Private Provision of Public Goods:

Uniform Price Mechanisms with a Threshold and Dynamics with a Tipping Point

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Abstract

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This dissertation comprises three manuscripts, all contributing to the literature of the private provision of public goods.

In Chapters 2 (complete information, CI) and 3 (private value information, PI), we introduce two novel mechanisms for provision point public goods: The uniform price auction mechanism (UPA) collects an endogenously determined uniform price from everyone offering at least that price, while the uniform price cap mechanism (UPC) collects the uniform price from everyone offering at least that price, plus the full offer of everyone offering less. With CI, UPC has the same undominated perfect equilibria as standard provision point (PPM) and proportional

rebate (PR) mechanisms—and UPA a somewhat broader set—but our mechanisms' wide-range-of-zero-marginal penalty structures may facilitate equilibrium selection and lead to higher contributions and more frequent provision. With PI, the uniform price mechanisms support Bayesian Nash equilibria (BNE) with higher contributions than the BNE of PPM or PR, potentially increasing efficiency. Our mechanisms outperform PR and PPM in both information environments in laboratory experiments: in general, UPC generates higher aggregate contributions and provision rates than PR and PPM; UPA attracts much higher contributions, although it provides less frequently. The ranking emerges because high offers are more common (especially among high-value people) in the uniform price mechanisms, where it is low cost to venture high offers to potentially meet other high offers to support provision.

In Chapter 4, we study durable public good games with a tipping point, below which collapsing the stock is optimal. With a payoff function linear in stock and income and a logistic growth function, we show the existence of a tipping point. Further, under a dynamic voluntary contribution mechanism (DVCM), both the open-loop equilibrium and the Markov perfect equilibrium (MPE) result in socially inefficiently low steady states and higher tipping points. Better outcomes can be supported in the MPE than in the open-loop solution and the highest stable steady state in the MPE approaches the efficient level asymptotically as the discount rate approaches zero. Lastly, we extend DVCM, introducing a provision point in a dynamic provision point mechanism (DPPM) and we show that the most rapid approach path to the efficient stock is supported in a MPE of DPPM.

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Chapter 1 Introduction

The central question in this dissertation is how to use market-based mechanisms to improve the private provision of environmental public goods, such as ecosystem services, including climate regulation, watershed services, biodiversity conservation, and recreation, etc., which are generally underprovided.

The underprovision is from both supply and demand. On the supply side, to induce potential providers to provide more public goods, there are many government supported environmental markets and programs. This is what most studies have focused on, for example, designing mechanisms to reveal providers' cost information. However, if you look through the reports from the United States Department of Agriculture, a common challenge to expanding these markets is lack of demand from the actual beneficiaries, which is a classic free rider problem. So, besides the government-supported programs, can we further increase the demand? Can we create a market to induce consumers to buy directly and buy more public goods? More generally, are there more effective and unsophisticated demand revelation mechanisms for public goods provision? The three manuscripts in this dissertation make progress in answering these questions. Some novel economic mechanisms were developed and were shown to be able to increase the public good provision in a simple market, where consumers directly pay producers.

This market has two key features. One is the discrete nature of the good: the good is provided only if a certain amount of money is raised, which is common in providing many public facilities or environmental conservation projects. For example, to provide adequate acres of wildlife habitat, farmland or forest, landowners need to be paid a minimum price. That price is a threshold and is called the provision point. The second feature is the durability of the good:

many ecosystem services depend on populations of animals or plants, and greenhouse gases have stock dynamics. Chapters 2 and 3 focus on the discrete feature in static games; Chapter 4 combines the two features in a dynamic game.

The discrete nature is a good property, because it induces provision in equilibrium: in a static setup, if the cost (provision point) is not reached, the good will not be provided, and no one will be better off from free riding. However, it is still difficult to provide the good in practice because people want to contribute as little as possible as long as the cost is covered. Therefore, there is a coordination or equilibrium selection problem which may discourage contribution. Partially, this is because people do not want to over-contribute in a standard provision point mechanism under which no contribution over the cost will be rebated. The concern of wasting over-contribution may discourage contribution in the first place, and generally we do not want to use the extra money to provide more goods either, to maintain the discrete nature. Thus, we need some effective rebate mechanisms. In Chapters 2 and 3, we introduce and characterize two novel mechanisms to encourage contribution by alleviating the over-contribution concern in complete and private value information, respectively. The idea is that if you offer high enough, you only pay a fixed endogenous price or something similar. We have two key results. First, they are indeed better in theory: they support higher contributions than no rebate and the most popular proportional rate in the Bayesian Nash equilibrium. Second, we do find they are better in lab experiments under both complete and private value information. Essentially, we get higher contributions from high value people.

In the past two decades, there have been many papers testing various rebate mechanisms, but to our best knowledge, Chapter 3 is the first one to systematically and clearly characterize the role of rebates in an appropriate incomplete information setup. Also, it suggests productive and promising avenues for designing new mechanisms.

Although durability is not new in the literature of public goods provision, many, if not most, of the existing studies rely on linear state equations, inconsistent with the nonlinearity of environmental public goods. Further, a major concern in providing these durable public goods is the existence of a tipping point (a threshold below which collapsing the stock is optimal), which is rarely captured endogenously in a game-theoretic model. Chapter 4 bridges these gaps by introducing and characterizing a model with a logistic growth function and an endogenously determined tipping point under various game-theoretic structures. We find, in general, that strategic behaviors result in inefficiently low steady states and high tipping points due to positive externalities, and that the socially efficient outcome can be approached only asymptotically when agents become infinitely patient. However, if a provision point is introduced, we show that the most rapid approach path to the efficient stock level is supported in a symmetric Markov perfect equilibrium, and the intuition is the same as in any provision point related mechanisms: the provision point breaks the free riding incentive by introducing a discontinuity in the payoff function. Methodologically, Chapter 4 also extends theoretical tools to solve Markov games in a singular optimal control problem. The key insight is the interdependence between state-dependent Markov contribution strategies and a non-singleton interval of singular stock levels.

Chapter 2 Uniform Price Mechanisms for Threshold Public Goods Provision with Complete Information: An Experimental Investigation

Abstract

We introduce two novel mechanisms for provision point public goods, motivated by the design of uniform price auctions: The uniform price auction mechanism (UPA) collects an endogenously determined uniform price from everyone offering at least that price, while the uniform price cap mechanism (UPC) collects the uniform price from everyone offering at least that price, plus the full offer of everyone offering less. UPC has the same undominated perfect equilibria as standard provision point (PPM) and proportional rebate (PR) mechanisms–and UPA a somewhat broader set—but our mechanisms' different marginal penalty structures may facilitate equilibrium selection and lead to higher contributions and more frequent provision. Through laboratory experiments, using both homogeneous (symmetric) and heterogeneous induced values, we show our mechanisms improve upon PR and PPM: UPC generates higher aggregate contributions than PR and PPM, leading to higher provision rates than PPM; UPA attracts much higher contributions, although it provides less frequently. This ranking emerges because high offers are more common (especially among high value people in the heterogeneous environment) in the uniform price mechanisms, where higher offers only increase the payment when needed for provision.

2.1 Introduction

A provision point public good is one that can be provided only when a threshold level of funding contributions is met. Canonical examples include bridges, parks and schools that require a discrete level of funding to cover building costs, but potentially continuous public goods such as public radio broadcasting and environmental conservation projects are sometimes marketed as provision point public goods in order to increase contributions. The public goods literature typically envisions determining outcomes through the provision point mechanism (PPM), in which people voluntarily and simultaneously contribute toward funding the good; if the total contribution reaches or exceeds the provision point (the threshold cost), the good is provided; otherwise contributions are refunded (money back guarantee). Because it is simpler than other public goods mechanisms in which provision is supported as a Nash equilibrium—in contrast to the unique zero-contribution prediction of the voluntary contribution mechanism—PPM has been systematically studied, both theoretically[1] and experimentally.[2]

However, PPM has a continuum of efficient Nash equilibria in which total contributions exactly equal the provision point (Bagnoli and Lipman, 1989), and theory is silent about the cost sharing rule. This equilibrium selection problem has led to an additional literature on whether and how different methods for rebating contributions in excess of the provision cost—attributable to imperfect coordination—affect incentives for making contributions. Marks and Croson (1998) compare no-rebate, proportional rebate (PR), and a "utilization" rebate in the form of higher public good levels, and find a utilization rebate leads to higher contributions, but no significant difference between no-rebate and PR under complete information. When value information is private, Gailmard and Palfrey (2005) find PR (called PCS in their paper) generates significantly higher contributions than no-rebate. Only a few of the possible factors affecting these mixed results have been explored. Rondeau et al. (1999) identify a significant group size effect, but no information (about group size and provision point) effects in PR; Rondeau et al.

[1] Bagnoli and Lipman (1989) study PPM under complete information; Nitzan and Romano (1990), McBride (2006), and Barbieri and Malueg (2010a) discuss threshold uncertainty; Alboth et al. (2001), Menezes et al. (2001), Laussel and Palfrey (2003), and Barbieri and Malueg (2008, 2010b) discuss PPM with private value information.

[2] See Chen (2008) for a recent review of related experimental studies; for earlier reviews see Davis and Holt (1993) and Ledyard (1995).

(2005) find PR generates higher contributions compared with the voluntary contribution mechanism; Spencer et al. (2009) show PR and five other lottery-based rebate rules result in full demand revelation with a group size of 45; all these studies are in one-shot games and do not provide comparisons between rebate mechanisms and PPM without rebate. We present a coherent framework for understanding how rebate rules affect contributions by differently "penalizing" higher offers when the provision point is exceeded, thus increasing or decreasing the chance of provision amid coordination failure, and use that to develop two novel mechanisms that outperform PPM and PR.

Our novel mechanisms are motivated by payment rules in multi-unit uniform price auctions (cf. Gailmard and Palfrey (2005) on excludable public goods). In our uniform price auction mechanism (UPA), everyone who pays, pays the same price: if there exists a price such that the number of contributions at or above that price multiplied by the price equals the provision point, then the good is provided, with only those offering at or above the uniform price paying the uniform price; the lowest such price will be chosen if more than one uniform price is possible. Our second mechanism addresses the inefficiency inherent in UPA, that contributions can exceed the provision cost, but still no uniform price meeting the provision rule exists. In the uniform price cap mechanism (UPC), no one pays more than the uniform price: if the provision point is exceeded, the lowest price cap will be calculated so whoever contributes above the cap only pays the cap, and those contributing less than the cap pay their full offer, such that the final collected payments equal the provision point.

The rebate rules in UPA and UPC present different incentives for making higher contributions than those of PPM and PR., Marks and Croson (1998) highlight the role of the "marginal penalty," the cost of contributing an additional dollar conditioned on provision, as differentiating among rebate rules since it captures how extra money is returned. For example, PPM has a marginal penalty of -1 since there is no rebate, and the marginal penalty in PR is, in general, between -1 and 0. Intuitively, in an environment with strategic uncertainty, lower marginal penalties will generate higher contributions and increase the likelihood of provision. However, the mixed results from Marks and Croson's (1998) and Gailmard and Palfrey's (2005) PR and PPM comparisons reveal that the role of marginal penalty is not fully understood. Our uniform price mechanisms introduce significant ranges with zero marginal penalty, extending the range of observable marginal penalty effects. Further, if larger regions of zero marginal penalty are important, UPA and UPC may improve performance over PPM and PR. This effect may be enhanced in the presence of other-regarding preferences as, relative to PPM and PR, higher offerors are not penalized with higher payments that lead to more unequal outcomes, unless the money is needed to secure provision.

Our experimental results show that, while average contribution behavior is consistent with the undominated perfect equilibrium predictions for each mechanism, the stable aggregate near the provision cost (or above in the case of UPA) belies a coordination process wherein subjects continuously adjust their offers in an effort to cheap ride. Thus, provision rates are ultimately determined by the willingness of other subjects to offer more than their "fair share" of the cost. The aggregate willingness to make these higher contributions is affected by the risk of venturing a higher offer, as captured by the marginal penalty. We find UPC generates higher contributions and provision rates than PR and PPM, and UPA generates much higher contributions.

The rest of the chapter is organized as follows. Section 2.2 defines precisely the four mechanisms to be compared. Section 2.3 characterizes the mechanisms' undominated perfect

Nash equilibria, and their respective marginal penalty structures. Section 2.4 describes the experimental design and procedures. Section 2.5 discusses the observed group and individual contributions. Section 2.6 synthesizes these results.

2.2 The Mechanisms

Suppose *N* agents each have endowment *I*. Each simultaneously chooses to make a contribution c_i to the provision of a threshold public good with a cost of *PP*. If the public good is provided, each agent receives a private value of v_i. If the public good is not provided, all contributions are refunded (money-back guarantee).

2.2.1 Provision Point Mechanism (PPM)

The payoff function for agent *i* under PPM is

$$(2.1) \qquad \pi_i = \begin{cases} I - c_i + v_i & if \quad \sum_{j=1}^{N} c_j \geq PP \\ I & otherwise \end{cases}$$

Under PPM, if *PP* is met or exceeded, each agent receives the initial endowment minus their contribution, plus their private value, v_i, for the public good; otherwise, they only get the initial endowment.

If *PP* is exceeded, PPM "burns" the excess contributions. Alternatively, excess contributions can be returned through rebate mechanisms, which may affect contribution strategies. PR, UPA, and UPC return excess contributions in different ways.

2.2.2 Proportional Rebate (PR)

Agent *i*'s payoff under PR is

$$(2.2) \qquad \pi_i = \begin{cases} I - c_i + v_i + \dfrac{c_i}{\sum_{j=1}^{N} c_j}\left(\sum_{j=1}^{N} c_j - PP\right) & if \ \sum_{j=1}^{N} c_j \geq PP \\ I & otherwise \end{cases}$$

Under PR, if $\sum_j c_j \geq PP$, the excess contribution ($\sum_j c_j - PP$) will be rebated. The rebate to each agent is proportional to the ratio of their individual contribution to the total contribution.

2.2.3 Uniform Price Auction (UPA)

Under UPA, a uniform price, *UP*, will be calculated. *UP* is the lowest price such that the number of contributions higher than that price times the price is equal to the provision point. The payoff under UPA is

$$(2.3) \qquad \pi_i = \begin{cases} I + v_i & \text{if } \sum_{j=1}^{N} c_j \geq PP,\ UP \text{ exists, and } c_i < UP \\ I - UP + v_i & \text{if } \sum_{j=1}^{N} c_j \geq PP,\ UP \text{ exists, and } c_i \geq UP \\ I & \text{otherwise} \end{cases}$$

where $UP = \min\{p > 0 : np = PP, n = |\{i : c_i \geq p\}|\}$. If an agent contributes less than *UP*, she pays nothing and the full c_i will be rebated. If an agent contributes *UP* or more, she will pay only the price *UP* and the excess contribution will be rebated. To provide the good, UPA requires not only that the total contribution meet or exceed *PP*, but also that the number of relatively high individual contributions be sufficient. More precisely, *PP* and the group size together determine a set of at most *N* possible prices, where *PP* is shared by $n \leq N$ agents offering at least PP/n. If the contributions in aggregate exceed *PP*, but cannot satisfy $np=PP$ *for some n*, the mechanism does not provide; UPA is not efficient.

2.2.4 Uniform Price Cap (UPC)

UPC is a modified version of UPA that ensures the good can be provided whenever total contributions exceed *PP*. The payoff under UPC is

$$(2.4)\quad \pi_i = \begin{cases} I - c_i + v_i & \text{if } \sum_{j=1}^{N} c_j \geq PP \text{ and } c_i < UC \\ I - UC + v_i & \text{if } \sum_{j=1}^{N} c_j \geq PP \text{ and } c_i \geq UC \\ I & \text{otherwise} \end{cases}$$

where $UC = \min\{p > 0 : \sum_{j \in \{j : c_j < p\}}^{N} c_j + np = PP, n = |\{i : c_i \geq p\}|\}$. Under UPC, if there are excess contributions, a uniform price cap (*UC*) will be calculated. If an agent contributes less than *UC*, she pays her full contribution c_i (under UPA they would pay nothing). If an agent contributes *UC* or more, she pays only the price cap and the excess contribution is rebated, just like under UPA. *UC* is calculated as the lowest price that could collect only the exact amount needed. Since the contributions lower than the price will not be rebated, the uniform cap *UC* always exists as long as *PP* is met or exceeded; UPC is efficient.

2.3 Theoretical Benchmarks: Nash Equilibrium and Marginal Penalty

To predict how the four mechanisms will affect individual and group contributions, we use undominated perfect equilibrium (UPE) and the marginal penalty associated with contribution beyond the provision point as theoretical benchmarks. UPE makes a precise prediction about group contributions, but includes a broad continuum of equilibria leading to that aggregate. We use marginal penalty to understand patterns of disequilibrium, which may be interpreted as a (non-refinement) selection process among UPE.

2.3.1 Undominated Perfect Equilibrium

Undominated perfect equilibria (UPE) eliminates dominated strategies before refining by trembling hand perfection, Bagnoli and Lipman (1989) show that combinations of contributions such that the provision point is exactly met and no one contributes more than their value, v_i are the UPE of PPM with complete information; these UPE are Pareto efficient. They also argue that, when rebate rules are incorporated into PPM, as long as the rebate scheme has the property that increasing one's contribution by $1 never increases one's rebate by more than $1, the resulting game has the same set of UPE. Since the rebate rules of PR and UPC both satisfy this property, and have the same condition for provision as PPM, they will have the same UPE as PPM (see appendix).

UPA has a different set of UPE from the other three mechanisms. Bagnoli and Lipman (1989) also require that the only condition of provision be that *PP* is met or exceeded, while UPA imposes constraints on the configurations of contributions that aggregate to meet the provision condition. We show in the appendix that **a UPE of UPA is any strategy profile such that one and only one uniform price of *PP/n* can be set, and no agent *i* chooses c_i greater than or equal to the lowest $PP/k \geq v_i$, for *k* in {*1,..,N*}**. Since the UPE of UPA are based on the possible uniform prices instead of group contributions, UPA has two main properties different from the other mechanisms. First, group contributions above *PP* are supported as equilibria. The only condition under which a uniform price *UP* exists when the provision point is exactly met is that $n=PP/UP$ is an integer number of agents each contributing *UP* and the other *N-(PP/UP)* agents choose $c_i=0$. There are at most *N* cases of UPE satisfying this condition; other UPEs of UPA involve group contributions strictly higher than *PP*. Second, the UPE of UPA does not exclude c_is that are greater than v_i (which are dominated in other mechanisms), as long as corresponding payments will not exceed v_i under any tremble. A contribution c_i above v_i is undominated as long as c_i is less than the lowest possible price higher than v_i. These two properties imply that the UPE of UPA includes aggregate and individual contributions that are not supported in the UPE of the other three mechanisms, which, respectively, include only the provision point and individual contributions not greater than v_i.

While the UPE refinement makes distinct predictions for UPA and the other mechanisms, it is inadequate in two ways. First, equilibrium predictions are not strongly predictive in existing PPM and PR experiments: Bagnoli and McKee (1991) report that the provision point is exactly met in only 54% of their PPM periods with five homogeneous subjects; Marks and Croson (1998) report 34% in PPM and 7% in PR. Second, there is still a wide continuum of individual UPE strategies in each mechanism, and three mechanisms have the same equilibrium strategy set, and so cannot explain differences in contributions among mechanisms. Therefore, we use the marginal penalty as a second theoretical benchmark.

2.3.2 Marginal Penalty of Overcontribution

The marginal penalty of overcontribution captures the private payoff loss associated with an additional unit of contribution, conditioned on provision. Marks and Croson (1998) argue that aggregate contributions will be higher when the marginal penalty is lower. The continuum of UPE makes selection among equally refined equilibria difficult, leading to a cost sharing coordination problem that makes excess contributions likely when provision occurs. The higher the loss associated with overcontribution, the more conservative agents may become about contributing more to increase the chance of provision in the face of strategic uncertainty about others' contributions, and thus the lower the aggregate contributions will be.

Figure 2.1 shows individual payoffs and the structures of marginal penalty for agent *i* under different mechanisms, conditioned on provision. For PPM (Panel A), since excess contributions will not be rebated, every dollar contributed to the public good beyond *PP* will be burned, and thus the marginal penalty of overcontribution is -1. For PR (Panel B), the marginal penalty is $\partial\pi_i / \partial c_i = -PP \cdot C_{-i} / C^2$ (Marks and Croson, 1998), where $C = \sum_j c_j$ and

$C_{-i} = \sum_{j \neq i} c_j$. This is bounded between -1 and 0, and almost always greater than -1. Marks and Croson argue the lower marginal penalty will lead to higher contributions in PR than PPM.

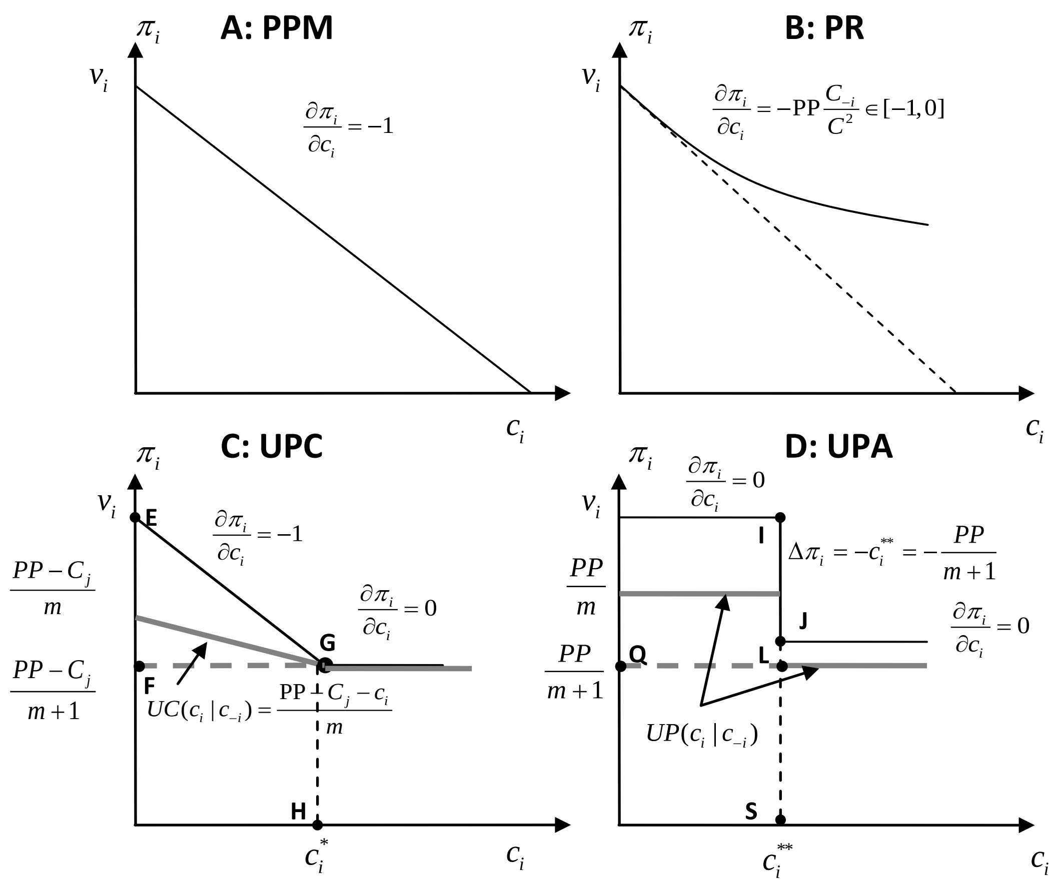

Figure 2.1 Individual Payoff and the Structures of Marginal Penalty, Conditioned on Provision

Note: π_i is agent *i*'s payoff; v_i is *i*'s induced value; c_i is *i*'s contribution; *PP* is provision point; *N* is the group size; $C_j = \sum_{j \in \{k: c_k < UC, k \neq i\}} c_j$; *m* indicates the number of agents contributing not less than the uniform price in UPC or UPA excluding agent i; $UC(c_i|c_{-i})$ and $UP(c_i|c_{-i})$ are respectively uniform cap and uniform price as a function of c_i given the others' contributions. c_i^* and c_i^{**} are the cutting points respectively for UPC and UPA; EF=FG=GH if assuming no price below $(PP - C_j)/(m+1)$ is possible in UPC; IJ=QL=LS; the initial endowment is set to be zero for convenience.

Our uniform price mechanisms aim to increase contributions by creating regions of zero marginal penalty. In UPC, if c_i is at or above $UC(c_i|c_{-i})$, the level of the cap that provides the good as a function of c_i given c_{-i}, then any incremental contribution will not change the uniform price and will be fully rebated, creating a marginal penalty of 0. If $c_i < UC(c_i|c_{-i})$, the marginal penalty is illustrated in Figure 2.1 (Panel C). Here, there exists a cutpoint, c_i^*, at which the marginal penalty changes from -1 to 0. In Figure 1, the intercept of the grey solid line with the *y*-axis represents the realized uniform price when c_i=0. When approaching c_i^* from below, $UC(c_i|c_{-i})$ decreases and the increased contribution is fully collected, leading to a marginal penalty of -1; when approaching c_i^* from above, $UC(c_i|c_{-i})$ stays constant at c_i^* and the marginal penalty is 0; at $c_i = c_i^*$, the marginal penalty is not defined. Based on beliefs about c_{-i}, agents calculate an expected marginal penalty between -1 and 0, implying higher group contributions in UPC than in PPM, while the ranking between UPC and PR would depend on beliefs.

The marginal penalty structure of UPA is similar to that of UPC, but with a critical difference. If $c_i < UP(c_i|c_{-i})$ (Figure 2.1, Panel D) where $UP(c_i|c_{-i})$ is the uniform price that provides the good through payments of *PP/m* by *m* other agents, there exists a cutpoint, c_i^{**}, at which *i*'s contribution is sufficient to be included in payments of the next lowest uniform price, and the final payment by *i* jumps from 0 to the new price, $UP(c_i^{**}|c_{-i}) = PP/(m+1)$. At all other points, the marginal penalty is 0, even when the contribution is lower than c_i^{**}. Thus, the marginal penalty of UPA is zero almost always, except at the cutpoint with a lump sum penalty. Given the broad range of values with no marginal penalty, we conjecture higher contributions in UPA than the other mechanisms.

Note that the marginal penalty structures of UPC and UPA not only suggest differences in aggregate contributions among mechanisms, but also if agents with higher v_is tend to make higher contributions, they are also suggestive of how incentives may differ across the range of values: agents with higher v_is may make higher contributions in UPC and UPA than in PPM and PR, because the marginal penalty for contributions typical of their value range is lower.

Similarly, we would also expect a higher contribution level from low value agents in UPA than in the other mechanisms, while the contribution level from low value agents in UPC could be higher or lower than that from PPM or PR.

2.4 Experimental Design and Procedures

To test the predictions of UPE and the effects of marginal penalty among the four mechanisms, we designed a controlled laboratory experiment in which subjects make contributions toward an induced value public good under complete information. Table 2.1 shows the mechanism treatments presented in each session: the first set of 3 treatments uses homogeneous induced values, and the second heterogeneous values. Within each set, the first treatment is always PPM (25 periods), to familiarize subjects with the baseline game; the following two treatments (25 periods each) apply the other mechanisms in a Latin Square to control for order effects.

Table 2.1 Treatment Arrangement of Experimental Sessions

Treatment Order	Homogeneous Induced Values (2 groups of size 5)			Heterogeneous Induced Values (1 group of size 10)		
	1st (25 pds)	2nd (25 pds)	3rd (25 pds)	4th (25 pds)	5th (25 pds)	6th (25 pds)
Session 1	PPM	PR	UPC	PPM	PR	UPC
Session 2	PPM	PR	UPA	PPM	PR	UPA
Session 3	PPM	UPC	PR	PPM	UPC	PR
Session 4	PPM	UPC	UPA	PPM	UPC	UPA
Session 5	PPM	UPA	PR	PPM	UPA	PR
Session 6	PPM	UPA	UPC	PPM	UPA	UPC

The homogeneous value treatments (v_i=10) have 2 groups of 5 subjects randomly regrouped among 10 session participants in each period, providing baseline comparisons among mechanisms. This setup is similar to that in Marks and Croson (1998), except we regroup each period to maintain the equilibrium selection problem that gives the best chance to observe the effects of marginal penalty differences among the mechanisms. The heterogeneous value treatments are designed to induce a broader range of contributions within each mechanism, to provide a yet stronger test of the marginal penalty effect. To have a wide range of possible uniform prices, we pool the 10 session participants in one group,[3] and assign one member to each induced value in {4, 4, 5, 5, 6, 6, 8, 8, 10, 12}, with the value-assignment reshuffled among group members in each period. The provision point is 30 in all treatments. These experimental parameters are chosen in a way that providing the good is always socially optimal, and no induced value is lower than the lowest UPA prices. All the information above is common knowledge.

The experimental software was developed in Z-Tree (Fischbacher, 2007). At the start of each treatment, the experimenter read the instructions aloud as subjects read along. Subjects were then given an initial budget of 15 experimental dollars. Subjects then simultaneously choose a contribution, $c_i \in [0,15]$ (with a precision of 0.1) towards the project. At the end of each period, subjects were informed whether the project is provided, and their earnings, payment and rebates. At the end of a session, earnings were totaled across all periods. Subjects were recruited through university-wide daily digest email server (mainly for undergraduates), and from an email list of students interested in participating in experiments. A total of 60 subjects participated in the six complete sessions, producing 9000 individual-level observations. Each session lasted roughly 1.5 hours with an average payment of $25.

2.5 Results

We measure the performance of the mechanisms by two indicators: aggregate group contribution and the provision rate. The provision rate reflects the efficiency of the mechanism, as provision is always efficient given our parameter values. Group contribution allows us to identify the role

[3] Our objective here is not to test for group size or heterogeneous value effects *per se*, but to provide a stronger test of differences among mechanisms in what we expect will be a more discriminating environment.

of marginal penalties, because they are meaningful only when there are excess contributions. In addition, group contribution is a measure of the extent of value revelation, which is of interest in cases where small-scale real money, real good provision point mechanisms are used to provide estimates of public value for non-market goods that are then applied over a broader population (e.g., Champ et al, 2002; Haskell et al., 2010; Bush et al., 2013; Swallow, 2013)

Figure 2.2 gives an overview of group contributions in each period, and five-period average provision rates, by mechanism in the homogeneous and heterogeneous induced value environments. Grey lines represent session-specific group contributions, dark lines represent averages over sessions, and dark dots represent five-period provision rates.[4]

2.5.1 Group Contributions

The uniform price mechanisms induce higher contributions—much higher in the case of UPA. In the homogeneous environment, UPC is also consistently above the provision point, PR oscillates around it, while PPM is consistently below it. In the heterogeneous environment, UPC is slightly higher than PR, and both of them are higher than PPM, which is in the neighborhood of the provision point. Importantly, across mechanisms, group contributions are generally not equal to the provision point, especially in the heterogeneous environment, reflecting the coordination problem.

[4] Each session-specific group contribution in a period in the homogeneous environment is the average contribution of two groups of size 5, since group members are reshuffled among 10 individuals in each period.

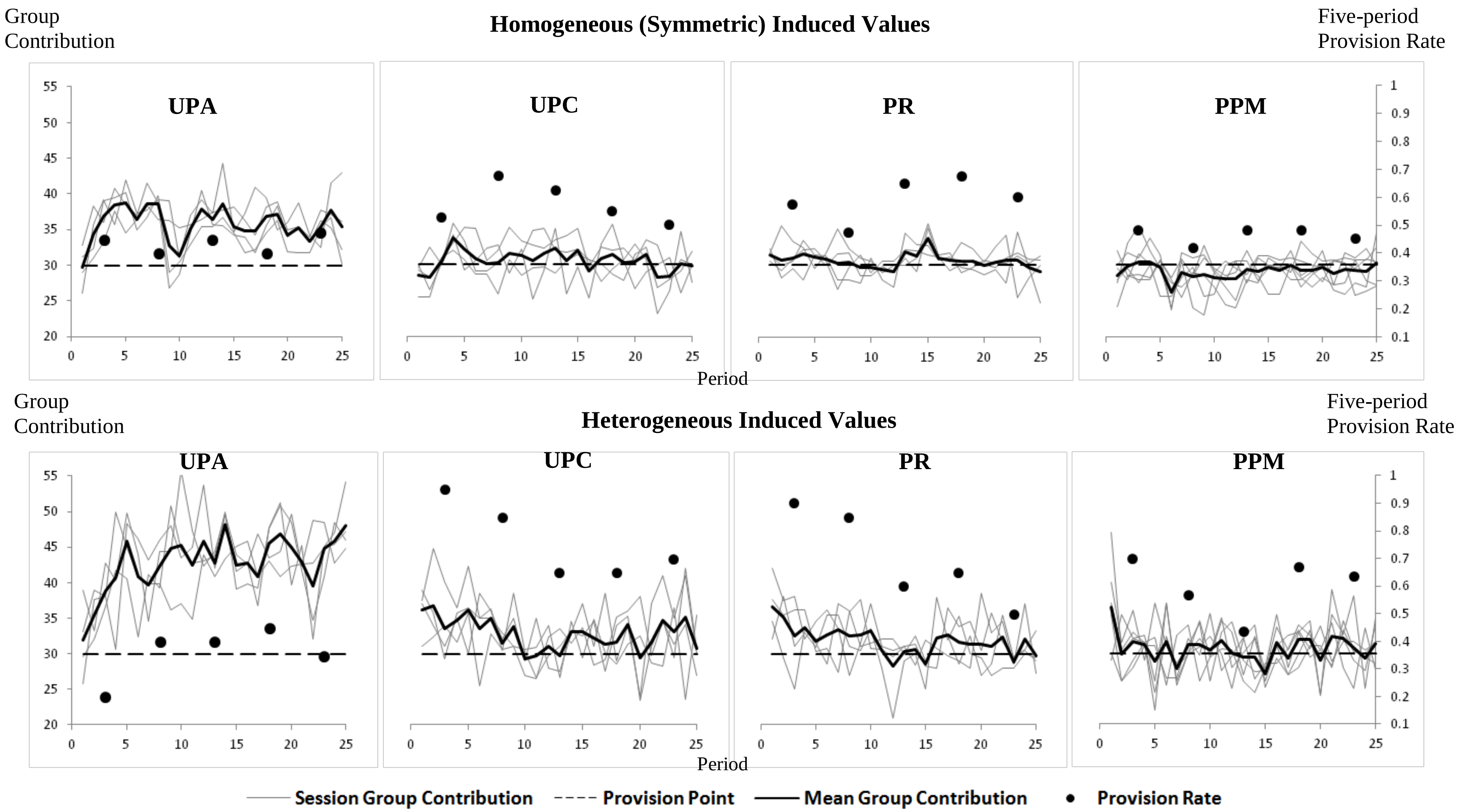

Figure 2.2 Group Contributions in Each Period and 5-Period Provision Rate by Mechanism

Table 2.2 presents results from 2-factor random effects regressions of group contributions in the last 20 periods of each treatment on mechanism dummies (group- and period-specific, cf. Marks and Croson, 1998). Models 1 and 3 each provide a baseline that includes only mechanism dummies, using PPM as the base, respectively for groups with homogeneous and heterogeneous values. Models 2 and 4 control for the previous two periods' provision rate, reflecting a Cournot-type response dynamic to minimize private costs based on previous provision outcomes (cf. Issac et al. (1989)'s notion of cheap riding).[5] Likelihood ratio tests advise using Models 2 and 4 for mechanism comparisons.

Table 2.2 Two-factor Random Effects Models of Group Contribution

Group Contribution	Homogeneous Values		Heterogeneous Values	
	(1)	(2)	(3)	(4)
PR	0.993***	0.925***	0.815	1.035*
	(0.346)	(0.344)	(0.542)	(0.538)
UPC	1.891***	1.955***	1.730***	2.036***
	(0.346)	(0.344)	(0.542)	(0.542)
UPA	7.053***	8.079***	13.37***	13.03***
	(0.346)	(0.545)	(0.542)	(0.544)
Provision Rate†				-1.955***
				(0.594)
Provision Rate × UPA		-2.387***		
		(0.990)		
Constant (PPM)	28.97***	28.97***	30.47***	31.59***
	(0.442)	(0.469)	(0.343)	(0.480)
Log-likelihood	-823.3	-820.5	-986.9	-981.6
Chi-square	435.2	448.4	716.8	749.2
R^2 overall	0.503	0.499	0.666	0.675
Number of observations	360	360	360	360
Number of periods (treatment-specific)	20	20	20	20

Standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1;
†: Provision rate over previous 2 periods, which yields the largest log-likelihood among 1 to 5-period lags.

Models 2 and 4 reflect an ordering of group contributions by mechanism that is broadly consistent with higher contributions occurring where the expected marginal penalty is lower.

[5] Since group members are reshuffled in the homogeneous environment and induced values are reshuffled in the heterogeneous environment, provision rate over several previous periods reflects how far away the average group contribution is from the provision point. In addition, we find the provision rate effect is significant only on UPA in Model 2 while on all mechanisms in Model 4. This seems to suggest that provision rate matters only when group contributions are not only statistically but also economically higher than the provision point, which makes sense since it may be too risky to "cheap ride" if group contributions are not way above the provision point.

UPA—with an almost-everywhere zero marginal penalty and expanded equilibrium outcome set (supporting contributions above *PP*)—is significantly higher than the others all with $p<0.001$ in both environments. Similarly, the lower expected marginal penalty from UPC leads to significantly higher group contributions than PPM and PR in both environments.[6] PR also generates contributions significantly higher than PPM, due to a smaller marginal penalty.[7]

These results contrast with Marks and Croson (1998), who find group contributions under PPM and PR are not statistically distinguishable. This may be attributable to the more challenging coordination process in our experiment providing a stronger test of the mechanisms: in their design, fixed groups of five homogeneous subjects were able to stabilize their total contributions at the provision point over 25 periods, while our groups were reshuffled every period, maintaining the strategic uncertainty and allowing the coordination problem to persist. Marks and Croson's group contributions are not significantly different from *PP* under both PPM and PR, based on a regression similar to our Model 1. However, in our data, aggregate contributions are generally not equal to the UPE predicted level of *PP*: in Model 1, the group contribution is significantly different from 30 in PPM (28.97; p=.020), and also borderline different in UPC (30.86; p=.070); similarly, in PR (31.28, p=.002) and UPC (32.20, p<.001) in Model 3. Although the group contributions in UPA—significantly higher than *PP* (homogeneous values: 36.02, p<.001; heterogeneous values: 43.84, p<.001)—are consistent with the UPE prediction for that mechanism, they still indicate that subjects cannot efficiently coordinate around one of a few possible prices, even a symmetric price (6) in the homogeneous environment. Therefore, our environment is more powerful for distinguishing among the mechanisms, and provides a better testbed for determining how the marginal penalty story

[6] Model 2: PPM vs. UPC (p<.001), PR vs. UPC (p=.010); Model 4: PPM vs. UPC (p<.001), PR vs. UPC (p=.087).
[7] Model 2: PPM vs. PR (p=.007); Model 4: PPM vs. PR (p=.054).

enhances the UPE prediction, and whether the uniform price mechanisms represent a practical improvement over PPM and PR for some applications.

2.5.2 Provision Rate

With the same provision condition among UPC, PR and PPM, the higher contributions associated with a lower expected marginal penalty generally result in a higher provision rate (Figure 2.2 dots). UPC—designed with marginal penalty in mind—performs weakly better than PR in the sense that UPC is significantly better than PPM in the heterogeneous environment while PR is not. Specifically, UPC has provision rates significantly higher than those for PPM in both environments (homogeneous: 58.8% vs. 45.8%, *z*-test p=0.011; heterogeneous: 71.3% vs. 57.5%, p=0.049). PR is significantly higher than PPM only with homogeneous values (60.0%, p=0.006; heterogeneous: 65.0%, p=0.288). With comparable expected marginal penalties, UPC and PR have similar provision rates in both environments (p=0.820 and p=0.396). Because the profile of offers, in addition to the total amount, affects the provision decision in UPA, it has provision rates significantly lower than the others in both homogeneous (43.1%) and heterogeneous (40.0%) environments all with p<0.01 except for PPM (45.8%, p=0.594) in the homogeneous environment.

To understand how a lower expected marginal penalty improves coordination, and ultimately induces higher group contributions and provision rates, we look into the differences among mechanisms at the individual contribution level. Figure 2.3 shows cumulative distributions of individual contribution by mechanism in the homogeneous and heterogeneous environments; these are essentially the selected strategy profiles.[8].

2.5.3 Distributions of individual contribution with homogeneous induced values

[8] In the heterogeneous environment, the distribution is over individual contributions from pooled induced values.

With homogeneous values, the observed distributions of contributions show how a wide continuum of efficient equilibria incentivizes people to deviate from the symmetric equilibrium ($c^*=6$) in hopes of favorably asymmetric cost sharing: observed individual contributions in all mechanisms have a wide range, inconsistent with playing the focal symmetric equilibrium, although the symmetric contribution of six is indeed the mode in all mechanisms (Figure 2.3, upper panel). This reveals the *fundamental coordination problem* of how much above six to contribute for subjects who are willing to compensate for those deviating below. With successful coordination admitting the possibility of excess group contributions attributable to the above-6 contributors, no rebate (-1 marginal penalty) may *hurt* the coordinating incentive, even to the point where average group contribution is below *PP* for homogenous PPM.

Therefore, the role of a lower expected marginal penalty in facilitating the coordination is to reduce the commitment associated with offering to coordinate on disadvantageous asymmetric equilibria, making the mechanism more robust to cheap riding. For conditional cooperators, an effective marginal penalty schedule will reduce the need to determine *ex ante* when coordination has failed in choosing a contribution, as the mechanism can choose the desired conditional contribution after observing all contributions. In fact, all rebate mechanisms attracted more individual contributions above 6 than PPM, led by the low-marginal-penalty uniform price mechanisms (Figure 2.3): 50% (z-test $p<0.001$) for UPA, followed by UPC (43%, $p<0.001$) and PR (37%, $p=0.041$), all compared to PPM (32%).[9]

[9] By Kolmogorov-Smirnov test, the cumulative distributions are indeed significantly different across mechanisms (PPM vs. PR with $p=.011$; all the other pairs with $p<.001$). Further, the significances are mainly due to the large differences of distributions in contribution above the symmetric level, which drives the mechanism ranking, except for the comparison between UPC and UPA, where the significance is due to the large difference in below-symmetric-level individual contribution.

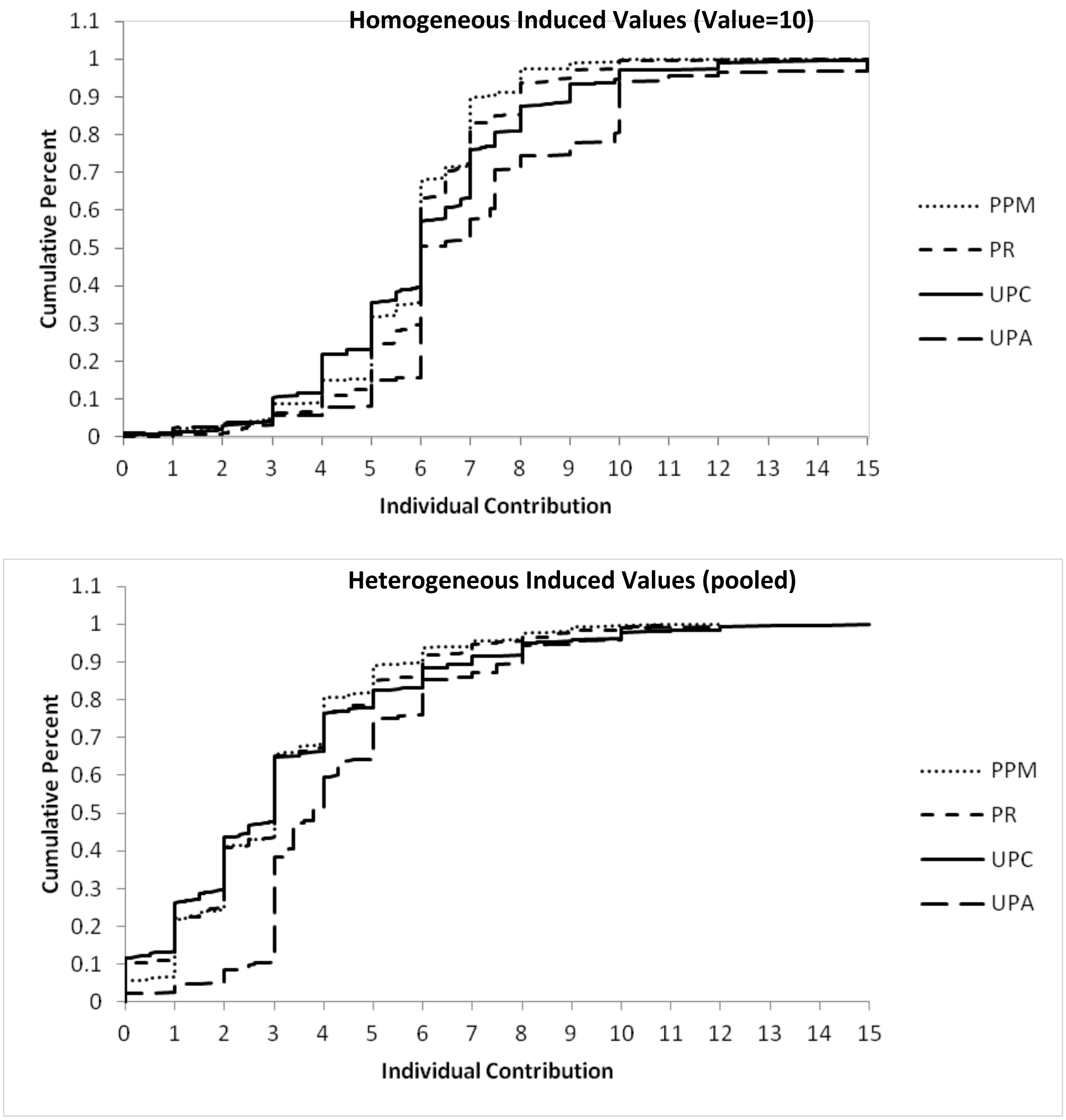

Figure 2.3 Cumulative Distribution of Individual Contribution by Mechanism

2.5.4 Distributions of individual contribution with heterogeneous induced values

With heterogeneous values, all rebate mechanisms generate a greater number of contributions above the symmetric UPE of 3 than PPM. They are ordered by marginal penalty, led by UPA, although the differences are smaller than in the homogeneous environment, except for UPA in the low-contribution range (Figure 2.3, lower panel).[10] Since the actual marginal penalty observed depends on the range of the contribution, and contribution is typically a function of value in heterogeneous environments, we compare mechanisms based on the marginal penalty structures which prevail in ranges of contributions observed to be associated with different induced values. Hence, we compare individual contributions by induced value in Figure 2.4, where average observed uniform prices from UPA and UPC are also included to show how being close to the kink in the marginal penalty function differentiates contributions across mechanisms.

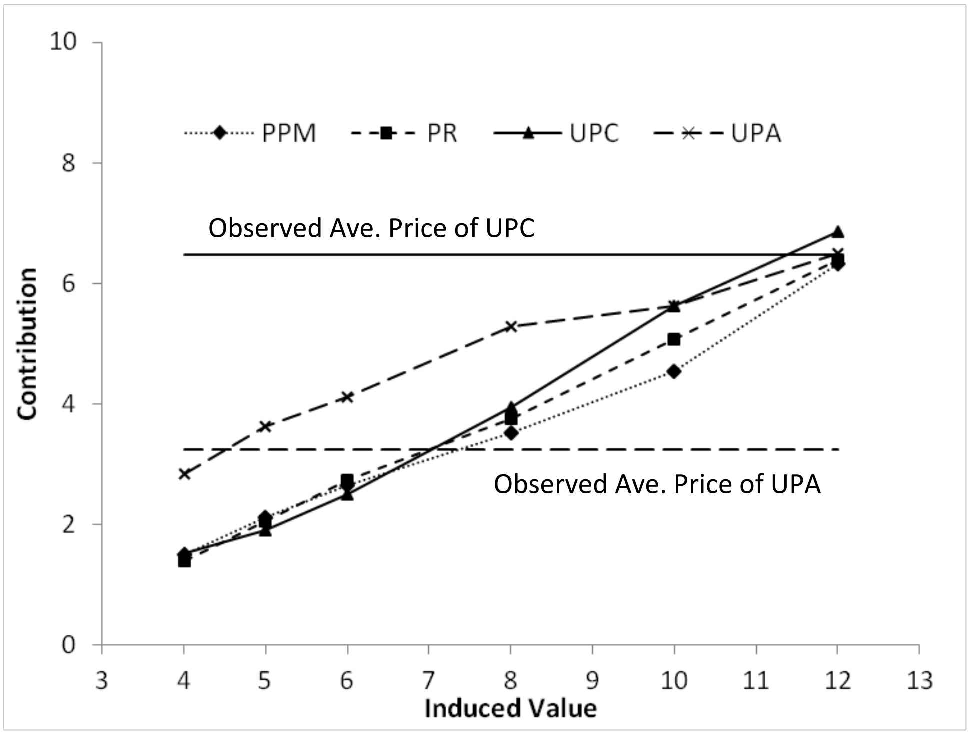

Figure 2.4 Mean Contributions by Induced Value

Average contributions increase with induced value in all mechanisms, but they are yet higher in the uniform price mechanisms (Figure 2.4): UPA stands out as generating much higher contributions, especially at low values, and UPC generates higher contributions at high values, both of which are consistent with the distribution of contributions in Figure 2.3.

[10] By Kolmogorov-Smirnov test, UPA is significantly ($p<0.001$) different from the others, and UPC is significantly different from PPM ($p=0.022$) but is similar to PR ($p=0.305$), with the latter two similar as well ($p=0.247$), all of which are consistent with the mechanism ranking.

The patterns of UPA and UPC in Figure 2.4 are supported by random effects tobit regressions of individual contributions on mechanism treatment (Table A2.3 in Appendices); further, they reflect subjects' responding differential marginal penalties within a mechanism across the value range. UPA's estimated contribution function has a significantly higher intercept and a significantly but slightly flatter slope than those for the other mechanisms, indicating that UPA generates higher contributions throughout the value range, more significantly at low values close to the observed average UPA price where a lump sum penalty occurs. With an intercept slightly lower (-0.746, p=0.023) than PPM, UPC's contribution function has a significantly steeper slope (0.697) than PPM (0.569, p<0.001) and PR (0.624, p=0.003), which implies UPC elicits higher contributions close to its observed uniform price than do PPM and PR, and above which the expected marginal penalty in UPC is lower.[11] PR generates meaningfully higher contributions than PPM only at the highest values, so differences in aggregate contributions are insignificant between PR and PPM.

2.6 Discussion

This chapter introduces two new mechanisms for threshold public goods, based on uniform price auctions: the uniform price auction mechanism (UPA) and the uniform price cap mechanism (UPC). It seeks to establish whether they perform better than the widely studied provision point mechanism without a rebate (PPM), and with a contribution-proportional rebate (PR). We first characterize these four mechanisms using the concepts of undominated perfect equilibrium (UPE) and the marginal payment penalty associated with over-contribution. We test the relative performance predictions through experiments characterized by complete information, with both homogeneous and heterogeneous values, and random regrouping between periods. Overall, the

[11] Across the values associated with contributions within one standard deviation of the average UPC uniform price (4.66-8.28) UPC generates contributions averaging 0.567 of induced value, significantly higher than PPM (0.491, p=0.037 by rank sum test); UPC is insignificantly higher than PR (0.519, p=0.506).

novel mechanisms improve upon those in the literature: UPA generated higher contributions, and UPC generated higher contributions and provision rates.

Understanding how the uniform price mechanisms generate higher contributions necessitates enhancing the equilibrium prediction provided by UPE: UPE establishes a continuum of equilibria for each mechanism, but coordinating on one among the equally refined equilibria causes groups to consistently make aggregate contributions that different from the provision point, with deviations that vary systematically by mechanism. It is in facilitating this coordination—off the equilibrium path—that differences among these mechanisms arise, and the differences that drive behavior are explained by the expected marginal penalty. The uniform price mechanisms UPA and UPC, both involving contribution ranges with zero-marginal penalty, generate significantly higher group contributions than PR, which has a marginal penalty between -1 and 0, which in turn attracts contributions higher than PPM, which has a marginal penalty of -1. These group contributions lead to a parallel order of provision rate, except for UPA which has more restrictive provision rules.

Examining the revealed strategy profiles, we find the coordination problem persists because many individual subjects are constantly adjusting their contributions in an effort to minimize their share of the cost when the good is provided. Provision, then, depends on some subjects offering above their equal share. The role of the rebate structure is to reduce the cost of making a high offer, so it may meet other high offers for purposes of coordination on disadvantageous asymmetric equilibria, making the mechanism more robust to cheap riding. Understanding this may be helpful in designing additional novel mechanisms, and identifying rebate structures best suited to specific applications with different information structures, distributions of values, and ratios between provision point and aggregate value.

That marginal penalty operates through facilitating coordination is supported by the observation that differences among mechanisms become more pronounced in environments where coordination is more difficult. In our experiment, the heterogeneous value treatment complicates coordination, since individual subjects may adopt different notions of “fair share” based on nominal contribution or share of value contributed. This also explains the mixed results comparing PPM and PR: Marks and Croson (1998) find no difference because they observe the same group of five homogeneous subjects playing the same game over 25 periods under complete information, a relatively easy coordination problem; while Gailmard and Palfrey (2005) find significant difference because they reshuffle group members playing a Bayesian game. Our environment is of intermediate complexity, two homogeneous groups of five people reshuffled among 10 in each period with complete information, and we have results consistent with Gailmard and Palfrey, suggesting Marks and Croson’s environment did not present a sufficient coordination challenge. Coordination difficulty also sheds light on the group size effect (6 vs. 50, Rondeau et al., 1999) and the full value (demand) revelation in large (45) groups (Rondeau et al., 1999; Spencer et al., 2009) in one-shot PR Bayesian games. Strategic uncertainty increases dramatically when group size changes from 6 to 45 or 50 in one-shot experiments with private information, and the rebate may facilitate the extremely challenging coordination problem by inducing much higher contributions.

Current work is extending this framework to games of incomplete information, where we conjecture not knowing where one falls in the realized value distribution yet further complicates coordination. Theoretically, this involves integrating over the distribution of others’ values in calculating the expected marginal penalty in order to enhance the Bayesian Nash equilibrium prediction. Empirically, this additional uncertainty could introduce enough noise into outcomes that subjects may have a hard time inferring the marginal penalty associated with different contributions, obscuring differences among mechanisms.

The novel uniform price UPA and UPC mechanisms do improve upon existing budget-balancing mechanisms in value revelation and provision of a threshold public good. If one is valuing a nonmarket good—whose eventual provision will be by an efficient coercive tax—through a small-scale pilot with real money, real public good to reduce hypothetical bias, then UPA minimizes the bias of welfare estimates caused by cheap riding (cf. Swallow et al, 2008). If efficient provision of the public good is the major concern, UPC represents a meaningful improvement over the well studied PPM and PR. This is especially true if the group being targeted for provision has a relatively large proportion of high value people, in which case the zero or low marginal penalty in the uniform price mechanisms allows them to make larger contributions, more fully revealing their values and increasing prospects for provision.

Chapter 3 Uniform Price Mechanisms for Threshold Public Goods Provision with Private Value Information: Theory and Experiment

Abstract

This chapter compares two novel uniform price mechanisms for provision point public goods to standard provision point (PPM) and proportional rebate (PR) mechanisms within a Bayesian game with private value information. The uniform price auction mechanism (UPA) collects an endogenously determined uniform price from everyone offering at least that price, while the uniform price cap mechanism (UPC) collects the uniform price from everyone offering at least that price, plus the full offer of everyone offering less. By rebating full amounts in excess of the price, the uniform price mechanisms create regions where the expected increase in payment associated with a higher offer is zero. We show that the uniform price mechanisms support Bayesian Nash equilibria (BNE) with higher contributions than BNE of PPM or PR, potentially increasing efficiency. We use laboratory experiments to test whether these more efficient BNE obtain, leading to higher contributions or more frequent provision. Our mechanisms outperform PR and PPM with private values: UPC generates higher aggregate contributions and provision rates than PR and PPM; UPA attracts much higher contributions, although it provides less frequently. This ranking emerges because high offers are more common (especially among high-value people) in the uniform price mechanisms, where it is low cost to venture high offers to potentially meet other high offers to support provision.

3.1 Introduction

The provision point mechanism (PPM) for public goods provision is one where the good can be provided only if a threshold level of funding contributions is met. After Bagnoli and Lipman (1989) showed that the equilibrium outcome is always efficient in undominated perfect equilibria with complete information, it has been systematically studied, both theoretically[12] and experimentally[13]. PPM's popularity in the economics literature can be attributed to the fact that many public goods have an inherent threshold or discrete nature for provision, such as parks, public radio broadcasting, and environmental conservation projects, where a minimum amount of funding is needed to provide one unit. The PPM requires only a slight modification—addition of the provision point—to the common open-ended donation solicitation used by charities, and hence many fundraisers use the PPM due to its simplicity and support for provision in equilibrium.[14]

One practical problem with the PPM is how to deal with the contribution in excess of the provision cost, attributable to a combination of incomplete value information and imperfect coordination within the continuum of efficient Nash equilibria. Without rebate, contributors may view the extra money as wasted, potentially creating a disincentive for contribution in the first place, and especially discouraging high offers. However, rebating extra money provides an opportunity to shift the off-path incentives of the PPM, and perhaps attract additional contributions: this has led to an additional literature on whether and how different methods for rebating contributions in excess of the provision cost affect contributions. The most popular rebate rule is the proportional rebate (PR), which rebates the extra contribution in proportion to

[12] Bagnoli and Lipman (1989) study PPM under complete information; Nitzan and Romano (1990), McBride (2006), and Barbieri and Malueg (2010a) discuss threshold uncertainty; Alboth et al. (2001), Menezes et al. (2001), Laussel and Palfrey (2003), and Barbieri and Malueg (2008, 2010b) discuss PPM with private value information.
[13] See Chen (2008) for a recent review of related experimental studies; for earlier reviews see Davis and Holt (1993) and Ledyard (1995).
[14] See real world examples in Bagnoli and Mckee (1991) and Marks and Croson (1998), or www.kickstarter.com.

the ratio of an individual's contribution to the total contribution.[15] The results in the experimental literature are mixed: Marks and Croson (1998) find no significant difference between PPM and PR under complete information, while Gailmard and Palfrey (2005) find PR (called PCS in their paper) generates significantly higher contributions than PPM when value information is private. Only a few of the possible factors affecting these mixed results have been explored (Rondeau et al.,1999; Rondeau et al, 2005; Spencer et al., 2009).

To provide a coherent framework for understanding how rebate rules affect contributions and also to improve upon PPM and PR, Li et al. (2014) introduce two novel uniform price mechanisms, the uniform price auction mechanism (UPA) and the uniform price cap mechanism (UPC). In UPA, everyone who pays, pays the same price: if there exists a price such that the number of contributions at or above that price multiplied by the price equals the provision point, then the good is provided, with only those offering at or above the uniform price paying the uniform price; the lowest such price will be chosen if more than one uniform price is possible. UPC addresses an inefficiency inherent in UPA, that contributions can exceed the provision cost, but still no uniform price meeting the provision rule exists. In UPC, no one pays more than the uniform price: if the provision point is exceeded, the lowest price cap will be calculated so whoever contributes above the cap pays only the cap, and those contributing less than the cap pay their full offer, such that the final collected payments equal the provision point.

The objective of the uniform price mechanisms is to design a rebate rule that induces higher overall contributions by alleviating participants' concern that contributing more to support provision may lead to losing more of the (over)contribution in the event of provision. This extent of concern is measured by the "marginal penalty," the cost of contributing an additional dollar conditioned on provision (Marks and Croson, 1998). In a true PPM, all overcontribution

[15] Marks and Croson (1998) and Gailmard and Palfrey (2005) provide comprehensive references on PR.

is wasted so the marginal penalty is -1, and in PR the marginal penalty is between -1 and 0. The two uniform price mechanisms, on the other hand, have a wide range of aggregate contributions where the marginal penalty is zero, and therefore they are expected to induce higher contributions. Through lab experiments, Li et al. (2014) find that the uniform price mechanisms do outperform PPM and PR under complete information. The insight is that the lower marginal penalty facilitates equilibrium selection by making it safer to tender higher offers, and therefore the uniform price mechanisms lead to higher contributions and more frequent provision.

A key feature missing in the application of these mechanisms in a complete information game is that the rebate is irrelevant in equilibrium, since excess contributions never occur in equilibrium,[16] so equilibrium theory is silent about how different rebate rules may affect the contribution behavior. In a Bayesian framework, however, excess contributions can occur when a value profile with higher-than-expected induced values is realized, and therefore the role of rebates can be explored by comparing the equilibria of each mechanism.

This chapter first elucidates how the alternative rebate rules affect the expected payoff function by examining the expected marginal penalty associated with higher offers in each mechanism. We then characterize the Bayesian Nash equilibria (BNE) of the two uniform price mechanisms, and compare the equilibrium sets with those of PPM and PR in an incomplete information setting. With an almost always zero-expected marginal penalty, UPA has a truth-telling BNE in a 2-player game, and depending on parameters, may support BNE where the expected group contribution is close to (more than 90% in our examples) the total expected induced value in a game with 3 or more players. UPC has a BNE characterization similar to PPM and PR as shown in Gailmard and Palfrey (2005). By a numerical example, we find

[16] Bagnoli and Lipman (1989) show in undominated perfect equilibria, the provision point is exactly met in PPM; and Li et al. (2014) verify that PR and UPC have the same set of undominated perfect equilibria as PPM.

rebates induce some BNE with higher contributions from high-value types: PR and UPC generate contributions comprising equilibria more efficient than PPM, and UPC induces higher contributions from high-value people than PPM and PR. These rebates work by reducing the expected payment cost for high-value people more than low-value people, since high-value people are more likely to face a value profile with higher-than-expected induced values and hence will experience overcontribution with a higher probability and pay more of the excess contribution in the absence of a rebate. Further, UPC has a marginal penalty structure that reduces the overcontribution cost more effectively than PR by only protecting high contributors: all excess contributions in UPC are returned to those contributing higher than the price-cap who are generally high-value people (as shown in **Proposition 3.2** below). These theoretical predictions are supported by our experimental results: UPC generates higher aggregate contributions and provision rates than PR and PPM; UPA attracts much higher contributions, although it provides less frequently.

The rest of the chapter is organized as follows. Section 2 defines precisely the four mechanisms to be compared, and analyzes the effect of their respective marginal penalty structures on their expected payments. Section 3 characterizes the BNE sets of UPA and UPC, demonstrates differences in the mechanisms' BNE sets with numerical examples, and explains the underlying role of marginal penalty in differentiating the sets of equilibria. Section 4 describes the experimental design and procedures. Section 5 discusses the observed group and individual contributions. Section 6 synthesizes these results.

3.2 The Mechanisms and Their Marginal-Penalty-Structure Effect on the Expected Payment

Suppose N agents each have endowment I. Each simultaneously chooses to make a contribution c_i to the provision of a threshold public good with a cost of PP. If the public good is provided, each agent receives a private value of v_i independently drawn from a common knowledge value distribution. If the public good is not provided, all contributions are refunded (money-back guarantee).

3.2.1 Provision Point Mechanism (PPM)

The payoff function for agent *i* under PPM is

$$(3.1) \quad \pi_i = \begin{cases} I - c_i + v_i & if \quad \sum_{j=1}^{N} c_j \geq PP \\ I & otherwise \end{cases}$$

Under PPM, if *PP* is met or exceeded, each agent receives the initial endowment *I* minus their contribution c_i, plus their private value, v_i, for the public good; otherwise, they only get *I*.

If contributions exceed *PP,* PPM "burns" the excess. Alternatively, excess contributions can be returned through rebate mechanisms, which may affect contribution strategies. PR, UPA, and UPC return excess contributions in different ways.

3.2.2 Proportional Rebate (3.PR)

Agent *i*'s payoff under PR is

$$(3.2) \quad \pi_i = \begin{cases} I - c_i + v_i + \frac{c_i}{\sum_{j=1}^{N} c_j}\left(\sum_{j=1}^{N} c_j - PP\right) & if \ \sum_{j=1}^{N} c_j \geq PP \\ I & otherwise \end{cases}$$

Under PR, if $\sum_j c_j \geq PP$, the excess contribution ($\sum_j c_j - PP$) will be rebated. The rebate to each agent is proportional to the ratio of their individual contribution to the total contribution.

3.2.3 Uniform Price Auction (UPA)

Under UPA, a uniform price, *UP,* will be calculated. *UP* is the lowest price such that the number of contributions higher than that price times the price is equal to *PP*. The payoff under UPA is

$$(3.3)\quad \pi_i = \begin{cases} I + v_i & \text{if } \sum_{j=1}^{N} c_j \geq PP,\ UP \neq \varnothing,\ and\ c_i < UP \\ I - UP + v_i & \text{if } \sum_{j=1}^{N} c_j \geq PP,\ UP \neq \varnothing,\ and\ c_i \geq UP \\ I & otherwise \end{cases}$$

where $UP = \min\{p > 0 : np = PP, n = |\{i : c_i \geq p\}|\}$. If an agent contributes less than *UP*, she pays nothing and the full c_i will be rebated. If an agent contributes *UP* or more, she will pay only the price *UP* and the excess contribution will be rebated. To provide the good, UPA requires not only that the total contribution meet or exceed *PP*, but also that the number of relatively high individual contributions be sufficient. More precisely, *PP* and the group size together determine a set of at most *N* possible prices, where *PP* is shared by $n \leq N$ agents offering at least *PP/n*. If the contributions in aggregate exceed *PP*, but cannot satisfy $np=PP$ for some *n*, the mechanism does not provide; with such an outcome, UPA is not efficient.

3.2.4 Uniform Price Cap (UPC)

UPC is a modified version of UPA that ensures the good can be provided whenever total contributions exceed *PP*. The payoff under UPC is

$$(3.4)\quad \pi_i = \begin{cases} I - c_i + v_i & \text{if } \sum_{j=1}^{N} c_j \geq PP\ and\ c_i < UC \\ I - UC + v_i & \text{if } \sum_{j=1}^{N} c_j \geq PP\ and\ c_i \geq UC \\ I & otherwise \end{cases}$$

where $UC = \min\{p > 0 : \sum_{j \in \{k : c_k < p\}} c_j + np = PP, n = |\{i : c_i \geq p\}|\}$. Under UPC, if there are excess contributions, a uniform price cap (*UC*) will be calculated. If an agent contributes less than *UC*, she pays her full contribution c_i (under UPA she would have paid nothing). If an agent contributes *UC* or more, she pays only the price cap and the excess contribution is rebated, just like under UPA. *UC* is calculated as the lowest price that could collect exactly *PP*. Since contributions lower than the price will not be rebated, the uniform cap *UC* always exists as long as contributions at least meet *PP*; with such an outcome, UPC is efficient.

3.2.5 The Effect of Marginal Penalty of Overcontribution on the Expected Payment

Marks and Croson (1998) use the concept of marginal penalty—the private payoff loss associated with an additional unit of contribution, conditioned on provision—to argue that rebate rules reduce the cost of making higher private offers in the absence of coordinating on a single equilibrium in their complete information game, increasing aggregate contributions and the likelihood of provision. It is insightful to translate their concept to an ex ante expected marginal penalty in our Bayesian framework, in order to better understand how rebate rules can influence BNE. Crucially, in a Bayesian game, a profile with high value-realizations will generate excess contributions in equilibrium, meaning the effect of lower marginal penalties can be identified within the equilibrium concept, as mechanisms with lower marginal penalties may support strategy profiles with higher contributions as equilibria.

To understand the marginal penalty associated with each of our mechanisms, we examine the expected payoff function, conditioned on others' strategies, of a 2-player Bayesian game with continuous types. Each agent has an induced value v_i, i=1, 2, independently drawn from $[\underline{v},\overline{v}]$ with a pdf $f(v_i) > 0, \forall\ v_i \in [\underline{v},\overline{v}]$, where $\overline{v} \in (PP/2, PP)$ to make a collective contribution necessary for provision. Without loss of generality, we discuss agent 1's expected payoff, conditioned on agent 2's contribution function $c_2(\cdot)$ of v_2. For simplicity, we assume $c_2(\cdot)$ is strictly increasing and differentiable over $[\underline{v},\overline{v}]$ and we omit the endowment *I* in the expected payoff function.

In PPM, agent 1's expected payoff of contributing c_1 conditioned on v_1 and $c_2(\cdot)$ is

(3.5) $$E\pi_1^{PPM}(c_1 \mid v_1, c_2(\cdot)) = v_1 \cdot \int_{v_2 \geq c_2^{-1}(PP-c_1)} f(v_2)\, dv_2 - c_1 \cdot \int_{v_2 \geq c_2^{-1}(PP-c_1)} f(v_2)\, dv_2 ,$$

where the integrations represent the probability of provision, and the two terms on the RHS are the expected benefit and cost, respectively. Take the derivative of (3.5) *w.r.t.* c_1, we have

(3.6) $$\frac{\partial E\pi_1^{PPM}(c_1 \mid v_1, c_2(\cdot))}{\partial c_1} = (v_1 - c_1) \cdot \frac{f\left(c_2^{-1}(PP-c_1)\right)}{c_2'\left(c_2^{-1}(PP-c_1)\right)} - \int_{v_2 \geq c_2^{-1}(PP-c_1)} 1 \cdot f(v_2)\, dv_2$$

On the RHS of (3.6), the first term represents the marginal net benefit due to an increased provision probability from a higher contribution. The second term, which is the one we are looking for, is the expected marginal penalty of overcontribution: it says given the same provision probability (the lower bound of the integration is unchanged), if c_1 increases by 1, the cost also increases by 1 (the integrand) conditioned on provision, reflecting a -1-marginal penalty in PPM. Rebates and hence a lower marginal penalty reduce ex ante this expected marginal penalty cost.

In PR, given the same $c_2(\cdot)$, we have

(3.7) $$E\pi_1^{PR}(c_1 \mid v_1, c_2(\cdot)) = v_1 \cdot \int_{v_2 \geq c_2^{-1}(PP-c_1)} f(v_2)\, dv_2 - \int_{v_2 \geq c_2^{-1}(PP-c_1)} \frac{c_1 \cdot PP}{c_1 + c_2(v_2)} f(v_2)\, dv_2$$

(3.8) $$\frac{\partial E\pi_1^{PR}(c_1 \mid v_1, c_2(\cdot))}{\partial c_1} = (v_1 - c_1) \cdot \frac{f\left(c_2^{-1}(PP-c_1)\right)}{c_2'\left(c_2^{-1}(PP-c_1)\right)} - \int_{v_2 \geq c_2^{-1}(PP-c_1)} \frac{PP \cdot c_2(v)}{(c_1 + c_2(v))^2} \cdot f(v_2)\, dv_2$$

Compared to (3.6), the only difference in (3.8) is the integrand on the RHS, which is the marginal penalty in PR (exactly the same as defined in Marks and Croson, 1998). Note that the marginal penalty is bounded between -1 and 0, almost always greater than -1, and becomes closer to 0 as c_1 increases. Given c_1 and $c_2(\cdot)$, the expected marginal penalty in PR is less than that in PPM, especially for higher contributors, and hence higher contributions may be supported in equilibrium in PR, consistent with Marks and Croson.

Our uniform price mechanisms aim to increase contributions by creating regions of zero-marginal penalty, which result in even lower expected payments. In a 2-player game of UPA, the good is provided only if both agents contribute *PP*/2 or above and each pays *PP*/2, so

$$(3.9)\ E\pi_1^{UPA}(c_1 \mid v_1, c_2(\cdot)) = \begin{cases} 0 & if\ \ c_1 < PP/2 \\ v_1 \cdot \int_{v_2 \geq c_2^{-1}(PP/2)} f(v_2)\, dv_2 - \dfrac{PP}{2} \cdot \int_{v_2 \geq c_2^{-1}(PP/2)} f(v_2)\, dv_2 & if\ \ c_1 \geq PP/2 \end{cases}$$

Compared to (3.5) and (3.7), the expected payoff in UPA is independent of c_1 within [0, *PP*/2) and [*PP*/2, *PP*). Since agent 1 pays either 0 or *PP*/2, the marginal penalty is always 0 except at *PP*/2 across which the payment jumps from 0 to *PP*/2. Thus, the expected marginal penalty is always zero except at *PP*/2, where the expected payment is $(PP/2)\int_{v_2 \geq c_2^{-1}(PP/2)} f(v_2)\, dv_2$. More generally in an *N*-player game (*N*>2), as shown in Li et al (2014), if $c_1 < UP(c_1|c_{-1})$ where $UP(c_1|c_{-1})$ is the uniform price that provides the good through payments of *PP/m* by *m* other agents, there exists a cutpoint, $c_{1,UPA}^{cp} \in (c_1, UP(c_1|c_{-1}))$ at which agent 1's contribution is sufficient to be included in payments of the next lowest uniform price, and the final payment jumps from 0 to the new price, $UP(c_{1,UPA}^{cp}|c_{-1}) = PP/(m+1)$. Then, the marginal penalty of UPA is zero almost always, except at the cutpoint with a lump sum penalty, resulting in an expected marginal penalty structure similar to the 2-player case. Given the broad range of values with no expected marginal penalty, we conjecture higher contributions in UPA than the other mechanisms.

In UPC, the uniform cap is $PP-c_1$ when $c_1<PP/2$, and is $\max\{PP-c_2(\cdot), PP/2\}$ when $c_1 \geq PP/2$, so we have

$$\text{(3.10)}\quad E\pi_1^{UPC}(c_1 \mid v_1, c_2(\cdot)) = \begin{cases} v_1 \cdot \int_{v_2 \ge c_2^{-1}(PP-c_1)} f(v_2)\, dv_2 - c_1 \cdot \int_{v_2 \ge c_2^{-1}(PP-c_1)} f(v_2)\, dv_2 & \text{if } c_1 < PP/2 \\ v_1 \cdot \int_{v_2 \ge c_2^{-1}(PP-c_1)} f(v_2)\, dv_2 - \int_{c_2^{-1}(PP-c_1)}^{c_2^{-1}(PP/2)} [PP - c_2(v_2)] f(v_2)\, dv_2 & \\ \qquad - \frac{PP}{2} \cdot \int_{v_2 \ge c_2^{-1}(PP/2)} f(v_2)\, dv_2 & \text{if } c_1 \ge PP/2 \end{cases}$$

(3.11)

$$\frac{\partial E\pi_1^{UPC}(c_1 \mid v_1, c_2(\cdot))}{\partial c_1} = \begin{cases} (v_1 - c_1)\dfrac{f\left(c_2^{-1}(PP-c_1)\right)}{c_2'\left(c_2^{-1}(PP-c_1)\right)} - \int_{v_2 \ge c_2^{-1}(PP-c_1)} 1 \cdot f(v_2)\, dv_2 & \text{if } c_1 < PP/2 \\ (v_1 - c_1)\dfrac{f\left(c_2^{-1}(PP-c_1)\right)}{c_2'\left(c_2^{-1}(PP-c_1)\right)} & \text{if } c_1 > PP/2 \end{cases}$$

UPC has a hybrid expected payoff structure of PPM (3.5) and UPA (3.9). Thus, when $c_1 < PP/2$, UPC has the same expected marginal penalty $-\int_{v_2 \ge c_2^{-1}(PP-c_1)} 1 \cdot f(v_2)\, dv_2$ as PPM. When $c_1 > PP/2$, UPC has a zero-expected marginal penalty, similar to UPA: the expected marginal cost of increasing c_1 is only due to the increased provision probability while not the marginal penalty of overcontribution (i.e., a zero-marginal penalty). At $c_1 = PP/2$, the expected marginal penalty is not defined. It is easy to verify that, in an N-player game ($N>2$), if c_1 is at or above $UC(c_1|c_{-1})$, the uniform cap that provides the good as a function of c_1 given c_{-1}, then any incremental contribution will not change the cap, creating a marginal penalty of 0. If $c_1 < UC(c_1|c_{-1})$, there exists a cutpoint, $c_{1,UPC}^{cp}$, at which the marginal penalty changes from -1 to 0 ($c_{1,UPC}^{cp} = PP/2$ in a 2-player game). Hence, UPC generally has a zero-expected marginal penalty when the individual contribution is high enough,[17] and based on beliefs about c_{-1}, agents calculate an expected

[17] Intuitively, this is due to our design of UPC to focus on protecting high contributors: the excess offer is only rebated to those contributing above the cap, in contrast to PR where all contributors share the excess offer.

marginal penalty between -1 and 0. Compared to PPM and PR, UPC may induce even higher contributions bearing in mind that the major excess offer would be from high contributors.

In contrast to in a complete information game where PPM, PR, and UPC have the same Pareto efficient equilibrium set, in Bayesian games these expected marginal penalties do differentiate the mechanisms' equilibrium sets and provide theoretical benchmarks for further experimental comparisons, as shown in the next section.

3.3 Bayesian Nash Equilibria

To proceed, we focus on Bayesian games with discrete types.[18] We first characterize some basic properties of the BNE sets of UPC and UPA as Gailmard and Palfrey (2005) do for PPM and PR. Then we solve for symmetric BNE of UPA in a 3-player game and discuss the general BNE solution structure and the value revelation property of UPA in an *N*-player game. Lastly, we differentiate the BNE sets of the four mechanisms using a numerical example.

3.3.1 Basic Properties of the BNE sets of UPC and UPA

Following the model setup in the beginning of Section 2, agent i's induced value v_i is independently drawn from a finite set V of real numbers with a common knowledge probability distribution function. V is the same for all agents. We assume $\max\{V\} \in \left(PP/N, PP\right)$, so, as in the interesting (and challenging real) context, provision is not optimal for any individual. All above information is common knowledge except that the realized v_i is only known to agent i.

Let $v = (v_1, \ldots, v_N) \in V^N$ denote a profile of values, $c(v) = (c_1(v_1), \ldots, c_N(v_N))$ denote a

[18] Although the role of rebates and the corresponding marginal penalty structure can be clearly revealed in a continuous-type analysis, these mechanisms except for UPA are much more difficult to analyze with private value information and continuous types: there is only a small body of literature on PPM (Alboth et al., 2001; Menezes et al., 2001; Laussel and Palfrey, 2003), and even no analysis on PR. Solving PR or UPC alone may need a full-length paper. On the other hand, it is easier to solve numerically a game with discrete types without losing the key insights about the mechanisms, as Gailmard and Palfrey (2005) do for PPM and PR. Therefore, we will focus on discrete types in the following sections of the paper.

contribution strategy profile with one contribution function for each agent, and

$v_{-1} = (v_1, \ldots, v_{i-1}, v_{i+1} \ldots, v_N)$ and similarly for $c_{-1}(v_{-1})$. Further, let $P_i(c_i \mid c_{-i})$ denote the provision probability when agent i contributes c_i given the contribution functions of the others, let $s_i(c(v))$ denote agent i's final payment as a function of $c(v)$, and let $S_i(c_i) = E_{v_{-i}}(s_i(c(v)) \mid c_i)$ denote i's expected payment given contribution c_i.

Gailmard and Palfrey (2005) present three common properties of the BNE sets of PPM and PR: no overbidding, payment monotonicity and contribution monotonicity. We show that the BNE sets of UPC and UPA have three similar properties with some regularity conditions.

Proposition 3.1 (No overbidding). In UPC, any strategy in which $c_i(v_i) > v_i$ for some v_i is ex post weakly dominated; in UPA, any strategy in which $c_i(v_i)$ is greater than the lowest $PP/k \geq v_i$ for $k \in \{1, \ldots, N\}$ and some v_i, is ex post weakly dominated.

Proof: See Appendix.

Lemma 3.1 (Payment monotonicity). In UPC, if $P_i(c_i \mid c_{-i}) > 0$ and c_i is less than *PP* and the cutpoint $c^{cp}_{i,UPC}$ associated with some uniform cap for some value profile v, $S_i(c_i)$ is strictly increasing at c_i; in UPA, if $P_i(c_i \mid c_{-i}) > 0$ and c_i is less than *PP* and the cutpoint $c^{cp}_{i,UPA}$ associated with some uniform price for some value profile v, $S_i(c_i)$ is strictly increasing at c_i in the sense that when c_i increases to a higher uniform price, $S_i(c_i)$ increases.

Proof: See Appendix.

Note that the cutpoints $c^{cp}_{i,UPC}$ and $c^{cp}_{i,UPA}$ are defined as in Section 2.5.

Proposition 3.2 (Contribution monotonicity). For both UPC and UPA, let $c^*(v)$ be a symmetric Bayesian Nash equilibrium contribution function.[19] If $P_i(c^*(v) \mid c^*_{-i}) > 0$ for all $v > \min\{V\}$ and $c^*(v)$ is less than the cutpoint associated with some uniform cap/price for some value profile v, then, for all v_i and v_j, $v_i > v_j \Rightarrow c^*(v_i) \geq c^*(v_j)$.

Proof: See Appendix.

The three properties of UPC are exactly the same as those of PPM and PR, except for the additional regularity condition that c_i or $c^*(v)$ is less than the cutpoint associated with some uniform cap for some value profile v. This regularity condition eliminates the uninteresting and extremely rare cases that c_i or $c^*(v)$ is always greater than any possible uniform cap for all value profiles; in these rare cases $S_i(c_i)$ is only weakly increasing at c_i. In addition to a similar regularity condition, the properties of UPA in **Proposition 3.1** and **Lemma 3.1** are restated to reflect the step-function form of the payment scheme, since in UPA realized payments can only be one of a finite number of uniform prices. With these adjustments, the proofs of the properties of UPC and UPA are similar to those for PR in Gailmard and Palfrey (2005).

These regularity conditions reflect some additional properties of UPC and UPA due to their marginal penalty structure. Next, we use symmetric BNE of UPA to show its advantage in generating higher contributions, and then demonstrate how UPC results in BNE sets different from PPM and PR and discuss the underling incentives.

3.3.2 Symmetric BNE of UPA

Since there are only finite possible prices in UPA, it is equivalent to a Bayesian game with discrete actions. It is known that a BNE solution to this kind of game has the form of a decision

[19] We abuse the notations here and in the following discussions to emphasize the symmetry: v and $c^*(v)$ are both scalars, representing one agent's value and equilibrium contribution strategy.

rule based on some critical values. Therefore, we can solve the Bayesian game for UPA by identifying the decision rules and the critical values. We will solve for symmetric BNE of UPA in a 3-player game first, and discuss the general BNE solution structure in an *N*-player game.

Without loss of generality, we assume agents only contribute the possible uniform prices. In a 3-player game, each agent has three contribution choices {0, *PP*/3, *PP*/2} and two critical values will be enough to characterize the decision rule. Let v_3^c and v_2^c denote the two critical values for a symmetric pure-strategy weakly monotonic BNE, assuming $0 \le v_3^c \le v_2^c \le \overline{v}$, where $0 = \min\{V\}$ and $\overline{v} = \max\{V\}$. The contribution function for each agent is in the general form

$$(3.12)\quad c(v) = \begin{cases} 0 & if\ \ v \le v_3^c \\ PP/3 & if\ \ v_3^c < v \le v_2^c \\ PP/2 & if\ \ v > v_2^c \end{cases}$$

Then the BNE of UPA in the 3-player game are as follows.

Proposition 3.3. In a 3-player Bayesian game, UPA has the following four categories of symmetric BNE: for *i* = 1, 2, 3,

a) $c_i^{BNE}(v_i) = 0$, with $v_3^c = v_2^c = \overline{v}$.

b) $c_i^{BNE}(v_i) = \begin{cases} 0 & if\ \ v_i \le PP/3 \\ PP/3 & if\ \ v_i > PP/3 \end{cases}$, with $v_3^c = PP/3$ and $v_2^c = \overline{v}$.

c) $c_i^{BNE}(v_i) = \begin{cases} 0 & if\ \ v_i \le \hat{v} \\ PP/2 & if\ \ v_i > \hat{v} \end{cases}$,

with $\overline{v} > \hat{v} = v_3^c = v_2^c > PP/2$ and $\hat{v}$ given by $(\hat{v} - PP/3)\ \Pr(v > \hat{v}) = (\hat{v} - PP/2)$.

d) $c_i^{BNE}(v_i) = \begin{cases} 0 & if\ \ v_i \le v_3^c \\ PP/3 & if\ \ v_3^c < v_i \le PP/2 \\ PP/2 & if\ \ v_i > PP/2 \end{cases}$, with $v_2^c = PP/2 > v_3^c > PP/3$ and v_3^c given by $v_3^c \Pr(v > v_2^c)^2 = (v_3^c - PP/3)\Pr(v > v_3^c)^2$.

Proof: See Appendix.

Note that $\Pr(v > \hat{v})$ denotes the probability of $v > \hat{v}$. Similarly for $\Pr(v > v_2^c)$ and $\Pr(v > v_3^c)$.

Remark 1. Given our parameter assumption that $\max\{V\} \in (PP/3, PP)$, solutions *a* and *b* always exist. The existence of *c* and *d* requires additional conditions on *PP*, $\overline{v}$, and the value distribution. First, *PP* should be less than $2\overline{v}$, otherwise *PP*/2 is not a feasible price. Also, *PP* should be high enough relative to $\overline{v}$ to support *c* and *d* (especially for *d*). We use a uniform value distribution over [0, 1] to demonstrate these additional conditions in the appendix.[20]

Remark 2. A "category" of symmetric BNE in UPA includes all the contribution strategies that are equivalent to a symmetric BNE where only the possible uniform prices are used as contribution choices, as one of the listed BNE in the proposition. For example, any strategy with individual contributions always below *PP*/3 is a symmetric BNE in category *a*, since contributing below *PP*/3 is equivalent to contributing 0. Because these small contributions cannot affect payments, reflecting the zero-marginal penalty structure, they cannot be excluded as equilibria. This feature advantages UPA in value revelation: in *d*, agents can reveal their true values without affecting their payments over the entire value range except for the interval [*PP*/3, v_3^c] where agents need to contribute less than *PP*/3 to follow the equilibrium strategy.

Generally, a BNE including more of the feasible uniform prices (*d* with two positive prices vs. *c* with only one) supports a larger truth-telling value range: *b* to *d* all support larger truth-telling value ranges than *a* (i.e., the non-truth-telling ranges of [*PP*/2, $\overline{v}$], [*PP*/2, $\hat{v}$] and [*PP*/3, v_3^c] are all smaller than [*PP*/3, $\overline{v}$]); the comparisons among *b* to *d* depend on parameters.

[20] These BNE also apply to a game with continuous types.

We show in the appendix that *d*, if it exits, has the largest truth-telling range. In the special case of a two-player game, truth-telling over the entire value range is supported in UPA as agents contribute *PP*/2 when $v_i \geq PP/2$ and 0 otherwise (See Appendix).

In an *N*-player game, UPA has a solution structure similar to that in the 3-player game, which is summarized in the following proposition.

Proposition 3.4. In an *N*-player Bayesian game, assuming $\max\{V\} \in (PP/N, PP)$,

- UPA always has the following two categories of symmetric BNE: for *i* = 1, …, *N*,

  e) $c_i^{BNE}(v_i) = 0$, for all $v_i \in V$.

  f) $c_i^{BNE}(v_i) = \begin{cases} 0 & if\ v_i \leq PP/N \\ PP/N & if\ v_i > PP/N \end{cases}$

- BNE with uniform prices higher than *PP*/*N* and/or with more than one uniform price may or may not exist, depending on whether the corresponding critical values can be identified from a system of polynomial equations given the parameters of *PP*, *V* and the value distribution. The BNE with the most numerous different prices, if it exits, is

  g) $$c_i^{BNE}(v_i) = \begin{cases} 0 & if\ v_i \leq v_N^c \\ PP/N & if\ v_N^c < v_i \leq v_{N-1}^c \\ \vdots & \vdots \\ PP/3 & if\ v_3^c < v_i \leq PP/2 \\ PP/2 & if\ v_i > PP/2 \end{cases},$$

with $v_2^c = PP/2 > v_k^c > PP/k$, for *k* = 3, …, *N*, and the critical values $v_3^c, \ldots, v_N^c$ are given by a system of *N*-2 polynomial equations.

Proof: See Appendix.

The general procedure to solve the N-player game and the general form of the expected payoff function at each possible uniform price used to construct the system of polynomial equations are given in the appendix.

Remark 3. Although it is not incentive compatible in general (*cf*. Borgers et al., 2015), UPA may still support relatively high value revelations for pure public goods due to the same reason discussed in *Remark* 2, in contrast to the direct serial cost sharing mechanism (Moulin, 1994) where incentive compatibility is obtained only for excludable public goods.[21] Therefore, we would expect relatively high contributions in UPA even in BNE. Note that this value revelation property is primarily due to the step-function style of payment scheme, which is fully captured by the almost always zero-expected marginal penalty structure. Also, the equilibrium *f* of UPA results in the same equilibrium outcome as the "conservative equal-costs" mechanism for pure public goods (Moulin, 1994), where everyone needs to pay PP/N and the good is not provided otherwise, i.e., the "conservative equal-costs" mechanism just implements a particular BNE category *f* of UPA.[22]

3.3.3 BNE-Set Comparisons by a Numerical Example

[21] If we have in equilibrium $v_2^c = PP/2 > v_3^c > PP/3 > \cdots > v_{N-1}^c > PP/(N-1) > v_N^c > PP/N$, the truth-telling value ranges supported by the symmetric BNE in UPA are as follows: let PP/k denote the highest uniform price in the set of BNE that exist given the model parameters, then the non-truth-telling value range is $[PP/m,\ v_m^c]$ or $[PP/(k\text{-}1),\ \max\{V\}]$, where $m \in \{k, k+1, \ldots, N\}$, v_m^c is a critical value defined in the same fashion as in the equilibrium *g*; alternatively, the truth-telling value range is $[\max\{PP/m, v_m^c\},\ \min\{PP/(m-1), \max\{V\}\}]$ and $[0, PP/N]$; and if k=N and N is large, the non-truth-telling value ranges will be relatively small.

[22] With the inefficiency of providing the good only when everyone has a value higher than the equal-cost share PP/N, the single uniform price guarantees that truth-telling is a dominant strategy in the "conservative equal-costs" mechanism, which is consistent with the characterization for dominant strategy implementation (Schwartz and Wen, 2013). This also explains why truth-telling is a dominant strategy in UPA in a 2-player game where $PP/2$ is the only positive uniform price.

To further illuminate how UPC and UPA differ from PPM and PR, we adopt the approach used in Gailmard and Palfrey (2005).[23] Through a well-constructed 3-player, 3-value numerical example, they find that, although they share some basic BNE properties, PPM and PR have different equilibrium sets, and they use those examples to motivate hypotheses for their subsequent experiment. We replicate their numerical results and show that our uniform price mechanisms have equilibrium sets different from those of PPM and PR using the same parameters. Further, there are systematic differences in the equilibrium sets that are explained by expected marginal penalties, which we use to predict how the four mechanisms will differ contributions: a lower expected marginal cost of contributing more (excessively) supports higher contributions in equilibrium.

In Gailmard and Palfrey's (2005) environment, each agent's value is independently drawn from a uniform distribution over a set of three values, $v_i \in \{29, 45, 90\}$, i=1, 2, 3. The provision point is 102. Let $c=\{c^L, c^M, c^H\}$ be the contribution function, where the superscripts L, M and H denote the contribution from v_i = 29, 45, and 90 respectively, and only integer-valued contributions are allowed. They find PPM has 35 nontrivial pure-strategy, symmetric, and weakly monotonic Bayesian Nash equilibria, and PR has 74.[24] All the nontrivial equilibria are categorized into four efficiency groups (*cf*. Gailmard and Palfrey's (2005) Table 1), measured by expected total net benefit (total induced values minus the cost). PR has four equilibria in the most efficient group, and PPM has none; PPM and PR respectively have 4 and 17 equilibria in

[23] A closed-form solution for UPC's BNE set is much more challenging to obtain, because UPC has a hybrid expected payoff structure between PPM and UPA (see Eq. (10) in Section 2.5). Nevertheless, we keep UPC in this chapter because its rebate structure provides an important point of comparison. As we will see below, the combination of the expected-payoff analysis and comparison of the numerically calculated BNE-set provides sufficient insights to understand the key role of rebates in attracting higher contributions in equilibrium, which may not be theoretically demonstrable even with a closed-form solution given the multiplicity of equilibria.
[24] Nontrivial means the provision probability is greater than zero.

the second most efficient group; 13 equilibria of PR and all the remaining 31 equilibria of PPM fall into the fourth group; the remaining equilibria of PR are in the third group.

In our uniform price mechanisms, UPC has 8 nontrivial symmetric, pure-strategy monotonic BNE, half in the most efficient group and half in the fourth group; UPA has two categories of BNE, both in the fourth group. Table 3.1 lists all the most efficient equilibria of PPM, PR and UPC and two equilibria for each category in UPA (see the appendix for a complete list of the nontrivial BNE of PPM, PR, and UPC).

Table 3.1 The most efficient Bayesian Nash equilibria under each mechanism*

PPM All in the 2nd efficient group	PR All in the 1st efficient group	UPC All in the 1st efficient group	UPA All in the 4th efficient group	
{23, 34, 45}	{21, 21, 60}	{20, 20, 62}	Category 1	{29, 45, 50}
{24, 34, 44}	{22, 22, 58}	{21, 21, 60}		{0, 34, 34}
{25, 35, 42}	{23, 23, 56}	{22, 22, 58}	Category 2	{29, 33, 90}
{26, 35, 41}	{24, 24, 54}	{23, 23, 56}		{0, 0, 51}

* {*X*, *Y*, *Z*} denotes the contribution from value 29, 45, and 90, respectively.

UPC induces much higher contributions from the highest value type than PPM: the lowest contribution from the high type in UPC (56) is much more than the high type's maximum contribution in PPM (45) in their most efficient equilibrium sets. Together with a similar pattern between PR and PPM, this comparison sheds light on how the rebate and its corresponding lower expected marginal penalty influence contributions of high types: the rebate in UPC and PR reduces the expected payment cost more significantly for high-value agents since they will experience overcontribution with the highest probability, and they could pay the most in the absence of a rebate. For example, {21, 21, 60} is a BNE in both PR and UPC, but is not an equilibrium in PPM since a high-value agent would be better off to deviate from 60 to 21.[25] This insight reflects the expected marginal penalty: PR and UPC both have lower expected marginal

[25] Gailmard and Palfrey (2005) show that if agent *i* with a value v_i has *any* interim profitable deviation, then one of these deviations is profitable: 0 or 102-x-y, where x, y are in {21, 21, 60}. This works for UPC as well, a fact used in the following paragraphs.

penalties than PPM, especially for high contributors who are generally high types based on **Proposition 3.2**.

Compared to PR with a rebate corresponding to a smaller but still not zero-expected marginal penalty, UPC—as designed—further reduces the expected cost with a zero-expected marginal penalty on high (enough) contributions, and hence induces even higher contributions from the high type than does PR.[26] Specifically, {20, 20, 62} is not a BNE in PR because an agent with value 90 has an incentive to deviate from 62 to 20: the agent would rather reduce her contribution than pay the expected proportional share. The realized value profile that drives the difference between mechanisms is when the other two agents are (29, 90), this agent's contribution of 62 will receive only a proportional rebate (18.1) in PR, while a full rebate above the uniform cap in UPC (21=62 – the uniform cap 41), i.e., the marginal penalty in UPC is low enough to eliminate the deviation incentive but that in PR is not.

With a much broader range of zero-expected marginal penalty, UPA generates the highest contributions in equilibrium, as demonstrated by the listed four BNE: in Category 1, {29, 45, 50} results in the same equilibrium outcome and payoff as {0, 34, 34} but supports truth-telling for low and medium types, and also supports higher contributions from high types since contributing 45 or 50 leads to the same payments as contributing 34 because the next highest possible uniform price is 51; in Category 2, {29, 33, 90} and {0, 0, 51} are similarly equivalent, and the former even supports truth-telling for the high type and on average results in group contributions quite close to the expected total induced values (93%), which without the cost of exclusion is not that much worse than the direct serial cost sharing mechanism in terms of value

[26] Note that if we take an average of all the nontrivial BNE within each mechanism, the two observations above about UPC still hold: the average contribution functions are {8, 25, 42}, {17, 27, 52}, and {12, 12, 54} for PPM, PR, and UPC, respectively. And the evidence becomes even stronger if we increase the contribution precision to 0.1: {11.3, 29.1, 40.8}, {18.6, 30.0, 52.6}, and {13.4, 13.4, 55.4} for PPM, PR, and UPC, respectively, and UPC has the highest equilibrium contribution from the high value type, 63.8, among all nontrivial BNE of the three mechanisms.

revelation. Also, note that this BNE set is consistent with **Proposition 3.3**: {0, 34, 34} and {0, 0, 51} correspond to the equilibrium *b* and *c* respectively.

The observations above support the role of the expected marginal penalty in differentiating the equilibrium sets and corresponding aggregate contribution levels. In UPC and PR, it is only when the contribution is high enough that the expected marginal penalty approaches zero, and hence we find the effect is the most significant for the high-value type in the numerical example.

Because more efficient equilibria comprise a larger portion of UPC's set of nontrivial BNE, we hypothesize that it will generate higher contributions than PR and PPM. Similarly, UPA has equilibria with much higher value revelation, but with lower provision rate due to the number of less efficient equilibria and the additional constraint on contribution profile as in the complete information case. For PPM and PR, based on the results from Gailmard and Palfrey (2005), we hypothesize PR is better than PPM in both contribution and provision rate.

3.4 Experimental Design and Procedures

In each period, 10 subjects each learn their random, private v_i, which was independently drawn from a uniform distribution on over a set of nine values, $v_i \in \{4, 5, 6, 7, 8, 9, 10, 11, 12\}$, i=1, …, 10. The provision point *PP* is 36, 45% the total expected induced value (80) in a group of size 10. Feasible positive UPA prices are {3.6, 4, 4.5, 5.2, 6, 7.2, 9, 12}. These experimental parameters are chosen in a way that providing the good is always socially optimal, and a wide range of uniform prices are possible. All the information above is common knowledge. Figure 3.1 presents the average equilibrium contribution at each induced value based on the nontrivial

BNE set of each mechanism with integer-valued contributions in this 10-player, 9-value game, and shows that the observations from the numerical example in Section 3.3 still hold.[27]

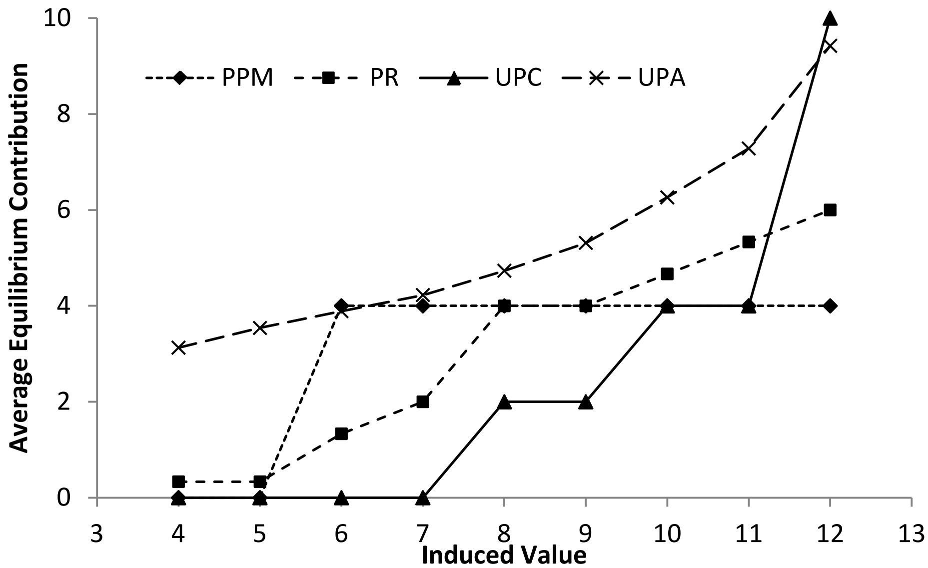

Figure 3.1 Average Equilibrium Contributions by Induced Value

Table 3.2 shows the mechanism treatments presented in each session: the first treatment is always PPM (25 periods), to familiarize subjects with the baseline game. The following treatments (25 periods each) apply the other mechanisms in a Latin Square to control for order effects.

[27] Note that the average equilibrium contributions in UPA are based on the nontrivial BNE that only uniform prices are chosen as contribution choices and hence are the lower bound. Also, although the contribution precision is 0.1 in the experiment, there are two justifications for integer precision we used here. First, as mentioned in the previous footnote, increasing precision will only make the differences among the mechanisms more significant, especially in favor of UPC, and hence will not change our main hypotheses. Second, a search with a 0.1 precision is impossible in terms of the computer runtime given our relatively large group size (10) and many value types (9). Even with an integer precision, it took a full day to search for UPC's equilibria on 24 nodes of a high-speed climate modeling cluster consisting of 200 nodes (5160 cores, 14.5 TB memory, 49.3 peak Tflops). With a precision of 0.1, the strategy space for each value type will be 10 times larger, which implies impossibility given that we have 9 different values. For UPA, however, we did obtain the complete BNE set since there are only 8 positive uniform price choices.

Table 3.2 Treatment Arrangement of Experimental Sessions

Treatment Order	1st (25 periods)	2nd (25 periods)	3rd (25 periods)
Session 1	PPM	PR	UPC
Session 2	PPM	PR	UPA
Session 3	PPM	UPC	PR
Session 4	PPM	UPC	UPA
Session 5	PPM	UPA	PR
Session 6	PPM	UPA	UPC

The experimental software was developed in z-Tree (Fischbacher, 2007). At the start of each treatment, the experimenter read the instructions aloud as subjects read along. Subjects were then given an initial budget of 15 experimental dollars. Subjects then simultaneously choose a contribution, $c_i \in [0,15]$ (with a precision of 0.1) towards the project. At the end of each period, subjects were informed whether the project is provided, and their earnings, payment and rebates. At the end of a session, earnings were totaled across all periods. Subjects were recruited through university-wide daily digest email server (mainly for undergraduates), and from an email list of students interested in participating in experiments. A total of 60 subjects participated in the six complete sessions, producing 4500 individual level observations.

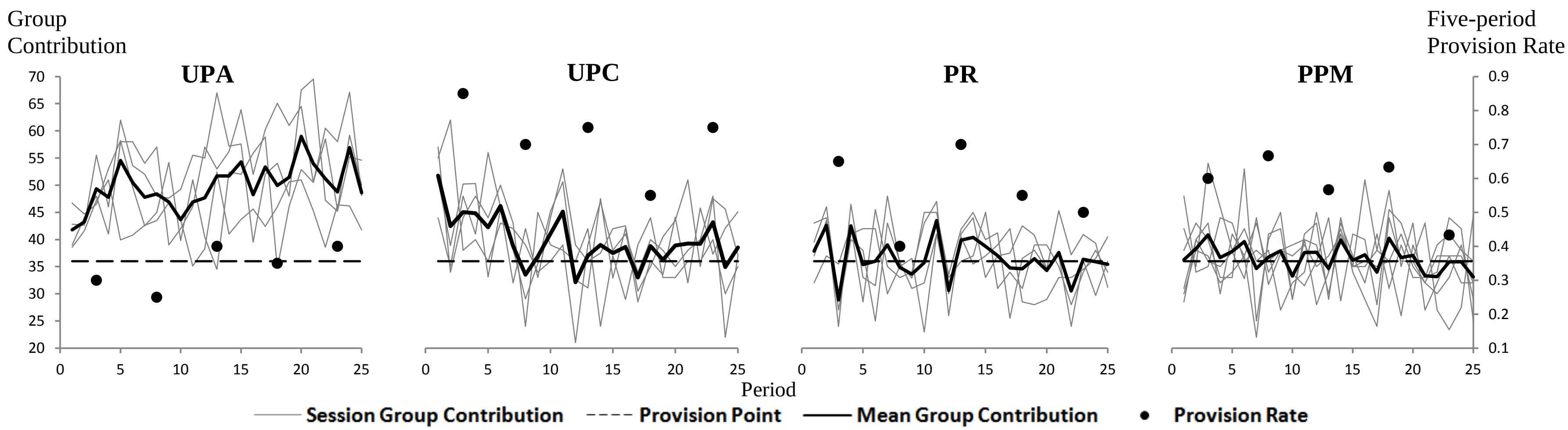

Figure 3.2 Group Contributions in Each Period and 5-Period Provision Rate by Mechanism

3.5 Results

We measure the performance of the mechanisms by two indicators: aggregate group contribution and the provision rate. The provision rate reflects the efficiency of the mechanism, as provision is always efficient given our parameter values; therefore, it is a direct test of our hypothesized differences in mechanism efficiency based on the BNE sets derived above. In addition, group contribution is a measure of the extent of value revelation. Although none of these mechanisms are incentive compatible, revelation properties may be of interest in applications where small-scale, real money, real good pilot programs are used to provide estimates of public value for non-market goods that are then applied over a broader population (e.g., Champ et al, 2002; Swallow et al., 2008; Haskell et al., 2010; Bush et al., 2013; Swallow, 2013).

Figure 3.2 gives an overview of group contributions in each period, and five-period provision rates, by mechanism. Grey lines represent session-specific group contributions, dark lines represent averages over sessions, and dark dots represent average five-period provision rates.

3.5.1 Group Contributions

Table 3.3 Two-factor Random Effects Models of Group Contribution

Group Contribution	(1)	(2)
PR	0.253	-0.0894
	(0.946)	(0.922)
UPC	1.770*	2.724***
	(0.946)	(0.943)
UPA	14.46***	12.42***
	(0.946)	(1.026)
Provision Rate†		-8.168***
		(1.831)
Constant (PPM)	36.24***	41.09***
	(0.712)	(1.286)
Log-likelihood	-1185	-1175
Chi-square (df)	272.4 (3)	308.9 (4)
R^2 overall	0.436	0.466
Number of observations	360	360
Number of periods (treatment-specific)	20	20

Standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1;

†: Provision rate over previous 5 periods, which yields the largest log-likelihood among 1 to 5-period lags.

In Figure 3.2, UPA generates much higher group contributions than the others, and UPC looks to be slightly higher than PR and PPM. Table 3.3 presents results from two-factor random effects regressions of group contribution on mechanism dummies (group- and period-specific, *cf*. Marks and Croson, 1998), focusing on the observations from the last 20 periods. [28]

Model 1 provides a baseline that includes only mechanism dummies, using PPM as the base. Average predicted group contributions are not distinguishable from the *PP* value of 36 in PPM (36.24, p=0.733) and PR (36.50, p=0.557), but are significantly higher than *PP* in UPC (38.01, p=0.017) and UPA (50.70, p<0.001).

Model 2 controls for the previous five periods' provision rate, since individuals' efforts to reduce their payments may influence the equilibrium selection process as they try to contribute just enough to obtain regular provision as a group.[29] It is significantly negative (p<0.001), which is evidence of "cheap riding" (*cf*. Issac et al. 1989) where individuals reveal less of their value when provision has been occurring. A likelihood ratio test advises using Model 2 for interpretation.

Model 2 reflects an ordering of group contributions by mechanism that is broadly consistent with higher contributions occurring where the expected marginal penalty is lower, especially for the marginal penalty structures of our new mechanisms. UPA—with an almost-everywhere zero-expected marginal penalty and a BNE set supporting group contributions close to the expected total induced values—is significantly higher than the others all with p<0.001 (likelihood ratio test). Similarly, the lower expected marginal penalty from UPC leads to significantly higher aggregate contributions than PPM (p=0.004) and PR (p=0.009). Increasing

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[28] We exclude the observations from the first five periods to isolate potential mechanism-learning or order effects in the early periods. We use the same rule for all the following regressions and statistical tests unless stated otherwise.
[29] Since induced values are randomly assigned in each period, provision rate over several previous periods reflects how far away the expected group contribution is from the provision point.

contributions for a higher probability of provision will not result in losing money due to overcontribution in a broad contribution range in these mechanisms. However, we cannot reject the hypothesis that PR and PPM generate the same group contributions, consistent with Marks and Croson (1998), but different from Gailmard and Palfrey (2005). The latter difference could be attributable to the lower cost-benefit ratio (the provision point divided by the total expected induced value; our 45% vs. Gailmard and Palfrey's 62%) or the larger group size (10 vs. 3) in our experiment.[30]

3.5.2 Provision Rate

The group contributions lead to a similar ordering of provision rates among the efficient mechanisms, as shown in Figure 3.2 (dots). With the same provision condition among UPC, PR and PPM, UPC—designed with marginal penalty in mind— performs better than both PPM and PR. Specifically, UPC has a provision rate 68.8%, which is significantly higher than those for PPM (57.5%, p=0.054 one-tailed z-test) and PR (53.8%, p=0.052 two-tailed), with the latter two not statistically distinguishable (p=0.601).[31] This ordering emphasizes the advantage of the zero-marginal penalty in UPC in inducing not only higher contributions, but also a larger proportion of higher group contributions. The similarity between PR and PPM in provision rate is still consistent with Marks and Croson (1998), but contrasts with Gailmard and Palfrey (2005) who found PR is significantly better than PPM. One may argue that the provision rate in PR is driven down by the low provision rate in the 5-period interval 6 to 10. However, PR and PPM are still not statistically different (p=0.638) when focusing on the last 15 periods' data, although PR has a nominally higher provision rate than PPM (58.3% vs. 54.4%). Because the profile of offers, in

[30] Marks and Croson (1998) has a cost-benefit ratio of 50% and a group of size 5.
[31] Wilcoxon rank sum test gives similar results: UPC vs. PPM (p=0.0545, one-tailed), UPC vs PR (p=0.0522, two-tailed), PPM vs. PR (p=0.602, two-tailed)

addition to the total amount, affects the provision decision in UPA, it has a provision rate (35.0%) significantly lower than the other mechanisms, all with $p<0.01$.

To understand how the marginal penalty structure of our uniform mechanisms induces incentives that result in higher group contributions and provision rates, we examine individual level contributions at different induced values, where BNE has different predictions across mechanisms.

3.5.3 Individual Contributions

Figure 3.3 shows average individual contributions at each induced value across mechanisms. Average observed uniform prices from UPA and UPC are also included to show how being close to where the marginal penalty changes sharply differentiates contributions across mechanisms.

Average contributions increase with induced value in all mechanisms, but they are yet higher in the uniform price mechanisms (Figure 3). Consistent with group contribution, UPA stands out as generating much higher contributions at all value levels; UPC has generally slightly higher contributions, especially at high values (10 to 12). Comparing Figures 3.1 and 3.3, we find the observed average contributions are following fairly closely the average contributions of the equilibria shown in Figure 3.1. Since Figure 3.1 only shows the lower bound of equilibrium contribution, consistent-with-equilibrium higher offers in UPA may make it more prominent in Figure 3.3.[32]

[32] The main difference between Figure 3.1 and 3 is that we do not observe zero average contributions from low value types in the lab data, which is quite normal in the public good experiments literature since usually subjects just do not contribute zero (Ledyard, 1995).

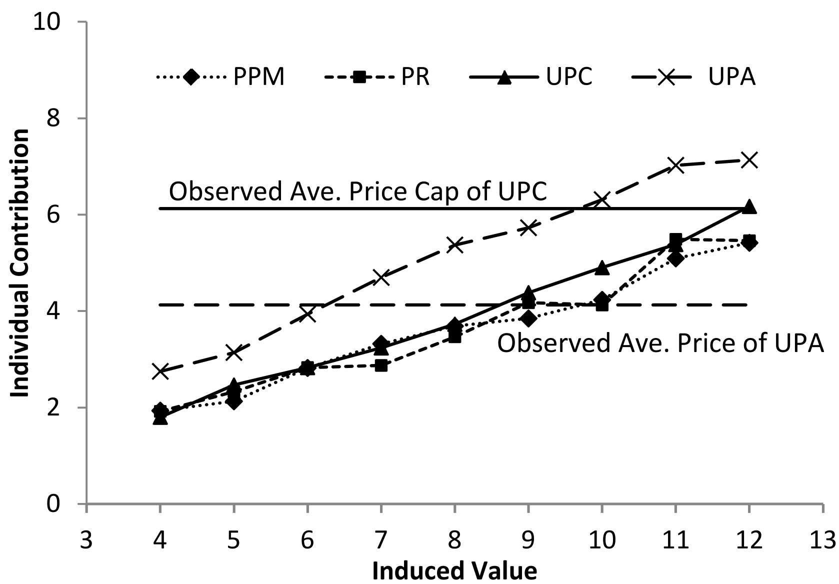


Figure 3.3 Mean Contributions by Induced Value

To investigate statistically how individual contribution varies with induced value and mechanism, we run a series of subject-treatment random effects tobit models of dollar amount contributed. Table 3.4 shows the results, using PPM as an excluded base mechanism. Model 1 is a baseline model which estimates mechanism-specific intercepts with mechanism dummies; variation in slope is captured by induced value. Provision rate and interaction terms among mechanisms and induced value, are added in Models 2 to 4, of which Model 4 is chosen for interpretation based on likelihood ratio tests.

An individual's value has a large and significantly positive (p<0.001) effect on her contribution, and there are no significant offsetting negative coefficients from the various mechanisms. This result provides strong statistical evidence of a positive relationship between individual contribution and induced value, which is consistent with **Proposition 3.2**, but has not been widely documented across provision point mechanisms, though Rondeau et al. (2005) and Spencer et al (2009) find similar effects in one-shot PR games, and Gailmard and Palfrey (2005) show a similar result based on median bid functions for PPM and PR. For PPM, the result is consistent with related theoretical predictions by Alboth et al. (2001) and Laussel and Palfrey (2003).

Table 3.4 Random Effects Tobit Models of Individual Contribution

Contribution	(1)	(2)	(3)	(4)
PR	-0.00425	0.0414	-0.0336	0.00336
	(0.348)	(0.415)	(0.348)	(0.415)
UPC	0.242	-0.272	0.318	-0.200
	(0.348)	(0.417)	(0.349)	(0.417)
UPA	1.546***	0.766*	1.383***	0.615
	(0.348)	(0.414)	(0.350)	(0.415)
Value	0.525***	0.490***	0.523***	0.489***
	(0.0103)	(0.0179)	(0.0103)	(0.0179)
PR × Value		-0.00568		-0.00455
		(0.0280)		(0.0279)
UPC × Value		0.0639**		0.0641**
		(0.0285)		(0.0284)
UPA × Value		0.0978***		0.0966***
		(0.0280)		(0.0280)
Provision Rate†			-0.669***	-0.659***
			(0.161)	(0.160)
Constant (PPM)	-0.676***	-0.394	-0.264	0.0121
	(0.236)	(0.263)	(0.255)	(0.281)
Log-likelihood	-6689	-6680	-6680	-6671
Chi-square (df)	2609 (4)	2644 (7)	2638 (5)	2673 (8)
R^2 overall	0.307	0.310	0.308	0.311
Number of observations	3600	3600	3600	3600
Number of groups	180	180	180	180
Number of periods	20	20	20	20

Standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1

†: Provision rate over previous 5 periods, which yields the largest log-likelihood among 1 to 5-period lags.

UPA has a larger intercept than the others and a significantly steeper slope than PPM and PR all with p<0.001. UPA's slope is also nominally steeper (p=0.293) than UPC. Combined, these results indicate that UPA generates higher contributions throughout the value range. This result reflects the prevalence of high value revelation BNE within UPA's equilibrium set.

With an intercept slightly, but not significantly, lower than those for PPM and PR, UPC has a significantly steeper slope than PPM (p=0.024) and PR (p=0.026), which implies UPC elicits higher contributions from higher valued people than do PPM and PR, consistent with our observation from the numerical example in Section 3.3. We cannot reject the hypothesis that PR and PPM have the same intercept and slope. These results indeed reflect the differences in the

BNE sets (Figure 1): UPC, PR and PPM differ the most at value 12, where UPC induces a contribution of 10, compared to 6 in PR, and only 4 in PPM. By Wilcoxon rank sum test, UPC does generate higher contributions at 12 (6.17) than PPM (5.42) and PR (5.46), both with borderline significance (p=0.093 and p=0.091, one-tailed), and no significant difference between PPM and PR (p=0.380, one-tailed).

Given the multiplicity of equilibria, the borderline significance suggests we refine the analysis to compare mechanisms where marginal adjustments to contributions are most consequential for subjects, in the range of the uniform cap. Specifically, we look for the range of induced values whose observed contributions are within one standard deviation of the average observed uniform cap (the flat line in Figure 3).[33] The observed average cap in UPC is 6.12 (s.d., 1.36), and contributions in this region come from subjects with values 10 to 12. To compare the individual contributions of UPC with those of PPM and PR within this value range, we use the ratio of contribution to value (i.e., value revelation) as the measure and pool the ratios at values 10 to 12.

We find individuals whose payoffs are most affected by their contributions are attentive to the incentives characterized by marginal penalty. Among subjects with values of 10 to 12, UPC induces a significantly higher value revelation (0.498) than PPM (0.447, p=0.037 by one-tailed Wilcoxon rank sum test) and borderline significantly higher than PR (0.456, p=0.059, one-tailed), consistent with the idea that subjects perceive a lower expected marginal penalty in this region. Lastly, UPC contributions are still significantly lower than UPA (0.621, p<0.001 two-tailed). The reason is that contributions of 10 to 12 are substantially above the typical range of UPA uniform prices (the observed average price is 4.13). Within the remaining value range (4 to

[33] Given the multiplicity of equilibria, we will not have a unique uniform cap even in equilibrium. Thus the observed average cap is the most reasonable and practical choice.

9), where the expected marginal penalties are similar, UPC has a similar value revelation (0.468) to those in PPM (0.463, p=0.944) and PR (0.452, p=0.913).

3.6 Discussion

This chapter compares two novel uniform price mechanisms for provision point public goods within a Bayesian game framework where each agent's induced value is randomly drawn from a known distribution but is private information. We analyze the effect of marginal penalty on the expected payment, characterize some basic properties of the BNE sets of UPC and UPA, solve for the symmetric BNE of UPA, and demonstrate their different equilibrium sets through a numerical example. Then we run experiments to evaluate the mechanisms' performance, against each other and the well studied PPM and PR mechanisms; these evaluations consider group contribution, provision rate, and individual contribution across the range of induced values. Overall, the novel mechanisms improve upon those in the literature with private value information: UPA generated significantly higher contributions than the other three mechanisms, and UPC was more efficient than the other three mechanisms.

Combining the marginal penalty effect on the expected payment and the BNE-set comparisons among mechanisms, this chapter extends our understanding of the role of the rebate rule in provision point public good provision. The concern in the standard PPM—that the excess contributions will be wasted—are captured by a marginal penalty of -1 associated with overcontribution. Alternative rebate mechanisms reduce the marginal penalty to be between -1 and 0. In a Bayesian game, it affects the equilibrium by changing the expected marginal cost of higher contributions in cases where aggregate value realizations are high. Thus, a lower marginal penalty from a rebate reduces the cost due to expected excess contribution and hence may facilitate higher contributions in equilibrium; the lower the marginal penalty is, the higher the contribution can be supported. With broad ranges of zero-marginal penalty, our uniform price mechanisms generally perform better than PPM and PR where marginal penalties are strictly less than 0.

Different zero-marginal penalty structures result in different strengths of the two uniform price mechanisms. UPA, with the broadest range of zero-marginal penalty (excepting cutpoints with a lump sum penalty), supports equilibria with high levels of value revelation: truth-telling is a dominant strategy in two-player games, and in our 10-player experimental environment, UPA has a BNE where 96.7% of the expected total induced value is revealed.[34] Therefore, even without incentive compatibility, UPA may perform nearly as well in a pure public good as Moulin's (1994) incentive compatible serial cost sharing mechanism in club good games. In contrast to its high contributions, the provision rate in UPA is significantly lower than in the other mechanisms, which may be attributable to the difficulty of coordinating to a particular number of payers. However, if we use one of the possible uniform prices as a coordination tool, the provision rate may increase reasonably. There does exist a real world example, where UPA was used with a suggested price, and the project was successfully funded.[35] A variation of this kind may improve the provision rate significantly since UPA has a finite number of prices.

With a zero-marginal penalty only for high contributions, UPC supports BNE with contributions higher than those of PPM, and especially higher than PR at high values. The reason is two-fold: first, it is the high-value people who are more likely to experience overcontributing *ex post*, and pay the most in the absence of a rebate; and second, UPC is more effective in protecting high-value people, as all excess contributions will be rebated to those

[34] The equilibrium strategy is {3.9, 5, 5.9, 5.9, 8, 8.9, 10, 10, 12} for value=4, …, 12, in our 10-player 9-value Bayesian game.

[35] The project was to prevent a ski facility near Boulder, Colorado from going into bankruptcy. Thanks to Bill Schulze for sharing this example from his personal experience.

contributing above the cap, and above-cap contributors are more likely to be high-value types according to **Proposition 3.2** and our data. The protection high-value people receive from PR is not as effective as that from UPC: the overcontribution is shared among all group members. Since there are multiple equilibria, some of which coincide, it is not clear this advantage of UPC will manifest in play based on theory alone. However, in our lab data, UPC performs significantly better than PPM and PR in both group contribution and provision rate. This could be partially attributed to the fact that UPC has a smaller nontrivial equilibrium set, making it easier to focus on the more efficient ones: if we increase the contribution precision from 1 to 0.1 in the 3-player 3-value example in Section 3.3, PR will have 5330 nontrivial equilibria, 40 of which are in the most efficient group, while UPC will have only 73 nontrivial equilibria, 42 of which in the most efficient group.[36] Additionally, in contrast to Gailmard and Palfrey (2005) where PR is more efficient than PPM, we find their difference is not significant. Since we used a quite different set of experimental parameters, these mixed results suggest that future research needs to further identify the conditions for various rebate mechanisms to work most efficiently. For example, group size and value distribution could be two important factors.

Within the incomplete information framework, the explicit integration of the effect of systematically varying the rebate (or equivalently, marginal penalty) structure into the Bayesian Nash equilibrium analysis allows us to assess rebates not just as an intuitive and empirically appealing method for attracting higher contributions from all participants, but rather as a way to refine mechanism design. Specifically, the combination of our theoretical and laboratory analysis indicates an important shift in the focus of the future research in the rebate mechanisms for provision point public good provision. Rather than simply using experiments to run horse

[36] See Appendix for a complete list of BNE of UPC and PR in the most efficient group with the contribution precision of 0.1.

races among different rebate rules, our environment shows that different rebate rules shift the equilibrium contributions of people with different values. This information can be used to tailor rebate rules to target most effectively populations with different value distributions, and may significantly improve provision. For example, for a good with values that are skewed low in the target population, UPC may be revised by allocating excess contributions disproportionately to lower contributors in order to attract slightly higher offers from the most numerous types. Thus, while we explore two specific rules in this paper, our approach suggests productive avenues for designing new mechanisms and supports new research at the nexus of mechanism design, experimental economics, and empirical applications that will enhance welfare improvements through private provision of public goods.

Chapter 4 Dynamic Public Goods with a Tipping Point

Abstract

We study a continuous-time, infinite-horizon, durable public good game with a tipping point, below which collapsing the stock is optimal. With a payoff function linear in stock and monetary income, and a logistic growth function, we show that a tipping point always exists in the game. We characterize how the equilibrium stock level and the tipping point change under various game structures. Under a dynamic voluntary contribution mechanism (DVCM), both the open-loop equilibrium and the Markov perfect equilibrium (MPE) result in inefficiently low steady states and higher tipping points. A higher stable steady state and a lower tipping point is shown to be supported in the MPE as compared to the open-loop solution and the highest stable steady state in MPE approaches the efficient stock level asymptotically as the discount rate approaches zero. Lastly, we extend DVCM by introducing a provision point to a dynamic provision point mechanism (DPPM). We characterize a class of MPE of DPPM and show that the *most rapid approach path* to the socially efficient stock level is supported in a symmetric MPE of DPPM.

4.1 Introduction

Many public goods have a durable nature, such as greenhouse gases (public "bad") and various ecosystem services that are dependent on populations of animals or plants naturally growing through stock dynamics. Of these important public goods, very few follow a linear state equation. However, in the existing literature for durable public good games, many, if not most, of the studies rely on a linear state equation (e.g., Fershtman and Nitzan, 1991; Dockner and Nishimura ,2001; Dutta and Radner, 2004; Arbel et al., 2014, Battaglini et al., 2014, and more in the survey by Long, 2010). Further, a major concern in providing these durable public goods is the existence of a tipping point, which is defined in Global Biodiversity Outlook 3 (2010) as "*a situation in which an ecosystem experiences a shift to a new state, with significant changes to biodiversity and the services to people it underpins, at a regional or global scale*". Global Biodiversity Outlook 4 (2014) even states "*there is evidence that several large-scale regime shifts have already started*", including degradation of coral reefs, and, more speculatively, collapse of some tropical fisheries. To our best knowledge, there are not many studies that capture an endogenously determined tipping point in a game-theoretic model for durable public goods.

To bridge these gaps, this chapter develops a continuous-time dynamic public good game with an endogenously determined tipping point (or extinction point as used in bioeconomics), below which collapsing the stock is optimal in the sense of maximizing the present value of total payoffs. In the game, N agents simultaneously contribute to sustain a public good stock at each instant in an infinite time horizon. The payoff function is linear in stock and monetary income. The public good stock follows a modified logistic growth function. The control variable is the group contribution provided to maintain the growth (surplus) of the stock, mimicking the protection of the juveniles in order to sustain a increased wildlife population that ultimately provides an ecosystem service public good.

In the social planner’s problem, we prove the existence of a tipping point under very general conditions and find a solution of a combination of bang-bang and singular controls. Then we characterize both the open-loop equilibrium and the Markov perfect equilibrium (MPE) for a dynamic voluntary contribution mechanism (DVCM), in which the achievable singular stock level becomes lower and the tipping point becomes higher. The MPE of DVCM generally supports a larger set of stable steady state singular stock levels including those higher than that from the open loop solution and with a lower tipping point. The socially efficient stock level can be approached only asymptotically in the MPE of DVCM when agents become infinitely patient (i.e., a zero-discount rate). In some cases, even though the efficient singular stock level is quite high, the stock collapses inevitably in both the open-loop and the MPE. However, when a provision point is introduced, i.e., in a dynamic provision point mechanism (DPPM), we characterize one class of MPE of DPPM and show that the socially efficient stock level can be supported in the MPE without any limiting conditions in a non-asymptotic sense, that is, via the *most rapid approach path*.

This chapter makes the following contributions to the literature. First, we use a more realistic logistic state equation to introduce a tipping point for a durable public good stock and characterize how the tipping point changes in various game-theoretic models. Polasky et al. (2006) use a logistic growth function in a common pool resource game and also find a singular solution for the social planner's problem. Their model contains simplifying assumptions which preclude consideration of tipping points however. Second, to solve a dynamic game with a nonlinear constraint and a tipping point—which imply non-unique singular paths in our case—is not trivial. In addition to Polasky et al. (2006), Clark (1976), in his well-known text, discussed a fishery model with two singular stocks, which is similar to ours. This work only mentioned the existence of a tipping point without providing a proof however.[37] In this chapter, we investigate the problem in a durable public good setup, prove the existence of a tipping point in Section 4.2.2, and extend the analysis to game-theoretic models. The interdependence between a state-dependent contribution function in the MPE and a non-singleton interval of singular stock levels, as discussed in Section 4.2.4, is relatively new in the literature of solving Markov games in a continuous-time singular optimal control problem. As far as the authors know, only Battaglini et al. (2014) find similar results, but their model is in a discrete time framework.

[37] The readers may find out why in Section 4.2.2.

This chapter provides more evidence of the role of irreversibility as a commitment instrument in providing dynamic public goods. In their classic paper, Fershtman and Nitzan (1991) compare the open-loop and the Markov solutions in a continuous-time DVCM and find that the free-riding problem is aggravated in the state-dependent Markov equilibrium with reversible investments. Recently, Battaglini et al. (2014) show that, with irreversibility, the MPE can result in better outcomes in a discrete-time model, and argue the reason is that the irreversibility constraint may limit the incentive of a current investor to free-ride on future investors by acting as a *commitment device*. The model in this chapter includes an irreversibility constraint and we find that our results in DVCM correspond as a continuous-time counterpart to the primary results with irreversibility obtained in Battaglini et al. (2014).

Our result for the DPPM extends to a dynamic framework for a durable public good the general result that the provision point mechanism (PPM) can induce more efficient outcomes than the traditional voluntary contribution mechanism (VCM) in static frameworks (Bagnoli and Lipman, 1989).[38] Although the PPM has been studied in a dynamic framework, most of these studies focus on environments in which the public good project and the benefit are provided only at the end of the game (Admati and Perry, 1991; Compte and Jehiel, 2004; Duffy et al., 2007; Marx and Matthews, 2000), and hence similar explorations for durable public goods are largely absent from the literature. As we will see in Section 4.3, the intuition for the efficient outcome that can be supported in the DPPM is the same as in any provision point related mechanisms: the provision point breaks the free riding incentive by introducing a discontinuity in the payoff function.

The rest of the chapter is organized as follows: Section 4.2.1 sets up the model; Section 4.2.2 solves the social planner's problem; Sections 4.2.3 and 4.2.4 characterize the open-loop

[38] For more recent references, see Li et at. (2014).

equilibrium and the Markov perfect equilibrium for the dynamic voluntary contribution mechanism, respectively; Section 4.2.5 gives numerical examples for cases discussed in Section 4.2.2-4.2.4; Section 4.3 extends the model to the dynamic provision point mechanism; and Section 4.4 concludes.

4.2 Dynamic voluntary contribution mechanism (DVCM)

4.2.1 The Model

Suppose *N* agents live infinitely with a flow of individual income w at each time *t*. Let $i \in I \equiv \{1, \ldots, N\}$ denote agent *i*. Each agent simultaneously chooses to make a contribution $c_{it} \in [0, w]$ to a durable public good. The durable public good follows a modified logistic growth function:

$$(4.1) \quad \dot{s}_t = \mathrm{d}s_t / \mathrm{d}t = r_{\text{int}} \cdot s_t (1 - \frac{s_t}{K}) \cdot \min\left(\sum_j c_{jt} \big/ PP, 1\right) - d \cdot s_t, \quad s_0 > 0$$

where s_t is the stock level of the public good at time t, and s_0 is the initial stock level. $d \in [0,1]$ denotes the depreciation rate or the natural mortality rate.

$r_{\text{int}} \cdot s_t (1 - \frac{s_t}{K}) \cdot \min\left(\sum_j c_{jt} \big/ PP, 1\right)$ is the conditional net growth. $r_{\text{int}} \cdot s_t (1 - s_t / K)$ specifies the logistic net growth, where $r_{\text{int}} > 0$ represents the intrinsic growth rate and $K > 0$ is the carrying capacity. $\min\left(\sum_j c_{jt} \big/ PP, 1\right)$ indicates the fraction of the net growth to be provided. A motivating example may be wildlife habitat which provides ecosystem services public goods by sustaining the wildlife population by conserving brood stock. To protect the entire habitat, a fixed cost *PP* (*provision point*) is required at each time instant. When the total contribution $\sum_j c_{jt} \in [0, PP]$, we assume a fraction $\sum_j c_{jt} \big/ PP$ of the habitat is provided and so is an identical fraction of the net growth. If the net growth represents the juveniles, then the total contribution essentially determines the survival rate. When $\sum_j c_{jt} > PP$, the habitat is fully provided and all the juveniles survive, but the survival rate cannot be greater than 1; that is, the net growth is bounded above by $r_{\text{int}} \cdot s_t(1-s_t/K)$. To make the problem more interesting, we assume $w \in (PP/N, PP)$, i.e., the fixed cost *PP* is affordable for a group but not any individual alone.

The instantaneous payoff of agent *i* at time *t* is given by

(4.2) $$\pi_{it}^{DVCM} = w + \alpha s_t - c_{it}$$

where α represents a constant marginal benefit from the stock level of the public good. The key tradeoff in the model is that a higher stock level will generate a higher payoff, but it is costly to sustain the stock level, especially when the stock is higher than *K*/2 and close to *K* where the natural growth becomes really low.

4.2.2 The social planner's problem

We solve the benevolent social planner's problem to provide a bench mark with which the equilibrium solutions can be compared. We will additionally use the social planner's case to demonstrate the general solution procedures which will be used to solve for the equilibria.

The social planner's problem is

$$\max_{\{\sum_j c_{jt}\}_{t=0}^{t=\infty}} \int_0^\infty [Nw + N\alpha \cdot s_t - \sum_j c_{jt}] \cdot e^{-\rho t} dt$$

(4.3) $$s.t.\ \dot{s}_t = r_{\text{int}} \cdot s_t(1-\frac{s_t}{K}) \cdot \min\left(\sum_j c_{jt}/PP, 1\right) - d \cdot s_t,$$

$$0 \le \sum_j c_{jt} \le Nw\ \forall\ t,\ 0 \le d \le 1,\ s_0 > 0$$

where ρ is the discount rate. First, note that the social planner will never have $\sum_j c_{jt} > PP$, since otherwise $\sum_j c_{jt} - PP$ is costly without bringing any benefit. As the constant term Nw is irrelevant in the optimization problem, we can simplify Problem (4.3) to:

$$\begin{aligned}
&\max_{\{\sum_j c_{jt}\}_{t=0}^{t=\infty}} \int_0^\infty [N\alpha \cdot s_t - \sum_j c_{jt}] \cdot e^{-\rho t} dt \\
(4.4)\quad & s.t.\ \dot{s}_t = r_{\text{int}} \cdot s_t (1 - \frac{s_t}{K}) \cdot \sum_j c_{jt} \Big/ PP - d \cdot s_t, \\
& 0 \le \sum_j c_{jt} \le PP\ \forall\ t,\ 0 \le d \le 1,\ s_0 > 0
\end{aligned}$$

The current value Hamiltonian H of Problem (4.4) is

$$\begin{aligned}
(4.5)\quad H(s_t, \sum_j c_{jt}, \lambda_t) &= N\alpha s_t - \sum_j c_{jt} + \lambda_t [r_{\text{int}} \cdot s_t (1 - \frac{s_t}{K}) \cdot \sum_j c_{jt} \Big/ PP - d \cdot s_t] \\
&= \frac{\sum_j c_{jt}}{PP} [\lambda_t \cdot r_{\text{int}} \cdot s_t (1 - \frac{s_t}{K}) - PP] + s_t (N\alpha - \lambda_t d)
\end{aligned},$$

where λ_t is the current-value costate variable associated with the state s_t. Since H is linear in the control $\sum_j c_{jt}$, generally the solution will be a combination of bang-bang and singular controls. Following standard procedures for a linear control problem (see Caputo, 2005 or Clark, 1976), we define a switching function $\sigma(\cdot)$ and then use $\sigma(\cdot)$, together with the necessary conditions from the Maximum principle, to characterize the optimal control.

Defining

$$(4.6)\quad \sigma(t, s_t, \lambda_t) \overset{\text{def}}{=} \lambda_t \cdot r_{\text{int}} \cdot s_t (1 - \frac{s_t}{K}) - PP$$

as the value of the switching function $\sigma(\cdot)$, the necessary condition $\max_{\sum_j c_{jt}} H(s_t, \sum_j c_{jt}, \lambda_t)$ can be written as the decision rule:

(4.7) $$\sum_j c_{jt} = \begin{cases} 0 & if\ \ \sigma(t, s_t, \lambda_t) < 0 \\ \in [0, PP] & if\ \ \sigma(t, s_t, \lambda_t) = 0 \\ PP & if\ \ \sigma(t, s_t, \lambda_t) > 0 \end{cases}$$

The remaining necessary conditions are given by

(4.8) $$\dot{\lambda}_t = \mathrm{d}\lambda_t / \mathrm{d}t = -H^*_{s_t} + \rho\lambda_t,$$

(4.9) $$\dot{s}_t = H^*_{\lambda_t},$$

(4.10) $$\lim_{t\to\infty} e^{-\rho t}\lambda_t s_t = 0$$

where $H^* \overset{\text{def}}{=} \max_{\sum_j c_{jt}} H$, $H^*_{s_t} = \dfrac{\partial H^*}{\partial s_t}$ and $H^*_{\lambda_t} = \dfrac{\partial H^*}{\partial \lambda_t}$. Eqs (4.8) and (4.9) are the costate and state equations, respectively. Eq (4.10) gives the transversality condition.

To identify the singular stock level, let $\sigma(t, s_t, \lambda_t) \equiv 0$, and take the derivative of both sides *w.r.t.* *t*, we will have

(4.11) $$\dot{\lambda}_t \cdot r_{\text{int}} \cdot s_t(1 - \frac{s_t}{K}) + \lambda_t \cdot r_{\text{int}} \cdot (1 - \frac{2s_t}{K}) \cdot \dot{s}_t \equiv 0$$

Substituting in $\dot{\lambda}_t$ and $\dot{s}_t$ given by Eqs (4.8) and (4.9), and noting that $\sigma(t, s_t, \lambda_t) \equiv 0$ implies $\lambda_t \equiv \dfrac{PP}{r_{\text{int}} \cdot s_t(1 - s_t/K)}$ by Eq (4.6), the singular stock level s^* can be characterized by the following equation:

(4.12) $$s^*(1 - s^*/K) = \frac{PP}{N r_{\text{int}} \alpha}[\rho + d \cdot \frac{s^*}{K - s^*}]$$

Eq (4.12) is equivalent to a cubic equation, potentially with three real roots. Given that it is costly to sustain the stock level, we will focus on $s^* \in (0, K)$, and so there will be at most two real roots. Figure 4.1 shows the case we are interested in. The horizontal axis represents the stock level, and the two conic curves are respectively based on $s(1 - s/K)$ (black curve) and

$[\rho + d \cdot s/(K-s)] \cdot PP/(Nr_{\text{int}}\alpha)$ (the blue curve). The two roots of Eq (4.12) s^* and s^{**} are represented by the intersections of the two curves (the red and the blue dots in Figure 4.1). By the Green's theorem approach as used in Sethi (1977), it is easy to show that s^* and s^{**} correspond to respectively the singular stock levels for maximum and minimum locally (**see Appendices**). The economic intuition is straightforward. Let the term $s(1-s/K)$ represent the marginal benefit (in a stock-related unit) due to the increased provision of the habitat, then the term $[\rho + d \cdot s/(K-s)] \cdot PP/(Nr_{\text{int}}\alpha)$ represents the marginal cost of increasing the stock level by taking account of the mortality rate (d) and the discount rate (ρ) which are transformed into a cost term by multiplying $PP/(Nr_{\text{int}}\alpha)$.[39] For a stock level $s \in (s^{**}, s^*)$, a relatively high growth rate results in the marginal benefit greater than the marginal cost, so it is optimal to increase the stock by protecting the juveniles. When the stock level s is greater than s^* or lower than s^{**}, the growth rate is so low such that the marginal benefit of sustaining the stock level is lower than the marginal cost, and hence it is not worth of increasing the stock level anymore.

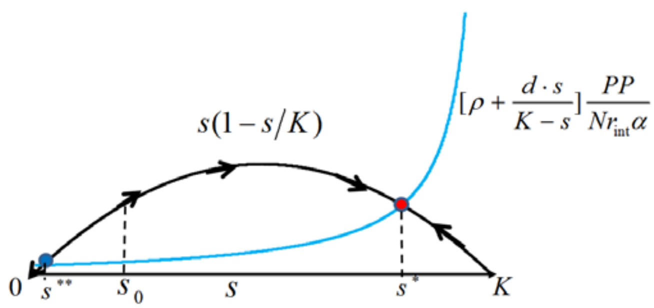


Figure 4.1 The relevant solutions of Eq (4.12)

[39] Technically and more intuitively, the marginal benefit of providing the habitat is represented by $N\alpha \cdot r_{\text{int}} \cdot s_t(1-s_t/K)/PP$., and the marginal cost by $\rho + d \cdot s/(K-s)$. Here, we rearrange the terms to simplify the graph and make it easier to use the graph to demonstrate some comparative statistics.

By Figure 4.1, s^* increases with *K*, *N*, r_{int}, α, and decreases with *PP*, ρ and *d*, which makes sense since the former set of parameters generates the benefit of a higher stock level while the latter set generates the cost.

If s^* is the only solution of Eq (4.12) for $s \in (0, K)$, then the *most rapid approach path* (*MRAP*, Spence and Starrett, 1975) to s^* is optimal. With an additional minimum solution s^{**}, it seems tempting to say that s^{**} represents the *tipping point*, below which it is optimal to allow the stock to collapse. However, as noted by Sethi (1977) and Rapaport and Cartigny (2005), when there are multiple singular solutions, the singular stock levels characterized by Eq (4.12) are only locally maxima or minima. Both studies discuss a case in which there are two local maxima and one local minimum, and show that generally the global maximum depends on the initial stock level. Unfortunately, their approach cannot be applied directly here since there is only one local maximum.

For our problem, we need to compare the singular stock s^* with the boundaries, 0 and *K*. It is easy to eliminate *K* by observing that when the stock is approaching *K*, the natural growth $s(1-s/K)$ is close to 0, the natural mortality $d \cdot s$ becomes larger, and it cannot be optimal to pay to have a negative growth rate $\dot{s}_t$. Thus, we only need to compare the *MRAP* to s^* with the *MRAP* to 0 for $s_0 \in (0, s^*]$.[40] Next, we will provide a proposition showing that, given some reasonable assumptions, there exists a tipping point s_{tip}, above which the *MRAP* to s^* is optimal for Problem (4.4).

[40] See Sethi (1977) for why only the most rapid approach path needs to be considered. For $s_0 \in (s^*, K)$, the two paths coincide until s^* is reached.

First, we will prove two useful lemmas.

Lemma 4.1. $\lim_{s_0 \to 0}\left[J(s_0; s^*) - J_0(s_0) \right] < 0$, assuming $0 < s^* < K(1 - d/r_{\text{int}})$,

where $J(s_0; s^*)$ *represents the group value of the MRAP from* s_0 *to* s^* *and then staying at* s^* *thereafter for* $s_0 \in (0, s^*]$*, and* $J_0(s_0)$ *is the group value of the MRAP from* s_0 *to 0.*

Proof. Along the *MRAP* to 0, the group contribution is always 0, the state equation becomes $\dot{s}_t = -d \cdot s_t$, and hence $s_t = s_0 e^{-dt}$. That is, the stock level 0 can be approached only asymptotically. Then the value of the *MRAP* to 0 is given by

(4.13) $$J_0(s_0) = \int_0^{\infty} [N\alpha s_t] \cdot e^{-\rho t} dt = \int_0^{\infty} [N\alpha s_0 e^{-dt}] \cdot e^{-\rho t} dt = \frac{N\alpha s_0}{d + \rho}$$

Along the *MRAP* to s^*, the group contribution is always *PP* before reaching s^*, so the state equation is $\dot{s}_t = r_{\text{int}} \cdot s_t (1 - \frac{s_t}{K}) - d \cdot s_t$. By separation of variables, it is easy to show that the stock level along the path is

(4.14) $$s_t = \frac{s_0 K(1 - d/r_{\text{int}})}{s_0 + \left[K(1 - d/r_{\text{int}}) - s_0 \right] e^{-(r_{\text{int}} - d)t}}$$

Let $s_{\tau(s_0; s^*)} = s^*$, that is, the stock level reaches s^* at $t = \tau(s_0; s^*)$ with an initial stock s_0, then

(4.15) $$\tau(s_0; s^*) = \frac{1}{(r_{\text{int}} - d)} \text{Ln} \frac{s^* / \left[K(1 - d/r_{\text{int}}) - s^* \right]}{s_0 / \left[K(1 - d/r_{\text{int}}) - s_0 \right]}$$

Thus, the value of the *MRAP* to s^* and staying at s^* thereafter for $s_0 \in (0, s^*]$ is

(4.16) $$J(s_0; s^*) = \int_0^{\tau(s_0; s^*)} [N\alpha s_t - PP] \cdot e^{-\rho t} dt + e^{-\rho \tau(s_0; s^*)} \int_0^{\infty} [N\alpha s^* - \sum_j c_{jt}^*] \cdot e^{-\rho t} dt$$

where s_t and $\tau(s_0;s^*)$ are given by Eqs (4.14) and (4.15), and

$\sum_j c_{jt}^* \overset{def.}{=} Nc^* \overset{def.}{=} PP \cdot \frac{d}{r_{\text{int}}(1-s^*K^{-1})}$ which is obtained by setting $\dot{s}_t = 0$ at s^*. Substituting s_t and c^* into Eq (4.16), we have

$$(4.17)\quad \begin{aligned} J(s_0;s^*) &= \underbrace{\int_0^{\tau(s_0;s^*)} \left[N\alpha \frac{s_0 K(1-d/r_{\text{int}})}{s_0 + \left[K(1-d/r_{\text{int}}) - s_0\right] e^{-(r_{\text{int}}-d)t}} \right] \cdot e^{-\rho t} dt - \frac{PP}{\rho}(1-e^{-\rho\tau(s_0;s^*)})}_{A} \\ &\quad + \underbrace{\frac{e^{-\rho\tau(s_0;s^*)}}{\rho}\left(N\alpha s^* - \frac{PP}{r_{\text{int}}} \frac{d}{1-s^*K^{-1}} \right)}_{B} \end{aligned}$$

where the term *A* represents the value of the *MRAP* to s^* and the term *B* represents the value of staying at s^* from $t = \tau(s_0;s^*)$. Note that, for s^* to be sustainable, we assume $r_{\text{int}} s^*(1-s^*K^{-1}) \geq d \cdot s^*$, i.e., $s^* < K(1-d/r_{\text{int}})$.[41] Then $\dot{s}_t > 0$ for all $s_t \in (0,s^*)$, and hence the stock level s_t along the *MRAP* to s^* in Eq (4.14) is greater than s_0 for any $s_0 \in (0,s^*)$.

When $0 < s^* < K(1-d/r_{\text{int}})$, $\lim_{s_0\to 0} \tau(s_0;s^*) = \infty$, and hence $\lim_{s_0\to 0} e^{-\rho\tau(s_0;s^*)} = 0$. Then we have

$$(4.18)\quad \lim_{s_0\to 0}\left[J(s_0;s^*) - J_0(s_0) \right] = \lim_{s_0\to 0} \int_0^{\tau(s_0;s^*)} \left[N\alpha \frac{s_0 K(1-d/r_{\text{int}})}{s_0 + \left[K(1-d/r_{\text{int}}) - s_0\right] e^{-(r_{\text{int}}-d)t}} \right] \cdot e^{-\rho t} dt - \frac{PP}{\rho}$$

The desired result will be obtained if the integration in (4.18) becomes strictly less than PP/ρ as s_0 goes to 0, which is true as we will argue next.

[41] That is, $\sum_j c_{jt}^* = PP \cdot \frac{d}{r_{\text{int}}(1-s^*K^{-1})} \leq PP$. Since $s^* > 0$, $s^* < K(1-d/r_{\text{int}})$ also implies $d < r_{\text{int}}$.

Given the state equation along the *MRAP* to s^*, $\dot{s}_t = r_{\text{int}} \cdot s_t(1 - \frac{s_t}{K}) - d \cdot s_t$, and an initial stock s_0 close to 0, there exists a time window [t_1, t_2] in which the stock grows faster than outside the time window, as illustrated in Figure 4.2 with the stock following Eq (4.14). That is, once the stock level becomes high enough, it only takes a short time period to reach s^*, no matter how low the initial stock level s_0 is.

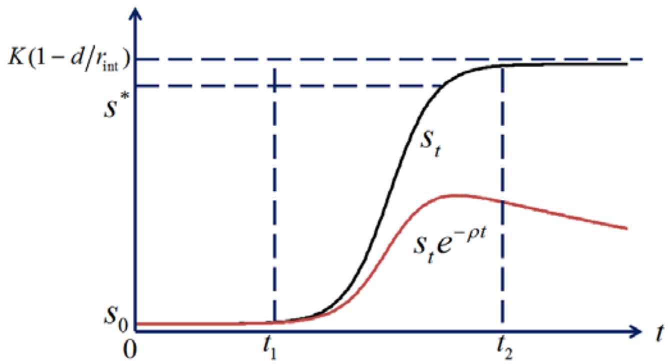


Figure 4.2 An illustration of Eq (4.14) with s_0 close to 0

As s_0 approaches 0, the fast-stock-increase time period t_1 to t_2 is postponed further and further (i.e., shifting to the right in Figure 4.2), and with an exponential discount factor $e^{-\rho t}$, the contribution of a higher stock level to the integration in (4.18) becomes trivial. In other words, during the period of 0 to $\tau(s_0; s^*)$, s_t is close to 0 most of the time. Hence, we should be able to construct an upper bound of the integration in (4.18) which is strictly less than PP/ρ (see Appendices for a rigorous construction). Graphically, the integration in *A* can be represented by the area below the red curve $s_t e^{-\rho t}$ in Figure 4.2.[42] As s_0 approaches 0, the red curve is not only pushed to the right, but also squeezed down because the maximum of $s_t e^{-\rho t}$ approaches 0 as *t*

[42] For convenience, the constant $N\alpha$ in the integration is dropped here. Also, it is easy to verify that the second order derivative of $s_t e^{-\rho t}$ *w.r.t. t* at its extreme point is negative, i.e., $s_t e^{-\rho t}$ has a maximum bounded above.

increases, and the area below the red curve will be strictly less than PP/ρ when s_0 is small enough. Thus, we have $\lim_{s_0 \to 0}\left[J(s_0; s^*) - J_0(s_0) \right] < 0$.

Lemma 4.1 guarantees that there always exists a small enough s_0 such that $J_0 > J_*$; that is, there will be a tipping point.[43] To make the model interesting, we also need a set of parameters such that $J_* > J_0$ for a large range of s_0. Intuitively, when *N*, *K*, α, and r_{int} are large, and *PP*, *d*, and ρ are small, it will be more attractive to reach a higher stock level and stay at that higher level. Actually, as we will illustrate by numerical examples later, such a set of parameters will result in $J_* - J_0$ is a concave function of s_0 for the majority of $s_0 \in (0, s^*)$. **Lemma 4.2** below demonstrates that only some very weak assumptions are needed for a non-singleton interval of s_0 in which $J_* > J_0$, and it also characterizes some curvature properties of $J_* - J_0$.

Lemma 4.2.

1) *For* $s \in (s_-^L, s_-^U)$, $J(s_0; s) - J_0(s_0)\big|_{s_0 = s} > 0$, *and* $\left.\frac{\partial (J(s_0; s) - J_0(s_0))}{\partial s_0}\right|_{s_0 = s} < 0$, *where* s_-^L *and* s_-^U *are the two roots of the equation* $s(1 - s/K) = \frac{PP}{N r_{\text{int}} \alpha}[\rho + d]$ *and* $s_-^L < s_-^U$, *assuming* $K/4 > \frac{PP}{N r_{\text{int}} \alpha}[\rho + d]$. *Further, for* $s \in (0, s_{--}^U)$, $\left.\frac{\partial^2 (J(s_0; s) - J_0(s_0))}{\partial {s_0}^2}\right|_{s_0 = s} < 0$, *where* s_{--}^U *is the real root of the equation* $s(1 - s/K) = \frac{PP}{N r_{\text{int}} \alpha}[\rho + d + r_{\text{int}}(2s/K - 1)]$, *assuming* $r_{\text{int}} > d + \rho$. *And* $s_-^L < s_{--}^U < s^* < s_-^U$ *for* $s \le s^* < K(1 - d/r_{\text{int}})$.

[43] To simplify the notation, we use J_* to denote $J(s_0; s^*)$, and J_0 to denote $J_0(s_0)$.

2) $\lim_{s_0 \to 0} \frac{\partial (J(s_0; s^*) - J_0(s_0))}{\partial s_0} > 0$, *assuming* $r_{\text{int}} > d + \rho$.

Proof. See Appendices.

Based on **Lemma 4.2. 1)**, as long as $K/4 > PP(\rho + d)/(Nr_{\text{int}}\alpha)$ we will have $J_* > J_0$ and $\partial(J_* - J_0)/\partial s_0 < 0$ when $s_0 = s^*$, which together imply $J_* > J_0$ at least for s_0 around s^*. Note that, $\rho + d < \rho + d \cdot s/(K - s)$ when $s > K/2$, hence the existence of s^* as a solution of Eq (4.12) implies $K/4 > PP(\rho + d)/(Nr_{\text{int}}\alpha)$, since $[\rho + d \cdot s/(K - s)] \cdot PP/(Nr_{\text{int}}\alpha)$ will be above $s(1 - s/K)$ otherwise. In other words, we always have $J_* > J_0$ for some s_0 as long as s^* exists. The property of the second derivative of $J_* - J_0$ in **Lemma 4.2. 1)** will be useful in Section 4.3 and will be discussed therein. **Lemma 4.2. 2)** implies that when s_0 is small enough, $J_* - J_0$ is an increasing function of s_0. The assumption of $r_{\text{int}} > d + \rho$ is easy to satisfy when r_{int} is greater than 1.

Based on **Lemmas 4.1** and **4.2**, we have the following conjecture which will be explained by intuition and supported by numerical examples later.

Conjecture 4.1.

There exits a $s_0^* \in (0, s^*)$ *such that*

$$\left. \frac{\partial (J(s_0; s^*) - J_0(s_0))}{\partial s_0} \right|_{s_0 = s_0^*} = 0 \text{ and } \left. \frac{\partial^2 (J(s_0; s^*) - J_0(s_0))}{\partial {s_0}^2} \right|_{s_0 = s_0^*} < 0;$$

$$\frac{\partial (J(s_0; s^*) - J_0(s_0))}{\partial s_0} > 0 \text{ for } s_0 \in (0, s_0^*) \text{ and } \frac{\partial (J(s_0; s^*) - J_0(s_0))}{\partial s_0} < 0 \text{ for } s_0 \in (s_0^*, s^*),$$

when N, K, α*, and* r_{int} *are large, and PP, d, and* ρ *are small, including the conditions required in* ***Lemmas 4.1*** *and* ***4.2****.*

A rigorous discussion of **Conjecture 4.1** involves the monotonicity of some *hypergeometric* functions (*thanks to* the integration in *A* of Eq 4.17), which is beyond the scope of this chapter, but the intuition is straightforward. Note that, given s^*, the shape of $J_* - J_0$ is essentially determined by two terms: the integration in *A* of Eq (4.17) and $e^{-\rho\tau(s_0;s)}$. By Eq (A4.15) in Appendices, we know $e^{-\rho\tau(s_0;s)}$ is monotonically increasing in s_0. Hence, we only need to consider the integration in *A* of Eq (4.17). Recall that the integration in *A* can be represented by the area under the red curve $s_t e^{-\rho t}$ in Figure 4.2. We have discussed the integration for the case of s_0 being close to 0, which can be arbitrarily small as shown in **Lemma 4.1**. Another extreme case is that when s_0 is close to s^*, the integration will also become arbitrarily small, actually the integration will be 0 when $s_0 = s^*$. If s_0 takes on an intermediate value, a high s^*, possibly near $K(1 - d/r_{\text{int}})$, will be reached within a reasonable time period $\tau(s_0; s^*)$. In this situation, the maximum of $s_t e^{-\rho t}$ will be higher and a relatively high $s_t e^{-\rho t}$ will be integrated over $\tau(s_0; s^*)$, and then the integration will be significantly greater than 0. Loosely speaking, when s_0 moves up from 0 in Figure 4.2, initially the bump represented by the red curve will move closer to the origin and become larger, and hence the integration increases; when s_0 is very close to s^*, the bump will still stay close to the origin but become smaller, and the integration decreases. Therefore, there may exist a $s_0^* \in (0, s^*)$ at which the integration is maximized and **Conjecture 4.1** is obtained.

Given **Lemma 4.1**, **Lemma 4.2**, and **Conjecture 4.1**, we have the following proposition.

Proposition 4.1.

There is always a tipping point $s_{tip} \in (0, s^*)$ *such that the MRAP to* s^* *is optimal for Problem (4.4) for any* $s_0 \in (s_{tip}, s^*]$, *and collapsing the stock is optimal for* $s_0 \in (0, s_{tip})$, *assuming* s^* *exists,* $r_{\text{int}} > d + \rho$, *and* ***Conjecture 4.1*** *holds.* s_{tip} *is defined as*

$$(4.19) \quad s_{tip} = \sup\{s_0 \in (0, s^*) \mid J(s_0; s^*) = J_0(s_0) \text{ and } J(s; s^*) < J_0(s) \text{ for any } s \in (0, s_0)\}$$

Proof. **Lemma 4.1 and Conjecture 4.1** imply the existence of $s_{tip} \in (0, s^*)$. **Lemma 4.1**, **Lemma 4.2** and **Conjecture 4.1** imply $J_* > J_0$ for any $s_0 \in (s_{tip}, s^*]$ and $J_* < J_0$ for $s_0 \in (0, s_{tip})$. Then the desired result is obtained.

By **Proposition 4.1**, we have the optimal solution to problem (4.4) as follows.

$$(4.20) \quad c_{it} = \begin{cases} 0 & \text{for } s_t \in (0, s_{tip}) \cup (s^*, K) \\ PP/N & \text{for } s_t \in [s_{tip}, s^*) \\ c^* = \dfrac{PP}{N} \dfrac{d}{r_{\text{int}}(1 - s^* K^{-1})} & \text{for } s_t = s^* \end{cases}$$

where s^* is the larger root of Eq (4.14) below *K*, as demonstrated in Figure 4.1. Note that Eq (4.20) satisfies the transversality condition Eq (4.10).

4.2.3 The open-loop equilibrium

In an open-loop equilibrium, agents only observe the initial stock level and make a strategic contribution plan for the entire planning horizon at *t*=0. This solution concept makes sense in cases where it is difficult or costly to update the information of the stock level along the path.

And, as we will show in Section 4.2.4, the open-loop equilibrium can be treated as a special case of the Markov perfect equilibrium and it will also help organize the solutions of the MPE. We will focus on the symmetric open-loop equilibrium.

Given the open-loop equilibrium contribution path of all other agents c_{jt}^{OL} for $j \neq i$, and $c_{-it}^{OL} = \sum_{j \neq i} c_{jt}^{OL}$, agent *i* solves

$$\max_{\{c_{it}\}_{t=0}^{t=\infty}} \int_0^{\infty} [\alpha \cdot s_t - c_{it}] \cdot e^{-\rho t} dt$$

$$\text{(4.21)} \quad s.t.\ \dot{s}_t = r_{\text{int}} \cdot s_t (1 - \frac{s_t}{K}) \cdot \frac{c_{it} + c_{-it}^{OL}}{PP} - d \cdot s_t ,$$

$$0 \leq c_{it} \leq \min\{w, PP - c_{-it}^{OL}\} \ \forall\ t,\ \ 0 < d < 1,\ \ s(0) = s_0 > 0$$

Again, the instantaneous endowment *w* is dropped from the objective function since a constant term is irrelevant in the optimization problem.

The current value Hamiltonian for agent *i* is

$$\text{(4.22)} \quad \begin{aligned} H_i &= \alpha s_t - c_{it} + \lambda_{it}[r_{\text{int}} \cdot s_t (1 - \frac{s_t}{K}) \cdot \frac{c_{it} + c_{-it}^{OL}}{PP} - d \cdot s_t] \\ &= \frac{c_{it}}{PP}[\lambda_{it} \cdot r_{\text{int}} \cdot s_t (1 - \frac{s_t}{K}) - PP] + s_t(\alpha - \lambda_{it} d) + \lambda_{it} r_{\text{int}} \cdot s_t (1 - \frac{s_t}{K}) \cdot \frac{c_{-it}^{OL}}{PP} \end{aligned}$$

where λ_{it} is the current-value costate variable associated with the state s_t for agent *i*. Since H_i is linear in c_{it}, following the same steps in Section 4.2.2, we will have the open-loop singular stock level s_{OL}^* characterized by the following equation (see Appendices for more details).

$$\text{(4.23)} \quad s_{OL}^* (1 - s_{OL}^* / K) = \frac{PP}{r_{\text{int}} \alpha} [\rho + d \cdot \frac{s_{OL}^*}{K - s_{OL}^*}]$$

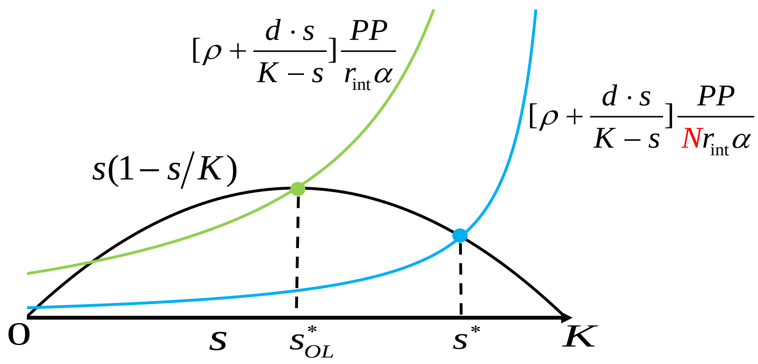


Figure 4.3 A comparison of the solutions of Eqs (4.12) and (4.23)

Comparing Eq (4.23) with Eq (4.12), the only difference is that there is a 1/*N* on the RHS of Eq (4.12), which implies that the marginal cost curve for the open-loop equilibrium is *N* times higher than that of the social planner's case, as illustrated in Figure 4.3. Therefore, the larger real root of Eq (4.23) less than *K*, s_{OL}^*, if it exists, is less than the social planner's singular stock level s^*. Then how will the tipping point s_{tip}^{OL} associated with s_{OL}^* change compared to s_{tip} of s^*? By **Lemma 4.3** to be shown next, we will find that s_{tip}^{OL} is strictly greater than s_{tip}.

Lemma 4.3. $\frac{\partial(J(s_0;s) - J_0(s_0))}{\partial s} > 0$ *for any* $s \in (s^{**}, s^*)$

Proof. See Appendices.

By Figure 4.3, $s_{OL}^* \in (s^{**}, s^*)$. Note that **Lemmas 4.1** and **4.2** can be applied to the *MRAP* to any general stock *s* less than s^*, and hence results similar to **Conjecture 4.1** and **Proposition 4.1** can be obtained for s_{tip}^{OL} and s_{OL}^*. Then **Lemma 4.3** implies $s_{tip}^{OL} > s_{tip}$, which is supported by a numerical example in Section 4.2.5. Essentially, targeting a lower stock level at which to stay will push $J(s_0;s) - J_0(s_0)$ down and the tipping point will become higher. We summarize these results of the open-loop equilibrium in **Proposition 4.2**.

Proposition 4.2

There is always a tipping point $s_{tip}^{OL} \in (0, s_{OL}^*)$ *such that the MRAP to* s_{OL}^* *is optimal for Problem (4.21) for any* $s_0 \in (s_{tip}^{OL}, s_{OL}^*]$*, and collapsing the stock is optimal for* $s_0 \in (0, s_{tip}^{OL})$*, assuming* s_{OL}^* *exists. Further,* $s_{OL}^* \le s^*$ *and* $s_{tip}^{OL} \ge s_{tip}$*, where equality holds only when N=1 for both.*

By **Proposition 4.2**, the symmetric open-loop equilibrium contribution function is

$$(4.24) \quad c_{it} = \begin{cases} 0 & \text{for } s_t \in (0, s_{tip}^{OL}) \cup (s_{OL}^*, K) \\ PP/N & \text{for } s_t \in [s_{tip}^{OL}, s_{OL}^*) \\ c_{OL}^* = \dfrac{PP}{N} \dfrac{d}{r_{\text{int}}(1 - s_{OL}^* K^{-1})} & \text{for } s_t = s_{OL}^* \end{cases}$$

Since $s_{OL}^* \le s^*$, we have a corollary that follows immediately.

Corollary 4.2. $c_{OL}^* \le c^*$*, where equality holds only when N=1.*

The intuition for the inefficient outcomes in the open-loop equilibrium—a lower steady state, a lower contribution, and easier to collapse—is straightforward: each agent only considers their own benefit from the stock level while ignoring the positive externality of their contribution, which is the same as in standard static public good games.

Note that, although the open-loop equilibrium contribution function in Eq (4.24) seems to depend on the stock level, the actual equilibrium contribution is still a time path which is calculated based on an initial stock level and Eq (4.24). Figure 4.4 compares the contribution functions of the social planner and the open-loop equilibrium.

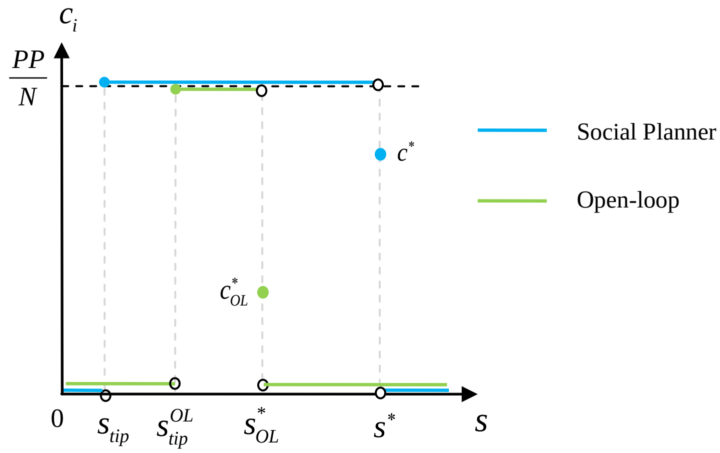


Figure 4.4 Contribution functions of the social planner and the open-loop equilibrium

4.2.4 The Markov perfect equilibrium

Once agents can update the stock level along the path and adjust their contributions accordingly, a Markov perfect equilibrium (MPE) solution is needed and is more appropriate for real cases. In the MPE, contributions depend only on the current stock level. Therefore, the equilibrium solution is a decision rule instead of a time path of contribution. We focus on symmetric MPE.

Given the MPE contribution function of all other agents $c_{jt}^{MPE}(s_t)$ for $j \neq i$, and $c_{-it}^{MPE}(s_t) = \sum_{j \neq i} c_{jt}^{MPE}(s_t)$, agent i solves

$$\max_{\{c_{it}\}_{t=0}^{t=\infty}} \int_0^\infty [\alpha \cdot s_t - c_{it}] \cdot e^{-\rho t} dt$$

(4.25) $$s.t.\ \dot{s}_t = r_{\text{int}} \cdot s_t (1 - \frac{s_t}{K}) \cdot \frac{c_{it} + c_{-it}^{MPE}(s_t)}{PP} - d \cdot s_t,$$

$$0 \leq c_{it} \leq \min\{w, PP - c_{-it}^{MPE}(s_t)\}\ \forall\ t,\ 0 < d < 1,\ s(0) = s_0 > 0$$

The key difference between the Markov equilibrium and the open-loop equilibrium is that $c_{jt}^{MPE}(s_t)$ is a function of the stock level while c_{jt}^{OL} is just a time path.

The current value Hamiltonian for agent i is

$$(4.26)\quad \begin{aligned} H_i &= \alpha s_t - c_{it} + \lambda_{it}[r_{\text{int}} \cdot s_t(1-\frac{s_t}{K})\cdot\frac{c_{it}+c_{-it}^{MPE}(s_t)}{PP} - d\cdot s_t] \\ &= \frac{c_{it}}{PP}[\lambda_{it}\cdot r_{\text{int}}\cdot s_t(1-\frac{s_t}{K}) - PP] + s_t(\alpha - \lambda_{it}d) + \lambda_{it}r_{\text{int}}\cdot s_t(1-\frac{s_t}{K})\cdot\frac{c_{-it}^{MPE}(s_t)}{PP} \end{aligned}$$

Since H_i is linear in c_{it}, technically we can follow the same steps as in the two previous sections to find the characteristic function for the singular stock level in MPE. However, the stock-dependent contribution function will change the costate equation and ultimately differentiate the MPE from the open-loop equilibrium.

The costate equation for the MPE is[44]

$$(4.27)\quad \dot{\lambda}_{it} = -\frac{\partial H_i^{MPE}}{\partial s_t} - \sum_{j\neq i}\frac{\partial H_i^{MPE}}{\partial c_{jt}^{MPE}}\frac{\partial c_{jt}^{MPE}(s_t)}{\partial s_t} + \rho\lambda_{it},$$

where $H_i^{MPE} = \max_{c_{it}} H_i$. The arguments highlighted in red in Eq (4.27) will result in the derivative of the contribution function *w.r.t.* the stock appearing in the costate equation and hence in the characteristic function for the singular stock level, which is not the case for either the social planner's problem or the open-loop equilibrium. Assuming a symmetric equilibrium contribution and writing out Eq (4.27), we will have

$$(4.28)\quad \dot{\lambda}_{it} = \lambda_{it}\left[\rho + d - r_{\text{int}}\cdot(1-\frac{2s_t}{K})\cdot\frac{Nc_{it}^{MPE}(s_t)}{PP} + r_{\text{int}}\cdot s_t(1-\frac{s_t}{K})\cdot\frac{(N-1)c_{it}^{MPE\prime}(s_t)}{PP}\right] - \alpha,$$

where $c_{it}^{MPE\prime}(s_t) = \dfrac{\partial c_{it}^{MPE}(s_t)}{\partial s_t}$.

With Eq (4.28) and following the same steps as in Section 4.2.2, we will have the characteristic function for a singular stock level in MPE

[44] See Mehlmann (1988, pg. 55) for a formal proof.

$$(4.29)\quad s_t(1-\frac{s_t}{K})=\frac{PP}{r_{\text{int}}\alpha}\left[\rho+d\frac{s_t}{K-s_t}-r_{\text{int}}\cdot s_t(1-\frac{s_t}{K})\cdot\frac{(N-1)c_{it}^{MPE\prime}(s_t)}{PP}\right]$$

Eq (4.29) is similar to the characteristic functions for both of the social planner's case and the open-loop equilibrium, except that the singular stock level in MPE depends on $c_{it}^{MPE\prime}(s_t)$. If $c_{it}^{MPE\prime}(s_t)=0$, Eq (4.29) becomes exactly the same as Eq (4.23), the characteristic function for the open-loop equilibrium, and we will have the same equilibrium solution as for the open-loop. In this sense, we say the open-loop equilibrium is a special case of the MPE.

However, by definition, generally $c_{it}^{MPE\prime}(s_t)\neq 0$ and is well defined, then we will have the following lemma.

Lemma 4.4. *A MPE contribution function* $c_{it}^{MPE}(s_t)$ *for Problem (4.25) with* $c_{it}^{MPE\prime}(s_t)\neq 0$ *and well defined at a singular stock level implies the existence of a non-singleton interval of singular stock levels, denoted by* $[s_2,s_3]$, *and Eq (4.29) holds for all* $s_t\in[s_2,s_3]$.

Proof.

We prove by contradiction. Assume there is a unique singular stock level s_{MPE1}^* determined by Eq (4.29) in the neighborhood of s_{MPE1}^*, then we may have an equilibrium contribution function similar to that in the open-loop equilibrium as illustrated in Figure 4.4 around the singular stock s_{OL}^*, where the contribution function is discontinuous at s_{OL}^*. But if so, $c_{it}^{MPE\prime}(s_t)$ cannot be well defined due to the discontinuity of c_{it}^{MPE} at $s_t=s_{MPE1}^*$. Therefore, a non-singleton interval of singular stock levels is necessary for a well-defined Markov perfect equilibrium contribution function for Problem (4.25), which means that the characteristic equation Eq (4.29) holds for a range of singular stock levels.[45] That is,

$$s_t(1-\frac{s_t}{K})=\frac{PP}{r_{\text{int}}\alpha}\left[\rho+d\frac{s_t}{K-s_t}-r_{\text{int}}\cdot s_t(1-\frac{s_t}{K})\cdot\frac{(N-1)c_{it}^{MPE\prime}(s_t)}{PP}\right] \text{ for any } s_t\in[s_2,s_3]$$

Thus, we have the desired result.

By **Lemma 4.4**, Eq (4.29) specifies the behavior of $c_{it}^{MPE}(s_t)$ within $[s_2,s_3]$ and hence can be used to solve for $c_{it}^{MPE}(s_t)$.

Before solving the contribution function, let us use Figure 4.3 to explain the underlying intuition for the interdependence between a state-dependent contribution function and a range of singular stock levels. As discussed earlier, if $c_{it}^{MPE\prime}(s_t)=0$, Eq (4.29) becomes exactly the same as Eq (4.23), and hence we can use Figure 4.3 to represent Eq (4.29), in which case there is a unique singular stock level s_{OL}^* for a local maximum determined by the intersection of the marginal benefit curve (black) and the marginal cost curve (green). If $c_{it}^{MPE\prime}(s_t)\neq 0$, we will have some extra freedom to change the green cost curve by choosing a non-zero $c_{it}^{MPE\prime}(s_t)$. In particular, we can construct a $c_{it}^{MPE}(s_t)$ such that whenever $s_t(1-\frac{s_t}{K})\neq\frac{PP}{r_{\text{int}}\alpha}[\rho+d\cdot\frac{s_t}{K-s_t}]$, $c_{it}^{MPE\prime}(s_t)$ satisfies $s_t(1-\frac{s_t}{K})=\frac{PP}{r_{\text{int}}\alpha}\left[\rho+d\frac{s_t}{K-s_t}-r_{\text{int}}\cdot s_t(1-\frac{s_t}{K})\cdot\frac{(N-1)c_{it}^{MPE\prime}(s_t)}{PP}\right]$. In this way, we can use $c_{it}^{MPE\prime}(s_t)$ to adjust the open-loop marginal cost curve to coincide with the marginal benefit curve for a range of stocks, and then the range of stock levels corresponding to

[45] The idea of a range of singular stock levels is similar to a flat-top objective function as introduced in Battaglini et al. (2014) where they discuss a dynamic public good game in a discrete time framework using the value function approach. Essentially, we introduce here their counterpart in a continuous time framework using the Hamiltonian approach in a more content-rich model setup.

the overlapping segment of the two curves are all singular solutions. You can imagine that with some non-zero $c_{it}^{MPE\prime}(s_t)$, the segment of the green cost curve for $s_t > s_{OL}^*$ can be bent down to the exact position of the black benefit curve, and the segment for $s_t < s_{OL}^*$ can be bent accordingly in a similar way. Further, depending on the location of the range relative to the intersections of the marginal benefit curve and the open-loop marginal cost curve (Figure 4.3), the singular solutions correspond to either local maxima or local minima. For example, a range of stock levels including s_{OL}^* but with a lower bound greater than the singular stock as obtained for the local minimum in the open-loop solution (i.e., the lower-stock intersection of the black curve and the green curve in Figure 4.3) will correspond to local maxima, while a range with the local minimum of the open-loop solution as the upper bound will be local minima.

To solve for $c_{it}^{MPE}(s_t)$, we rearrange Eq (4.29) to obtain the differential equation

$$(4.30)\quad c_{it}^{MPE\prime}(s_t) = \frac{1}{N-1}\left[\frac{PP}{r_{\text{int}}}\left(\frac{\rho}{s_t(1-s_t/K)} + \frac{d}{K(1-s_t/K)^2}\right) - \alpha\right]$$

Integrating $c_{it}^{MPE\prime}(s_t)$ with respect to s_t, we will have

$$(4.31)\quad c_{it}^{MPE}(s_t) = \frac{1}{N-1}\left[\frac{PP}{r_{\text{int}}}\left(\rho \ln\frac{s_t}{1-s_t/K} + \frac{d}{1-s_t/K}\right) - \alpha s_t\right] + const$$

Eq (4.31) gives the contribution function that ensures Eq (4.29) holds for any $s_t \in [s_2, s_3]$. The constant *const* in Eq (4.31) can be determined by choosing a steady state $s_{MPE}^* \in [s_2, s_3]$ at which $\dot{s}_t = 0$, i.e.,

$$\dot{s}_t = r_{\text{int}} \cdot s_{MPE}^*(1 - \frac{s_{MPE}^*}{K}) \cdot \frac{Nc^{MPE}(s_{MPE}^*)}{PP} - d \cdot s_{MPE}^* = 0$$

where we use $c^{MPE}(s_{MPE}^*)$ to denote the symmetric MPE contribution function. Thus,

$$(4.32)\quad c^{MPE}(s_{MPE}^*) = \frac{PP \cdot d}{N \cdot r_{\text{int}}(1 - s_{MPE}^*/K)}$$

Plug Eq (4.32) and $s_t = s_{MPE}^*$ into Eq (4.31), we will have

$$(4.33)\quad const(s_{MPE}^*) = -\frac{1}{N-1}\left[\frac{PP}{r_{\text{int}}}\left(\rho \ln \frac{s_{MPE}^*}{1 - s_{MPE}^*/K} + \frac{1}{N}\frac{d}{1 - s_{MPE}^*/K}\right) - \alpha s_{MPE}^*\right]$$

The MPE contribution function supporting a range of singular stock levels with a steady state $s_{MPE}^* \in [s_2, s_3]$ is fully characterized by Eq (4.31) and (4.33).

So far, we have constructed the solution framework for a Markov perfect equilibrium. It seems that a MPE contribution function gives us considerable freedom to choose a singular stock level. Then, can a MPE support a higher singular stock which results in a lower tipping point than the open-loop equilibrium? How close to the efficient stock s^* can a singular stock be supported by a MPE? To answer these questions, we need to identify some reasonable boundaries of the range of the singular stock levels, denoted by s_2 and s_3. As in Battaglini et al. (2014), we will focus on stable steady states supported by monotonic contribution functions (i.e., non-decreasing with stock).

For s_{MPE}^* to be a stable steady state, we need $\partial \dot{s}_t / \partial s_t \big|_{s_t = s_{MPE}^*} < 0$, that is,

$$\left.\frac{\partial \dot{s}_t}{\partial s_t}\right|_{s_t = s_{MPE}^*} = r_{\text{int}}(1 - \frac{2s_t}{K}) \cdot \frac{Nc_{it}^{MPE}(s_t)}{PP} + r_{\text{int}} \cdot s_t(1 - \frac{s_t}{K}) \cdot \frac{Nc_{it}^{MPE\prime}(s_t)}{PP} - d\,\Bigg|_{s_t = s_{MPE}^*} < 0\,.$$

Plug in $c_{it}^{MPE}(s_{MPE}^*)$ (Eq 4.32) and $c_{it}^{MPE\prime}(s_{MPE}^*)$ (Eq 4.31), we will have

$$s_t(1 - \frac{s_t}{K}) > \frac{PP}{r_{\text{int}}\alpha}\left[\rho + \frac{d}{N}\frac{s_t}{K - s_t}\right] \text{ at } s_t = s_{MPE}^*\,.$$

Then, the boundaries of stable steady states in MPE are characterized by the following equation

$$(4.34)\quad s_{st}(1-\frac{s_{st}}{K})=\frac{PP}{r_{\text{int}}\alpha}\left[\rho+\frac{d}{N}\frac{s_{st}}{K-s_{st}}\right]$$

with s_{st}^U and s_{st}^L denoting the two real roots that are less than *K*, and $s_{st}^U \geq s_{st}^L$. Noticing that Eqs (4.34), (4.12) and (4.23) share the same LHS, we add a curve representing the RHS of Eq (4.34) to Figure 4.3 for comparisons of the solutions of the three equations (Figure 4.5).

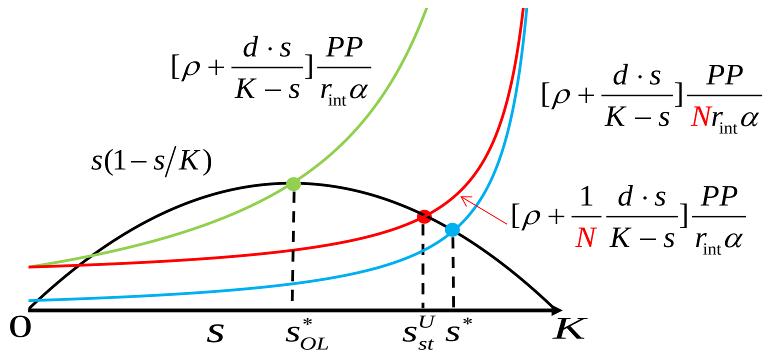


Figure 4.5 Comparisons of the solutions of Eqs (4.12), (4.23) and (4.34)

Observing Figure 4.5, we have the following lemma.

Lemma 4.5. $s_{OL}^* \leq s_{st}^U \leq s^*$, *where* s^*, s_{OL}^*, *and* s_{st}^U *are defined in Eqs (4.12), (4.23), and (4.34), respectively. And* $s_{OL}^* = s_{st}^U$ *when N = 1 or d = 0;* $s_{st}^U = s^*$ *when N = 1 or* $\rho = 0$.

Proof. All results are straightforward based on Figure 4.5.

By **Lemma 4.5**, the upper bound of the stable singular stocks supported in the MPE is generally lower than the socially efficient singular stock level s^*, but higher than the singular level in the open-loop equilibrium s_{OL}^*. And when *N* > 1, the efficient singular stock level can be achieved only asymptotically in the MPE as agents become infinitely patient, which is similar to results obtained in the existing literature (Battaglini et al, 2014; Marx and Matthew, 2000).

Note, s_{st}^L (the unlabeled intersection of the black curve and the red curve in Figure 4.5) is lower than the singular stock for the local minimum in the open-loop solution, and hence it is irrelevant for optimality.

In terms of monotonicity, we have the following lemma.

Lemma 4.6. s_{OL}^* *is the lower bound of the monotonic singular stocks for local maxima in MPE for Problem (4.25).*

Proof. The desired result can be obtained immediately by rearranging Eq (4.30).

Lemma 4.6 is intuitive based on our previous explanation of the role of a stock-dependent contribution function in supporting a range of singular stocks: it is straightforward to see that $c_{it}^{MPE\prime}(s_t) > 0$ when $s_t(1 - s_t/K) < \frac{PP}{r_{\text{int}}\alpha}\left(\rho + d\frac{s_t}{K - s_t}\right)$, i.e., for the stock levels in the segments of the black marginal benefit curve lower than the green marginal cost curve in Figure 4.5.

Let s_2 and s_3 denote respectively the lower bound and the upper bound of monotonic stable singular stock levels in MPE, we will have $s_2 = s_{OL}^*$ and $s_3 = s_{st}^U$ by **Lemmas 4.5** and **4.6**. Once a singular stock level s_{MPE}^* is chosen, a tipping point $s_{tip}^{MPE}(s_{MPE}^*)$ can be identified following the same steps as in the previous sections, and an equilibrium contribution function can be specified. For example, for any $s_{MPE}^* \in [s_{OL}^*, s_{st}^U]$, the contribution function

$$(4.35)\quad c_{it} = \begin{cases} 0 & \text{for } s_t \in (0, s_{tip}^{MPE}) \cup (s_{st}^U, K) \\ PP/N & \text{for } s_t \in [s_{tip}^{MPE}, s_{OL}^*) \\ c_{it}^{MPE}(s_t; s_{MPE}^*) & \text{for } s_t \in [s_{OL}^*, s_{st}^U] \end{cases} \quad \text{for all } i,$$

where $c_{it}^{MPE}(s_t; s_{MPE}^*) = \frac{1}{N-1}\left[\frac{PP}{r_{\text{int}}}\left(\rho \ln\frac{s_t}{1 - s_t/K} + \frac{d}{1 - s_t/K}\right) - \alpha s_t\right] + const(s_{MPE}^*)$,

is a Markov perfect equilibrium which supports s_{MPE}^* as a monotonic stable steady state.

Next, we construct a particular MPE in which s_{st}^{U} is supported as a steady state singular stock and we show that the associated tipping point $s_{tip}^{MPE}(s_{st}^{U}) \in (s_{tip}, s_{tip}^{OL})$. Figure 4.6 shows the constructed MPE contribution function with $s_{MPE}^{*} = s_{st}^{U}$, combined with typical contribution functions of the social planner and the open-loop equilibrium.

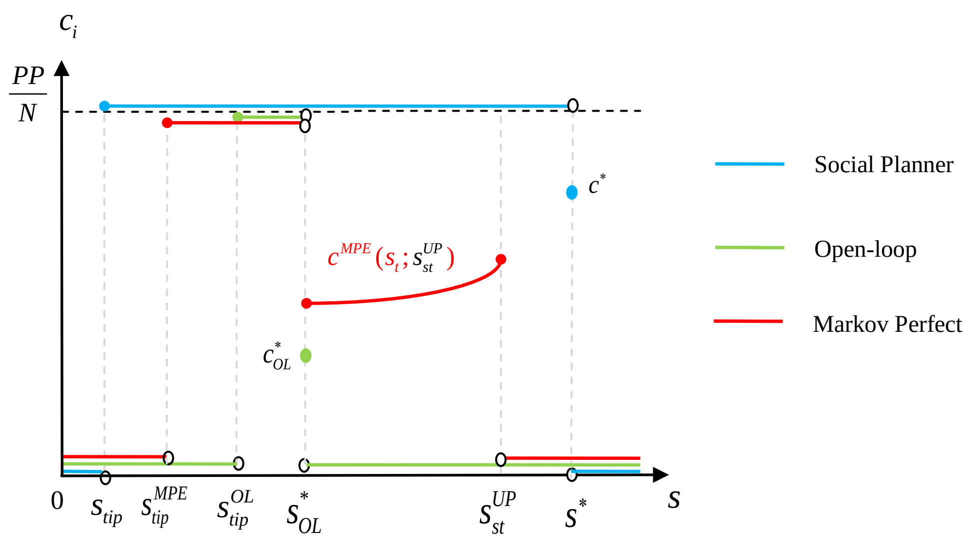


Figure 4.6 Contribution functions of the social planner, the open-loop and the MPE equilibria

To support Figure 4.6, we need the following three lemmas.

Lemma 4.7. $\dfrac{\partial c_{it}^{MPE}(s_t; s_{MPE}^{*})}{\partial s_{MPE}^{*}} \geq 0$ *for all* $s_{MPE}^{*} \in [s_{OL}^{*}, s_{st}^{U}]$

Proof.

Note that each stable steady state $s_{MPE}^{*} \in [s_{OL}^{*}, s_{st}^{U}]$ is supported by a unique equilibrium contribution path within the singular range, which is determined by the constant term $const(s_{MPE}^{*})$. Essentially, $const(s_{MPE}^{*})$ moves the contribution path up or down within the singular range. Further, take the derivative of $const(s_{MPE}^{*})$ (Eq 4.33) *w.r.t.* s_{MPE}^{*}, we will have

$$(4.36)\quad \begin{aligned}\frac{\partial const(s^*_{MPE})}{\partial s^*_{MPE}} &= -\frac{1}{N-1}\left[\frac{PP}{r_{\text{int}}}\left(\frac{\rho}{s^*_{MPE}(1-s^*_{MPE}/K)}+\frac{1}{N}\frac{d}{K(1-s^*_{MPE}/K)^2}\right)-\alpha\right]\\ &= \frac{1}{N-1}\frac{1}{s^*_{MPE}(1-s^*_{MPE}/K)}\left[\alpha s^*_{MPE}(1-s^*_{MPE}/K)-\frac{PP}{r_{\text{int}}}\left(\rho+\frac{d}{N}\frac{s^*_{MPE}}{K-s^*_{MPE}}\right)\right]\end{aligned}$$

Since the red terms in Eq (4.36) can be rearranged to an equation similar to Eq (4.34) and hence are greater than or equal to zero for any $s^*_{MPE}\in[s^*_{OL},s^U_{st}]$ with equality only at $s^*_{MPE}=s^U_{st}$, we will have $\frac{\partial const(s^*_{MPE})}{\partial s^*_{MPE}}\geq 0$, and hence $\frac{\partial c^{MPE}_{it}(s_t;s^*_{MPE})}{\partial s^*_{MPE}}\geq 0$ for all $s^*_{MPE}\in[s^*_{OL},s^U_{st}]$.

Lemma 4.7 means that the higher a monotonic stable steady state to be supported, the higher the equilibrium contribution that is needed within the singular range.

Lemma 4.8. $J(s_0;s^U_{st})\in\left(J(s_0;s^*_{OL}),J(s_0;s^*)\right)$ *for all* $s_0\in(0,s^*_{OL}]$

Proof.

When $s_0\leq s^*_{OL}$, $J(s_0;s^U_{st})>J(s_0;s^*_{OL})$ due to the following additional integration over the stock level $[s^*_{OL},s^U_{st}]$ in $J(s_0;s^U_{st})$

$$(4.37)\quad e^{-\rho\tau(s_0;s^*_{OL})}\int_0^\infty[N\alpha s(c^{MPE}_{it}(s_t;s^U_{st}))-Nc^{MPE}_{it}(s_t;s^U_{st})]\cdot e^{-\rho t}dt$$

where $c^{MPE}_{it}(s_t;s^U_{st})$ is defined in (4.35), and $s(c^{MPE}_{it}(s_t;s^U_{st}))$ is the stock level following the state equation

$$(4.38)\quad \dot{s}_t=r_{\text{int}}\cdot s_t(1-\frac{s_t}{K})\cdot\frac{Nc^{MPE}_{it}(s_t;s^U_{st})}{PP}-d\cdot s_t \text{ for } s_t\in[s^*_{OL},s^U_{st}].$$

Note that the integration (4.37) is greater than $\frac{e^{-\rho\tau(s_0;s^*_{OL})}}{\rho}\left(N\alpha s^*_{OL}-\frac{PP}{r_{\text{int}}}\frac{d}{1-s^*_{OL}K^{-1}}\right)$, the value of staying at s^*_{OL} once s^*_{OL} is reached in $J(s_0;s^*_{OL})$. The reason is based on one insight from using the Green's theorem to solve for the social planner's problem in Section 4.2.2, namely, as long as the stock level is above the tipping point and below the singular stock level, it is optimal to reach the singular stock as fast as possible. Similarly, it is straightforward to show it is also better to reach a stock level closer to the singular stock from below as fast as possible and then stay there, than to reach that stock level later.[46] By **Lemma 4.7**, the stock path given by $c_{it}^{MPE}(s_t; s_{st}^U)$ (as shown in Figure 4.6) in (4.37) is closer to the socially efficient stock path than staying at s_{OL}^*, therefore, we will have $J(s_0; s_{st}^U) > J(s_0; s_{OL}^*)$ for $s_0 \leq s_{OL}^*$.

Similarly, since the stock path given by $c_{it}^{MPE}(s_t; s_{st}^U)$ in (4.37) is still lower or slower than the socially efficient stock path, we have $J(s_0; s_{st}^U) < J(s_0; s^*)$ for $s_0 \leq s_{OL}^*$. And thus the desired result is obtained.

Lemma 4.8 essentially shows that the MPE contribution function over the singular range pushes $J(s_0; s)$ up for $s_0 \leq s_{OL}^*$ when the targeted stock level increases from s_{OL}^* to s_{st}^U. Then we will have the following result immediately.

Lemma 4.9. $s_{tip}^{MPE}(s_{st}^U) \in (s_{tip}, s_{tip}^{OL})$, *where* s_{st}^U *is supported as a steady state singular stock by the constructed MPE shown in Figure 4.6.*

Proof.

By **Lemma 4.3** and **Proposition 4.2**, we know that the higher s the *MRAP* to for $s \in (s^{**}, s^*)$, the lower the associated tipping point, and in particular, we have $s_{tip} < s_{tip}^{OL}$. By **Lemma 4.8**, we will have $J(s_0; s_{st}^U) - J_0(s_0) \in \left(J(s_0; s_{OL}^*) - J_0(s_0), J(s_0; s^*) - J_0(s_0)\right)$, and hence $s_{tip}^{MPE}(s_{st}^U)$ $\in (s_{tip}, s_{tip}^{OL})$.

[46] Use the same trick of transforming a line-integration over a simple closed curve to a double integration over an area and show that the integrand is negative over the integration area.

Lastly, we summarize all the above observations about the MPE by the following proposition.

Proposition 4.3

The Markov perfect equilibrium can support more efficient outcomes than the open-loop equilibrium, in terms of stable steady states, tipping points, and equilibrium contributions, by allowing a non-singleton interval of singular stock levels. The socially efficient stock level can be achieved asymptotically from below when agents are infinitely patient.

Proof. See **Lemmas 4.4** to **4.9**.

A similar analysis can show that a stable steady state lower than s_{OL}^{*} can be supported by a contribution function monotonically decreasing with stock, but the tipping point will be higher.

4.2.5 Numerical Examples

We use numerical examples to show the differences among the solutions of the social planner, the open-loop equilibrium, and the Markov perfect equilibrium. Particularly, we are interested in the singular stock level and the tipping point in each of the three cases.

Table 4.1 shows the singular stock levels and the tipping points for the social planner, the open-loop equilibrium and the Markov perfect equilibrium with the parameters: $N = 5$, $PP = 60$; $K = 100$; $d = 0.1$; $\alpha = 0.3$; $r_{\text{int}} = 1.2$; $\rho = 0.05$. As expected, the social planner has the highest singular stock level and the lowest tipping point, and the open-loop equilibrium results in the lowest singular stock level and the highest tipping point. By introducing a stock-dependent contribution function as in the MPE, a much higher singular stock level can be sustained as a monotonic stable steady state, with a tipping point lower than that in the open-loop solution.

Table 4.1 Numerical examples of singular levels and tipping points*

	Singular Stock Level	Tipping Point
Social Planner	81	8×10^{-8}
Open-Loop	50	4×10^{-6}
Markov Perfect	75**	5×10^{-7}

*N = 5, PP = 60; K = 100; d = 0.1; α = 0.3; r_{int} = 1.2; ρ = 0.05

**: the upper bound of the monotonic stable steady state in MPE

If we change ρ from 0.05 to 0.19 and keep the other parameters the same, the social planner's singular stock level s^* will decrease to 77 and the tipping point will increase to 0.3, and in both the open-loop equilibrium and the Markov perfect equilibrium, no (stable) singular stock level can be supported, i.e., there is no real root less than K in Eqs (4.23) and (4.34).

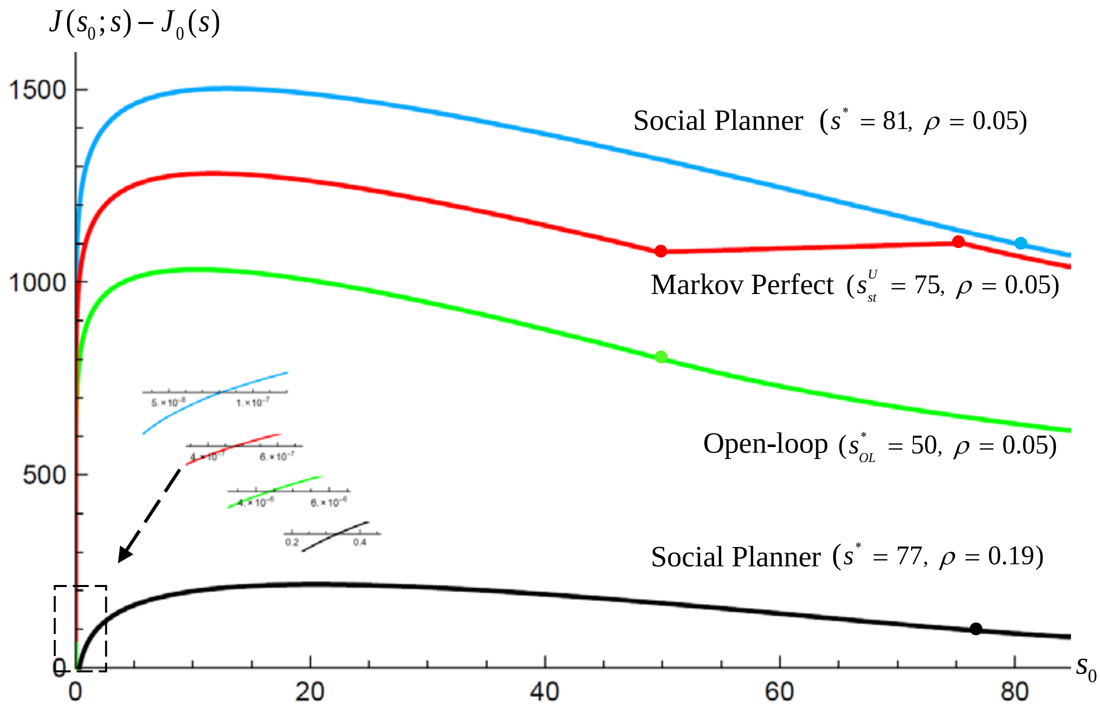


Figure 4.7 Numerical examples of $J(s_0;s)-J_0(s)$ as a function of s_0.

N = 5, PP = 60; K = 100; d = 0.1; α = 0.3; r_{int} = 1.2; color dots represents targeted singular stock levels.

Figure 4.7 shows how $J(s_0;s)-J_0(s)$ changes with s_0 for the four singular stock levels mentioned above. Note that when s_0 is greater than the targeted stock level s, $J(s_0;s)-J_0(s)$ is given by

(4.39) $J(s_0;s) - J_0(s_0) = e^{-\frac{\rho}{d}\left[\ln\frac{s_0}{s}\right]}\left[\frac{\left(N\alpha s - PP\cdot d/\left[r_{\text{int}}(1-sK^{-1})\right]\right)}{\rho} - \frac{N\alpha s}{d+\rho}\right]$ for $s_0 > s$.

(4.39) represents the value of the *MRAP* of s_0 being reduced to s and then staying at s. Note that it takes $\ln(s_0/s)/d$ of time to decrease s_0 to s before which $J(s_0;s)$ and $J_0(s_0)$ coincide. Also, for the MPE, when $s_0 \in [s^*_{OL}, s^U_{st}]$, the integration (4.37) is used for $J(s_0;s)$. Color dots in Figure 4.7 are used to represent the targeted stock levels. And, as expected, $J(s_0;s) - J_0(s)$ is consistent with **Conjecture 4.1** in all four cases except for $s_0 \in [s^*_{OL}, s^U_{st}]$ where a particular constructed MPE contribution function is used.

4.3 Dynamic Provision Point Mechanism (DPPM)

As shown in Section 4.2, a dynamic voluntary contribution mechanism can only achieve the socially efficient outcome asymptotically as a stable steady state in a Markov perfect equilibrium when agents become infinitely patient (ρ= 0). In this section, we will show when a provision point is introduced, the *most rapid approach path* to the efficient singular stock level s^* can be supported in a symmetric MPE. The intuition is the same as in static games: a provision point breaks the free riding incentive by introducing a discontinuity in the payoff function.

Next, we first explain the social planner's problem with a provision point and show how the solution may look different from that in Section 4.2.2 and then we will characterize one class of MPE of DPPM including an efficient symmetric MPE in a non-asymptotic sense.

4.3.1 The social planner's problem with a provision point

In the dynamic provision point mechanism, the logistic net growth $r_{\text{int}} \cdot s_t(1 - s_t/K)$ is either fully provided when the total contribution $\sum_j c_{jt} \geq PP$, or not provided at all when $\sum_j c_{jt} < PP$.

There is no partial provision like one half of the net growth if $\sum_j c_{jt} = PP/2$. When the net growth is not provided, no contribution will be collected. In the context of our motivation example in Section 4.2.1, this provision point setup mimics a situation where the public good stock depends on an indivisible habitat which cannot be provided at all if the cost is not covered and no juveniles (growth or surplus) can survive without a protected area. For example, for some grassland nesting birds (Bobolinks), almost all (99%) of the breeding effort will be lost without the provision of a farmland-based habitat (e.g. Bollinger et al., 1990).

The social planner's problem with a provision point is

$$\begin{aligned} &\max_{\{\delta_t\}_{t=0}^{t=\infty}} \int_0^\infty [N\alpha \cdot s_t - PP \cdot \delta_t] \cdot e^{-\rho t} dt \\ (4.40) \quad &s.t.\ \dot{s}_t = r_{\text{int}} \cdot s_t(1-\frac{s_t}{K}) \cdot \delta_t - d \cdot s_t, \\ &\quad \delta_t \in \{0,1\}\ \forall\ t,\ 0 \le d \le 1,\ s_0 > 0 \end{aligned}$$

where $\delta_t \in \{0,1\}$ indicates whether or not the net growth will be provided. $\delta_t = 1$ if $\sum_j c_{jt} \ge PP$, $\delta_t = 0$ otherwise. Still, we assume $w \in (PP/N, PP)$ and we drop the term Nw in (4.40) since it is irrelevant in the optimization problem.

Problem (4.40) is exactly the same as Problem (4.4) except that the control in (4.40) becomes discrete (binary) $\delta_t \in \{0,1\}$, while in (4.4) the corresponding control $\sum_j c_{jt} \big/ PP \in [0,1]$. Therefore, the singular solution in (4.4) is generally no longer admissible in (4.40). However, we have the following lemma.

Lemma 4.10. *The singular stock level in Problem (4.4) can be approximated in Problem (4.40) as closely as desired by "chattering" around the singular level if the control can be switched at an infinite rate in Problem (4.40).*

Proof. Marchal (1973) gives some sufficient conditions for the desired result to hold. Specifically, the following three conditions are required:

1) $\dot{s}_t$ is bounded;
2) The control function is a measurable function in the restricted meaning of Borel;
3) The differential system is canonical in the meaning of Pontryagin.

The three conditions are all quite general, and based on Marchal (1973)'s explanation in his paper, they are all satisfied in Problem (4.40). Hence, we have the desired outcome.

The intuition to support **Lemma 4.10** is quite clear. Let us have a thought experiment as follows. Starting with a stock level lower than s^*, we can follow the *MRAP* to some stock right below s^* by setting $\delta_t = 1$. Since δ_t can only take the value of 1, instead of $Nc^*/PP = d/\left[r_{\text{int}}(1-s^*K^{-1})\right]$ for being able to stay at s^*, the stock level will be higher than s^* at the next moment if it takes a time interval $\Delta t > 0$ to switch from 1 to 0. Similarly, when the stock is reduced from a stock level higher than s^* to just above s^* by setting $\delta_t = 0$, the stock level will be lower than s^* after Δt. Thus, we will have a stock path "chattering" around s^*. If the time length of switching Δt becomes smaller, the stock level will overshoot less and be closer to s^*. As Δt goes to zero, i.e., δ_t can be switched at an infinite rate, the amplitude of the oscillation (chattering) would be arbitrarily close to zero and hence the singular level can be achieved in the limit. The sufficient conditions required by Marchal (1973) just make sure the above argument can hold rigorously.

Although we do not have a constraint in (4.40) to restrict δ_t from being switched between 0 and 1 at an infinite rate, an infinite switching rate is not possible in practice. To make the problem more realistic and more intuitive to explain, i.e., to allow for $\Delta t > 0$, we can discretize (4.40). For simplicity, let $\Delta t = 1$, then we will just have the same problem in a standard discrete time framework. Based on **Lemma 4.10**, we have the following conjecture.

Conjecture 4.2.

The optimal stock path of the discretized version of Problem (4.40) is the MRAP to a stock level closer to s^* than otherwise, and then chattering around s^* if s_0 is higher than the tipping point, otherwise the MRAP to 0.[47]

Due to the binary control, we cannot solve the problem analytically in the discrete time and prove **Conjecture 4.2** directly. However, if this problem can be converted to a well-defined dynamic programming problem, we will be able to solve it numerically and verify our conjecture by numerical examples, as we will show next.

The social planner's problem of DPPM in the discrete time framework is

$$\begin{aligned} &\max_{\{\delta_t\}_{t=0}^{t=\infty}} \sum_{t=0}^{\infty} \beta^t [N\alpha \cdot s_t - PP \cdot \delta_t] \\ (4.41) \quad & s.t. \quad s_{t+1} = r_{\text{int}} \cdot s_t (1 - \frac{s_t}{K}) \cdot \delta_t + s_t (1-d), \\ & \qquad \delta_t \in \{0,1\} \ \forall \ t, \ 0 \le d \le 1, \ s_0 > 0 \end{aligned}$$

where β is the discount factor and $\beta = 1/(1+\rho)$. The recursive representation of (4.41) is

$$(4.42) \quad V^P(s) = \max_{\delta} \left\{ \begin{aligned} & N\alpha \cdot s - PP \cdot \delta + \beta \cdot V^P(s_+) \\ & s.t. \quad s_+ = r_{\text{int}} \cdot s(1 - \frac{s}{K}) \cdot \delta + s(1-d), \\ & \qquad \delta \in \{0,1\} \ \forall \ t, \ 0 \le d \le 1, \ s > 0 \end{aligned} \right\}$$

Following standard procedures in Stokey et al. (1989), it is easy to verify that (4.42) is a well-defined dynamic programming problem and hence we would expect a convergent numerical solution.

47 Our argument about the existence of a tipping point still holds in this problem.

Figure 4.8 shows a numerical solution of (4.42). The conditional value functions are given in *Panel A* and the simulated optimal stock path is in *Panel B*. The parameters are N = 5, PP = 60; K = 100; d = 0.1; α = 0.3; r_{int} = 1.2; β = 0.84(corresponding to ρ = 0.19), s_0 = 10.[48]

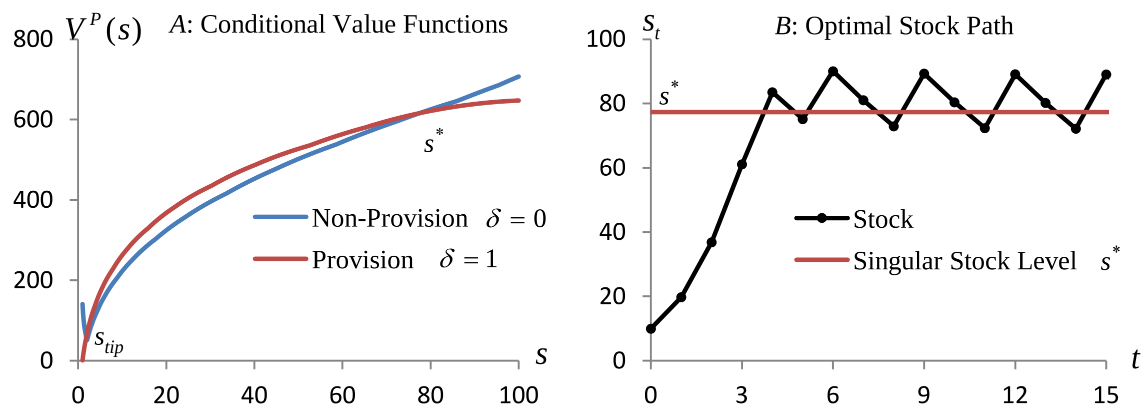


Figure 4.8 A numerical solution of the problem (4.42)

N = 5, PP = 60; K = 100; d = 0.1; α = 0.3; r_{int} = 1.2; β = 0.84; s_0 = 10

In *Panel A* of Figure 4.8, the two conditional value functions represent respectively the value of the value function with δ = 1 (red) and δ = 0 (blue) at each stock level, and their intersections are the singular stock level (the higher one with s = 77) and the tipping point (the lower one with s = 1). Compared to the example with the same parameters in Section 4.2.5, where the singular stock level is 77 and the tipping point is 0.3, the numerical results in the discrete time are quite accurate for the singular stock level, but are a bit off for the tipping point, which makes sense since the approximation error is generally large near the boundaries and the singular level is not so close to the boundary (K) while the tipping point is quite close to 0. Bearing in mind the approximation error, the numerical results in the discrete time given here are quite consistent with **Conjecture 4.2**.

Given the conditional value functions in Figure 4.8, when the stock level is between the tipping point and the singular stock level, it is optimal to choose δ = 1, which corresponds to a

[48] The CompEcon Toolbox provided by Miranda and Frackler (2002) is used for the numerical solution.

higher value function, and when the stock is higher than the singular level or lower than the tipping point, $\delta = 0$ becomes optimal. Hence, consistent with **Conjecture 4.2**, the optimal stock path will be the *MRAP* to a stock level as close as possible to the singular level and then chattering around it, as shown in *Panel B* of Figure 4.8.

4.3.2 The Markov perfect equilibrium

Since the open-loop equilibrium can be treated as a special case of the Markov perfect equilibrium as shown in Section 4.2.4, we will just focus on the MPE for DPPM. And we will continue to assume $\Delta t > 0$ to make the discussion more intuitive. Taking Δt to 0 in the limit will give the solution when δ_t can be switched at an infinite rate.

As in the static provision point mechanism (SPPM), any equilibrium of the dynamic provision point mechanism needs to be constructed due to the discontinuity of the payoff function. Even for the efficient outcome, Bagnoli and Lipman (1989) shows that SPPM has a continuum of Nash equilibria, you can imagine how many different classes of equilibrium DPPM may have. Therefore, instead of categorizing all the possible equilibria, we will only characterize one class of MPE of DPPM in which not only is the socially efficient singular stock level achievable, but the *most rapid approach path* to the singular stock can also be supported in a symmetric MPE. Especially, unlike the majority of the literature in dynamic public good games (Battaglini et al, 2014; Lockwood and Thomas, 2002; Marx and Matthew, 2000; Matthews, 2013), this efficient MPE in DPPM does not require any limiting conditions on model parameters, such as, the discount rate and the depreciation (mortality) rate.

Proposition 4.4

*Given **Conjectures 4.1** and **4.2** hold for* $s_0 \in (0, s^*)$*, there exists a class of MPE of DPPM such that, the most rapid approach path to and then chattering around any stock level between a tipping point* s_{tip}^{DPPM} *higher than* s_{tip} *(Eq 4.19) and the socially efficient singular stock* s^* *(determined by Eq 4.12 or the problems 4.40-4.42), with* s^* *included, is an equilibrium stock path in MPE of DPPM. The corresponding equilibrium contribution function is: whenever the stock can be increased to be closer to the targeted stock level than otherwise, each agent contributes PP/N; and whenever the stock can be decreased to be closer to the targeted stock than otherwise, each agent contributes 0.* s_{tip}^{DPPM} *is equal to* s_-^L *as defined in **Lemma 4.2** in Section 4.2.2, that is, the smaller root less than K of the following equations*

$$J(s_0; s_{tip}^{DPPM}) = J_0(s_0)\Big|_{s_0 = s_{tip}^{DPPM}} \quad or \quad s_{tip}^{DPPM}(1 - s_{tip}^{DPPM}/K) = \frac{PP}{Nr_{\text{int}}\alpha}[\rho + d],$$

where $J(\cdot\,;\,\cdot)$ *is defined by Eq (4.17) with the first argument representing the initial stock level* s_0 *and the second argument representing the targeted stock level to stay at or around.*

Proof.

We first explain why s_{tip}^{DPPM} is needed, and then show that the *MRAP* to any stock level between s_{tip}^{DPPM} and s^* can be supported by a symmetric MPE.

We note in Section 4.2 that the lower the singular stock to be stayed at, the higher the associated tipping point, as shown in Figure 4.7 where the curve $J(s_0; s) - J_0(s)$ shifts down as the targeted singular stock decreases. Given **Conjecture 4.1** and the property that $J(s_0; s) > J_0(s_0)\big|_{s_0 = s}$ for all $s \in (s_-^L, s_-^U)$ as discussed in **Lemma 4.2**, we will have $J(s_0; s) > J_0(s_0)$ for all $s_0 \in (s_{tip}(s), s)$ as long as $s \in (s_-^L, s_-^U)$, where $s_{tip}(s)$ is the tipping point associated with staying at a stock level *s*. This observation has been demonstrated in the numerical examples in Section 4.2.5 and will be supported by another numerical example in the end of this section. Then, when $s = s_-^L$, we will have $J(s_0;s) = J_0(s_0)\big|_{s_0=s}$,

$\left.\frac{\partial(J(s_0;s) - J_0(s_0))}{\partial s_0}\right|_{s_0=s} = 0$, and $\left.\frac{\partial^2(J(s_0;s) - J_0(s_0))}{\partial s_0{}^2}\right|_{s_0=s} < 0$, which, combined with **Conjecture 4.1** imply that the curve $J(s_0;s_-^L) - J_0(s_0)$ is tangent to the horizontal s_0 axis from below. Hence, for a targeted stock level *s* to be admissible to be stayed at or round (instead of being collapsed) for some non-empty set of initial stocks s_0, *s* should be greater than s_-^L and we define $s_{tip}^{DPPM} = s_-^L$. Note, s_{tip}^{DPPM} is also the highest tipping point among those for the stock levels admissible to stay at.

When s_0 is greater than the targeted stock level *s*, $J(s_0;s) - J_0(s_0)$ follows Eq (4.37), which can be simplified to $J(s_0;s) - J_0(s_0) = \left(s_0/s\right)^{-\frac{\rho}{d}}\left[J(s;s) - J_0(s)\right]$. Obviously, $J(s_0;s) - J_0(s_0)$ decreases with s_0 and is always greater than zero since $J(s_0;s) > J_0(s_0)\big|_{s_0=s}$ for all $s \in (s_-^L, s_-^U)$ when $s_0 > s$. So $J(s_0;s)$ and $J_0(s_0)$ still intersect only once, that is, at the tipping point $s_{tip}(s)$.

Next, we show why the *MRAP* to and then staying at any *s* between s_{tip}^{DPPM} and s^* can be supported in MPE of DPPM when *s* is greater than $s_{tip}(s)$. First, as in **Lemma 4.8**, we will apply again the insight from using the Green's theorem to solve for the social planner's problem in Section 4.2.2, which is, it is better to reach a stock level closer to the efficient singular stock from below as fast as possible and then stay there, than to reach that stock level later. In other words, if we compare the values of the social planner's objective function from two stock paths to reach a targeted stock level s between s_{tip}^{DPPM} and s^* from below (but above $s_{tip}(s)$) and then stay there, and one path is faster than the other, then the faster stock path will result in a higher value of the objective function. The same result holds when the control becomes binary in the discrete time framework, as supported by the consistency of the singular solutions of (4.4) and (4.42) shown above.

Now, assume all the other agents $-i = \{j \in I : j \neq i\}$ follow the constructed contribution function specified above to reach a targeted stock level s. Then agent i can only change the stock path when the other agents each contribute PP/N: agent i can contribute 0 to slow down the process of increasing the stock to the targeted one. When the other agents contribute 0 to decrease the stock, agent i cannot increase the stock by contributing PP since the period or instantaneous income w is less than PP. If agent i deviates once by contributing 0, then the targeted stock level will be reached just one period later. Considering 1) all agents use the same symmetric contribution function, and 2) even when agent i deviates, everyone still has the same value of the conditional objective function, all agents will always share equally the value of the group's objective function. That is, the group acts like the social planner, and one step deviation of agent i is equivalent to the social planner's delaying one period to reach s. As discussed above, however, the value of the social planner's objective function becomes lower when it takes a longer time to reach a stock level closer to the efficient singular stock. Then the value of agent i's objective function (i.e., one Nth of the value of the group's objective function) also becomes lower. Thus, agent i will not deviate from the constructed contribution function. Since this game is stationary (i.e., time independent in an infinite time horizon, see Dockner et al., 2000), and the contribution function does not depend on the stock level as long as it is above the associated tipping point, the contribution strategy specified above is a symmetric Markov perfect equilibrium. And thus, we have the desired outcome.

It is easy to verify that the socially efficient singular stock is supported as an equilibrium stock level in this class of MPE. Actually, each agent will just face one *N*th of the conditional value functions shown in *Panel A* of Figure 4.8. Therefore, the *MRAP* to the socially efficient singular stock level are supported in the MPE of DPPM. The "*contribution goal equilibrium*" in Marx and Matthews (2000) is proved in a similar way as in **Proposition 4.4**.

Recall the numerical examples in Section 4.2.5 with N = 5, PP = 60, K = 100, d = 0.1, α = 0.3, and r_{int} = 1.2, when ρ= 0.19, no symmetric open-loop equilibrium or Markov perfect equilibrium in DVCM can support any stable singular stock level. In the DPPM, however, we just showed that any stock between s_{tip}^{DPPM} and s^* can be supported in the MPE. Using this set of parameters, we have s_{tip}^{DPPM} = 11 and s^* = 77. Then any stock level between 11 and 77 can be supported in the MPE based on equilibrium stock paths similar to that in *Panel B* of Figure 4.8. Figure 4.9 shows how $J(s_0;s) - J_0(s_0)$ changes with s_0 for s = 11, 22, 33, 44, 55, 66, and 77. As expected again, $J(s_0;s) - J_0(s_0)$ is consistent with **Conjecture 4.1** for $s_0 \in (0,s)$, and $J(s_0;s)$ and $J_0(s_0)$ intersect only once except for s = 11 where $J(s_0;s) = J_0(s_0)$ for all $s_0 \geq s$. When ρ= 0.05, s_{tip}^{DPPM} = 5 and s^* = 81, we will have a graph similar to Figure 4.9.

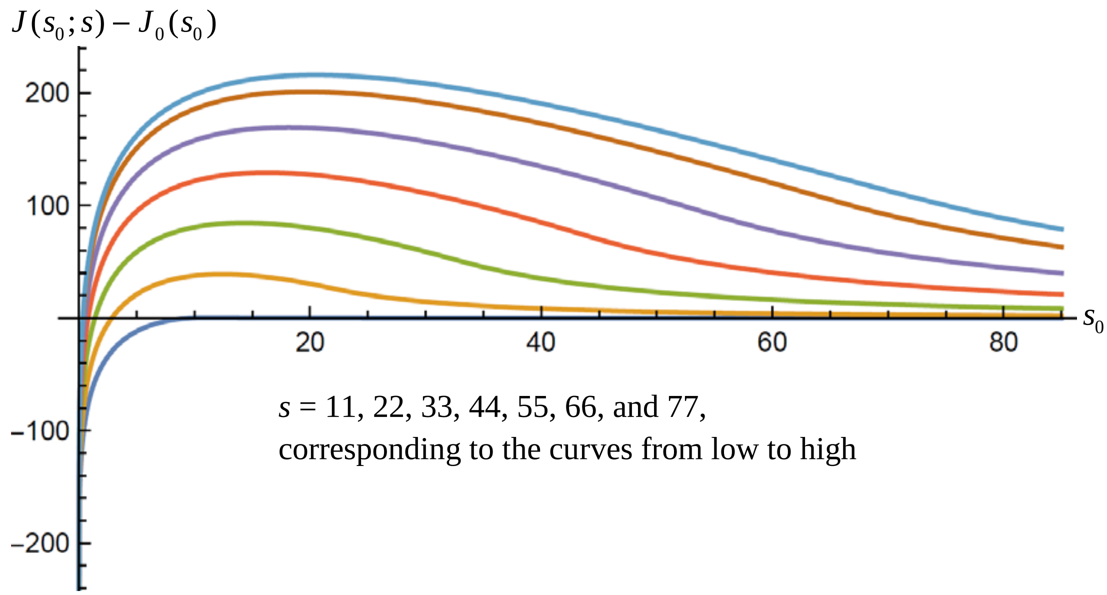


Figure 4.9 $J(s_0; s) - J_0(s_0)$ as a function of s_0, varying with *s*.

N = 5, PP = 60; K = 100; d = 0.1; α = 0.3; r_{int} = 1.2; ρ = 0.19.

4.4 Conclusions

In this chapter we have studied a continuous-time durable public good game with a tipping point, below which collapsing the stock is optimal. The public good stock follows a modified logistic growth function motivated by the fact that many important public goods provision problems involve biological or ecological processes (e.g., various ecosystem services). First, we show that a tipping point always exists in the game, which is generally missing in the current durable public goods literature where only a linear state equation is used. Then we characterize the open-loop equilibrium and the Markov perfect equilibrium (MPE) in a dynamic voluntary contribution mechanism (DVCM) setup. Both equilibrium concepts result in inefficiently low steady states and higher tipping points. However, by constructing a stock-dependent MPE contribution function, we show a unique singular solution in the open-loop equilibrium can be extended to a non-singleton interval of singular stock levels in MPE. As a result, a higher stable steady state and a lower tipping point can be supported in MPE than in the open-loop. The highest stable steady state in MPE approaches the efficient stock level asymptotically as the discount rate becomes 0. Lastly, we extend the DVCM by introducing a provision point, to a dynamic provision point mechanism (DPPM). We characterize a class of MPE of DPPM and show that the *most rapid approach path* to the socially efficient stock level is supported in a symmetric MPE of DPPM.

We have two suggestions for future research. One is to test our theoretical predictions about the DVCM and DPPM directly by lab experiments. Based on symmetric Markov strategies, the DPPM obviously has a larger equilibrium set supporting better outcomes, including the socially efficient one, than the DVCM. Thus, even if we assume agents treat all equilibria unselectively, we would expect the DPPM will generate a higher level of contributions, and sustain a higher level of the public good stock than the DVCM, especially with parameters that result in the collapse of the stock in the MPE of the DVCM, as in the numerical examples. On the other hand, since there is a continuum of equilibrium stocks supported in the class of MPE in DPPM that can be reached in a finite time, we will have an *equilibrium stock selection* problem, which may jeopardize the advantage of the DPPM, considering the discreteness of the provision process. These contradicting predictions will make an experimental comparison very interesting. Another interesting direction is to further explore the solution techniques we used to solve the MPE in the singular solution setup, since a singular solution is quite common in many applied optimal control problems.

Appendices

Chapter 2
A2.1 Undominated Perfect Equilibrium
A2.1.1 PR and UPC

To show that PR and UPC have the same set of UPE as PPM, we only need to show that in equilibrium, the group contribution under PR and UPC cannot be greater than the provision point (*PP*). When no excess contribution is involved, the same argument for PPM in Bagnoli and Lipman (1989) is applicable to PR and UPC, since the three mechanisms differ only in how to rebate excess contributions. To simplify the discussion, we assume no one can afford *PP* alone.

Under PR, any agent contributing above 0 has incentive to deviate if there is excess contribution. Let *TC* denote the group contribution greater than *PP*, c_i denote agent *i*'s contribution greater than 0, and *x* denote the ratio of excess contribution to TC, i.e., $x = (TC\text{-}PP) / PP$. Then agent *i* would be strictly better off by decreasing the contribution by the minimum R_i of $\{c_i, xTC\}$, since given $1>x>0$ and $TC>PP>c_i$, R_i is greater than the original rebate xc_i that *i* will receive and the good is still provided. Intuitively, because the excess contribution will be shared among all positive contributors, then as long as the provision point is met, one can always be better off by reducing contribution as much as possible since it is equivalent to receiving the excess contribution as much as possible while not sharing with the others.

Similarly, if there is excess contribution under UPC, whoever contributes above 0 and is not the only one contributing above the uniform cap would be better off by reducing contribution to zero or until the excess contribution is exhausted. Intuitively, with excess contribution, those contributing below the cap are giving up the rebate to those contributing above the cap; if there are more than one contributing higher than the cap, the rebate is shared among the above-cap contributors; in both cases, agents have incentive to reduce contributions.

A1.2 UPA

Although UPA is different from the other three mechanisms in terms of the provision condition and how final payments are calculated, the nature of provision point or threshold for provision makes the set of UPE in UPA parallel to that in PPM in the sense that after adjusting the provision condition and the calculation of payment, the proof for the UPE of PPM in Bagnoli and Lipman (1989) can be similarly used to identify the UPE of UPA. Therefore, we provide a sketch of the proof for UPA, and a more rigorous proof can be easily obtained by following the proof of Theorem 1 in Bagnoli and Lipman (1989).

To keep the problem interesting and simplify the discussion, we assume it is feasible and socially optimal to provide the good in UPA and no one can afford *PP* alone. We want to argue that an undominated perfect equilibrium (UPE) of UPA is any strategy profile such that 1) one and only one uniform price of PP/n can be set, and 2) no agent *i* chooses c_i greater than or equal to the lowest $PP/k \geqslant v_i$, for *k* in $\{1,..,N\}$. The condition 1) is equivalent to the statement in PPM that *the provision point is exactly met*; the condition 2) is equivalent to that *no one contributes more that their value* in PPM, i.e., the 'undominated' has a different meaning in UPA. The condition 2) is straightforward, since in UPA only a finite number of uniform prices could be possibly charged, any contribution below the lowest possible price that is higher than the contributor's induced value will not result in a negative payoff and hence is not dominated. To prove the UPE of UPA, we show 1) that any strategy profile that more than one uniform price can be set is not a Nash equilibrium, then argue 2) that any strategy profile that no uniform price can be set (i.e., non-provision) is not a UPE, and finally conclude 3) that the remaining undominated strategy profiles are all UPE.

For 1), if more than one uniform price can be set, for example, two prices $UP_1 > UP_2$ are available to charge, whoever contributes above UP_1 but below UP_2 would be better off to reduce contribution below UP_1, because then the realized uniform price becomes UP_2 and they will receive their full induced value without paying anything. Anticipating this, whoever contributes above UP_2 would at least reduce their contributions lower than UP_2 but still above UP_1 to avoid ending up with paying higher prices. Thus, any strategy profile consisting of more than one matched price is not a Nash equilibrium and hence not a UPE of UPA.

For 2), we first eliminate the cases that are not Nash equilibria, and then show the remaining Nash equilibria are not perfect equilibria. Given a strategy profile that no uniform price can be set, if there exists an agent *i* who could increase c_i to make the strategy profile match one price $UP_i < v_i$, *i*' induced value, then *i* would be better off and this strategy profile is not a Nash equilibrium. Let us call this situation Case 1. Then, for a strategy profile resulting in no existence of any uniform price to be a Nash equilibrium, the strategy profile should satisfy the negation of the condition for Case 1, that is, for all subjects, no price can be set without increasing any subject's contribution to a level greater than or equal to that subject's induced value (Case 2). In other words, in Case 2, no one would like to provide the good by increasing contributions since they will not be better off.

However, any Nash equilibrium in Case 2 is a not perfect equilibrium and hence not a UPE. First, based on our assumption that it socially optimal to provide the good, there exists some price *UP* such that *PP*/*UP* is less than the number of induced values greater than *UP*. Then, that *UP* cannot be set in any equilibrium strategy profile in Case 2 means the number of individual contributions $\geq UP$ is less than *PP*/*UP*. Let $n \geq 1$ denote the number of subjects with induced values higher than *UP* but contributing below *UP* in this strategy profile. Since this strategy profile is a Nash equilibrium, we have $n>1$; otherwise $n=1$, and this strategy profile is not a Nash equilibrium because this agent can be better off by contributing *UP*. If $n>1$, however, this Nash equilibrium cannot be perfect. Because, if anyone of the *n* agents changes to contribute *UP*, when the others tremble to contribute not less than *UP*, this agent will be strictly better off. Since this is true for all Nash equilibria without provision, we just show that any Nash equilibrium without provision is not a perfect equilibrium and hence not a UPE.

Finally, after eliminating the strategies discussed above that are either not Nash equibliria or not UPE, the remaining undominated strategy profiles are what we want. It is easy to see that this kind of strategy is a Nash equilibrium. Moreover, since all of these equilibria lead to the core outcomes (i.e., the good is provided, only *PP* is collected, and no one pays more than their induced value) as defined in Bagnoli and Lipman (1989), and hence are strong equilibria (van Damme, 1983), they also satisfy the robustness requirements of UPE. Therefore, we have the UPE of UPA as desired.

A2.2 Random Effects Tobit Models of Individual Contribution with Heterogeneous Values

Table A2.3 Random Effects Tobit Models of Individual Contribution with Heterogeneous Values

Contribution	(1)	(2)	(3)	(4)
PR	0.0474	-0.330	0.0705	-0.307
	(0.290)	(0.328)	(0.290)	(0.328)
UPC	0.108	-0.779**	0.140	-0.746**
	(0.290)	(0.329)	(0.290)	(0.329)
UPA	1.381***	2.122***	1.346***	2.086***
	(0.289)	(0.327)	(0.290)	(0.327)
Value	0.584***	0.569***	0.584***	0.569***
	(0.00830)	(0.0141)	(0.00829)	(0.0141)
PR × Value		0.0550**		0.0549**
		(0.0225)		(0.0225)
UPC × Value		0.128***		0.128***
		(0.0225)		(0.0225)
UPA × Value		-0.109***		-0.109***
		(0.0222)		(0.0222)
Provision Rate†			-0.202***	-0.202***
			(0.0635)	(0.0627)
Constant (PPM)	-0.985***	-0.878***	-0.868***	-0.761***
	(0.192)	(0.207)	(0.195)	(0.210)
Log-likelihood	-5874	-5824	-5868	-5819
Chi-square	4977	5193	5003	5221
R^2 overall	0.450	0.456	0.451	0.457
Number of observations	3600	3600	3600	3600
Number of groups	180	180	180	180
Number of periods	20	20	20	20

Standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1

†: Provision rate over previous 2 periods, which yields the largest log-likelihood among 1 to 5-period lags.

PPM is excluded as the base mechanism. Model 1 is a baseline model which estimates mechanism-specific intercepts with mechanism dummies; variation in slope captured by induced value. Interaction terms among mechanisms and induced value, and provision rate are added in Models 2 to 4, of which Model 4 is the most reliable one we interpret.

A2.3 Sample Experimental Instructions

Experimental Instructions for PPM (Homogeneous)

This is an experiment in the economics of decision-making. During the experiment, you will be asked to make a series of decisions. If you follow the instructions and make careful decisions, you can earn a considerable amount of money.

Experiment Overview

Each decision that you make is considered one *period* of the experiment. In each of these periods, you will be randomly assigned to a group of size 5. You will be asked to decide how much money you will offer towards the cost of a project. This cost is predetermined and known to you. All members of your group receive a benefit when the project is implemented, which occurs only when the total offers of all members in your group meet or exceed the cost of the project.

How You Earn Money

At the beginning of each period, you will be told the individual value (benefit) you will receive if the project is implemented. This value will be 10 experimental dollars. You will then be asked to make an offer from zero up to 15 experimental dollars.

You will be working with experimental dollars. Fifteen experimental dollars are equal to $1. Your initial fund will be 15 experimental dollars, which represents part of your fee for showing up today. Your earnings for each period will be added to this amount.

The Process

You will be asked to decide how much money you will offer towards the cost of a project.

- If your group's total offers are **sufficient** for the project to be implemented, you will receive your individual value for the project, minus a contribution in the amount of your offer.

- If the group's total offers are **not sufficient** to implement the project, your offer will not be collected and you will receive no additional earnings.

<u>Examples</u>
There are two possible outcomes in each period:

(Outcome 1) The group offers **do** allow the project to be implemented.

Project Cost (*known to you*)	$100	
Your Individual Value	$11	*Others in your group will have the same value as yours*
Your Offer	$1	*Others in your group may offer more or less than you do*
Total Offers of Your Group	$110	*Project cost exceeded*
Your Earnings for This Period	$10	*$11 Value* *<u>-$1 Contribution</u>* *=$10 Earnings*

The total offers of your group are sufficient for the project to be implemented. In this case, the project cost is exceeded. Your earnings ($10) are your individual value ($11) minus your contribution ($1).

(Outcome 2) The group offers **do not** allow the project to be implemented.

In this example, your offer will not be collected and you will not receive any additional earnings.

Project Cost (*known to you*)	$100	
Your Individual Value	$12	*Others in your group will have the same value as yours*
Your Offer	$12	*Others in your group may offer more or less than you do*
Total Offers of Your Group	$85	*Does not meet project cost*
Your Earnings for This Period	$0	*No additional value received*

The project cost is not met, so the project is not implemented. You do not receive your individual value for this period. Your $12 offer is not collected.

Please do not speak to other participants during the experiment. Follow the instructions to the best of your ability. If you have questions, raise your hand and the administrator will assist you.

Instructions At-A-Glance

- In each period you will be randomly assigned to a group of 5 participants.
- You will be asked to decide how much money you will offer towards the cost of a project.
- Based on the offers of everyone in your group, the administrator will determine whether the project can be implemented.
- If the total offers meet or exceed the project cost, the project is implemented and your earnings will be your individual value minus your contribution.
- If the group's total offers are not sufficient to implement the project, your offer will not be collected and you will receive no additional earnings.
- If you offer more, in exchange for incurring some of the costs, you increase the chance that the project is implemented.

At the end of the experiment, your earnings will be totaled across all periods and converted from experimental dollars to real dollars. You will be paid as you leave.

Experimental Instructions for UPA (Homogeneous)

Experiment Overview
The overall process for this treatment is the same as for the previous treatment in terms of group size (5), individual value (10), conversion rate (15 to 1), and the steps you will take. The difference in this treatment is the method by which your net contribution or possible rebate is calculated.

The Process
Again, you will be asked to decide how much money you will offer towards the cost of a project.

- If your group's total offers are sufficient to implement the project, a *Uniform Contribution* will be calculated for your group.
- If your offer is above the *Uniform Contribution*, you pay only the uniform contribution and we will rebate any amount you offered above the uniform contribution. Your earnings will be your individual value for the project minus the *Uniform Contribution*.
- If your offer is lower than the *Uniform Contribution*, you pay nothing. Your earnings will be your individual value for the project.
- If the group's total offers are not sufficient to implement the project, your offer will not be collected and you will receive no additional earnings. .

The *Uniform Contribution* will be the lowest amount that allows us to collect <u>only the exact amount needed</u> to implement the project.

Examples
There are three possible outcomes in each period:

(Outcome 1) The group offers **do** allow the project to be implemented and the *Uniform Contribution* is *equal to or lower* than your offer.

In this example, your offer is higher than the *Uniform Contribution*, so you pay <u>only</u> the *Uniform Contribution*. Your earnings are your individual value for the project, minus your offer, plus a rebate of your offer above the uniform contribution.

Project Cost (*known to you*)	$110	
Your Individual Value	$12	*Others in your group will have the same value as yours*
Your Offer	$11	*Others in your group may offer more or less than you do*
Total Offers of Your Group	$150	*Project cost exceeded*
Calculated Uniform Contribution	$7.50	
Your Rebate for this Period	$3.50	*$11 Offer - $7.50 Contribution*
Your Earnings for This Period	$4.50	*$12 Value* *-$11 Offer* <u>*+ $3.50 Rebate*</u> *= $4.50 Earnings*

The Uniform Contribution is lower than your offer. Even though you offered $11, you pay only the Uniform Contribution of $7.50. Your earnings ($4.50) are your individual value ($12) minus your offer ($11), plus the rebate of your offer above the Uniform Contribution (3.50).

(Outcome 2) The group offers **do** allow the project to be implemented and the *Uniform Contribution* is *higher than* your offer.

In this example, your offer is lower than the *Uniform Contribution*, so you do not contribute. Your earnings are your individual value for the project.

Project Cost (*known to you*)	$80	
Your Individual Value	$11	*Others in your group will have the same value as yours*
Your Offer	$1	*Others in your group may offer more or less than you do*
Total Offers of Your Group	$100	*Project cost exceeded*
Calculated Uniform Contribution	$9	
Your Earnings for This Period	$11	*$11 Value* *- $0 Contribution* *=$11 Earnings*

Your offer of $1 is lower than the Uniform Contribution of $9, so you pay nothing. Your earnings ($11) are your individual value ($11).

(Outcome 3) The group offers **do not** allow the project to be implemented.

In this example, your offer will not be collected and you will not receive any additional earnings.

Project Cost (*known to you*)	$100	
Your Individual Value	$12	*Others in your group will have the same value as yours*
Your Offer	$12	*Others in your group may offer more or less than you do*
Total Offers of Your Group	$85	*Uniform contribution cannot be calculated*
Your Earnings for This Period	$0	*No additional value received*

The project cost is not met, so the project is not implemented. You do not receive your individual value for this period. Your $12 offer is not collected.

Note that, given the project cost and your group size, there are only a few possible Uniform Contribution levels. For example, if the project cost is $30 and the group size is 3, then the possible Uniform Contribution levels are as follows.

- If 3 members contribute at least 10 (=30/3), then the project will be implemented and the Uniform Contribution is 10;

- If 2 members contribute at least 15 (=30/2) (and the other less than 10) then the project will be implemented and the Uniform Contribution is 15;
- If 1 member contributes at least 30 (=30/1) (and both other less than 15 and at least one other below 10), then the calculated Uniform Contribution is 30. However, if no one has an individual value ≥ 30, no one would contribute 30, and hence this Uniform Contribution is not feasible.

The feasible Uniform Contribution levels and the numbers of contributors needed to implement them in this treatment are given at the end of this instruction.

Please do not speak to other participants during the experiment. Follow the instructions to the best of your ability. If you have questions, raise your hand and the administrator will assist you.

Instructions At-A-Glance

- In each period you will be randomly assigned to a group of 5 participants.
- You will be asked to decide how much money you will offer towards the cost of a project.
- Based on the offers of everyone in your group, the administrator will determine whether the project can be implemented.
- If total offers are such that a uniform contribution can be calculated, the project is implemented and any excess funds are rebated. The *Uniform Contribution* will be calculated as the lowest amount that allows us to collect only the exact amount needed to implement the project.
    - If the project is implemented and your offer is above the *Uniform Contribution*, you pay only the uniform contribution. Your earnings will be your individual value minus the *Uniform Contribution*.
    - If the project is implemented and your offer is lower than the *Uniform Contribution*, you pay nothing. Your earnings will be your individual value.
- If the group's total offers are not sufficient to implement the project, your offer will not be collected and you will receive no additional earnings.
- If you offer more, in exchange for incurring some of the costs, you increase the chance that the project is implemented.

At the end of the experiment, your earnings will be totaled across all periods and converted from experimental dollars to real dollars. You will be paid as you leave.

The table below shows the feasible Uniform Contribution levels and the number of contributors needed to implement them in this treatment, given that the project cost is 30, the group size is 5, and the individual value is 10.

Feasible Uniform Contribution	Number of contributors needed
6	5
7.5	4
10	3

Experimental Instructions for UPC (Homogeneous)

Experiment Overview

The overall process for this treatment is the same as for the previous treatment in terms of group size (5), individual value (10), conversion rate (15 to 1), and the steps you will take. The difference in this treatment is the method by which your net contribution or possible rebate is calculated.

The Process

Again, you will be asked to decide how much money you will offer towards the cost of a project.

- If your group's total offers are sufficient to implement the project, a *Contribution Cap* will be calculated for your group.
    - If your offer is above the *Contribution Cap*, you pay only the contribution cap and we will rebate any amount you offered above the contribution cap.
    - If your offer is lower than the *Contribution Cap*, you pay your offer amount.
- Your earnings will be your individual value for the project minus your contribution in both cases.
- If the group's total offers are not sufficient to implement the project, your offer will not be collected and you will receive no additional earnings.

The *Contribution Cap* will be the lowest amount that allows us to collect only the exact amount needed to implement the project.

Examples

There are three possible outcomes in each period:

(Outcome 1) The group offers **do** allow the project to be implemented and the *Contribution Cap* is *equal to or lower* than your offer.

In this example, your offer is higher than the *Contribution Cap*, so you pay only the *Contribution Cap*. Your earnings are your individual value for the project, minus your offer, plus a rebate of your offer above the contribution cap.

Project Cost (*known to you*)	$110	
Your Individual Value	$12	*Others in your group will have the same value as yours*
Your Offer	$11	*Others in your group may offer more or less than you do*
Total Offers of Your Group	$150	*Project cost exceeded*
Calculated Contribution Cap	$7.50	
Your Rebate for this Period	$3.50	*$11 Offer - $7.50 Contribution Cap*
Your Earnings for this Period	$4.50	*$12 Value* *-$11 Offer* *+ $3.50 Rebate* *= $4.50 Earnings*

The Contribution Cap is lower than your offer. Even though you offered $11, you pay only the Contribution Cap of $7.50. Your earnings ($4.50) are your individual value ($12) minus your offer ($11), plus the rebate of your offer above the Contribution Cap ($3.50).

(Outcome 2) The group offers **do** allow the project to be implemented and the *Contribution Cap* is *higher than* your offer.

In this example, your offer is lower than the *Contribution Cap*, so your contribution is the full amount of your offer. Your earnings are your individual value for the project, minus your contribution.

Project Cost (*known to you*)	$80	
Your Individual Value	$11	*Others in your group will have the same value as yours*
Your Offer	$1	*Others in your group may offer more or less than you do*
Total Offers of Your Group	$100	*Project cost exceeded*
Calculated Contribution Cap	$9	*Higher than your offer*
Your Earnings for This Period	$10	*$11 Value* *- $1 Contribution (as Offered)* *=$10 Earnings*

Your offer of $1 is lower than the Contribution Cap of $9, so you pay your offer amount. Your earnings ($10) are your individual value ($11) minus your contribution at your original offer amount ($1).

(Outcome 3) The group offers **do not** allow the project to be implemented.

In this example, your offer will not be collected and you will not receive any additional earnings.

Project Cost (*known to you*)	$100	
Your Individual Value	$12	*Others in your group will have the same value as yours*
Your Offer	$12	*Others in your group may offer more or less than you do*
Total Offers of Your Group	$85	*Does not meet project cost*
Your Earnings for This Period	$0	*No additional value received*

The project cost is not met, so the project is not implemented. You do not receive your individual value for this period. Your $12 offer is not collected.

Please do not speak to other participants during the experiment. Follow the instructions to the best of your ability. If you have questions, raise your hand and the administrator will assist you.

Instructions At-A-Glance

- In each period you will be randomly assigned to a group of 5 participants.
- You will be asked to decide how much money you will offer towards the cost of a project.
- Based on the offers of everyone in your group, the administrator will determine whether the project can be implemented.
- If total offers are such that a contribution cap can be calculated, the project is implemented and any excess funds are rebated. The *Contribution Cap* will be calculated as the lowest amount that allows us to collect <u>only the exact amount needed</u> to implement the project.
    - If the project is implemented and your offer is above the *Contribution Cap,* you pay only the contribution cap. Your earnings will be your individual value minus the *Contribution Cap*.
    - If the project is implemented and your offer is lower than the *Contribution Cap,* you pay only your offer. Your earnings will be your individual value minus your contribution.
- If the group's total offers are not sufficient to implement the project, your offer will not be collected and you will receive no additional earnings.
- If you offer more, in exchange for incurring some of the costs, you increase the chance that the project is implemented.

At the end of the experiment, your earnings will be totaled across all periods and converted from experimental dollars to real dollars. You will be paid as you leave.

Experimental Instructions for PPM (Heterogeneous)

<u>Experiment Overview</u>

Now we have run through a series of treatments involving 3 different mechanisms: PPM without rebate, UPA with *Uniform Contribution*, and UPC with *Contribution Cap*. The following treatments will repeat the three mechanisms but with different parameter setups.

The overall process for the following treatments is the same as for the previous treatments in terms of conversion rate (15 to 1) and the steps you will take.

However, the changes are

- Group size: you will be in a group of **10** participants instead of 5.
- Individual value:
  - In previous treatments, everyone in your group has the same individual value over periods;
  - In the following treatments, individual values in your group will be exactly the following **10** numbers: {**4, 4, 5, 5, 6, 6, 8, 8, 10, 12**}.
  - Each one of the 10 numbers will be assigned to exactly one of the 10 members in your group. So you **know** everyone's individual value in your group, although you **do not know** their identities.
  - The 10 individual values will be the same in each period for your group.
  - However, the 10 values and your 10 group members will be **randomly matched in each period**. In other words, your individual value may be different period by period, since each of the 10 values may be assigned to a different group member every period.

This treatment will repeat the mechanism PPM without rebate, i.e., if your group's total offers **exceed** the cost of the project, the excess funds will **NOT** be rebated.

Please do not speak to other participants during the experiment. Follow the instructions to the best of your ability. If you have questions, raise your hand and the administrator will assist you.

Experimental Instructions for UPA (Heterogeneous)

Experiment Overview

The overall process for this treatment is the same as for the previous treatment in terms of
group size (10),
individual value (one of the 10 values in each period {**4, 4, 5, 5, 6, 6, 8, 8, 10, 12**}),
conversion rate (15 to 1), and
the steps you will take.

The difference in this treatment is the method by which your net contribution or possible rebate is calculated.

This treatment will repeat the mechanism UPA with *Uniform Contribution*, i.e.,

- If your offer is above the *Uniform Contribution*, you pay only the uniform contribution and we will rebate any amount you offered above the uniform contribution. Your earnings will be your individual value for the project minus the *Uniform Contribution*.
- If your offer is lower than the *Uniform Contribution*, you pay nothing. Your earnings will be your individual value for the project.

Please do not speak to other participants during the experiment. Follow the instructions to the best of your ability. If you have questions, raise your hand and the administrator will assist you.

The table below shows the feasible Uniform Contribution levels and the number of contributors needed to implement them in this treatment, given that the project cost is 30, the group size is 10, and the individual values in your group are {4, 4, 5, 5, 6, 6, 8, 8, 10, 12}.

Feasible Uniform Contribution	Number of contributors needed
3	10
3.4	9
3.8	8
4.3	7
5	6
6	5
7.5	4

Experimental Instructions for UPC (Heterogeneous)

<u>Experiment Overview</u>

The overall process for this treatment is the same as for the previous treatment in terms of

group size (10),
individual value (one of the 10 values in each period {**4, 4, 5, 5, 6, 6, 8, 8, 10, 12**}),
conversion rate (15 to 1), and
the steps you will take.

The difference in this treatment is the method by which your net contribution or possible rebate is calculated.

This treatment will repeat the mechanism UPC with *Contribution Cap*, i.e.,

- If your offer is above the *Contribution Cap*, you pay only the contribution cap and we will rebate any amount you offered above the contribution cap.
- If your offer is lower than the *Contribution Cap*, you pay your offer amount.

Please do not speak to other participants during the experiment. Follow the instructions to the best of your ability. If you have questions, raise your hand and the administrator will assist you.

Experimental Instructions for PR (Homogeneous)

Experiment Overview
The overall process for this treatment is the same as for the previous treatment in terms of group size (5), individual value (10), conversion rate (15 to 1), and the steps you will take. The difference in this treatment is the method by which your net contribution or possible rebate is calculated.

The Process
Again, you will be asked to decide how much money you will offer towards the cost of a project.

- If your group's total offers **equal** the cost of the project, the project will be implemented. Your earnings will be your individual value for the project minus your contribution.
- If your group's total offers **exceed** the cost of the project, the project will be implemented and excess funds will be rebated. Your earnings will be your individual value for the project minus your offer, plus your rebate. Your rebate will be in proportion to the excess funds offered by your group. So, if X% of your group's total offers is not needed, your rebate will be X% of your offer.
- If the group's total offers are not sufficient to implement the project, your offer will not be collected and you will receive no additional earnings.

Examples
There are three possible outcomes in each period:

(Outcome 1) The group offers are **exactly equal** to the project cost and the project is implemented.

In this example, all of your offer is needed, so your earnings are your individual value for the project, minus your contribution in the amount of your offer.

Project Cost (*known to you*)	$100	
Your Individual Value	$12	*Others in your group will have the same value as yours*
Your Offer	$2	*Others in your group may offer more or less than you do*
Total Offers of Your Group	$100	*Exactly meets project cost*
Your Earnings for This Period	$10	*$12 Value* *-$2 Contribution (as Offered)* *=$10 Earnings*

The project cost is exactly met, so the project is implemented. Your earnings ($10) are your individual value ($12) minus your contribution ($2).

(Outcome 2) The group offers **exceed** the project cost and the project is implemented.

In this example, total offers exceed the amount needed, so a portion of each offer will be rebated. Your earnings are your individual value for the project, minus your offer, plus your *rebate*. This rebate is based upon the proportion of total offers represented by excess funds offered by your group.

Project Cost (*known to you*)	$150	
Your Individual Value	$10	*Others in your group will have the same value as yours*
Your Offer	$10	*Others in your group may offer more or less than you do*
Total Offers of Your Group	$200	*Exceeds project cost*
Total Excess Contributions	$50	*$200 offered - $150 needed*
Your Rebate from Excess Contributions	$2.50	*We need 75% of the money offered, so you get 25% of your money back*
Your Earnings for This Period	$2.50	*$10 Value* *- $10 Offer* *+ $2.50 rebate* *=$2.50 earnings*

The project cost is exceeded, so the project is implemented. Because we only need 75% of the money offered, you get a 25% of your money back. Your earnings ($2.50) are your individual value ($10) minus your offer ($10), plus your rebate ($2.50).

(Outcome 3) The group offers **do not** allow the project to be implemented.

In this example, your offer will not be collected and you will not receive any additional earnings.

Project Cost (*known to you*)	$100	
Your Individual Value	$12	*Others in your group will have the same value as yours*
Your Offer	$10	*Others in your group may offer more or less than you do*
Total Offers of Your Group	$85	*Does not meet project cost*
Your Earnings for This Period	$0	*No additional value received*

The project cost is not met, so the project is not implemented. You do not receive your individual value for this period. Your $10 offer is not collected.

Please do not speak to other participants during the experiment. Follow these instructions to the best of your ability. If you have questions, raise your hand and the administrator will assist you.

Instructions At-A-Glance

- In each period you will be randomly assigned to a group of 5 participants.
- You will be asked to decide how much money you will offer towards the cost of a project.
- Based on the offers of everyone in your group, the administrator will determine whether the project can be implemented.
- If the total offers of your group **equal** the cost of the project, the project is implemented. Your earnings will be your individual value minus your contribution.
- If the total offers of your group **exceed** the cost of the project, the project is implemented and excess contributions are rebated so that <u>only the exact amount needed</u> for the project is actually collected. Your earnings will be your individual value minus your offer, plus a rebate based upon the proportion of total offers represented by excess funds offered by your group.
- If the group's total offers are not sufficient to implement the project, your offer will not be collected and you will receive no additional earnings.
- If you offer more, in exchange for incurring some of the costs, you increase the chance that the project is implemented.

At the end of the experiment, your earnings will be added across all periods and converted from experimental dollars to real dollars. You will be paid as you leave.

Experimental Instructions for PR (Heterogeneous)

Experiment Overview

The overall process for this treatment is the same as for the previous treatment in terms of

group size (10),
individual value (one of the 10 values in each period {**4, 4, 5, 5, 6, 6, 8, 8, 10, 12**}),
conversion rate (15 to 1), and
the steps you will take.

The difference in this treatment is the method by which your net contribution or possible rebate is calculated.

This treatment will repeat the mechanism PR with proportional rebate, i.e., if X% of your group's total offers is not needed, your rebate will be X% of your offer.

Please do not speak to other participants during the experiment. Follow the instructions to the best of your ability. If you have questions, raise your hand and the administrator will assist you.

Chapter 3

A3.1 Proofs of Propositions 3.1 to 3.2, and Lemma 3.1.

Proof of Proposition 3.1. In UPC, any strategy in which $c_i(v_i) > v_i$ for some v_i is ex post weakly dominated; in UPA, any strategy in which $c_i(v_i)$ is greater than the lowest $PP/k \geq v_i$ for $k \in \{1,\ldots,N\}$ and some v_i, is ex post weakly dominated.

Proof. Consider agent i. For UPC, suppose $c_i > v_i$, let $c_i' = v_i$ and fix $c_{-i}(v_i)$.
If $c_{-i} + c_i \geq PP$, and $c_{-i} + v_i < PP$ at any value profile v, agent i suffers a loss under c_i. Note, when $c_{-i} + c_i > PP$, even if all the excess contribution is rebated to agent i, she still suffers a loss since $c_{-i} + v_i < PP$.

If $c_{-i} + c_i \geq PP$, and $c_{-i} + v_i \geq PP$, c_i is weakly dominated by c_i'. Since $c_i > v_i$, if $c_{-i} + v_i = PP$, agent i suffers a loss or breaks even under c_i in case of provision; if $c_{-i} + v_i > PP$, c_i' results in the same payoff as c_i if c_i' is greater than or equal to the uniform price cap, and c_i' leads to a higher payoff if c_i' is lower than the cap. If $c_{-i} + c_i < PP$, agent i has a 0-payoff under both c_i' and c_i. This finishes the proof for UPC.

Similarly, in UPA, suppose $c_i > \min\{PP/k : PP/k \geq v_i, \text{for } k = 1,\ldots,N\}$, let $c_i' = v_i$ and fix $c_{-i}(v_i)$. If the good is provided under c_i while not under c_i', then the uniform price *UP* is great than v_i and agent i suffers a loss under c_i. If the good can be provided under both c_i and c_i', c_i is weakly dominated by c_i': c_i' results in the same payoff as c_i if c_i' is greater than or equal to *UP*, and c_i' leads to a higher payoff if c_i' is lower than *UP*. Lastly, if the good cannot be provided under either c_i or c_i', agent i has a 0-payoff under both c_i' and c_i. This finishes the proof for UPA and hence for this proposition.

Note that $\min\{PP/k : PP/k \geq v_i, \text{for } k = 1,\ldots,N\}$ in UPA is equivalent to v_i in UPC. In UPA only a finite number of uniform prices could be possibly charged, and hence any contribution below the lowest possible price that is higher than v_i will not result in a negative payoff.

Proof of Lemma 3.1. In UPC, if $P_i(c_i \mid c_{-i}) > 0$ and c_i is less than *PP* and the cutpoint $c_{i,UPC}^{cp}$ associated with some uniform cap for some value profile v, $S_i(c_i)$ is strictly increasing at c_i; in UPA, if $P_i(c_i \mid c_{-i}) > 0$ and c_i is less than *PP* and the cutpoint $c_{i,UPA}^{cp}$ associated with some uniform price for some value profile v, $S_i(c_i)$ is strictly increasing at c_i in the sense that when c_i increases to a higher uniform price, $S_i(c_i)$ increases. The cutpoints $c_{i,UPC}^{cp}$ and $c_{i,UPA}^{cp}$ are defined as in Section 2.5.

Proof. Consider agent i, given the contribution functions of the others. $P_i(c_i \mid c_{-i}) > 0$ implies the good is provided at some value profiles, all of which constitute a set denoted by $\hat{V}^N \subseteq V^N$. For UPC, If c_i is less than the cutpoint $c_{i,UPC}^{cp}$ (as defined in Section 2.5) associated with the uniform price cap for some value profile $v \in \hat{V}^N$, we will have $\partial s_i / \partial c_i = 1$ at c_i under that value profile, i.e., the marginal penalty is -1; $\partial s_i / \partial c_i = 0$ otherwise. Since $S_i(c_i) = E_{v_{-i}}(s_i(c(v)) \mid c_i)$, which is the expected payment over all the possible value profiles, $S_i(c_i)$ is strictly increasing at c_i. Note that, it is only when $c_i \geq c_{i,UPC}^{cp}$ for all $v \in \hat{V}^N$ that $S_i(c_i)$ becomes weakly increasing at c_i. Given the broad range of the value profiles, this case, if not impossible, is really rare (e.g., everyone contributes the equal share of the cost no matter what values they have), and hence we will not miss much by focusing on the regular case where $S_i(c_i)$ is strictly increasing at c_i.

For UPA, since only a finite number of uniform prices can be possibly charged, a meaningful increase of c_i has to reach the next higher price. Then, If c_i is less than the cutpoint $c_{i,UPA}^{cp}$ (as defined in Section 2.5) associated with the uniform price for some value profile $v \in \hat{V}^N$, agent i will pay higher—due to a lump sum penalty—with a meaningful increase of c_i, and hence $S_i(c_i)$ is strictly increasing at c_i. Similar to the case in UPC, it is only when $c_i \geq c_{i,UPA}^{cp}$ for all $v \in \hat{V}^N$ that $S_i(c_i)$ becomes weakly increasing at c_i. So we can just focus on the case where $S_i(c_i)$ is strictly increasing at c_i without loss of generality. This finishes the proof.

Proof of Proposition 3.2. For both UPC and UPA, let $c^*(v)$ be a symmetric Bayesian Nash equilibrium contribution function. If $P_i(c^*(v) \mid c_{-i}^*) > 0$ for all $v > \min\{V\}$ and $c^*(v)$ is less than the cutpoint associated with some uniform cap/price for some value profile v, then, for all v_i and v_j, $v_i > v_j \Rightarrow c^*(v_i) \geq c^*(v_j)$.

Proof. Given that **Lemma 3.1** holds, the proof is exactly the same as that of Proposition 3.3 in Gailmard and Palfrey (2005). We give the proof for UPC here; the proof for UPA is similar. Consider two different values $v_i > v_i'$ for agent i, and some contribution function such that $c(v_i)$ $= c$ and $c(v_i') = c'$. For i to optimize, we will have

$$v_i P_i(c \mid c_{-i}) - S_i(c) \geq v_i P_i(c' \mid c_{-i}) - S_i(c') \text{ and } v_i' P_i(c' \mid c_{-i}) - S_i(c') \geq v_i' P_i(c \mid c_{-i}) - S_i(c)$$

Rearrange the two inequalities, we will have

$$(v_i - v_i')(P_i(c \mid c_{-i}) - P_i(c' \mid c_{-i})) \geq 0$$

If $P_i(c \mid c_{-i}) > P_i(c' \mid c_{-i})$, then $v_i > v_i' \Rightarrow c > c'$. If $P_i(c \mid c_{-i}) = P_i(c' \mid c_{-i}) > 0$, $c' > c \Rightarrow$ $S_i(c') > S_i(c)$ by **Lemma 3.1**, which implies $v_i' P_i(c' \mid c_{-i}) - S_i(c') < v_i' P_i(c \mid c_{-i}) - S_i(c)$, contradicting the assumption that i optimizes. This finishes the proof.

A3.2 Symmetric Bayesian Nash Equilibria of UPA

Since there are only finite prices agents will actually pay, UPA is equivalent to a Bayesian game with discrete actions. It is known that a BNE solution to this kind of games is just a decision rule based on some critical values. Therefore, we can solve the Bayesian game for UPA by identifying the decision rules and the critical values. We first solve for symmetric BNE of UPA in both 2-player and 3-player games to show the procedures, and then extend to an *N*-player game.

A3.2.1 A 2-player game

Without loss of generality, we assume agents only contribute the possible uniform prices. Thus, in a 2-player game, there are only two possible contribution choices: 0 or *PP*/2.[49] The contribution function for each agent is in the general form

(A3.1) $c(v) = \begin{cases} 0 & if\ \ v \le v_2^c \\ PP/2 & if\ \ v > v_2^c \end{cases}$, where v_2^c is the critical value.

At v_2^c, an agent is indifferent between contributing *PP*/2 and contributing 0, which can be identified by the following equation

(A3.2) $(v_2^c - PP/2)\Pr(v \ge v_2^c) = 0$

where $\Pr(v \ge v_2^c)$ denotes the probability of $v \ge v_2^c$ for $v \in V$, and the two sides of (A3.2) represent the expected payoffs of contributing *PP*/2 and 0 respectively. Note that when one agent contributes *PP*/2, the good is provided iff the other agent has an induced value $v \ge v_2^c$.

To solve (A3.2), we can first check whether v_2^c could be on the boundaries. If $v_2^c = 0$, then the LHS is less than 0 and hence $v_2^c \ne 0$. If $v_2^c = \overline{v}$, then $\Pr(v \ge v_2^c) = 0$ and (A3.2) holds. So $v_2^c = \overline{v}$ is one solution, which is the pure free-rider BNE with the good never provided. If $v_2^c \in (0, \overline{v})$, we will have $v_2^c = PP/2$ since $\Pr(v \ge v_2^c) > 0$. Thus, $v_2^c = PP/2$ is another solution, which is the nontrivial BNE

(A3.3) $c_i^{BNE}(v_i) = \begin{cases} 0 & if\ \ v_i \le PP/2 \\ PP/2 & if\ \ v_i > PP/2 \end{cases}$, for *i* = 1, 2.

It is easy to verify that this symmetric BNE is the only nontrivial BNE category in the 2-player game, and truth-telling is supported over the entire value range.

A3.2.2 A 3-player game (Proof of Proposition 3.3)

In a 3-player game, each player has three contribution choices {0, *PP*/3, *PP*/2} and two critical values will be sufficient to characterize the decision rule. Let v_3^c and v_2^c denote the two critical values for a symmetric pure-strategy weakly monotonic BNE, assuming $0 \le v_3^c \le v_2^c \le \overline{v}$, where $0 = \min\{V\}$ and $\overline{v} = \max\{V\}$. The contribution function for each agent is in the general form

(A3.4) $c(v) = \begin{cases} 0 & if\ \ v \le v_3^c \\ PP/3 & if\ \ v_3^c < v \le v_2^c \\ PP/2 & if\ \ v > v_2^c \end{cases}$

Since players can choose some subset of {0, *PP*/3, *PP*/2} to contribute, more than one decision rule could be supported in equilibrium and can be represented generally by (A3.4). For example, when $v_3^c = v_2^c < \overline{v}$, only 0 or *PP*/2 will be chosen; and when $v_3^c < v_2^c = \overline{v}$, only 0 or *PP*/3 will be chosen; and when $v_3^c = \overline{v}$, only 0 will be chosen.

The expected payoff for each contribution choice for agent i is as follows.

(A3.5) $E\pi(0) = v_i \Pr(v > v_2^c)^2$

[49] Since $\overline{v} < PP$, the only positive price is *PP*/2 and contributions less than *PP* all collapse to one of the two final payments: 0 or *PP*/2.

(A3.6) $E\pi(PP/3) = (v_i - PP/3)\Pr(v > v_3^c)^2$

(A3.7) $E\pi(PP/2) = (v_i - PP/3)\Pr(v > v_3^c)^2 + 2(v_i - PP/2)\Pr(v > v_2^c)\Pr(v \le v_3^c)$

To solve for the critical values, we need to use the fact that at each critical value, two of the three expected payoffs are equal. Thus, two distinct critical values will result in two equations, and if the two critical values are the same, one payoff function becomes irrelevant and only one equation will be obtained. The critical value(s) in either case can be identified. Note that $v_3^c \neq 0$, otherwise $E\pi(0) = 0 \neq -PP/3 = E\pi(PP/3)$. The following three cases capture all nontrivial BNE.

Case I: $v_2^c = \overline{v}$

If $v_2^c = \overline{v}$, $\Pr(v > v_2^c) = 0$ and (A3.7) collapses to (A3.6). Then $E\pi(0) = E\pi(PP/3)$ at v_3^c implies

(A3.8) $(v_3^c - PP/3)\Pr(v > v_3^c)^2 = 0$

Therefore, $v_3^c = PP/3$ or $\overline{v}$ corresponding to two BNE

(A3.9) $c_i^{BNE}(v_i) = \begin{cases} 0 & if\ v_i \le PP/3 \\ PP/3 & if\ v_i > PP/3 \end{cases}$, for i = 1, 2, 3, with $v_3^c = PP/3$ and $v_2^c = \overline{v}$.

(A3.10) $c_i^{BNE}(v_i) = 0$, for i = 1, 2, 3, with $v_3^c = v_2^c = \overline{v}$.

As long as $PP<3\overline{v}$, (A3.9) is independent of value distribution and always exits.

Case II: $0 < v_3^c = v_2^c = \hat{v} < \overline{v}$

In this case, (A3.6) becomes irrelevant since $E\pi(c=0) = E\pi(PP/3)$ at $v = \hat{v}$ will lead to $\hat{v} = \overline{v}$. Then $E\pi(0) = E\pi(PP/2)$ at $v = \hat{v}$ implies

(A3.11) $[(\hat{v} - PP/3)\ \Pr(v > \hat{v}) - (\hat{v} - PP/2)]\ \Pr(v > \hat{v}) = 0$

If $\hat{v} = \overline{v}$, (A3.11) goes to (A3.10); if $0 < \hat{v} < \overline{v}$, (A3.11) becomes

(A3.12) $(\hat{v} - PP/3)\ \Pr(v > \hat{v}) = (\hat{v} - PP/2) \Leftrightarrow \Pr(v > \hat{v}) = (\hat{v} - PP/2)/(\hat{v} - PP/3) \in (0,1)$

For (A3.12) to hold, we need the solution of $\hat{v} > PP/2$, otherwise this case does not exist in equilibrium. If we do have $\hat{v} > PP/2$ given the value distribution and the provision point level, the resulting BNE is

(A3.13) $c_i^{BNE}(v_i) = \begin{cases} 0 & if\ v_i \le \hat{v} \\ PP/2 & if\ v_i > \hat{v} \end{cases}$, for i = 1, 2, 3, with $\hat{v}$ given by (A3.12).

Case III: $0 < v_3^c < v_2^c < \overline{v}$

In this case, $E\pi(0) = E\pi(PP/3)$ at v_3^c and $E\pi(PP/3) = E\pi(PP/2)$ at v_2^c lead to the following two equations

(A3.14) $v_3^c \Pr(v > v_2^c)^2 = (v_3^c - PP/3)\Pr(v > v_3^c)^2$

(A3.15) $2(v_2^c - PP/2)\Pr(v > v_2^c)\Pr(v \le v_3^c) = 0$

(A3.15) implies that $v_2^c = PP/2$, which can be plugged into (A3.14) to solve for v_3^c. However, v_3^c needs to be greater than *PP*/3 for (A3.14) to hold. If we do have $v_3^c > PP/3$ given the value distribution and the provision point level, the resulting BNE is

(A3.16) $$c_i^{BNE}(v_i) = \begin{cases} 0 & \text{if } v_i \le v_3^c \\ PP/3 & \text{if } v_3^c < v_i \le PP/2 \\ PP/2 & \text{if } v_i > PP/2 \end{cases}$$, for *i*=1, 2, 3, with v_3^c given by (A3.14).

Thus we have just proved **Proposition 3.3**, and (A3.10), (A3.9), (A3.13), (A3.16) correspond to the BNE *a* to *d*, respectively. Note that within the three nontrivial BNE, only (A3.9) always exists, while the existence of the other two depends on the value distribution and the provision point level.

Next, we use some numerical examples to demonstrate Cases II and III. Let the value distribution be uniform over [0, 1].

Case II

(A3.12) becomes

(A3.17) $$(\hat{v} - PP/3)\,(1-\hat{v}) = (\hat{v} - PP/2)$$

Then we have

(A3.18) $$\hat{v} = \frac{PP}{6} + \frac{\sqrt{PP^2/9 + 2\,PP/3}}{2}$$

It is easy to check that when 1/4<*PP*<2, $\hat{v}$ is between *PP*/2 and *PP*. Figure A1 below shows the function $f(\hat{v}\,|\,PP) = (\hat{v} - PP/3)\,(1-\hat{v}) - (\hat{v} - PP/2)$ with *PP* =0.2, 0.4, …, 1.4, where $\hat{v}$ is between *PP*/2 and *PP* except for *PP* =0.2.

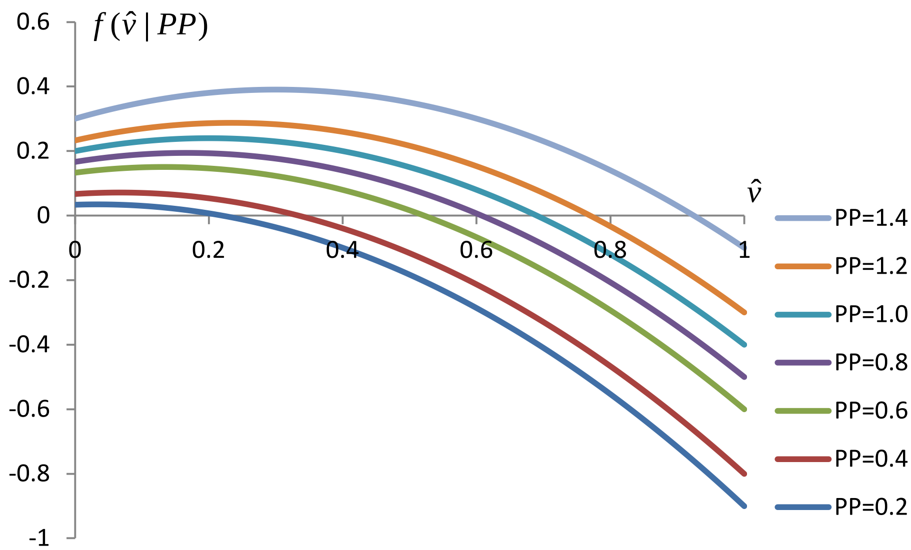


Figure A3.1 The function $f(\hat{v}\,|\,PP)$ with a series of *PP* levels

Case III

(A3.14) becomes

(A3.19) $$v_3^c(2 - PP/2 - v_3^c)(PP/2 - v_3^c) = (PP/3)(1-v_3^c)^2$$

Figure A2 below shows the function $f(v_3^c \mid PP) = v_3^c(2 - PP/2 - v_3^c)(PP/2 - v_3^c) - (PP/3)(1 - v_3^c)^2$ with PP =1.3, 1.4, …, 1.9. A solution of v_3^c between PP/3 and PP/2 can be obtained only when PP =1.8 and 1.9, i.e., when PP is high enough. Actually, when PP=1.8, there are two solutions between PP/3 and PP/2, which are 0.6551 and 0.8. Checking the two possible solutions by comparing (A3.5) and (A3.6), we find 0.6551 is the actual solution.

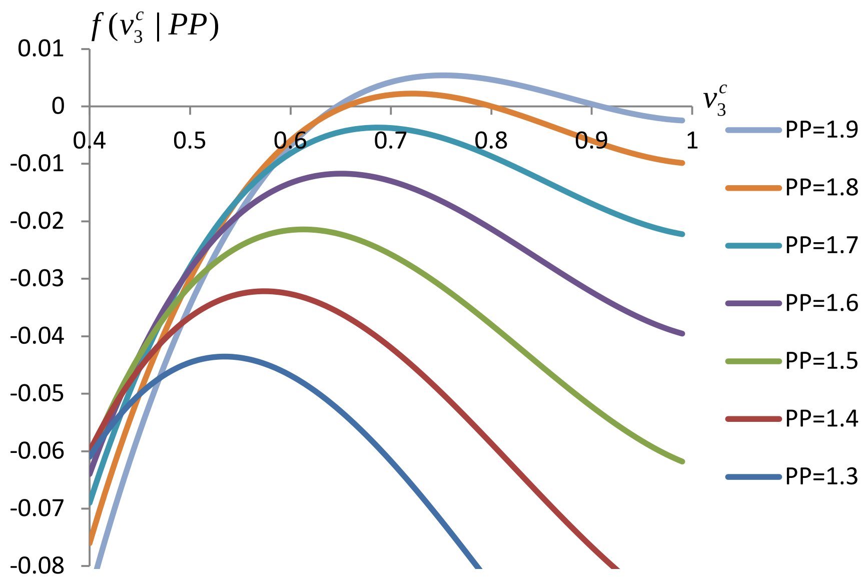


Figure A3.2 The function $f(v_3^c \mid PP)$ with a series of PP levels

When PP=1.8, the three nontrivial BNE (A3.9), (A3.13), (A3.16) all exits, and we can compare their truth-telling properties. With PP=1.8, the critical values are 0.6 in (A3.9) and 0.9245 in (A3.13). The two critical values in (A3.16) are 0.6551 (v_3^c) and 0.9 (v_2^c). The non-truth-telling value ranges are [0.9, 1], [0.6, 0.9245], and [0.6, 0.6551] in (A3.9), (A3.13), (A3.16), respectively. Obviously, the Case III, where there are two distinct uniform prices, has the shortest non-truth-telling value range.

A3.2.3 An *N*-player game (Proof of Proposition 3.4)

It can be seen from the 3-player game that as the number of players increases, the solution could become intractable quickly in terms of all the possible equilibrium cases. However, the procedure to solve the game is similar: we first construct each possible BNE by specifying a decision rule, then use the relevant expected payoff functions to form a system of polynomial equations of the critical values, and if we can find the real roots satisfying all the equations and constraints, we obtain one BNE given the model parameters. Since in general we cannot explicitly solve a system of polynomial equations when N is large, we cannot specify the conditions of the existence of a particular BNE. But we do notice some patterns, which are summarized in **Proposition 3.4**. Next, we give the general form of the expected payoff function in an N-player game, and then prove the proposition.

In an N-player game of UPA, the possible uniform prices and hence the possible contribution choices are {0, PP/N, $PP/(N$-1), …, PP/2}. The contribution function for each agent is in the general form

(A3.20) $$c_i^{BNE}(v_i)=\begin{cases} 0 & if\ \ v_i \le v_N^c \\ PP/N & if\ \ v_N^c < v_i \le v_{N-1}^c \\ \vdots & \vdots \\ PP/3 & if\ \ v_3^c < v_i \le v_2^c \\ PP/2 & if\ \ v_i > v_2^c \end{cases},$$

where $v_2^c, v_3^c, \ldots, v_N^c$ are the critical values.

The expected payoffs for contribution choices of 0 and *PP*/2 for agent i are as follows.

(A3.21) $$E\pi(0)=v_i\sum_{j=1}^{N-2}\left\{\binom{N-1}{N-j}\Pr(v>v_{N-j}^c)^{N-j}\prod_{l=1}^{j-1}\binom{j-l}{1}\Pr(v\le v_{N-(j-l)}^c)\right\}$$

(A3.22) $$E\pi(\frac{PP}{2})=\sum_{j=1}^{N-1}\left\{(v_i-\frac{PP}{N-j+1})\binom{N-1}{N-j}\Pr(v>v_{N-j+1}^c)^{N-j}\prod_{l=1}^{j-1}\binom{l}{1}\Pr(v\le v_{N-l+1}^c)\right\}.$$

Given (A3.21) and (A3.22), we have the following general form of expected payoff for contributing between 0 and *PP*/2,

(A3.23)

$$E\pi(\frac{PP}{m})=\left\{\begin{aligned} &\sum_{j=1}^{N-(m-1)}\left\{(v_i-\frac{PP}{N-j+1})\binom{N-1}{N-j}\Pr(v>v_{N-j+1}^c)^{N-j}\prod_{l=1}^{j-1}\binom{l}{1}\Pr(v\le v_{N-l+1}^c)^1\right\} \\ &+v_i\left(\sum_{i=2}^{m-2}\left[\binom{N-1}{i}\Pr(v>v_i^c)^i\prod_{h=i}^{m-3}\binom{N-1-h}{1}\Pr(v\le v_{h+1}^c)\right]\right)\prod_{k=m-2}^{N-2}\binom{N-1-k}{1}\Pr(v\le v_{k+2}^c)\end{aligned}\right\}$$

for $m = 3, 4, \ldots, N$.[50]

Given (A3.21) – (A3.23) and a list of critical values for a BNE, we will be able to construct a system of polynomial equations by equating two relevant expected payoffs at each of the critical values. If the solution exists, we will have a BNE following the decision rule defined by the list of critical values. Note that in general the expected payoffs for two consecutive uniform prices just have two different terms, one term from the first summation of the RHS of (A3.23), and the other from the second summation. This property extremely simplifies the system of polynomial equations if there are many consecutive prices used in the to-be-solved equilibrium.

Clearly, the existence of the solution depends on the parameters such as *PP*, the value distribution, and the selected decision rule. However, as we show next, there are two BNE which are easy to confirm, and in the BNE where *PP*/2 and all the uniform prices lower than *PP*/2 are supported, we will have $v_2^c = PP/2 > v_k^c > PP/k$, for $k = 3, \ldots, N$, which reflects some general feature of the critical values and the truth-telling property of the BNE in UPA.

First, if all critical values are equal to $\max\{V\} \in (PP/N, PP)$, (A3.21) – (A3.23) all become zero, and the whole system of equations is satisfied, in which case we just show the BNE *e* in **Proposition 3.4**.

[50] Note that if a>b, $\sum_{k=a}^{b}(\cdot)=0$ and $\prod_{k=a}^{b}(\cdot)=1$.

Second, if we have $0 < v_N^c < v_{N-1}^c = v_{N-2}^c = \cdots = v_3^c = v_2^c = \overline{v}$, it is easy to check that $E\pi(0) = 0$, and all the other expected payoffs collapse to the same nonzero term, which is the expected payoff with the contribution *PP*/*N*

$$E\pi(\frac{PP}{N}) = (v_i - \frac{PP}{N})\binom{N-1}{N-1}\Pr(v > v_N^c)^{N-1} \tag{A3.24}$$
.

Since $E\pi(0) = E\pi(PP/N)$ at v_N^c, we have $v_N^c = PP/N$, and the BNE *f* in **Proposition 3.4** is obtained.

Lastly, if all the *N* – 1 critical values are distinct and less than $\overline{v}$, they can be solved by equating $E\pi(PP/n)$ to $E\pi(PP/(n+1)))$ at v_n^c for n = 2, 3, …, *N*-1, and equating $E\pi(PP/n)$ to $E\pi(0)$ at n = *N*, where we have *N*-1 equations for *N*-1 unknowns. Writing them out, we find $E\pi(PP/2)$ only has one more term than $E\pi(PP/3)$, and all the other terms are exactly the same. That extra term is $(v_i - PP/2)(N-1)\Pr(v > v_2^c)\prod_{l=1}^{N-2} l\Pr(v \le v_{N-l+1}^c)$ which equals zero at v_2^c. Given that all the probabilities are strictly positive, we will have $v_2^c = PP/2$. For the other equations, each side has only two different terms, which can be rearranged into a general form of $(v_k^c - PP/k)\cdot(+) = (+)$, for *k* = 3, …, *N*, where (+) denotes a positive number. Thus, we have $v_k^c > PP/k$, for *k* = 3, …, *N*. This finishes the proof of **Proposition 3.4**.

Note that generally, if there is a BNE with $0 < v_N^c < \cdots < v_m^c < v_{m-1}^c = v_{m-2}^c = \cdots = v_2^c = \overline{v}$, then we will have $v_m^c = PP/m$, and $v_k^c > PP/k$, for *k* = *m*+1, …, *N*. This relationship between a uniform price and the critical value corresponding to the lower bound of contributing at that price reflects the general feature of UPA, in which there are value intervals where truth telling is not supported in equilibrium, except at value intervals that are either below the lowest price entirely or above the highest price supported in equilibrium entirely in this category of BNE.

A3.3 The nontrivial BNE of PPM, PR, and UPC in the 3-player 3-value example (Section 3.3)

A3.3.1 The nontrivial BNE based on integer-valued contributions

PPM (Gailmard and Palfrey)			PR (Gailmard and Palfrey, 2005)						UPC		
N	BNE	Efficiency Group	N	BNE	Efficiency Group	N	BNE	Efficiency Group	N	BNE	Efficiency Group
1	{26,35,41}	2	1	{24,24,54}	1	38	{16,29,57}	3	1	{23,23,56}	1
2	{25,35,42}	2	2	{23,23,56}	1	39	{17,29,56}	3	2	{22,22,58}	1
3	{24,34,44}	2	3	{22,22,58}	1	40	{18,29,55}	3	3	{21,21,60}	1
4	{23,34,45}	2	4	{21,21,60}	1	41	{19,29,54}	3	4	{20,20,62}	1
5	{18,30,42}	4	5	{22,36,44}	2	42	{20,29,53}	3	5	{6,6,48}	4
6	{18,18,42}	4	6	{23,36,43}	2	43	{13,28,61}	3	6	{4,4,49}	4
7	{16,16,43}	4	7	{24,36,42}	2	44	{14,28,60}	3	7	{2,2,50}	4
8	{14,29,44}	4	8	{25,36,41}	2	45	{15,28,59}	3	8	{0,0,51}	4
9	{14,14,44}	4	9	{26,36,40}	2	46	{16,28,58}	3			
10	{12,12,45}	4	10	{20,35,47}	2	47	{17,28,57}	3			
11	{10,28,46}	4	11	{21,35,46}	2	48	{18,28,56}	3			
12	{10,10,46}	4	12	{22,35,45}	2	49	{19,28,55}	3			
13	{8,8,47}	4	13	{23,35,44}	2	50	{13,27,62}	3			
14	{6,34,34}	4	14	{24,35,43}	2	51	{14,27,61}	3			
15	{6,33,36}	4	15	{25,35,42}	2	52	{15,27,60}	3			
16	{6,27,48}	4	16	{19,34,49}	2	53	{16,27,59}	3			
17	{6,6,48}	4	17	{20,34,48}	2	54	{17,27,58}	3			
18	{5,34,34}	4	18	{21,34,47}	2	55	{18,27,57}	3			
19	{5,33,36}	4	19	{22,34,46}	2	56	{19,27,56}	3			
20	{4,34,34}	4	20	{23,34,45}	2	57	{15,26,61}	3			
21	{4,33,36}	4	21	{24,34,44}	2	58	{16,26,60}	3			
22	{4,4,49}	4	22	{22,33,47}	3	59	{17,26,59}	3			
23	{3,34,34}	4	23	{23,33,46}	3	60	{18,26,58}	3			
24	{3,33,36}	4	24	{20,32,50}	3	61	{17,25,60}	3			
25	{2,34,34}	4	25	{21,32,49}	3	62	{6,27,48}	4			
26	{2,33,36}	4	26	{22,32,48}	3	63	{0,23,56}	4			
27	{2,26,50}	4	27	{18,31,53}	3	64	{2,26,50}	4			
28	{2,2,50}	4	28	{19,31,52}	3	65	{0,24,54}	4			
29	{1,34,34}	4	29	{20,31,51}	3	66	{0,25,52}	4			
30	{1,33,36}	4	30	{21,31,50}	3	67	{14,14,44}	4			
31	{0,34,34}	4	31	{16,30,56}	3	68	{12,12,45}	4			
32	{0,33,36}	4	32	{17,30,55}	3	69	{10,10,46}	4			
33	{0,25,52}	4	33	{18,30,54}	3	70	{8,8,47}	4			
34	{0,24,54}	4	34	{19,30,53}	3	71	{6,6,48}	4			
35	{0,0,51}	4	35	{20,30,52}	3	72	{4,4,49}	4			
			36	{14,29,59}	3	73	{2,2,50}	4			
			37	{15,29,58}	3	74	{0,0,51}	4			

Note: {X, Y, Z} denotes the contribution from value 29, 45, and 90, respectively. The cost is 102.

A3.3.2 The nontrivial BNE in the most efficient group with a contribution precision of 0.1

N	UPC	PR
1	{19.1, 19.1, 63.8}	{20.3, 20.3, 61.4}
2	{19.2, 19.2, 63.6}	{20.4, 20.4, 61.2}
3	{19.3, 19.3, 63.4}	{20.5, 20.5, 61}
4	{19.4, 19.4, 63.2}	{20.6, 20.6, 60.8}
5	{19.5, 19.5, 63}	{20.7, 20.7, 60.6}
6	{19.6, 19.6, 62.8}	{20.8, 20.8, 60.4}
7	{19.7, 19.7, 62.6}	{20.9, 20.9, 60.2}
8	{19.8, 19.8, 62.4}	**{21, 21, 60}**
9	{19.9, 19.9, 62.2}	{21.1, 21.1, 59.8}
10	**{20, 20, 62}**	{21.2, 21.2, 59.6}
11	{20.1, 20.1, 61.8}	{21.3, 21.3, 59.4}
12	{20.2, 20.2, 61.6}	{21.4, 21.4, 59.2}
13	{20.3, 20.3, 61.4}	{21.5, 21.5, 59}
14	{20.4, 20.4, 61.2}	{21.6, 21.6, 58.8}
15	{20.5, 20.5, 61}	{21.7, 21.7, 58.6}
16	{20.6, 20.6, 60.8}	{21.8, 21.8, 58.4}
17	{20.7, 20.7, 60.6}	{21.9, 21.9, 58.2}
18	{20.8, 20.8, 60.4}	**{22, 22, 58}**
19	{20.9, 20.9, 60.2}	{22.1, 22.1, 57.8}
20	**{21, 21, 60}**	{22.2, 22.2, 57.6}
21	{21.1, 21.1, 59.8}	{22.3, 22.3, 57.4}
22	{21.2, 21.2, 59.6}	{22.4, 22.4, 57.2}
23	{21.3, 21.3, 59.4}	{22.5, 22.5, 57}
24	{21.4, 21.4, 59.2}	{22.6, 22.6, 56.8}
25	{21.5, 21.5, 59}	{22.7, 22.7, 56.6}
26	{21.6, 21.6, 58.8}	{22.8, 22.8, 56.4}
27	{21.7, 21.7, 58.6}	{22.9, 22.9, 56.2}
28	{21.8, 21.8, 58.4}	**{23, 23, 56}**
29	{21.9, 21.9, 58.2}	{23.1, 23.1, 55.8}
30	**{22, 22, 58}**	{23.2, 23.2, 55.6}
31	{22.1, 22.1, 57.8}	{23.3, 23.3, 55.4}
32	{22.2, 22.2, 57.6}	{23.4, 23.4, 55.2}
33	{22.3, 22.3, 57.4}	{23.5, 23.5, 55}
34	{22.4, 22.4, 57.2}	{23.6, 23.6, 54.8}
35	{22.5, 22.5, 57}	{23.7, 23.7, 54.6}
36	{22.6, 22.6, 56.8}	{23.8, 23.8, 54.4}
37	{22.7, 22.7, 56.6}	{23.9, 23.9, 54.2}
38	{22.8, 22.8, 56.4}	**{24, 24, 54}**
39	{22.9, 22.9, 56.2}	{24.1, 24.1, 53.8}
40	**{23, 23, 56}**	{24.2, 24.2, 53.6}
41	{23.1, 23.1, 55.8}	
42	{23.2, 23.2, 55.6}	

Note: {*X*, *Y*, *Z*} denotes the contribution from value 29, 45, and 90, respectively. The cost is 102.

With a contribution precision of 0.1, PPM has 584 nontrivial equilibria, none of which are in the most efficient group; PR has 5330, 40 of which in the most efficient group; UPC has only 73 , 42 of which in the most efficient group. Also, notice that the eight most efficient BNE in UPC and PR (four for each) based on the integer-valued contributions are still included (in bold).

A3.4 Sample Experimental Instructions

Experimental Instructions for PPM

This is an experiment in the economics of decision-making. During the experiment, you will be asked to make a series of decisions. If you follow the instructions and make careful decisions, you can earn a considerable amount of money.

Experiment Overview

Each decision that you make is considered one *period* of the experiment. In each of these periods, you will be randomly assigned to a group of size 10. You will be asked to decide how much money you will offer towards the cost of a project. This cost is predetermined and known to you. All members of your group receive a benefit when the project is implemented, which occurs only when the total offers of all members in your group meet or exceed the cost of the project.

How You Earn Money

At the beginning of each period, you will be told the individual value (benefit) you will receive if the project is implemented. This value will be randomly chosen from the following set of nine numbers: {**4, 5, 6, 7, 8, 9, 10, 11, 12**}, period by period. The **chance** of **each** number being assigned to you is **1/9**, i.e., you have an equal chance for each number, although you may have a different individual value each period. You will then be asked to make an offer from zero up to 15 experimental dollars.

You will be working with experimental dollars. Fifteen experimental dollars are equal to $1. Your initial fund will be 15 experimental dollars, which represents part of your fee for showing up today. Your earnings for each period will be added to this amount.

The Process

You will be asked to decide how much money you will offer towards the cost of a project.

- If your group's total offers are **sufficient** for the project to be implemented, you will receive your individual value for the project, minus a contribution in the amount of your offer.

- If the group's total offers are **not sufficient** to implement the project, your offer will not be collected and you will receive no additional earnings.

Examples
There are two possible outcomes in each period:

(Outcome 1) The group offers **do** allow the project to be implemented.

Project Cost (*known to you*)	$100	
Your Individual Value	$11	*Others in your group may have values higher or lower than yours*
Your Offer	$1	*Others in your group may offer more or less than you do*
Total Offers of Your Group	$110	*Project cost exceeded*
Your Earnings for This Period	$10	*$11 Value* *-$1 Contribution* *=$10 Earnings*

The total offers of your group are sufficient for the project to be implemented. In this case, the project cost is exceeded. Your earnings ($10) are your individual value ($11) minus your contribution ($1).

(Outcome 2) The group offers **do not** allow the project to be implemented.

In this example, your offer will not be collected and you will not receive any additional earnings.

Project Cost (*known to you*)	$100	
Your Individual Value	$12	*Others in your group may have values higher or lower than yours*
Your Offer	$12	*Others in your group may offer more or less than you do*
Total Offers of Your Group	$85	*Does not meet project cost*
Your Earnings for This Period	$0	*No additional value received*

The project cost is not met, so the project is not implemented. You do not receive your individual value for this period. Your $12 offer is not collected.

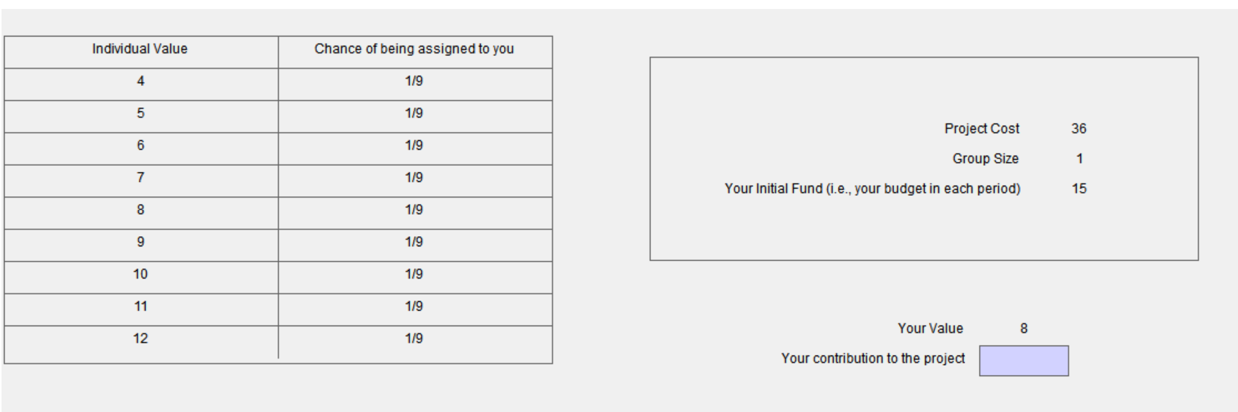
Please do not speak to other participants during the experiment. Follow the instructions to the best of your ability. If you have questions, raise your hand and the administrator will assist you.

Instructions At-A-Glance

- In each period you will be randomly assigned to a group of 10 participants.
- You will be asked to decide how much money you will offer towards the cost of a project.
- Based on the offers of everyone in your group, the administrator will determine whether the project can be implemented.
- If the total offers meet or exceed the project cost, the project is implemented and your earnings will be your individual value minus your contribution.
- If the group's total offers are not sufficient to implement the project, your offer will not be collected and you will receive no additional earnings.
- If you offer more, in exchange for incurring some of the costs, you increase the chance that the project is implemented.

At the end of the experiment, your earnings will be totaled across all periods and converted from experimental dollars to real dollars. You will be paid as you leave.

Contribution stage:



Earning display stage:

Experimental Instructions for UPA

Experiment Overview

The overall process for this treatment is the same as for the previous treatment in terms of group size (10), individual value (randomly chosen in each period from the set {**4, 5, 6, 7, 8, 9, 10, 11, 12**} **with an equal chance**), conversion rate (15 to 1), and the steps you will take. The difference in this treatment is the method by which your net contribution or possible rebate is calculated.

The Process

Again, you will be asked to decide how much money you will offer towards the cost of a project.

- If your group's total offers are sufficient to implement the project, a *Uniform Contribution* will be calculated for your group.
- If your offer is above the *Uniform Contribution*, you pay only the uniform contribution and we will rebate any amount you offered above the uniform contribution. Your earnings will be your individual value for the project minus the *Uniform Contribution*.
- If your offer is lower than the *Uniform Contribution*, you pay nothing. Your earnings will be your individual value for the project.
- If the group's total offers are not sufficient to implement the project, your offer will not be collected and you will receive no additional earnings. .

The *Uniform Contribution* will be the lowest amount that allows us to collect only the exact amount needed to implement the project.

Examples

There are three possible outcomes in each period:

(Outcome 1) The group offers **do** allow the project to be implemented and the *Uniform Contribution* is *equal to or lower* than your offer.

In this example, your offer is higher than the *Uniform Contribution*, so you pay only the *Uniform Contribution*. Your earnings are your individual value for the project, minus your offer, plus a rebate of your offer above the uniform contribution.

Project Cost (*known to you*)	$110	
Your Individual Value	$12	*Others in your group may have values higher or lower than yours*
Your Offer	$11	*Others in your group may offer more or less than you do*
Total Offers of Your Group	$150	*Project cost exceeded*
Calculated Uniform Contribution	$7.50	
Your Rebate for this Period	$3.50	*$11 Offer - $7.50 Contribution*
Your Earnings for This Period	$4.50	*$12 Value* *-$11 Offer* *+ $3.50 Rebate* *= $4.50 Earnings*

The Uniform Contribution is lower than your offer. Even though you offered $11, you pay only the Uniform Contribution of $7.50. Your earnings ($4.50) are your individual value ($12) minus your offer ($11), plus the rebate of your offer above the Uniform Contribution (3.50).

(Outcome 2) The group offers **do** allow the project to be implemented and the *Uniform Contribution* is *higher than* your offer.

In this example, your offer is lower than the *Uniform Contribution*, so you do not contribute. Your earnings are your individual value for the project.

Project Cost (*known to you*)	$80	
Your Individual Value	$11	*Others in your group may have values higher or lower than yours*
Your Offer	$1	*Others in your group may offer more or less than you do*
Total Offers of Your Group	$100	*Project cost exceeded*
Calculated Uniform Contribution	$9	
Your Earnings for This Period	$11	*$11 Value* *- $0 Contribution* *=$11 Earnings*

Your offer of $1 is lower than the Uniform Contribution of $9, so you pay nothing. Your earnings ($11) are your individual value ($11).

(Outcome 3) The group offers **do not** allow the project to be implemented.

In this example, your offer will not be collected and you will not receive any additional earnings.

Project Cost (*known to you*)	$100	
Your Individual Value	$12	*Others in your group may have values higher or lower than yours*
Your Offer	$12	*Others in your group may offer more or less than you do*
Total Offers of Your Group	$85	*Uniform contribution cannot be calculated*
Your Earnings for This Period	$0	*No additional value received*

The project cost is not met, so the project is not implemented. You do not receive your individual value for this period. Your $12 offer is not collected.

Note that, given the project cost and your group size, there are only a few possible Uniform Contribution levels. For example, if the project cost is $30 and the group size is 3, then the possible Uniform Contribution levels are as follows.

- If 3 members contribute at least 10 (=30/3), then the project will be implemented and the Uniform Contribution is 10;
- If 2 members contribute at least 15 (=30/2) (and the other less than 10) then the project will be implemented and the Uniform Contribution is 15;
- If 1 member contributes at least 30 (=30/1) (and both other less than 15 and at least one other below 10), then the calculated Uniform Contribution is 30. However, if no one has an individual value ≥ 30, no one would contribute 30, and hence this Uniform Contribution is not feasible.

The feasible Uniform Contribution levels and the numbers of contributors needed to implement them in this treatment are given at the end of this instruction.

Please do not speak to other participants during the experiment. Follow the instructions to the best of your ability. If you have questions, raise your hand and the administrator will assist you.

<u>Instructions At-A-Glance</u>

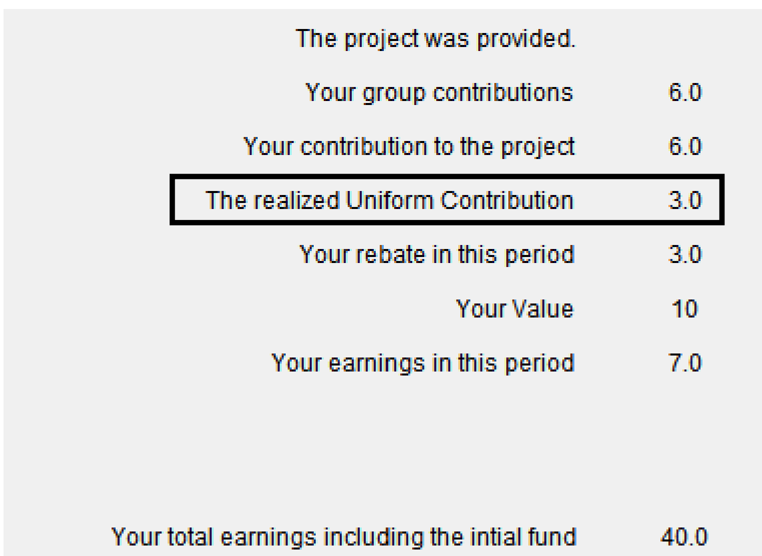
- In each period you will be randomly assigned to a group of 10 participants.
- You will be asked to decide how much money you will offer towards the cost of a project.
- Based on the offers of everyone in your group, the administrator will determine whether the project can be implemented.
- If total offers are such that a uniform contribution can be calculated, the project is implemented and any excess funds are rebated. The *Uniform Contribution* will be calculated as the lowest amount that allows us to collect <u>only the exact amount needed</u> to implement the project.
    - If the project is implemented and your offer is above the *Uniform Contribution*, you pay only the uniform contribution. Your earnings will be your individual value minus the *Uniform Contribution*.
    - If the project is implemented and your offer is lower than the *Uniform Contribution,* you pay nothing. Your earnings will be your individual value.
- If the group's total offers are not sufficient to implement the project, your offer will not be collected and you will receive no additional earnings.
- If you offer more, in exchange for incurring some of the costs, you increase the chance that the project is implemented.

At the end of the experiment, your earnings will be totaled across all periods and converted from experimental dollars to real dollars. You will be paid as you leave.

The table below shows the feasible Uniform Contribution levels and the number of contributors needed to implement them in this treatment, given that the project cost is **36**, the group size is **10**, and the individual value distribution (randomly chosen in each period from the set {**4, 5, 6, 7, 8, 9, 10, 11, 12**} **with an equal chance**).

Feasible Uniform Contribution	Number of contributors needed
12	3
9	4
7.2	5
6	6
5.2	7
4.5	8
4	9
3.6	10

Earning display stage:



Experimental Instructions for UPC

Experiment Overview

The overall process for this treatment is the same as for the previous treatment in terms of group size (10), individual value (randomly chosen in each period from the set {**4, 5, 6, 7, 8, 9, 10, 11, 12**} **with an equal chance**), conversion rate (15 to 1), and the steps you will take. The difference in this treatment is the method by which your net contribution or possible rebate is calculated.

The Process

Again, you will be asked to decide how much money you will offer towards the cost of a project.

- If your group's total offers are sufficient to implement the project, a *Contribution Cap* will be calculated for your group.
    - If your offer is above the *Contribution Cap*, you pay only the contribution cap and we will rebate any amount you offered above the contribution cap.
    - If your offer is lower than the *Contribution Cap*, you pay your offer amount.
- Your earnings will be your individual value for the project minus your contribution in both cases.
- If the group's total offers are not sufficient to implement the project, your offer will not be collected and you will receive no additional earnings.

The *Contribution Cap* will be the lowest amount that allows us to collect only the exact amount needed to implement the project.

Examples

There are three possible outcomes in each period:

(Outcome 1) The group offers **do** allow the project to be implemented and the *Contribution Cap* is *equal to or lower* than your offer.

In this example, your offer is higher than the *Contribution Cap*, so you pay only the *Contribution Cap*. Your earnings are your individual value for the project, minus your offer, plus a rebate of your offer above the contribution cap.

Project Cost (*known to you*)	$110	
Your Individual Value	$12	*Others in your group may have values higher or lower than yours*
Your Offer	$11	*Others in your group may offer more or less than you do*
Total Offers of Your Group	$150	*Project cost exceeded*
Calculated Contribution Cap	$7.50	
Your Rebate for this Period	$3.50	*$11 Offer - $7.50 Contribution Cap*
Your Earnings for this Period	$4.50	*$12 Value* *-$11 Offer* *+ $3.50 Rebate* *= $4.50 Earnings*

The Contribution Cap is lower than your offer. Even though you offered $11, you pay only the Contribution Cap of $7.50. Your earnings ($4.50) are your individual value ($12) minus your offer ($11), plus the rebate of your offer above the Contribution Cap ($3.50).

(Outcome 2) The group offers **do** allow the project to be implemented and the *Contribution Cap* is *higher than* your offer.

In this example, your offer is lower than the *Contribution Cap*, so your contribution is the full amount of your offer. Your earnings are your individual value for the project, minus your contribution.

Project Cost (*known to you*)	$80	
Your Individual Value	$11	*Others in your group may have values higher or lower than yours*
Your Offer	$1	*Others in your group may offer more or less than you do*
Total Offers of Your Group	$100	*Project cost exceeded*
Calculated Contribution Cap	$9	*Higher than your offer*
Your Earnings for This Period	$10	*$11 Value* *- $1 Contribution (as Offered)* *=$10 Earnings*

Your offer of $1 is lower than the Contribution Cap of $9, so you pay your offer amount. Your earnings ($10) are your individual value ($11) minus your contribution at your original offer amount ($1).

(Outcome 3) The group offers **do not** allow the project to be implemented.

In this example, your offer will not be collected and you will not receive any additional earnings.

Project Cost (*known to you*)	$100	
Your Individual Value	$12	*Others in your group may have values higher or lower than yours*
Your Offer	$12	*Others in your group may offer more or less than you do*
Total Offers of Your Group	$85	*Does not meet project cost*
Your Earnings for This Period	$0	*No additional value received*

The project cost is not met, so the project is not implemented. You do not receive your individual value for this period. Your $12 offer is not collected.

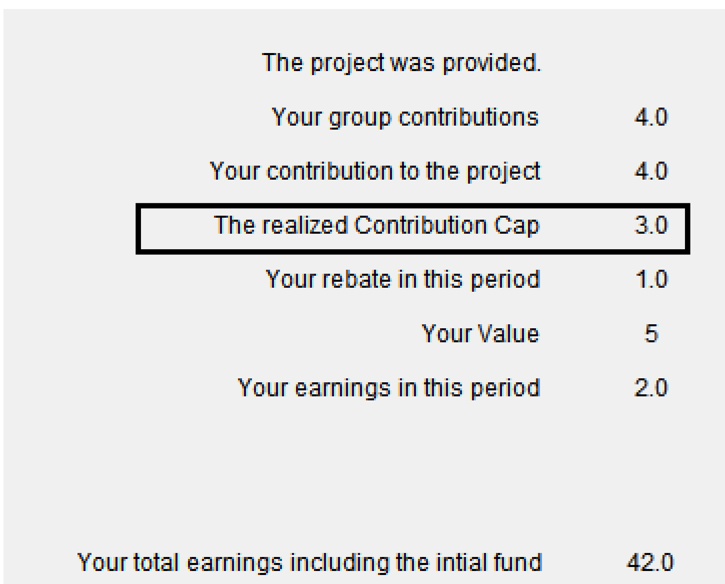
Please do not speak to other participants during the experiment. Follow the instructions to the best of your ability. If you have questions, raise your hand and the administrator will assist you.

Instructions At-A-Glance

- In each period you will be randomly assigned to a group of 10 participants.
- You will be asked to decide how much money you will offer towards the cost of a project.
- Based on the offers of everyone in your group, the administrator will determine whether the project can be implemented.
- If total offers are such that a contribution cap can be calculated, the project is implemented and any excess funds are rebated. The *Contribution Cap* will be calculated as the lowest amount that allows us to collect <u>only the exact amount needed</u> to implement the project.
  - If the project is implemented and your offer is above the *Contribution Cap*, you pay only the contribution cap. Your earnings will be your individual value minus the *Contribution Cap*.
  - If the project is implemented and your offer is lower than the *Contribution Cap,* you pay only your offer. Your earnings will be your individual value minus your contribution.
- If the group's total offers are not sufficient to implement the project, your offer will not be collected and you will receive no additional earnings.
- If you offer more, in exchange for incurring some of the costs, you increase the chance that the project is implemented.

At the end of the experiment, your earnings will be totaled across all periods and converted from experimental dollars to real dollars. You will be paid as you leave.

Earning display stage:



Experimental Instructions for PR

<u>Experiment Overview</u>
The overall process for this treatment is the same as for the previous treatment in terms of group size (10), individual value (randomly chosen in each period from the set {**4, 5, 6, 7, 8, 9, 10, 11, 12**} **with an equal chance**), conversion rate (15 to 1), and the steps you will take. The difference in this treatment is the method by which your net contribution or possible rebate is calculated.

<u>The Process</u>
Again, you will be asked to decide how much money you will offer towards the cost of a project.

- If your group's total offers **equal** the cost of the project, the project will be implemented. Your earnings will be your individual value for the project minus your contribution.
- If your group's total offers **exceed** the cost of the project, the project will be implemented and excess funds will be rebated. Your earnings will be your individual value for the project minus your offer, plus your rebate. Your rebate will be in proportion to the excess funds offered by your group. So, if X% of your group's total offers is not needed, your rebate will be X% of your offer.
- If the group's total offers are not sufficient to implement the project, your offer will not be collected and you will receive no additional earnings.

<u>Examples</u>
There are three possible outcomes in each period:

(Outcome 1) The group offers are **exactly equal** to the project cost and the project is implemented.

In this example, all of your offer is needed, so your earnings are your individual value for the project, minus your contribution in the amount of your offer.

Project Cost (*known to you*)	$100	
Your Individual Value	$12	*Others in your group may have values higher or lower than yours*
Your Offer	$2	*Others in your group may offer more or less than you do*
Total Offers of Your Group	$100	*Exactly meets project cost*
Your Earnings for This Period	$10	*$12 Value* *<u>-$2 Contribution (as Offered)</u>* *=$10 Earnings*

The project cost is exactly met, so the project is implemented. Your earnings ($10) are your individual value ($12) minus your contribution ($2).

(Outcome 2) The group offers **exceed** the project cost and the project is implemented.

In this example, total offers exceed the amount needed, so a portion of each offer will be rebated. Your earnings are your individual value for the project, minus your offer, plus your *rebate*. This rebate is based upon the proportion of total offers represented by excess funds offered by your group.

Project Cost (*known to you*)	$150	
Your Individual Value	$10	*Others in your group may have values higher or lower than yours*
Your Offer	$10	*Others in your group may offer more or less than you do*
Total Offers of Your Group	$200	*Exceeds project cost*
Total Excess Contributions	$50	*$200 offered - $150 needed*
Your Rebate from Excess Contributions	$2.50	*We need 75% of the money offered, so you get 25% of your money back*
Your Earnings for This Period	$2.50	*$10 Value* *- $10 Offer* *+ $2.50 rebate* *=$2.50 earnings*

The project cost is exceeded, so the project is implemented. Because we only need 75% of the money offered, you get a 25% of your money back. Your earnings ($2.50) are your individual value ($10) minus your offer ($10), plus your rebate ($2.50).

(Outcome 3) The group offers **do not** allow the project to be implemented.

In this example, your offer will not be collected and you will not receive any additional earnings.

Project Cost (*known to you*)	$100	
Your Individual Value	$12	*Others in your group may have values higher or lower than yours*
Your Offer	$10	*Others in your group may offer more or less than you do*
Total Offers of Your Group	$85	*Does not meet project cost*
Your Earnings for This Period	$0	*No additional value received*

The project cost is not met, so the project is not implemented. You do not receive your individual value for this period. Your $10 offer is not collected.

Please do not speak to other participants during the experiment. Follow these instructions to the best of your ability. If you have questions, raise your hand and the administrator will assist you.

<u>Instructions At-A-Glance</u>

- In each period you will be randomly assigned to a group of 10 participants.
- You will be asked to decide how much money you will offer towards the cost of a project.
- Based on the offers of everyone in your group, the administrator will determine whether the project can be implemented.
- If the total offers of your group **equal** the cost of the project, the project is implemented. Your earnings will be your individual value minus your contribution.
- If the total offers of your group **exceed** the cost of the project, the project is implemented and excess contributions are rebated so that <u>only the exact amount needed</u> for the project is actually collected. Your earnings will be your individual value minus your offer, plus a rebate based upon the proportion of total offers represented by excess funds offered by your group.
- If the group's total offers are not sufficient to implement the project, your offer will not be collected and you will receive no additional earnings.
- If you offer more, in exchange for incurring some of the costs, you increase the chance that the project is implemented.

At the end of the experiment, your earnings will be added across all periods and converted from experimental dollars to real dollars. You will be paid as you leave.

Earning display stage:

The project was provided.	
Your group contributions	5.0
Your contribution to the project	5.0
Your rebate this period	2.0
Your Value	6
Your earnings in this period	3.0
Your total earnings including the intial fund	33.0

Chapter 4

A4.1 The local maximum and minimum stocks of (4) by Green's Theorem Approach
The key idea of the Green's theorem approach is to transfer a line integral over a simple closed curve into an integral over the area bounded by the closed curve, and then by making use of some simple properties of the integrand of the area integral to show the *MRAP* to some singular stock level is the optimal path (Sethi, 1977).

Specifically, we rearrange the state equation in problem (4) and use s_t and $\dot{s}_t$ to represent the group contribution $\sum_j c_{jt}$ as follows

(A4.1) $\sum_j c_{jt} = \dfrac{PP\left[d \cdot s_t + \dot{s}_t\right]}{r_{\text{int}} \cdot s_t(1-s_t K^{-1})}$

Plug (A4.1) into (4), we will have

$$\text{(A4.2)} \quad \begin{aligned} &\max_{\{s_t\}_{t=0}^{t=\infty}} J = \int_0^\infty \left\{ [N\alpha \cdot s_t - \frac{PP}{r_{\text{int}}(1-s_t K^{-1})} \cdot d] - \frac{PP}{r_{\text{int}} \cdot s_t(1-s_t K^{-1})} \cdot \dot{s}_t \right\} \cdot e^{-\rho t} dt \\ &s.t.\ 0 \le \frac{d}{1-s_t K^{-1}} + \frac{1}{s_t(1-s_t K^{-1})} \cdot \dot{s}_t \le 1\ \forall\ t,\ 0 \le d \le 1,\ s_0 > 0 \end{aligned}$$

Let $G(s_t, \dot{s}_t) = M(s_t) + N(s_t) \cdot \dot{s}_t$, where $M(s_t) = N\alpha \cdot s_t - \dfrac{PP}{r_{\text{int}}(1-s_t K^{-1})} \cdot d$, and

$N(s_t) = -\dfrac{PP}{r_{\text{int}} \cdot s_t(1-s_t K^{-1})}$, then we have

(A4.3) $J_\Gamma = \int_\Gamma \left\{M(s)dt + N(s)ds\right\} \cdot e^{-\rho t}$,

where Γ is a curve in (t, s) space.[51] By the Green's theorem,

$$\text{(A4.4)} \quad \begin{aligned} J_\Gamma &= \iint_R \left\{(\partial/\partial t)[e^{-\rho t} N(s)] - (\partial/\partial s)[e^{-\rho t} M(s)]\right\} dtds \\ &= \iint_R e^{-\rho t} I(s) dtds = \Delta_R, \end{aligned}$$

where $I(s) = -[\rho N(s) + M'(s)]$ and Δ_R denotes the double integral over the area R .

Plug $N(s_t)$ and $M'(s)$ into $I(s)$, where $M'(s) = N\alpha - \dfrac{PP \cdot d}{K r_{\text{int}}(1-s_t K^{-1})^2}$, we will have

[51] Rigorously, we need to solve a finite horizon problem first, and then take the planning horizon to infinity in the limit to show the desired results. We omit these intermediate steps here and in the application of the Green's theorem as well. For the details, see Sethi (1977).

$$I(s)=\frac{\rho PP}{r_{\text{int}}\cdot s_t(1-s_tK^{-1})}-N\alpha+\frac{PP\cdot d}{Kr_{\text{int}}(1-s_tK^{-1})^2}$$

(A4.5)
$$=\frac{1}{s_t(1-s_tK^{-1})}\left\{\frac{PP}{r_{\text{int}}}\left[\rho+d\cdot\frac{s_t}{K-s_t}\right]-N\alpha s_t(1-s_t/K)\right\}$$

$$=\frac{1}{s_t(1-s_tK^{-1})}\cdot\frac{\mathrm{d}\,\sigma}{\mathrm{d}\,t}$$

To max J_Γ, or to specify the optimal control, we need to partition the (t,s)-space into the regions where the integrand $I(s)$ takes positive or negative values. Let $I(s)=0$, we will have the same equation as Eq (4.12), and it is easy to verify that

(A4.6) $$I(s)\begin{cases}<0 & if\ s_t\in(s^{**},s^*)\\ >0 & if\ s_t>s^*\text{ or }s_t<s^{**}\end{cases}$$

Figure A4.1 shows how the sign of $I(s)$ changes in the (t,s)-space.

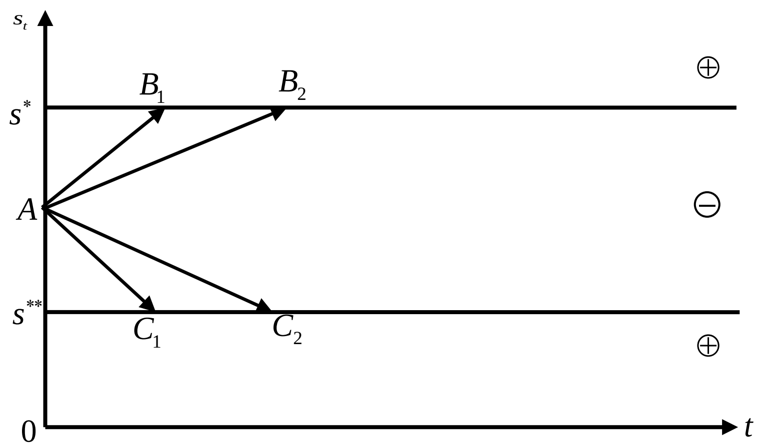


Figure A4.1 Case of multiple equilibria

To show the *MRAP* to and then staying at s^* and s^{**} are the local maximum and the local minimum, respectively, we need to show the two stock paths correspond to respectively the highest and the lowest line integral of (A4.3) locally. Since the proof is similar when *A* is above s^* or below s^{**}, we just need to show $J_{AB_1B_2}>J_{AB_2}$ and $J_{AC_1C_2}<J_{AC_2}$, where *A* is an initial stock level between s^* and s^{**} as shown in Figure A4.1, and the stock paths AB_1 and AC_1 represent the *MRAP* to s^* and s^{**}, respectively.

Note that $J_{AB_1B_2}-J_{AB_2}=\Delta_{AB_1B_2A}=-\Delta_{AB_2B_1A}$. Observing the counterclockwise convention for the line integrals, we have $\Delta_{AB_2B_1A}<0$ due to $I(s)<0$ for $s_t\in(s^{**},s^*)$. Hence, $J_{AB_1B_2}>J_{AB_2}$. Similarly, $J_{AC_1C_2}-J_{AC_2}=\Delta_{AC_1C_2A}<0$ due to $I(s)<0$ for $s_t\in(s^{**},s^*)$, i.e., $J_{AC_1C_2}<J_{AC_2}$. Further, if we take the limit of B_2 and C_2 to infinity, we will have $J_{AB_2}=J_{AC_2}$ in the limit, and hence $J_{AB_1B_2}>J_{AC_1C_2}$. In other words, the *MRAP* to s^* is always better than the *MRAP* to s^{**}.

A4.2 An upper bound of the integration in (4.18) strictly less than PP/ρ

First, given an initial stock $s_0 = s_0^1$ close enough to 0, we can choose a $t_1(s_0^1)$ in Figure 4.2 low enough such that $\partial s_t/\partial t > 0$ and $\partial^2 s_t/\partial t^2 > 0$ for t between 0 and t_1, i.e., s_t is convex in t from 0 to t_1. Let $s_1 = s_{t_1(s_0^1)}$. Then we will have $\lim_{s_0 \to 0} \tau(s_0; s_1) = \infty$ and

$$\lim_{s_0 \to 0} \int_0^{\tau(s_0;s^*)} N\alpha s_t e^{-\rho t} dt < N\alpha \lim_{s_0 \to 0} \left(\int_0^{\tau(s_0;s_1)} s_1 e^{-\rho t} dt + e^{-\rho\tau(s_0;s_1)} \int_0^{\tau(s_1;s^*)} K(1 - d/r_{\text{int}}) e^{-\rho t} dt \right)$$

$$= N\alpha \lim_{s_0 \to 0} \left(\frac{s_1}{\rho} + e^{-\rho\tau(s_0;s_1)} \frac{K(1 - d/r_{\text{int}})}{\rho} \int_0^{\tau(s_1;s^*)} e^{-\rho t} dt \right)$$

Since $\tau(s_1; s^*)$ is a finite number, we have $\lim_{s_0 \to 0} \int_0^{\tau(s_0;s^*)} N\alpha s_t e^{-\rho t} dt < N\alpha \frac{s_1}{\rho}$. Therefore, as long as $s_1 < \frac{PP}{N\alpha}$, we will have $\lim_{s_0 \to 0} \left[J(s_0; s^*) - J_0(s_0) \right] < 0$.

A4.3 Proof of Lemma 4.2

1) *For* $s \in (s_-^L, s_-^U)$, $J(s_0; s) - J_0(s_0)\big|_{s_0 = s} > 0$, *and* $\left.\frac{\partial (J(s_0; s) - J_0(s_0))}{\partial s_0}\right|_{s_0 = s} < 0$, *where* s_-^L *and* s_-^U *are the two different roots of the equation* $s(1 - s/K) = \frac{PP}{N r_{\text{int}} \alpha}[\rho + d]$ *and* $s_-^L < s_-^U$, *assuming* $K/4 > \frac{PP}{N r_{\text{int}} \alpha}[\rho + d]$. *Further, for* $s \in (0, s_{--}^U)$, $\left.\frac{\partial^2 (J(s_0; s) - J_0(s_0))}{\partial {s_0}^2}\right|_{s_0 = s} < 0$, *where* s_{--}^U *is the real root of the equation* $s(1 - s/K) = \frac{PP}{N r_{\text{int}} \alpha}[\rho + d + r_{\text{int}}(2s/K - 1)]$, *assuming* $r_{\text{int}} > d + \rho$. *And* $s^{**} < s_-^L < s_{--}^U < s^* < s_-^U$ *for* $s \le s^* < K(1 - d/r_{\text{int}})$.

2) $\lim_{s_0 \to 0} \frac{\partial (J(s_0; s^*) - J_0(s_0))}{\partial s_0} > 0$, assuming $r_{\text{int}} > d + \rho$.

We use $J(s_0; s)$ to represent the group value of the *MRAP* from s_0 to s and then staying at s thereafter for $s_0 \in (0, s]$, and $J_0(s_0)$ the group value of the *MRAP* to 0. Then, we have

(A4.7)
$$J(s_0; s) - J_0(s_0) = \int_0^{\tau(s_0;s)} N\alpha s_t e^{-\rho t} dt - \frac{PP}{\rho}(1 - e^{-\rho\tau(s_0;s)}) + \frac{e^{-\rho\tau(s_0;s)}}{\rho} \left(N\alpha s - \frac{PP}{r_{\text{int}}} \frac{d}{1 - sK^{-1}} \right) - \frac{N\alpha s_0}{d + \rho}$$

where

(A4.8)
$$\tau(s_0; s) = \frac{1}{(r_{\text{int}} - d)} \text{Ln} \frac{s / \left[K(1 - d/r_{\text{int}}) - s \right]}{s_0 / \left[K(1 - d/r_{\text{int}}) - s_0 \right]}$$

(A4.9) $s_t = \dfrac{s_0 K(1-d/r_{\text{int}})}{s_0 + \left[K(1-d/r_{\text{int}}) - s_0\right] e^{-(r_{\text{int}}-d)t}}$

Next, we take three steps to prove property 1).

Step 1

When $s_0 = s$, $\tau(s_0;s)$ =0, (A4.7) becomes

(A4.10) $$J(s;s) - J_0(s) = \frac{1}{\rho}\left(N\alpha s - \frac{PP}{r_{\text{int}}}\frac{d}{1-sK^{-1}}\right) - \frac{N\alpha s}{d+\rho}$$

For $J(s;s) - J_0(s) > 0$, we need $\frac{1}{\rho}\left(N\alpha s - \frac{PP}{r_{\text{int}}}\frac{d}{1-sK^{-1}}\right) - \frac{N\alpha s}{d+\rho} > 0$, which can be simplified to $s(1-s/K) > \frac{PP}{Nr_{\text{int}}\alpha}[\rho + d]$. Let s_-^L and s_-^U denote the two real roots of the equation

(A4.11) $$s(1-s/K) = \frac{PP}{Nr_{\text{int}}\alpha}[\rho + d]$$

Then, we will have $J(s_0;s) > J_0(s_0)\big|_{s_0=s}$ for $s \in (s_-^L, s_-^U)$. Note that for the existence of two different roots of (A4.11), we need the model parameters to satisfy $K/4 > \frac{PP}{Nr_{\text{int}}\alpha}[\rho + d]$.

Further, since $[\rho + d] < [\rho + d\, s/(K-s)]$ when $s > K/2$, $s^{**} < s_-^L < s^* < s_-^U$, where s^* and s^{**} are the singular stock levels defined in Eq (4.12). Thus, for all $s \in (s_-^L, s^*]$, $J(s;s) > J_0(s)$.

Step 2

Take the derivative of $J(s_0;s) - J_0(s_0)$ *w.r.t.* s_0,

(A4.12) $$\frac{\partial(J(s_0;s) - J_0(s_0))}{\partial s_0} = \underbrace{\frac{\partial \tau(s_0;s)}{\partial s_0} N\alpha s_{\tau(s_0;s)} e^{-\rho\tau(s_0;s)}}_{D_1} + \underbrace{\int_0^{\tau(s_0;s)} N\alpha \frac{\partial s_t}{\partial s_0} e^{-\rho t} dt}_{D_2} + \underbrace{\frac{1}{\rho}\left(N\alpha s + PP\frac{K\left(1-d/r_{\text{int}}\right) - s}{K-s}\right)\frac{\partial e^{-\rho\tau(s_0;s)}}{\partial s_0}}_{D_3} - \underbrace{\frac{N\alpha}{d+\rho}}_{D_4}$$

where

(A4.13) $$\frac{\partial \tau(s_0;s)}{\partial s_0} = -\frac{1}{(r_{\text{int}} - d)}\frac{K(1-d/r_{\text{int}})}{s_0\left[K(1-d/r_{\text{int}}) - s_0\right]} = \frac{-K/r_{\text{int}}}{s_0\left[K(1-d/r_{\text{int}}) - s_0\right]} < 0$$

(A4.14) $$\frac{\partial s_t}{\partial s_0} = \frac{\left[K(1-d/r_{\text{int}})\right]^2 e^{-(r_{\text{int}}-d)t}}{\left(s_0 + \left[K(1-d/r_{\text{int}}) - s_0\right] e^{-(r_{\text{int}}-d)t}\right)^2} > 0$$

(A4.15) $$\frac{\partial e^{-\rho\tau(s_0;s)}}{\partial s_0} = -\rho e^{-\rho\tau(s_0;s)}\frac{\partial \tau(s_0;s)}{\partial s_0} > 0$$

When $s_0 = s$, $\tau(s_0;s)$ =0, and (A4.12) with (A4.13)-(A4.15) plugged in, becomes

(A4.16)
$$\left.\frac{\partial(J(s_0;s)-J_0(s_0))}{\partial s_0}\right|_{s_0=s} = \frac{PP}{r_{\text{int}}s(1-s/K)} - \frac{N\alpha}{\rho+d}$$
$$= \frac{N\alpha}{(\rho+d)s(1-s/K)}\left[\frac{PP}{Nr_{\text{int}}\alpha}(\rho+d) - s(1-s/K)\right]$$

Comparing (A4.11) and the RHS of (A4.16), we have $\left.\frac{\partial(J(s_0;s)-J_0(s_0))}{\partial s_0}\right|_{s_0=s} < 0$ for $s \in (s_-^L, s_-^U)$.

Step 3

Take the second derivative of $J(s_0;s)-J_0(s_0)$ *w.r.t.* s_0, we will have

(A4.17)
$$\frac{\partial D_1}{\partial s_0} = \frac{\partial^2\tau(s_0;s)}{\partial {s_0}^2} N\alpha s_{\tau(s_0;s)} e^{-\rho\tau(s_0;s)}$$
$$+\frac{\partial\tau(s_0;s)}{\partial s_0} N\alpha\left[\left(\frac{\partial s(s_0,\tau(s_0;s))}{\partial s_0} + \frac{\partial s(s_0,\tau(s_0;s))}{\partial t}\frac{\partial\tau(s_0;s)}{\partial s_0}\right)e^{-\rho\tau(s_0;s)} + s_{\tau(s_0;s)}\frac{\partial e^{-\rho\tau(s_0;s)}}{\partial s_0}\right]$$

(A4.18)
$$\frac{\partial D_2}{\partial s_0} = \frac{\partial\tau(s_0;s)}{\partial s_0} N\alpha \frac{\partial s(s_0,\tau(s_0;s))}{\partial s_0} e^{-\rho\tau(s_0;s)} + \int_0^{\tau(s_0;s)} N\alpha \frac{\partial^2 s_t}{\partial {s_0}^2} e^{-\rho t} dt$$

(A4.19)
$$\frac{\partial D_3}{\partial s_0} = \frac{1}{\rho}\left(N\alpha s + PP\frac{K\left(1-d/r_{\text{int}}\right)-s}{K-s}\right)\frac{\partial^2 e^{-\rho\tau(s_0;s)}}{\partial {s_0}^2}$$

(A4.20)
$$\frac{\partial D_4}{\partial s_0} = 0$$

where

(A4.21)
$$\frac{\partial^2\tau(s_0;s)}{\partial {s_0}^2} = \frac{1}{(r_{\text{int}}-d)}\frac{K(1-d/r_{\text{int}})\left[K(1-d/r_{\text{int}})-2s_0\right]}{\left(s_0\left[K(1-d/r_{\text{int}})-s_0\right]\right)^2}\begin{cases} >0 & \text{if } s_0 < \frac{1}{2}K(1-d/r_{\text{int}}) \\ <0 & \text{if } s_0 > \frac{1}{2}K(1-d/r_{\text{int}}) \end{cases}$$

(A4.22)
$$\frac{\partial s_t}{\partial t} = \frac{s_0 K(1-d/r_{\text{int}})\left[K(1-d/r_{\text{int}})-s_0\right](r_{\text{int}}-d)e^{-(r_{\text{int}}-d)t}}{\left(s_0+\left[K(1-d/r_{\text{int}})-s_0\right]e^{-(r_{\text{int}}-d)t}\right)^2} > 0$$

(A4.23)
$$\frac{\partial^2 s_t}{\partial {s_0}^2} = \frac{-2\left[K(1-d/r_{\text{int}})\right]^2 e^{-(r_{\text{int}}-d)t}\cdot\left(1-e^{-(r_{\text{int}}-d)t}\right)}{\left(s_0+\left[K(1-d/r_{\text{int}})-s_0\right]e^{-(r_{\text{int}}-d)t}\right)^3} < 0$$

(A4.24)
$$\frac{\partial^2 e^{-\rho\tau(s_0;s)}}{\partial {s_0}^2} = \rho e^{-\rho\tau(s_0;s)}\left[\rho\left(\frac{\partial\tau(s_0;s)}{\partial s_0}\right)^2 - \frac{\partial^2\tau(s_0;s)}{\partial {s_0}^2}\right]$$

Plug (A4.13) and (A4.21) into (A4.24), we will have

(A4.25) $$\frac{\partial^2 e^{-\rho\tau(s_0;s)}}{\partial s_0^{\,2}} = \frac{K(1-d/r_{\text{int}})\left[2s_0 - K(1-\dfrac{d+\rho}{r_{\text{int}}})\right]}{(r_{\text{int}}-d)\left(s_0\left[K(1-d/r_{\text{int}})-s_0\right]\right)^2}\begin{cases} >0 & if\ \ s_0 > \dfrac{1}{2}K(1-\dfrac{d+\rho}{r_{\text{int}}}) \\ <0 & if\ \ s_0 < \dfrac{1}{2}K(1-\dfrac{d+\rho}{r_{\text{int}}}) \end{cases}$$

Note, for clarity, $s_{\tau(s_0;s)}$ in (A4.17) and (A4.18) is rewritten as $s(s_0,\tau(s_0;s))$ explicitly to emphasize the two separate arguments s_0 and $\tau(s_0;s)$.

When $s_0 = s$ and $\tau(s_0;s)=0$, (A4.15) and (A4.24) becomes

(A4.26) $$\left.\frac{\partial e^{-\rho\tau(s_0;s)}}{\partial s_0}\right|_{s_0=s} = -\rho\frac{\partial\tau(s_0;s)}{\partial s_0}$$

(A4.27) $$\left.\frac{\partial^2 e^{-\rho\tau(s_0;s)}}{\partial s_0^{\,2}}\right|_{s_0=s} = \rho\left[\rho\left(\frac{\partial\tau(s_0;s)}{\partial s_0}\right)^2 - \frac{\partial^2\tau(s_0;s)}{\partial s_0^{\,2}}\right]$$

Then (A4.17) – (A4.19) can be simplified to

(A4.28) $$\left.\frac{\partial D_1}{\partial s_0}\right|_{s_0=s} = N\alpha\begin{pmatrix} \dfrac{\partial^2\tau(s_0;s)}{\partial s_0^{\,2}}s + \dfrac{\partial\tau(s_0;s)}{\partial s_0}\dfrac{\partial s(s_0,\tau(s_0;s))}{\partial s_0} \\ +\left(\dfrac{\partial\tau(s_0;s)}{\partial s_0}\right)^2\left[\dfrac{\partial s(s_0,\tau(s_0;s))}{\partial t} - \rho s\right] \end{pmatrix}$$

(A4.29) $$\left.\frac{\partial D_2}{\partial s_0}\right|_{s_0=s} = N\alpha\frac{\partial\tau(s_0;s)}{\partial s_0}\frac{\partial s(s_0,\tau(s_0;s))}{\partial s_0}$$

(A4.30) $$\left.\frac{\partial D_3}{\partial s_0}\right|_{s_0=s} = \left(N\alpha s + PP\frac{K\left(1-d/r_{\text{int}}\right)-s}{K-s}\right)\left[\rho\left(\frac{\partial\tau(s_0;s)}{\partial s_0}\right)^2 - \frac{\partial^2\tau(s_0;s)}{\partial s_0^{\,2}}\right]$$

Then,

(A4.31)

$$\left.\frac{\partial D_1}{\partial s_0}+\frac{\partial D_2}{\partial s_0}+\frac{\partial D_3}{\partial s_0}\right|_{s_0=s} = N\alpha\left(2\frac{\partial\tau(s_0;s)}{\partial s_0}\frac{\partial s(s_0,\tau(s_0;s))}{\partial s_0} + \left(\frac{\partial\tau(s_0;s)}{\partial s_0}\right)^2\frac{\partial s(s_0,\tau(s_0;s))}{\partial t}\right) + PP\frac{K\left(1-d/r_{\text{int}}\right)-s}{K-s}\left[\rho\left(\frac{\partial\tau(s_0;s)}{\partial s_0}\right)^2 - \frac{\partial^2\tau(s_0;s)}{\partial s_0^{\,2}}\right]$$

Note that (A4.14) and (A4.22) can be simplified to

(A4.32) $$\left.\frac{\partial s_t}{\partial s_0}\right|_{\substack{s_0=s,\\ t=\tau(s_0;s)}} = \left.\frac{\partial s(s_0,\tau(s_0;s))}{\partial s_0}\right|_{s_0=s} = 1$$

(A4.33) $$\left.\frac{\partial s(s_0,t)}{\partial t}\right|_{\substack{s_0=s,\\ t=\tau(s_0;s)}} = \frac{s\left[K(1-d/r_{\text{int}})-s_0\right](r_{\text{int}}-d)}{K(1-d/r_{\text{int}})}$$

Combining (A4.13) at $s_0 = s$ and (A4.33), we have

(A4.34) $$\left.\frac{\partial s(s_0,\tau(s_0;s))}{\partial t}\frac{\partial \tau(s_0;s)}{\partial s_0}\right|_{s_0=s} = -1$$

Plug (A4.32) and (A4.34) into (A4.31), we will have

(A4.35)

$$\left.\frac{\partial\left(D_1+D_2+D_3\right)}{\partial s_0}\right|_{s_0=s} = N\alpha\frac{\partial \tau(s_0;s)}{\partial s_0} + PP\frac{K\left(1-d/r_{\text{int}}\right)-s}{K-s}\left[\rho\left(\frac{\partial\tau(s_0;s)}{\partial s_0}\right)^2 - \frac{\partial^2\tau(s_0;s)}{\partial {s_0}^2}\right]$$

Plug (A4.13) and (A4.21) into (A4.35) with $s_0 = s$, and note that, $\partial^2(J(s_0;s)-J_0(s_0))/\partial {s_0}^2 =$ $\partial\left(D_1+D_2+D_3\right)/\partial s_0$ with $\partial D_4/\partial s_0 = 0$. Thus, we have

(A4.36)

$$\left.\frac{\partial^2(J(s_0;s)-J_0(s_0))}{\partial {s_0}^2}\right|_{s_0=s} = \frac{K}{r_{\text{int}}s\left[K(1-\frac{d}{r_{\text{int}}})-s\right]}\left(\frac{PP}{r_{\text{int}}s(1-\frac{s}{K})}\left[\rho+d+r_{\text{int}}(\frac{2s}{K}-1)\right]-N\alpha\right)$$

Assuming $r_{\text{int}} > d+\rho$, then the equation

(A4.37) $$s(1-s/K) = \frac{PP}{Nr_{\text{int}}\alpha}[\rho+d+r_{\text{int}}(2s/K-1)]$$

only has one real root greater than 0, denoted by $s^U_{-\,-}$. And it is easy to verify that

$$\left.\frac{\partial^2(J(s_0;s)-J_0(s_0))}{\partial {s_0}^2}\right|_{s_0=s} < 0 \text{ for } s\in(0,s^U_{-\,-}).$$

Note, when $s = K/2$, (A4.37) is equal to (A4.11). With $K/4 > \frac{PP}{Nr_{\text{int}}\alpha}[\rho+d]$, $s^U_{-\,-} > K/2$.

Also, rearrange the characteristic equation for the singular stock level, Eq (4.12) as follows

(A4.38) $$s(1-s/K) = \frac{PP}{Nr_{\text{int}}\alpha}[\rho+d+d\cdot\frac{2s-K}{K-s}]$$

Comparing the RHS of (A4.11), (A4.37) and (A4.38), and noting that $s < K(1-d/r_{\text{int}})$ implies $r_{\text{int}}(2s/K-1) > d\cdot\left(2s-K\right)/\left(K-s\right)$ for $s > K/2$, we will have

(A4.39) $$K/2 < s^U_{-\,-} < s^* < s^U_-$$

Together with (A4.11), we have $s^{**} < s^L_- < s^U_{-\,-} < s^* < s^U_-$ *for* $s \le s^* < K(1-d/r_{\text{int}})$.

Proof of Property 2)

Plug (A4.15) into (A4.12), we have

(A4.40) $$\frac{\partial(J(s_0;s)-J_0(s_0))}{\partial s_0} = \underbrace{\int_0^{\tau(s_0;s)} N\alpha\frac{\partial s_t}{\partial s_0}e^{-\rho t}dt}_{D_2} + \underbrace{C(s)\frac{Ke^{-\rho\tau(s_0;s)}}{r_{\text{int}}s_0\left[K(1-d/r_{\text{int}})-s_0\right]}}_{D_5} - \underbrace{\frac{N\alpha}{d+\rho}}_{D_4}$$

where $C(s) = PP\dfrac{K\left(1-d/r_{\text{int}}\right)-s}{K-s}$>0.

Using (A4.14) and noticing $\lim_{s_0\to 0}\tau(s_0;s)=\infty$, we have

(A4.41)

$$\lim_{s_0\to 0} D_2 = \lim_{s_0\to 0}\int_0^{\tau(s_0;s)} N\alpha \frac{\left[K(1-d/r_{\text{int}})\right]^2 e^{-(r_{\text{int}}-d)t}}{\left(s_0+\left[K(1-d/r_{\text{int}})-s_0\right]e^{-(r_{\text{int}}-d)t}\right)^2} e^{-\rho t}dt = N\alpha\int_0^{\infty} e^{(r_{\text{int}}-d-\rho)t}dt = \infty$$

assuming $r_{\text{int}} > d+\rho$.

Using Eq(4.15) to write out $e^{-\rho\tau(s_0;s)}$,

(A4.42) $$e^{-\rho\tau(s_0;s)} = \left(\frac{s_0/\left[K(1-d/r_{\text{int}})-s_0\right]}{s/\left[K(1-d/r_{\text{int}})-s\right]}\right)^{\frac{\rho}{(r_{\text{int}}-d)}}$$

Then D_5 can be rewritten as

(A4.43) $$D_5 = C(s)\frac{K}{r_{\text{int}}}\left(\frac{s}{\left[K(1-d/r_{\text{int}})-s\right]}\right)^{\frac{-\rho}{(r_{\text{int}}-d)}}\left[K(1-d/r_{\text{int}})-s_0\right]^{\frac{d-\rho-r_{\text{int}}}{(r_{\text{int}}-d)}} s_0^{\frac{\rho+d-r_{\text{int}}}{(r_{\text{int}}-d)}}$$

Thus,

(A4.44) $$\lim_{s_0\to 0} D_5 = C(s)\frac{K}{r_{\text{int}}}\left(\frac{s}{\left[K(1-d/r_{\text{int}})-s\right]}\right)^{\frac{-\rho}{(r_{\text{int}}-d)}}\left[K(1-d/r_{\text{int}})\right]^{\frac{d-\rho-r_{\text{int}}}{(r_{\text{int}}-d)}} \lim_{s_0\to 0} s_0^{\frac{\rho+d-r_{\text{int}}}{(r_{\text{int}}-d)}} = \infty$$

assuming $r_{\text{int}} > d+\rho$.

Given (A4.41) and (A4.44), we will have

(A4.45) $$\lim_{s_0\to 0}\frac{\partial(J(s_0;s)-J_0(s_0))}{\partial s_0} = \infty \text{ for } 0 < s < K(1-d/r_{\text{int}}).$$

Since $0 < s^* < K(1-d/r_{\text{int}})$, we have $\lim_{s_0\to 0}\dfrac{\partial(J(s_0;s^*)-J_0(s_0))}{\partial s_0} > 0$.

A4.4 The characteristic equation for the open-loop singular stock level s_{OL}^*

By the Maximum Principle and Eq (4.22),

$$\begin{cases} H_i^{OL} = \max_{c_{it}} H_i \quad 0 \le c_{it} \le \min\{w, PP - c_{-it}^{OL}\} \\ \dot{\lambda}_t^i = -H_{s_t}^{OL} + \rho\lambda_{it}, \\ \dot{s}_t = H_{\lambda_t}^{OL}, \ \lim_{t\to\infty} e^{-\rho t}\lambda_{it}s_t = 0 \end{cases}$$

Since H_i is linear in c_{it}, we use a switching function to specify the decision rule for c_{it}.

$$\sigma_i(t, s_t, \lambda_{it}) \overset{\text{def}}{=} \lambda_{it}\cdot r_{\text{int}}\cdot s_t(1-\frac{s_t}{K}) - PP$$

$$c_{it} = \begin{cases} 0 & if \;\; \sigma_i < 0 \\ \in [0, PP - c_{-it}^{OL}] & if \;\; \sigma_i = 0 \\ PP - c_{-it}^{OL} & if \;\; \sigma_i > 0 \end{cases}$$

$$\sigma_i = 0 \text{ implies } \lambda_{it} = \frac{PP}{r_{\text{int}} \cdot s_t (1 - s_t / K)}$$

$$\begin{aligned} \dot{\lambda}_{it} &= -H_{s_t}^{OL} + \rho\lambda_{it} \\ &= -\left[\alpha + \lambda_{it} \left(r_{\text{int}} \cdot (1 - \frac{2s_t}{K}) \cdot \frac{c_{it}^{OL} + c_{-it}^{OL}}{PP} - d \right) \right] + \rho\lambda_{it} \\ &= \lambda_{it} \left[\rho + d - r_{\text{int}} \cdot (1 - \frac{2s_t}{K}) \cdot \frac{c_{it}^{OL} + c_{-it}^{OL}}{PP} \right] - \alpha \end{aligned}$$

Assume $\sigma_i \equiv 0$, then $\dfrac{\mathrm{d}\,\sigma_i}{\mathrm{d}\,t} \equiv 0$, and we will have

$$\dot{\lambda}_{it} \cdot r_{\text{int}} \cdot s_t (1 - \frac{s_t}{K}) + \lambda_{it} \cdot r_{\text{int}} \cdot (1 - \frac{2s_t}{K}) \dot{s}_t = 0$$

Plug in λ_{it}, $\dot{\lambda}_{it}$ and $\dot{s}_t$, we will have

$$\begin{aligned} &\left\{ \lambda_{it} \left[\rho + d - r_{\text{int}} \cdot (1 - \frac{2s_t}{K}) \cdot \frac{c_{it}^{OL} + c_{-it}^{OL}}{PP} \right] - \alpha \right\} \cdot r_{\text{int}} \cdot s_t (1 - \frac{s_t}{K}) \\ &+ \lambda_{it} \cdot r_{\text{int}} \cdot (1 - \frac{2s_t}{K}) \left[r_{\text{int}} \cdot s_t (1 - \frac{s_t}{K}) \cdot \frac{c_{it} + c_{-it}^{OL}}{PP} - d \cdot s_t \right] \\ &= r_{\text{int}} \left\{ \begin{aligned} &\lambda_{it} \rho s_t (1 - \frac{s_t}{K}) - \alpha s_t (1 - \frac{s_t}{K}) - \lambda_{it} r_{\text{int}} \cdot (1 - \frac{2s_t}{K}) \cdot \frac{c_{it}^{OL} + c_{-it}^{OL}}{PP} + \lambda_{it} r_{\text{int}} \cdot (1 - \frac{2s_t}{K}) \cdot \frac{c_{it}^{OL} + c_{-it}^{OL}}{PP} \\ &+ \lambda_{it} d s_t (1 - \frac{s_t}{K}) - \lambda_{it} d s_t (1 - \frac{2s_t}{K}) \end{aligned} \right\} \\ &= r_{\text{int}} \left\{ \lambda_{it} \left[\rho s_t (1 - \frac{s_t}{K}) + d s_t \frac{s_t}{K} \right] - \alpha s_t (1 - \frac{s_t}{K}) \right\} \\ &= r_{\text{int}} \left\{ \frac{PP}{r_{\text{int}} \cdot s_t (1 - s_t / K)} \left[\rho s_t (1 - \frac{s_t}{K}) + d s_t \frac{s_t}{K} \right] - \alpha s_t (1 - \frac{s_t}{K}) \right\} \\ &= r_{\text{int}} \left\{ \frac{PP}{r_{\text{int}}} \left[\rho + d \frac{s_t}{K - s_t} \right] - \alpha s_t (1 - \frac{s_t}{K}) \right\} = 0 \end{aligned}$$

Hence, we can solve for the open-loop singular stock level s_{OL}^* by the following equation.

$$s_{OL}^* (1 - s_{OL}^* / K) = \frac{PP}{r_{\text{int}} \alpha} [\rho + d \cdot \frac{s_{OL}^*}{K - s_{OL}^*}]$$

A4.5 Proof of Lemma 4.3

$$\frac{\partial (J(s_0;s)-J_0(s_0))}{\partial s}>0 \text{ for any } s\in(s^{**},s^{*})$$

Proof.

Using (A4.7), we have

$$\frac{\partial (J(s_0;s)-J_0(s_0))}{\partial s}=\frac{\partial \tau(s_0;s)}{\partial s}N\alpha s_{\tau(s_0;s)}e^{-\rho\tau(s_0;s)}$$

(A4.46)

$$+\frac{1}{\rho}\left(N\alpha s+PP\frac{K\left(1-d/r_{\text{int}}\right)-s}{K-s}\right)\frac{\partial e^{-\rho\tau(s_0;s)}}{\partial s}$$

$$+\frac{e^{-\rho\tau(s_0;s)}}{\rho}\left(N\alpha-\frac{PP}{r_{\text{int}}}\frac{d\cdot K^{-1}}{(1-sK^{-1})^2}\right)$$

Note that $s_{\tau(s_0;s)}=s$, $\frac{\partial e^{-\rho\tau(s_0;s)}}{\partial s}=-\rho e^{-\rho\tau(s_0;s)}\frac{\partial \tau(s_0;s)}{\partial s}$, and $\frac{\partial \tau(s_0;s)}{\partial s}=\frac{K/r_{\text{int}}}{s\left[K(1-d/r_{\text{int}})-s\right]}$, then we will have

(A4.47)

$$\frac{\partial (J(s_0;s)-J_0(s_0))}{\partial s}=\frac{e^{-\rho\tau(s_0;s)}}{\rho}\left(N\alpha-\frac{PP}{r_{\text{int}}}\frac{d\cdot K^{-1}}{(1-sK^{-1})^2}-\frac{PP\cdot\rho}{r_{\text{int}}s(1-sK^{-1})}\right)$$

For the RHS of (A4.47) to be greater than 0, we need

(A4.48)

$$N\alpha-\frac{PP}{r_{\text{int}}}\frac{d\cdot K^{-1}}{(1-sK^{-1})^2}-\frac{PP\cdot\rho}{r_{\text{int}}s(1-sK^{-1})}>0$$

which can be rearranged to $s(1-s/K)>\frac{PP}{Nr_{\text{int}}\alpha}[\rho+d\cdot\frac{s}{K-s}]$

Noting Eq (4.12), the desired result is obtained.

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