On Managing Stochastic Decentralized Projects

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Abstract

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Managing decentralized projects effectively is a critical issue today as projects have become increasingly complex, costly, and strategically important (especially IT and product development projects). In this dissertation, we analyze a decentralized project that is composed of *n* serial stages with stochastic durations; the project is planned, organized, and funded by a client organization that contracts the work at each stage to independent subcontractors. The focus of this dissertation is to build a game theory framework to examine the performances of various types of contracts.

In chapter two, we propose an exponential incentive payment contract. Our goal is to maximize the client's expected discounted profit. Our proposed contract reflects the convex time-cost trade-off that is well known in the project scheduling literature. We show that this type of contract dominates a fixed price contract with respect to expected client's profit and schedule performance, regardless of payment timing considerations. Using a piece-wise

linear approximation, we show that our contract is a generalization of an incentive/disincentive contract that is frequently used in practice. We show how our contract can be used to find the optimal due date and penalties/bonuses in an incentive/disincentive contract. We compare this contract with several variations and discuss implications for both the client and subcontractors.

In chapter three, we analyze the case where both the client and subcontractors incur an overhead/indirect cost, an important cost element in projects, in addition to resource related direct costs and possible penalty/delay costs. We propose an Exponential Incentive Contract (EIC) that coordinates a decentralized project with risk neutral subcontractors under discounting. In the case where discounting is minimal and can be neglected, we show that the first order Taylor series approximation of EIC is a Linear Incentive Contract (LIC) and it coordinates the decentralized project with risk neutral and risk-averse subcontractors. We also discuss how EIC can be better implemented in practice by approximating it by contracts that are commonly used.

In chapter four, we numerically analyze the cost plus contract and show that the cost plus contract actually creates more uncertainties as it led subcontractors to have non-unique optimal work rates. The performance of the cost plus contractor falls in between that of a fixed price contract and an incentive contract regarding the level of project coordination, client's and subcontractors' expected profits. Moreover, in a dynamic setting where subcontractor work rates can be adjusted without incurring significant additional costs, the cost plus contract optimal work rate will converge to a fixed price contract optimal work rate.

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DEDICATION

In memory of my grandfather, Yu-Guan Chen.

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1 Introduction

The strategic importance of projects in both the public and private sectors has increased greatly in recent years; these projects include many new product development (NPD) projects as well as infrastructure and IT projects. For example, the American Society of Civil Engineers (ASCE) issued a report in 2013 that gave the U.S. infrastructure an overall grade of D+, citing an urgent need for improvement and upgrade projects to address increasing congestion, pollution, and safety concerns caused by a growing population and neglected repair (*www.asce.org*). The estimated price tag from the ASCE for these investments: $3.6 trillion USD needed by 2020. Noting similar concerns in most other countries, the World Economic Forum (2013) estimated that the cost for world-wide infrastructure investment projects might range as high as $5 trillion USD per year. The importance and complexity of private sector projects have likewise increased; for example, the average cost to develop a new drug in the U.S. is now estimated at $868 million USD with potential returns in excess of $5 billion USD (Adams and Brantner 2006).

However, the failure of many of these projects to meet their stated goals appears to be widespread; Flyvbjerg *et al*. (2002) reported that ninety percent of 258 transportation infrastructure projects they studied in Europe, North America, and ten developing nations exceeded their estimated cost by an average of 28 percent. With respect to NPD projects, Tatikonda and Rosenthal (2000) reported that the "average company….had achieved the objectives for past development projects only to a low or moderate extent." Empirical evidence gathered by the Standish Group (2009) indicates that less than thirty-five percent of recent IT projects could be classified as successful. Empirical evidence also suggests that there is significant room for improving the management of both public and private sector projects (Tatikonda and Rosenthal 2000; Flyvbierg *et al*. 2002). While the reasons for schedule delays and cost overruns are varied, some observers have associated these problems with the structure and coordination of decentralized projects. As noted in a recent issue of the Bloomberg News (2012), "agencies can't keep their private contractors in check. Starved of funds and expertise..., officials contract out the project management ... to private companies that have little incentive to keep costs down…." This observation emphasizes the importance of designing proper coordination mechanisms in stochastic decentralized projects; the design of such contracts is the topic of this dissertation.

Many of these public and private projects share a number of common characteristics. First, most of these projects are decentralized; that is, there is a project owner or client organization (*e.g.*, Department of Transportation) who receives a payoff when the project is successfully completed and defines, plans, and funds the stages of the project that are performed by independent subcontractors. Second, these projects are frequently designed as a series of sequential stages so the client organization can review the project at various milestones or stage-gates as the project progresses.

We assume that a project consists of *n* sequential stages; without loss of generality, we assume that the stages are sequentially indexed $i = 1, \ldots, n$ from the start to the completion of the project. We consider serial projects to reflect the fact that many projects characterized by general precedence networks can be subdivided into sequential groups of tasks that are separated by review points or "stage gates" to improve project monitoring and control (Santiago and Vakili, 2005). In some projects (*e.g.*, new product development or construction projects), the project naturally defines a sequential series of tasks or stages.

Following previous research (*e.g.*, Kamien and Schwartz 1972, Buss and Rosenblatt 1997, Kwon *et al*. 2010, Chen *et al*. 2015), we assume that the client receives a fixed payment Q when the project is completed. This amount may represent the expected value of future profits earned by a completed new product development project or the social welfare accrued by a completed public infrastructure project. We assume that the client's payment Q and costs are continuously discounted at an exogenous rate $\alpha \geq 0$.

Following previous research and practice, we assume that there are two types of costs incurred by the client. First, there is a fixed overhead/indirect cost per time unit that reflects security costs, utility expenses, managerial costs, etc. We let H denote the overhead/indirect cost rate incurred by the client during the entire project duration and C_i denote any overhead/indirect cost rate that is related to a specific i^{th} stage. Second, there are resource related costs at each stage that are defined by $k_i r_i^2$ where r_i is the work rate at stage i and

k_i is the resource cost parameter that reflects the complexity and difficulty of the i^{th} stage (Kwon *et al*. 2010; Chen *et al.* 2015). The work rate r_i is a decision variable that is set by the client in a centralized project (or a subcontractor in a decentralized project) to optimize some objective.

The duration of stage i is denoted by t_i, where t_i is a non-negative random variable that is stochastically non-increasing in the work rate $r_i > 0$ (*i.e*., a larger work rate r_i will lead to a higher probability of completing stage i in a shorter time span). Following previous research (*e.g*., Buss and Rosenblatt 1997; Tavares 2002; Klastorin and Mitchell 2007), we assume that t_i are independent and exponentially distributed with density $f(t) = r_i e^{-r_i t}$. Following previous research (Kwon *et al*., 2010), we model this process as a Stackelberg game and assume that the client subcontracts the work at each stage to an independent subcontractor and sets the contract terms (*i.e*., payment amounts and timing) with each subcontractor at the start of the project. Each subcontractor subsequently determines the work rate that optimizes her goals. An important characteristic of this game is that work rates are not observable or verifiable by the client; hence, directly contracting on work rate is infeasible. The contracts issued by the client will directly affect subcontractors' work rates and these work rates will directly affect the profitability of the client via the completion time at each stage. We study this problem from the client's perspective, the goal is to maximize the client's expected discounted profit by issuing an appropriate contract form. Although we have assumed that a project is outsourced to n

independent subcontractors, our results still hold under partial outsourcing (only outsource some stages and complete the rest in-house). The decision to outsource or complete in-house is out of the scope of this dissertation, the focus of this dissertation is to study how contracting affects the project's performance.

1.1 Literature Review

In this section, we provide an overview of the building blocks of this dissertation. Researches that are most related to this dissertation can be categorized into three streams: Decision of outsourcing, managing centralized projects, and managing decentralized projects.

1.1.1 Decision Of Outsourcing

The scope and complexity of projects have increased significantly in recent years, outsourcing a part of tasks to independent subcontractors has become a dominant trend. Which tasks to outsource and whom to outsource to are the focus in this area.

Gutierrez and Paul (2000) discussed the problem of partitioning a project into stages that are outsource to independent subcontractors to minimize project risks. Their model minimized the expected project makespan; when there were multiple optimal solutions, they selected the partition that minimized the variance of project makespan. In a related paper, Paul and Gutierrez (2005) used the concept of stochastic ordering to analyze the case when a client wants to select a single subcontractor from a pool of possible subcontractors. They show that a fixed price contract minimizes the expected cost to the client when the subcontractors are risk neutral (that may not hold when the subcontractors are risk averse). Kwon *et al*. (2011) examined the impact of different sourcing decisions of a project that consists of two tasks with exponential completion times on operating profits. The decision is to outsource some or all the stages in a project when the client pays each subcontractor a fixed amount that is negotiated at time zero. Their models analyze the trade-off between efficiency (outsourced tasks) and control (tasks performed in-house) under both parallel and serial project networks. Their results showed that when the revenue of the organization is relatively small (large), it's beneficial for the organization to keep the project in-house (outsource).

1.1.2 Managing Centralized Projects

The majority of project management related researches fall into this stream. A centralized project, loosely speaking, is a project where the client organization has absolute control over all the tasks, including the control of the starting time of each task and the resources used in each task. Researches in this area focus on scheduling problems (optimal starting times) and time-cost trade-off problems (optimal resources allocation).

Buss and Rosenblatt (1997) considered the problem of finding optimal task start times in projects with parallel precedence networks when task durations are exponential with a goal to maximize the expected discounted profit (a fixed revenue is earned when the project is completed). Their research uncovers a major difference between deterministic and stochastic durations of the same project; that is, when durations are stochastic, delaying the start time of a task on the critical path may increase the expected discounted profit. However in a deterministic setting, only slack tasks are candidates for delay. Our research complements their research by considering the optimal payments to subcontractors which takes into account of the impact of these payments on subcontractors' levels of effort. Elmaghraby (2005) considered a two stage serial project with work contents that are exponential distributed. The duration of a stage is then defined as the work content divided by the amount of resources selected. The objective of their model was to maximize the expected profit by selecting optimal amount of resources in each stage; optimality is achieved through balancing the resource costs and the tardiness cost for a given due date. Their result shows that a dynamic solution can be better than a static solution. In general, their research focuses on a client's resources decision problem under a due date contract. Klastorin and Mitchell (2007) presented an effective algorithm for the stochastic compression problem under a general project network that assumed exponential task duration times. In their research the compression cost is linear in compression time and the overhead cost is linear in project makespan. The objective is to find the optimal compression time for each task that maximizes the expected profit. Papers that applied robust optimization to the stochastic time-cost trade-off problem included Goh and Hall (2013), who developed a satisficing time-cost trade-off model, and Cohen *et al*. (2007) who minimized the total cost.

1.1.3 Managing Decentralized Projects

In a decentralized project, where the client does not have direct control over the subcontractor's actions, contract becomes an important method used to ensure the success of a project. However, for a contract to be enforceable by law, the agreement must be measurable and verifiable and this creates a significant problem in projects. Recall in 1.1.2, a majority of researches seek to find the optimal level of resource that should be allocated in each task. However, in a decentralized project, the client is unable to directly control the level of resource allocated in each task, especially when this resource allocation is not observable to the client. Therefore, directly contract on "resources" is usually not feasible in practice. As result, researchers have been designing contracts to indirectly affect how subcontractors allocated their resources. These incentive problems have been widely studied (Weitzman, 1980), generally in the context of principle-agent theory. However, specific research that combines this area with project management literature has been limited.

Dayanand and Padman (2001) formulated a mix integer programming (MIP) problem that solves for optimal payment amounts and timing to maximize the discounted client's profit. Their results indicates that the client benefits by paying small amounts and/or paying the subcontractors as late as possible. While their analysis considered general project networks, their models were limited to deterministic task durations and neglect the situation where a subcontractor would respond differently to client's decisions. Bayiz & Corbett (2005) analyzed the case when a linear incentive contract was used to coordinate the relative efforts of all subcontractors when a subcontractor's work effort could not be observed by the client. Their research suggested that contracts that increase payments to subcontractors if their relative tasks are completed before a given due date are weakly superior to fixed-price contracts in terms of a shorter expected project makespan and higher expected profits for the client.

Kwon *et al*. (2010) examine delayed payment contracts in the context of projects with parallel tasks. Their model uses an "imputed" continuous-time discount rate to capture the view that both suppliers and manufacturers discount the value of future payments. Their work also assumes that all subprojects are of equal difficulty so all subcontractors' total cost per unit time are equal (thus the work rate is the same for all subcontractors).

Kwon *et al*. (2010*b*) applied the concepts of supply chain coordination to project management. Assuming that duration times are exponentially distributed and the cost of a subcontractor is a quadratic function of the work rate, they showed that time-based and cost sharing contracts can achieve optimal channel coordination when there is a single subcontractor. Chen & Lee (2015) showed that the delivery-schedule-based contracts are able to coordinate the decentralized supply chain in a project management context. In their work, they assumed that payments, penalty rates, and bonus rates are exogenously given; their main focus is on the subcontractor's optimal decision about the targeted material delivery schedule, as well as the subcontractor's optimal decision about her own production schedule.

1.2 Scope and Contributions of this Research

In chapter 2, we propose an incentive payment contract for a serial stochastic decentralized project and analyze its expected profitability (for both the client and subcontractors) and makespan by modeling this contract as a Stackelberg game. We compare this contract to several variations, including a fixed price contract (when subcontractors are guaranteed a fixed payment regardless of stage duration), a hybrid contract (that places a lower bound on the subcontractor payments), a dynamic contract (when subcontractors negotiate with the client only after observing the performance of previous subcontractors), and two payment timing options. Our analysis indicates a number of significant implications, including the non-intuitive result that the incentive payment contract is equivalent with respect to expected client and subcontractor profits and project makespan for both the delayed and non-delayed payment options (a result that differs from the results found by Kwon *et al*. (2010) when tasks are performed in parallel). We show that our results extend to fixed price contracts (a special case of the incentive payment contract), hybrid and dynamic contracts. We also show how certain contractual arrangements can encourage undesired results (*e.g.*, delays and lower expected profits). Our work shows that "pay for increased effort" contracts are always superior to fixed price contracts in terms of shorter expected project duration and higher expected profits for the client. While we prove this result analytically when task durations are exponential, we show numerically that this result also holds when task durations follow both normal and gamma distributions. Recognizing that a contract based on a nonlinear function might be difficult to implement in practice, we show how a client can use the equilibrium solution to the (exponential) contract to define a comparable Incentive/Disincentive (I/D) contract (*i.e.*, define due dates, tardiness penalties, and earliness bonuses) by using a piece-wise linear approximation.

In chapter 3, we focus on coordinating contracts, where a coordinating contract is defined as a contract that induces the subcontractors to select the centralized work rates (centralized work rates maximize the project's collective profit). We show that an Exponential Incentive Contract (EIC) will coordinate decentralized projects while providing the client with the maximum expect discounted profit. To better implement EIC, we propose two approximations by contracts that are commonly used. The first approximation is a general piece-wise linear approximation of EIC, also known as the Incentive/Disincentive (I/D) contract. In the second approximation, we study the case where discounting can be neglected and show that the first order Taylor series approximation of EIC is able to achieve coordination and provides an unambiguous property to handle risk averse subcontractors. This approximation is known as the "lane rental" contract, commonly used in transportation (highway) projects.

In chapter 4, we analyze the cost plus contract and show that this type of contract induces multiple optimal solutions in the subcontractor's problem. We propose a probabilistic approach for a risk neutral client and show that at one extreme a cost plus contract is equivalent to a fixed price contract and at the other extreme a cost plus contract is equivalent to a linear incentive contract. We also proposed a maximum minimum approach for a risk averse client and show that the client's optimal decision is equivalent to that under a fixed price contract.

Last, in chapter 5, we summarize our findings and provide potential research extensions.

2 Incentive Contracts in Serial Stochastic Projects

Empirical evidence comparing contract type and project outcome is limited but generally supports the conclusion that incentive contracts can have a significant and positive impact on project outcomes (Meng and Gallagher, 2012). Several case studies (Bubshait, 2003; Berends, 2000) support these conclusions. Many state transportation departments use incentive/disincentive (I/D) contracts, including the Washington State Department of Transportation (WSDOT) that reports it has used I/D contracts with favorable outcomes to reward subcontractors for early completion of a project phase and/or penalize a subcontractor for late completion or failure to meet quality standards (Walker, 2010).

To better understand how to improve outcomes in real-world projects, we have been studying the design and implications of various contracts and propose a new contract that we denote an incentive payment contract. This contract generalizes several types of contracts observed in practice today and allows tractable analysis. Our analysis is based on a project that consists of $n \geq 1$ sequential stages where the duration of each stage is characterized by a non-negative random variable. Many organizations structure large risky projects as a series of sequential subprojects or stages with numerous review points (or "stage gates") between stages (Santiago & Vakili, 2005), and outsource many or all of these stages to subcontractors. In this way, organizations can focus on managing the overall project and leave specialized functions to experienced subcontractors.

Following previous research (Kwon *et al*., 2010), we model this process as a Stackelberg game and assume that the client subcontracts the work at each stage to an independent subcontractor and sets the contract terms (*i.e*., payment amounts and timing) with each subcontractor at the start of the project. Each subcontractor subsequently determines the work rate that maximizes her respective expected discounted profit. The client receives a fixed payment when the project is completed and sets the contract terms to maximize his expected discounted profit.

The contract that we propose assumes that the client sets values $p_i > 0$ and $\beta_i \geq 0$ for the i^{th} subcontractor at the start of the project; each subcontractor receives a payment equal to $r_i(t_i) = p_i e^{-\beta_i t_i}$, where t_i represents the realized duration of stage *i*. The exponential form of these contracts reflects both the inverse relationship between direct costs and task durations that is generally accepted in most project management literature (Klastorin, 2010) as well as the convexity of this time-cost trade-off (Elmaghraby, 1977). In this way, the contract sets payment terms that reflect the non-decreasing marginal costs associated with reducing stage duration (a similar mechanism was suggested by Bernstein and Federgruen 2005 for coordinating decentralized supply chains).

The variable β_i set by the client represents an incentive that impacts each subcontractor's work rate and performance. Furthermore, we assume that each subcontractor has an opportunity cost $O_i \geq 0$ that represents alternative investment opportunities (that, in turn, also reflects general economic conditions). Given values $p_i > 0$ and $\beta_i \geq 0$, the subcontractor sets the work rate to maximize her expected profit; however, if this expected profit is less than O_i, the subcontractor would decide to not participate in this project. Knowing the value of O_i, we show how this opportunity cost can influence the contract terms offered by the client.

We also examine the timing of subcontractor payments as part of the contract definition; for example, the client can pay a subcontractor when she completes her work or when the entire project is completed. The former payment mechanism is similar to most payment schemes in current practice (Dayanand and Padman, 2001; Meng and Gallagher, 2012) and includes payments made at defined milestones or fixed intervals. Alternatively, the client can pay all subcontractors when the entire project is completed; Kwon *et al.* (2010) labeled this type of contract as a "delayed payment contract". In contrast to the results reported by Kwon *et al*. (2010) for a project when all tasks can be performed simultaneously (*i.e*., a parallel precedence network), we show that there is no difference at equilibrium between delayed and non-delayed payments for all firms in a sequential project (a result that holds for a variety of modeling assumptions).

In this paper, we describe our proposed incentive payment contract and several variations:

- A fixed payment contract that occurs when $\beta_i = 0$. In this case, the subcontractor is paid an amount p_i that is independent of the stage duration. Fixed payment contracts are widely used in practice.

- A delayed payment contract when the client pays each subcontractor when the project is completed.
- A hybrid contract that consists of the proposed incentive payment contract with a guaranteed minimum payment that is independent of stage duration.
- A dynamic incentive payment contract when subcontractors determine their respective work rates after observing the realized duration of preceding stages.

In all cases, we assume that the client sets the payments p_i and values of β_i that maximize his expected discounted profit subject to subcontractor participation constraints; the subcontractors respond by setting their work rates that maximize their expected discounted profits. We compare the various contract types with respect to the expected project makespan and discounted profits for the client and subcontractors. Given the general form of the incentive payment, $r_i(t_i) = p_i e^{-b_i t_i}$, we can analytically compare alternative contracts. For example, we show that the incentive payment contract with $\beta_i > 0$ always dominates a fixed price contract ($\beta_i = 0$) with respect to a client's expected discounted profit and expected project makespan. We describe implications for other contract definitions as well.

In addition, we show that the general form of the incentive payment contract generalizes many I/D contracts that include penalties for tardiness as well as rewards for early completion (Shr, 2004). Specifically, we show how a client can derive an optimal I/D contract (including due dates, tardiness penalties, and earliness rewards) that approximates the equilibrium incentive payment contract but would be easier to implement. To our knowledge, this is the first work that analytically compares fixed price and incentive contracts in a project environment, and presents a structured methodology for setting due dates, tardiness penalties, and earliness bonuses in an I/D contract.

2.1 Incentive Payment Contract and Stackelberg Game Defined

We assume that the client is managing a complex project that is defined by a series of sequential stages that are outsourced to independent subcontractors. Without loss of generality, we assume that the subcontractors are sequentially indexed from the start of the project; that is, the subcontractors proceed in order $i = 1,\ldots, n$. Following Kamien and Schwartz (1972), Buss and Rosenblatt (1997), Kwon *et al*. (2010), and others, we assume that the client receives a fixed payment Q when the project is completed. We assume that the duration t_i of stage i is exponentially distributed with density $f(t) = e^{-r_i t}\, dt$ where the parameter $r_i > 0$ defines the work rate set by the i^{th} subcontractor. The exponential completion time assumption has been widely used in previous project management research (*e.g.*, Buss and Rosenblatt, 1997; Tavares, 2002; Klastorin and Mitchell, 2007; Kwon *et al.*, 2010). We assume that the subcontractors incur an operating cost *k(r)* per unit time; generalizing Kwon *et al*. (2010), we let $k_i(r_i) = K_i + k_i r_i^2$ with $K_i \geq 0$ and $k_i > 0$ although we could use any non-decreasing convex function of the work rate r. Assuming a positive discount rate $\alpha > 0$, subcontractor

i's discounted cost at time 0 is defined by $e^{-\alpha \sum_{j=1}^{i-1} t_j} \int_0^{t_i} \left(K_i + k_i \, r_i^2 \right) e^{-\alpha t} \, dt$ where stage i starts at time $\sum_{j=1}^{i-1} t_j$. Our primary analytical results are based on the assumption that $K_i = 0$ although we study the impact of positive $K_i > 0$ numerically. Furthermore, we assume that the parameters K_i and k_i are common knowledge for all participants in the Stackelberg game, which is reasonable in many scenarios (e.g., the client and subcontractors interact repeatedly).

In this game, the client initially sets $p_i > 0$ and $\beta_i \geq 0$ for all subcontractors to maximize his discounted profit, where each i^{th} subcontractor receives an amount $p_i \, e^{-\beta_i t_i}$ when stage i (or the project) is completed. Given the values of β_i and p_i, each subcontractor determines her equilibrium work rate (and expected stage duration) that maximizes her discounted profit.

In our analysis, we assume that each subcontractor will only accept the contract terms from the client if she can earn a discounted profit that equals or exceeds her opportunity cost $O_i \geq 0$, where we define the opportunity costs as a function H_i of the expected task duration $E[t_i]$, $O_i = H_i\left(E\left[t_i\right]\right)$ where $E\left[t_i\right] = r_i^{-1}$. Opportunity costs could indicate varying economic environments; for example, if $H_i\left(E\left[t_i\right]\right) = a_i + b_i \, E\left[t_i\right]$, small values of $a_i > 0$ and b_i could indicate a difficult economic environment (where subcontractors have limited choices) while larger values of a_i and b_i may indicate multiple alternatives made possible by a strong or improving economic environment. Alternatively, an opportunity cost could indicate the reputation and quality standards of the subcontractor (*e.g.*, a highly respected subcontractor would have more alternatives and could charge a higher rent). Opportunity costs could also reflect expected indirect and overhead costs; for example, a greater value of b_i may indicate the case where a subcontractor expects her task to incur a substantial time commitment with a diminished appeal of project participation.

The notation used in the remainder of the paper is summarized below.

<u>Client Decision Variables</u>:

p_i	payment amount to subcontractor *i*
β_i	incentive factor for subcontractor *i*

<u>Subcontractor Decision Variable</u>:

r_i	work rate chosen by subcontractor *i*

<u>Parameters</u>:

K_i	subcontractor *i*'s fixed resource cost per time period
k_i	subcontractor *i*'s variable resource cost parameter
α	(continuous) discount rate
O_i	subcontractor *i*'s opportunity cost

2.1.1 Basic Incentive Payment Contract Defined

In the basic incentive payment contract, the client sets p_i and $\beta_i \geq 0$ at time 0 and pays an amount $r_i(t_i) = p_i e^{-\beta_i t_i}$ at the conclusion of the i^{th} subcontractor's stage (when stage duration

t_i is realized). We denote this as contract 𝕀; the expected discounted profit for a subcontractor in this case is given by

$$E[\pi_i^{\mathbb{I}}] = E\left[(p_i e^{-\beta_i t_i}) e^{-\alpha \sum_{j=1}^{i} t_j} - e^{-\alpha \sum_{j=1}^{i-1} t_j} \int_0^{t_i} (K_i + k_i r_i^2) e^{-\alpha t} dt \right] = \left(\frac{p_i r_i}{\alpha + r_i + \beta_i} - \frac{(K_i + k_i r_i^2)}{\alpha + r_i} \right) \prod_{j=1}^{i-1} \frac{r_j}{\alpha + r_j}. \tag{2.1}$$

The expected profit defined by (2.1) indicates how changes in other subcontractors' work rates impact the expected profit of subcontractor i. Specifically, when the work rates of predecessor subcontractors k (for $k < i$) increase, the profits of the i^{th} subcontractor increase. With respect to successor subcontractors, changes in their work rates have no impact on subcontractor i's expected work rates or expected payoff.

The subcontractor's expected profit defined by (2.1) is concave with respect to r_i; this observation leads to the result in Proposition 2.1.

Proposition 2.1. Given contract 𝕀, the equilibrium work rate, r_i^*, for each subcontractor is given by the unique positive solution to the equation:

$$r_i^* = \sqrt{\alpha^2 + \frac{p_i(\alpha + \beta_i)(\alpha + r_i^*)^2}{k_i(\alpha + \beta_i + r_i^*)^2} + \frac{K_i}{k_i}} - \alpha. \tag{2.2}$$

Subcontractor i will participate in contract 𝕀 if and only if $\left(\frac{p_i r_i^*}{\alpha + r_i^* + \beta_i} - \frac{(K_i + k_i r_i^{*2})}{\alpha + r_i^*} \right) \prod_{j=1}^{i-1} \frac{r_j^*}{\alpha + r_j^*} \geq O_i$.

For positive $K_i > 0$, we cannot guarantee that the subcontractor will earn positive profit;

however, when $K_i = 0$, we can show that $E\left[\pi_i^{II}\right] > 0$ for the equilibrium work rate r_i^* found by solving (2.2), as stated in the following proposition.

Proposition 2.2. If $K_i = 0$, given values of $p_i > 0$ and $\beta_i \geq 0$, $E\left[\pi_i^{II}\right] > 0$ when the work rate for the i^{th} subcontractor r_i^* is determined by the unique solution to (2.2).

This proposition implies that a subcontractor would always participate in the project for any $p_i > 0$ and $\beta_i \geq 0$ set by the client when opportunity costs $O_i = 0$ (in 2.2, we discuss the case when $O_i > 0$ and $K_i > 0$,). For the remainder of this section, we assume that $O_i = K_i = 0 \ \forall \ i$ so that all subcontractors participate.

We can characterize the relationship between the subcontractor's equilibrium work rate and the value of β_i assuming a fixed payment p_i. Using (2.2), we can show, using implicit differentiation and algebraic manipulation, that

$$\frac{\partial r_i^*}{\partial \beta_i} = \frac{\left(\alpha + r_i^*\right)\left[\dfrac{r_i^* - \left(\alpha + \beta_i\right)}{\alpha + \beta_i + r_i^*}\right]}{2\left(\alpha + \beta_i\right)\left[\dfrac{\alpha^2}{r_i^*\left(2\alpha + r_i^*\right)} + \dfrac{\alpha + r_i^*}{\alpha + \beta_i + r_i^*}\right]};$$

since the denominator is positive, the sign of $\dfrac{\partial r_i^*}{\partial \beta_i}$ depends on the term $r_i^* - \left(\alpha + \beta_i\right)$. This observation allows us to provide the following result, which links the effect of β_i with the value of the payment chosen.

Corollary 2.1. The following conditions hold if $K_i = 0$:

- *If* $p_i < 3\alpha k_i$, *there does not exist any* $\beta_i \geq 0$ *where* $r_i^* = \alpha + \beta_i$, *which implies* $\frac{\partial r_i^*}{\partial \beta_i} < 0$, *for* $\beta_i \geq 0$.
- *If* $p_i = 3\alpha k_i$, *then only* $\beta_i = 0$ *satisfies* $r_i^* = \alpha + \beta_i$, *which implies* $\frac{\partial r_i^*}{\partial \beta_i} < 0$, *for* $\beta_i > 0$.
- *If* $p_i > 3\alpha k_i$, *there exists a unique value* $\beta_i = \hat{\beta}_i > 0$ *that satisfies* $r_i^* = \alpha + \hat{\beta}_i$, *which implies* $\frac{\partial r_i^*}{\partial \beta_i} > 0$, *for* $0 \leq \beta_i < \hat{\beta}_i$ *and* $\frac{\partial r_i^*}{\partial \beta_i} < 0$, *for* $\beta_i > \hat{\beta}_i$.

These results indicate that the relationship between the incentive factor β_i and a subcontractor's equilibrium work rate r_i^* (and expected duration) depends on the value of p_i. Specifically, if $p_i \leq 3\alpha k_i$, increasing β_i will result in a subcontractor reducing her work rate (and increasing her expected stage duration). This occurs when p_i is relatively small (representing an unappealing project); in this case, increasing b_i will further reduce the motivation for the subcontractor to participate in the project. On the other hand, if $p_i > 3\alpha k_i$, increasing β_i will result in a subcontractor increasing her work rate (and reducing the expected duration) as long as $b_i < \hat{b}_i$. The relationship between p_i and b_i values is further discussed in 2.1.2 that considers the equilibrium value of β_i that maximizes the client's expected discounted profit.

We can also analyze the relationship between b_i and a subcontractor's expected discounted profit. Our results are indicated in Corollary 2.2 that shows subcontractor *i*'s expected profit will decrease as β_i increases, given that the subcontractor sets an equilibrium work rate defined by (2.2).

Corollary 2.2. If $K_i = 0$ for a subcontractor operating with contract $\mathbb{I}$, then $\frac{\partial E\left[\pi_i^{\mathbb{I}}\right]}{\partial \beta_i} < 0$, where $E\left[\pi_i^{\mathbb{I}}\right]$ is defined by (1) and the equilibrium work rate is defined by (2).

2.1.2 Maximizing Expected Client Payoff

In this section, we continue to assume that the subcontractor opportunity costs $O_i = 0$ and $K_i = 0$ for all i; all subcontractors will participate for all values of p_i and β_i since they are guaranteed to earn a positive profit (Proposition 2.2). Using contract $\mathbb{I}$, the client (knowing subcontractor cost parameters k_i) sets p_i and β_i (where $\beta_i > 0$) at time 0 and pays $p_i e^{-\beta_i t_i}$ to the i^{th} subcontractor when her stage is completed at time $\sum_{k=1}^{i} t_k$. As previously indicated, each subcontractor sets her equilibrium work rate using (2.2). When the project is completed at time $T = \sum_{k=1}^{n} t_k$, the client receives a fixed payment Q.

Under contract $\mathbb{I}$, the expected discounted profit earned by the client is equal to

$$E\left[\Pi_C^{\mathbb{I}}\right] = E\left[Q e^{-\alpha T} - \sum_{i=1}^{n}\left(p_i e^{-\beta_i t_i}\right) e^{-\alpha \sum_{j=1}^{i} t_j}\right] = Q \prod_{j=1}^{n} \frac{r_j}{\alpha + r_j} - \sum_{i=1}^{n} \frac{p_i r_i}{\alpha + \beta_i + r_i} \prod_{j=1}^{i-1} \frac{r_j}{\alpha + r_j}, \tag{2.3}$$

where r_j is defined by (2.2) and Proposition 2.1. Given values of r_i, values of p_i can be defined by inverting equation (2.2); that is,

$$p_i = \frac{\left(r_i^2 + 2\alpha r_i\right)\left(\alpha + \beta_i + r_i\right)^2 k_i}{\left(\alpha + \beta_i\right)\left(\alpha + r_i\right)^2} . \qquad (2.4)$$

Using (2.3) and (2.4), the problem for the client is then to find the equilibrium values of r_i^* and β_i^* that maximize his expected discounted profit,

$$E\left[\Pi_C^{II}\right] = Q\prod_{j=1}^{n}\frac{r_j}{\alpha + r_j} - \sum_{i=1}^{n}\frac{\left(r_i^3 + 2\alpha r_i^2\right)\left(\alpha + \beta_i + r_i\right)k_i}{\left(\alpha + \beta_i\right)\left(\alpha + r_i\right)^2}\prod_{j=1}^{i-1}\frac{r_j}{\alpha + r_j} . \qquad (2.5)$$

To find the values of r_i^* and β_i^* (and therefore p_i^*), we initially assume that the values of β_i are fixed and use the variable transformation $g_i = \frac{\alpha}{\alpha + r_i}$. For given β_i , we will denote the expected client's profit by $E\left[\Pi_C^{II}\left(\beta_1,...,\beta_n\right)\right]$. Using the g_i variables, the expected client's profit can be stated as

$$E\left[\Pi_C^{II}\left(\beta_1,...,\beta_n\right)\right] = Q\prod_{j=1}^{n}\left(1 - g_j\right) - \sum_{i=1}^{n}\left(\frac{\alpha k_i}{\alpha + \beta_i}\right)\left(1 - g_i\right)^2\left(\frac{1}{g_i} + 1\right)\left(\frac{\alpha}{g_i} + \beta_i\right)\prod_{j=1}^{i-1}\left(1 - g_j\right). \qquad (2.6)$$

Since (2.5) is not jointly concave in r_i^* and β_i^*, we define a search procedure with order O(n) to find the equilibrium values of g_i^* and r_i^* and subsequently find β_i^*. We denote this search procedure as Algorithm *SIP* ("Search for Incentive Payments") to find the unique equilibrium values of g_i that we denote as $\hat{g}_i$. This algorithm is described and its correctness proved in Appendix A.

To find the equilibrium values of $\beta_i \geq 0$, we note that equation (2.6) indicates that the client's expected profit increases monotonically with increasing values of β_i and, in fact, approaches an asymptote as $\beta_i \to \infty$. Support for this statement is given in Proposition 2.3.

Proposition 2.3. If $O_i = K_i = 0\ \forall i$, for unique equilibrium values $(\hat{\gamma}_1, ..., \hat{\gamma}_n)$ defined for given $\{\beta_1, ..., \beta_n\}$, the expected client's profit increases with increasing values of $\beta_i \geq 0$.

Similar to Proposition 2.2, we can prove that the expected client's profit is always positive for equilibrium values of $r_i^* > 0$; that is, the client would always participate in this game when using contract Ⅰ. In the next section, we explore the client's trade-offs between increasing values of β_i and expected payoff, as well as positive values of $O_i > 0$ and $K_i > 0$.

The following proposition characterizes the project makespan's dependence on the incentive parameters β_i. We utilize the notation r_i^{**} to denote the subcontractor i's best-response function *induced* by the prices given by Algorithm SIP.

Proposition 2.4. If $K_i = 0$, a subcontractor's equilibrium work rate r_i^{**} increases in β_i. Furthermore, r_j^{**} increases in β_i for all predecessor subcontractors $j < i$.

Proposition 2.4 indicates that greater values of β_i will result in a greater work rate for subcontractor i (and smaller expected duration). In addition, the work rate for all subcontractors that precede the i^{th} subcontractor will also increase, thereby further reducing the project makespan. These observations result in two significant implications for the client. First, increasing the value of β_n will influence all subcontractors while a comparable increase in β_{n-1}, for example, will only influence *n*-1 subcontractors. Second, the project makespan is decreasing in each incentive parameter β_i.

2.1.3 Variations of the Basic Incentive Payment Contract $\mathbb{I}$

2.1.3.1 Fixed Payment Contracts: Incentive Payment Contracts with $\beta_i = 0$

When $\beta_i = 0$, the client pays subcontractor *i* an amount p_i regardless of t_i (the realized duration of stage *i*). We denote this as contract $\mathbb{F}$. The expected NPV for subcontractor *i* for contract $\mathbb{F}$ can be defined as follows:

$$E[\pi_i^{\mathbb{F}}] = E\left[p_i e^{-\alpha\sum_{j=1}^{i} t_j} - e^{-\alpha\sum_{j=1}^{i-1} t_j} \int_0^{t_i} (K_i + k_i r_i^2) e^{-\alpha t} dt \right] = \left(\frac{p_i r_i}{\alpha + r_i} - \frac{(K_i + k_i r_i^2)}{\alpha + r_i} \right) \prod_{j=1}^{i-1} \frac{r_j}{\alpha + r_j}. \quad (2.7)$$

The expected profit defined by (2.7) is strictly concave in the work rate, r_i; as a result, we can find a closed-form solution for a subcontractor's equilibrium work rate when contract $\mathbb{F}$ is used. This result is given in Proposition 2.5.

Proposition 2.5. Given contract $\mathbb{F}$, the unique equilibrium work rate, r_i^*, for each subcontractor $i = 1,\ldots,n$ is given by $r_i^* = \sqrt{a^2 + \frac{p_i a + K_i}{k_i}} - a$. Subcontractor *i* will participate in Contract $\mathbb{F}$ if and only if $p_i \prod_{k=1}^{i} \frac{r_k^*}{\alpha + r_k^*} - \frac{(K_i + k_i r_i^{*2})}{\alpha + r_i^*} \prod_{j=1}^{i-1} \frac{r_j^*}{\alpha + r_j^*} \geq O_i$.

Assuming that the subcontractor participates in the project (that is, the resulting profits exceed her opportunity cost), the results from Proposition 2.5 indicate that each subcontractor acts independently when setting their own work rate (and expected task duration) under contract $\mathbb{F}$. This observation however, does not apply to their expected profits defined by (2.7) which indicates that subcontractor *i*'s expected profit $E\left[\pi_i^{\mathbb{F}}\right]$ increases (decreases) as the work rate r_j of its predecessors (that is, for $j < i$) increase (decrease). In contrast, the work rates of successor subcontractors (for $j > i$) have no impact on the profits of subcontractor i. Under contract $\mathbb{F}$, the outcome of each individual subcontractor varies with the performance of preceding subcontractors, although such performance is outside of any individual subcontractor's control.

2.1.3.2 Delayed Payment Contracts

In contracts $\mathbb{I}$ or $\mathbb{F}$, the client may specify that the subcontractors are paid at the completion of the project (*i.e.*, at time $T = \sum_{j=1}^{n} t_j$). These delayed payment contracts are analogous to the contracts used by the Boeing Company for the suppliers of the Boeing 787 Dreamliner and a possible factor behind the development delays of that airliner (Greising and Johnsson, 2007). We denote these contracts by $\mathbb{I}_{\mathbb{D}}$ or $\mathbb{F}_{\mathbb{D}}$, respectively.

In contract $\mathbb{I}_{\mathbb{D}}$, the client sets p_i and $b_i > 0$ at time 0 and pays $p_i e^{-\beta_i t_i}$ when the project is completed. Subcontractor *i*'s expected profit in this case is given by:

$$E[\pi_i^{\mathbb{I}_{\mathbb{D}}}] = E\left[(p_i e^{-\beta_i t_i}) e^{-\alpha T} - e^{-\alpha \sum_{j=1}^{i-1} t_j} \int_0^{t_i} (K_i + k_i r_i^2)\, e^{-\alpha t} dt \right]. \qquad (2.8)$$

The expected payoffs at equilibrium for the client and subcontractors under contracts $\mathbb{I}$ and $\mathbb{I}_{\mathbb{D}}$ are equivalent (as well as the expected makespan), even for non-exponentially distributed task durations; this can be shown by rewriting subcontractor *i*'s expected payoff under contract $\mathbb{I}$ as defined by equation (2.1) as:

$$E[\pi_i^{\mathbb{I}}] = E\left[e^{-\alpha\sum_{j=1}^{i-1} t_j}\right] \times E\left[(p_i e^{-\beta_i t_i})e^{-\alpha t_i} - \int_0^{t_i} (K_i + k_i r_i^2)\, e^{-\alpha t} dt\right].$$

Under contract $\mathbb{I}_{\mathbb{D}}$, subcontractor *i*'s payoff as defined by (2.8) can be rewritten as

$$E[\pi_i^{\mathbb{I}_{\mathbb{D}}}] = E\left[e^{-\alpha\sum_{j=1}^{i-1} t_j}\right] \times E\left[\left(e^{-\alpha\sum_{j=i+1}^{n} t_j} p_i e^{-\beta_i t_i}\right)e^{-\alpha t_i} - \int_0^{t_i} (K_i + k_i r_i^2)\, e^{-\alpha t} dt\right].$$

If we let $p_i' = p_i \; E\left[e^{-a\sum_{j=i+1}^{n} t_j}\right]$, then the expected discounted client's profit for the delayed payment contract $\mathbb{I}_{\mathbb{D}}$ can be rewritten as $E\left[\Pi_C^{\mathbb{I}_{\mathbb{D}}}\right] = E\left[Q e^{-\alpha T} - \sum_{i=1}^{n}\left(p_i' e^{-\beta_i t_i}\right) e^{-\alpha\sum_{j=1}^{i} t_j}\right]$ that is equivalent to the expected client's profit under contract $\mathbb{I}$ as defined by (2.3).

Likewise, the unique equilibrium work rates for all subcontractors are equal under the two contracts as stated and proved in Proposition 2.6. This result does not require the task durations to be exponentially distributed, and only requires the non-negative random variables to be statistically independent.

Proposition 2.6. For each subcontractor, the unique equilibrium work rate under contracts $\mathbb{I}$ and $\mathbb{I}_{\mathbb{D}}$ are equal; that is, $r_j^{\mathbb{I}*} = r_j^{\mathbb{I}_{\mathbb{D}}*}$ for *j = 1,…, n*. Furthermore, the expected client's profit, expected subcontractors' profit, and the expected makespan are equal under contracts $\mathbb{I}$ and $\mathbb{I}_{\mathbb{D}}$

given independent task durations.

Proposition 2.6 also holds under positive opportunity costs. If subcontractors willingly participate in the project, opportunity costs are a non-binding constraint and the analysis of Proposition 2.6 is applicable. If subcontractors are not willing to participate, an "adjustment'' is necessary, which is discussed in detail in section 2.2.2. However, since the functional forms of the subcontractors' profits are essentially the same, the adjustments will be identical under both contracts, and Proposition 6 remains applicable.

Note that the equivalence of delayed and non-delayed contracts only holds for sequential projects; Kwon *et al*. (2010) showed that there is no equivalence in projects where tasks are performed in parallel. The intuition behind this observation is differing motivations: (1) under a delayed parallel contract, a subcontractor has a motivation to slow down if other subcontractors are slow, whereas (2) there is no such motivation under a delayed sequential project or any non-delayed (sequential or parallel) project.

2.1.3.3 Hybrid Contracts: Incentive Payment Contracts with Minimum Guaranteed Payments

To reduce subcontractor risk, a client may offer a subcontractor an incentive payment contract with a guaranteed amount $g_i > 0$ that is paid regardless of the stage duration t_i. We denote this (combination of a fixed and incentive payment contract) as a hybrid contract $\mathbb{I}_{\mathbb{H}}$. To avoid trivial cases, we assume that $g_i < p_i$. In a hybrid contract $\mathbb{I}_{\mathbb{H}}$, a subcontractor is paid an amount equal to $\max\left\{p_i e^{-b_i t_i}, g_i\right\}$ following the realization of t_i.

We assume that the amount g_i is negotiated at time $t = 0$ with the values of p_i and b_i. A subcontractor under incentive contract $\mathbb{I}$ would have to complete their stage no later than time q_i to earn at least an amount g_i, where $q_i = -b_i^{-1} \ln\left(\frac{g_i}{p_i}\right)$. In this hybrid contract $\mathbb{I}_{\mathbb{H}}$, the expected discounted profit for subcontractor i is

$$E[\pi_i^{\mathbb{I}_{\mathbb{H}}}] = E\left[e^{-\alpha\sum_{j=1}^{i} t_j} \max\left(p_i e^{-\beta_i t_i}, \gamma_i\right) - e^{-\alpha\sum_{j=1}^{i-1} t_j} \int_0^{t_i} (K_i + k_i r_i^2) e^{-\alpha t} dt \right]$$
$$= \left\{ \frac{p_i r_i \left[1 - e^{-(\alpha+\beta_i+r_i)\theta_i}\right]}{\alpha+\beta_i+r_i} + \frac{\gamma_i r_i e^{-(\alpha+r_i)\theta_i}}{\alpha+r_i} - \frac{K_i + k_i r_i^2}{\alpha+r_i} \right\} \prod_{j=1}^{i-1} \frac{r_j}{\alpha+r_j}.$$

The definition of $E\left[\pi_i^{\mathbb{I}_{\mathbb{H}}}\right]$ indicates that the hybrid contract approaches a pure incentive payment contract as $g_i \to 0$ and a fixed price contract as $g_i \to p_i$. While we find analytical results intractable, numerical experiments indicate that the expected subcontractor profits and expected project makespan increase monotonically with g_i at equilibrium, while the expected client profit decreases. Overall, the hybrid contract falls between the pure incentive payment contract $\mathbb{I}$ and the fixed price contract $\mathbb{F}$.

2.1.3.4 Dynamic Incentive Payment Contracts

In a dynamic contract, each subcontractor waits until preceding subcontractors have completed their respective stages before negotiating the terms of their contract with the client. (In previous discussions, we assumed that static contracts were used where negotiations between the client and subcontractors occurred at time 0.) When using an incentive payment contract 𝕀, however, the optimal values of p_i and b_i are the same in both the dynamic and static contracts for all subcontractors and, therefore, the expected client's profit is the same as well.

To understand the equivalence between static and dynamic contracts, assume that *(m-1)* stages have been completed at time $t = S$, where S is the realized sum of the random durations of the *(m-1)* preceding stages (that subcontractor m observes). Subcontractor m's expected payoff at time $t = S$ then becomes

$$E[\pi_m^{\mathbb{I}}] = E\left[(p_m e^{-\beta_m t_m}) e^{-\alpha t_m} - \int_0^{t_m} (K_m + k_m r_m^2) e^{-\alpha t} dt \right] e^{-\alpha S} = \left(\frac{p_m r_m}{\alpha + r_m + \beta_m} - \frac{(K_m + k_m r_m^2)}{\alpha + r_m} \right) e^{-\alpha S}.$$

Since $e^{-\alpha S}$ is a constant, previously completed stages do not affect subcontractor m's first-order condition, resulting in the same work rate that maximizes (1) at time $t = 0$. Intuitively, the dynamic contract changes the time at which subcontractor m makes a decision (time S versus time 0), but the decision itself is the same since the subcontractor can only *directly* influence her discounted profit over the interval $(t_{m-1}, t_m]$. Her expected profit, discounted to time zero, depends only on the decisions of predecessor subcontractors that are outside of her control in both the static and dynamic contracts. The subsequent analysis for the client, which only requires the best-response functions of the subcontractors, is identical. Thus, static and dynamic incentive payment contracts are equivalent.

2.2 Implications of Incentive Payment Contracts

We investigate the impact of positive fixed costs $K_i > 0$ in 2.2.1, positive opportunity costs $O_i > 0$ in 2.2.2, and profit-makespan trade-offs in 2.2.3.

2.2.1 Positive Fixed Costs ($K_i > 0$)

While our analytical results do not extend to cases with positive $K_i > 0$, we investigated the impact of positive fixed costs numerically (including subcontractor participation constraints). In this section, we assume zero opportunity costs and the following parameters for $n = 3$ subcontractors:

$$Q = \$1000,$$
$$a = 0.1,$$
$$k_1 = k_2 = k_3 = 200.$$

When $K_i > 0$, Proposition 2.3 no longer necessarily holds although we observe that the client's equilibrium profit is unimodal in each b_i for most K_i. Furthermore, we find that the client's profit is maximized at finite values of b_i for the above parameters when $K_i > 5$. If $K_i < 5$, the client's equilibrium profit is still strictly increasing in the b_i parameters. These results are summarized in Table 2.1, which provides the equilibrium client profit and maximizing values of b_i^* as a function of K, where $K_i = K$ for all i. We found similar results for varying values of k_i, as well as the cases when task durations follow both the normal and gamma distributions.

K	$E[\Pi_C^{\mathbb{I}^*}(\beta)]$	β_1^*	β_2^*	β_3^*
0	338.5	Inf	Inf	Inf
5	351.1	10.9	Inf	Inf
10	332.3	1.8	3.8	7.5
15	312.7	0.9	1.9	3.4
20	293.5	0.6	1.3	2.2

TABLE 2.1: NUMERICAL EXAMPLE (CONTRACT $\mathbb{I}$) WITH N = 3 SUBCONTRACTORS.

Comparing contracts $\mathbb{F}$ and $\mathbb{I}$, Proposition 2.3 shows that an incentive payment contract (contract $\mathbb{I}$) dominates a fixed price contract (contract $\mathbb{F}$) with respect to the client's expected profit; Table 2.1 indicates this result continues to hold when $K_i > 0$. With respect to a subcontractor's expected profit, however, the reverse appears to hold; that is, a subcontractor's expected profit is greater with a fixed price contract than an incentive payment contract given the same payments p_i to each subcontractor.

2.2.2 Subcontractor Opportunity Costs Considered

When subcontractors have positive opportunity costs O_i, they may not participate if offered the prices from the SIP Algorithm (described in Appendix A). In this case, we propose a procedure that appropriately modifies the SIP prices to guarantee subcontractor participation, while sacrificing a minimum amount of the client's profit. We discuss contracts $\mathbb{F}$ and $\mathbb{I}$ in detail; other contracts can be analyzed in an analogous fashion.

Using contract $\mathbb{F}$, subcontractor i's expected profit, defined by (2.7) and evaluated at the equilibrium work rate r_i^* of Proposition 2.5, can be written as

$$\left(p_i - \frac{K_i}{r_i^*} - k_i r_i^*\right)\left(\frac{r_i^*}{\alpha + r_i^*}\right)\prod_{j=1}^{i-1}\frac{r_j^*}{\alpha + r_j^*}.$$

This expression is strictly increasing in p_i (easily seen by calculating its derivative and noting that $\frac{\P r_i^*}{\P p_i} > 0$); when the price is zero, the subcontractor profit is also zero. Conversely, the opportunity costs are decreasing in p_i assuming $O_i = a_i + b_i\, r_i^{-1}$. Thus, there exists a unique price $\underline{p}_i(O_i)$ where the subcontractor's profit is exactly equal to its opportunity cost O_i. This is the minimum price that must be offered to subcontractor i to induce participation; our heuristic adjustment is basically to offer the subcontractor a price equal to $\max\left\{p_i^{SIP}, \underline{p}_i(O_i)\right\}$ where p_i^{SIP} is the price determined by the SIP algorithm.

A similar case exists under contract $\mathbb{I}$ when $O_i > 0$, although the adjustment procedure is more involved. Basically, if the subcontractor's expected profit is too low when $b = 0$ to entice her to participate in the project, prices are adjusted upwards; if the expected profit exceeds O_i, the value of b_i is increased (to a finite value) to reduce the subcontractor's expected profit to O_i. The adjusted SIP algorithms for both contracts $\mathbb{F}$ and $\mathbb{I}$ are fully described in Appendix A.

To further illustrate the impact of opportunity costs, we modified the example in Table 2.1 to include positive subcontractor opportunity costs. For simplicity, we assume that all subcontractors have the same positive opportunity cost structure: $O_i = a + \frac{b}{r_i}$ for i =1, …, 3. We consider a set of values for the parameter $a \in \{0, 2, 4, 6\}$ and let $b = 1$. Recall that cost values for the three subcontractors are $k_1 = k_2 = k_3 = 200$ and we let $K_1 = K_2 = K_3 = 3$ (the common fixed cost value of 3 is selected to preserve the monotonicity of Proposition 2.3).

Case	O_i	Client's Profit	Makespan	System Profit
1	$0 + r_i^{-1}$	349.0	6.1	359.6
2	$2 + r_i^{-1}$	270.5	9.6	300.9
3	$4 + r_i^{-1}$	182.0	12.1	239.5
4	$6 + r_i^{-1}$	139.2	17.4	191.2

TABLE 2.2: NUMERICAL EXAMPLE (CONTRACT 𝕀) WITH OPPORTUNITY COSTS.

In case 1, the opportunity costs, at equilibrium, are smaller than the corresponding subcontractor profits under contract 𝔽. Therefore, each subcontractor is offered contract 𝕀, and, per the *Opportunity-Cost Adjustment (*contract 𝕀*)*, the b_i parameters are adjusted upwards for each subcontractor to lower the subcontractor profits to their opportunity costs, while simultaneously increasing the client's profit. In case 2, the opportunity cost is larger than Subcontractor 1's profit under contract 𝔽, but lower than Subcontractors 2 and 3's profit. Contract 𝔽 is offered to Subcontractor 1, with the p_1^{SIP} price appropriately adjusted upwards, per the *Opportunity-Cost Adjustment (*contract 𝔽*)*, to guarantee participation. In contrast, contract 𝕀 is offered to Subcontractors 2 and 3, and the b_i parameters are adjusted upwards for these subcontractors to lower their profits to their opportunity costs, while simultaneously increasing the client's profit. Case 3 is similar to case 2 with the difference that contract 𝕀 is only offered to Subcontractor 3. In case 4, the opportunity costs at equilibrium are larger than the corresponding subcontractor profits under contract 𝔽. Therefore, each subcontractor is offered contract 𝔽, and the p_i^{SIP} prices are adjusted upwards for each subcontractor to raise the subcontractor profits to the level of their opportunity costs, thereby guaranteeing their participation.

The numerical example in Table 2.2 leads to several general insights. For simplicity we fix the *b* parameter and vary the *a* parameter. In strong economic climates (suggested by high values of the parameter a_i), contract 𝕀 is not feasible and contract 𝔽, with prices adjusted upwards, must be used. In weak economic environments (suggested by low values of the parameter a_i), contract 𝕀 dominates contract 𝔽, and the opportunity costs lead to finite subcontractor-dependent incentive parameters b_i. In the intermediate economic climates, our numerical results suggest that both contracts 𝔽 and 𝕀 can be utilized. Therefore, it appears that contract 𝕀 is more appropriate in weaker economic environments. Our analysis indicates that the client benefits as the economy weakens: the client's profit is decreasing in the parameter a and the project makespan is increasing in a. However, the entire system's profit (client and all subcontractors) decreases as the parameter a increases. This observation can be understood intuitively if we consider the incentive parameters b_i as proxies for the level of coordination in the project: as the parameters are increased, the subcontractor incentives become more aligned with that of the project (and client), resulting in more system profit. However, opportunity costs restrict the values of b_i, and consequently limit the level of coordination. Therefore, while the subcontractors' profits increase due to higher opportunity costs, the client's profit decreases by an amount that is (much) more than all subcontractor increases combined. Similar behaviors are observed by varying the parameter *b*.

2.2.3 Expected Profit/Makespan Trade-offs

As indicated in the previous discussion, there is a trade-off between the client and subcontractor profits, as well as project makespan. The concept of opportunity cost allows us to compare the fixed price contract $\mathbb{F}$ with the incentive payment contract $\mathbb{I}$ when each subcontractor earns an amount exactly equal to their respective opportunity cost (in this example, we set $K = 0$ and $O_i = 24$ for all $i = 1,2,3$). Specifically, we modified our algorithm to find the equilibrium solution when the expected makespan must be less than or equal to a given parameter, *X*. By solving our model for varying values of *X*, we derived the results given in Figure 2.1, which indicate that the incentive payment contract is clearly superior to a fixed price contract in this example. Furthermore, our analysis showed that the two curves (and contracts) retain the shapes indicated in Figure 2.1 but converge as the opportunity cost *O* increases (since the optimal values of β_i converge to zero).

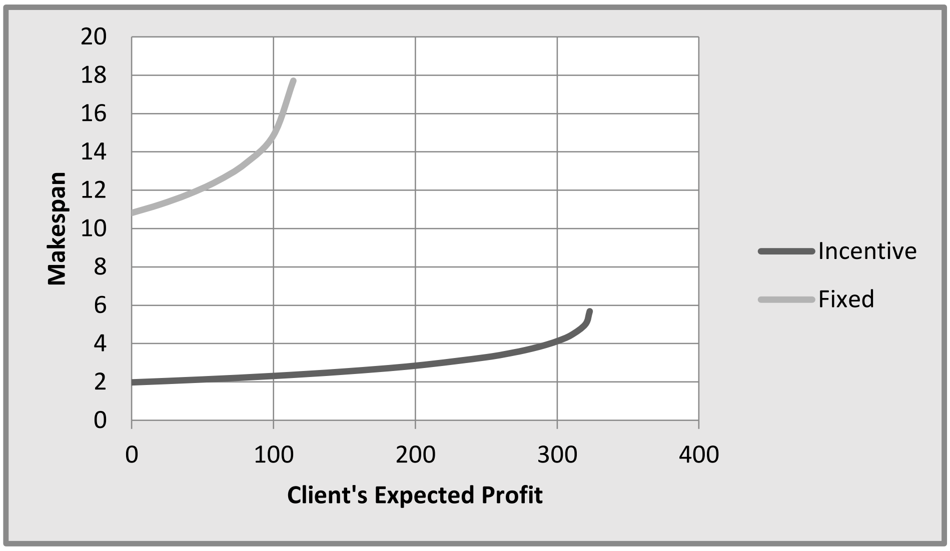

FIGURE 2.1: EXPECTED CLIENT'S PROFIT VERSUS MAKESPAN (O = 24).

2.3 Using an Incentive Payment Contract to Define an Optimal I/D Contract

Given an equilibrium solution to the (exponential) form of contract $\mathbb{I}$, we can use this solution to generate piece-wise linear contracts that incorporate a deadline, penalties for late completion, and rewards for early completion. These linear contracts are generally known as I/D contracts in practice. By using linear approximations of contract $\mathbb{I}$, we are able to derive an I/D contract that is more likely to be adopted in practice, yet retain many of the benefits of the nonlinear incentive payment contract.

Recall that the duration of stage *i* is a non-negative random variable with pdf $f_i(t) = r_i e^{-r_i t}$ and CDF $F_i(t) = 1 - e^{-r_i t}$. Contract $\mathbb{I}$'s payment function is $r_i(t_i) = p_i e^{-b_i t_i}$ where the equilibrium parameters p_i and b_i are found using the SIP algorithm (modified appropriately if subcontractor opportunity costs are positive) and t_i is the realized duration of stage *i*. To convert the incentive payment contract to a form that is more likely to be adopted in practice, we want to use our results to define an incentive contract with the general form:

$$\text{payment} \quad + \quad \text{Bonus} \ \times \ (\text{Due Date} - t_i)^+ \quad - \quad \text{Penalty} \ \times \ (t_i - \text{Due Date})^+ \ .$$

To derive such a contract, we initially define a maximum possible duration $\hat{T}_i$ for each i^{th} subcontractor that could occur with high probability. To determine a value for $\hat{T}_i$, we let $0 < \omega < 1$ denote the probability that the duration of stage *i* is within time $\hat{T}_i$. Given a value of ω (*e.g.*, 0.95), we let $\hat{T}_i = F_i^{-1}(\omega) = \dfrac{-\ln(1-\omega)}{r_i}$.

Next, let τ_i denote the due date for the i^{th} subcontractor, that we derive from the equilibrium form of Contract II: The piece-wise linear approximation to $r_i(t_i) = p_i e^{-b_i t_i}$ will share three points with the original function, namely $t_i \in \{0, \tau_i, \hat{T}_i\}$. Therefore, the linear approximation is a global over-estimator of $r_i(t_i) = p_i e^{-b_i t_i}$, as indicated in Figure 2.2.

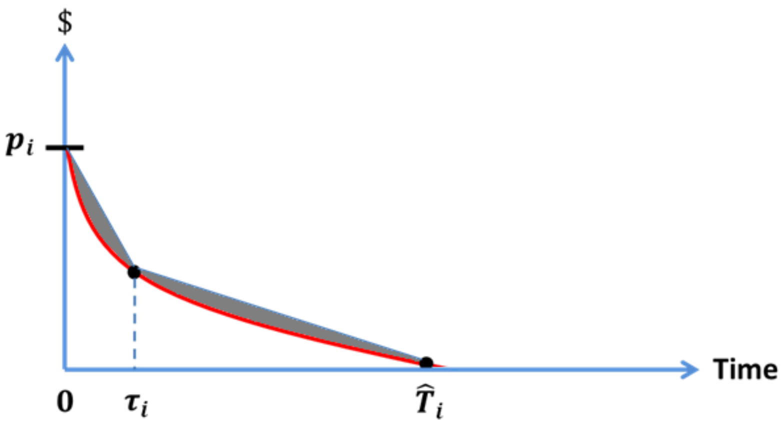

FIGURE 2.2: PIECE-WISE LINEAR APPROXIMATION OF INCENTIVE PAYMENT CONTRACT SOLUTION.

The quality of the approximation can be measured by comparing the areas under the two payment curves; since the approximation is an over estimator, we can simply subtract the area under $r_i(t_i) = p_i e^{-b_i t_i}$ (from 0 to $\hat{T}_i$) from that of the linear approximation (this is the L_1

function space norm of the non-negative difference). This results in the following non-negative expression for the difference in areas:

$$A_i(t_i) = t_i p_i + \left(\hat{T}_i - 2t_i\right) r_i(t_i) + t_i r_i(\hat{T}_i).$$

To find t_i, we want to minimize $A_i(t_i)$. We can show that the function $A_i(t_i)$ is convex and using the first order condition, the minimizing value of t_i satisfies $t_i = -b_i^{-1} \ln\left[\frac{1+e^{-b_i\hat{T}_i}}{2-2t_i b_i + \hat{T}_i b_i}\right]$. There is no closed form solution for t_i; however, given that $A_i(t_i)$ is convex, we can use a gradient search procedure to find t_i easily.

The base payment is defined as $r_i(t_i)$, the bonus is then defined as $\frac{p_i - r_i(t_i)}{t_i}$ (the absolute value of the first segment's slope) and the penalty is defined as $\frac{r_i(t_i) - r_i(\hat{T}_i)}{\hat{T}_i - t_i}$ (the absolute value of the second segment's slope). Therefore, the piece-wise linear approximation to Contract 𝕀, at equilibrium, is defined as

$$\textit{Payment to } i^{th} \textit{ subcontractor } = r_i(t_i) + \left(t_i - t_i\right)^+ \left[\frac{p_i - r_i(t_i)}{t_i}\right] - \left(t_i - t_i\right)^+ \left[\frac{r_i(t_i) - r_i(\hat{T}_i)}{\hat{T}_i - t_i}\right]$$

We can extend this approach to n+1 segments, with n "deadlines," that would allow multiple levels of penalties and rewards. Finding these values requires the solution of a convex optimization problem that can be solved efficiently using standard algorithms and programs.

2.4 Conclusions

We proposed and analyzed an "incentive payment" contract for a stochastic project that consists of $n \geq 1$ serial stages, where each stage is completed by an independent subcontractor. In the basic form of the "incentive payment" contract, the client pays each subcontractor an amount $p_i e^{-\beta_i t_i}$ at the conclusion of a subcontractor's stage or the entire project. The parameters $p_i > 0$ and $\beta_i \geq 0$ are revealed to each subcontractor at the beginning of the project.

The primary contribution of this chapter is to analytically demonstrate the superiority of an incentive contract over a fixed price contract from the perspective of a client who wants to maximize his expected discounted profit in serial stochastic projects. We also showed how a client can calculate optimal parameters for these incentive contracts. Our analysis revealed several other significant implications as well. For example, we showed that the two incentive payment contracts (contracts $\mathbb{I}$ and $\mathbb{I}_{\mathbb{D}}$) are equivalent with respect to expected profit for the client and subcontractors, as well as the expected makespan (we showed that this result also holds for the fixed price contracts $\mathbb{F}$ and $\mathbb{F}_{\mathbb{D}}$). However, we showed that there are significant differences between the incentive payment contracts (contracts $\mathbb{I}$ and $\mathbb{I}_{\mathbb{D}}$) and the fixed payment contracts (contracts $\mathbb{F}$ and $\mathbb{F}_{\mathbb{D}}$). The client will always have a greater expected profit with an incentive payment contract although subcontractors will have a greater expected profit with a fixed price contract. We also showed that the expected makespan is always less with an incentive type contract than with a fixed price contract. Unfortunately, the client is not always able to utilize an incentive payment contract; if subcontractors have large opportunity costs, then only contract $\mathbb{F}$ (with appropriate adjustments) can be utilized; if subcontractors have small opportunity costs, then contract 𝕀 can be used, but the choice of parameters (*i.e.*, β_i) is restricted. We also showed how an incentive payment contract can be applied in practice, by deriving a piece-wise linear approximation. In its simplest form, this approximation allows for a deadline, a penalty rate for late completion, and a reward rate for early completion.

Our results remain applicable if subcontractors and the client discount cash flows at different rates. Specifically, Proposition 2.1 still holds when the discount rate is replaced by subcontractor-dependent discount rates α_i. We found a generalization of the analysis of the client's equilibrium profit intractable, but we have confirmed via computational studies that our main findings still hold. Our results also remain applicable if the subcontractors discount their respective opportunity costs, O_i, from time $\sum_{j=1}^{i} t_j$ (assuming contract 𝕀) based on the expected duration of preceding stages calculated using equation (2.2).

We also studied the effect of changing subcontractors' risk preferences. While we were unable to derive analytical results, our numerical studies indicated that when subcontractors are risk averse, the equilibrium work rates are strictly less than that of risk-neutral subcontractors. On the other hand, we observed that risk-taking subcontractors' equilibrium work rates are strictly greater than the risk-neutral counterparts.

Appendix A

A.1 Algorithm SIP ("Search for Incentive Payments")

In Proposition A1, we show that the expected client's profit defined by (2.6) is concave for each value of g_i (holding all values of g_j constant for $j \neq i$) such that the problem of finding equilibrium $\hat{g}_i$ values can be decomposed into n subproblems if solved in a backward sequence n, n-1, n-2,…,1. The latter result leads to the development of an efficient backward search procedure for finding the equilibrium values $\hat{g}_i$.

Proposition A1. Given values of $\beta_i \geq 0$, the expected client's profit $E\left[\Pi_C^{\mathbb{I}}\left(\beta_1,\ldots,\beta_n\right)\right]$ defined by (2.6) is concave for $g_i = \dfrac{\alpha}{\alpha + r_i}$ when g_j (for all $j \neq i$) are fixed. Furthermore, the problem of finding unique equilibrium $\hat{g}_i$ values that maximize (2.6) can be decomposed into n subproblems.

Proof: See **A.3**.

Using the results from Proposition A1, we can define a search procedure that we denote as Algorithm *SIP* ("Search for Incentive Payments") to find the unique equilibrium values of g_i. The correctness of Algorithm *SIP* is implied by the proof of Proposition A1. While the algorithm is presented for finding equilibrium subcontractor payments when contract $\mathbb{I}$ is used, it is immediately applicable for contract $\mathbb{F}$ by setting $\beta_i = 0$ for all i, and it can be easily modified to find equilibrium subcontractor payments for other related contracts as well.

Algorithm SIP.

1) Set $j = n$.
2) Use binary search on g_j to find the value $\hat{g}_j$ that satisfies

$$\frac{a k_j}{a + b_j}\left[b_j \left(1 + \frac{1}{\left(g_j\right)^2} - 2 g_j \right) + a \left(\frac{2}{\left(g_j\right)^3} - \frac{1}{\left(g_j\right)^2} - 1 \right) \right] = \text{Constant}(j)$$

where

$$\text{Constant}(j) = \begin{cases} Q\prod_{h=j+1}^{n}\left(1-\hat{g}_h\right) - \sum_{m=j+1}^{n}\frac{ak_m}{a+b_m}\left(1-\hat{g}_m\right)^2\left(\frac{1}{\hat{g}_m}+1\right)\left(\frac{a}{\hat{g}_m}+b_m\right)\prod_{h=j+1}^{m-1}\left(1-\hat{g}_h\right), & \text{for } j<n \\ Q, & \text{for } j=n. \end{cases}$$

3) Calculate $\hat{r}_j = \frac{a\left(1-\hat{g}_j\right)}{\hat{g}_j}$ and $\hat{p}_j$ using (2.4).

4) Set $j = j$-1 and go to step 2, letting $g_k = \hat{g}_k$ and $p_k = \hat{p}_k$ for all $k = j+1,...,n.$

The equation used in step 2 to find $\hat{g}_j$ is defined by setting $\frac{\partial E\left[\Pi_C^{\mathbb{I}}\left(\beta_1,...,\beta_n\right)\right]}{\partial g_j} = 0$; details are given in the proof of Proposition A1 in A3. When searching for $\hat{g}_j$ in step 2 of the algorithm, it should be noted that the term Constant(j) is defined by the values $\left(\hat{g}_{j+1},...,\hat{g}_n\right)$ that were calculated in the previous iterations (since we use a backward search method). Given values of $\hat{g}_i$, the equilibrium expected client's profit $E\left[\Pi_C^{\mathbb{I}*}\left(\beta_1,...,\beta_n\right)\right]$ can be found using (2.6); conversely, we can find values $\hat{r}_i = \frac{a\left(1-\hat{g}_i\right)}{\hat{g}_i}$ and $\hat{p}_i$ from (4) and the maximum expected client's profit using (2.5).

A.2 Modified SIP Algorithm When Opportunity Costs $O_i > 0$

A.2.1 Modifications for Contract $\mathbb{F}$

When opportunity costs are positive, the SIP Algorithm (described in A.1) can be modified for contract $\mathbb{F}$ as follows:

1) Calculate the prices $p_n^{SIP}, p_{n-1}^{SIP}, \dots, p_1^{SIP}$ from Algorithm SIP.

2) For j=1,…,n, $p_i^{\mathbb{F}} = \max\left\{p_i^{SIP}, \underline{p}_i(O_i)\right\}$.

Recall that Algorithm SIP necessarily calculates the prices in reverse order: $p_n^{SIP}, p_{n-1}^{SIP}, \dots, p_1^{SIP}$. However, the second step in our procedure must be processed in the natural order $p_1^{\mathbb{F}}, p_2^{\mathbb{F}}, \dots, p_n^{\mathbb{F}}$ since increasing subcontractor i's price will also increase her work rate, which will result in a *decrease* in the minimum price $\underline{p}_j(O_j)$ for all successors $j > i$ (due to the structure of the profit function). Therefore, there is a "backwards pass" to determine the SIP prices, and then a "forward pass" to determine the adjusted prices.

A.2.2 Modifications for Contract 𝕀

In this section, for simplicity, we assume that $K_i = 0$ for all i. Under contract 𝕀, the price adjustments require a more involved procedure. Subcontractor i's expected profit, given in Equation (2.1), evaluated at the equilibrium work rate r_i^* of Proposition 1, can be written as

$$E[\pi_i^{\mathbb{I}}(r_i^*,\beta_i)] = \left(\frac{k_i r_i^*(r_i^{*2} + 2r_i^*\alpha)}{(\alpha+\beta_i)(\alpha+r_i^*)} - \frac{k_i r_i^{*2}}{\alpha + r_i} \right) \prod_{j=1}^{i-1} \frac{r_j^*}{\alpha + r_j^*} .$$

Next, we use the substitution $g_i = \dfrac{\alpha}{\alpha + r_i^*}$, that results in the profit expression

$$E[\pi_i^{\mathbb{I}}(g_i,\beta_i)] = \alpha k_i \left(\frac{\alpha}{(\alpha+\beta_i)} \left(\frac{1}{g_i^2} - 1 - \frac{1}{g_i} + g_i \right) - \left(\frac{1}{g_i} - 2 + g_i \right) \right) \prod_{j=1}^{i-1} \left(1 - g_j\right).$$

For a given β_i, Algorithm SIP gives $\hat{g}_i(\beta_i)$, which maximizes the client's profit; suppressing the dependence on β_i for clarity (i.e., $\hat{g}_i = \hat{g}_i(\beta_i)$), the subcontractor's (maximized) profit is now only a function of β_i: $E[\pi_i^{\mathbb{I}}(\beta_i)]$. The derivative of this profit with respect to β_i can be written as

$$\frac{\partial E[\pi_i^{\mathbb{I}}(\beta_i)]}{\partial \beta_i} = \frac{\alpha k_i}{(\alpha+\beta_i)\hat{g}_i^2} \left[(1-\hat{g}_i) \left(\frac{\beta_i \hat{g}_i (1+\hat{g}_i) - 2\alpha}{\hat{g}_i} \right) \frac{\partial \hat{g}_i}{\partial \beta_i} - \frac{\alpha}{(\alpha+\beta_i)} (1+\hat{g}_i)(1-\hat{g}_i)^2 \right] \prod_{j=1}^{i-1} \left(1 - g_j\right).$$

The one unknown quantity in this expression is the partial derivative $\dfrac{\partial \hat{g}_i}{\partial \beta_i}$. However, recalling step 2 of SIP Algorithm, we see that $\hat{g}_i$ is determined by setting

$$\frac{\alpha k_j}{\alpha + \beta_j} \left[\beta_j \left(1 + \frac{1}{\left(g_j\right)^2} - 2g_j \right) + \alpha \left(\frac{2}{\left(g_j\right)^3} - \frac{1}{\left(g_j\right)^2} - 1 \right) \right]$$

equal to a constant; the derivative of this expression with respect to β_i can be shown to be negative, which implies that $\dfrac{\partial \hat{g}_i}{\partial \beta_i} < 0$. In addition, since $\hat{g}_i \in (0,1)$ for all $\beta_i \geq 0$, the derivative $\dfrac{\partial \hat{g}_i}{\partial \beta_i} > -1$ for all $\beta_i \geq 0$, except possibly on a set of b_i with measure strictly less than 1; in other words, $\hat{g}_i$ decreases more slowly than b_i increases. Therefore, for a large enough b_i, the derivative $\frac{\partial E[\pi_i^{\mathbb{I}}(\beta_i)]}{\partial \beta_i}$ is negative.

When opportunity costs are positive, the SIP Algorithm (described in A.1) can be modified for contract 𝕀 as follows:

In the natural order i =1,…,n, we evaluate the subcontractor's expected profit under the SIP prices when $b_i = 0$.

1) If $E[\pi_i^{\mathbb{I}}(0)] < O_i$, then apply the adjustment analysis from Contract 𝔽 to raise subcontractor i's profit to O_i by determining the adjusted price $p_i^{\mathbb{F}} = \max\left\{p_i^{SIP}, \underline{p}_i(O_i)\right\}$.

2) Alternatively, if $E[\pi_i^{\mathbb{I}}(0)] > O_i$, b_i is increased to lower subcontractor i's profit to O_i.

A.3 Technical Proofs

Proof of Proposition 2.1.

Given that $E[\pi_i^{\mathbb{I}}] = \left(\frac{p_i r_i}{\alpha+\beta_i+r_i} - \frac{K_i + k_i r_i^2}{\alpha + r_i}\right)\prod_{j=1}^{i-1}\frac{r_j}{\alpha+r_j}$,

setting the first order condition (FOC) to zero and holding all values of $r_l \; (l \neq i)$ constant, we get

$\frac{\partial E\left[\pi_i^{\mathbb{I}}\right]}{\partial r_i} = \left[\frac{p_i(\alpha+\beta_i)}{(\alpha+\beta_i+r_i)^2} - \frac{k_i(r_i^2+2\alpha r_i) - K_i}{(\alpha+r_i)^2}\right]\prod_{j=1}^{i-1}\frac{r_j}{\alpha+r_j} = 0$, implying that the equilibrium work rate for subcontractor i is equal to

$$r_i^* = \sqrt{\alpha^2 + \frac{p_i}{k_i}\frac{(\alpha+\beta_i)(\alpha+r_i^*)^2}{(\alpha+\beta_i+r_i^*)^2} + \frac{K_i}{k_i}} - \alpha. \qquad \text{(A0)}$$

The second order condition (SOC) confirms that this maximizes $E[\pi_i^{\mathbb{I}}]$; that is,

$$\frac{\partial^2 E\left[\pi_i^{\mathbb{I}}\right]}{\partial r_i^2} = \left[-\frac{2(\alpha+\beta_i)p_i}{(\alpha+\beta_i+r_i)^3} - \frac{2\alpha^2 k_i + K_i}{(\alpha+r_i)^3}\right]\prod_{j=1}^{i-1}\frac{r_j}{\alpha+r_j} < 0.$$

To prove uniqueness, rewrite (A0) as

$$\frac{p_i}{k_i}\frac{(\alpha+\beta_i)(\alpha+r_i)^2}{(\alpha+\beta_i+r_i)^2}+\frac{K_i}{k_i}=(r_i)^2+2\alpha r_i \tag{A1}$$

Denote the left-hand-side of (A1) as LHS, and similarly define the RHS. From (A1) the $LHS_{(r_i=0)} > RHS_{(r_i=0)}$ *and* $\lim_{r_i\to\infty} LHS < \lim_{r_i\to\infty} RHS$. Moreover, the derivative of $LHS_{(r_i)}$ is positive and strictly decreasing in r_i while the derivative of $RHS_{(r_i)}$ is positive and strictly increasing in r_i. Consequently, for a given β_i and p_i the positive solution r_i^* that satisfies (A0) is unique. *Q.E.D.*

Proof of Proposition 2.2

When subcontractor *i* selects its optimal work rate as defined in (2.2), we can rewrite p_i as a function of r_i^*:

$$p_i=\frac{\left[\left(r_i^*\right)^2+2ar_i^*\right]\left(a+b_i+r_i^*\right)^2 k_i}{\left(a+b_i\right)\left(a+r_i^*\right)^2}. \tag{A2}$$

Substituting this expression for p_i into (2.1) we have

$$E\left[\pi_i^{II}\right]=\left[\frac{k_i(r_i^*)^2\left((r_i^*)^2+2\alpha r_i^*+\alpha^2+\alpha\beta\right)}{\left(\alpha+\beta_i\right)\left(\alpha+r_i^*\right)^2}\right]\prod_{j=1}^{i-1}\frac{r_j}{\alpha+r_j}>0. \tag{A3}$$

Q.E.D

Proof of Corollary 2.1

Replace all r_i^* with $\alpha+\beta_i$ in the expression for $\frac{\partial r_i^*}{\partial \beta_i}$, and we have

$$\frac{\alpha^2}{(2\alpha+\beta_i)^2}+\frac{p_i}{4k_i}\frac{1}{(\alpha+\beta_i)}=1.$$

Define $f(\beta_i)=\frac{\alpha^2}{(2\alpha+\beta_i)^2}+\frac{p_i}{4k_i}\frac{1}{(\alpha+\beta_i)}$, which is strictly decreasing in β_i. Note that β_i is defined for $\beta_i \geq 0$. If $f(0)<1$, then $p_i<3\alpha k_i$. Hence, there does not exist a value of β_i where $r_i^*=\alpha+\beta_i$. Alternatively, if $p_i \geq 3\alpha k_i$, then $f(0)\geq 1$. If $f(0)=1$, since $f(\beta_i)$ is strictly decreasing, $\hat{\beta}_i=0$ is the only value of β_i that allows $r_i^*=\alpha+\beta_i$. If $f(0)>1$, we need to make sure that there exists a value of $\beta_i>0$ where $f(\beta_i)\leq 1$; this is easily seen by taking the limit

$$\lim_{\beta_i \to \infty} f(\beta_i) = 0.$$

Therefore, there exists a unique $\hat{\beta}_i > 0$ where $r_i^* = \alpha + \hat{\beta}_i$. *Q.E.D.*

Proof of Corollary 2.2

We provide this proof by contradiction. Taking the derivative of (C3) with respect to β_i , we have

$$\frac{\partial E\left[\pi_i^{\mathbb{I}*}\right]}{\partial \beta_i} = 2k_i r_i^* \left[\frac{\left[(r_i^*)^3 + 3(r_i^*)^2 + 3\alpha^2 r_i^* + \alpha^2(\alpha+\beta_i)\right](\alpha+\beta_i)\frac{\partial r_i^*}{\partial \beta_i} - (\alpha + \frac{r_i^*}{2})(\alpha + r_i^*)(r_i^*)^2}{(\alpha+\beta_i)^2(\alpha+r_i^*)^3} \right] \prod_{j=1}^{i-1} \frac{r_j}{\alpha + r_j},$$

which implies

$$\frac{\partial E\left[\pi_i^{\mathbb{I}^*}\right]}{\partial \beta_i} > 0 \quad \mathit{iff} \quad \frac{\partial r_i^*}{\partial \beta_i} > \frac{(\alpha + \frac{r_i^*}{2})(\alpha + r_i^*)(r_i^*)^2}{[(r_i^*)^3 + 3(r_i^*)^2 + 3\alpha^2 r_i^* + \alpha^2(\alpha+\beta_i)](\alpha+\beta_i)}. \qquad \text{(A4)}$$

Given that $\frac{\partial r_i^*}{\partial \beta_i} = \frac{(\alpha + r_i^*)\left[\frac{r_i^* - (\alpha+\beta_i)}{\alpha+\beta_i+r_i^*}\right]}{2(\alpha+\beta_i)\left[\frac{\alpha^2}{r_i^*(2\alpha+r_i^*)} + \frac{\alpha+r_i^*}{\alpha+\beta_i+r_i^*}\right]}$,

we can substitute $\frac{\partial r_i^*}{\partial \beta_i}$ from this expression into (A4) and after a couple of iterations of rearranging terms we can observe the following equivalence:

$$\frac{\partial E\left[\pi_i^{\mathbb{I}^*}\right]}{\partial \beta_i} > 0 \quad \mathit{iff} \quad -\frac{1}{2}\frac{(\alpha+\beta_i)(2\alpha + r_i^*)r_i^*}{(r_i^*)^3 + 3(r_i^*)^2 + 3\alpha^2 r_i^* + \alpha^2(\alpha+\beta_i)} > 0.$$

However, the later inequality is impossible, since $\alpha, \beta_i, r_i^* \geq 0$. Hence

$$\frac{\partial E\left[\pi_i^{\mathbb{I}^*}\right]}{\partial \beta_i} < 0 \quad \forall \beta_i \geq 0. \qquad \mathit{Q.E.D.}$$

Proof of Proposition A1.

Taking the partial derivative of (2.6) with respect to g_n and setting it to zero, we have

$$\frac{\partial E[\Pi_C^{\mathbb{I}}(\beta_1,\ldots,\beta_n)]}{\partial g_n} =$$

$$\left\{\frac{\alpha k_n}{\alpha+\beta_n}\left[\beta_n\left(1+\frac{1}{(g_n)^2}-2g_n\right)+\alpha\left(\frac{2}{(g_n)^3}-\frac{1}{(g_n)^2}-1\right)\right]-Q\right\}\prod_{j=1}^{n-1}(1-g_j)=0,$$

implying that

$$b_n\left(1+\frac{1}{(g_n)^2}-2g_n\right)+a\left(\frac{2}{(g_n)^3}-\frac{1}{(g_n)^2}-1\right)-\frac{Q(a+b_n)}{ak_n}=0. \qquad \text{(A5)}$$

We denote the left-hand-side of (A5) as LHS. From (A5) $\lim_{g_n\to 0}\text{LHS}=\infty>0$ $\text{LHS}_{(g_n=1)}=-\frac{Q(\alpha+\beta_n)}{\alpha k_n}<0,$

and LHS is strictly decreasing in g_n. We can conclude that $E[\Pi_C^{\mathbb{I}}(\beta_1,\ldots,\beta_n)]$ is concave in g_n and that there exists a unique optimal solution $\hat{g}_n$ that maximizes the Client's expect profit. Note that $\hat{g}_n$ does not depend on g_i for any $i < n$.

Next, we use strong induction to show that $E[\Pi_C^{\mathbb{I}}(\beta_1,\ldots,\beta_n)]$ is concave in g_i, with a unique optimal solution $\hat{g}_i$ for all i, where $\hat{g}_i$ depends only on g_j for $j>i$. This characteristic implies that we are able to decompose the Client's problem into n sub-problems and solve them in reverse order $j=n,\ldots,1$. The base case, where $E[\Pi_C^{\mathbb{I}}(\beta_1,\ldots,\beta_n)]$ is concave in g_n with a unique optimal solution $\hat{g}_n$, has already been proven. For the induction step, we assume that $E[\Pi_C^{\mathbb{I}}(\beta_1,\ldots,\beta_n)]$ is concave in g_j for all $j>i$, with unique optimal solutions $\hat{g}_j$; furthermore $\hat{g}_j$ does not depend on g_k, for $k \le i$ and we set $g_j=\hat{g}_j$ for $j \ge i+1$. We then show that $E[\Pi_C^{\mathbb{I}}(\beta_1,\ldots,\beta_n)]$ is concave in g_i with a unique optimal solution $\hat{g}_i$. Take the derivative of $E[\Pi_C^{\mathbb{I}}(\beta_1,\ldots,\beta_n)]$ with respect to g_i, and letting it equal zero, we have (for $i < n$):

$$\frac{\partial E[\Pi_C^{\mathbb{I}}(\beta_1,\ldots,\beta_n)]}{\partial g_i}=\frac{\alpha k_i}{\alpha+\beta_i}\left[\beta_i\left(1+\frac{1}{(g_i)^2}-2(g_i)\right)+\alpha\left(\frac{2}{(g_i)^3}-\frac{1}{(g_i)^2}-1\right)\right]$$

$$\underbrace{-Q\prod_{j=i+1}^{n}\left(1-\hat{g}_j\right)+\sum_{m=i+1}^{n}\frac{\alpha k_m}{\alpha+\beta_m}\left(1-\hat{g}_m\right)^2\left(\frac{1}{\hat{g}_m}+1\right)\left(\frac{\alpha}{\hat{g}_m}+\beta_m\right)\prod_{j=i+1}^{m-1}\left(1-\hat{g}_m\right)}_{f(\hat{g}_{i+1},\cdots,\hat{g}_n)}=0.$$

(A6)

Here we invoke the inductive hypothesis: at the optimal $\hat{g}_{i+1}$, we have that $\frac{\partial E[\Pi_C^{\mathbb{I}}(\beta_1,...,\beta_n)]}{\partial g_{i+1}}=0$.

Multiplying this equality (Equation (A6) with i replaced by $i+1$) by $(1-\hat{g}_{i+1})$, we have

$$\frac{\alpha k_{i+1}}{\alpha+\beta_{i+1}}(1-\hat{g}_{i+1})\left[\beta_{i+1}\left(1+\frac{1}{(\hat{g}_{i+1})^2}-2\hat{g}_{i+1}\right)+\alpha\left(\frac{2}{(\hat{g}_{i+1})^3}-\frac{1}{(\hat{g}_{i+1})^2}-1\right)\right]$$
$$-Q\prod_{j=i+1}^{n}\left(1-\hat{g}_j\right)+\sum_{m=i+2}^{n}\frac{\alpha k_m}{\alpha+\beta_m}(1-\hat{g}_m)^2\left(\frac{1}{\hat{g}_m}+1\right)\left(\frac{\alpha}{\hat{g}_m}+\beta_m\right)\prod_{j=i+1}^{m-1}\left(1-\hat{g}_j\right)=0$$

(A7)

Denote the left-hand-side of (A7) as LHS2; note that LHS2 = 0. Let $f(\hat{g}_{i+1},\cdots,\hat{g}_n)$ denote the constant

$$\left\{-Q\prod_{j=i+1}^{n}\left(1-\hat{g}_j\right)+\sum_{m=i+1}^{n}\frac{ak_m}{a+b_m}\left(1-\hat{g}_m\right)^2\left(\frac{1}{\hat{g}_m}+1\right)\left(\frac{a}{\hat{g}_m}+b_m\right)\prod_{j=i+1}^{m-1}\left(1-\hat{g}_j\right)\right\}$$ in Equation (A6).

Subtracting $f(\hat{g}_{i+1},\cdots,\hat{g}_n)$ from LHS2, we get the result that

$$\frac{b_{i+1}(\hat{g}_{i+1}^2+1)\left(1-\hat{g}_{i+1}\right)}{\hat{g}_{i+1}^2}+\frac{2a\left(1-\hat{g}_{i+1}\right)}{\hat{g}_{i+1}^3}>0.$$

Since LHS2 = 0 and the difference of LHS2 and $f(\hat{g}_{i+1},\cdots,\hat{g}_n)$ is positive, we can infer that

$$-Q\prod_{j=i+1}^{n}\left(1-\hat{g}_j\right)+\sum_{m=i+1}^{n}\frac{\alpha k_m}{\alpha+\beta_m}(1-\hat{g}_m)^2\left(\frac{1}{\hat{g}_m}+1\right)\left(\frac{\alpha}{\hat{g}_m}+\beta_m\right)\prod_{j=i+1}^{m-1}\left(1-\hat{g}_j\right)<0.$$

Since $\lim_{g_i\to 0}\frac{\partial E[\Pi_C^{\mathbb{I}}(\beta_1,...,\beta_n)]}{\partial g_i}=\infty>0$, $\lim_{g_i\to 1}\frac{\partial E[\Pi_C^{\mathbb{I}}(\beta_1,...,\beta_n)]}{\partial g_i}=f(\hat{g}_{i+1},\cdots,\hat{g}_n)<0$ and

$\frac{\partial E[\Pi_C^{\mathbb{I}}(\beta_1,...,\beta_n)]}{\partial g_i}$ is strictly decreasing in g_i, we can infer that $E[\Pi_C^{\mathbb{I}}(\beta_1,...,\beta_n)]$ is concave in g_i

with a unique optimal solution $\hat{g}_i \; \forall i$. Since g_i has a one-to-one correspondence with r_i, $E[\Pi_C^{\mathbb{I}}(\beta_1,...,\beta_n)]$ has a set of unique optimal solutions $r_1^*,...,r_n^*$. *Q.E.D.*

Proof of Proposition 2.3

The derivative of (2.6) with respect to b_i satisfies

$$\frac{\partial E[\Pi_C^{\mathbb{I}}(\beta_1,...,\beta_n)]}{\partial \beta_i} = \frac{\alpha^2 k_i \left(1-g_i\right)^3}{(\alpha+\beta_i)^2 g_i} \prod_{j=1}^{i-1}\left(1-g_j\right) > 0.$$

This result implies that $E[\Pi_C^{\mathbb{I}}(\beta_1,...,\beta_n)]$ is increasing in β_i, $\forall i$ and hence the profit function can be reduced to

$$E\left[\lim_{\beta_1,...,\beta_n\to\infty} E[\Pi_C^{\mathbb{I}}(\beta_1,...,\beta_n)]\right] = Q\prod_{j=1}^{n}\left(1-g_j\right) - \sum_{m=1}^{n}\alpha k_m \left(1-g_m\right)^2\left(\frac{1}{g_m}+1\right)\prod_{j=1}^{m-1}\left(1-g_j\right).$$

Then from (C6), the optimality condition for $i = 1$ (with $b_i \to \infty \; " \; i$), namely

$$\frac{\partial E\left[\lim_{\beta_1,...,\beta_n\to\infty} E[\Pi_C^{\mathbb{I}}(\beta_1,...,\beta_n)]\right]}{\partial g_1} = 0,$$ is given by

$$Q\prod_{j=2}^{n}\left(1-\hat{g}_j\right) - \sum_{m=2}^{n} a k_m \left(1-\hat{g}_m\right)^2\left(\frac{1}{\hat{g}_m}+1\right)\prod_{j=2}^{m-1}\left(1-\hat{g}_j\right) = a k_1\left(1+\frac{1}{\hat{g}_1^{\,2}} - 2\hat{g}_1\right).$$

Multiply both sides by $(1-\hat{g}_1)$ and then subtract $\alpha k_1 \left(1-\hat{g}_1\right)^2\left(\frac{1}{\hat{g}_1}+1\right)$, we have

$$\underbrace{Q\tilde{O}_{j=1}^{n}\left(1-\hat{g}_j\right) - \mathring{a}_{m=1}^{n} a k_m\left(1-\hat{g}_m\right)^2\left(\frac{1}{\hat{g}_m}+1\right)\tilde{O}_{j=1}^{m-1}\left(1-\hat{g}_j\right)}_{E[\mathrm{P}_C^{I}(\hat{g}_1,...,\hat{g}_n)]} = a k_1\left(1-\hat{g}_1\right)^2\left(1+\frac{1}{\hat{g}_1^2}\right) > 0.$$

where the left-hand side of this equation is the optimized Client's profit. *Q.E.D.*

Proof of Proposition 2.4

From (C6), the optimality condition for i is given by

$$\frac{\alpha k_i}{\alpha+\beta_i}\underbrace{\left[\beta_i\left(1+\frac{1}{(\gamma_i)^2}-2(\gamma_i)\right)+\alpha\left(\frac{2}{(\gamma_i)^3}-\frac{1}{(\gamma_i)^2}-1\right)\right]}_{\text{strictly decreasing in } \gamma_i}$$

$$=\Theta\prod_{j=i+1}^{n}\left(1-\hat{\gamma}_j\right)-\sum_{m=i+1}^{n}\frac{\alpha k_m}{\alpha+\beta_m}\left(1-\hat{\gamma}_m\right)^2\left(\frac{1}{\hat{\gamma}_m}+1\right)\left(\frac{\alpha}{\hat{\gamma}_m}+\beta_m\right)\prod_{j=i+1}^{m-1}\left(1-\hat{\gamma}_j\right).$$

Since

$\frac{\partial \text{LHS}}{\partial \beta_i}=\frac{2\alpha^2 k_n\left(1-\hat{g}_i\right)\left(\left(\hat{g}_i\right)^3-1\right)}{\left(\alpha+\beta_i\right)\left(\hat{g}_i\right)^3}<0$, the LHS is strictly decreasing in β_i, and the RHS is not affected by β_i; hence r_i^* is increasing in β_i.

We next examine the influence of successors. Again, from (A6), the optimality condition for g_i is

$$\frac{\alpha k_i}{\alpha+\beta_i}\underbrace{\left[\beta_i\left(1+\frac{1}{(\gamma_i)^2}-2(\gamma_i)\right)+\alpha\left(\frac{2}{(\gamma_i)^3}-\frac{1}{(\gamma_i)^2}-1\right)\right]}_{\text{strictly decreasing in } \gamma_i}$$

$$=\Theta\prod_{j=i+1}^{n}\left(1-\hat{\gamma}_j\right)-\sum_{m=i+1}^{n}\frac{\alpha k_m}{\alpha+\beta_m}\left(1-\hat{\gamma}_m\right)^2\left(\frac{1}{\hat{\gamma}_m}+1\right)\left(\frac{\alpha}{\hat{\gamma}_m}+\beta_m\right)\prod_{k=i+1}^{m-1}\left(1-\hat{\gamma}_k\right).$$

From (2.6), we can interpret the RHS as the expected profit of a reduced problem consisting of only subcontractors $i+1$ to n. Hence we can denote the RHS as $E[\Pi_C^{\mathbb{I}}(\beta_{i+1},...,\beta_n)]$. From Proposition 2.3**,** we can conclude that the RHS is increasing in β_j, $\forall j=i+1,...,n$, that is

$$\frac{\partial E[\Pi_C^{\mathbb{I}}(\beta_{i+1},...,\beta_n)]}{\partial \beta_j}=\frac{\alpha^2 k_j\left(1-g_j\right)^3}{(\alpha+\beta_j)^2 g_j}\prod_{k=1}^{j-1}\left(1-g_k\right)>0, \quad \forall j=i+1,\ ...,\ n.$$

Since the LHS is not affected by β_j, $j=i+1,...,n$ and is strictly decreasing in g_i, decreasing $\hat{g}_i$ increases r_i^*. *Q.E.D.*

Proof of Proposition 2.5.

Given that $E[\pi_i^{\mathbb{F}}]=p_i\prod_{k=1}^{i}\frac{r_k}{\alpha+r_k}-\frac{K_i+k_i r_i^2}{\alpha+r_i}\prod_{j=1}^{i-1}\frac{r_j}{\alpha+r_j}$,

setting the first order condition (FOC) to zero and holding all values of r_l $(l\neq i)$ constant, we get

$$\frac{\partial E\left[\pi_i^{\mathbb{F}}\right]}{\partial r_i}=\left(\frac{\alpha p_i-2\alpha k_i r_i-k_i r_i^2+K_i}{(\alpha+r_i)^2}\right)\prod_{j=1}^{i-1}\frac{r_j}{\alpha+r_j}=0,$$

implying that the optimal work rate for subcontractor i is equal to

$$r_i^*=\sqrt{\alpha^2+\frac{p_i\alpha+K_i}{k_i}}-\alpha.$$

The second order condition (SOC) confirms that this maximizes $E\left[\pi_i^{\mathbb{F}}\right]$; that is,

$$\frac{\partial^2 E\left[\pi_i^{\mathbb{F}}\right]}{\partial r_i^2}=-\frac{2\alpha\left(p_i+\alpha k_i+K_i\right)}{\left(\alpha+r_i\right)^3}\prod_{j=1}^{i-1}\frac{r_j}{\alpha+r_j}<0.$$

Q.E.D.

Proof of Proposition 2.6.

From (2.1) and (2.9) we have,

$$E\left[\pi_i^{\mathbb{I}}\right]=E\left[e^{-\alpha\sum_{j=1}^{i-1}t_j}\right]\times E\left[\left(p_i\ e^{-\beta_i t_i}\right)e^{-\alpha t_i}-\int_0^{t_i}(K_i+k_i r_i^2)e^{-\alpha t}\,dt\right].$$

and

$$E\left[\pi_i^{\mathbb{I}_{\mathbb{D}}}\right]=E\left[e^{-\alpha\sum_{j=1}^{i-1}t_j}\right]\times E\left[\left(e^{-\alpha\sum_{j=i+1}^{n}t_j}\ p_i\ e^{-\beta_i t_i}\right)e^{-\alpha t_i}-\int_0^{t_i}(K_i+k_i r_i^2)e^{-\alpha t}\,dt\right].$$

Letting $p_i'=p_i\ E\left[e^{-a\sum_{j=i+1}^{n}t_j}\right]$, then $E\left[\Pi_C^{\mathbb{I}_{\mathbb{D}}}\right]=E\left[Q\,e^{-\alpha T}-\sum_{i=1}^{n}\left(p_i' e^{-\beta_i t_i}\right)e^{-\alpha\sum_{j=1}^{i}t_j}\right]$, that is equivalent to the expected client's profit under contract I as defined by (3). Since, $E[\Pi_C^{\mathbb{I}_{\mathbb{D}}}]=E[\Pi_C^{\mathbb{I}}]$ and $E[\pi_i^{\mathbb{I}_{\mathbb{D}}}]=E[\pi_i^{\mathbb{I}}]\quad\forall i$, we can conclude that the equilibrium solutions of p_i and $p_i'\quad\forall i$ are also equivalent. Therefore, the equilibrium work rates for all subcontractors are equal under the two contracts. *Q.E.D.*

3 On Coordinating Contracts in Serial Decentralized Projects

Following work in supply chain management (Cachon, 2006), we define a coordinating contract in a decentralized project as a contract that results in a Nash equilibrium when no stakeholder in the project has any incentive to deviate from the project optimal actions. Before considering a decentralized project, we initially analyze a centralized project where the client organization owns and controls all work in the project. Using the results from the centralized project as a baseline, we then consider a decentralized project and show that an Exponential Incentive Contract (EIC) coordinates a decentralized project.

This chapter is organized as follows. In 3.1, we analyze the case of a centralized project where the client owns and/or controls all stages of the project. We derive closed form solutions that maximize the expected discount centralized profit. In the case when discounting is minimal and can be neglected, we show that there exist unique work rates that first-order stochastically maximize the client's profit. This result provides an unambiguous work rate (for $\alpha = 0$) for both risk neutral and risk-averse clients. In 3.2, we analyze a decentralized project where the client sets the contract terms and independent subcontractors react by setting the work rates that optimize their objectives. Modeling the decentralized project as a Stackelberg game, we show that EIC coordinates a decentralized project with risk neutral subcontractors under discounting. In 3.3 we discuss how EIC can be better implemented in practice by approximating it by contracts that are commonly used. In 3.4 we summarize our results and discuss the managerial and practical implications of our findings.

3.1 Centralized Project

We assume that a project consists of n sequential stages; without loss of generality, we assume that the stages are sequentially indexed $i = 1, \ldots, n$ from the start to the completion of the project. We consider serial projects to reflect the fact that many projects characterized by general precedence networks can be subdivided into sequential group of tasks that are separated by review points or "stage gates" to improve project monitoring and control (Santiago and Vakili, 2005). In some projects (*e.g.*, new product development or construction projects), the project naturally defines a sequential series of tasks or stages.

Following previous research (*e.g.*, Kamien and Schwartz 1972, Buss and Rosenblatt 1997, Kwon *et al*. 2010, Chen *et al*. 2015), we assume that the client receives a fixed payment Q when the project is completed. This amount may represent the expected value of future profits earned by a completed new product development project or the social welfare accrued by a completed public infrastructure project. We assume that the client's payment Q and costs are continuously discounted at an exogenous rate $\alpha \geq 0$. We assume that there are two types of costs incurred by the client. First, there is a fixed overhead/indirect cost per time unit that reflects security costs, utility expenses, managerial costs, etc. We let H denote the overhead/indirect cost rate incurred by the client during the entire project duration and C_i denote any overhead/indirect cost rate that is related to a specific i^{th} stage. Second, there are resource related costs at each stage that are defined by $k_i r_i^2$ where r_i is the work rate at stage i and k_i is the resource cost parameter that reflects the complexity and difficulty of the i^{th} stage (Kwon *et al*. 2010; Chen *et al* 2015). The work rate r_i is a decision variable that is set by the client in a centralized project (or a subcontractor in a decentralized project) to optimize some objective.

The duration of stage i is denoted by t_i, where t_i is a non-negative random variable that is stochastically non-increasing in the work rate $r_i > 0$ (*i.e*., a larger work rate r_i will lead to a higher probability of completing stage i in a shorter time span). Following previous research (*e.g*., Buss and Rosenblatt 1997; Tavares 2002; Klastorin and Mitchell 2007), we assume that t_i are independent and exponentially distributed with density $f(t) = r_i e^{-r_i t}$. The notation we use in this chapter is summarized below.

<u>Decision Variables</u>:

r_i	work rate for stage i

<u>Parameters</u>:

Q	fixed payment to the client when project is completed,
H	indirect/overhead cost rate incurred during project makespan,
C_i	indirect/overhead cost rate incurred at stage i,
α	non-negative discount rate,
k_i	variable resource cost parameter at stage i,
t_i	duration of stage i (non-negative random variable),
T	project duration or makespan where $T = \sum_{i=1}^{n} t_i$.

In a centralized project, the client controls all stages of the project and sets the work rates r_i at each stage. Letting $\Pi_C^{CP}(r_1,...r_n)$ denote the client's profit for a centralized project,

$$\Pi_C^{CP}(r_1,...,r_n) = Qe^{-\alpha T} - \sum_{i=1}^{n} e^{-\alpha\sum_{j=1}^{i-1} t_j} \int_0^{t_i} (H + C_i + k_i r_i^2) e^{-\alpha t} dt \tag{3.1}$$

where t_i is the realized duration of stage *i*. Since the duration of each stage is $t_i \sim \exp(r_i)$, the project profit $\Pi_C^{CP}(r_1,...r_n)$ is also a random variable. In this chapter, we focus on risk neutral clients (but also discuss risk aversion) who consider the goal of maximizing their expected profit $E\left[\Pi_C^{CP}(r_1,...,r_n)\right]$. In 3.1.1 we start our analysis of a centralized project with a general discount rate ($\alpha > 0$) and we provide closed-form solution for the optimal work rate at each stage; this problem serves as the benchmark for the decentralized project studied in 3.2. In 3.1.2 we analyze a centralized project where the discount rate is 0 and we derive a stochastic dominance result, which has implications for a risk-averse client.

3.1.1 Centralized Project (CP) Model $(\alpha > 0)$

Recalling (3.1), the expected discounted centralized profit at time 0 is calculated as

$$E\left[\Pi_C^{CP}(r_1,...,r_n)\right] = Q\prod_{i=1}^{n}\frac{r_i}{\alpha + r_i} - \sum_{i=1}^{n}\frac{\left(H + C_i + k_i r_i^2\right)}{\alpha + r_i}\prod_{j=1}^{i-1}\frac{r_j}{\alpha + r_j}. \tag{3.2}$$

The goal is to select a work rate in each stage so that $E\left[\Pi_C^{CP}(r_1,...,r_n)\right]$ is maximized. The unique closed-form solutions are provided in the following Proposition.

Proposition 3.1. When $\Pi_C^{CP}(r_1,...,r_n)$ is defined by (3.1),

$$E\left[\Pi_C^{CP}\left(r_1^*,...,r_n^*\right)\right] \geq E\left[\Pi_C^{CP}\left(r_1,...,r_n\right)\right] \text{ for all } r_1,\ldots,r_n > 0 \quad \text{if } r_i^* = \sqrt{\alpha^2 + \frac{\hat{Q}_i \alpha + H + C_i}{k_i}} - \alpha$$

where $\hat{Q}_i = Q\prod_{j=i+1}^{n} \frac{r_j^*}{\alpha + r_j^*} - \sum_{j=i+1}^{n} \frac{\left(H + C_j + k_j\left(r_j^*\right)^2\right)}{\alpha + r_j^*} \prod_{m=i+1}^{j-1} \frac{r_m^*}{\alpha + r_m^*}$ $\quad$ *for* $i = n-1,...,1$ and $\hat{Q}_n = Q.$

Proof: See Appendix.

Proposition 3.1 provides closed-form solutions for the centralized project. This set of work rates guarantees that the expect centralized profit is maximized and will be used as a benchmark in a decentralized setting, where the client has no direct control over the work rate at each stage.

3.1.2 Centralized Project (CP) Model $(\alpha = 0)$

In 2015, the savings interest rate in Washington State was, in general less than 0.03% at banks including JPMorgan Chase, Bank of America, Wells Fargo…etc. A CBS news article indicates that the low interest rates are not likely to rise anytime soon (Gibson 2015). From the perspective of managing a project, it is also interesting to study how a marginal interest rate affects the decisions of a client. When $\alpha = 0$, the client's profit defined by (3.1) can be simplified as

$$\Pi_C^{CP}\left(r_1,...,r_n\right) = Q - \sum_{i=1}^{n}\left(H + C_i + k_i r_i^2\right) t_i \ . \tag{3.3}$$

In Proposition 3.2, we show that there exist unique values r_i^* that first-order stochastically maximize the client's profit, where for random variables X and Y, $X \geq_{FSD} Y$ is defined as $P(X > t) \geq P(Y > t),\ \forall t.$

Proposition 3.2. When $\Pi_C^{CP}\left(r_1,...,r_n\right)$ is defined by (3.3),

$\Pi_C^{CP}\left(r_1^*,...,r_n^*\right) \geq_{FSD} \Pi_C^{CP}\left(r_1,...,r_n\right)$ for all $r_1,\ldots,r_n > 0$ if $r_i^* = \sqrt{\dfrac{H+C_i}{k_i}}$ for all $i = 1,\ldots,n$.

Proof: See Appendix.

Proposition 3.2 shows that $r_i^* = \sqrt{\dfrac{H+C_i}{k_i}}$ first order stochastically dominates all other work rates $r_i > 0$. Note that the work rates from Proposition 3.2 are the same work rates from Proposition 3.1, evaluated at $\alpha = 0$. From Proposition 3.2, we can derive Corollaries 3.1 and 3.2 which convey important managerial insights.

Corollary 3.1. When $\Pi_C^{CP}\left(r_1,...,r_n\right)$ is defined by (3.3),

$E\left[\Pi_C^{CP}\left(r_1^*,...,r_n^*\right)\right] \geq E\left[\Pi_C^{CP}\left(r_1,...,r_n\right)\right]$ for all $r_1,\ldots,r_n > 0$ if $r_i^* = \sqrt{\dfrac{H+C_i}{k_i}}$ for all $i = 1,\ldots,n$.

Corollary 3.2 When $\Pi_C^{CP}\left(r_1,...,r_n\right)$ is defined by (3.3),

$E\left[u\left(\Pi_C^{CP}\left(r_1^*,...,r_n^*\right)\right)\right] \geq E\left[u\left(\Pi_C^{CP}\left(r_1,...,r_n\right)\right)\right]$ for all $r_1,\ldots,r_n > 0$ and any monotone increasing and strictly concave utility functions u if $r_i^* = \sqrt{\dfrac{H+C_i}{k_i}}$ for all $i = 1,\ldots,n$.

(For a complete review of stochastic dominance, please refer to chapter 4 of "Topics in

Microeconomics" (Wolfstetter 1999), including the proofs of Corollaries 3.1 and 3.2.)

Corollary 3.1 defines the decision of a risk neutral client. In order to maximize the expected profit, the client must set work rates at each stage based on the formula in Proposition 3.1 to achieve this goal.

In the literature on project management, research that studies risk aversion is rare. The common method is to assume a stylized concave utility function and maximize the expected utility of the profit. However, the optimal work rate will typically depend on the utility function chosen and the level of risk aversion. In practice, it is difficult to not only specify the form of the concave utility function but also correctly define the level of risk aversion. In this paper, from Corollary 3.2, we show that r_i^* provides an unambiguous work rate (for $\alpha = 0$); it is the optimal solution for any monotone increasing and strictly concave utility functions (i.e., any risk-averse utility functions), which significantly increases the applicability of selecting r_i^* in practice.

However, in the general discounting case this property is not preserved and we provide a simple example to show that with discounting, stochastic dominance can not be achieved, meaning the optimal solution depends on both the utility function and the level of risk aversion selected. Consider a project with 2 stages and parameters $Q = 350, H = 20, \alpha = 0.1, k_1 = k_2 = 20, C_1 = C_2 = 5.$ From Proposition 3.1, we can calculate $r_1^* = 1.54$ and $r_2^* = 1.63$; this is the work rate that maximizes the expected discounted profit.

y	Risk Neutral Work Rate		Risk Averse Work Rate		
	$r_1^* = 1.54$	$r_2^* = 1.63$	Pr(Profit < y)	r_1	r_2
	Pr(Profit < y)				
-85	0.31%		0.27%	1.11	1.33
-25	1.07%		1.00%	1.30	1.50
35	3.06%		3.02%	1.34	1.52
95	8.17%		8.13%	1.43	1.60

TABLE 3.1: COUNTEREXAMPLE OF UNANIMOUS WORK RATES WHEN $\alpha = 0.1$

From Table 3.1, we can see that the optimal risk neutral work rates do not minimize the probability of the centralized profit being less than a value *y*. The results from Table 3.1 imply that $r_i^* = \sqrt{\alpha^2 + \frac{\hat{Q}_i \alpha + H + C_i}{k_i}} - \alpha$ does not first-order stochastically dominate other work rates $r_i > 0$ in the discounting case. Next we need to see whether there exists any other work rate that stochastic dominates all others in each stage. The answer is negative. Recall that a necessary condition for a random variable X to stochastic dominate another random variable Y is $\mathrm{E}[X] \geq E[Y]$. In Proposition 3.1, we have shown that $r_i^* = \sqrt{\alpha^2 + \frac{\hat{Q}_i \alpha + H + C_i}{k_i}} - \alpha$ uniquely maximizes the expected discounted profit; hence if there exists a work rate that stochastically dominates all other work rates in each stage it has to be

$r_i^* = \sqrt{\alpha^2 + \frac{\hat{Q}_i \alpha + H + C_i}{k_i}} - \alpha$, which we have shown can not be the case.

3.2 Decentralized Project

As the complexity of projects increases, project owners typically outsource many or all stages of a project to independent subcontractors who act in their own interest. Aligning the goals of the client and subcontractors in a decentralized project is one of the major challenges facing project owners and the goal of most contracts.

Given that the client outsources the *n* stages of a project to independent subcontractors, we assume that the client sets the contract terms with each subcontractor at the start of the project and model the contracting process as a Stackelberg game. Given the contract terms, each i^{th} subcontractor subsequently decides if she will participate and, if so, determines the work rate $\hat{r}_i$ that optimizes her expected profit. We assume that the client has perfect information about subcontractor cost parameters; this assumption is based on the observation that many clients have previously worked with most subcontractors or know their general cost structures based on reviews and shared information.

Assume that the client offers each i^{th} subcontractor a payment $\beta_i > 0$ at the beginning of the project and each subcontractor reacts by setting an appropriate work rate $\hat{r}_i$. Using the same cost structure for a centralized project, the client would realize a discounted profit equal to

$$\Pi_C^{DP}\left(\hat{r}_1,...,\hat{r}_n\right)=Qe^{-\alpha\sum_{i=1}^{n}t_i}-\sum_{i=1}^{n}\left[\beta_i e^{-\alpha\sum_{j=1}^{i}t_j}\right]-\sum_{i=1}^{n}\left[e^{-\alpha\sum_{j=1}^{i-1}t_i}\int_0^{t_i}He^{-\alpha x}dx\right]$$

while each i^{th} subcontractor would earn a discounted profit

$$\pi_i(\hat{r}_i)=\left[\beta_i e^{-\alpha t_i}-\int_0^{t_i}\left(C_i+k_i r_i^2\right)e^{-\alpha x}dx\right]e^{-\alpha\sum_{j=1}^{i-1}t_j}$$

where H (or C_i) is the indirect/overhead cost rate incurred by the client (or subcontractor) and t_i is the realized stage duration. The decentralized project's collective profit, denoted by $\Psi^{DP}\left(\hat{r}_1,...,\hat{r}_n\right)$, is then

$$\Psi^{DP}\left(\hat{r}_1,...,\hat{r}_n\right)=\Pi_C^{DP}\left(\hat{r}_1,...,\hat{r}_n\right)+\sum_{i=1}^{n}\pi_i(\hat{r}_i)=Q\,e^{-\alpha T}-\sum_{i=1}^{n}e^{-\alpha\sum_{j=1}^{i-1}t_j}\int_0^{t_i}(H+C_i+k_i r_i^2)\,e^{-\alpha t}dt. \tag{3.4}$$

Note that the collective profit of the decentralized project defined by (3.4) is equal to the profit for the centralized project defined by (3.1) when $\hat{r}_i=r_i$ for all $i=1,...,n.$ As a result, $E\left[\Psi^{DP}\left(\hat{r}_1,...,\hat{r}_n\right)\right]$ is maximized by Proposition 3.1 when $\hat{r}_i=\sqrt{\alpha^2+\dfrac{\hat{Q}_i\alpha+H+C_i}{k_i}}-\alpha$.

When this occurs, we say that the decentralized project is coordinated. To achieve coordination in a decentralized project, we propose an exponential incentive contract (EIC) that pays each subcontractor an amount equal to $\beta_i = p_i - e^{s_i t_i}$, where p_i is a fixed payment, s_i is a penalty cost per time unit, and t_i is the realized duration of subcontractor i's stage. If s_i = 0, we note that the contract is a fixed price contract that is widely used in practice and pays a subcontractor a fixed amount p_i regardless of the stage duration.

3.2.1 Subcontractor Response to EIC

Using an EIC contract, a subcontractor's profit (a random variable) is now defined as

$$\pi_i(\hat{r}_i) = \left[(p_i - e^{s_i t_i}) e^{-\alpha t_i} - \int_0^{t_i} \left(C_i + k_i \hat{r}_i^2 \right) e^{-\alpha x} dx \right] e^{-\alpha \sum_{j=1}^{i-1} t_j} . \quad (3.5)$$

Since $t_i \sim exp(\hat{r}_i)$, the expected profit for subcontractor *i* is equal to

$$E\left[\pi_i(\hat{r}_i)\right] = \left[\frac{p_i \hat{r}_i - C_i - k_i \hat{r}_i^2}{\alpha + \hat{r}_i} - \frac{\hat{r}_i}{\alpha - s_i + \hat{r}_i} \right] \prod_{j=1}^{i-1} \frac{\hat{r}_j}{\alpha + \hat{r}_j} . \quad (3.6)$$

Assuming that a subcontractor is willing to participate in the project, a risk neutral subcontractor's objective is to maximize her expected discounted profit. The unique solution to the subcontractor's problem is provided in the following Proposition.

Proposition 3.3. Given $p_i > 0, s_i \geq 0$, $\hat{r}_i^* = \sqrt{\frac{\left[k_i\left(\hat{r}_i^*\right)^2 + 2k_i \hat{r}_i^* \alpha - \left(p_i \alpha + C_i\right)\right]\left(\alpha - s_i + \hat{r}_i^*\right)^2}{\left(s_i - \alpha\right)}} - \alpha > 0$.

$\hat{r}_i^*$ uniquely maximizes $E\left[\pi_i(\hat{r}_i)\right]$ as defined in (3.6), where $\hat{r}_i^*$ is the unique solution to

$$\hat{r}_i^* = \sqrt{\frac{\left[k_i\left(\hat{r}_i^*\right)^2 + 2k_i \hat{r}_i^* \alpha - \left(p_i \alpha + C_i\right)\right]\left(\alpha - s_i + \hat{r}_i^*\right)^2}{\left(s_i - \alpha\right)}} - \alpha.$$

Proof: See Appendix.

Proposition 3.3 provides an implicit expression for subcontractor *i*'s optimal work rate for any given pair of $p_i > 0, s_i \geq 0$. Moreover, we have shown that this work rate always exists ($\hat{r}_i^*\left(p_i, s_i\right) > 0$) and is unique.

3.2.2 Subcontractor Participation Constraints

The previous analysis is based on the assumption that subcontractors are willing to participate in the project. In practice, this may not be the case as subcontractors often have a bound on their objectives that must be satisfied for them to participate. In the case of a risk neutral subcontractor, this constraint could be stated as a lower bound on the expected profit; denoting this lower bound by an opportunity cost O_i, the i^{th} subcontractor would only participate in the project if $E\left[\pi_i\left(\hat{r}_i^*\right)\right] \geq O_i$, where O_i reflects current economic conditions and alternative investment opportunities for the i^{th} subcontractor. Subcontractor participation constraints are summarized in the following Corollary.

Corollary 3.3. To ensure subcontractor participation we must have

$E\left[\pi_i\left(\hat{r}_i^*\right)\right] = \left[\frac{k_i\left(\hat{r}_i^*\right)^2 - C_i}{\alpha} - \frac{s_i\left(\hat{r}_i^*\right)^2}{\alpha\left(\alpha - s_i + \hat{r}_i^*\right)^2}\right]\prod_{j=1}^{i-1}\frac{r_j}{\alpha + r_j} \geq O_i$ for all $i = 1,\ldots,n$, where $\hat{r}_i^*$ is given in Proposition 3.3.

Proof: See Appendix.

3.2.3 Client Response to EIC

Using an EIC contract, a client's profit (a random variable) is now defined as

$$\Pi_C^{DP} = Qe^{-\alpha\sum_{i=1}^{n}t_i} - \sum_{i=1}^{n}\left[\left(p_i - e^{s_i t_i}\right)e^{-\alpha\sum_{j=1}^{i}t_j}\right] - \sum_{i=1}^{n}\left[e^{-\alpha\sum_{j=1}^{i-1}t_i}\int_0^{t_i} He^{-\alpha x}dx\right].$$

The client's optimization problem can then be formulated as

$$\max_{p_i, s_i} E\left[\Pi_C^{DP}\right] = Q\prod_{i=1}^{n}\frac{\hat{r}_i^*}{\alpha+\hat{r}_i^*} - \sum_{i=1}^{n}\left[\left(\frac{p_i\hat{r}_i^*}{\alpha+\hat{r}_i^*} - \frac{\hat{r}_i^*}{\alpha - s_i + \hat{r}_i^*}\right)\prod_{j=1}^{i-1}\frac{\hat{r}_j^*}{\alpha+\hat{r}_j^*}\right] - \sum_{i=1}^{n}\left[\frac{H}{\alpha+\hat{r}_i^*}\prod_{j=1}^{i-1}\frac{\hat{r}_j^*}{\alpha+\hat{r}_j^*}\right]$$

$s.t.$

$$\hat{r}_i^* = \sqrt{\frac{\left[k_i\left(\hat{r}_i^*\right)^2 + 2k_i\hat{r}_i^*\alpha - \left(p_i\alpha + C_i\right)\right]\left(\alpha - s_i + \hat{r}_i^*\right)^2}{\left(s_i - \alpha\right)}} - \alpha \qquad \forall i$$

$$\left[\frac{k_i\left(\hat{r}_i^*\right)^2 - C_i}{\alpha} - \frac{s_i\left(\hat{r}_i^*\right)^2}{\alpha\left(\alpha - s_i + \hat{r}_i^*\right)^2}\right]\prod_{j=1}^{i-1}\frac{\hat{r}_j^*}{\alpha+\hat{r}_j^*} \geq O_i \qquad \forall i\,.$$

(P3.1)

The first set of constraints are often referred to as Individual Compatibility (IC) constraints and the second set of constraints are referred to as the Individual Rationality (IR) constraints in the principle-agent problem literature. To solve P3.1, we are able to substitute the constraints into the objective function and convert P3.1 into an unconstrained optimization problem. Then we show that this unconstrained problem has the exact same solutions as the centralized case described in Proposition 3.1. Based on these insights, we provide an algorithm to solve P3.1 as follows

Algorithm 3.1

1. Set $\hat{r}_i^{**} = \sqrt{\alpha^2 + \frac{\hat{Q}_i\alpha + H + C_i}{k_i}} - \alpha$ where

$$\hat{Q}_i = Q\prod_{j=i+1}^{n}\frac{\hat{r}_j^{**}}{\alpha+\hat{r}_j^{**}} - \sum_{j=i+1}^{n}\frac{\left(H + C_j + k_j\left(\hat{r}_j^{**}\right)^2\right)}{\alpha+\hat{r}_j^{**}}\prod_{m=i+1}^{j-1}\frac{\hat{r}_m^{**}}{\alpha+\hat{r}_m^{**}} \qquad \textit{for } i = n,\ldots,1.$$

2. Set $s_i^* = \frac{\xi_i\alpha\left(\alpha+\hat{r}_i^{**}\right)}{\xi_i\alpha + \left(\hat{r}_i^{**}\right)^2}$, where $\xi_i = \frac{k_i\left(\hat{r}_i^{**}\right)^2 - C_i}{\alpha} - \frac{O_i}{\prod_{j=1}^{i-1}\frac{\hat{r}_j^{**}}{\alpha+\hat{r}_j^{**}}}$ $\qquad$ *for* $i = n,\ldots,1.$

3. Set $p_i^* = \frac{k_i\left(\hat{r}_i^{**}\right)^2 - C_i + 2k_i\hat{r}_i^{**}\alpha}{\alpha} - \frac{\left(s_i^* - \alpha\right)\left(\alpha + \hat{r}_i^{**}\right)^2}{\alpha\left(\alpha - s_i^* + \hat{r}_i^{**}\right)^2}$ *for* $i = n, \ldots, 1.$

Proposition 3.4. Algorithm 3.1 provides an optimal solution to P3.1.

Proof: See Appendix.

This algorithm first calculates the optimal work rate at each stage and then solves for the optimal penalty s_i^* and optimal initial payment p_i^* respectively. The client will then issue an EIC with each of the subcontractors with the payment form $p_i^* - e^{s_i^* t_i}$. An important result obtained from Algorithm 3.1 is that the payment form $p_i^* - e^{s_i^* t_i}$ coordinates the decentralized project; this result is summarized in the following Corollary.

Corollary 3.4. EIC, with p_i^* and s_i^* specified by Algorithm 3.1, coordinates the decentralized project. The client obtains $E\left[\Pi_C^{CP}\left(r_1^*, \ldots, r_n^*\right)\right] - \sum_{i=1}^{n} O_i$ and subcontractor *i* obtains O_i expected profit for every *i*.

Corollary 3.4 indicates that EIC is an optimal contract for the client. Recall that coordination implies that the collective expected profit of the decentralized project is maximized $E\left[\Psi^{DP}\left(\hat{r}_1^{**}, \ldots, \hat{r}_n^{**}\right)\right] = E\left[\Pi_C^{CP}\left(r_1^*, \ldots, r_n^*\right)\right]$. Moreover, the client is, on average, only giving the subcontractors their minimum bound on the expected profit, O_i. Hence we conclude that this is the optimal contract for the client.

3.3 Implementing EIC in Practice

In this section, we discuss how EIC can be better implemented in practice by approximating it by contracts that are commonly used. In 3.3.1 we show a general piece-wise linear approximation of EIC is also known as the Incentive/Disincentive (I/D) contract. In 3.3.2, we study the case where discounting can be neglected and show that the first order Taylor series approximation of EIC is able to achieve coordination and provides an unambiguous property to handle risk averse subcontractors. This approximation is known as the "lane rental" contract, commonly used in transportation (highway) projects.

3.3.1 General Approximation

The exponential form is the key to achieve coordination under discounting; however, clients may not be comfortable issuing an exponential penalty. A figure of the EIC payment is illustrated as follows:

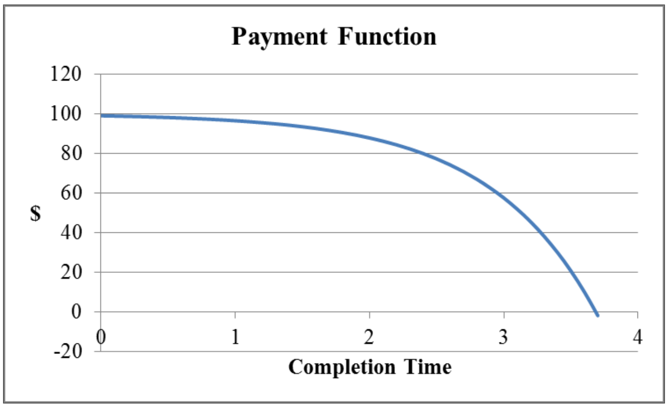

FIGURE 3.1: ILLUSTRATION OF AN EIC PAYMENT FUNCTION

To convert the exponential incentive payment contract to a form that is more likely to be adopted in practice, we apply the piece-wise linear approximation that Chen et al. (2015) suggested. The piece-wise linear approximation of the payment in Figure 3.1 with two linear segments is illustrated as follows

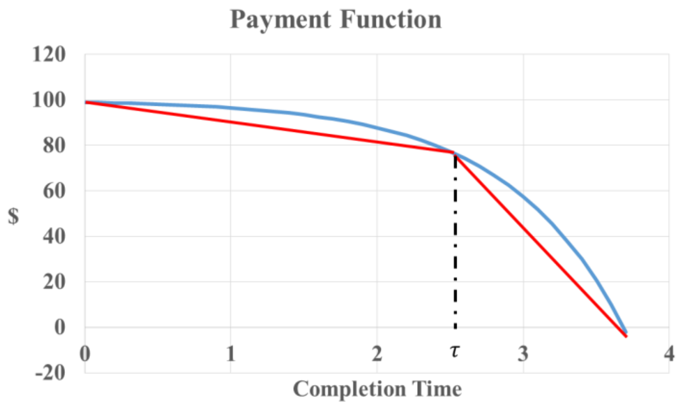

FIGURE 3.2: PIECE-WISE LINEAR APPROXIMATION OF THE EIC PAYMENT FUNCTION

The red lines in Figure 3.2 can be used to formulate an I/D contract, where the initial payment is the intercept with the y-axis, the penalty before time τ is the slope of the first line segment and the penalty after time τ is the slope of the second line segment. The intuition of this approximation is to minimize the area between the approximated linear functions and the EIC payment function for a given number of linear segments. Obviously, the quality of this approximation increases as more linear segments are used. However, since the linear functions are approximations, project coordination is not guaranteed.

3.3.2 First Order Taylor series Approximation (Linear Incentive Contract)

Recall that EIC's payment to each subcontractor is a function of their completion time $\beta(t_i) = p_i - e^{s_i t_t}$. The first order Taylor series approximation of $\beta(t_i) = p_i - e^{s_i t_t}$ at time 0 is

$$\beta(t_i) \approx g(t_i) = (p_i - 1) - s_i t_i = \hat{p}_i - s_i t_i ,$$

where $\hat{p}_i$ is a fixed payment, s_i is a penalty cost per time unit, and t_i is the realized duration of subcontractor *i*'s stage. If $s_i = 0$, we note that the contract is a fixed price contract that is widely used in practice and pays a subcontractor a fixed amount $\hat{p}_i$ regardless of the stage duration. Linear incentive contracts (LIC) are most notably used in transportation (highway) projects where they are sometimes referred to as "lane rental" contracts (since a subcontractor must "rent" a lane to close it). According to the Washington State Department of Transportation, "the intent is to minimize the impacts of a project on the traveling public…[by creating] a monetary incentive for the subcontractor to be innovative and minimize the duration of lane closures" (WSDOT 2015). Interestingly, we can show that this first order Taylor series approximation of EIC, LIC, has important managerial implications in the case where discounting is a minor issue and can be neglected.

3.3.2.1 Subcontractor Response to LIC $(\alpha = 0)$

Using a LIC contract, a subcontractor's profit (a random variable) is now defined as

$$\pi_i(\hat{r}_i) = (p_i - s_i t_i) - (C_i + k_i \hat{r}_i^2) t_i .$$

Our analysis of a subcontractor's response when a client offers a LIC contract is based on Proposition 3.2 that defines a unique work rate that stochastically dominates (maximizes) a client's profit.

Proposition 3.5. Given realized profit $\pi_i(\hat{r}_i) = p_i - (C_i + s_i + k_i\hat{r}_i^2)t_i$ where p_i, s_i, C_i, and k_i are known values, setting $\hat{r}_i^* = \sqrt{\frac{s_i + C_i}{k_i}}$ will result in $\pi_i(\hat{r}_i^*) \geq_{FSD} \pi_i(\hat{r}_i)$ for all $\hat{r}_i > 0$.

Proof: See Appendix.

The direct result from Proposition 3.5 is that setting $\hat{r}_i^* = \sqrt{\frac{s_i + C_i}{k_i}}$ will not only maximize the subcontractor's expected profit (for risk neutral goals) but also maximize any monotone increasing and strictly concave utility functions (for risk averse goals); c.f., Corollaries 3.1 and 3.2. These results have clear implications for practice and present a strong case for the use of linear incentive contracts that simultaneously optimize goals for both risk neutral and risk averse subcontractors.

3.3.2.2 Subcontractor Participation Constraints under LIC $(\alpha = 0)$

The previous analysis is based on the assumption that subcontractors are willing to participate in the project. In practice, this may not be the case as subcontractors often have a bound on their objectives that must be satisfied for them to participate. For example, this constraint could be stated as a lower bound on the expected profit; denoting this lower bound by an opportunity cost O_i, the i^{th} subcontractor would only participate in the project if

$$E\left[\pi_i\left(\hat{r}_i^*\right)\right] = p_i - \sqrt{4(C_i + s_i)k_i} \geq O_i, \tag{3.7}$$

where O_i reflects current economic conditions and alternative investment opportunities for the i^{th} subcontractor.

3.3.2.3 Client's optimization problem under LIC $(\alpha = 0)$

Using LIC in a decentralized project, the client will earn a profit (for realized values of t_i) equal to

$$\Pi_C^{DP}\left(p_1,\ldots,p_n,s_1,\ldots,s_n\right) = Q - \sum_{i=1}^{n}\left(p_i - s_i t_i\right) - HT. \tag{3.8}$$

The client's problem is then to maximize his expected profit,

$$\begin{aligned} &\max_{p_i, s_i} \; E\left[\Pi_C^{DP}\right] = Q - \sum_{i=1}^{n} p_i - \sum_{i=1}^{n}\left(\frac{H - s_i}{\hat{r}_i}\right) \\ &s.t \\ &\hat{r}_i = \sqrt{\frac{C_i + s_i}{k_i}} \qquad\qquad \forall i \\ &p_i - \sqrt{4(C_i + s_i)k_i} \geq O_i \qquad \forall i. \end{aligned} \tag{P3.2}$$

In problem P3.2, the client wants to find the non-negative values of p_i^* and s_i^* that maximize his expected profit subject to subcontractors' best responses (Proposition 3.5) and participation constraints (3.7). Since $\hat{r}_i = \sqrt{\frac{C_i + s_i}{k_i}}$ (by Proposition 3.5) are independent of p_i, the client would always set the values of p_i such that the subcontractors' participation constraints are satisfied at equality to maximize his expected profit defined by (3.7), $p_i = \sqrt{4(C_i + s_i)k_i} + O_i$. Substitute $\hat{r}_i = \sqrt{\frac{C_i + s_i}{k_i}}$ and $p_i = \sqrt{4(C_i + s_i)k_i} + O_i$ into the objective function of P3.2, the client's optimization problem becomes

$$\max_{s_i} \; E\left[\Pi_C^{DP}\right] = Q - \sum_{i=1}^{n}\left(O_i + \sqrt{4(C_i + s_i)k_i}\right) - \sum_{i=1}^{n}\left[\left(H - s_i\right)\sqrt{\frac{k_i}{C_i + s_i}}\right]. \tag{P3.3}$$

Proposition 3.6. The project is coordinated and the risk neutral client's expected profit defined by (3.8) is maximized when $s_i^* = H$ for all $i = 1,\ldots,n$.

Proof: See Appendix.

Proposition 3.6 indicates that the client should set the penalty costs s_i for each subcontractor equal to his overhead cost per time unit H and the fixed payment p_i equal to the subcontractor's opportunity cost O_i plus a value $\sqrt{4(C_i + H)k_i}$. Furthermore, if $s_i^* = H$, $\hat{r}_i^* = r_i^*$ for all $i = 1,\ldots,n$ and the project is coordinated when the client and subcontractors want to maximize their respective expected profits; the results also hold when the subcontractors are maximizing their expected risk-averse utility, and we have shown coordination is possible for risk-averse subcontractors under LIC. These results are summarized in Table 3.2

Optimal Penalty Rate	$s_i^* = H$
Optimal Initial Payment	$p_i^* = O_i + \sqrt{4(C_i + H)k_i}$
Equilibrium Work Rate	$\hat{r}_i^* = \sqrt{\dfrac{H + C_i}{k_i}} = r_i^*$
Subcontractor's Profit	$E\left[p_i\left(\hat{r}_i^*\right)\right] = O_i$
Client's Expected Profit	$E\left[\mathrm{P}_C^{DP}\left(\hat{r}_1^*,\ldots,\hat{r}_n^*\right)\right] = Q - \sum_{i=1}^{n}\left(O_i + \sqrt{4(C_i + H)k_i}\right)$
Total Coordinated (Max) Expected Profit	$E\left[\mathrm{Y}^{DP}\left(\hat{r}_1^*,\ldots,\hat{r}_n^*\right)\right] = E\left[\mathrm{P}_C^{CP}\left(r_1^*,\ldots,r_n^*\right)\right] = Q - \sum_{i=1}^{n}\sqrt{4(C_i + H)k_i}$

TABLE 3.2: DECENTRALIZED PROJECT RESULTS WITH LIC

Table 3.2 concludes that LIC is also an optimal contract for the client when $\alpha = 0$. However, LIC has two advantages over EIC when discounting is negligible. The first is that LIC is easy to implement (set $s_i^* = H$) and has a simple linear penalty function. The second is that LIC processes the stochastic dominance property such that setting $\hat{r}_i^* = \sqrt{\frac{H + C_i}{k_i}}$ optimizes both risk neutral and risk averse subcontractors' goals.

3.4 Conclusions

In this chapter, we propose an Exponential Incentive Contract with payment function $p_i - e^{s_i t_i}$, that coordinates a decentralized project with *n* subcontractors under continuous discounting. We show that this is the optimal contract for the client as the client obtains the maximum expected discounted profit. The contribution of this chapter is twofold. The first is that we have provided a framework to analyze a stochastic decentralized project for arbitrary *n* subcontractors; whereas in the project management literature, researches that study contract coordination under discounting are limited by two subcontractors. The second is that we have provided a link from the theoretical EIC to the commonly used contracts in practice. In this chapter, we have also provided analysis to the case where discounting is minimal. We show that the first order Taylor series approximation of EIC, LIC is also a coordinating contract;

Moreover, LIC processes the stochastic dominance property that ensures the adoption of $\hat{r}_i^* = \sqrt{\dfrac{H + C_i}{k_i}}$ for any risk-averse utility function.

Appendix B

Proof of Proposition 3.1

The centralized discounted profit at time 0 is formulated as

$$\Pi_C^{CP} = Qe^{-\alpha\sum_{i=1}^{n} t_i} - \sum_{i=1}^{n} e^{-\alpha\sum_{j=1}^{i-1} t_i} \int_0^{t_i} \left(H + C_i + k_i \left(r_i\right)^2\right) e^{-\alpha x} dx .$$

The expected discounted profit at time 0 is

$$E\left[\Pi_C^{CP}\right] = QE\left[e^{-\alpha\sum_{i=1}^{n} t_i}\right] - \sum_{i=1}^{n} E\left[e^{-\alpha\sum_{j=1}^{i-1} t_j}\left(\frac{\left(H + C_i + k_i r_i^2\right)\left(1 - \mathrm{e}^{-at_i}\right)}{a}\right)\right]$$

$$= Q\prod_{i=1}^{n}\frac{r_i}{\alpha + r_i} - \sum_{i=1}^{n}\frac{\left(H + C_i + k_i r_i^2\right)}{\alpha + r_i}\prod_{j=1}^{i-1}\frac{r_j}{\alpha + r_j}.$$

We solve from the last stage *n*. We isolate the terms in $E\left[\Pi_C^{CP}\right]$ that only includes r_n as follows

$$E\left[\hat{\Pi}_C^{CP}(n)\right] = \left(Q\frac{r_n}{\alpha + r_n} - \frac{\left(H + C_n + k_n r_n^2\right)}{\alpha + r_n}\right)\prod_{j=1}^{n-1}\frac{r_j}{\alpha + r_j} .$$

$$\frac{\partial E\left[\Pi_C^{CP}\right]}{\partial r_n} = \frac{\partial E\left[\hat{\Pi}_C^{CP}(n)\right]}{\partial r_n} = \left(\frac{-k_n r_n^2 - 2k_n r_n a + \left(Qa + H + C_n\right)}{\left(a + r_n\right)^2}\right)\prod_{j=1}^{n-1}\frac{r_j}{\alpha + r_j} .$$

$$\frac{\partial^2 E\left[\Pi_C^{CP}\right]}{\partial r_n^2} = \left(-\frac{2\left(Qa + ka^2 + H + C\right)}{\left(a + r\right)^3}\right)\prod_{j=1}^{n-1}\frac{r_j}{\alpha + r_j} < 0.$$

Setting $\frac{\partial E\left[\Pi_C^{CP}\right]}{\partial r_n} = 0$, we have $r_n^* = \sqrt{\alpha^2 + \frac{Q\alpha + H + C_n}{k_n}} - \alpha$ and the second order condition confirms that r_n^* maximizes $E\left[\Pi_C^{CP}\right]$. Note that $r_n^*\left(Q, \alpha, H, C_n, k_n\right)$ is a function of the project overall parameters and specific cost parameters only related to stage *n*. Next we solve for stage *n -1*. We isolate the terms in $E\left[\Pi_C^{CP}\right]$ that only includes r_{n-1} as follows

$$E\left[\hat{\Pi}_C^{CP}(n-1)\right]=\left(Q\frac{r_{n-1}}{\alpha+r_{n-1}}\frac{r_n^*}{\alpha+r_n^*}-\frac{\left(H+C_{n-1}+k_{n-1}r_{n-1}{}^2\right)}{\alpha+r_{n-1}}-\frac{\left(H+C_n+k_n\left(r_n^*\right)^2\right)}{\alpha+r_n^*}\frac{r_{n-1}}{\alpha+r_{n-1}}\right)\prod_{j=1}^{n-2}\frac{r_j}{\alpha+r_j}$$

$$=\left(\left(\underbrace{Q\frac{r_n^*}{\alpha+r_n^*}-\frac{\left(H+C_n+k_n\left(r_n^*\right)^2\right)}{\alpha+r_n^*}}_{\text{contant}}\right)\frac{r_{n-1}}{\alpha+r_{n-1}}-\frac{\left(H+C_{n-1}+k_{n-1}r_{n-1}{}^2\right)}{\alpha+r_{n-1}}\right)\prod_{j=1}^{n-2}\frac{r_j}{\alpha+r_j}.$$

Clearly $r_{n-1}^*=\sqrt{\alpha^2+\frac{\hat{Q}_{n-1}\alpha+H+C_{n-1}}{k_{n-1}}}-\alpha$, where $\hat{Q}_{n-1}=Q\frac{r_n^*}{\alpha+r_n^*}-\frac{\left(H+C_n+k_n\left(r_n^*\right)^2\right)}{\alpha+r_n^*}$. Then

through induction, we can show that $r_i^*=\sqrt{\alpha^2+\frac{\hat{Q}_i\alpha+H+C_i}{k_i}}-\alpha$, where

$$\hat{Q}_i=Q\prod_{j=i+1}^{n}\frac{r_j^*}{\alpha+r_j^*}-\sum_{j=i+1}^{n}\frac{\left(H+C_j+k_j\left(r_j^*\right)^2\right)}{\alpha+r_j^*}\prod_{m=i+1}^{j-1}\frac{r_m^*}{\alpha+r_m^*}\quad for\ i=1,\ldots,n-1.$$

Q.E.D

Proof of Proposition 3.2

Since the duration of tasks are independent of each other and Q is constant, stochastically maximizing

$\Pi_C^{CP}\left(r_1,\ldots,r_n\right)=Q-\sum_{i=1}^{n}\left[\left(H+C_i+k_ir_i^2\right)t_i\right]$ is equivalent to stochastically minimizing the costs at each stage, namely, $\max_{r_i>0} Pb\left[\left(H+C_i+k_ir_i^2\right)t_i\le y\right]$ for all y. Recall that t_i is exponentially distributed, it is equivalent to $\max_{r_i>0}\left[1-e^{-\frac{r_i}{H+C_i+k_ir_i^2}y}\right]$. Note that $1-e^{-\frac{r_i}{H+C_i+k_ir_i^2}y}$ is maximized when the exponent $\frac{r_i}{H+C_i+k_ir_i^2}y$ is maximized. From $\frac{\partial}{\partial r_i}\frac{r_i}{H+C_i+k_ir_i^2}y=\frac{y\left(H+C_i-k_ir_i^2\right)}{\left(H+C_i+k_ir_i^2\right)^2}$, we can see that

$\frac{\partial}{\partial r_i}\frac{r_i}{H+C_i+k_ir_i^2}y>0\quad\forall\, 0<r_i<\sqrt{\frac{H+C_i}{k_i}}$ and $\frac{\partial}{\partial r_i}\frac{r_i}{H+C_i+k_ir_i^2}y<0\quad\forall\, r_i>\sqrt{\frac{H+C_i}{k_i}}$. This result implies that $\frac{r_i}{H+C_i+k_ir_i^2}y$ is unimodal in r_i and has a maximum at $r_i^*=\sqrt{\frac{H+C_i}{k_i}}$, which does not depend on y. Thus, the cost of each stage is stochastically minimized when $r_i^*=\sqrt{\frac{H+C_i}{k_i}}$. By Theorem

1.A.3 (Shaked and Shanthikumar 2007), the total cost $\sum_{i=1}^{n}\left(H+C_i+k_i r_i^2\right)t_i$ is stochastically minimized

when $r_i^* = \sqrt{\dfrac{H+C_i}{k_i}} \quad for \quad i=1,...,n.$ Since Q is constant , we have

$$Pb\left(Q-\sum_{i=1}^{n}\left(H+C_i+k_i\left(r_i^*\right)^2\right)t_i \le y\right) \le Pb\left(Q-\sum_{i=1}^{n}\left(H+C_i+k_i\left(r_i\right)^2\right)t_i \le y\right), \quad \forall y \quad and \quad \forall r_i \ge 0 \quad i=1,...,n.$$

Thus $\Pi_C^{CP}\left(r_1^*,...,r_n^*\right) \ge_{FSD} \Pi_C^{CP}\left(r_1,...,r_n\right), \quad \forall r_i > 0.$ *Q.E.D*

Proof of Proposition 3.3

Here, we assume $s_i > \alpha$, note that s_i is the client's decision variable and can be viewed as a constant in the subcontractor *i*'s problem. In Proposition 4, we will then show that, at the equilibrium, $s_i > \alpha$ is always true $\left(s_i^* > \alpha\right)$.

Subcontractor *i*'s optimization problem is

$$\max_{\hat{r}_i} E\left[\pi_i\right] = \left[\frac{p_i\hat{r}_i - C_i - k_i\hat{r}_i^2}{\alpha+\hat{r}_i} - \frac{\hat{r}_i}{\alpha - s_i + \hat{r}_i}\right]\prod_{j=1}^{i-1}\frac{\hat{r}_j}{\alpha+\hat{r}_j}.$$

Since $\dfrac{\partial^2 E\left[\pi_i\right]}{\partial \hat{r}_i^2} = \left[\dfrac{-k_i\alpha^2 - 2\alpha p_i - 2C_i}{\left(\alpha+\hat{r}_i\right)^3} - \dfrac{2\left(s_i-\alpha\right)}{\left(\alpha-s_i+\hat{r}_i\right)^3}\right]\prod_{j=1}^{i-1}\dfrac{\hat{r}_j}{\alpha+\hat{r}_j} < 0, \quad \left(s_i > \alpha\right)$

to find the maximizer r_i^*, we set $\dfrac{\partial E\left[\pi_i\right]}{\partial \hat{r}_i} = \left[\dfrac{\left(p_i - 2k_i\hat{r}_i\right)\alpha + C_i - k_i\hat{r}_i^2}{\left(\alpha+\hat{r}_i\right)^2} + \dfrac{s_i-\alpha}{\left(\alpha-s_i+\hat{r}_i\right)^2}\right]\prod_{j=1}^{i-1}\dfrac{\hat{r}_j}{\alpha+\hat{r}_j} = 0$. Which

implies $\dfrac{\left(p_i - 2k_i\hat{r}_i^*\right)\alpha + C_i - k_i\left(\hat{r}_i^*\right)^2}{\left(\alpha+\hat{r}_i^*\right)^2} + \dfrac{s_i-\alpha}{\left(\alpha-s_i+\hat{r}_i^*\right)^2} = 0$,

$$\hat{r}_i^* = \sqrt{\frac{\left[k_i\left(\hat{r}_i^*\right)^2 + 2k_i\hat{r}_i^*\alpha - \left(p_i\alpha + C_i\right)\right]\left(\alpha - s_i + \hat{r}_i^*\right)^2}{\left(s_i-\alpha\right)}} - \alpha > 0.$$

Moreover,

$\lim_{\hat{r}_i\to 0}\dfrac{\partial E\left[\pi_i\right]}{\partial \hat{r}_i} = \left[\dfrac{p_i\alpha + C_i}{\alpha^2} + \dfrac{1}{\left(\alpha-s_i\right)^2}\right]\prod_{j=1}^{i-1}\dfrac{\hat{r}_j}{\alpha+\hat{r}_j} > 0$ and $\lim_{\hat{r}_i\to\infty}\dfrac{\partial E\left[\pi_i\right]}{\partial \hat{r}_i} = \left[-k_i\right]\prod_{j=1}^{i-1}\dfrac{\hat{r}_j}{\alpha+\hat{r}_j} < 0$ we can conclude that $\hat{r}_i^* > 0$ (existence). *Q.E.D*

Proof of Corollary 3.3

Rearrange $\hat{r}_i^* = \sqrt{\frac{\left[k_i\left(\hat{r}_i^*\right)^2 + 2k_i\hat{r}_i^*\alpha - \left(p_i\alpha + C_i\right)\right]\left(\alpha - s_i + \hat{r}_i^*\right)^2}{\left(s_i - \alpha\right)}} - \alpha$, we have

$p_i = \frac{k_i\left(\hat{r}_i^*\right)^2 - C_i + 2k_i\hat{r}_i^*\alpha}{\alpha} - \frac{\left(s_i - \alpha\right)\left(\alpha + \hat{r}_i^*\right)^2}{\alpha\left(\alpha - s_i + \hat{r}_i^*\right)^2}$. Substitute this expression into (3.6) we obtain

$$E\left[\pi_i\left(\hat{r}_i^*\right)\right] = \left[\frac{k_i\left(\hat{r}_i^*\right)^2 - C_i}{\alpha} - \frac{s_i\left(\hat{r}_i^*\right)^2}{\alpha\left(\alpha - s_i + \hat{r}_i^*\right)^2}\right]\prod_{j=1}^{i-1}\frac{r_j}{\alpha + r_j}. \qquad Q.E.D$$

Algorithm 3.1

Re-write the first set of constraints in P3.1 by isolating p_i we have the equivalent problem

$$\max_{p\geq 0, s\geq 0} E\left[\Pi_C^{DP}\right] = Q\prod_{i=1}^{n}\frac{\hat{r}_i^*}{\alpha + \hat{r}_i^*} - \sum_{i=1}^{n}\left[\left(\frac{p_i\hat{r}_i^*}{\alpha + \hat{r}_i^*} - \frac{\hat{r}_i^*}{\alpha - s_i + \hat{r}_i^*}\right)\prod_{j=1}^{i-1}\frac{\hat{r}_j^*}{\alpha + \hat{r}_j^*}\right] - \sum_{i=1}^{n}\left[\frac{H}{\alpha + \hat{r}_i^*}\prod_{j=1}^{i-1}\frac{\hat{r}_j^*}{\alpha + \hat{r}_j^*}\right]$$

$$s.t$$

$$p_i = \frac{k_i\left(\hat{r}_i^*\right)^2 - C_i + 2k_i\hat{r}_i^*\alpha}{\alpha} - \frac{\left(s_i - \alpha\right)\left(\alpha + \hat{r}_i^*\right)^2}{\alpha\left(\alpha - s_i + \hat{r}_i^*\right)^2} \qquad \forall i \tag{P3.2}$$

$$\left[\frac{k_i\left(\hat{r}_i^*\right)^2 - C_i}{\alpha} - \frac{s_i\left(\hat{r}_i^*\right)^2}{\alpha\left(\alpha - s_i + \hat{r}_i^*\right)^2}\right]\prod_{j=1}^{i-1}\frac{\hat{r}_j^*}{\alpha + \hat{r}_j^*} \geq O_i \qquad \forall i\,.$$

Next substitute p_i in the objective of P3.2, we have

$$\max_{\hat{r}^*, s} E\left[\Pi_C^{DP}\right] = Q\prod_{i=1}^{n}\frac{\hat{r}_i^*}{\alpha + \hat{r}_i^*} - \sum_{i=1}^{n}\left[\left(\frac{\left(k_i\left(\hat{r}_i^*\right)^2 - C_i + 2k_i\hat{r}_i^*\alpha\right)\hat{r}_i^*}{\alpha\left(\alpha + \hat{r}_i^*\right)} - \frac{s_i\left(\hat{r}_i^*\right)^2}{\alpha\left(\alpha - s_i + \hat{r}_i^*\right)^2}\right)\prod_{j=1}^{i-1}\frac{\hat{r}_j^*}{\alpha + \hat{r}_j^*}\right]$$
$$- \sum_{i=1}^{n}\left[\frac{H}{\alpha + \hat{r}_i^*}\prod_{j=1}^{i-1}\frac{\hat{r}_j^*}{\alpha + \hat{r}_j^*}\right] \tag{P3.3}$$

$$s.t$$

$$\left[\frac{k_i\left(\hat{r}_i^*\right)^2 - C_i}{\alpha} - \frac{s_i\left(\hat{r}_i^*\right)^2}{\alpha\left(\alpha - s_i + \hat{r}_i^*\right)^2}\right]\prod_{j=1}^{i-1}\frac{\hat{r}_j^*}{\alpha + \hat{r}_j^*} \geq O_i \qquad \forall i\,.$$

We first assume that the constraints are all binding,

$\left[\frac{k_i\left(\hat{r}_i^*\right)^2 - C_i}{\alpha} - \frac{s_i\left(\hat{r}_i^*\right)^2}{\alpha\left(\alpha - s_i + \hat{r}_i^*\right)^2}\right]\prod_{j=1}^{i-1}\frac{\hat{r}_j^*}{\alpha + \hat{r}_j^*} = O_i \quad \forall i$, and then show that this is always the case at equilibrium. The binding constraints imply that

$\frac{s_i\left(\hat{r}_i^*\right)^2}{\alpha\left(\alpha - s_i + \hat{r}_i^*\right)^2}\prod_{j=1}^{i-1}\frac{\hat{r}_j^*}{\alpha + \hat{r}_j^*} = \frac{k_i\left(\hat{r}_i^*\right)^2 - C_i}{\alpha}\prod_{j=1}^{i-1}\frac{\hat{r}_j^*}{\alpha + \hat{r}_j^*} - O_i \quad \forall i.$ Substitute this expression into the objective of P3.3 we have

$$\max_{\hat{r}^*} E\left[\Pi_C^{DP}\right] = Q\prod_{i=1}^{n}\frac{\hat{r}_i^*}{\alpha+\hat{r}_i^*} - \sum_{i=1}^{n}\left[\left(\frac{\left(k_i\left(\hat{r}_i^*\right)^2 - C_i + 2k_i\hat{r}_i^*\alpha\right)\hat{r}_i^*}{\alpha\left(\alpha+\hat{r}_i^*\right)} - \frac{k_i\left(\hat{r}_i^*\right)^2 - C_i}{\alpha}\right)\prod_{j=1}^{i-1}\frac{\hat{r}_j^*}{\alpha+\hat{r}_j^*} + O_i\right]$$
$$-\sum_{i=1}^{n}\left[\frac{H}{\alpha+\hat{r}_i^*}\prod_{j=1}^{i-1}\frac{\hat{r}_j^*}{\alpha+\hat{r}_j^*}\right].$$

Which can be simplified as

$$\max_{\hat{r}^*} E\left[\Pi_C^{DP}\right] = \underbrace{Q\prod_{i=1}^{n}\frac{r_i}{\alpha+r_i} - \sum_{i=1}^{n}\left[\left(\frac{H+C_i+k_i r_i^2}{\alpha+r_i}\right)\prod_{j=1}^{i-1}\frac{r_j}{\alpha+r_j}\right]}_{Centralized\ profit\ E\left[\Pi_C^{CP}\right]} - \sum_{i=1}^{n} O_i. \tag{P3.4}$$

Since $\sum_{i=1}^{n} O_i$ is a constant, the solution to P3.4 is the same as the centralized solution, namely $\hat{r}_i^{**} = r_i^*$. Previously, we have assumed that the constraints in P3.3 are binding, here we provide the reasons. The maximum expected profit for the whole system is at most $E\left[\Pi_C^{CP}\left(r_1^*,...,r_n^*\right)\right]$, since the subcontractors, in total, wants $\sum_{i=1}^{n} O_i$, the maximum that the client could obtain is $E\left[\Pi_C^{CP}\left(r_1^*,...,r_n^*\right)\right] - \sum_{i=1}^{n} O_i$. This is exactly what the client has obtained at equilibrium when the constraints are binding. Note that in P3.1, the first set of constraint $\hat{r}_i^* = \sqrt{\frac{\left[k_i\left(\hat{r}_i^*\right)^2 + 2k_i\hat{r}_i^*\alpha - \left(p_i\alpha + C_i\right)\right]\left(\alpha - s_i + \hat{r}_i^*\right)^2}{\left(s_i-\alpha\right)}} - \alpha$ were obtained from Proposition 3.3, where we have assumed $s_i > \alpha \ \forall i$. Since this assumption also leads the client to receiving the maximum expected profit, hence we can conclude that at the equilibrium $s_i^* > \alpha \ \forall i$. Thus far, we have shown that the solution to P1 is equivalent to the solution that maximizes (1). Hence the first step is to set Set

$\hat{r}_i^{**} = \sqrt{\alpha^2 + \frac{\hat{Q}_i\alpha + H + C_i}{k_i}} - \alpha$ where

$\hat{Q}_i = Q\prod_{j=i+1}^{n}\frac{\hat{r}_j^{**}}{\alpha+\hat{r}_j^{**}} - \sum_{j=i+1}^{n}\frac{\left(H + C_j + k_j\left(\hat{r}_j^{**}\right)^2\right)}{\alpha+\hat{r}_j^{**}}\prod_{m=i+1}^{j-1}\frac{\hat{r}_m^{**}}{\alpha+\hat{r}_m^{**}} \quad for\ i = n,...,1.$ Since

$\left[\frac{k_i\left(\hat{r}_i^{**}\right)^2 - C_i}{\alpha} - \frac{s_i\left(\hat{r}_i^{**}\right)^2}{\alpha\left(\alpha - s_i + \hat{r}_i^{**}\right)^2}\right]\prod_{j=1}^{i-1}\frac{\hat{r}_j^{**}}{\alpha+\hat{r}_j^{**}} = O_i \quad \forall i$. Then it is easy to show that

$s_i^* = \frac{\xi_i\alpha\left(\alpha+\hat{r}_i^{**}\right)}{\xi_i\alpha + \left(\hat{r}_i^{**}\right)^2}$, where $\xi_i = \frac{k_i\left(\hat{r}_i^{**}\right)^2 - C_i}{\alpha} - \frac{O_i}{\prod_{j=1}^{i-1}\frac{\hat{r}_j^{**}}{\alpha+\hat{r}_j^{**}}} \quad for\ i = n,...,1.$ Last step, from the first set of constraints in P3.2 we can conclude that

$$p_i^* = \frac{k_i\left(\hat{r}_i^{**}\right)^2 - C_i + 2k_i\hat{r}_i^{**}\alpha}{\alpha} - \frac{\left(s_i^* - \alpha\right)\left(\alpha + \hat{r}_i^{**}\right)^2}{\alpha\left(\alpha - s_i^* + \hat{r}_i^{**}\right)^2} \qquad \textit{for } i = n,\ldots,1. \qquad \textit{Q.E.D}$$

Proof of Proposition 3.4

Follows directly from Algorithm 3.1.

Proof of Corollary 3.4

Follows directly from Algorithm 3.1.

Proof of Proposition 3.5

The objective is to find a $\hat{r}_i^*$ such that $\pi_i(\hat{r}_i^*) \geq_{FSD} \pi_i(\hat{r}_i)$ for all $\hat{r}_i > 0$. Since p_i is constant, it is equivalent to $\max_{\hat{r}_i \geq 0} Pb\left[\left(s_i + C_i + k_i\hat{r}_i^2\right)t_i \leq y\right]$ for all *y*. Since t_i is exponentially distributed, it is equivalent to $\max_{\hat{r}_i \geq 0}\left[1 - e^{-\frac{\hat{r}_i}{s_i + C_i + k_i\hat{r}_i^2}y}\right]$. Similar to the proof of Proposition 3.2, the exponent $\frac{\hat{r}_i}{s_i + C_i + k_i\hat{r}_i^2}y$ is uniquely maximized when $\hat{r}_i^* = \sqrt{\frac{s_i + C_i}{k_i}}$, which does not depend on *y*.

Proof of Proposition 3.6

Take the derivative of the objective function of P3.3 with respect to s_i we have

$\frac{\partial}{\partial s_i}E\left[\Pi_C^{DP}\right] = \frac{(H - s_i)\sqrt{k_i}}{2\left(C_i + s_i\right)^{3/2}}$. Given that $s_i \geq 0$, the sign of $\frac{\partial}{\partial s_i}E\left[\Pi_C^{DP}\right]$ depends only on the numerator $g(s_i) = (H - s_i)\sqrt{k_i}$. Since $g(0) > 0$, $\lim_{s_i \to \infty} g(s_i) = -\infty < 0$, and $g(s_i)$ is strictly decreasing in s_i, we can conclude that $E\left[\Pi_C^{DP}\right]$ is unimodal in s_i and has a maximum at $s_i^* = H$.

Q.E.D

4 Cost Plus Contract

In this chapter, we analyze another type of contract that is also commonly used in practice, the cost plus contract. To be specific, we show how this type of contract affects the Client's expected profit, subcontractors' expected profits, and the level of system coordination. Following the framework in chapter 3 (the case where $\alpha = 0$), we analyze the cost plus contract and compare it with the fixed price contract and incentive contract to provide managerial insights.

4.1 Cost plus contract formulation

Following previous chapters, we focus on sequential networks and assume that the client subcontracts all *n* stages to *n* independent subcontractors. We begin the analysis by defining the contract parameters in a cost plus contract. The client pays subcontractor *i* a payment amount equal to:

$$PA_i = \min\left(\left(1+\alpha_i\right)TC_i, B_i\right),$$

where $\alpha_i \geq 0$ is the subcontractor *i*'s "cost plus" multiplier, TC_i is the total cost incurred in stage *i*, and $B_i \geq 0$ is the maximum payment that subcontractor *i* receives. This type of contract indicates that if the total cost is below a budget, the client will pay the subcontractor the total cost plus a certain percentage of the total cost (cost plus). If the total cost exceeds the budget, the subcontractor has to absorb the loss.

Following the cost structure and duration distribution in chapter 3, we assume that the total cost in stage i is $\left(C_i + k_i r_i^2\right)t_i$ and duration t_i is exponential distributed with work rate r_i. For a given work rate r_i, there exists a unique threshold $\hat{t}_i$ such that for $t_i \le \hat{t}_i$, $P_i = \left(1+\alpha_i\right)\left(C_i + k_i r_i^2\right)t_i$ and for $t_i > \hat{t}_i$, $PA_i = B_i$. The following figure illustrates this threshold $\hat{t}_i$:

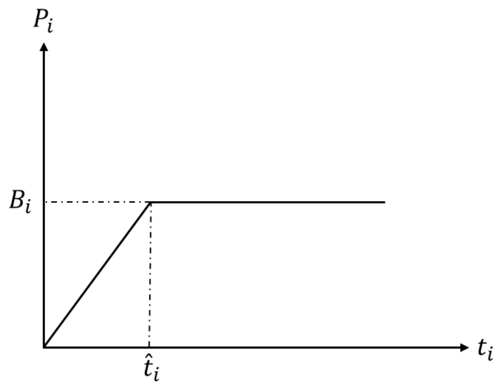

FIGURE 4.1: THE COST PLUS PAYMENT FUNCTION.

We can solve $\hat{t}_i$ by setting $\left(1+\alpha_i\right)\left(C_i + k_i r_i^2\right)t_i = B_i$ and we have

$$\hat{t}_i = \frac{B_i}{\left(1+\alpha_i\right)\left(C_i + k_i r_i^2\right)}.$$

Given any pair of $\left(\alpha_i, B_i\right)$, the subcontractor i's expected profit is formulated as:

$$\begin{aligned}\pi_i &= \int_0^{\hat{t}_i} \left(1+\alpha_i\right)\left(C_i + k_i r_i^2\right)t_i \cdot r_i e^{-r_i t_i} dt_i + \int_{\hat{t}_i}^{\infty} B_i \cdot r_i e^{-r_i t_i} dt_i - \int_0^{\infty} \left(C_i + k_i r_i^2\right)t_i \cdot r_i e^{-r_i t_i} dt_i \\ &= \frac{\left(C_i + k_i r_i^2\right)\left(\alpha_i - \left(1+\alpha_i\right)e^{-r_i \hat{t}_i}\right)}{r_i}.\end{aligned}$$

In the first line, the first integral is the subcontractor i's expected revenue in interval $[0, \hat{t}_i)$, the second component is the subcontractor i's expected revenue in interval $[\hat{t}_i, \infty)$, and the third component is the expected total cost incurred. Hence, for a given pair of $\left(\alpha_i, B_i\right)$, the subcontractor i's optimization problem is:

$$\max_{r_i \geq 0} \frac{\left(C_i + k_i r_i^2\right)\left(\alpha_i - \left(1+\alpha_i\right)e^{-r_i \hat{t}_i}\right)}{r_i}.$$

This problem turns out to be extremely difficult to analyze analytically, instead we analyze it numerically and provide insights about this contract. In practice, parameter α_i is normally based on industry standards, hence in the analysis we put more focus on the budget B_i. One of the reasons that cost plus contract is difficult to analyze is that there exists a $\hat{B}_i$ such that $\forall\, 0 \leq B_i \leq \hat{B}_i,\ r_i^*(B_i) = \sqrt{\frac{C_i}{k_i}}$ and $\forall\, B_i > \hat{B}_i,\ r_i^*(B_i)$ is not unique and in fact there are two solutions, one greater (upper branch), $r_i^{U*}(B_i)$, and one smaller (lower branch), $r_i^{L*}(B_i)$, than $\sqrt{\frac{C_i}{k_i}}$. Note that $\sqrt{\frac{C_i}{k_i}}$ is the fixed price contract equilibrium work rate that we derived in chapter 3.

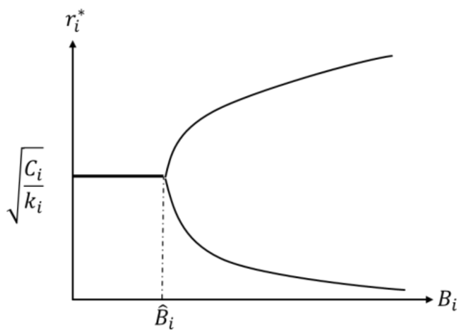

FIGURE 4.2: OPTIMAL WORK RATES.

A direct consequence of this non-unique optimal work rate is that a risk neutral client would be unsure of how to set B_i that maximizes its expect profit in the Stackelberg game. Since the two optimal work rates are indifferent to the subcontractors, a natural approach is to assign a probability *q* to the upper branch work rate and 1-*q* to the lower branch (for illustration purposes, here we assume that all subcontractors have the same probability *q*). The client's expected profit maximization problem is formulated as follows:

$$\max_{B_1,\ldots,B_n} \quad E\left[\Pi_C^{DP}\left(B_1,\ldots,B_n\right)\right] =$$
$$qE\left[\Pi_C^{DP}\left(B_1,\ldots,B_n,r_1^{U*}(B_1),\ldots,r_n^{U*}(B_n)\right)\right]+(1-q)E\left[\Pi_C^{DP}\left(B_1,\ldots,B_n,r_1^{L*}(B_1),\ldots,r_n^{L*}(B_n)\right)\right]$$
where
$$E\left[\Pi_C^{DP}\left(B_1,\ldots,B_n,r_1^{*}(B_1),\ldots,r_n^{*}(B_n)\right)\right] =$$
$$Q-\sum_{i=1}^{n}\left[\int_0^{\hat{t}_i}(1+\alpha_i)\left(C_i+k_i\left(r_i^*\right)^2\right)t_i\cdot r_i^* e^{-r_i^* t_i}dt_i+\int_{\hat{t}_i}^{\infty}B_i\cdot r_i^* e^{-r_i^* t_i}dt_i\right]-H\sum_{i=1}^{n}\frac{1}{r_i^*}.$$

To understand how this probability q affects the client's and systems expected profits, we generate various sets of Q,H,k_i,C_i,α_i for $i=1,2,3$ to study its impact. All of our numerical results support that as q increases, the system's expected profit, the client's expected profit, and the subcontractors' expected profits increase. Moreover, under the mix strategy scenario the maximum system profit that can be achieved (when q = 1) is equivalent to that of LIC (recall chapter 3, LIC is a coordinating contract when discounting can be neglected); when q = 0, the maximum system profit that can be achieved is equivalent to that of an fixed price contract (recall chapter 3, fixed price contract is not a coordinating contract). As a result, the probability q directly affects the coordination ability of a cost plus contract. However, q is very difficult to observe in practice and a misleading prior distribution (an incorrect estimation of q) of q may lead to a suboptimal decision B which in turn has catastrophic effects on the client's expected profit.

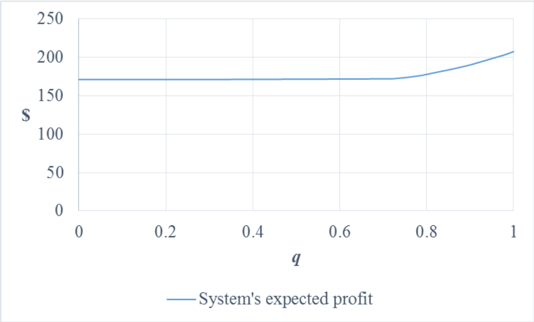

FIGURE 4.3: *Q* AND COORDINATION.

A better approach is to apply the maxmin approach, which is to maximize the worst expected possible outcome that results from the subcontractor choosing one of the two optimal work rates. This approach is more realistic since a prior distribution is not needed. From our numerical results, we observe that the client's expected profit is greater if subcontractors work at the upper branch rate $r_i^{U*}(B_i)$, namely

$$E\left[\Pi_C^{DP}\left(B_1,...,B_n,r_1^{U*}(B_1),...,r_n^{U*}(B_n)\right)\right] \geq E\left[\Pi_C^{DP}\left(B_1,...,B_n,r_1^{L*}(B_1),...,r_n^{L*}(B_n)\right)\right] \quad \forall B_i \geq 0,\ i=1,...,n.$$

Therefore, the maxmin optimization problem becomes

$$\max_{B_1,...,B_n} \quad E\left[\Pi_C^{DP}\left(B_1,...,B_n,r_1^{L*}(B_1),...,r_n^{L*}(B_n)\right)\right].$$

In this case, all of our results show that as the budget B increases, the worst case outcome becomes even worse; Hence, the client should set B as small as possible, just enough to ensure the participation of the subcontractors.

$_{r_i}$

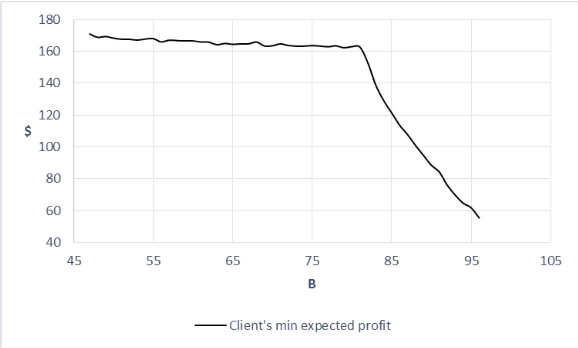

FIGURE 4.4: MAXMIN RESULTS.

From the numerical analysis of the maxmin approach above, we show that system coordination under a cost plus contract is never achieved. The client's optimal expected profit is equivalent to a fixed price contract which is dominated by incentive contracts (proved in chapter 3). Moreover, the structure of cost plus contracts induces non-unique work rates which lead to even more uncertainty.

4.2 The dynamics of a Cost plus contract

An important question is to understand how subcontractors would react when work rates are adjustable. Recall chapter 3, under an incentive contract, a subcontractor's profit is $\pi_i = p_i - (C_i + s_i + k_i r_i^2)t_i$. Since $t_i \sim exp(r_i)$, the memoryless property implies that a subcontractor will not adjust her work rate once she starts the task. For example, assume subcontractor *i* is at time $\tilde{t}$, then the total cost spent is $(C_i + s_i + k_i \hat{r}_{i(\text{old})}^2)\tilde{t}$, a constant. If subcontractor *i* is able to adjust her work rate she would maximize her expected profit at time $\tilde{t}$, which is $\max_{r_i} \quad E\left[\pi_i \middle| t = \tilde{t}\right] = p_i - (C_i + s_i + k_i r_i^2)\mathrm{E}\left[t_i \middle| t > \tilde{t}\right] - (C_i + s_i + k_i \hat{r}_{i(\text{old})}^2)\tilde{t}$.

Since $(C_i + s_i + k_i \hat{r}^2_{i(\text{old})})\tilde{t}$ is a constant and $\mathrm{E}\left[t_i \middle| t > \tilde{t}\right] = \mathrm{E}\left[t_i\right] = \frac{1}{r_i}$ (memoryless property), we can conclude that $r_i^*(t=0) = r_i^*(t=\tilde{t}) = \sqrt{\frac{s_i + C_i}{k_i}}$ (Recall Proposition 3.5). Hence the static solution is equivalent to the dynamic solution for a subcontractor under incentive contract.

However, this is not the case with a cost plus contract. Assume the initial budget is $B_i^*(t=0)$; then at time $\tilde{t}$, the remaining budget becomes $B_i^*(t=\tilde{t}) = B_i^*(t=0) - \left(C_i + k_i r_i^2\right)\tilde{t} \le B_i^*(t=0)$. Illustrated in figure 4.5, as the remaining budget gets smaller, the two optimal work rates get closer to the fixed price contract equilibrium work rate $\sqrt{\frac{C_i}{k_i}}$. This result shows that under a cost plus contract

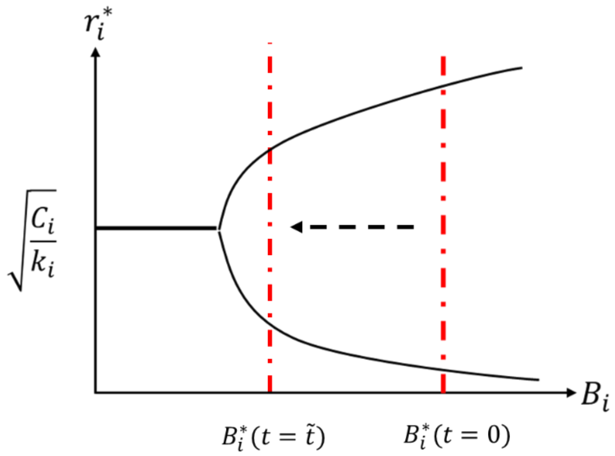

FIGURE 4.5: DYNAMIC SOLUTION FOR COST PLUS CONTRACT.

, if work rates are adjustable, the optimal work rate will converge to $\sqrt{\frac{C_i}{k_i}}$ as the remaining budget gets smaller.

4.3 Conclusion

At first glance, the cost plus contract seems to provide an advantage for the client as it imposes a maximum budget for each subcontractor. This budget limits the client's risks by transferring them to the subcontractors. This may be one of the reasons why clients are using the cost plus contracts commonly in practice. However from our analysis, we show that the cost plus contract actually creates more uncertainties as it led subcontractors to have non-unique optimal work rates. The performance of the cost plus contractor falls in between that of a fixed price contract and an incentive contract regarding the level of project coordination, client's and subcontractors' expected profits.

Moreover, in a dynamic setting where subcontractor work rates can be adjusted without incurring significant additional costs, the cost plus contract optimal work rate will converge to a fixed price contract optimal work rate.

5 Dissertation Conclusions and Extensions

In Chapter two, we proposed and analyzed an "exponential incentive payment" contract for a stochastic project that consists of $n \geq 1$ serial stages, where each stage is completed by an independent subcontractor. In the basic form of the "incentive payment" contract, the client pays each subcontractor an amount $p_i e^{-\beta_i t_i}$ at the conclusion of a subcontractor's stage or the entire project. The parameters $p_i > 0$ and $\beta_i \geq 0$ are revealed to each subcontractor at the beginning of the project. The primary contribution of this chapter is to analytically demonstrate the superiority of an incentive contract over a fixed price contract from the perspective of a client who wants to maximize his expected discounted profit in serial stochastic projects. We also showed how a client can calculate optimal parameters for these incentive contracts. Our analysis revealed several other significant implications as well. For example, we showed that the two incentive payment contracts (contracts $\mathbb{I}$ and $\mathbb{I}_{\mathbb{D}}$) are equivalent with respect to expected profit for the client and subcontractors, as well as the expected makespan (we showed that this result also holds for the fixed price contracts $\mathbb{F}$ and $\mathbb{F}_{\mathbb{D}}$). However, we showed that there are significant differences between the incentive payment contracts (contracts $\mathbb{I}$ and $\mathbb{I}_{\mathbb{D}}$) and the fixed payment contracts (contracts $\mathbb{F}$ and $\mathbb{F}_{\mathbb{D}}$). The client will always have a greater expected profit with an incentive payment contract although subcontractors will have a greater expected profit with a fixed price contract. We also showed that the expected makespan is always less with an incentive type contract than with a fixed price contract. Unfortunately, the client is not always able to utilize an incentive payment contract; if subcontractors have large opportunity costs, then only contract $\mathbb{F}$ (with appropriate adjustments) can be utilized; if subcontractors have small opportunity costs, then contract $\mathbb{I}$ can be used, but the choice of parameters (*i.e.*, b_i) is restricted. We also showed how an incentive payment contract can be applied in practice, by deriving a piece-wise linear approximation. In its simplest form, this approximation allows for a deadline, a penalty rate for late completion, and a reward rate for early completion.

In chapter three, we propose an Exponential Incentive Contract (EIC) with payment function $p_i - e^{s_i t_i}$,that coordinates a decentralized project with *n* subcontractors under continuous discounting. We show that this is the optimal contract for the client as the client obtains the maximum expected discounted profit. The contribution of this chapter is twofold. The first is that we have provided a framework to analyze a stochastic decentralized project for arbitrary *n* subcontractors; whereas in the project management literature, researches that study contract coordination under discounting are limited by two subcontractors. The second is that we have provided a link from the theoretical EIC to the commonly used contracts in practice. In this chapter, we have also provided analysis to the case where discounting is minimal. We show that the first order Taylor series approximation of EIC, LIC is also a coordinating contract; Moreover, LIC processes the stochastic dominance property that ensures the adoption of $\hat{r}_i^* = \sqrt{\frac{H + C_i}{k_i}}$ for any risk-averse utility function. Another advantage of LIC is its simple coordination solution, setting the linear penalty as the client's overhead cost $s_i = H$.

In chapter four, we show that the cost plus contract actually creates more uncertainties as it led subcontractors to have non-unique optimal work rates. The performance of the cost plus contractor falls in between that of a fixed price contract and an incentive contract regarding the level of project coordination, client's and subcontractors' expected profits. Moreover, in a dynamic setting where subcontractor work rates can be adjusted without incurring significant additional costs, the cost plus contract optimal work rate will converge to a fixed price contract optimal work rate. Whereas, in a dynamic setting, the incentive contract's and fixed price contract's remains stable.

There are a number of important extensions that should be considered in future work. First, we are continuing to analyze various contracts in decentralized projects, including the case when the project can be characterized by a general network topologies. Preliminary numerical results suggest that many of the results reported in this dissertation continue to hold in this case; We observe that the performance of an incentive contract is superior than that of a cost plus contract and the performance of a cost plus contract is superior than that of a fixed price contract, regarding the level of project coordination, client's and subcontractors' expected profits. Second, incentive contracts frequently include quality and/or scope incentives as well as budget and schedule incentives. While these incentives are not part of this work, we are currently investigating how quality and/or scope goals can be included in an incentive payment contract and resultant I/D contract.

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