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Macroeconomic Dynamics of Market Risk Capital Requirements and Credit Supply Interdependence

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Abstract

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The 2008 global financial crisis revealed serious weaknesses in the worldwide banking system and financial regulatory regime. Concerns arose about the possible procyclical effects of risk-sensitive capital requirements that rely on Value-at-Risk (VaR) for managing market risks. The first chapter of this dissertation begins with a brief history of the Basel Committee on Banking Supervision, its contribution in designing capital requirements, and some basic examples to illustrate how VaR contributes to the procyclicality of bank leverage and its amplification effects on financial markets. It then reviews the importance of financial factors, such as leverage and risk, on the business cycle and how the macroeconomic literature has attempted to account for these factors. The second and third chapters analyze how market risk-sensitive capital requirements can affect credit supply and amplify business cycles. These requirements effectively risk-constrain leveraged financial institutions and induce feedback effects on the macroeconomy when banks adjust their balance sheets to comply with these capital requirements.

The second chapter analyzes the procyclical effects of Basel II's VaR-based capital requirements within a fully dynamic general equilibrium macroeconomic model with a financial sector. The model is calibrated to U.S. data and estimated with Bayesian techniques to pin down the dynamics and is able to capture four important business cycle correlations between

financial factors and macroeconomic activity. The results suggest that when leveraged financial institutions are constrained by VaR-based capital requirements, increased financial market volatility forces banks to adjust their balance sheets to comply with higher capital charges by selling assets at reduced prices. This action depletes bank capital and deteriorates risk-weighted balance sheet positions, raising their perceived probability of default and interbank borrowing costs. Ultimately, these effects restrict the financial sector's ability to supply credit to the productive sectors to finance investment. Additionally, if financial markets become illiquid and banks are unable to sell assets and violate their risk constraints, the effects become amplified. These results provide some rationale for the Federal Reserve taking on the buyer of last resort role in the asset-backed securities market during the 2008 financial crisis. Thus, credit supply and market risk capital requirements are shown to be interdependent through risk constraints.

The third chapter analyzes how Basel III's proposed switch from Value-at-Risk to stressed Conditional Value-at-Risk (CVaR) to measure market risks might affect the procyclicality of bank risk constraints on the macroeconomy. CVaR may reduce the spillover effects of market risk-sensitive capital requirements on credit supply and aggregate investment compared to the current VaR regime if regulation abandons the efficient markets hypothesis in favor of the fractal markets hypothesis and calibrates risks to stressed market conditions. Stressed CVaR can reduce banks' balance sheet response to increases in perceived volatility, which should reduce the risk-constrained feedback effects as banks are forced to comply with increased capital charges. However, because asset returns are generally non-normally distributed, if CVaR is locally calibrated to current market conditions it may amplify these effects in response to changes in perceived tail risks. These results provide some supporting evidence for Basel III's proposed stressed CVaR market risk regime.

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DEDICATION

To my parents, sister, and family,
To my grandfather, James Zupke, who was unable to see this finished work but who I know
would be proud,
And especially to my Alex.

Chapter 1

A BRIEF HISTORY OF CAPITAL REQUIREMENTS AND FINANCIAL SECTOR MODELING IN MACROECONOMICS: VALUE-AT-RISK AND LEVERAGE

Abstract

This chapter briefly reviews the history of the Basel Committee on Banking Supervision and its contribution in designing capital requirements. The empirical literature documenting the importance of financial factors in the modern business cycle is reviewed as well as the theoretical literature that aims to capture some of the reported stylized facts. The review focuses on models describing Value-at-Risk, bank leverage, and includes some basic examples to illustrate the procyclicality and amplification effects of these two factors before concluding with suggestions for future research.

1.1 A Brief History of Capital Requirements

1.1.1 Origins of the Basel Committee on Banking Supervision

Banks can finance their operations in two ways: with borrowed money, such as deposits or debt, or with funds provided by the owners, called equity capital. Together, these two sources of funds are used to finance banks' activities such that assets equal liabilities plus equity ($A=L+E$). Commercial banks have often been subject to reserve requirements, which dictate banks to hold a specified fraction of deposits as cash so that they may have enough money on hand to meet customer withdrawal demands on a typical day. These are different from capital requirements, which dictate banks to finance a specific fraction of assets with capital. The greater the fraction of assets financed with capital, the more likely a bank will be able to withstand adverse economic conditions and stay in business. The goal of such requirements is to promote safe and sound banking practices and promote system-wide financial stability. However, capital requirements also come with a cost. They restrict banks' ability to borrow and provide credit to the productive sectors of the economy.¹

Before the 1980s, banks in the United States were not required to use specific numerical capital adequacy rules.² Bank capital managers relied on internal assessments and subjective measures about the soundness of their own firm, mainly emphasizing the quality of the loans held on the balance sheet. In the 1930s and 1940s, capital ratios were considered for regulation, but ultimately dismissed as ineffective measures of actual capital adequacy.³ More efforts were made in the 1950s to use capital-to-risk ratios to measure capital adequacy but these did not gain widespread acceptance as it was thought that strict adherence to numerical ratios would detract from more comprehensive capital analysis. Between World War II and the 1970s, there was very little reason to doubt the effectiveness of such an approach as the financial system was considered very stable and experienced very few bank failures. It was

¹Burhouse et al. (2003).

²Norton (1995).

³Ryon (1969).

not until the 1970s that the regulatory environment began to change.⁴

The origin of the Basel Committee on Banking supervision can be traced back to the disruption in the financial system following the collapse of the Bretton Woods system in 1973.⁵ The Bretton Woods system, named for the meeting place of the conference in Bretton Woods, New Hampshire in July 1944, was a set of rules designed to regulate the international monetary system, exchange rates, and monetary policy among the United States, Canada, Western Europe, and Japan. The objective was to address a lack of cooperation between countries and to prevent competitive devaluation of currencies, which had occurred in the 1930s and 1940s as many countries left the gold standard during the Great Depression. The conference established the International Monetary Fund (IMF) and the International Bank for Reconstruction and Development (IBRD), which is now part of the World Bank. The major feature of the Bretton Woods system was to peg the value of the participating countries' currencies to a reserve currency, and which the value of the reserve currency was tied to gold. The U.S. dollar was selected as the reserve currency for the strength, size, and perceived stability of the U.S. economy. Additionally, most international transactions at the time were settled using the U.S. dollar, and the U.S. controlled almost two-thirds of the world's gold supply at the time. In the early 1960s, the U.S. dollar's fixed value against gold had become overvalued due to a large increase in government spending on social and military programs. In August 1971, President Nixon temporarily suspended the dollar's convertibility into gold, which marked the beginning of the end for the Bretton Woods system.⁶

Following the Bretton Woods collapse, many banks suffered large losses from exposures to foreign exchange as exchange rates became much more volatile than they had been in prior years. In June 1974, it was discovered that Bankhaus Herstatt, a West German Bank, had foreign exchange exposures that were three times its capital. After the discovery, West Germany's regulatory authority withdrew Herstatt's banking license. Consequently, numerous

⁴Burhouse et al. (2003).

⁵Basel Committee on Banking Supervision (2015).

⁶Asher and Mason (1973).

banks outside West Germany faced large losses from exposures to Herstatt and its inability to repay its debts. In response to this international banking panic, the G10 countries established the Committee on Banking Regulations and Supervisory Practices in 1974, which later became known as the Basel Committee on Banking Supervision. The objective of the Basel Committee was to improve financial stability through cooperation of its member countries and the quality of worldwide banking supervision. To achieve its goals, the Basel Committee has resorted to setting minimum requirements, such as minimum capital ratios, for the regulation of financial institutions and standardizing them across countries. Since the first meeting in February 1975, the Basel Committee has grown to over twenty-eight participating countries, where each country is represented by their central bank or other regulatory authority. The Basel Committee and its decisions have no legal force. Thus, the Basel Committee acts more like a supervisory body, and it is up its members to implement and enforce suggested policies in their respective countries.⁷

In 1975 the Basel Committee issued one of its first papers known as the “Concordat.” This document set out to close gaps in the international supervision of financial institutions so that “(i) no foreign banking establishment would escape supervision; and (ii) supervision would be adequate and consistent across member jurisdictions.”⁸ This document established principles for sharing supervisory authority over foreign bank branches between the home and foreign country. Unfortunately, these regulatory standards were not airtight. Over time, certain financial institutions and activities have been able to escape regulatory oversight as financial innovation outpaced regulation. These institutions and activities have become known as the shadow banking system. Shadow banks are financial institutions that act like banks and look like banks but are not regulated like banks. Examples of the shadow banking system that have become synonymous with the 2008 global financial crisis include asset-backed commercial paper conduits, money market funds, repurchase agreements, mortgage

⁷Basel Committee on Banking Supervision (2015).

⁸Basel Committee on Banking Supervision (2015) and Committee on Banking Regulations and Supervisory Practices (1975).

companies, and structured investment vehicles as well as the investment banks that helped facilitate the markets for these activities.⁹ By September 2012, the Basel Committee had formed twenty-nine basic principles on which they believed were important for an effective financial regulatory system.¹⁰

1.1.2 From Basel I to Basel III

Beginning with the Basel Accord

In the 1970s the U.S. experienced a period of stagflation: high inflation and economic stagnation. Concurrently, Latin America experienced a debt crisis where debt owed to commercial banks had increased at a cumulative annual rate of 20.4%. Latin American countries quadrupled their external debt from 1975 to 1983.¹¹ These factors eventually caught the attention of the Basel Committee with concerns that the capital ratios of major international banks had deteriorated too severely.¹² Figure 1.1 shows that the capital-asset ratio for U.S. chartered depository institutions (financial intermediaries that operate like commercial banks) fell from 8% (which would eventually become the minimum) in 1960 to below 6% by 1983.¹³

By the mid-1980s, The Basel Committee recognized a need to converge international regulations governing capital adequacy and thereby strengthen the international banking system. Its members began working towards a better measurement of capital adequacy ratios based on a risk-weighted approach to measure balance sheet risk as the quality of bank assets were considered an important determinant of bank health.¹⁴ Regulators wanted a better definition of capital adequacy in order to address two concerning trends in the

⁹Bernanke (2013).

¹⁰Basel Committee on Banking Supervision (2012b).

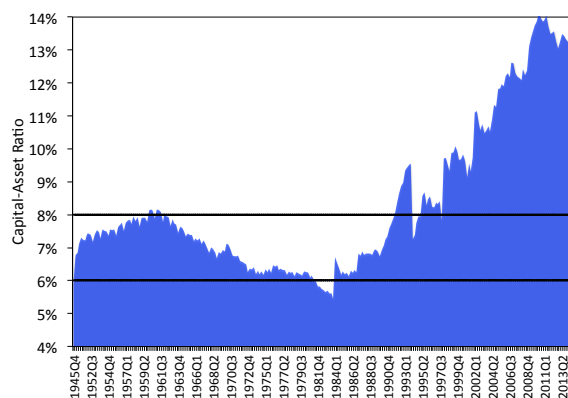
¹¹Institute of Latin American Studies (1986).

¹²Basel Committee on Banking Supervision (2015).

¹³Burhouse et al. (2003).

¹⁴Basel Committee on Banking Supervision (2015).

Figure 1.1: Capital-Asset Ratio of U.S. Chartered Depository Institutions



Note: Data is from 1945-2014 found at the Board of Governors of the Federal Reserve System Financial Accounts of the United States.

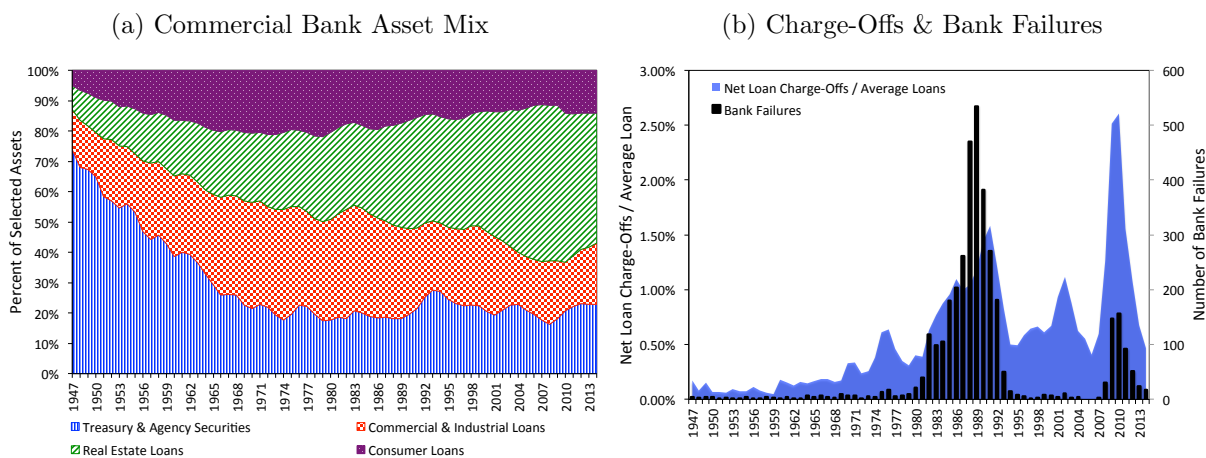
banking industry at the time. Banks had been investing in riskier, high yield, and less liquid assets while also increasing off balance sheet risks.¹⁵ Figure 1.2a illustrates the growing importance of credit risk (the risk that a borrower might default on a loan), where the mix of commercial banks' assets shifted away from safe Treasury securities towards commercial and industrial, real estate, and consumer loans. Figure 1.2b shows that the build-up of credit risk, coupled with rising interest rates as the Federal Reserve attempted to curb inflation, cascaded into a severe episode of loan losses and bank failures beginning in the early 1980s. A total of 1,043 of 3,234 savings and loans institutions failed during what is now known as the savings and loans crisis.¹⁶ It was not until 1981 when the Federal Reserve Board and the Office of the Comptroller of the Currency in the U.S. announced a minimum capital ratio of 6% for community banks and 5% for larger regional banks.¹⁷ But by then, it was already too late for the savings and loans institutions to shore up their balance sheets as the failures

¹⁵Burhouse et al. (2003).

¹⁶Curry and Shibut (2000).

¹⁷Burhouse et al. (2003).

Figure 1.2: Shifting Commercial Bank Risks



Note: Data on commercial banks are from 1947-2014 found at the Board of Governors of the Federal Reserve System and FDIC databases. The asset mix is relative to the four assets listed.

had already begun. However, as figure 1.1 shows, U.S. chartered depository institutions have steadily increased their capital-asset ratios since.

The resulting document that was released in July 1988 became known as the “Basel Capital Accord,” or Basel I, and had two fundamental objectives: “to strengthen the soundness and stability of the international banking system; and... be fair and have a high degree of consistency in its application to banks in different countries with a view to diminishing an existing source of competitive inequality among international banks.”¹⁸ The Basel I framework was introduced not only to banks in member countries, but also in almost all other countries with international banks. The framework adopted a minimum capital-to-risk-weighted assets (a weighted measure of total assets where the weight depends on the perceived riskiness of the asset) ratio of 8% with a very coarse weighting system that existed of only five weighting buckets: 0, 10, 20, 50, and 100 percent.¹⁹ However, the accord was intended to be flexible

¹⁸Basel Committee on Banking Supervision (1988).

¹⁹Basel Committee on Banking Supervision (1988).

and underwent three major changes in 1991, 1995, and 1996. The 1991 amendment allowed loan-loss reserves to be included in the capital ratio, while the 1995 amendment attempted to account for credit exposures from derivative contracts (a type of contract between two or more parties that is based on the value of an underlying asset such as stocks, bonds, or commodities).²⁰

The original Basel Accord was mainly focused on assessing capital adequacy in regards to credit risk, leaving market risks (risks associated with changes in interest rates, exchange rates, asset prices, traded debt, commodities, and options). However, the third amendment that was made in 1996 set out to address market risk considerations for capital adequacy. This amendment was called the “Market Risk Amendment.”²¹ In the years following the 1987 market crash, there had been a strong effort within the financial industry to quantify market risks. This effort began as a research project within J.P. Morgan led by Chairman Dennis Weatherstone and research chief Till Guldemann. They created a statistic, which they called Value-at-Risk (VaR), that was designed to capture the largest loss on a portfolio of trading positions with a given level of confidence and holding period. Weatherstone began using the concept to aggregate risks across all trading desks in his department and had this number included in his “4:15 report,” a report that provided him with an estimate of potential losses just fifteen minutes after the market closed. This statistic gave Weatherstone information he had not previously known and used to it make judgments about how to adjust his firm’s future trading positions. This research group began providing consultative services, teaching the VaR method throughout the financial industry in the late 1980s and 1990s and was later spun off as RiskMetricsTM. Its teachings became so pervasive in the financial industry in the 1990s that the Basel Committee decided to accept VaR as the standard and regulatory market risk measure.²² This decision would go on to fuel large amounts of research and much controversy about VaR as a credible risk measure.

²⁰Basel Committee on Banking Supervision (2015).

²¹Basel Committee on Banking Supervision (2015).

²²Taleb (2007).

An important aspect of this amendment was that it allowed banks to use their own internal models, with the approval of their regulators, to calculate the VaR which they would have to hold capital against. If banks decided to use their own models, they were required to use a minimum of the past year of historical data. Thus, banks were, for the first time, allowed to determine their own capital charges with their own models.²³ These developments would provide opportunities for banks to arbitrage regulatory risk requirements by allowing them to design their models to fit their needs and to choose a window of data history that could minimize capital charges if they could also satisfy the strict quantitative standards set forth under the new amendment.

Basel II Revisions

In June 1999, the Basel Committee began to work on a replacement to the original capital adequacy framework, which was released in June 2004 as the “Revised Capital Framework”, or Basel II. A key component to the revisions was that they were based on the concepts of three pillars:

- minimum capital requirements,
- supervisory review process, and
- market discipline

The first pillar included three major components in the minimum capital requirements: credit risk, market risk, and operational risk (risks associated with the internal processes of running the business such as legal risks). Under Basel II, VaR remained the preferred approach to quantify market risks, but the original five weighting buckets that were used in Basel I’s credit risk model gave way to three methods of varying degrees of sophistication: the standardized approach, the foundation internal ratings-based (F-IRB) approach, and the advanced internal ratings-based (A-IRB) approach. The standardized approach changed the

²³Basel Committee on Banking Supervision (1996).

weighting buckets and based risk weights on the credit ratings of the borrower.²⁴ It also allowed credit ratings to be determined by credit ratings agencies such as the three major firms Fitch, Moody's, and Standard & Poor's. This would become a problem in the run-up to the 2008 financial crisis, as investment banks were allowed to shop the ratings agencies to find the best possible rating for credit related assets like mortgage-backed securities (MBS) they were trying to sell. By the time the crisis hit, ratings agencies were giving AAA ratings on MBS's that should have been BBB rated or worse.²⁵ The use of credit ratings agencies in credit risk calculations is now prohibited in the U.S. under the Dodd-Frank Wall Street Reform and Consumer Protection Act of 2010.

If banks chose to use the F-IRB approach, they would be required to use their regulator's model to calculate inputs into the credit risk capital formula including the probability of default (PD) for loans held on the balance sheet, exposure at default (ED) which estimates the degree to which a bank is exposed to a borrower in the event of default, loss given default (LGD) which measures the share of a loan lost when a borrower defaults, and the effective maturity of the loan (M). However, if banks chose the A-IRB approach, this allowed them to develop and use their own models to quantify credit risk by estimating the parameters above with approval from their regulator, similar to what the Market Risk Amendment allowed them to do with VaR.²⁶

The second pillar was supposed to give regulators better tools to oversee the capital adequacy of financial institutions and set forth a procedure for regulatory review and internal assessment of banks' capital adequacy. This procedure became known as the Internal Capital Adequacy Assessment Process (ICAAP). The third pillar aimed to complement the first two pillars by instituting disclosure and transparency requirements to strengthen market discipline and encourage sound banking practices. The idea was to provide investors and other market participants with enough information about banks' activities to reward those

²⁴Basel Committee on Banking Supervision (2004).

²⁵Cohen (2011).

²⁶Basel Committee on Banking Supervision (2004).

institutions with sound balance sheets and penalize those that took excessive risks.²⁷

The 2004 release focused mainly on the banking book (the part of the balance sheet where loans and other securities are expected to be held to maturity), as opposed to the trading book (the part of the balance sheet where assets are expected to be held for short-term trading and profit opportunities). In July 2005, the Basel Committee published a document focusing on the treatment of the trading book. Member countries and some other non-member countries that agreed to adopt the new set of capital standards did so on varying time tables.²⁸ Basel II was to be implemented in 2008 in most major economies, but the financial crisis that hit that year prevented the new framework from being fully implemented or effective.²⁹

On to Basel 2.5 and the Basel III Capital Framework

Even before the crisis hit in 2008 and interrupted the implementation of Basel II, it was clear that Basel II had some fundamental issues. Financial institutions, specifically security broker dealers (financial intermediaries such as investment banks that operate primarily in capital markets and facilitate the flow of funds through the financial sector), had built up an inordinate amount of leverage (defined as total assets over equity) on the back of short-term liabilities. This meant that broker dealers had relied heavily on short-term borrowing to finance asset growth, reducing the liquidity of their balance sheets, and leaving them susceptible to failure if adverse price movements negatively impacted asset values and their ability to repay debt. Figure 1.3a shows that broker dealers have operated with higher leverage than chartered depository banks since at least since 1990. Whereas chartered depository banks have not operated with a leverage ratio above 14.4 since the first quarter of 1990, broker dealers amassed a leverage ratio of 47.9 in the first quarter of 2008 compared to 28.3 in the fourth quarter of 2000. Much of the leverage growth for broker dealers was the re-

²⁷Basel Committee on Banking Supervision (2004, 2015).

²⁸Basel Committee on Banking Supervision (2015).

²⁹Office of the Comptroller of the Currency (2007).

sult of highly active balance sheet management as broker dealers financed asset growth with short-term, often overnight, repurchase agreements (or repos, which are a type of financial contract where the borrower exchanges a security with a lender for cash and promises to buy the security back in the future at an agreed upon date and price).³⁰ Figure 1.3b illustrates this financing strategy for broker dealers where percent changes in total liabilities and leverage have a correlation coefficient of 0.57 but percent changes in repo liabilities and leverage have an even stronger correlation coefficient of 0.87 between 1990:Q1 and 2014:Q4. It is this type of balance sheet management that results in the procyclicality (increases and decreases with the business cycle) of broker dealer leverage observed in figure 1.3a. If broker dealers were not so active in their balance sheet management, these correlations would be much closer to zero or even negative since positive changes in asset prices, without any changes in debt, cause leverage to fall.

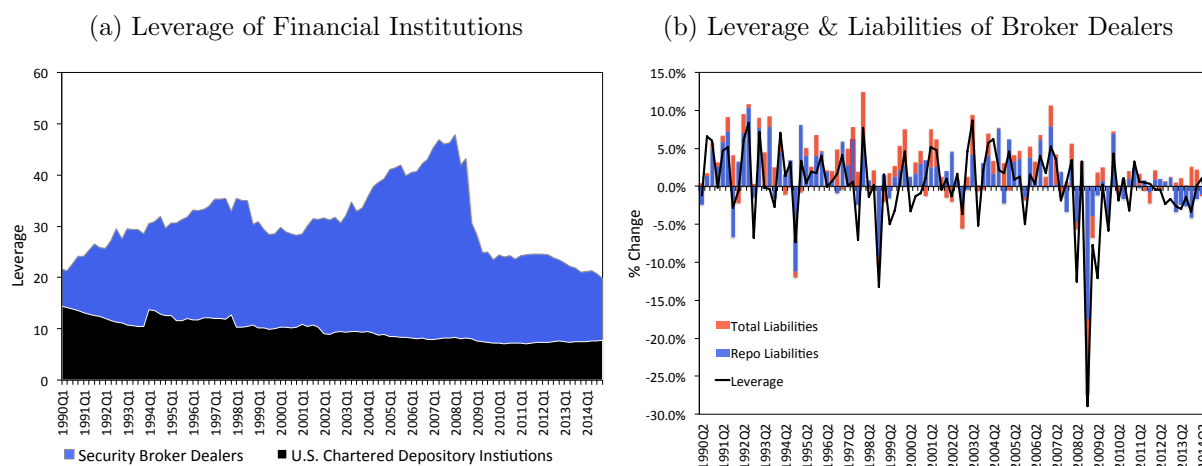
The procyclicality of leverage became a cause for concern in contributing to the build-up of risks within the financial system prior to the crisis and to the severity of the crash.³¹ Small capital charges that tend to occur in prolonged expansion periods when risk appears low leave banks with excess capital, allowing them to expand their balance sheets with risky asset purchases and to leverage up. However, large capital charges that tend to occur in recessions when risk appears high, leave banks with a shortage of capital, forcing them to contract their balance sheets by selling assets, reducing debt, and to deleverage. Poor risk management and perverse incentive structures installed in the search for profit are also likely to be partially responsible for the leverage behavior of financial institutions. Investment banks became infamous for the payout of large bonuses to their traders who made large short-term profits. This scheme incentivized traders to make big bets with the possibility of huge payoffs in the short-term at the expense of long-term risks with no real punishment if the bets failed in the future.³² Consequently, if and when those risky bets failed, banks

³⁰Adrian and Shin (2010).

³¹Basel Committee on Banking Supervision (2013).

³²Taleb (2007).

Figure 1.3: Leverage and Liabilities



Note: Data on financial institutions are from 1990-2014 found at the Board of Governors of the Federal Reserve System.

would be unable to maintain the current state of their balance sheets, because they would be so highly leveraged and dependent on short-term financing. The higher the leverage, the less the bank is able to withstand sudden withdrawals of funding. This is what happened to Northern Rock, a U.K. bank that failed in September 2007, and Lehman Brothers, a high profile investment bank that failed in September 2008, after mounting losses in the financial sector caused interbank lending to dry up as many financial institutions attempted to shore up their own balance sheets leaving Northern Rock and Lehman Brothers unable to fund their activities.³³ The failure of these two banks, even though a year apart, became beacons for the start of the global financial crisis.

The Basel Committee responded to the growing liquidity and leverage concerns in the same month that Lehman Brothers failed with a document entitled “Principles for Sound Liquidity Risk Management” and again in July 2009 when they attempted to address some issues that have often been cited as key elements of the turmoil including the treatment of

³³Shin (2010).

market risks associated with complex securitized positions, off balance sheet vehicles, and other trading book exposures.³⁴ With respect to securitized positions, the revisions required banks to use a standardized measurement method for the specific risks associated with each securitized product on the balance sheet. These additions also introduced a stressed VaR capital charge on top of the original VaR charge, because VaR was especially poor at determining appropriate levels of capital needed to withstand losses during financial crises. The addition of the stressed VaR charge, which significantly increased capital requirements for the trading book, was meant reduce this concern by requiring its calculation “to reflect historical data from a continuous 12-month period that reflects a period of significant financial stress appropriate to the bank’s current portfolio...and must be larger than VaR.”³⁵ Together, the 2008 and 2009 market risk revisions became known as Basel 2.5.

In September 2010, the Basel Committee announced that it would be increasing the minimum capital standards and introducing a new 2.5% capital conservation buffer as well as new regulatory ratios. The goal was to promote a more resilient banking sector, improve its ability to absorb adverse shocks, and to prevent the financial sector from disrupting economic activity.³⁶ These ratios included the liquidity coverage ratio (LCR), the net stable funding ratio (NSFR), a countercyclical capital buffer, and a supplemental leverage ratio (SLR) to complement the existing risk-weighted capital ratio. The LCR established a minimum amount of cash and near-cash assets that must be held on the balance sheet to cover at least 30-days of net cash outflows, while the NSFR is intended to address maturity mismatches and the liquidity of bank balance sheets.³⁷ The countercyclical capital buffer was put in place to reduce the procyclicality of bank balance sheet adjustments with the aim of curtailing credit booms during economic expansions and speed up the capital rebuilding process during a recession. The new leverage ratio was designed to prevent the massive leverage boom

³⁴Basel Committee on Banking Supervision (2015).

³⁵Federal Register Vol. 77 No. 169 (2012).

³⁶Basel Committee on Banking Supervision (2009).

³⁷Basel Committee on Banking Supervision (2012b).

experienced during prolonged expansion periods and the subsequent deleveraging process at the onset of recessions that have been features of many financial crises.³⁸ Additionally, in October 2013 the Basel Committee suggested switching from Value-at-Risk to stressed Conditional Value-at-Risk (CVaR, or sometimes referred to as Expected Shortfall (ES) or Tail VaR (TVaR)), to measure market risks.³⁹ By this time, CVaR had been known to possess better properties than VaR and so had the potential to reduce the use of risky trading strategies by financial institutions. Finally, special considerations were allowed for systemically important financial institutions (SIFIs), financial institutions that are deemed too important to the financial sector and economy as a whole, to help prevent these “too big to fail” banks from having to be rescued as was done in the U.S. under the Emergency Economic Stabilization Act of 2008.⁴⁰

These new ratios combined with the new market risk measure have come to compose what is now Basel III. The framework was formally introduced in 2013 and is scheduled to be being fully implemented, after a gradual phase in of the new requirements, by the beginning of 2019.⁴¹

1.2 Leverage, Value-at-Risk, and Asset Price Dynamics

Financial risk, rather than being exogenous (arising from some outside source), is endogenous: it is generated by the actions of the agents within the system. Financial institutions are active players since they make adjustments to their balance sheets in response to the changing economic environment. When balance sheets are continuously marked-to-market, any changes in asset prices must be immediately reflected on the balance sheet. Asset prices not only indicate the strength of financial institutions, they also drive the institutions to take action. Basic economic theory suggests that when financial markets are highly competitive,

³⁸Jordà et al. (2013).

³⁹Basel Committee on Banking Supervision (2013).

⁴⁰Basel Committee on Banking Supervision (2012b).

⁴¹Basel Committee on Banking Supervision (2015).

then prices will serve to create an efficient allocation of wealth. However, when binding constraints are introduced into financial markets, prices may become distorted and wealth may become inefficiently allocated, which can have serious consequences for a financial system.⁴²

Take for example, an asset price bubble. By definition, an asset that is experiencing a price bubble is valued above what its economic fundamentals suggest due to what some would call “irrational exuberance.” However, there will usually be uncertainty surrounding the true value of the asset and disagreement about the existence of a bubble as not everyone will be able to “see the forest for the trees.” When financial institutions own inflated assets, their marked-to-market wealth increases, and they will tend to take more risks in search of higher profits. Increased demand can then lead to even higher prices inflating the bubble further. If the bubble pops, marked-to-market wealth will crash as the risks those financial institutions took materialize into large losses and bank failures. Thus, the problem arises from relying on market prices to make decisions that then distort those same prices.⁴³ Had the bubble not existed in the first place, the misallocation of wealth and its devastating consequences could have been avoided altogether. As will be discussed in the following sections, regulations can play an important part in introducing binding constraints and price distortions into the financial system.

1.2.1 Procyclical Leverage and Asset Price Dynamics

In section 1.1.2 and figure 1.3a, it was shown that chartered depository banks have maintained a fairly stable leverage ratio while broker dealers have exhibited procyclical leverage. Adrian and Shin (2010) argue that the reason for this falls on the degree to which these financial institutions manage their balance sheets in response to changes in asset prices. Accounting standards often dictate that bank balance sheets must be continuously marked to market. As a result, changes in asset prices will have an immediate impact on the equity

⁴²Shin (2010).

⁴³Shin (2010).

position of financial institutions. And the more leveraged financial institutions are, the more sensitive equity becomes to marked-to-market balance sheet changes. If these banks were simply passive balance sheet managers, then changes in asset prices would result in a decline in leverage. However, Adrian and Shin (2010) observe the opposite, implying that these financial institutions actively manage their balance sheets.

To see why, take Adrian and Shin's (2010) balance sheet example where a financial intermediary holds \$100 worth of securities financed with \$90 in debt and \$10 in equity. This bank's leverage is then $L = 100/10 = 10$.

Assets	Liabilities	Equity
Securities: 100	Debt: 90	10

Let assets be A , equity be E , and debt be D so that $A = D + E$, then:

$$L = \frac{A}{E} = \frac{A}{A - D}$$

To see how leverage changes with asset and debt sizes, take the partial derivative of L with respect to A and then again with respect to D :

$$\begin{aligned} \frac{\partial L}{\partial A} &= -\frac{D}{E^2} < 0 \\ \frac{\partial L}{\partial D} &= \frac{D + E}{E^2} > 0 \end{aligned}$$

Increases in total assets without any changes in debt will cause equity to increase and leverage to fall, but an increase in debt without any changes in assets will cause equity to fall and leverage to increase. Thus, an increase in total assets that is funded completely with debt will cause leverage to increase since $|\partial L/\partial D| > |\partial L/\partial A|$. If bank leverage remains constant or increases with total assets, then this is a signal that balance sheets are actively managed. Let us see what happens to this bank's leverage when asset prices rise by 1% to 101.

As the balance sheet is marked-to-market, the increase in total assets without any change in debt leads to an immediate increase in equity to 11. Thus, leverage of the passive balance

Assets	Liabilities	Equity
Securities: 101	Debt: 90	11

sheet manager falls to $L = 101/11 = 9.18$, because equity rose at a faster 10% rate compared the 1% growth rate of assets. If this bank wanted to maintain a constant leverage ratio of 10, it would have to take on an additional \$10 of debt to finance another \$10 worth of assets so that leverage is able to rise back to $L = 111/11 = 10$. So, if the bank targets a fixed leverage ratio, that requires it to increase the size of its balance sheet by purchasing more assets when asset prices rise. This is what Shin (2010) refers to as an upward-sloping demand response. This mechanism becomes even stronger when banks allow leverage to be procyclical: that is allow leverage to increase when asset prices increases. In this case, Tte bank would purchase, say \$20 in assets instead of \$10, financed with \$20 in debt and leverage would rise above 10 to $L = 121/11 = 11$. Thus, for the active balance sheet manager, as asset prices increase, leverage falls, and the bank holds excess capital. To use up the surplus capital, the bank expands its balance sheets by purchasing more assets financed with debt. If financial markets are not perfectly liquid so that increased demand puts upward pressure on the price, then this mechanism creates extra feedback where initial increases in asset prices cause banks to purchase more of the asset and leverage up, which increases the asset price even more, and so on. It also works in reverse as a process Bernanke (2013) refers to as a fire sale. Initial decreases in asset prices cause deleveraging and asset prices to spiral downward, negatively impacting the balance sheets of other financial intermediaries and destabilizing the financial system.⁴⁴

⁴⁴Adrian and Shin (2010).

1.2.2 Risk Measures

After the stock market crashed on Black Monday 1987, Dennis Weatherstone and Till Guldemann of J.P. Morgan began their search for a method to quantify a realistic worst case loss that could happen over a specified time period. The keyword here is “realistic” as the absolute worst case loss is clearly to lose everything. However, losing everything does not happen very often, and operating a financial institution on the belief that everything could be lost tomorrow would prevent the company from making any investments. Their answer to this problem was Value-at-Risk (VaR).

VaR is a statistical concept designed to provide a quantifiable measure of how much a financial institution can expect to lose from changes in market prices with a certain degree of confidence over a given time period. Mathematically, VaR for a given confidence level c is defined as:

$$VaR_c = \{v : Pr(x \leq v) = 1 - c\}$$

for $c \in (0, 1)$. If the distribution of price changes is continuous and denoted $f(x)$, then VaR_c is the quantile that solves:

$$1 - c = \int_{-\infty}^{VaR_c} f(x) dx$$

When $f(x)$ is assumed to be normal, VaR_c can be written as:

$$VaR_c = \mu - Z_c \sigma$$

where μ is the expected return, σ is the standard deviation, and Z_c is a constant that depends on the confidence level c . Deciding what is a realistic loss is quite arbitrary. The notion of “realistic” here is the statistical concept of a confidence level, and it is typically chosen to be 99%. $VaR_{99\%}$ says that there is only a 1% chance that losses will be larger than $\mu - Z_{99\%} \sigma$ over the specified time period. Take for example, a portfolio worth \$1 million today. If the expected return on this portfolio is 2% over ten days with a standard deviation of 10%, then

at a 99% confidence level, the portfolio could expect to lose almost \$213,000 leaving the value of the portfolio at \$747,000 in a “realistic” worst case scenario. There is only a 1% chance that the portfolio suffers a loss larger than \$213,000 such that the portfolio is worth less than \$747,000. Figure 1.4a illustrates this concept for a standard normal distribution. The vertical red line indicates the potential losses represented by VaR where the shaded area to the left indicates the probability of a loss larger than VaR .

A question one might have about Value-at-Risk is whether or not it is the correct risk measure. VaR happens to have a number of undesirable properties that make it less than an ideal risk measure. In fact, it is not a coherent risk measure; because, under some scenarios, VaR can be poorly behaved. It may provide multiple solutions making it hard for a financial institution to choose the optimal mix of assets in a portfolio. There are also cases when the sum of the VaR’s of two portfolios considered separately can be lower than the VaR of the combined portfolio.⁴⁵ This violates the principle that a well-diversified portfolio should carry lower risk. Since VaR is a quantile, it also ignores everything in the left tail of the distribution (the shaded area in figure 1.4a) which makes it possible for a financial institution to manipulate its VaR by “stuffing” risk in the tail.⁴⁶ So, if a financial institution wants an idea about how much it can expect to lose under normal market conditions, then VaR might be a reasonable measure to use. Otherwise, VaR can be misleading.

One alternative to VaR, is Conditional Value-at-Risk (CVaR). CVaR measures how bad losses can get when losses are larger than expected with a given level of confidence: i.e. how bad losses can get if they exceed the VaR level. Thus, if a financial institution wants an idea about the potential losses in the case that something unusually bad happens, then CVaR is a more appropriate risk measure. CVaR is better behaved in terms of finding the optimal mix of assets and is also a coherent risk measure.⁴⁷ For continuous distributions and a given

⁴⁵Rockafellar and Uryasev (2000).

⁴⁶Danielsson (2002).

⁴⁷Rockafellar and Uryasev (2000).

confidence level c , CVaR is defined mathematically as:

$$CVaR_c = E[x|x \leq VaR_c]$$

If the returns distribution is denoted $f(x)$, then $CVaR_c$ is the solution to:

$$CVaR_c = \frac{1}{1-c} \int_{-\infty}^{VaR_c} xf(x)dx$$

If the distribution has discontinuities, then Rockafellar and Uryasev (2002) show that CVaR is a weighted average of VaR and the expected loss given that the loss strictly exceeds VaR:

$$CVaR_c = \lambda_c VaR_c + (1 - \lambda_c)E[x|x < VaR_c]$$

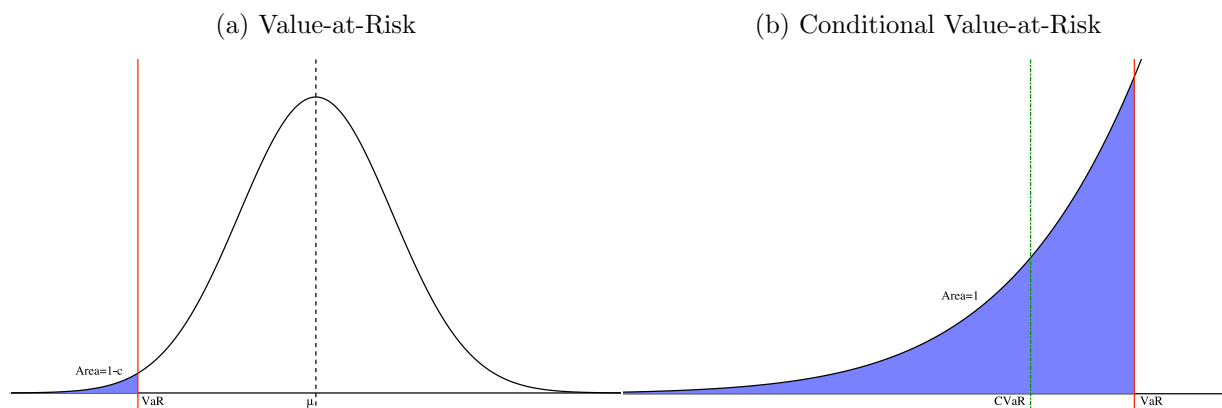
where λ_c is the weight, which is zero for continuous distributions.⁴⁸ Returning to the \$1 million portfolio example above, if losses do happen to exceed \$213,000, CVaR would suggest that, with 99% confidence, expected losses would be about \$247,000 leaving the value of the portfolio at \$713,000. Thus, CVaR will always show larger potential losses than VaR at the same confidence level. This is illustrated in figure 1.4b where the red line is the VaR level, the vertical green dash-dot line is the $CVaR$ level, and the density function has been rescaled so that the area to the left of VaR is one.

The major rationale for any risk measure is to quantify how much capital a financial institution must hold to cover expected losses so the institution can remain solvent. If x is the value of a firm's assets and the firm has capital equal to $\mu - VaR_c$, then the firm can remain solvent as long as the value of assets does not fall below VaR_c . The probability of insolvency for the firm is then $1 - c$. If the firm holds capital against VaR_c to prevent insolvency, then $CVaR_c$ would provide a measure of the value of recoverable assets in the case the firm does become insolvent.⁴⁹ Alternatively, if the firm holds capital equal to $\mu - CVaR_c$, then the firm can remain solvent as long as the value of assets does not fall below $CVaR_c$. The probability of insolvency would then be less than $1 - c$.

⁴⁸See Rockafellar and Uryasev (2002) for the derivation of λ_c .

⁴⁹Shin (2010).

Figure 1.4: Risk Measures for a Standard Normal Distribution



1.2.3 Value-at-Risk Dictates Procyclical Leverage and Asset Price Dynamics

Shin (2010) asserts that “one of the paradoxes of the recent global financial crisis is that it erupted in an era when risk management was at the heart of the management of the largest and most sophisticated financial institutions.” As will be seen, the upward-sloping demand and downward-sloping supply responses discussed in section 1.2.1 that can destabilize the financial system can be amplified by regulations and risk management techniques such as Value-at-Risk. VaR was included in the Basel regulations for banking supervision under the assumption that making each individual bank safe will help make the entire financial system stable. However, this assumption neglects the concept that risk is endogenous and that the actions of one institution responding to a changing economic environment under regulatory constraints can create a pecuniary externality: their actions may negatively affect asset prices and the soundness of other institutions, thus jeopardizing the stability of the entire financial system.⁵⁰

To illustrate this point, take the simple example provided in Shin (2010). In this example, there are two investors, one unleveraged and one leveraged investor, who trade a risky

⁵⁰Daniélsson et al. (2004); Bianchi (2011); and Mendoza (2010).

security. The returns from holding this security are known to be risky with payoffs denoted by the random variable w that is uniformly distributed over $[q - z, q + z]$. The expected return is $E[w] = q > 0$, the variance of returns is $\sigma^2 = z^2/3$, and $z > 0$. For an investor with equity e who holds cash and y units of the risky asset at price p , the expected return of the portfolio is $W = qy + (e - py)$, where $e - py$ is the amount of cash holdings.

The unleveraged investor is not constrained in anyway and is assumed to have mean-variance preferences:

$$U = E[W] - \frac{1}{2\tau}\sigma_w^2$$

where $\tau > 0$ is a constant referred to as the unleveraged investor's "risk tolerance," where a higher τ implies a higher risk tolerance, and the variance of returns is σ_w^2 . After substituting the expected return and variance into the utility function and maximizing for y , the solution to the unleveraged investor's problem is:

$$y_u = \begin{cases} \frac{3\tau}{z^2}(q - p) & \text{if } q > p \\ 0 & \text{otherwise} \end{cases} \quad (1.1)$$

The leveraged investor is assumed to be risk neutral but is constrained by the VaR of his assets as is the case for financial institutions under Basel II style regulations. This VaR constraint dictates that the leveraged investor must have enough equity so that the probability of default is kept below some target level. In the example considered here, a default probability of zero is achievable because the returns are chosen to be uniform. The leveraged investor's problem is to:

$$\max_{\{y\}} qy + (e - py) \text{ subject to } VaR \leq e$$

The balance sheet of the leveraged investor that is marked-to-market is $py = d + e$ where d is the amount of debt. The VaR constraint with zero probability of default dictates that the leveraged investor's assets valued in the worst case scenario ($py - (q - z)y$) needs to be at least as large as his equity e so that the investor can fully repay his debt. Thus, holdings of

the risky security must satisfy:

$$py - (q - z)y \leq e \quad (1.2)$$

which defines the VaR constraint. This constraint will always bind since the leveraged investor will not hold any of the risky security if $p > q$, but will hold no cash if $p < q$ since the expected return is increasing in y . Thus, the optimal holdings of the risky security can be solved by setting $VaR = e$. Rearranging this constraint for y yields the solution to the leveraged investor's problem:

$$y_L = \frac{e}{p - q + z} \quad (1.3)$$

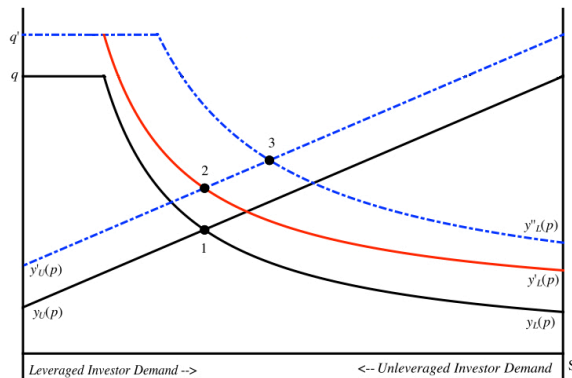
The market clearing condition with a limited supply of the risky security is given by $y_u + y_l = S$ where S is the total amount of the security available. The solution is then fully determined if e is taken to be a state variable. Additionally, it can be seen that leverage is inversely related to unit-VaR:

$$L = \frac{py}{e} = \frac{p}{p - q + z} \quad (1.4)$$

where $VaR = py - (q - z)y = (p - q + z)y$ so unit-VaR is $VaR/y = p - q + z$. Therefore, leverage will be procyclical: when risk and VaR are low, leverage will be high and when risk and VaR are high, leverage will be low.

Figure 1.5 illustrates the case when investors expect the return of the risky security to increase from q to q' and demonstrates the asset price dynamics when at least one investor is leveraged and VaR-constrained. The original equilibrium is denoted by the intersection of the solid black lines representing the leveraged investors demand $y_L(p)$ and the unleveraged investors demand $y_U(p)$ at point 1. With an improved outlook on the return of the risky security, both demands increase to $y'_L(p)$ (solid red line) and $y'_U(p)$ (blue dash-dot line) with the leveraged investor's holdings of the risky security being nearly unchanged. However, because the leverage investor's balance sheet is marked-to-marked, the increase in the asset price will immediately increase his equity. This acts to loosen the VaR constraint, allowing

Figure 1.5: Shin (2010) Upward-Sloping Demand Response



Note: Recreated from Shin (2010) with additional intermediate step denoted by the solid red line $y'_L(p)$.

the leveraged investor to bear more risk and purchase more of the risky security, increasing the demand to $y''_L(p)$ (blue dash-dot line) with the final equilibrium at point 3 where the leveraged investor's holdings of the risky security has increased. This response from the VaR-constrained leveraged investor is the embodiment of Shin's (2010) upward-sloping demand response when the market is not fully liquid (i.e. the unleveraged investor's demand is downward sloping) and actually increases the more leveraged the investor is. When this process works in reverse, Shin (2010) refers to it as a downward-sloping supply response and is represented as the movement from point 3 to point 1 in the figure. This is reminiscent of Bernanke's (2013) fire sales in which decreased capital forces banks to sell assets at depressed prices.

To gain further insight into the dynamics, examine the resulting Lagrange multiplier from the leveraged investor's optimization. The Lagrange multiplier in this context is the rate of change in the expected return on the portfolio with respect to a change in equity. Solving for this variable gives:

$$\lambda = \frac{dE[W]}{de} = \frac{dE[W]}{dy} \frac{dy}{de} = \frac{q - p}{p - q + z} \tag{1.5}$$

This shows that an increase in unit-VaR or decrease in the expected return $q - p$ will cause λ to fall. The implication is that the leveraged investor's profit opportunities have been reduced as he is required to hold more equity for the same holdings of the risky security in order to stay solvent with the same probability. Furthermore, note that $VaR = py - (q - z)y$ from the VaR constraint, then VaR can be written as a multiple of the standard deviation of the portfolio such that $VaR = py - (q - z)y = (p - q + z)y = \alpha zy/\sqrt{3} = \alpha\sigma_w$ and the constant $\alpha = \sqrt{3}(p - q + z)/z$, where $p - q + z > 0$. Now, the VaR constraint can be rearranged to show that the variance of the portfolio must be below some level:

$$\sigma_w^2 \leq \left(\frac{e}{\alpha}\right)^2 \quad (1.6)$$

and rewrite the optimization problem as:

$$\mathcal{L} = E[W] + \lambda \left[\left(\frac{e}{\alpha}\right)^2 - \sigma_w^2 \right] = E[W] + \lambda \left(\frac{e}{\alpha}\right)^2 - \lambda\sigma_w^2 \quad (1.7)$$

First, notice that this setup is identical to a mean-variance optimization setup except for the constant $\lambda(e/\alpha)^2$, which drops out after taking the first-order condition, and λ , which has replaced $1/2\tau$. The resulting first-order condition is identical to equation (1.1) after the substitution for τ is made:

$$y = \frac{3}{2\lambda z^2}(q - p) \quad (1.8)$$

λ is related to the leveraged investors risk tolerance, which Shin (2010) calls "risk appetite." A higher risk tolerance translates to a smaller λ , so a lower λ implies a higher risk appetite in this case. The leveraged investor's risk appetite will change over time depending on the economic environment as can be seen after combining the new first-order condition (1.8), the new VaR constraint (1.6), and α :

$$\lambda = \frac{3(q - p)(p - q + z)}{2ez^2} \quad (1.9)$$

A lower unit-VaR and higher equity e all cause λ to fall increasing the investor's risk appetite. Thus, a leveraged VaR-constrained investor behaves like a mean-variance investor with a

time-varying risk appetite. This characteristic will cause these investors to chase higher expected returns when equity increases and risk is low in good times fueling asset price booms, but will cause them to flee the market when equity falls and risk is high in bad times fueling asset price crashes. A financial system composed of these investors can create a feedback loop in which balance sheet sizes adjust in the same direction as asset prices, inducing further price changes which affects all investors through changes in equity, inducing further balance sheet adjustments, and so on destabilizing the financial system.⁵¹

This concept was taken further in Daniélsson et al. (2004). They develop a general equilibrium framework where only traders exist, who maximize a constant absolute risk aversion utility function subject to a VaR constraint to make portfolio decisions. They investigate the model's outcome when traders do not know the true process for the evolution of prices and treat market risk as exogenous: i.e. they do not take into account the effects of their own or other market participants behavior on asset prices. This differs from the Basak and Shapiro (2001) model where traders know the true underlying process for the evolution of prices. Traders in the Daniélsson et al. (2004) model use VaR to restrict their portfolio choice and forecast risk in a backward-looking manner as is standard under regulatory risk practices. They show that in their standard asset pricing framework, when the VaR constraint binds, the associated Lagrange multiplier does indeed indicate the degree of the trader's risk appetite, which varies depending on market outcomes and the slackness of the constraint. When volatility is high in their model, the VaR constraint is more binding, and low risk appetite leads to asset sales amplifying market volatility. They find that prices are lower on average when the VaR constraint is used but are also more volatile, illustrating the destabilizing nature of VaR constraints on prices in a dynamic model.

⁵¹Shin (2010).

1.3 The Importance of the Financial Sector in Business Cycles: Empirical Evidence

Prior to the 2008 global financial crisis, many models of the macroeconomy ignored the financial sector, believing intermediated finance to be a veil for channeling household savings to the productive sector and often citing the Modigliani-Miller (1958) theorem to support this assumption.⁵² This theorem suggests that banks' capital structure is irrelevant to the real economy if financial markets are perfectly competitive. However, that assumption ignores reasons for the existence of financial intermediaries, which range from market imperfections arising from asymmetric information, moral hazard, or monitoring costs, among others. Banks help solve these issues by specializing in credit monitoring and pooling funds to reduce the cost of supplying credit through risk sharing.⁵³ Significant empirical evidence has begun to mount that contradicts this theorem implying that financial factors, such as leverage and VaR, are important to business cycle analysis.

Much of the recent empirical literature regarding the importance of financial factors in business cycles is addressed by Adrian and Shin along with their coauthors. As was seen in section 1.1.2 and discussed in Adrian and Shin (2010, 2013), financial intermediary leverage tends to be procyclical and as section 1.2 showed, these leverage responses can be a result of VaR-based balance sheet management. Using a panel regression consisting of a number of broker dealer balance sheets overtime, Adrian and Shin (2010) find that leverage is negatively related to the lagged growth of trading book VaR and positively related to the growth in total assets and repos on a quarterly basis. In addition, using weekly data on broker dealer repos they show that changes in collateralized borrowing can be a significant forecasting variable for changes in aggregate market volatility and the price of risk as measured by the VIX (the Chicago Board Options Exchange implied stock market volatility). Adrian and Shin (2013) further show that these intermediaries target a fixed VaR-equity ratio, where

⁵²He and Krishnamurthy (2013b).

⁵³Calomiris and Gorton (1991).

the VIX and intermediary unit-VaR are strongly positively correlated. Thus, broker dealers tend to adjust balance sheet size on the margin using repos. When volatility and VaR are low, broker dealers tend to increase short-term borrowing and leverage up but reduce short-term borrowing and deleverage when volatility and VaR are high. These intermediaries chase yield and flee from risk in a coordinated fashion such that changes in intermediary balance sheets become reflected in asset prices and volatility. Hence, changes in intermediary balance sheets are important risk pricing factors.

If financial intermediary balance sheet adjustments can affect volatility and asset prices, then it should be expected that changes in intermediary balance sheets can have an effect on the real economy. Adrian et al. (2010) find evidence to support this. Using aggregate broker dealer data, they use predictive regressions to show that broker dealer leverage is a significant forecasting variable for the excess return of multiple assets including equities, corporate bonds, and Treasury securities, even after controlling for some possibly confounding variables. The rationale being that changes in leverage indicate changes in the willingness of these intermediaries to take on risk. Furthermore, after supplementing the data set with standard aggregate macroeconomic data (GDP, inflation, and the target federal funds rate) and estimating a vector autoregression, they show that as intermediary balance sheets suffer capital losses, macroeconomic activity tends to slow. Thus, the functioning of the financial sector is closely tied to the dynamics of the macroeconomy as intermediary balance sheets are indicative of market risk premia, which in turn affects the supply of credit, consumption, and investment decisions.

When the financial sector fails to operate efficiently, it can have devastating consequences on the economy. Jordà et al. (2013) study the role of credit in business cycles and note two key observations about recessions associated with financial crises by exploiting a panel data set over 14 countries and 140 years. The first is that financial crisis recessions tend to be more costly than other types of recessions in terms of losses in output and employment. And the second is that more credit intensive expansions tend to be followed by much deeper recessions and more prolonged recoveries as a decline in credit amplifies the downturn. Highly

leveraged expansion periods appear to be associated with slower credit, investment, and output growth following the bust as households, firms, and financial institutions are forced to deleverage. These periods are also accompanied with deflationary pressures making it more costly for debtors to repay as the real value of debt rises. Credit availability contracts as banks attempt to rid their balance sheets of risk and see less profitable lending opportunities with high unemployment, rising delinquencies, and debt overhang in the productive sectors, which then feeds into declining investment and output. Adrian et al. (2012) note that firms will tend to substitute bond finance for bank lending during recessions but are typically unable to make up for the fall in bank lending as both bond and loan interest rates rise. Financial recessions can then be seen as the consequences of deteriorating lending standards that follow credit booms. Since there is only a limited supply of prime borrowers, once that supply dries up banks take more risks by lending to less credit worthy borrowers to increase profits. Accordingly, the financial sector can essentially sow the seeds of its own demise making financial business cycles endogenous.

Adrian and Brunnermeier (2014) note that this type systemic risk and leverage builds up in times of low volatility and that those risks materialize during crises, which Brunnermeier and Sannikov (2014) call the “volatility paradox.” In the most simple models, risk is assumed to follow a normal distribution. Markowitz (1952) showed that under the normality assumption, standard deviation can be used as a measure of risk and the covariances of returns can be used to explain how diversification reduces the aggregate risk of a portfolio. Fama’s (1965) efficient markets hypothesis (EMH) embodies this concept, where price changes follow a normal distribution due to the assumption that all market participants have full information. However, risk is not always normal as pointed out by Mandelbrot (1963), where he observed that the tail of returns distributions were much thicker than the assumed normal distribution. As a consequence, the standard deviation is no longer the correct risk measure, and these fat tail distributions predict that extreme events should occur more often than what is predicted by a normal distribution. In fact, the tails of returns distributions appear thin during normal low volatility periods and fat during high volatility crisis periods, mean-

ing downside risk appears low during booms and high during recessions. An alternative to the EMH, known as the fractal markets hypothesis (FMH), predicts such behavior in returns distributions. It stresses that in normal times, investment horizons vary among all types of investors causing markets to be very liquid and asset price movements to be relatively small. However, during a crisis, investment horizons converge causing markets to become illiquid and asset prices to make extreme jumps. The existence of risk-sensitive capital requirements like VaR can also contribute to such behavior. In times of low risk, capital charges are low freeing up bank capital and allowing banks to purchase assets, but in times of high risk, capital charges increase requiring banks to sell assets. This can initiate the VaR feedback effects described in section 1.2.3, which may give rise to large price movements and fat tailed returns distributions.

Gorton and Metrick (2012) provide further insight into the linkages within the modern financial system that can precipitate Shin's (2010) VaR feedback effects and Bernanke's (2013) fire sales. They provide evidence that the crisis was initiated by a run on short-term wholesale funding markets, the repo markets that broker dealers relied so heavily on to finance balance sheet growth, causing the financial sector to become insolvent for the first time since the Great Depression. For these events to have unfolded the way they did, three things had to have occurred. First, the collateral value underlying repos declined rapidly with an increase in the uncertainty surrounding the value of that same collateral, effectively tightening risk constraints. Second, this uncertainty caused the interest rate on repos to spike making it more costly for banks to borrow against that collateral. And third, haircuts on repos, the difference between the amount of cash borrowed and the collateral value, also spiked as concerns arose about the solvency of banks and the value of seizable assets in the event of default. According to Acharya and Mora (2012), banks were not passive to funding losses. Those that faced the greatest liquidity concerns actively sought to attract deposits by raising rates, essentially self-signaling their financial distress. The end result was a withdrawal in funding that these banks needed to maintain their current balance sheet activities and remain solvent. Thus, runs on wholesale funding can trigger the VaR feedback effect and

asset fire sales that deplete bank capital and restrict the financial sector's ability to supply credit.

The evidence to support the claim that credit supply factors are key drivers of the financial business cycle is immense. Thus, models of the macroeconomy need to be supplemented with credit supply factors if they are to adequately capture the properties of the modern business cycles.

1.4 Towards a Better Model of the Financial Sector: Theoretical Macroeconomics

Two of the first papers to explore financial factors and their amplification effects in general equilibrium macroeconomic models were the seminal papers by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). Bernanke and Gertler (1989) was one of the first papers to show how borrower balance sheets can affect output dynamics. This analysis was later implemented in a full business cycle model in Bernanke et al. (1999), which became known as the financial accelerator.⁵⁴ The idea behind the financial accelerator is that changes in the market for credit amplify and propagate shocks to real economy through endogenous developments in the external finance premium: i.e. the difference between the cost of external funding and the opportunity cost of using internal funds for financing investment decisions. The external finance premium has an inverse relationship with borrowers' net worth. When net worth is high, borrowers are closer to fully collateralizing external funds implying less risk for the lender. When net worth is low, borrowers cannot fully collateralize external funds, implying more risk for the lender and a larger premium. Because borrower net worth is procyclical, the external finance premium exacerbates shocks to borrower net worth affecting investment, consumption, and output.

This mechanism was studied further by Kiyotaki and Moore (1997) who use limited-liability collateral constraints to generate an additional balance sheet effect on the business cycle based on the relationship between credit limits and asset prices. With this, they show

⁵⁴Bernanke et al. (1996).

that persistent shocks can amplify and spill over to the rest of the economy. In their model, lenders cannot force borrowers to repay unless loans are secured with durable assets which take on a dual role. Durable assets such as capital are factors in production and also serve as collateral for loans. When shocks hit borrowing constrained firms, they have to cut their demand for capital as their net worth falls, causing a fall in capital prices that further tightens borrowing constraints. This mechanism builds on the Bernanke-Gertler financial accelerator and shows that shocks to net worth act through changes in the value of borrowers' assets in a forward-looking manner. However, neither the Bernanke-Gertler nor the Kiyotaki-Moore mechanisms are able to generate the procyclical bank leverage that is observed in the data, because they apply constraints to the demand-side of credit rather than to the supply-side.

The existence of the leverage cycle was studied even before the 2008 financial crisis by Geanakoplos (2003); however, its potential was not fully appreciated until after the crisis unfolded. He was able to show that when heterogeneous lenders are subject to collateral constraints, his model gave rise to a procyclical leverage cycle. In the model, booms are driven by loosening collateral constraints and increased leverage, while busts are driven by tightening collateral constraints and deleveraging. The amplification feature of procyclical leverage has also been shown to exist in models with VaR constraints including Danielsson et al. (2011) and Adrian and Boyarchenko (2015). They develop frameworks with leveraged banks who are subject to a VaR constraint that requires them to maintain a level of capital that limits their probability of default to a constant. The degree to which the constraint binds is determined by market outcomes, effectively making risk appetite time-varying. When the VaR constraint binds more in bad times, risk appetite decreases as risk premiums increase, whereas the VaR constraint binds less in good times and risk appetite increases as risk premiums decrease. More important is that these models that include VaR constraints are able to generate procyclical leverage as the constraint tightens and loosens over the course of the business cycle.

Researchers have also begun to take the study of the financial business cycle further by implementing a financial sector directly into full scale macroeconomic models. Authors

such as Gertler and Kiyotaki (2010); Curdia and Woodford (2011); He and Krishnamurthy (2013a, 2014); Gertler et al. (2012); Gertler and Karadi (2011); Iacoviello (2005, 2014); and Iacoviello and Neri (2010) have developed models of constrained financial intermediation and have shown that constrained financial intermediaries can disrupt economic activity when negative shocks hit the financial sector. Gertler et al. (2012) focus on how liquidity and asset return risk affects the degree of bank risk exposure and find that lower perceived risk increases the financial sector's susceptibility to crises through increased leverage decisions. Iacoviello (2005, 2014) and Iacoviello and Neri (2010) studies the role of exogenously imposed capital adequacy constraints on perfectly competitive banks. They find that when banks face capital adequacy constraints, exogenous loan defaults can trigger deleveraging and credit crunches when banks hold too little capital, which spill over and affect the rest of the economy. Another portion of the financial intermediation literature studies the effects of what policy interventions can do to relieve stress on financial institutions. Curdia and Woodford (2011); Gertler and Karadi (2011); and He and Krishnamurthy (2013a, 2014) develop general equilibrium models with perfectly competitive banks and find that targeted central bank asset purchases, like those undertaken by the Federal Reserve during the 2008 financial crisis, direct central bank credit intermediation, and capital infusions can be effective, because they help alleviate capital constraints.

Other authors such as Goodfriend and McCallum (2007) and Gerali et al. (2010) have stepped away from the perfect competition assumption and modify the financial sector to be monopolistically competitive to be more in line with evidence that suggests that banks have some degree of market power.⁵⁵ Goodfriend and McCallum (2007) emphasize the demand-side of credit where their model features both a banking accelerator effect, which works similarly to Bernanke-Gertler financial accelerator in the presence of sticky interest rates, and a banking attenuator effect that oppose each other as changes in monetary policy can affect

⁵⁵For further discussion on market power in the banking sector see Freixas and Rochet (1997); Diamond (1984); Greenbaum et al. (1989); Sharpe (1990); Kim et al. (2003); Thadden (2004); Demirgüç-Kunt et al. (2004); Berger et al. (2004); and Degryse and Ongena (2008).

the price of capital and the demand for bank deposits simultaneously. Gerali et al. (2010), however, emphasize the supply-side of credit by introducing bank balance sheet positions directly into interbank market conditions. They also find both a banking accelerator and attenuator with an overall attenuating effect compared to a perfectly competitive banking sector with fully flexible rates. The existence of bank market power here alters the pass-through of policy rate changes on interest rates in the economy: a loan rate markup amplifies changes in monetary policy for borrowers, but a deposit rate markdown dampens changes in the policy rate for depositors resulting in countercyclical credit spreads. Darracq Pariès et al. (2011) take the Gerali et al. (2010) model further by supplementing the balance sheet effects with Basel II and Basel III credit risk capital requirements and find that these risk-sensitive capital charges can have procyclical amplification effects on business cycle dynamics as banks expand and contract credit to comply with a regulatory capital-asset ratio.

The above models all focus on linearized models; however, it can be argued that linearized models, which only study the dynamics of the economy near the steady state, will not be able to adequately capture the dynamics of financial crises. As financial crises are severe by nature, the economy is unlikely to be near the steady state when it is experiencing a crisis rendering the dynamics of linearized financial crisis models inaccurate. Thus, authors such as Brunnermeier and Sannikov (2014) as well as Boissay et al. (2016) have taken financial sector modeling even further, developing non-linear models to study how the dynamics change when the economy moves away from the steady state. Brunnermeier and Sannikov (2014) use a non-linear macroeconomic model to show that the economy can become unstable when shocks are large enough to move the economy far enough away from the steady state. Their economy also reacts asymmetrically to shocks, where positive shocks have small effects, but negative shocks have large effects. Rather counterintuitively, regulatory restrictions can do more harm than good as these constraints tend to bind in downturns, exacerbating recessions, but have little effect on behavior in boom times, adding fuel to the fire. More recently, Boissay et al. (2016) have included moral hazard and asymmetric information into a modeled financial sector. They find that these two market imperfections can lead to sudden interbank market

freezes as documented by Gorton and Metrick (2012), banking crises, and credit crunches that cause the deep long-lasting recessions documented by Jordà et al. (2013).

Some authors like Gennaioli et al. (2013) as well as Wickens (2011) have suggested that the inability to correctly price the risk of default can sow the seeds of a financial crisis. Wickens (2011), with a model based on the Curdia and Woodford (2011) model, is able to price default risk in a general equilibrium model in which the interest rate spread between deposits and loans reflects the risk of default. He also presents a way to study the effects of systemic versus idiosyncratic bank risk shocks. Gennaioli et al. (2013) also study the trade-off between systemic and idiosyncratic risk in a model of non-traditional financial intermediation. The novelty of the Gennaioli et al. (2013) model is their departure from rational expectations and the use of securitization to derive a borrowing constraint in which banks can only borrow against the value of riskless and securitized risky assets in the believed worst state of the world. Banks securitize and sell risky loans to diversify idiosyncratic risk at the expense of building up systemic risk as banks become more interconnected. They show that securitization is welfare enhancing under rational expectations. However, when banks neglect tail risks (i.e. neglect the possibility that the worst possible state can occur), they take on too much balance sheet risk. Once the neglected state is realized, banks suffer a large capital loss and a financial crisis results. Due to the existence of the risk-based borrowing constraint, their model is also able to generate the procyclical bank leverage documented in Adrian and Shin (2010) where some of the more complicated models like He and Krishnamurthy (2013a); Bernanke and Gertler (1989); Kiyotaki and Moore (1997); and Gertler and Kiyotaki (2015) are unable to do so.

1.5 Summary and Future Research

As a result of the 2008 financial crisis, risk-sensitive capital regulations came under scrutiny for their potential procyclical and amplifying effects. As a response, the Basel Committee on Banking Supervision met to address the concerns that excessive credit growth, deteriorating credit standards, use of risky trading strategies, and the excessive leverage

build-up likely contributed to the severity of the crisis. Notable among the Basel Committee's concerns are the procyclicality and amplification effects of Value-at-Risk, which have been illustrated in a number of models including Shin (2010); Daniélsson et al. (2011); and Adrian and Boyarchenko (2015).⁵⁶ In the hopes of promoting individual bank soundness and a more resilient banking sector, the Basel Committee has proposed new regulations that modify and add to the existing credit risk and market risk capital standards. One of the new proposals that may have important implications is the suggested switch of the market risk measure to the more credible CVaR, with the intention to prevent the misperception and underpricing of risk that underlined many of the problems prior to the crisis.

The new research on financial intermediation has produced models that capture one aspect or another of financial business cycles or financial crises documented in the empirical literature. However, a more complete and unified model of financial intermediation in business cycles has yet to be developed. Adrian et al. (2012) pose a challenge for the development of this macroeconomic model to capture five stylized facts about the modern macroeconomy:

- coexistence of bank lending and bond finance,
- substitution from bank to bond finance during recessions,
- countercyclical credit spreads,
- stickiness of bank capital, and
- procyclical bank leverage.

In addition, this unified model must also be able to capture two other stylized facts that have been documented by He and Krishnamurthy (2013a) and Adrian and Brunnermeier (2014):

- countercyclical risk premia, and
- negative correlation between volatility and bank leverage.

⁵⁶Basel Committee on Banking Supervision (2013).

An added bonus would also be to capture elements of fire sales and the dependence on wholesale funding discussed by Bernanke (2013) and Gorton and Metrick (2012) respectively. Common ingredients in many of the models that have been able to capture countercyclical risk premia, procyclical bank leverage, and the negative correlation between volatility and bank leverage are capital constraints or VaR-based risk constraints. Furthermore, countercyclical credit spreads have been replicated by modeling a monopolistically competitive financial sector. Thus, market power and risk constraints appear to be important modeling devices for capturing the empirical regularities of the financial business cycle. However, more work needs to be done on bank and bond financing as well as integrating the results from the models described in the previous section.

While the Adrian and Shin literature has used VaR to model risky asset demand, those assets have typically been interpreted as loans. Thus, they simplify the use of VaR as a constraint by imposing it on bank lending decisions where VaR is actually used among the major banks to manage trading book decisions as is required under the Basel regulations. Banks are then required to manage credit risk using credit risk specific models and VaR to manage market risks. The rest of this dissertation will take the VaR concept more seriously, building on the research of the Adrian and Shin literature. The objective is to gain further understanding into how a leveraged financial system subject to risk constraints interacts with the business cycle of the economy as a whole when VaR is used in a manner that is consistent with capital regulations. With VaR being used to model banks' market risk decisions rather than credit risk decisions, it is not clear whether VaR will have the same procyclical effects on leverage and lending as it does in the Adrian and Shin literature. This is explored in chapter 2, while chapter 3 aims to be more forward-looking and examines some potential consequences of Basel III's proposed switch from VaR to stressed CVaR to manage market risks. Apart from the mathematical benefits and coherence of CVaR as a risk measure, it is unclear at this point what unintended consequences, positive or negative, this new regulatory restriction may have on the financial sector and macroeconomy.

Chapter 2

**CREDIT SUPPLY AND FINANCIAL MARKET VOLATILITY
UNDER VALUE-AT-RISK-BASED CAPITAL
REQUIREMENTS****Abstract**

Following the 2008 financial crisis, concerns arose about the possible procyclical effects of risk-sensitive capital requirements that rely on Value-at-Risk (VaR) for determining the market risk capital charge on a portfolio of trading securities. To analyze this issue, this chapter modifies a fully dynamic general equilibrium macroeconomic model with a financial sector and subjects it to a VaR constraint that is consistent with risk-weighted capital requirements. The model is calibrated to U.S. data and estimated with Bayesian techniques to pin down the dynamics and is able to capture four important business cycle correlations between financial factors and macroeconomic activity. The results suggest that when leveraged financial institutions are constrained by VaR-based capital requirements, increased financial market volatility forces banks to adjust their balance sheets to comply with higher capital charges by selling assets at reduced prices. This action depletes bank capital and deteriorates risk-weighted balance sheet positions, raising banks' perceived probability of default and interbank borrowing costs. Ultimately, these effects restrict the financial sector's ability to supply credit to the productive sectors to finance investment. Additionally, if financial markets become illiquid and banks are unable to sell assets such that they violate their risk constraints, the effects become amplified. These results provide some rationale for the Federal Reserve taking on the buyer of last resort role in the asset-backed securities market during the 2008 financial crisis. Thus, credit supply and market risk capital requirements are shown to be interdependent through risk constraints.

2.1 Introduction

Risk management is an essential part of the operation of a financial institution, and the value-added of a good risk management system can indeed be substantial. But there may be a divergence of interests between an individual firm and the system as a whole. Exploring exactly how the divergence of interests plays out in the economy is an urgent modeling task for economists. As a first step, putting Value-at-Risk into a general equilibrium context is an important conceptual task that has barely begun. More needs to be done.

— Hyun Song Shin, *Risk and Liquidity* (2010)

The 2008 financial crisis sparked new research topics in macroeconomics, but more importantly, provided significant empirical evidence against the Modigliani-Miller (1958) theorem. This theorem suggests that banks' capital structure is irrelevant to the real economy if financial markets are perfectly competitive; however, evidence in favor of imperfect competition in the banking sector contradicts this theorem implying that financial factors, such as leverage and risk, are important for the analysis of business cycle dynamics.

A major contributing factor to the financial crisis was the inability to correctly price the risk of asset-backed securities (ABS).¹ ABS's were designed to create a category of safe assets by pooling loans and taking advantage of diversification benefits. Because many ABS's were considered nearly riskless, and investors are highly interested in riskless debt, ABS's were often used by banks as collateral to raise short-term debt via repurchase agreements (repos), a contract in which the borrowing party sells a security for cash with the promise to buy it back in the future at an agreed upon price.² When more loans defaulted than expected, a large adverse shock to the price of ABS's occurred along with uncertainty about the value of the collateral underlying many of these assets. This made it difficult for banks to refinance

¹Wickens (2011) and Gennaioli et al. (2013).

²Bernanke et al. (2011).

short-term debt when repo markets froze, resulting in a run on wholesale funding markets.³ The crisis did not stop there, however. Troubles in the ABS markets spilled over in to other markets, including equities markets, as fire sales began with banks attempting to sell assets and remove risk from their balance sheets, causing asset prices to fall and volatility to spike.⁴ Ultimately, banks lost a significant amount of equity capital, leaving many highly under capitalized and on the brink of failure.

During the aftermath of the financial crisis, bank capital regulations came under scrutiny. Risk-sensitive capital regulations as suggested by the Basel Committee on Banking Supervision require banks to hold capital against three types of risk: credit risk, market risk, and operational risk. Credit risk applies to default risk on loans; market risk applies to price, interest rate, and exchange rate volatility associated with trading securities; and operational risk applies to losses that may be sustained from failed internal processes. The intention of capital requirements is to ensure banks' ability to honor debt repayment in the face of operating losses and remain solvent. However, because capital requirements are risk-sensitive, concerns arose about potential procyclical and amplifying effects, notably with market risk capital requirements.⁵ As part of market risk capital regulations, banks are required to hold capital against the Value-at-Risk (VaR) of their portfolio of trading securities, which may consist of those ABS's that became so problematic during the crisis, or equity securities among others. VaR is a statistical measure aimed at quantifying the largest loss a trader can expect with a given level of confidence. It was created as a response to the 1987 market crash and has been a risk management standard in the financial industry since the 1990s before being adopted as regulation in 1996. It is typically calculated using a minimum the past year of historical data with a 10-day holding period. But because VaR is calculated with such an arguably short time interval, it is essentially a local measure of risk. This opens the door for perceived risk to vary with the business cycle. For example, when asset prices

³Acharya and Mora (2012) and Gorton and Metrick (2012).

⁴Bernanke (2013).

⁵Basel Committee on Banking Supervision (2013)).

are expected to decline and markets become more volatile, which typically coincide at the onset of a downturn, banks' financial assets appear riskier. VaR will increase and banks will be required to hold more capital, which could cause or amplify a credit contraction as banks adjust their balance sheets to comply with the higher capital requirements. Another concern about risk-sensitive capital requirements as it applies to market risk is that all banks under this type of regulation may act uniformly to market conditions causing asset prices to be more volatile.

While each of the three types of risk is important for financial operations, this paper will focus solely on market risk because of the concerns noted by the Basel Committee and modeling urgency noted by Shin (2010) in the opening quotation. If the Basel Committee's concerns are legitimate, then high volatility episodes that have occurred after VaR-based capital regulations were instituted (like those that occurred during the dot com bubble in 2000, the Enron scandal in 2002, and the financial crisis in 2008), could have contributed negatively to credit availability. Thus, this paper is motivated to answer the question of why financial market volatility can affect credit market conditions even though the two markets may seem disconnected. To do so, the monopolistically competitive banking sector developed by Gerali et al. (2010) is modified, and this paper proposes that VaR-based capital regulations create a link that propagates changes in financial market volatility to credit markets and the real economy. The model is calibrated to U.S. data over the period 1997:Q2-2007:Q4 and includes a number of shocks to facilitate Bayesian estimation of the model to pin down the dynamics.

The major contribution of this paper identifies Value-at-Risk as applied to the risk management of banks' trading books under risk-sensitive capital requirements to be a supply-side factor in credit market dynamics. The model is able to replicate four empirical correlations between financial factors and macroeconomic activity. It captures the correlations of financial institution leverage with financial market volatility, leverage with loans, trading book size with volatility, and leverage with aggregate investment. It also captures the relationship between banks' trading book size and leverage in response to volatility shocks. Most impor-

tantly, the model is able to replicate the procyclicality of financial institution leverage with respect to financial market volatility and aggregate investment, which many recent macro models are unable to do. This result arises because banks' asset purchases are connected to total debt through a Value-at-Risk constraint and a marked-to-market balance sheet identity, plus aggregate investment is connected to bank lending through a collateral constraint. In addition, the volatility of investment and loans increase when banks are allowed to manage a trading book under VaR-based capital regulations.

Two results arise from the impulse response analysis. The first suggests that an increase in financial market volatility can be transmitted through the financial sector to the real economy through a risk-constrained feedback effect. Increased volatility raises VaR and requires banks to adjust their balance sheet positions to comply with higher capital charges. Higher volatility also affects risk-weighted balance sheet positions and the cost of interbank funds in the sense that banks' risk-weighted capital-asset ratio is a good predictor for its perceived default probability. Banks respond to a rise in volatility by selling risky assets as their VaR constraint tightens, putting downward pressure on asset prices. This results in a loss in banks' equity capital, which generates further selling pressures that spill over into a decline in credit supply and aggregate investment. The second result suggests that these effects can be amplified if financial markets become illiquid and banks are unable to sell assets such that they violate their VaR constraints. Key to this result is the inability to offload risk when markets are illiquid, which induces a spike in the interbank interest rate spread due to deteriorating risk-weighted balance sheet conditions. Thus, asset liquidity is an important factor in minimizing macroeconomic fluctuations emanating from higher financial market volatility and provides some rationale for the Federal Reserve taking on the role of buyer of last resort in the ABS market during the 2008 financial crisis.

The rest of this paper is organized as follows: section 2.2 reviews some empirical facts about financial business cycles, section 2.3 reviews other related literature on the financial accelerator and relevant macroeconomic models with a financial sector. Sections 2.4 and 2.5 presents the model, section 2.6 discusses the calibration and Bayesian estimation of the

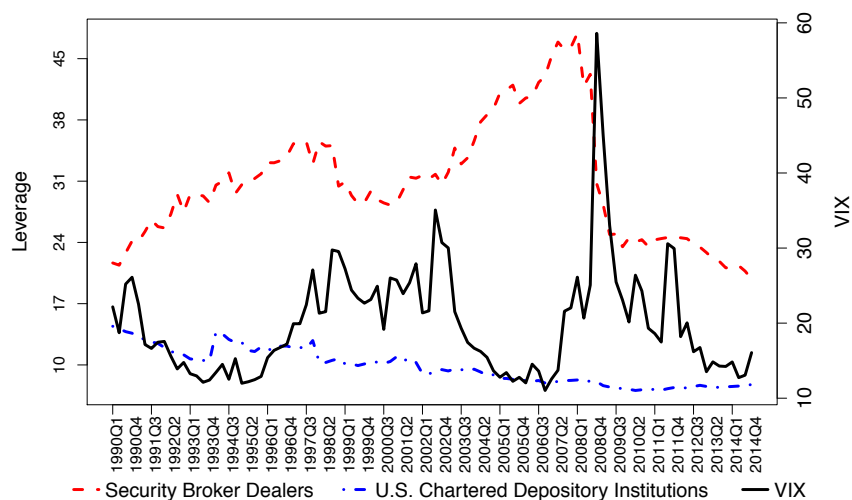
model, and section 2.7 presents the results from impulse responses to a volatility shock. Finally, section 2.8 concludes.

2.2 *Empirical Relevance of Volatility in Business Cycles*

This section documents some empirical evidence regarding the cyclical nature of the financial sector, financial market volatility, and their connection to the real economy for which the model constructed in this paper will account for. Adrian and Shin (2010) and Adrian et al. (2010) note some empirical facts about financial sector leverage adjustments and their role in affecting macroeconomic dynamics. They show that there is a strong positive relationship between changes in asset prices, leverage, and balance sheet size. When balance sheets are continuously marked to market, changes in asset prices have an immediate impact on bank capital. Financial intermediaries are not passive, and adjust balance sheets in such a way that leverage, defined as the ratio of total assets over equity, is generally procyclical. The evidence suggests that financial institutions actively manage their balance sheets. U.S. chartered depository institutions (financial intermediaries such as commercial banks as well as savings and loan associations) tend to target a fixed leverage ratio; however, security broker dealers (financial intermediaries such as investment banks that operate primarily in capital markets) display even stronger balance sheet management through procyclical leverage choices. Adrian and Shin (2013) suggest that these financial institutions manage leverage in the short-term to maintain a constant VaR-equity ratio, even as market conditions deteriorate, by financing changes in asset growth with changes in debt.

Leverage is inversely related to total assets. When asset prices rise, net worth increases, and leverage falls. Adrian and Shin (2010) show that for leverage to remain constant or increase with a rise in asset prices, banks must expand total assets through security purchases or loan creation to use up excess capital. Financing these moves is often done through short-term debt issuance or repurchase agreements which causes leverage to increase. When this process works in reverse and an excess supply of securities is not met with sufficient demand, it puts downward pressure on prices adding feedback to the leverage process by weakening

Figure 2.1: Leverage of Financial Institutions and Market Volatility



Note: Data on financial institutions are from 1990-2014 found at the Board of Governors of the Federal Reserve System Financial Accounts of the United States; the VIX is found at the Chicago Board Options Exchange.

balance sheets. Adrian et al. (2010) further show that as financial intermediary balance sheets become weaker, macroeconomic activity tends to slow. In fact, regulatory risk models actually dictate active management of Value-at-Risk through balance sheet adjustments. Shin (2010) is able to illustrate this in a simple example where the leverage of a VaR-constrained investor is inversely related to VaR, whereas Adrian and Shin (2010) use a panel regression to empirically show that lagged VaR is negatively related to leverage. So, when banks face the possibility of losing more at the same confidence level, they adjust their balance sheet to reduce leverage.

These observations can be seen in figures 2.1 and 2.2 which span the time period 1990:Q1-2014:Q4. Figure 2.1 displays the volatility paradox coined by Brunnermeier and Sannikov (2014) and empirically documented in Adrian and Brunnermeier (2014) in which low levels of risk tends to lead to higher leverage. Looking at two different types of financial institutions,

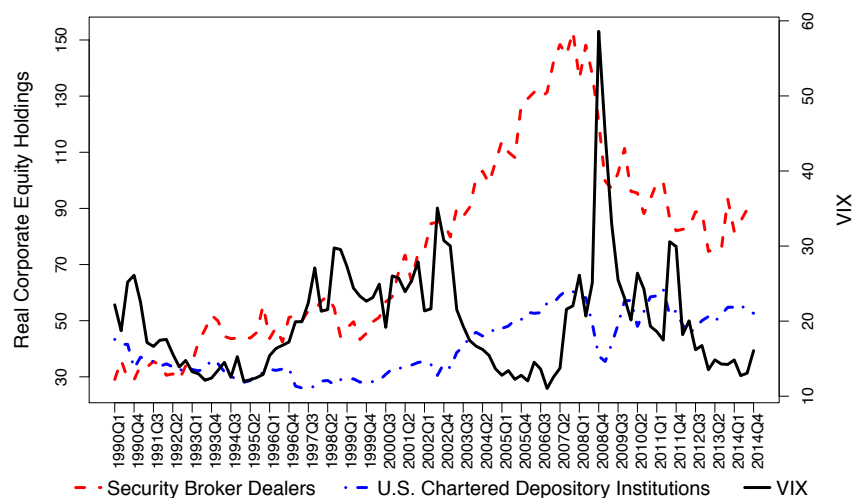
U.S. chartered depository institutions and security broker dealers, the former has maintained a fairly stable leverage ratio that has been declining since 1990.⁶ Security broker dealers, however, have had a pattern of increasing leverage in low volatility periods measured by the Chicago Board of Exchange Volatility Index (VIX). This happened in the U.S. in the 1990s and early 2000s. There are two noted exceptions to this: 1997-98 and 2008-09. 1997-98 corresponds to the Asian and Russian financial crises and 2008-09 corresponds to the most recent financial crisis. One aspect common to both periods was an increased level of financial market volatility and perceived risk. When financial institutions calculate their VaR, they are required to do so using a minimum of the past 250 trading days of price movements. When this local measure of volatility is low, perceived risk measured by VaR tends to be low. The correlation coefficient between security broker dealer leverage and the VIX between 1990:Q1-2010:Q2 is -0.31, suggesting a negative relationship between leverage and volatility.⁷

One asset class subject to financial market volatility and VaR-based capital regulations is corporate equities. While equities were not at the cause of the 2008 financial crisis, these markets were affected after ABS markets froze. Figure 2.2 shows security broker dealers' corporate equity holdings normalized by the S&P 500 price began increasing around 1999 when volatility was relatively high but continued to increase in the early 2000s as volatility

⁶U.S. chartered depository institutions are financial intermediaries that raise funds mainly through demand and time deposits to fund loans and invest in securities. These firms include national commercial banks, state-chartered commercial banks, federal savings banks, state-chartered savings banks, cooperative banks, and savings and loan associations. Security broker dealers are firms that register with the Securities and Exchange Commission who buy and sell securities or hold an inventory of securities for resale. For more information, see the Board of Governors of the Federal Reserve Financial Accounts Guide.

⁷Determining the endpoint for the time period used to measure the correlation is difficult. On the one hand, it is important to include as many business cycles as possible. On the other hand, relationships among time series change as the macroeconomic environment changes and there is likely a structural break in the relationship between security broker dealer leverage and the macroeconomy after 2008 with the collapse of the large investment banks and the restructuring of the U.S. financial system. Using the Bai and Perron (1998) method for estimating structural breakpoints, 2010:Q2 is suggested to be a structural breakpoint in the relationship between security broker dealer leverage, commercial and industrial loans, and investment. The Johansen (1998) test for cointegration suggests one cointegrating vector and is statistically significant at the 99% level. The correlation coefficient between security broker dealer leverage and the VIX is calculated using logged and HP-filtered data except for the VIX which is logged and demeaned. The Pearson's product-moment correlation test suggests that this correlation is significant at the 99% level.

Figure 2.2: Corporate Equity Holdings of Financial Institutions and Market Volatility



Note: Data on financial institutions are from 1990-2014 found at the Board of Governors of the Federal Reserve System Financial Accounts of the United States, the fix VIX is found at the Chicago Board Options Exchange. Corporate equity holdings are normalized by the S&P 500 index.

fell. U.S. chartered depository institutions also exhibit this pattern but not nearly to the same degree. Security broker dealers slightly decreased their holdings of these securities around the time of the 1997-98 crises. However, both types of financial institutions decreased their equity holdings much more when volatility and risk spiked in 2008-09 during the financial crisis. Because VaR-based capital requirements increase with risk, banks are required to hold more capital during high volatility episodes. To meet the higher capital requirements, banks can raise more capital or lower their VaR by selling risky securities. The correlation coefficient over the period 1990:Q1-2010:Q2 between corporate equity holdings and the VIX for U.S. chartered depository institutions and security broker dealers is -0.55 and -0.21 respectively. This suggests that financial institutions subject to VaR-based capital requirements buy assets when their perceived risk is low and sell the same assets when their perceived risk is high. During the upward phase of the leverage cycle, security broker dealers increased their holdings

of corporate equities, but did the opposite during the deleveraging phase as they removed risk from their balance sheets. The correlation coefficient between security broker dealer leverage and corporate equity holdings is 0.49 over this same period.⁸

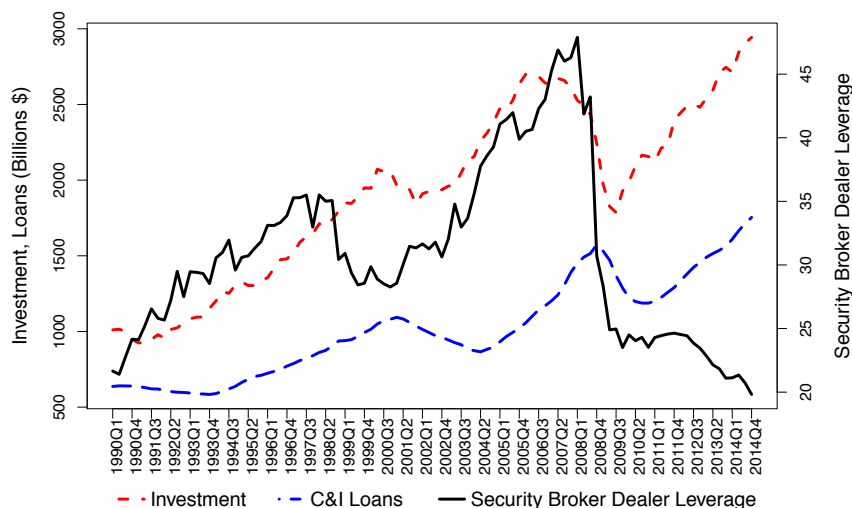
Jordà et al. (2013) study the role of credit in business cycles and note two additional key observations about recessions associated with financial factors (i.e. financial crises) by exploiting a panel data set over 14 countries and 140 years. The first is that financial crisis recessions tend to be more costly than other types of recessions, with output and employment declining more than in other types of recessions. And the second is that more credit intensive expansions tend to be followed by much deeper recessions and more prolonged recoveries as a decline in credit amplifies the downturn. Highly leveraged expansion periods appear to be associated with slower credit, investment, and output growth following the bust as households, firms, and financial institutions are forced to deleverage. These periods are also accompanied with deflationary pressures making it more costly for debtors to repay as the real value of debt rises. Thus, credit availability inevitably contracts as banks attempt to rid their balance sheets of risk and see less profitable lending opportunities with high unemployment, rising delinquencies, and debt overhang in the productive sectors, which then feeds into declining investment and output. Additionally, Adrian et al. (2012) suggest that credit supply shocks are key drivers of financial business cycles.

As securitization has become more prominent within the financial sector (more than \$2 trillion of mortgage related securitized assets were issued in 2007 compared to just over \$1 trillion of Treasury securities while they were both around \$500 billion in 1995), security broker dealers have begun to play a more central role in credit intermediation as they are often the market-making institutions for securitized products.⁹ Total financial assets for security broker dealers as a percent of total financial assets for U.S. chartered depository

⁸All correlation coefficients are calculated using logged and HP-filtered data except for the VIX which is logged and demeaned. The Pearson's product-moment correlation test suggests that both correlations are significant at the 95% level.

⁹Gorton and Metrick (2012).

Figure 2.3: Investment, Loans, and Leverage



Note: Data are from 1990-2014 found at the Board of Governors of the Federal Reserve System and the Organization for Economic Co-operation and Development.

institutions has increased from 12.7% in 1990:Q1 to 46.4% at the peak in 2007:Q2.¹⁰ In the sense that increased leverage indicates a higher appetite for risk, security broker dealer leverage has become a good indicator for credit availability and macroeconomic activity. Thus, financial factors are hugely important for explaining the modern business cycle, so models of the macroeconomy need to be supplemented with them.

Figure 2.3 displays the connection between financial intermediary leverage and real activity: i.e. credit and investment. Commercial and industrial (C&I) loans, investment, and security broker dealer leverage all exhibit procyclicality. However, financial market volatility is countercyclical as the correlation coefficient between the VIX and investment is -0.38 between 1990:Q1-2010:Q2.. When volatility and perceived risk was low in the mid-1990s and mid-2000s security broker dealer leverage was increasing along with C&I loans and investment as the two have a 0.11 correlation coefficient. However, when volatility increased

¹⁰Board of Governors of the Federal Reserve System (2015a).

around 1997-98 and 2008, security broker dealer leverage fell. Investment and C&I loans did not begin declining until leverage bottomed out in 2001 but declined almost simultaneously with leverage around 2008. Because volatility affects banks' VaR, capital requirements, and leverage, volatility may also impact credit availability and investment since security broker dealer leverage appears to be a good predictor of macroeconomic activity. The correlation coefficient of security broker dealer leverage with C&I loans and security broker dealer leverage with investment is 0.11 and 0.61 respectively.¹¹ The model developed in this paper is able to qualitatively replicate these observations about financial business cycles. Namely, increased levels of financial market volatility and perceived risk are correlated with deleveraging, a reduction in risky security holdings as dictated by VaR-based capital regulations, and a reduction in credit supply and investment.

2.3 Other Related Literature

This paper contributes to the macro-financial, capital requirements, and business cycle literature by analyzing the role that bank balance sheets play in propagating changes in financial market volatility to the real economy when banks are subject VaR-based capital requirements. The importance of the balance sheet strength in propagating shocks has been known for quite some time. The seminal paper by Bernanke and Gertler (1989) was one of the first to show how borrower balance sheets affect output dynamics. The analysis was later implemented in a full business cycle model in Bernanke et al. (1999), which became known as the financial accelerator.¹² The idea of the financial accelerator is that changes in the market for credit amplify and propagate shocks to real economy through endogenous developments in the external finance premium: i.e. the difference between the cost of external funding and

¹¹All correlation coefficients are calculated using logged HP-filtered data except for the VIX which is logged and demeaned. The Pearson's product-moment correlation test suggests that all correlations are significant at the 99% level except for the correlations between C&I loans and investment and between security broker dealer leverage and C&I loans, which are not statistically significant at the 95% confidence level.

¹²Bernanke et al. (1996).

the opportunity cost of using internal funds for financing investment decisions. The external finance premium has an inverse relationship with borrowers net worth. When net worth is high, borrowers are closer to fully collateralizing external funds implying less risk for the lender. When net worth is low, borrowers cannot fully collateralize external funds, implying more risk for the lender and a larger premium. Because borrower net worth is procyclical, the external finance premium exacerbates shocks to borrower net worth affecting investment, consumption, and output.

This mechanism was studied further by Kiyotaki and Moore (1997) who use limited-liability collateral constraints to generate a transmission mechanism based on the relationship between credit limits and asset prices. With this, they show that persistent shocks can amplify and spill over to the rest of the economy. Lenders cannot force borrowers to repay unless loans are secured with durable assets which take on a dual role. Durable assets, such as capital, are factors in production and also serve as collateral for loans. When shocks hit borrowing constrained firms, they have to cut their demand for capital as their net worth falls, causing a fall in capital prices and further tightening borrowing constraints. This mechanism builds on the Bernanke-Gertler financial accelerator and shows that shocks to net worth act through changes in the value of borrowers' assets in a forward-looking manner. However, neither the Bernanke-Gertler nor the Kiyotaki-Moore mechanisms are able to generate the procyclicality of bank leverage that is observed in the data, because they apply constraints to the demand-side of credit rather than to the supply-side.

The existence of the leverage cycle was studied even before the 2008 financial crisis by Geanakoplos (2003); however, its potential was not fully appreciated until after the crisis unfolded. He was able to show that when heterogeneous agents are subject to collateral constraints, his model gave rise to a procyclical leverage cycle. In the model, booms are driven by loosening collateral requirements and increased leverage, while busts are driven by tightening collateral requirements and forced deleveraging. The amplification feature of procyclical leverage has also been shown to exist in models with Value-at-Risk constraints including Danielsson et al. (2011) and Adrian and Boyarchenko (2015). They develop frame-

works with leveraged banks, who are subject to a VaR constraint that requires them to maintain a level of capital that limits their probability of default to a constant, and unleveraged risk-averse value investors. They show that the associated Lagrange multiplier on the VaR constraint can be interpreted as a degree of risk appetite. The degree to which the constraint binds is determined by market outcomes, effectively making risk appetite time-varying. When the VaR constraint binds more in bad times, risk appetite decreases as risk premiums increase, whereas the VaR constraint binds less in good times and risk appetite increases as risk premiums decrease. What's more important, is that these models that include VaR constraints are able to generate procyclical leverage as the constraint tightens and loosens over the course of the business cycle. Using a VaR constraint gives the model the ability to determine the steady state portfolio size when agents maximize a risk-neutral objective function, which would not otherwise be possible.

A key difference between the two papers is that Daniélsson et al. (2011) abstracts from the effects of bank balance sheets on the rest of the macroeconomy, while Adrian and Boyarchenko (2015) include both the consumption and investment decisions in their model economy. However, Adrian and Boyarchenko (2015) simplify the use of VaR as a constraint by imposing it on bank lending decisions where VaR is actually used among the major banks to allocate capital to the trading book as is required by regulation. Banks are then required to allocate capital to loans using credit risk models. This is a departure made from Adrian and Boyarchenko (2015), as VaR is used in this paper to allocate capital to the trading book to be more consistent with bank behavior under capital regulations.

The introduction of the Basel II risk-sensitive capital adequacy framework introduced concerns that the financial sector could provide substantial financial accelerator type effects on the real economy, some of which are studied by Darracq Pariès et al. (2011). They use a DSGE model augmented with a banking sector developed by Gerali et al. (2010) (discussed in further detail below) to show that these regulations are a cause for concern. Using the credit risk requirements of Basel II and a quadratic adjustment cost applied to the risk-weighted capital-asset ratio, they show that risk-sensitive capital requirements imply a higher volatility

to output growth and inflation. While increasing credit risk played a major factor in loan contraction during the 2008-09 crisis, especially with regards to mortgage finance, securities markets experienced extreme volatility and asset price declines as well. Since banks hold securities for profit and balance sheet management reasons, and because many loans were held as asset-backed securities in the trading book rather than in the loan book, their analysis needs to be supplemented with market risk considerations. This is one area that this paper contribute to the literature.

The model used to analyze the procyclical effects of Value-at-Risk to financial market volatility shocks essentially builds on the work of Gerali et al. (2010). They build a financial sector into a standard DSGE model and include a number of different channels for shocks to propagate through including the interest rate channel, a nominal debt channel, a collateral channel, and an asset price channel, all of which have been shown to amplify technology shocks compared to frictionless financial models. The financial sector developed in their paper is monopolistically competitive, which allows banks to set interest rates that adjust sluggishly to changes in the central bank policy rate. Because banks are price setters due to credit market power, the resulting loan rate markup amplifies changes in monetary policy for borrowers. However, the resulting deposit rate markdown dampens changes in the policy rate for depositors. The presence of credit market power and interest rate frictions alters the pass-through of policy rate changes, dampening the effect compared to the cases where interest rates are fully flexible and a model with perfectly competitive banks. They also introduce capital requirements in such a way that changes in leverage will either amplify or dampen changes in monetary policy. If leverage increases with the policy rate, then the transmission of shocks will be amplified and vice versa. Overall, they find that due to the presence of interest rate frictions, credit market power dampens the effect of monetary policy and technology shocks to real variables. Their model does not qualitatively change the response of the main macroeconomic variables of interest compared to more standard New Keynesian models. Therefore, this makes for a good model in which to embed risk-based capital regulations.

Gerali et al. (2010) note that their model omits some elements of the 2008 financial crisis, including the increase in risk observed in financial markets. This is the omission tackled in this paper, which contributes to the literature by analyzing the procyclical effects of VaR-based capital requirements in response to financial market volatility shocks from the supply-side of credit. While the potential procyclical concerns of VaR-based capital requirements have been noted in other studies, to the best of my knowledge, this is the first paper to illustrate this concern within a fully dynamic general equilibrium model of market risk capital requirements. Risk-weighted capital requirements are implemented in the way Darracq Pariès et al. (2011) do within the Gerali et al. (2010) framework. However, as they study the effects of credit risk capital regulations on business cycle fluctuations, this paper studies the effects of market risk capital regulations. A VaR constraint is also used to derive banks' demand function for assets subject to VaR-based capital regulations in a manner similar to the those employed in Shin (2010); Daniélsson et al. (2011); and Adrian and Boyarchenko (2015). The combination of the VaR constraint and risk-weighted capital requirements allows for the analysis of time-varying volatility effects on financial business cycles when banks are subject to VaR-based capital requirements in a meaningful way. The model is then able to replicate the procyclicality of bank leverage and the volatility paradox where many recent macro models such as Brunnermeier and Sannikov (2014); He and Krishnamurthy (2013a); Bernanke and Gertler (1989); Kiyotaki and Moore (1997); and Gertler and Kiyotaki (2015) are unable to do so.

2.4 The Supply and Demand Sectors of the Model Economy

The model economy is a standard New Keynesian model augmented with a financial sector. It consists of two agents who differ only in their degrees of patience: households and entrepreneurs. The assumption that the discount factor for households (β_H) is higher than that for entrepreneurs (β_E) ensures that households are more patient than entrepreneurs and will choose to save while entrepreneurs will choose to borrow.

Households consume, supply differentiated labor, set their desired wage, and save via

bank deposits and a portfolio of equity securities. Entrepreneurs consume and produce a homogeneous intermediate good using household labor and capital. Capital is purchased from perfectly competitive capital goods producers and financed with collateralized bank loans. The intermediate good is then sold to monopolistically competitive retailers who costlessly differentiate it, set the retail price, and sell it to households and entrepreneurs as the final consumption good. Capital goods producers are included to derive a market price of capital.

The main financial assets, one period deposits and loans, are supplied by monopolistically competitive banks.¹³ This allows banks to set interest rates in order to maximize profits. Heterogeneity in the rate of time preference between households and entrepreneurs will ensure the flow of funds through the financial sector from depositor households to borrower entrepreneurs. Households face no financial constraints but when financing capital purchases, entrepreneurs are constrained by the future value of undepreciated capital according to a Kiyotaki and Moore (1997) type collateral constraint. This assumption is meant to be consistent with some empirical evidence that suggests that firm balance sheet conditions are important for investment decisions and credit availability.¹⁴

Apart from deposits and loans, households and banks participate in a secondary market for risky equity securities, which is the new asset introduced into the model. Households purchase a portfolio of equity securities to help smooth intertemporal consumption by equating the rate of return on deposits (the deposit interest rate) to the rate of return on equity securities (the dividend yield paid out of retailer profits). On the other side of this market, banks purchase a portfolio of equity securities (the trading book) to maximize profits subject to a Value-at-Risk constraint that explicitly takes into account price volatility and limits the

¹³An endogenous motive for the existence of banks is not modeled here. However, the model implicitly assumes that there is some form of market imperfection, such as asymmetric information or monitoring costs that prevents households from directly lending to entrepreneurs. Banks solve this issue by specializing in credit monitoring and pooling funds to reduce the cost of supplying credit through risk sharing among households. See Calomiris and Gorton (1991) for more information.

¹⁴See Kiyotaki and Moore (1997); Bernanke et al. (1999); Flannery and Öztekin (2012); Berger et al. (2008); and Faulkender et al. (2012) for further discussion.

amount of risk they can hold on their balance sheet.

The economy is also subject to a number of nominal frictions that are used to generate the persistence to various shocks that is observed in the data. Nominal rigidities are an important foundation for New Keynesian models as they allow for the transmission of monetary policy shocks and are a tractable way to improve the model performance relative to the data. Retailers and households are responsible for setting the consumption good price and the nominal wage subject to Rotemberg (1982) style quadratic adjustment costs. Since retail interest rates are essentially another nominal goods price, banks will also be subject to a quadratic adjustment cost on interest rate setting. Some authors have shown wage rigidities to be more important than price rigidities in explaining business cycles. Christiano et al. (2005) find wage rigidities to be crucial to their model's performance, whereas price rigidities play a much smaller role. Price frictions alone cannot generate enough persistence in output unless price contracts are assumed to be extremely long. However, their model with only wage frictions does not have this problem. It is also an important feature of Gerali et al. (2010) where the estimated wage adjustment cost parameter is about three-and-half times larger than the price adjustment cost parameter. It is also important to note that the use of adjustment costs to achieve nominal rigidities is not microfounded and is somewhat of an ad hoc assumption. However, it is no more ad hoc than the Calvo (1983) style price adjustment mechanism, and there is a relationship between the two. Nominal rigidities are not imposed a priori but all adjustment cost parameters will be estimated to match the persistence that best fits the data.

The following sections describe the setup of the entrepreneurs, households, and financial sector. The rest of the model, which is more or less standard, is described in more detail in appendix A.

2.4.1 Entrepreneurs

The supply-side of the economy is described first as it will inform the setup of the household problem. There is a continuum of measure one of entrepreneurs indexed by i that

maximize utility by choosing the final goods consumption bundle $c_t^E(i)$, loans $b_t(i)$ that cost the interest rate r_t^b , labor input $l_t(i)$ that costs the real wage rate w_t , and capital input $k_t(i)$ which is purchased at the price q_t^k . Utility from consumption is assumed to depend on deviations from lagged external group specific habits in consumption, where h represents the degree of habit formation.¹⁵ Labor and capital are combined to produce a homogeneous intermediate good, y_t , using a Cobb-Douglas technology production function where total factor productivity (A_t^E) is assumed to be an exogenously given stochastic process.

Entrepreneurs are financially constrained and must borrow from banks in order to finance capital purchases. They can only borrow a fraction (the loan-to-value ratio, M_t^E) of the expected future value of their undepreciated capital stock when loans come due in the next period. This assumption creates a connection between the investment decision and bank lending. M_t^E will also be modeled as an exogenously given stochastic process. It is assumed that entrepreneurs always repay in full so the model remains in a neighborhood of the steady state and the collateral constraint always binds.¹⁶ Capital is assumed to depreciate at rate δ and entrepreneurs resell undepreciated capital to capital goods producers at the end of each period.

¹⁵Habit formation introduces non-separability of preferences over time. An increase in current consumption lowers the marginal utility of consumption and increases it in the next period, implying households would prefer to consume more tomorrow when consumption increases today. In the business cycle literature, habit formation is used to capture the hump-shaped response of consumption and will tend to smooth consumption. Multiplying by $(1 - h)^\gamma$ offsets the impact of habit formation on the steady state marginal utility of consumption.

¹⁶To assume the collateral constraint always binds near the steady state, the size of shocks must be sufficiently small. The implicit assumption behind the collateral constraint is that banks cannot force entrepreneurs to foreclose on output, so debt must be secured for repossession in the case of default. If entrepreneurs choose to default, banks know they may be able to renegotiate debt down to the value of capital, so banks formulate the collateral constraint as an expected break-even condition.

The entrepreneur problem is then to maximize utility:¹⁷

$$\max_{\{c_t^E(i), b_t(i), k_t(i), l_t(i)\}} E_0 \sum_{t=0}^{\infty} \beta_E^t \left[(1-h)^\gamma \frac{(c_t^E(i) - hc_{t-1}^E)^{1-\gamma}}{1-\gamma} \right]$$

Subject to the budget constraint (BC), collateral constraint (CC), and production function (PF):

$$[BC] : c_t^E(i) + w_t l_t(i) + \frac{(1+r_{t-1}^b)b_{t-1}(i)}{\pi_t} + q_t^k k_t(i) \leq \frac{y_t(i)}{x_t} + b_t(i) + q_t^k(1-\delta)k_{t-1}(i) \quad (2.1)$$

$$[CC] : (1+r_t^b)b_t(i) \leq M_t^E E_t [q_{t+1}^k \pi_{t+1} k_t(i)(1-\delta)] \quad (2.2)$$

$$[PF] : y_t(i) = A_t^E k_{t-1}(i)^\alpha l_t(i)^{1-\alpha} \quad (2.3)$$

where $\pi_t = P_t/P_{t-1}$ is the gross inflation rate and $x_t = P_t/P_t^W$ is the markup of the retail price over the wholesale price.¹⁸

This setup includes three channels for shocks to propagate through. The first, since debt is defined in nominal terms, is the nominal debt channel. When interest and debt payments are denominated in nominal terms, changes in inflation effectively redistribute wealth between borrowers and lenders. The incorporation of the collateral constraint links entrepreneur balance sheets to credit and makes capital play a dual role: it is a factor in production and collateral for loans. The collateral constraint also introduces two other transmission channels. The second channel is the collateral channel whereby changes in the interest rate affect the shadow value of borrowing. The third is the asset price channel in which changes in the capital price affects the value of collateral entrepreneurs can borrow

¹⁷The specification of CRRA utility implies that consumption increases in the same proportion as income and that the intertemporal elasticity of substitution will be equal to the inverse of the relative risk aversion parameter γ . Thus, the risk aversion and intertemporal elasticity of substitution will be constant with respect to the level of wealth and consumption. The intertemporal elasticity of substitution is defined as $-U'(c_t)/(c_t U''(c_t)) = 1/RRA = 1/\gamma$.

¹⁸Debt is denominated in nominal terms to ensure the inclusion of the gross inflation variable, π_t . If instead debt were denominated in real terms, critical nominal price and wage frictions would not play a role in the model.

against. When credit limits and asset prices interact, shocks can become persistent, amplify, and spill over to other sectors as a result of the collateral constraint.

Once the intermediate output is produced, entrepreneurs sell it to a measure one of monopolistically competitive retailers indexed by j at the wholesale price P_t^W . Retailers then costlessly differentiate it and sell it to households at the retail price P_t . Retail profits, written without subscripts j , will be:

$$\Pi_t^R = y_t \left[1 - \frac{1}{x_t} - \frac{\kappa_p}{2} (\pi_t - \pi_{t-1}^\iota \pi^{1-\iota})^2 \right] \quad (2.4)$$

where $(\kappa_p/2)(\pi_t - \pi_{t-1}^\iota \pi^{1-\iota})^2$ is the adjustment cost on price setting.

2.4.2 Households

There is a continuum of measure one of households indexed by i that maximize utility by choosing deviations of their final goods consumption bundle $c_t^H(i)$ from lagged external group habits, deposits $d_t(i)$ that pay the interest rate r_t^d , and equity securities $s_{j,t}^H(i)$ at price $q_{j,t}^e(i)$. Each retailer j pays an exogenous fraction δ_j^e of profits $\Pi_{j,t}^R$ as dividends to holders of its equity shares $s_{j,t}$. It will be assumed that it is costly for households to manage a portfolio of equity securities of size $s_t^H(i) = \int_j s_{j,t}^H(i) dj$ in that a fee equal to a fraction of the value of assets under management, $Fq_t^e s_t^H(i)$, is paid to the financial sector as revenue for providing this service. This fee will prove to be important once the financial sector is discussed in section 2.5. Its inclusion will allow for the arbitrage conditions for households and banks to hold simultaneously so that the VaR constraint will bind in steady state.¹⁹ It will be assumed that retailers will not issue new shares or buy back any existing shares, so all transactions happen in the secondary market.

Households earn income by providing differentiated labor services, $l_t(i)$, that pay the individual real wage rate $w_t(i)$. Each household i also owns one retailer j so that profits net of dividend payments are paid to households as a lump sum. The household problem is then

¹⁹See section 2.5.3 for a discussion about the binding VaR constraint.

to:²⁰

$$\max_{\{c_t^H(i), d_t(i), s_{j,t}^H(i)\}} E_0 \sum_{t=0}^{\infty} \beta_H^t \left[\epsilon_t^c (1-h)^\gamma \frac{(c_t^H(i) - hc_{t-1}^H)^{1-\gamma}}{1-\gamma} - \frac{l_t(i)^{1+\phi}}{1+\phi} \right]$$

Subject to the budget constraint:

$$\begin{aligned} c_t^H(i) + d_t(i) + \int_j q_{j,t}^e s_{j,t}^H(i) dj \leq \\ w_t l_t(i) + \frac{(1+r_{t-1}^d)d_{t-1}(i)}{\pi_t} + \int_j \left(q_{j,t}^e(i)(1-F) + \delta_j^e \frac{\Pi_{j,t-1}^R}{\pi_t} \right) s_{j,t-1}^H dj + (1-\delta_j^e) \Pi_{j,t}^R \end{aligned} \quad (2.5)$$

where ϵ_t^c represents a consumption demand shock. Debt is again denominated in nominal terms for depositors so the nominal debt channel also affects households. Households are assumed to be passive in the equity securities market in that financial wealth does not directly enter the utility function. As a result, households will view bank deposits and equity securities as perfect substitutes for savings. This assumption will create a link between banks' asset purchases and debt, which will help the model generate procyclical bank leverage.²¹ When banks buy equity shares from households, households will increase deposits as they reduce equity holdings.

To illustrate why the portfolio management fee is important when examining the bank's problem, the first-order conditions for deposits and equity securities from the household's first-order conditions will be needed:²²

$$[s_t^H] : q_t^e = \beta_H E_t \left[\frac{\lambda_{t+1}^H}{\lambda_t^H} \left(q_{t+1}^e (1-F) + \delta_j^e \frac{\Pi_t^R}{\pi_{t+1}} \right) \right] + \epsilon_t^q \quad (2.6)$$

²⁰The specification of the disutility of labor implies that ϕ is the inverse of the Frisch elasticity of labor supply and that it is constant with respect to the real wage rate holding the marginal utility of wealth constant. The Frisch elasticity of labor is defined as $U'(l_t)/(l_t U''(l_t)) = 1/\phi$ and measures the substitution effect of a change in the wage rate on the labor supply.

²¹A more realistic assumption to link bank asset purchases to bank debt would be to explicitly model the use of repurchase agreements by banks. However, this is outside the scope of this paper.

²²Modeling the equity price as a function of the fundamental and transitory components can be rationalized from Carlson and Sargent (1997) where they use a similar formulation for the equity price and find that no single fundamental is able to explain the high stock prices observed in the 1990s.

$$[d_t] : \lambda_t^H = \beta_H E_t \left[\lambda_{t+1}^H \frac{(1 + r_t^d)}{\pi_{t+1}} \right] \quad (2.7)$$

written without subscripts i or j . λ_t^H is the Lagrange multiplier on the budget constraint and ϵ_t^q is a shock to the equity price. In steady state, these arbitrage conditions require that the rates of return from the two assets be equalized i.e. the deposit rate must equal the dividend yield. Since banks will also hold a portfolio of equity securities, the portfolio management fee will be needed to equalize the steady state rates of return for banks and ensure that the Value-at-Risk constraint binds at the steady state.

If the first-order condition for household equity holdings is rewritten using forward iteration, it can be seen that:

$$q_t^e = \sum_{k=0}^{\infty} E_t \left[\beta_H^{k+1} (1 - F)^k \frac{\lambda_{t+1+k}^H}{\lambda_t^H} \left(\frac{\delta^e \Pi_{t+k}^R}{\pi_{t+1+k}} \right) \right] + \lim_{T \rightarrow \infty} (\beta_H (1 - F))^T E_t \left[\frac{\lambda_{t+1+T}^H}{\lambda_t^H} q_{t+T}^e \right] \\ + \sum_{k=0}^{\infty} E_t \left[(\beta_H (1 - F))^k \frac{\lambda_{t+k}^H}{\lambda_t^H} \epsilon_{t+k}^q \right]$$

The first term on the right hand side represents the fundamental value equal to the discounted sum of all expected future dividend payments net of the portfolio management fee. The second term is the transversality condition that must go to zero as $T \rightarrow \infty$. The third term on the right hand side is an addition to the transversality condition representing a transitory shock to the equity price. It is bounded since $\beta_H (1 - F) < 1$ and ϵ_t^q is assumed to be a covariance stationary process. Thus, the equity price shock can be thought of as a shock that temporarily sets the price above or below its fundamental value similar to a pure expectations or bubble shock.

2.5 The Financial Sector of the Model Economy

The financial sector is modeled after Gerali et al. (2010) which offers a way to study credit intermediation from the supply-side. Banks will play a key role in the model economy as they act as an intermediary for all financial transactions and are assumed to be monopolistically competitive. This allows banks to set and adjust interest rate spreads over the course of

the business cycle. Theoretical reasons for the existence of market power in the banking industry range from the presence of switching costs, asymmetric information, menu costs for opening accounts, regulatory restrictions, market contestability, and customer relationships. The degree to which these affect market power affects interest rate spreads.²³

The modeled financial sector consists of a measure one of banks each with three branches: two retail branches and a wholesale branch. The retail branches include a deposit branch and a loan branch. The financial sector will be responsible for transforming household deposits into loans to entrepreneurs and it will also help households manage a portfolio of equity securities. The branch of the bank that helps manage the household portfolio will receive the fee, $Fq_t^e s_{t-1}^H(i)$, as a lump sum transfer and incur a cost exactly equal to the fee for providing the service so that it does not affect the bank's decision problem. This setup allows for an interbank market to be modeled, a market that is very important in the credit intermediation process. Balance sheet conditions in one part of the financial sector can disrupt the flow of funds through interbank lending and changing interest rate spreads, affecting borrowing costs and credit supply to end borrowers.

The deposit branch will be responsible for collecting debt from households by setting the deposit rate and channeling funds to the wholesale branch sector through the first part of the interbank market as wholesale deposits. The wholesale branch then lends wholesale deposits to the loan branch sector as wholesale loans through the second part of the interbank market. Finally, the loan branch takes wholesale loans and provides credit to entrepreneurs by setting the loan rate. The central bank will be assumed to interact with the interbank market by setting the policy rate using a naive Taylor rule. The central bank policy rate will act as the base interest rate in which all other interest rates are set relative to. By funneling wholesale funds through the interbank market, the wholesale branch is in charge of managing the capital (equity) position of the entire bank. It chooses how much debt to take on, how much credit

²³For further discussion on market power in the banking sector see Freixas and Rochet (1997); Diamond (1984), Greenbaum et al. (1989); Sharpe (1990); Kim et al. (2003); Thadden (2004); Demirgüç-Kunt et al. (2004); Berger et al. (2004); and Degryse and Ongena (2008).

to make available, and how much to invest in a portfolio of risky equity securities subject to its balance sheet identity and regulatory risk-weighted capital requirements. Risk-weighted capital requirements are implemented via a quadratic adjustment cost on a risk-weighted capital-asset ratio, where the target ratio is set by the regulator. The trading desk of the wholesale branch is then subject to a Value-at-Risk constraint designed to limit the amount of risk it can hold on its balance sheet and be consistent with these capital requirements.

As mentioned previously, quadratic adjustment costs on interest rate setting in the loan and deposit branch optimization will be used to capture the persistence in interest rate markups (or markdowns) from some base interest rate that is observed in the data, which tends to be the federal funds rate in the U.S. Theoretical and empirical reasons for interest rate frictions include switching costs, menu costs, maintaining customer relations, adverse selection, uncertainty about future monetary policy actions, and monopolistic competition. Although, the practice of indexing deposit and loan rates to some current market rate may make interest rate setting more flexible.²⁴

A quadratic adjustment cost on banks' risk-weighted capital-asset ratio is also used in the wholesale branch optimization to be consistent with capital regulations and capture another empirical observation that financial institutions tend to target a fixed VaR-equity ratio but also allows for leverage to be procyclical as is observed of security broker dealers. In addition, others have found that firms that target a leverage ratio and the speed at which they converge to it depends on a number of factors including whether they are under- or over-leveraged, cash flows, access to capital markets, size, and the institutional environment in which they operate in. Firms that are larger, more financially constrained, under-leveraged, well-capitalized, under heavy-regulations, or operating in less developed institutions are all

²⁴For further discussion on interest rate setting frictions see Berger and Hannan (1991); Berger and Udell (1992); Calem et al. (2006); de Bondt et al. (2005); Gambacorta (2008); Driscoll and Judson (2013); Kok Sørensen and Werner (2006); Gropp et al. (1989); Nakajima and Teranishi (2009); and Adrian and Shin (2013).

found to adjust leverage more slowly than their counterparts.²⁵

The addition of risky equity securities as part of bank balance sheets in conjunction with risk-weighted capital requirements is the main contribution made to the literature. Adding this asset allows for the study of how changes in financial market volatility transmit from the financial sector to the real economy. The transmission mechanism relies on the risk-weighted capital-asset ratio adjustment cost and the VaR constraint that is consistent with risk-weighted capital regulations. The VaR constraint limits the amount of risk banks can hold on their balance sheet and effectively works as their demand for assets subject to VaR-based capital regulations. The risk-weighted capital-asset ratio adjustment cost introduces risk and balance sheet conditions into the interbank market. This mechanism works by altering the risk-weighted capital-asset ratio and regulatory capital in response to volatility shocks, interest rate spreads, and credit flows through the financial sector.

2.5.1 Retail Branches

This section will start with the description of the financial sector and a brief overview of the deposit and loan branches, which are identical to those used by Gerali et al. (2010). The description of these two branches will help inform the derivation of the Value-at-Risk constraint and the wholesale branch problem.

Deposit Branch

There is a measure one of monopolistically competitive deposit branches indexed by i that collect deposits from households $d_t(i)$, pay out the deposit rate $r_t^d(i)$, and remits the deposits to the wholesale branch sector as wholesale deposits $D_t(i)$ at the central bank policy rate r_t^{cb} . It is implicitly assumed that the wholesale branch can borrow directly from the central bank at this rate and is used to close the model. Deposit branches maximize period profits subject to adjustment costs on interest rate setting (κ_d) and a Dixit-Stiglitz type

²⁵For further discussion on leverage frictions see Adrian and Shin (2013); Elliot et al. (2012); Faulkender et al. (2012); and Flannery and Öztekin (2012); and Berger et al. (2008).

CES demand curve for deposits. The interest rate elasticity for deposits, ϵ_t^d , is assumed to be time-varying and modeled as an exogenous stochastic process. Since deposit branches only have one source of revenue, wholesale deposits must be equal to household deposits: $D_t(i) = d_t(i)$.

The maximization program for the deposit branch can be written as:²⁶

$$\max_{\{r_t^d(i)\}} E_0 \sum_{t=0}^{\infty} \lambda_t^H \beta_H^t \left[r_t^{cb} D_t(i) - r_t^d(i) d_t(i) - \frac{\kappa_d}{2} \left(\frac{r_t^d(i)}{r_{t-1}^d(i)} - 1 \right)^2 r_t^d(i) d_t \right]$$

subject to the deposits demand curve:

$$d_t(i) = \left(\frac{r_t^d(i)}{r_t^d} \right)^{-\epsilon_t^d} d_t \quad (2.8)$$

Under completely flexible rates, the deposit rate is determined by:

$$r_t^d = \frac{\epsilon_t^d}{\epsilon_t^d - 1} r_t^{cb}$$

where $\epsilon_t^d/(\epsilon_t^d - 1)$ is the markdown of the deposit rate compared to the central bank policy rate.

Loan Branch

There is a measure one of monopolistically competitive loan branches indexed by i that takes wholesale funding $B_t(i)$ from the wholesale sector at the interbank rate r_t^{ib} and issues loans $b_t(i)$ to entrepreneurs at the loan rate $r_t^b(i)$ to maximize period profits subject to adjustment costs on interest rate setting (κ_b) and a Dixit-Stiglitz type CES demand curve for loans. The interest rate elasticity for loans, ϵ_t^b , is also assumed to be time-varying and modeled as an exogenous stochastic process. Since loan branches only have one source of revenue they will issue all its wholesale borrowing as loans so that $B_t(i) = b_t(i)$.

²⁶The use of $\lambda_t^H \beta_H^t$ as the discount factor for banks connects the bank's rate of time preference to the household's through the Euler equation: $1 = \beta_H E_t [(\lambda_{t+1}^H / \lambda_t^H)((1 + r_t^d) / \pi_{t+1})]$. It essentially sets the intertemporal discount factor for banks to be the deposit rate.

The maximization program for the loan branch can be written as:

$$\max_{\{r_t^b(i)\}} E_0 \sum_{t=0}^{\infty} \lambda_t^H \beta_H^t \left[r_t^b(i) b_t(i) - r_t^{ib} B_t(i) - \frac{\kappa_b}{2} \left(\frac{r_t^b(i)}{r_{t-1}^b(i)} - 1 \right)^2 r_t^b(i) b_t \right]$$

subject to the loan demand curve:

$$b_t(i) = \left(\frac{r_t^b(i)}{r_t^b} \right)^{-\epsilon_t^b} b_t \quad (2.9)$$

Under completely flexible rates, the loan rate is determined by:

$$r_t^b = \frac{\epsilon_t^b}{\epsilon_t^b - 1} r_t^{ib}$$

where $\epsilon_t^b/(\epsilon_t^b - 1)$ is the markup of the loan rate compared to the interbank rate.

At this point it is important to note that the use of the CES demand curve for deposits and loans may be an unrealistic assumption. Since the U.S. financial sector contains a few very large banks among other smaller more regional banks, an oligopoly structure may be more realistic. However, the monopolistic competition framework provides a convenient way to capture the existence of market power in the financial sector and generate non-zero steady state interest rate spreads.

2.5.2 Value-at-Risk and Risk-Weighted Capital Regulations

Two new features will be added to the wholesale branch compared to the one used by Gerali et al. (2010). The first is the inclusion of the risk-weighted capital-asset ratio, and the second is the trading desk that is subject a Value-at-Risk constraint. However, before the wholesale branch problem can be fully defined, some background on Value-at-Risk and risk-weighted capital regulations is necessary.

Value-at-Risk

Value-at-Risk originated as concept during a research effort within J.P. Morgan in the late 1980s. It was spearheaded by Chairman Dennis Weatherstone and research chief Till

Guldimann to quantify and manage potential risks as a response to the 1987 stock market crash. The concept became part of Weatherstone's "4:15 report", which gave him an estimated measure of risks comparable to profit and loss aggregated from all trading desks just fifteen minutes after the market closed. VaR provided Weatherstone with information on trading activities he had not known previously and used it to make judgments about how to adjust the firm's future trading positions. This research group was later spun off and became known as RiskMetricsTM. It began providing consultative services on the VaR method, eventually leading to its adoption into financial risk management in the 1990s and by the Basel Committee on Banking Supervision in 1996 as regulation. VaR then gained widespread acceptance after the collapse of Long Term Capital Management (LTCM), a hedge fund that employed Nobel Prize winners Myron Scholes and Robert Merton, in 1998 after the Asian and Russian financial crises. LTCM used VaR as a risk management technique, so its failure naturally sparked many narratives as to why. However, more emphasis was put into LTCM's misuse of VaR rather than to any of VaR's shortcomings at the time.²⁷

VaR is a statistical concept designed to provide a quantifiable measure about how much an investor can expect to lose from market fluctuations with a certain degree of confidence over a given time period. Expected losses give an idea about how much capital the portfolio manager must hold to cover losses from market volatility so the organization can remain solvent. Mathematically, Value-at-Risk for a given confidence level c is defined as:

$$VaR_c = \{v : Pr(x \leq v) = 1 - c\}$$

for $c \in (0, 1)$. So, if the distribution of price changes is continuous and denoted $f(x)$, then VaR_c is the quantile that solves:

$$1 - c = \int_{-\infty}^{VaR_c} f(x)dx$$

²⁷Taleb (2007).

When $f(x)$ is assumed to be normal, VaR_c can be written as:

$$VaR_c = \mu - Z_c\sigma$$

where μ is the expected return, σ is the standard deviation, and Z_c is a constant that depends on the confidence level c . At a 99% confidence level, $VaR_{99\%}$ says that there is only a 1% chance that losses will be larger than $\mu - Z_c\sigma$ over the specified time period.

VaR is really better thought of as possible losses from normal market conditions and has been known to have fairly poor performance during times of financial stress. Thus, it is important to distinguish between normal and crisis periods and remember that VaR is still a very limited risk measure. It is especially limited when thought of in terms of a normal distribution when fluctuations in stock returns have been shown to be non-normal. Price fluctuations tend to appear normally distributed in times of calm markets but become more non-normally distributed depending on the severity of market stress. VaR also has nothing to say about risks in the tail of the distribution: i.e. how bad things can get if losses exceed the VaR level. Consequently, it can be a dangerous risk measure if misinterpreted as the worst possible loss rather than as its strict definition.

Conditional Value-at-Risk (CVaR, or sometimes referred to as Expected Shortfall (ES) or Tail VaR (TVaR)) is an alternative measure that aims to capture tail risks, and its benefits over VaR have been well documented.²⁸ Most notably, VaR has been proven not to be a coherent risk measure: i.e. there are cases when the sum of the VaR's of two portfolios considered separately (the concept Weatherstone used to aggregate risks across trading desks) can be lower than the VaR of the combined portfolio, violating the diversification principle that a well-diversified portfolio carries lower risk. CVaR, however, is coherent and does not suffer from this problem.

Financial regulators are currently contemplating moving market risk regulations from VaR to CVaR, but it appears as though they will adopt a 97.5% confidence level instead of

²⁸Rockafellar and Uryasev (2000, 2002).

the 99% confidence level used for VaR.²⁹ The difference between VaR and CVaR at these confidence levels will be negligible since it will be assumed that equity price fluctuations are normally distributed and the problem will be reduced to a representative equity price. These two measures of risk are essentially the same in this case and removes any of the differences between VaR and CVaR. CVaR is likely to have different properties compared to VaR in more micro-level scenarios when multiple assets are included in the portfolio and asset price movements are non-normally distributed. However, this paper focuses on the macroeconomic implications of risk-based capital requirements and not about the differences between these two risk measures. All that is needed to study this is a risk measure that depends on volatility. VaR accomplishes exactly this in very simple manner. Macroprudential effects of CVaR compared to VaR are studied in chapter 3.

One further note about risk management systems that rely on statistical measures of risk is that they treat volatility as exogenous. This neglects any behavior that market participants may have on market prices which could create extra feedback³⁰. Relying on these types of risk practices will always be imperfect since they do not account for this pecuniary externality.

Risk-Weighted Capital Regulations

Risk-based capital regulations require banks to calculate the denominator of their risk-based capital-asset ratio as the sum of its risk-weighted assets. Assets are converted to risk-weighted assets by multiplying the measure of risk for each asset by the inverse of the target risk-weighted capital-asset ratio. The Federal Reserve now considers the minimum risk-weighted capital-asset ratio to be 8% but could also include an extra 2.5% capital conservation buffer so that the minimum is 10.5%.³¹

Market risk is defined as losses to trading positions that could result from market movements that affect interest rates, credit spreads, equity prices, exchange rates, or commodity

²⁹Basel Committee on Banking Supervision (2013).

³⁰Danielsson et al. (2004) and Shin (2010).

³¹Federal Register Vol. 77 No. 169 (2012).

prices. A trading position is defined as “a position that is held by a bank for the purpose of short-term resale or with the intent of benefiting from actual or expected short-term price movements or to lock in arbitrage profits.” In what follows, this paper focuses solely on market risk applied to equity prices. The market risk capital rule, which applies to any bank with “aggregate trading assets and liabilities equal to: i) 10 percent or more of quarter-end total assets, or ii) \$1 billion or more...or any bank deemed necessary or appropriate because of the level of market risk of the bank or to ensure safe and sound banking practices,” states that “a bank’s VaR-based capital charge be equal to the greater of 1) the previous day’s VaR-based measure or 2) the average of the daily VaR-based measures for each of the preceding 60 days multiplied by 3 or a higher factor based on the back-testing of the bank’s modeling of its VaR.”³² Banks are allowed to use internal models to calculate its VaR with approval from its regulator, in which case VaR must be calculated with a 10-day holding period and one-tail 99% confidence level estimated from a period of at least the past year of historical data. While many internal models tend to be proprietary to the individual firm, there are three basic approaches to compute VaR: the variance-covariance method, the historical simulation method, and the Monte Carlo simulation method.

Under the variance-covariance method, assets are standardized and the variance and covariances of the assets in the trading book are calculated using historical data. However, some assumptions about how returns are distributed are needed, and a convenient one that is often used is normality. The strength in this approach is its simplicity. Although, it suffers from some major weaknesses in assuming normality and homoskedasticity in the variance of returns. If these are wrong, and often are, VaR will understate risks. Extended models attempt to use non-normal distributions and GARCH (generalized autoregressive conditional heteroskedasticity) approaches to address these weakness. However, non-normal approaches tend to still require that transformations of the returns distribution fall into a multivariate

³²Federal Register Vol. 77 No. 169 (2012) describes the total market risk capital charge being equal to the sum of the VaR-based capital charge, stressed VaR-based capital charge, specific risk capital charge, incremental risk capital requirement, comprehensive risk capital requirement, and capital charge for *de minimis* exposures. The only capital charged considered in this paper is the VaR-based capital charge.

normal distribution.

The historical simulation method requires creating a time series of the returns of the portfolio using historical data. This approach does not require calculating the variance and covariances of each asset, because changes in the portfolio provide all the required information needed to calculate VaR and thus, does not rely on the normality assumption. However, it does rely on the assumption of the past being a good predictor for the future and also requires choosing the appropriate length of data history. Choosing the length of data history can be a difficult task, because it should be representative of the business cycle but also needs to reflect current economic conditions. This approach obviously has a difficult time evaluating risks for new assets with very little or no historical data. Attempts to improve this approach have considered weighting the recent past more heavily to reflect current economic conditions, using ARIMA models to forecast VaR as they are more sensitive to volatility changes, and scaling data to reflect volatility changes using GARCH.

The last method, Monte Carlo simulation, requires specifying probability distributions for all market risk factors relevant to the portfolio and how they move together then simulating the outcomes. The power in this method is the freedom to use distributions other than the normal distribution, where historical data is often used to inform the appropriate distributional choice if the normal distribution is not assumed. It is also flexible enough to cover non-linearity problems that options induce into returns distributions. This approach can become very difficult if the number of risk factors becomes large as the number of distributions to estimate and simulate becomes burdensome. Attempts to improve upon this approach include running simulations on a limited number of scenarios to reduce the computational burden and applying the variance-covariance method with the normality assumption to speed up the computation.³³

Each of these approaches has strengths in either speed of calculation or distributional freedom, but all rely on selecting the appropriate historical data window to calibrate some

³³Damodaran (2007).

aspect of the model, subjecting them to business cycle movements to varying degrees. Because VaR is calculated in this manner, it is essentially a local measure of risk and has the potential to have procyclical and amplifying effects on bank balance sheets since financial markets are generally more stable in expansions but more volatile in downturns. Plus, internal VaR models have the potential to be very similar across banks with the standardization of the methodology, creating very little model risk diversification. This could cause banks to act in a coordinated fashion to changes in financial market volatility and amplify price volatility.

The model estimated here is done using data at a quarterly frequency so VaR will be calculated according to quarterly price fluctuations. The calculation of the risk-weighted capital-asset ratio in the model is as follows. Loans are assumed not to be risky in the model; however, banks will still be required to hold capital against them. The capital charge on loans at steady state will be the target risk-weighted capital-asset ratio multiplied by the size of the loan book:

$$K^{lb} = v_b B$$

The capital charge on the trading book at steady state is calculated using the market risk capital rule described above:³⁴

$$K^{tb} = M \cdot VaR$$

Total bank capital at steady state is then the sum of the two capital charges:

$$K^b = K^{lb} + K^{tb} = v_b B + M \cdot VaR$$

Risk-weighted assets at any time t must be:

$$RWA_t = B_t + \frac{M}{v_b} \cdot VaR_t \tag{2.10}$$

³⁴ M can be thought of as a safety factor. If the holding period is 10 days and the confidence level is 99%, then a loss greater than VaR would be expected to occur once every 4 years. Arguably, financial stress events would happen far too often under this scenario. Thus, multiplying by M should lessen the likelihood of such losses. M can also be thought of as increasing the time horizon for the holding period or the confidence level.

to be consistent with the steady state risk-weighted capital-asset ratio being equal to the target:

$$\frac{K^b}{RWA} = v_b$$

Equation (2.10) implies a risk-weight on entrepreneur loans of one, which is consistent with the Basel Committee standards for corporate exposures.

Bank leverage is related to the inverse of the risk-weighted capital-asset ratio and will be defined using Adrian and Shin's definition as total assets over equity:

$$L_t = \frac{B_t + q_t^e s_t^b}{K_t^b} = \frac{B_t + q_t^e s_t^b}{B_t + q_t^e s_t^b - D_t} = \frac{A_t^b}{A_t^b - D_t^b} \quad (2.11)$$

where s_t^b is the size of the bank's trading book. To see how leverage changes with bank asset size (A_t^b) and debt, take the partial derivative of L_t with respect to A_t^b and then again with respect to D_t :

$$\begin{aligned} \frac{\partial L_t}{\partial A_t^b} &= -\frac{D_t}{(K_t^b)^2} < 0 \\ \frac{\partial L_t}{\partial D_t} &= \frac{D_t + K_t^b}{(K_t^b)^2} > 0 \end{aligned}$$

Thus, increases in total assets will cause leverage to fall, but an increase in debt will cause leverage to increase. An increase in total assets that is funded completely with debt will cause leverage to increase as $|\partial L_t / \partial D_t| > |\partial L_t / \partial A_t^b|$. If bank leverage remains constant or increases with total assets, then this is a signal of active balance sheet management that is observed of both U.S. chartered depository institutions and security broker dealers.

2.5.3 Wholesale Branch

The wholesale branch is perfectly competitive and manages the capital position of its combined bank: deposit branch i , loan branch i , and wholesale branch i . It combines bank capital K_t^b (bank equity) with wholesale deposits D_t received from the deposit branch sector at the central bank policy rate r_t^{cb} and issues wholesale loans B_t to the loan branch sector

at the interbank rate r_t^{ib} . It also holds a portfolio of equity securities $s_{j,t}^b$. The central bank policy rate can be thought of as the Federal Reserve's target federal funds rate, while the interbank rate can be thought of as the effective federal funds rate.

From the perspective of the wholesale branch, bank capital and wholesale deposits are perfect substitutes as sources of funds. Gerali et al. (2010) use a quadratic adjustment cost on the capital-asset ratio to pin down the steady state choice of bank capital. In their model, the capital-asset ratio consists of loans and bank capital only and is not risk-weighted since risk is absent from the model. This paper builds on this concept by making capital requirements risk-weighted according to current regulatory capital requirements in conjunction with a Value-at-Risk constraint. The target risk-weighted capital-asset ratio will be regulator determined and denoted v_b . The VaR constraint is used to pin down the size of banks' trading books, but it also pins down the steady state level of bank capital adding an extra degree of freedom to the steady state calibration. The quadratic adjustment cost on the risk-weighted capital-asset ratio will link risk-weighted balance sheet conditions and the interbank market.

The addition of these two elements allows for the analysis of the contribution of banks' VaR on the dynamics of the model economy through its interaction with the risk-weighted capital-asset ratio. This mechanism also helps capture the trade-off involved with managing bank resources. If the capital-asset ratio is too high, banks may be able to earn higher profits by reducing it, or if the capital-asset ratio is too low, banks may face punishment from financial markets in terms of higher borrowing costs on the interbank market or fines from regulators.

The maximization program for the wholesale branch is to maximize period profits:

$$\max_{\{B_t, D_t, s_{j,t}^b\}} r_t^{ib} B_t - r_t^{cb} D_t + \int_j E_t [q_{j,t+1}^e \pi_{t+1} - q_{j,t}^e + \delta_j^e \Pi_{j,t}^R] s_{j,t}^b dj - \frac{\kappa_{Kb}}{2} \left(\frac{K_t^b}{RWA_t} - v_b \right)^2 K_t^b \quad (2.12)$$

subject to the marked-to-market balance sheet identity:

$$B_t + \int_j q_{j,t}^e s_{j,t}^b dj = D_t + K_t^b \quad (2.13)$$

where equity securities are valued at the market price q_t^e . The index j denotes the equity price and shares from retailer j . RWA_t stands for risk-weighted assets and is defined as in the previous section.

The problem for this branch can be boiled down to how much to invest in riskless loans (the loan book) and a portfolio of risky equity securities (the trading book) given its deposits and capital level. The first step is to substitute the balance sheet identity in for D_t to get:

$$\begin{aligned} \max_{\{B_t, s_{j,t}^b\}} & (r_t^{ib} - r_t^{cb}) + r_t^{cb} K_t^b B_t + \int_j E_t [q_{j,t+1}^e \pi_{t+1} - q_{j,t}^e + \delta_j^e \Pi_{j,t}^R - r_t^{cb} q_{j,t}^e] s_{j,t}^b dj \\ & - \frac{\kappa_{Kb}}{2} \left(\frac{K_t^b}{RWA_t} - v_b \right)^2 K_t^b \end{aligned}$$

The wholesale branch will be assumed to make two decisions. The first is to choose the size of its loan book (B_t). The second is to choose the size of its trading book (s_t^b) given market prices subject to risk limits set by wholesale branch management. This means that only the trading desk, and hence the trading book first-order condition, will be subject to the VaR constraint.³⁵

Loan Book

This section defines the major contribution made in this paper. It will be shown that risk-weighted capital requirements and Value-at-Risk can affect the flow of funds through the interbank market by affecting the interbank interest rate spread. This section also derives a VaR constraint similar to those used in Shin (2010); Danielsson et al. (2011); and Adrian

³⁵This assumption prevents the Lagrange multiplier from appearing in the first-order condition for wholesale loans and deposits. With the VaR constraint assumed to bind in the steady state, this Lagrange multiplier will be positive. Thus, excluding it from the loan book optimization ensures that the steady state target rate equals the steady state interbank rate to be consistent with the observation that the Federal Reserve's target rate equals the effective federal funds rate on average.

and Boyarchenko (2015) that will act as the the bank's trading book demand. This equation will determine banks' risk management strategy in response to changes in asset prices and financial market volatility.

The first-order condition for wholesale loans (the loan book) is:

$$[B_t, D_t] : r_t^{ib} = r_t^{cb} - \kappa_{Kb} \left(\frac{K_t^b}{RWA_t} - v_b \right) \left(\frac{K_t^b}{RWA_t} \right)^2 \quad (2.14)$$

Equation (2.14) determines the interbank rate spread $r_t^{ib} - r_t^{cb}$ and is similar to Gerali et al. (2010) with the exception of risk-weighted assets. The appearance of risk-weighted assets in this condition allows for risk and bank balance sheet conditions to affect the cost of interbank funds. Kapan and Minoiu (2013) and Goldberg et al. (2010) find evidence that banks' balance sheet structure affects their cost of funds and ability to access wholesale funding markets. Banks with higher, less risky, and better quality capital are better able to maintain access to wholesale funding and at a lower cost. In addition, Brei et al. (2013) show that banks with higher regulatory capital ratios increase lending during normal markets. Equation (2.14) captures these observations.

In steady state, the interbank rate spread will be zero. If the risk-weighted capital-asset ratio falls below the target, the interbank rate spread will increase as banks' balance sheets appear riskier when they are undercapitalized on a risk-weighted basis. One way this happens is with an increase in financial market volatility that raises the trading book variance. When volatility increases, then before any other balance sheet adjustments occur, RWA increases as VaR increases, lowering the risk-weighted capital-asset ratio below the target v_b . Thus, in the sense that the risk-weighted capital-asset ratio is an indicator for bank soundness, changes in this ratio will reflect changes in a bank's perceived default probability, affect its interbank borrowing costs, and make it more difficult for the bank to borrow.

Again, it is worth noting that the use of adjustment costs to derive the interbank rate condition is ad hoc. Although, it could be seen as a shorthand to a more formal microfounded approach. For example, Adrian and Shin (2013) set up a contracting problem between a bank and creditor to derive a rationale for the behavior of banks that can be described with VaR

constraints. Their solution endogenously solves for bank leverage, asset size, and the repo interest rate while determining the conditions that give rise to VaR as a contracting outcome.

Value-at-Risk Constraint

With the appropriate capital allocated for wholesale loans determined by equation (2.14), wholesale branch management then uses the rest of the capital for the trading desk to invest in a portfolio of risky equity securities subject to a Value-at-Risk constraint. Banks' equity holdings are inherently risky due to price fluctuations from market activity. Because the equity price is determined in a stochastic general equilibrium framework, the model implies an endogenous equity price variance, which can be derived from the first-order Taylor approximation of the equity price condition (2.6) around the steady state:

$$\begin{aligned} q_t^e - q^e &\approx \frac{q^e}{\lambda^H}(\lambda_{t+1}^H - \lambda^H) - \frac{q^e}{\lambda^H}(\lambda_t^H - \lambda^H) + \beta_H(1 - F)(q_{t+1}^e - q^e) + \beta_H\delta^e(\Pi_t^R - \Pi^R) \\ &\quad - \beta_H\delta^e\Pi^R(\pi_{t+1} - \pi) + \epsilon_t^q \end{aligned}$$

Squaring both sides and taking the expectation produces the variance of q_t^e as a function of other variances and covariances:

$$\begin{aligned} \sigma_{t,q^e}^2 &\approx \left(\frac{(\beta_H\delta^e)^2}{1 - (\beta_H(1 - F))^2} \right) \sigma_{\Pi^R}^2 + \left(\frac{(\beta_H\delta_e\Pi_R)^2}{1 - (\beta_H(1 - F))^2} \right) \sigma_\pi^2 + \sigma_{\epsilon_t^q}^2 \\ &\quad + 2 \left(\frac{\beta_H^2(1 - F)\delta_e}{1 - (\beta_H(1 - F))^2} \right) \cdot Cov(q^e, \Pi^R) - 2 \left(\frac{\beta_H^2(1 - F)\delta^e\Pi^R}{1 - (\beta_H(1 - F))^2} \right) \cdot Cov(q^e, \pi) \\ &\quad - 2 \left(\frac{(\beta_H\delta^e)^2\Pi^R}{1 - (\beta_H(1 - F))^2} \right) \cdot Cov(\Pi^R, \pi) \end{aligned}$$

where ϵ_t^q is assumed to be independent of all other variables, and variables without a subscript t represent steady state values. Covariances can be solved for in a similar fashion.³⁶ Since the model is stationary, this variance (and all other covariances and variances) will be constant at any point in time unless there is a shock to $\sigma_{\epsilon_t^q}$. Once the complete model is linearized,

³⁶Instead of squaring both sides, multiply both sides by $(x_t - x)$ and then take expectations to get the covariance between any variable x_t and q_t^e .

all endogenous variances and covariances can be solved for and σ_{q^e} can be determined.³⁷ The equity price shock, ϵ_t^q , then creates excess volatility in the equity price beyond what the fundamental value is responsible for and can be broken down into two terms: the fundamental variance and the transitory variance represented by $\sigma_{\epsilon_t^q}^2$. Therefore, banks have an endogenous motive to manage risks with VaR and financial regulators also have motive to institute risk-weighted capital requirements to ensure safe and sound banking practices.

This idea can be illustrated in figure 2.4. If each period t is one quarter of a year, then there are higher frequency price movements that happen in the secondary market between periods. The equity price q_t^e , containing both the fundamental and transitory components, can be thought of at the weekly frequency (solid black line), while the fundamental value can be thought of as the quarterly average (dashed blue line). Financial market volatility shocks then can be imagined to come from exogenous market activity not explained by the fundamental component of the equity price.

To see how the VaR constraint can be applied to the model at hand, first combine the profits from all three branches:

$$\Pi_t^b = r_t^b b_t - r_t^d d_t + \int_j (q_{j,t+1}^e \pi_{t+1} - q_{j,t}^e + \delta_j^e \Pi_{j,t}^R) s_{j,t}^b dj - Adj_t^B \quad (2.15)$$

where Adj_t^B contains all bank adjustment cost terms. Bank capital will be assumed to be accumulated out of retained earnings according to:

$$K_{t+1}^b \pi_{t+1} = (1 - \delta^b) K_t^b + \Pi_t^b + \epsilon_t^{Kb} \quad (2.16)$$

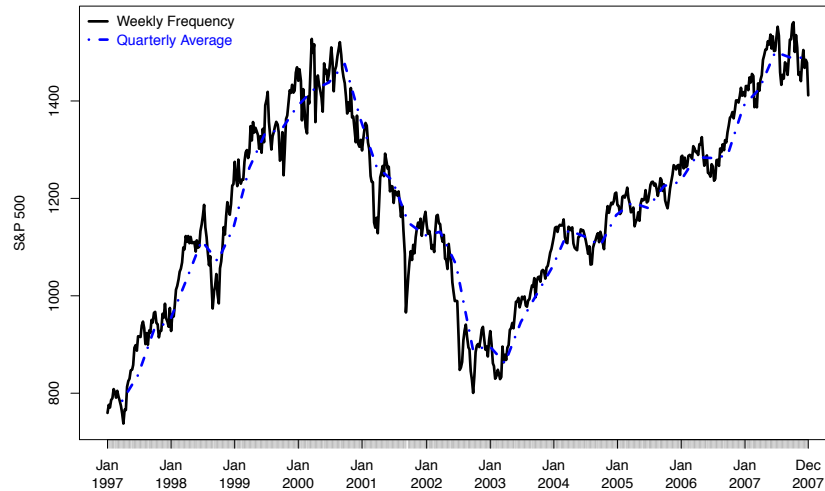
and is assumed to depreciate at a constant rate δ^b meant to capture bank management costs.

Without this assumption, banks can accumulate an infinite amount of capital and become self financing.³⁸ ϵ_t^{Kb} represents a shock to the bank's ability to retain earnings and can be

³⁷The variance of the representative equity price is not endogenized since the interest is in shocking this variable and not much would be gained from doing so in this context. However, if one were to do so, shocking $\sigma_{\epsilon_t^q}$ could create extra feedback effects increasing σ_{t,q^e} more than the shock itself.

³⁸One other method that has been used in the literature to avoid this issue is to assume that banks are finitely lived and exit the market with some exogenously given probability. Following Gerali et al. (2010), the bank capital depreciation method is used for its simplicity and intuitive interpretation.

Figure 2.4: S&P 500



Note: Data are from 1997-2007 found at Yahoo! Finance.

thought of as unforeseen internal losses from operational risk if the shock is negative.

In order for banks to remain in operation, a bank's capital must remain positive so that the value of its assets is larger than its liabilities:

$$K_{t+1}^b \geq 0 \implies \Pi_t^b \geq -(1 - \delta^b)K_t^b$$

Combine bank profits (2.15) with this last constraint and subtract expected losses from the loan book ($v_b B_t$) and the trading book ($Z\sigma_{t,pf}$). Finally, trading profits and expected losses need to be multiplied by M to arrive at the VaR constraint consistent with the risk-weighted

capital-asset ratio:

$$r_t^b b_t - v_b B_t - M \cdot VaR_t - r_t^d d_t + \int_j \delta_j^e \Pi_{j,t}^R s_{j,t}^b dj - Adj_t^B \geq -(1 - \delta^b) K_t^b$$

$$VaR_t = \int_j E_t [q_{j,t}^e - q_{j,t+1}^e \pi_{t+1}] s_{j,t}^b dj + Z \sigma_{t,pf}$$

$$\sigma_{t,pf}^2 = \int_j \int_i \rho_{ji} \sigma_j \sigma_i s_{j,t}^b s_{i,t}^b di dj$$

where ρ_{ji} is the correlation between equities i and j and σ_j is the standard deviation of equity price j . $\int_j \delta_j^e \Pi_{j,t}^R s_{j,t}^b dj$ are the riskless profits from dividend payments, while $v_b B_t$ and $M \cdot VaR_t$ are expected losses on the loan book and the trading book consistent with risk-weighted capital requirements in steady state. These regulations effectively reduce the amount of risk that financial institutions are allowed to take for any level of capital. The VaR constraint then acts like the bank's risk management strategy and implies that the bank targets a fixed probability of default, which is determined by the target risk-weighted capital-asset ratio, v_b , the confidence level associated with Z and the regulatory multiple M .

For simplicity, equity price fluctuations are assumed to be normally distributed, so VaR and the trading book variance can be written as above. Under this specification, VaR is positive when trading losses are expected. Z is a constant determined by the relevant confidence level c which is set to 99% according to regulatory risk practices. This implies that $Z = 2.3264$.

Combining bank profits (2.15), bank capital accumulation (2.16), and the VaR constraint and then evaluating the result at the steady state, reveals:

$$K^b \geq v_b B + M \cdot VaR$$

showing that bank capital in steady state must be at least as large as the total capital charge. The VaR constraint will be assumed to hold with equality in the region near the steady state so $K^b = v_b B + M \cdot VaR$, which is consistent with the risk-weighted capital-asset ratio.

Trading Book

With the VaR constraint now derived, the trading desk optimization of the wholesale branch can be defined. The trading desk's problem is to maximize period profits (2.12) subject to the VaR constraint and the balance sheet identity (2.13). Each equity share choice of the trading desk depends on the idiosyncratic characteristics of each equity security (δ_j^e , q_j^e , σ_j , ρ_{ji}). To simplify the analysis, a symmetric equilibrium will be assumed for all sectors of the economy resulting in conditions:

$$\begin{aligned} \delta_i^e = \delta_j^e = \delta^e &\implies q_{i,t}^e = q_{j,t}^e = q_t^e \\ \sigma_i = \sigma_j = \sigma, \rho_{ij} = \rho_{ik} = \rho &\implies s_{i,t}^b = s_{j,t}^b = s_t^b \end{aligned}$$

This assumption reduces the problem to a representative equity security corresponding to a market index (i.e. S&P 500), thus abstracting from portfolio shuffling and allows wholesale bank profits and the VaR constraint to be written more succinctly as:

$$\max_{\{B_t, s_{j,t}^b\}} (r_t^{ib} - r_t^{cb}) B_t + E_t [q_{t+1}^e \pi_{t+1} - q_t^e + \delta^e \Pi_t^R - r_t^{cb} q_t^e] s_t^b - \frac{\kappa_{Kb}}{2} \left(\frac{K_t^b}{RWA_t} - v_b \right)^2 K_t^b + r_t^{cb} K_t^b$$

$$r_t^b b_t - v_b B_t - M \cdot VaR_t - r_t^d d_t + \delta^e \Pi_t^R s_t^b - Adj_t^B \geq -(1 - \delta^b) K_t^b \quad (2.17)$$

$$VaR_t = E_t [q_t^e - q_{t+1}^e \pi_{t+1}] s_t^b + Z \sigma_{t,pf} \quad (2.18)$$

$$\sigma_{t,pf} = \sigma_t s_t^b (1 + 2\rho)^{1/2} \quad (2.19)$$

where ρ will be set to 0 for convenience. The volatility term σ_t that shows up in the VaR equation (2.18) represents the standard deviation of the representative equity price and will be assumed to change only if there is a shock to transitory volatility. Assuming a symmetric equilibrium in the wholesale market of the financial sector implies that this paper models an exaggerated case in which there is zero diversification of internal VaR-models across banks. The consequence of this is that all banks will react identically to changes in financial market

volatility, which is one of the concerns about VaR-based capital regulations, and is not that unrealistic of an assumption.

The first-order condition for the bank's trading book size is:

$$\begin{aligned} [s_t^b] : E_t [q_{t+1}^e \pi_{t+1} - q_t^e + \delta^e \Pi_t^R] - r_t^{cb} q_t^e - \frac{\partial Adj_t^{Kb}}{\partial s_t^b} \\ + \lambda_t^b \left(-M \cdot (E_t [q_t^e - q_{t+1}^e \pi_{t+1}] + Z \sigma_t) + \delta^e \Pi_t^R - \frac{\partial Adj_t^{Kb}}{\partial s_t^b} \right) = 0 \end{aligned} \quad (2.20)$$

$$\frac{\partial Adj_t^{Kb}}{\partial s_t^b} = -\kappa_{Kb} \left(\frac{K_t^b}{RWA_t} - v_b \right) \left(\frac{K_t^b}{RWA_t} \right)^2 \frac{M}{v_b} (E_t [q_t^e - q_{t+1}^e \pi_{t+1}] + Z \sigma_t)$$

where λ_t^b is the Lagrange multiplier on the VaR constraint. This shows that the the first-order condition for the bank's trading book size does not determine the size of the trading book in steady state. Instead, the trading book size is determined by the VaR constraint, which will be assumed to hold in the neighborhood of the steady state.

Rearranging this first-order condition for λ_t^b yields:

$$\lambda_t^b = \frac{E_t [q_{t+1}^e \pi_{t+1} - q_t^e + \delta^e \Pi_t^R] - r_t^{cb} q_t^e - \frac{\partial Adj_t^{Kb}}{\partial s_t^b}}{M (E_t [q_t^e - q_{t+1}^e \pi_{t+1}] + Z \sigma_t) - \delta^e \Pi_t^R + \frac{\partial Adj_t^{Kb}}{\partial s_t^b}} \quad (2.21)$$

which has the economic interpretation as the rate of change in expected profits with respect to having to hold another unit of regulatory capital, or the bank's expected return on equity (ROE).³⁹ To see how expected profits change with volatility, take the partial derivative of λ_t^b with respect to σ_t around the steady state:

$$\begin{aligned} \frac{\partial \lambda_t^b}{\partial \sigma_t} &\approx \frac{(r^{cb} q^e - MZ\sigma) \frac{\partial^2 Adj_t^{Kb}}{\partial s_t^b \partial \sigma_t} - MZ q^e (r^b - r^{cb})}{(MZ\sigma - \delta^e \Pi^R)^2} < 0 \\ \frac{\partial^2 Adj_t^{Kb}}{\partial s_t^b \partial \sigma_t} &\approx \kappa_{Kb} \left(\frac{K_b^3}{RWA^4} \right) \left(\frac{MZ}{vb} \right)^2 \sigma > 0 \end{aligned}$$

where $MZ\sigma > r^{cb} q^e$, $\delta^e \Pi^R = q^e r^b$, and $r^b > r^{cb}$ under the steady state conditions which will be discussed in the following section. So, when financial market volatility increases, higher

³⁹Danielsson et al. (2011).

capital is required under VaR-based capital regulations, and λ_t^b declines reflecting a decline in banks' expected ROE. Banks will need to adjust their balance sheets to meet the higher capital requirements and could sell risky securities, decrease lending, or both. With lower market volatility, less bank capital is required under risk-weighted capital regulations and banks will search for more profitable, and possibly risky, uses of its excess capital. λ_t^b then can also be interpreted as the bank's risk appetite, taking into account expectations about future prices and is time varying.⁴⁰

Binding VaR Constraint

Because the first-order condition for the trading book size does not determine the size of the trading book in steady state, the VaR constraint will. This means that the VaR constraint needs to bind in steady state. To ensure this, first note that the expected profits from investing in a portfolio of equity securities is $E_t[q_{t+1}^e \pi_{t+1} - q_t^e + \delta^e \Pi_t^R - q_t^e r_t^{cb}] s_t^b$. The wholesale branch will only have incentive to invest in a portfolio of equity securities in steady state if $\delta^e \Pi^R \geq q^e r^{cb}$.

Now, examining the household's first-order conditions (2.6) and (2.7) in steady state, these equations reduce to:

$$\begin{aligned} [d] : 1 &= \beta_H(1 + r^d) \\ [s^H] : q^e &= \frac{\beta_H \delta^e \Pi^R}{1 - \beta_H(1 - F)} \end{aligned}$$

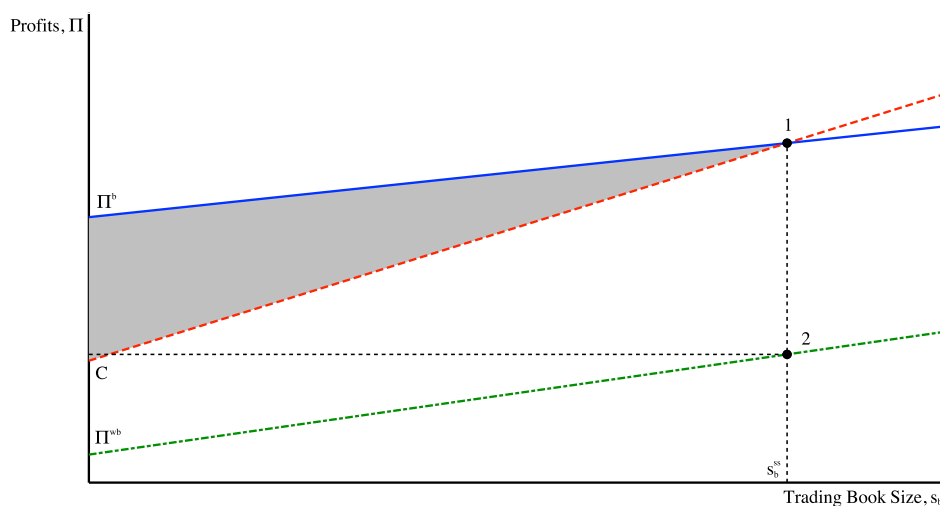
Combining these two conditions implies that:

$$r^d = \frac{\delta^e \Pi^R}{q^e} - F$$

so that the return on deposits equals the dividend yield net of the portfolio management fee. If F were 0, this would imply that $q^e r^d = \delta^e \Pi^R$, but it is required that $q^e r^{cb} \leq \delta^e \Pi^R$. However, with $F = 0$, the household's arbitrage condition would prevent the wholesale branch from

⁴⁰A similar result is found in Daniélsson et al. (2004, 2011) and Shin (2010).

Figure 2.5: Value-at-Risk Constraint



Note: Figure drawn using steady state values.

having an incentive to invest in the risky asset since $r^d < r^{cb}$ if the deposit branch is to have positive profits. Then $\delta^e \Pi^R = r^d q^e < r^{cb} q^e$ violating the necessary condition.

This is where the portfolio management fee becomes important. With $F > 0$, it is possible to obtain a steady state calibration with $\delta^e \Pi^R \geq q^e r^{cb}$. F is calibrated so that the dividend yield equals the return on lending ($\delta^e \Pi^R / q^e = r^b$). This way, the wholesale branch has an incentive to invest in a portfolio of risky equity securities in steady state, but it provides no added benefit over lending to entrepreneurs for the bank as a whole. Setting $F = r^b - r^d$ achieves the desired result. This ensures that $\lambda_t^b > 0$ in steady state, but there is still one more condition required for the VaR constraint to bind in steady state.⁴¹

Figure 2.5 illustrates the bank's problem of allocating its assets between loans to entrepreneurs and investing in a portfolio of risky equity securities subject to the VaR con-

⁴¹Setting $\delta^e \Pi^R = q^e r^{cb}$ so that the wholesale branch is indifferent between lending and investing in a portfolio of risky equity securities in steady state would set $\lambda^b = 0$ in steady state, implying that the VaR constraint is not binding.

straint given the level of deposits. Π^b (the blue line) is total bank profits (2.15), Π^{wb} (the green dot-dashed line) represents wholesale bank profits, and C (the red dashed line) is the effect of the constraint. The equation for C is derived as the right hand side after rearranging the VaR constraint (2.17) to look like $\Pi^b \geq v_b B + MZ\sigma s^b - (1 - \delta^b)K^b$. The equations are then written as a function of the trading book size by taking into account the bank's balance sheet identity (2.13) and the bank capital accumulation equation (2.16) and substituting them into the relevant equations for B , noting that in steady state $K^b = v_b B + MZ\sigma S^b$, $\delta^e \Pi^R = r^b q^e$, and $r^{cb} = r^{ib}$:

$$\begin{aligned}\Pi^b &= \left(\frac{r^b}{1 - v_b} - r^d \right) D + \left(\frac{r^b}{1 - v_b} \right) (MZ\sigma - v_b q^e) s^b \\ \Pi^{wb} &= \left(\frac{r^{cb} v_b}{1 - v_b} \right) D + \left(q^e (r^b - r^{cb}) + \left(\frac{r^{cb}}{1 - v_b} \right) (MZ\sigma - v_b q^e) \right) s^b \\ C &= \left(\frac{\delta^b v_b}{1 - v_b} \right) D + \left(\frac{\delta^b}{1 - v_b} \right) (MZ\sigma - v_b q^e) s^b\end{aligned}$$

The equations show that these functions will increase in the trading book size only if the risk per dollar invested is larger than the ratio of regulatory parameters, $Z\sigma/q^e > v_b/M$, and the constraint will start below bank profits and increase at a faster rate only if $r^b < \delta^b < v_b$, which is satisfied at the steady state with the parameters chosen in this model. If these do not hold, both the constraint and bank profits will be decreasing or non-intersecting and the constraint will not bind. Additionally, wholesale bank profits as a function of the trading book size will increase at a faster rate compared to total bank profits if $q^e > MZ\sigma$, which is also satisfied at the steady state in this paper.

The VaR constraint is satisfied with a trading book size left of the vertical dotted line and above the constraint represented by the grey shaded region. Point 1 denotes the bank's steady state trading book size (s_b^{ss}), and point 2 denotes wholesale bank profits at this point. As a result of the separation of decisions among the three branches, the wholesale branch has an incentive to increase the size of its trading book with less benefit to overall profits and with the added cost of extra balance sheet risk. The VaR constraint then effectively limits the market risk the bank is able to hold on its balance sheet and prevents one branch of

the bank from endangering the operations of the entire bank. This is potentially a concern for financial institutions with a separation-of-decisions structure similar to this model. The trading desk of an institution like this may not take into account its decisions on the entire institution and could take on excess risk, putting the financial stability of the institution in question. Capital regulations, like the ones considered in this paper, are designed with this mind.

Brunnermeier and Sannikov (2014) note that capital constraints may not always bind: they tend to bind in downturns when capital is scarce but have little effect on behavior during expansions when it is much easier to raise capital. In support of this, Lambertini and Uysal (2015) find that banks tend have regulatory capital ratios above the minimum requirements. However, Shin (2010) shows that a simple VaR constraint will always bind if the expected return on the risky asset is above its price. In the model considered here, the VaR constraint will always bind if the expected return on the risky asset is larger than the expected return on interbank loans. Otherwise, banks will not hold any of the risky asset. If the VaR constraint is not binding, banks can take on more balance sheet risk and increase expected profits. Thus, if banks are selecting a mix of assets to maximize profits, their VaR constraint should be binding.

Procyclical Leverage and Asset Demand-Supply Responses

With the VaR constraint (2.17) now shown to bind, rearranging it to get the bank's demand function for the trading book size results in:

$$s_t^b = \frac{(r_t^b - r_t^d - v_b) D_t + (1 + r_t^b - \delta^b - v_b) K_t^b - Adj_t^B}{M (E_t [q_t^e - q_{t+1}^e \pi_{t+1}] + Z \sigma_t) + (r_t^b - v_b) q_t^e - \delta^e \Pi_t^R} \quad (2.22)$$

after substituting in the balance sheet identity (2.13) for B_t . From this, it can be seen that the trading book size depends positively on the overall intermediation spread ($r_t^b - r_t^d$) and the return on bank capital ($1 + r_t^b - \delta^b - v_b$). Higher unit-VaR ($U_t^{VaR} = E_t [q_t^e - q_{t+1}^e \pi_{t+1}] + Z \sigma_t$) also reduces the size of the bank's trading book, which can be seen by differentiating equation

(2.22) with respect to U_t^{VaR} around the steady state:

$$\frac{\partial s_t^b}{\partial U_t^{VaR}} \approx \frac{Ms^b}{v_b q^e - MZ\sigma} < 0$$

This shows that banks will reduce the size of their trading book when financial market volatility increases if $v_b/M < Z\sigma/q^e$, which is satisfied when the VaR constraint binds. Additionally, leverage is inversely related to unit-VaR as in Shin (2010), because $\partial s_t^b/\partial U_t^{VaR} < 0$ and K_t^b is correlated with lagged unit-VaR (U_{t-1}^{VaR}) through the bank capital accumulation equation (2.16). To see this, differentiate the leverage equation (2.11) with respect to U_t^{VaR} around the steady state:

$$\frac{\partial L_t}{\partial U_t^{VaR}} \approx \frac{q^e}{K^b} \frac{\partial s_t^b}{\partial U_t^{VaR}} < 0$$

Therefore, leverage will tend to be procyclical (high when volatility is low, and low when volatility is high), because the wholesale branch is subject to the VaR constraint and actively manages its balance sheet in the context of this model.

Next, to see how the trading book size responds to an increase in the expected future asset price, examine figure 2.6 where the original equilibrium is denoted by point 1. First, it can be seen from the household's first-order condition for q_t^e (2.6) that an increase in the expected period $t + 1$ asset price of one will cause the period t asset price to increase by:

$$\frac{\partial q_t^e}{\partial q_{t+1}^e} \approx \beta_H(1 - F) > 0$$

This corresponds to the increase in the household's asset demand from q_e (black line) to q_e' (blue dash-dot line) in figure 2.6. With improved expectations about future returns, banks' asset demand increases from $s_b(q_e)$ (solid black line) to $s_b'(q_e)$ (solid red line), and the trading book size increases minimally as represented by the movement from point 1 to point 2. However, because bank balance sheets are marked to market and changes in asset prices affect bank capital, changes in bank capital will affect banks' risk appetite. Banks' take advantage of the capital gains that occur in period $t + 1$, and this corresponds to the increase in banks' asset demand from $s_b'(q_e)$ (solid red line) to $s_b''(q_e)$ (blue dash-dot line).

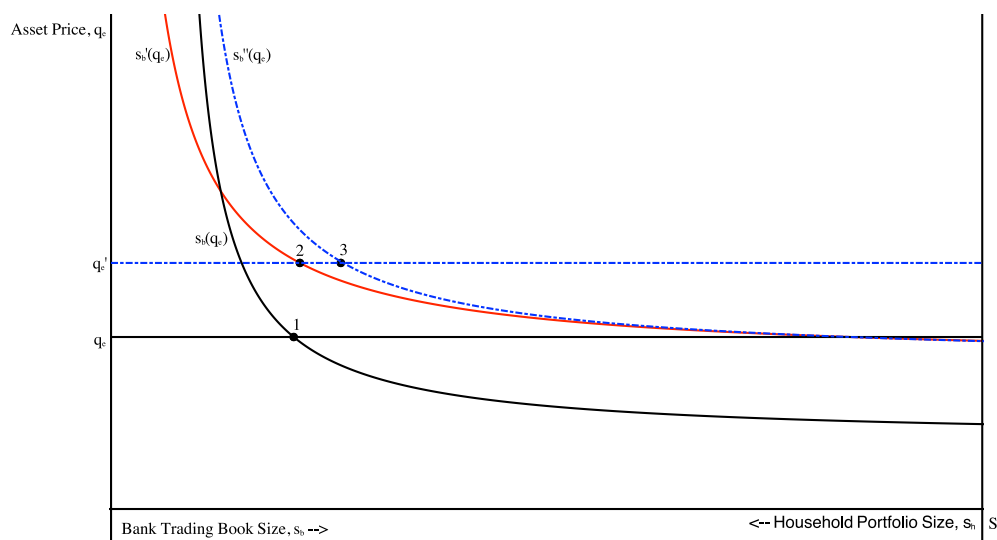
The increase in the period t asset price that results from the increase in the expected period $t + 1$ asset price induces banks to purchase more of the asset and is represented by the movement from point 2 to point 3 in the figure. After taking the partial derivative of s_t^b with respect to q_{t+1}^e around the steady state, it can be seen that banks' asset demand increases in total by:

$$\frac{\partial s_t^b}{\partial q_{t+1}^e} \approx \frac{s^b M}{MZ\sigma - q^e v_b} > 0$$

where s^b , q^e , and σ are steady state values and $MZ\sigma > q^e v_b$ in steady state. Thus, VaR-constrained financial institutions exhibit what Shin (2010) calls an upward-sloping demand response: leveraged financial institutions will purchase more of an asset when the price rises, instead of less, because of the effects that asset price changes have on capital. When this process works in reverse, Shin (2010) refers to this as a downward-sloping supply response since the decrease in the period t price reduces banks capital, risk appetite, and asset holdings and is represented in the movement from point 3 to point 1 in the figure. Since households are assumed to be passive in the equity securities market in this model, changes in banks' asset demand will not affect the price. Only changes in expectations and retailer profits will affect the price, reducing the strength of Shin's (2010) VaR feedback effect here. Thus, this model can be seen as a lower bound case.

What is noteworthy here is that increased volatility in financial markets can induce a feedback effect when leveraged financial institutions are VaR-constrained. Increased financial market volatility raises capital charges and could lower asset prices if expectations about the future equity price falls. Lower asset prices impair balance sheets by reducing bank capital, leading to further selling and downward price pressures. In addition, if liquidity for these

Figure 2.6: Upward-Sloping Demand Response



Note: Figure drawn using steady state values.

risky assets dries up, the effect can be magnified as the bank may violate its VaR constraint.⁴²

⁴²An alternative to using the VaR constraint would be to assume banks use portfolio theory and maximize mean-variance returns. The resulting condition for the trading book size would be: $s_t^b = (E_t [q_{t+1}^e \pi_{t+1} - q_t^e] + \delta^e \Pi^R - r^{cb} q_t^e - \frac{\partial Adj_t^{kb}}{\partial s_t^b}) / (\tau \sigma_t^2)$. There are two drawbacks to this approach. The first is that this condition is not necessarily consistent with capital regulations. Second, the risk aversion parameter τ needs to be calibrated and will strongly affect the model dynamics. The VaR constraint has the advantage of taking into account capital requirements and also provides easier calibration of risk aversion parameters M and Z as they are set by regulation. Shin (2010) also shows that a VaR-constrained investor acts like a mean-variance optimizer but with time-varying risk appetite.

2.6 Calibration and Estimation

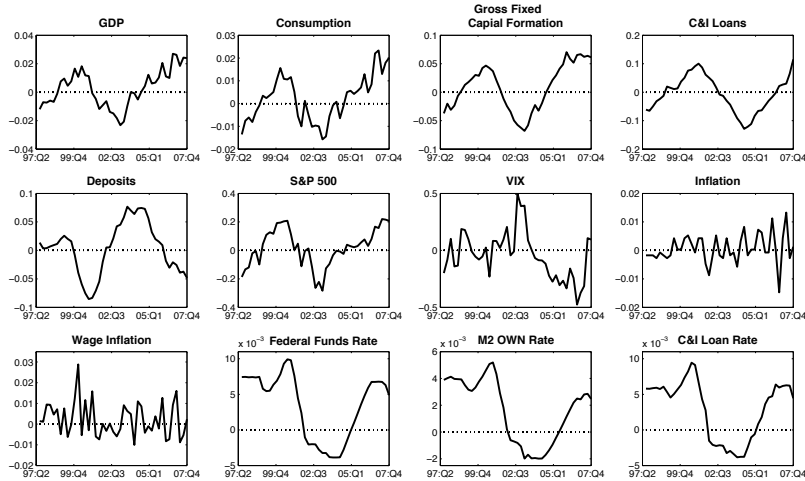
2.6.1 Data and Methodology

The model is log-linearized around the steady and the resulting state-space is used to compute the likelihood function. Bayesian techniques are used to estimate the parameters that affect the dynamics of the model which include the standard deviations and autoregressive coefficients of the shock processes as well as all parameters affecting adjustment costs and the Taylor rule for monetary policy. All shocks ϵ_t^x are assumed to follow AR(1) processes of the form $\epsilon_t^x = (1 - \rho_x)\epsilon_{ss}^x + \rho_x\epsilon_{t-1}^x + e_t^x$. Estimation of the posterior distribution is done using the Metropolis-Hastings Markov Chain Monte Carlo algorithm similar to Smets and Wouters (2007).⁴³ The observables are real output, real consumption, real investment, real deposits, real loans to entrepreneurs, the real equity price, and financial market volatility as well as inflation, wage inflation, the interbank policy rate, the deposit rate, and the entrepreneur loan rate.

All macroeconomic data are taken from the St. Louis Federal Reserve Economic Data (FRED) over the time period 1997:Q2 to 2007:Q4 and include seasonally adjusted gross domestic product, personal consumption expenditures, gross fixed capital formation, commercial and industrial loans, total savings deposits at all depository institutions, the consumer price index, nonfarm business sector compensation per hour, the effective federal funds rate, the M2 OWN rate, and the weighted-average effective loan rate of all commercial and industrial loans. The weighted average rate on commercial and industrial loans limits the time frame since its first observation is 1997:Q2. Data is included up until 2007:Q4 to try to incorporate as much information as possible while this is also near the end of the time frame that a Taylor rule can approximate movements in the federal funds rate reasonably well. This time frame corresponds to the period after which VaR-based capital regulations were implemented and before the start of the 2008 financial crisis. Financial data regarding the

⁴³Calculation of the steady state and Bayesian estimation are computed using Dynare 4.4.3 in conjunction with Matlab R2015b.

Figure 2.7: Observable Macroeconomic Data



Note: Data are from 1997:Q2-2007:Q4. Real variables are logged and detrended using the HP-filter with smoothing parameter set at 1,600 as suggested by Ravn and Uhlig (2002) except for the VIX which is logged and demeaned. All rates are expressed on a quarterly basis and demeaned.

S&P 500 is taken from Robert Shiller's *Irrational Exuberance Online Data* which includes the historical S&P 500 price as well as dividends and earnings data. All nominal data are converted into billions of U.S. dollars and deflated by the consumer price index to convert them into real terms. Variables that exhibit trends are logged and detrended using the Hodrick-Prescott filter with smoothing parameter set at 1600 following the Ravn and Uhlig (2002) suggestion, while all annual rates are demeaned and converted into quarterly rates. More information about the data can be found in appendix B.

2.6.2 Calibrated Parameters

The set of calibrated parameters includes the household and entrepreneur discount factors (β_H, β_E) , the coefficient of relative risk aversion (γ) , the inverse Frisch elasticity of labor supply (ϕ) , the steady state values of all price elasticities $(\epsilon^d, \epsilon^b, \epsilon^y, \epsilon^l)$, the depreciation rates

Table 2.1: Calibrated Parameters

Parameter	Description	Value
β_H	Household Discount Factor	0.995
β_E	Entrepreneur Discount Factor	0.975
γ	Coefficient of Relative Risk Aversion	1.38
ϕ	Inverse Frisch Elasticity of Labor Supply	1.83
α	Capital's Share of Output	0.25
δ	Depreciation Rate of Physical Capital	0.025
δ^b	Bank Capital Management Cost	0.04
δ^e	Dividend Rate	0.51
ϵ^y	Price Elasticity	6
ϵ^l	Wage Elasticity	5
ϵ^d	Deposit Rate Elasticity	-1.21
ϵ^b	Loan Rate Elasticity	3.03
M^E	Firm Loan-to-Value Ratio	0.46
v_b	Target Risk-Weighted Capital-Asset Ratio	0.105
σ	Volatility	0.67
M	Regulatory VaR Multiple	3
Z	VaR _{99%} Constant	2.326

of physical capital and bank capital (δ , δ^b), the target risk-weighted capital-asset ratio (v_b), VaR-based capital charges (M , Z), the dividend rate (δ^e), the steady state loan-to-value ratio for entrepreneurs (M^E), and the share of output paid to capital (α). These are summarized in table 2.1. The household discount rate, β_H , is set to 0.995 in order to obtain a steady state deposit rate equal to the mean of the M2 OWN rate over the sample period which is 2.02% on an annual basis. This also implies that the steady state deposit rate elasticity, ϵ^d , must be set to -1.21. To set the steady state entrepreneur loan rate, the loan rate elasticity, ϵ^b , is set to 3.03 to match the mean of the weighted-average rate on commercial and industrial loans over the sample period, which is 5.52% on an annual basis. To ensure a borrowing motive for entrepreneurs, the calibration follows Gerali et al. (2010) setting the discount factor for entrepreneurs, β_E , to 0.975.⁴⁴

⁴⁴This is also in the range suggested by Iacoviello (2005) and Iacoviello and Neri (2010).

The consumption price elasticity, ϵ^y , is set to 6 to deliver a steady state markup ($\epsilon^y/(\epsilon^y - 1)$) of 20%, and the wage elasticity, ϵ^l , is set to 5 to get a steady state markup ($\epsilon^w/(\epsilon^w - 1)$) of 25% following Gerali et al. (2010). Capital's share of income in the production function, α , is set to 0.25. The values of the coefficient of relative risk aversion (γ), and the inverse Frisch elasticity of labor supply (ϕ) are taken to be the posterior means of Smets and Wouters (2007) estimates which are 1.38 and 1.83 respectively.⁴⁵ Using data from the Board of Governors of the Federal Reserve System on the ratio of total liabilities to total assets for nonfinancial corporate businesses, the loan-to-value ratio for entrepreneurs, M^E , is inferred to be 0.46.

The capital depreciation rate, δ , is set according to what is standard in the business cycle literature at 0.025. Gerali et al. (2010) set the depreciation rate on bank capital, δ^b , to ensure the steady state capital-asset ratio is exactly equal to the target, v_b . However, the presence of the VaR constraint ensures that the risk-weighted capital-asset ratio exactly equals the target in steady state, so there is more freedom to set δ^b . The depreciation rate of bank capital is then set to match the ratio of loans-to-assets for U.S. chartered commercial banks which is 0.67. This ensures that the bank's trading book in the model is not given too much weight on the bank's balance sheet. As a result, δ^b is set to 0.04.

In regards to the financial parameters, the target risk-weighted capital-asset ratio is set to 0.105. This corresponds to the minimum risk-weighted capital-asset ratio of 8% under current capital regulations plus the 2.5% capital conservation buffer. VaR-based capital charge parameters are also set according to current regulations where the multiple on VaR, M , is set to 3, and the corresponding constant associated with a 99% confidence level, Z is set to 2.326. The standard deviation of the representative equity price is calibrated so that σ/q^e equals the standard deviation of the quarterly percentage change of the S&P 500 price. This results in σ being about 8% of the steady state equity price with a value of 0.67. And finally, the dividend rate δ_e is determined by the ratio of dividends-per-share over

⁴⁵Gandelman and Hernández-Murillo (2015) estimate the coefficient of relative risk aversion for multiple countries using GMM techniques and find it to be 1.39 for the U.S.

earnings-per-share which is 0.51.

2.6.3 Prior Distributions

The set of estimated parameters includes all adjustment costs ($\kappa_i, \kappa_p, \kappa_w, \kappa_{Kb}, \kappa_b, \kappa_d$), parameters of the Taylor rule for monetary policy (ϕ_R, ϕ_π, ϕ_y), the inflation and wage indexation parameters (ι_p, ι_w), the degree of consumption habit formation (h), and the standard error and autoregressive coefficients of all shocks ($\epsilon_t^c, \epsilon_t^r, \epsilon_t^b, \epsilon_t^d, \epsilon_t^y, \epsilon_t^l, \epsilon_t^i, \epsilon_t^q, \epsilon_t^{Kb}, A_t^E, M_t^E, \sigma_t$). Prior distributions are chosen to be similar to Gerali et al. (2010) and Smets and Wouters (2007) which can be found in tables 2.2 and 2.3. For the autoregressive coefficients on the shock processes and the degree of habit formation, a beta distribution with a prior mean of 0.5 and standard deviation of 0.15 is used so as to not falsely identify persistence based on prior specification. The inverse gamma distribution with a mean of 0.01 and standard deviation of 0.1 is chosen for the standard error of all shocks. The parameters in the Taylor rule for monetary policy, ϕ_R, ϕ_π , and ϕ_y , are given prior means of 0.75, 2.0, and 0.1 with standard deviations 0.1, 0.5, and 0.15 respectively. The prior distribution for ϕ_π is chosen to be a gamma distribution, for ϕ_R , a beta distribution, and for ϕ_y , a normal distribution. The priors for the inflation and wage inflation indexation parameters, ι_p and ι_w , are chosen to be beta distributions centered at 0.5 with a standard deviation of 0.15 following Smets and Wouters (2007).

Prior distributions for adjustment cost parameters are chosen to be gamma distributions following Gerali et al. (2010). For the investment adjustment cost, κ_i is set with a prior mean of 4 and standard deviation of 1.0 according to Smets and Wouters (2007). The adjustment cost parameters for price and wage setting, κ_p and κ_w , are set with a prior mean of 50 and standard deviation of 20 to be fairly uninformative.⁴⁶ The mean of interest rate setting costs, κ_d and κ_b , are chosen to be 5 with standard deviations of 2.5, and the mean of the

⁴⁶Keen and Wang (2007), who attempt to relate Rotemberg (1982) adjustment costs to Calvo (1983) style frictions, suggest that κ_p depends on the steady state markup and percent of firms reoptimizing prices in a given period. With a steady state markup of 20% as calibrated in this model, they suggest that κ_p fall between 10 and 100 if 20% or more of firms are reoptimizing in a given period.

Table 2.2: Prior and Posterior Distributions of Structural Parameters

Parameter	Description	Prior Distribution			Posterior Distribution				
		Distribution	Mean	Std. Dev.	5%	Mean	Median	95%	Std. Dev
κ_w	Wage Adj. Cost	Gamma	50	20	12.6076	33.4180	31.0991	54.2423	15.0928
κ_p	Price Adj. Cost	Gamma	50	20	11.1650	23.9495	22.9599	35.6001	7.4613
κ_i	Investment Adj. Cost	Gamma	4	1	7.6899	9.9354	9.8792	12.0151	1.3309
κ_{Kb}	Risk-Weighted Capital-Asset Ratio Adj. Cost	Gamma	10	5	1.6616	7.2969	6.6991	12.5668	3.7449
κ_d	Deposit Rate Adj. Cost	Gamma	5	2.5	0.1251	1.1263	0.8916	2.2248	0.9235
κ_b	Loan Rate Adj. Cost	Gamma	5	2.5	0.1011	0.6311	0.4543	1.1256	0.8350
ϕ_π	Taylor Rule Coef. on Inflation	Gamma	2	0.5	2.2712	2.9284	2.8878	3.5414	0.4022
ϕ_R	Taylor Rule Coef. on Lagged Policy Rate	Beta	0.75	0.1	0.8605	0.8926	0.8943	0.9254	0.0201
ϕ_y	Taylor Rule Coef. on Lagged Output	Normal	0.1	0.15	0.1513	0.3756	0.3756	0.6035	0.1398
l_p	Price Inflation Indexation	Beta	0.5	0.15	0.0924	0.2670	0.2541	0.4312	0.1085
l_w	Wage Inflation Indexation	Beta	0.5	0.15	0.2526	0.4859	0.4843	0.7234	0.1403
h	Consumption Habits	Beta	0.5	0.15	0.4309	0.5978	0.6112	0.7700	0.1002

Note: Results based on 16 chains of 100,000 draws each from the Metropolis-Hastings Markov Chain Monte Carlo algorithm.

capital-asset ratio cost, κ_{Kb} , is set to 10 with a standard deviation of 5 to also be fairly uninformative and similar to Gerali et al. (2010).

2.6.4 Posterior Estimation

Results from the posterior estimation can be found in tables 2.2 and 2.3. Draws from the posterior distribution for all estimated parameters are obtained using the Metropolis-Hastings Markov Chain Monte Carlo method of the algorithm where the scale factor is calibrated to achieve an acceptance rate of about 33%.⁴⁷ Sixteen parallel chains with length 100,000 each are used, and convergence is assessed using diagnostics suggested by Brooks and Gelman (1998), which can be found in appendix C.⁴⁸

Most shocks exhibit persistence with AR(1) coefficients above 0.5. The shocks with AR(1) coefficients below 0.5 include the wage elasticity, investment, and equity price shocks. The two inflation indexation parameters are very similar to the estimates found by Smets and Wouters (2007). However, neither the wage or price inflation indexation parameter

⁴⁷In Dynare 4.4.3, this is done using a combination of the `mode compute 6` and `9` options in the estimation command.

⁴⁸These are automatically provided from the estimation command in Dynare 4.4.3. Convergence is determined if the pooled draws from all chains converges to the draws from within individual chains and settles around a particular value.

Table 2.3: Prior and Posterior Distributions of Shock Parameters

Parameter	Description	Prior Distribution			Posterior Distribution				
		Distribution	Mean	Std. Dev.	5%	Mean	Median	95%	Std. Dev
Autoregressive Coefficients									
ρ_{AE}	Technology	Beta	0.5	0.15	0.9408	0.9669	0.9736	0.9943	0.0253
ρ_c	Consumption Preference	Beta	0.5	0.15	0.3914	0.5557	0.5605	0.7242	0.1024
ρ_i	Investment Efficiency	Beta	0.5	0.15	0.2166	0.4069	0.4045	0.6012	0.1162
ρ_y	Price Elasticity	Beta	0.5	0.15	0.9769	0.9859	0.9868	0.9953	0.0058
ρ_l	Wage Elasticity	Beta	0.5	0.15	0.3262	0.4957	0.4978	0.6733	0.1055
ρ_{ME}	Firm Loan-to-Value Ratio	Beta	0.5	0.15	0.9009	0.9334	0.9350	0.9659	0.0197
ρ_d	Deposit Rate Elasticity	Beta	0.5	0.15	0.7606	0.8309	0.8353	0.9033	0.0482
ρ_b	Loan Rate Elasticity	Beta	0.5	0.15	0.7438	0.8443	0.8663	0.9558	0.1106
ρ_{Kb}	Bank Capital	Beta	0.5	0.15	0.3591	0.5193	0.5228	0.6737	0.0966
ρ_σ	Volatility	Beta	0.5	0.15	0.4764	0.6224	0.6257	0.7697	0.0888
ρ_{q^e}	Equity Price	Beta	0.5	0.15	0.1617	0.2459	0.2458	0.3263	0.0504
Shock Standard Deviations									
e^{AE}	Technology	Inverse Gamma	0.01	0.1	0.0125	0.0165	0.0163	0.0203	0.0024
e^r	Monetary Policy	Inverse Gamma	0.01	0.1	0.0015	0.0019	0.0019	0.0023	0.0002
e^c	Consumption Preference	Inverse Gamma	0.01	0.1	0.0233	0.0325	0.0318	0.0416	0.0058
e^i	Investment Efficiency	Inverse Gamma	0.01	0.1	0.0145	0.0205	0.0200	0.0264	0.0036
e^y	Price Elasticity	Inverse Gamma	0.01	0.1	0.2151	0.3902	0.3770	0.5584	0.1047
e^l	Wage Elasticity	Inverse Gamma	0.01	0.1	0.8364	1.8648	1.7395	2.8464	0.7390
e^{ME}	Firm Loan-to-Value Ratio	Inverse Gamma	0.01	0.1	0.0108	0.0144	0.0141	0.0180	0.0022
e^d	Deposit Rate Elasticity	Inverse Gamma	0.01	0.1	0.0578	0.0957	0.0896	0.1333	0.0310
e^b	Loan Rate Elasticity	Inverse Gamma	0.01	0.1	0.0735	0.1092	0.1012	0.1405	0.0402
e^{Kb}	Bank Capital	Inverse Gamma	0.01	0.1	0.0217	0.0273	0.0270	0.0330	0.0035
e^σ	Volatility	Inverse Gamma	0.01	0.1	0.1265	0.1539	0.1522	0.1806	0.0167
e^{q^e}	Equity Price	Inverse Gamma	0.01	0.1	0.6095	0.7538	0.7453	0.8998	0.0877

Note: Results based on 16 chains of 100,000 draws each from the Metropolis-Hastings Markov Chain Monte Carlo algorithm.

exhibits persistence with estimated posterior medians of 0.48 and 0.25 respectively. That inflation persistence is estimated to be so low is not necessarily surprising given that the gross inflation rate in figure 2.7 appears strongly mean reverting. The degree of consumption habit formation is fairly strong with an estimated posterior median of 0.61. For monetary policy, there appears to be a high degree of persistence as ϕ_R has a posterior median of 0.89. The posterior median of the monetary policy response to inflation, ϕ_π , is estimated to be 2.89 while the response to output fluctuations, ϕ_y , is estimated to be 0.38.

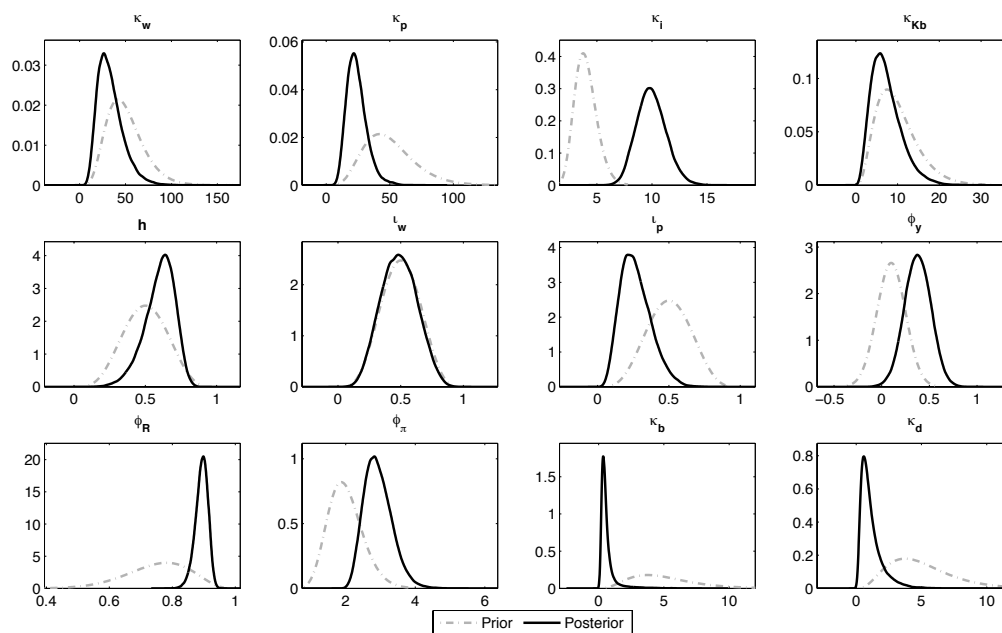
Estimation of the financial adjustment cost parameters suggest very little frictions at the aggregate level for both deposit and loan rate setting. The posterior medians of κ_d and κ_b are estimated to be 0.89 and 0.45 respectively. This is not necessarily surprising since both rates appear to move closely with the federal funds rate as can be seen in figure 2.7. The

posterior median of adjustment costs on the risk-weighted capital-asset ratio, κ_{Kb} , is much higher than that of interest rate setting costs at 6.7. However, this could be due to weak identification, a problem Gerali et al. (2010) also face.⁴⁹ Identification analysis did indicate that κ_{Kb} had some identification power but that it has the weakest identification of all the adjustment cost parameters. Figure 2.8 shows the posterior distribution did move away from the prior distribution somewhat. After experimenting with smaller and larger prior means for κ_{Kb} , the log posterior likelihood is found to be relatively flat for this parameter. The posterior mean moved to the original estimated mean when the prior mean was set at 15 but moved lower when set at 5. The log data density at the original posterior estimation was 1439 but was slightly lower at 1438 when the prior mean was set at 5 or 15. Therefore, setting κ_{Kb} at 6.7 seems to best fit the data.

Investment adjustment costs have an estimated posterior median of 9.88, somewhat higher than the Smets and Wouters (2007) estimate. And as for nominal rigidities on price and wage setting, wage frictions are estimated to be stronger than price frictions. κ_w has an estimated posterior median of 31.10 whereas κ_p has an estimated posterior median of 22.96. The findings that wage rigidities are larger than that of price rigidities is consistent with results found in other studies mentioned in 2.4.

⁴⁹Identification strength for an estimated parameter is assessed using Dynare's identification command and by the movement of the posterior distribution away from the prior distribution. The output is shown in figure ?? in appendix in C and in figure 2.8 respectively.

Figure 2.8: Posterior Distribution of Structural Parameters



Note: Results based on 16 chains of 100,000 draws each from the Metropolis-Hastings Markov Chain Monte Carlo algorithm.

2.7 Performance and Simulations

2.7.1 Performance Relative to the Data

The performance of the model relative to the data is assessed from the matrix of correlations for variables discussed in the empirical section between 1990:Q1-2014:Q4. The relationships of interest include the correlations of security broker dealer leverage with investment, leverage with commercial and industrial loans, leverage with financial market volatility represented by the VIX, leverage with the size of security broker dealers' corporate equity holdings (trading book size), and volatility with trading book size.

Overall, the model matches the target correlations fairly well, getting the sign right for

Table 2.4: Correlations: Model vs. Data

	Leverage		Investment		Loans		Volatility	
	Model	Data	Model	Data	Model	Data	Model	Data
Investment	0.44	0.61						
Loans	0.64	0.11	0.72	0.11				
Volatility	-0.20	-0.31	-0.01	-0.34	-0.01	0.58		
Trading Book Size	-0.60	0.49	-0.42	0.27	-0.60	-0.25	-0.29	-0.22

Note: Data correlations are logged and HP-filtered between 1990:Q1-2010:Q2.

four of them. Importantly, the model captures the procyclicality of leverage documented by Adrian and Shin (2010) evidenced by the positive correlation between leverage and investment. However, the model does not quite capture the magnitude of this relationship, nor does it quite capture the magnitude of the relationship between leverage and volatility. The model is able to fully capture the relationship between volatility and trading book size but does not get the direction correct for the relationship between leverage and trading book size. However, as will be shown in the next subsection, leverage and trading book size will move together when the model is simulated in response to a volatility shock. This suggests that the Value-at-Risk constraint is able to capture the relationship between volatility and trading book size. In addition, the volatility of investment and loans compared to a model where banks do not hold a trading book, and hence a model without the VaR constraint, is increased by 25.3% and 31.2% respectively. Therefore, leveraged VaR-constrained financial institutions may contribute to higher business cycle volatility.

The model overshoots the relationship between leverage and loans and between loans and investment. This is likely due to the fact the model only provides entrepreneurs with one source of outside funding: bank loans. The model does not allow entrepreneurs to raise funds via capital markets, which is an important source of funding for many firms. In fact, Adrian et al. (2012) note that firms which had access to capital markets were able to make up a large portion of the decline in lending by issuing corporate debt following the 2008

financial crisis. The model falls short in two other areas as well, namely the correlations of leverage with trading book size and trading book size size with investment. In these two cases, the model produces correlations with the opposite signs as those observed in the data. This may be due to oversimplified bank balance sheets in the model as they only have two assets and one source of debt to manage leverage. In the model, deposits are the only source of funding since short-term debt like repos, which are an important source of funding for financial institutions, are not modeled. These results suggest that modeling the trading book demand with a Value-at-Risk constraint and including risk-weighted capital requirements via the adjustment cost mechanism might be a reasonable way to capture the stylized facts discussed previously. This will be explored further in the next subsection.

2.7.2 Simulations

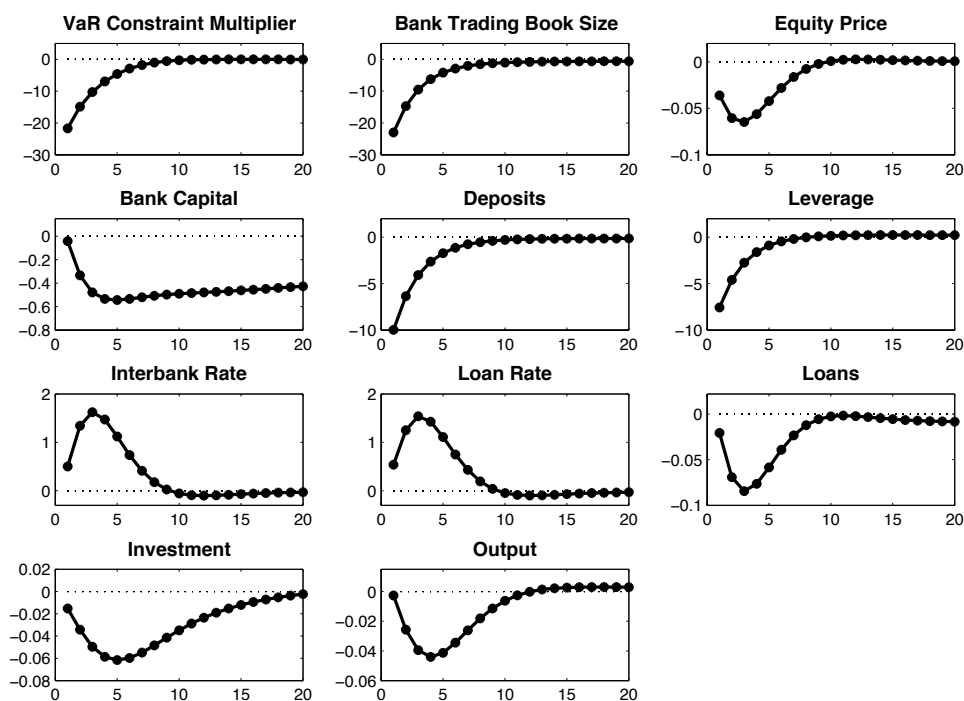
To study the dynamics of the linearized model in response to a volatility shock, the focus will be on an unanticipated one standard deviation shock to σ_t with parameter values set at their posterior median.⁵⁰ The goal is to assess how increased financial market volatility can affect credit supply and investment when financial institutions are subject to VaR-based capital requirements. The first experiment will consider the full model discussed above. Then to assess the effect of the VaR constraint, it will be turned off.

Transitory Volatility Shock

The transmission of a transitory volatility shock is studied by analyzing the impulse response of a positive unanticipated one standard deviation shock to σ_t , which corresponds to 22% increase in volatility, and is shown in figure 2.9. The size of this shock is about one-third the size of the 62% increase in the VIX between 2002:Q2 and 2002:Q3.

⁵⁰Simulations are computed under the assumption that the expected value of all future shocks is zero. Since the model is stochastic and log-linearized around the steady state, agents will behave as if the value of future shocks is zero due to certainty equivalence. This also implies that perturbation methods are valid only in a neighborhood of the steady state. As shocks become too large, the linear approximation becomes less accurate and the conditions that ensure that all constraints bind may no longer hold.

Figure 2.9: Positive Transitory Volatility Shock



Note: Impulse responses are in percent deviation from steady state values.

The volatility shock originates in the financial system through the VaR constraint and market risk capital requirements. Upon impact of the shock, the VaR constraint tightens and banks will need to sell securities, reduce loans, or reduce debt to ease the constraint. The VaR constraint multiplier decreases representing a loss in banks' expected ROE and a decrease in risk appetite as banks are forced to hold more regulatory capital. Banks respond by unwinding their trading positions, selling risky equity securities, and lowering their debt level as total assets fall. Because households view deposits and equity shares as perfect substitutes, households play the role of backstop in a way that makes the equities market very liquid and purchase the excess supply of these securities. The selling pressure that is

exerted by banks having to comply with higher capital charges and dictated by the VaR constraint puts downward force on the equity price as households lower their expectation about the future equity price. Banks then sell the equity securities at a loss, resulting in a loss in their equity capital, and further selling pressures. Households switch their savings channel towards equity securities and decrease the amount of bank deposits, effectively making banks smaller. This decrease in bank debt is comparable to the decline in wholesale funding that banks experienced during the crisis and causes banks to deleverage.⁵¹

The increase in volatility initially increases VaR and risk-weighted assets (RWA), decreasing the risk-weighted capital-asset ratio (RW-CAR). Since banks are now undercapitalized on a risk-weighted basis, potential lenders see an increased default probability associated with lending to banks on the interbank market. Together with the loss in equity capital, this puts upward pressure on the interbank rate and loan rate, because the loan rate is a markup over the interbank rate. Thus, when banks' risk-weighted balance sheet positions deteriorate, the effects are passed through to entrepreneurs in the form of higher borrowing costs and amplified by the degree of the markup. However, banks are able to remove risk from their balance sheets very efficiently following their risk management strategy represented by the VaR constraint, because the equity securities market is very liquid.

The act of removing risk from balance sheets eases the VaR constraint and improves banks' risk-weighted balance sheet positions and interbank market conditions. This helps banks substitute balance sheet assets away from the high capital burden asset towards loans, which now require relatively less regulatory capital. Because banks are forced to deleverage, suffering a capital and deposit loss from selling equity securities, credit supply and total loans decrease. Finally, with loans falling and becoming more expensive, firms decrease the

⁵¹Acharya and Mora (2012).

use of capital in production, which results in a decrease in investment and output.⁵²

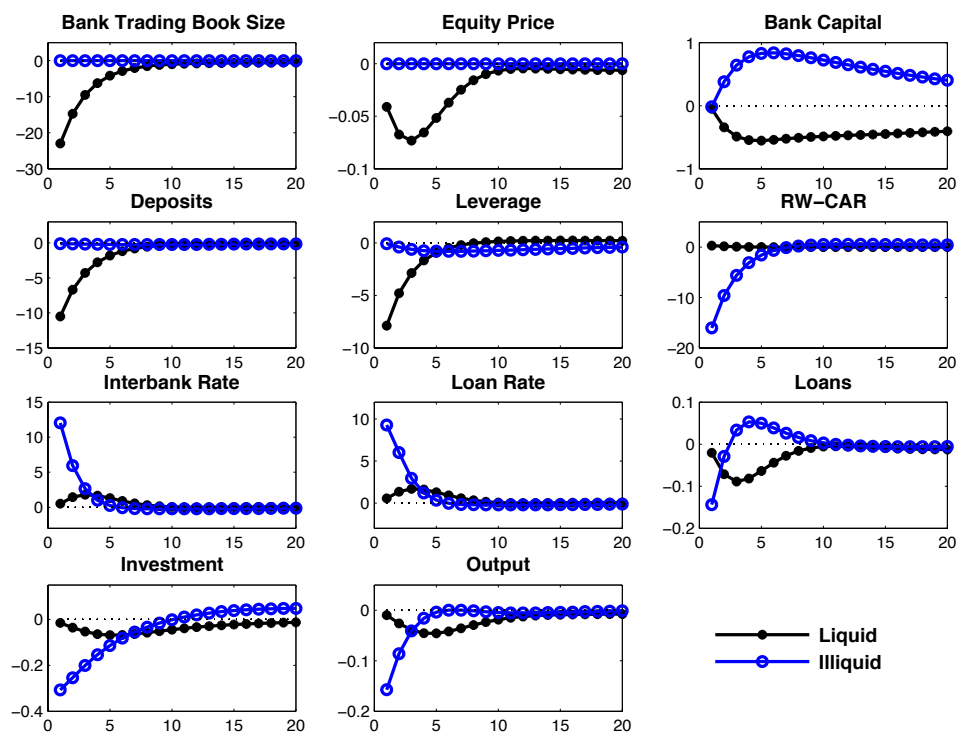
The model's response to a positive volatility shock captures all four target correlations. A positive volatility shock initiated a decline in banks' trading book size and leverage resulting from a decline in debt followed by decreased lending and investment. This shock also generates a feedback effect as asset prices decline from selling pressures dictated by a tightening VaR constraint, a larger VaR-based capital charge, and a more pessimistic view about the future asset price. The decline in the asset price results in a loss in banks' equity capital, which spills over into higher borrowing costs and reduced credit supply. Even though the overall model does not capture the correlation between trading book size and leverage observed in the data, the model's response to a volatility shock alone does show banks' trading book size and leverage decreasing together.

Effect of the Value-at-Risk Constraint

To assess the effect of the VaR constraint, this next experiment turns this equation off. This allows for the analysis of the effect of the risk-weighted capital-asset ratio adjustment cost alone on the model. It can be interpreted as a pure risk shock to balance sheet assets or a case in which banks are unable to satisfy their VaR constraint, which may occur when asset markets become illiquid and banks are forced to keep the size of their trading book at its current level. The effect of the VaR constraint is analyzed by the difference between the liquid markets benchmark case (black line with • markers) analyzed in the previous subsection and the illiquid markets case (blue line with ○ markers) in figure 2.10. The illiquid markets case, in which the VaR constraint is turned off, is simulated with the assumption that exogenous market activity between periods $t - 1$ and t causes perceived volatility to increase and then

⁵²This can be interpreted as an aggregate supply decrease that outweighs the decrease in aggregate demand causing inflation to rise, something that also occurs with the negative bank capital shock in Gerali et al. (2010). The central bank responds with an increase in the policy rate by naively following the Taylor rule, driving up both the deposit and loan rates. This shock is not meant to capture everything associated with financial crises. There are likely other aggregate demand factors and credit losses at play that could result in deflationary pressures in contrast to what was produced here. So, there is still some to be desired since the data suggests that investment and the effective federal funds rate are positively correlated.

Figure 2.10: Effect of the Value-at-Risk Constraint



Note: Impulse responses are in percent deviation from steady state values.

households are assumed to exogenously halt trading so that the equity price remains at its steady state value.⁵³ This will illustrate the effect of risk-weighted capital requirements alone on the dynamics of the model economy.

The effects of a volatility shock become amplified in the illiquid markets case compared to the liquid markets case. Upon impact of the shock VaR, RWA, and the capital charge on the trading book increase as volatility increases leading to a decrease in the RW-CAR.

⁵³Operationally, this is done by treating the household portfolio and banks' trading book size as well as the equity price as constants. This requires shutting off the equity price, the VaR constraint, and equities market clearing conditions.

The key here is that banks no longer have the same freedom to adjust their RWA. Instead of having two instruments, loans and equity securities, to adjust their RWA, banks are left with only loans. Banks are forced to draw down loans when they cannot sell their other assets in order to bring down their RW-CAR ratio, but do so more inefficiently. With banks now having an elevated perceived default probability, these interbank conditions push the interbank and loan rates up.⁵⁴ With credit less available and more expensive, the reduction in investment and output are magnified immediately upon impact of the shock.

The central bank responds to the decrease in aggregate demand by lowering the policy rate. Since the deposit rate is a markdown compared to the policy rate, the deposit rate also falls by the degree of the markdown. A rise in the loan rate and fall in the deposit rate results in an increase in the intermediation spread $r_t^b - r_t^d$, which helps banks recapitalize on interest income to meet the higher VaR-based capital charge. With a lower deposit rate, households choose to reduce deposits but by much less so compared to the liquid markets case, because deposits are the only adjustable asset by which households can save. The smaller reduction in deposits along with the reduction in loans leads to a muted deleveraging process.

One key feature of the illiquid markets scenario is the rise in the interbank rate spread $r_t^{ib} - r_t^{cb}$, which signals distress in the interbank market comparable to the spike in repo rates for collateralized wholesale funds observed during the 2008 crisis.⁵⁵ Even with a decline in the central bank policy rate, the interbank rate increases. The central bank is unable to move the interbank rate in the desired direction, because risk-weighted balance sheet effects outweigh the decline in the policy rate. This result stems from banks' inability to remove risk from their balance sheets in the illiquid markets environment.⁵⁶ The resulting amplified credit contraction is mainly due to the effects of risk-weighted balance sheet positions on

⁵⁴This is consistent with observations from Adrian et al. (2012).

⁵⁵Gorton and Metrick (2012).

⁵⁶This result depends on the size of the balance sheet adjustment cost κ_{Kb} , which was more weakly identified compared to the other adjustment cost parameters. Smaller values of κ_{Kb} dampen the risk effects on interest rates; however, with κ_{Kb} as low as 1, the qualitative results upon impact of the shock remain the same but with diminished persistence.

the interbank market and borrowing costs. Thus, asset liquidity is an important factor in banks' leverage management and in minimizing macroeconomic fluctuations emanating from financial market volatility. These results provide some rationale for the Federal Reserve taking on the role of buyer of last resort in the ABS market during the 2008 financial crisis. Doing so likely helped ease risk constraints and concerns about the soundness of many financial institutions.

Why is the interpretation of this exercise as illiquid markets important? One assumption built into trading book regulations is that a bank's trading book can be liquidated within the 10-day holding period, an assumption that proved false during the financial crisis. As asset markets became less liquid, banks were forced to hold risk positions much longer or were unable to unwind risk positions without substantially affecting market prices. Banks incurred large capital charges and marked-to-market balance sheet losses as markets became more volatile and asset prices declined.⁵⁷

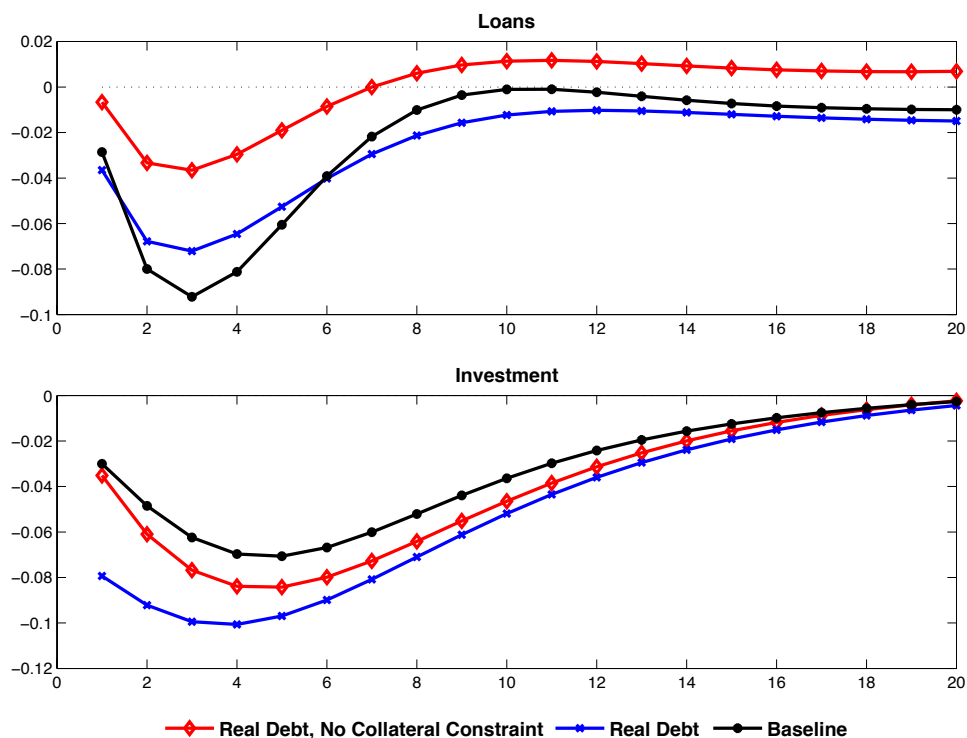
Effect of Model Features on the Dynamics

Now that the impulse responses to a volatility shock have been analyzed, it is important to understand how some of the model features are affecting the dynamics of the model. Specifically, how are the assumptions of financially constrained entrepreneurs and nominal debt contributing?⁵⁸ To answer this, the impulse responses of the baseline model described in this paper are compared to the model responses after progressively shutting down each feature. Figure 2.11 focuses only on the real variables of interest: loans and investment. The black line with • markers plots the baseline model's responses, the blue line with × markers plots the model's response after the nominal debt channel is turned off and all debt

⁵⁷Gorton and Metrick (2012).

⁵⁸The effect of price, wage, and interest rate rigidities are left out of the analysis. The literature on price and wage rigidities have shown them to be important modeling devices that help capture features in the data. The effect of interest rate rigidities are left out of the analysis as well since Gerali et al. (2010) discuss their effects in detail. Briefly, interest rate frictions are found to dampen the economy's response to monetary policy shocks. The interest rate frictions in this model were estimated to be quite small and are unlikely to have a significant effect on the model economy.

Figure 2.11: Decomposition of the Dynamics



Note: Impulse responses are in percent deviation from steady state values.

is denominated in real terms, and finally, the red line with \diamond markers plots the model's response after the collateral constraint has been turned off.

Here, the attention is placed on the responses of loans and investment as the responses of the other variables are not significantly affected. First, the nominal debt channel appears to have a strong effect on investment in the model. When debt is denominated in nominal terms, there is an overall dampening effect on investment and an increase in the persistence of the shock. However, the collateral constraint has an amplification effect on investment and loans. When it is removed from the model, both the responses of loans and investment fall and the persistence is reduced. The important result from this exercise is to note that

neither feature qualitatively affects the direction of the model's responses. The dynamics are driven by the VaR constraint and its effects on the interbank market via the risk-weighted capital-asset ratio adjustment cost.

2.8 Conclusion

Concerns about the potential procyclical effects of VaR-based capital requirements have been raised by the Basel Committee on Banking Supervision and were illustrated in this paper. This paper developed a DSGE model to document the structural links that Value-at-Risk-based capital requirements create between financial market volatility and the macroeconomy. Including risk-weighted capital regulations along with a VaR constraint allowed the model to capture four important correlations observed of financial institution leverage with financial market volatility, leverage with loans, trading book size with volatility, and leverage with aggregate investment. In response to a volatility shock alone, the model also captured the relationship between leverage and trading book size. Most importantly, the model was able to capture the procyclicality of financial institution leverage with respect to volatility in financial markets and aggregate investment. In addition, the volatility of investment and loans increased when banks were allowed to manage a trading book under VaR-based capital regulations. These results contribute to the growing literature on the importance of financial factors in business cycles by identifying the VaR of banks' trading books to be a link between financial markets and credit markets, suggesting that VaR-based capital requirements can be a supply-side factor in credit markets and leveraged VaR-constrained financial institutions may contribute to higher business cycle volatility.

The model was estimated using Bayesian techniques with U.S. data over the period 1997:Q2-2007:Q4. A VaR constraint that accounted for risk-weighted capital regulations in conjunction with an adjustment cost on the risk-weighted capital-asset ratio was used as a mechanism to transmit financial market volatility to the real economy in a fully dynamic general equilibrium model that included a monopolistically competitive financial sector with financial and nominal frictions. The use of the adjustment cost on the risk-weighted capital-

asset ratio was included to be consistent with the observation that banks tend to target a constant VaR-equity ratio while also allowing for procyclical leverage. It also enabled the model to generate a mechanism for risk and balance sheet positions to affect the interbank market that is consistent with empirical observations. However, the use of an adjustment cost mechanism is rather ad hoc so further research on a more microfounded approach, such as a contracting problem, to interbank conditions needs to be conducted.

Analysis of the impulse responses revealed the important characteristics of the model. VaR-based capital regulations can affect credit supply and aggregate investment along two dimensions: the impact of risk-weighted balance sheet positions on the interbank market that raise borrowing costs as well as balance sheet adjustments that result in a loss in banks' overall funding base. Two modeling devices were also necessary for the model's response to a positive volatility shock to capture the target correlations: asset purchases needed to be linked to bank debt and investment needed to be connected to bank lending. The VaR constraint and marked-to-marked balance sheet identity satisfied the first. With households assumed to view deposits and equity securities as perfect substitutes, the equities market was very liquid, which applied very little friction to banks' ability to change debt and risk positions. The second was satisfied with the use of a collateral constraint on entrepreneur borrowing. While, the collateral constraint had an amplifying effect on the model's responses, removal of the collateral constraint did not qualitatively impact the results.

A positive volatility shock initiated a risk-constrained feedback effect. As VaR-based capital charges increased with higher volatility, the VaR constraint tightened and incentivized banks to sell risky assets to ease their risk-weighted balance sheet position. This put downward pressure on asset prices. Selling assets at a loss resulted in a loss in banks' equity capital generating further selling pressures that spilled over into reduced debt, deleveraging, and a reduction in credit supply and aggregate investment. This response captured the procyclicality of bank leverage, trading book size, and leverage with respect to financial market volatility. However, with households being passive in asset markets, this model can be viewed as a lower bound case. Had households been modeled as active investors, the effects of the

volatility shock would likely have been larger.

Then, to assess the impact of the Value-at-Risk constraint on the model, an experiment that compared the model's response with and without it was conducted. The response without the VaR constraint was interpreted as an aggregate shock to balance sheet assets when banks were unable to satisfy their VaR constraint, such as in cases when asset markets become illiquid. This analysis displayed some important differences between these two polar cases. When asset markets were illiquid, banks had less freedom to adjust their RW-CAR. The result was an amplified impact on the interbank market as risk-weighted balance conditions deteriorated. The absence of liquidity was responsible for a spike in the interbank rate spread, signaling distress in the interbank market, which is an important feature of financial crises. The central bank was unable to move the interbank bank rate in the desired direction, because risk-weighted balance sheet effects outweighed the policy rate response. Loan rates rose more when markets were illiquid, which resulted in a magnified decline in credit and investment. Crucial to these results was banks' inability to remove risk from their balance sheets when asset markets were illiquid. Asset liquidity can then be seen as important factor in minimizing macroeconomic fluctuations emanating from volatility shocks and provides some rationale for the Federal Reserve taking on the role of buyer of last resort in the ABS market during the 2008 crisis. Thus, countercyclical financial market volatility can amplify business cycles.

This stylized exercise is in no way a complete analysis of asset liquidity, which is arguably an important aspect of financial markets as illustrated by the turmoil caused by the 2008 financial crisis when repo markets froze. As a result of uncertainty surrounding the collateral value behind many mortgage-backed securities used in repos, interbank lenders began charging higher rates and higher haircuts as concerns arose about banks' ability to repay repos due to their exposure to risky subprime mortgage-backed securities.⁵⁹ This made it difficult for banks to roll over short-term debt and maintain the heightened leverage that was built

⁵⁹Gorton and Metrick (2012).

in the run-up to the crisis. Financial institutions became stuck with high risk assets on their balance sheets and were forced to deleverage. The model developed here could be a decent starting point to build a more structural model to study asset liquidity conditions. In this model, households were assumed to be passive investors. As a consequence, banks were able to shed risk without much resistance. Allowing for households to respond directly to changes in risk could make asset markets less liquid by amplifying changes in asset prices or reducing the speed with which banks can sell risky assets in response changes in volatility.

Even though there is evidence that Value-at-Risk can have negative consequences on business cycle volatility, the rationale for regulatory capital is still valid as moral hazard and agency problems that VaR could help mitigate were not considered in this paper. As Danielsson et al. (2004) note, “The overall case for risk regulation must be based on a cost-benefit analysis in which the limitations to risk-taking behavior is set against the possible damage done by the endogenously generated risk that arises from risk regulation.” Policymakers interested in reducing the effects of procyclical capital charges on credit supply and aggregate investment could consider increasing the minimum data history requirement for calculation from the current 250 trading days to a fuller data history including multiple business cycles to reduce exposure to the effects of time-varying volatility. Disallowing regulatory capital to be marked-to-market could also limit some the procyclical effects as it should reduce the effects of the upward-sloping demand and downward-sloping supply responses that result from marked-to-market changes in capital on risk constraints. Alternative risk measures, such as the stressed VaR and stressed CVaR have been suggested with Basel 2.5 and Basel III, where risk measures are to be calibrated to the worst known financial stress periods, could also be a step in the right direction. The stressed risk measure approach will likely calibrate risk-based capital charges using a large volatility measure and help ensure banks are adequately capitalized in normal times to withstand a market crash in the event that one occurs. The procyclical effects of these alternative risk measures are considered in chapter 3.

Chapter 3

**CREDIT SUPPLY AND THE REVELATION OF TAIL RISK
UNDER CONDITIONAL VALUE-AT-RISK-BASED CAPITAL
REQUIREMENTS**

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Abstract

The 2008 global financial crisis revealed serious weaknesses in the worldwide banking system and financial regulatory regime. The Basel Committee on Banking Supervision met to address these concerns and has suggested switching the measurement for market risk capital requirements from Value-at-Risk (VaR) to stressed Conditional Value-at-Risk (CVaR) as a microprudential policy to help ensure the soundness of individual financial institutions. This paper finds CVaR may reduce the spillover effects of market risk capital requirements on credit supply and aggregate investment compared to the current VaR regime if regulation abandons the efficient markets hypothesis in favor of the fractal markets hypothesis and calibrates risks to stressed market conditions. Stressed CVaR can reduce banks' balance sheet response to increases in perceived volatility, which should reduce the risk-constrained feedback effect from falling asset prices and depleted bank capital that is induced when banks have to sell assets to comply with increased capital charges. However, because asset returns are generally non-normally distributed, if CVaR is not calibrated to stressed market conditions it may amplify these effects in response to changes in perceived tail risks. Thus, these results provide some supporting evidence for Basel III's proposed stressed CVaR market risk regime.

3.1 Introduction

The 2008 financial crisis revealed some serious weaknesses in the worldwide banking system and financial regulatory regime. When asset returns are assumed or locally estimated to be normally distributed, as is suggested under the dominant paradigm of the efficient markets hypothesis (EMH), risks associated with trading strategies appear relatively low due to nearly non-existent tail risks (i.e. low downside risk).¹ As perceived downside risks were low in the run-up to the recent crisis, banks built up excessive leverage by funding asset growth with with short-term repurchase agreements (repos) that were often backed by securitized assets.² After a wave of unexpected defaults occurred beginning in the summer of 2007, the value of the collateral underlying many repo agreements collapsed, destroying wealth throughout the financial system. Concerns over increased market risks and mounting losses forced many financial institutions to take shorter-term investment horizons to stave off insolvency. These financial institutions began unwinding complex, interconnected trading positions causing the interbank repo market to freeze and a liquidity crisis ensued.³ As a result, highly leveraged financial institutions were unable to rollover the short-term debt that was needed to maintain current balance sheet positions, forcing fire sales of assets and a massive deleveraging process.⁴ Asset returns took on non-normal, heavy-tailed distributions that are more consistent with the alternative, and more general, fractal markets hypothesis (FMH) than the EMH.⁵ This event revealed unforeseen tail risks that banks were ill-equipped to protect against and a full blown financial crisis had begun.

As a response, the Basel Committee on Banking Supervision met to address the concerns that the procyclicality of risk-based capital requirements may have contributed to the build-

¹Locally estimated risks as those estimated using a limited window of data history, i.e. the past 250 trading days, rather than the full, global set of data history.

²Adrian and Shin (2010).

³Gorton and Metrick (2012).

⁴Bernanke (2013) and Adrian and Shin (2010).

⁵See section 3.3.2 for a description of the EMH and FMH.

up of risks within the financial system prior to the crisis and to the severity of the crash. The stated goal of the meeting was to promote a more resilient banking sector and “improve the banking sector’s ability to absorb shocks arising from financial and economic stress, whatever the source, thus reducing the risk of spillover from the financial sector to the real economy”.⁶ The focus of these reforms at a microprudential level is to improve overall risk management and transparency. At a macroprudential level, the reforms attempt to lessen systemic risk and weaken the procyclicality of risk-sensitive capital charges. These regulations, now known as Basel III, modify the existing credit risk and market risk capital standards to address the excessive credit growth, deteriorating credit standards, and the use of risky trading strategies that were observed of financial institutions in the run-up to the recent crisis. Basel III also introduced some new regulations, including a liquidity coverage ratio (LCR), the net stable funding ratio (NSFR), a countercyclical capital buffer, and a supplemental leverage ratio (SLR) to complement the existing risk-weighted capital ratio. The new liquidity framework is meant to help ensure that banks maintain adequate access to cash, enough to meet 30 days of net cash outflows and address maturity mismatches on bank balance sheets, in case of a rapid reversal in the liquidity of financial markets. The countercyclical capital buffer is designed to reduce the procyclicality of bank balance sheet adjustments by limiting the effects of adverse shocks to bank capital and speed up the rebuilding of capital during an economic recovery.⁷ Finally, the addition of the leverage ratio is intended to prevent the excessive build-up of leverage during expansions and the successive deleveraging at the onset of a crisis that have been symptomatic of many financial crises throughout history.⁸

One particularly interesting suggestion made by Basel III is to switch from the current market risk regime, which requires banks to supplement their Value-at-Risk-based (VaR) capital charge with a stressed VaR charge introduced with Basel 2.5, to a single stressed Conditional Value-at-Risk (CVaR, or sometimes referred to as Expected Shortfall (ES) or

⁶Basel Committee on Banking Supervision (2009).

⁷Basel Committee on Banking Supervision (2012a).

⁸Jordà et al. (2013).

Tail VaR (TVaR)) regime. Stressed risk measures are to be calculated using data from known financial stress periods that would maximize the risk measure for a given portfolio. The reason for the proposed switch stems from some of VaR's undesirable mathematical properties, which prevent it from being a coherent risk measure. Because VaR is a quantile of a returns distribution measuring the expected loss on a portfolio of trading positions with a given confidence level and holding period, it is possible to find a portfolio in which its VaR is larger than the sum of the VaR's of two sub-portfolios.⁹ This is not a desirable property since financial theory tends to stress the benefits of diversification. Another significant shortcoming is its inability to capture tail risks (i.e. the risks beyond the VaR level), which can incentivize VaR-constrained institutions to arbitrage their regulatory VaR measure with risky options trading strategies. One anecdotal reason that has often been quoted for why the financial crisis was so severe was the widespread use, and misuse, of VaR to manage risks and its inability to correctly account for how bad losses can get when markets crash.

CVaR, on the other hand, is a coherent risk measure and its benefits over VaR have been well documented.¹⁰ CVaR measures the expected loss on a portfolio of trading securities given that the loss exceeds the VaR threshold and is specifically designed to capture tail risks. Because of this, CVaR has some important microprudential benefits in that it may help limit the incentives for financial institutions to arbitrage their regulatory risk measure as is possible under the VaR regime. However, CVaR also has some possible macroprudential benefits as well. One in particular is that it may be less responsive to time-varying volatility if asset returns are assumed to be non-normally distributed. This property can help limit the spillover effects of banks' balance sheet adjustments in response to changes in market risk capital charges on credit supply and investment. A potential drawback for CVaR, however, is that it is more responsive to time-varying tail risks compared to VaR, which could amplify these same spillover effects. In suggesting the switch from VaR to CVaR to measure market risks, Basel III appears to consider the credibility of CVaR as a risk measure to be an

⁹Rockafellar and Uryasev (2000, 2002).

¹⁰Rockafellar and Uryasev (2000, 2002).

important factor in promoting financial stability.

While these new regulations suggested by Basel III all have important implications in their own right, this paper will focus solely on the differences between CVaR and VaR for market risk capital regulations to build on the work in chapter 2. The major contribution made in this paper is the analysis of market risk capital charges on credit supply and investment under stressed and locally calibrated VaR- and CVaR-based market risk regimes in response to two types of time-varying risk. The first is time-varying volatility that affects the dispersion of returns distributions. It has been discussed in a wide range of literature and explored in fully dynamic general equilibrium model in chapter 2. The second is the novel time-varying tail risk that affects the tail thickness of the returns distribution and has only recently begun to be studied in the depth that it deserves.¹¹ This paper finds that implementing CVaR to calculate market risk capital charges can reduce the procyclical effects of risk-sensitive capital charges on credit supply and investment compared to the current VaR regime if regulation abandons the efficient markets hypothesis in favor of the fractal markets hypothesis and calibrates risks to stressed market conditions. Stressed CVaR can reduce banks' balance sheet response to increases in perceived volatility, which should reduce the risk-constrained feedback effect from falling asset prices and depleted bank capital that is induced by banks having to sell assets to comply with increased capital charges. However, because asset returns are generally non-normally distributed, if CVaR is not calibrated to stressed market conditions, it may amplify these effects in response to changes in perceived tail risks. These results provide some supporting evidence for Basel III's proposed stressed CVaR market risk regime.

The rest of this paper is organized as follows: section 3.2 discusses some relevant papers in the macroeconomic literature, section 3.3 compares VaR to CVaR as risk metrics while also documenting the empirical non-normality of asset returns, section 3.4 reviews the model used to analyze the policy implications of Basel III's proposed switch from VaR to CVaR to

¹¹Gennaioli et al. (2013).

measure market risks, section 3.5 analyzes the impulse responses of the model economy to volatility and tail risk shocks, and finally section 3.6 provides some concluding remarks.

3.2 *Macroeconomic Literature*

This paper contributes to the macro-financial, capital requirements, and business cycle literature by analyzing the role that bank balance sheets play in propagating changes in financial market volatility and tail risks to the real economy when banks are subject to risk-sensitive capital requirements under two competing market risk capital regimes: VaR and CVaR. One paper that discusses the role of tail risk is Gennaioli et al. (2013). They present a partial equilibrium model of shadow banking that supports the notion that the inability to correctly assess and price risk can sow the seeds of a financial crisis. In their model, depositors are interested in riskless debt, which is in line with evidence presented by Bernanke et al. (2011). The novelty of Gennaioli et al. (2013) is their use of securitization to back the issuance of riskless deposits within the model. Intermediaries use raised funds to finance safe and risky loans subject to a borrowing constraint in which they can only borrow against the value of riskless loans and securitized risky assets in the believed worst state of the world. Banks securitize and sell risky loans to diversify idiosyncratic risk at the expense of building up systemic risk. Gennaioli et al. (2013) show that securitization is welfare enhancing under rational expectations. However, when banks and investors neglect tail risks (i.e. neglect the possibility that the worst possible state can occur), banks take on too much debt and make too many risky loans. Once the neglected state is realized, banks suffer a large loss in equity capital and are unable to repay the riskless debt in full and a financial crisis results.

The model used here to analyze the effects of time-varying volatility and tail risk under both stressed and locally calibrated VaR- and CVaR-based capital regimes is the DSGE model used in chapter 2. This model embeds a monopolistically competitive financial sector that is subject to risk-based capital requirements into an otherwise standard fully dynamic general equilibrium model. The financial sector in that paper is largely based on the work

of Gerali et al. (2010), which omits the issue of financial risk that was so important to the 2008 financial crisis. This issue is addressed in chapter 2 under VaR-based capital requirements and the assumption that asset returns are normally distributed. The major contribution of chapter 2 identifies the VaR of banks' trading books to be a supply-side factor in credit market dynamics. The results suggest that when leveraged financial institutions are constrained by VaR-based capital requirements, increased financial market volatility can induce a feedback effect as banks adjust their balance sheets to comply with higher capital charges resulting in a fall in asset prices and depleted bank capital. This spills over into credit markets as risk-weighted balance sheet positions deteriorate and raise the cost of interbank funding. In addition, the model shows that leveraged VaR-constrained financial institutions can contribute to higher business cycle volatility.

The model is also able to replicate some empirical correlations between financial institution leverage, volatility in financial markets, banks' trading book size, and aggregate investment that are important to understanding financial business cycles. Namely, financial institution leverage is found to be negatively correlated with financial market volatility measured by the Chicago Board of Exchange Volatility Index (VIX): i.e. lower volatility tends to be related with higher leverage and vice versa. When volatility is low, risk-based capital charges tend to be low, which allows financial institutions to allocate excess capital to risky investments. Financial institutions tend to increase the size of their balance sheets by increasing the size of their trading books and loan books by raising debt, ultimately leading to increased leverage, credit supply, and aggregate investment.

This paper examines the concept of risk from a different perspective. The normality assumption is abandoned and asset returns are allowed to follow non-normal, heavy-tailed distributions. Instead of analyzing the effects of changes in financial market volatility alone on bank balance sheet adjustments and macroeconomic activity, these effects are also analyzed in response to changing tail risks under both stressed and locally calibrated VaR- and CVaR-based capital regimes. While the procyclical concerns of VaR-based capital requirements are examined in other studies including chapter 2, to the best of my knowledge

this is the first paper to illustrate the different macroeconomic consequences between VaR and CVaR within a fully dynamic general equilibrium model. To do so, the risk constraint developed in chapter 2 is modified for more general distributions and risk measures. This allows for the analysis of the effect of time-varying financial market volatility and tail risks on business cycles in a meaningful way.

3.3 Risk Metrics and Empirical Non-Normality

3.3.1 Value-at-Risk vs Conditional Value-at-Risk: A Microprudential Benefit

Value-at-Risk has been a financial industry standard for risk management since the 1990s and adopted as regulation in 1996 as an amendment to the original Basel Accords. VaR is a statistical concept that measures the largest loss on a portfolio of trading positions that can be expected with a given confidence level and holding period.¹² The idea behind it is to give a quantifiable measure of how much capital a financial institution needs to hold in order to withstand large fluctuations in market prices so that it can remain solvent. Thus, VaR describes the probability of default given by the confidence level with which it is calculated. Mathematically, VaR for a given confidence level c is defined as:

$$VaR_c = \{v : Pr(x \leq v) = 1 - c\}$$

for $c \in (0, 1)$. If the returns distribution is continuous and denoted $f(x)$, then VaR_c is the quantile that solves:

$$1 - c = \int_{-\infty}^{VaR_c} f(x) dx$$

At a 99% confidence level, $VaR_{99\%}$ says that there is only a 1% chance that losses will be larger than the $VaR_{99\%}$ value and a 1% probability of default.

There are three basic approaches to calculate VaR: the variance-covariance method, the historical simulation method, and the Monte Carlo simulation method.¹³ While each has

¹²See chapters 1 and 2 for more detail about VaR-based capital regulations.

¹³See chapter 2 for more detail about VaR estimation methods.

strengths and weaknesses, playing on the trade-offs between calculation speed, simplicity, and flexibility, each relies on selecting an appropriate historical data window to calibrate some aspect of the model. Under Basel II regulations, a minimum of the past year of historical data is required for calibration. Because VaR is calculated in this manner, it is essentially a local measure of risk subject to business cycle movements. These approaches also fall back on the normality assumption for asset returns to simplify the model and speed up numerical calculations. However, as will be shown below, the normality assumption can severely underestimate risk.

Markowitz (1952) showed that under the normality assumption, the standard deviation can be used as a measure of risk and the covariances of returns could be used to explain how diversification reduces the aggregate risk of a portfolio. In this case, VaR for a normal distribution can be written analytically as:

$$VaR_c^N = \mu + \text{erf}^{-1}(2(1-c) - 1) \sqrt{2}\sigma$$

$$\sigma^2 = \int_i (\omega_i \sigma_i)^2 di + 2 \int_i \int_{j>i} \omega_i \omega_j \text{Cov}(x_i, x_j) dj di$$

where μ is the mean of returns, σ is the standard deviation of the portfolio, ω_i is the weight of asset i in the portfolio, σ_i is the standard deviation of asset i , and $\text{Cov}(x_i, x_j)$ is the covariance between assets i and j . This concept is still widely used in risk management. Thus, VaR has a tendency to be thought of in terms of a normal distribution, and in that case it is really better thought of as possible losses from normal market conditions when tail thickness is small and downside risk is low. VaR has been known to have poor performance during times of financial stress, mostly because returns distributions are not normal during crisis periods.

Even though VaR has become the financial industry standard risk measure, it has a number of shortcomings that make it less than ideal. The financial industry has been slow in changing this standard, because VaR has an easy interpretation and is relatively easy to calculate and backtest for accuracy. Unfortunately, its limitations may outweigh the benefits. VaR's most notable shortcomings stem from its rather undesirable mathematical properties

that prevent it from being a coherent risk measure: it lacks subadditivity and convexity.¹⁴ The only case in which VaR is coherent is when it is based on the standard deviation of normal distributions. VaR can be poorly behaved as it may exhibit multiple local extrema making it difficult to determine the optimal mix of assets, especially when losses are not normally distributed or non-linear. There are also cases when the sum of the VaR's of two portfolios considered separately can be lower than the VaR of the combined portfolio.¹⁵ This violates the diversification principle that a well-diversified portfolio should carry lower risk. VaR also has nothing to say about risks in the tail of the distribution: i.e. how bad things can get if losses exceed the VaR level. This makes it possible for a financial institution to manipulate its VaR by stuffing risk in the tail.

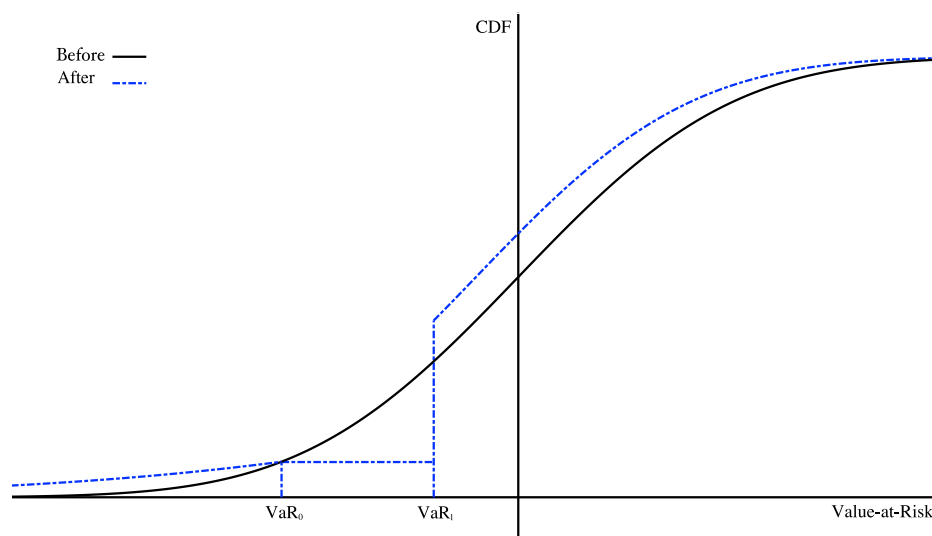
Daniélsson (2002) provides an example of how a financial institution can manipulate VaR by stuffing the tails. Say a financial institution would like to reduce its VaR from VaR_0 to VaR_1 such that $|VaR_1| < |VaR_0|$. The institution could write a put option with a strike price right below VaR_0 and buy a put option with a strike price right above VaR_1 . This strategy puts a kink into the cumulative returns distribution as illustrated in figure 3.1. This strategy does lower VaR and required capital but perversely increases tail downside risk so the financial institution is required to hold less capital against more risk. Because a VaR-constrained institution is blind to tail risks, incentives to use such a strategy are strong especially to arbitrage regulatory capital.

This is one area where CVaR can improve upon VaR as an alternative risk measure. CVaR is designed to capture tail risks by measuring the expected loss given that losses are equal to or exceed the VaR threshold. Because VaR describes the probability of default given by the confidence level with which it is calculated, CVaR then describes the expected value of recoverable assets in the case of default. Consequently, CVaR first gained traction

¹⁴A subadditive function is one in which $f : A \rightarrow B$, having a domain A and ordered codomain B that are both closed under addition, with the following property: $\forall x, y \in A, f(x + y) \leq f(x) + f(y)$. A convex function is one in which $f : X \rightarrow \mathbb{R}$, having convex domain X , and the following property: $\forall x_1, x_2 \in X, \forall t \in [0, 1] : f(tx_1 + (1 - t)x_2) \leq tf(x_1) + (1 - t)f(x_2)$.

¹⁵Rockafellar and Uryasev (2000).

Figure 3.1: Value-at-Risk Manipulation



Note: Recreated from Daniélsson (2002).

in the insurance industry and credit risk evaluations before catching the attention of the rest of the financial industry.¹⁶Rockafellar and Uryasev (2000, 2002) document the benefits of CVaR over VaR. Most notably, CVaR is a coherent risk measure satisfying both subadditivity and convexity, among others, which VaR fails to do. CVaR also has some computational advantages over VaR. Rockafellar and Uryasev (2000, 2002) develop an efficient CVaR minimization technique for continuous, discrete, and non-normal distributions that has improved in- and out-of-sample properties compared to other VaR optimization routines. These results have sparked the development of the CVaR methodology since efficient algorithms for VaR optimization have been lacking. Thus, CVaR succeeds where VaR fails.

Intuitively, CVaR measures how bad losses can get when losses are larger than expected with a given level of confidence. For continuous distributions and a given confidence level c ,

¹⁶Rockafellar and Uryasev (2000).

CVaR is defined as:

$$CVaR_c = E [x|x \leq VaR_c]$$

If the returns distribution is denoted $f(x)$, then $CVaR_c$ is the solution to:

$$CVaR_c = \frac{1}{1-c} \int_{-\infty}^{VaR_c} xf(x)dx$$

If the distribution has discontinuities, then Rockafellar and Uryasev (2002) show that CVaR is a weighted average of VaR and the expected loss given that the loss strictly exceeds VaR:

$$CVaR_c = \lambda_c VaR_c + (1 - \lambda_c)E [x|x < VaR_c]$$

where λ_c is the weight, which is zero for continuous distributions.¹⁷ Returning to the example above, if CVaR was the risk measure used instead of VaR, CVaR would pick up on the increased tail risk and actually show an increase in risk, preventing the financial institution from using the tail stuffing strategy.

At this point, there are a couple of important issues to note about CVaR. First, because CVaR attempts to account for the entire tail of a distribution, it requires a lot observations to be able to model the tail accurately. Unfortunately, the once-in-a-hundred year event that capital regulations strive to protect against happens too rarely to get a full picture of the tail. This creates some computational challenges associated with calculating CVaR and backtesting it. Thus, in order to use CVaR, some assumptions about the distribution may be necessary. Second, the example given above is only one case in which CVaR can prevent the manipulation of the returns distributions and risk measure. However, financial institution management is usually savvy and can be very motivated to arbitrage regulatory capital requirements in search of higher profits. Thus, they will likely find ways to manipulate CVaR which have not yet been thought of. Despite these concerns, the Basel Committee on Banking Supervision has agreed that the net benefits of CVaR are greater than those of VaR and has suggested switching to CVaR for determining market risk capital charges.

¹⁷See Rockafellar and Uryasev (2002) for the derivation of λ_c .

3.3.2 *There is (Almost) No Such Thing as a Normal Market*

Traditional finance theory is based on Louis Bachelier's (1900) theory of stochastic processes (also known as Brownian motion) and Fama's (1965) random walk hypothesis to evaluate asset prices.¹⁸ These theories imply that financial markets are efficient: i.e. the efficient markets hypothesis (EMH). Fama describes an efficient market as "a market where there are large numbers of rational, profit-maximizers actively competing, with each trying to predict future market values of individual securities, and where important current information is almost freely available to all participants." As a consequence, asset prices at any given time should reflect all publicly available information of events that have occurred and expectations of future events. But because the world is highly uncertain, market participants are likely to have differing beliefs about future events, which should cause actual asset prices to fluctuate randomly around some fundamental value as an independent and identically distributed normal random process.

However, many empirical studies have shown asset returns to be anything but normally distributed. Particularly, Mandelbrot (1963) observed that the tail of returns distributions were much thicker than the assumed normal distribution, implying that extreme events should occur with a much higher probability than what the normal distribution predicts. According to Mandelbrot and Hudson (2004), theories based on the normal distribution suggest that there should be only one day in 300,000 years for daily stock price movements to be larger than 7%. Yet, there have been at least 48 such events since 1916 including Black Tuesday and Black Thursday in 1929, Black Monday in 1987, the collapse of Long Term Capital Management (LTCM) in 1998, the Enron scandal in 2001, and the mortgage crisis in 2008. Clearly, this is at odds with the efficient markets hypothesis. One of the most famous equations based on this theory is the Black-Scholes option pricing formula, which many financial pundits have claimed to be at least partially responsible for the crash as many

¹⁸Davis and Etheridge (2006).

financial institutions took the normality assumption for granted.¹⁹

One alternative to the EMH is the fractal markets hypothesis (FMH), which is largely based on Mandelbrot's work. When Mandelbrot calculated the variance of cotton prices with sample sizes of different lengths, he observed that the variance did not settle down around a limiting value. Instead, the variance behaved as if it were infinite.²⁰ This led him to propose modeling asset returns with stable distributions, a more general family of random variables that includes the normal distribution as a limiting case. Stable distributions are characterized by four parameters: the location parameter $\mu \in (-\infty, \infty)$, the volatility parameter $\gamma \in (0, \infty)$, the skewness parameter $\beta \in [-1, 1]$, and most importantly the stability parameter $\alpha \in (0, 2]$.²¹ α governs the tail thickness of the distribution with tail thickness increasing as α approaches zero. The general form of the probability density function is not analytically expressible except when $\alpha \in \{0.5, 1, 2\}$ where $\alpha = 0.5$ corresponds to a Levy distribution, $\alpha = 1$ corresponds to a Cauchy distribution, and $\alpha = 2$ corresponds to a normal distribution. The probability density function is then defined by its characteristic equation:

$$\phi(t) = \begin{cases} \exp \{it\mu - |\gamma t|^\alpha [1 - i\beta \operatorname{sgn}(t) \tan(\frac{\pi\alpha}{2})]\} & \text{if } \alpha \neq 1 \\ \exp \{it\mu - \gamma t [1 + i\beta \operatorname{sgn}(t) (\frac{2}{\pi}) \log |t|]\} & \text{if } \alpha = 1 \end{cases} \quad (3.1)$$

The stable distribution also has some desirable transformation properties. If x_1, \dots, x_n are independent stable random variables with the same α where $x_i \sim S_\alpha(\mu_i, \beta_i, \gamma_i)$, then $X = \sum_{i=1}^n \omega_i x_i$ is a stable random variable such that $X \sim S_\alpha(\mu, \beta, \gamma)$ where:

a) if $\alpha \neq 1$

¹⁹Taleb (2007).

²⁰Fama (1963).

²¹The location parameter is often referred to as δ rather than μ . μ is used to be consistent with the terminology of the normal distribution. γ is related to σ of the normal distribution as $\gamma = \sigma/\sqrt{2}$.

$$\begin{aligned}\gamma &= ((|\omega_1|\gamma_1)^\alpha + \dots + (|\omega_n|\gamma_n)^\alpha)^{\frac{1}{\alpha}} \\ \beta &= \frac{\text{sgn}(\omega_1)\beta_1(|\omega_1|\gamma_1)^\alpha + \dots + \text{sgn}(\omega_n)\beta_n(|\omega_n|\gamma_n)^\alpha}{(|\omega_1|\gamma_1)^\alpha + \dots + (|\omega_n|\gamma_n)^\alpha} \\ \mu &= \omega_1\mu_1 + \dots + \omega_n\mu_n\end{aligned}$$

b) if $\alpha = 1$

$$\mu = \omega_1\mu_1 + \dots + \omega_n\mu_n - \frac{2}{\pi}(\omega_1\ln|\omega_1|\gamma_1\beta_1 + \dots + \omega_n\ln|\omega_n|\gamma_n\beta_n)$$

Unlike Markowitz's work with the normal distribution, the relevant portfolio risk measure is no longer the variance as stable distributions have infinite variances. Instead, the portfolio risk measure is deemed the variation defined as γ^α , which reduces to half the variance ($\sigma^2/2$) in the normal distribution case. If dependence between the random variables x_i is assumed, γ^α no longer has a closed form solution, and is written as a function of the linear combination of random variables and denoted $\gamma^\alpha(\omega_1x_1 + \dots + \omega_nx_n)$. Then, if covariance is the indicator of dependence, the covariation can be used to estimate dependence between two stable random variables with the same α as:

$$[x_1; x_2]_\alpha = \frac{1}{\alpha} \frac{\partial \gamma^\alpha(\omega_1x_1 + \omega_2x_2)}{\partial \omega_1} \Big|_{\omega_1=0; \omega_2=1}$$

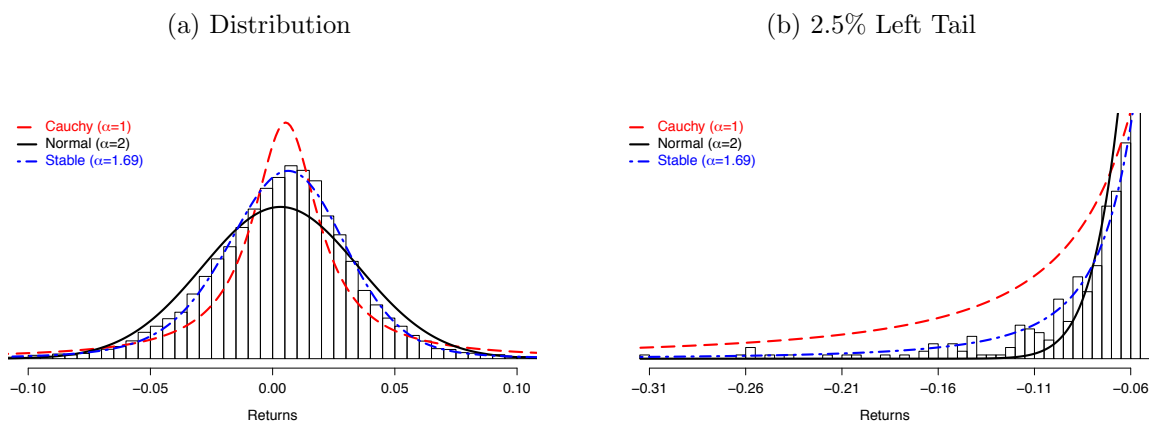
The dependence structure between individual returns in the portfolio is determined by the matrix of covariations.²² Thus, risk now depends on volatility parameters γ_i and on the degree of stability α such that VaR for a portfolio of independent stable distributions will be a nonlinear function of these parameters:

$$\begin{aligned}VaR_c^S &= \mu - f(\gamma, \alpha) \\ \gamma^\alpha &= \int_i (|\omega_i|\gamma_i)^\alpha di\end{aligned}$$

Mandelbrot's theory proposed that returns distributions are such that $\alpha \in (1, 2)$, which happens to encompass, as limiting cases, the normal distribution (and the EMH) at one

²²Khindanova et al. (2001).

Figure 3.2: 10-Day S&P 500 Returns Distribution



Note: Data is from 1970-2015 found at Yahoo! Finance and the density functions are drawn with estimated parameter values. The 2.5% left tail is chosen to encompass both the 99% and 97.5% confidence levels required for Basel II's VaR and Basel III's CVaR market risk measures.

extreme when $\alpha = 2$ (and β has no effect) and the Cauchy distribution at the other when $\alpha = 1$ (and $\beta = 0$). This class of distributions has a defined mean equal to μ when $\alpha > 1$ but undefined variance unless $\alpha = 2$, in which case the variance is $2\gamma^2$. In the limiting case when $\alpha = 2$, both the mean and variance are defined, because the tails decay exponentially. However, when $\alpha = 1$, the mean and variance are undefined, because the tails decay according to a power law. In addition, α is known as the fractal dimension of the probability space, which is where the FMH gets its name.

The fit of three distributions (normal, Cauchy, and stable) to the 10-day S&P 500 returns in figure 3.2a from January 1970 to December 2015, where 10 days is the holding period required under the current regulatory regime, displays evidence that the actual historical returns distribution lies somewhere between normal and Cauchy.²³ The normal distribution estimates the standard deviation to be $\sigma = 0.03$ with a slightly positive mean. The Cauchy

²³The parameters for the 10-day S&P 500 returns between January 1970 and December 2015 are estimated using the maximum likelihood approach with the 'fitdistr' function in the "MASS" package and the 'stableFit' function in the "fBasics" package available for R.

distribution estimates the volatility parameter to be $\gamma = 0.02$. The best fit, however, is a stable distribution with an estimated stability parameter $\alpha = 1.69$, volatility parameter $\gamma = 0.02$, and skewness parameter $\beta = -0.29$ with a slightly positive mean.²⁴ This is in line with many studies that find asset returns to have excess kurtosis and negative skew.²⁵ The 2.5% left tail of this distribution, the tail representing losses and the part of the distribution most important to risk measures, is displayed in figure 3.2b. From this, it is clear that the normal distribution severely underestimates the the probabilities of extreme losses, while the Cauchy distribution overestimates these probabilities. However, the best fitting stable distribution does a much better job at estimating the probabilities of extreme losses.

Two major concepts that the FMH emphasizes are investment horizons and market liquidity. Markets are considered stable when composed of investors of different investment horizons. However, when information arises that causes investors to converge to short-term horizon strategies, financial markets become illiquid, inefficient, and unstable causing crashes and crises.²⁶ The implication is that, under normal market conditions returns distributions will tend to appear normal. In crisis periods however, returns distributions can be far from normal. Figure 3.3 illustrates this point. The estimated stability parameter for daily returns of the S&P 500 within a year tends to be close to 2 for the relatively calm historical periods.²⁷ However, during four known market crashes (1987, 1998, 2001, and 2008), the stability parameter falls indicating thicker tails and higher downside risk. 2008 experienced the largest decline in the stability parameter of these four market crashes, while 2001's increased tail risk was the smallest. The FMH is also consistent with jump diffusion models of asset price

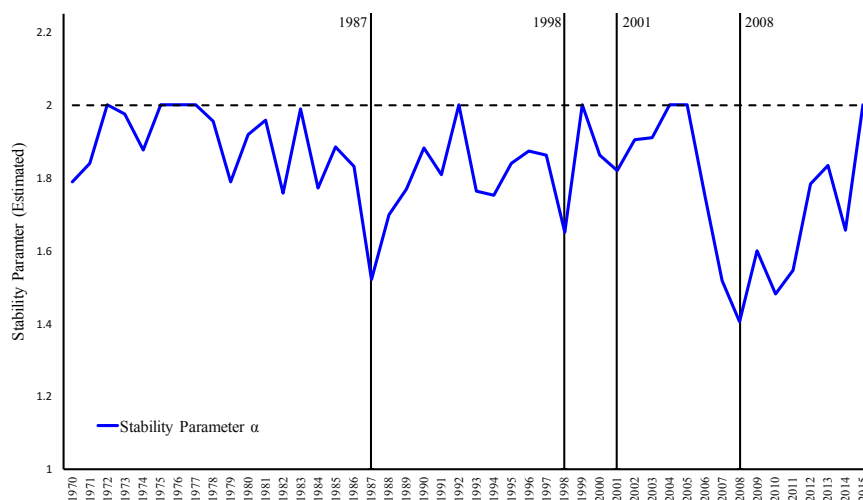
²⁴A negative skewness parameter β indicates a long left tail, while a positive β indicates a long right tail.

²⁵Mandelbrot (1963); Singleton and Wingender (1986); Lai (1991); Corrado and Su (1996); Peiró (2002); Jondeau and Rockinger (2003); Kim and White (2004); León et al. (2005); Ekholm and Pasternack (2005); Bae et al. (2006); Xu (2007); Hutson et al. (2008); Albuquerque (2009); and Wen and Yang (2009) among others.

²⁶Peters (1994).

²⁷The parameters of the stable distribution for the daily S&P 500 returns are estimated for each year using the maximum likelihood approach in the 'stableFit' function in the "fBasics" package available for R. Daily rather than the 10-day changes are used to increase the sample size and improve accuracy of the estimation.

Figure 3.3: Instability of the Daily S&P 500 Returns



Note: Data is from 1970-2015 found at Yahoo! Finance. The stability parameter is estimated for daily returns of the S&P 500 within each year.

dynamics, because stable distributions describe fractional diffusion processes when $\alpha \in (1, 2)$ instead of the traditional diffusion process produced by the normal distribution. Jump diffusion models describe asset price dynamics with small continuous movements interspersed with large jumps, whereas the traditional diffusion process consists of only small continuous movements without any jumps.²⁸ This is the embodiment of Crockett’s (2000) quote: “The received wisdom is that risk increases in recessions and falls in booms. In contrast... think of risk as *increasing* during upswings, as financial imbalances build-up, and *materialising* in recessions.”

If risk is measured locally using at least the past 250 trading days as is required under current regulation, it can lead financial institutions to severely underestimate risk. Risk will appear normal and tail downside risk will appear small before an inevitable increase in tail risk at the start of a crisis. Using an empirical approach to measure risk that relies on

²⁸Merton (1976).

historical data will cause not only volatility, but also tail risk to be time-varying and will contribute to the procyclicality of capital charges. As was shown in chapter 2, procyclical market risk capital charges can have negative spillover effects on credit supply and aggregate investment as a consequence of bank balance sheet adjustments that are needed to comply with higher capital charges at the onset of a market crash. Even small changes in the stability parameter can lead to big changes in tail risks and capital charges. So, exactly when local risk appears small, potential risks are large due to unmodeled or poorly forecasted tail risks.

Returning to the daily returns of the S&P 500 within a year, the standard deviation of the volatility parameter γ relative to its mean is estimated to be 33.2% between 1970 and 2015, which illustrates the traditional time-varying volatility. However, as was just discussed, a second source of risk is present in the form of time-varying tail risk where the standard deviation of the stability parameter α is estimated to be 0.16.²⁹ Changes in the volatility parameter and the stability parameter tend to occur simultaneously as they have an estimated correlation coefficient of -0.27.³⁰ Thus, increased levels of volatility are positively correlated with increased levels of tail risk (volatility increases as γ increases and tail risk increases as α decreases). Many risk models do not account for the time-varying nature of tail risk in returns distribution, although GARCH methods do attempt to account for the time-varying nature of the variance of returns and some even allow for excess kurtosis using t-distributions.³¹ However, the financial econometrics literature has recognized the importance of skewness and kurtosis for the performance of financial models and have begun to integrate them into the GARCH framework.³²

To see why changes in the stability parameter can cause large changes in measured risk, consider a one standard deviation increase in γ (i.e. time-varying volatility) and a one

²⁹Skewness β is also time-varying, but this third source of risk is not considered in this paper.

³⁰The Pearson's product-moment correlation test suggests that the correlation is significant at the 90% level.

³¹Bollerslev (1986); Nelson (1991); Engle and Ng (1991); Glosten et al. (1993); Zakoian (1994); Hentschel (1995); Jondeau and Rockinger (2003); and Klüppelberg et al. (2004).

³²Jondeau and Rockinger (2003); León et al. (2005); Kim and White (2004); and Kim et al. (2008).

standard deviation decrease in α (i.e. time-varying tail risk) at the daily frequency of the S&P 500. The starting point will be from normal market conditions in which the distribution appears normal. In this case, both VaR and CVaR will have nearly identical starting values when risk is measured at the 99% confidence level for VaR (the Basel II requirement) and 97.5% confidence level for CVaR (the Basel III requirement). A one standard deviation increase in volatility increases both VaR and CVaR by 33.2%.³³ This is a result of the normal distribution producing linear risk measures that have nearly identical proportionality to the standard deviation of returns for these two confidence levels. However, a one standard deviation decrease in the stability parameter increases VaR by 21% and CVaR by 46%. Thus, VaR and CVaR are nearly identically sensitive to volatility under normality, but CVaR is much more sensitive to time-varying tail risk when starting from a normal distribution. VaR is unable to capture changes in tail risks and could cause banks to be severely undercapitalized when markets are calm. CVaR on the other hand, does capture these risks, but if estimated locally (i.e. with a short, current data window), CVaR can amplify the procyclical spillover effects of market risk capital charges on credit supply and investment due to time-varying tail risks. Consequently, calibrating risks to known financial stress periods using CVaR could help banks be adequately capitalized in normal times to withstand the occurrence of a financial crisis while also minimizing the procyclical effects of time-varying risks on capital charges.

3.3.3 *Value-at-Risk vs Conditional Value-at-Risk: A Potential Macroprudential Benefit*

Instead of measuring risks with CVaR at the 99% confidence level, which would necessarily be larger than VaR at the 99% level and require financial institutions to hold more capital

³³In order to numerically calculate the density function, the Matlab package “STBL” written by Mark Veillette is used and can be downloaded from the Matlab associated website at <http://www.mathworks.com/matlabcentral/fileexchange/37514-stbl-alpha-stable-distributions-for-matlab>. With the density function found, VaR is calculated by minimizing equation (3.6) using Matlab’s ‘fsolve’ function with a function tolerance of $1e^{-10}$. To calculate CVaR from the numerical density function after VaR is calculated, Matlab’s ‘integral’ function is used on equation (3.7) with the ‘trust-region-dogleg’ algorithm that is specially designed for nonlinear problems and an absolute tolerance of $1e^{-10}$. The Matlab function designed to accomplish this can be found at <https://sites.google.com/site/hubbardalex/research>.

for the same portfolio, Basel III has agreed to implement CVaR at the 97.5% confidence level so that the two risk measures capture a similar portion of the distribution.³⁴ To see how VaR and CVaR differ, consider their analytical representations for a normal distribution:

$$\begin{aligned} VaR_c^N &= \mu + \text{erf}^{-1}(2(1-c) - 1) \sqrt{2}\sigma \\ CVaR_c^N &= \mu - \frac{1}{(1-c)\sqrt{2\pi}} \exp\left\{-\left(\text{erf}^{-1}(2(1-c) - 1)\right)^2\right\} \sigma \end{aligned}$$

In this case, both VaR and CVaR depend on a constant multiple of the standard deviation where the constant for VaR is:

$$Z_c^{VaR} = \text{erf}^{-1}(2(1-c) - 1) \sqrt{2}$$

and the constant for CVaR is:

$$Z_c^{CVaR} = -1 / \left((1-c)\sqrt{2\pi} \right) \exp\left\{-\left(\text{erf}^{-1}(2(1-c) - 1)\right)^2\right\}$$

Under the current regulatory regime with VaR evaluated at the 99% confidence level, $Z_{99\%}^{VaR} = -2.3263$. However, under the proposed CVaR regime with a 97.5% confidence level, $Z_{97.5\%}^{CVaR} = -2.3378$. Thus, $CVaR_{97.5\%}$ and $VaR_{99\%}$ are nearly equivalent, implying the capital charge for market risks will be nearly the same under either regime for a representative portfolio. Chapter 2 found that increased perceived market risks can have negative spillover effects on credit supply and aggregate investment with the implication here being that if markets become more volatile, rising volatility in crisis periods will require a nearly equivalent rise in the market risk capital charge under VaR or CVaR, regardless of the level of σ . Rising capital charges will require financial institutions to adjust their balance sheets to meet the higher capital requirements and may reduce credit supply. CVaR provides no macroprudential benefit under assumed normality, because VaR and CVaR are nearly identical and linear in the standard deviation of the portfolio.

As discussed in section 3.3.2, returns distributions are generally non-normal. Asset returns are better described by a stable distribution with stability parameter $\alpha \in (1, 2)$. Unfortunately, the density functions of most stable distributions are not analytically tractable,

³⁴Basel Committee on Banking Supervision (2013).

and none in Mandelbrot's range are. The limiting case when the distribution is Cauchy at $\alpha = 1$ is, and this special case will be used to illustrate a potential macroprudential benefit of CVaR. This case coincides with the most stressed financial conditions possible under the FMH (see figure 3.3) and can be thought to represent Basel III's stressed CVaR measure.

The probability density function of the Cauchy distribution is:

$$f(x) = \frac{1}{\pi\gamma \left[1 + \left(\frac{x-\mu}{\gamma}\right)^2\right]}$$

where the skewness parameter of the stable distribution is $\beta = 0$. The quantile function, and therefore VaR, is defined for the Cauchy distribution, but CVaR is not because the mean of a Cauchy distribution is undefined. In order to derive an analytically tractable expression for CVaR, a symmetrically truncated Cauchy distribution is used. The integrating factor in this case is $F = \pi / (2 \tan^{-1}(d/\gamma))$ where d is some distance between μ and the chosen endpoints. Returns distributions have a natural lower bound truncation at -1, because the maximum possible loss in terms of the percentage of the value of a portfolio is 100%. For the S&P 500 between 1970 and 2015, the largest observed 10-day decline was 31.5% ending October 5, 1987, and the largest observed 10-day gain was 21.6% ending March 9, 2009. So, it seems reasonable to truncate the Cauchy distribution with $d = 1$ so the domain is $x \in [-1, 1]$.

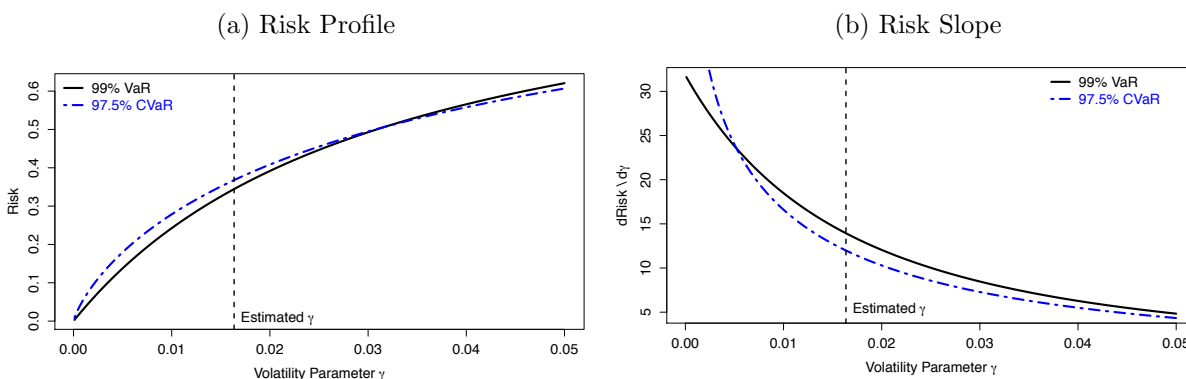
Under the truncated Cauchy case, VaR and CVaR for a given confidence level c can be analytically expressed as:

$$VaR_c^C = \mu + \gamma \tan \left[\tan^{-1} \left(\frac{d}{\gamma} \right) (2(1-c) - 1) \right]$$

$$CVaR_c^C = \mu + \frac{\gamma}{4(1-c) \tan^{-1} \left(\frac{d}{\gamma} \right)} \log \left(\frac{\gamma^2 \sec^2 \left[\tan^{-1} \left(\frac{d}{\gamma} \right) (2(1-c) - 1) \right]}{\gamma^2 + d^2} \right)$$

Thus, under the FMH, risk measures no longer depend linearly on the volatility parameter. VaR is nonlinear in γ and CVaR is even more so. At the worst case, these risk measures are highly nonlinear, but as α approaches 2, these risk measure become more linear as the distribution approaches a normal distribution. The best way to illustrate the differences

Figure 3.4: VaR vs CVaR for a Truncated Cauchy Distribution



Note: Figure drawn using estimated values from the 10-Day S&P 500 returns distribution between 1970-2015 found at Yahoo! Finance.

between the two risk measures is to examine how the risk profile changes with respect to γ , which is analogous to how these risk measures change with financial market volatility.³⁵

In figure 3.4a, for low values of γ , $\text{CVaR}_{97.5\%}$ for the Cauchy distribution produces larger measures of risk than $\text{VaR}_{99\%}$. However, for some γ large enough, this relationship switches. At the estimated γ , $\text{CVaR}_{97.5\%} > \text{VaR}_{99\%}$, which would entail a higher market risk capital charge. The major implication comes from figure 3.4b. For γ large enough, the change in CVaR with respect to the volatility parameter is smaller than the change in VaR. Therefore, stressed CVaR will produce smaller changes in measured risk compared to stressed VaR for the same change in volatility. This feature stems from the fact that CVaR is a conditional average measure rather than a single point like VaR. A smaller increase in measured risk resulting from increased financial market volatility should dampen the risk-constrained feedback effect that result from banks having to sell assets at a loss and depleting bank capital to satisfy their risk constraint and capital requirements. In the sense that banks' reported risk-

³⁵Parameters for the Cauchy distribution of the 10-day S&P 500 returns are estimated using the 'fitdistr' function of the "MASS" package in R.

weighted capital-asset ratio is good proxy for their probability of default, a smaller increase in measured risk could also minimize the effects of risk-weighted balance sheet positions on the cost of borrowing allowing banks to more easily maintain access to interbank funding.³⁶ If both of these effects are minimized with CVaR, then their spillover effects on credit supply and aggregate investment that were documented in chapter 2 should also be dampened.

This potential macroprudential benefit is negated if regulation continues to allow historical calculation of risk measures using a minimum of the past 250 trading days, as is suggested under Basel II. As seen earlier, the revelation of tail risks at the onset of a crisis increases CVaR substantially more than VaR and could increase spillover effects on credit supply and investment. As of Basel 2.5, a stressed VaR measure, which attempts to account for stressed market conditions by calibrating VaR to the worst known financial stress period to ensure that the regulatory capital charge is sufficient under both calm and crisis periods, has been implemented in addition to the VaR requirement. Basel III suggests switching from the combined VaR and stressed VaR to a single stressed CVaR measure to simplify regulations and reduce duplicative capital requirements.

Calculating a stressed CVaR measure entails some practical difficulties, however. Identifying stressed periods that account for the full set of relevant risk factors is practical only for a short windows of historical data. As a result, the Basel Committee is suggesting that the observation window go back to at least to 2005 but notes that this is still likely to require significant approximations and computational burdens. To overcome these issues, the committee suggests using the indirect method for calculating maximum stress. To do this, “banks must specify a reduced set of risk factors that are relevant for their portfolios and for which there is a sufficiently long history of observations so that no approximations are required. . . the identified set of risk factors must meet a range of criteria on data availability and quality” such that “these risk factors explain at least 75% of the variation of the full model.”³⁷ CVaR for banks’ trading books using the reduced set of risk factors ($CVaR_{R,S}$)

³⁶Kapan and Minoiu (2013); Goldberg et al. (2010); and Brei et al. (2013).

³⁷Basel Committee on Banking Supervision (2013).

is calibrated to the most severe one year period over the specified historical window. This value is then scaled up by the ratio of the current CVaR using the full set of risk factors ($CVaR_{F,C}$) to the current CVaR measure using the reduced set of risk factors ($CVaR_{R,C}$). Basel III's suggested risk measure is:³⁸

$$CVaR = \frac{CVaR_{F,C}}{CVaR_{R,C}} \cdot CVaR_{R,S}$$

Figure 3.3 showed that the returns distribution for the S&P 500 fluctuated between normal and Cauchy, where Cauchy is the limit but is never actually reached and represents a worst case financial stress scenario under the FMH. Thus, modeling returns with a Cauchy distribution is comparable to Basel III's proposed stressed CVaR measure for determining market risk capital charges, although this will likely overstate the degree of potential stress and required capital. An important implication of this is that if there is insufficient data to satisfy Basel III's CVaR calculation requirements, modeling returns with a Cauchy distribution would be a safe assumption. Using a Cauchy distribution better captures the tail thickness of actual returns under the worst case financial stress conditions compared to a normal distribution and would maximize the risk measure. It also facilitates analytical expressions for the risk measures that would ease computational burdens. This technique could potentially have been useful for dealing with risks associated with mortgage-backed securities prior to the 2008 financial crisis. These assets were relatively new and had not experienced a period of significant financial stress before 2008, and as a result, their risks were severely underestimated.³⁹

3.4 *The Model Economy*

The model used here to analyze the effects of time-varying volatility and tail risk under both stressed and locally calibrated VaR- and CVaR-based capital regimes is the same as the one used in chapter 2. The financial sector consists of a continuum of three branch banks.

³⁸Basel Committee on Banking Supervision (2013).

³⁹Wickens (2011).

There are two monopolistically competitive retail branch sectors, a deposit branch sector and a loan branch sector, as well as a perfectly competitive wholesale branch sector.⁴⁰ The deposit branch sector sets the deposit rate r_t^d , takes household deposits d_t , and remits them to the wholesale branch sector as wholesale deposits D_t at the central bank policy rate r_t^{cb} . The loan branch sector sets the loan rate r_t^b , takes wholesale loans B_t from the wholesale branch sector at the interbank rate r_t^{ib} , and makes loans to entrepreneurs b_t . The wholesale branch manages the capital position of the combined bank subject to regulator determined risk-weighted capital requirements and its balance sheet identity:

$$B_t + q_t^e s_t^b = D_t + K_t^b \quad (3.2)$$

The wholesale branch sector takes wholesale deposits from the deposit branch sector at the central bank policy rate, makes loans to the loan branch sector at the interbank rate, and invests in a portfolio of risky equity securities s_t^b at the real price q_t^e subject to a risk constraint. From chapter 2, it was shown that deposit branch profits Π_t^{db} , wholesale branch profits Π_t^{wb} , and loan branch profits Π_t^{lb} are:

$$\begin{aligned} \Pi_t^{db} &= r_t^{cb} D_t - r_t^d d_t - \frac{\kappa_d}{2} \left(\frac{r_t^d}{r_{t-1}^d} - 1 \right)^2 r_t^d d_t \\ \Pi_t^{wb} &= r_t^{ib} B_t - r_t^b B_t + (q_{t+1}^e \pi_{t+1} - q_t^e + \delta^e \Pi_t^R) s_t^b - \frac{\kappa_{Kb}}{2} \left(\frac{K_t^b}{RWA_t} - v_b \right)^2 K_t^b \\ \Pi_t^{lb} &= r_t^b b_t - r_t^{ib} B_t - \frac{\kappa_b}{2} \left(\frac{r_t^b}{r_{t-1}^b} - 1 \right)^2 r_t^b b_t \end{aligned}$$

where δ^e is the dividend rate paid out of retailer profits Π_t^R to share holders, π_t is the gross inflation rate, and the last term in each profit equation is a cost associated with interest rate setting or with the actual risk-weighted capital-asset ratio deviating from the regulator

⁴⁰For further discussion on market power in the banking sector see Freixas and Rochet (1997); Diamond (1984), Greenbaum et al. (1989); Sharpe (1990); Kim et al. (2003); Thadden (2004); Demirgüç-Kunt et al. (2004); Berger et al. (2004); and Degryse and Ongena (2008).

determined target ratio v_b .⁴¹ The risk-weighted capital asset ratio is the ratio of bank capital to risk-weighted assets, where risk-weighted assets is defined as:

$$RWA_t = B_t + \frac{M}{v_b} \cdot R_t^{isk} \quad (3.3)$$

where R_t^{isk} is the market risk measure, which will be either VaR_t or $CVaR_t$. RWA_t weights the capital charge for loans and the trading book to be consistent with the steady state risk-weighted capital-asset ratio being exactly equal to the target:

$$\frac{K^b}{RWA} = v_b$$

v_b is set at 10.5% to be consistent with the minimum 8% plus an optional 2.5% capital conservation buffer.⁴² Adding the three branch profits together gives total bank profits:

$$\Pi_t^b = r_t^b b_t - r_t^d d_t + (q_{t+1}^e \pi_{t+1} - q_t^e + \delta^e \Pi_t^R) s_t^b - Adj_t^B \quad (3.4)$$

where Adj_t^B is the combined adjustment costs for the three branches.

The following sections describe the results for the wholesale branch, which will be important for interpreting the impulse responses in section 3.5. The complete structural details and results for the rest of the financial sector and model economy, which is fairly standard, can be found in chapter 2 and appendix A.

3.4.1 Loan Book

The first-order condition for the loan book is:

$$[B_t, D_t] : r_t^{ib} = r_t^{cb} - \kappa_{Kb} \left(\frac{K_t^b}{RWA_t} - v_b \right) \left(\frac{K_t^b}{RWA_t} \right)^2 \quad (3.5)$$

⁴¹For further discussion on interest rate setting frictions see Berger and Hannan (1991); Berger and Udell (1992); Calem et al. (2006); de Bondt et al. (2005); Gambacorta (2008); Driscoll and Judson (2013); Kok Sørensen and Werner (2006); Gropp et al. (1989); Nakajima and Teranishi (2009); and Adrian and Shin (2013). For further discussion on leverage frictions see Adrian and Shin (2013); Elliot et al. (2012); Faulkender et al. (2012); and Flannery and Öztekin (2012); and Berger et al. (2008).

⁴²Federal Register Vol. 77 No. 169 (2012).

Equation (3.5) determines the interbank rate spread $r_t^{ib} - r_t^{cb}$ and is similar to Gerali et al. (2010) with the exception of risk-weighted assets. The appearance of risk-weighted assets in this condition allows for risk and bank balance sheet conditions to affect the cost of interbank funds. In steady state, the interbank rate spread will be zero, consistent with the observation that the effective federal funds rate tends to equal the target federal funds rate on average. If the risk-weighted capital-asset ratio falls below the target, the interbank rate spread will increase as banks' balance sheets appear riskier when they are undercapitalized on a risk-weighted basis. One way this happens is with an increase in market risks due to either increased volatility or tail risk. When market risk increases, then before any other balance sheet adjustments occur, RWA increases as the market risk measure increases, lowering the risk-weighted capital-asset ratio below the target v_b . Thus, in the sense that the risk-weighted capital-asset ratio is an indicator for bank soundness, changes in this ratio will reflect changes in the bank's perceived default probability, affect its interbank borrowing costs, and make it more difficult for the bank to borrow.⁴³

3.4.2 Risk Constraint

There is one important difference between the model used here and the one used in chapter 2. In order to meaningfully analyze the policy implications of Basel III's proposed switch from VaR to stressed CVaR for determining market risk capital charges, a normal distribution can no longer be used to model market risks. Instead, market risks will be modeled with a more general framework using stable distributions, which will allow the observed volatility and tail risk of asset returns to vary exogenously with the business cycle. This modeling choice will allow for the analysis of the differential effects between banks' response to changes in volatility and tail risk on credit supply and aggregate investment

⁴³Kapan and Minoiu (2013) and Goldberg et al. (2010) find evidence that banks' balance sheet structure affects their cost of funds and ability to access wholesale funding markets. Banks with higher, less risky, and better quality capital are better able to maintain access to wholesale funding and at a lower cost. In addition, Brei et al. (2013) show that banks with higher regulatory capital ratios increase lending during normal markets. Equation (3.5) captures these observations.

when subject to competing market risk capital regimes in a meaningful way.

This one difference affects the modeling of banks' capital requirements and the risk constraint that was derived in chapter 2 where market risks were measured with VaR and an assumed normal distribution. In that chapter, the risk measure was defined as:

$$VaR_t = (E_t [q_t^e - q_{t+1}^e \pi_{t+1}] + Z_{99\%} \sigma_t) s_t^b$$

where $Z_{99\%}$ is the constant associated with the required 99% confidence level and σ_t is the standard deviation of price changes for the representative asset. Now, risks will be measured under four different regimes, stressed and locally calibrated VaR and CVaR using stable distributions. As mentioned earlier, the density functions of most stable distributions are not analytically tractable, which poses some modeling complications. VaR and CVaR will have to be numerically calculated according to the general expressions:

$$1 - c = \int_{-\infty}^{VaR_c} f(x) dx \quad (3.6)$$

$$CVaR_c = \frac{1}{1 - c} \int_{-\infty}^{VaR_c} x f(x) dx \quad (3.7)$$

where $c = 0.99$ for the VaR measure (3.6) according to the Basel II risk regime, and $c = 0.975$ for the CVaR measure (3.7) according to the Basel III risk regime. $f(x)$ is the returns distribution assumed to be in the family of stable distributions as defined in equation (3.1) where the location parameter $\mu_t = E_t [q_t^e - q_{t+1}^e \pi_{t+1}] s_t^b$. The skewness parameter is set to $\beta = 0$ to simplify the calculation but the stability parameter α and the volatility parameter γ will be allowed to vary. More detail about the chosen values for α and γ will be given in section 3.5. In order to work with the numerical algorithm, a truncated returns distribution needs to be implemented. As discussed 3.3.3, it is reasonable to symmetrically truncate the returns distribution so that the lower bound is the maximum possible loss equal to the total value invested such that the domain is $x \in [-q_t^e s_t^b, q_t^e s_t^b]$.

To derive the risk constraint relevant here, bank capital is assumed to be accumulated out of retained earnings according to:

$$K_{t+1}^b = (1 - \delta^b)K_t^b + \Pi_t^b + \epsilon_t^{K^b} \quad (3.8)$$

where K_t^b is total bank capital and δ^b is an assumed cost associated with managing the capital position of the bank, which is used to prevent the bank from accumulating an infinite amount of capital and becoming self financing. Then, in order for banks to remain in operation, a bank's capital must remain positive so that the value of its assets is larger than its liabilities:

$$K_{t+1}^b \geq 0 \implies \Pi_t^b \geq -(1 - \delta^b)K_t^b$$

Combining bank profits (3.4) with this last constraint gives:

$$r_t^b b_t - r_t^d d_t + (q_{t+1}^e \pi_{t+1} - q_t^e + \delta^e \Pi_t^R) s_t^b - Adj_t^B \geq -(1 - \delta^b)K_t^b$$

Finally, to arrive at the risk constraint, which will be assumed to bind in the neighborhood of the steady state, subtract expected losses from the loan book and the trading book:⁴⁴

$$r_t^b b_t - v_b B_t - r_t^d d_t - M \cdot R_t^{isk} + \delta^e \Pi_t^R s_t^b - Adj_t^B = -(1 - \delta^b)K_t^b \quad (3.9)$$

where R_t^{isk} will either be VaR_t or $CVaR_t$. $v_b B_t$ is the capital charge on the loan book and $M \cdot R_t^{isk}$ is the capital charge on the trading book. M is a scaling factor determined by the regulator which is typically set to 3 under the current VaR-based regulatory regime.⁴⁵ Under the Basel III regime, the multiple on stressed CVaR is the ratio of the current CVaR using the full set of risk factors over the current CVaR measure using the reduced set of risk factors ($CVaR_{F,C}/CVaR_{R,C}$). For simplicity, it will be assumed that this scaling factor continues to be 3 to ease comparisons across regimes.

⁴⁴See section 3.4.4 for a discussion about the risk constraint binding.

⁴⁵ M can be thought of as a safety factor. If the holding period is 10 days and the confidence level is 99%, then a loss greater than VaR would be expected to occur once every 4 years. Arguably, financial stress events would happen far too often under this scenario. Thus, multiplying by M should lessen the likelihood of such losses. M can also be thought of as increasing the time horizon for the holding period or the confidence level.

3.4.3 Trading Book

The only first-order condition to differ from the model in chapter 2 is the the first-order condition for the trading book, which becomes:

$$[s_t^b] : E_t [q_{t+1}^e \pi_{t+1} - q_t^e + \delta^e \Pi_t^R] - r_t^{cb} q_t^e - \frac{\partial Adj_t^{Kb}}{\partial s_t^b} + \lambda_t^b \left(-M \frac{\partial R_t^{isk}}{\partial s_t^b} + \delta^e \Pi_t^R - \frac{\partial Adj_t^{Kb}}{\partial s_t^b} \right) = 0$$

$$\frac{\partial Adj_t^{Kb}}{\partial s_t^b} = -\kappa_{Kb} \left(\frac{K_t^b}{RWA_t} - v_b \right) \left(\frac{K_t^b}{RWA_t} \right)^2 \frac{M}{v_b} \frac{\partial R_t^{isk}}{\partial s_t^b}$$

where λ_t^b is the Lagrange multiplier on the risk constraint. λ_t^b has the interpretation as the rate of change in expected profits with having to hold another unit of regulatory capital, or the expected return on equity (ROE), as well being the time-varying risk appetite of the bank.⁴⁶ Rearranging the first order condition for the trading book size gives an equation for λ_t^b :

$$\lambda_t^b = \frac{E_t [q_{t+1}^e \pi_{t+1} - q_t^e + \delta^e \Pi_t^R] - r_t^{cb} q_t^e - \frac{\partial Adj_t^{Kb}}{\partial s_t^b}}{M \frac{\partial R_t^{isk}}{\partial s_t^b} - \delta^e \Pi_t^R + \frac{\partial Adj_t^{Kb}}{\partial s_t^b}} \quad (3.10)$$

Since, the trading book size s_t^b affects the volatility of the bank's returns distribution, an increase in s_t^b will increase the measured risk of the trading book so that $\partial R_t^{isk} / \partial s_t^b > 0$. This partial derivative will also increase if γ_t increases or α_t decreases for any level of s_t^b . Chapter 2 found that with VaR and a normal distribution, $\partial \lambda_t / \partial \sigma_t < 0$. It should then be the case that $\partial \lambda_t^b / \partial \gamma_t < 0$ since γ_t of the stable distribution is related to σ_t of the normal distribution by $\sigma_t = \gamma_t \sqrt{2}$. By the same logic, $\partial \lambda_t^b / \partial \alpha_t > 0$, because both a decrease in α_t and an increase in γ_t increase the measured risk of the returns distribution. This implies that an increase in measured risk reduces banks' expected ROE as they are required to hold an extra unit of regulatory capital.

⁴⁶Danielsson et al. (2011).

3.4.4 Binding Risk Constraint

Because the first-order condition for the trading book size does not determine the size of the trading book in steady state, the risk constraint will. This means that the risk constraint needs to bind in steady state. To ensure this, first note that the expected profits from investing in a portfolio of equity securities is $E_t[q_{t+1}^e \pi_{t+1} - q_t^e + \delta^e \Pi_t^R - q_t^e r_t^{cb}] s_t^b$ once the balance sheet identity (3.2) is substituted into wholesale branch profits. The wholesale branch will only have incentive to invest in a portfolio of equity securities in steady state if $\delta^e \Pi^R \geq q^e r^{cb}$. Thus, the model is calibrated such that the dividend yield equals the return on lending, $\delta^e \Pi^R / q^e = r^b$, so that the bank as a whole is indifferent between making another loan and increasing its trading book size by another unit in steady state.

To see how the conditions for a binding risk constraint change from chapter 2, first rearrange the risk constraint (3.9) to look like $\Pi^b \geq v_b B + M \cdot R_t^{isk} - (1 - \delta^b) K^b$, where the right hand side is the effect of the constraint (C). The equation is then written as a function of the trading book size by taking into account the bank's balance sheet identity (3.2) and the bank capital accumulation equation (3.8) and substituting them into the above equation for B , noting that in steady state $K^b = v_b B + M \cdot R^{isk}$ and $\delta^e \Pi^R = r^b q^e$:

$$\begin{aligned} \Pi^b &= \left(\frac{r^b}{1 - v_b} - r^d \right) D + \left(\frac{r^b}{1 - v_b} \right) (M \cdot R^{isk} - v_b q^e s^b) \\ C &= \left(\frac{\delta^b v_b}{1 - v_b} \right) D + \left(\frac{\delta^b}{1 - v_b} \right) (M \cdot R^{isk} - v_b q^e s^b) \end{aligned}$$

The equations show that these functions will increase in the trading book size only if the risk per dollar invested is larger than the ratio of regulatory parameters, $R^{isk} / (q^e s^b) > v_b / M$, and the constraint will start below bank profits and increase at a faster rate only if $r^b < \delta^b < v_b$, which is satisfied at the steady state with the parameters chosen in this model. If these do not hold, both the constraint and bank profits will be decreasing or non-intersecting and the constraint will not bind. However, it has been noted that capital constraints may not always bind: they tend to bind in downturns when capital is scarce but have little effect on

behavior during expansions when it is much easier to raise capital.⁴⁷ The risk constraint in this model will always bind, as it does in Shin (2010), if the expected return on the risky asset is larger than the expected return on interbank loans. Otherwise, banks will not hold any of the risky asset. If the risk constraint is not binding, banks can take on more balance sheet risk and increase expected profits. Thus, if banks are selecting a mix of assets to maximize profits, their risk constraint should be binding.

3.4.5 Procyclical Leverage and Asset Demand-Supply Responses

With the risk constraint (3.9) now shown to bind, rearranging it to get the bank's demand function for the trading book size results in:

$$s_t^b = \frac{(r_t^b - r_t^d - v_b) D_t + (1 + r_t^b - \delta^b - v_b) K_t^b - Adj_t^B}{M \frac{R_t^{Risk}}{s_t^b} + (r_t^b - v_b) q_t^e - \delta^e \Pi_t^R} \quad (3.11)$$

$$R_t^{Risk} = R_t^{Risk}(\alpha_t, \Gamma_t)$$

$$\Gamma_t = \gamma_t s_t^b$$

after substituting in the balance sheet identity (3.2) for B_t where $R_t^{Risk}(\alpha_t, \Gamma_t)$ is a function of the stability parameter α_t and portfolio volatility Γ_t , which is equal to the representative asset volatility γ_t weighted by the trading book size s_t^b . With non-linear risk equations, the trading book demand equation does not have a simple analytical expression as it did in chapter 2 under VaR and the normality assumption. However, it can be seen that the trading book size still depends positively on the overall intermediation spread ($r_t^b - r_t^d$) and the return on bank capital ($1 + r_t^b - \delta^b - v_b$). As in chapter 2, higher unit-risk ($U_t^{Risk} = R_t^{Risk}/s_t^b$) also reduces the size of the bank's trading book, which can be seen by differentiating equation (3.11) with respect to U_t^{Risk} around the steady state:

$$\frac{\partial s_t^b}{\partial U_t^{Risk}} \approx \frac{M s_t^b}{v_b q^e - M \frac{R_t^{Risk}}{s_t^b}} < 0$$

⁴⁷Brunnermeier and Sannikov (2014).

This shows that banks will reduce the size of their trading book when market risk increases, whether it is due to volatility or tail risk, if $v_b/M < R^{risk}/(q^e s^b)$, which is satisfied when the risk constraint binds.

Finally, leverage defined as total assets over bank capital:

$$L_t = \frac{B_t + q_t^e s_t^b}{K_t^b} \quad (3.12)$$

will be continue to be inversely related to unit-risk as in Shin (2010). This occurs because $\partial s_t^b / \partial U_t^{Risk} < 0$ and K_t^b is correlated with lagged unit-risk (U_{t-1}^{Risk}) through the bank capital accumulation equation (3.8). To see this, differentiate the leverage equation (3.12) with respect to U_t^{Risk} around the steady state:

$$\frac{\partial L_t}{\partial U_t^{Risk}} \approx \frac{q^e}{K^b} \frac{\partial s_t^b}{\partial U_t^{Risk}} < 0$$

Therefore, leverage will tend to be procyclical (high when volatility is low, and low when volatility is high), because the wholesale branch is subject to the risk constraint and actively manages its balance sheet in the context of this model. More importantly, CVaR has the potential to reduce the procyclical leverage response to a change in risk by reducing the bank's change in perceived risk, capital charges, and demand-supply responses when risks are calibrated to stressed market conditions as was illustrated in figure 3.4b.

3.5 Simulations

To study the dynamics of the linearized model, the focus will be on unanticipated shocks to γ_t (volatility shock) and α_t (tail risk shock) with parameter values for the model set at their calibrated or estimated values in chapter 2.⁴⁸ The only exception is for the parameters involving the returns distributions. In chapter 2, the steady state volatility parameter

⁴⁸Simulations are computed under the assumption that the expected value of all future shocks is zero. Since the model is stochastic and log-linearized around the steady state, agents will behave as if the value of future shocks is zero due to certainty equivalence. This also implies that perturbation methods are valid only in a neighborhood of the steady state. As shocks become too large, the linear approximation becomes less accurate and the conditions that ensure that all constraints bind may no longer hold.

associated with the assumed normal returns distribution was calibrated to be $\sigma = 0.67$. In order to map σ of the normal distribution to γ of the stable distribution, the relationship is $\gamma = \sigma/\sqrt{2}$, implying $\gamma = 0.48$.

The first simulation will compare the model economy's response to a tail risk shock under the local VaR and CVaR market risk regimes relative to a volatility shock when returns are initially perceived to be normally distributed as is typically observed during calm market periods just prior to a recession or financial crisis. The second simulation will compare the model economy's response to a volatility shock under the stressed VaR and CVaR market risk regimes as has been implemented under Basel 2.5 (stressed VaR regime) and suggested under Basel III (stressed CVaR regime). Stressed markets will be modeled using a Cauchy distribution, noting that the Cauchy case is the worst possible financial stress conditions allowed under the FMH. There are two objectives for the simulations as there are two parts to the Basel III market risk regime: the switch from VaR_{99%} to CVaR_{97.5%} and a switch from local risk calibration to calibrating risk to stressed market conditions. The first objective is to assess how the CVaR_{97.5%} market risk regime affects the procyclicality of market risk capital charges and its spillover effects on credit supply and aggregate investment compared to the VaR_{99%} market risk regime that was studied in chapter 2. The second is to analyze these same concerns comparing stressed CVaR to stressed VaR for calculating regulatory capital.

3.5.1 Transitory Volatility and Tail Risk Shocks Under Local VaR and CVaR Market Risk Capital Regimes

To simulate the economy's response to a volatility and tail risk shock with the same persistence, it will be assumed that the steady state of the economy is one in which banks locally estimate the returns distributions to be normal.⁴⁹ One concern here is how to calibrate the shock sizes so they are comparable and representative of a typical business cycle. Because

⁴⁹The persistence is estimated using Bayesian techniques and an AR(1) process in chapter 2. The estimated posterior median for the AR(1) coefficient on the σ_t shock is 0.6257.

the volatility parameter γ is only bounded below by 0, but the stability parameter α is bounded between 1 and 2, the question becomes how to best equate the size of the shocks for comparison. In order to address this, the size of the shocks are calibrated to match one standard deviation movements of the estimated stable distribution parameters, where the estimates are obtained using the maximum likelihood technique to fit a stable distribution to the daily returns distribution of the S&P 500 within a year between 1970-2015.⁵⁰ Thus, the shocks used are a one standard deviation percent increase relative to the mean in γ for the volatility shock and a one standard deviation decrease in α for the tail risk shock. The size of the volatility shock is estimated to be a 33% increase in volatility, the same as the relative standard deviation for the quarterly VIX and about one-half the size of the 62% increase in the VIX between 2002:Q2 and 2002:Q3. The size of the tail risk shock is estimated to be a 7.5% decrease in the stability parameter α , or a decrease from 2 to 1.84 and representative of the decrease in α observed between 1999 and 2001. Since shocking the stability parameter will result in a stable distribution with $\alpha \in (1, 2)$ where the density function is not analytically expressible, a numerical algorithm is used to find the density function and risk measures.⁵¹

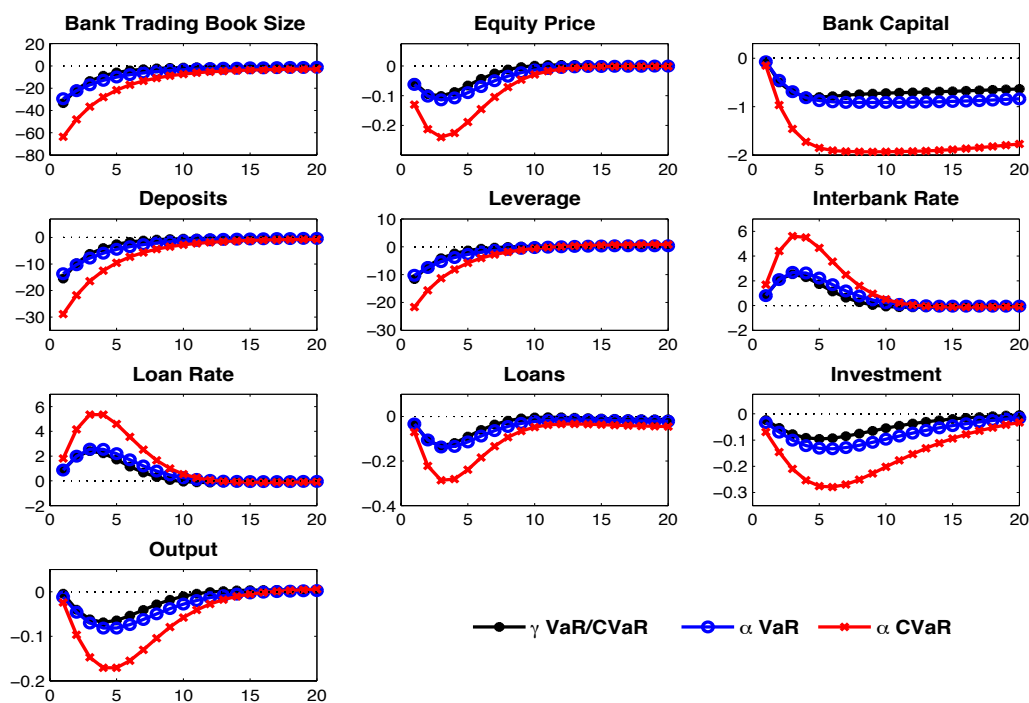
The first shock plotted in figure 3.5 labeled γ VaR/CVaR (black line with \bullet markers) is a positive shock to γ_t (volatility shock) under the VaR or CVaR regime as they are nearly identical. The second shock plotted labeled α VaR (blue line with \circ markers) is a negative shock to the stability parameter α_t (tail risk shock) under the VaR regime. Finally, the third shock plotted labeled α CVaR (red line with \times markers) is a negative shock to α_t under the CVaR regime.

Each shock originates in the financial system through the risk constraint and market risk capital requirements. Upon impact of the shock, the risk constraint tightens and banks will need to sell securities, reduce loans, or reduce debt to ease the constraint. Banks' expected

⁵⁰See section 3.3.2

⁵¹The impulse response functions are calculated using Dynare 4.4.3 in conjunction with Matlab R2015b. The risk measure is calculated numerically within Dynare using an external Matlab function designed for this model that can be found at <https://sites.google.com/site/hubbardalex/research>.

Figure 3.5: 1 Std. Dev. Positive Transitory Volatility & Tail Risk Shocks Under Local Risk Regimes



Note: Impulse responses are in percent deviation from steady state values.

ROE and risk appetite falls as banks are forced to hold more regulatory capital. Banks respond by unwinding their trading positions, selling risky equity securities, and lowering their debt level as total assets fall. Households view deposits and equity shares as perfect substitutes in this model and play the role of backstop in a way that makes the equities market very liquid, purchasing the excess supply of these securities. The selling pressure that is exerted by banks having to comply with higher capital charges and dictated by the risk constraint puts downward force on the equity price as households lower their expectation about the future equity price. Banks then sell the equity securities at a loss, resulting in a loss in their equity capital. Households switch their savings channel towards equity securities

and decrease the amount of bank deposits, effectively making banks smaller. This decrease in bank debt is comparable to the run in wholesale funding banks experienced during the crisis and causes banks to deleverage.⁵²

The increase in market risk initially increases banks' risk measure and risk-weighted assets (RWA), decreasing the risk-weighted capital-asset ratio (RW-CAR). Since banks are now undercapitalized on a risk-weighted basis, potential lenders see an increased default probability associated with lending to banks on the interbank market. Together with the loss in equity capital, this puts upward pressure on the interbank rate and loan rate, because the loan rate is a markup over the interbank rate. Thus, when banks' risk-weighted balance sheet positions deteriorate, the effects are passed through to entrepreneurs in the form of higher borrowing costs and amplified by the degree of the markup. However, banks are able to remove risk from their balance sheets very efficiently following their risk management strategy that is represented by the risk constraint, because the equity securities market is very liquid.

The act of removing risk from balance sheets eases the risk constraint and improves banks' risk-weighted balance sheet positions and interbank market conditions. This helps banks substitute balance sheet assets away from the high capital burden asset towards loans, which now require relatively less regulatory capital. Because banks are forced to deleverage, suffering a capital and deposit loss from selling equity securities, credit supply and total loans decrease. Finally, with loans falling and becoming more expensive, firms decrease the use of capital in production, which results in a decrease in investment and output.⁵³

The notable results here concern the amplification effects of a tail risk shock compared

⁵²Acharya and Mora (2012).

⁵³This can be interpreted as an aggregate supply decrease that outweighs the decrease in aggregate demand causing inflation to rise, something that also occurs with the negative bank capital shock in Gerali et al. (2010). The central bank responds with an increase in the policy rate by naively following the Taylor rule, driving up both the deposit and loan rates. This shock is not meant to capture everything associated with financial crises. There are likely other aggregate demand factors and credit losses at play that could result in deflationary pressures in contrast to what was produced here. So, there is still some to be desired since the data suggests that investment and the effective federal funds rate are positively correlated.

to a volatility shock and the amplification effects of a tail risk shock under the CVaR regime compared to the VaR regime when risks are initially perceived to be normal. With the shock size representative of typical business cycle movements, the model's response under the VaR regime is very similar to both a volatility and tail risk shock. However, under the CVaR regime, the tail risk shock significantly amplifies the model's response compared to the VaR regime and a volatility shock. Upon impact of the shocks, the tail risk shock amplifies banks' initial asset selling response compared to the volatility shock and the VaR regime. The percentage point decline in banks' trading book size is nearly twice as large. This balance sheet response amplifies the risk-constrained feedback effect that results from the equity price decline and bank capital loss, causing a more severe deleveraging event as well as a decline in investment and output that are also nearly twice as large in terms of percentage points compared to the other two cases at the peak, which occurs 5 quarters after the shock.

When risks are locally calibrated and the returns distribution appears normal, both the VaR and CVaR regime generate nearly identical responses to increased financial market volatility. This is because the risk measures are linear and have nearly the same proportionality to the standard deviation of the portfolio when the perceived returns distribution is normal. The important differences appear when the tail thickness of the returns distribution increases. Under the VaR regime, volatility and tail risk shocks have nearly identical effects. Although, under the CVaR regime, the response to a tail risk shock is amplified, because CVaR is designed specifically to capture changes in tail thickness. Since Basel III is suggesting a switch from VaR to CVaR to measure market risks for required capital, this policy has the potential to amplify banks' balance sheet response and the risk-constrained feedback effect on business cycle volatility. However, using a stressed risk measure to determine market risk capital charges can eliminate some of the effects of procyclical capital charges by limiting tail risk exposure. This is analyzed in the next section.

3.5.2 Model Properties Under Stressed VaR & Stressed CVaR Regimes

To calibrate risks to financial stress periods, it is necessary to have some theory about what a stressed market is. According to the FMH and the the results displayed in figure 3.3, a stressed market tends to have returns distributions that are heavy-tailed, meaning the distribution is non-normal with a stability parameter $\alpha \in (1, 2)$. As market stress worsens, returns distributions move away from a normal distribution towards a Cauchy distribution where $\alpha = 1$. While it does not appear that markets fully reach a Cauchy distribution during stressed periods, the Cauchy distribution represents the most stressed scenario and eliminates the possibility of any emerging tail risks that were seen in the previous section to have an amplification effect on banks' balance sheet response and the risk-constrained feedback effect on credit supply and aggregate investment under the CVaR regime. What is not as easily pinned down is the size of the volatility parameter γ . In theory, calibrating risk to the worst stressed period should capture the highest observed γ . However, it is still possible to underestimate this parameter as there are only a limited number of stressed market periods available for estimation and some assets may not have a long enough data history to observe a γ representative of a stressed period (i.e. mortgage backed securities at the time of the financial crisis), leaving stressed risk measures vulnerable to increases in estimated γ .⁵⁴ A negative consequence of measuring stressed risks using a Cauchy distribution is that it may impose excess capital burden on banks and reduce the steady state investment level, especially if markets never actually reach the Cauchy case.

With this in mind, in order to evaluate the procyclical differences between Basel 2.5's stressed VaR and Basel III's stressed CVaR, the steady state of the model is now calibrated assuming that banks model market risks using a Cauchy distribution to measure stressed risks. The volatility parameter γ for the Cauchy distribution is set to match the calibrated

⁵⁴Basel III's stressed CVaR measure is scaled up by locally estimated CVaR measures ($CVaR_{F,C}/CVaR_{R,C}$), this multiple on stressed CVaR will be subject to time-varying risks.

Table 3.1: Steady State Changes to Bank Balance Sheets

	Stressed VaR Regime	Stressed CVaR Regime
Trading Book Value	-34.4%	-48.9%
Total Assets	-12.0%	-17.1%
Deposits	-12.0%	-17.1%
Bank Capital	-12.0%	-17.1%
Risk	-16.6%	-23.7%

Note: Percent changes are relative to the original VaR-based capital regime. Loans, investment, and output are expected to remain nearly unchanged from the original VaR regime

standard deviation of the assumed normal distribution in chapter 2.⁵⁵ This results in a steady state $\gamma = 0.08$.

Steady State Changes Under Stressed VaR & Stressed CVaR Regimes

The first consideration is to examine how banks respond to the new regime in the long run by evaluating changes to some steady state variables relative to the original VaR regime, which are reflected in table 3.1. First considering banks' balance sheets, the model predicts almost no change in lending with all balance sheet adjustments occurring on the trading book and debt level since households are fully willing to take up the slack in the equities market. This is a result of households viewing bank deposits and equity securities as perfect substitutes for savings here. The model predicts a 34.4% reduction in banks' trading book value when subject to the stressed VaR regime and a 48.9% reduction if subject to the stressed CVaR regime, with nearly all the decline resulting from banks' decision to hold less securities. The value of these securities are determined purely by the present value of all future dividend payments, which is unaffected. As banks' hold less risky securities, their total reported risk measure falls by 16.6% and 23.7% respectively. With a smaller trading

⁵⁵To match the standard deviations, a truncated Cauchy distribution is used as in section 3.3.3, otherwise the Cauchy distribution has an infinite variance. The variance for the truncated Cauchy distribution is $\sigma^2 = (\gamma d)/(\tan^{-1}(d/\gamma)) - \gamma^2$ where d is a distance from the mean, taken to be equal to the value of the portfolio $q^e s^b$, used to symmetrically truncate the distribution.

book, banks do not need as much debt to finance their asset holdings so banks' debt level also declines but by a smaller 12% and 17.1% respectively. The net effect is a 12% and 17.1% decline in required capital.

These adjustments also result in nearly zero change in bank leverage and risk-equity ratio, consistent with the notion that financial institutions tend to target a fixed risk-equity ratio.⁵⁶ Since the model predicts almost no change in overall lending, aggregate investment and output are also unaffected. In this model, supply-side fundamentals defined by the entrepreneur's problem, which include a Kiyotaki and Moore (1997) type collateral constraint, determine how much aggregate borrowing and investment take place in steady state. The parameters affecting the collateral constraint and the steady state capital stock (the loan-to-value ratio, physical capital depreciation rate, steady state inflation rate, the steady state cost of capital, the steady state loan rate elasticity, and the steady state loan rate) remain unchanged, and therefore, do not affect credit or investment demand fundamentals.

In a world where securitization is a large part of the financial intermediation process as it is in the U.S., changes in the market risk capital regime may reduce the amount of securitized assets held on bank balance sheet. This could reduce credit supply as financial intermediation would be restricted with an increased capital charge on securitized assets held in the trading book. This issue is not considered here and left for further research.⁵⁷

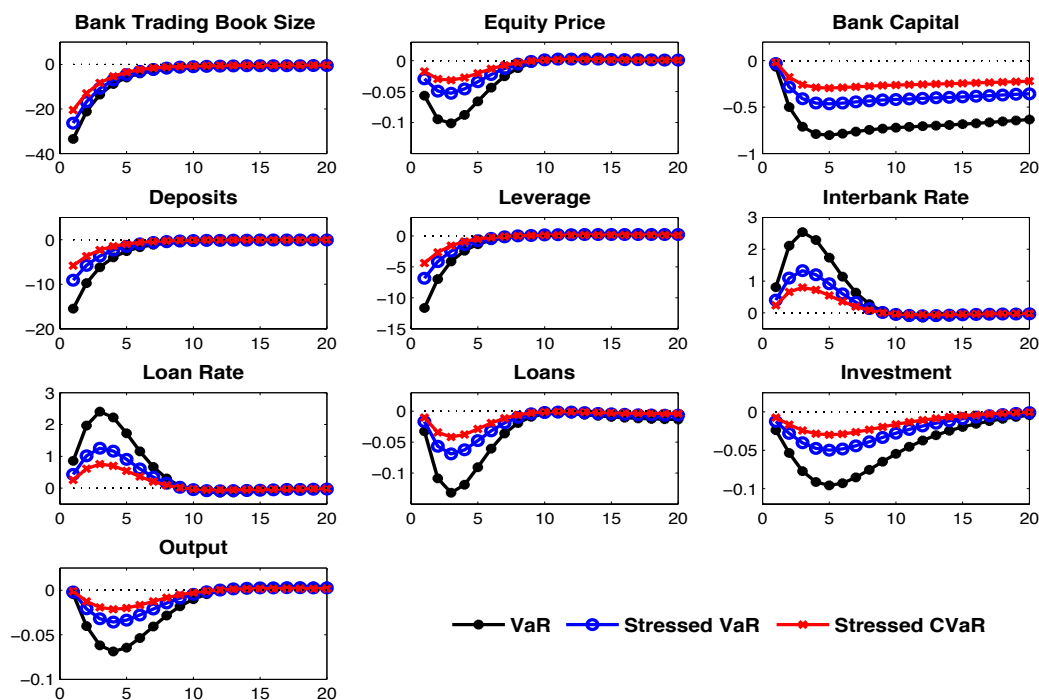
Transitory Volatility Shock Under Stressed VaR & Stressed CVaR Regimes

The final consideration is to examine the cyclical properties of the stressed VaR and stressed CVaR market risk capital regimes compared to the original VaR regime in response

⁵⁶Adrian and Shin (2010, 2013).

⁵⁷Also not considered here is any endogenous feedback in the volatility of financial markets that may occur as a result of the new market risk capital requirements. If these effects were included, expected changes to bank balance sheets could be larger. The assumption that household's view bank deposits and equity securities as perfect substitutes is likely to be unrealistic. If households were to view these two assets as imperfect substitutes, supply and demand interactions would likely lead to a decrease in the equity price. Finally, changes in credit-risk capital requirements are also abstracted from here, which would likely have stronger effects on steady state lending and aggregate investment.

Figure 3.6: 1 Std. Dev. Positive Transitory Volatility Shock Under Stressed Risk Regimes



Note: Impulse responses are in percent deviation from steady state values.

to an unanticipated one standard deviation shock (the same 33% increase as in the previous section) to the volatility parameter γ_t . The black line in figure 3.6 with \bullet markers represents the simulation under the original VaR regime. The blue line with \circ markers represents the simulation under the stressed VaR regime. Finally, the red line with \times markers represents the simulation under the stressed CVaR regime.

The first notable observation is that both the stressed VaR and stressed CVaR regimes are less responsive to the increase in volatility compared to the original VaR regime. Under the stressed VaR regime, banks' initial selling response is reduced by 8 percentage points and by another 6 percentage points under the stressed CVaR regime compared to the original

VaR regime. This property dampens the risk-constrained feedback effect, because the equity price decline and bank capital loss are less severe leading to a deleveraging event as well as a decline in investment and output that are about half the size in terms of percentage points when under the stressed VaR regime compared to the local VaR regime at the peak, which occurs 5 quarters after the shock. Under the stressed CVaR regime, these responses are further dampened by almost half in terms of percentage points compared to the stressed VaR regime.

If stressed risk measures are still vulnerable to time-varying volatility, both the stressed VaR and stressed CVaR regime appear to dampen the spillover effects of procyclical capital charges on credit supply and aggregate investment compared to the local VaR regime. On top of that, Basel III's proposed stressed CVaR measure appears to dampen these spillover effects even further compared to the stressed VaR regime. The results from these simulations provide some supporting evidence for Basel III's proposed market risk changes for two reasons. One, stressed risk measures reduce banks' exposure to time-varying tail risk that can have a large impact on banks' balance sheet response and the risk-constrained feedback effect. And two, stressed CVaR dampens these effects compared to the local VaR regime and the stressed VaR regime.

3.6 Conclusion

This paper expanded on the analysis from chapter 2 to evaluate the potential macroeconomic implications of Basel III's proposed market risk capital regime. Chapter 2 documented the structural links that Value-at-Risk-based capital requirements create between volatility in financial markets and the macroeconomy. That chapter also illustrated that the concerns raised by the Basel Committee on Banking Supervision about the procyclical effects of VaR-based capital charges were indeed well founded. The results suggested that capital regulators and risk managers should seek an alternative market risk measure that could reduce the procyclicality of risk-based capital charges and limit the spillover effects of banks' balance sheet responses on credit supply and aggregate investment. The Basel Committee has taken on

this challenge and suggested moving away from the current regime, which requires banks to calculate their market risk capital charge using VaR at a one-tail 99% confidence level supplemented with a stressed VaR measure calibrated to known financial stress periods, to a single stressed CVaR measure at a one-tail 97.5% confidence level to determine the market risk capital charge. The intention is to remove duplicative capital charges, limit banks' ability to arbitrage regulatory capital requirements, and help ensure a more stable financial system.

To evaluate whether Basel III's proposed switch from VaR to stressed CVaR for measuring market risks can reduce the procyclical effects of risk-based capital charges, this paper modified the DSGE model from chapter 2 that included a monopolistically competitive financial sector subject to risk-based capital requirements. However, in order to make the comparison between VaR- and CVaR-based capital regimes meaningful, some ingenuity was needed. Both $\text{VaR}_{99\%}$ and $\text{CVaR}_{97.5\%}$ produce nearly identical risk measures if the efficient markets hypothesis and the normality of asset returns is assumed as was done in chapter 2. Instead, asset returns are modeled with stable distributions following the fractal markets hypothesis, which is a more accurate and flexible assumption. Using stable distributions to model daily returns of the S&P 500, it was shown that the returns distribution tends to appear normal and exhibit thin tails when markets are relatively calm. However, when markets become stressed, the returns distribution tends to move away from the normal distribution towards a Cauchy distribution, exhibiting much thicker tails and revealing unforeseen tail risks. The implication is that calm markets may be modeled with a normal distribution, while stressed markets may be modeled with a Cauchy distribution.

When markets are calm and normal, banks are exposed to two sources of risk: volatility and tail risk. This triggered the question of how VaR and CVaR respond to changes in the parameters that govern these two types of risk. It was then shown that VaR is more responsive to time-varying volatility than to time-varying tail risk. It was also shown that CVaR is more responsive to time-varying tail risk than VaR. If risks are calibrated to stressed market periods, which is the approach taken with Basel 2.5 using an additional stressed VaR

measure, and Basel III, which plans to institute a stressed CVaR measure, exposure to time-varying tail risks can be reduced leaving exposure to time-varying volatility as the main concern. Thus, stressed CVaR has the potential to dampen the negative spillover effects of market risk capital charges on the real economy. Stressed CVaR can reduce banks' balance sheet response to changes in perceived risk, which should reduce the risk-constrained feedback effect on credit supply and aggregate investment that results from falling asset prices and depleted bank capital as banks sell assets to comply with higher capital requirements.

Determining capital charges from calibrated financial stress periods raised the concern that higher capital burdens could lower the steady state levels of lending and aggregate investment. However, the model predicts nearly no negative effects on lending and investment in the long run in terms of reduced steady state levels. The modeled banks' response to the implementation of stressed market risk capital regimes relative to the local VaR regime predicted that almost all balance sheet adjustments occur on banks' trading books and debt levels. This is a result of households being fully willing to take up the slack in asset markets as they view bank deposits and equity securities as perfect substitutes for saving here, an assumption that likely needs to be reconsidered in future research. The model also predicted a reduction in banks' trading book value when subject to the stressed VaR regime and an even larger reduction if subject to the stressed CVaR regime, with nearly all the decline resulting from banks' decision to hold less securities. As banks' held less risky securities, their total reported risk measure fell along with total debt. The net effect was a decline in banks' required capital. These adjustments also left bank leverage and the risk-equity ratio unaffected. As the model predicted almost no change in long run lending, long run aggregate investment was also unaffected as the supply-side fundamentals that determined the steady state level of borrowing and investment were unaffected.

Finally, to perform impulse response analysis and evaluate whether stressed CVaR can reduce the procyclical spillover effects of market risk capital charges on macroeconomic activity compared to stressed and local VaR, the model economy was simulated in response to two types of shocks: a volatility shock and a novel tail risk shock. This required formulating

a numerical algorithm for use inside the analytical framework in order to work with stable distributions. The results from these simulations suggest that Basel III's proposed stressed CVaR capital charge can dampen the spillover effects for two reasons. One, calibrating the risk measure to stressed market periods, which tend to exhibit heavy-tailed returns distributions, should help banks be adequately capitalized in normal times to withstand a financial crisis in the event that one occurs, because it reduces tail risk exposure leaving time-varying volatility as the main source of instability. And two, stressed CVaR is less responsive to time-varying volatility compared to stressed VaR. These properties can reduce the size of changes to regulatory capital charges and dampen banks' balance sheet response as well as the risk-constrained feedback effect on credit supply and aggregate investment.

Overall, these results provide some supporting evidence for Basel III's proposed switch to a stressed CVaR market risk regime. However, this paper abstracted from securitized financial intermediation, which may be negatively impacted if increased trading book capital requirements restrict the securitization process. This paper also did not consider any features CVaR may have if bank creditors use it to determine the value of recoverable assets in the event of default when VaR is used to determine the probability of default as suggested by Shin (2010). CVaR also does not provide a solution to some of the concerns raised by Daniélsson et al. (2001) about the Basel II risk-sensitive capital framework:

- *The proposed regulations fail to consider the fact that risk is endogenous. Value-at-Risk can destabilize and induce crashes when they would not otherwise occur, and*
- *Financial regulation is inherently procyclical. . . this set of proposals [Value-at-Risk] will, overall, exacerbate this tendency significantly. In so far as the purpose of financial regulation is to reduce the likelihood of systemic crises, these proposals [Value-at-Risk] will actually tend to negate, not promote this useful purpose.*

CVaR still treats risk as exogenous. It does not account for how market participants' actions will affect price changes and risk. Thus, CVaR can still destabilize and induce crashes when they would not otherwise occur. CVaR as financial regulation is also still procyclical;

however, this paper showed that CVaR can reduce some of the procyclicality induced by VaR. CVaR may then be able to reduce the severity of systemic crises, but more research needs to be done to understand if CVaR has any effect on limiting the likelihood of systemic crises. Until financial regulation and risk management can fully understand and account for the endogenous nature of financial risk, neither VaR nor CVaR will be ideal market risk capital standards from a macroprudential perspective.

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Appendix A

MODEL ECONOMY DETAILS

A.1 *Entrepreneurs and Households*

The first-order conditions for the entrepreneur are:

$$[c_t^E] : (1 - h)^\gamma (c_t^E - hc_{t-1}^E)^{-\gamma} = \lambda_t^E \quad (\text{A.1})$$

$$[k_t] : \lambda_t^E q_t^k = E_t [\psi_t^E M_t^E q_{t+1}^k \pi_{t+1} (1 - \delta) + \beta_E \lambda_{t+1}^E (r_{t+1}^k + q_{t+1}^k (1 - \delta))] \quad (\text{A.2})$$

$$[b_t] : \lambda_t^E = \psi_t^E (1 + r_t^b) + \beta_E E_t \left[\lambda_{t+1}^E \frac{(1 + r_t^b)}{\pi_{t+1}} \right] \quad (\text{A.3})$$

$$[l_t] : w_t = \frac{(1 - \alpha) y_t}{x_t} \frac{y_t}{l_t} \quad (\text{A.4})$$

written without subscripts i . $r_t^k = (\alpha/x_t) A_t^E k_{t-1}^{\alpha-1} l_t^{1-\alpha}$ is the rental rate of capital and $x_t = P_t/P_t^w$ is the retail markup over the wholesale price. λ_t^E is the Lagrange multiplier on the budget constraint and ψ_t^E is the Lagrange multiplier on the collateral constraint.

The remaining first-order condition for the household is:

$$[c_t^H] : (1 - h)^\gamma (c_t^H - hc_{t-1}^H)^{-\gamma} = \lambda_t^H \quad (\text{A.5})$$

A.2 *Capital Goods Producers*

Perfectly competitive capital goods producers buy undepreciated capital from entrepreneurs (who also own the capital producers) and combine it with final goods purchased from retailers (defined below) to maximize profits. Old capital is converted one-to-one into new capital, but the final good is converted into new capital subject to a quadratic adjustment cost (κ_i). They then sell newly produced capital goods back to entrepreneurs. The capital goods producer's problem is then to choose the level investment

i_t to:

$$\max_{\{i_t\}} E_0 \sum_{t=0}^{\infty} \lambda_t^E \beta_E^t \left[q_t^k (k_t - (1 - \delta)k_{t-1}) - i_t \right]$$

Subject to the capital accumulation equation:

$$k_t = (1 - \delta)k_{t-1} + \left[1 - \frac{\kappa_i}{2} \left(\frac{i_t \epsilon_t^i}{i_{t-1}} - 1 \right)^2 \right] i_t \quad (\text{A.6})$$

where ϵ_t^i is a shock to the productivity of investment goods. The functional form of investment adjustment costs used here assumes that adjustment costs depend on the growth rate of investment rather than its level, and, up to a first-order, adjustments costs are zero in the neighborhood of the steady state. This specification helps match the response of investment in the model to that observed in the data to monetary policy and technology shocks.

The maximization results in:

$$\begin{aligned} [i_t] : 1 = & q_t^k \left(1 - \frac{\kappa_i}{2} \left(\frac{i_t \epsilon_t^i}{i_{t-1}} - 1 \right)^2 - \kappa_i \left(\frac{i_t \epsilon_t^i}{i_{t-1}} - 1 \right) \frac{i_t \epsilon_t^i}{i_{t-1}} \right. \\ & \left. + \beta_E E_t \left[\frac{\lambda_{t+1}^E}{\lambda_t^E} q_{t+1}^k \epsilon_{t+1}^i \kappa_i \left(\frac{i_{t+1} \epsilon_{t+1}^i}{i_t} - 1 \right) \left(\frac{i_{t+1}}{i_t} \right)^2 \right] \right) \end{aligned} \quad (\text{A.7})$$

which gives the condition for the price of capital is and is identical to the condition derived in Gerali et al. (2010).

The investment problem is separated from the entrepreneur's problem as a matter of convenience. Gerali et al. (2010); Christiano et al. (2005); and Smets and Wouters (2007) all use a similar formulation of adjustment costs to generate a hump-shaped response in investment. Bernanke et al. (1999) note that the same condition for the capital price can be derived if this problem is folded into the capital choice of the entrepreneur's problem.

A.3 Retailers

There is a continuum of measure one of monopolistically competitive retail goods producers indexed by j that buy intermediate goods from entrepreneurs at the wholesale

price P_t^W , costlessly differentiates it, and sells it to households, entrepreneurs, and capital goods producers at price $P_t(j)$, but with a markup x_t over the wholesale price. Retailers maximize profits by setting prices subject to a Dixit-Stiglitz type CES demand curve where the price-elasticity for their product, ϵ_t^y , is assumed to be time-varying and modeled as an exogenous stochastic process. Prices are also assumed to be sticky and indexed to a combination of steady state and past inflation (π and π_{t-1} respectively). If retailers want to change their price to something other than what the index allows, they face a Rotemberg-type adjustment cost with parameter κ_p . Retailers are also assumed to have issued a measure one of equity securities that promise to pay a fraction of retail profits as dividends in the following period to the holder (households and banks) of the security. The dividend rate δ_j^e is assumed to be exogenously determined and retailers will not issue new equity shares or buyback any existing shares in this model.

The retailer's maximization problem is then:

$$\max_{\{P_t(j)\}} E_0 \sum_{t=0}^{\infty} \lambda_t^H \beta_H^t \left[P_t(j) y_t(j) - P_t^W y_t(j) - \frac{\kappa_p}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - \pi_{t-1}^{\iota_p} \pi^{1-\iota_p} \right)^2 P_t y_t \right]$$

subject to the demand curve from households who purchase the differentiated goods:

$$y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon_t^y} y_t \quad (\text{A.8})$$

where the price index is given by:

$$P_t = \left[\int_0^1 P_t(j)^{1-\epsilon_t^y} dj \right]^{\frac{1}{1-\epsilon_t^y}}$$

The first step to derive the first-order condition is to substitute the consumption goods demand equation (A.8) in for $y_t(j)$ to get:

$$\begin{aligned} \max_{\{P_t(j)\}} E_0 \sum_{t=0}^{\infty} \lambda_t^H \beta_H^t & \left[P_t(j) \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon_t^y} y_t - P_t^W \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon_t^y} y_t \right. \\ & \left. - \frac{\kappa_p}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - \pi_{t-1}^{\iota_p} \pi^{1-\iota_p} \right)^2 P_t y_t \right] \end{aligned}$$

Differentiating with respect to $P_t(j)$, and after assuming a symmetric equilibrium which results in $P_t(j) = P_t$, the first-order condition for price setting is:

$$\begin{aligned}
[P_t] : 1 - \epsilon_t^y + \frac{\epsilon_t^y}{x_t} - \kappa_p (\pi_t - \pi_{t-1}^{\iota_p} \pi^{1-\iota_p}) \pi_t \\
+ \beta_H E_t \left[\frac{\lambda_{t+1}^H}{\lambda_t^H} \kappa_p (\pi_{t+1} - \pi_t^{\iota_p} \pi^{1-\iota_p}) \pi_{t+1} \frac{y_{t+1}}{y_t} \right] = 0
\end{aligned} \tag{A.9}$$

which produces a non-linear Phillips curve identical to the condition derived in Gerali et al. (2010).

A.4 Wage Setting

Each household i is assumed to supply differentiated labor and is able to set its nominal wage rate $W_t(i)$ through a wage setting process that maximizes utility subject to a Dixit-Stiglitz type CES demand curve from entrepreneurs who hire household labor. The wage-elasticity, ϵ_t^l , is assumed to be time-varying and modeled as an exogenous stochastic process. Wages are thus assumed to be sticky and indexed to a combination of steady state and past inflation (π and π_{t-1} respectively). If households want to change their wage to something other than what the index allows, they face a Rotemberg-type adjustment cost with parameter κ_w . This is achieved by:

$$\max_{\{W_t(i)\}} E_0 \sum_{t=0}^{\infty} \lambda_t^H \beta_H^t \left[\frac{W_t(i)}{P_t} l_t(i) - \frac{\kappa_w}{2} \left(\frac{W_t(i)}{W_{t-1}(i)} - \pi_{t-1}^{\iota_w} \pi^{1-\iota_w} \right)^2 \frac{W_t}{P_t} \right] - \frac{l_t(i)^{1+\phi}}{1+\phi}$$

subject to the labor demand curve:

$$l_t(i) = \left(\frac{W_t(i)}{W_t} \right)^{-\epsilon_t^l} l_t \tag{A.10}$$

where the wage index is given by:

$$W_t = \left[\int_0^1 W_t(i)^{1-\epsilon_t^l} di \right]^{\frac{1}{1-\epsilon_t^l}}$$

Nominal wage inflation can then be defined as:

$$\pi_t^w = \frac{w_t}{w_{t-1}} \pi_t$$

Following the same procedure as in the retailer problem, substitute the labor demand equation (A.10) in for $l_t(i)$ to get:

$$\max_{\{W_t(i)\}} E_0 \sum_{t=0}^{\infty} \lambda_t^H \beta_H^t \left[\frac{W_t(i)}{P_t} \left(\frac{W_t(i)}{W_t} \right)^{-\epsilon_t^l} l_t - \frac{\kappa_w}{2} \left(\frac{W_t(i)}{W_{t-1}(i)} - \pi_{t-1}^{\iota_w} \pi^{1-\iota_w} \right)^2 \frac{W_t}{P_t} \right] - \frac{\left[\left(\frac{W_t(i)}{W_t} \right)^{-\epsilon_t^l} l_t \right]^{1+\phi}}{1+\phi}$$

Differentiating with respect to $W_t(i)$, and after assuming a symmetric equilibrium which results in $W_t(i) = W_t$, the first-order condition for wage setting is:

$$\begin{aligned} [W_t] : (1-\epsilon_t^l)l_t + \frac{\epsilon_t^l l_t^{1+\phi}}{w_t \lambda_t^H} - \kappa_w (\pi_t^w - \pi_{t-1}^{\iota_w} \pi^{1-\iota_w}) \pi_t^w \\ + \beta_H E_t \left[\frac{\lambda_{t+1}^H}{\lambda_t^H} \kappa_w (\pi_{t+1}^w - \pi_t^{\iota_w} \pi^{1-\iota_w}) \frac{\pi_{t+1}^w{}^2}{\pi_{t+1}} \right] = 0 \end{aligned} \quad (\text{A.11})$$

which produces a non-linear wage-Phillips curve and is identical to the condition derived in Gerali et al. (2010).

A.5 Wholesale Branch

The problem for the wholesale branch is to maximize discounted cash flows which can be written as:

$$\begin{aligned} \max_{\{B_t, D_t, S_{j,t}^b\}} E_0 \sum_{t=0}^{\infty} \lambda_t^H \beta_H^t \left[(1+r_t^{ib})B_t - B_{t+1}\pi_{t+1} + D_{t+1}\pi_{t+1} - (1+r_t^{cb})D_t \right. \\ \left. + \int_j (q_{j,t+1}^e \pi_{t+1} + \delta_j^e \Pi_{j,t}^R) S_{j,t}^b dj - \int_j q_{j,t+1}^e S_{j,t+1}^b \pi_{t+1} dj + K_{t+1}^b \pi_{t+1} \right. \\ \left. - K_t^b - \frac{\kappa_{Kb}}{2} \left(\frac{K_t^b}{RWA_t} - v_b \right)^2 K_t^b \right] \end{aligned}$$

where the index j denotes the equity price and shares from retailer j . RWA_t stands for risk-weighted assets and is defined as:

$$RWA_t = B_t + \frac{M}{v_b} VaR_t$$

Combine wholesale branch cash flows with the balance sheet identity:

$$B_t + \int_j q_{j,t}^e S_{j,t}^b dj = D_t + K_t^b$$

which shows that the wholesale branch problem reduces to maximizing period profits:

$$\max_{\{B_t, D_t, S_{j,t}^b\}} r_t^{ib} B_t - r_t^{cb} D_t + \int_j E_t [q_{j,t+1}^e \pi_{t+1} - q_{j,t}^e + \delta_j^e \Pi_{j,t}^R] S_{j,t}^b dj - \frac{\kappa_{Kb}}{2} \left(\frac{K_t^b}{RWA_t} - v_b \right)^2 K_t^b$$

The conditions used to reduce the problem to a representative equity in section 2.5.3 arise from the first-order conditions for the trading desk of wholesale branch, which are:

$$\begin{aligned} [s_{j,t}^b] : E_t [q_{j,t+1}^e \pi_{t+1} - q_{j,t}^e + \delta_j^e \Pi_{j,t}^R] - r_t^{cb} q_{j,t}^e - \frac{dAdj_t^{Kb}}{ds_{j,t}^b} = \\ \lambda_t^b \left(M \cdot \left(E_t [q_{j,t}^e - q_{j,t+1}^e \pi_{t+1}] + \frac{Z}{\sigma_{pf}} \left(\sigma_j^2 s_{j,t}^b + \int_{i \neq j} \rho_{ij} \sigma_i \sigma_j s_{i,t}^b di \right) \right) - \delta_j^e \Pi_{j,t}^R + \frac{dAdj_t^{Kb}}{ds_{j,t}^b} \right) \\ \frac{dAdj_t^{Kb}}{ds_{j,t}^b} = -\kappa_{Kb} \left(\frac{K_t^b}{RWA_t} - v_b \right) \left(\frac{K_t^b}{RWA_t} \right)^2 \frac{dRWA_t}{ds_{j,t}^b} \\ \frac{dRWA_t}{ds_{j,t}^b} = \frac{M}{v_b} \left(E_t [q_{j,t}^e - q_{j,t+1}^e \pi_{t+1}] + \frac{Z}{\sigma_{pf}} \left(\sigma_j^2 s_{j,t}^b + \int_{i \neq j} \rho_{ij} \sigma_i \sigma_j s_{i,t}^b di \right) \right) \end{aligned}$$

and the first-order condition for the household's equity choice equation (2.6).

A.6 Deposit Branch

The first-order condition is found by maximizing deposit branch profits subject to the deposit demand equation. The first step is to substitute the deposit demand equation (2.8) in for $d_t(i)$ in deposit branch profits to get:

$$\max_{\{r_t^d(i)\}} E_0 \sum_{t=0}^{\infty} \lambda_t^H \beta_H^t \left[r_t^{cb} \left(\frac{r_t^d(i)}{r_t^d} \right)^{-\epsilon_t^d} d_t - r_t^d(i) \left(\frac{r_t^d(i)}{r_t^d} \right)^{-\epsilon_t^d} d_t - \frac{\kappa_d}{2} \left(\frac{r_t^d(i)}{r_{t-1}^d(i)} - 1 \right)^2 r_t^d(i) d_t \right]$$

Differentiating with respect to $r_t^d(i)$, and after assuming a symmetric equilibrium which results in $r_t^d(i) = r_t^d$, the first-order condition is:

$$\begin{aligned}
[r_t^d] : & -(1-\epsilon_t^d) - \epsilon_t^d \frac{r_t^{cb}}{r_t^d} - \kappa_d \left(\frac{r_t^d}{r_{t-1}^d} - 1 \right) \frac{r_t^d}{r_{t-1}^d} \\
& + \beta_H E_t \left[\frac{\lambda_{t+1}^H}{\lambda_t^H} \kappa_d \left(\frac{r_{t+1}^d}{r_t^d} - 1 \right) \left(\frac{r_{t+1}^d}{r_t^d} \right)^2 \frac{d_{t+1}}{d_t} \right] = 0
\end{aligned} \tag{A.12}$$

which is identical to the condition derived in Gerali et al. (2010). Given the deposit demand, the corresponding Dixit-Stiglitz deposit rate index is:

$$r_t^d = \left[\int_0^1 r_t^d(i)^{1-\epsilon_t^d} di \right]^{\frac{1}{1-\epsilon_t^d}}$$

A.7 Loan Branch

The first-order condition is found by maximizing loan branch profits subject to the loan demand equation. Again, the first step is to substitute the loan demand equation (2.9) in for $b_t(i)$ into loan branch profits to get:

$$\max_{\{r_t^b(i)\}} E_0 \sum_{t=0}^{\infty} \lambda_t^H \beta_H^t \left[r_t^b(i) \left(\frac{r_t^b(i)}{r_t^b} \right)^{-\epsilon_t^b} b_t - r_t^{ib} \left(\frac{r_t^b(i)}{r_t^b} \right)^{-\epsilon_t^b} b_t - \frac{\kappa_b}{2} \left(\frac{r_t^b(i)}{r_{t-1}^b(i)} - 1 \right)^2 r_t^b(i) b_t \right]$$

Differentiating with respect to $r_t^b(i)$, and after assuming a symmetric equilibrium which results in $r_t^b(i) = r_t^b$, the first-order condition is:

$$\begin{aligned}
[r_t^b] : & (1-\epsilon_t^b) + \epsilon_t^b \frac{r_t^{ib}}{r_t^b} - \kappa_b \left(\frac{r_t^b}{r_{t-1}^b} - 1 \right) \frac{r_t^b}{r_{t-1}^b} \\
& + \beta_H E_t \left[\frac{\lambda_{t+1}^H}{\lambda_t^H} \kappa_b \left(\frac{r_{t+1}^b}{r_t^b} - 1 \right) \left(\frac{r_{t+1}^b}{r_t^b} \right)^2 \frac{b_{t+1}}{b_t} \right] = 0
\end{aligned} \tag{A.13}$$

which is again identical to the condition derived in Gerali et al. (2010). Given the loan demand, the corresponding Dixit-Stiglitz loan rate index is:

$$r_t^b = \left[\int_0^1 r_t^b(i)^{1-\epsilon_t^b} di \right]^{\frac{1}{1-\epsilon_t^b}}$$

A.8 Monetary Policy

The central bank is assumed to set its policy rate r_t^{cb} according to a Taylor rule given by:

$$(1 + r_t^{cb}) = (1 + r^{cb})^{1-\phi_R} (1 + r_{t-1}^{cb})^{\phi_R} \left(\frac{\pi_t}{\pi}\right)^{\phi_\pi(1-\phi_R)} \left(\frac{y_t}{y_{t-1}}\right)^{\phi_y(1-\phi_R)} \epsilon_t^r \quad (\text{A.14})$$

where ϕ_R , ϕ_π , and ϕ_y are the weights assigned to interest rate persistence, inflation, and output responses of monetary policy respectively. ϵ_t^r is assumed to be an exogenous stochastic process meant to capture monetary policy shocks. The steady state policy rate (r^{cb}) is pinned down through the rate of time preference and the steady state markup over the deposit rate.

A.9 Market Clearing

Market clearing for the model is given by:

$$y_t + \delta^e \frac{\Pi_{t-1}^R}{\pi_t} = c_t + q_t^k [k_t - (1 - \delta)k_{t-1}] + \delta^b \frac{K_{t-1}^b}{\pi_t} + \delta^e \Pi_t^R + F q_t^e s_{t-1}^H + Adj_t$$

$$1 = s_t^H + s_t^b \quad (\text{A.15})$$

which states that income is given in period t from retailer distributed dividends out of period $t - 1$ profits but a fraction of retail profits today must be stored to be distributed out as dividends in the next period. The fraction of bank capital that depreciates, the portfolio management fee, as well as any adjustment costs that occur take away from consumption and investment in capital in period t .

Appendix B

DATA

B.1 Data Series

Output: Gross domestic product, quarterly, nominal, billions of dollars, seasonally adjusted (St. Louis Federal Reserve Economic Data)

Consumption: Personal consumption expenditures, quarterly, nominal, billions of dollars, seasonally adjusted annual rate (St. Louis Federal Reserve Economic Data)

Investment: Gross fixed capital formation, quarterly, nominal, billions of dollars, seasonally adjusted (St. Louis Federal Reserve Economic Data)

Loans to Entrepreneurs: Commercial and industrial loans, quarterly, nominal, billions of dollars, seasonally adjusted (St. Louis Federal Reserve Economic Data)

Deposits: Total savings deposits at all depository institutions, quarterly, nominal, billions of dollars, seasonally adjusted (St. Louis Federal Reserve Economic Data)

Equity Price: S&P 500 price, quarterly, nominal, not seasonally adjusted, (Robert Shiller's *Irrational Exuberance* Online Data)

Consumer Price Index: Consumer price index for all urban consumers: all items, quarterly, seasonally adjusted (St. Louis Federal Reserve Economic Data)

Gross Inflation Rate: Ratio CPI_t / CPI_{t-1} , quarterly, seasonally adjusted (St. Louis Federal Reserve Economic Data)

Wage: Nonfarm business sector compensation per hour, nominal, quarterly, seasonally adjusted (St. Louis Federal Reserve Economic Data)

Gross Wage Inflation Rate: Ratio W_t / W_{t-1} , quarterly, seasonally adjusted (St. Louis Federal Reserve Economic Data)

Central Bank Policy Rate: Effective federal funds rate, quarterly, not seasonally adjusted (St. Louis Federal Reserve Economic Data)

Interest Rate on Loans to Entrepreneurs: Weighted-average effective loan rate of all C&I loans, quarterly, not seasonally adjusted (St. Louis Federal Reserve Economic Data)

Interest Rate on Deposits: M2 OWN rate, quarterly, not seasonally adjusted (St. Louis Federal Reserve Economic Data)

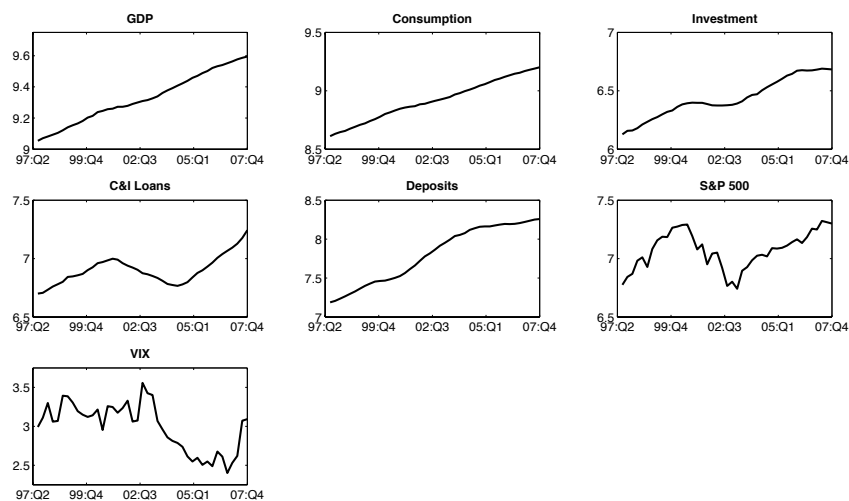
Dividend Rate: Dividends-per-share over earnings-per-share, quarterly, nominal, not seasonally adjusted (Robert Shiller's *Irrational Exuberance* Online Data)

Entrepreneur Loan-to-Value Ratio: Inferred from the ratio of total liabilities over total assets for nonfinancial corporate businesses in the U.S., quarterly, not seasonally adjusted (Board of Governors of the Federal Reserve System)

Volatility: VIX, quarterly, not seasonally adjusted (Chicago Board Options Exchange)

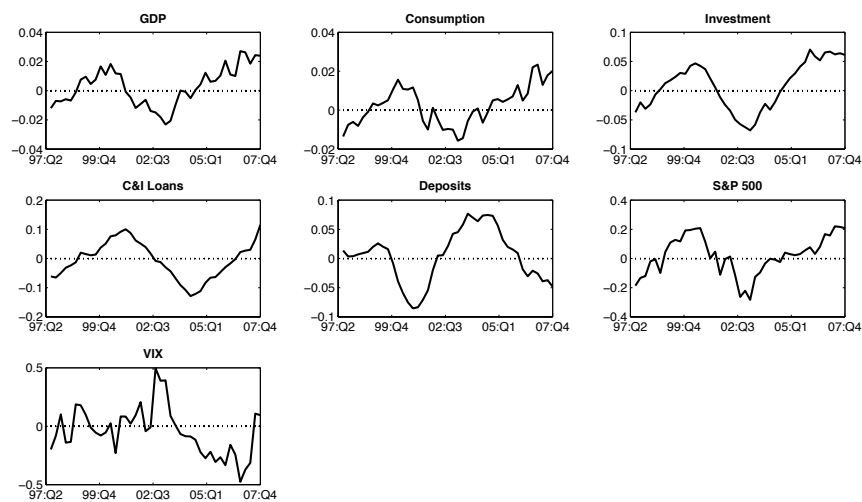
All nominal data are converted to real data by dividing by the consumer price index in decimal form found by dividing the consumer price index by 100.

Figure B.1: Raw Macroeconomic Data



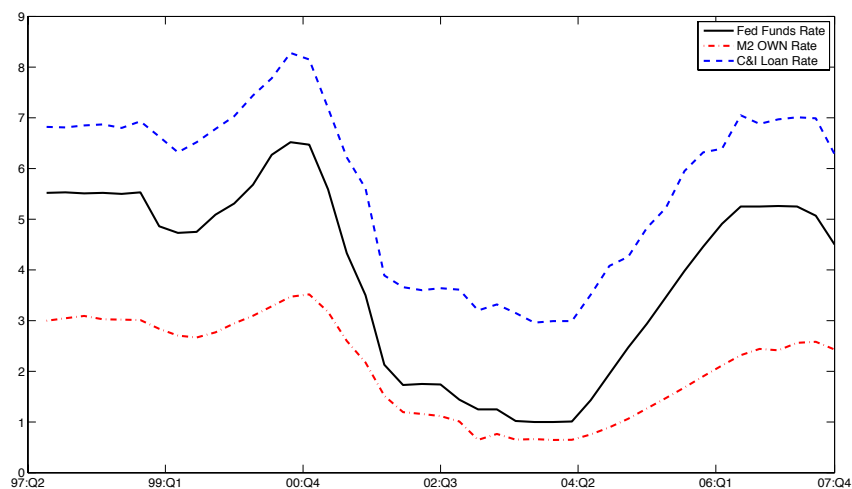
Note: Data are from 1997:Q2-2007:Q4 and are logged.

Figure B.2: Transformed Macroeconomic Data



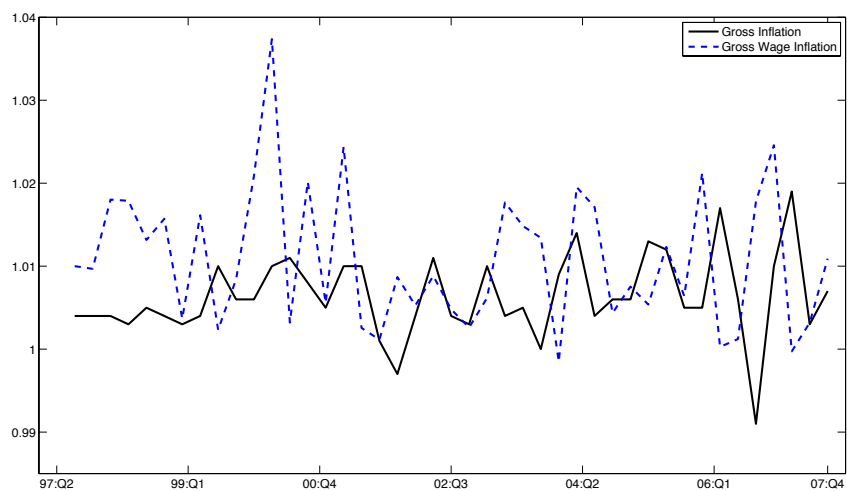
Note: Data are from 1997:Q2-2007:Q4 and are logged and detrended using the HP-filter with smoothing parameter set at 1,600 as suggested by Ravn and Uhlig (2002) except for the VIX which is logged and demeaned.

Figure B.3: Interest Rates



Note: Data are from 1997:Q2-2007:Q4. Interest rates are expressed on an annual basis.

Figure B.4: Inflation Rates



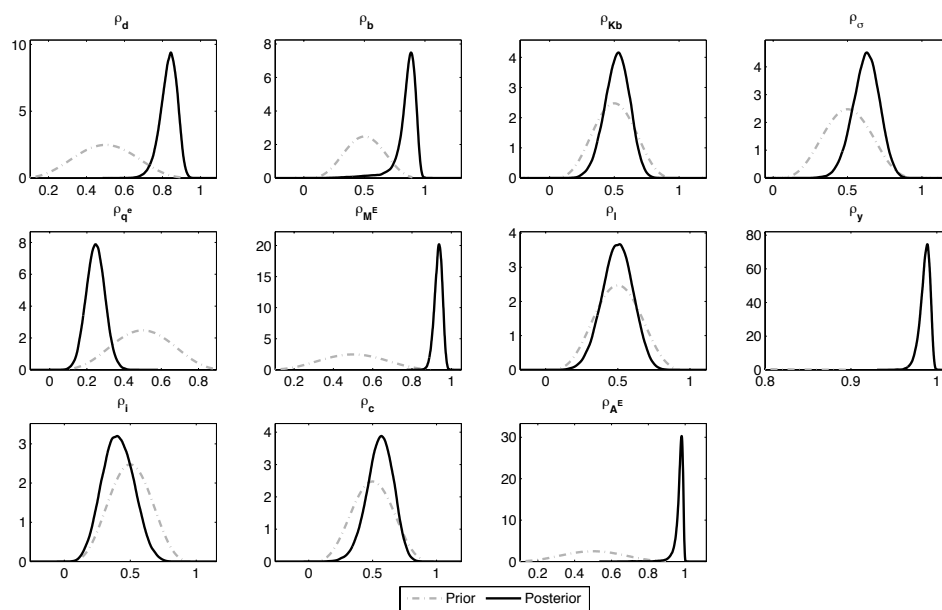
Note: Data are from 1997:Q2-2007:Q4. Inflation rates are expressed on a quarterly basis.

Appendix C

BAYESIAN ESTIMATION

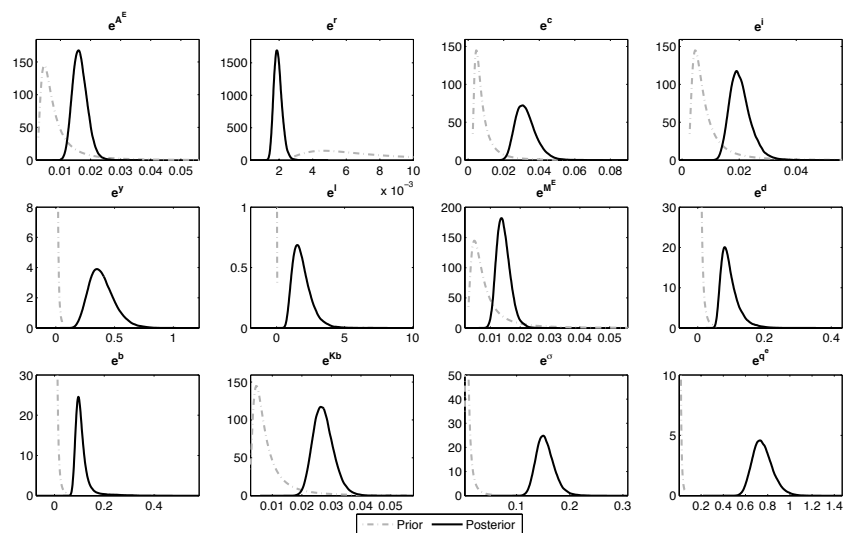
C.1 Shock Posterior Distributions

Figure C.1: Posterior Distribution of AR(1) Coefficients



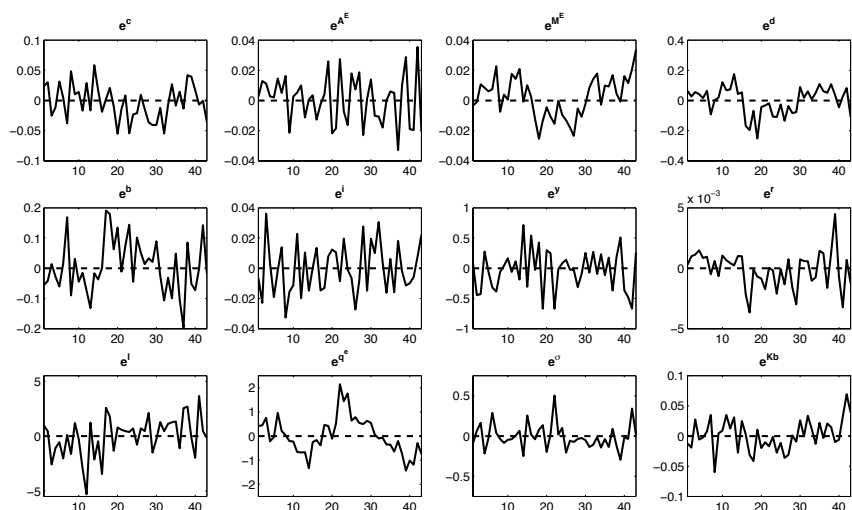
Note: Results based on 16 chains of 100,000 draws each from the Metropolis-Hastings Markov Chain Monte Carlo algorithm.

Figure C.2: Posterior Distribution of Shock Standard Deviations



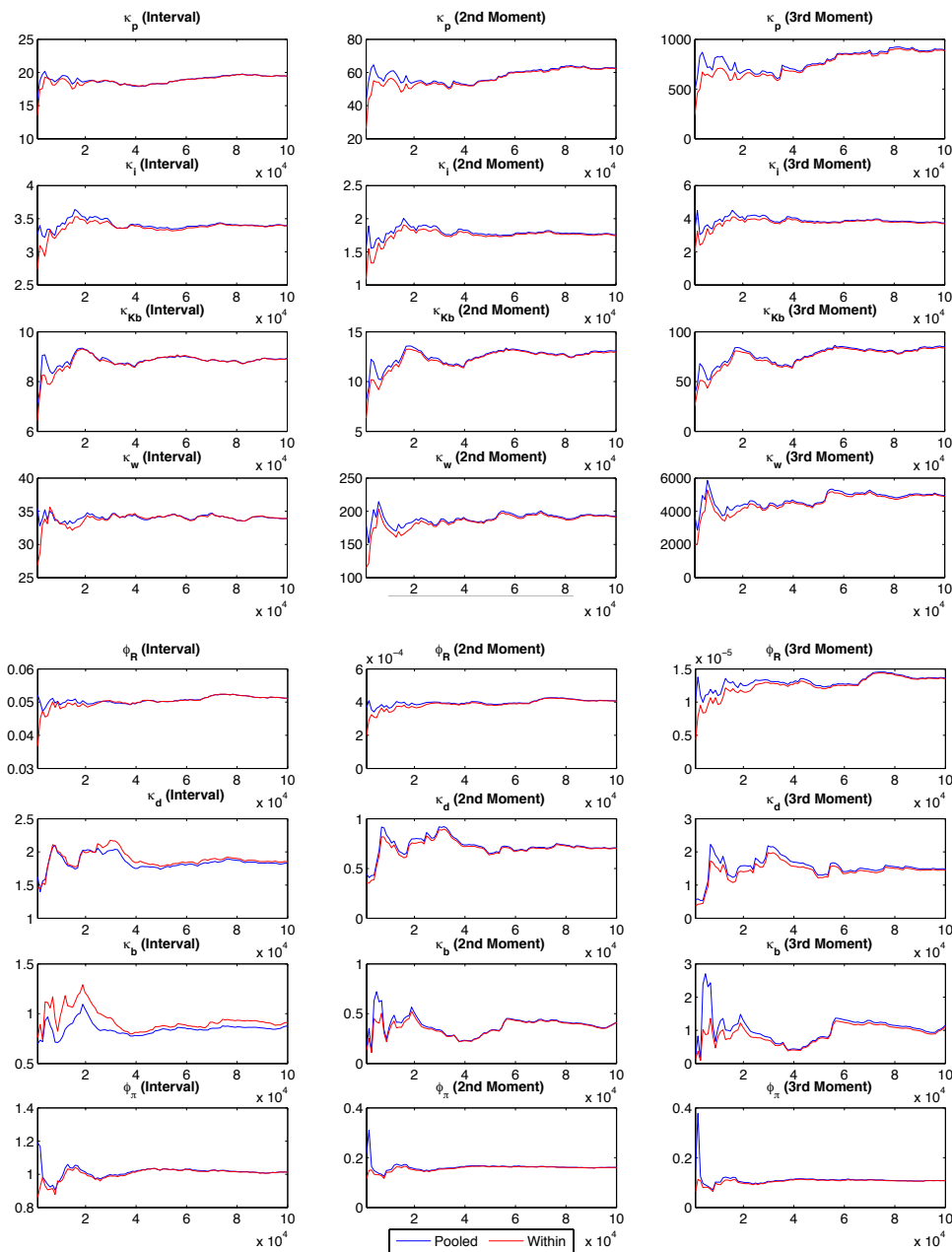
Note: Results based on 16 chains of 100,000 draws each from the Metropolis-Hastings Markov Chain Monte Carlo algorithm.

Figure C.3: Smoothed Shocks



Note: Results based on 16 chains of 100,000 draws each from the Metropolis-Hastings Markov Chain Monte Carlo algorithm.

Figure C.6: Univariate Convergence Diagnostics: Structural Parameters 1



Note: Results based on 16 chains of 100,000 draws each from the Metropolis-Hastings Markov Chain Monte Carlo algorithm.

Figure C.7: Univariate Convergence Diagnostics: Structural Parameters 2

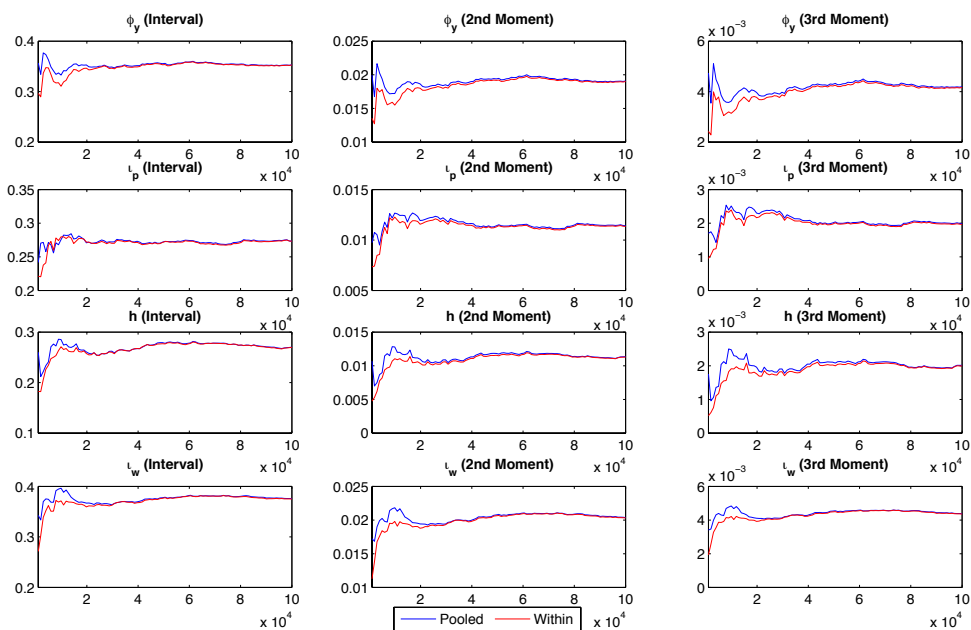
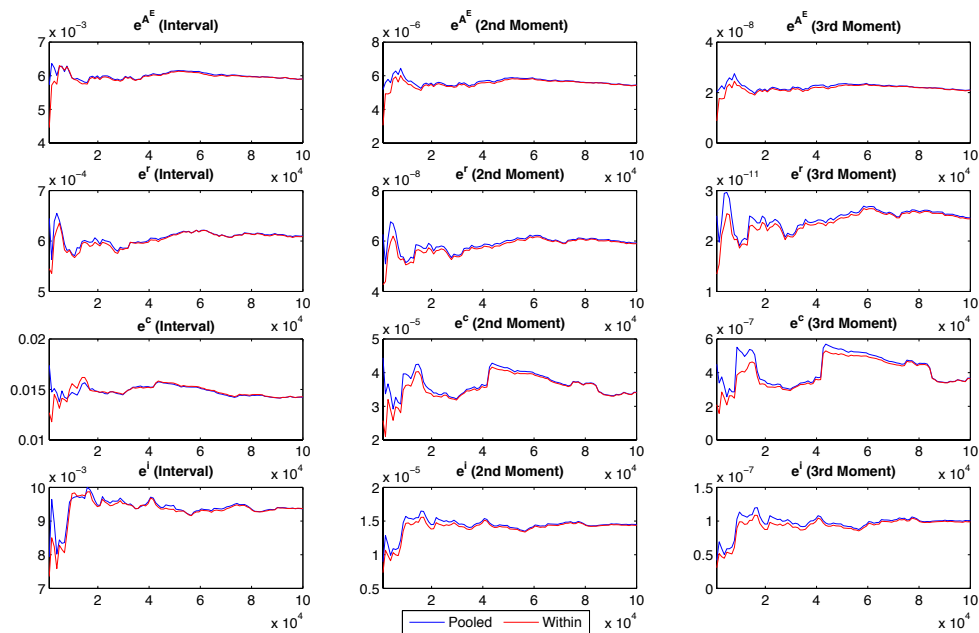
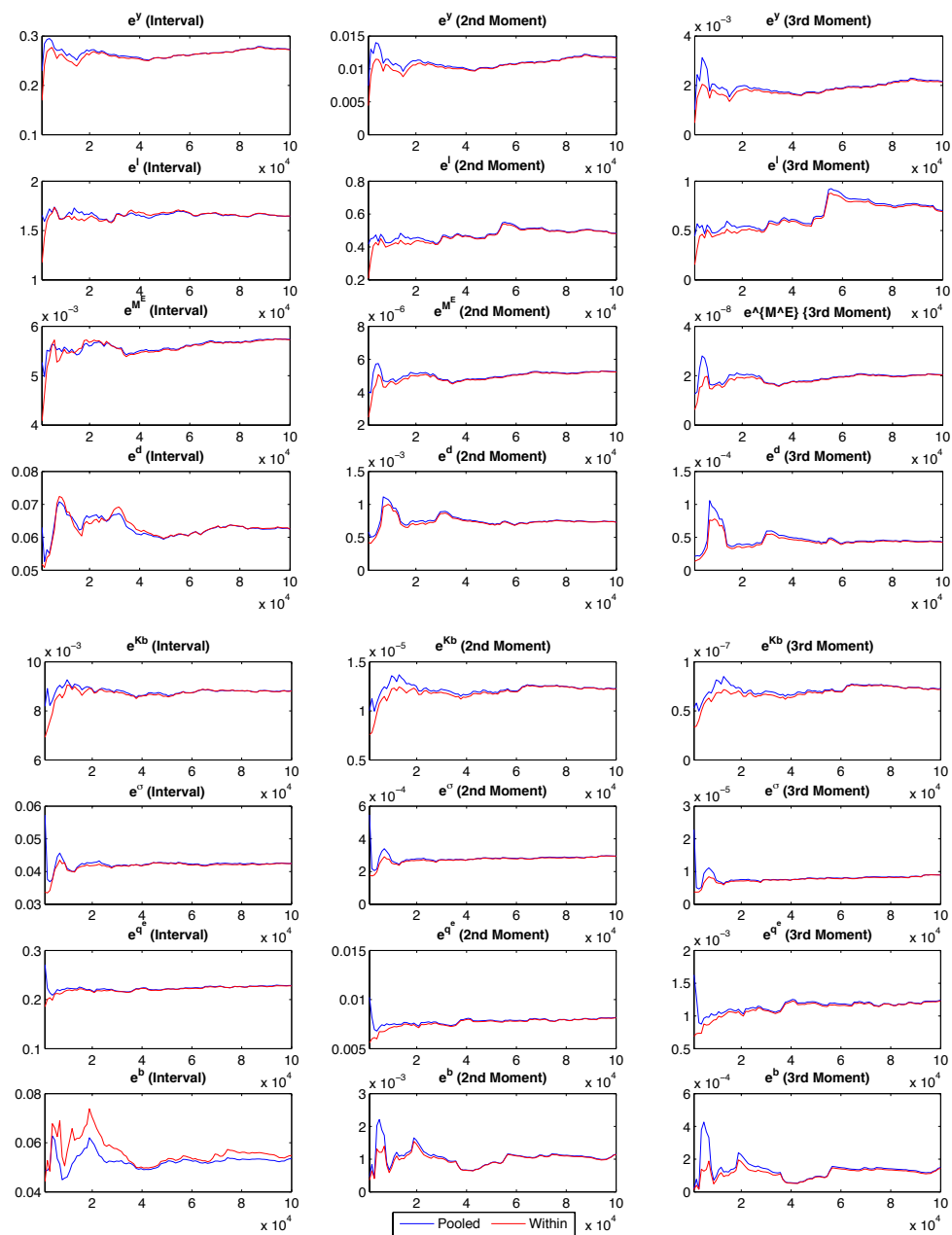


Figure C.8: Univariate Convergence Diagnostics: Shock Standard Deviations 1



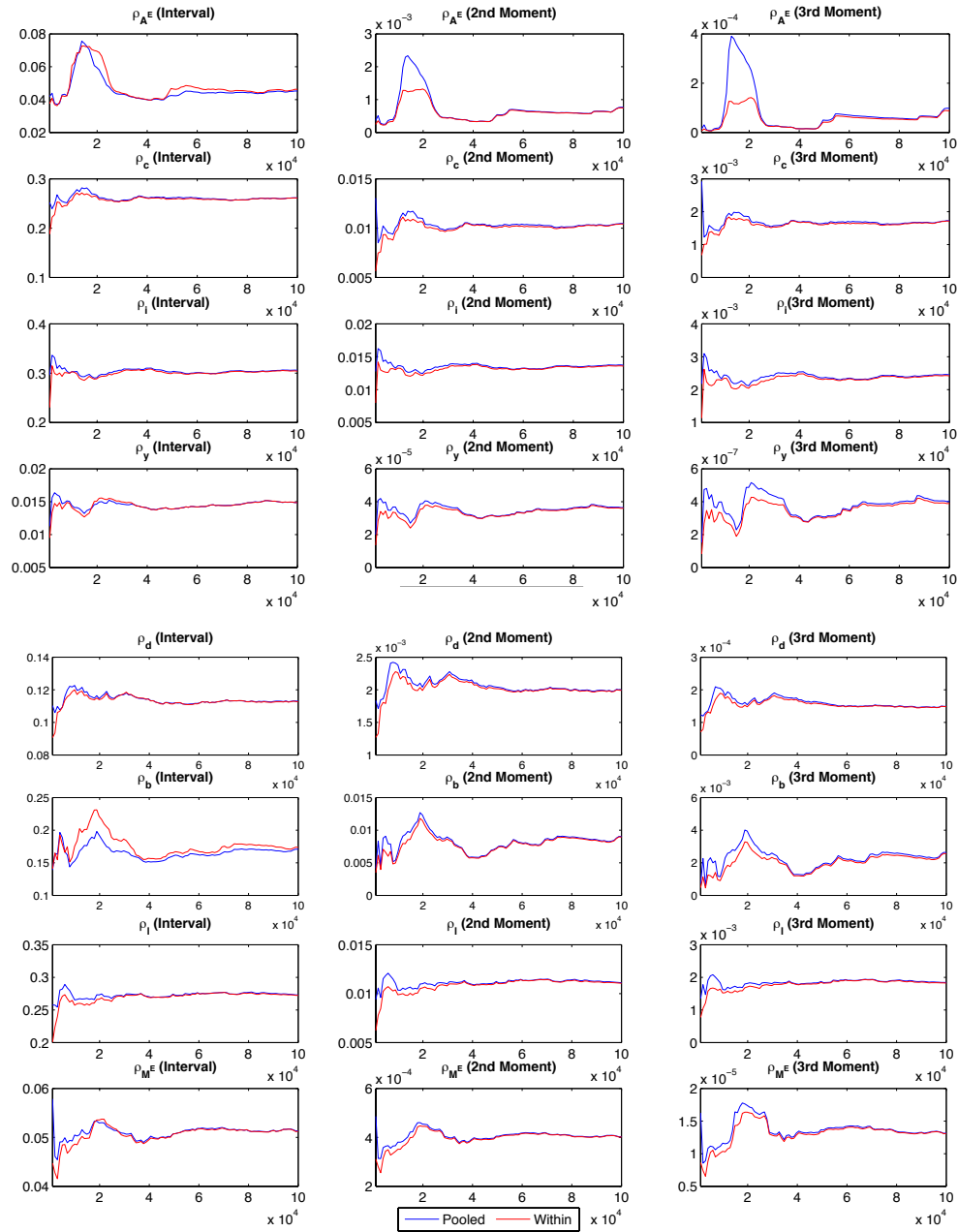
Note: Results based on 16 chains of 100,000 draws each from the Metropolis-Hastings Markov Chain Monte Carlo algorithm.

Figure C.9: Univariate Convergence Diagnostics: Shock Standard Deviations 2



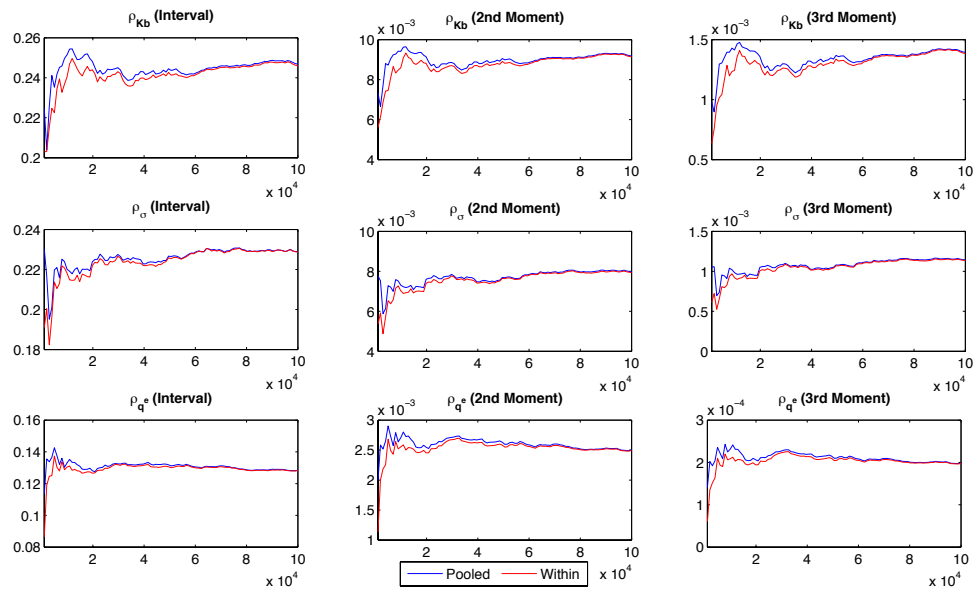
Note: Results based on 16 chains of 100,000 draws each from the Metropolis-Hastings Markov Chain Monte Carlo algorithm.

Figure C.10: Univariate Convergence Diagnostics: AR(1) Coefficients 1



Note: Results based on 16 chains of 100,000 draws each from the Metropolis-Hastings Markov Chain Monte Carlo algorithm.

Figure C.11: Univariate Convergence Diagnostics: AR(1) Coefficients 2



Note: Results based on 16 chains of 100,000 draws each from the Metropolis-Hastings Markov Chain Monte Carlo algorithm.