Essays on Financial Econometrics

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Abstract

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This dissertation studies the U.S. stock market. The first chapter explores a four-moment CAPM under regime switching which incorporates the risk premia for skewness and kurtosis. As expected, estimates of risk premia for covariance, co-skewness and co-kurtosis risks are different across regimes. By allowing time-varying (regime-specific) volatility correlations among asset specific innovations it captures strong volatility correlations in a crash state. As the market evolves to a more bullish state the volatility correlations weaken. Large changes in skewness and kurtosis are linked to regime switching. Optimal weights within a portfolio of small-caps, large-caps, and a risk-free asset are different across regimes when skewness and kurtosis preferences are considered.

The second chapter investigates price discovery and information revealing patterns during the two recent volatile U.S. stock market periods (Tech bust 2000 and credit crisis 2008). In volatile markets, a large-cap stock (MSFT) reveals a considerable amount of private information through trades before the open and during trading hours while a small-cap stock (OPNT) shows little trade-correlated information over the trading day. This is contrary to a typical expectation that small firms' trades are more informative. Information interactions between two stocks are most active during the first-half and the last-half of trading hours while dormant during mid-day trading. This implies information revealing patterns could be different in volatile markets.

The third chapter provides a framework to improve risk-adjusted returns in tactical asset allocation. Tactical asset allocation involves judgments of the future asset returns in a portfolio and thus it is important to identify the market turns. With this motivation, a Markov switching model is applied to the spread returns between small-caps and large-caps to specify market states. Next, a dynamic ordered probit model is adopted to estimate market bullishness (latent variable) and forecasts of the latent variable are

used to get tactical tilts between two opposing assets. To get better forecasts of asset returns, the higher moments of the regime switching model are incorporated since changes in skewness and kurtosis reflect changing market conditions. Finally, it is observed that higher moments can provide better downside protections in tactical asset allocation.

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Dedication

To my parents and my wonderful two daughters - Clare and Francis

Chapter I

Higher moments CAPM under regime switching and multivariate mixture of normals

1. Introduction

The standard mean-variance CAPM is based on the normality assumption of asset return distributions and under this setup investors have preferences over the mean and the variance of asset returns. However, some stylized features in asset returns distribution are asymmetry and leptokurtosis, i.e. skewness and positive excess kurtosis. Given this presence of skewness and excess kurtosis, it is reasonable to assume that investors also have preferences for skewness and kurtosis.

Some limitations of mean-variance analysis in portfolio selection are addressed in Borch (1969) and Feldstein (1969). They indicate that, in general, if a portfolio includes more than one risky asset with positive variance, it is not possible to make a preference ordering solely based on mean and variance. A preference ordering based on mean and variance alone is possible under a very restrictive assumption. Feldstein (1969) noted that "If we exclude linear and quadratic utility functions, such a preference ordering can be defined if and only if each asset has a distribution such that any linear combination of these variables (assets) has a distribution with only two independent parameters" (p.10). However, it is unreasonable to hold this restrictive assumption with contrary evidence of skewed and fat-tailed asset return distributions. A higher moments CAPM is an attractive alternative in the sense it allows investors to make a preference ordering of a portfolio without being confined to the normality assumption since it considers effects of higher moments (skewness and kurtosis) beyond mean and variance.

There exists a literature on the incorporation of skewness into risk premia (Kraus and Litzenberger, 1976; Sears and Wei, 1985, 1988). More recently, Harvey and Siddique (2000) emphasize conditional skewness in an asset pricing model and explain the time variations in expected market risk premiums. Their intuition is that if investors know that the asset returns show conditional skew at time t, the expected excess returns should include rewards for accepting the asset with skewness. In a similar way, this intuition can be extended to kurtosis risk; if time-varying kurtosis of the asset is known to investors, investors require compensation for holding the asset with excess kurtosis. Previous research has documented that U.S. common stock returns are distributed with more returns in the extreme tails (for example, see Fama (1965)). Also, Fang and Lai (1997) show that, in the presence of skewness and kurtosis in asset returns distribution, the expected excess return is related not only to the systematic variance but also to the systematic skewness and systematic kurtosis. Usual risk averse investors prefer assets with positively skewed returns and avoid assets with high volatility and fatter tails. To make investors hold assets with very low or negative skewness, expected returns need to be high enough. Assets with high kurtosis have sharper peaks and longer, fatter tails, while assets with low kurtosis have more rounded peaks and shorter, thinner tails and thus investors are compensated in higher expected return for holding assets with high kurtosis.

One of the controversies with the capital asset pricing model (CAPM) is whether it is a conditional or an unconditional model. That is, whether coefficients of risk premia are constant or change at each time period as conditioning information changes. Jagannathan and Wang (1996) shows the conditional CAPM performs better than the unconditional CAPM in explaining the cross-section of average returns on stocks. Empirical results on unconditional three-moment CAPM are noticeable. Kraus and Litzenberger (1976) find that coefficients on covariance and co-skewness are significant under the assumption that all investors hold the market portfolio and have a preference for positive return skewness in their portfolios. Friend and Westerfield (1980) point out that Kraus and Litzenberger's conclusion possibly comes from a selection of the particular period since the significance of coefficients is more sensitive to the sample periods and statistical procedures used, rather than to the market index. However, empirical results on conditional three-moment CAPM are more convincing. Harvey and Siddique (2000), by introducing conditional skewness, offer an explanation for time-variations in the ex-ante market risk premium. Most recently, Guidolin and Timmermann (2008) use a four-moment conditional international CAPM to explain the strong home bias observed in US investor's international asset allocation in the world capital market. They find evidence of distinct bull and bear state in the global market and show that large changes in conditional skew and kurtosis are linked to regime switching.

In this chapter, I explore a conditional four-moment CAPM under regime switching for the US stock market data. The four moment CAPM prices covariance, co-skewness and co-kurtosis risks and regime switching allows state dependent risk prices which can explain time variations in expected market risk premium. For assets other than the market, it explains the cross-sectional variation in expected excess asset returns. This modeling approach is most closely related to Guidolin and Timmermann (2008) in the sense it addresses skew and kurtosis preferences under regime switching. However, this paper accommodates time-varying (regime-specific) volatility correlations, i.e. the correlations between asset-specific innovations are different across regimes while Guidolin and Timmermann (2008) assume constant volatility correlations across regimes. By finding evidence of four different regimes described as crash, low growth, bull and recovery states, this chapter captures the rich dynamics of US stock market through time varying skewness and kurtosis under regime switching which explain the differences in optimal asset holdings across regimes.

2. Skewness and Kurtosis in conditional asset pricing model and regime switching

One general consensus about the conditional CAPM is that it has relatively strong empirical support as compared to the unconditional CAPM. It is not surprising since the unconditional CAPM was driven under the hypothetical economy where investors live for only one period. In the real world investors live for more than one period and thus it is reasonable to hold the conditional CAPM in empirical examination. The conditional CAPM provides explanation for the time-variations of market risk premium through time-varying market beta. More interestingly, Harvey and Siddique (2000) show that conditional skewness helps explain the cross-sectional variation of expected returns across assets. They show that low expected return portfolios have higher skewness than high expected return portfolios.

2.1 Skewness and Kurtosis in conditional asset pricing model

The basic first order condition for an investor to hold a risky asset for one period is that the asset's price should be equal to the expected discounted value of the asset's payoff. Under no-arbitrage, the gross return for an arbitrary asset i, $1+R_{t+1}^{i}$ satisfies

$$E[(1+R_{t+1}^{i})m_{t+1} \mid I_t]=1 \qquad i=1,\ldots,k \tag{1.1}$$

where non-negative m_{t+1} is the investor's inter-temporal marginal rate of substitution between time t and $t+1$, and I_t is the information set available at time t. The marginal rate of substitution m_{t+1} can be thought as a pricing kernel or stochastic discount factor that prices all risky asset payoffs. Expanding equation (1) gives us

$$E_t[(1+R_{t+1}^{i})m_{t+1}]=Cov_t[(1+R_{t+1}^{i}),m_{t+1}]+E_t[(1+R_{t+1}^{i})]E_t[m_{t+1}]=1 \tag{1.2}$$

which can be written as

$$E_t[(1+R_{t+1}^i)] = \frac{1}{E_t[m_{t+1}]} - \frac{Cov_t[(1+R_{t+1}^i), m_{t+1}]}{E_t[m_{t+1}]} . \quad (1.3)$$

The standard two-moment CAPM comes from the above equation (1.3) and the linearity assumption of the pricing kernel (m_{t+1}) in the market return (R_{t+1}^M)

$$m_{t+1} = a_t + b_t R_{t+1}^M . \quad (1.4)$$

Given equation (3), (4) and assuming the existence of a risk-free asset gives rise to the standard two-moment CAPM

$$E_t[R_{t+1}^i] - R_t^f = \frac{E_t[R_{t+1}^M] - R_t^f}{Var_t[R_{t+1}^M]} Cov_t[R_{t+1}^i, R_{t+1}^M]. \quad (1.5)$$

The higher moments CAPM departs from the standard CAPM by assuming that the pricing kernel m_{t+1} is nonlinear in the market return. Harvey and Siddique (2000) present a nonlinear model where the pricing kernel m_{t+1} is quadratic in the market return to accomodate the effect of skewness in the asset pricing model. Consistent with previous research, to accommodate the effects of higher moments up to the forth, suppose that the pricing kernel m_{t+1} is cubic in the market return

$$m_{t+1} = a_t + b_t R_{t+1}^M + c_t (R_{t+1}^M)^2 + d_t (R_{t+1}^M)^3 . \quad (1.6)$$

If we relate the pricing kernel to the marginal rate of substitution, a third-order Taylor expansion gives us

$$m_{t+1} = 1 + \frac{W_t U''(W_t)}{U'(W_t)} R_{t+1}^M + \frac{W_t^2 U'''(W_t)}{2U'(W_t)} (R_{t+1}^M)^2 + \frac{W_t^3 U''''(W_t)}{6U'(W_t)} (R_{t+1}^M)^3 . \quad (1.7)$$

Since a usual risk averse investor is assumed to have positive marginal utility of wealth $(U'(W_t) > 0)$, this implies decreasing marginal utility of wealth $(U''(W_t) < 0)$, non-increasing absolute risk aversion

$(U'''(W_t) > 0)$ and decreasing absolute prudence(the precautionary saving motive) $(U''''(W_t) < 0)$, comparing (1.6) and (1.7) gives $b_t < 0, c_t > 0$ and $d_t < 0$. As equation (1.4) implies equation (1.5), similarly the cubic pricing kernel (1.6) leads to a four-moment CAPM

$$\begin{aligned} E_t[R_{t+1}^i] - R_t^f &= -b_t R_t^f Cov_t[R_{t+1}^i, R_{t+1}^M] - c_t R_t^f Cov_t[R_{t+1}^i, (R_{t+1}^M)^2] - d_t R_t^f Cov_t[R_{t+1}^i, (R_{t+1}^M)^3] \\ &= \beta_t Cov_t[R_{t+1}^i, R_{t+1}^M] + \gamma_t Cov_t[R_{t+1}^i, (R_{t+1}^M)^2] + \delta_t Cov_t[R_{t+1}^i, (R_{t+1}^M)^3] \end{aligned} \quad (1.8)$$

where $\beta_t (= -b_t R_t^f) > 0$, $\gamma_t (= -c_t R_t^f) < 0$ and $\delta_t (= -d_t R_t^f) > 0$, assuming the existence of a conditionally risk-free asset.

In the standard CAPM, the expected risk premium of an asset i is the product of the conditional covariance with the market return and the price of covariance. In the four-moment CAPM, the expected return of an asset is also determined by its conditional co-skewness, conditional co-kurtosis and the prices of co-skewness and co-kurtosis with the market where co-skewness of an asset is a covariance of an asset's return with the square of the market returns and co-kurtosis of an asset is a covariance of an asset's return with the cubic of the market returns.

When the market returns distribution is positively skewed, if an asset returns show positive co-skewness with the market, this means that an asset's returns distribution is skewed to the right of the market returns. In this case, all other things equal, investors would prefer a positive co-skewness because this represents a higher probability of extreme positive returns in the asset over market returns. The price of co-skewness is negatively related to risk premium since non-increasing absolute risk aversion implies that investors prefer increases in total skewness of a portfolio. When investors add an asset with a negative co-skew to a portfolio, it reduces the total skewness of the portfolio. Therefore, assets with negative co-skew must have higher expected return as compared to assets with positive or zero co-skewness. Therefore, the expected excess return of an asset is negatively related to its market co-skewness. If the market had negative skewness, investors would be averse to positive skewness with the market.

Co-kurtosis measures the degree of peakedness of an asset returns in relation to market returns peakedness. A higher co-kurtosis of an asset with the market means that the asset returns distribution has heavier tails as compared to the market returns. For risk-adverse investors, a lower co-kurtosis is preferred as the asset's returns would not be much different from the market's returns. The price of co-kurtosis is positively related to risk premium since decreasing absolute prudence implies that temperance is greater than prudence. If investors have jobs, unavoidable risk such as labor income risk might lead investors to reduce exposure to another risk which is related to asset even though the two risks are independent and this can be thought as temperance in terms of moderation of accepting risk. Investors prefer decreases in total kurtosis and when they add an asset with high co-kurtosis to a portfolio, it increases the total kurtosis of the portfolio. Therefore, assets with high co-kurtosis must have higher expected return than those with low co-kurtosis and thus the expected excess return is positively correlated with its market co-kurtosis.

2.2 Regime switching

It is well known that regime switching model can capture many properties of an asset returns distribution. Regime switching model identifies regimes with different means and variances and this allows us to capture time-variations of asset return as the underlying state probabilities change over time. Univariate regime switching model has some advantages in asset return modeling: it can accommodate some stylized features in asset returns such as serial correlation in returns and volatility clustering. Multivariate regime switching model allows us to capture time-varying volatilities and correlations through regime-dependent covariance. There are more empirical evidences that asset returns follow regime switching process. Turner, Startz and Nelson (1989) report the evidence of two regimes in US stock market and Kim, Nelson and Startz (1998) presents the evidence of three regimes in the mean and volatility of US asset returns. Timmermann (2000) presents regime switching model can capture heteroskedasticity, fat tails and skews in the underlying distribution of returns. Hamilton (2005) points out that portfolio decisions are likely to be closely related to the evolving uncertainty about the underlying state of the economy which is captured through a regime switching process and such regimes could be linked to business cycle variations in economic growth associated with the economic cycle.

Four-moment CAPM under regime switching and Multivariate location-scale mixture of normals

In a framework of the four-moment CAPM, the expected return of an asset depends on its conditional covariance, co-skewness and co-kurtosis risks with the market. To incorporate regime-dependence in the four-moment CAPM, the price of risk associated with these moments is allowed to depend on a latent state variable, S_t, which is assumed to follow a k-state Markov process. The state variable is unobservable to investors, but they observe the behavior of an asset return, r_t, and they infer the evolution of S_t through the observed behavior of an asset return.

The empirical analogue of the four-moment CAPM under regime switching is

$$\begin{aligned} r_{t+1}^{i} &= \alpha_{s_{t+1}}^{i} + \beta_{s_{t+1}} Cov_t[r_{t+1}^{i}, r_{t+1}^{M}] + \gamma_{s_{t+1}} Cov_t[r_{t+1}^{i}, (r_{t+1}^{M})^2] + \delta_{s_{t+1}} Cov_t[r_{t+1}^{i}, (r_{t+1}^{M})^3] + \varepsilon_{t+1}^{i} \\ r_{t+1}^{M} &= \alpha_{s_{t+1}}^{M} + \beta_{s_{t+1}} Var_t[r_{t+1}^{M}] + \gamma_{s_{t+1}} Skew_t[r_{t+1}^{M}] + \delta_{s_{t+1}} Kurt_t[r_{t+1}^{M}] + \varepsilon_{t+1}^{M} \end{aligned} \qquad (1.9)$$

where $r_{t+1}^{i} = R_{t+1}^{i} - R_t^f \ (i = 1,...,h)$ are excess returns of individual assets and $r_{t+1}^{M} = R_{t+1}^{M} - R_t^f$ is the market excess return, $\alpha_{s_{t+1}}^{i}$ is asset-specific intercept and the time-varying (regime-specific) risk premia $\beta_{s_{t+1}}$, $\gamma_{s_{t+1}}$ and $\delta_{s_{t+1}}$ are common among assets, and the $h+1$ dimension error term vector $\varepsilon_{t+1} = [\varepsilon_{t+1}^{1},...,\varepsilon_{t+1}^{h},\varepsilon_{t+1}^{M}] \sim N(0,\Omega_{S_{t+1}})$. The state variable S_t follows a k-state Markov process with constant transition probability matrix, P

$$P[i,j] = \Pr(s_t = j \mid s_{t-1} = i) = p_{ij}, \qquad i,j = 1,...,k. \qquad (1.10)$$

An advantage of the regime switching model is that if we assume the state of next period is known to investors, the return distribution conditional on the current information is normal. To make it simple, consider a univariate regime-switching model. If investors know the state of next period is state l, then the return distribution of the next period is $N(\mu_l, \sigma_l^2)$. However, since investors are unsure about future states, the return distribution is a mixture of normals with weights which reflect current state probabilities, i.e. $\sum_{i=1}^{k} \pi_i N(\mu_i, \sigma_i^2)$ where π_i is the state probability which can be found from its relation to transition probability matrix($\pi' P = \pi'$)[1].

Now we can relate the four-moment CAPM under regime switching with multivariate location-scale mixture of normals. Since the intercepts and all coefficients of the four moment CAPM are regime dependent and all these higher moment factors are conditional moments of regime-switching model (which can be computed as functions of the regime-specific mean and variance parameters and state

[1] The fundamental limit theorem for regular Markov chains states that if P is the transition matrix for a regular Markov chain, then $\lim_{n\to\infty} P^n = W$ where W is a matrix with each row equal to the unique fixed probability row vector w for P. A Markov chain is called a regular chain if some power of the transition matrix has only positive elements. Furthermore, all entries in W are strictly positive. Then $wP = w$, and any row vector v such that $vP = v$ is a constant multiple of w.

probabilities), the conditional higher moments of regime switching model can be mapped to the regime dependent mean $\mu_{s_{t+1}}$ which can be defined as $\mu_{s_{t+1}} = \alpha_{s_{t+1}} + \beta_{s_{t+1}} CV + \gamma_{s_{t+1}} CS + \delta_{s_{t+1}} CK$ where $\alpha_{s_{t+1}} = [\alpha^1_{s_{t+1}}, \alpha^2_{s_{t+1}}, ..., \alpha^h_{s_{t+1}}, \alpha^M_{s_{t+1}}]'$ is a vector of intercepts, $\beta_{s_{t+1}}$, $\gamma_{s_{t+1}}$ and $\delta_{s_{t+1}}$ are regime dependent scalars which imply prices for systematic covariance, co-skewness and co-kurtosis risks and CV , CS and CK are $h+1$ vectors of conditional covariance, conditional co-skew and conditional co-kurtosis of regime-switching mode. Therefore, the four-moment CAPM under regime switching can be represented as

$$y_{t+1} = \mu_{S_{t+1}} + \varepsilon_{t+1}$$
$$\varepsilon_{t+1} = [\varepsilon^1_{t+1}, ..., \varepsilon^h_{t+1}, \varepsilon^M_{t+1}] \sim N(0, \Omega_{S_{t+1}}) \qquad (1.11)$$

where $y_{t+1} = [r^1_{t+1}, \cdots, r^h_{t+1}\ , r^M_{t+1}]'$ is a $h+1$ vector of excess returns and the state variable S_t follows a k-state Markov process with constant transition probability P. Therefore, the distribution of excess returns is a k-component multivariate location-scale mixture of normals,

$$y = \pi_1 N(\mu_1, \Omega_1) + \pi_2 N(\mu_2, \Omega_2) + \cdots + \pi_k N(\mu_k, \Omega_k) \qquad (1.12)$$

where $\pi = [\pi_1, \pi_2, \cdots, \pi_k]'$ is the vector of mixing proportions such that $\pi' 1 = 1$.

2.3 Data

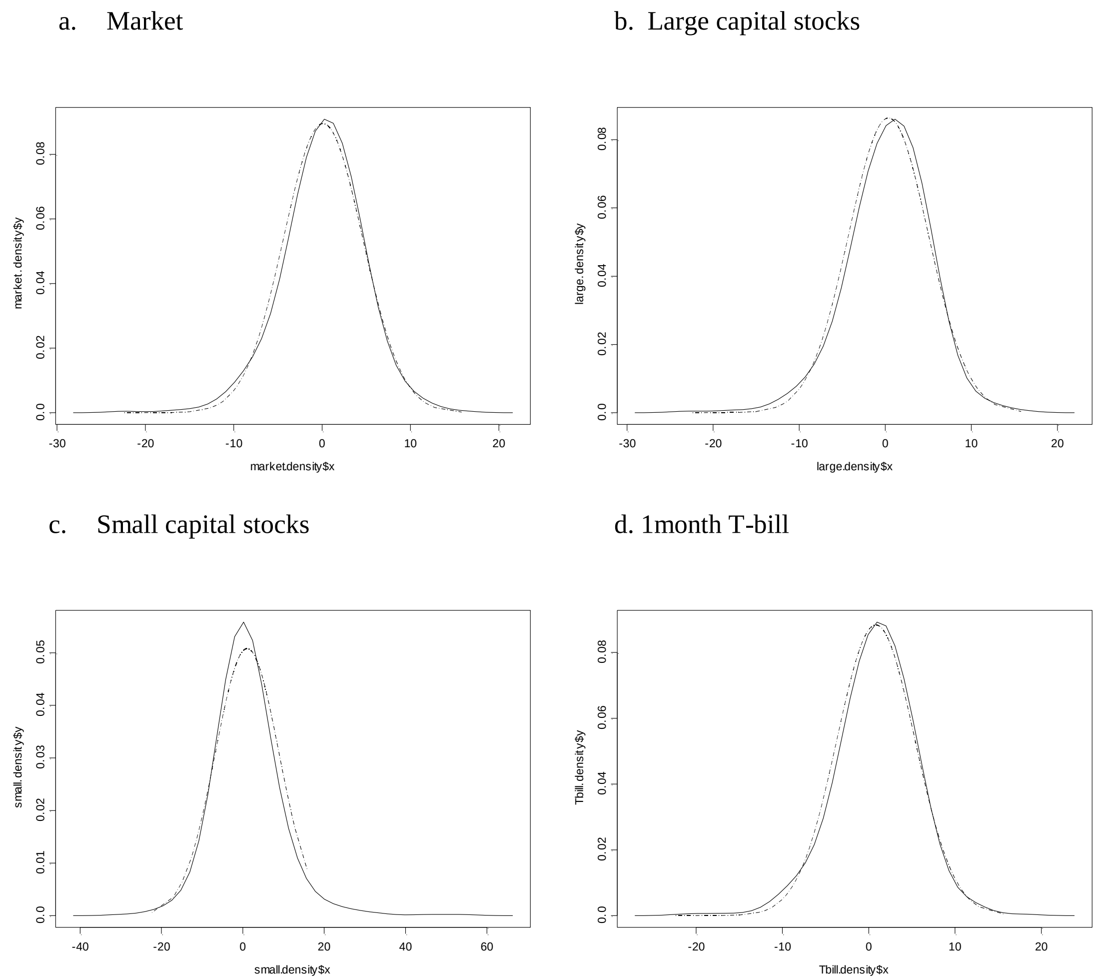
This chapter analyses monthly U.S. stock market returns of all common stocks listed on NYSE/AMEX/NASDAQ. For this analysis, monthly data are provided by Center for Research in Stock Prices (CRSP) and the sample period covers 1970:01~2008:12, a total of 468 observations. Based on market capitalization, the first size-sorted CRSP decile portfolio is used for small firms stocks and the decile 10 portfolio is used for large firms stocks. Excess returns are obtained by subtracting the risk free rate from these returns and the risk free rate is measured by the 1month T-bill rate.

Table 1.1 Summary statistics for US monthly stock (excess) returns

	Small-cap stocks	Large-cap stocks	Market returns (Value weighted)	1-month T-bill
Mean	1.0650	0.3573	0.3770	0.4740
Median	0.3875	0.7974	0.7970	0.4344
Maximum	53.7681	17.3802	16.0331	1.5158
Minimum	-29.1043	-21.7631	-23.1403	0.0029
Std. Dev.	7.8541	4.5216	4.6205	0.2438
Skewness	1.3793	-0.4991	-0.6099	0.9845
Kurtosis	10.6772	5.0302	5.1863	4.7421
Co-skewness	-0.6825	-0.4029		
Co-kurtosis	5.4919	4.8835		
Jarque-Bera	1297.7010	99.8043	122.2205	134.7776
Probability	0.0000	0.0000	0.0000	0.0000

Stock returns are in excess of the 1-month US T-bill rate and include dividends. The sample period is 1970:01 –2008:12. Value- weighted index returns including all dividends are calculated on each of the portfolios.

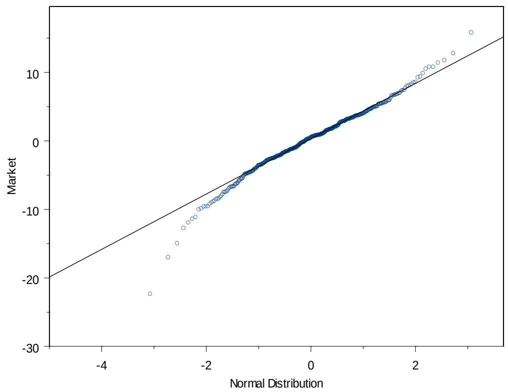
As summarized in table 1.1, small-cap stocks returns show positive skewness while large-cap stocks and market returns are slightly negatively skewed. Generally if the distribution of data is skewed to the left, the mean is less than the median, which is less than the mode (mean<median<mode) and the coefficient of skewness is less than zero. If the distribution of data is skewed to the right, the mode is less than the median, which is less than the mean (mean>median>mode) and the coefficient of skewness is positive. A normal curve is symmetrical and the coefficient of skewness in a normal curve is zero. Therefore, skewness is a measure of departure from a normal curve and the coefficient of skewness is a statistical measure of asymmetry of a curve. When it comes to kurtosis, small-cap stocks show the highest kurtosis though all other data show excess kurtosis. Kurtosis is a measure of whether the data are peaked or flat relative to a normal distribution. That is, data sets with high kurtosis tend to have a distinct peak near the mean, decline rather rapidly, and have heavy tails. Data sets with low kurtosis tend to have a flat top near the mean rather than a sharp peak. The kurtosis for a standard normal is three and thus excess kurtosis (K-3) can be used as a measure of departure from normal distribution. None of data in table1.1 is close to normal and positive excess kurtosis indicates a "peaked" distribution as compared to normal. Even though the normal distribution itself is symmetric, location-scale mixture of normals can accommodate asymmetric distributions. Skewness and excess kurtosis in asset returns distribution can be introduced through differences between location (mean) parameters and scale (variance) parameters.



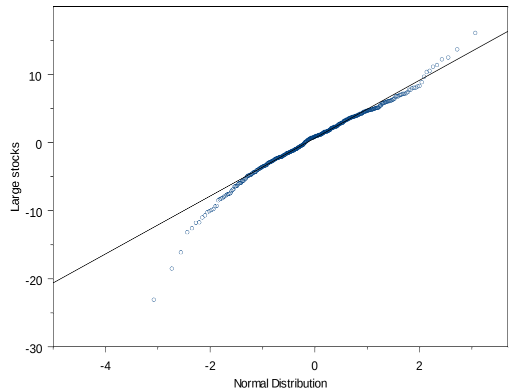
Note: —— Smoothed histogram - - - Normal density

Figure 1.1 Comparison of smoothed histograms and normal densities of US monthly stock returns

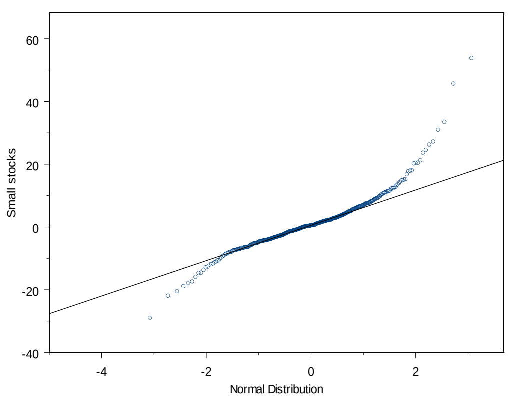
a. Market



b. Large capital stocks



c. Small capital stocks



d. 1month T-bill

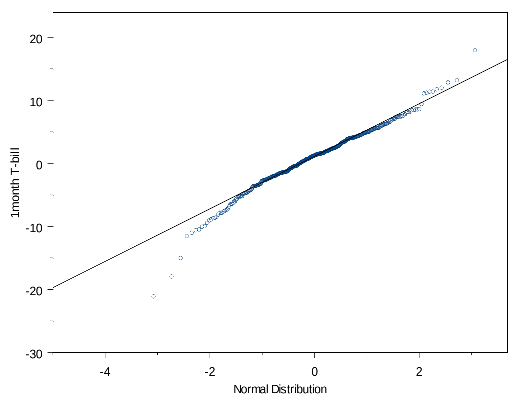


Figure 1.2 Normal QQ plot of US monthly stock returns

2.4 Model specification- Testing the number of regimes

This section tests the number of regimes which reflect the number of components in the multivariate mixture of normals. To gain intuition, first consider a simple univariate case. Let $H_k(r;\vartheta)$ be a distribution corresponding finite k-state regime switching model with probability density function $h(r;\vartheta)=\sum_{i=1}^{k}\pi_i f_i(r;\mu_i,\sigma_i^2)$ where π_i is the state probability ($\pi' P = \pi'$), $f_i(\cdot)$ is a normal probability density function with mean μ_i $(\mu_1 < \mu_2 < \cdots < \mu_k)$, variance σ_i^2 and $\vartheta = (p_{11}, \cdots, p_{1k}, \cdots, p_{k1}, \cdots, p_{kk}, \mu_1, \sigma_1^2, \cdots, \mu_k, \sigma_k^2)$ is the vector of unknown parameters with dimension $k(k+1)$ since there are $k(k-1)$ unknown parameters in transition probability matrix, k unknown means and k unknown variances. Consider a test of the null hypothesis that the sample data comes from a k_0-component normal mixture versus the alternative hypothesis that the sample data comes from a k_1-component normal mixture where $k_1 > k_0$. Let $\hat{\vartheta}^{k_0}$ be the maximum-likelihood estimator of ϑ over H_{k_0}. Ryden (1998) proposes a specification test based on a penalized likelihood criteria. A penalized likelihood estimator selects the family H_{k_0}, where k_0 maximizes $\log\ L(\hat{\vartheta}_n^{k_0}; r_1, \cdots, r_n) - \omega_{n,k_0}$ over k_0 =1, 2, 3, … by introducing a penalty term ω_{n,k_0} which satisfies $\omega_{n,k_0} \geq 0$ and $\omega_{n,k_0} < \omega_{n,k_1}$. This prevents an overly large model from being selected by considering decreasing parsimony of parameters. The two most common choices for ω_{n,k_0} are the AIC ($\omega_{n,k_0} = m$ number of parameters) and the BIC ($\omega_{n,k_0} = -\frac{1}{2} m \ln(n)$). Define the LR statistic as $LR_n^{k_0} = 2\{\log L(\vartheta_n^{k_1}; r_1, \cdots, r_n) - \log L(\vartheta_n^{k_0}; r_1, \cdots, r_n)\}$ and if $LR_n^{k_0} > 2(\omega_{n,k_1} - \omega_{n,k_0})$ then select H_{k_1}. In other words, this test chooses the model with the lowest AIC or BIC value.

To determine the number of regimes for the multivariate joint distribution, this chapter undertakes specification test for the values of k =1, 2, 3, 4, 5. Specification results are presented in table 1.2. Both the AIC and the HQ support the model with four states.

Table 1.2 Model selection for regime switching

Model	Number of parameters	Log likelihood	LR test (Linearity)	AIC	HQ
1	9	-3111.90	N/A	6241.80	6223.87
2	20	-2966.28	291.26 (0.000)	5972.56	5932.72
3	33	-2957.64	308.54 (0.000)	5981.28	5915.54
4	48	-2909.07	405.68 (0.000)	5914.14	5818.51
5	60	-2886.19	451.42 (0.000)	5934.38	5824.89

2.5 Estimation results

First, as a benchmark, consider an empirical analogue of standard CAPM which assumes normality of asset returns, i.e. a single state model and thus the price of covariance risk is constant

$$\begin{aligned} r_{t+1}^{i} &= \alpha^{i} + \lambda\, Cov[r_{t+1}^{i}, r_{t+1}^{M} \mid I_t] + \varepsilon_{t+1}^{i} \\ r_{t+1}^{M} &= \alpha^{M} + \lambda\, Var[r_{t+1}^{M} \mid I_t] + \varepsilon_{t+1}^{M} \end{aligned} \quad . \qquad (1.13)$$

By imposing the restriction $\alpha^{i} = \alpha^{M} = 0$ on equation (1.13), it becomes the standard CAPM in which $\lambda = E_t[r_{t+1}^{M} \mid I_t] / Var_t[r_{t+1}^{M} \mid I_t]$. Following the instrumental variables approach of Campbell (1987) and Harvey (1989), Hansen's generalized method of moments (GMM) is adopted in estimating equation (1.13) and equally weighted market returns are used as an instrument variable. Let z_t denote equally weighted market return at time t, then the orthogonality conditions are $E[z_t \varepsilon_{t+1}] = 0$ where ε_{t+1} is a $h+1$ dimensional vector of errors. The results are shown in table 1.3 below. All Intercepts are negative and statistically significant. The estimated U.S. stock market price of covariance risk is positive and statistically significant. The J-statistic shows that the overidentifying restrictions are satisfied.

Table 1.3 Parameter estimates for a two-moment CAPM

Price of covariance(λ)	0.1763*		
	(0.0719)		
	Market	Small-cap stocks	Large-cap stocks
Intercept(α)	-3.6966*	-3.6404*	-3.5404*
	(1.6893)	(1.7925)	(1.6224)
J-statistic	0.0753		

* denotes the parameter estimate is significant at 5% significance level.

Since errors are drawn from single state (time-invariant distribution) under the above two-moment CAPM with constant covariance price, we can consider returns distribution as multivariate normal distribution and the results are in table 1.4.

Table 1.4 Estimates of a single state model

	Market	Small-caps	Large-caps
Mean excess return(%)	0.3770	1.0652	0.3572
	(0.2042)	(0.3494)	(0.2005)
Variance			
Market	21.3018 (1.3917)		
Small-caps	20.7552 (1.9282)	61.5501 (4.0230)	
Large-caps	20.7239 (1.3580)	18.1355 (1.8390)	20.4000 (1.3329)
Correlations			
Market	1.0000		
Small-caps	0.5741	1.0000	
Large-caps	0.9940	0.5127	1.0000

Now, consider the four-moment CAPM under four-state regime switching (1.9), where the prices of covariance, co-skewness and co-kurtosis risks are regime dependent. Following Timmermann (2000), the conditional moments of regime-switching model are computed as functions of the (regime-specific) mean and variance parameters $\{\mu_l^1, \cdots, \mu_l^h, \mu_l^M, \Omega_l\}_{l=1}^k$ and state probabilities, π_t. This model-based estimates are more accurate, especially for the third and forth moments, than estimates obtained directly from realized returns which tend to be affected by outliers.[2] First, to get mean excess returns under four-state regime-switching model, the model (1.11) is estimated by maximum likelihood estimation and the results are in table 1.6. The mean excess returns under regime switching can be computed as the weighted average of regime dependent mean returns

$$\bar{y}_{t+1} = E[y_{t+1} \mid I_t] = \sum_{l=1}^{k} (\pi_t P e_l) \mu_l \,, \qquad (1.14)$$

where π_t is the vector of state probabilities, e_l is a vector of zeros with a one in the lth position so $(\pi_t' P e_l)$ is the ex-ante probability of being in state l at time $t+1$ given information at time t and μ_l is a mean return vector in state l.

---

[2] For comparison, see Figure1.6 and Figure1.7.

Then the conditional variance, skewness and kurtosis of (excess) returns on the market portfolio, r_{t+1}^{M}, can be computed as follows:

$$Var_t[r_{t+1}^{M}] = \sum_{l=1}^{k}(\pi_t' Pe_l)[\mu_l^{M} - e_{h+1}'\bar{y}_{t+1}]^2 + \sum_{l=1}^{k}(\pi_t' Pe_l)Var[\varepsilon_{t+1}^{M} \mid S_{t+1} = l]$$

$$Skew_t[r_{t+1}^{M}] = \sum_{l=1}^{k}(\pi_t' Pe_l)[\mu_l^{M} - e_{h+1}'\bar{y}_{t+1}]^3 + 3\sum_{l=1}^{k}(\pi_t' Pe_l)[\mu_l^{M} - e_{h+1}'\bar{y}_{t+1}]Var[\varepsilon_{t+1}^{M} \mid S_{t+1} = l]$$

$$Kurt_t[r_{t+1}^{M}] = \sum_{l=1}^{k}(\pi_t' Pe_l)(\mu_l^{M} - e_{h+1}'\bar{y}_{t+1})^4 + 6\sum_{l=1}^{k}(\pi_t' Pe_l)[(\mu_l^{M} - e_{h+1}'\bar{y}_{t+1})^2 Var[\varepsilon_{t+1}^{M} \mid S_{t+1} = l]]. \quad (1.15)$$

Similarly, the conditional covariance, co-skewness and co-kurtosis between individual asset returns, r_{t+1}^{i} , and the market returns, r_{t+1}^{M} , are

$$Cov_t[r_{t+1}^{i}, r_{t+1}^{M}] = \sum_{l=1}^{k}(\pi_t' Pe_l)[(\mu_l^{i} - e_i'\bar{y}_{t+1})(\mu_l^{M} - e_{h+1}'\bar{y}_{t+1}] + \sum_{l=1}^{k}(\pi_t' Pe_l)Cov[\varepsilon_{t+1}^{i}, \varepsilon_{t+1}^{M} \mid S_{t+1} = l]$$

$$\begin{aligned} Coskew[r_{t+1}^{i}, r_{t+1}^{M}] = Cov_t[r_{t+1}^{i}, (r_{t+1}^{M})^2] = & \sum_{l=1}^{k}(\pi_t' Pe_l)[(\mu_l^{i} - e_i'\bar{y}_{t+1})(\mu_l^{M} - e_{h+1}'\bar{y}_{t+1})^2] \\ & + \sum_{l=1}^{k}(\pi_t' Pe_l)[(\mu_l^{i} - e_i'\bar{y}_{t+1})Var[\varepsilon_{t+1}^{M} \mid S_{t+1} = l]] \\ & + 2\sum_{l=1}^{k}(\pi_t' Pe_l)[(\mu_l^{M} - e_{h+1}'\bar{y}_{t+1})Cov[\varepsilon_{t+1}^{i}, \varepsilon_{t+1}^{M} \mid S_{t+1} = l]] \end{aligned}$$

$$\begin{aligned} Cokurt[r_{t+1}^{i}, r_{t+1}^{M}] = Cov_t[r_{t+1}^{i}, (r_{t+1}^{M})^3] = & \sum_{l=1}^{k}(\pi_t' Pe_l)[(\mu_l^{i} - e_i'\bar{y}_{t+1})(\mu_l^{M} - e_{h+1}'\bar{y}_{t+1})^3] \\ & + 3\sum_{l=1}^{k}(\pi_t' Pe_l)[(\mu_l^{i} - e_i'\bar{y}_{t+1})(\mu_l^{M} - e_{h+1}'\bar{y}_{t+1})Var[\varepsilon_{t+1}^{M} \mid S_{t+1} = l]] \\ & + 3\sum_{l=1}^{k}(\pi_t' Pe_l)[(\mu_l^{M} - e_{h+1}'\bar{y}_{t+1})^2 Cov[\varepsilon_{t+1}^{i}, \varepsilon_{t+1}^{M} \mid S_{t+1} = l]] . \quad (1.16) \end{aligned}$$

In estimating the four moment CAPM under regime switching (1.9), GMM estimation is adopted. Equally weighted market returns are used as an instrument variable. Let z_t denote equally weighted market return at time t, then the orthogonality conditions are $E[z_t \varepsilon_{t+1}] = 0$ where ε_{t+1} is a $h+1$ dimensional vector of regime-dependent errors, i.e. $\varepsilon_{t+1} = [\varepsilon_{t+1}^{1}, ..., \varepsilon_{t+1}^{h}, \varepsilon_{t+1}^{M}] \sim N(0, \Omega_{S_{t+1}})$. Therefore we have $4(h+1)$ orthogonality conditions. The estimates of the model (1.9) are shown in table 1.5. Intercepts estimates are significant in all states. In crash and low growth state, intercepts are negative while intercepts are mostly positive in bull and recovery states. As expected, estimates of risk premia for covariance, co-skewness and co-kurtosis risks are different across regimes. Covariance risk premium is positive in all states, however, significant only in a low growth and recovery states. Co-skewness risk premium is negative and significant in all states. Co-kurtosis risk premium is positive but insignificant in crash state, however, negative and significant in other positive growth states.

Table 1.5 Parameter estimates of a four-moment CAPM

		Cross sectional		
	Crash state	Low growth state	Bull state	Recovery state
Covariance	0.3534	0.9325*	0.3581	3.5006*
	(0.7082)	(0.2626)	(0.3377)	(0.3639)
Co-skew	-1.6629*	-1.7753*	-2.6947*	-4.9266*
	(0.2067)	(0.0874)	(0.2504)	(0.0744)
Co-kurtosis	0.0716	-0.1996*	-0.6761*	-1.9141*
	(0.1732)	(0.0354)	(0.0747)	(0.2482)
Intercept				
Market	-2.2711*	-0.7331*	2.0018*	2.2932*
	(0.5608)	(0.1708)	(0.3604)	(0.2879)
Small-caps	-4.9574*	-1.0076*	5.4632*	-2.3397*
	(0.6311)	(0.2918)	(0.8834)	(0.7377)
Large-caps	-2.1292*	-0.6580*	1.8084*	2.4644*
	(0.5493)	(0.1581)	(0.3312)	(0.2757)
J-statistic	0.3571	0.1405	0.1396	0.7863

* denotes the parameter estimate is significant at 5% significance level.

Table 1.6 summarizes estimates of a four-state regime-switching mean returns model. Two regimes capture periods with high volatility and low persistence and two regimes capture periods with relatively higher persistence. The first regime can be characterized as crash state which shows negative excess mean returns and high volatility and correlations are relatively high as compared to other states. The persistence of the first state is considerably low as compared to low growth and bull states. The second regime is low growth state with small positive excess mean returns and low volatility but small caps show small negative excess mean returns. The third regime is bull state and more volatile as compared to the low growth state, especially small stocks are most volatile with notably positive excess returns. The fourth state is recovery state with strong rally and high volatility and this state is transitory.

The correlations between asset-specific innovations are different across regimes. The correlation between small-caps and large-caps is most strengthened during the crash state and the correlation is getting weaker as market evolves to more bullish states and lowest in recovery state. This implies that diversification only through equities (small-caps and large-caps) may not work well in the crash state. In general, large-caps are more closely correlated to the market as compared to small-caps.

Table 1.6 Parameter estimates of a four-state regime-switching model

		Market	Small-caps	Large-caps
Mean excess return				
Crash		-3.4128(1.1344)	-4.7255(1.2983)	-3.2075(1.0953)
Low growth		0.3580(0.2446)	-0.1758(0.3076)	0.3818(0.2413)
Bull		1.7313(0.4450)	5.4327(0.9981)	1.5557(0.4360)
Recovery		5.1355(0.8393)	8.6023(3.2083)	4.7952(0.9807)
Variance				
Crash		44.4438 (9.3857)	55.5473 (10.7366)	42.5981 (9.1075)
Low growth		10.2146 (1.1377)	13.8148 (1.7161)	10.0664 (1.1353)
Bull		17.0642 (2.5523)	79.8928 (13.1779)	16.0518 (2.4066)
Recovery		13.4170 (4.2468)	212.7829 (68.7256)	18.4545 (5.8426)
Correlations				
Crash				
Market		1.0000		
Small-caps		0.8112	1.0000	
Large-caps		0.9964	0.7757	1.0000
Low growth				
Market		1.0000		
Small-caps		0.5769	1.0000	
Large-caps		0.9949	0.5169	1.0000
Bull				
Market		1.0000		
Small-caps		0.5195	1.0000	
Large-caps		0.9884	0.4265	1.0000
Recovery				
Market		1.0000		
Small-caps		0.3928	1.0000	
Large-caps		0.9992	0.4128	1.0000
Transition probabilities				
	Crash	Low growth	Bull	Recovery
Crash	0.7432	0.0000	0.0000	0.2568
Low growth	0.0650	0.8538	0.0812	0.0000
Bull	0.0000	0.2764	0.7236	0.0000
Recovery	0.1270	0.0003	0.6952	0.1775

Transitions between regimes show a specific pattern that once it exits from highly volatile crash state it goes to the volatile recovery state with high expected excess returns and this feature captures volatility clustering in asset returns.

The state probabilities indicate the long-run probabilities of the various states as the process continues for a very long time, i.e. the long-run expected fraction of visits that the Markov chain makes to the various states. The state probabilities are 15.58% for state1 (crash), 52.04% for state2 (low growth), 27.52% for state3 (bull) and 4.86% for state4 (recovery). Hence, although the crash state is clearly not visited as often as the other states, it does not only pick up extremely rare events.

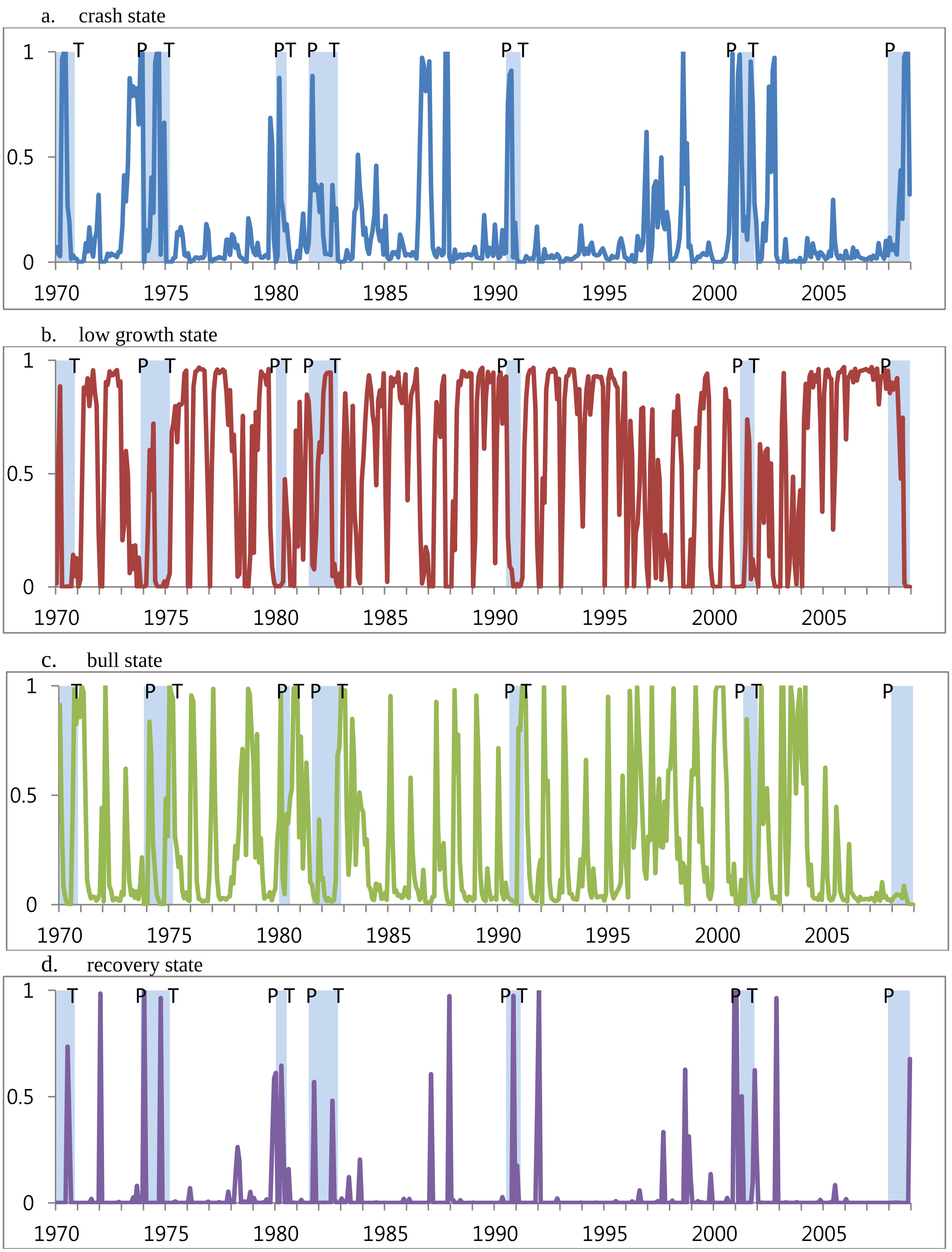
Figure1.3 shows state probabilities in each state and crash state includes the two oil shocks (1973-1974, 1979-1980), Black Monday (October 1987), the doldrums of 1990, the Asian Financial Crisis (1997), the internet bubble burst (2002) and the global financial crisis of 2008. Usually recovery state follows after crash state.

Figure1.4 shows mean excess returns of US stock market and portfolios of small-cap stocks and large-cap stocks. Large negative excess returns are associated with the crash state. However, in most of times US stock returns are positive.

Figure1.5 presents mean excess returns, volatility, skew and kurtosis of the U.S. stock market and the times with negative skewness, high volatility and high kurtosis coincide with crash state. This is reassuring since large changes in skewness and kurtosis are linked to regime switching. Moreover, even within a regime, we can observe a fair bit of changes in these higher moments and this implies market conditions change consistently even in a given regime.

Figure1.6 shows covariance, co-skewness and co-kurtosis of portfolios of small-cap stocks and large-cap stocks in relation to US stock market. Mostly the portfolio of small-cap stocks is the most volatile and associated with more extreme uncertainty (i.e. higher co-kurtosis) even though it is more negatively coskewed to the market which shows negative skew most of the time.

Figure1.7 shows conditional covariance, co-skewness and co-kurtosis obtained by recursive expansion from sample period of returns. These estimates are heavily affected by outliers (while those estimates of regime-switching model are less sensitive to outliers) and thus they are not good enough to reflect changing market conditions.



Note: Shaded areas are contraction periods determined by the NBER Business Cycle Determining Committee.

Figure 1.3 State probabilities

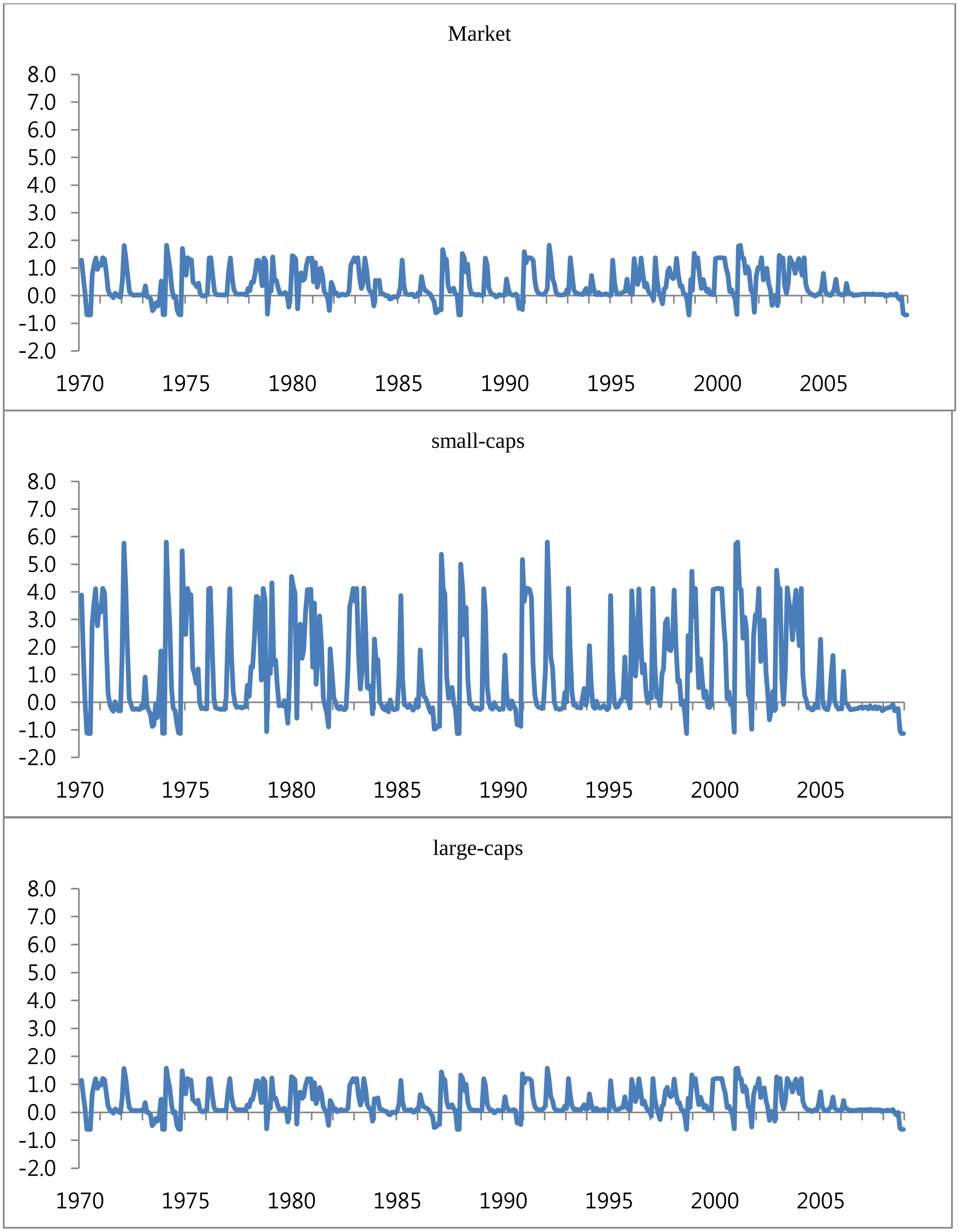


Figure 1.4 Mean excess returns implied by the four-state regime-switching model

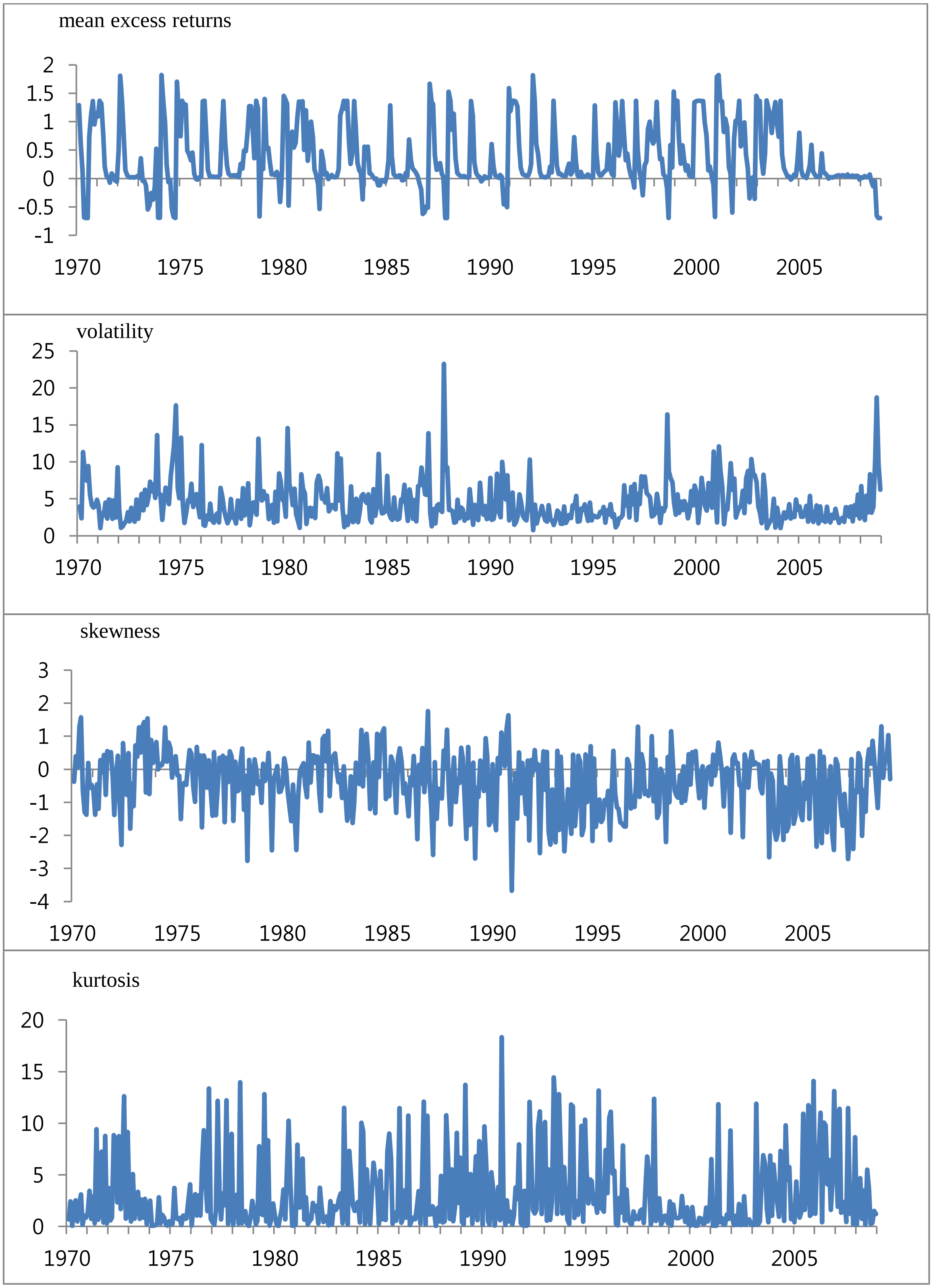
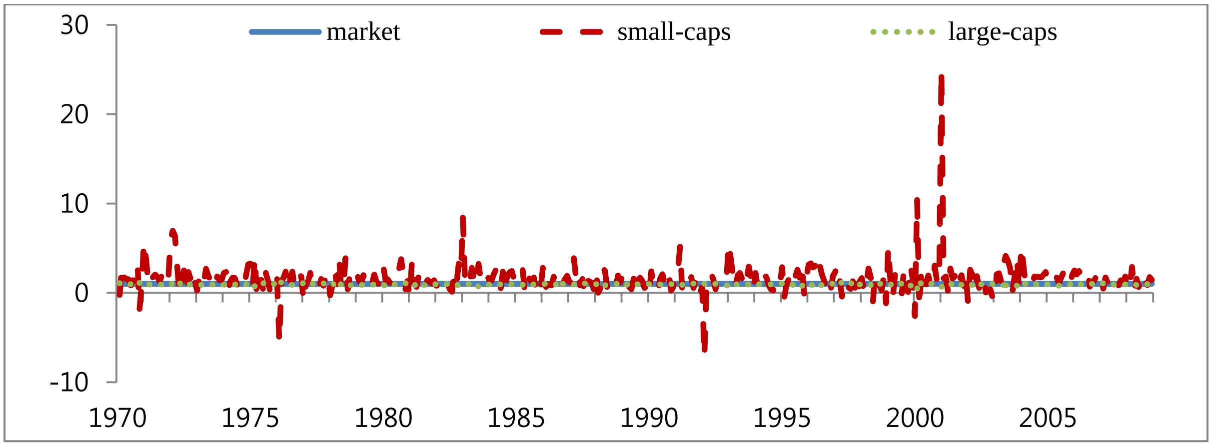
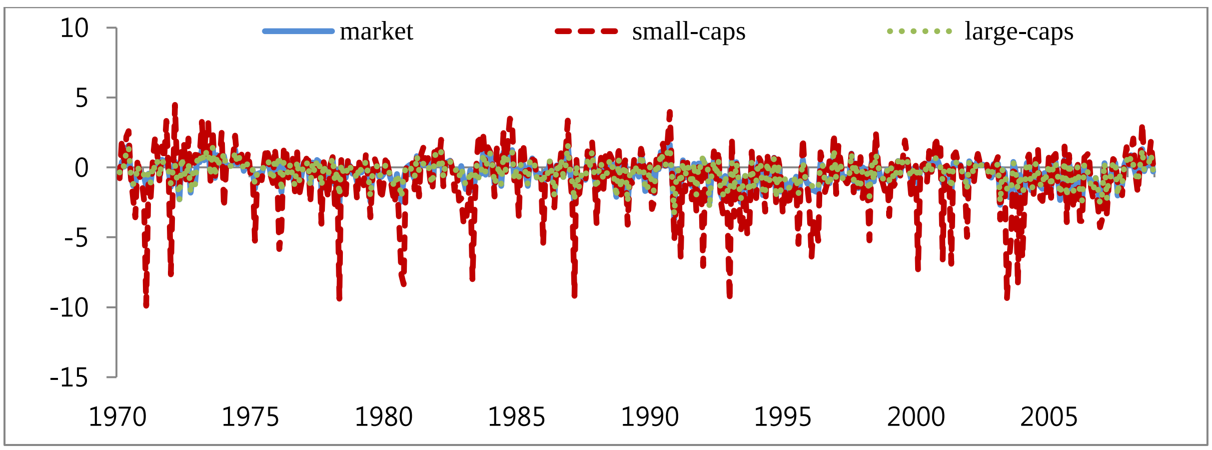


Figure 1.5 Mean excess returns, volatility, skew and kurtosis of the U.S. stock market implied by the four-state regime-switching model

a. co-variance



b. co-skewness



c. co-kurtosis

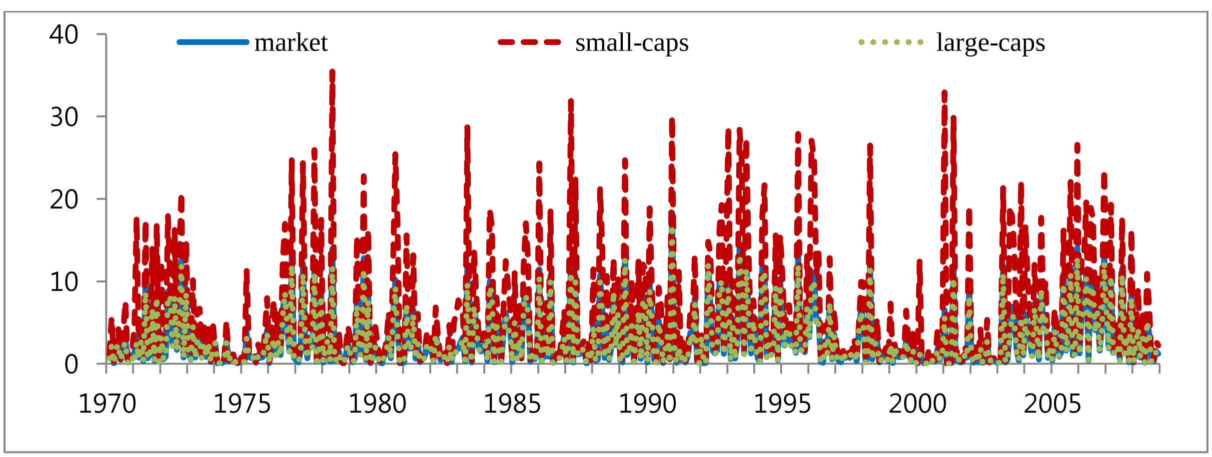


Figure 1.6 Covariance, co-skewness, co-kurtosis implied by the four-state regime-switching model

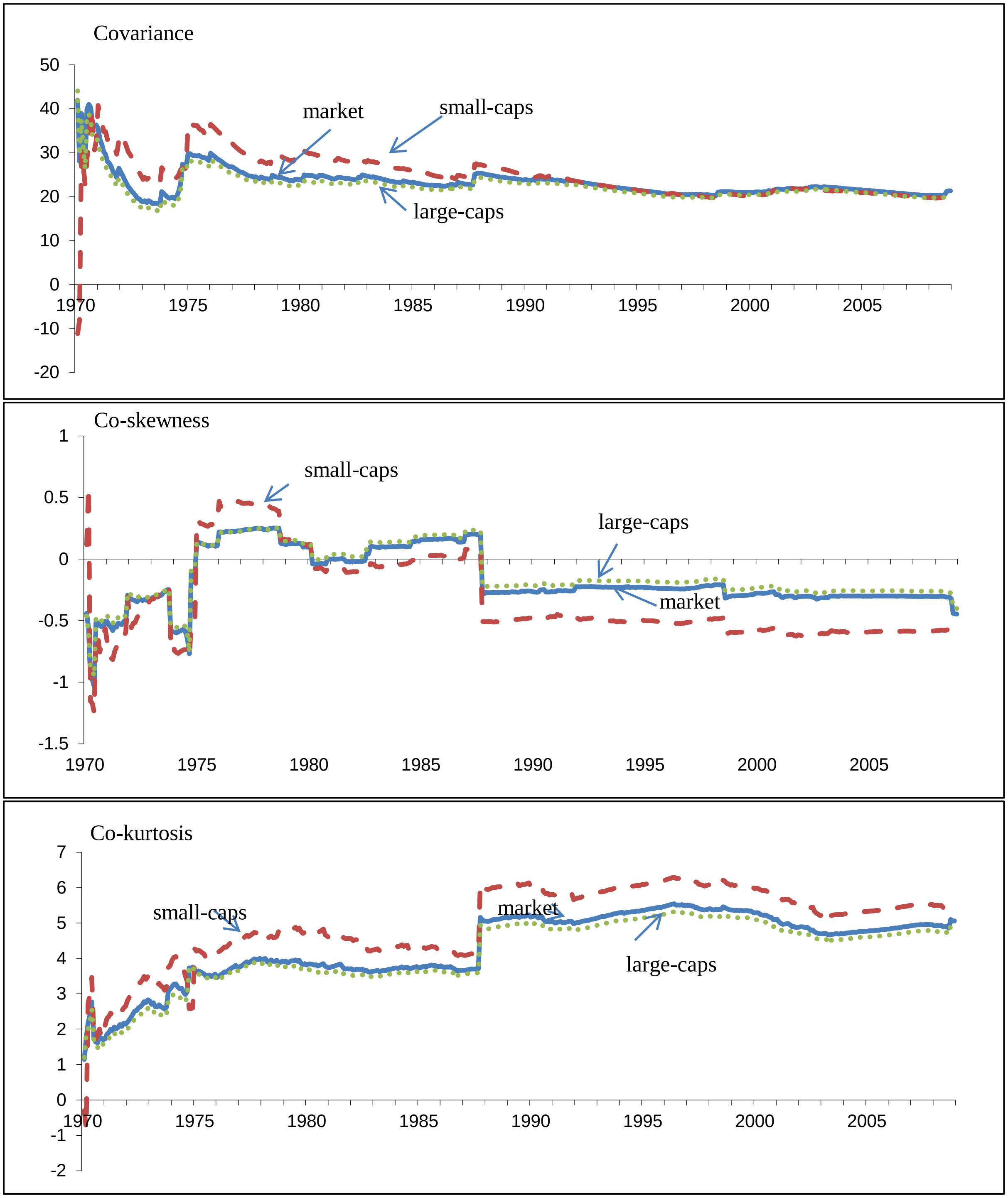


Figure 1.7 Conditional covariance, co-skew and co-kurtosis derived by recursive expansion

3. Portfolio allocation under Skew and Kurtosis Preferences

3.1 Preferences over moments and solution to portfolio allocation

Now this chapter turns to its interest to the investor's asset allocation problem. Investor's preferences depend on skew and kurtosis of returns and this chapter allows these preferences vary depending on regimes. Consider an investor whose wealth at time t is W_t and if the investor allocates his/her wealth among the available assets, then his/her wealth at time $t+1$ can be expressed as $W_{t+1} = W_t(1 + r_{t+1}^f + \omega_t' y_{t+1})$ where r_{t+1}^f is the risk free rate, ω_t is the vector of allocations to the risky assets and y_{t+1} is the vector of excess returns.

The investor will choose the allocation that maximizes the expected utility at time $t+1$. That is

$$\omega_t^* = \arg \max_{\omega_t} E[U(W_{t+1}(\omega_t);\theta) \mid I_t]$$

$$\text{s.t. } W_{t+1} = W_t(1 + r_{t+1}^f + \omega_t' y_{t+1}) \qquad (1.16)$$

where θ is a set of shape parameters.

To accommodate moment effect in utility function, Guidolin and Timmermann (2008) present a four-moment preference function with power utility and this chapter adopts their approach and introduce here. To begin with, consider an *m*-th order Taylor expansion of utility function around some wealth level x,

$$E_t[U^m(W_{t+1}(\omega_t);\theta)] \approx \hat{E}_t[U^m(W_{t+1}(\omega_t);\theta)] = \sum_{n=0}^{m} \frac{1}{n!} U^{(n)}(x;\theta) E_t[(W_{t+1} - x)^n] \qquad (1.17)$$

and $x = E_t[W_{t+1}]$ without loosing generality.

Under the specification of *k* states, the n-th non-central moment of the returns is given by

$$M_{t+1}^{(n)} = E[r_{t+1}^n \mid I_t] = \sum_{S_{t+1}=1}^{k} E[r_{t+1}^n \mid S_{t+1}, I_t] \Pr(S_{t+1} \mid I_t)$$
$$= M_{1t+1}^{(n)} + M_{2t+1}^{(n)} + \cdots + M_{kt+1}^{(n)} \qquad (1.18)$$

where $r_t = y_t + r_f$.

As most studies on optimal asset allocation, they use power utility function which can be written as

$$U(W_{t+1}) = \frac{W_{t+1}^{1-\theta}}{1-\theta} \quad for \;\; \theta > 0, \;\; \theta \neq 1 \qquad (1.19)$$

where θ is a coefficient of relative risk aversion. For a given coefficient of relative risk aversion, to accommodate the effect of skew and kurtosis preferences, forth order expansion of the utility function is considered and by taking expectations, they present the following four-moment preference function

$$\hat{E}_t[U^4(W_{t+1}(\omega_{t+1});\theta)] = c_0(\theta) + c_1(\theta)E_t[W_{t+1}] + c_2(\theta)E_t[W_{t+1}^2] + c_3(\theta)E_t[W_{t+1}^3] + c_4(\theta)E_t[W_{t+1}^4],$$

where

$$c_0(\theta) = x^{1-\theta}[(1-\theta)^{-1} - 1 - \frac{1}{2}\theta - \frac{1}{6}\theta(\theta+1) - \frac{1}{24}\theta(\theta+1)(\theta+2)]$$

$$c_1(\theta) = \frac{1}{6}x^{-\theta}[6 + 6\theta + 3\theta(\theta+1) + \theta(\theta+1)(\theta+2)] > 0$$

$$c_2(\theta) = -\frac{1}{4}\theta x^{-(1+\theta)}[2 + 2(\theta+1) + (\theta+1)(\theta+2)] < 0$$

$$c_3(\theta) = \frac{1}{6}\theta(\theta+1)(\theta+3)x^{-(2+\theta)} > 0$$

$$c_4(\theta) = -\frac{1}{24}\theta(\theta+1)(\theta+2)x^{-(3+\theta)} < 0. \qquad (1.20)$$

Expected utility from the next period wealth increases in $E_t[W_{t+1}]$ and $E_t[W_{t+1}^3]$, decreases in $E_t[W_{t+1}^2]$ and $E_t[W_{t+1}^4]$. A solution to the optimal asset allocation problem can be found by solving a system of equations in $\hat{\omega}_t$ derived from the first order conditions

$$\nabla_{\omega_t}\hat{E}_t[U^4(W_{t+1}(\omega_t);\theta)]|_{\hat{\omega}_t} = 0'. \qquad (1.21)$$

3.2 Empirical results- Investor's perception of the state probability and asset allocation

The state variable S_t is unobservable and thus investors are uncertain about current state of the economy. However, investors have beliefs about state probabilities and these perceptions are important to asset allocation.

State1-Crash state

Once investors believe the economy is in early stage of crash state, they sell stocks in the market since unavoidable risk (related to investors' labor income) makes investors to reduce the portion of risky asset of precautionary saving motive and thus stock prices fall. Therefore, large negative excess returns will be realized which causes decrease in skewness and kurtosis will rise due to the increased uncertainty related to unavoidable risk and volatility is likely to be high. In this state, the intensity of temperance exceeds the intensity of precautionary saving and this will speed up investor's selling motive of risky assets and thus volatility correlations among risky assets are likely to be strengthened. Investors may prefer to invest risk-free asset in this state or to cash risky assets for future investment since risky assets like stocks are less attractive. If stock market volatility is higher in a crash state than in expansionary states, equity investments are much less attractive in this state unless their mean returns rise commensurably. This regime is not likely to be persistent. Since the duration of the regime is positively related to the probability of the regime and the investor's perception of the crash state probability is not likely high from historical learning.

State2-Low growth state

This state is characterized by low volatility and small positive excess returns on assets and very persistent as compared to other states. When investors perceive the current state is likely to be in low growth state, the portion of risky assets is likely to be less than in bull or recovery state.

State3-Bull state

This state is characterized by high volatility and strong positive excess returns on assets, especially prices of small stocks rise rapidly. Knowing that the current state is a persistent bull state will make most risky assets more attractive than in a crash or low growth state. Risky assets are more attractive in this state and investors expect to have good investment opportunity.

State4-Recovery state

After crash state, investors will face recovery state which is likely to be transitory as long as the magnitude of crash is not so large enough that policy makers' (the central bank or government) effort also reduces the duration of this state. Recovery state is characterized by high volatility and positive excess returns on assets. If investors believe the economy is in recovery state, they will increase the portion of risky assets, especially assets with strong positive expected excess returns.

Table1.7 reports equity allocations for the single state and four-state model using 1-month investment horizon. Steady-state holdings represent the asset allocation when investors have very imprecise information about the current state. Under mean-variance preference, allocations to risky asset is relatively small as compared to skew and skew-kurtosis preference, especially crash and low growth state-stocks are much less attractive and risk-free asset is preferred. However, when skew preference or skew-kurtosis preferences are considered, the allocations to stocks are much higher and optimal asset allocation differs across regimes. Under skew preference and skew-kurtosis preferences, stocks are less attractive in crash state and more attractive in bull and recovery states. Especially, under skew preference small-cap stocks are most attractive in bull and recovery state. However, considering skew-kurtosis preferences together brings up notable decrease in the proportion of small-cap stocks. Optimal weights of small-cap stocks are smaller under skew-kurtosis preference as compared to those obtained under skew preference. This implies small-cap stocks become less attractive as compared to large-cap stocks due to the relatively high kurtosis when kurtosis preference is considered together.

Table 1.7 Optimal portfolio weights

a. mean-variance preference (m=2)					
	steady-state	crash	low growth	bull	recovery
Small-caps*	0.2112	0.1641	0.2620	0.1540	0.1429
Large-caps*	0.7915	0.8359	0.7380	0.8560	0.8571
T-bill	0.3621	0.8997	0.3860	0.0760	0.0035
b. skew preference (m=3)					
	steady-state	crash	low growth	bull	recovery
Small-caps*	0.6805	0.1853	0.6299	1.0000	1.0000
Large-caps*	0.3195	0.8147	0.3701	0.0000	0.0000
T-bill	0.2871	0.7182	0.2927	0.0830	0.0000
c. skew-kurtosis preferences (m=4)					
	steady-state	crash	low growth	bull	recovery
Small-caps*	0.4194	0.0769	0.5895	0.4515	0.6552
Large-caps*	0.5806	0.9231	0.4105	0.5885	0.3447
T-bill	0.3344	0.8889	0.2980	0.1143	0.1927

*Stock holdings are shown as a fraction of total equity portfolio, while the T-bill holdings are reported as a proportion of the total portfolio (Weights calculated under power utility with $\theta = 2$ which is compatible with much empirical evidence).

4. Conclusion

This chapter explores a four-moment CAPM under regime switching and the implication of skewness and kurtosis preferences on asset allocation with US stock market data. This model captures four regimes (the crash, low growth, bull and recovery) and finds time-varying skewness and kurtosis play a role in explaining cross-sectional variations in asset risk premium in combination with regime-dependent risk prices. By considering higher moments (skewness and kurtosis) preferences, this chapter provides explanation for a considerable holding portion of stocks in asset portfolio. Investors prefer positively skewed returns and dislike extreme uncertainty (fat tails) and this skew-kurtosis preference explains considerable proportion of large-cap stocks in the portfolio.

Chapter II

Price discovery and information revealing in volatile markets

1. Introduction

The standard two-moment CAPM is based on mean-variance preference and assumes symmetric information world where all traders share same information about the asset's expected risk and return. Previous chapter shows a four-moment CAPM under regime switching which additionally considers skewness and kurtosis preferences can explain the preference of large-cap stocks over small-cap stocks with U.S. stock market data. This chapter provides some empirical evidence which suggests the standard CAPM needs to be recast again to incorporate information asymmetry. O'Hara (2003) proposes an asymmetric information asset-pricing model that incorporates the transaction costs of liquidity and the risks of price discovery. If information risk matters in asset pricing, i.e. if price discovery role matters, it will make effect on the returns that investors want in equilibrium. This chapter examines price discovery role by market capitalization in relation to information risk, especially for volatile US market periods since price discovery role (i.e. information risk) matters much more when the market is volatile.

Price discovery is one of the important market functions. Price reveals information as a form of interaction between decision makers and information make impacts on price discovery. With less information, price variance is likely to increase and this will decrease the efficiency of price discovery. In a rational expectation equilibrium, where decision makers optimize based on all available information, agents must combine the information in prices with their private information. Private information can be revealed through trading and much of microstructure literature tells about it. Kyle (1985) shows informed traders move first with signaling and all private information is incorporated into prices by the end of trading. Glosten and Milgrom (1985) finds positive bid-ask spread are related to the presence of informed traders and thus, the resulting trade prices convey information. Harsbrouck (1991) provides a measure of trade informativeness by decomposing the variance of changes in the efficient price into trade-correlated and –uncorrelated components. Barclay and Hendershott (2003) report that the probability of informed traders are quite different before the open and after the close by using Easley, Kiefer, and O'Hara's model (1996, 1997a, b). They also find that relatively low after-hours trading volume can generate significant price discovery. Previous studies suggest that information asymmetry will be highest before the open and lowest after the close as information accumulates overnight when there is little trading-i.e. information asymmetry decays over the day.

Harsbrouck (1991) shows, for a sample of NYSE listed companies, trades are found to be more informative for small firms. However, this chapter suggests that this is not the case on days of large price changes during highly volatile market periods. It compares intraday trade informativeness between a large-cap stock (Microsoft corporation, MSFT) and a small-cap stock (OPNET technologies, Inc., OPNT) for highly volatile days by using NYSE TAQ data. This comparison shows MSFT reveals a considerable amount of private information through trades before the open and this private information decays over the day. However, a small cap stock (OPNT) doesn't have enough quotes and trades before the open and even reveals little trade-correlated information during the trading hours. O'Hara (2003) shows that the equilibrium risk premium is higher for assets in which a larger fraction of the information is private rather than public. I am sympathetic to this idea and at the same time consider a possibility that private information plays significant role in price discovery when little public information is available. In this situation, stocks which reveal information through trades can be preferred over other stocks which convey little information through trades since private information revealed through trades can mitigate price uncertainty by allowing faster price discovery.

In a next step, this chapter investigates information interactions between stocks (cross effects) for trading hours through cross-price and cross-trading volume to examine whether the prices and order flows from one stock convey information that is relevant to value other stock within same industry level. Furthermore, to examine how trade-informativeness makes effect on the price efficiency, this chapter compares the price efficiency during two volatile market periods within a framework of "unbiasedness regression" (Biais, Hillion and Spatt, 1999). The tech bust in 2000 and financial crisis in 2008 are two recent volatile periods in the US stock market and these periods are considered to compare price discovery through information revealing. Opening price efficiency is analyzed for both stocks from August 2000 to December 2008 with daily data. Intraday price efficiency on the closing price and post-close price efficiency on the opening price are examined for two recent volatile US stock market periods (2000:4Q and 2008:4Q) with MSFT trade data. Finally, this chapter compares the efficiency of after-hours price during the two periods and investigates how trading volume and price efficiency are related.

2. Trade- Informativeness

2.1 Public versus Private Information

Following Hasbrouck (1991), this chapter measures the informativeness of stock trades with a bivariate vector auto-regression (VAR) model of prices and trades. This econometric analysis decomposes the variance of the permanent component of price changes into two parts, one is attributable to public information and another is attributable to private information which is revealed through trades. For this analysis, quote mid-points are used instead of trade prices. Quotes have some advantages over trade prices. First, they are usually considered as active until revised. Second, quotes can be revised even in the absence of trades because a dealer can update the bid and ask quotes when new public information arrives. Thus, quotes are assumed to be more reliable as compared to last-traded prices since they reflect most current public information. Finally, in the short-run, mid-quotes are relatively less volatile than trade prices which are subject to bid-ask bounce (a source of transitory volatility) although we admit quote midpoints are also affected from transitory components.

Data alignment

Quote revisions and transactions (market events) are defined on a sequence of event times indexed by t. The time scale t can be defined as transaction sequence. The price variable p_t is the logarithm of the mid-quote at time t and prevailing mid-quote at the beginning of time t is the preceding mid-quote p_{t-1}. The trade variable x_t is trade sign at time t and is solely determined by comparing trade prices to prevailing mid-quotes: x_t =+1 if a trade price is greater than a prevailing mid-quote (a buy order), x_t =-1 if a trade price is less than a prevailing mid-quote(a sell order) and x_t = 0 if a trade price is equal to a prevailing mid-quote or in the absence of trade between quotes. To identify prevailing mid-quote, most recent mid-quote before a trade execution is considered and Lee and Ready (1991)'s 5-seconds rule is not applied here since the different reporting-lag problem between quotes and trades is not likely to be a matter in recent years[3]. To sum up data alignment process, the prevailing mid-quote at time t is p_{t-1} and

[3] As the NYSE abolished the mark-sense card system for trade recording in 1994, the floor reporter began to use a band-held device to report trades. Also, in 2000, 91% of all trades were reported through the Display Book (the floor reporter position is eliminated). On July 24 2001, all trade reporting is done directly through display book. On May 27 2003, the NYSE introduce 'auto-quoting' for all stocks. This procedure implies that the NYSE automatically updates the NYSE's best bid or offer whenever a limit order is transmitted to the Display Book at a better price than the previous best bid or offer (Vergote (2005)).

a trade x_t is executed and new public information arrives and the market maker posts a new quote p_t . In cases where more than one quote is observed at the same time to the second unit, the average of mid-quotes is considered for the prevailing mid-quote and this rule also applies to trade prices.

Model

Let p_t be the logarithm of the mid-quote at time t and it is comprised of two parts-permanent component (m_t) and stationary component (s_t).

$$p_t = m_t + s_t \qquad (2.1)$$

where m_t is considered as the efficient price and s_t is interpreted as transitory discrepancy.

The efficient price evolves as a random walk

$$m_t = m_{t-1} + v_t \qquad (2.2)$$

where $v_t \sim (0, \sigma_v^2)$ with $Ev_t v_s = 0$ for $t \neq s$. Also it is assumed that s_t is zero-mean stochastic process and jointly covariance stationary with v_t. The return at time t is defined as $r_t = p_t - p_{t-1}$, i.e. the percentage change in the mid-quote. To decompose the variance of return into its public and private information, a structural VAR needs to be estimated.

The structural VAR (SVAR) model of returns and trades is

$$\begin{aligned} r_t &= \sum_{i=1}^{p} \alpha_i r_{t-i} + \sum_{i=0}^{p} \beta_i x_{t-i} + \varepsilon_{1,t} \\ x_t &= \sum_{i=0}^{p} \gamma_i r_{t-i} + \sum_{i=1}^{p} \delta_i x_{t-i} + \varepsilon_{2,t} \end{aligned} \quad , \qquad (2.3)$$

where $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are structural shocks and $E(\varepsilon_{1,t}^2) = \sigma_{\varepsilon 1}^2$, $E(\varepsilon_{2,t}^2) = \sigma_{\varepsilon 2}^2$ and $Cov(\varepsilon_{1,t}, \varepsilon_{2,t}) = 0$.

The above SVAR model is slightly different from Hasbrouck (1991)'s model in that it allows contemporaneous effect of returns (price change) on trades to accommodate changes in market events environment. Recently market events (quote revisions and transactions) occur much more frequently as compared to Hasbrouck (1991)'s sample period (the first quarter of 1989)[4].

[4] For example, on Dec 1 2008, MSFT quote revisions occur about 1.5million times during the day and some times more than 200 quote revisions fall on the same time to the second. Especially during the trading hour, quote revision occurs every 1 second and trades are executed every 1.5 second on average.

The SVAR can be represented in matrix form,

$$\begin{pmatrix} 1 & -\beta_0 \\ \gamma_0 & 1 \end{pmatrix}\begin{pmatrix} r_t \\ x_t \end{pmatrix} = \sum_{i=1}^{p}\begin{pmatrix} \alpha_i & \beta_i \\ \gamma_i & \delta_i \end{pmatrix}\begin{pmatrix} r_{t-i} \\ x_{t-i} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} \tag{2.4}$$

and then a reduced form VAR is written as

$$\begin{pmatrix} r_t \\ x_t \end{pmatrix} = \sum_{i=1}^{p}\begin{pmatrix} 1 & -\beta_0 \\ \gamma_0 & 1 \end{pmatrix}^{-1}\begin{pmatrix} \alpha_i & \beta_i \\ \gamma_i & \delta_i \end{pmatrix}\begin{pmatrix} r_{t-i} \\ x_{t-i} \end{pmatrix} + \begin{pmatrix} 1 & -\beta_0 \\ \gamma_0 & 1 \end{pmatrix}^{-1}\begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix}. \tag{2.5}$$

Let $y_t = \begin{pmatrix} r_t \\ x_t \end{pmatrix}$, $\mathrm{B} = \begin{pmatrix} 1 & -\beta_0 \\ \gamma_0 & 1 \end{pmatrix}$, $\Gamma^i = \begin{pmatrix} \alpha_i & \beta_i \\ \gamma_i & \delta_i \end{pmatrix}$ and $\varepsilon_t = \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix}$, then equation (2.5) becomes

$$\begin{aligned} y_t &= \sum_{i=1}^{p} \mathrm{B}^{-1}\Gamma^i y_{t-1} + \mathrm{B}^{-1}\varepsilon_t \\ &= \sum_{i=1}^{p} \mathrm{A}^i y_{t-1} + u_t \end{aligned} \tag{2.6}$$

where $\mathrm{A}^i = \mathrm{B}^{-1}\Gamma^i$ and $u_t = \mathrm{B}^{-1}\varepsilon_t$ are the reduced form shocks, i.e. linear combinations of structural shocks.

To identify parameters in the SVAR, a reduced form VAR is estimated and structural identification is obtained through a long-run restriction which based on the causal structure between prices and trades. The long run restriction can take two mechanisms. One possible mechanism is that a strong price movement due to public information may draw traders to buy or sell on what they perceive to be a price trend, i.e. trade shock $\varepsilon_{2,t}$ has no long run cumulative effect on price change (return) r_t. Another mechanism is the opposite direction: from trades to prices, i.e. price shock $\varepsilon_{1,t}$ has no long run cumulative effect on trade x_t. This chapter analyzes trade informativeness under both mechanisms though first mechanism is more likely to hold when we consider the evolution of price over time. The lag length for the VAR is determined by using model selection criteria such as likelihood ratio (LR), Akaike information criterion (AIC), Schwarz information criterion (SC) and Hannan-Quinn information criterion (HQ).

The structural moving average (SMA) representation for this bivariate VAR is

$$\begin{pmatrix} r_t \\ x_t \end{pmatrix} = \begin{pmatrix} a(L) & b(L) \\ c(L) & d(L) \end{pmatrix}\begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix}, \tag{2.7}$$

where $a(L)$, $b(L)$, $c(L)$ and $d(L)$ are the lag polynomial operators.

Then the variance of return is

$$\sigma_v^2 = \left(\sum_{i=0}^{\infty} a_i\right)^2 \sigma_{\varepsilon 1}^2 + \left(\sum_{i=0}^{\infty} b_i\right)^2 \sigma_{\varepsilon 2}^2 , \tag{2.8}$$

where the second term, $\left(\sum_{i=0}^{\infty} b_i\right)^2 \sigma_{\varepsilon 2}^2$, is the variance component attributable private information revealed through trades.

2.2 Empirical analysis: Trade-informativeness by market capitalization in volatile periods

This section examines intraday trade-informativeness by market capitalization for volatile U.S. stock market periods. For this analysis, the Trade and Quote (TAQ)[5] data is used. Microsoft corporation (MSFT) is used for a large-cap stock and OPNET technologies, Inc. (OPNT) is used for a small-cap stock among stocks in application software industry sector of NASDAQ. To capture the information revealing pattern in volatile market, over the period from August 2000 to December 2008, the days where the largest price changes (top 1%) occurred on both stocks are considered. Most strong positive daily returns are observed on October 19, 2000 and October 28, 2008. Large negative daily returns are realized on November 30, 2000 and December 1, 2008. It's not surprising because fourth quarters of 2000 and 2008 are the most volatile periods during the sample period.

In many cases, companies report earnings before the open and after the close. These reports will make effect on intrinsic values of stocks and people want to access the market when they perceive changes in intrinsic values of stocks. Therefore, it is meaningful to examine how trade-informativeness evolves over a day including pre-market and after-hours trading period. In NASDAQ, pre-market stock trading takes place from 4:00 a.m. to 9:30 a.m. Eastern Time and after-hours stock trading takes place from 4:00 p.m. to 8:00 p.m. Eastern Time. Barclay and Hendershott (2003) report larger bid-ask spreads during the pre-open and after-hours trading due to relatively thin trading. An insufficient number of buyers and sellers which makes it harder to get the price for a stock, severe price swings and limited information about price quotes are some factors that contribute to higher costs in after-hours trading. Many of the after-hours traders are professionals with large institutions. Therefore they may have better access to more information than individual investors. It is expected after-hours trading is likely to be dominated by these

[5] The Trade and Quote (TAQ) database contains intraday transactions data (trades and quotes) for all securities listed on the New York Stock Exchange (NYSE) and American Stock Exchange (AMEX), as well as NASDAQ National Market System (NMS).

professionals who have strong incentives to trade after hours even with larger trading costs and lack of liquidity.

For intraday analysis, this chapter divides the trading data into pre-open (8:00 a.m. ~ 9:30 a.m.), the first half-hour of trading (9:30 a.m. ~ 10:00 a.m.), the middle day (10:00 a.m. ~ 3:30 p.m.), the last half-hour of trading (3:30 p.m.~4:00 p.m.), post-close(4:00 p.m.~6:30 p.m.) and overnight (6:30 p.m.~8:00 a.m.). The trading process is assumed to restart at the beginning of each time period. Table 2.1 provides ratio of private information to total information for each intraday time period for days realized strong positive return. Table 2.2 provides the ratio of private information to total information for days of large negative return. In general, MSFT (a large-cap stock) shows a considerable portion of variance is attributable to private information in times of volatile market while OPNT (a small-cap stock) does not and this implies that information revealing pattern could be different in volatile periods. Furthermore, information asymmetry does not seem to be resolved as quickly as in normal times as we see a fair amount of variance is attributable to private information even during the mid-day and the last half-hour trading. The information asymmetry is expected to be highest during the pre-opens and declines as time approaches the end of trading day in normal times.

One notable phenomenon is that the pre-open information asymmetry is much lessened during the days in 2008 as compared to those in 2000. Cao, Ghysels and Hatheway (2000) argue that market makers have incentives to communicate each other to make sure that the opening bid and ask quotes reflect all available information since informed traders (non-market makers) can exploit their private information against market makers. For example, MSFT shows more frequent quote revisions during the pre-opens in 2008 and this supports the argument (10/19/2000: 211 quotes, 11/30/2000: 260 quotes, 10/28/2008: 1414 quotes and 12/01/2008: 414 quotes). This implies quote revisions can resolve information asymmetry to some extent by updating information more frequently. Meanwhile, OPNT (a small cap stock) doesn't have enough quotes and trades during the pre-opens. Barclay and Henderscott (2004) find a reason in larger trading cost during the pre-open (three to four times larger than trading cost during the trading hour). A considerable trading activity is needed to move price discovery from the trading day to pre-open. For less active stocks like OPNT, the high cost of pre-open trading may deter pre-open trading and thus trading volume may not be sufficient to draw this move in price discovery. Also it shows little trade-correlated information is revealed during the trading hours. However, relatively high portion of trade-correlated information is observed during the post-closes in 2008. If information risk matters in asset pricing, i.e. if price discovery role matters, it will make effect on the return that investors want in equilibrium.

Table 2.1 Ratio of return variance attributable to private information for days of strong positive return

a. MSFT

Date	daily return(%)	7am-8am		Pre-open		9:30am-10am		10am-3:30pm		3:30pm-4pm		post-close	
2000.10.19	19.57	-		0.6209	[3]	0.1572	[13]	0.3153	[16]	0.3402	[8]	0.1444	[13]
2008.10.28	9.07	0.7731	[14]	0.0525	[6]	0.1387	[9]	0.0991	[12]	0.0531	[2]	0.0636	[7]
2000.12.22	6.91	-		0.5123	[7]	0.3914	[9]	0.2719	[9]	0.3736	[8]	0.0000	[2]
2008.10.16	6.75	0.0076	[3]	0.0288	[9]	0.3276	[9]	0.5027	[27]	0.0802	[17]	0.1140	[17]
2000.12.5	6.09	-		0.0231	[1]	0.1006	[6]	0.4178	[12]	0.1106	[4]	0.0135	[1]
2008.12.16	5.62	0.8443	[9]	0.0093	[3]	0.1009	[8]	0.0826	[5]	0.0650	[5]	0.0004	[3]
2000.10.20	5.35	-		0.1287	[2]	0.1228	[4]	0.3433	[11]	0.3270	[7]	0.0000	[2]
2008.10.26	5.21	-		0.0532	[2]	0.3996	[7]	0.4425	[15]	0.1184	[6]	0.0025	[1]
2001.8.24	4.96	-		0.0310	[2]	0.0286	[6]	0.0799	[14]	0.0543	[5]	0.0022	[1]
2001.2.26	4.95	-		0.0035	[3]	0.4213	[8]	0.4897	[9]	0.4015	[8]	0.0021	[3]
2001.1.18	4.84	-		0.0315	[1]	0.2646	[7]	0.4987	[12]	0.0527	[4]	0.0331	[4]
2001.3.23	4.74	-		0.2320	[7]	0.3477	[7]	0.4751	[9]	0.4973	[6]	0.0048	[1]
2001.9.24	4.63	-		0.3220	[6]	0.0164	[3]	0.0554	[9]	0.0409	[5]	0.0133	[1]
2008.6.12	4.13	-		0.0178	[5]	0.2737	[12]	0.0678	[17]	0.0053	[6]	0.2598	[6]
2001.1.11	4.01	-		0.0454	[1]	0.2745	[10]	0.3755	[12]	0.2147	[5]	0.0018	[5]
Average	6.46	0.5417		0.1408		0.2244		0.3012		0.1823		0.0437	

Lag orders are presented in [ ].

b. OPNT

Date	daily return(%)	7am-8am	Pre-open	9:30am-10am		10am-3:30pm		3:30pm-4pm		post-close	
2000.10.19	14.12	-	-	0.0103	[3]	0.0134	[1]	0.0266	[1]	-	
2008.10.28	20.55	-	-	0.0004	[6]	0.0346	[18]	0.0384	[9]	0.1131	[13]
2000.12.22	24.22	-	-	0.0540	[1]	0.0010	[1]	0.1561	[1]	-	
2008.10.16	20.11	-	-	0.0521	[6]	0.0406	[18]	0.1242	[13]	-	
2000.12.5	15.10	-	-	0.0044	[1]	0.0055	[2]	-	-	-	
2008.12.16	10.92	-	-	0.0630	[7]	0.0702	[16]	0.0000	[13]	0.0694	[18]
2000.10.20	16.49	-	-	0.1998	[1]	0.0137	[3]	0.0050	[1]	-	
2008.10.26	12.40	-	-	0.1869	[1]	0.0013	[1]	0.1085	[1]	-	
2001.8.24	11.26	-	-	0.0362	[1]	0.0791	[6]	0.0030	[1]	-	
2001.2.26	11.68	-	-	0.0102	[1]	0.0915	[2]	0.0008	[1]	-	
2001.1.18	12.72	-	-	0.0368	[1]	0.0533	[7]	0.3696	[3]	-	
2001.3.23	7.58	-	-	0.6439	[2]	0.0312	[2]	0.4996	[4]	-	
2001.9.24	8.33	-	-	0.0174	[1]	0.0002	[1]	0.0152	[1]	-	
2008.6.12	8.51	-	-	0.1718	[4]	0.1892	[17]	0.1720	[5]	0.0177	[1]
2001.1.11	21.56	-	-	0.0019	[1]	0.1564	[3]	0.1837	[1]	-	
Average	14.37	-	-	0.0993		0.0521		0.1216		0.0667	

Lag orders are presented in [ ].

Table 2.2 Ratio of return variance attributable to private information for days of large negative return

a. MSFT

Date	daily return(%)	7am-8am		Pre-open		9:30am-10am		10am-3:30pm		3:30pm-4pm		post-close	
2000.11.30	-11.80	-		0.9290	[16]	0.1364	[5]	0.1931	[12]	0.1403	[7]	0.0106	[5]
2000.12.15	-11.37	-		0.0700	[2]	0.3037	[4]	0.3882	[12]	0.1162	[8]	0.0267	[5]
2000.12.20	-7.39	-		0.2575	[6]	0.1480	[6]	0.4045	[11]	0.1810	[7]	0.0420	[6]
2008.12. 1	-7.96	0.0092	[1]	0.2333	[4]	0.1528	[17]	0.1912	[16]	0.1836	[5]	0.4349	[6]
2002. 7.25	-7.35	-		0.0513	[3]	0.0813	[7]	0.0704	[19]	0.0362	[2]	0.0683	[5]
2008.10. 7	-6.74	0.0000	[4]	0.0005	[9]	0.0545	[4]	0.1324	[19]	0.0117	[8]	0.0001	[8]
2000.10.16	-6.27	-		0.2408	[3]	0.0693	[7]	0.4070	[12]	0.2822	[8]	0.0578	[7]
2008.10.15	-5.98	0.0623	[1]	0.0708	[6]	0.3710	[9]	0.1754	[16]	0.0631	[3]	0.1910	[8]
2008.12.11	-5.63	0.0008	[4]	0.0751	[6]	0.2457	[11]	0.1890	[18]	0.1042	[8]	0.0264	[8]
2008.11.14	-5.60	0.0572	[1]	0.1506	[4]	0.1129	[10]	0.1707	[5]	0.0962	[4]	0.1191	[4]
2001. 8.30	-5.49	-		0.5085	[6]	0.0956	[8]	0.0829	[10]	0.1027	[5]	0.0014	[1]
2008.10.14	-5.49	0.0317	[9]	0.0666	[8]	0.0609	[12]	0.0496	[22]	0.0382	[4]	0.0366	[6]
2008.9.17	-5.46	-		0.0014	[4]	0.1211	[5]	0.1271	[19]	0.2430	[7]	0.0770	[9]
2001.3.21	-4.99	-		0.0151	[3]	0.4306	[8]	0.5963	[12]	0.4605	[6]	0.0526	[2]
2002.9.23	-4.70	-		0.4445	[5]	0.1652	[4]	0.2049	[11]	0.2068	[5]	0.0322	[2]
2000.10.23	-4.69	-		0.0088	[6]	0.2207	[9]	0.3536	[9]	0.4176	[7]	0.0008	[1]
2001.3.2	-4.50	-		0.0196	[7]	0.3371	[10]	0.4643	[10]	0.1316	[4]	0.0843	[1]
2000.10.3	-4.35	-		0.0413	[1]	0.0172	[2]	0.2480	[8]	0.3838	[9]	0.0148	[3]
2007.2.27	-4.13	-		0.0064	[2]	0.1431	[6]	0.2217	[15]	0.1159	[5]	0.0185	[3]
2001.12.20	-3.93	-		0.0021	[1]	0.0460	[4]	0.0780	[9]	0.0100	[6]	0.0157	[2]
2008.2.5	-3.71	-		0.0114	[10]	0.2539	[14]	0.3423	[20]	0.0171	[5]	0.0371	[11]
Average	-6.07	0.0269		0.1526		0.1699		0.2424		0.1591		0.0642	

Lag orders are presented in [ ].

b. OPNT

Date	daily return(%)	7am-8am	Pre-open		9:30am-10am		10am-3:30pm		3:30pm-4pm		post-close	
2000.11.30	-20.93	-	-		0.0004	[1]	0.2084	[3]	0.0011	[1]	0.1697	[1]
2000.12.15	-12.06	-	-		-		0.0000	[2]	0.0057	[1]	-	
2000.12.20	-37.57	-	-		-		0.1127	[5]	0.0226	[1]	0.1665	[2]
2008.12. 1	-12.23	-	-		0.0266	[4]	0.0178	[17]	0.1451	[16]	0.9981	[4]
2002. 7.25	-7.55	-	-		0.3422	[4]	0.0675	[1]	0.0168	[1]	-	
2008.10. 7	-8.02	-	0.0004	[1]	0.0952	[8]	0.0672	[7]	0.0047	[4]	0.0148	[7]
2000.10.16	-19.80	-	-		0.4641	[3]	0.0070	[1]	0.0111	[1]	-	
2008.10.15	-6.77	-	-		0.0047	[7]	0.0001	[17]	0.0235	[7]	0.0198	[7]
2008.12.11	-7.37	-	-		0.1066	[9]	0.0310	[18]	0.1020	[11]	0.0082	[14]
2008.11.14	-7.24	-	-		0.0072	[6]	0.0127	[18]	0.0308	[7]	0.0001	[6]
2001. 8.30	-7.69	-	0.1860	[1]	0.1349	[1]	0.0051	[1]	0.0297	[1]	-	
2008.10.14	-10.38	-	-		0.0010	[2]	0.0184	[8]	0.0334	[5]	0.2612	[4]
2008.9.17	-7.87	-	-		0.0209	[8]	0.0791	[14]	0.1192	[5]	-	
2001.3.21	-11.52	-	-		0.0034	[2]	0.0043	[1]	0.0203	[1]	-	
2002.9.23	-9.14	-	-		0.6726	[7]	0.0456	[7]	0.5525	[3]	-	
2000.10.23	-9.58	-	-		0.1382	[1]	0.1105	[2]	0.0301	[1]	-	
2001.3.2	-6.73	-	-		0.0005	[4]	0.0322	[1]	-		-	
2000.10.3	-8.41	-	-		0.9852	[2]	0.0228	[2]	0.0997	[3]	-	
2007.2.27	-12.41	-	0.0018	[1]	0.1672	[4]	0.0667	[11]	0.0576	[2]	0.0009	[5]
2001.12.20	-12.83	-	-		0.1403	[4]	0.1633	[3]	0.2233	[2]	-	
2008.2.5	-9.09	-	-		0.0389	[3]	0.1122	[18]	0.0392	[4]	-	
Average	-11.68	-	0.0627		0.1763		0.0564		0.0784		0.1821	

Lag orders are presented in [ ].

2.3 Cross effect-Information interactions

Through previous variance decomposition analysis, it is observed that OPNT (a small cap stock) shows little pre-open trading. Its trades are much less informative and most of its return variance is attributable to public information. It is of interest to consider possibility that the prices and order flows from one stock convey information that is relevant to value other stock within a same industry category. This section examines information interactions across stocks through cross-stock prices and cross-order flows.

To analyze cross effects between MSFT and OPNT, the raw data are regularized to 1-minute intervals. The quotes are aligned to each 1-minute sampling interval by using the most recent observation. The trades during each 1-minute interval are aggregated into the signed volume (the buying motive trade volumes minus the selling motive trading volumes). A four-variate structural VAR model is considered for cross effects analysis; to control own effect, own returns and own trades (signed volume) are included in the model with cross stock returns and cross stock trades (signed volume).

Model

$$\begin{aligned}
r_{1,t} &= \sum_{i=1}^{p} \alpha_{1,i}^{1} r_{1,t-i} + \sum_{i=0}^{p} \alpha_{2,i}^{1} r_{2,t-i} + \sum_{i=0}^{p} \beta_{1,i}^{1} x_{1,t-i} + \sum_{i=0}^{p} \beta_{2,i}^{1} x_{2,t-i} + \eta_{1,t} \\
r_{2,t} &= \sum_{i=0}^{p} \alpha_{1,i}^{2} r_{1,t-i} + \sum_{i=1}^{p} \alpha_{2i}^{2} r_{2,t-i} + \sum_{i=0}^{p} \beta_{1,i}^{2} x_{1,t-i} + \sum_{i=0}^{p} \beta_{2,i}^{2} x_{2,t-i} + \eta_{2,t} \\
x_{1,t} &= \sum_{i=0}^{p} \gamma_{1,i}^{1} r_{1t-i} + \sum_{i=0}^{p} \gamma_{2i}^{1} r_{2,t-i} + \sum_{i=1}^{p} \delta_{1,i}^{1} x_{1,t-i} + \sum_{i=0}^{p} \delta_{2,i}^{1} x_{2,t-i} + \eta_{3,t} \\
x_{2,t} &= \sum_{i=0}^{p} \gamma_{1,i}^{2} r_{1,t-i} + \sum_{i=0}^{p} \gamma_{2,i}^{2} r_{2,t-i} + \sum_{i=0}^{p} \delta_{1,i}^{2} x_{1t-i} + \sum_{i=1}^{p} \delta_{2,i}^{2} x_{2,t-i} + \eta_{4,t}
\end{aligned} \tag{2.9}$$

where $r_{1,t}$ and $r_{2,t}$ are stock returns, $x_{1,t}$ and $x_{2,t}$ are signed volumes, $\eta_{1,t}$, $\eta_{2,t}$, $\eta_{3,t}$ and $\eta_{4,t}$ are structural shocks such that $E(\eta_{1,t}^2) = \sigma_{\eta 1}^2, E(\eta_{2,t}^2) = \sigma_{\eta 2}^2, E(\eta_{3,t}^2) = \sigma_{\eta 3}^2$ and $E(\eta_{4,t}^2) = \sigma_{\eta 4}^2$ and $Cov(\eta_{j,t}, \eta_{k,t}) = 0 \quad for \quad j \neq k \quad j,k = 1,2,3,4$.

The above SVAR can be represented in matrix form,

$$\begin{pmatrix} 1 & \alpha_{2,0}^{1} & \beta_{1,0}^{1} & \beta_{2,0}^{1} \\ \alpha_{1,0}^{2} & 1 & \beta_{1,0}^{2} & \beta_{2,0}^{2} \\ \gamma_{1,0}^{1} & \gamma_{2,0}^{1} & 1 & \delta_{2,0}^{1} \\ \gamma_{1,0}^{2} & \gamma_{2,0}^{2} & \delta_{1,0}^{2} & 1 \end{pmatrix} \begin{pmatrix} r_{1,t} \\ r_{2,t} \\ x_{1,t} \\ x_{2,t} \end{pmatrix} = \sum_{i=1}^{p} \begin{pmatrix} \alpha_{1,i}^{1} & \alpha_{2,i}^{1} & \beta_{1,i}^{1} & \beta_{2,i}^{1} \\ \alpha_{1,i}^{2} & \alpha_{2,i}^{2} & \beta_{1,i}^{2} & \beta_{2,i}^{2} \\ \gamma_{1,i}^{1} & \gamma_{2,i}^{1} & \delta_{1,i}^{1} & \delta_{2,i}^{1} \\ \gamma_{1,i}^{2} & \gamma_{2,i}^{2} & \delta_{1,i}^{2} & \delta_{2,i}^{2} \end{pmatrix} \begin{pmatrix} r_{1,t-i} \\ r_{2,t-i} \\ x_{1,t-i} \\ x_{2,t-i} \end{pmatrix} + \begin{pmatrix} \eta_{1,t} \\ \eta_{2,t} \\ \eta_{3,t} \\ \eta_{4,t} \end{pmatrix} \qquad (2.10)$$

and then a reduced form VAR is written as

$$\begin{pmatrix} r_{1,t} \\ r_{2,t} \\ x_{1,t} \\ x_{2,t} \end{pmatrix} = \sum_{i=1}^{p} \begin{pmatrix} 1 & \alpha_{2,0}^{1} & \beta_{1,0}^{1} & \beta_{2,0}^{1} \\ \alpha_{1,0}^{2} & 1 & \beta_{1,0}^{2} & \beta_{2,0}^{2} \\ \gamma_{1,0}^{1} & \gamma_{2,0}^{1} & 1 & \delta_{2,0}^{1} \\ \gamma_{1,0}^{2} & \gamma_{2,0}^{2} & \delta_{1,0}^{2} & 1 \end{pmatrix}^{-1} \begin{pmatrix} \alpha_{1,i}^{1} & \alpha_{2,i}^{1} & \beta_{1,i}^{1} & \beta_{2,i}^{1} \\ \alpha_{1,i}^{2} & \alpha_{2,i}^{2} & \beta_{1,i}^{2} & \beta_{2,i}^{2} \\ \gamma_{1,i}^{1} & \gamma_{2,i}^{1} & \delta_{1,i}^{1} & \delta_{2,i}^{1} \\ \gamma_{1,i}^{2} & \gamma_{2,i}^{2} & \delta_{1,i}^{2} & \delta_{2,i}^{2} \end{pmatrix} \begin{pmatrix} r_{1,t-i} \\ r_{2,t-i} \\ x_{1,t-i} \\ x_{2,t-i} \end{pmatrix} + \begin{pmatrix} 1 & \alpha_{2,0}^{1} & \beta_{1,0}^{1} & \beta_{2,0}^{1} \\ \alpha_{1,0}^{2} & 1 & \beta_{1,0}^{2} & \beta_{2,0}^{2} \\ \gamma_{1,0}^{1} & \gamma_{2,0}^{1} & 1 & \delta_{2,0}^{1} \\ \gamma_{1,0}^{2} & \gamma_{2,0}^{2} & \delta_{1,0}^{2} & 1 \end{pmatrix}^{-1} \begin{pmatrix} \eta_{1,t} \\ \eta_{2,t} \\ \eta_{3,t} \\ \eta_{4,t} \end{pmatrix} \qquad (2.11).$$

Let $z_t = \begin{pmatrix} r_{1,t} \\ r_{2,t} \\ x_{1,t} \\ x_{2,t} \end{pmatrix}$, $\Lambda = \begin{pmatrix} 1 & \alpha_{2,0}^{1} & \beta_{1,0}^{1} & \beta_{2,0}^{1} \\ \alpha_{1,0}^{2} & 1 & \beta_{1,0}^{2} & \beta_{2,0}^{2} \\ \gamma_{1,0}^{1} & \gamma_{2,0}^{1} & 1 & \delta_{2,0}^{1} \\ \gamma_{1,0}^{2} & \gamma_{2,0}^{2} & \delta_{1,0}^{2} & 1 \end{pmatrix}$, $\mathrm{H}^i = \begin{pmatrix} \alpha_{1,i}^{1} & \alpha_{2,i}^{1} & \beta_{1,i}^{1} & \beta_{2,i}^{1} \\ \alpha_{1,i}^{2} & \alpha_{2,i}^{2} & \beta_{1,i}^{2} & \beta_{2,i}^{2} \\ \gamma_{1,i}^{1} & \gamma_{2,i}^{1} & \delta_{1,i}^{1} & \delta_{2,i}^{1} \\ \gamma_{1,i}^{2} & \gamma_{2,i}^{2} & \delta_{1,i}^{2} & \delta_{2,i}^{2} \end{pmatrix}$ and $\eta_t = \begin{pmatrix} \eta_{1,t} \\ \eta_{2,t} \\ \eta_{3,t} \\ \eta_{4,t} \end{pmatrix}$,

then equation (2.11) becomes

$$\begin{aligned} z_t &= \sum_{i=1}^{p} \Lambda^{-1} \mathrm{H}^i z_{t-1} + \Lambda^{-1} \eta_t \\ &= \sum_{i=1}^{p} \Phi^i z_{t-1} + \omega_t \end{aligned} \qquad (2.12)$$

where $\Phi^i = \Lambda^{-1}\mathrm{H}^i$ and $\omega_t = \Lambda^{-1}\eta_t$ are reduced form shocks, i.e. linear combinations of structural shocks.

To identify parameters in the SVAR, a reduced form VAR is estimated and structural identification is obtained by imposing long-run restrictions based on relative exogeneity among variables. This chapter assumes the long-run response pattern takes a form of lower triangular matrix: in the long run, stock1 return ($r_{1,t}$) is not affected by stock2 return shock ($\varepsilon_{2,t}$), own(stock1) trade shock ($\varepsilon_{3,t}$) and cross(stock2) trade shock ($\varepsilon_{4,t}$), stock2 return ($r_{2,t}$) is not affected by cross-trade shock($\varepsilon_{3,t}$) and own trade shock($\varepsilon_{4,t}$), stock1 trades ($x_{1,t}$) is not affected by stock2 trade shock $\varepsilon_{4,t}$. The lag length for the

VAR is determined by using model selection criteria as mentioned before.

The structural moving average (SMA) representation for this multivariate VAR is

$$\begin{pmatrix} r_{1,t} \\ r_{2,t} \\ x_{1,t} \\ x_{2,t} \end{pmatrix} = \begin{pmatrix} a_1(L)\ b_1(L)\ c_1(L)\ d_1(L) \\ a_2(L)\ b_2(L)\ c_2(L)\ d_2(L) \\ a_3(L)\ b_3(L)\ c_3(L)\ d_3(L) \\ a_4(L)\ b_4(L)\ c_4(L)\ d_4(L) \end{pmatrix} \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \\ \varepsilon_{4,t} \end{pmatrix}, \tag{2.13}$$

where $a_j(L)$, $b_j(L)$, $c_j(L)$ and $d_j(L)$ for $i = 1,2,3,4$ are the lag polynomial operators.

Then the return variance of stock h (h =1, 2) is

$$\sigma_{rh}^2 = \left(\sum_{l=0}^{\infty} a_{hl}\right)^2 \sigma_{\varepsilon 1}^2 + \left(\sum_{l=0}^{\infty} b_{hl}\right)^2 \sigma_{\varepsilon 2}^2 + \left(\sum_{l=0}^{\infty} c_{hl}\right)^2 \sigma_{\varepsilon 3}^2 + \left(\sum_{l=0}^{\infty} d_{hl}\right)^2 \sigma_{\varepsilon 4}^2, \tag{2.14}$$

where the first term $\left(\sum_{j=0}^{\infty} a_{1j}\right)^2 \sigma_{\varepsilon 1}^2$ is the variance component to its stock 1 return shock, the second term, $\left(\sum_{i=0}^{\infty} b_i\right)^2 \sigma_{\varepsilon 2}^2$, is the variance component attributable to stock 2 return shock, the third term, $\left(\sum_{j=0}^{\infty} c_{1j}\right)^2 \sigma_{\varepsilon 3}^2$, is the variance component to its stock 1 trade shock and the last term $\left(\sum_{j=0}^{\infty} d_{1j}\right)^2 \sigma_{\varepsilon 4}^2$, is the variance component to stock 2 trade shock.

For this analysis, this chapter assumes prices are relatively more exogenous than trades and, under this assumption, allows two possible cases: one case is a small firm stock (OPNT) is stock1 whose return is relatively more exogenous and the other case is the opposite, i.e. a large firm stock (MSFT) is stock1. However, the empirical data analysis tells the second case is not the case and hence both table 2.3 and table 2.4 report cross effects under the first case. Mostly, information interactions (cross effects) are most active during the first-half (9:30 a.m.-10:00 a.m.) and the last-half (3:30 p.m.-4 p.m.) trading hour. Especially, cross-price (return) effects are most pronounced during the last half-hour of trading in 2008. However, information interactions are dormant during the middle day of trading.

Table 2.3 Ratios of return variance attributable to cross effects for days of strong positive return

a. First-half trading hour (9:30am-10am)

Date	Lags	MSFT Own Returns	MSFT Own Trades	MSFT Cross Returns	MSFT Cross Trades	OPNT Own Returns	OPNT Own Trades	OPNT Cross Returns	OPNT Cross Trades
2000.10.19	5	0.3992	0.0006	0.4833	0.1169	0.4312	0.0226	0.3681	0.1781
2008.10.28	5	0.7040	0.0258	0.0002	0.2700	0.0011	0.5829	0.2245	0.1914
2000.12.22	5	0.0756	0.0180	0.0031	0.9033	0.0014	0.0394	0.9375	0.0217
2008.10.16	5	0.4209	0.0001	0.4097	0.1693	0.6379	0.0509	0.2898	0.0215
2000.12.5	5	0.0002	0.0006	0.1110	0.8883	0.3425	0.3578	0.2764	0.0233
2008.12.16	5	0.0824	0.4654	0.3257	0.1266	0.0040	0.0351	0.9545	0.0065
2000.10.20	5	0.0001	0.0097	0.7808	0.2094	0.7975	0.0267	0.0968	0.0790
2008.10.26	1	0.8049	0.0177	0.1773	0.0000	0.8645	0.0073	0.1282	0.0001
2001.8.24	5	0.0267	0.2897	0.2386	0.4449	0.6961	0.0121	0.2840	0.0078
2001.2.26	1	0.9511	0.0006	0.0156	0.0327	0.8396	0.0411	0.1115	0.0078
2001.1.18	5	0.5736	0.0381	0.1311	0.2572	0.0640	0.3836	0.4055	0.1470
2001.3.23	5	0.4119	0.0013	0.5776	0.0092	0.2550	0.4236	0.1737	0.1476
2001.9.24	5	0.9635	0.0011	0.0001	0.0353	0.5525	0.4207	0.0204	0.0064
2008.6.12	3	0.7181	0.2020	0.0753	0.0045	0.7186	0.0641	0.0127	0.2047
2001.1.11	5	0.4209	0.2484	0.0765	0.2542	0.7964	0.0027	0.1948	0.0062
Average		0.4369	0.0879	**0.2271**	**0.2481**	0.4668	0.1647	**0.2986**	**0.0699**

b. Mid-day trading (10am-3:30pm)

Date	Lags	MSFT Own Returns	MSFT Own Trades	MSFT Cross Returns	MSFT Cross Trades	OPNT Own Returns	OPNT Own Trades	OPNT Cross Returns	OPNT Cross Trades
2000.10.19	7	0.9180	0.0073	0.0536	0.0211	0.6779	0.1075	0.0140	0.2006
2008.10.28	6	0.8958	0.0103	0.0931	0.0008	0.7416	0.0186	0.0088	0.2310
2000.12.22	1	0.9749	0.0100	0.0124	0.0026	0.9640	0.0001	0.0102	0.0257
2008.10.16	5	0.8026	0.1495	0.0477	0.0002	0.9635	0.0028	0.0311	0.0026
2000.12.5	1	0.9940	0.0036	0.0017	0.0008	0.9843	0.0114	0.0023	0.0020
2008.12.16	7	0.0000	0.9997	0.0003	0.0000	0.9798	0.0000	0.0022	0.0180
2000.10.20	6	0.9316	0.0604	0.0064	0.0016	0.9065	0.0002	0.0125	0.0808
2008.10.26	4	0.9860	0.0019	0.0053	0.0067	0.9457	0.0501	0.0018	0.0023
2001.8.24	1	0.9574	0.0037	0.0359	0.0030	0.9664	0.0005	0.0331	0.0000
2001.2.26	1	0.9871	0.0069	0.0045	0.0014	0.9834	0.0144	0.0007	0.0014
2001.1.18	1	0.9744	0.0197	0.0036	0.0023	0.9951	0.0001	0.0001	0.0046
2001.3.23	1	0.9631	0.0014	0.0128	0.0227	0.9938	0.0012	0.0042	0.0008
2001.9.24	4	0.9981	0.0002	0.0014	0.0003	0.9904	0.0001	0.0001	0.0094
2008.6.12	4	0.9867	0.0009	0.0096	0.0028	0.9972	0.0000	0.0014	0.0013
2001.1.11	2	0.9434	0.0002	0.0555	0.0009	0.9437	0.0390	0.0101	0.0072
Average		0.8875	0.0850	**0.0229**	**0.0045**	0.9356	0.0164	**0.0089**	**0.0392**

Table 2.3 continued

c. Last-half trading hour (3:30pm-4pm)

Date	Lags	MSFT Own Returns	MSFT Own Trades	MSFT Cross Returns	MSFT Cross Trades	OPNT Own Returns	OPNT Own Trades	OPNT Cross Returns	OPNT Cross Trades
2000.10.19	1	0.8642	0.0002	0.0089	0.1268	0.9864	0.0052	0.0065	0.0020
2008.10.28	1	0.7679	0.0179	0.1919	0.0222	0.5656	0.0279	0.1929	0.2136
2000.12.22	1	0.9644	0.0014	0.0338	0.0004	0.5267	0.1784	0.0227	0.2722
2008.10.16	5	0.0592	0.0405	0.7410	0.1593	0.0406	0.2560	0.5194	0.1840
2000.12.5	1	0.9244	0.0014	0.0742	0.0000	0.9323	0.0145	0.0293	0.0239
2008.12.16	2	0.0024	0.1167	0.4709	0.4100	0.0331	0.0730	0.8901	0.0038
2000.10.20	1	0.7015	0.0000	0.2059	0.0926	0.8602	0.0149	0.1078	0.0170
2008.10.26	5	0.7556	0.0951	0.0189	0.1304	0.5456	0.2619	0.0022	0.1903
2001.8.24	4	0.4914	0.0872	0.2883	0.1331	0.4899	0.1960	0.0385	0.2755
2001.2.26	1	0.6804	0.2732	0.0024	0.0440	0.9759	0.0178	0.0039	0.0025
2001.1.18	1	0.8195	0.0021	0.1725	0.0059	0.4195	0.2533	0.2636	0.0636
2001.3.23	5	0.3564	0.0227	0.1064	0.5145	0.5782	0.0182	0.2493	0.1544
2001.9.24	1	0.3039	0.0058	0.5155	0.1748	0.4023	0.0194	0.5067	0.0716
2008.6.12	1	0.8122	0.0003	0.1857	0.0019	0.9505	0.0010	0.0016	0.0469
2001.1.11	1	0.9963	0.0024	0.0012	0.0002	0.9998	0.0000	0.0000	0.0002
Average		0.6333	0.0445	**0.2012**	**0.1211**	0.6204	0.0892	**0.1890**	**0.1014**

Table 2.4 Ratios of return variance attributable to cross effects for days of large negative return

a. First-half trading hour (9:30am-10am)

Date	Lags	MSFT Own Returns	MSFT Own Trades	MSFT Cross Returns	MSFT Cross Trades	OPNT Own Returns	OPNT Own Trades	OPNT Cross Returns	OPNT Cross Trades
2000.11.30	1	0.4063	0.0117	0.5254	0.0566	0.6808	0.3143	0.0046	0.0003
2000.12.15	-	-	-	-	-	-	-	-	-
2000.12.20	1	0.9074	0.0121	0.0645	0.0160	0.9414	0.0010	0.0416	0.0160
2008.12. 1	5	0.0001	0.9442	0.0015	0.0543	0.4100	0.1711	0.3467	0.0722
2002. 7.25	5	0.8527	0.0000	0.1436	0.0037	0.7028	0.2473	0.0499	0.0000
2008.10. 7	2	0.7261	0.1553	0.0075	0.1111	0.8182	0.0118	0.0000	0.1700
2000.10.16	5	0.3900	0.2779	0.2173	0.1148	0.0162	0.0014	0.1316	0.8507
2008.10.15	5	0.8361	0.0002	0.0302	0.1336	0.9940	0.0023	0.0009	0.0028
2008.12.11	5	0.0000	0.0096	0.0004	0.9900	0.3083	0.2565	0.4294	0.0058
2008.11.14	5	0.6413	0.0059	0.1428	0.2100	0.1136	0.6735	0.1566	0.0563
2001. 8.30	5	0.3047	0.0918	0.6004	0.0918	0.2238	0.7101	0.0294	0.0367
2008.10.14	5	0.0122	0.2434	0.0632	0.6812	0.8569	0.0576	0.0681	0.0174
2008.9.17	5	0.1499	0.0061	0.2672	0.5768	0.2889	0.3936	0.3175	0.0000
2001.3.21	5	0.0513	0.3893	0.0194	0.5401	0.9076	0.0454	0.0005	0.0465
2002.9.23	5	0.0321	0.3189	0.3425	0.3065	0.2711	0.0448	0.1019	0.5823
2000.10.23	5	0.9303	0.0038	0.0251	0.0408	0.0090	0.2438	0.7461	0.0011
2001.3.2	5	0.7296	0.0033	0.2664	0.0007	0.9285	0.0478	0.0111	0.0127
2000.10.3	5	0.0108	0.0216	0.9442	0.0216	0.0435	0.2209	0.7055	0.0300
2007.2.27	5	0.4186	0.1744	0.3613	0.0457	0.4572	0.5279	0.0141	0.0009
2001.12.20	5	0.0835	0.6970	0.1912	0.0283	0.6662	0.0607	0.0527	0.2203
2008.2.5	5	0.3135	0.0337	0.5719	0.0809	0.3918	0.0010	0.5520	0.0552
Average		0.3898	0.1700	**0.2393**	**0.2009**	0.5015	0.2016	**0.1880**	**0.1089**

Table 2.4 continued

b. Mid-day trading (10am-3:30pm)

Date	Lags	MSFT Own Returns	MSFT Own Trades	MSFT Cross Returns	MSFT Cross Trades	OPNT Own Returns	OPNT Own Trades	OPNT Cross Returns	OPNT Cross Trades
2000.11.30	1	0.9849	0.0063	0.0000	0.0087	0.9855	0.0116	0.0001	0.0028
2000.12.15	2	0.9651	0.0157	0.0045	0.0147	0.9943	0.0039	0.0017	0.0000
2000.12.20	1	0.9837	0.0125	0.0009	0.0028	0.9894	0.0091	0.0007	0.0007
2008.12. 1	3	0.9291	0.0095	0.0559	0.0054	0.9682	0.0061	0.0142	0.0115
2002. 7.25	1	0.9982	0.0002	0.0011	0.0005	0.9983	0.0010	0.0003	0.0004
2008.10. 7	2	0.9568	0.0000	0.0432	0.0000	0.9843	0.0074	0.0041	0.0043
2000.10.16	1	0.9661	0.0242	0.0045	0.0051	0.9815	0.0032	0.0116	0.0037
2008.10.15	1	0.9861	0.0092	0.0037	0.0010	0.9811	0.0095	0.0000	0.0093
2008.12.11	6	0.8603	0.1248	0.0019	0.0130	0.9695	0.0017	0.0149	0.0138
2008.11.14	6	0.8896	0.1087	0.0007	0.0010	0.9882	0.0019	0.0051	0.0049
2001. 8.30	1	0.9848	0.0007	0.0005	0.0141	0.9963	0.0011	0.0000	0.0026
2008.10.14	1	0.9728	0.0074	0.0170	0.0028	0.9257	0.0700	0.0015	0.0027
2008.9.17	1	0.9831	0.0135	0.0023	0.0011	0.9730	0.0000	0.0000	0.0269
2001.3.21	1	0.9743	0.0057	0.0032	0.0168	0.9993	0.0002	0.0004	0.0000
2002.9.23	3	0.8996	0.0014	0.0833	0.0158	0.9307	0.0000	0.0363	0.0330
2000.10.23	1	0.9887	0.0085	0.0018	0.0010	0.9910	0.0026	0.0036	0.0028
2001.3.2	1	0.9543	0.0015	0.0423	0.0019	0.9916	0.0000	0.0065	0.0019
2000.10.3	2	0.9534	0.0003	0.0318	0.0145	0.9679	0.0021	0.0300	0.0001
2007.2.27	1	0.8792	0.0000	0.1204	0.0004	0.9866	0.0003	0.0123	0.0008
2001.12.20	7	0.8221	0.0602	0.0010	0.1167	0.9182	0.0096	0.0079	0.0643
2008.2.5	1	0.9819	0.0021	0.0136	0.0023	0.9905	0.0092	0.0002	0.0001
Average		0.9483	0.0196	**0.0207**	**0.0114**	0.9767	0.0072	**0.0072**	**0.0089**

c. Last-half trading hour (3:30pm-4pm)

Date	Lags	MSFT Own Returns	MSFT Own Trades	MSFT Cross Returns	MSFT Cross Trades	OPNT Own Returns	OPNT Own Trades	OPNT Cross Returns	OPNT Cross Trades
2000.11.30	6	0.8238	0.0008	0.0609	0.1145	0.9614	0.0136	0.0189	0.0061
2000.12.15	2	0.2935	0.0415	0.6304	0.0346	0.8939	0.0084	0.0972	0.0005
2000.12.20	1	0.5241	0.0001	0.4629	0.0129	0.9444	0.0048	0.0236	0.0272
2008.12. 1	6	0.0001	0.9948	0.0030	0.0021	0.0000	0.0033	0.0000	0.9966
2002. 7.25	2	0.6032	0.0858	0.2316	0.0794	0.6300	0.0014	0.3369	0.0316
2008.10. 7	1	0.7574	0.0061	0.0596	0.1769	0.9965	0.0007	0.0019	0.0010
2000.10.16	2	0.5467	0.0422	0.4108	0.0003	0.8821	0.0336	0.0384	0.0459
2008.10.15	5	0.7291	0.0880	0.1795	0.0034	0.5530	0.0071	0.1003	0.3397
2008.12.11	1	0.9435	0.0185	0.0324	0.0056	0.7378	0.1244	0.0041	0.1338
2008.11.14	1	0.2231	0.7715	0.0007	0.0047	0.4088	0.0203	0.3466	0.2243
2001. 8.30	2	0.9622	0.0114	0.0052	0.0212	0.9690	0.0007	0.0282	0.0021
2008.10.14	2	0.9512	0.0040	0.0018	0.0431	0.9224	0.0229	0.0218	0.0329
2008.9.17	1	0.9609	0.0198	0.0193	0.0000	0.8035	0.1607	0.0000	0.0358
2001.3.21	1	0.8147	0.0004	0.1403	0.0447	0.8726	0.0153	0.0468	0.0653
2002.9.23	1	0.9688	0.0101	0.0172	0.0038	0.9632	0.0255	0.0002	0.0111
2000.10.23	1	0.9582	0.0238	0.0174	0.0006	0.9736	0.0071	0.0081	0.0112
2001.3.2	-	-	-	-	-	-	-	-	-
2000.10.3	1	0.8315	0.0526	0.0062	0.1098	0.9530	0.0008	0.0429	0.0032
2007.2.27	1	0.5067	0.3073	0.1187	0.0673	0.7011	0.0363	0.2307	0.0319
2001.12.20	1	0.9292	0.0597	0.0104	0.0007	0.9783	0.0024	0.0169	0.0025
2008.2.5	3	0.4623	0.4822	0.0378	0.0178	0.4536	0.1493	0.2229	0.1742
Average		0.6895	0.1510	**0.1223**	**0.0372**	0.7799	0.0319	**0.0793**	**0.1088**

3. The efficiency of price

This section examines how trades make impacts on the efficiency of price. Biais, Hillion and Spatt (1999) propose a regression ("Unbiasedness regressions") method to test the efficiency of stock prices by inferring signal to noise ratio. The model is briefly touched as following:

Model

$$ret_{cc} = \alpha + \beta ret_{ci} + \varepsilon_i \qquad (2.6)$$

where ret_{cc} is the close-to-close return and ret_{ci} is the return from the close to the end of time period i. In this model, the slope β can be considered as the signal to noise ratio, since if stock returns are serially uncorrelated and measured without error, then the slope of the unbiased regression would be equal to one. However, we can not observe the true return process and the observed return includes noise. Therefore, the observed close-to-close return is represented as $ret_{cc} = ret^*_{cc} + v$, where ret^*_{cc} denotes the true close-to-close return and v is noise. In a similar way, the observed close-to-the end of the time period i return is $ret_{ci} = ret^*_{ci} + u$, where ret^*_{ci} is the true close-to-the end of the time period i return and u is noise. Both u and v are zero mean and variances are σ^2_u and σ^2_v, respectively. By using simple ordinary least square estimation, a slope estimates b is obtained where

$$p\lim b = \beta \left(\frac{\sigma^2_{ret^*_{ci}}}{\sigma^2_{ret^*_{ci}} + \sigma^2_u} \right). \qquad (2.7)$$

The term $\left(\frac{\sigma^2_{ret^*_{ci}}}{\sigma^2_{ret^*_{ci}} + \sigma^2_u} \right)$ can be interpreted as the signal to noise ratio, $\sigma^2_{ret^*_{ci}}$ measures the information discovered from the close to the end of the time period i, σ^2_u is the noise in the price of the period.

3.1 Opening price efficiency

To begin with, this section examines how the efficiency of opening price evolves during our sample period (8/2/2000~12/31/2008) with MSFT and OPNT trade data. To test the efficiency of opening prices, the close-to-close return (ret_{cc}) is regressed on the close-to-open return (ret_{co}). Figure 2.1 shows the unbiasedness slope coefficient and average trading volume by year. The efficiency of MSFT opening prices has been close to one and stable during the sample period. The efficiency of OPNT opening prices has been less than one most of the period and it fluctuates as the average trading volume varies. This provides evidence that trading activity plays important role in price discovery. However, it is surprising an increase in average trading volume can improve opening price efficiency even without pre-open trading in case of less active small cap stock (OPNT).

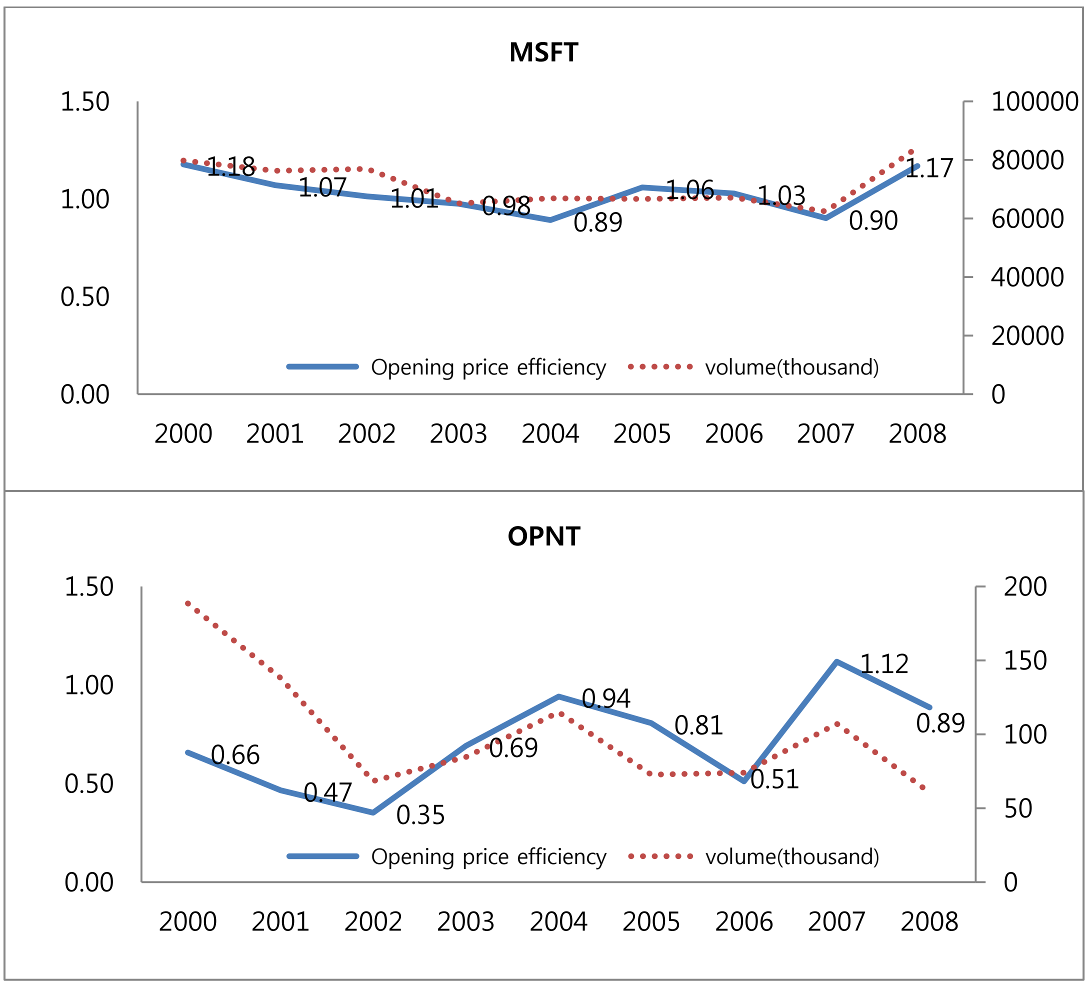


Figure 2.1 The efficiency of opening price and average trading volume by year (MSFT, OPNT)

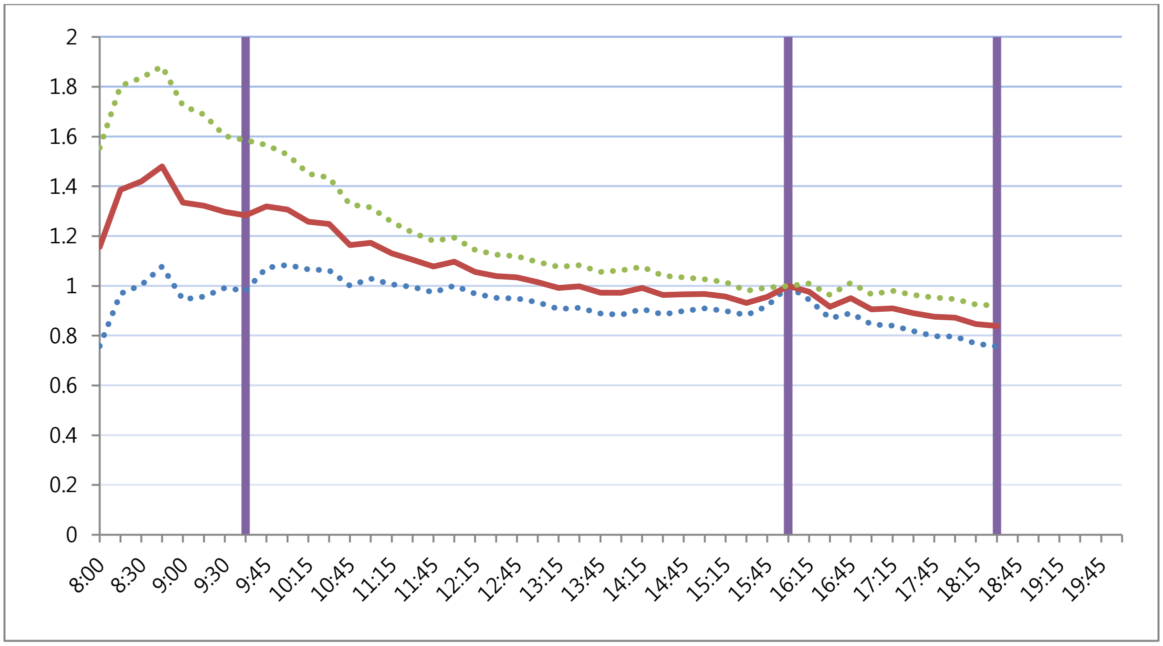
3.2 Intraday price efficiency

Trades in the pre-open are more informative than trades in the post-close. Post-close trades are large but mostly presumed as liquidity motivated since the amount of information revealed is low. To examine how price efficiency evolves over a day, especially for those highly volatile periods (2000:4Q and 2008:4Q), "unbiasedness regressions" are done for each 15-minute intervals with MSFT trade prices. The close-to-close return (ret_{cc}) is regressed on the return from close to the end of time period i (ret_{ci}). Figure 2.2 shows intraday price efficiency (the slope coefficient estimates) with confidence intervals which are calculated using the standard errors of the slope coefficient estimates.

During the fourth quarter of 2000, the signal to noise ratio is higher than one in the pre-open and it starts to decrease and touches one around 1 pm. Then, it stays around one until the close and dwindles as the post-close progresses. During the pre-open, though confidence interval is wider than other time period, the lower bound moves around one. During the trading hours, confidence band continues to shrinks and the lower bound moves around one (from 0.9 to 1.1) most of the time. Post-close price efficiency declines after the close and this shows after-hours stock prices may not track with stock's closing price.

However, the fourth quarter of 2008 shows somewhat different intraday price efficiency pattern. The slope coefficient is slightly higher than one during the pre-open and records an intraday high at 10 a.m. and falls and rises during the trading hours and stays one after the close. It is interesting, again the slope estimate most closely approaches one around 1pm. Overall, confidence interval is much wider as compared to that of 2000. The lower bound ranges from 0.55 to 0.80 during the pre-open and is much less than one most of the trading hours. However, during the post-close and overnight, the slope stays one which shows after-hours stock prices closely track with stock's closing price.

a. 4^{th} quarter 2000



b. 4^{th} quarter 2008

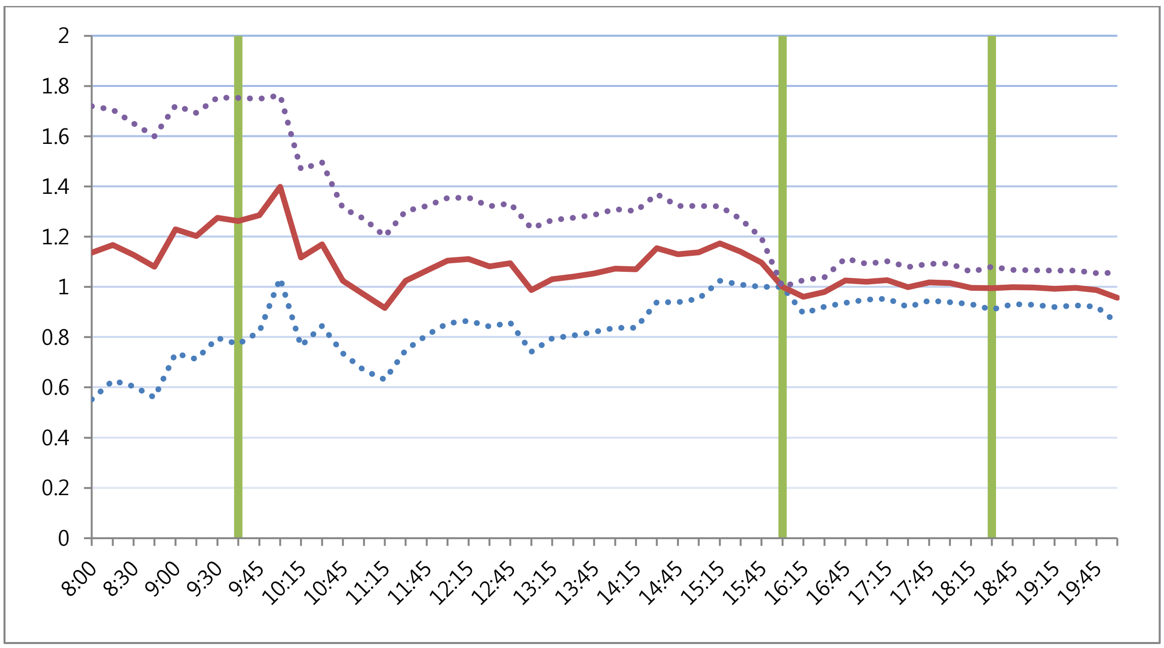
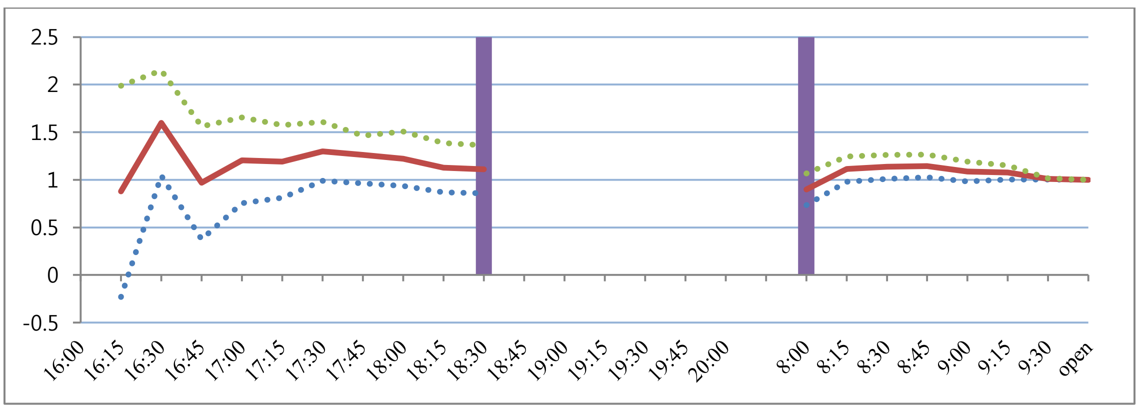


Figure 2.2 Intraday price efficiency on the closing price by time period (MSFT)

3.3 After-hours[6] and pre-open price efficiency on the opening price

This section investigates how after-hours and pre-open prices closely mirror the following open prices with MSFT trade data. To test the after-close price efficiency, the close-to-open return (ret_{co}) is regressed on the return from close-to-the after-close time period i (ret_{ci}). Figure 2.3 shows the after-hours price efficiency with confidence intervals. During the fourth quarter 2000, the efficiency estimate stays a bit above one most of the post-close time period. Meanwhile, during the fourth quarter 2008, the efficiency estimate is much lower, ranges from 0.4 to 0.8. It increases slowly during the post-close and starts to fluctuate during the overnight and this implies portfolio or inventory motives for trades are greater during this period. As time approaches the beginning of the pre-open, the efficiency estimates jumps to the point which is very close to one. As information accumulates during the overnight, a relatively higher probability of informed traders is expected during the pre-open. Overall, after-hours price efficiency on the opening price dropped during the fourth quarter 2008.

a. 4th quarter 2000



b. 4th quarter 2008

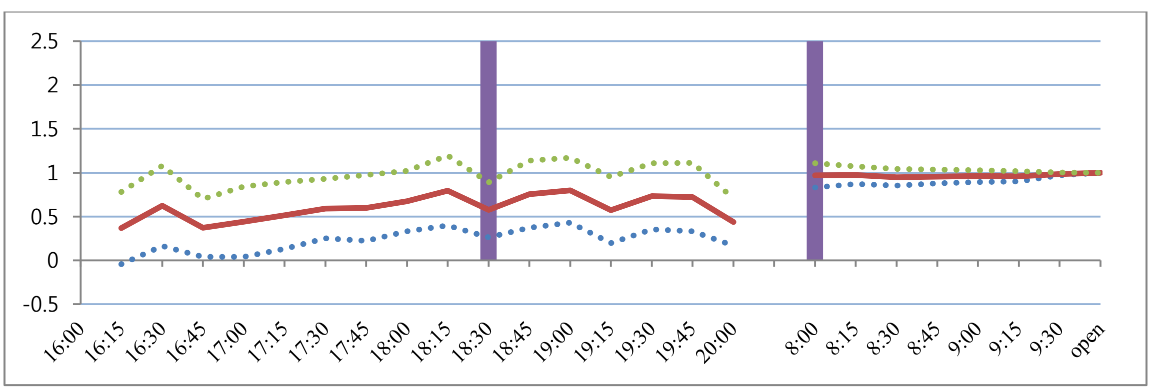


Figure 2.3 After-hours price efficiency on the opening price (MSFT)

[6] After-close hours include post-close (4:00 p.m.~6:30 p.m.), overnight (6:30 p.m.~8:00 a.m.), and pre-open(8:00 a.m.~9:30a.m.)

3.4 Trading volume and the efficiency of price discovery

Barclay et al. (1990) show that a significant amount of trading volume is required before private information is revealed through trading. To some extent, trading volume may have positive effect on the efficiency of price discovery. Previous analysis with MSFT trade data shows overall efficiency of price discovery is lower during the fourth quarter 2008 as compared to the fourth quarter 2000.

Table 2.5 summarizes MSFT after-hours trading for each period. In absolute terms, fourth quarter 2008 reports twice as much total trading volume and seven times as much total number of trades per day as fourth quarter 2000 did. However, pre-open trades declines significantly both in absolute and relative terms. During the pre-opens, trading volume falls from 1.20% to 0.22% and number of trades declines from 0.80% to 0.12%. Post-close trading volume increases a little bit from 4.25% to 4.75% though number of trades declines significantly from 1.26% to 0.16%.

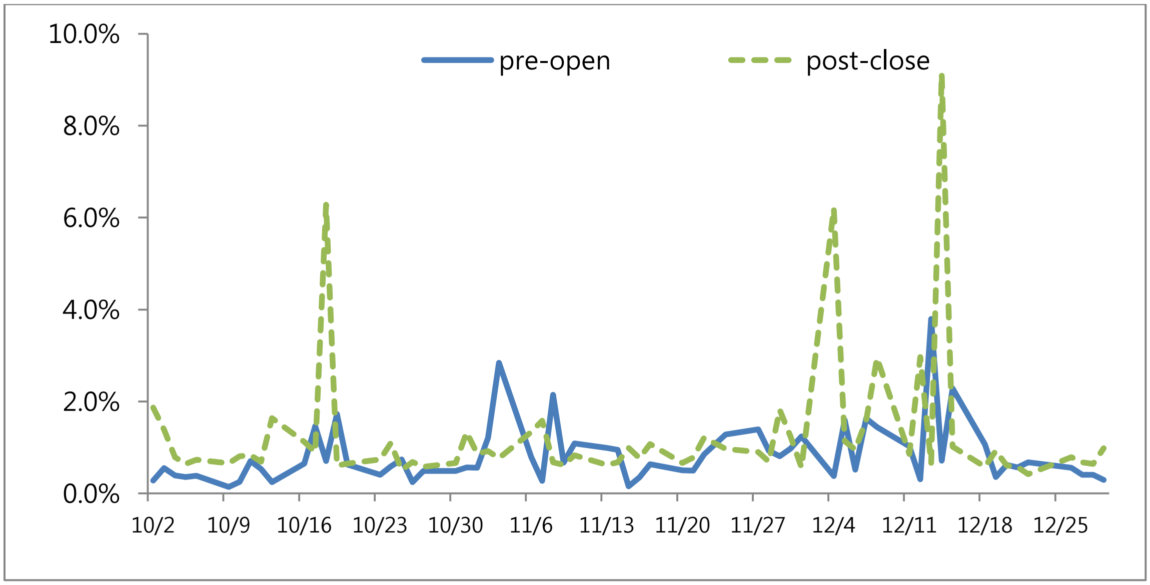
Table 2.5 MSFT After-hours trading: Average trading volume and average number of trades per day

	Pre-open (8:00a.m.~9:30a.m.)		Post-close (4p.m.~6:30p.m.)		Trading day	
	Trading volume (thousand)	Number of trades	Trading volume (thousand)	Number of Trades	Trading volume (thousand)	Number of trades
2000:4Q	689.2(1.20%)	416.1(0.80%)	2246.8(4.25%)	576.6(1.26%)	50375.9(94.5%)	45484.1(97.8%)
2008:4Q	254.9(0.22%)	376.3(0.12%)	4283.8(4.75%)	583.8(0.16%)	95365.4(94.9%)	313769.6(99.7%)

Figure 2.4 also presents daily after-hours trading proportion in terms of trade volume and number of trades. A comparison of two periods shows a trade shift from pre-open to trading day during fourth quarter 2008. Usually, trade shift from trading day to pre-open is more likely happen when liquidity and trading costs in the pre-open are closer to those during the trading day. However, a backward shift (i.e. from pre-open to trading day) in price discovery is observed during fourth quarter of 2008 and weakened trading activity over the latter period might have brought this backward shift since pre-open trading cost can rise with weakened trading activity. Bid-ask spreads[7] are used as a proxy for transaction costs and an increase in transaction cost from 0.99% to 1.31% (of the mid-quotes) supports the literature.

[7] Bid-ask spread is a positive function of price level and return variance, a negative function of measures of market activity, depth, and continuity, and negatively correlated with the degree of competition(Copeland and Galai (1983)).

a. Number of trades



b. Trade volume

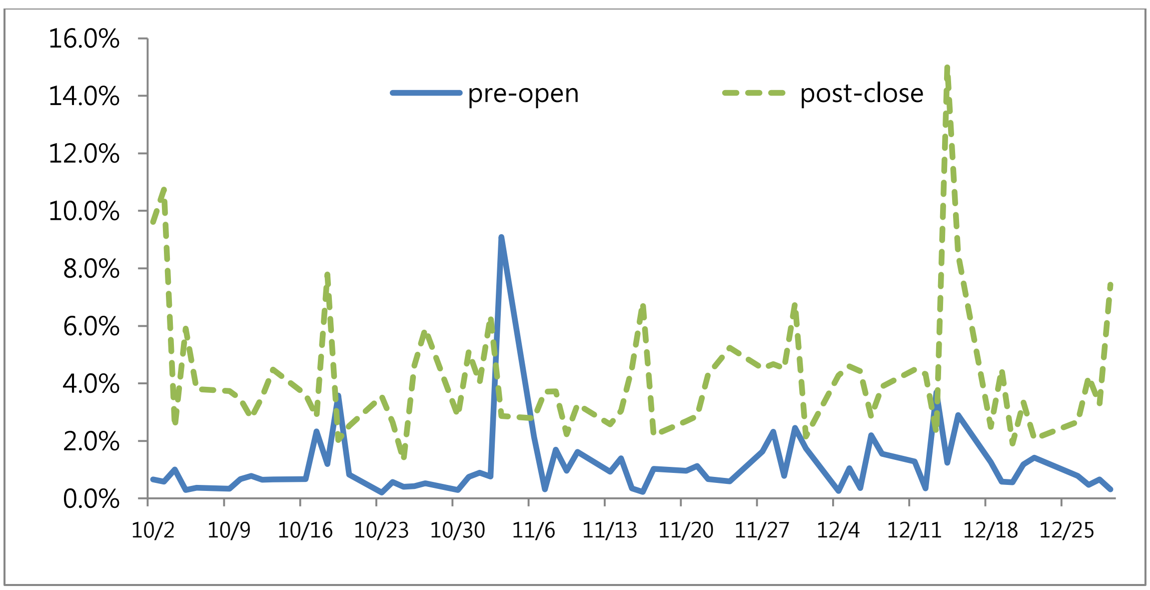
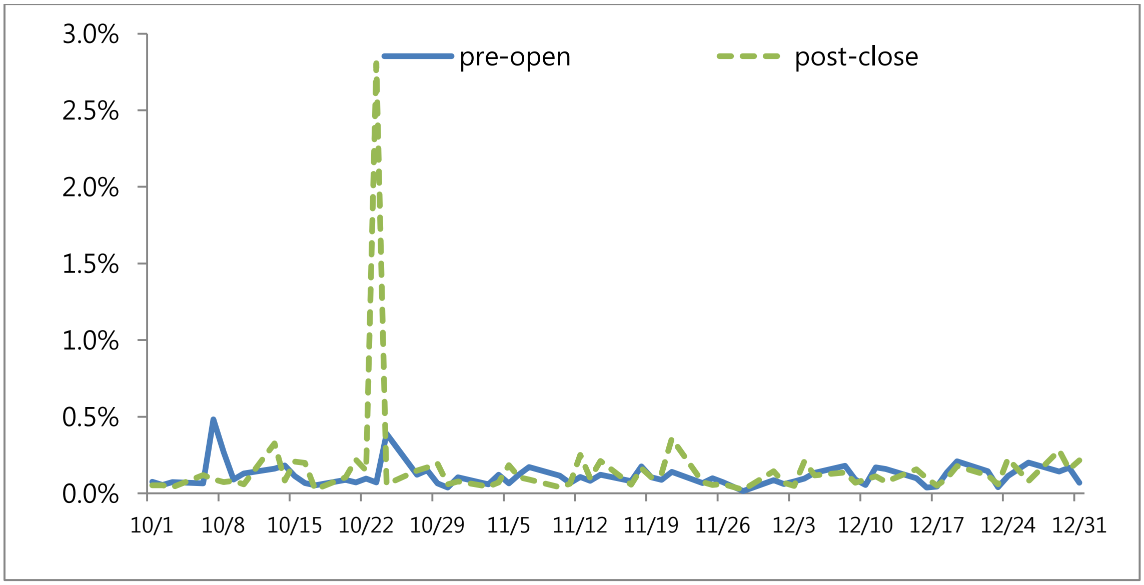


Figure 2.4. MSFT after-hours trades and trading volume during the fourth quarter 2000

a. Number of trades



b. Trade volume

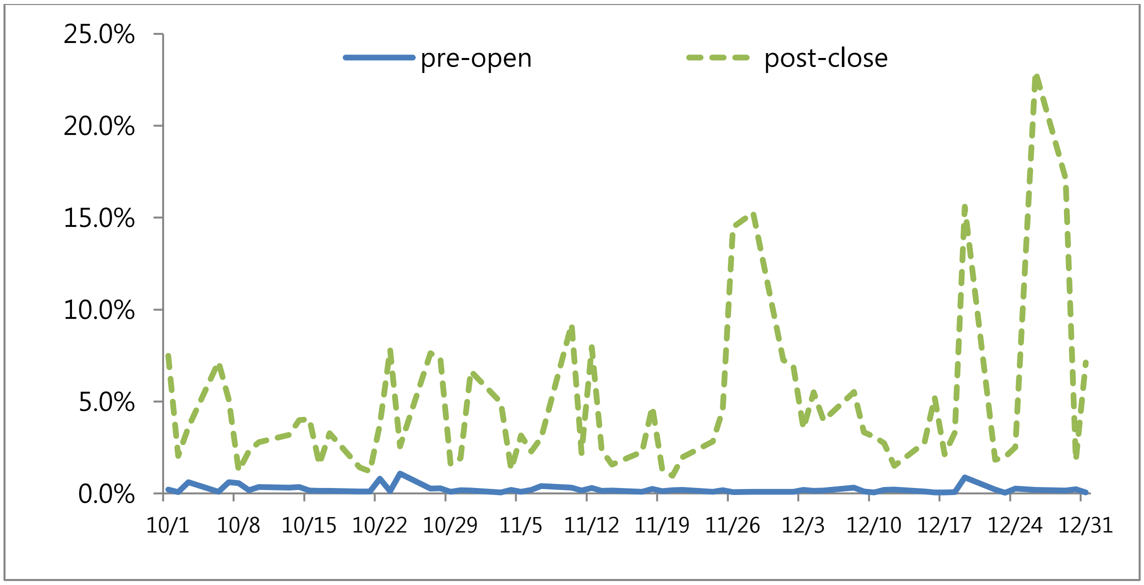


Figure 2.5. MSFT after-hours trades and trading volume during the fourth quarter 2008

4. Concluding remarks

This chapter investigates price discovery and information revealing pattern in times of volatile U.S. stock market by market capitalization. Contrary to our general belief, information asymmetry is more clearly observed in a large-cap stock (MSFT) than in a small-cap stock (OPNT) in volatile markets. Information interactions (cross effects) between two stocks are most pronounced during the first and the last half-hour of trading and dormant during the middle of trading hours. When it comes to price efficiency, price efficiency is improved with an increase in trading volume. Especially, opening price efficiency is closely related to pre-open trading volume and weakened trading activity can lower the efficiency of price discovery because of larger trading cost as examined. However, it is encouraging that when pre-open trading is not active like OPNT (a small-cap stock), an increase in trading-hour trading volume can improve the efficiency of opening-price. A large-cap stock like MSFT could be attractive in the sense that price discovery is more efficient and shows relatively high trade-informativeness (backed up with enough trading volume) in volatile markets when uncertainty level is expected to be high.

Chapter III

Do the higher moments provide better downside protections in tactical asset allocation?

1. Introduction

The ultimate goal of a tactical asset allocation strategy is to achieve acceptable risk-adjusted returns, i.e. improve the risk-adjusted returns of passive management investing. One of the common risk measures is the volatility of the portfolio and this can be calculated by various methods. Typically, investors use standard deviation and beta to measure the risk. However, the maximum dropdown could be an alternative risk measure and this chapter suggests a method to reduce this dropdown in tactical asset allocation in an attempt to improve risk-adjusted returns.

Tactical asset allocation (TAA) is an active management strategy that rebalances a portfolio's asset allocation depending on the market situation. In this strategy, an investor adjusts his/her asset allocation based on the valuations of the market in which he/she invested. For example, we can consider a simple portfolio consisting of two assets. When an investor expects one asset will outperform, he may increase his position toward the outperforming asset. Tactical asset allocation involves judgments on the future returns of assets and thus it is important to identify time periods when an asset generally outperformed or underperformed. To identify these regimes, this chapter applies a Markov switching model to the spread returns between two opposing assets, small cap stocks and large cap stocks. In a next step, a dynamic ordered probit model (Dueker, 2005) is adopted to forecast asset returns to implement a tactical asset allocation between large cap stocks and small cap stocks on a continual basis as market conditions evolve. The idea is that if an investor has some information or perception about how deeply the market embedded in the current regime, it would be helpful to get tactical tilts in asset allocation. Also, as reported in the first chapter, large changes in skewness and kurtosis are strongly correlated to regime shifts in the U.S. stock market. Therefore, it is meaningful to incorporate these higher moments in predicting the future asset returns. Furthermore, even within a regime, we do see relatively moderate changes in skewness and kurtosis. This is not surprising since we expect a fair bit of heterogeneity in asset returns in each regime.

If these changes within a regime do signal gradual shift to another regime, this is very encouraging since it will help us to get better prediction of asset returns even within a regime. Therefore, this chapter investigates whether the higher moments improve the explanatory power of the model and provide better downside protections. Assuming that investors have preferences over skewness and kurtosis of asset returns, it is natural to incorporate these preferences in asset allocation. Also, since changes in skewness and kurtosis of asset returns reflect changes in market conditions, it is expected to get better performances as higher moments are incorporated in estimating asset returns.

2. Regime Specification and Asset Return Forecasts

2.1 Data

This chapter aims to discern regime probabilities and get equity tilts toward outperforming stocks in a portfolio consists of two opposing assets, small cap stocks and large cap stocks which commonly listed on NYSE/AMEX/NASDAQ. For this analysis, monthly data are provided by Center for Research in Stock Prices (CRSP) and the sample period covers 1995:01~2008:12, a total of 168 observations. Based on market capitalization, the first size-sorted CRSP decile portfolio is used for small cap stocks and the decile 10 portfolio is used for large cap stocks.

2.2 Model

Regime Specification: Mean-Variance Markov switching model

As a first step to specify regimes, spread returns are constructed to measure the return performance of small cap stocks relative to large cap stocks. The spread return is positive when the small cap stocks return exceeds the large cap stocks return. In general, small-caps are expected to outperform large-caps in bull markets and thus the spread returns can imply the degree of market bullishness. The model for regime specification is a mean-variance Markov switching model

$$X_t = \mu_{s_t} + \eta_t \qquad (3.1)$$

where X_t is spread return and $\eta_t \sim N(0, \sigma^2_{S_t})$. The state variable S_t follows a k-state Markov process with constant transition probability matrix, P

$$P[i, j] = \Pr(s_t = j \mid s_{t-1} = i) = p_{ij}, \qquad i, j = 1, ..., k . \qquad (3.2)$$

Asset Return forecast: Dynamic Ordered Probit Model (DOPM)

The model to forecast asset returns is a dynamic ordered probit model (Dueker, 2005). Ordered probit model can be considered as a generalized version of probit model since it allows dependent variable to take more than two outcomes in an ordered fashion from lowest to highest. Suppose that we are trying to find relationship between the spread return and exact market bullishness in an ordered probit model, then the relationship can be characterized as

$$y^*_{t+1} = Z_t\beta + \varepsilon_{t+1} \qquad (3.3)$$

where y^* is the exact but unobserved (latent) dependent variable (the exact level of market bullishness), Z is the vector of independent variables (including spread return and possibly other explanatory variables which are assumed to influence the outcome of y), β is the vector of regression coefficients which to be estimated and $\varepsilon_{t+1} \sim N(0,1)$.

A dynamic ordered probit model has a serially correlated measure of dependent variable in addition to measures of a set of explanatory variables. In a framework of a dynamic ordered probit model, the relationship between the spread return and exact market bullishness can be mapped as

$$y^*_{t+1} = \rho y^*_t + Z_t\beta + \varepsilon_{t+1}. \qquad (3.4)$$

Since we cannot observe y^*, instead we can only observe the categories of outcomes:

$$y = \begin{cases} 0 \quad if \quad y^* \le 0, \\ 1 \quad if \quad 0 < y^* \le \mu_1, \\ 2 \quad if \quad \mu_1 < y^* \le \mu_2, \\ \vdots \\ N \quad if \quad \mu_{N-1} < y^* \end{cases} \qquad (3.5)$$

where the constants $\mu_1, \cdots, \mu_{N-1}$ are cut-off coefficients that separate the categories in an ordered fashion.

In this chapter, the specified regimes through the Markov switching model (3.1) are used as the observations on y which can be served as categories in which an unobserved variable of asset market bullishness (y^*) falls into those categories. For explanatory variables, some macro-economic variables are considered such as business cycle index (BCI), the growth rate of total nonfarm employees, the growth rate of real personal consumption expenditures, the spread between 3-Month London Interbank Offered Rate based on U.S. Dollar (LIBOR) and 3-month Treasury bill, and the spread between Moody's Seasoned BAA corporate bond yield and the 10-year Treasury constant maturity rate. The BCI data is provided by Russell Investments and all other data are available on Federal Reserve Economic Data (FRED).

In a dynamic ordered probit model, regime probabilities depend on not only past values of the explanatory variables but the degree of regime embeddedness inherited from the past random shocks as well. Thererfore, past shocks are remembered and affect the regime forecasts going forward and allows us to discern how deeply embedded we are in the current regime.

2.3 Improving Downside Protection: Timing Market Turns with higher moments

The main difference between tactical asset allocation and strategic asset allocation is the frequency in which the portfolios are rebalanced. When market conditions mandate a more active approach like tactical asset allocation, investors are proactive in adjusting investments to current market conditions without constraining themselves to certain time lines such as quarterly or annually adjusting. Therefore, timing market turns is important in tactical asset allocation. To improve this ability, this chapter incorporates higher moments of spread returns to predict market bullishness in the future. Since large changes in skewness and kurtosis imply regime shifts, when these moments are utilized in predicting asset returns, these can provide better downside protection while allowing the investor's portfolio run for the upside.

The moments of Markov switching model are calculated the same way as presented the first paper. The mean spread returns under regime switching can be computed as the weighted average of regime dependent mean returns

$$\bar{X}_{t+1} = E[X_{t+1} \mid I_t] = \sum_{l=1}^{k} (\pi_t P e_l) \mu_l \qquad (3.6)$$

where π_t is the vector of state probabilities, e_l is a vector of zeros with a one in the l^{th} position so $(\pi_t' P e_l)$ is the ex-ante probability of being in state l at time $t+1$ given information at time t and μ_l is a mean return in state l. Then the other moments are calculated as following:

$$Var_t[X_{t+1}] = \sum_{l=1}^{k} (\pi_t' P e_l)[\mu_l - \bar{X}_{t+1}]^2 + \sum_{l=1}^{k} (\pi_t' P e_l) Var[\eta_{t+1} \mid S_{t+1} = l]$$

$$Skew_t[X_{t+1}] = \sum_{l=1}^{k} (\pi_t' P e_l)[\mu_l - \bar{X}_{t+1}]^3 + 3\sum_{l=1}^{k} (\pi_t' P e_l)[\mu_l - \bar{X}_{t+1}] Var[\eta_{t+1} \mid S_{t+1} = l]$$

$$Kurt_t[X_{t+1}] = \sum_{l=1}^{k} (\pi_t' P e_l)[\mu_l - \bar{X}_{t+1}]^4 + 6\sum_{l=1}^{k} (\pi_t' P e_l)[\mu_l - \bar{X}_{t+1}]^2 Var[\eta_{t+1} \mid S_{t+1} = l] \ . \qquad (3.7)$$

The asset return forecasting model with higher moments can be written as

$$y_{t+1}^* = \rho y_t^* + Z_t \beta + \gamma S_t[X_{t+1}] + \delta K_t[X_{t+1}] + \varepsilon_{t+1} \qquad (3.8)$$

where $S_t[X_{t+1}] = Skew_t[X_{t+1}] / (Var_t[X_{t+1}])^{1.5}$, $K_t[X_{t+1}] = Kurt_t[X_{t+1}] / (Var_t[X_{t+1}])^2$,

and $\varepsilon_{t+1} \sim N(0,1)$.

3. Estimation Results

3.1 Market Regime Specification – Bear, Neutral, Bull

As a first step in tactical asset allocation, we specified regimes based on the spread return which is positive when small-caps return exceeds large-caps return. The degree of market bullishness can be reflected in spread returns since bull markets are typically good for small-caps relative to large-caps. The observed spread returns fall into three categories which characterized as bear, neutral and bull market depending on asset market bullishness. Table 3.1 presents the parameter estimates of a three-state regime switching model. Figure 3.1 shows plots of spread returns, regime probabilities for bear, neutral and bull states.

Tabel 3.1 Parameter estimates of a 3-state mean-variance regime-switching model

	Bear state	Neutral state	Bull state
1. Mean Spread Return(%)	-0.2769	0.7409	0.9801
	(0.4974)	(1.1968)	(0.3853)
2. Variance	9.0637	40.4117	501.6206
	(1.7698)	(6.8331)	(278.8633)
3. Transition Probabilities			
Bear	0.9597	0.0403	0.0000
Neutral	0.0000	0.9602	0.0398
Bull	0.4018	0.0000	0.5982

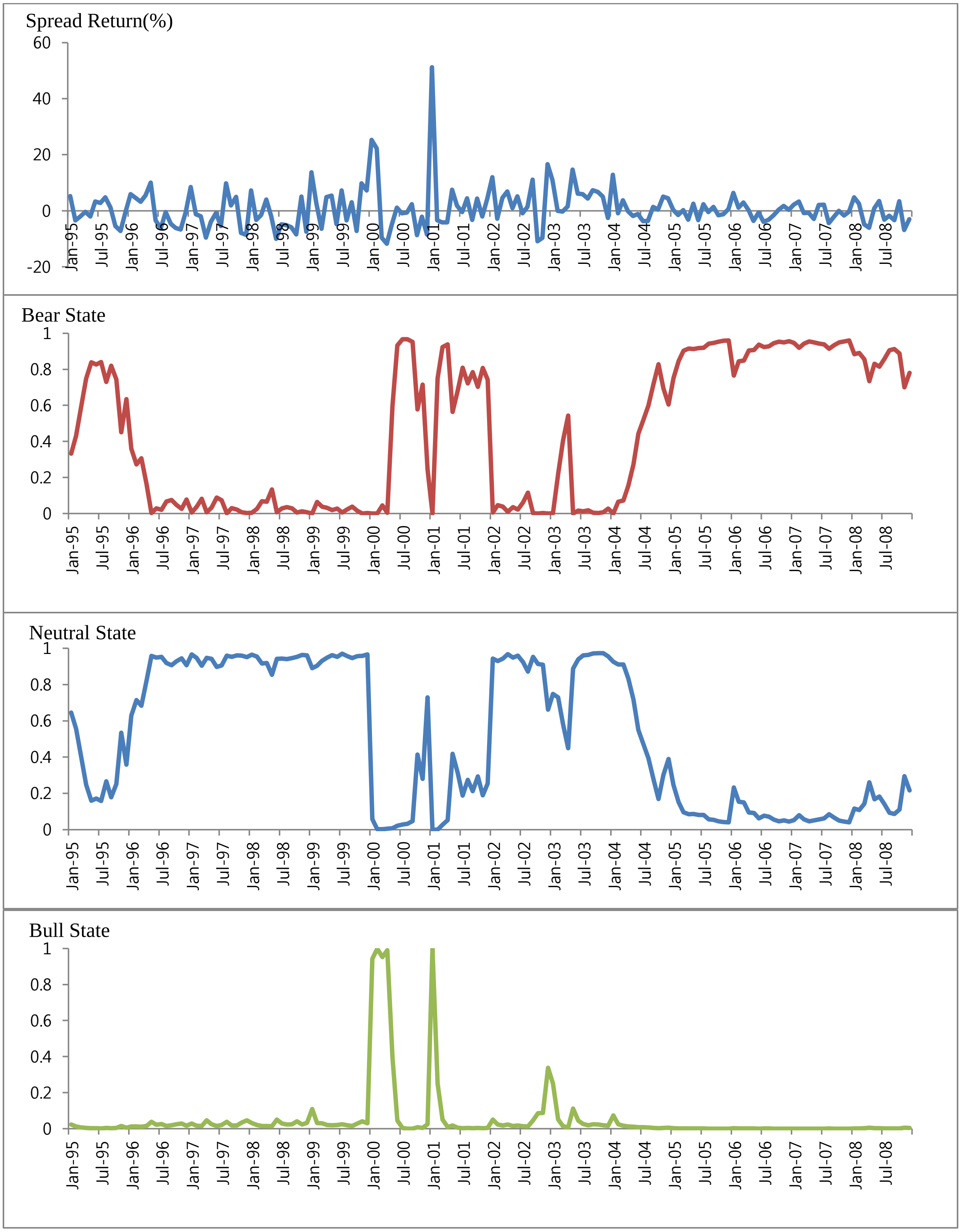


Figure 3.1 Spread return (small/large caps) and state probabilities

3.2 Return Forecasts and Tactical Tilts

The dynamic ordered probit is useful in tactical asset allocation since the level of unobserved (latent) variable can play a role in signaling the degree to which an asset return is in a bull, bear and neutral regime. Also, gradual regime change implied by latent variable can lead investors to re-allocate their portfolios on a continual basis as market conditions evolve. As an example, we can look at dynamic re-allocation, based on signals from a dynamic ordered probit model, across two opposing assets (small-caps and large-caps). It is a stylized fact, based on historical performances, that small-cap stocks often perform well in bull markets and large cap stocks tend to outperform in bear markets. To demonstrate the general correctness of the model's signals, this chapter measures tilt profits relative to formation of a 40-60 static allocation between small-cap stocks and large-cap stocks which is a steady-state optimal allocation under skew-kurtosis preferences (table 1.7, p27). If an investor has some information (or perception) about the upcoming state of the market through the level of latent variable, he/she can utilize this information to re-allocate assets across small-cap stocks and large-cap stocks. For example, when the spread return is large enough to signal bull market, the investor would tilt toward small-cap stocks exposure.

To get equity factor tilts between small-caps and large-caps, the specified regimes with Markov switching model are used as the observations on y which can be served as categories in which an unobserved variable of market bullishness (y^*) falls into those categories. Figure 3.2 illustrates a Bull-Bear-Neutral classification for the spread return between small-caps and large-caps. Second, values of latent variable are estimated for this classification through the dynamic ordered probit model. As explanatory variables, we considered a set of macroeconomic variables such as business cycle index, Figure 3.3 shows market bullishness (the estimates of latent variable) implied by the spread returns. Figure 3.4 indicates how well the latent variable captures the dynamics of the spread returns between small-caps and large-caps. Third, to get equity tilts, I ran the model with 60 vintages to get real-time forecasts of the latent variable and ranked these 60 (monthly) forecasts into quantiles. To determine the relative standing of each forecast in a population of 60 forecasts, a percentile ranking system is considered. The percentile ranks of the values of latent forecasts are used to get tilts against a 40-60 static allocation:

Tilt toward small-cap stocks (small-cap stocks allocation in excess of 40%) = q - 0.4,

where q is the percentile of the forecast.

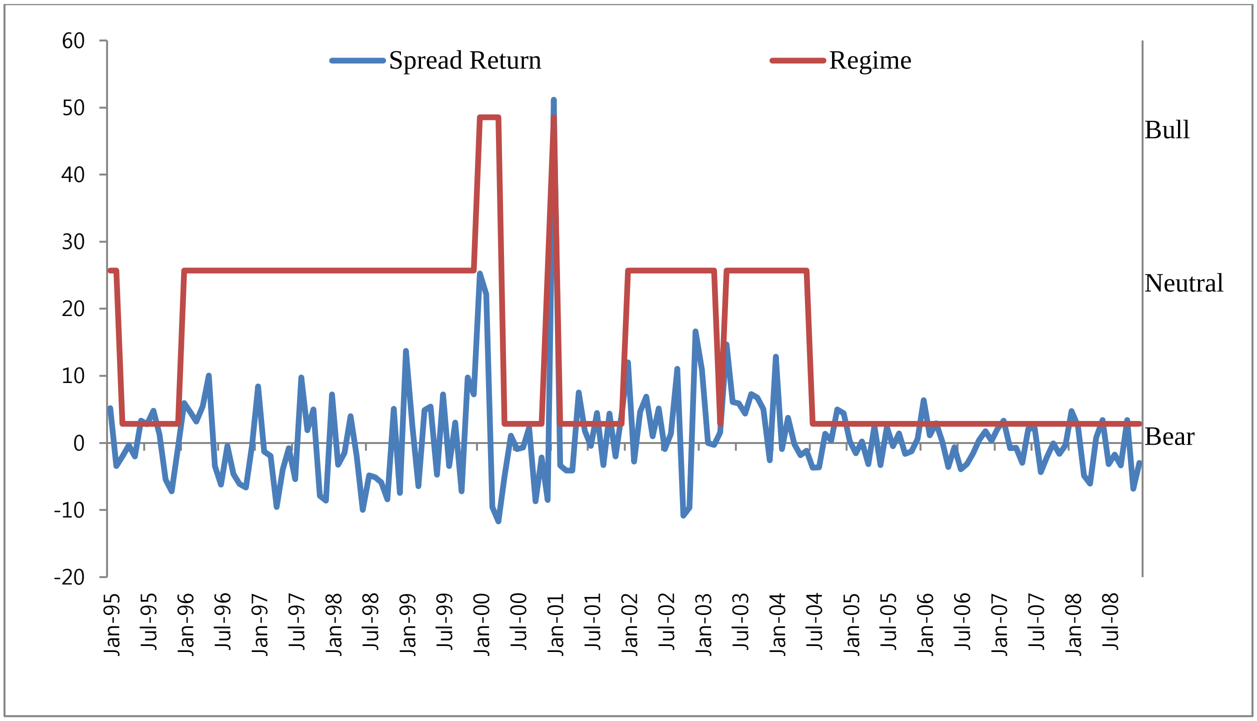


Figure 3.2 Spread return with regime classification

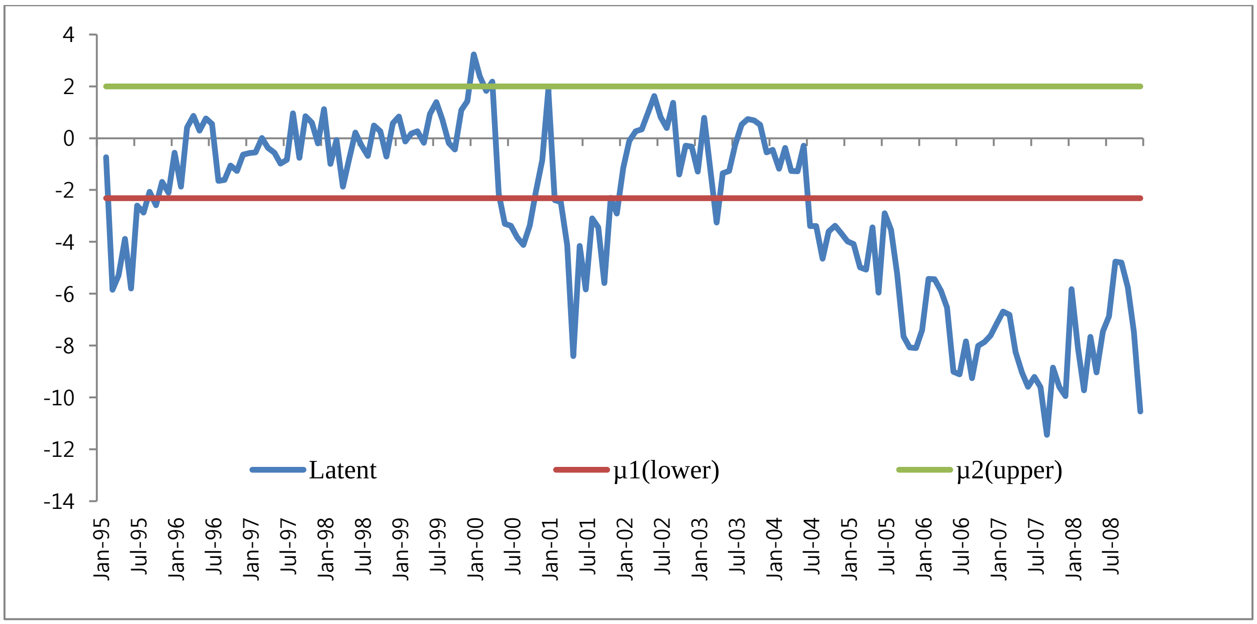


Figure 3.3 Market bullishness (latent: y^*) implied by spread returns with cut-off coefficients

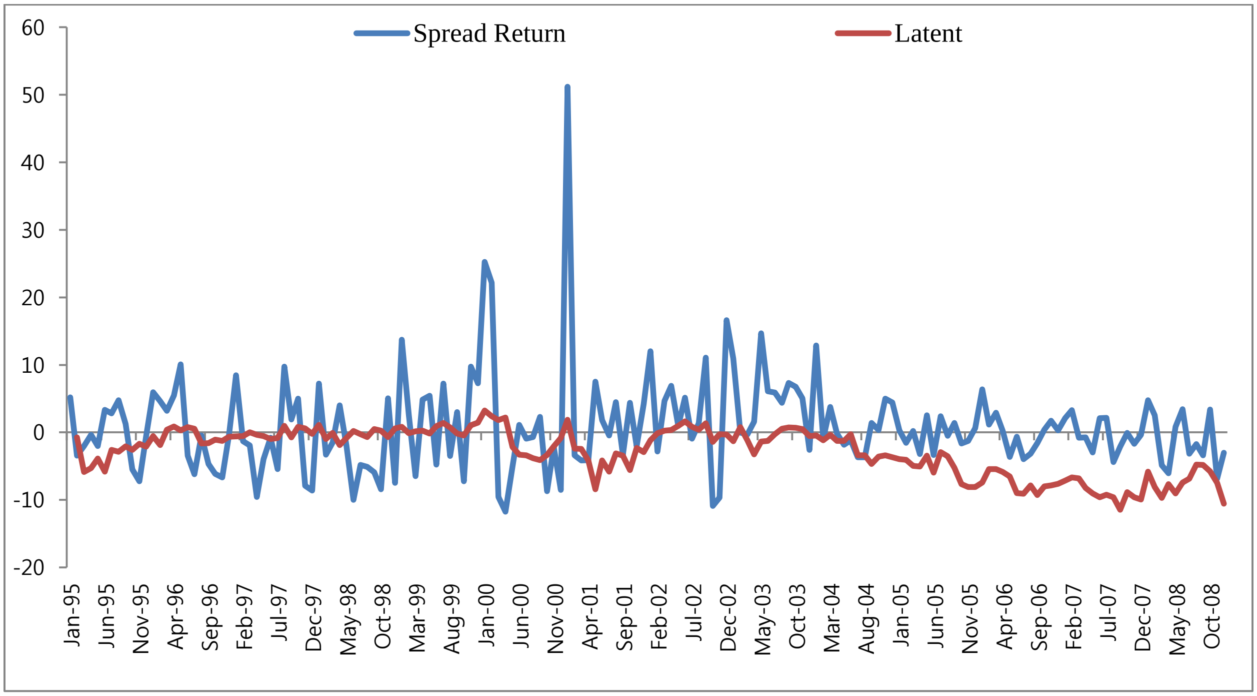


Figure 3.4 Spread return (small/large caps) vs. latent (y^*) variable

Figure 3.5 presents dynamic tilts and subsequent spread returns across the population of 60 vintage forecasts. Figure 3.6 presents cumulative excess returns (relative to the 40-60 static benchmark) with dynamic tilts toward outperforming equity for 60 months of back-test period. In general, the cumulative excess returns slope upward, indicating that the model is leading to a tilt in the right direction.

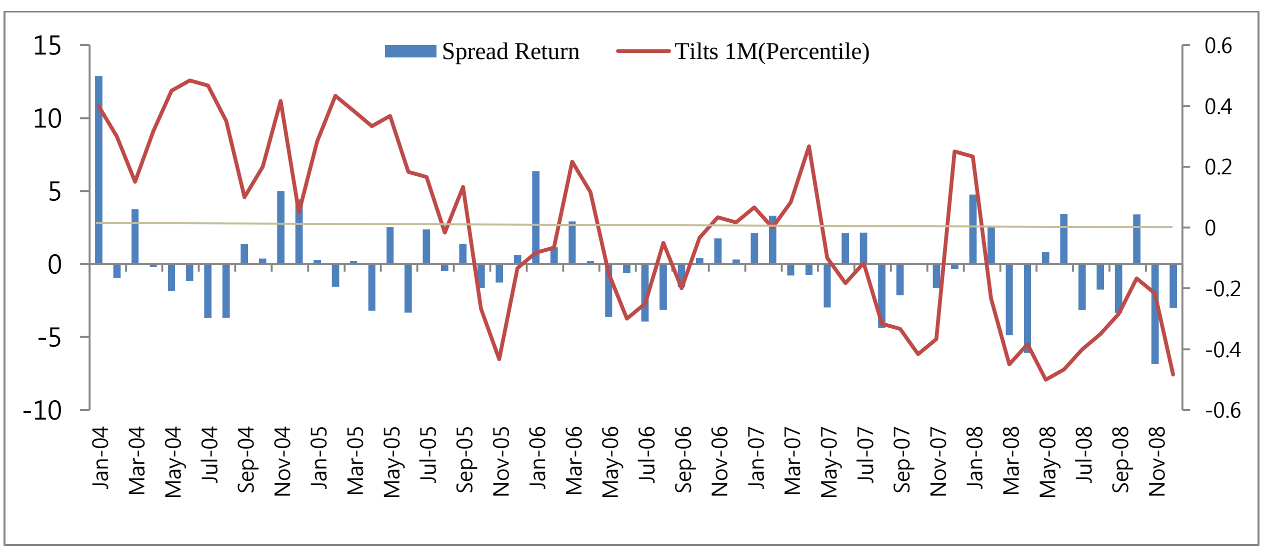


Figure 3.5 Spread return (small/large caps) vs. dynamic tilts against a 40-60 static allocation

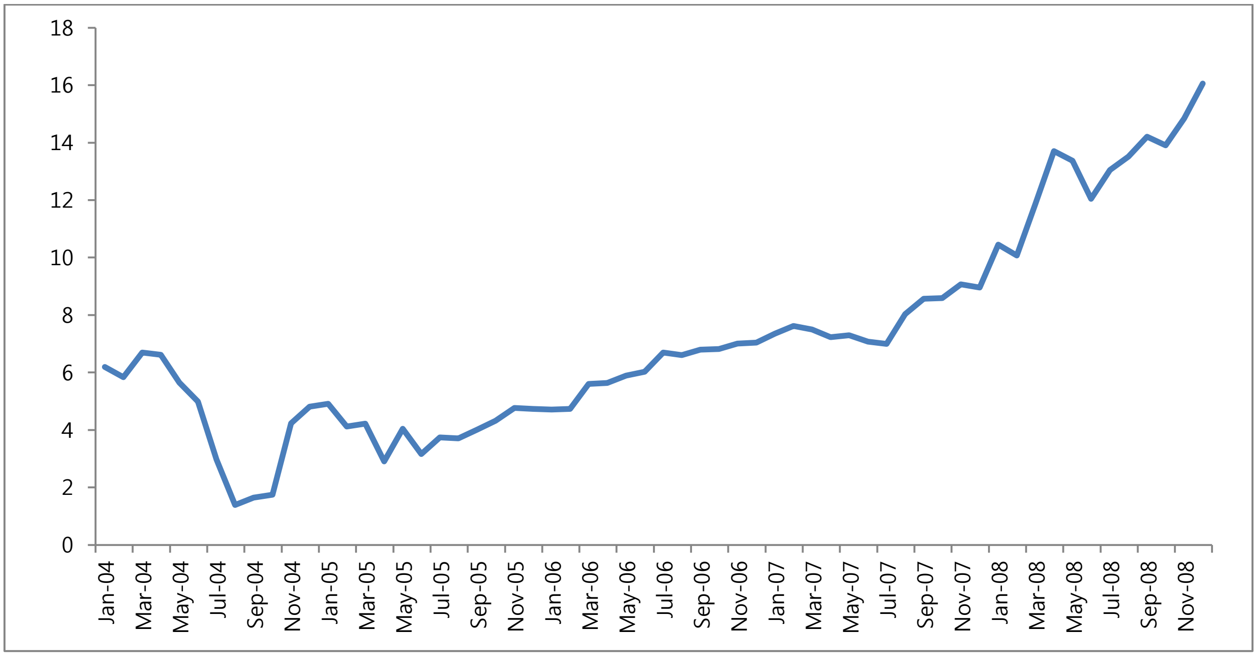


Figure 3.6 Cumulative excess returns of dynamic tilts against a 40-60 static allocation

3.3 Return Forecasts with Higher Moments and Downside Protections

Previously, the model-based forecasts of a latent variable which measure market bullishness are used to determine the tilts toward stocks that are expected to outperform. In this section, I test whether higher moments of Markov switching model, skewness and kurtosis which are expected to give signals in times of regime shifts, can improve the forecasts of market bullishness. Skewness signals the direction of the market: A spike indicates that tide is turning in a bullish direction while a dip suggests that the market is moving to a bearish direction. Kurtosis means the possibility of extreme situation such as downturns and upturns and large values imply high probability of extreme values. Figure 3.7 presents state probabilities and dynamics of higher moments during the whole sample period. This shows that large changes in skewness and kurtosis are linked to regime shifts. Also when we look at the movements of higher moments during the recent five years which is specified as bear regime, we do see a considerable amount of variation.

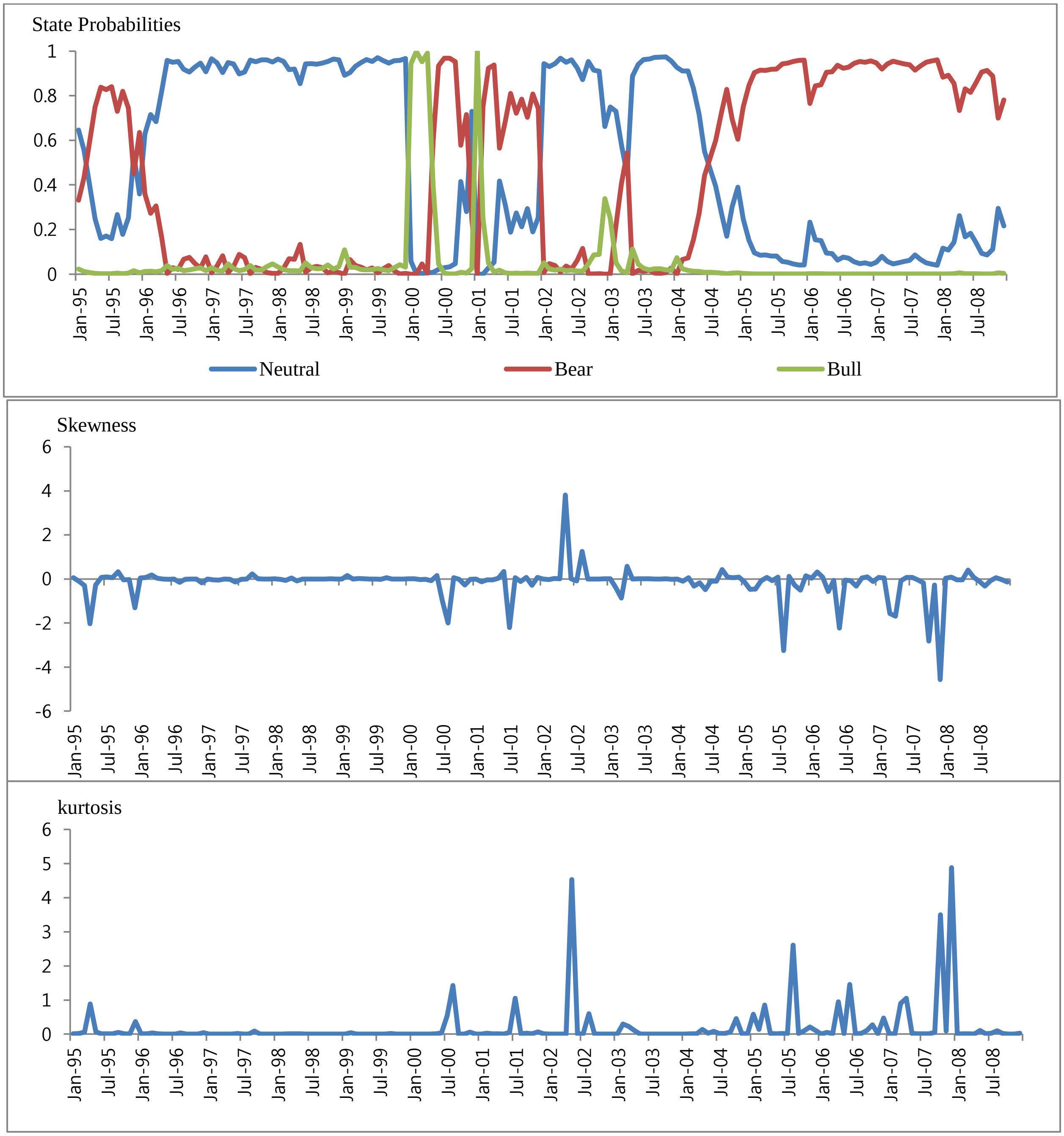


Figure 3.7 State probabilities and higher moments of spread return

To test whether higher moments improve our forecasts of market bullishness, I incorporated these moments in the dynamic ordered probit model. Figure 3.8 compares cumulative excess returns of higher moments against a 40-60 static allocation. When skewness (of spread returns) are added to the model with other explanatory variables, cumulative excess returns started to improve during the 2006 and exceeded the cumulative returns with regime specification only. When both skewness and kurtosis added to the model, this gave us much more enhanced cumulative excess returns during the whole back-test period. Figure 3.9 compares dynamic tilts (toward small cap stocks) of higher moments against a 40-60 static allocation. In general, a tilt toward small-cap stocks is made when spread return is positive and this is the right direction. There is a backsliding in cumulative excess returns in 2004 which stayed tilted toward small-caps with negative spread returns implying bear market. It stayed too long after the high in March 2004. However, the backsliding is mitigated with higher moments and this is attractive investors who want downside protections.

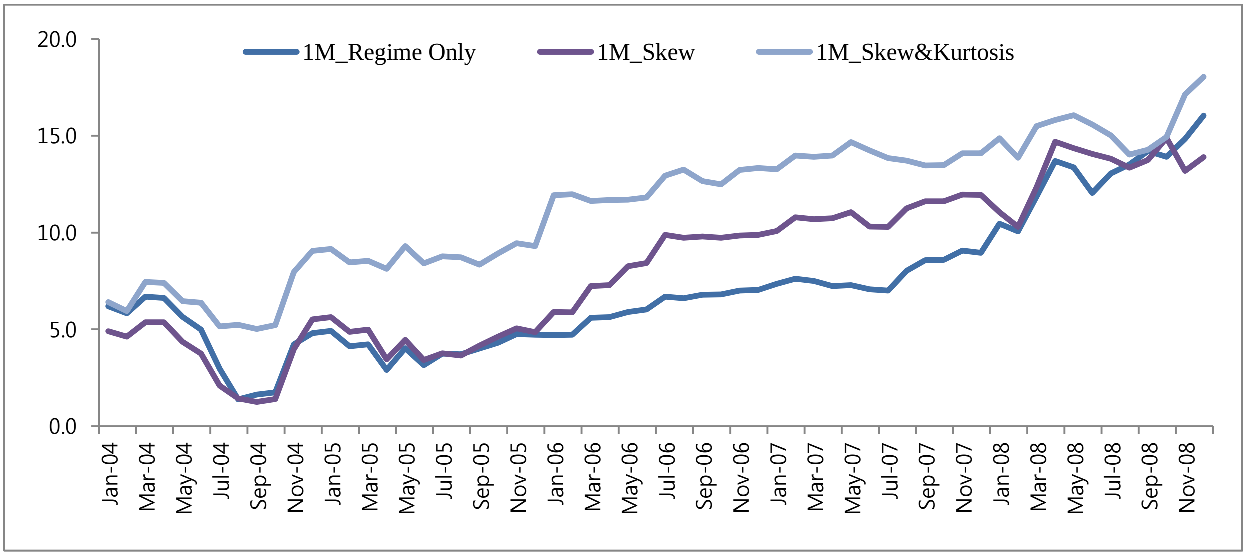


Figure 3.8 Cumulative excess returns of dynamic tilts with higher moments against a 40-60 static allocation

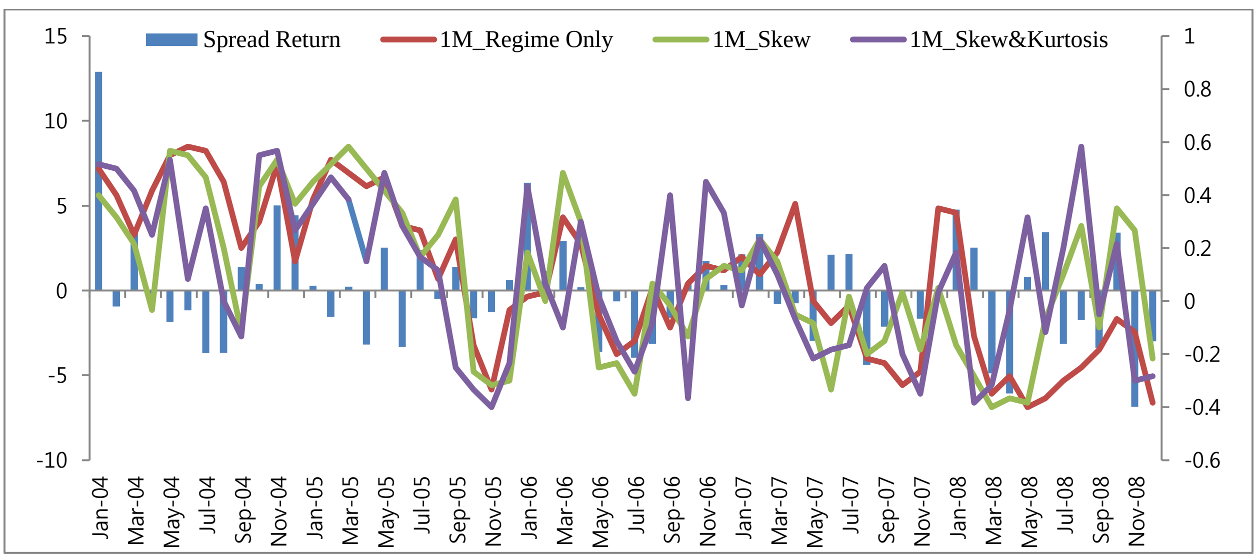


Figure 3.9 Dynamic tilts with higher moments against a 40-60 static allocation

4. Conclusion

This chapter presents a framework for tactical asset allocation between two opposing equities. In this framework, market regimes are specified based on spread returns and forecasts of a latent variable (market bullishness implied by spread returns) are used to get tactical tilts. To improve our ability to time the market turns, higher moments of spread returns under regime switching are incorporated in predicting asset returns. The back-test results support that this approach provides better downside protections in tactical asset allocation.

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Appendix A

Estimation of a dynamic ordered probit model (3.8) in chapter III

A augmented dynamic ordered probit model

$$y^*_{t+1} = \rho y^*_t + Z_t\beta + \gamma S_t[X_{t+1}] + \delta K_t[X_{t+1}] + \varepsilon_{t+1} \qquad (3.8)$$

can be written as

$$\Phi(L)Y_t = \mu + \varepsilon_t$$

where

$$Y_t = \begin{pmatrix} \tilde{Z}_t \\ y^*_t \end{pmatrix}$$

and $\tilde{Z}_t = \begin{pmatrix} Z_t & S_t[X_{t+1}] & K_t[X_{t+1}] \end{pmatrix}$ is a set of explanatory variables, *(h+2)* , including the conditional skewness and kurtosis of spread returns driven under 3-state regime switching model, $\Phi(L)$ is a set of regression coefficients matrices, *(h+2)* x*(h+2)* , with lag order *L*=0,…, p, μ is a set of intercepts, and ε_t are normally distributed with zero means. The covariance matrix of forecast errors is denoted as Σ .

For estimation Markov Chain Monte Carlo (MCMC) algorithm is considered and thus conditional distributions for the regression coefficents (Φ), the covariance matrix (Σ), and the latent variable (y^*_t) are need to be specified. For a given set of values of y^*_t and Σ , the regression coeffcients Φ are normally distributed. The covariance matrix is sampled from the inverted Wishart distribution ($\Sigma^{-1} \mid \Phi, Y \sim W_{h+2}(T, \sum_t \hat{\varepsilon}_t \hat{\varepsilon}'_t)$). The latent variable(y^*_t) follows a truncated normal distribution for a single period and the full conditional distribution of $y^*_t \mid \{Y_{-t}\}, \tilde{Z}_t$ can be expressed as $f(y^*_t \mid \{Y_{-t}\}, \tilde{Z}_t) \propto f(y^*_t \mid \{Y_{-t}\})$. The conditional distribution of $y^*_t \mid \{Y_{-t}\}$ follows $N(C^{-1}D, C^{-1})$ where $C = (\Sigma^{-1} + \phi'_1\Sigma^{-1} + \cdots + \phi'_p\Sigma^{-1})$ and $D = (-\Sigma^{-1}k_t + \phi'_1\Sigma^{-1}k_{t+1} + \cdots + \phi'_p\Sigma^{-1}k_{t+p})$ where k_t is the known part of forecasting error $\varepsilon_t = Y_t - \mu - \phi_1 Y_{t-1} - \cdots - \phi_p Y_{t-p}$. The full conditional distribution of $y^*_t \mid \{Y_{-t}\}, \tilde{Z}_t$ is driven by the Metropolis-Hastings(M-H) algorithm. For more details, please refer to Dueker (2005, p5~p10).

Vita

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