Essays on Growth, Human Capital, and Income Distribution

by

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A dissertation submitted in partial fulfillment

of the requirements for the degree of

Doctor of Philosophy

University of Washington

1996

Approved by [signature]
(Chairperson of Supervisory Committee)

Program Authorized
to Offer Degree Economics

Date 5/30/96

UMI Number: 9637911

University of Washington

Abstract

Essays on Growth, Human Capital, and Income Distribution

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The first two chapters present a pair of endogenous growth models with both physical and human capital and a public education input. These papers contribute to the literature on endogenous growth by analyzing a nonrepresentative agent model. That is, agents are permitted to differ in their initial endowment of both types of capital. Using this framework I study the dynamics of the income and wealth distribution and optimal fiscal policy. Existing models of growth and income distribution that are modeled using only human capital typically find a counterfactual income convergence. This pair of models illustrates how convergence is sensitive to the extent of factor markets and to the existence of nonhuman wealth.

In regard to fiscal policy, I find that the optimal income tax rate is constant over time and the same for all agents regardless of initial endowments. However, if the tax rates on capital and effective labor income can differ, individuals would be made better off with a low (high) tax on the factor in which they have a relatively large (small) endowment.

The third chapter expands the set of policy instruments to include a consumption tax and government debt. It also parameterizes the extent to which the productivity of public

educational expenditures depends on the individual's effective learning. The second contribution of this paper is to determine the dynamic path of private debt between individuals in the economy when the degree of congestion in learning varies by region.

Finally, empirical studies have shown that credit constraints are prevalent in many developing economies. The existence of imperfect credit markets implies that some agents who would otherwise invest in human capital will remain uneducated due to the inability to borrow or self-finance out of wealth. My fourth chapter investigates the consequence of moving from autarky to free trade in an economy with credit constraints. In particular, I look at how trade impacts wealth, income, and investment in human capital.

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ACKNOWLEDGMENTS

I wish to express my sincere appreciation to Professor Stephen J. Turnovsky, chairperson of the advisory committee, for encouraging me to pursue this topic and for guiding me to the completion of this thesis. Without his help this work would not have been possible. I am indebted also to Professor Richard Hartman and Professor Charles Engel for their many invaluable comments and suggestions during all stages of this dissertation.

Special thanks go to my colleague, Sailesh Jha, for helping me to clarify and develop these topics and for recommending several useful journal references. I also wish to thank the participants of the Macro/International Brown Bag Seminars for their many insights.

Finally, my sincerest gratitude beiongs to my friends, colleagues, and students for their unwavering support and encouragement. Among those, special appreciation owes to Jill Gill, Sweta Saxena, Katie Allen, and Mike Hendrickson.

Introduction

The basic purpose of this dissertation is to study the accumulation of human capital and its impact on growth or development of an economy in a framework with heterogeneous agents. Although preferences are assumed to be similar across individuals, this work recognizes that people in the economy differ along important dimensions, notably their inherited wealth or initial stocks of capital. These differences may alter their behavior or their choices for government policy. Given a framework of heterogeneous agents, this dissertation also seeks to study the relationship between growth or development and the distribution of income and wealth. This relationship has received considerable attention in the literature dating back to Kaldor (1956) and Kuznets (1966), for instance. However, new theoretical models allow us to reexamine this issue in a fresh and rigorous manner.

The first three models consist of infinite horizon intertemporal optimizing models where the economy is characterized by ongoing growth. The neoclassical growth models of the 1950s and 1960s such as the models of Solow (1956) and Swan (1956) suffered from the drawback that the economy reaches a steady state in which only such factors as population growth and exogenous technological change are able to generate growth. The new endogenous growth literature instead investigates endogenous sources of growth. That is, growth rates respond to preference and technology parameters but also to government policy variables such as fiscal policy, commercial policy, R&D subsidies and so forth. The first three chapters fit in with this recent literature. In particular, they are closely related to the work of Lucas (1988) in which human capital accumulation is the engine of growth. They are also similar to the Barro (1990) article that specifies a productive use for government spending in addition to accounting for the distortionary effects of taxation. But whereas in the Barro article, the government provides physical capital infrastructure, in these papers the government contributes to human capital accumulation through educational services.

The main contribution to this literature is to investigate growth in the context of non-representative agent models. I analyze the interaction between growth and the distribution of income. Three main issues of concern are 1) income convergence, 2) the relationship between optimal fiscal policy and growth maximization, and 3) whether there is a conflict between agents on the timing or level of the preferred tax rate and if so what is the political resolution.

For the first issue, Chapters 1 and 2 will discuss the conditions under which inequality shrinks or persists. Essentially, if factor markets are not complete, income and wealth converge. This is the result typically found in the literature, for example by Tamura (1991). On the other hand, when factor markets are complete, the distribution of income and wealth do not converge. Inequality persists despite diminishing returns to private human capital. Furthermore, when there is variance in abilities, those with greater learning ability initially devote a greater amount of time to learning and consequently they have permanently greater human capital and wage earnings. So this result contradicts the results of these other articles that find income convergence and is consistent with the US data on the postwar income distribution which shows that inequality has been fairly static overall.

Second, I am able to provide explicit closed-form solutions to the transitional dynamics of the models in Chapters 1-3. Other two sector growth models such as Mulligan and Sala-i-Martin (1993) required numerical simulations to study the transition. The advantage of explicit solutions is that they facilitate the comparison between growth versus welfare maximization. Chapters 1 and 2 study the optimal income tax while Chapter 3 broadens the policy variables to include a consumption tax and government debt.

In regard to the third issue, Chapters 1 and 2 look at whether the agents concur about the tax rate that maximizes each of their indirect utility functions. Chapter 1 will show that when there is a proportional tax on all income, agents agree on the rate that should be set. In contrast, in Chapter 2 the tax on wage versus capital income differs. In this case, the tax rates preferred by an agent depends on the initial endowment of each capital stock and on the ability to learn. Nevertheless, it is shown that the median voter theorem can be used to derive the political equilibrium.

The final chapter is an adaptation of the two period overlapping generation model. It focuses on investment in skills in an economy with imperfect credit markets. The empirical literature has documented the existence of credit constraints, particularly for developing countries. The consequence is that in the absence of government intervention, inherited wealth becomes critical to the educational decision. Chapter 4 thus presents a model of an economy with skilled and unskilled labor. The acquisition of skills requires payment of tuition and is therefore partially dependent on initial wealth. The dynamics of wealth for a specific lineage and for the aggregate are analyzed. A major concern of this chapter is the impact that trade has on the fraction of unskilled workers in each generation. The chapter discusses the channels through which an increase in the world relative price of the low-tech good will act upon the decision to invest in education.

Chapter 1

Income Distribution, Transitional Dynamics, and Optimal Fiscal Policy in a Growing Economy with Human and Physical Capital

1.1 Introduction

Empirical evidence has shown that while there is a tendency for similar growth rates across the industrialized countries, rates of growth in GNP across all countries are diverse over sustained periods. Furthermore, the industrialized countries do not seem to be trending to a steady state level of capital as predicted by the old growth models. This evidence has generated a lot of discussion lately about the determinants of ongoing economic growth. Several new models have been suggested. Romer (1986) concentrates on the possibility of increasing returns resulting from physical capital spillovers. Rebelo (1991) describes a class of models that have constant return to scale production functions. He shows that if there is a core of capital goods whose production does not depend on nonreproducible factors, constant returns to scale production functions are compatible with endogenous growth. Some models analyze the effects of government policy. It is often concluded that distortionary taxation reduces growth. An exception is Barro (1990) who constructs a productive role for government in providing physical infrastructure. He finds that there is a positive tax rate that balances the productive use of the tax revenue with the distortionary impact on private incentives for capital accumulation in order to achieve maximum growth. Furthermore, this tax rate that maximizes growth also maximizes welfare. Lucas (1988) suggests that human capital accumulation can be an engine for growth. This idea has particular appeal as we enter an age dominated by growth in services and information-based industries. This paper concurs with Lucas on the importance of human capital accumulation, but also recognizes the widespread phenomenon of public education and other government services such as research grants that assist in the accumulation and spread of knowledge.

As in Barro, this paper investigates the issue of optimal taxation when taxes are distortionary but the revenue is used productively. Yet, unlike in Barro, the tax rate that maximizes growth does not maximize welfare. In addition, while Barro and the above papers are confined to a representative agent model, this paper allows the initial endowments of physical and human capital to vary across agents. Thus the model provides an explicit treatment for the income/ wealth distribution. Furthermore, in contrast to the Mulligan and Sala-i-Martin (1993) article that analyzes the transitional dynamics of two sector growth models using numerical methods, this paper presents a closed-form solution for the transitional dynamics as well as the balanced growth path. The solution facilitates determination of optimal fiscal policy and also provides a transparent explanation for why growth maximization and welfare maximization fail to be equivalent policies.

The remainder of this chapter is organized as follows. Section 1.2 solves the optimization problem faced by the individual agent. Section 1.3 traces out the dynamics for the aggregate capital stocks and the income/ wealth distribution while Section 1.4 does so for the individual capital stocks. Based on these results, an agent's indirect utility function is constructed in Section 1.5 and used to determine optimal fiscal policy. Section 6 compares this optimal policy to the one that maximizes long-run growth. Section 1.7 discusses some alternative production technologies. Finally, Section 1.8 concludes.

1.2 The Individual Agent's Problem

There is a continuum of agents indexed i $\in$ [0,1]. Each agent has access to a similar constant returns to scale private production function for the production of physical output, y_{it}^k :

$$y_{it}^k = \theta\, k_{it}^a \left(u_{it} h_{it}\right)^{1-a} \qquad 0<a<1 \qquad (1.1a)$$

where k_{it} is physical capital owned by agent i, h_{it} is her human capital, and u_{it} is the proportion of her unitary time endowment devoted to the production of physical goods or capital. As in Lucas (1988) and Rebelo (1991) human capital increases effective labor supply and a higher effective labor supply raises the productivity of physical capital.

The agent can accumulate human capital by spending time in the learning sector and benefits from education services provided by the government, G.

$$y_{it}^h = \eta \left[\left(1-u_{it}\right)h_{it}\right]^{1-b} G_t^b \qquad 0<b<1 \qquad (1.1b)$$

For analytical tractability I assume full depreciation in each sector each period. Output in the physical sector can be either consumed by individual i at time t, c_{it}, or saved for next period's stock of physical capital, whereas output in the learning sector contributes fully to the agent's stock of human capital in period t+1. The government collects a proportional tax, τ, only on the production of physical output. The idea here is that learning is a personal non-market activity which is not subject to taxation. Thus human capital is taxed only as it contributes to the production of physical output.

$$k_{i,t+1} = (1-\tau_t)y_{it}^k - c_{it} \tag{1.2}$$

$$h_{i,t+1} = y_{it}^h \tag{1.3}$$

Each agent acts to maximize discounted logarithmic utility $\Omega_i = \sum_{t=0}^{\infty} R^t \ln c_{it}$ over an infinite horizon subject to (1.2) and (1.3) where 0<R<1 is the identical discount factor across agents. Substituting for consumption this leads to the Lagrangian:

$$L_i = \sum_{t=0}^{\infty} R^t \left\{ \ln\left[(1-\tau_t)\theta\, k_{it}^a (u_{it}h_{it})^{1-a} - k_{i,t+1}\right] + \lambda_{it}\left[\eta\left((1-u_{it})h_{it}\right)^{1-b} G_t^b - h_{i,t+1}\right]\right\} \tag{1.4}$$

The standard optimality conditions are:

$$\lambda_{it} c_{it} \frac{(1-b)y_{it}^h}{(1-u_{it})h_{it}} = \frac{(1-\tau_t)(1-a)y_{it}^k}{u_{it}h_{it}} \tag{1.5}$$

$$\frac{c_{i,t+1}}{c_{it}} = \frac{(1-\tau_{t+1})R\, a\, y_{i,t+1}^k}{k_{i,t+1}} \tag{1.6}$$

$$\lambda_{it} = \frac{R(1-a)(1-\tau_{t+1})y_{i,t+1}^k}{c_{i,t+1}h_{i,t+1}} + \lambda_{i,t+1}\frac{R(1-b)y_{i,t+1}^h}{h_{i,t+1}} \tag{1.7}$$

Equation (1.5) describes the optimal allocation of human capital between production of market goods and investment in next period's human capital. Equation (1.6) is the standard Keynes-Ramsey rule equating the marginal rate of substitution between consumption at times t and t+1 to the after-tax rate of return to private physical capital. Equation (1.7) relates the shadow price of human capital to its marginal rate of return.

Finally, the following transversality conditions must be imposed in order to ensure the agent's intertemporal budget constraint is met:

$$\lim_{t\to\infty} \frac{R^t\, k_{i,t+1}}{c_{it}} = 0; \quad \lim_{t\to\infty} R^t\, \lambda_{it}\, h_{i,t+1} = 0 \tag{1.8}$$

Combining (1.5) and (1.7) eliminates λ_{it} and $\lambda_{i,t+1}$. Then using (1.3), cancelling some terms and rearranging leads to:

$$\frac{c_{i,t+1}}{c_{it}} = \frac{R(1-b)u_{it}\left(1-\tau_{t+1}\right)y^k_{i,t+1}}{u_{i,t+1}\left(1-u_{it}\right)\left(1-\tau_t\right)y^k_{it}} \tag{1.9}$$

A solution is a sequence $\{c_{it}\}_{t=0}^{\infty}$, $\{u_{it}\}_{t=0}^{\infty}$, $\{k_{it}\}_{t=0}^{\infty}$, $\{h_{it}\}_{t=0}^{\infty}$, $\{\lambda_{it}\}_{t=0}^{\infty}$ which satisfies equations (1.2), (1.3), and (1.5) - (1.8) or equivalently (1.2), (1.3), (1.6), (1.8) and (1.9). The unique solution is derived formally in the appendix. Heuristically, we can hypothesize that the solution will take the form of a constant proportional after-tax savings rate, s: $c_{it} = (1-s)\left(1-\tau_t\right)y^k_{it}$. Using this form and optimality condition (1.6):

$$\frac{c_{i,t+1}}{c_{it}} = \frac{(1-\tau_{t+1})y^k_{i,t+1}}{(1-\tau_t)y^k_{it}} = \frac{(1-\tau_{t+1})Ra\, y^k_{i,t+1}}{k_{i,t+1}} \tag{1.10}$$

Equation (1.10) implies $k_{i,t+1} = (1-\tau_t)Ra\, y^k_{it}$ while the budget constraint implies $k_{i,t+1} = s(1-\tau_t)\, y^k_{it}$. Equating constants yields the marginal propensity to consume out of physical production $(1-s) = (1-Ra)$. Using the hypothesized form and optimality condition (1.9) produces the constant proportion of time devoted to physical production, $u_{it} = u_{i,t+1} = \bar{u}$.

$$\frac{c_{i,t+1}}{c_{it}} = \frac{(1-\tau_{t+1})y^k_{i,t+1}}{(1-\tau_t)y^k_{it}} = \frac{R(1-b)u_{it}(1-\tau_{t+1})\, y^k_{i,t+1}}{u_{i,t+1}(1-u_{it})(1-\tau_t)y^k_{it}} \quad \Rightarrow \quad \bar{u} = 1 - R(1-b)$$

Finally, we can verify the transversality conditions (1.8) are met.

Thus the solution to the individual agent's optimization problem consists of the following:

$$\bar{u} = 1 - R(1-b) \tag{1.11}$$

$$c_{it} = (1-\tau_t)(1-Ra)\theta\, k^a_{it}(\bar{u}h_{it})^{1-a} \tag{1.12}$$

$$k_{i,t+1} = (1-\tau_t)Ra\theta\, k^a_{it}(\bar{u}h_{it})^{1-a} \tag{1.13}$$

$$h_{i,t+1} = \eta\left[(1-\bar{u})h_{it}\right]^{1-b} G^b_t \tag{1.14}$$

With logarithmic utility, the savings rate and the proportion of time in physical production are constant.[1] The explanation of constant s and u consists of offsetting wealth and substitution effects. When physical capital is low relative to human capital, it can be raised either by saving more (higher s), or by spending more time producing physical output rather than learning (higher u). Since agents like to smooth consumption, they dislike higher savings. This is the consumption smoothing or wealth effect. However, when k/h is low, the return to applying one's human capital to physical output is small compared with using it for additional learning. So agents want smaller u when k/h is low. This is the substitution effect. When these two effects cancel, u and s are constant. Also notice that $\partial\bar{u}/\partial R = -(1-b)$. The larger is R (the more patient is the agent), the more time is spent learning and less time spent in physical production. This is particularly true the less spillover occurs. Greater spillovers induce less time spent learning. Agents are able to quickly capture the knowledge from basic research and thus choose to spend more of their time producing final goods.

An interesting aspect of the solution to this model is that it does not require an assumption of balanced growth. Although u is constant, with logarithmic utility the physical and human capital stocks can be growing at arbitrary relative rates. In contrast, if utility was nonlogarithmic-CES, a constant u could be obtained only on a balanced growth path. However, the balanced growth condition would impose an initial ratio of human to physical capital. Furthermore, in this model since the private capital stocks would need to grow at the same rate as basic research, the magnitudes of initial private capital stocks would be fixed relative to the aggregate capital stocks. But confining the initial factors to specific values would defeat the intent of allowing heterogeneity.

[1] Mulligan and Sala-i-Martin (1993) provide a result for the more general constant elasticity of

The solution and equation (1.6) show that the gross growth rates of consumption (lagged one period) and physical capital are equal: $\frac{c_{it}}{c_{i,t-1}} = \frac{k_{i,t+1}}{k_{it}}$. The physical capital growth rate is diminishing in k_{it} but increasing in h_{it} while the human capital growth rate, $\frac{h_{i,t+1}}{h_{it}}$ is diminishing in h_{it}. Substituting (1.13) and (1.14) into (1.6) we can find the initial growth rate of consumption:

$$\frac{c_{i1}}{c_{i0}} = (1-\tau_1)(1-\tau_0)^{a-1}(Ra\theta)^a \bar{u}^{(1-a)a} \eta^{(1-a)} (1-\bar{u})^{(1-a)(1-b)} G_0^{(1-a)b} k_{i0}^{-a(1-a)} h_{i0}^{(a-b)(1-a)} \tag{1.15}$$

Agents with higher physical capital will have lower initial consumption growth. Agents with higher human capital will have lower initial consumption growth if 1-*a*>1-*b*, that is, if the output sector is the human capital intensive sector.

1.3 Initial Distribution and Aggregate Dynamics

The previous section described the solution to an individual's optimization problem. Given initial stocks of human and physical capital, equations (1.11) - (1.14) show that each agent's choices of consumption and capital accumulation depend on the tax rate and level of government educational outlays. In order to determine the pattern of taxation and government spending that maximizes welfare, I construct the indirect utility function for each individual. However, since each period's government spending level requires the extraction of a portion of aggregate output, and aggregate output each period in turn results

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substitution utility function rather than the logarithmic utility function assumed here and by Tamura.

from the tax distorted accumulation of both capital stocks, it is first necessary to solve for the aggregate dynamics. In this model, the aggregate dynamics rely on the assumption about the distribution of factors.

Many empirical studies of the distribution of wage and salary earnings, and wealth conclude that each can be approximated by a lognormal distribution. However, there is some evidence of leptokurtosis. In particular, the upper tail sometimes contains more frequency than expected from the lognormal and a Pareto distribution provides a closer fit for this region only. Lydall (1968), Aitchison and Brown (1969), and Thatcher (1976) discuss evidence on the distribution of wage and salary earnings while Sargan (1957) and Atkinson (1975) study data on the distribution of wealth. Extrapolating from the complexity of the upper tail, I will assume that the initial distribution of factor endowments across agents is jointly lognormally distributed. This is consistent with lognormally distributed wealth and earnings from human capital. This assumption has the additional advantage of adding dispersion and skewness to this model (whose solutions depend on Cobb Douglas production functions) in a manner which allows tractable dynamics.

Let $$\begin{bmatrix} \ln k_{i0} \\ \ln h_{i0} \end{bmatrix} \sim N\left[\begin{pmatrix} \mu_{k0} \\ \mu_{h0} \end{pmatrix}, \begin{pmatrix} \sigma^2_{k,0} & \sigma_{kh,0} \\ \sigma_{kh,0} & \sigma^2_{h,0} \end{pmatrix}\right] \tag{1.16}$$

where $\mu_{k0} = E(\ln k_{io})$, $\mu_{h0} = E(\ln h_{io})$, $\sigma^2_{k0} = Var(\ln k_{i0})$, $\sigma^2_{h0} = Var(\ln h_{i0})$, and $\sigma_{kh,0} = Cov(\ln k_{i0}, \ln h_{i0})$.

The government maintains a balanced budget constraint. Since the number of individuals in the economy sum to unit mass, total physical output is equal to average physical output. Therefore,

$$G_t = E\left[\tau_t\ \theta\, k_{it}^{a}\left(uh_{it}\right)^{1-a}\right] = \tau_t\ \theta\, u^{1-a} E\left[e^{a\ln k_{it}+(1-a)\ln h_{it}}\right]$$

Using the property that $E\left[e^{x}\right] = e^{E[x]+1/2Var[x]}$ and taking logs:

$$\ln G_t = \ln \tau_t \theta u^{1-a} + a\mu_{kt} + (1-a)\mu_{ht} + \frac{a^2}{2}\sigma_{k,t}^2 + \frac{(1-a)^2}{2}\sigma_{h,t}^2 + a(1-a)\sigma_{kh,t} \tag{1.17}$$

Taking logs of (1.13) and (1.14) and using (1.17), the evolution of the log capital stocks follow:

$$\ln k_{i,t+1} = \ln(1-\tau_t)Ra\theta u^{1-a} + a\ln k_{it} + (1-a)\ln h_{it} \tag{1.18a}$$

$$\begin{aligned}\ln h_{i,t+1} = \ln \tau_t^b \theta^b \eta u^{b(1-a)}(1-u)^{1-b} + (1-b)\ln h_{it} + ba\mu_{kt} + b(1-a)\mu_{ht} \\ + \frac{ba^2}{2}\sigma_{k,t}^2 + \frac{b(1-a)^2}{2}\sigma_{h,t}^2 + ba(1-a)\sigma_{kh,t}\end{aligned} \tag{1.18b}$$

The evolution of aggregate log capital stocks can be found by taking the expectation of equations (1.18a) and (1.18b). These can be represented in matrix notation for the period t = 1 in terms of the initial period t = 0 as follows:

$$\mu_1 = \mathbf{A}\mu_0 + \mathbf{V}\sigma_0 + \mathbf{T}_0 + \mathbf{D} \tag{1.19}$$

where $\mu_t = \begin{bmatrix} \mu_{kt} \\ \mu_{ht} \end{bmatrix}$, $\sigma_t = \begin{bmatrix} \sigma^2_{k,t} \\ \sigma^2_{h,t} \\ \sigma_{kh,t} \end{bmatrix}$, $\mathbf{A} = \begin{bmatrix} a & 1-a \\ ab & 1-ab \end{bmatrix}$,

$$\mathbf{V} = \begin{bmatrix} 0 & 0 & 0 \\ \frac{a^2 b}{2} & \frac{(1-a)^2 b}{2} & a(1-a)b \end{bmatrix},$$

$$\mathbf{T_t} = \begin{bmatrix} \ln(1-\tau_t) \\ b \ln \tau_t \end{bmatrix}, \text{ and } \mathbf{D} = \begin{bmatrix} \ln Ra\theta u^{1-a} \\ \ln \eta \theta^b u^{b(1-a)}(1-u)^{1-b} \end{bmatrix}.$$

Likewise the evolution of the variances of log capital stocks can be derived by taking the variances and covariance of equations (1.18a) and (1.18b).

$$\sigma_1 = \mathbf{S}\sigma_0 \tag{1.20}$$

where $\mathbf{S} = \begin{bmatrix} a^2 & (1-a)^2 & 2a(1-a) \\ 0 & (1-b)^2 & 0 \\ 0 & (1-a)(1-b) & a(1-b) \end{bmatrix}$.

For any time period t, $\sigma_t = S^t \sigma_0$. (1.21)

The appendix shows the matrix $\mathbf{S}^t$. Since $\mathbf{S}^t \to \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ as $t \to \infty$, $\lim_{t\to\infty} \sigma_t = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

provided that $0<a<1$ and $0<(1-b)<1$.

Tamura (1991) finds a similar convergence result in a model with only human capital, h_t, but with human capital spillovers, H_t, across agents. However, convergence is not necessarily driven by an externality or transfer across agents such as Tamura's human capital spillover, H_t, or this model's public good, G_t. The same result could be obtained by replacing the spillover/public good with any factor which is constant across agents. Let $h_{i,t+1} = \eta\left[(1-u_{it})h_{it}\right]^{1-b} L^b$ where L is some fixed factor. Then the matrix S would be identical. Convergence is driven by the decreasing returns to private capital in the learning sector. This is similar to the neoclassical result of convergent economies. However, the use of H or G does allow ongoing growth since H and G grow over time. It is obvious from the matrix S^t that the speed of convergence is determined by the magnitude of a and $1-b$. The closer these production parameters are to one, the longer it takes for income to equalize. If human capital production was linear in own human capital, $b=0$, but $a<1$, then the variance of log human capital would not change over time and the variance of log physical capital would converge to that of log human capital. On the other hand, if $1-b<1$ but $a=1$, then human capital would eventually equalize but the variance of log physical capital would remain unchanged.[2] Transforming from the behavior of log capital (ln x) to the behavior of capital (x) using the properties of the lognormal distribution shown in the appendix, if $\mu = E(\ln x)$ is growing but $\sigma^2 = Var(\ln x)$ remains constant, then both E(x) and Var(x) are growing. However, they grow in a manner such that the standard deviation of capital relative to the mean remains constant.

This model has heretofore excluded shocks to the production functions and agent specific productivity differences. Amending the model in either of these directions would clearly give rise to permanent inequality. For example, if human capital accumulation

2 This is a trivial case since there would be no reason to accumulate human capital if labor is not used in

includes ability differences, $h_{i,t+1} = A_i\, \eta \left[(1-u_{it})h_{it}\right]^{1-b} G_t^b$, then the variance of log human and physical capital would converge to the variance of log A_i.

Finally, using recursive substitution in equation (1.19) we can find the pattern for the average log capital stocks through time:

$$\mu_t = \mathbf{A}^t \mu_0 + \left(\sum_{j=0}^{t-1} \mathbf{A}^{t-1-j} \mathbf{V}\mathbf{S}^j \right) \sigma_0 + \left(\sum_{j=0}^{t-1} \mathbf{A}^j \right) \mathbf{D} + \sum_{j=0}^{t-1} \mathbf{A}^{t-1-j}\, \mathbf{T}_j \qquad t>0 \qquad (1.22)$$

Using results shown in the appendix, it can be demonstrated that for large t:

$$\begin{aligned} \mu_t \approx {} & \tfrac{1}{1-a+ab} \begin{bmatrix} ab & 1-a \\ ab & 1-a \end{bmatrix} \mu_0 + \tfrac{b(1-a)}{2(1-a+ab)} \begin{bmatrix} a^2 & (1-a)^2 & 2a(1-a) \\ a^2 & (1-a)^2 & 2a(1-a) \end{bmatrix} (\mathbf{I}-\mathbf{S})^{-1} \sigma_0 \\ & + \left\{ \left(\tfrac{1}{1-a+ab}\right)^2 \begin{bmatrix} 1-a & a-1 \\ -ab & ab \end{bmatrix} + \tfrac{t}{1-a+ab} \begin{bmatrix} ab & 1-a \\ ab & 1-a \end{bmatrix} \right\} \mathbf{D} + \sum_{j=0}^{t-1} \mathbf{A}^{t-1-j}\, \mathbf{T}_j \end{aligned} \qquad (1.23)$$

From this formulation for μ_t we can observe two implications for long-run aggregate growth rates. First, for large t, the difference in log capital stocks $(\mu_{ht} - \mu_{kt})$ converge to a function of the parameters a, b, R, θ, η, and $\{\tau_j\}_{j=0}^{t-1}$ and since the variances of log capital stocks go to zero. $\mu_{kt} \approx \ln E(k_{it})$ and $\mu_{ht} \approx \ln E(h_{it})$. Then, $E(k_{it}) \approx \phi E(h_{it})$ where ϕ is a constant. In the long-run human and physical capital become proportional. They converge to a balanced growth path. Second, the long-run vector of aggregate log factor growth rates can be represented as

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production. This case would degenerate to Rebelo's (1991) 'AK' model.

$$\lim_{t\to\infty} \ln\begin{bmatrix} E(k_{i,t+1})/E(k_{it}) \\ E(h_{i,t+1})/E(h_{it}) \end{bmatrix} = \lim_{t\to\infty} \mu_{t+1} - \mu_t = \frac{1}{1-a+ab}\begin{bmatrix} ab & 1-a \\ ab & 1-a \end{bmatrix}\mathbf{D}$$
$$+\left(\sum_{j=0}^{t} \mathbf{A}^{t-j}\,\mathbf{T}_j - \sum_{j=0}^{t-1} \mathbf{A}^{t-1-j}\,\mathbf{T}_j \right) \qquad (1.24)$$

The long run capital growth rates do not depend on initial averages or variances of the capital stocks but only on a, b, R, θ, η, and $\{\tau_j\}_{j=0}^{t-1}$. However, the levels do depend on these initial conditions. Furthermore, in the long run $E(k_{it}) \approx e^{\mu_{kt}}$ and $E(h_{it}) \approx e^{\mu_{ht}}$ and the appendix re-expresses equation (1.23) in terms of the means and variances of initial capital. We can make the following generalization of the impact of initial conditions on the levels of aggregate capital stocks in the distant future. If the correlation between initial human and physical capital is zero or negative, or if the ratio of the mean to the standard deviation is approximately equal between the two capital stocks, then future means are increasing in initial means and decreasing in initial variances.

1.4 The Dynamics of an Individual's Factor Ownership

Using the solutions for the aggregate variables μ_t and σ_t , we can now obtain expressions for k_{it} and h_{it} in terms of k_{i0}, h_{i0}, the parameters of the initial distribution μ_0 and σ_0 , and the tax rates. The solutions for k_{it} and h_{it} will be used in the next section to construct the indirect utility function.

Factor stocks in period t=1 from equations (1.18a) and (1.18b) can be represented in matrix notation in terms of the initial period t=0.

$$\mathbf{F}_{i1} = \mathbf{B}\,\mathbf{F}_{i0} + \mathbf{M}\,\mu_0 + \mathbf{V}\,\sigma_0 + \mathbf{D} + \mathbf{T}_0 \tag{1.25}$$

where $\mathbf{F}_{\text{it}} = \begin{bmatrix} \ln k_{it} \\ \ln h_{it} \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} a & 1-a \\ 0 & 1-b \end{bmatrix}$, and $\mathbf{M} = \begin{bmatrix} 0 & 0 \\ ab & (1-a)b \end{bmatrix}$.

Using recursive substitution, equations (1.19), (1.20) and (1.25), and the fact that **B+M=A**:

$$\mathbf{F}_{it} = \mathbf{B}^t\mathbf{F}_{i0} + \left(\sum_{j=0}^{t-1}\mathbf{B}^{t-1-j}\mathbf{M}\mathbf{A}^j\right)\mu_0 + \left(\sum_{j=0}^{t-1}\mathbf{A}^{t-1-j}\mathbf{V}\mathbf{S}^j\right)\sigma_0 + \left(\sum_{j=0}^{t-1}\mathbf{A}^j\right)\mathbf{D} + \sum_{j=0}^{t-1}\mathbf{A}^{t-1-j}\,\mathbf{T}_j \tag{1.26}$$

Using the results in the appendix it can be shown that for large t, $\mathbf{F}_{\text{it}} \approx \mu_t$. If $0<a,b<1$ we have seen that the variance of each capital stock approaches zero. Then in the long run each agent's capital stock is nearly identical to the average capital stock. Therefore, the initial averages and variances as well as parameters a, b, R, θ, η, and $\{\tau_j\}_{j=0}^{t-1}$ affect each agent's long-run capital stock in the same manner as they affect the economy-wide averages.

1.5 Welfare Maximization and Tax Policy

We can use the results from the previous section to finally construct the indirect utility function and examine the optimal tax policy for each individual.

$$\text{Let } \Omega_i\left(\tau_t;a,b,R,\theta,\eta\right) = \max_{\tau_t} \sum_{t=0}^{\infty} R^t \ln c_{it} \tag{1.27}$$

But substituting for the individual's optimal consumption at each point in time from (1.12), noting the fact that $\ln y_{it}^k = \ln \theta \bar{u}^{1-a} + [a \quad 1-a]\mathbf{F}_{it}$, and using (1.26), some tedious algebra reveals that:

$$\sum_{t=0}^{\infty} R^t \ln c_{it} = \sum_{t=0}^{\infty} R^t \ln\left(1-\tau_t\right) + [a \quad 1-a]\left(\sum_{j=0}^{\infty} R^j \mathbf{A}^j\right) \sum_{t=0}^{\infty} R^{t+1}\, \mathbf{T}_t + Q \tag{1.28}$$

where Q denotes all terms not involving τ_t. Maximization with respect to τ_t yields the welfare maximizing tax rate:

$$\tau_t^* = \tau^* = \frac{Rb(1-a)}{1-R(1-b)} = \frac{Rb(1-a)}{\bar{u}} \tag{1.29}$$

Thus the tax rate which maximizes welfare is the same for all agents and is constant over time. A priori, we may have expected individuals with low human capital or low total wealth to desire higher initial taxes and educational outlays compared to wealthier individuals. Yet, high initial taxes discourage everyone from accumulating physical capital. This leads to lower future wealth from which to extract future government revenue. Consequently, all agents weigh this distortion against current government services and agree on the optimal income tax. While this result is likely to depend on the assumption of logarithmic utility, it does provide a contrast to the optimal policy results in Alesina and Rodrick (1991), Persson and Tabellini (1994), Perotti (1993), and Saint-Paul and Verdier (1991). These papers find that the optimal tax rate is decreasing in an agent's positon in the income/wealth distribution. The reason is that these models use public funds collected from taxation at least in part for redistribution from the rich to the poor. However, Glomm and Ravikumar (1992) use tax revenue only for productive government spending. Consequently they also find a common tax rate preference.

From (1.29) we find the following. The more human capital contributes to physical production, the greater should be the tax rate: $\partial\tau^*/\partial(1-a) > 0$. The larger the spillover, the greater the tax: $\partial\tau^*/\partial b > 0$. We saw earlier that the larger the spillover, the greater is the tendency for agents to free ride on basic research. They spend more time in physical production. A higher tax rate removes more resources from physical production and channels them into basic research which then translate into higher human capital growth. Finally, the more patient is the agent, the higher the tax rate should be: $\partial\tau^*/\partial R > 0$. Impatient agents want to consume now. Therefore, they dislike high taxes which remove resources from final goods production even if it contributes to higher human capital growth.

1.6 The Growth Rate of an Individual's Consumption

Barro (1990) found that maximization of long-run growth and maximization of welfare were equivalent policies. In the previous section we saw that the optimal tax rate was constant over time and equal across individuals. Furthermore, Sections 1.3 and 1.4 found that there is a trend toward long-run balanced growth. This section investigates whether this welfare optimal tax rate is indeed equal to the tax rate that maximizes long-run growth. Taking logs of equation (1.6) we can find the gross rate of consumption growth (in logs).

$$\ln\left(\frac{c_{it}}{c_{i,t-1}}\right) = \ln(1-\tau)Ra\theta u^{1-a} + (1-a)\left(\ln h_{it} - \ln k_{it}\right) \tag{1.30}$$

But $\left(\ln h_{it} - \ln k_{it}\right)$ can be written as $[-1 \quad 1]\mathbf{F}_{it}$ and since $\mathbf{F}_{it} \approx \mu_t$ for large t, then

$$\begin{aligned}\ln\left(\frac{c_{it}}{c_{i,t-1}}\right) &= \ln(1-\tau)Ra\theta u^{1-a} + (1-a)\left(\mu_{ht} - \mu_{kt}\right)\\ &= \tfrac{1}{\text{1-a+ab}}\ln(1-\tau)^{ab}\tau^{(1-a)b}\theta^{b}\eta^{1-a}(Ra)^{ab}\bar{u}^{(1-a)b}(1-\bar{u})^{(1-a)(1-b)}\end{aligned} \tag{1.31}$$

Since the variances approach zero in the long run, aggregate and individual consumption growth rates are the same. This growth rate depends only on the parameters a, b, R, θ, η, and τ. The tax rate which maximizes long run consumption growth is $\tau=1-a$. This tax rate also maximizes long run factor growth rates as expected since there is balanced growth in the long run. However, it is different from the welfare maximizing tax rate.

Intuitively, the difference between growth maximization and welfare maximization is analogous to the difference between the golden rule and modified golden rule capital stocks of the old growth models. In the neoclassical growth models, one could obtain greater short-run consumption by reducing the capital stock from the golden rule level and sacrificing some long-run consumption. With impatient agents, it was therefore suboptimal to try to achieve the maximum steady state consumption. Likewise, in this endogenous growth model with transitional dynamics, initial consumption and short-run consumption growth can be increased by reducing the tax rate from $\tau=1\text{-}a$ and sacrificing some long-run growth. Equation (1.12) with t=0 shows that initial consumption is decreasing in the income tax. Substituting for G_0 in equation (1.15) leads to the initial growth rate of consumption $\frac{c_{i1}}{c_{i0}} = (1-\tau)^a \tau^{b(1-a)} f(a,b,R,\theta,\eta,k_{i0},h_{i0},\mu_0,\sigma_0)$. A reduction in the tax rate from τ=(1-a) would increase this initial growth rate and a similar result obtains for all future growth rates. Barro's (1990) one sector model finds growth and welfare maximization to be equivalent policies because there are no transitional dynamics. The economy in effect jumps from one steady state growth rate to another when the tax rate is changed.

Barro's one sector model of endogenous growth has the production function $y = Ak^{1-\alpha}g^{\alpha}$ where g is per capita public services. He finds that the growth maximizing tax rate is $\tau=\alpha$, the exponent on government services. The growth maximizing tax rate here is the exponent on human capital in the final goods production function. Based on Barro's result, we may intuitively expect the growth maximizing tax rate to equal $b(1\text{-}a)$: the share of government services in learning multiplied by the share of human capital in production of output. However, if we examine the optimal tax rate as people become increasingly patient, $\lim_{R\to 1} \tau^* = \frac{b(1-a)}{b}$, we see that the share of government services in learning has two offsetting effects. In the numerator we find Barro's effect. However, the denominator shows that the larger the government induced spillover the more time is spent in production. A larger tax base from increased production allows for a smaller tax rate to collect the same revenue for basic research.

1.7 Alternative Endogenous Growth Production Technologies

This model is very adaptable to changes in the assumptions of the production technology along the lines of other endogenous growth models. The general solution forms given by equations (1.21), (1.22), and (1.26) are robust to such changes although the elements of the matrices **A**, **B**, **V**, **S**, **M**, and **D** may vary. For example, the human capital spillover model of Tamura and others would replace G_t with $H_t = E(h_{it}) = E\left(e^{\ln h_{it}}\right) = \exp\left\{\mu_{ht} + \frac{1}{2}\sigma_{ht}^2\right\}$. Then equations (1.18a) and (1.18b) would change to:

$$\ln k_{i,t+1} = \ln Ra\theta u^{1-a} + a\ln k_{it} + (1-a)\ln h_{it} \tag{1.18a'}$$

$$\begin{aligned}\ln h_{i,t+1} &= \ln \eta(1-u)^{1-b} + (1-b)\ln h_{it} + b\ln H_t \\ &= \ln \eta(1-u)^{1-b} + (1-b)\ln h_{it} + b\mu_{ht} + \frac{b}{2}\sigma_{h,t}^2\end{aligned} \tag{1.18b'}$$

Given these changes the matrices **A**, **V**, **M**, and **D** would become

$$\mathbf{A} = \begin{bmatrix} a & 1-a \\ 0 & 1 \end{bmatrix}, \mathbf{V} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{b}{2} & 0 \end{bmatrix}, \mathbf{M} = \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix}, \text{ and } \mathbf{D} = \begin{bmatrix} \ln Ra\theta u^{1-a} \\ \ln \eta(1-u)^{1-b} \end{bmatrix}$$

but **B** and **S** would remain the same as in the public good model.

The model can also accomodate the use of own physical capital in the learning sector. This alteration would result in a constant proportion of physical capital devoted to each sector in addition to the constants u and s. The solution method and generation of dynamics would proceed as above. However, the matrix **S** would become much more complicated requiring the use of a cubic formula to find the eigenvalues.

It is also trivial to add physical capital externalities (or human capital externalities) to the physical output production function as in Romer. These extensions do not affect the general solutions for the dynamics (1.21), (1.22), and (1.26) but merely complicate the matrices **A**, **B**, **V**, **S**, **M**, and **D**.

1.8 Conclusions

This paper has derived explicit paths for individual and aggregate consumption and capital stocks. Since agents may have any arbitrary endowments of physical and human capital, their capital stocks will likely be growing at different initial rates. However, in the long-run there is a trend toward balanced growth. The dynamics for the distribution of capital ownership were also presented. There will be convergence in the distribution only if the human capital production function exhibits decreasing returns to private capital, human capital is essential to output production, and there are no production shocks or individual specific fixed factors. If these conditions hold, the speed of convergence is increasing in the contribution of human capital to output production and the contribution of government education services to learning, or $(1-a)$ and b respectively.

This paper also finds that the growth maximizing tax rate is somewhat analogous to Barro's result. However, in contrast to the one-sector model of Barro, the share of government in production (of output in Barro's model and learning in this model) has offsetting effects. Furthermore, this tax rate is not welfare maximizing. The tax rate which maximizes welfare is smaller than that which maximizes consumption growth except for the case when agents are completely patient (if R=1). This result is analogous to the difference between the golden rule and modified golden rule capital stocks of the old growth models. In the present context, it is optimal to sacrifice some long-run growth in order to obtain higher short-run growth. A second surprising result is that there is no conflict among agents over the amount or timing of the income tax. The optimal income tax rate is constant over time and across individuals. This finding contrasts with the results of others, notably Alesina and Rodrick (1991), Persson and Tabellini (1994), Perotti (1993), and Saint-Paul and Verdier (1991), who find that the optimal tax rate is decreasing in an agent's positon in the income/wealth distribution.

Chapter 2

Growth, Welfare, and Persistent Inequality in a Model with Differential Taxation of Human and Physical Capital

2.1 Introduction

Economists have long been interested in the relationship between growth and the distribution of income. Kaldor (1956) emphasizes the causal effect of income distribution on economic growth and capital accumulation. On the other hand, the opposite causal link, from growth to income distribution is most notably embodied in the Kuznets curve: the hypothesis that income inequality first increases and then decreases with development.

The recent advancement of new growth theory has spawned a literature that, like this paper, examines the linkages between endogenous growth and income distribution. For example, Perotti (1993), and Persson and Tabellini (1994) model growing economies where the agents differ in the size of their income endowment. Human capital accumulation is the source of growth and some form of inheritance is the mechanism that generates dynamics in income distribution. In fact, human capital accumulation is also the engine of growth in papers by Galor and Zeira (1993), Glomm and Ravikumar (1992), Saint-Paul and Verdier (1991), and Tamura (1991). Galor and Zeira model credit market imperfections and indivisibilities in human capital accumulation. Mean income and inequality determine the number of people with income above the threshold required for investment in human capital. Glomm and Ravikumar, and Saint-Paul and Verdier examine the public versus private education decision. Tamura as well as Glomm and Ravikumar examine the convergence properties of the economy and obtain the result that income inequality, which derives from heterogeneity of human capital, declines over time.

Table 2.1 presents empirical evidence on the postwar income distribution in the U.S. Although there are some demographic trends and measurement issues underlying the data, the overall picture emerging from the table is the constancy of the distribution over time. The counterfactual income convergence result found by Tamura (1991) and Glomm and Ravikumar (1992) occurs due to decreasing returns to private human capital in models with human capital alone. The assumption of decreasing returns to private human capital is preserved in this paper. However, agents also own physical capital. I show that when the returns to each factor in the production of output are market determined and individuals differ in their ability to learn, inequalities persist through time in a manner consistent with the evidence of Table 2.1.

This paper assumes an endogenous growth structure similar to the Lucas (1988) learning or doing model. Moreover, as in Barro (1990), there is a government sector that imposes distortionary taxation but uses the revenue for productive purposes. In this paper, the public input is for educational services that enhance human capital accumulation rather than physical infrastructure as in Barro.

Given a role for the public sector, I also analyze optimal taxation. I do not find growth and welfare maximization to be equivalent policies. Furthermore, Barro and many others explore government policy questions in terms of a representative agent model so that it is not necessary to worry about the weights assigned to each individual in a social welfare function. By assuming an economy of identical individuals, the weights are irrelevant since each individual wants the same policy. But this approach merely assumes away potential political conflicts. In contrast, I model an economy of heterogeneous agents and investigate the circumstances under which political conflicts do or do not emerge. The aforementioned papers about growth and income distribution also investigate political conflict over policy. Yet these papers contain only one factor of production and therefore cannot address questions about optimal taxation by income source. Alesina and Rodrick (1991) do investigate a model with both physical capital and unskilled labor. Their model is one of wealth heterogeneity. However, labor is supplied inelastically in their model and there is no tax on labor.

The remainder of this chapter is organized as follows. Section 2.2 presents the individual's optimization problem. The individual and aggregate equilibria are derived in Section 2.3. In Section 2.4 the indirect utility function is constructed and used to evaluate optimal fiscal policy while Section 2.5 analyzes growth maximization. The political equilibrium and its implications for growth is discussed in Section 2.6. Finally, Section 2.7 concludes.

2.2 The Agent's Problem

There is a continuum of agents indexed i $\in$ [0,1]. Each agent owns physical capital, k_{it}, and human capital, h_{it}, and devotes the proportion, u_{it}, of her unitary time endowment to the production of physical goods or capital. Agents rent out their factors in aggregate factor markets for which they receive a market wage or interest payment. Income earned from physical production can be either consumed by individual i at time t, c_{it}, or saved for next period's stock of physical capital. The government imposes the tax rate τ_r on interest earnings and the tax rate τ_w on wage earnings. Thus the agent's budget constraint is given by (2.1):

$$k_{i,t+1} = (1-\tau_r)r_t k_{it} + (1-\tau_w)w_t u_{it} h_{it} - c_{it} \tag{2.1}$$

The agent can accumulate human capital by spending time in the learning sector and benefits from a nonrival public good of education technology or basic research, G.

$$h_{i,t+1} = \eta\, I_i^b \left[(1-u_{it})h_{it}\right]^{1-b} G_t^b \tag{2.2}$$

Furthermore, there are individual specific differences in the ability to attain human capital. For later convenience, an agent's specific factor is I_i^b. One can think of these factors as differences in intelligence or any other personal attribute that may contribute to an agent's ability to acquire human capital. For analytical tractability I assume full depreciation in each sector each period.

Factor payments are determined by firms operating in complete factor markets. Firms maximize profits $\Pi_t = Y_t - w_t U_t H_t - r_t K_t$ where $Y_t = \theta K_t^a (U_t H_t)^{1-a}$. The production function has the interpretation that human capital increases effective labor supply and a higher effective labor supply raises the productivity of physical capital. The return on physical (human) capital is equal to the marginal product of physical (human) capital. Variables lacking subscript i denote aggregate, or equivalently, average quantities.

$$r_t = \partial Y_t \big/ \partial K_t = a\theta\left(\frac{U_t H_t}{K_t}\right)^{1-a} = \frac{aY_t}{K_t} \tag{2.3a}$$

$$w_t = \partial Y_t \big/ \partial U_t H_t = (1-a)\theta\left(\frac{K_t}{U_t H_t}\right)^{a} = \frac{(1-a)Y_t}{U_t H_t} \tag{2.3b}$$

Each agent acts to maximize discounted logarithmic utility $\Omega_i = \sum_{t=0}^{\infty} R^t \ln c_{it}$ over an infinite horizon subject to (2.1) and (2.2) where 0<R<1 is the identical discount factor across agents.

Substituting for consumption leads to the Lagrangian:

$$L_i = \sum_{t=0}^{\infty} R^t \left\{ \ln\left[(1-\tau_r)r_t k_{it} + (1-\tau_w)w_t u_{it} h_{it} - k_{i,t+1}\right] + \lambda_{it}\left[\eta I_i^b \left((1-u_{it})h_{it}\right)^{1-b} G_t^b - h_{i,t+1}\right]\right\} \tag{2.4}$$

The standard optimality conditions are:

$$\lambda_{it} = \frac{(1-\tau_w)w_t(1-u_{it})h_{it}}{c_{it}(1-b)y_{it}^h} \tag{2.5}$$

$$\frac{c_{i,t+1}}{c_{it}} = (1-\tau_r)Rr_{t+1} \tag{2.6}$$

$$\lambda_{it} = \frac{R(1-\tau_w)w_{t+1}u_{i,t+1}}{c_{i,t+1}} + \lambda_{i,t+1}\frac{R(1-b)y_{i,t+1}^h}{h_{i,t+1}} \tag{2.7}$$

Equation (2.5) describes the optimal allocation of human capital between production of market goods and investment in next period's human capital. Equation (2.6) is the standard Keynes-Ramsey rule equating the marginal rate of substitution between consumption at times t and t+1 to the after-tax rate of return to private physical capital.

Equation (2.7) relates the shadow price of human capital to its marginal rate of return. The return to physical and human capital in the physical goods sector is given by r and w respectively.

Finally, the following transversality conditions must be imposed in order to ensure the agent's intertemporal budget constraint is met:

$$\lim_{t\to\infty} \frac{R^t\, k_{i,t+1}}{c_{it}} = 0; \quad \lim_{t\to\infty} R^t\, \lambda_{it}\, h_{i,t+1} = 0 \tag{2.8}$$

A solution is a sequence $\{c_{it}\}_{t=0}^{\infty}$, $\{u_{it}\}_{t=0}^{\infty}$, $\{k_{it}\}_{t=0}^{\infty}$, $\{h_{it}\}_{t=0}^{\infty}$, $\{\lambda_{it}\}_{t=0}^{\infty}$ which satisfies equations (2.1), (2.2), and (2.5) -(2.8). Combining (2.5) and (2.7) leads to:

$$\frac{c_{i,t+1}}{c_{it}} = \frac{R(1-b)w_{t+1}y_{it}^{h}}{(1-u_{it})w_t h_{it}} \tag{2.9}$$

Eliminating the growth of consumption from equations (2.6) and (2.9) and using (2.2):

$$\frac{(i-u_{it})h_{it}}{I_i} = \left[\frac{w_{t+1}(1-b)}{w_t(1-\tau_r)r_{t+1}}\right]^{1/b} G_t \tag{2.10}$$

Since the right hand side is constant for all agents and normalizing the average ability $I_i = 1$, we can compare an individual's learning time to the average.

$$\frac{(1-u_{it})h_{it}}{I_i} = (1-U_t)H_t \tag{2.11}$$

Equation (2.11) shows that an agent will spend more initial time learning in order to make up for a deficit in acquired human capital. However, if the agent is less intelligent than average, she will spend less initial time learning. At time t=1, substituting for $(1-u_{i0})h_{i0}$, an agent will accumulate

$$h_{i1} = \eta I_i \left[(1-U_0)H_0\right]^{1-b} G_0^b = I_i H_1 \tag{2.12}$$

and combining (2.11) and (2.12) leads to

$$(1-u_{i1})h_{i1} = I_i(1-U_1)H_1 \qquad \text{or} \tag{2.13a}$$

$$(1-u_{i1}) = (1-U_1) \tag{2.13b}$$

Equations (2.11) - (2.13) imply that after the initial period, learning time is equalized for all agents.

$$1-u_{it} = 1-U_t \qquad \forall i,\ t>0 \tag{2.14}$$

Therefore, after the initial time an agent's labor income is a proportion, I_i, of average labor earnings.

$$w_t u_{it} h_{it} = I_i(w_t U_t H_t) \qquad t>0 \tag{2.15}$$

This result assumes that the bound $0<u<1$ is not violated. The bound will hold provided that the agent's human capital relative to her ability is not too low compared to average human capital, or

$$\frac{h_{it}}{I_i} \geq \left(1-U_t H_t\right).$$

2.3 Individual and Aggregate Equilibrium

When human capital is distributed in a manner such that a non-negligible fraction of agents have very low human capital, agents in the lower tail of the distribution will be at a corner solution. They will spend their entire initial time allocation learning and yet will still be unable to catch up to the more educated agents. The proportion of agents at a corner solution affects aggregate production, wages and rental rates and therefore individual behavior as well.

However, while the model is intractable when many agents are at a corner solution, it is easily solved for a condense population. I define a condense population as one in which the human capital of the least knowledgeable agents is large enough relative to average human capital to allow for an interior optimum. That is, I will hereafter assume that the number of agents with critically low human capital is negligible.

Given this assumption, (2.10) holds for all agents and therefore $\left(1-u_{i0}\right)h_{i0} = I_i\left(1-U_0\right)H_0 \;\forall i$. Aggregating (2.1), (2.2), (2.6), and (2.9) by integrating over individuals and using (2.3) we can derive a set of equations describing the aggregate equilibrium.

$$\frac{C_{t+1}}{C_t} = (1-\tau_r)R\, r_{t+1} = \frac{(1-\tau_r)R\, aY_{t+1}}{K_{t+1}} \tag{2.16}$$

$$\frac{C_{t+1}}{C_t} = \frac{R(1-b)w_{t+1}H_{t+1}}{(1-U_t)w_t H_t} = \frac{R(1-b)U_t\, Y_{t+1}}{(1-U_t)U_{t+1}\, Y_t} \tag{2.17}$$

$$K_{t+1} = (1-\tau_r)r_t K_t + (1-\tau_w)w_t U_t H_t - C_t = (1-a\tau_r-(1-a)\tau_w)Y_t - C_t \tag{2.18}$$

$$H_{t+1} = \eta\left[(1-U_t)H_t\right]^{1-b} G_t^b \tag{2.19}$$

The government maintains a balanced budget. Thus:

$$G_t = \tau_r\, r_t K_t + \tau_w\, w_t U_t H_t = (a\tau_r + (1-a)\tau_w)\, Y_t \tag{2.20}$$

I hypothesize that aggregate consumption is proportional to aggregate production.[1] Using the hypothesis along with (2.16) and (2.17) respectively leads to:

$$K_{t+1} = (1-\tau_r)RaY_t = (1-\tau_r)R\, r_t K_t \tag{2.21}$$

$$1-U_t = R(1-b) \tag{2.22}$$

Finally, combining (2.18) and (2.21):

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[1] See the appendix for a formal discussion.

$$C_t = \left[1 - Ra - (1-R)a\tau_r - (1-a)\tau_w\right] Y_t \tag{2.23}$$

Thus aggregate production is divided into aggregate consumption, investment, and government revenue in proportions which are constant over time. Aggregate learning time is a constant over time. Since this is true for the initial period as well, we can solve for an individual's initial learning time.

$$\left(1 - u_{i0}\right) = \frac{I_i R(1-b)H_0}{h_{i0}} \tag{2.24}$$

The individual's initial consumption can be derived using (2.1), (2.6), and (2.8) in the following manner. Use recursive substitution in equation (2.6) to get the expression:

$$c_{it} = \left(1-\tau_r\right)^t R^t \left(\prod_{s=1}^{t} r_s\right) c_{i0} \tag{2.25}$$

Likewise, use recursive substitution in budget constraint (2.1) along with (2.25) to express $k_{i,t+1}$ in terms of k_{i0}, c_{i0}, and a stream of wage earnings. Then substitute these expressions for $k_{i,t+1}$ and c_{it} into the first transversality condition (2.8). Further rearrangement and cancellation yield a result for initial consumption.

$$c_{i0} = (1-R)\left\{\left(1-\tau_r\right)r_0 k_{i0} + \left(1-\tau_w\right)\left(\prod_{s=1}^{\infty} r_s^{-1}\right)\sum_{t=0}^{\infty}\left(1-\tau_r\right)^{-t}\left(\prod_{s=t+1}^{\infty} r_s\right) w_t u_{it} h_{it}\right\} \tag{2.26}$$

Initial consumption is equal to a fraction, 1-R, of initial income plus future wage earnings, or in other words, each agent consumes a fraction of permanent income.

Recall that although agents have different amounts of human capital initially, the uneducated study longer initially and everyone ends up with the same human capital from t=1 onwards adjusting for the ability factor. Therefore, wage earnings differ at time t=0 but are proportionate for everyone thereafter: $w_t u_{it} h_{it} = I_i w_t \overline{U} H_t \quad t > 0$.

From (2.3) and (2.23) we see that total wage earnings and aggregate consumption, respectively, are proportional to output. Therefore these three aggregate quantities grow at the same rate and using (2.16):

$$w_t u_{it} h_{it} = I_i \left(1 - \tau_r\right)^t R^t \left(\prod_{s=1}^{t} r_s \right) w_0 \overline{U} H_0 \tag{2.27}$$

Substituting this into (2.26) for t>0 and substituting in for u_{i0} from (2.24) we find:

$$c_{i0} = (1-R)\left[\left(1-\tau_r\right) r_0 k_{i0} + \left(1-\tau_w\right) w_0 h_{i0}\right] + Rb\left(1-\tau_w\right) I_i w_0 H_0 \tag{2.28}$$

$$k_{i1} = R\left(1-\tau_r\right) r_0 k_{i0} + R\left(1-\tau_w\right) w_0 \left(h_{i0} - I_i H_0\right) \tag{2.29}$$

Next period's physical capital ownership is a fraction, R, of the difference between the agent's initial after-tax total income and a proportion, I_i, of the economy average initial after-tax wage earnings. Agents with low initial human capital or high ability must borrow only if they also own very little initial physical capital. Thus the credit market requirements in this economy depend on the variance of ability and initial human capital, the relative magnitudes of rental vs. wage earnings, and the correlation between the initial ownership of factors. In contrast to the Tamura (1991) consumption loan market model which requires uneducated agents to borrow from the more educated ones in order to equalize their human capital while at the same time maintain a smooth consumption flow, in this model it is possible that the uneducated agents can self-finance. Credit markets are not required a priori.

Finally, we can analyze the dynamics of inequality. Equation (2.15) shows that the variance of labor earnings is related to the variance of abilities.

$$Var\left(w_t u_{it} h_{it}\right) = \left(w_t U H_t\right)^2 Var\left(I_i\right) \tag{2.30}$$

Using equations (2.25), (2.27) - (2.29), and the budget constraint (2.1) we can show that an individual's capital, consumption, and labor earnings grow at a common rate for t>0:

$$\frac{k_{i,t+2}}{k_{i,t+1}} = \frac{c_{i,t+1}}{c_{it}} = \frac{w_{t+1} u_{i,t+1} h_{i,t+1}}{w_t u_{it} h_{it}} = \left(1-\tau_r\right) R r_{t+1} \tag{2.31}$$

Then from (2.31), the wealth and labor income distributions evolve according to (2.32) and (2.33):

$$Var\left(w_{t+1} u_{i,t+1} h_{i,t+1}\right) = \left[\left(1-\tau_r\right) R r_{t+1}\right]^2 Var\left(w_t u_{it} h_{it}\right) \tag{2.32}$$

$$Var\left(k_{i,t+1}\right) = \left[\left(1-\tau_r\right) R r_t\right]^2 Var\left(k_{it}\right) \tag{2.33}$$

Physical capital wealth and labor income remain permanently unequal as long as the long-run balanced growth rate is positive, that is, as long as $\lim_{t\to\infty}(1-\tau_r)R\, r_t > 1$. A positive long-run balanced growth rate will occur if the productivity parameters are large enough, as can be seen in Section 2.5.

If we restrict this model such that there are no ability differences, (2.11) implies that human capital and thus wage earnings would equalize after one period. This case would be similar to Tamura (1991). However, from (2.29) the variance of physical capital at time t=1 would equal zero only in the special case that initial physical and human capital were perfectly negatively correlated and $(1-\tau_r)^2 r_0^2\, Var(k_{i0}) = (1-\tau_w)^2 w_0^2\, Var(h_{i0})$. On the other hand, the larger is the correlation between the initial capital stocks, the larger is $Var(k_{i1})$. So in general total income would not exhibit immediate convergence. Furthermore, (2.33) would continue to hold so that wealth and total income inclusive of capital earnings would remain unequal.

2.4 Welfare Maximization and Tax Policy

This section solves for the indirect utility function and evaluates the optimal taxes on each factor income source. Taking logs of (2.25) and substituting into the objective function we have:

$$\Omega_i = \frac{R\ln\left(1-\tau_r\right)R}{(1-R)^2} + \sum_{t=0}^{\infty} R^t \sum_{s=1}^{t} \ln r_s + \frac{\ln c_{i0}}{1-R} \tag{2.34}$$

Initial consumption was derived in the previous section (2.28). The rental rate on capital at each time period remains to be determined. Taking logs of (2.19) and (2.21) and substituting for G_t from (2.20), we can describe the evolution of the aggregate capital stocks (in logs):

$$\mathbf{N}_1 = \mathbf{A}\mathbf{N}_0 + \mathbf{D} \qquad \text{where} \qquad \mathbf{N}_t = \begin{bmatrix} \ln K_t \\ \ln H_t \end{bmatrix},$$

$$\mathbf{D} = \begin{bmatrix} \ln\left(1-\tau_r\right)Ra\theta u^{1-a} \\ \ln\left(a\tau_r + (1-a)\tau_w\right)^b \eta\theta^b u^{b(1-a)}(1-u)^{1-b} \end{bmatrix} \text{ and } \mathbf{A} = \begin{bmatrix} a & 1-a \\ ab & 1-ab \end{bmatrix}.$$

Recursive substitution leads to:

$$\mathbf{N}_t = \mathbf{A}^t\mathbf{N}_0 + \left(\sum_{j=0}^{t-1}\mathbf{A}^{\mathbf{j}}\right)\mathbf{D} \tag{2.35}$$

Using the fact that $\ln r_t = \ln a\theta u^{1-a} + (1-a)[-1 \quad 1]\mathbf{N}_t$ along with results from the appendix, some additional algebra reveals the indirect utility function:

$$\Omega_i = \frac{Ra(1-R(1-b))}{(1-R)^2(1-Ra(1-b))}\ln\left(1-\tau_r\right) + \frac{R(1-a)}{(1-R)^2(1-Ra(1-b))}\ln\left(a\tau_r + (1-a)\tau_w\right)^b + \left(\tfrac{1}{1-R}\right)\ln c_{i0} + Q \tag{2.36}$$

where Q denotes terms not involving the tax rates and c_{i0} is given by (2.28).

As a useful reference point, let us examine the optimal policy when the tax rates on each income source are constrained to be equal. Setting $\tau_r = \tau_w$ and maximizing (2.36) results in the optimal tax rate: $\tau^* = \dfrac{b(1-a)R}{1-R(1-b)}$. Consequently, the relative magnitudes of an agent's initial factors are unimportant when there is no tax rate differentiation. In addition, the more human capital contributes to physical production and the public input contributes to learning, the greater the optimal tax rate: $\dfrac{\partial\tau^*}{\partial(1-a)} > 0$ and $\dfrac{\partial\tau^*}{\partial b} > 0$. Furthermore, the more patient is the agent, the higher the tax should be: $\dfrac{\partial\tau^*}{\partial R} > 0$.

In contrast let us now consider optimal tax policy when the two tax rates are not confined to be equal. In this case the resultant optimal tax policy does depend on the agent's initial factors. The optimal tax rates can be expressed as functions of $\omega_i = \dfrac{(1-R)w_0 h_{i0} + RbI_i w_0 H_0}{(1-R)r_0 k_{i0}}$. Through comparative static analysis we find that $\partial\tau_r^*/\partial\omega_i > 0;\ \ \partial\tau_w^*/\partial\omega_i < 0$. Differential factor taxation has a redistributive effect. Therefore agents who are initially physical capital rich relative to their human capital ownership and ability prefer greater wage tax (smaller interest tax) than agents who are initially human capital rich relative to their physical capital.

2.5 Growth Maximization and Tax Policy

Having evaluated the optimal tax policy from the viewpoint of each individual, I now analyze how tax policy impacts long-run growth. The long-run percentage growth rate of individual and aggregate consumption can be expressed as $\lim_{t\to\infty} \ln\left(\dfrac{c_{it}}{c_{i,t-1}}\right) = \lim_{t\to\infty} \ln\left(\dfrac{C_t}{C_{t-1}}\right) = \lim_{t\to\infty}\left[\ln R + \ln(1-\tau_r) + \ln r_t\right]$ using results (2.6) and (2.16). Substituting for $\ln r_t$ as in the previous section leads to:

$$\lim_{t\to\infty} \ln\left(\frac{C_t}{C_{t-1}}\right) = \frac{1}{1\text{-a+ab}} \ln\left(1-\tau_r\right)^{ab}\left[a\tau_r + (1-a)\tau_w\right]^{(1-a)b} (Ra)^{ab} \theta^b \eta^{(1-a)} \bar{u}^{(1-a)b} (1-\bar{u})^{(1-a)(1-b)} \tag{2.37}$$

Maximizing this long-run growth rate results in a corner solution. The optimal policy is to completely tax labor earnings but keep earnings on capital untaxed (i.e. $\tau_w = 1$ and $\tau_r = 0$).[2] The intuition is as follows. From (2.21) we see that the proportion of current aggregate production which becomes next period's physical capital stock is negatively related to the tax on capital, but the tax on labor is irrelevant. Furthermore the aggregate amount of time spent in production (and in learning) is unaffected by either tax rate. On the other hand, tax revenue collected from either source is used for basic research which contributes to growth of human capital. In summary, both taxes have a positive effect on growth but only the tax on capital has a countervailing negative effect. Confining the tax rates to be equal we find that the growth maximizing tax rate is $\tau = 1 - a$. Furthermore, $\lim_{t\to\infty} \ln\left(\frac{C_t}{C_{t-1}}\right) = \lim_{t\to\infty} \mathrm{N}_{t+1} - \mathrm{N}_t$. So the long-run growth of aggregate consumption equals that of aggregate capital stocks.

These growth maximizing tax rates do not match the welfare maximizing rates of the previous section for two reasons. First, maximizing long-run growth ignores the fact that future consumption is discounted. Second, the two tax rates impact an individual's initial level of consumption differently depending on the relative magnitudes of initial factors. Taxing labor but not capital may maximize consumption growth but at the expense

2 This result depends on the lack of a labor-leisure decision. If agents value leisure, a 100% tax on labor would result in zero production or human capital accumulation.

of those who start out physical capital poor but human capital rich. Thus changing taxes to maximize growth may lead to a deterioration of welfare for some agents.[3]

2.6 The Political Equilibrium and Growth

Welfare analysis produced the result that the optimal tax on capital (labor) is increasing (decreasing) in ω_i . For an agent with an initial ratio ω_i , the tax rate preferences τ_r^*, τ_w^* are single-peaked. Eliminating ω_i in the expressions for the optimal taxes, we can derive a locus of optimal policies.

$$\tau_r^* = \frac{Rb(1-a)}{a\bar{u}} - \left(\frac{1-a}{a}\right)\tau_w^* \tag{2.38}$$

Each point on this line in τ_r^*, τ_w^* space corresponds to a different ω_i. Every individual in the economy will desire a policy package (τ_r^*, τ_w^*) somewhere on this line. See Figure 2.1. If the political equilibrium is determined by majority voting, then despite the fact that there are two policy issues to be decided, the median voter theorem can be applied. All policy packages that lie off of the line (2.38) can be Pareto improved upon by at least one policy package that lies on the line. Therefore, voters with the median ω_i will chose the optimal tax package.

Since it can be shown that the growth rate $\gamma\left(\tau_r^*(\omega_i), \tau_w^*(\omega_i)\right)$ is decreasing in ω_i, skewness in the distribution of abilities and initial factor endowments will impact growth.

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[3] This analysis has been concerned with differential taxation by earnings source not size. Obviously a progressive income tax structure would be prefered by those with low total earnings due to the redistributive nature of such a structure.

The higher is the median ω_i, the lower the growth rate will be. Thus, it can be emphasized that growth depends on skewness in factor sources rather than size since an agent with high I_i, h_{i0}, and k_{i0} may have the same ω_i as an agent with low I_i, h_{i0}, and k_{i0} .

2.7 Conclusions

This paper shows that even if there are decreasing returns to private reproducible factors in the learning sector of standard endogenous growth models, wealth and total income will remain permanently unequal when factor markets are complete. If there are ability differences across agents, wage income will also be permanently unequal since more able individuals will devote a greater amount of initial time to learning. This persistence of inequality is consistent with U.S. data on the income distribution.

The paper also analyzes the growth and welfare implications of differential tax rates on interest and labor income. I find that growth would be maximized by completely taxing wage earnings but keeping interest earnings tax-free. However, welfare maximization differs from growth maximization. Agents who are human capital rich (relative to their physical capital) prefer larger interest tax and smaller wage tax rates compared to agents who are physical capital rich (relative to their human capital). If the political equilibrium is determined by majority voting, the median voter will set the tax rates because preferences are single-peaked and the pair of tax issues can be converted to a single dimension. Consequently, growth is decreasing in the ratio of the human to physical capital endowment of the median voter.

An extension to this paper would be to incorporate leisure into the utility function. Growth would not be monotonically increasing in the tax on wage income since labor supply would be affected. This could produce a potentially complicated relationship between growth and the median value of ω_i.

Table 2.1

Share of Aggregate Income Received by Each Fifth and Top 5% of Families

	Percentage Share						
Year	Low Fifth	Second Fifth	Middle Fifth	Fourth Fifth	Highest Fifth	Top 5%	Gini
1992	4.4	10.5	16.5	24.0	44.6	17.6	0.403
1987	4.6	10.8	16.8	24.0	43.8	17.2	0.393
1982	4.8	11.2	17.1	24.2	42.7	15.9	0.380
1977	5.3	11.6	17.5	24.2	41.4	15.7	0.363
1972	5.5	11.9	17.5	23.9	41.4	15.9	0.359
1967	5.4	12.2	17.5	23.5	41.4	16.4	0.358
1962	5.0	12.1	17.6	24.0	41.3	15.7	0.362
1957	5.1	12.7	18.1	23.8	40.4	15.6	0.351
1952	4.9	12.3	17.4	23.4	41.9	17.4	0.368
1947	5.0	11.9	17.0	23.1	43.0	17.5	0.376

Source: Current Population Reports, Series P-60, Table B-7, and Blinder (1980)

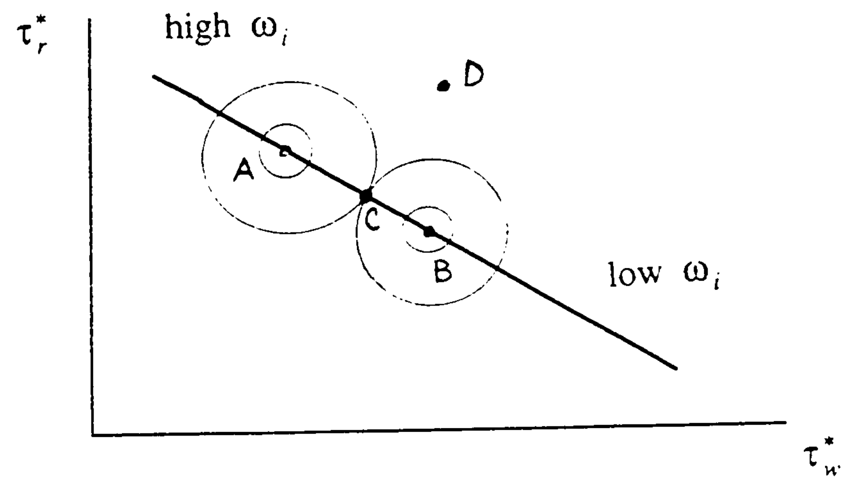

Figure 2.1

The Political Equilibrium and Policy Packages

Chapter 3

Optimal Fiscal Policy and Regional Debt in the presence of Human Capital Externalities

3.1 Introduction

There has been a lot of recent research geared toward explaining cross-country differences in growth rates. Empirical evidence suggests that human capital accumulation is a major factor contributing to these growth differentials. Consequently, a number of articles investigate human capital as the engine for growth, notably Lucas (1988). In addition, the role of the government in providing public services has been explored by some authors. Turnovsky (1995) and Turnovsky (1996) analyze expenditures on infrastructure. Other papers have explored the role that government provided educational services play in promoting human capital accumulation. For example, Glomm and Ravikumar (1992) study the private versus public education decision. This paper also investigates optimal government fiscal policy in a framework where government spending contributes to learning. The model is an extension of the endogenous growth models pioneered by Barro (1990) , Lucas (1988), and Rebelo (1991).

The policy instruments chosen for consideration include an income tax, a consumption tax, and government debt. In most countries the income tax is the major source of government revenue. However, there is frequent debate about the potential merits of switching in part to a consumption tax. While the income tax distorts the savings decision, the consumption tax introduces a distortion into the labor-leisure choice. However, while this labor-leisure distortion is fairly well known, the impact of the consumption tax on the allocation of time between production and the accumulation of human capital has received less attention. Furthermore, the impact of debt on economic growth and welfare has also generated ample research interest in the literature lately, due in part to the large deficits and debt experienced by the U.S. in recent history.

These fiscal issues are addressed in the framework of an endogenous growth model with human capital. It is shown that as in Turnovsky (1996) government debt and consumption taxation are redundant policies if there are no externalities in the human capital sector. However, if human capital externalities are introduced, the first best optimum no longer is attainable with the three policy instruments of income tax, consumption tax, and government debt. Instead the second best policies are examined. These optimal fiscal policies are also compared to those that maximize long-run growth.

Finally, this paper considers the pattern of inter-agent debt that would emerge if two regions were linked by government expenditures and perfect capital mobility but had different technologies for learning. That is, one region is subject to externalities but the other region is not.

The rest of this chapter proceeds as follows. Section 3.2 derives the equilibrium in a centrally planned economy. The purpose is to provide a benchmark against which the decentralized equilibrium can be compared. Section 3.3 considers the decentralized equilibrium with no externalities. We show how the available policy instruments can be used by the government to replicate the first best optimum. Optimal fiscal policy is then compared to the policy that maximizes long-run growth in Section 3.4. We then turn to a decentralized economy characterized by externalities in learning in Section 3.5. Section 3.6 analyzes the consequences of differing degrees of learning congestion faced by agents in two separate regions of an economy. The concluding section provides an overview of our results.

3.2 The Analytical Framework and First Best Optimum

We consider an economy populated by identical representative agents who consume a single consumption good c, deriving intertemporal utility represented by the logarithmic utility function

$$W = \sum_{t=0}^{\infty} R^t \ln c_t \qquad (3.1)$$

where 0<R<1 is the discount factor.

Output of the physical good is produced by a constant returns to scale production function using physical capital and human capital

$$y_t^k = k_t^a \left(u_t h_t\right)^{1-a} \qquad (3.2)$$

where k denotes the stock of physical capital, h denotes human capital, and u denotes the fraction of the agent's unitary time endowment devoted to physical production.

For convenience the size of the population is normalized to be one. Furthermore, there are no adjustment costs in converting output to the capital good and capital is assumed to depreciate fully each period. Since the government extracts G amount of physical output from the economy, the economy-wide resource constraint is:

$$k_{t+1} = k_t^a (u_t h_t)^{1-a} - c_t - G_t \tag{3.3}$$

The representative agent accumulates human capital by spending the fraction of time *1-u* learning. Learning is augmented by prior human capital at a decreasing rate and also is a positive function of the amount of government educational services provided to the agent.

$$y_t^h = [(1-u_t)h_t]^{1-b} G_t^b \tag{3.4}$$

$$h_{t+1} = y_t^h \tag{3.5}$$

As a benchmark, we consider the case where the government acts as a central planner and chooses *c*, *u*, *G*, *k*, and *h* directly to maximize the representative agent's utility. Substituting for consumption leads to the Lagrangian:

$$L = \sum_{t=0}^{\infty} R^t \left\{ \ln\left[k_t^a (u_t h_t)^{1-a} - G_t - k_{t+1}\right] + \lambda_t \left[\left[(1-u_t)h_t\right]^{1-b} G_t^b - h_{t+1}\right]\right\} \tag{3.6}$$

After eliminating the multiplier, the optimality conditions become:

$$\frac{c_{t+1}}{c_t} = \frac{Ra\, y_{t+1}^k}{k_{t+1}} \tag{3.7}$$

$$\frac{c_{t+1}}{c_t} = \frac{R(1-b)u_t\, y_{t+1}^k}{(1-u_t)u_{t+1}\, y_t^k} \tag{3.8}$$

$$G_t = \frac{(1-u_t)b(1-a)}{u_t(1-b)} y_t^k \tag{3.9}$$

Equation (3.7) is the Keynes-Ramsey rule equating the marginal rate of substitution between consumption at times t and t+1 to the return on physical capital. Equation (3.8) is derived from the optimal allocation of human capital to each sector. Equation (3.9) describes the optimal amount of government expenditures trading off the resource drain such expenditures constitute versus their productive benefit to education.

Finally, in order to ensure that the economy's intertemporal budget constraint is met, the following transversality conditions must be met:

$$\lim_{t \to \infty} R^t \frac{k_{t+1}}{c_t} = 0 \tag{3.10a}$$

and $$\lim_{t \to \infty} R^t \lambda_t h_{t+1} = 0 \tag{3.10b}$$

The solution to this problem with logarithmic utility consists of constant fractions of output being devoted to government expenditures (3.11), consumption (3.12), and investment (3.13), respectively, and a constant fraction of time allocated to production (3.14). Human capital accumulates in accordance with (3.15).

$$G_t = \xi y_t^k \quad \text{where } \xi = \left[\frac{Rb(1-a)}{1-R(1-b)}\right] \tag{3.11}$$

$$c_t = (1 - Ra - \xi)y_t^k \tag{3.12}$$

$$k_{t+1} = Ra\, y_t^k \tag{3.13}$$

$$u_t = \bar{u} = 1 - R(1-b) \tag{3.14}$$

$$h_{t+1} = \left[(1-\bar{u})h_t\right]^{1-b} G_t^b \tag{3.15}$$

3.3 The Decentralized Equilibrium with No Externalities

We now focus on the behavior of a representative agent operating in a decentralized economy. The objective of the agent is to maximize the utility function (3.1) subject to the budget constraint (3.16) and the accumulation equation (3.5) for human capital.

$$k_{t+1} + B_{t+1} - B_t = (1-\tau)k_t^a (u_t h_t)^{1-a} + (1-\tau)i_t B_t - (1+\omega)c_t \tag{3.16}$$

The agent produces output using physical and human capital. The individual also earns interest on prior ownership of government bonds. These resources are used for consumption, investment in physical capital, the accumulation of bonds, and the payment of taxes. We assume there is a proportional tax on output τ and a tax on consumption ω.[1] The agent must choose consumption, the amount of time allocated to production, investment in each capital stock, and investment in bonds each period. Substituting for consumption in (3.1) leads to the Lagrangian for the agent's problem:

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[1] Prior analysis indicates that the first and second best fiscal policies do not vary through time. Thus without loss of generality, this analysis assumes constant tax rates.

$$L=\sum_{t=0}^{\infty}R^{t}\left\{\begin{array}{l}\ln\left[\frac{(1-\tau)}{(1+\omega)}k_t^{a}(u_t h_t)^{1-a}+\frac{(1+(1-\tau)i_t)}{(1+\omega)}B_t-\frac{1}{(1+\omega)}k_{t+1}-\frac{1}{(1+\omega)}B_{t+1}\right]\\+\lambda_t\left[\left[(1-u_t)h_t\right]^{1-b}(G_t)^{b}-h_{t+1}\right]\end{array}\right\}$$

(3.17)

The optimality conditions for the agent's problem after eliminating the multiplier consist of (3.18) - (3.20), the budget constraints (3.16) and (3.5), and transversality conditions (3.21).

$$\frac{c_{t+1}}{c_t}=\frac{(1-\tau)Ra\,y_{t+1}^{k}}{k_{t+1}} \tag{3.18}$$

$$\frac{c_{t+1}}{c_t}=\frac{R(1-b)u_t\,y_{t+1}^{k}}{(1-u_t)u_{t+1}\,y_t^{k}} \tag{3.19}$$

$$\frac{c_{t+1}}{c_t}=R(1+(1-\tau)i_{t+1}) \tag{3.20}$$

$$\lim_{t\to\infty}R^{t}\frac{k_{t+1}}{(1+\omega)c_t}=0 \tag{3.21a}$$

$$\lim_{t\to\infty}R^{t}\,\lambda_t h_{t+1}=0 \tag{3.21b}$$

and $$\lim_{t\to\infty}R^{t}\frac{B_{t+1}}{(1+\omega)c_t}=0 \tag{3.21c}$$

Equations (3.18) and (3.19) have similar interpretations as in the central planner's problem except that (3.18) is now a function of the income tax rate. Equation (3.20) ensures that the marginal rate of substitution is equal to the after-tax return on government bonds. Comparing (3.18) and (3.20) we can see that there is an arbitrage condition that requires the after-tax returns on bonds and capital to be equal (3.22).

$$\left(1+(1-\tau)i_{t+1}\right)=\frac{(1-\tau)a\,y^{k}_{t+1}}{k_{t+1}} \tag{3.22}$$

Consider a solution to this problem of the form that consumption is a constant proportion of output net of tax effects and the agent's ownership of government bonds is a constant proportion of her capital ownership.

$$c_t=\frac{(1-\tau)}{(1+\omega)}\phi\, y^{k}_{t} \tag{3.23}$$

$$\text{and}\quad B_t=\eta\, k_t \tag{3.24}$$

Then for consumption to take this form, equation (3.19) and the transversality conditions imply that the fraction of time devoted to production should be constant (3.21).[2]

$$u_t=\bar{u}=1-R(1-b) \tag{3.25}$$

2 In fact, if the allocation of time between sectors is constant, the transversality conditions require that the ratio of bonds to physical capital also should not vary.

Comparing (3.18) and (3.19), we consequently find that capital investment is proportional to output net of the income tax (3.26).

$$k_{t+1} = (1-\tau)Ra\, y_t^k \tag{3.26}$$

From (3.22) we can write:

$$\left(1+(1-\tau)i_t\right)B_t = (1-\tau)a\, y_t^k \frac{B_t}{k_t} \tag{3.27}$$

Using equations (3.24) and (3.27), the agent's budget constraint can be written as:

$$(1+\eta)k_{t+1} = (1-\tau)k_t^a\left(uh_t\right)^{1-a}(1+a\,\eta) - (1+\omega)c_t \tag{3.28}$$

Combining (3.28) with (3.26), we can determine how consumption relates to output and the fiscal policy variables.

$$c_t = \frac{(1-\tau)}{(1+\omega)}\left(1 - Ra + a(1-R)\eta\right) u^{1-a}k_t^a h_t^{1-a} \tag{3.29}$$

Finally, given a constant amount of time devoted to learning, human capital accumulates according to (3.30).

$$h_{t+1} = \left[(1-\bar{u})h_t\right]^{1-b} G_t^b \tag{3.30}$$

To close the model and analyze the optimal fiscal policy, we must consider the government's budget constraint. The amount of expenditures on educational services and the payment of interest on prior debt must equal the tax revenue collected from the income tax and consumption tax plus the issuance of new debt.

$$B_{t+1} = G_t + \left[1+(1-\tau)i_t\right]B_t - \tau\, y_t^k - \omega\, c_t \tag{3.31}$$

We can substitute for bonds, interest payments, and consumption from the agent's solution above. We can also write that the amount of government expenditures is a constant fraction of output. Thus the government budget constraint can be rewritten in terms of output.

$$\eta(1-\tau)Ra\, y_t^k = g\, y_t^k + \eta(1-\tau)a\, y_t^k - \tau\, y_t^k - \omega\frac{(1-\tau)}{(1+\omega)}\left(1 - Ra + a(1-R)\eta\right) y_t^k \tag{3.32}$$

We now address the key question, namely how can the government set fiscal policy in the decentralized economy so as to replicate the first best optimum. If the government acts as central planner to maximize welfare, it will compare the agent's solution to the decentralized optimization as summarized by equations (3.24) - (3.26), (3.29), and (3.30) with the solution to the command optimum as summarized by equations (3.11) - (3.15). It will determine the optimal share of government, the optimal income and consumption tax rates, and the optimal amount of debt relative to physical capital, or g^*, τ^*, ω^*, and η^*, respectively, so that the agent's choices given these policies will coincide with the choices the central planner would directly have made.

From (3.11) we can see that the government should set its expenditures proportional to output, and comparing (3.13) with (3.26) reveals that there should be no tax or subsidy on output.

$$g^* = \xi \text{ and } \tau^* = 0 \tag{3.33}$$

Comparing (3.12) with (3.29), we see that the use of debt and consumption taxes is redundant. Either one could be used alone to restore the first best optimum.

$$\text{If } \eta^* = 0, \text{ then } \omega^* = \frac{\xi}{1 - Ra - \xi} = \frac{Rb(1-a)}{(1 - Ra(1-b))(1-R)} \;. \tag{3.34}$$

$$\text{If } \omega^* = 0, \text{ then } \eta^* = \frac{\xi}{a(R-1)} = \frac{Rb(1-a)}{a(R-1)(1-R+Rb)}. \tag{3.35}$$

That is, the optimal fiscal policy is a zero tax on output so that there will be no distortion of physical capital accumulation. Government spending consists of a constant share of output which is fully funded through the consumption tax and/or debt policy. The tax on consumption is not distortionary since there is no labor-leisure choice. Notice that if expenditures are not funded through consumption or income taxes, the government must be a creditor to the economy. That is, at some initial time it levies lump-sum taxes on the economy which it uses to acquire claims on capital. Future expenditures on educational services and additional claims on the growing capital stock are financed through the interest payments the government receives from the private agents.

3.4 Balanced Growth Analysis

Now we turn to an analysis of how tax policy can be used to maximize the long-run growth rate of the economy and compare that to the optimal policy. The economy is on a balanced growth path if physical capital, human capital, and consumption are all growing at the same rate. Time allocated to each sector should be constant.

$$\frac{k_{t+1}}{k_t} = \frac{h_{t+1}}{h_t} = \frac{c_{t+1}}{c_t} \tag{3.36}$$

From equations (3.26) and (3.30) we can find the growth rates of physical capital and human capital, respectively, in terms of the ratio of human to physical capital.

$$\frac{k_{t+1}}{k_t} = (1-\tau)Rau^{1-a}\left(\frac{h_t}{k_t}\right)^{1-a} \tag{3.37}$$

$$\frac{h_{t+1}}{h_t} = (1-u)^{1-b}\left[\frac{g\, k_t^a (uh_t)^{1-a}}{h_t}\right]^b \tag{3.38}$$

Equating the right-hand sides of (3.37) and (3.38), we can solve for the ratio of human to physical capital required on a balanced growth path in terms of government policy variables.

$$\frac{h_t}{k_t} = \left[\frac{g^b (1-u)^{1-b} u^{(1-a)(b-1)}}{(1-\tau)Ra}\right]^{\frac{1}{1-a+ab}} \tag{3.39}$$

Substituting (3.39) into (3.37) allows us to solve for the balanced growth rate.

$$\gamma = (1-\tau)Rau^{1-a}\left[\frac{g^b(1-u)^{1-b}u^{(1-a)(b-1)}}{(1-\tau)Ra}\right]^{\frac{1-a}{1-a+ab}} \quad (3.40)$$

Finally using the government's budget constraint (3.32), the growth rate can be expressed as a function of the tax and debt policy variables and the technology and preference parameters:

$$\gamma = (1-\tau)^{\frac{ab}{1-a+ab}}\left[\frac{(1-\tau)}{(1+\omega)}(\omega(1-Ra)+(R-1)a\eta)+\tau\right]^{\frac{(1-a)b}{1-a+ab}}\chi(a,b,R) \quad (3.41)$$

If the government is interested in promoting the highest level of long-run growth then it will maximize (3.41). It is interesting to compare growth maximization with the optimal tax policy discussed above. For example, we found that replication of the first best optimum required a zero tax on output. Consider setting $\tau = 0$. Then it is evident from (3.41) that the long-run growth rate is strictly increasing in ω and strictly decreasing in η. This result contrasts with the welfare maximizing policy for the consumption tax which occurred at a value between zero and one and the welfare maximizing policy for debt which also took on a specific value. The intuition for the divergence is that maximization of the long-run growth rate ignors the welfare effects that occur during the transition. Transitional growth rates will be affected by the tax in a different manner from the growth rate at infinity.

Finally we can consider the case of no consumption tax or debt. If we let $\omega = 0$ and $\eta = 0$, then growth is maximized at $\tau = 1 - a$.

3.5 The Decentralized Equilibrium with Human Capital Externalities

The previous section considered a baseline case where human capital is accumulated using learning time, prior human capital, and government provided educational services. However, it was assumed that there were no externalities in learning either due to direct spillovers across agents or due to spillovers induced by the government expenditures. In this section, in contrast, we will investigate how optimal policy is changed if the services provided by the government to aid learning are subject to externalities.

The agent continues to maximize the utility function represented by (3.1). As above, output and consumption are taxed at different rates and the budget constraint (3.16) includes the accumulation of government debt and the interest earnings received from owning that asset.

Human capital accumulates using learning time, prior knowledge, and the educational services provided by the government. It is assumed that learning is a private non-market activity and is not subject to direct taxation. Eventually, learning will lead to higher output which is then taxed at the rate τ.

$$h_{t+1} = \left[(1-u_t)h_t\right]^{1-b}\left(G_t^s\right)^b \tag{3.42}$$

However, the amount of services actually obtained by the agent are subject to externalities based on the behavioral spillovers from other agents in the economy.

$$G_t^s = G_t\left[\frac{(1-u_t)h_t}{(1-U_t)H_t}\right]^{\delta} \tag{3.43}$$

In equation (3.43), δ parameterizes the amount of spillover. If δ is positive, the services are subject to a negative externality that could be interpreted as congestion resulting from other agents' use of the services. On the other hand, if δ is negative, it represents a positive externality as would occur if the individual benefits from interacting with more knowledgeable agents in attempting to utilize the services.

The Lagrangian for this problem is:

$$L=\sum_{t=0}^{\infty}R^{t}\left\{\begin{array}{l}\ln\left[\frac{(1-\tau)}{(1+\omega)}k_t^{a}(u_th_t)^{1-a}+\frac{(1+(1-\tau)i_t)}{(1+\omega)}B_t-\frac{1}{(1+\omega)}k_{t+1}-\frac{1}{(1+\omega)}B_{t+1}\right]\\ +\lambda_t\left[\left[(1-u_t)h_t\right]^{1-b+b\delta}(G_t)^{b}\left[(1-U_t)H_t\right]^{-b}-h_{t+1}\right]\end{array}\right\} \tag{3.44}$$

The optimality conditions for this problem with externalities consist of the same equations as in Section 3.3, namely (3.18), (3.20), and (3.21), except that (3.19) should be revised to (3.45):

$$\frac{c_{t+1}}{c_t}=\frac{R(1-b+b\delta)u_t}{(1-u_t)u_{t+1}}\frac{y_{t+1}^{k}}{y_t^{k}} \tag{3.45}$$

The solution to this problem is again of the form that consumption is a constant proportion of output and the agent's ownership of government bonds is a constant proportion of her capital ownership. However, we find that the time allocation decision now depends on the amount and type of the externality.

$$u_t=\hat{u}=1-R(1-b+b\delta) \tag{3.46}$$

If there is a negative externality due to congestion, the agent would try to overaccumulate human capital by increasing learning time (decreasing production time). If there is a positive externality, the opposite would hold.

Solving as before we find that capital investment and consumption relate to output in the same manner as earlier (3.47) and (3.48). However, since production time is lower, output is also lower relative to the economy with no externalities for given stocks of capital.

$$k_{t+1} = (1-\tau)Ra\,\hat{u}^{1-a}k_t^a h_t^{1-a} \tag{3.47}$$

$$c_t = \frac{(1-\tau)}{(1+\omega)}(1-Ra+a(1-R)\eta)\,\hat{u}^{1-a}k_t^a h_t^{1-a} \tag{3.48}$$

Since in equilibrium everyone has the same learning time and prior knowledge, human capital accumulation is governed by equation (3.49).

$$h_{t+1} = \left[(1-\hat{u})h_t\right]^{1-b} G_t^b \tag{3.49}$$

If the government acts as central planner by attempting to replicate the first best optimum through its selection of the fiscal policy instruments $\hat{g}$, $\hat{\tau}$, $\hat{\omega}$, and $\hat{\eta}$, it now finds that the first best is unattainable in the presence of these externalities as explained below.

Comparing (3.13) with (3.47), the optimal output tax can be derived.

$$1-\hat{\tau} = \left(\frac{\overline{u}}{\hat{u}}\right)^{1-a} \tag{3.50}$$

As opposed to optimal policy when externalities are absent, the output tax must now be used to correct for the distortion to time allocation. For instance, if there is a negative externality, we saw that the agent spent too little time in the production of output. In this case, the output must actually be subsidized in order to provide resources for physical capital investment. That is, $\hat{\tau} < 0$. It should be noted that the allocation of time itself is not affected by government policy variables. However, physical investment and consumption are each proportional to output net of taxes or subsidies.

Now equating the human capital accumulation equations (3.15) and (3.49),

$$(1-\bar{u})^{1-b}\left(\xi\,\bar{u}^{1-a}k_t^a h_t^{1-a}\right)^b = (1-\hat{u})^{1-b}\left(\hat{g}\,\hat{u}^{1-a}k_t^a h_t^{1-a}\right)^b \tag{3.51}$$

we can solve for the optimal amount of government services as a proportion of output that will restore the first best amount of human capital investment.

$$\Rightarrow \quad \hat{g} = \left(\frac{1-\bar{u}}{1-\hat{u}}\right)^{\frac{1-b}{b}}\left(\frac{\bar{u}}{\hat{u}}\right)^{1-a}\xi \tag{3.52}$$

From the government's budget constraint, the amount of debt/credit to the economy as a proportion of output depends on the expenditure and tax policies as given by (3.53).

$$\hat{\eta} = \frac{1}{a(R-1)}\left[\frac{(\hat{g}-\hat{\tau})(1+\hat{\omega})}{1-\hat{\tau}} - \hat{\omega}(1-Ra)\right] \tag{3.53}$$

Substituting (3.53) into (3.48) leads to (3.54).

$$c_t = (1-\hat{\tau})\left(1 - Ra - \frac{(\hat{g}-\hat{\tau})}{1-\hat{\tau}}\right)\hat{u}^{1-a}k_t^a h_t^{1-a} \tag{3.54}$$

Thus the for the first best optimum to hold it must be the case that (3.55) holds, which in general will not be the case.

$$\frac{(\hat{g}-\hat{\tau})}{1-\hat{\tau}}=\xi \tag{3.55}$$

The essential difficulty in replicating the first best optimum stems from the inability of fiscal variables to impact the time allocation decision. However, if the government services provided to the agent are excludable so that the amount of services can vary individually, then the government could counteract the impact of the externality by providing a level of services that depends on the amount of effective learning that the agent expends.

Although the first best optimum cannot be achieved in general, we can still ask about optimal fiscal policy that will be second best. In order to analyze second best policy, we can use the solution to the decentralized economy to construct the agent's indirect utility function. The tax rates and amount of debt can be chosen to maximize this indirect utility function.

Taking logs of (3.47) and (3.49) and substituting for government expenditures from the government's budget constraint, we can write:

$$\begin{bmatrix} ln\,k_{t+1} \\ ln\,h_{t+1} \end{bmatrix} = \begin{bmatrix} a & 1-a \\ ab & 1-ab \end{bmatrix} \begin{bmatrix} ln\,k_t \\ ln\,h_t \end{bmatrix} + \begin{bmatrix} ln(1-\tau) \\ b\,ln\left(\frac{1-\tau}{1+\omega}(\omega(1-Ra)+(R-1)\eta)+\tau\right) \end{bmatrix} + \begin{bmatrix} ln\,Ra\hat{u}^{1-a} \\ ln(1-\hat{u})^{1-b}\,\hat{u}^{(1-a)b} \end{bmatrix} \tag{3.56}$$

In matrix notation this equation can be written as

$$X_{t+1} = AX_t + T + D \tag{3.57}$$

or using recursive substitution

$$X_t = A^t X_0 + \left(\sum_{j=0}^{t-1} A^j \right)(T + D) \tag{3.58}$$

for any t>0.

Now, substituting the solution for consumption (3.48) into (3.1) we have

$$\sum_{t=0}^{\infty} \mathrm{R}^t \, ln c_t = \sum_{t=0}^{\infty} \mathrm{R}^t \, ln \frac{(1-\tau)}{(1+\omega)}(1 - Ra + a(1-R)\eta)\, \hat{u}^{1-a} + \sum_{t=0}^{\infty} \mathrm{R}^t [a \quad 1-a] X_t \tag{3.60}$$

After some extensive algebra, the indirect utility function can be written as

$$\sum_{t=0}^{\infty} R^t \ln \frac{(1-\tau)}{(1+\omega)}(1 - Ra + a(1-R)\eta) + \sum_{t=0}^{\infty} \frac{R^{t+1}}{(1 - Ra(1-b))(1-R)} [a(1 - R + Rb) \quad 1-a] \, T \tag{3.61}$$

plus some terms not involving the fiscal variables.

Maximizing (3.61) by choosing the income tax and consumption tax and debt, we find that it is optimal to set a zero income tax as in (3.33). There is the same tradeoff between consumption taxes and debt as in Section 3.2. We consequently find that (3.34) and (3.35) describe the optimal policy for the consumption tax and debt, respectively, when used alone. Thus the second best policy in the presence of externalities is to set fiscal policy the same as the first best policy when there are no externalities.

Finally, when neither the consumption tax nor debt is used as an instrument, the optimal income tax policy is to set:

$$\tau^* = \xi \tag{3.62}$$

Output is taxed at the rate that equals the government's share of output from the command optimum.

3.6 Two Regions

The sections above examined the behavior of individuals when there are no externalities in human capital accumulation as well as when learning is subject to externalities. In this section we consider an economy populated by two types of agents. Half of the agents face these externalities while the other half does not. It may be due to a technological, institutional, or cultural innovation in one region or school district versus the other that impacts the way in which government services are able to be used by those agents studying in that region. It is further assumed that all of the agents are under the same fiscal authority and capital is perfectly mobile.

Type 1 agents face no externalities, $\delta = 0$. Their human capital accumulates in accordance with (3.30) and their economic choices will be denoted by variables with a bar. Type 2 agents are assumed to face congestion in their learning, that is $\delta > 0$. Their learning can be described by (3.42) and (3.43) and their choices will be denoted by variables with a hat.

In this section government debt is removed as a policy instrument. However, since capital is perfectly mobile, there will be private debt between agents of different types. Furthermore, to simplify the analysis we also eliminate the consumption tax. Since in equilibrium type 2 agents will likely end up with net asset claims on the type 1 agents, we can define N as the claim owned by a type 2 agent on capital operated by type 1 agent. Therefore the wealth of a type 1 agent is given by $\overline{W}_t = \bar{k}_t$ while the wealth of a type 2 agent is given by $\hat{W}_t = N_t + \hat{k}_t$. Type 1 individuals operate the amount of capital equal to $\bar{k}_t + N_t$ while type 2 individuals operate the amount $\hat{k}_t$. Hence, the budget constraint of a type 1 agent consists of (3.63)

$$\bar{k}_{t+1} = (1-\tau)\left(\bar{k}_t + N_t\right)^a \left(\bar{u}_t \bar{h}_t\right)^{1-a} - \left(1 + (1-\tau)i_t\right)N_t - \bar{c}_t \tag{3.63}$$

while the budget constraint of a type 2 agent consists of (3.64).

$$\hat{k}_{t+1} + N_{t+1} = (1-\tau)\hat{k}_t^a \left(\hat{u}_t \hat{h}_t\right)^{1-a} + \left(1 + (1-\tau)i_t\right)N_t - \hat{c}_t \tag{3.64}$$

If N is positive it implies that type 1 agents owe debt to type 2 agents for the use of capital in production of output. The optimality conditions for type 1 agents are described by the equations in Section 3.3, namely (3.18) - (3.21), except that private bonds now replace the government bonds and the consumption tax is zero. The optimality conditions for type 2 agents are the same except that (3.45) now replaces (3.19) as in Section 3.5.

Since a condition like (3.20) holds for each agent type, the growth rate of consumption of the two types must be equal. Therefore, from (3.18) we find that the marginal product of capital must be equal for each type's production (3.65).

$$\frac{a\,\bar{y}_t^k}{\bar{k}_t + N_t} = \frac{a\,\hat{y}_t^k}{\hat{k}_t} \tag{3.65}$$

Equalization of marginal products implies a relationship between the capital and effective labor operated by each type (3.66).

$$\frac{\bar{k}_t + N_t}{\hat{k}_t} = \frac{\bar{u}_t \bar{h}_t}{\hat{u}_t \hat{h}_t} \tag{3.66}$$

Given that consumption grows at the same rate for each type, we can also equate the right-hand sides of (3.19) and (3.45). Then using (3.66) we can derive an equation involving only the time allocation and human capital variables.

$$\frac{(1-\hat{u}_t)\hat{h}_t}{(1-\bar{u}_t)\bar{h}_t} = \left(\frac{1-b+b\delta}{1-b}\right)^{\frac{1-b}{b}} \tag{3.67}$$

In equilibrium everyone in each region has the same human capital and learning time. Therefore, the agent's effective learning is equal to the average effective learning for that region. So, using the equations for human capital accumulation, we find that the ratio of human capital of type 2 to type 1 for all periods following the initial period depends on the degree of congestion faced by type 2 agents.

$$\frac{\hat{h}_t}{\bar{h}_t} = D^{\frac{1-b}{b}} \qquad t>0 \tag{3.68}$$

where $D = \dfrac{1-b+b\delta}{1-b}$. Thus the greater is the externality, the greater will be the human capital of type 2 agents relative to type 1 agents.

Define $n_t = \frac{N_t}{\hat{k}_t}$ as the ratio of type 2's claim on capital of type 1 relative to the capital directly operated by type 2 and define $w_t = \frac{\overline{W}_t}{\hat{W}_t}$ as the wealth of type 1 relative to type 2. Then

$$\bar{k}_t = w_t(1+n_t)\hat{k}_t \tag{3.69}$$

Given (3.68) and (3.69) it can be verified that a solution to this problem consists of constant n and w following the initial period and the following set of choices for learning time, consumption, and investment for t>0:

$$1-\bar{u}_t = R(1-b) \tag{3.70}$$

$$\bar{c}_t = (1-\tau)((1-Ra)w(1+n)+(1-a)n)\hat{y}_t^k \tag{3.71}$$

$$\bar{k}_{t+1} = (1-\tau)Raw(1+n)\hat{y}_t^k \tag{3.72}$$

$$1-\hat{u}_t = R(1-b+b\delta) \tag{3.73}$$

$$\hat{c}_t = (1-\tau)(1-Ra+a(1-R)n)\hat{y}_t^k \tag{3.74}$$

$$\hat{k}_{t+1} = (1-\tau)Ra\hat{y}_t^k \tag{3.75}$$

$$N_{t+1} = (1-\tau)Ran\hat{y}_t^k \tag{3.76}$$

Therefore, for all t>0

$$\frac{\bar{k}_t + N_t}{\hat{k}_t} = \frac{\bar{u}_t \bar{h}_t}{\hat{u}_t \hat{h}_t} = \frac{1-R(1-b)}{1-R(1-b+b\delta)} D^{\frac{b-1}{b}} \equiv Q \tag{3.77}$$

The ratio of private debt to the capital operated by type 2 can be written in terms of the relative wealth ratio.

$$n = \frac{Q-w}{1+w} \tag{3.78}$$

We can solve for n, w, and the initial variables n_0, $\bar{u}_0$, and $\hat{u}_0$ using equations (3.78) - (3.82). Equalization of the marginal products of capital ties the initial debt and initial relative production times to the exogenously given initial relative wealth and human capital of each type.

$$w_0(1+n_0)+n_0 = \frac{\bar{u}_0}{\hat{u}_0}\frac{\bar{h}_0}{\hat{h}_0} \tag{3.79}$$

The initial relative learning time of each type depends on the degree of congestion and on the initial relative human capital of each type (3.80). Since the initial relative human capital stocks are not constrained to be given by (3.68), the initial period differs from the remaining infinite horizon in the sense that time allocated to learning is not governed by (3.70) and (3.71) but must adjust.

$$\frac{1-\hat{u}_0}{1-\bar{u}_0} = D^{\frac{1}{b}} \frac{\bar{h}_0}{\hat{h}_0} \tag{3.80}$$

Finally, the initial debt, subsequent debt, the ensuing wealth ratio, and initial time allocations must be consistent with the optimality conditions that determine the growth rate of consumption.

$$\frac{\hat{c}_1}{\hat{c}_0} = \left(\frac{1-Ra+a(1-R)n}{1+an_0-(1+n)\frac{a}{1-b+b\delta}\frac{\hat{u}_1(1-\hat{u}_0)}{\hat{u}_0}} \right) \frac{\hat{y}_1}{\hat{y}_0} \tag{3.81}$$
$$= \left(\frac{R(1-b+b\delta)\hat{u}_0}{\hat{u}_1(1-\hat{u}_0)} \right) \frac{\hat{y}_1}{\hat{y}_0}$$

$$\frac{\bar{c}_1}{\bar{c}_0} = \left(\frac{(1-Ra)w(1+n)+(1-a)n}{\left(w_0(1+n_0)+(1-a)n_0\right) - \frac{w(1+n)}{w(1+n)+n}\left(\frac{a}{1-b}\right)\frac{\bar{u}_1(1-\bar{u}_0)}{\bar{u}_0}\left(\left(w_0(1+n_0)+n_0\right)\right)} \right) \frac{\hat{y}_1}{\hat{y}_0}$$
$$= \left(\frac{R(1-b+b\delta)\hat{u}_0}{\hat{u}_1(1-\hat{u}_0)} \right) \frac{\hat{y}_1}{\hat{y}_0} \tag{3.82}$$

From (3.79) and (3.80) we can observe the following. Suppose that the physical wealth and human capital of each type are equal initially. Then a type 2 would spend relatively more time learning while a type 1 would spend relatively more time producing output. Since an increase in production time increases the marginal product of capital and if human and nonhuman wealth is initially the same for each type, capital should flow from type 2 to type 1 for use in production. Thus type 1 receives capital and owes a debt to type 2. Furthermore, holding constant initial relative wealth, higher initial human capital of type 1 relative to type 2 implies a greater amount of debt incurred. More capital would be induced to flow to type 1's production as an outcome of the higher marginal product of capital. On the other hand, greater initial physical wealth of type 1 relative to type 2 has the opposite effect of reducing type 1's debt to type 2.

3.7 Conclusions

This paper has shown that if there are no externalities in the accumulation of knowledge, the first best optimum can be reproduced using some combination of consumption taxation and government debt policy where in general the government should actually be a creditor to the private economy. For this case, output should not be taxed so that no distortions to investment are introduced. When there are externalities in the human capital sector, however, the allocation of time between production versus learning becomes distorted. The available policy instruments of the income tax, consumption tax, and government debt are not able to impact this choice and are not able to correct its repercussions. Nevertheless, while the first best cannot be achieved, the second best optimum is for the government to set policy as if it were trying to attain first best with no externalities. The impact of these policy choices on long-run growth was also examined. The maximization of growth involves a larger income or consumption tax compared to the optimal policies since it ignores the short-run welfare implications such taxes create.

We also analyzed the consequences of an externality in the use of government educational services in terms of the induced capital flows and debt between regions or agents experiencing different degrees of congestion. Agents who do not take into account the effect of their behavior on the services received by others thereby overaccumulate human capital by sending too much time learning. This spurs capital to flow to equalize marginal products.

Chapter 4

Credit Constraints, Skill Acquisition, and Trade

4.1 Introduction

Early trade theory has articulated the static gains that can be achieved through trade. Recently, researchers have been interested in the relationship between dynamic gains from trade and the development process or growth. Openness has been credited with the dramatic performance of the East Asian economies. In addition, human capital accumulation has contributed to higher growth rates across countries.

This paper studies the relationship between trade, the acquition of skills, and the development process. However, in contrast to previous literature, the focus here is on the role that imperfect credit markets play in the decision to acquire higher education. The inability to borrow or the possibility that borrowing occurs at high interest rates means that the initial wealth bequested from parents contributes to occupational choice. That is, an individual with enough initial wealth to fund tuition payments without borrowing may find lifetime earnings from skilled labor to be much larger than lifetime earnings from unskilled labor.

There is considerable empirical evidence that imperfect credit markets constrain human capital investment and also entrepreneurial activity. Becker (1975) and Atkinson (1974) have proposed that the acquisition of human capital is prone to credit market imperfections since it is a nontraded asset that depends on inherited wealth. More recently, Parish and Willis (1992) suggested that in Taiwan attempts by altruistic parents to finance optimal investment in their children's human capital may be frustrated by credit constraints. Ryoo and Nam (1993) proposed that imperfect credit markets may be important in South Korea. Blanchflower and Oswald (1990) find that the distribution of inherited wealth is a major factor in determining the level of entrepreneurial activity. Evans and Jovanovic (1989) document that entrepreneurs in the U.S. are capital constrained while Levy (1993) finds the same result for Tanzania and Sri Lanka.

This paper draws on the credit market structure of Galor and Zeira (1990). The interest rate for borrowing is greater than the lending rate. The difference reflects some resource cost required to monitor borrowers to ensure repayment. The cost is proportional to the amount borrowed. Consequently, inherited wealth becomes important to the decision to acquire education since those with insufficient wealth to pay for tuition must borrow at this higher rate.

Following this introduction, Section 4.2 lays out the basic framework for the evolution of a closed economy. Potential steady state equilibria are examined in Section 4.3. Trade in goods is introduced in Section 4.4. Section 4.5 describes how the initial conditions contribute to the behavior of the economy even in the long-run and considers the impact of a change in the world relative price of low-tech goods. Finally, some extensions to this model are discussed in the concluding Section 4.6.

4.2 The Model

4.2.1 Utility Maximization

The economy consists of overlapping generations with two period lives. Each parent has one child and the population of each generation is normalized to unity. Individuals consume two goods when they are old: a high-tech good, c^h, and a low-tech good, c^l, such as agriculture. In the second period of their life they also bequeath some resources to their offspring, b. For simplicity, there is no consumption during youth.

Agents maximize utility

$$U = \eta \ln c_{t+1}^{h} + \theta \ln c_{t+1}^{l} + \phi \ln b_{t+1} \tag{4.1}$$

subject to the budget constraint

$$c_{t+1}^{h} + p_{t+1}^{l} c_{t+1}^{l} + b_{t+1} = I_L \tag{4.2}$$

where I_L is lifetime income, $\eta + \theta + \phi = 1$, and the high-tech good is the numeraire. Optimization yields the result that expenditures on each consumption good and the bequest are each a constant fraction of lifetime income.

$$c_{t+1}^{h} = \eta\, I_L \tag{4.3}$$

$$p_{t+1}^{l} c_{t+1}^{l} = \theta\, I_L \tag{4.4}$$

$$b_{t+1} = \phi\, I_L \tag{4.5}$$

$$\text{or } b_{t+1} = \frac{\phi}{\eta} c_{t+1}^{h} = \frac{\phi}{\theta} p_{t+1}^{l} c_{t+1}^{l} \tag{4.6}$$

In addition to deciding how to partition lifetime earnings between the consumption goods and the bequest, a young agent must decide whether to spend the first period becoming educated in order to be a skilled worker in the second period or rather to work as an unskilled laborer in both periods. In order to acquire education, the agent must pay a tuition charge of z. If the amount of the initial wealth inherited from the parent is not high enough to cover tuition, the youth may borrow funds. However, whereas the lending rate at time t is r_t, the borrowing rate is $r_t + \Delta$ where Δ represents the proportionate resource cost that must be expended in order to monitor the agent to ensure repayment of the loan. Individuals with inherited wealth less than the amount of tuition must compare the lifetime earnings inclusive of borrowing costs from each occupational opportunity.

If an agent is an unskilled worker, she will earn low-skilled wages in both periods and interest on savings from first period bequests and wages.

$$\left(b_t + w_t^l\right)(1 + r_{t+1}) + w_{t+1}^l \qquad (4.7)$$

If she becomes skilled, she earns the skilled wage in the second period but must pay interest on the amount borrowed to fund tuition.

$$w_{t+1}^h + (b_t - z)(1 + r_{t+1} + \Delta) \qquad (4.8)$$

Therefore, the critical value for the individual's occupational decision is:

$$f_t = \frac{w_t^l(1 + r_{t+1}) + w_{t+1}^l - w_{t+1}^h + z(1 + r_{t+1} + \Delta)}{\Delta} \qquad (4.9)$$

Any youth with inherited wealth below this critical value finds lifetime earnings to be greater when remaining unskilled, while those with inherited wealth above this value would chose to aquire education.

Furthermore, by assumption unconstrained skilled workers make at least as much as unskilled workers. Otherwise there is no incentive for anyone to aquire skills.

$$w_{t+1}^{h} - z(1 + r_{t+1}) \geq w_t^{l}(1 + r_{t+1}) + w_{t+1}^{l} \tag{4.10}$$

Therefore, we can divide agents into three types or classes depending on the size of inherited wealth. The first and second period budget constraints are shown below for each type.

Budget Constraints for low-skilled workers $b_t \in [0, f_t]$

$$\sigma_t = b_t + w_t^{l} \tag{4.11}$$

$$c_{t+1}^{h} + p_{t+1}^{l} c_{t+1}^{l} + b_{t+1} = (1 + r_{t+1})\sigma_t + w_{t+1}^{l} \tag{4.12}$$

Workers that do not aquire education earn a first period wage. The wage and inherited wealth are saved and earn the lending rate. Savings along with second period unskilled wages can be divided into consumption of the two goods and bequests to the next generation.

Budget Constraints for educated borrowers $b_t \in [f_t, z]$

$$\sigma_t = b_t - z \tag{4.13}$$

$$c_{t+1}^{h} + p_{t+1}^{l} c_{t+1}^{l} + b_{t+1} = (1 + r_{t+1} + \Delta)\sigma_t + w_{t+1}^{h} \tag{4.14}$$

Those wishing to aquire education with inherited wealth less than the tuition payment must take out a loan. In the second period they pay back the loan at the borrowing rate using some of the skilled wage income. The remaining income goes toward consumption and bequests.

Budget Constraints for educated lenders $b_t \in [z, \infty)$

$$\sigma_t = b_t - z \tag{4.15}$$

$$c_{t+1}^h + p_{t+1}^l c_{t+1}^l + b_{t+1} = (1 + r_{t+1})\sigma_t + w_{t+1}^h \tag{4.16}$$

Individuals inheriting enough wealth for tuition payments can save the remainder of their bequest at the lending rate. These savings supplement the skilled wage income in the second period. The individuals use the resources for consumption and bequests.

4.2.2 Production

The production of high-tech output is accomplished through the use of a Cobb Douglas production function using capital, K, and skilled labor, H.

$$Y_t^h = AK_t^{\alpha} H_t^{1-\alpha} \tag{4.17}$$

Low-tech output is produced using unskilled labor alone.

$$Y_t^l = BL_t \tag{4.18}$$

Profit maximization requires that the interest rate equals the marginal product of capital and the skilled (unskilled) wage equals the marginal product of labor in the production of high-tech (low-tech) output.

$$n = \alpha A\left(\frac{H_t}{K_t}\right)^{1-\alpha} \tag{4.19}$$

$$w_t^h = (1-\alpha)A\left(\frac{K_t}{H_t}\right)^{\alpha} \tag{4.20}$$

$$w_t^l = p_t^l B \tag{4.21}$$

4.2.3 Labor Markets

We normalize the population of each generation to equal unity. Thus the number of low-skilled workers plus the number of students of a particular generation born at time t must sum to one.

$$l_t + s_t = 1 \tag{4.22}$$

The total number of unskilled workers producing low-tech output at time t equals the sum of the agents born at times t and t-1 that chose to remain uneducated.

$$L_t = l_t + l_{t-1} \tag{4.23}$$

The number of skilled workers available to produce high-tech output at time t equals the number of students at t-1.

$$H_t = s_{t-1} = 1 - l_{t-1} \tag{4.24}$$

Since f is the critical value of bequests that determines whether an agent aquires education, the number of unskilled workers versus students from generation born at time t can also be represented in terms of the cumulative density function.

$$l_t = \int_0^{f_t} g(b_t)db_t = G(f_t) \tag{4.25}$$

$$s_t = \int_{f_t}^{\infty} g(b_t)db_t = 1 - G(f_t) \tag{4.26}$$

4.2.4 Credit Markets

In the credit markets, the saving by the poor (those choosing no education) plus the saving by the rich (those agents wealthy enough to self-finance education) fund borrowing by the middle class and the ownership of capital.

$$\int_0^{f_t} \left(w_t^l + b_t\right) g(b_t)db_t + \int_z^{\infty} (b_t - z) g(b_t)db_t = \int_{f_t}^{z} (z - b_t) g(b_t)db_t + K_{t+1} \tag{4.27}$$

Simplifying (4.27) we find that the total wages earned by young laborers plus the total bequests received by the youth must equal the total tuition costs plus the capital stock.

$$w_t^l l_t + \bar{b}_t = z\, s_t + K_{t+1} \tag{4.28}$$

4.2.5 Economy's Resource Constraints

In the high-tech sector, resources consists of capital and high-tech output which can be used for consumption of the high-tech good, capital accumulation, tuition, and monitoring services for loans.

$$K_{t+1} + \int_0^{\infty} c_t^h dG(b_{t-1}) + z\, s_t = K_t + Y_t^h + \Delta \int_{f_{t-1}}^{z} \sigma_{t-1} dG(b_{t-1}) \tag{4.29}$$

In the low-tech sector, output is used for consumption of the low-tech good.

$$p_t^l \int_0^{\infty} c_t^l dG(b_{t-1}) = p_t^l Y_t^l = w_t^l (l_t + l_{t-1}) \tag{4.30}$$

The relative price can be determined from (4.6) by aggregating across individuals.

$$p_t^l = \frac{\theta}{\eta}\frac{\int_0^{\infty} c_t^h dG(b_{t-1})}{\int_0^{\infty} c_t^l dG(b_{t-1})} = \frac{\theta}{\phi}\frac{\bar{b}_t}{\int_0^{\infty} c_t^l dG(b_{t-1})} \tag{4.31}$$

4.2.6 The Evolution of Wealth

From utility maximization (4.5) we know that individuals in each of the classes bequeath a fraction of their lifetime earnings to their children. Thus we can represent the evolution of aggregate wealth by integrating over the bequests of individuals in each of the three classes.

$$\bar{b}_{t+1} = \int_0^{\infty} b_{t+1} dG(b_t) = \phi\int_0^{f_t}\left[\left(w_t^l + b_t\right)(1+r_{t+1}) + w_{t+1}^l\right]dG(b_t)$$
$$+\phi\int_{f_t}^{z}\left[(b_t - z)(1+r_{t+1}+\Delta) + w_{t+1}^h\right]dG(b_t) + \phi\int_z^{\infty}\left[(b_t - z)(1+r_{t+1}) + w_{t+1}^h\right]dG(b_t) \tag{4.32}$$

The dynamics of inherited wealth of a specific lineage can be analyzed by examining Figure 4.1. An individual born at time t inherits b_t. If this inherited wealth is below the critical value f, the individual chooses not to study. For bequests in the amount between f and z, the individual borrows to fund education. Bequests over the amount of z allow self-financed education. For a given value of b_t, the vertical axis measures the bequest that will be given to the agent's offspring. The dynamics of bequests are represented by the arrows. There are two stable and one unstable steady state. However, the slopes and intercepts of the lines describing the evolution of bequests are themselves endogenously changing over time.

4.3 Steady State in the Autarkic Economy

The above section considered the basic evolution of the economy for the case when there is no trade in goods. In this section we analyze the properties of the economy in steady state with no goods trade. Steady state is defined as the condition of economy in which capital, dynastic bequests, and the number of unskilled workers are all constant over time, that is, $K_{t+1} = K_t = K$, $b_{t+1} = b_t = b$, and $l_{t+1} = l_t = l$. This also implies constant wages, interest rate, relative price, output, and consumption.

Given the above definition, in steady state the slopes and intercepts of the lines describing the evolution of bequests must be constant. Since in equilibrium both goods must be consumed, the steady state diagram cannot look like Graphs 1, 2, or 3 in Figure 4.2. Possible steady state representations of the dynamics of inherited wealth consist of Graphs 4, 5, and 6 in Figure 4.2. The following analysis will look closely at the case illustrated in Graph 4 in which the higher borrowing rate produces a wedge between the lifetime earnings of unconstrained skilled and unskilled workers and in which there are no borrowers in steady state equilibrium.

Define $\kappa = \dfrac{K}{1-l}$ as the steady state capital to labor ratio in the high-tech sector. Then the steady state resource constraints are equations (4.33) and (4.34) for the high-tech sector and low-tech sector, respectively.

$$\int_0^\infty c^h dG(b) = Y^h - zs = A\kappa^\alpha(1-l) - z(1-l) \qquad (4.33)$$

$$\int_0^\infty c^l dG(b) = Y^l = 2Bl \tag{4.34}$$

The steady state interest rate, wages, and relative price are (4.35) - (4.38).

$$r = \alpha A \kappa^{\alpha-1} \tag{4.35}$$

$$w^h = (1-\alpha) A \kappa^{\alpha} \tag{4.36}$$

$$w^l = \frac{\theta}{2\eta}\left(A\kappa^{\alpha} - z\right)\frac{1-l}{l} \tag{4.37}$$

$$p^l = \frac{\theta}{\eta}\frac{A\kappa^{\alpha}(1-l) - z(1-l)}{2Bl} \tag{4.38}$$

From the equation for the dynamics of bequests we can find the steady state aggregate inherited wealth.

$$\bar{b} = \frac{\phi}{\eta + \theta - \phi r}\left[(2+r)w^l l + \left(w^h - z(1+r)(1-l)\right)\right] \tag{4.39}$$

From utility maximization and the resource constraint we find another expression for aggregate wealth.

$$\bar{b} = \frac{\phi}{\eta}\left(A\kappa^{\alpha} - z\right)(1-l) \tag{4.40}$$

Finally, the credit market clearing condition gives us (4.41).

$$\bar{b} = (\kappa + z)(1-l) - \frac{\theta}{2\eta}\left(A\kappa^{\alpha} - z\right)(1-l) \tag{4.41}$$

Combining (4.40) and (4.41) produces the implicit solution for the capital to labor ratio in terms of the tuition cost.

$$A\kappa^{\alpha} - z = \frac{2\eta}{\theta + 2\phi}(\kappa + z) \tag{4.42}$$

If credit markets were perfect ($\Delta = 0$), (4.10) would hold with equality as a career arbitrage condition generating another relationship between capital and labor (4.43).

$$\alpha A\kappa^{\alpha-1} = \frac{2\left(l(1-\phi) - \theta\right)}{\theta + 2\phi l} \tag{4.43}$$

These two equations could be solved for the steady state amount of capital and unskilled or skilled labor. This scenario corresponds to Graph 6 in Figure 4.2. However, when credit markets are imperfect (4.43) may not hold. For a steady state resembling Graph 4 to hold, we instead have the inequality condition (4.44).

$$\frac{\phi}{\eta+\theta-\phi r}(2+r)w^{l} < z < \frac{\phi}{\eta+\theta-\phi r}\left(w^{h} - z(1+r)\right) \tag{4.44}$$

This condition imposes a lower bound on the capital to labor ratio and an upper bound on the ratio of skilled to unskilled workers in a generation. In general, the specific values of steady state capital and labor are not determined since as we shall see below, they depend on the initial distribution of wealth.

4.4 Trade in Goods

We will assume that the country is small and therefore takes the relative price of the low-tech good as given in world markets. Furthermore, assume that the world price, $\bar{p}$, is greater than the price that would prevail in autarky, p^{l}, so that the home country imports the high-tech good and exports the low-tech good when the country initially opens to free trade. Let m be the import of the high-tech good and let x be the export of the low-tech good.

Although free trade in goods prevails, we assume that the capital account is closed. Therefore, trade must be balanced in every period (4.45).

$$m_t = \bar{p}x_t \tag{4.45}$$

The market clearing conditions for the high-tech and low-tech sectors must change to (4.46) and (4.47), respectively, to account for the imports and exports of goods.

$$K_{t+1} + \int_{0}^{\infty} c_t^h dG(b_{t-1}) + z\, s_t = K_t + Y_t^h + m_t + \Delta \int_{f_{t-1}}^{z} \sigma_{t-1} dG(b_{t-1}) \tag{4.46}$$

$$\int_0^{\infty} c_t^l dG(b_{t-1}) = Y_t^l - x_t \tag{4.47}$$

The unskilled wage now depends directly on the world relative price (4.48). The remainder of the model follows Section 4.2.

$$w_t^l = \bar{p}B \tag{4.48}$$

The steady state equilibrium corresponding to Graph 4 in Figure 4.2 consists of two classes of agents. The rich self-finance their education and the poor choose to remain uneducated. There are no borrowers in steady state for this scenario. Therefore the market clearing conditions are given by (4.49) and (4.50) and the ratio of aggregate high-tech consumption to low-tech consumption must be consistent with the world relative price (4.51).

$$\int_0^{\infty} c^h dG(b) = A\kappa^{\alpha}(1-l) - z(1-l) + m \tag{4.49}$$

$$\int_0^{\infty} c^l dG(b) = 2Bl - x \tag{4.50}$$

$$\bar{p} = \frac{\theta}{\eta} \frac{\int_0^{\infty} c^h dG(b)}{\int_0^{\infty} c^l dG(b)} = \frac{\theta}{\eta} \frac{A\kappa^{\alpha}(1-l) - z(1-l) + m}{2Bl - x} \tag{4.51}$$

From utility maximization we have:

$$\bar{b} = \frac{\phi}{\eta}\left(\left(A\kappa^{\alpha} - z\right)(1-l) + m\right) \tag{4.52}$$

The credit market clearing condition gives us:

$$\bar{b} = (\kappa + z)(1-l) - B\bar{p}l \tag{4.53}$$

Using (4.52), (4.53), and the balanced trade condition leads to an equation relating capital and labor to the tuition cost, world price, and other parameters.

$$(\eta + \theta)\kappa - \phi A\kappa^{\alpha} + z = (1+\phi)\bar{p}B\frac{l}{1-l} \tag{4.54}$$

As in the autarkic steady state, if credit is unconstrained, the career arbitrage condition will generate another relationship between capital and labor that will in combination with (4.54) allow the determination of each of these variables. Such a situation corresponds to Figure 4.2, Graph 6. However, credit constraints may be binding as in Figure 4.2, Graph 4. As above this case requires only (4.44) to hold and is therefore consistent with a range of equilibria.

4.5 Initial Conditions and Short-run Dynamics

4.5.1 The Autarkic Economy

In the above sections we examined the behavior of potential steady state equilibria. We found that if credit imperfections are not binding the steady state would be characterized by a career arbitrage condition which determines capital and labor. On the other hand, credit constaints may lead to a many potential equilibria characterized by a wedge between the earnings of unskilled and skilled labor for any given amount of bequest. The steady state equilibrium depends on the dynamic path of bequests, capital, interest rates, prices, and wages that emerge from the initial conditions faced by the economy. Therefore, we now turn to the analysis of the initial conditions and initial dynamics.

First, let us consider the autarkic economy. At some initial point in time the capital stock, the number of old unskilled workers, and the number of skilled workers are given by K_0, L_{-1}, and $H_0 = 1 - L_{-1}$, respectively. Consequently, the initial interest rate, skilled wage rate, and high-tech output are given. However, the number of initial youth that study versus work as unskilled, l_0, in the initial period must be determined to be consistent with utility maximization and the resource constraints. The initial low-tech output, unskilled wage, and the initial relative price are all functions of l_0. Also, the bequests to the initial youth are exogenously determined for the middle and upper class families but the bequests from the initial unskilled old depends on the wage and thus depends on l_0. The capital stock in the next period in turn depends on l_0 through the credit market clearing condition:

$$K_1(l_0) = \bar{b}_0(l_0) + w_0^l(l_0) - (1 - l_0)z \tag{4.55}$$

The period 1 high-tech output, interest rate, skilled wage rate, and bequests are all therefore functions of l_0. Likewise, the period 1 low-tech output, unskilled wage, and relative price are functions of l_0 and l_1.

To determine the evolution of unskilled labor, we can find the critical value from (4.9).

$$f_0 = \frac{w_0^l(l_0)(1+n(l_0)) + w_1^l(l_0,l_1) - w_1^h(l_0) + z(1+n(l_0)+\Delta)}{\Delta} \tag{4.56}$$

Therefore, if the initial distribution of inherited wealth is continuous, the number of initial youth remaining uneducated is given by (4.57).

$$l_0 = \int_0^{f_0} g(b_0)db_0 \quad \text{or} \quad l_0 = G_0(f_0(l_0,l_1)) \tag{4.57}$$

Equation (4.57) implicitly defines a dynamic relationship in the fraction of youth for each generation that remain uneducated. It is a function of initial bequests, capital, the tuition cost and other parameters. Likewise, there is an implied dynamic relationship between the initial unskilled old and the endogenously determined initial unskilled youth (4.58).

$$l_{-1} = G_{-1}(f_{-1}(l_{-1},l_0)) \tag{4.58}$$

As an experiment, consider an initial distribution that consists of some fraction, l_{-1}, of the initial youth inheriting the same amount of bequest, $\underline{\underline{b_0}}$, that is much lower than tuition costs while the remaining fraction of initial youth, $1-l_{-1}$, receive the same amount of bequest, $\overline{\overline{b_0}}$, that is much higher than the tuition charge. The fraction l_{-1} also represents the number of initial old that are unskilled workers. We investigate whether an equilibrium consists of $l_0 = l_{-1}$. If this condition holds, low-tech output would be

$$Y_0^l = 2BL_{-1} \tag{4.59}$$

Next periods capital stock could be obtained from the credit market clearing condition

$$K_1\left(L_{-1}, \underline{\underline{b}}_0, \overline{\overline{b}}_0\right) = L_{-1}\underline{\underline{b}}_0 + (1 - L_{-1})\overline{\overline{b}}_0 + w_0(L_{-1}) - z(1 - L_{-1}) \tag{4.60}$$

Having derived the period 1 capital stock, we could also find the period 1 interest rate, and skilled wage rate in terms of L_{-1}, $\underline{\underline{b}}_0$, and $\overline{\overline{b}}_0$. Then substituting these quantities into the expression for the critical value of wealth that separates unskilled workers from students (4.56), we can compare the critical value to $\underline{\underline{b}}_0$. As long as the critical value is larger, $l_0 = L_{-1}$ is a consistent equilibrium for the initial period. It can be verified that such a situation holds for some parameter values, in particular if z is large enough. Thus there may be no change in the number of unskilled versus skilled workers when each group is separated by a wide margin and tuition costs are prohibitive for the poor group.

4.5.2 The Open Economy

We consider now the initial dynamics for the scenario that the country opens for free trade with the world and takes the relative price as exogenous. In the initial period we have K_0, L_{-1}, and $H_0 = 1 - L_{-1}$ given as before and therefore also the initial interest rate, skilled wage rate, and high-tech output. In addition, the unskilled wage rate is fixed by the relative price as in (4.48). The initial bequests are likewise exogenous for all three classes of agents with the bequests from unskilled parents depending on the world relative price.

The critical value of inherited wealth that now separates uneducated initial youth from those seeking an education is a function of the world price and the total number of initial youth that forgo education (4.61).

$$f_0 = \frac{B\bar{p}(1+n(l_0)) + B\bar{p} - w_1^h(l_0) + z(1+n(l_0)+\Delta)}{\Delta} \tag{4.61}$$

As above the total number of initial youth with bequests too low to justify aquiring an education depends also on the distribution function as in (4.57).

An important question to investigate is the impact of a change in the world price on the initial number of unskilled labor. Suppose for example that the initial bequests are uniformly distributed over some interval and are held constant as the world price changes. That is, let $b_0 \in [b_L, b_U]$. From (4.57), the initial number of unskilled is related to the critical value of wealth as in (4.62).

$$l_0 = \frac{f_0 - b_L}{b_U - b_L} \tag{4.62}$$

Combining (4.61) and (4.62) leads to the number of initial youth remaining as unskilled as an implicit function of the world price and other parameters. Differentiating this implicit function with respect to the world price produces an ambiguous result that depends on the parameters. However, some intuition for the conflicting effects can be extracted from an examination of (4.61). An increase in the world relative price of low-tech goods will increase the initial unskilled wage. This creates an incentive for young individuals to remain unskilled to benefit from the higher wage. However, greater unskilled wages also lead to more savings and thus a higher capital stock for the subsequent period. This higher capital stock decreases the interest rate, r_1, and increases the skilled wage rate, w_1^h, prevailing in period 1. Both of these changes encourage the initial youth to become students. Finally note that this example holds constant the distribution of initial bequests. But as mentioned earlier, the bequests from unskilled parents would actually rise since such parents would earn more income from the increase in the initial unskilled wage. Intuitively, larger bequests received by poorer agents should relax their borrowing constraints and enable more of them to acquire education.

4.6 Conclusions and Extensions

This paper has investigated the impact of imperfect credit markets on the incentives to acquire education. In contrast to many models that impose an arbitrage condition on returns to working in one sector versus another, the divergence of the borrowing rate from the lending rate creates a wedge between the earnings from each sector. For a given amount of inherited wealth, most agents will be strictly better off working in one of the two sectors. High borrowing costs prevent poor agents from becoming educated and obtaining the high skilled wage. Thus the distribution of wealth plays an important role in determining the number of unskilled versus skilled workers as well as determining capital accumulation and the production structure. The evolution of the economy therefore depends on the initial conditions including the distribution of wealth. The economy can consequently arrive at a range of steady state equilibria.

When the country opens to free trade, the short-run dynamics and steady state again depend on initial conditions. In addition, a change in the world relative price of low-tech goods impacts the incentive to invest in education through several channels.

One important direction this model could be extended is to embed this framework in a growth model. We could add learning by doing such as $A_{t+1} - A_t = \gamma A_t Y_t^h$ or

$A_{t+1} - A_t = \gamma A_t s_t$ in order to link trade and skill acquisition with growth. If trade increases the incentive for low skill work by raising the wage more than it reduces the costs of becoming skilled, trade may involve underdevelopment in a growth context. That is, the shift to more low-tech production due to the initial comparative advantage in trade can amplify this initial comparative advantage due to forgone learning by doing in the high-tech sector. However, if opening to trade in the presence of credit constraints actually assists in relaxing those constraints in a manner that dominates the unskilled wage incentive effect, growth through learning by doing could be enhanced.

This model has the property that there must be a steady state with unskilled workers since low-tech output must be consumed and can be produced only with unskilled workers. An alternative development framework may be to allow for nonconvexities in the production of both goods. The framework may retain the credit structure outlined above but involve substitution of high-tech production for low-tech production for both goods or substitution of a more advanced industry structure as in Banerjee and Newman (1993) for example. This alternative framework could be consistent with everyone obtaining education in the long-run.

An additional extension would be to provide a richer description of the credit sector and the evolution of monitoring costs over time. Finally, government policy plays a vital role in educational achievement through tuition or loan subsidies as well as redistribution of resources to the poor. Such policies allow the poor to circumvent high market borrowing rates required to fund tuition costs. Thus a more complete analysis of the development process would include government.

Unskilled Lender: $b_{t+1} = \phi\left[b_t(1+r_{t+1}) + w_t^l(1+r_{t+1}) + w_{t+1}^l\right]$

Skilled Borrower: $b_{t+1} = \phi\left[b_t(1+r_{t+1}+\Delta) - z(1+r_{t+1}+\Delta) + w_{t+1}^h\right]$

Skilled Lender: $b_{t+1} = \phi\left[b_t(1+r_{t+1}) - z(1+r_{t+1}) + w_{t+1}^h\right]$

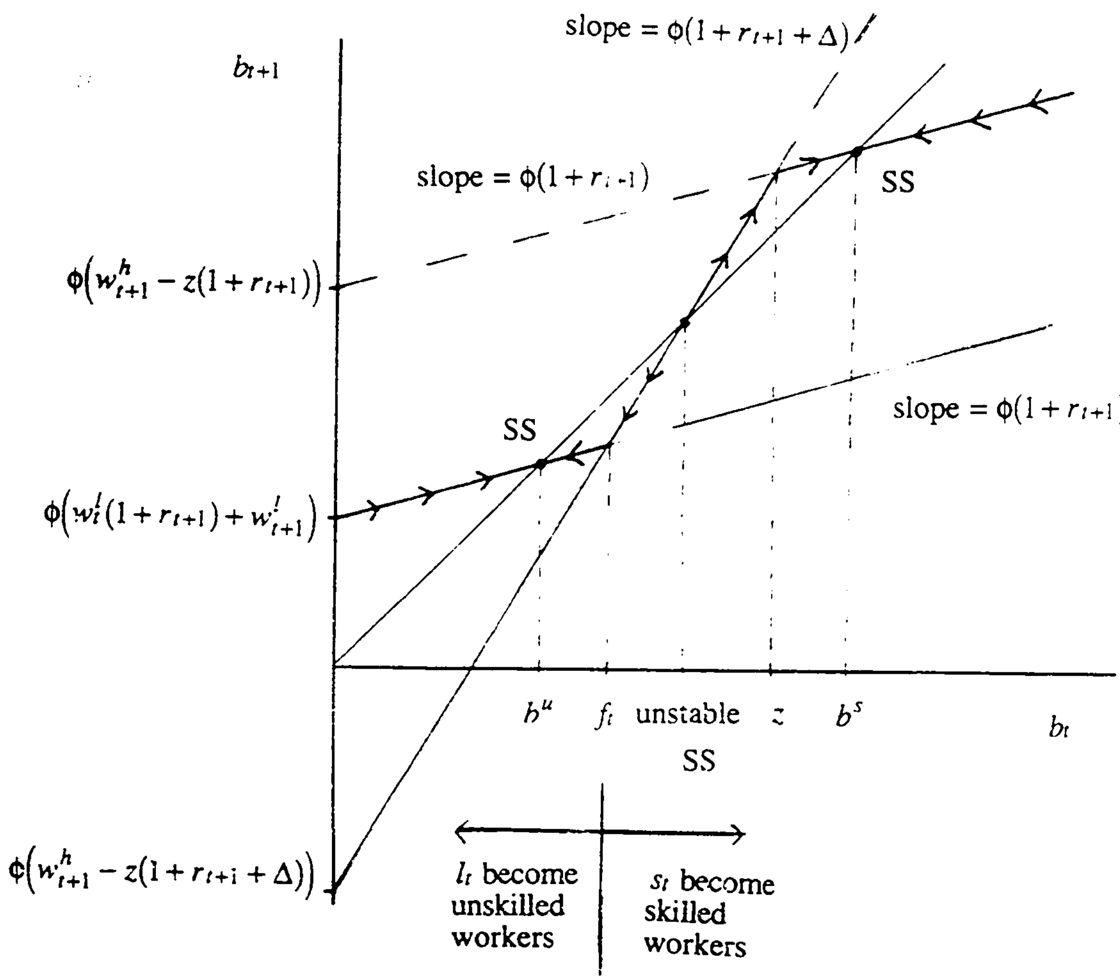

Figure 4.1

Dynamics of Wealth

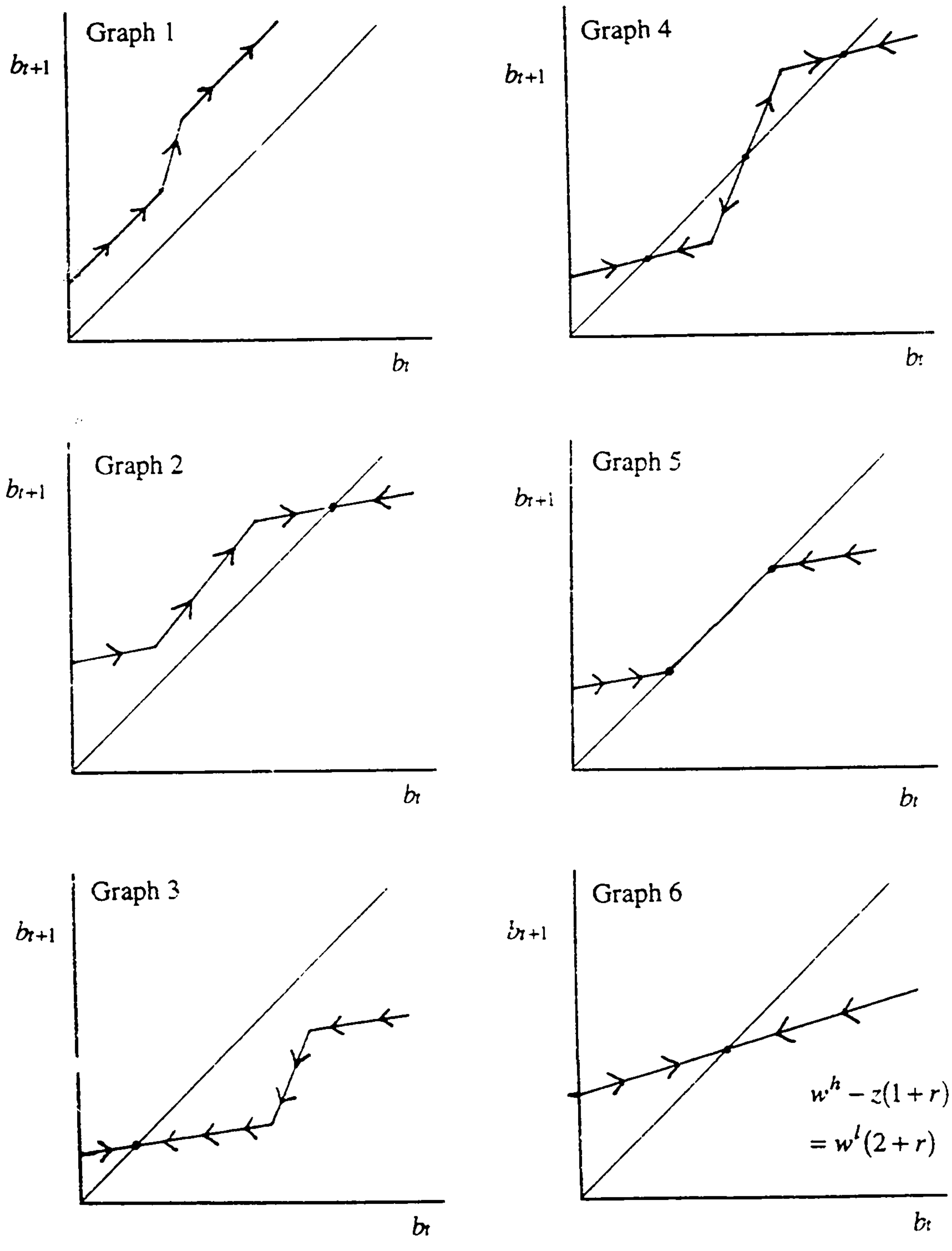

Figure 4.2

Steady State Diagrams

Conclusions

This dissertation has studied the interaction between human capital acquisition, growth and development, and the distribution of income. The first three chapters developed similar endogenous growth models with human and physical capital. The fourth chapter was set in a developing country context whereby inherited wealth is important to the decision to acquire skills due to credit constraints.

Chapter 1 analyzes optimal fiscal policy and the dynamics of the income distribution in a framework whereby heterogeneous agents endogenously accumulate human and physical capital. Agents differ in their initial endowment of each factor. The chapter provides an explicit closed-form solution for the entire transition path of individual and aggregate income, wealth , and consumption and show the long-run trend toward balanced growth. Based on this solution, I am able to construct the indirect utility function, evaluate optimal fiscal policy, and clearly show why focusing on growth maximization can produce misleading conclusions about welfare.

Chapter 2 presents a related endogenous growth model with human and physical capital. The economy is populated with agents who have different initial capital endowments and also vary in their ability to learn. Despite decreasing returns to private human capital, the model exhibits persistent inequality of income and wealth. The paper also contrasts optimal fiscal policy with growth maximization. Finally, when wage and capital tax rates can differ, the growth rate depends on the median voter's relative ratio of initial human capital endowment rather than the combined size of the initial endowments.

An extension to these first two chapters would be to introduce leisure into the utility function. This modification could have a particularly important impact on growth when there is differential tax of labor and capital earnings. It would also be interesting to study a more general specification of utility. We could learn how the elasticity of substitution impacts the result of time invariant tax rate preferences from Chapter 1, for instance. Furthermore, Chapter 2 assumed that the tax rates were constant over time. Allowing them to vary could link this work to the contribution by Chamley (1986) which found the optimal long-run tax on capital to be zero.

Chapter 3 augmented the analysis of optimal fiscal policy by introducing a consumption tax and government debt in addition to the income tax. However, in the presence of externalities in the human capital sector, this set of instruments is not able to correct for the distortion to time allocation. The first best could not be replicated. Future research could consider additional types of instruments such as a tax or subsidy to effective learning.

Finally, the last chapter featured overlapping generations with two period lives. Since individuals are constrained by high interest rates when they borrow to finance tuition, inherited wealth plays a critical role in the agent's decision to become educated. The initial distribution of wealth impacts the aggregate dynamics for capital accumulation and the production structure. The chapter presented some simple examples of the dynamics resulting from given initial distributions. This research could be extended by simulating more general distributions over longer time horizons. Several other extensions have been mentioned in the conclusion to Chapter 4. Additionally, related research may consider the possibility of efficiency wages in the skilled labor sector, a frequent assumption made when modeling development. When addressing the desirability of free trade, we could also examine the role of a nontraded goods sector or consider the middle country in a three country model.

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Appendix A

Solution to the Agent's Optimization Problem of Chapter 1

The first order conditions together with budget constraints (1.2) can be combined to derive a second order nonlinear difference equation for u as follows below (dropping the subscript i for convenience). From (1.6) and (1.9):

$$\frac{c_{t+1}}{c_t} = \frac{(1-\tau_{t+1})Ra\, y_{t+1}^k}{k_{t+1}} = \frac{R(1-b)u_t}{(1-u_t)u_{t+1}} \frac{(1-\tau_{t+1})}{(1-\tau_t)} \frac{y_{t+1}^k}{y_t^k} \tag{A1}$$

(A1) thus implies:

$$k_{t+1} = \frac{a(1-u_t)u_{t+1}}{(1-b)u_t}(1-\tau_t)y_t^k \tag{A2}$$

Using budget constraint (1.2) along with (A2) leads to:

$$\begin{aligned} c_t &= (1-\tau_t)y_t^k - k_{t+1} \\ &= \left[1 - \frac{a(1-u_t)u_{t+1}}{(1-b)u_t}\right](1-\tau_t)y_t^k \end{aligned} \tag{A3}$$

Combining (A1) and (A3):

$$\frac{c_{t+1}}{c_t}=\left[\frac{1-\dfrac{a(1-u_{t+1})u_{t+2}}{(1-b)u_{t+1}}}{1-\dfrac{a(1-u_t)u_{t+1}}{(1-b)u_t}}\right]\frac{(1-\tau_{t+1})y^k_{t+1}}{(1-\tau_t)y^k_t}=\left[\frac{R(1-b)u_t}{(1-u_t)u_{t+1}}\right]\frac{(1-\tau_{t+1})}{(1-\tau_t)}\frac{y^k_{t+1}}{y^k_t} \quad \text{(A4)}$$

Cancelling terms in the middle and right-hand side of (A4) produces the difference equation

$$\frac{R(1-b)u_t}{(1-u_t)u_{t+1}}+\frac{a(1-u_{t+1})u_{t+2}}{(1-b)u_{t+1}}=1+Ra \quad \text{(A5)}$$

Any solution to the agent's optimization problem must involve a path for u that satisfies (A5).

First define $X_t=\dfrac{u_{t+1}(1-u_t)}{u_t}$. (A6)

Then from (A5) we have a first order difference equation in X.

$$\frac{R(1-b)}{X_t}+\frac{a}{1-b}X_{t+1}=1+Ra \qquad \text{or}$$

$$X_{t+1}=\frac{1-b}{a}\left[1+Ra-\frac{R(1-b)}{X_t}\right] \quad \text{(A7)}$$

This equation has one stable solution $X=\dfrac{1-b}{a}$ and one unstable solution $X=R(1-b)$. See Graph 1. The first transversality condition in (1.8) requires $\lim_{t\to\infty}X_t<1-b$.

Therefore, X must begin and remain at the unstable solution, If $X_0 > R(1-b)$, X will converge to $X = \frac{1-b}{a}$ which violates (1.8). Likewise if $X_0 < R(1-b)$, X will become negative which is ruled out by the definitions of u and X. Given that $X_t = R(1-b)$ for all t, (A6) describes another first order nonlinear equation.

$$u_{t+1} = R(1-b)\frac{u_t}{1-u_t} \tag{A8}$$

This has an unstable solution $u = 1 - R(1-b)$ and a trivial solution u=0. See Graph 2. If $u_0 < 1 - R(1-b)$, u will converge to zero. But the second transversality condition in (1.8) can be used to show the restriction $\lim_{t \to \infty} u_t > b$. On the other hand, if $u_0 > 1 - R(1-b)$, u will become larger than one. This is ruled out by definition of u. Therefore, the unique solution to the agent's optimization problem of model 1 is $u_t = \bar{u} = 1 - R(1-b)$.

Appendix B

Solution to the Aggregate Equilibrium of Chapter 2

Equations (2.16)-(2.18) can be combined to derive a second order nonlinear difference equation for U as follows below. From (2.16) and (2.17):

$$\frac{C_{t+1}}{C_t} = \frac{\left(1-\tau_r\right) Ra\, Y_{t+1}}{K_{t+1}} = \frac{R(1-b)U_t}{\left(1-U_t\right)U_{t+1}} \frac{Y_{t+1}}{Y_t} \tag{B1}$$

(B1) thus implies:

$$K_{t+1} = \frac{a\left(1-U_t\right)U_{t+1}}{(1-b)U_t}\left(1-\tau_r\right)Y_t \tag{B2}$$

Using the aggregate resource constraint (2.18) along with (B2) leads to:

$$\begin{aligned} C_t &= \left(1-a\tau_r-(1-a)\tau_w\right)Y_t - K_{t+1} \\ &= \left[1-a\tau_r-(1-a)\tau_w - \frac{a\left(1-U_t\right)U_{t+1}}{(1-b)U_t}\left(1-\tau_r\right)\right]Y_t \end{aligned} \tag{B3}$$

Combining (B1) and (B3):

$$\frac{C_{t+1}}{C_t} = \left[\frac{1 - a\tau_r - (1-a)\tau_w - \dfrac{a(1-U_{t+1})U_{t+2}}{(1-b)U_{t+1}}(1-\tau_r)}{1 - a\tau_r - (1-a)\tau_w - \dfrac{a(1-U_t)U_{t+1}}{(1-b)U_t}(1-\tau_r)} \right] \frac{Y_{t+1}}{Y_t} = \left[\frac{R(1-b)U_t}{(1-U_t)U_{t+1}} \right] \frac{Y_{t+1}}{Y_t} \tag{B4}$$

Cancelling terms in the middle and right-hand side of (B4) produces the difference equation

$$\frac{R(1-b)U_t}{(1-U_t)U_{t+1}} \left[1 - a\tau_r - (1-a)\tau_w \right] + \frac{a(1-U_{t+1})U_{t+2}}{(1-b)U_{t+1}}(1-\tau_r) = 1 - a\tau_r - (1-a)\tau_w - Ra(1-\tau_r) \tag{B5}$$

Any solution to the agent's optimization problem must involve a path for U that satisfies (B5).

First define $X_t = \dfrac{U_{t+1}(1-U_t)}{U_t}$. (B6)

Then from (B5) we have a first order difference equation in X.

$$X_{t+1} = \frac{1-b}{a(1-\tau_r)} \left[1 - a\tau_r - (1-a)\tau_w + Ra(1-\tau_r) - \frac{R(1-b)}{X_t} \left[1 - a\tau_r - (1-a)\tau_w \right] \right] \tag{B7}$$

This equation has one stable solution $X = \dfrac{1-b}{a} \left[\dfrac{1 - a\tau_r - (1-a)\tau_w}{1-\tau_r} \right]$ and one unstable solution $X = R(1-b)$. Since the transversality conditions hold for all agents, they should hold for a reference agent with average ability and average initial capital endowments. Moreover, since the number of individuals in the economy sum to unit mass, aggregate variables are equal to average variables. Hence there is a pair of transversality conditions that the aggregate equilibrium must satisfy. The first transversality condition in (2.8) requires $\lim_{t\to\infty} X_t < 1-b$. Therefore, X must begin and remain at the unstable solution, If $X_0 > R(1-b)$, X will converge to $X = \frac{1-b}{a}\left[\frac{1-a\tau_r-(1-a)\tau_w}{1-\tau_r}\right]$ which violates (2.8) as long as τ_r, $\tau_w < 1$. Likewise if $X_0 < R(1-b)$, X will become negative which is ruled out by the definitions of U and X. Given that $X_t = R(1-b)$ for all t, (B6) describes another first order nonlinear equation.

$$U_{t+1} = R(1-b)\frac{U_t}{1-U_t} \tag{B8}$$

This has an unstable solution $U = 1-R(1-b)$ and a trivial solution U=0. If $U_0 < 1-R(1-b)$, U will converge to zero. But the second transversality condition in (2.8) can be used to show the restriction $\lim_{t\to\infty} U_t > b$. On the other hand, if $U_0 > 1-R(1-b)$, U will become larger than one. This is ruled out by definition of U. Therefore, the unique solution to the aggregate equilibrium is $U_t = \overline{U} = 1-R(1-b)$.

Appendix C

Matrix Results

The eigenvalues of **A** are *a(1-b)* and *1*. Therefore we can use the property of matrix powers that $\mathbf{X^n} = \mathbf{P}_X \mathbf{D}_X^n \mathbf{P}_X^{-1}$ where $\mathbf{D_X}$ is a diagonal matrix with the eigenvalues of **X** as the elements and $\mathbf{P_X}$ is a matrix of eigenvectors corresponding to each eigenvalue. Therefore,

$$\mathbf{A}^t = \mathbf{P}_A \begin{bmatrix} a^t(1-b)^t & 0 \\ 0 & 1 \end{bmatrix} \mathbf{P}_\mathbf{A}^{-1} \text{ where}$$

$$\mathbf{P_A} = \begin{bmatrix} \frac{a-1}{ab} & 1 \\ 1 & 1 \end{bmatrix} \text{ and } \mathbf{P}_\mathbf{A}^{-1} = \frac{1}{1-a+ab}\begin{bmatrix} -ab & ab \\ ab & 1-a \end{bmatrix}$$

Multiplying this out

$$\mathbf{A^t} = \frac{a^t(1-b)^t}{1-a+ab}\begin{bmatrix} 1-a & a-1 \\ -ab & ab \end{bmatrix} + \frac{1}{1-a+ab}\begin{bmatrix} ab & 1-a \\ ab & 1-a \end{bmatrix}. \text{ In addition,}$$

$$\sum_{j=0}^{t-1} \mathbf{A^j} = \left(\frac{1-a^t(1-b)^t}{(1-a(1-b))^2}\right)\begin{bmatrix} 1-a & a-1 \\ -ab & ab \end{bmatrix} + \frac{t}{1-a+ab}\begin{bmatrix} ab & 1-a \\ ab & 1-a \end{bmatrix}.$$

The eigenvalues of **S** are a^2, $(1-b)^2$, and *a(1-b)* thus

$$\mathbf{S^t} = \mathbf{P_S}\begin{bmatrix} a^{2t} & 0 & 0 \\ 0 & (1-b)^{2t} & 0 \\ 0 & 0 & a^t(1-b)^t \end{bmatrix}\mathbf{P_S^{-1}} \text{ where}$$

$$\mathbf{P_S} = \begin{bmatrix} 1 & \frac{1-a}{1-a-b} & \frac{2(1-a)}{1-a-b} \\ 0 & \frac{1-a-b}{1-a} & 0 \\ 0 & 1 & 1 \end{bmatrix} \text{ and } \mathbf{P_S^{-1}} = \begin{bmatrix} 1 & \left(\frac{1-a}{1-a-b}\right)^2 & -2\left(\frac{1-a}{1-a-b}\right) \\ 0 & \left(\frac{1-a}{1-a-b}\right) & 0 \\ 0 & -\left(\frac{1-a}{1-a-b}\right) & 1 \end{bmatrix}$$

Therefore, $$\mathbf{S^t} = \begin{bmatrix} a^{2t} & \left(\frac{1-a}{1-a-b}\right)^2\left[(1-b)^t - a^t\right]^2 & 2\left(\frac{1-a}{1-a-b}\right)a^t\left[(1-b)^t - a^t\right] \\ 0 & (1-b)^{2t} & 0 \\ 0 & \left(\frac{1-a}{1-a-b}\right)(1-b)^t\left[(1-b)^t - a^t\right] & a^t(1-b)^t \end{bmatrix}$$

For large t:

$$\sum_{j=0}^{t-1}\mathbf{A}^{t-1-j}\mathbf{V}\mathbf{S}^j \approx \frac{b(1-a)}{2(1-a+ab)}\begin{bmatrix} a^2 & (1-a)^2 & 2a(1-a) \\ a^2 & (1-a)^2 & 2a(1-a) \end{bmatrix}(\mathbf{I}-\mathbf{S})^{-1}$$

The eigenvalues of **B** are a and $(1\text{-}b)$ thus

$$\mathbf{B^t} = \mathbf{P_B}\begin{bmatrix} a^t & 0 \\ 0 & (1-b)^t \end{bmatrix}\mathbf{P_B^{-1}} \text{ where}$$

$$\mathbf{P_B} = \begin{bmatrix} 1 & \frac{1-a}{1-a-b} \\ 0 & 1 \end{bmatrix} \text{ and } \mathbf{P_B^{-1}} = \begin{bmatrix} 1 & -\left(\frac{1-a}{1-a-b}\right) \\ 0 & 1 \end{bmatrix}.$$

Therefore, $\mathbf{B}^{\mathbf{t}} = \begin{bmatrix} a^t & \left(\frac{1-a}{1-a-b}\right)\left((1-b)^t - a^t\right) \\ 0 & (1-b)^t \end{bmatrix}$

For large t:

$$\sum_{j=0}^{t-1} \mathbf{B}^{t-1-j}\mathbf{M}\mathbf{A}^{j} \approx \frac{1}{1-a+ab}\begin{bmatrix} ab & 1-a \\ ab & 1-a \end{bmatrix}$$

Appendix D

Properties of the Lognormal Distribution

Let x be a random variable and let the notation m, v, μ, and σ^2 be defined in the following manner. Let $m = E(x)$, $v = Var(x)$, $\mu = E(\ln x)$, and $\sigma^2 = Var(\ln x)$. Then following relationships hold:

$$m = e^{\mu + \frac{1}{2}\sigma^2}$$

$$v = e^{2\mu+\sigma^2}\left(e^{\sigma^2} - 1\right) = m^2\left(e^{\sigma^2} - 1\right)$$

$$\sigma^2 = \ln\left(1 + \frac{v}{m^2}\right)$$

$$\mu = \ln m - \frac{1}{2}\ln\left(1 + \frac{v}{m^2}\right) = \ln m - \frac{1}{2}\sigma^2$$

Furthermore, the matrixes μ_0 and σ_0 can be re-expressed as:

$$\mu_0 = \begin{bmatrix} \mu_{k0} \\ \mu_{h0} \end{bmatrix} = \begin{bmatrix} \ln m_{k0} - \frac{1}{2}\ln\left(1 + \frac{v_{k0}}{m_{k0}^2}\right) \\ \ln m_{h0} - \frac{1}{2}\ln\left(1 + \frac{v_{h0}}{m_{h0}^2}\right) \end{bmatrix}$$

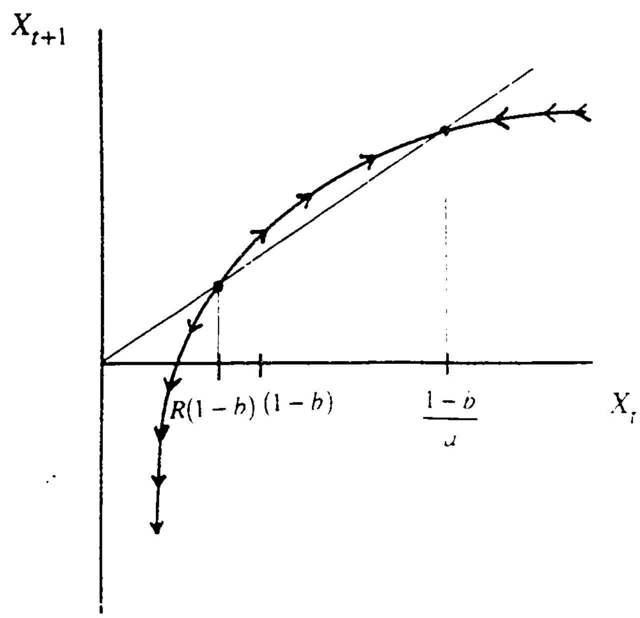
$$\sigma_0 = \begin{bmatrix} \sigma_{k,0}^2 \\ \sigma_{h,0}^2 \\ \sigma_{kh,0} \end{bmatrix} = \begin{bmatrix} \ln\left(1 + \frac{v_{k0}}{m_{k0}^2}\right) \\ \ln\left(1 + \frac{v_{h0}}{m_{h0}^2}\right) \\ \ln\left(1 + \frac{\rho_{kh,0}\sqrt{v_{k0}v_{h0}}}{m_{k0}m_{h0}}\right) \end{bmatrix}$$

where $\rho_{kh,0} = Corr(k_{i0}, h_{i0})$. Then equation (1.21b) can be transformed from the means and variances of log capital to the means and variances of capital.

$$\begin{bmatrix} \ln m_{kt} \\ \ln m_{ht} \end{bmatrix} = \left\{ \left(\tfrac{1}{1-a+ab}\right)^2 \begin{bmatrix} 1-a & a-1 \\ -ab & ab \end{bmatrix} + \tfrac{t}{1-a+ab} \begin{bmatrix} ab & 1-a \\ ab & 1-a \end{bmatrix} \right\} \mathbf{D} + \tfrac{1}{1-a+ab} \Big\{ ab \ln m_{k0} + (1-a) \ln m_{h0}$$

$$- \tfrac{ab}{2(1+a)} \ln\left(1 + \frac{v_{k0}}{m_{k0}^2}\right) - \left(\tfrac{1-a}{2}\right)\left[1 - \tfrac{(1-a)(1+a-ab)}{(1-a+ab)(1+a)(2-b)}\right] \ln\left(1 + \frac{v_{h0}}{m_{h0}^2}\right) + \tfrac{ba(1-a)}{(1+a)(1-a+ab)} \ln\left(1 + \frac{\rho_{kh,0}\sqrt{v_{k0}v_{h0}}}{m_{k0}m_{h0}}\right) \Big\},$$

Graph 1

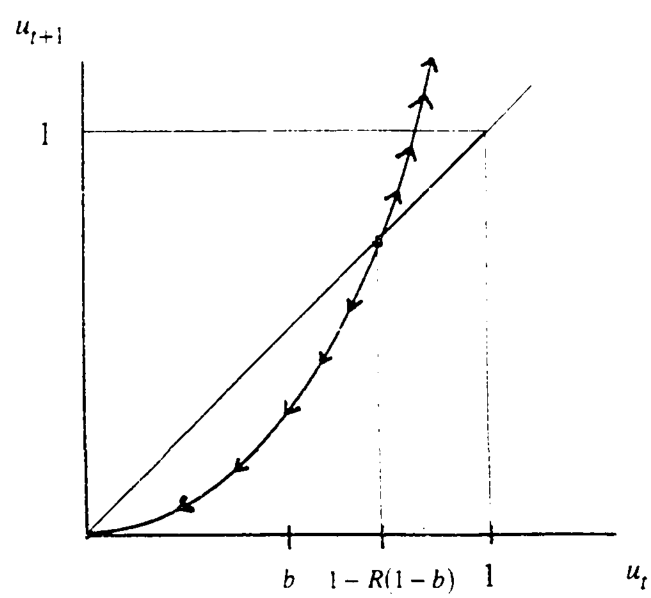


$$X_{t+1} = \frac{1-b}{a}\left[1 + Ra - \frac{R(1-b)}{X_t}\right]$$

$$X_t \geq 0$$

$$\lim_{t\to\infty} X_t < 1-b$$

Graph 2



$$u_{t+1} = R(1-b)\frac{u_t}{1-u_t}$$

$$0 \leq u_t \leq 1$$

$$\lim_{t\to\infty} u_t > b$$

Figure A.1

Dynamics of Time Allocation

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