on the basis of self-efficacy perceptions” (Zimmerman, 1989, p. 329). Therefore three elements: (a) students’ self-regulated learning strategies, (b) self-efficacy perceptions of performance skill, and (c) commitment to academic/behavioral goals will be further investigated to elicit metacognitive processes involved in regulating not only academic skills, but also behavioral performance.

Implementation of Self-Regulation Strategies. Research reviewing self-regulation strategies that address behavioral and personal determinants focus on self-determined behaviors, self-efficacy, self-control and self-management. Particularly, self-determination has frequently been highlighted in special education as integral in promoting positive school outcomes and is viewed as a best-practice within the field (Carter, Lane, Pierson, Glaeser, 2006; Wehmeyer & Palmer, 2003). In a review of the literature focusing on self-determination interventions for students with EBD, Carter et al. (2011) found a majority of the reviewed studies (n=81) focused primarily on self-management and self-regulation strategies (65.4%), with goal setting and attainment representing 29.6% of the reviewed studies and only 7.4% focusing on choice-making. Within the self-management and self-regulation category, several components were further described including the most frequently researched: self-monitoring/recording (31%), self-evaluating (24%), self-instruction/talk (21%) and self-reinforcement (9%). Related to mathematics, other reviews of the literature conclude that self-management interventions produce large effects on mathematics achievement for students with EBD (Mooney et al., 2005). Further, the literature has established that self-monitoring, self-instruction, and strategy instruction improve computation skills for students with EBD (Mooney et al., 2005; Ryan et al., 2008). For example, in two studies the target student was appointed in charge of his or her own intervention, specifically self-monitoring during mathematics computation. Levendoski and Cartledge (2000) had students assess whether they were working by marking on a self-monitoring sheet using 10 minute intervals, an example of self-monitoring of attention (SMA). A variation of this is self-monitoring of performance (SMP), which has students record the total number of mathematics problems, the number of problems completed, and the number correct on a self-monitoring sheet. One recent study compared the positive outcomes of SMA versus SMP,

and SMP appeared to produce better outcomes on computation measures of accuracy and productivity, as well as increased levels of on-task behavior for students with EBD (Rafferty & Raimondi, 2009). It should be noted, that the success of student-directed instruction is a result of targeting skill fluency.

While these studies are promising for producing positive effects on student outcomes, self-monitoring alone is best for building fluency of already acquired skills since it is restricted in instructional capacity (Hodge et al., 2006). That is, students benefit from self-directed interventions when targeting skills already in their repertoire and are inappropriate for new skills that have not been taught. In regards to this study, self-directed strategies alone would be unsuitable without an instructional component since this study intends to target skill acquisition. Therefore, embedding student-directed strategies within an instructional component warrants further discussion of studies successful in this combined approach.

Self-Regulation Strategy Development (SRSD) in writing is a well-documented, effective teaching approach. (Zito and colleagues (2007) description of SRSD in writing as incorporating six instructional stages where collaboration between teachers and students address key components of the writing process by focusing on academic and self-regulation strategies (Zito et al., 2007). Studies using SRSD in mathematics instruction have targeted problem-solving strategy development where students self-regulate their academic behavior relative to following steps in a procedure. For example, a multiple-baseline-across subjects design was conducted to investigate SRSD across 4 fourth and fifth grade students with LD who had difficulty solving word problems. to determine students' effectiveness at solving word problems using the following steps: 1) read the problem; 2) circle important words; 3) draw a picture; 4) write a mathematics sentence; and 5) solve (Case, Harris, Graham, 1992). While student performance

improved, the focus was on following steps to solve problem situations (procedural knowledge) with limited attention to successful mathematics behaviors such as making sense of the problem. Current research implementing self-regulation strategies within mathematics instruction have a limited scope focusing primarily on self-monitoring of computation and self-regulation of procedural strategies (Fuchs, Fuchs, Prentice, Burch, & Paulsen 2002; Mooney et al., 2005).

Given the context of mathematics reform curricula, current instructional practices place a greater focus on problem-solving often by providing small groups of students complex problem situations in which mathematical discourse and strategy development are central (Sayeski & Paulsen, 2010; Templeton et al., 2008). Therefore, to support the success of students with EBD within this curriculum, research is needed to investigate self-regulation strategies in which students regulate levels of participation/engagement, positive peer/adult interactions, and mathematic behaviors such as persistence in solving complex tasks; thus going beyond current practices of regulating steps in a procedure or monitoring correct responses to basic computation equations. Self-regulated learning entails learners “attending to instruction, processing information, rehearsing and relating new learning to prior knowledge, believing that one is capable of learning and establishing productive social relationships and work environments” (Schunk, 2001). Further, including a motivational component within mathematics instruction has been recommended by Gersten et al. (2009) as a way to increase student engagement, completion and achievement within mathematics instruction (Recommendation 8). Motivation is especially important for students who have a history of anxiety, learned helplessness, and academic failure within mathematics instruction (Allsopp, Kyger, & Lovin, 2007).

Support for teaching behaviors used in mathematics comes from the *NCTM Process Standards* targeting: problem solving, reasoning and proof, communication, connections and

representation (NCTM, 2000). The *Process Standards* were used to guide the development of *Common Core State Standards for Mathematical Practice* which articulate behaviors that mathematicians engage in and provides specific behaviors which can be addressed through student-directed instruction. Relevant to this study, two Common Core Mathematical Practice Standards (2011) are highlighted: 1) make sense of problems and persevere in solving them and 2) models with mathematics. Making sense of problems, preserving, and using representation are three behaviors which will aid in the development of mathematical reasoning skills taught during mathematics instruction.

Empirical evidence of multicomponent interventions

Reviews of the literature resulted in identifying one study investigating the effects of a multicomponent intervention targeting mathematics and behavioral support for students with EBD. The nature of the identified study is described in detail since it shares similar features to this study. Mulcahy and Kremin (2009) studied their intervention effects across four middle school students with EBD receiving mathematics instruction in a self-contained classroom. The intervention was developed to teach area and perimeter targeting procedural knowledge (cue cards) and conceptual knowledge (concrete and pictorial representation) using a problem-solving context and included self-monitoring of academic performance (problems completed and problems correct) and self-evaluation of behavioral expectations. The intervention was implemented one-on-one in a pullout setting for 45 minutes across 11 sessions (one student received 2 repeated sessions n=13). Booster sessions were also incorporated after the first two students performed poorly on the first post intervention measure. Results from this study indicated improvement on postintervention means for all four students (90% or greater) and an average of 97% was achieved across all four students and all sessions on daily objective probes

(lesson mastery). However, performance on maintenance and transfer measures was mixed indicating limited retention/transfer. Unfortunately, no behavioral data was collected to demonstrate a change in behavior due to the self-monitoring intervention. Anecdotal notes indicated positive levels of student interest in completing daily behavior checklists, however one student was described as having behavioral challenges including work refusal for novel and challenging tasks demonstrating “limited persistence to task and self-efficacy” (p.145) and for all students the following was true, “when a solution did not reflect what they expected, they did not attempt to find a more accurate solution” (p. 146). These behavioral descriptions are quite useful in designing similar studies. For example, students self-evaluated their behavior according to “appropriate behavior and examples and non-examples were provided of desired behaviors” (p.140). Further, students were all on behavior contracts for individual behaviors but were not explicitly linked to the students’ self-evaluation checklists.

Implications from this study compel us to pay attention to the unique, complex characteristics of students with EBD. Refusal to attempt or explore multiple solutions with unknown or challenging tasks even in a highly structured, one-on-one environment makes salient the context of teaching mathematics to students with EBD. It does not appear enough to implement self-monitoring of performance or on-task behavior when encouraging students to engage in complex, higher-level thinking often perceived as aversive. It is reasonable to understand why direct instruction in which procedures are first modeled is advocated for within the field of EBD. However, by engaging in such prescribed practices students with EBD are at a greater disadvantage in which limited opportunities are available to develop mathematical reasoning skills and build their repertoire of self-regulation strategies necessary for mathematical achievement and positive post-school outcomes. Designing instruction that provides access to

high quality mathematics instruction by meeting *NCTM Standards* and the needs of students with EBD requires an innovative way of thinking.

Chapter 3: Method

Setting

This study was conducted at a public, therapeutic day school for students with Emotional and Behavioral Disabilities (EBD) located within an urban school district in the Puget Sound area serving elementary, middle and high school students. Students served at this school have been unsuccessful in comprehensive school settings and require the most intensive behavioral support within the district. The school includes a diverse group of students representing a range in cultures, ethnicities, and socio-economic status. The 2011-2012 demographic data for the school district is presented in Table 4 along with the most current school enrollment data (November 2011). Mobility rate for this school averages 57.8 percent during the school year.

Table 4

<i>Demographics by Percentage</i>		
	School District	School
	2011-2012	Nov. 2011
African American	18.6	47.6
American Indian/Alaskan Native	0.8	0.0
Asian	24.8	0.0
Hispanic	19.7	19.0
Pacific Islander	0.7	0.0
White	31.1	33.3
Multiracial	--	--
Free/Reduced Lunch	54.4	68.9
Total Population	14,591	42

The participants for this study were in two elementary classrooms within this school. One classroom served students from grades 1st through 3rd and the other classroom served students from grades 4th through 5th. The classroom management strategies implemented in both classrooms draw from the school's philosophical framework of Re-EDucation (Hobbs, 1982), a strength-based model targeted towards the Re-Education of students with Emotional Disturbance. Specific to this study, classroom management techniques for the participants included a continuum of consequence starting with warnings and ending in a time-out from the classroom setting. In addition, each student received a daily report card based on personal goals identified by the students and staff. Students earned points for meeting goals and progressed through a leveled system called the *Circle of Courage* (Brendtro, Brokenleg, & Van Bockern, 2005). In mathematics, all of the students participated in traditional mathematics instruction consistent with the curriculum the school district adopted (*Everyday Mathematics*). The instructional setting for mathematics was small group, ranging from a 1:2 to 2:7 adult, student ratio.

Participants

Screening to identify participants that met inclusion criteria began in the August 2012. Establishing criteria for inclusion began by identifying elementary students from grades third through fifth who were identified as EBD on the student's Individualized Education Plan (IEP) or had another disability category (e.g., Other Health Impaired, Speech and Language Impairment) and were placed at one of the classrooms at the school. Next, classroom teachers were asked to nominate students who would benefit from receiving additional mathematics instruction. The primary classroom teacher nominated only one student and the upper elementary teacher nominated seven. A recruitment letter was then sought to gain permission to screen

students. Eight parents of students gave approval for their children to be screened for potential participation in the study. Of the eight students, five were then screened using the *Test of Early Mathematics Ability*, second edition-2 (TEMA-2; Ginsburg & Baroody, 1990). For students to be included in the study they needed to perform below the 25th percentile on the TEMA-2 and/or have goals and objectives on their IEP in the area of mathematics to qualify for this intervention. Of the five students screened, three students met study criteria and parental permission was then sought using an informed consent approved by the University of Washington's Institutional Review Board. Parental consent for and student assent from all three students was gained.

The inclusion criteria (identified for EBD and ≤ 25 on the TEMA-2) was established to ensure the sample met the target objectives of the proposed intervention by exhibiting emotional/behavior deficits and mathematics difficulty (MD). The cutoff score of the 25th percentile or below has a strong research backing (Murphy, Mazzocco, Hanich, & Early, 2007). Table 5 summarizes the demographic information for each of the three participants.

Molly. Molly (pseudonym) is a Caucasian, eight year old female student at the time of the study. She is in the third grade and has attended the school for 1 year. She receives special education services in the area of Social/Emotional/Behavioral under the category of Other Health Impaired. Behaviorally Molly's IEP identified the following behaviors as areas of concern: a) having an unsafe body 86% of the time, b) not completing 56% of daily assignments, c) displaying off-task behaviors 51% of the time, and d) difficulty following directions with 2 reminders 66% of the time. According to the TEMA-2 she is performing at the 5th percentile for students ages 3 through 8 and the item profile shows relative strength with informal assessment items such as using manipulatives to solve math problems. However, weaknesses were displayed

include difficulty counting, reading and writing numbers and mentally solving computation problems.

Eva. Eva (pseudoname) is a Hispanic, eleven year old female student at the time of the study. She is in the fifth grade and receives special education services in the area of Mathematics, Reading and Social/Emotional/Behavioral under the category of EBD.

Behaviorally Eva's IEP identified the following behaviors as areas of concern: a) inappropriately gaining attention 1 out of 5 opportunities, b) harming self an average of once per day, c) refusal to participate in academic expectations 2 class periods per day, d) off-task during independent work 20 out of 25 minutes and e) not completing 65% of daily assignments. According to the TEMA-2 she is performing at the 50th percentile for students ages 3 through 8 and the item profile shows relative strength with formal and informal assessment items. Despite her relative strengths, weaknesses were displayed such as mentally solving basic facts, errors with number sequencing, estimation and solving math problems involving multiples of tens.

Kyle. Kyle (pseudoname) is a Caucasian, ten year old male student at the time of the study. He is in the fourth grade and receives special education services in the area of Reading and Social/Emotional/Behavioral under the category of multiple disabilities including ADHD and speech and language. Behaviorally Kyle's IEP identified the following behaviors as areas of concern: a) having an unsafe body 4 out of 10 days and b) difficulty following directions with 2 reminders when escalated 20% of the time. According to the TEMA-2 he is performing at the 57th for students ages 3 through 8 and the item profile shows relative strength with formal and informal assessment items. Despite his relative strengths, weaknesses were displayed such as mentally computing basic facts and solving math problems involving multiples of tens.

Table 5

Participant Demographics

Participant	Gender	Ethnicity	Age	Grade	Special Education Category	TEMA-2 Percentile
Molly	Female	Caucasian	8 yrs 1 month	3 rd	OHI	5 th
Eva	Female	Hispanic	10 yrs 10 months	5 th	EBD	50 th
Kyle	Male	Caucasian	10 yrs 0 months	4 th	Multiple Disabilities	57 th

Note. OHI – Other Health Impaired; EBD – Emotional/Behavioral Disabilities

Instructional Procedures

Baseline: During the baseline data collection period, students participated in one-on-one instruction for 4 days per week when present in school for 20 minute sessions over 3 weeks. Given attendance and participation rates, the range of sessions was 7-13 sessions for the three students. The type of instruction during baseline included worksheets and math fact flash cards provided by the classroom teachers and designated as typical extra practice that the teachers provide the students. Intervention intensity and duration was selected based on previous mathematics interventions implementing instruction that is supplemental to the core curriculum or Tier 2 interventions within a multi-tiered intervention model such as Response to Intervention (RtI; Bryant et al., 2008). Instruction was provided by the investigator of this study.

Intervention Design. The mathematics intervention component was developed by the investigator and is based on principles of effective instruction as suggested by Nelson and colleagues (2008). Developed sessions followed a modified explicit lesson framework (advanced organizer, teacher modeling/facilitation, guided practice and feedback) exploring the inclusion of student discourse (opportunities to share thinking) as an additional component. All lessons utilized multiple modes of representation with concrete manipulative devices, visual representation and abstract symbols available for students to engage in the mathematics presented. Table 6 displays the lesson framework for each instructional session utilized for each

of the following instructional tasks described. Lessons were provided during the following times of day: Molly (1:00), Eva (12:30) and Kyle (11:45).

Table 6

Lesson Framework

Lesson Component	Time	Description
Advanced Organizer	1-2 min.	Provides a link between the current and previous lesson instruction, identifies lesson objective, and a rationale for the skill being taught.
Teacher Modeling/ Facilitation	2-3min.	The teacher presents students with a learning task using designated materials ensuring students understand the problem context/ mathematical language (manipulatives, visual imagery, numerals/symbols). Use of real-life, meaningful context for problem-situations were included when applicable.
Guided Practice	15 min.	The students use provided materials to demonstrate/talk through their strategies for solving lesson tasks. The teacher asks students about strategies, and prompts with varying degrees of support based on student needs.
Discussion	On-going	Student discusses strategies and the teacher facilitates to make explicit student thinking.
Feedback Routine	On-going	The teacher probes student thinking throughout instruction and provides explicit feedback/support on student's use of strategies.

Student interest and motivation to engage in mathematical tasks was considered to be a critical element for the intervention implementation, given the characteristics of students with EBD. As part of increasing student motivation, students were provided choices in the use of modes of representation across learning tasks. Learning tasks included counting collections, problem solving situations and an activity involving groupings of tens and ones. All learning tasks (counting collections, problem-solving situations and grouping activity) utilized modes of representation (concrete manipulative devices, visual images, abstract symbols, and oral

language) and CGI practices such as teacher talk moves (e.g. revoicing, reasoning, adding-on) related to making sense of student thinking along a learning trajectory.

Counting Collections. Students were provided with collections of objects ranging from 40 to 200. Objects included bead necklaces, toy lizards, glittery snowflakes, pom-poms and marbles. Each session began with the collection of objects presented as one group and students were encouraged to develop counting strategies that would help organize and track their count. As they counted, they were asked to count out loud and record either during or after they completed their count.

Problem-Solving Situations. Problem-Solving situations were contextualized (real-life, meaningful context) as supported in the literature for students with disabilities (Griffin, 2007). The problems posed in the problem situations were linked to previous counting collection tasks focusing on groupings of tens. This was hypothesized to increase transfer and provide further opportunities for students to practice the targeted skill.

Grouping Activity. This task was a replication of a bundling sticks activity used during a study to improve students conceptual place value understanding (Ellemor-Collins & Wright, 2011). For this task, the student is shown a set of bundles of tens and ones. After viewing, the teacher covers the sticks with a sheet and either adds or removes tens and/or ones. The student was then challenged to determine how many sticks were then under the sheet.

Mathematical Content. Each session addressed higher-order reasoning of place value concepts with the target of building proficiency in composing and decomposing numbers and number composition reasoning skills ultimately encouraging more efficient strategy use in solving problems using the four operations. The lessons were aligned with Washington State's Common Core Standards to provide a basis for lesson content. Depending on student screening

data, multiplication and division were incorporated as appropriate. The standards addressed were:

Operations and Algebraic Thinking:

2.OA.1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).

and

3.OA.3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).

Numbers and Operations in Base Ten

2.NBT.1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

- 100 can be thought of as a bundle of ten tens — called a “hundred.”
- The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

2.NBT.3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

2.NBT.7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting

three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

and/or

3.NBT.2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

2.NBT.9. Explain why addition and subtraction strategies work, using place value and the properties of operations.

3.NBT.3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80 , 5×60) using strategies based on place value and properties of operations.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them
2. Use appropriate tools strategically

Instruction was designed to elicit a specific strategy use by building on student representations and teacher-added contributions. Strategies thought to develop place value reasoning included: making landmark or friendly numbers, breaking each number into its place value, and adding up in chunks/removal (Parrish, 2010). A sample lesson of counting collections is provided.

Sample Lesson

Lesson Objective	The student will demonstrate counting skills by counting objects, organizing objects and making a record of the count.
Advanced Organizer	T: <i>I'm going to need your help with something. I'm having a party with some friend this weekend and I have some beads I want to give them. But, I have more beads than I want to bring so I need to count them first and decide what I should bring.</i>
Teacher Facilitation –	T: Presents a bag full of beads and asks the student to count how many beads there are.

Task 1	
Student Problem Solving	S: The student will count the beads using any strategy that makes sense. T: Actively asks questions about individual thinking and/or prompts when necessary. If student has difficulty organizing beads ask, “how do you know which you have counted?”
Strategy Discussion	T: Explicitly discuss the strategy the student used to count beads and utilize representations to support mathematical ideas.
Teacher Facilitation – Task 2	T: <i>Next, record how many beads you counted by drawing.</i> T: Teacher presents student with material to record count and provides support as needed.
Student Problem Solving	S: The student will use materials to record count T: Actively asks questions about individual thinking and/or prompts when necessary.
Closure	T: <i>Tell me about what you drew on the paper and how it matches the way you counted the beads.</i> S: Student shares their recording sheet and explains strategy for counting. T: Make explicit student’s thinking and brings closure to lesson.
Feedback Routine	On-going throughout the lesson based on students’ thinking and behavioral reinforcement for desired behaviors.

Note. *Focus on counting #s with 6 & 7, and crossing decades

Behavioral Component. In addition to the mathematics component, instruction included key behavioral supports some of which were embedded into the mathematics instruction and others which were supplemental to instruction. A menu of student-directed self-regulation strategies are outlined in Table 7. Baseline behavioral data were utilized to determine appropriate self-regulation strategies necessary to meet the individual needs of the students. Selected self-regulation strategies were taught individually to each participating student by the research investigator. Participants not responding to mathematical instruction during the intervention phase were provided with behavioral coaching of target behaviors and asked to set goals, self-monitor and self-evaluate their behavioral performance during instruction.

Table 7

Menu of Self-Regulation Strategies

Components	Example Strategies
Attitudes	Academic Choices
	Meaningful Context for Instruction
	Incorporate Student Interests into Instruction
Beliefs (Self-Efficacy)	Customize Level of Challenge for Tasks
	Self-Monitoring of Tasks/Behavior & Graph
	Self-Evaluation
	Goal Setting
Domain Knowledge	Students identify areas of interest for future learning
	Hands-on activities
Flexibility of Strategy Use	Self-Record strategy use

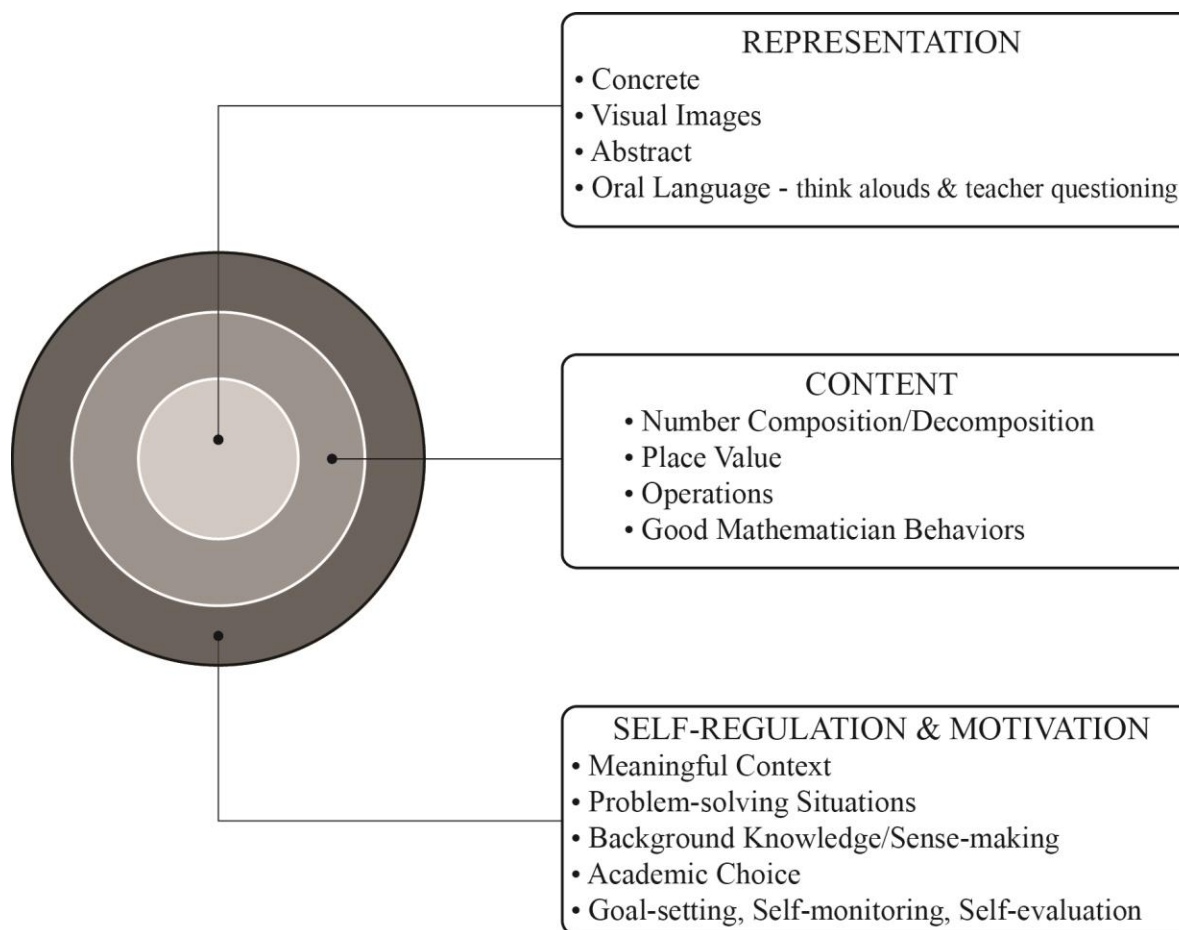
Modified from Wery, J. J., & Nietfeld, J. L. (2010). Supporting self-regulated learning with exceptional children. *Teaching Exceptional Children*, 42(4), 70-78.

Sequencing the Intervention Components. The study intervention integrated multiple modes of representation shown in Figure 3. As students developed fluency with representing strategies for solving problems it was hypothesized that they would advance their use to more fluent numerical approaches that communicated understandings of place value, number composition and operations. During instruction students were encouraged to use concrete manipulatives, visual imagery, abstract numerals and symbols, and oral language deepening their internal representations. As students gained confidence in their learning and developed deeper understanding of mathematical concepts, it was the intention of the intervention to guide students through the phases of instruction and progress them through a continuum of teacher-directed to student-directed instruction.

Further, it was anticipated that self-regulation strategies would initially improve on-task behaviors and decrease disruptive behaviors. This was accomplished by selecting strategies that

address behavioral and personal determinants, specifically looking to enhance motivation, persistence in completing mathematical tasks, and self-confidence by incorporating such strategies as academic choice, goal setting, self-monitoring/observation, and self-evaluation. Teacher instruction and feedback was integral throughout all intervention sessions promoting the use of three key behaviors good mathematicians use: a) making sense of problems, b) persevering during difficult tasks and c) using multiple modes of representation. In addition to the mathematics intervention components, self-regulation strategies are also graphically represented in Figure 2.

Figure 2



Measures

Screening and outcome measures aimed to measure mathematical understanding, use of problem-solving strategies and behavioral performance of the participants. Mathematical content in which problem solving and reasoning skills were measured address concepts of place value, number composition/decomposition and efficient strategy use in solving problems across the four operations. All measures were administered individually following the outlined procedures for each assessment. Table 8 displays measures, purpose, and implementation schedule.

Table 8

Proposed Measures

Measure	Purpose	Schedule
Test of Early Mathematics Ability, 2nd edition	Screening	October 2012
Objective Probe - Problem Solving Task	Effectiveness	Daily
Place Value Interview	Effectiveness	October 2012, November 2012
Direct Observation of Behavior	Effectiveness	Daily

Test of Early Mathematics Ability, 2nd Edition (TEMA-2; Ginsburg & Baroody, 1990). The TEMA-2 is a norm-referenced, standardized test for children ages 3-8 or for older students who have difficulty in mathematics as a criterion referenced or diagnostic tool. The TEMA-2 identifies students' strengths and weaknesses in both informal and formal mathematical knowledge. For example, concepts of counting, quantity, magnitude, calculation and written representation of numbers are measured. Internal reliability across parallel forms of the test has a mean of .95. The TEMA-2 was administered to screen students to see if they met participation criteria.

Objective Probes - Problem Solving Task. Each session concluded with the student solving one problem-solving task. Problem-solving tasks aligned with the intervention's mathematical focus as follows: concepts of place value, number composition, and efficient and effective strategy use for solving across operations. Objective probes measured mathematical reasoning growth related to conceptual and procedural knowledge. The use of objective probes provided lesson progress monitoring of student performance on lesson objectives (Hudson & Miller, 2006; Mulchany & Krezmien, 2009). Student performance on objective probes were assessed using the rubric presented in Table 9.

Table 9
Problem Solving Rubric

	5	4	3	2	1	0
Place Value/ Number Composition*	Composes & decomposes tens & ones (understanding of part-whole relationships)	Breaks apart number into its place value. Demonstrates partial understanding of part-whole relationships. Some errors still present.	Student has face value understanding of digits. Inconsistent understanding of part-whole relationships.	Positional understanding of digits (identifies ones & tens place). Attempts an inaccurate use of part-whole relationships.	Multi-digit number is seen as the whole number it represents. No evidence of number composition	Does not use written or spoken language to express understanding of place value
Efficient Strategy Use	Invented algorithm	Counting strategy	Direct modeling by 10s or higher	Direct modeling by 1s	Inaccurate strategy attempted	None
Total _____						

*Verbally stated or evidenced in written work/use of materials.

Algorithm Use

	5	4	3	2	1	0
Standard Algorithm	standard algorithm with conceptual explanation	standard algorithm with procedural explanation	standard algorithm with no explanation	standard algorithm with errors	Invalid algorithm	Did not attempt
Total _____						

Place Value Interview (Adapted from Philipp, Cabral, & Schappelle, 2012).

Interviews were conducted and video recorded individually two times during the study, providing pre- and post-intervention data. Tasks were related to place value concepts, number composition and decomposition, word and computation problem solving, and extension problems requiring conceptual and procedural knowledge of number concepts. Similar interviews have been conducted and validated in numerous studies to support their effectiveness in measuring students' mathematical reasoning skills (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Carpenter, Franke, Jacobs, Fennema, & Empson, 1997; Carpenter & Moser, 1984; San Diego State University Research Foundation, 2012; Villasenor & Kepner, 1993).

Direct Observation Data. Behaviors during baseline, intervention and post-intervention were video recorded and later and coded as: on-task and disruptive behaviors (see Table 8). All behaviors were coded using 10 second interval data. Two data collectors completed the coding of the videos and the study investigator conducted reliability checks. Training began during September utilizing video taken from the regularly occurring mathematics instruction. Initial behaviors and definitions were modified until data collectors reached inter-observer coefficients above 90 percent with practice sessions. Inter-observer reliability checks occurred for approximately 10 percent of observations. Inter-observer reliability coefficients for data collection in this study ranged from 85 to 97 percent with an average of 91percent.

Table 8

Defined Behaviors

Behavior Codes	Defined
On-task	<ul style="list-style-type: none"> • Attending to assigned activity • Appropriate motor responses <ul style="list-style-type: none"> ○ Writing ○ Looking at teacher ○ Eyes on materials ○ Using math materials to solve problems
Off-Task	<ul style="list-style-type: none"> • Staying in assigned area • Out of seat/assigned area • Making noises • Using materials inappropriately (flicking pencils, throwing materials on the ground) • Comments not related to math or behavior
Off-topic comments (negative talk, making noises)	<ul style="list-style-type: none"> • Statements/vocalizations to provoke, annoy, mock, threaten, or tease others (i.e. derogative name-calling) • Use of profanity • Verbal sounds • Purposefully wrong answers
Out of Seat/ Assigned Area	<ul style="list-style-type: none"> • Moves about room, away from lesson materials and teacher without permission, spins in chair
Noncompliance	<ul style="list-style-type: none"> • Refusal to follow directions, work refusal, arguing or verbal noncompliance
Misuse of Materials	<ul style="list-style-type: none"> • Uses materials inappropriately (flips pencil, plays with math materials)
Verbal participation	Asking or commenting on mathematics skill
Appropriate Use of Materials	Uses math materials to show thinking

Study Design/Data Analysis

Mathematics and behavioral performance on several dependent variables were evaluated using a multiple-baseline time-series design across 3 intervention groups. The single-subject design adhered to the quality indicators as outlined by Horner et al. (2005). Also evaluated was students' mathematical understanding through clinical interviews prior to the first intervention with the first participant and at the end of the last intervention session of the study. Conducting the interviews at the same time controlled for differential effects of elapsed time within the

school year. Finally, videos of the mathematics/behavior intervention implementation were coded and analyzed for trends in students' performance. This study design addresses Research Questions 1 through 3 as displayed in Table 9.

Table 9

Research Questions and Data Collection

Research Question	Data Gathered
	Quantitative
Q1: What effect does the multi-component mathematics/behavior intervention have on student understanding of place value, number composition and efficient strategy use for problem-solving tasks?	Place Value interview Problem-solving Probes
Q2: What effect does the multi-component mathematics/behavior intervention have on increasing on-task behaviors and decreasing disruptive behaviors?	Direct observation data of video recordings
Q3: What effect does the multi-component mathematics/behavior intervention have on the frequency of statements students make articulating mathematical reasoning and appropriate use of mathematical materials?	Direct observation data

Study Phases. The study began in October of the 2012-2013 school year, and was conducted over a period of 7 weeks. The baseline phase was a minimum of 4 days and a maximum of 10 days. The intervention phase lasted 5 weeks and was implemented for approximately 20 minutes/day, 4 days/week. Follow-up data were collected during the last week of November 2013.

Student Screening. Student screening began during the beginning of the school year (September 2012) and resulted in identifying 3 students meeting the criteria for inclusion. Students meeting criteria were offered an invitation to participate in the study. In addition,

students were interviewed using task-based mathematics interviews and administered domain probes measuring place value (see measures).

Baseline Phase. A minimum of 4 days baseline with continuous assessment of 4/week on problem-solving tasks and behavior was conducted. Participating students were randomly ordered to begin instruction. The baseline phase began in October and continued for 3 weeks. Behavioral data collection for baseline took place in the same setting as the pull-out intervention sessions and consisted of the investigator completing teacher provided extra practice typical of what would be carried out in class.

Intervention Phase. Participants were randomly assigned a time to begin implementation with 3 waves being conducted. The mathematics/behavior intervention was implemented in three waves with the intervention start dates beginning every week: (a) Individual 1: October 22, (b) Group/Individual 2: October 29, and (c) Individual 3: November 1. The participants received the supplemental mathematics/behavior intervention approximately 4 days a week for 20 minutes, averaging a total of 3 weeks of instruction or approximately 10 sessions. Observation of on-task behavior was collected from video recording of the invention sessions. At the end of the intervention phase for Individual 3, all participating students were administered the post-test assessment and clinical interviews.

Data Collection. This study collected quantitative data on several outcomes measures related to mathematics and behavioral performance. For mathematics: (a) continuous daily problem-solving tasks (1 per probe) measured by rubric and (b) pre-post clinical interviews. For behavior continuous direct observations of on-task behavior, disruptive behavior, and participation (see Measures for observable definitions) was continuously collected via video recordings. Interrater reliability checks, established before baseline, continued 1/week.

Follow-up Data Collection. Follow-up data were collected during the last week of November 2012 to measure generalization and maintenance effects. During this time, students were observed in their classrooms during mathematics instruction. Eva and Molly received instruction in a small group of approximately 5 students and Kyle's instructional group included one other student. Mathematics instruction was scheduled for 45 minutes and included whole group guided practice activities followed by independent paper-pencil tasks. Observations were conducted for the entire mathematics block which lasted for 20 minutes for Molly and 36 minutes for Kyle and Eva. During this time, behavioral data was coded and an objective probe was administered following instruction.

Data Analysis. Adhering to the quality indicators for single-subject research, visual analysis of the data involved systematic visual inspection for within and across phases of change in level, trend, or variability (Horner et al., 2005). Student's mathematical growth on quantitative measures must be at a level of mastery set at 80% or higher.

Chapter 4: Results

The data were interpreted using a combination of visual inspection and item analysis. Data analyses were conducted in the following steps in accordance with the research questions as outlined in the method section: a) mathematical understanding, b) behavioral change and c) mathematical behaviors. For the place value interviews percentage correct and item analysis was conducted. For the on-task, disruptive behaviors and mathematical behaviors measures, the median percentage across phases were compared to evaluate either the increase or decrease in percentage of intervals or frequency of behavior. The results are displayed in Figures 3, 5-9.

Mathematical Understanding

Research Question 1: What effect does the multi-component mathematics/behavior intervention have on student understanding of place value, number composition and efficient strategy use for problem-solving tasks?

Place Value Interviews. All participants improved their percentage correct from pre-baseline to postintervention on the place value interviews that were conducted. Figure 3 displays the results of the interviews for the three participants. As a group, the mean score on the baseline and post-intervention place value interviews were 56 and 80.67 respectively with an overall group increase on 24.67 percentage points. Molly had the greatest percent increase of 42 percentage points, increasing from 32 percent to 74 percent. Eva increased her performance by 18 percentage points, increasing from 62 to 80 percent. Finally, Kyle had a 14 percent increase, improving from 74 percent correct to 88 percent correct.

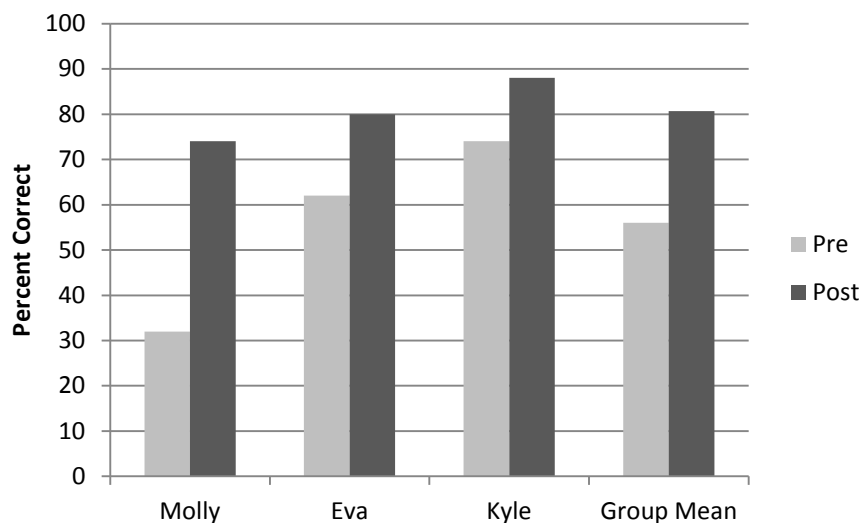


Figure 3. Accuracy on place value interview assessment for Molly, Eva and Kyle.

Item Analysis. The post-intervention place value interview consisted of twelve areas of questioning some with sub-questions resulting in 25 questions for Eva and Kyle. Two question areas (n=6 questions) were not posed to Molly since they were not addressed during the intervention phase, resulting in a total of 19 questions. An item analysis was conducted of all twelve question areas, 5 of which are highlighted based on student growth.

Counting (by ones and tens). Two questions involved having the students count; one was to rote count by 10s and the other to count the total number of objects provided in a bag. Prior to and during baseline instruction, Molly consistently made counting errors beyond 15. For example she would often skip numbers ending in 6 and 7 and decade numbers (e.g. 16, 26, 37, 60, 70, 77, 80 and 87). However, during the post-assessment she accurately counted objects 1 to 68. Further when asked to rote count by tens during the post-assessment she counted from 10 to 190 with only one error (skipping 110) improving from her pre-assessment performance where she did not count beyond 100. Eva also showed improvement in counting by tens where initially she counted 10 to 100, 200, 300, 400 and 500 improving during the post-assessment by

demonstrating a count from 10 to 260 with zero errors. Kyle had accurate counts during pre and post assessment sessions.

Digit Correspondence. For this task, students were presented with a number and asked how many tens and how many ones. Molly provided incorrect answers by guessing during the pre-assessment and was accurate on all responses during the post-assessment demonstrating an emergent understanding of place value initially not present. Eva and Kyle were consistently accurate during pre and post-assessment sessions.

Number Representation. Students were presented with the number 65 visually represented with base-ten blocks, two different ways (see Figure 4). They were then asked “There are 65 squares on the paper. Using your pencil draw a circle around the squares that this part of the number 65 stands for. Draw a circle around the squares that this part of the number 65 stands for.” Molly initially had difficulty identifying the 6 in Figure 4.a and by the post-assessment could identify the 6 and 5 for the image in Figure 4.a. Eva improved by accurately identifying the 6 and 5 for both images (Figure 4 a and b) during the post-assessment whereas initially she only accurately identified the 5 in Figure 4a. Student C accurately identified the 6 and 5 during both assessment sessions.

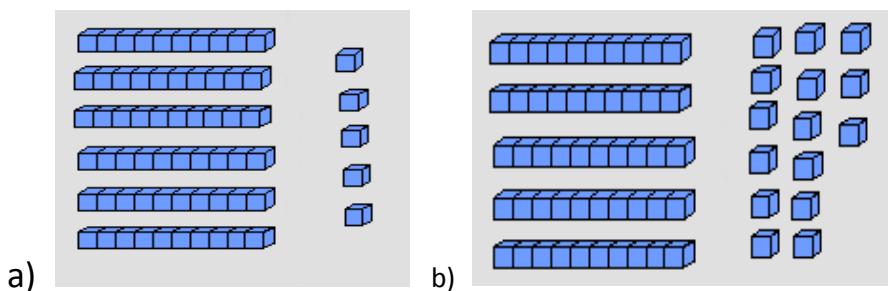


Figure 4. Number representation task presented to students during the place value interview

Building a Number Multiple Ways. The students were orally given a number (32 and 74) and asked to use base-ten blocks to represent the number two different ways. During the pre-

assessment, Molly and Kyle both had difficulty conceptualizing a second way to represent the number beyond the more traditional response of “three tens and two ones” or “seven tens and four ones.” However, both demonstrated improvement with this task by accurately providing another representation during the post-assessment. For Molly, she showed 2 tens and 12 ones for 32 and Kyle showed 5 tens and 24 ones for 74. Eva was accurate in her representations for both assessment sessions.

Written Story Problems. Three types of story problems were presented to the students: a) Separate Result Unknown, b) Division and c) Multiplication. Molly had the greatest success with completing the three presented tasks using manipulative devices to direct model all three problems. She would accurately make groups of ten and then proceed to model the problem resulting in 2 out of 3 correct during pre-assessment and 3 out of 3 correct during the post-assessment. Eva also showed improvement from 1 out of 3 correct on the pre-assessment to 2 out of 3 on the post-assessment. Further, Eva was motivated to use manipulative devices to try a valid direct model strategy, but had some difficulty understanding the semantics of the problem resulting in an incorrect response. Finally, Student C demonstrated the greatest difficulty improving from 0 out of 3 correct on the pre-assessment to 1 out of 3 correct on the post-assessment. He was more motivated to solve using written numbers and symbols and if prompted would use visual representations. His difficulty during the post-assessment may have been due to semantics as well for example when presented with the problem “*Mrs. Hall has 142 pencils and wants to put them into boxes of 10. How many boxes will she need? What is leftover?*” he drew 10 groups of 10 and said 42 would be leftover. Similarly, on the multiplication problem “*Ms. Molly has 14 bags with 10 marbles in each bag and 4 extra. How many marbles did she have?*”

he mentally computed the answer 190. When asked to explain he said 14 bags of 10 marbles equals 140 plus 4 extra which is 40. He then inaccurately added 140 and 40 to get 190.

Strategy Use. All participants displayed little to no change in strategy use during the baseline phase. When the intervention was implemented for all three participants, a change in strategy use was noted. Following the intervention, all participants continued to display efficient strategy use when solving objective probes. The following section details the results of each participant displayed in Figure 5.

Molly. During baseline sessions (n=4), Molly displayed a lack of understanding of place value when using a strategy to solve problems 75% of the time. For example, she would select an invalid strategy such as the standard algorithm for addition when the problem was a *Result Unknown* problem type ($62-28=?$). However, Molly did display competence in counting by tens during one baseline session (session 4), which she had not performed prior to this point in time. During the pre-assessment interview and the baseline phase, it appeared that the function of Molly's behavior was work avoidance. Once the intervention was implemented, the student continued to display behaviors that impeded her ability to attend to the mathematics. Her strategy use continued to remain stable with little change or improvement in place value understanding. Therefore, the behavioral component of goal setting, self-monitoring and self-evaluation was implemented in addition to the mathematics intervention resulting in an intervention plus phase of instruction. During the intervention plus phase, there was a clear and instant change in level, trend and stability. Molly consistently demonstrated improved strategy use 100% of the time by moving between direct modeling by ones (n=4 sessions) and by tens (n=5 sessions). Direct modeling was displayed either by using manipulative devices or by drawing a picture. After nine days of intervention plus, the intervention was stopped and one follow-up probe was

administered resulting in a continuation of improved strategy use. Finally, the accuracy of correctly solving objective probes increased from 50% during baseline to 100% during intervention plus. It should be noted too that Molly solved zero problems correctly (see Figure 6) during the intervention phase (n=4 sessions).

Eva. During baseline sessions (n=7), Eva displayed a lack of understanding of place value when using a strategy to solve problems 100% of the time (Figure 5). She would consistently use an invalid standard algorithm to solve problems presented, and would try various ways of plugging in numbers to find a reasonable answer. During the intervention phase, there was a clear and instant change in level. Eva consistently demonstrated improved strategy use 88.89% of the time typically using the direct modeling by tens strategy 66.67% of the time (n=6 sessions). While Eva did successfully use the standard algorithm during session 5, the problem presented could have been solved in a more efficient way using understanding of place value. Conversely, Eva used an invented algorithm 2 sessions later, demonstrating a deeper level of place value understanding. After nine days of the intervention, instruction was stopped and one follow-up probe was administered resulting in a continuation of improved strategy use. Finally, the accuracy of correctly solving objective probes increased from 28.57% during baseline to 88.89% during the intervention phase and 100% during the follow-up probe.

Kyle. Kyle displayed a similar lack in place value understanding during baseline sessions (n=10), fluctuating between using an invalid strategy to solve problems 60% of the time and an accurate standard algorithm 40% of the time (Figure 5). During the intervention phase, there was a clear and instant change in level. Kyle consistently demonstrated improved strategy use 100% of the time moving between direct modeling by ones 42.86% of the time (n=3 sessions) and direct modeling by tens 42.86% of the time (n=3 sessions). During the fourth intervention

session, Kyle used a counting strategy, demonstrating a heightened level of place value understanding. After seven days of the intervention, instruction was stopped and one follow-up probe was administered resulting in a continuation of improved strategy use. Finally, the accuracy of correctly solving objective probes increased from 40% during baseline to 100% during the intervention phase and 100% during the follow-up probe.

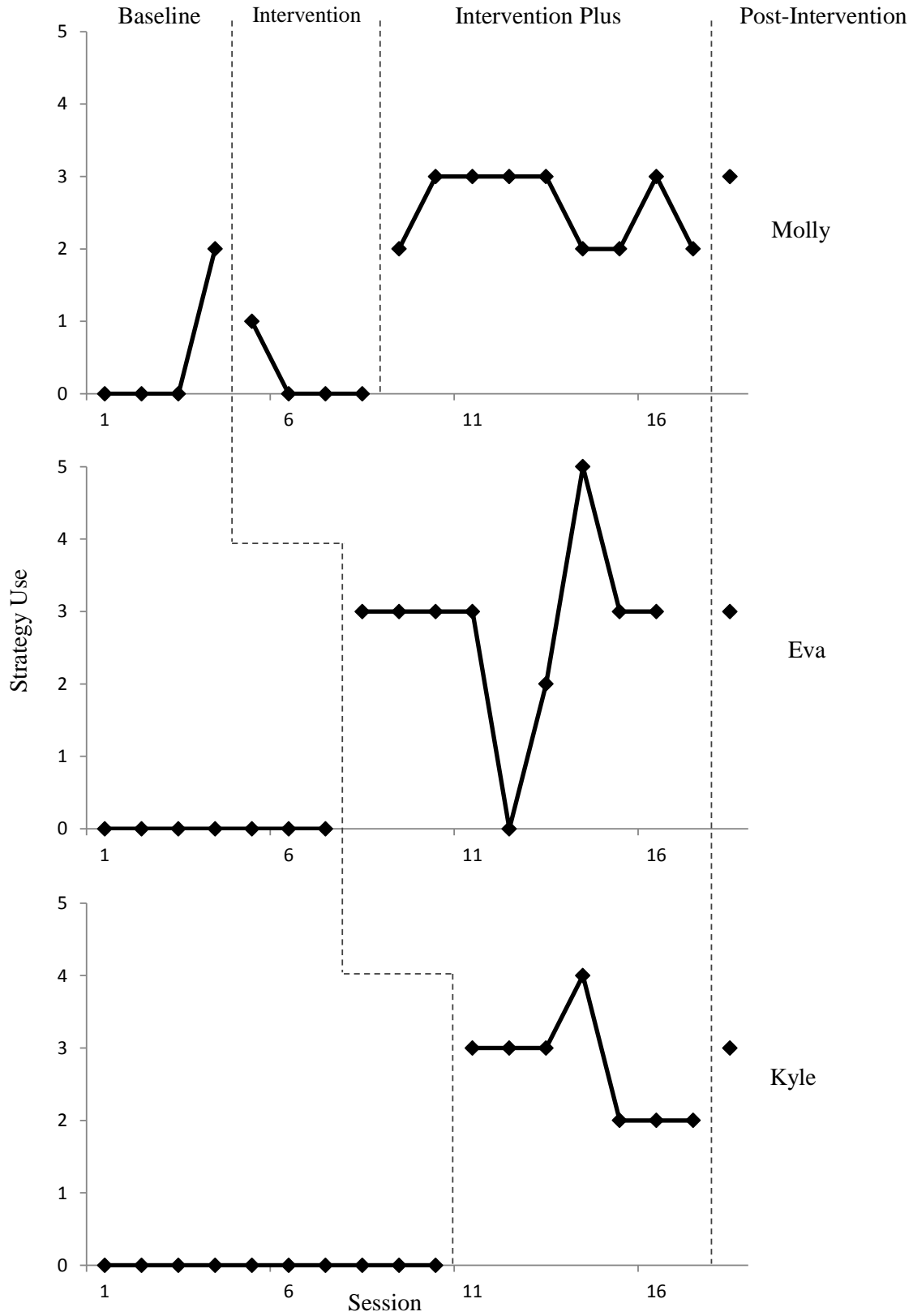


Figure 5. Efficient strategy use.

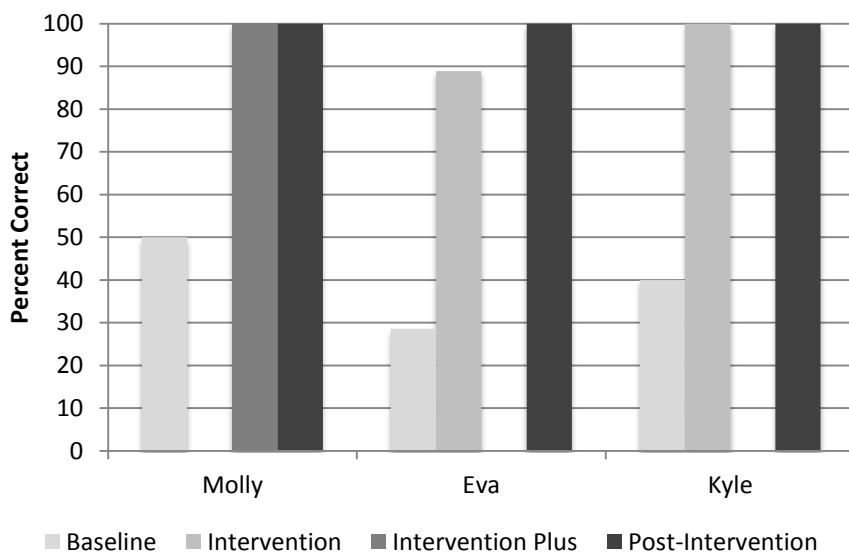


Figure 6. Accuracy on problem-solving probes

Behavioral Change

Research Question 2: What effect does the multi-component mathematics/behavior intervention have on increasing on-task behaviors and decreasing disruptive behaviors?

Molly.

On-task behavior. Molly displayed on-task behavior in 39.5% of baseline intervals (range = 38%-75%) and after implementing the intervention the on-task percentage decreased to 21% of intervention intervals (range = 0%-58%; see Figure 7). After the intervention plus was introduced, there was an immediate increase in on-task behavior (median = 67%, range = 21%-91%). The third session of intervention plus occurred on Halloween which may explain why Molly had a low percentage of on-task intervals (21%), her lowest during any intervention plus session. Despite this data point, Molly's magnitude of change was 27.5% or roughly an increase of 4.5 minutes of instructional time during a 20 minute intervention. Further, there is a decreasing trend during baseline and intervention, followed up an upward trend during intervention plus and maintained during follow-up data collection. The stability of the baseline

data is effected by the outlier of 75% on session one of baseline, leading to overlapping data points.

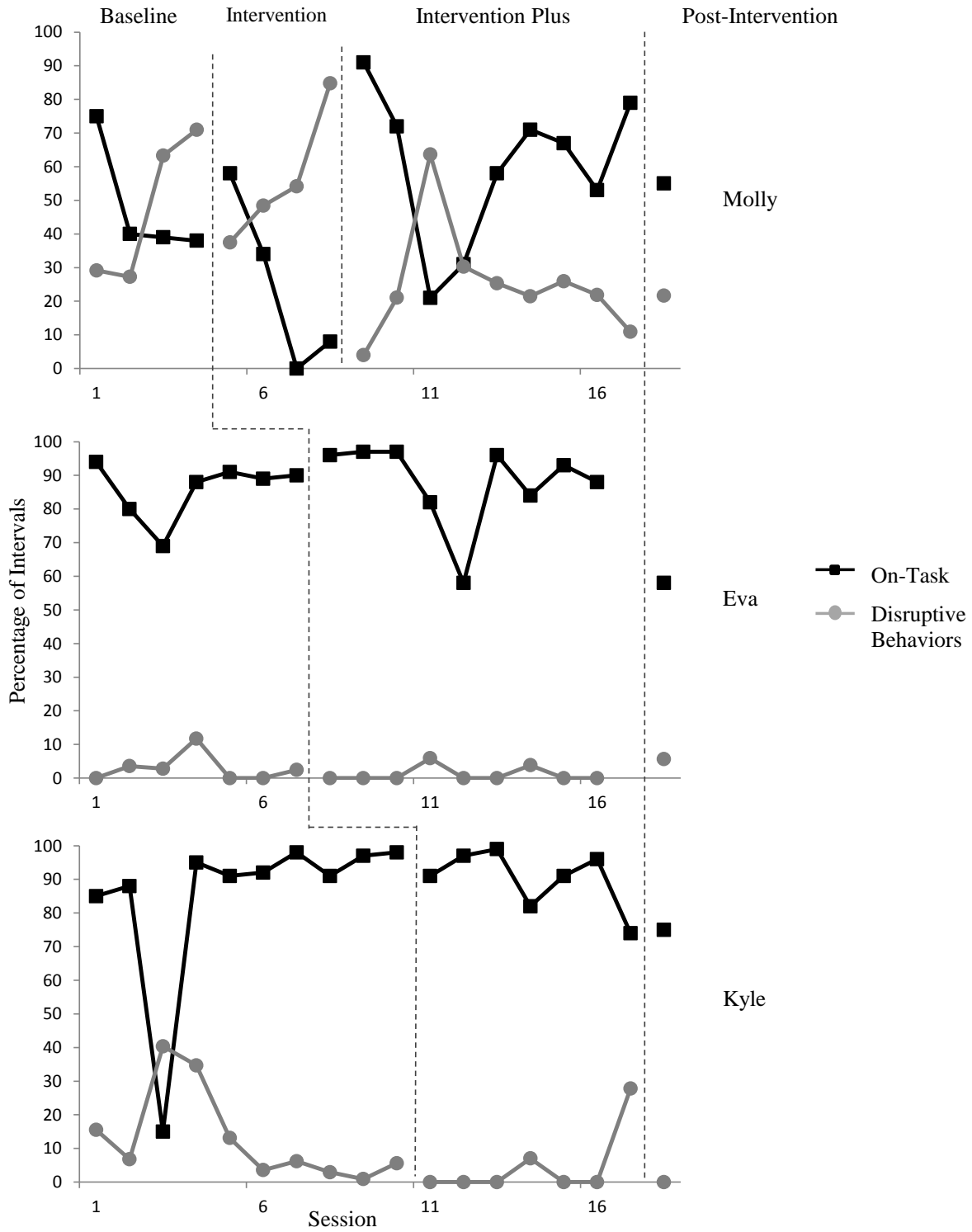


Figure 7. On-task and Disruptive Behaviors.

Disruptive behavior. During baseline, Molly displayed disruptive behaviors in 46.25% of intervals (range = 27.27% – 63.33%; see Figure 7). Following implementation of the intervention, Molly’s disruptive behaviors increased to 51.31% of intervals (range = 37.5% - 84.78%). Once the intervention plus phase was implemented, there was a decrease in disruptive behavior (median = 21.88%, range = 3.97% – 63.64%). Again, session 11 (session 3 of the intervention plus phase) resulted in the highest percentage of disruptive behavior intervals (63.64%) possibly due to the excitement of Halloween. Molly’s magnitude of change was 24.37% or roughly an decrease of 4 minutes of disruption during a 20 minute intervention. The trend during baseline and intervention is a steep increasing slope, compared to the slightly increasing line at a lower percentage during intervention plus. The outlier of 63.64% affects the stability of the data, otherwise the intervention plus phase only has one data point overlapping (30.30%). Upon further analysis of Molly’s disruptive behavior, it appears that the intervention plus was most impactful on her non-compliance shown in Figure 8. During baseline, Molly displayed a median of 16 noncompliant behaviors and a median of 18 during intervention. This percentage dropped to a median of 3 non-compliant behaviors during the intervention plus phase and went back up during follow-up data collection (median = 16) with a magnitude of change of 13 fewer non-compliant behaviors per session from baseline to intervention plus.

Self-monitoring sheets. Molly completed 9 self-monitoring sheets during all intervention plus sessions. Of the 9 sheets completed, Molly self-selected to goal behavior of *following directions* 90% of the time. She identified *count number aloud* 10% of the time and during the ninth session she selected three behaviors (*following directions, complete work and stay at table*). The goal for frequency of behaviors ranged from 7 to 14 with a median of 8 desired

behaviors per session. Following each session Molly self-evaluated her performance and met or exceeded her goal during all nine sessions.

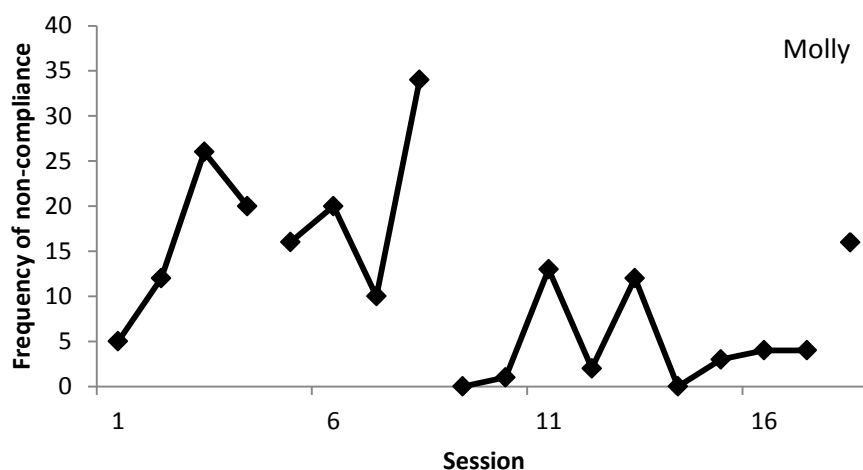


Figure 8. Molly's frequency of non-compliant behaviors

Eva.

On-task behavior. On-task behavior was displayed in 89% of baseline intervals (range = 69%-94%) and after implementing the intervention the on-task percentage slightly increased to 93% of intervention intervals (range = 58%-97%; see Figure 7). As noted earlier, this student did not appear to need behavioral modification beyond the original intervention design which is further supported by the relatively high on-task percentage during baseline. Eva's magnitude of change was 4% or roughly an increase of 40 seconds of instructional time. The trend and stability of the data during baseline and intervention shows little change in slope and percentage. The outlier of 58 during session 5 of intervention (session 12 on figure 7) somewhat affects the stability of the data. On this day, the student complained of a headache and nausea, but verbalized a desire to continue with the lesson.

Disruptive behavior. During baseline, Eva displayed minimal disruptive behaviors resulting in only 2.43% of intervals (range = 0% – 11.73%; Figure 7). Little change occurred following the implementation of the intervention, where Eva's disruptive behaviors decreased to

0% of intervals (range = 0% - 5.91%). Eva's magnitude of change was 2.43% or roughly a decrease of 24 seconds during instruction. The trend and stability during baseline and intervention is a flat line very near 0% with no outliers.

Kyle.

On-task behavior. During baseline, on-task behavior was displayed in 91% of intervals (range = 15%-98%) and after implementing the intervention the on-task percentage remained at 91% of intervention intervals (range = 74%-99%; Figure 7). Like Eva, Kyle did not appear to need behavioral modification beyond the original intervention design, again supported by the relatively high on-task percentage during baseline. No magnitude of change was found (0%) and the trend of the data has a slight increase during baseline and intervention with a few outliers (n=3). These outliers impact the stability of the data during baseline and intervention and lead to overlapping data. The outlier of 15 during session 3 of baseline affects the stability of the data.

Disruptive behavior. During baseline, Kyle displayed relatively few disruptive behaviors resulting in 6.19% of intervals (range = 0.89% – 40.38%; Figure 7). Following this initial spike in disruptiveness, Kyle's behavior stabilized to below 10% of interval and following the implementation of the intervention, his disruptive behaviors decreased to 0% of intervals (range = 0% - 27.78%). Kyle's magnitude of change was 6.19% or roughly a decrease of about 1 minute of disruption during instruction. There is a slight decreasing trend during baseline which remains flat and close to 0% during intervention except for the final intervention session (session 17) which spikes up to 27.78% impacting the stability of the intervention data.

Mathematical Behaviors

Research Question 3: What effect does the multi-component mathematics/behavior intervention have on the frequency of statements students make articulating mathematical reasoning and appropriate use of mathematical materials?

Two mathematical behaviors were coded during sessions, verbal statements articulating mathematical reasoning (including oral counting) and appropriate use of mathematics materials. Data reported are frequency counts of the total number of behaviors during each phase displayed in Figure 9.

Molly. A median of 40.5 mathematical behaviors were displayed during baseline (range = 31-56) and after implementing the intervention the frequency of behaviors decreased to a median of 24.5 behaviors (range = 0-47). After the intervention plus was introduced, there was a gradual increase in mathematical behaviors (median = 47, range = 9-119). Molly's magnitude of change was 6.5 from baseline to intervention plus and 22.5 from intervention to intervention plus. Further, there is a slight increasing slope during baseline and decreasing slope during the intervention phase, followed up an upward trend during intervention plus. Baseline and intervention phase data are fairly stable with intervention plus data less stable.

Eva. During baseline, a median of 54 mathematical behaviors were displayed during baseline (range = 23-79). After implementing the intervention, there was an increase in mathematical behaviors (median = 94, range = 12-164). Molly's magnitude of change was 40 from intervention to intervention plus. Little trend in data during baseline exists and a somewhat increasing trend is displayed during intervention. The outlier of 12 and 49 during session 5 and 6 of the intervention (session 12 and 13) affects the stability of the data. Again this is when Eva complained of a headache and nausea.

Kyle. A median of 49 mathematical behaviors were displayed during baseline (range = 2-99) and after implementing the intervention the frequency of behaviors increased to a median of 104 behaviors (range = 56-188). Kyle's magnitude of change was 55 from baseline to intervention. Further, there is a slight decreasing trend during baseline. As a result of an initial jump in behaviors during the first session of the intervention the trend appears to decrease but at a higher score. While the data is fairly stable during the baseline phase, there is little to no stability during the intervention phase with a high level of overlap.

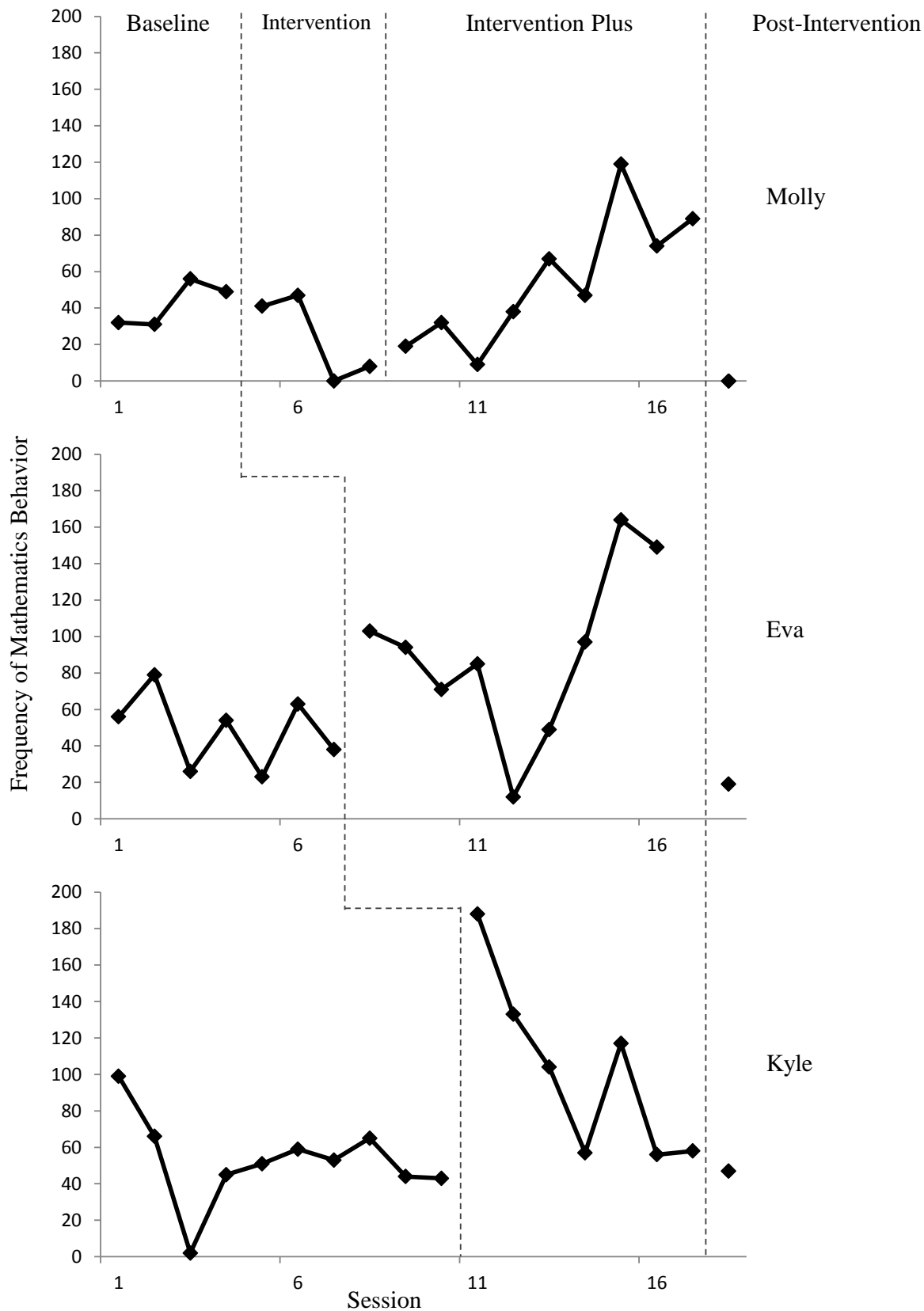


Figure 9. *Mathematical Statements and Appropriate use of Mathematical Materials.*

Chapter 5: Discussion

In this single-subject study, the effects of a multi-component mathematics and behavior intervention were examined to determine if mathematical performance, on-task behaviors and mathematical behaviors would improve while decreasing disruptive behaviors. The study operated under the belief that for all students improving mathematical performance involves making sense of student thinking around mathematical concepts (Clements & Sarama, 2007). This study hypothesized that unique to students with EBD, an additional focus of addressing disruptive behaviors is also necessary. Given the need for targeting mathematics and behavioral performance for students with EBD, results of this study are promising for several reasons.

Mathematical Understanding

Three important findings are highlighted with regards to improved mathematical understanding: a) the success of a modified explicit lesson framework and the use of representation, b) improved place value understanding and c) changes in the strategy use initiated by participants.

Lesson Framework. The instructional strategies employed during this study appear to have influenced the mathematical performance of students with EBD. If this finding holds true in future research, this will be noteworthy since this adds to the limited body of research investigating CGI methods for student with disabilities. Prior to this study all three students received traditional mathematics instruction consistent with teacher directed lessons, a few guided practice problems and independent paper-pencil math problems. During the intervention however, the instruction provided to the participants incorporated a modified explicit instruction format by allotting more opportunities for students to initiate the process of sharing their thinking. In doing so, instruction deviated from the more traditional communication pattern of

IRE (Initiation-Response-Evaluation) often displayed in special education settings (Griffin et al., 2009). By encouraging students to share their thinking and not immediately correcting misunderstandings, teachers gain insight into how students are processing mathematical concepts and can then build upon these misunderstandings during later instruction. The instructional approach of not initially correcting incorrect responses is further supported by research findings which suggest incorrect responses are often correlated with low rates of teacher praise leading to student frustration (Gagnon, Wehby, Strong, & Falk, 2006). In not responding with correction, the role of the teacher is varied in this modified explicit instruction format – moving away from modeling *how* to solve a problem towards *facilitating* students understanding of the problems posed. In varying this role, the teacher works to guide and further develop student thinking as they make sense of problems. Further, during each session participants were encouraged to use representation to display their thinking. All participants used manipulative devices show their thinking and Eva and Kyle often moved beyond the use of materials to include drawings as well. As the participants worked to use representation they would share their thinking. Eva was exceptionally verbal compared to her peers and rarely needed any prompting to share how she was understanding a problem. For her, it appeared as though she was highly motivated by the one-on-one instructional setting which provided her with a risk-free space to share her thinking, free from the negative comments she received from peers which were observed during her regular mathematics block (pre-baseline and follow-up sessions). This is consistent with the literature on establishing positive sociomathematical norms in which students must feel they are in a supportive environment before they are willing to engage in risk-taking behaviors such as sharing how they understand a problem (Yackel & Cobb, 1996). However, this step is necessary because as students engage in the process of verbalizing how they make sense of problems,

students' misunderstandings are revealed and become the source for further investigation. In this way, the use of CGI and representation resulted in 100% accuracy for Molly and Kyle and nearly 90% accuracy for Eva on solving objective probes. As student accuracy improved, so did their knowledge of place value. This finding adds to the body of research supporting the use of representation during mathematics instruction as best-practice for students with disabilities (Gersten et al., 2009).

Place Value Understanding. All participants increased their understanding of place value concepts beyond that of simply naming the place value of digits. For Molly and Eva, this began with improved counting skills which emerged within the first few intervention sessions (intervention plus sessions for Molly). The development of their counting skills is a critical aspect of developing greater understanding of place value (Cawley et al., 2007). For example, Molly who consistently had difficulty naming and writing numbers early on during instruction became more fluent during the later lessons as her counting skills improved. Further, Eva who demonstrated difficulty with counting by tens beyond 100 became very fluent with counting as she began working with visual representations of groups of ten. These two examples align with research on number composition in which the mental activity of forming units is central to counting. During the act of counting, countable units are grouped together (first with tens) aiding student understanding of our number system (Schwartz, 2010).

By representing numbers in a variety of ways during intervention sessions, all participants demonstrated greater flexibility in understanding numbers. In general, representation of numbers often resulted in groupings of tens but at times students were encouraged to compose numbers using various grouping strategies (e.g. displaying 24 with 4 groups of six or 1 group of ten and 14 ones). In doing so, participants demonstrated emergent understanding that numbers

can be composed in a variety of ways. The process of representing numbers using manipulative devices and drawings is related to the strategies the participants selected to solve problems.

Strategy Use-Direct Modeling. Each of the three participants were highly successful with making sense of the story problems presented during the intervention sessions as evidenced by the percent of accuracy when solving problems (group average 96.3%). The accuracy rate of participants is linked to the efficiency in strategy use. During baseline, all participants would read the story problem and then record the numbers embedded in the text into a standard algorithm. This resulted in approximately 40% accuracy, as the participants used invalid algorithms 47% of the time. Once the intervention began (intervention plus for Molly), direct modeling of the story problem was the most commonly used strategy (87.83% of the time), either by direct modeling by ones or by tens. While this is not the most efficient strategy to solve problems, it is a valid strategy to use to make sense of problems (Carpenter et al., 1999). With direct modeling, the problem is modeled either with manipulative devices or drawings and is a tool for understanding the problem situations. When participants in this study engaged in the process of understanding a problem posed by modeling the problem, they were more likely to solve the problem correctly and showed greater levels of perseverance often double checking their work with a variety of representations. Eva in particular, would attempt to solve problems first with direct modeling and then at times would follow this with an invalid algorithm or an algorithm with errors (44.44% of the time). She would then return back to her direct modeling visual as a guide for understanding the problem situation, abandoning the algorithm if it did not result in the same number as the direct modeling strategy. Findings from this study suggest that encouraging students to model problem situations first before solving, results in high levels of accuracy. Research conducted by Alter et al. (2011), had similar findings in which accuracy rates

improved as students drew pictures to solve word problems and teachers engaged in probing students as they worked. For participants to engage in modeling problems, they needed to display key mathematical behaviors.

It should be noted that both Eva and Kyle demonstrated a change in strategy use that was unexpected. For Eva, the change occurred for only one session and happened on a day when she was experiencing a headache and nausea. Her classroom teacher also added that she may have been experiencing some disruption in the home setting which may have led to feeling sick. This is consistent for students with EBD who often experience somatic symptoms when dealing with emotional challenges (citation). Similarly, Kyle's last three sessions resulted in a decrease in efficient strategy use. He too was dealing with changes in the home in which his mother had become unemployed and he expressed frequent concern about his mother's wellbeing. During this timeframe, Kyle refused to participate in his regular classroom program although with the permission of his teacher was willing to continue with the intervention. However, sessions became more brief and he expressed a desire to finish quickly which is hypothesized to have impacted his strategy use and frequency of mathematical behaviors.

Improved Behavioral Performance

The hypothesis that participants would increase on-task behaviors while decreasing disruptive behaviors was true for Molly. Of the three study participants, two demonstrated behaviors that were motivated by gaining adult attention and the other's behavior was motivated by work avoidance. For Eva and Kyle, their behavior was perceived to be a function of seeking adult attention observed during pre-baseline observations and shared informally when classroom teachers nominated potential students for this study. Therefore, the instructional setting naturally accommodated their need for attention in which one-on-one instruction was provided. It is

hypothesized that had these students (Eva and Kyle) received their instruction in a small group setting where adult attention is less frequent, they would have benefited from additional self-regulation strategies (goal setting, self-monitoring, and self-evaluation). Evidence of the attention seeking behavioral function was displayed during follow-up data collection for Eva who received mathematics instruction with a group of approximately 5 other students. During the follow-up session, Eva demonstrated a low level of on-task behavior not exhibited during baseline or intervention. In addition to Eva's need for attention, it is further hypothesized that she is more motivated by adult attention compared to peer attention. This may be due to the belief that her peers are more negative towards her during mathematics instruction where it is apparent that she is "behind" the group, often requiring more time to process content and complete work. Comments and gestures from her peers suggested they were annoyed with how long it would take her to respond (e.g. eye rolling, "hurry up," "you're so slow"). In these situations, Eva was less likely to volunteer answers or participate during group discussions.

In contrast, Molly's disruptive behavior was most likely a function of work avoidance and therefore was not addressed by the one-on-one interaction. Molly's disruptive behaviors actually increased during the intervention prior to the introduction of the self-regulation component. The increase in behavior may have been a result of the introduction of new mathematical materials and new expectations for engaging in mathematical tasks. This finding is similar to that of Alter et al. (2011) in which a change in instruction impacted behavioral performance. This is noteworthy, as teachers of students with EBD are encouraged to implement structured, predictable instruction (Jones et al. (2004). It is widely expected as best practice within the field of EBD that teachers establish classroom environments that are highly structured (Jones et al., 2004). Therefore, deviating from the predictable mathematics instruction may have

been difficult for Molly, but was also necessary in order to achieve the mathematical goals of this study. Yet, once Molly was introduced to the self-regulation component, her on-task behaviors improved and her disruptive behaviors decreased.

Of the disruptive behaviors coded for (off-topic comments, out of seat, noncompliance and misuse of materials), noncompliant behaviors was most responsive to the self-regulation strategies employed. This makes sense since Molly self-selected the goal of following directions 90% of the time. For Molly, had the additional self-regulation component not been added, she would probably not have improved her mathematical performance evidenced by the data collected during the intervention phase. This supports the literature on the importance of self-regulating behavior that is incompatible with the challenging behavior (Maag, 2004). Incompatible behaviors are behaviors that when exhibited prevent the learner from engaging in the problem behavior. In the case of Molly, when she was following teacher directives (incompatible behavior) she was unable to be non-compliant (problem behavior). This finding is important in developing interventions for students with EBD in two ways.

First, as described above, it appears that changing instructional approaches is difficult for students with EBD and including a behavioral component to address a rise in anticipated behaviors is necessary. Second, current research seems to suggest that while students with EBD may improve academic performance without a behavioral component (Alter et al., 2011), an added emphasis on behavior can enhance academic performance or produce increases above and beyond expectations (Cook et al., 2012). For example, findings from Alter et al. (2011), found that a mathematics intervention for students with EBD improved both accuracy and time on-task. And in Cook et al. (2011), when reading and behavioral components were combined participants' reading and behavioral performance improved beyond what they had when reading

and behavior were targeted independently. However, findings from this study seem to suggest that this may not be true for all students with EBD or for all forms of academic instruction such as CGI methods, seeing as Molly's academic performance remained unchanged without the additional behavioral component. Also, since Eva and Kyle did not receive the self-regulation component of the intervention it is unknown what impact this would have had on their academic performance.

Mathematical Behaviors

A key finding from this study is that promoting positive, pro-social behaviors specific to mathematics is effective in addressing mathematical & behavioral performance. Findings from previous studies have suggested that targeting replacement behaviors such as on-task behavior is sufficient in decreasing problem behaviors (Mooney, Ryan, Uhing, Reid, Epstein, 2005). This study extends the literature on types of behaviors to monitor. In each intervention session in this study, students were encouraged to verbalize their thinking and make use of mathematical materials available (manipulative devices, white board, paper and writing utensils). In doing so, all students received positive, behavior specific praise such as, "*I really like the way you used the cubes to understand what the problem was asking.*" Only Molly received additional, earned incentives (tangibles) for meeting self-selected goals for demonstrating these behaviors; for Eva and Kyle, positive praise was sufficient in reinforcing desired behaviors.

Maag, (2004) reviewed research and recommended that reinforcing positive behaviors during academic instruction is related to improvement in behavior and academic performance. For example, Kraemer et al. (2012) found that two Tier 2 behavioral interventions (*Mystery Motivator* and *Get 'Em On Task*) decreased off-task behavior for participants in an elementary English and Mathematics class. However unique to this study was moving beyond general

classroom behaviors such as paying attention during instruction, using an appropriate voice level and completing assigned work. While these are necessary behaviors, they are generic and do not focus on what is unique about each academic subject. This study extends the literature on reinforcing positive behaviors by incorporating a behavioral component of targeting behaviors specific to mathematics, and as a result all three participants increased the frequency of verbalizing their thinking and appropriately using mathematics materials. This study provides evidence that these behaviors are connected to helping students make sense of the mathematics they are engaged in. For example, results from this study suggest that the use of such behaviors may be linked to the accuracy of students solving the objective probes at the end of each session. Also, for Kyle who experienced a decrease in mathematical behaviors during the last three sessions, it is noted that his strategy use also changed to a less efficient strategy (direct modeling by ones) during those same sessions. It is therefore plausible that changes in mathematical behavior are related to mathematical understanding. As students worked to complete problems, they were encouraged to slow down and demonstrate (verbally and/or visually) what the problem was asking. In doing so, they would often self-correct leading to increased rates of problems solved correctly, and as a group averaged an increase in accuracy of 56.77 %.

While the use of these behaviors did not appear to affect the percentage of Eva or Kyle's on-task or disruptive behaviors, which were already at acceptable levels, it is possible that Molly's behavior did improve by focusing on these behaviors. This is evidenced in the decrease of non-compliant behaviors where she began increasing compliance with the self-selected target behaviors of following directions, counting numbers aloud, completing work and staying in the assigned area. However, as previously mentioned, what did occur for all participants was an increase in efficient strategy use, connected to both the instruction received and the

encouragement of behaviors specific to mathematics. What is unknown at the conclusion of this study is what if any benchmark should be established for the frequency of mathematical behaviors displayed during instruction. For example, as a group the participants averaged a median score of 82 behaviors per sessions but no benchmark has been established to draw meaning from this number. Therefore at this time it is only hypothesized that this average is sufficiently high enough to impact mathematical strategy use and accuracy rates.

Implications

Recommendations for Future Research. Future studies should implement this intervention under typical classroom conditions including small group instruction and in a classroom setting with peers present. Further, replication of this study should also be considered for a broader group of students including: a) students with EBD who struggle in mathematics, but are performing at grade level, b) students whose behavior is a function of work avoidance, c) students with EBD whose behavior is not as severe (e.g. those that are educated in general education classrooms) and c) students at-risk of EBD. Since the instruction of this study is designed to meet individual student needs, future instruction should still promote mathematical growth in the same way and potential results should indicate the flexibility of this intervention for a broader population. Following the examination of this intervention with various populations, future research should also be conducted to examine more closely the strength of each intervention component. Ultimately, replication studies will need to be conducted to establish an empirical base for its effectiveness in improving outcomes for students with EBD. In developing future studies, it would be helpful to conduct observations during participants' regular mathematics block to assist in drawing conclusions from intervention results. Further, since relatively few research studies have investigated CGI for students with disabilities, more

research is needed. While this study provides evidence of the effectiveness of CGI for the participants in this study, more research is needed before any broad conclusions can be drawn about its success for students with EBD.

Recommendations for Practice. The results of this study reveal three important practices for teachers working with students with EBD. First, implementing interventions that focus on mathematics and behavior is an effective approach to improving mathematical and behavioral performance. While this is not an easy endeavor, taking a more holistic approach to addressing the unique needs of students with EBD results in simultaneously improving performance across mathematics and behavior. Given the pressure on teachers to maximize instructional time, this study supports the efficiency of addressing behavior while teaching mathematics. In doing so, this ultimately saves instructional time by not needing to stop instruction to address disruptive behaviors and reduces the amount of time teachers need to spend teaching behavior as a separate instructional block of time.

As previously noted, taking a dual approach to teaching behavior and mathematics does require a concerted effort towards both arenas. As teachers consider incorporating behavioral support into their mathematics instruction it is useful to target content specific behaviors. For example, using the Common Core Mathematical Practice Standards (2011) to identify what behaviors might be useful for their students during mathematics is one approach. In doing so, teachers may ask themselves the following questions: *What makes mathematics unique from the other subjects I teach? What do good mathematicians do? What is it that I want my students to do during mathematics instruction?* Teachers should also consider student IEP goals and determine if there are areas of need such as students who have difficulty accepting fault. For such students, getting them to persevere after their first attempt at solving a problem is incorrect

may be a reasonable area of focus. Once target behaviors are identified, teachers will need to develop a plan for incorporating behavioral instruction into their mathematics instruction.

Research has found that students with EBD receive low quality mathematics instruction such as paper pencil tasks and limited levels of active engagement (Jackson & Neel, 2006). However, this study provides promising evidence for the use of CGI for students with EBD. CGI methods provide ample opportunities for students to engage in mathematical tasks and in turn numerous opportunities to practice target behaviors. As teachers evolve in their role as a facilitator, they will see their students level of engagement increase as they begin modeling and verbalizing their thinking. Although explicit instruction is grounded in the belief that all skills must be modeled first, this study provides evidence of students' capability for solving problems correctly without direct teacher modeling. Further, in students gaining opportunities to express their thinking, they deepen their knowledge of mathematical concepts.

Limitations

Interpretation of findings from this study must be considered with the following limitations in mind: sample size, instructor bias and intervention setting. First, this study sought to include up to six participants, but only three met study criteria. This small sample size limits the types of analyses that can be conducted as well as the ability to generalize findings. Since the study criteria was set to include a sub-population within the EBD category (students who function below grade level in math), a larger sample sizes was difficult to obtain. Second, since the author of this study implemented all instruction, instructor bias cannot be ruled out. To address this concern, two observers were utilized to code behavioral data limiting the instructor bias to that of scoring the objective probes. Future research should consider conducting reliability checks on the scoring of all objective probes to further reduce instructor bias. Third,

the intervention sessions were conducted in two locations within the school. Molly received instruction in a conference room, limiting distractions of peers and classroom objects/materials. Eva and Kyle received instruction in an empty classroom utilized for teacher professional development. Both locations were away from peers and typical classroom distractions. In addition, instruction was a one-to-one student-to-teacher ratio. This interaction is atypical of a classroom environment and future research should be conducted to determine the effectiveness of instruction in a group setting or in typical classrooms.

Conclusion

Limited research exists on providing high quality mathematics instruction for students with EBD, and even fewer studies have explored the interaction of mathematics and behavior instruction. Prior to this study, research on implementing instruction on both mathematics and behavior for elementary students with EBD was absent from the research base. This study provides preliminary research on the effectiveness of multi-component interventions addressing mathematics and self-regulation strategies. Continued research is necessary to identify specific populations that benefit from this dual instructional approach and the effectiveness of each instructional component. Further, absent from the research base is the implementation of CGI instructional methods for students with EBD and this study provides seminal evidence on its effectiveness for the study participants. The promising outcomes of this study support the need for a continuation of research on CGI and multi-component interventions for students with EBD.

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