Impact of Wi-Fi deployments and imperfect knowledge of Radar systems on Radar Wi-Fi coexistence

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Abstract

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Spectrum sharing between dis-similar wireless communication systems has been a focus of on-going research to provide potential solutions for the lack of spectrum availability in the current usable range. This arises from the growing usage in mobile service applications that require very high data rates. In this thesis, we examine the coexistence between wireless local area networks (WLAN) and Radars. A large amount of licensed spectrum is allocated for the use of Radars \(^1\) which allows for sufficient re-use of spectrum due to the spatially and temporally sporadic nature of Radar operation. We present the determination of protection regions around Radar primary users so that the aggregate interference from active Wi-Fi secondary users is kept within acceptable limits. The impact of Wi-Fi deployments - modelled as suitable random processes on the statistics of the protection region is studied. Using the Wi-Fi users, the location and mainbeam direction of the Radar is predicted. Also, a sophisticated software program to compute protection regions in the US is described.

\(^1\)In the US, over 1700 MHz spectrum in the 225 MHz - 3.7 GHz range is allocated for Radar operations.
<table>
<thead>
<tr>
<th>Table of Contents</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>List of Figures</td>
<td>ii</td>
</tr>
<tr>
<td>Glossary</td>
<td>iv</td>
</tr>
<tr>
<td>Chapter 1:</td>
<td>1</td>
</tr>
<tr>
<td>Chapter 2:</td>
<td>6</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>6</td>
</tr>
<tr>
<td>2.2 Search Radar: Noise limited operation</td>
<td>6</td>
</tr>
<tr>
<td>2.3 Cumulative interference to Radar from randomly deployed Wi-Fi APs</td>
<td>10</td>
</tr>
<tr>
<td>2.4 Simulation Results for Omni Antenna Gain Pattern</td>
<td>15</td>
</tr>
<tr>
<td>2.5 Simulation Results for typical Radar gain pattern</td>
<td>26</td>
</tr>
<tr>
<td>2.6 Statistics of optimal protection region</td>
<td>33</td>
</tr>
<tr>
<td>2.7 WLAN Network Activity and its Implications</td>
<td>36</td>
</tr>
<tr>
<td>2.8 Real-world example</td>
<td>43</td>
</tr>
<tr>
<td>Chapter 3:</td>
<td>47</td>
</tr>
<tr>
<td>3.1 Introduction</td>
<td>47</td>
</tr>
<tr>
<td>3.2 Prediction of Main Beam Direction</td>
<td>47</td>
</tr>
<tr>
<td>3.3 Prediction of Location</td>
<td>54</td>
</tr>
<tr>
<td>Chapter 4:</td>
<td>61</td>
</tr>
<tr>
<td>4.1 Conclusions</td>
<td>61</td>
</tr>
<tr>
<td>4.2 Future Work</td>
<td>62</td>
</tr>
<tr>
<td>Appendix A:</td>
<td>67</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>The original protection regions computed by the NTIA (outer yellow) were revised recently in [18] (inner blue). Even the revised region encompasses many population centers such as San Francisco and New York.</td>
<td>3</td>
</tr>
<tr>
<td>2.1</td>
<td>Aggregate interference from multiple Wi-Fi access points to radar receiver</td>
<td>12</td>
</tr>
<tr>
<td>2.2</td>
<td>Mean of aggregate interference (1000 iterations)</td>
<td>16</td>
</tr>
<tr>
<td>2.3</td>
<td>Variance of aggregate interference (1000 iterations)</td>
<td>16</td>
</tr>
<tr>
<td>2.4</td>
<td>$P_{out}$ vs. $d(\theta)$ - Numerical brute force algorithm ($I_{\text{max}} = -50\text{dBm}$)</td>
<td>18</td>
</tr>
<tr>
<td>2.5</td>
<td>Update of $d(\theta)$ vs. Time (s) - Iterative Monte Carlo simulation ($I_{\text{max}} = -50\text{dBm}$)</td>
<td>20</td>
</tr>
<tr>
<td>2.6</td>
<td>Gaussian fit is not a good approximation</td>
<td>23</td>
</tr>
<tr>
<td>2.7</td>
<td>Gaussian fit is a reasonable approximation</td>
<td>23</td>
</tr>
<tr>
<td>2.8</td>
<td>Mean of aggregate interference (1000 iterations)</td>
<td>25</td>
</tr>
<tr>
<td>2.9</td>
<td>One realization of the Thomas cluster process</td>
<td>25</td>
</tr>
<tr>
<td>2.10</td>
<td>One realization of the inhomogeneous PPP</td>
<td>27</td>
</tr>
<tr>
<td>2.11</td>
<td>$\lambda(x)$</td>
<td>27</td>
</tr>
<tr>
<td>2.12</td>
<td>Protection region for the inhomogeneous PPP Wi-Fi deployment, compared with the homogeneous PPP Wi-Fi deployment ($\lambda_0 = 2\times10^{-6}, I_{\text{max}} = -50\text{dBm}$)</td>
<td>28</td>
</tr>
<tr>
<td>2.13</td>
<td>Radar antenna gain vs. Azimuth</td>
<td>29</td>
</tr>
<tr>
<td>2.14</td>
<td>For $I_{\text{max}} = -50\text{dBm}$, the Gaussian assumption is not a good approximation for distribution of $I_{\text{aggr}}$</td>
<td>31</td>
</tr>
<tr>
<td>2.15</td>
<td>For $I_{\text{max}} = -70\text{dBm}$, the Gaussian assumption is a good approximation for distribution of $I_{\text{aggr}}$</td>
<td>32</td>
</tr>
<tr>
<td>2.16</td>
<td>Gaussian fit is not a good approximation</td>
<td>32</td>
</tr>
<tr>
<td>2.17</td>
<td>Gaussian fit is a reasonable approximation</td>
<td>32</td>
</tr>
<tr>
<td>2.18</td>
<td>Protection region for the inhomogeneous PPP Wi-Fi deployment, compared with the homogeneous PPP Wi-Fi deployment ($\lambda_0 = 2\times10^{-6}, I_{\text{max}} = -60\text{dBm}$)</td>
<td>33</td>
</tr>
<tr>
<td>2.19</td>
<td>Numerical Brute Force: P.D.F. of $d_{opt}$ with mean 637.64 km and standard deviation 74.41 km</td>
<td>35</td>
</tr>
</tbody>
</table>
GLOSSARY

AP: Access Point

ATC: Air Traffic Control

CA: Collision Avoidance

CDF: Cumulative Distribution Function

CW: Contention Window

CSMA: Carrier Sense Multiple Access

DCF: Distributed Coordination Functions

DFS: Dynamic Frequency Selection

DOD: Department of Defense

FCC: Federal Communications Commission

FDR: Frequency Dependent Rejection

INR: Interference-to-Noise Ratio

ISM: Industrial, Scientific, Medical band

ITU: International Telecommunication Union

MAC: Medium Access Control

PDF: Probability Density Function

NTIA: National Telecommunications and Information Administration
OFDM: Orthogonal Frequency Division Multiplexing

PPP: Poisson Point Process

QPSK: Quadrature Phase Shift Keying

RMSE: Minimum Mean Squared Error

SINR: Signal-to-Interference-plus-Noise Ratio

SNR: Signal-to-Noise Ratio

TDMA: Time Division Multiple Access

U-NII: Unlicensed National Information Infrastructure

WLAN: Wireless Local Area Network

WI-FI: WLAN based on the IEEE 802.11 Standard
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Chapter 1

INTRODUCTION

The novelty of the proposed concepts center around spectrum sharing between dis-similar systems, i.e. with designated primary users sharing the frequency bands with secondary networks, unlike sharing in the unlicensed bands. The first instance of such primary-secondary sharing was the unlicensed reuse of local TV spectrum pursuant to the TV White Spaces ruling by the FCC in 2008 [1]. Pursuant to a Presidential directive [5], the FCC has undertaken that 150 MHz of spectrum between 3550-3700 MHz be made available for wireless broadband applications for Citizens Band Radio Service [6]. A comprehensive listing of U.S. Federal spectrum allocation for various radars is shown in Table 1.1. This is not, however, the first instance of spectrum sharing between a (licensed) radar and an unlicensed network. The globally unlicensed bands according to Article 5 of the ITU regulations are shown in Table 1.2. Wi-Fi operation in the 5 GHz (U-NII unlicensed bands in North America but not elsewhere) are already subject to Dynamic Frequency Select (DFS) requirements (requiring channel sensing and avoidance) to avoid interfering with various radars operational worldwide [10], [11]. Currently, only 36% of 5 GHz channels are unencumbered by radar protection requirements.

The potential of Radar-LTE coexistence in the 2700-2900 MHz bands was studied in [2] for different LTE deployment scenarios. However, for multiple secondary users it is assumed that the distance to the Radar is equal which is not practical. There have been some DoD studies [3], [4] in the US for Radars in 2700-2900 MHz bands. Protection for the Radar is determined by experimental measurements of interference from continuous wave, CDMA-QPSK and TDMA-QPSK waveform emissions.

Recent Radar coexistence studies with LTE and Wi-Fi using lab tests and field tests have
Table 1.1: NTIA Federal Spectrum Requirement Forecast for Radar Bands

<table>
<thead>
<tr>
<th>Frequency band</th>
<th>Federal Government use</th>
</tr>
</thead>
<tbody>
<tr>
<td>92–100 GHz</td>
<td>Airborne fire-control, beacons, atmospheric research, cloud detection, and synthetic vision radars</td>
</tr>
<tr>
<td>31.8–36 GHz</td>
<td>Airborne navigational, mapping, weather, beacon, terrain following and avoidance; aircraft carrier PAR, test range, atmospheric and oceanic research, altimeter, scatterometer, and synthetic vision radars</td>
</tr>
<tr>
<td>24.05–24.65 GHz</td>
<td>Doppler radiolocation, vehicle speed detection, scatterometer, and precipitation radars</td>
</tr>
<tr>
<td>15.4–17.3 GHz</td>
<td>Airborne and shipborne multimode search, battlefield, aircraft carrier PAR, fire-control, test range, ASDE, scatterometer, precipitation, atmospheric research, and spaceborne radars</td>
</tr>
<tr>
<td>13.25–14.2 GHz</td>
<td>Airborne and shipborne search and acquisition Doppler, airborne weather, altimeters, scatterometer, precipitation, environmental research, and spaceborne radars</td>
</tr>
<tr>
<td>8.5–10.55 GHz</td>
<td>Airborne and shipborne surveillance and navigation, fire-control, battlefield, maritime, weather, test range, airborne radionavigation, ATC, SAR’s, altimeters, ASDE, scatterometer, vehicle speed detection, and spaceborne radars</td>
</tr>
<tr>
<td>5250–5925 MHz</td>
<td>NOAA weather radars, FAA TDWR, surveillance and air defense (airborne, shipborne, land-based), fire-control, maritime, test range, SAR’s, altimeters, airborne, and spaceborne radars</td>
</tr>
<tr>
<td>4200–4400 MHz</td>
<td>Aircraft radar altimeters</td>
</tr>
<tr>
<td>3100–3650 MHz</td>
<td>DOD surveillance and air defense (airborne, shipborne, land-based), ATC, SAR’s, altimeters, test range, and spaceborne radars</td>
</tr>
<tr>
<td>2700–3100 MHz</td>
<td>ATC, maritime, and weather radars; DOD shipborne, airborne, ground air surveillance radars; range control, and spaceborne radars</td>
</tr>
<tr>
<td>2310–2385 MHz</td>
<td>Planetary and lunar radar</td>
</tr>
<tr>
<td>1215–1390 MHz</td>
<td>ATC, SAR’s, and DOD early warning air defense, battlefield, shipborne long-range surveillance, and spaceborne radars</td>
</tr>
<tr>
<td>890–942 MHz</td>
<td>Navy shipborne long-range surveillance, test range, NASA research, and wind profiler radars</td>
</tr>
<tr>
<td>420–450 MHz</td>
<td>DOD early warning and long-range surveillance radars; and wind profiler radars</td>
</tr>
<tr>
<td>216–220 MHz</td>
<td>DOD space surveillance radar</td>
</tr>
<tr>
<td>3–30 MHz</td>
<td>DOD OTH and surface wave radar</td>
</tr>
</tbody>
</table>
Figure 1.1: The original protection regions computed by the NTIA (outer yellow) were revised recently in [18] (inner blue). Even the revised region encompasses many population centers such as San Francisco and New York.

been performed for the SPN-43 Radar in the 3500-3700 MHz bands [14]. The distance to the Radar was assumed constant in all the experiments. The exclusion zones are determined using dedicated sensors to protect the Radar system. It is concluded that fixed protection regions by the FCC are unnecessary but it lacks the characterization of the interference at the Radar. The Radar Wi-Fi coexistence scenario must consider two important factors:

1. The impact of interference from Radar systems on the performance of WLANs, and

2. Measures needed to protect Radars from unlicensed wireless users.

In this work, we focus exclusively on the second factor; this is typically done by enforcing a protection region that represents a spatial domain surrounding the protected Radar location.
in which co-channel transmission by WLAN users is forbidden\(^1\). We assume all radars are mono-static and the protection regions are determined by regulatory policy-making as a collaboration between NTIA and FCC, representing the need to balance incumbent protection and the needs of secondary civilian networks to set up an effective service \cite{12}. Often, these protection regions can extend for hundreds of kilometers from the radar location, which if located near population centers, makes deployment of secondary systems inefficient. The NTIA in its initial Fast Track report \cite{13}, initially inferred that large geographical separation and frequency offsets were required to minimize interference from higher power macro cell networks to federal radar systems in the 3.5 GHz band. However, this analysis was later revised as the initial assumptions were significantly conservative (for example, 4G and beyond networks use smaller cells with lower power Base Stations) as shown in Figure 1.1. The above example and our own subsequent analysis emphasizes this important underlying point - that the determination of protection regions is significantly impacted by a multiplicity of factors on both the secondary (notably, the transmit power, deployment geometry, transmit antenna patterns) and primary (receiver sensitivity and operational margins, receive antenna patterns) side as well as the channel characteristics (distance and frequency dependant path loss, and any statistical effects such as small-scale fading and large-scale shadowing).

\(1\)Outside such regions, WLAN operation is allowed as it is deemed to offer only acceptable interference to primary receivers.
Table 1.2: Unlicensed bands per Article 5 of ITU regulations (Primary allocation in capital, secondary allocation in small letters)
Region 1: Europe, Africa, the former Soviet Union, Mongolia, and the Middle East west of the Persian Gulf.
Region 2: Americas including Greenland, and some of the eastern Pacific Islands.

<table>
<thead>
<tr>
<th>Frequency range</th>
<th>Availability</th>
<th>Licensed users</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.765-6.795 MHz</td>
<td>Subject to local acceptance</td>
<td>FIXED SERVICE and Mobile service</td>
</tr>
<tr>
<td>13.553-13.567 MHz</td>
<td>Worldwide</td>
<td>FIXED and Mobile services except Aeronautical mobile (R) service</td>
</tr>
<tr>
<td>26.957-27.283 MHz</td>
<td>Worldwide</td>
<td>FIXED and MOBILE SERVICE except Aeronautical mobile service, CB Radio</td>
</tr>
<tr>
<td>40.66-40.7 MHz</td>
<td>Worldwide</td>
<td>Fixed, Mobile services and Earth exploration-satellite service</td>
</tr>
<tr>
<td>433.05-434.79 MHz</td>
<td>only in region 1, subject to local acceptance</td>
<td>AMATEUR SERVICE and RADIOLOCATION SERVICE</td>
</tr>
<tr>
<td>902-928 MHz</td>
<td>Region 2 only</td>
<td>FIXED, Mobile except aeronautical mobile and Radiolocation service</td>
</tr>
<tr>
<td>2.4-2.5 GHz</td>
<td>Worldwide</td>
<td>FIXED, MOBILE, RADIOLOCATION, Amateur and Amateur-satellite service</td>
</tr>
<tr>
<td>5.725-5.875 GHz</td>
<td>Worldwide</td>
<td>FIXED-SATELLITE, RADIOLOCATION, MOBILE, Amateur and Amateur-satellite service</td>
</tr>
<tr>
<td>24-24.25 GHz</td>
<td>Worldwide</td>
<td>AMATEUR, AMATEUR-SATELLITE, RADIOLOCATION and Earth exploration-satellite service (active)</td>
</tr>
<tr>
<td>61-61.5 GHz</td>
<td>Subject to local acceptance</td>
<td>FIXED, INTER-SATELLITE, MOBILE and RADIOLOCATION SERVICE</td>
</tr>
<tr>
<td>122-123 GHz</td>
<td>Subject to local acceptance</td>
<td>EARTH EXPLORATION-SATELLITE, FIXED, INTER-SATELLITE, MOBILE, SPACE RESEARCH (passive) and Amateur service</td>
</tr>
<tr>
<td>244-246 GHz</td>
<td>Subject to local acceptance</td>
<td>RADIOLOCATION, RADIO ASTRONOMY, Amateur and Amateur-satellite service</td>
</tr>
</tbody>
</table>
Chapter 2

COMPUTATIONS FOR VARIOUS WI-FI DEPLOYMENTS

2.1 Introduction

In this thesis, the role of the distribution of secondary users in determining the protection region for a Radar source is discussed. From [1] we know that the protection region for a Radar is a function of the time-spatial density. This does not give us information about the change in the protection region due to the density of secondary users. If one assumes that the time-spatial density increases linearly with the density of secondary users, then the aggregate interference and the protection region is an overestimate of the true values. Therefore, the relationship between the time-spatial density and the density of Wi-Fi networks is found using the analysis of the distributed coordination function discussed in [2]. Specifically, the role of the number of users associated with co-channel access points is investigated. By considering a Poisson point process (PPP) for the distribution of Wi-Fi APs, the results obtained in [2] allow us to characterize the change in the protection region as a function of the intensity of the PPP.

Section 2 uses the results obtained in [2] and Section 3 characterizes the Wi-Fi network distribution and the aggregate interference at a radar source.

2.2 Search Radar: Noise limited operation

Consider a rotating search radar limited where all interfering (Wi-Fi) sources lie in the same plane as the radar, and the desired target is at an elevation\(^1\). The performance characteristics

\(^1\)Hence, in the sequel, consider a general radar antenna pattern \(G(\theta, \phi)\) where these represent the azimuth and elevation angles, respectively; all interference calculations use the horizontal antenna pattern corresponding to \((\phi = 0)\) plane.
for the purely noise limited radar operation case is quantified in terms of the (desired) probability of detection $P_D$ and acceptable false alarm rate $P_{FA}$. In turn, this determines the minimum SNR requirements at the radar receiver input (corresponding to a maximum operating range) and sets the baseline for comparison with any spectrum sharing regime. In order to enable cognitive reuse by secondary transmitters outside the protection region, we need to define rules for ‘acceptable’ interference at the radar. A recent DARPA program [7] suggested that a drop of 5% in $P_D$ for fixed $P_{FA}$ at the edge of radar operating range as being acceptable, which we adopt. This margin can be allocated to aggregate interference produced by a (random) spatial distribution of multiple Wi-Fi networks that are operational outside of the protection region.

At the monostatic radar receiver, the power of the reflected signal from the target, assuming free space propagation is given by the Radar Equation [15]:

$$P_R = \frac{P_T G^2 \lambda^2}{(4\pi)^3 d^4 \sigma}$$

where the radar’s instantaneous transmit power is $P_T$, $G$ is the radar’s antenna gain on both transmit and receive in the direction of the target, $\lambda$ is the wavelength and $d$ the distance from source to the target of interest, and $\sigma$ represents the target’s radar cross section.

Using the above, the effective SNR ($\text{SNR}_{\text{eff}}$) for reliable operation can be computed for a given probability of detection $P_D$ and probability of false alarm $P_{FA}$. The minimum required SNR per pulse ($\text{SNR}_{\text{min}}$) is determined by the nature of any pulse integration employed in the detection chain, namely coherent and non-coherent. Using $\text{SNR}_{\text{min}}$, we can find the maximum interference level that can be tolerated by the radar, which we denote by $I_{\text{max}}$. In turn, this determines the minimum separation between the radar and secondary users.

\subsection{2.2.1 Noise-limited Radar Operation: Design Equations}

For a radar transmitting a pulse train $x(t) = \sum_n \sqrt{P_T}s(t-nT)$ with pulse repetition frequency of $f_R$, the power of reflected signal from the target at the radar receiver is given by (2.1).
For a single received pulse, the signal-to-noise ratio (SNR) at the receiver input is

$$\text{SNR}_p = \frac{P_T G^2 \lambda^2}{(4\pi)^3 d^4 N_0 f_{BW}} \sigma$$  \hspace{1cm} (2.2)

with $f_{BW}$ representing the pulse bandwidth and $N_0$ being the one-sided noise spectral density,

$$N_0 = FKT_E$$  \hspace{1cm} (2.3)

where $F$ is the receiver noise figure and $T_E$ is the ambient temperature. Radar detection typically operates based on processing of multiple pulses received from the target. For a coherent receiver that integrates a number of pulses $M$ reflected from the target, the resulting SNR at the detector input is increased by a factor of $M$, i.e.,

$$\text{SNR}_{\text{eff}} = M \frac{P_T G^2 \lambda^2}{(4\pi)^3 d^4 N_0 f_{BW}} \sigma$$  \hspace{1cm} (2.4)

where $M = T_I f_R$, product of illumination time $T_I$ and pulse repetition frequency $f_R$. The target illumination time $T_I$ depends on radar scan rate as well as antenna pattern. Let $\theta_V$ and $\theta_H$ (in radian) denote the vertical and horizontal antenna beam width, respectively, then the antenna gain can be approximated as

$$G \approx \frac{4\pi}{\theta_H \theta_V} \rho_A$$  \hspace{1cm} (2.5)

where $\rho_A$ is the antenna efficiency, i.e., the radar concentrates the emitted power into an area of $\theta_V \theta_H$ with efficiency of $\rho_A$ where the latter is typically 0.5. If the radar scans an area of $\Omega$ (steradians) (full search of azimuth/elevation implies $\Omega = 4\pi$) within a scan time of $T_S$, the illumination time is determined as:

$$T_I \approx T_S \frac{\theta_H \theta_V}{\Omega} \approx T_S \frac{4\pi \rho_A}{\Omega G}$$  \hspace{1cm} (2.6)

Using (2.4)-(2.6) yields

$$\text{SNR}_{\text{eff}} = \frac{T_S}{\Omega} \frac{P_T G \lambda^2 f_R}{(4\pi)^3 d^4 N_0 f_{BW} L} \sigma$$  \hspace{1cm} (2.7)

Note that the antenna efficiency $\rho_A$ has been replaced by equivalent $L$ that represents total losses in the system (e.g. transmission lines mismatch, loss of perfect coherence in pulse detector etc.).
The relationship between \( P_D, P_{FA} \) and \( SNR_p \) for a single pulse is fairly accurately expressed via the following equation [16]:

\[
SNR_p = \ln \left( \frac{0.62}{P_{FA}} \right) + 0.12 \ln \left( \frac{0.62}{P_{FA}} \right) \ln \left( \frac{P_D}{1 - P_D} \right) + 1.7 \ln \left( \frac{P_D}{1 - P_D} \right)
\]  

\[ (2.8) \]

For a coherent receiver integrating \( M \) pulses, the SNR-performance relationship described in (2.8) with \( SNR_p \) should be replaced with the effective SNR at the detector input (\( SNR_{\text{eff}} \)), given by (2.7). Overall, the baseline performance of a noise-limited radar can be evaluated in two ways:

- Assuming that maximum operational range of the radar is known, calculate SNR from (2.2) or (2.7) for the maximum distance \( d \). Then, using (2.8), we can trade-off between \( P_D \) and \( P_{FA} \).

- Assuming that target \( P_D \) and \( P_{FA} \) are specified, estimate required SNR from (2.8) and then determine maximum range from (2.2) or (2.7).

As discussed subsequently, the interference from a Wi-Fi transmitter can be modelled as a Gaussian process, which is additive to the (thermal) AWGN noise. Thus radar system performance in the presence of (Wi-Fi) interference is determined by Signal-to-Interference-plus-Noise ratio (SINR) at receiver input. Using (2.7), this is given by

\[
SINR = \frac{T_S}{\Omega} \frac{P_T G \lambda^2 f R}{(4\pi)^2 d^4 L N_0 f_{BW} + I_{aggr}} \sigma
\]

\[ (2.9) \]

where \( I_{aggr} \) represents total interference power from secondary user(s). Therefore, maximum additional interference level \( I_{\text{max}} \) that can be tolerated is determined as follows:

\[
SINR_{\text{min}} \leq \frac{T_S}{\Omega} \frac{P_T G \lambda^2 f R \sigma}{(4\pi)^2 d^4 L (N_0 f_{BW} + I_{aggr})}
\]

\[
I_{aggr} \leq \frac{T_S}{\Omega} \frac{P_T G \lambda^2 f R \sigma}{(4\pi)^2 d^4 L SINR_{\text{min}}} - N_0 f_{BW} = I_{\text{max}}
\]

\[ (2.10) \]
For a given deployment of Wi-Fi networks, the resulting aggregate interference at the radar receiver input $I_{agg}$ is modelled (statistically) in detail in Sec. 4. The difference between the received interference from secondary users ($I_{agg}$) and the maximum tolerable interference at the radar receiver ($I_{max}$) denotes the available margin (a positive margin implies secondary operation is possible).

For a single Wi-Fi AP, the interference power at the radar is given by

$$I_{SU\rightarrow Radar} = \frac{P_{SU} G(\theta)}{L_{Rd-SU}(d_{Rd-SU})FDR(\Delta f)} \quad (2.11)$$

where $L_{Rd-SU}(.)$ is the path-loss between radar and secondary user (SU) as a function of distance and $G(\theta)$ defines radar’s antenna gain in the direction of SU (function of azimuth angle). $FDR(\Delta f)$ is frequency dependent rejection factor that accounts for the mismatch between the spectral support of the transmitted signal and receiver input band; depending on spectral shape of transmitted signal $P(f)$ and receiver input filter $H(f)$, this is computed as

$$FDR(\Delta f) = \frac{\int_{0}^{\infty} P(f)df}{\int_{0}^{\infty} P(f)H(f + \Delta f)df} \quad (2.12)$$

Since Wi-Fi and Radar are assumed to be spectrally flat and overlapping, the FDR is calculated as the ratio of corresponding bandwidths $FDR = \frac{20\text{MHz}}{653\text{KHz}} = 30.6$ [3]. Note that we have used IF 3-dB bandwidth for radar’s receiver which is significantly smaller than RF 3-dB bandwidth of 10 MHz, since radar signal detection happens at IF.

### 2.3 Cumulative interference to Radar from randomly deployed Wi-Fi APs

Wi-Fi transmissions use OFDM signals, where each OFDM symbol is a linear combination of many randomly modulated sub-carriers. Hence, using central limit theorem, the transmitted OFDM signal in time-domain over the air is well-approximated as a Gaussian random process [8]. Thereafter, the transmitted signal undergoes linear time-invariant filtering - the equivalent of the channel and any receiver front-end processing such as the matched filter - that preserves the Gaussianity of the resulting interference process at the detector input.
Thus, the power of any interference variance ($I_{aggr}$) at the detector input is additive to AWGN noise power, effectively raising the noise floor and system performance is determined by Signal-to-Interference-plus-Noise ratio (SINR) at the receiver input.

If multiple secondary users coexist with the radar simultaneously, the accumulated interference power at radar location can only be characterized statistically. The aggregate interference seen by the radar receiver is determined by several factors, notably the stochastic geometry of the deployment of the Wi-Fi networks (influences the distances and hence the component contributed to $I_{aggr}$) and the multiple access protocol employed (that determines which users are simultaneously active). Since users within a single Wi-Fi network share the common channel in time based on CSMA-CA, only (upto) one source can be active at any instant within one network. However, different Wi-Fi networks can simultaneously operate in the vicinity of a radar. In this work, we assume that each Wi-Fi network is simultaneously active (i.e. there is an active user), which represents a worst-case scenario and acts as upper bound on $I_{aggr}$, and focus exclusively on the impact of the geometry of Wi-Fi network deployment.

The stochastic geometry [21] of the Wi-Fi network deployments impact the aggregate interference at the radar receiver. Let the Wi-Fi APs and their associated users be randomly distributed in 2-D space according to a point process.

Without loss of generality, assume a radar located at the 2-D origin, with APs randomly distributed over Cartesian locations $\mathbf{x} \in \mathbb{R}^2$. The aggregate interference from secondary users at the radar receiver is described as [20]:

$$I_{aggr} = \sum_{\mathbf{x} \in \Phi} \frac{P_{su} G(\theta_{\mathbf{x}})}{FDR(\Delta f)} l(||\mathbf{x}||)$$  \hspace{1cm} (2.13)

where $P_{su}$ is the secondary user transmit power, $||\mathbf{x}||$ is the distance between radar and secondary access point, $l(.)$ describes the impulse response function that models signal attenuation (inverse of path loss) between the source and receiver, and $G(.)$ is radar’s antenna gain in the azimuthal direction of interferer $\theta_{\mathbf{x}}$. Note that (2.13) is the aggregate interference from multiple Wi-Fi sources that are simultaneously active, and is the summation of
components (2.11) that represent the interference from a single user. The random spatial deployment of Wi-Fi sources in turn implies the aggregate interference $I_{aggr}$ at the radar must also be statistically characterized and plays an important role in performance analysis of radar detection under Wi-Fi interference. When Wi-Fi deployment may be modelled as a homogeneous Poisson point process with intensity $\lambda$, assuming propagation with pathloss exponent $\alpha = 4$ \cite{20}, it results in a Levy p.d.f for $I_{aggr}$ \cite{20}:

$$
 f_{I_{aggr}}(x) = \frac{\pi \lambda}{2x^{3/2}} e^{x} p\left(-\frac{\pi^{3} \lambda^{2}}{4x}\right) 
$$

(2.14)

It is worthwhile noting that the Levy p.d.f has a heavier tail than the Gaussian distribution and was derived \cite{20} for the scenario of no protection region around the radar, i.e. nearby Wi-Fi sources contribute significantly to $I_{aggr}$.

\footnote{There exists no closed form solution other than $\alpha = 4$; the p.d.f for these must be obtained via numerical simulations.}
2.3.1 Average and Variance of Aggregate Interference ($\mu_I, \sigma^2_I$): Campbell's Theorem

Given the fact that the p.d.f of $I_{aggr}$ is available in closed form for only a few specific cases, we now resort to the use of a basic result that allows characterization of the first and second moments of $I_{aggr}$ for a broad class of spatial point processes. For a broad class of point processes with intensity function $\Lambda(x)$, the mean and variance of the random sum $\sum_{x \in \Phi} f(x)$ is given by Campbell’s theorem [20] as

$$E\left[\sum_{x \in \Phi} f(x)\right] = \int_{\mathbb{R}^2} f(x)\Lambda(x)dx \quad (2.15)$$

$$\text{var}\left[\sum_{x \in \Phi} f(x)\right] = \int_{\mathbb{R}^2} f^2(x)\Lambda(x)dx \quad (2.16)$$

Identifying $f(x) = \frac{P_{su}G(\theta_x)}{FDR(\Delta f)}l(||x||)$ from (2.13) allows us to compute the statistics of the aggregate interference at the radar input, as derived below.

**Homogeneous Poisson Point Process (PPP)**

We first consider the homogeneous PPP, for which $\Lambda(x) = \lambda$. The average interference received at radar is calculated from (2.15) by integrating over the $\mathbb{R}^2$ plane, i.e.,

$$\mu_I = E[I_{aggr}] = \frac{\lambda P_{SU}K_0}{FDR(\Delta f)} \int_{\mathbb{R}^2} G(\theta_x)l(||x||)dx \quad (2.17)$$

To incorporate the protection region, we assume there is a minimum separation distance between radar and active SUs, denoted as a function of azimuth angle $\theta$, $d(\theta)$. Using polar coordinates and considering $l(r) = K_\theta r^{-\alpha}$ for a general path loss model with radial symmetry, we obtain:

$$\mu_I = \frac{\lambda P_{SU}K_0}{FDR(\Delta f)} \int_\theta \int_{d(\theta)}^{\infty} G(\theta)r^{1-\alpha}drd\theta = C_{\mu_I} \int_\theta G(\theta)d^{2-\alpha}(\theta)d\theta$$

$$C_{\mu_I} = \frac{\lambda P_{SU}K_0}{FDR(\Delta f)(\alpha - 2)} \quad (2.18)$$

where $\alpha > 2$ is necessary to guarantee convergence of the integral over $\theta$.

It is evident that the mean total interference is dependant on the choice of function $d(\theta)$.
which in turn depends on the input radar antenna pattern \( G(\theta) \). As we show in the next section, the choice of \( d(\theta) \) can be optimized subject to some constraints.

The variance of aggregate interference - calculated from (2.16) - follows a similar approach to that for \( \mu_I \), resulting in:

\[
\sigma^2_I = \frac{\lambda P^2_{SU} K_0^2}{\text{FDR}^2(\Delta f)} \int_{\theta} \int_{d(\theta)} G^2(\theta) r^{-2\alpha} r dr d\theta = C_{\sigma^2_I} \int_{\theta} G^2(\theta) d^2 - 2\alpha(\theta) d\theta
\]

\[
C_{\sigma^2_I} = \frac{\lambda P^2_{SU} K_0^2}{\text{FDR}^2(\Delta f)(2\alpha - 2)}
\]

(2.19)

where \( \alpha > 1 \) ensures convergence of the integration over \( r \).

**Non-homogeneous Poisson Process**

We consider the Thomas cluster process, which belongs to the class of Neyman-Scott cluster processes\[20\] and is characterized by

\[
\Lambda(x) = \lambda_p \bar{c}
\]

where \( \lambda_p \) is the intensity of a parent standard Poisson process that identifies a cluster head (AP); the number of (client) points associated with a cluster is independently Poisson distributed with density \( \bar{c} \). Around every parent point, each daughter point is distributed according to a symmetric normal distribution in 2-D with variance \( \zeta^2 \), i.e.,

\[
f_{cl}(x) = \frac{1}{2\pi\zeta^2} \exp\left( - \frac{||x||^2}{2\zeta^2} \right)
\]

(2.20)

where \( \lambda = \lambda_p \bar{c} \), and \( ||x|| \) is the radial distance of the client with respect to cluster head (AP). The average interference is given by \[20\]

\[
E[I_{agg}] = E[I_{agg-PPP(\lambda)}] + \frac{\bar{c} P_{su}}{\text{FDR}(\Delta f)(4\pi\zeta^2)} \int_{R^2} G(\theta_x) l(||x||) \exp\left( - \frac{||x||^2}{4\zeta^2} \right) dx
\]

(2.21)

where \( E[I_{agg-PPP(\lambda)}] = \mu_I \) is the average interference at the radar receiver when the Wi-Fi nodes are distributed according to a homogeneous PPP with density \( \lambda \). The closed-form solution for the above integral is generally not available in closed form and must be computed numerically.
Note that the average interference at the radar is identical at each azimuth, when the Wi-Fi deployment follows a homogeneous PPP or the Thomas cluster process. We later will consider an inhomogeneous process in azimuth for the Wi-Fi deployment and explore its impact on the aggregate interference via simulation.

2.4 Simulation Results for Omni Antenna Gain Pattern

For a radar receiver located at the origin, Wi-Fi nodes are scattered according to a PPP with density $\lambda = 10^{-6}$. The total area of deployment is a circle with outer radius of 20,000 km. In this section, we assume an omni-directional antenna pattern for radar with a gain of $G(\theta) = 10$ dBi. Secondary user power (including its antenna gain) is set to $P_{su} = 1.0$ watt, in line with commercial Wi-Fi devices. We consider a path-loss component of $\alpha = 4$ and outage probability of $P_{out,max} = 0.1$. Aggregate interference power $I_{aggr}$ at the radar receiver is simulated via the following steps:

i Initially, for a given omni-protection region ($d(\theta) = R$), the mean and variance of the $I_{aggr}$ at the radar is calculated using Campbell’s theorem.

ii For each $d(\theta)$, the numerically computed mean ($\mu_{theory}$) and variance ($\sigma_{theory}^2$) based on Campbell’s theorem are validated by Monte-Carlo simulations reported next. For every simulated case, a total of 1000 snapshots are generated and then sample mean/variance of aggregated interference is calculated.

2.4.1 Campbell’s Theorem verification

The protection region in this case is circular, i.e. $d(\theta) = R$, where $R$ is the radius of the protection region. Our simulation showed that sufficient number of snapshots must be generated to achieve reasonable accuracy. Figure 2.2 and 2.3 shows calculated mean and variance for a total of 1000 snapshots. It can be seen from the figures that Campbell’s theorem is corroborated for a range of distances from 35 km up to 4500 km. The results are tabulated in Table 2.1.
2.4.2 Optimizing the protection region for homogenous PPP

In this section we explore the optimization of $d(\theta)$ subject to a constraint on $I_{\text{aggr}}$ and outage probability, i.e.,

$$P(I_{\text{aggr}} > I_{\text{max}}) \leq P_{\text{out,max}}$$

where a typical value is $P_{\text{out,max}} = 0.1$. We choose specific values of $I_{\text{max}}$ (for example -50 dBm and -60 dBm) for which the protection region is optimized, i.e we find $d_{\text{opt}}(\theta)$ for which this constraint is satisfied, as described next. In following sections, we use three different methods for calculating $d(\theta)$, namely brute-force simulation, iterative Monte-Carlo and an
Table 2.1: Campbell’s theorem verification table

<table>
<thead>
<tr>
<th>$d(\theta) = R$ (km)</th>
<th>$\mu_{\text{theory}}$ (dBm)</th>
<th>$\mu_{\text{sample-1000}}$</th>
<th>$\sigma^2_{\text{theory}}$ (dB)</th>
<th>$\sigma^2_{\text{sample-1000}}$</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>35.6</td>
<td>-36.7733</td>
<td>-37.0256</td>
<td>-114.3099</td>
<td>-116.2947</td>
<td>22.22</td>
</tr>
<tr>
<td>50.3</td>
<td>-39.7836</td>
<td>-39.4451</td>
<td>-123.3408</td>
<td>-122.9496</td>
<td>21.81</td>
</tr>
<tr>
<td>71.1</td>
<td>-42.7939</td>
<td>-43.2891</td>
<td>-132.3717</td>
<td>-133.4190</td>
<td>22.24</td>
</tr>
<tr>
<td>100.6</td>
<td>-45.8042</td>
<td>-45.8034</td>
<td>-141.4026</td>
<td>-141.7799</td>
<td>21.73</td>
</tr>
<tr>
<td>142.3</td>
<td>-48.8145</td>
<td>-48.8520</td>
<td>-150.4335</td>
<td>-150.7537</td>
<td>21.87</td>
</tr>
<tr>
<td>201.2</td>
<td>-51.8248</td>
<td>-51.5716</td>
<td>-159.4644</td>
<td>-159.2130</td>
<td>22.07</td>
</tr>
<tr>
<td>284.5</td>
<td>-54.8351</td>
<td>-54.6039</td>
<td>-168.4953</td>
<td>-168.2962</td>
<td>22.10</td>
</tr>
<tr>
<td>402.4</td>
<td>-57.8454</td>
<td>-58.0629</td>
<td>-177.5262</td>
<td>-177.6571</td>
<td>22.11</td>
</tr>
<tr>
<td>569.0</td>
<td>-60.8557</td>
<td>-60.8903</td>
<td>-186.5571</td>
<td>-186.6209</td>
<td>22.00</td>
</tr>
<tr>
<td>804.8</td>
<td>-63.8660</td>
<td>-63.9500</td>
<td>-195.5880</td>
<td>-196.1037</td>
<td>21.81</td>
</tr>
<tr>
<td>1138.1</td>
<td>-66.8763</td>
<td>-66.8855</td>
<td>-204.6189</td>
<td>-204.6284</td>
<td>21.79</td>
</tr>
<tr>
<td>1609.5</td>
<td>-69.8866</td>
<td>-69.8694</td>
<td>-213.6498</td>
<td>-213.5399</td>
<td>21.69</td>
</tr>
<tr>
<td>2276.2</td>
<td>-72.8969</td>
<td>-72.9489</td>
<td>-222.6807</td>
<td>-222.8702</td>
<td>21.70</td>
</tr>
<tr>
<td>3219.0</td>
<td>-75.9072</td>
<td>-76.0084</td>
<td>-231.7116</td>
<td>-231.7877</td>
<td>20.88</td>
</tr>
<tr>
<td>4552.4</td>
<td>-78.9175</td>
<td>-79.1378</td>
<td>-240.7425</td>
<td>-240.7864</td>
<td>20.40</td>
</tr>
</tbody>
</table>

analytical method based on Gaussian assumption.

**Numerical brute-force algorithm for omni-protection region**

For simulating the brute-force method, we utilize parameters in Section 2.4, with $I_{\text{max}} = -50$ dBm. The steps described below are followed to calculate the optimal protection distance:

i Initially, pick $d(\theta) = 100$ km. For a given value of $d(\theta)$, a histogram of $I_{\text{aggr}}$ is generated for 1000 snapshots.
ii Using the $I_{\text{max}}$ value, the outage probability $P_{\text{out}} = P[I_{\text{aggr}} > I_{\text{max}}]$ is calculated and compared against $P_{\text{out, max}}$.

If $P_{\text{out}} \leq P_{\text{out, max}}$, then stop; the current value of $d(\theta)$ is the optimum $d_{\text{opt}}(\theta)$.

iii If $P_{\text{out}} > P_{\text{out, max}}$, then repeat experiment after incrementing $d(\theta)$ by $\Delta = 0.5$ km.

Figure 2.4 shows the variation of $P_{\text{out}}$ vs. $d(\theta)$, from which we observe that $P_{\text{out}}$ oscillates around its true value for averaging over 1000 snapshots. We then use a cubic spline function for smoothing with smoothing parameter of 0.001; and also tabulate the results in Table 2.2. The brute-force method takes extraordinary time for completion; in the next section, we utilize an iterative Monte-Carlo method that is significantly faster.

Figure 2.4: $P_{\text{out}}$ vs. $d(\theta)$ - Numerical brute force algorithm ($I_{\text{max}} = -50$dBm)

Iterative Monte Carlo simulation to determine $d_{\text{opt}}(\theta)$

In this section, we adopt an adaptive on-line algorithm for updating the values of $d(\theta)$ based on the Adam algorithm proposed in [27]. Adaptive moment estimation (Adam) computes an adaptive learning rates for each parameter by using an exponentially decaying average of
Table 2.2: Effect of smoothing in Figure 2.4

<table>
<thead>
<tr>
<th></th>
<th>$d_{opt}(\theta)$ (km)</th>
<th>$P_{out}$</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without smoothing</td>
<td>252.5</td>
<td>0.100</td>
<td>4082.4</td>
</tr>
<tr>
<td>With smoothing</td>
<td>269.9</td>
<td>0.098</td>
<td>4082.4</td>
</tr>
</tbody>
</table>

past squared gradients ($v_t$) and the past gradients ($m_t$). The algorithm steps are outlined below:

i The gradient is chosen as $g_t = P_{out} - P_{out,max}$.

ii Initialize: Stepsize $\alpha = 100$, $\beta_1 = 0.9$, $\beta_2 = 0.999$ and $\epsilon = 10^{-8}$.

iii The first moment vector ($m$), second moment vector ($v$) and timestep vector ($t$) are initialized as
$m \leftarrow 0, v \leftarrow 0, t \leftarrow 0$ and $d(\theta) = 10$ km.

iv **while** $d(\theta)$ has not converged, **do**

\[
t \leftarrow t + 1
\]

**run** Monte Carlo experiment 1000 times to obtain $P_{out-t}$

\[
g_t \leftarrow P_{out-t} - P_{out,max}
\]

\[
m_t \leftarrow \beta_1 m_{t-1} + (1 - \beta_1) g_t
\]

\[
v_t \leftarrow \beta_2 v_{t-1} + (1 - \beta_2) g_t^2
\]

\[
\alpha_t \leftarrow \alpha (1 - \beta_2^t)^{\frac{1}{\beta_1}}/(1 - \beta_1^t)
\]

\[
m_{\hat{t}} \leftarrow \beta_1 m_{t-1} + (1 - \beta_1) g_t
\]

\[
\hat{v}_t \leftarrow \beta_2 v_{t-1} + (1 - \beta_2) g_t^2
\]

\[
d(\theta)_t \leftarrow d(\theta)_{t-1} - \alpha_t \hat{m}_t/\left(\hat{v}_t\right)^{\frac{1}{2}} + \epsilon)
\]

**end while**

v **stop**: $d(\theta)_t$ is optimal protection distance
In order to check convergence at each iteration, a histogram of $I_{aggr}$ is generated, using 1000 snapshot results for a given value of $d(\theta)$. Using $I_{\text{max}}$ value, the outage probability ($P_{\text{out}} = P[I_{aggr} > I_{\text{max}}]$) is calculated and compared against $P_{\text{out,max}}$. If $|P_{\text{out}} - P_{\text{out,max}}| \leq \delta = 0.005$, then algorithm stops; keeping current value of $d(\theta)$ as $d_{\text{opt}}(\theta)$. Figure 2.5 shows the updating process of $d(\theta)$ in time (seconds) from initialization to convergence. The benefits, in terms of time to convergence, of using the iterative update is captured in Table 5 which shows very significant gains.

![Figure 2.5: Update of $d(\theta)$ vs. Time (s) - Iterative Monte Carlo simulation ($I_{\text{max}} = -50$dBm)](image)

Table 2.3: Comparison of processing time for finding $d_{\text{opt}}$

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>$d_{\text{opt}}(\theta)$ (km)</th>
<th>$P_{\text{out}}$</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numerical brute-force</td>
<td>269.9</td>
<td>0.100</td>
<td>4082.4</td>
</tr>
<tr>
<td>Iterative Monte Carlo</td>
<td>268.8</td>
<td>0.099</td>
<td>66.2</td>
</tr>
</tbody>
</table>
**Gaussian assumption for distribution of $I_{aggr}$**

In [9], the distribution of $I_{aggr}$ was approximated as a normal distribution $N(\mu_I, \sigma_I^2)$, as it allows further inroads towards an analytical determination of the optimum protection region as summarized below. With the Gaussian assumption, the outage probability can be determined as

$$P_{outage} = \Pr\{I_{aggr} > I_{\text{max}}\} = Q\left(\frac{I_{\text{max}} - \mu_I}{\sigma_I}\right)$$

(2.22)

where $Q(.)$ function is the tail probability of the standard normal distribution. It is desired to set an upper bound for probability of outage, $P_{out,\text{max}}$, i.e.,

$$P_{outage} \leq P_{out,\text{max}} \Rightarrow Q\left(\frac{I_{\text{max}} - \mu_I}{\sigma_I}\right) \leq P_{out,\text{max}}$$

$$I_{\text{max}} \geq \mu_I + \sigma_I Q^{-1}(P_{out,\text{max}})$$

(2.23)

Here, the coexistence criteria is defined by (3.4). This inequality is trivially achieved by letting $d(\theta) \to \infty$ (apparent from (2.18) for example). In order to avoid this, we minimize the total protection area subject to net interference limit as formulated in following optimization problem:

$$d_{opt} = \arg \min_{d(\theta)} \int_{0}^{2\pi} d^2(\theta) d\theta$$

subject to:

$$\mu_I + \sigma_I Q^{-1}(P_{out,\text{max}}) \leq I_{\text{max}}$$

(2.25)

For a general antenna pattern $G(\theta)$, it is proven in the appendix that optimum protection distance $d_{opt}(\theta)$ is proportional to $G^{1/\alpha}(\theta)$ with a constant $\gamma$ that is determined by numerically solving following equation:

$$d_{opt}(\theta) = \gamma G^{\frac{1}{\alpha}}(\theta) \quad \text{where}$$

$$A\gamma^{2-\alpha} + B\gamma^{1-\alpha} - I_{\text{max}} = 0$$

(2.26)
in which $A$ and $B$ are determined by:

$$A = C_{\mu_1} \int_0^{2\pi} G_{\frac{\alpha}{2}}(\theta) d\theta$$

$$B = Q^{-1}(P_{\text{out, max}}) \sqrt{C_{\sigma^2} \int_0^{2\pi} G_{\frac{\alpha}{2}}(\theta) d\theta}$$

(2.27)

### 2.4.2.0.1 Comparison of the Gaussian assumption with Monte Carlo Simulation:

In order to evaluate accuracy of Gaussian assumption, here we compare calculated protection distance with that of Monte-Carlo simulations. The maximum interference level $I_{\text{max}}$ is varied from -70 dBm to -40 dBm to cover both large and small cases of protection distance. Table 2.4 compares resulting distances as well as simulated outage probability for both methods.

Table 2.4: Comparison of calculated protection distance and resulting outage probability for Gaussian versus Monte-Carlo

<table>
<thead>
<tr>
<th>$I_{\text{max}}$(dBm)</th>
<th>-40</th>
<th>-50</th>
<th>-60</th>
<th>-70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iterative Monte-Carlo</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d_{\text{opt}}$ km</td>
<td>41.41</td>
<td>261.49</td>
<td>654.7</td>
<td>1787.5</td>
</tr>
<tr>
<td>($P_{\text{out}}$)</td>
<td>(0.098)</td>
<td>(0.102)</td>
<td>(0.100)</td>
<td>(0.101)</td>
</tr>
<tr>
<td>Gaussian assumption</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d_{\text{opt}}$ km</td>
<td>112.08</td>
<td>262.45</td>
<td>659.0</td>
<td>1809.0</td>
</tr>
<tr>
<td>($P_{\text{out}}$)</td>
<td>(0.057)</td>
<td>(0.098)</td>
<td>(0.106)</td>
<td>(0.089)</td>
</tr>
</tbody>
</table>

Monte-Carlo method achieves the desired outage probability of $P_{\text{out}} \approx 0.1$ for all cases of $I_{\text{max}}$. For smaller values of $I_{\text{max}}$, Gaussian assumption results in similar $P_{\text{out}}, d_{\text{opt}}$, providing an accurate analytical solution. For large values such as $I_{\text{max}} = -40$ dBm, Gaussian assumption resulted in an outage probability of $P_{\text{out}} \approx 0.057$ that is significantly smaller than the allowed bound. Therefore, the corresponding protection distance is significantly larger than
Monte-Carlo method (112 km versus 41 km) \(^3\).

In Figures 2.6 and 2.7, we plot the histogram of \(I_{\text{agg}}\) obtained from the above simulation scenario (for two cases of \(I_{\text{max}} = -50, -70\) dBm), and compare with the Gaussian, Inverse Gaussian and Levy p.d.f. In [20], the Inverse Gaussian distribution is shown to be a better fit when compared to the Levy distribution in the case of active Wi-Fi nodes outside the protection region, with p.d.f given by (2.28).

\[
f_I(x) = \left[ \frac{\kappa}{2\pi x^3} \right]^{1/2} \exp \left( -\frac{\kappa(x - \nu)^2}{2\nu^2x} \right) \tag{2.28}\]

where \(E[I] = \nu\) and \(\text{var}(I) = \nu^3/\kappa\).

### 2.4.3 Poisson Cluster Process

In this and next sections, we consider Wi-Fi deployments that are spatially inhomogeneous point processes. Here, we have focused on Poisson Cluster Processes for spatial distribution

---

\(^3\)The approximate value from Gaussian method in such cases can be used to initialize the iterative Monte-Carlo simulation, which will then find the accurate value quickly.
of nodes. A cluster process is created using a parent process of density \( \lambda_p \) that defines the location of the clusters; and a child process that is created around each cluster point. The number of points in a cluster is a Poisson random variable with a mean of \( \bar{c} \). These points are scattered around the cluster center, with a Gaussian distribution of zero mean and variance of \( \zeta \).

Similar to previous results, we use a radar antenna pattern of \( G(\theta) = 10 \text{ dBi} \), along with \( \lambda_p = 10^{-7} \text{ km}^{-2}, \bar{c} = 100 \) and \( \zeta = 800 \) for the Poisson cluster (Thomas) process. Path-loss exponent is set to \( \alpha = 4 \), \( I_{\text{max}} = -60 \text{ dBm} \) and \( P_{\text{out, max}} = 0.1 \) is utilized. The simulations are conducted for an area with outer radius of 20,000 km and 1000 snapshots are used for averaging. The protection distance is still a circle with radius \( d_{\text{opt}}(\theta) \) and results from the fact that the spatial process, while inhomogeneous, exhibits radial symmetry of cluster/node locations (as in Figure 2.9) and all sources use omni-directional antenna pattern.

### 2.4.3.1 Campbell’s theorem verification

Similar to Section 2.4.1, Campbell’s theorem is verified numerically using new spatial distribution and for various protection distances. Figure 2.8, shows average interference measured at the radar receiver from Wi-Fi nodes with a Poisson cluster distribution. It presents a perfect match between what Campbell predicts for average interference, and our simulation results.

It is insightful to compare average interference from a Poisson cluster process with a corresponding homogeneous PPP. From (2.21), we know that the increase in average interference power, compared to the homogeneous PPP, is given by

\[
\frac{\bar{c}P_{SU}}{FDR(\Delta f)(4\pi\zeta^2)} \int_{R^2} G(\theta_x) l(||x||) \exp\left(-\frac{||x||^2}{4\zeta^2}\right) dx
\]

This value tends to 0 for large values of protection regions, therefore the increase in interference power due to clustering is negligible. We have simulated a cluster process with \( \lambda_p = 10^{-7} \) and \( \bar{c} = 2000 \), and for \( \zeta \in (0, 300] \); no visible difference was observed compared to corresponding homogeneous PPP.
The optimum protection region can be found using the same methods described for the homogeneous PPP. A fair comparison of the Thomas cluster process with the homogeneous PPP is made by setting $\lambda = \lambda_p \bar{c}$. Table (2.5) shows optimum protection regions, using the iterative Monte Carlo simulation, for both cases of spatial distributions. The resulting
distances are very close, which is in line with our previous observation about average $I_{aggr}$.

Table 2.5: Comparison of protection distance ($I_{\text{max}} = -60 \text{ dBm}, \lambda_p = 10^{-7}, \bar{c} = 10$)

<table>
<thead>
<tr>
<th>Point process</th>
<th>Intensity</th>
<th>$d_{\text{opt}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homogeneous PPP</td>
<td>$\lambda = \lambda_p \bar{c}$</td>
<td>662.8</td>
</tr>
<tr>
<td>Thomas cluster proc</td>
<td>$\lambda_p, \bar{c}$</td>
<td>663.2</td>
</tr>
</tbody>
</table>

2.4.4 **Inhomogeneous PPP**

In this section, we explore an inhomogeneous PPP that is not symmetric in azimuth angle. From [21], initially a homogeneous PPP with intensity $\lambda_0$ is simulated. Then, points are randomly removed with a *spatially dependant* probability $1 - \frac{\lambda(x)}{\lambda_0}$. This is done by assigning every point with a random number uniformly distributed in $[0, 1]$. Let random number $u_n$ be assigned to the $n^{th}$ point. The point is deleted if $u_n < 1 - \frac{\lambda(x)}{\lambda_0}$. The resulting point process has an intensity $\lambda(x)$ and a realization is shown in Figure 2.10. The resulting density function $\lambda(x)$ of the point process is shown in Figure 2.11.

For such an inhomogeneous point process, the protection region is found for small sectors assuming a homogeneous PPP. For a given sector, Campbell’s theorem is verified and a protection distance is obtained. This is repeated locally for all (sufficiently small) sectors to obtain the overall protection region. The result is shown in Figure 2.12 and compared with the protection region for a corresponding homogeneous PPP with intensity $\frac{\lambda_0}{2}$. In this case, the protection region is not circular. However, the total area of protected region for both cases is very similar, as shown in Table 2.6.

2.5 **Simulation Results for typical Radar gain pattern**

In this section, we investigate the case of realistic (non-omni) antenna gain patterns at the radar receiver, using the model in [19]. The maximum gain of the antenna (main lobe) is
Figure 2.10: One realization of the inhomogeneous PPP

Figure 2.11: $\lambda(x)$

typically in the range of $22 < G_{\text{max}} < 48$ dBi. The following piece-wise function describes
Figure 2.12: Protection region for the inhomogeneous PPP Wi-Fi deployment, compared with the homogeneous PPP Wi-Fi deployment ($\lambda_0 = 2 \times 10^{-6}, I_{\text{max}} = -50 \text{ dBm}$)

Table 2.6: Fair comparison with a homogeneous PPP having $\frac{\lambda_0}{2}$ intensity for $I_{\text{max}} = -50 \text{ dBm}$

<table>
<thead>
<tr>
<th>Poisson Point Process</th>
<th>Intensity</th>
<th>Area of protection region (sq. km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inhomogeneous</td>
<td>$\lambda(x)$</td>
<td>$2.2733 \times 10^5$</td>
</tr>
<tr>
<td>Homogeneous</td>
<td>$\frac{\lambda_0}{2}$</td>
<td>$2.2770 \times 10^5$</td>
</tr>
</tbody>
</table>

Antenna gain $G(\theta)$ versus azimuth angle:

$$
G(\theta) = \begin{cases} 
G_{\text{max}} - 0.0004 \times 10^{G_{\text{max}}/10} \theta^2 & \theta \in [0, \theta_M] \\
0.75G_{\text{max}} - 7 & \theta \in [\theta_M, \theta_R] \\
53 - G_{\text{max}}/2 - 25 \log(\theta) & \theta \in [\theta_R, \theta_B] \\
11 - \frac{G_{\text{max}}}{2} & \theta \in [\theta_B, 180]
\end{cases}
$$

(2.29)

where $\theta_M = 50\sqrt{0.25G_{\text{max}} + \frac{7}{10}G_{\text{max}}/20}$, $\theta_R = 250/10^{G_{\text{max}}/20}$ and $\theta_B = 48$. Figure 2.13 shows antenna gain versus azimuth with a main lobe value of $G_{\text{max}} = 33.5 \text{ dBi}$.

Following the same approach as the previous sections, the results for the homogeneous PPP are used as a baseline for the non-omni gain pattern. It is important to note that the protection region is now not radially symmetric. It is larger along the main beam direction.
of the radar, and significantly smaller in the other directions, as in Figure 2.14. Clustering in
the deployment geometry by itself has little effect on the protection region (same as what was
observed in Section 2.4.3) but inhomogeneous PPP models that are not radially symmetric
does shift the protection region towards the denser portion of the Wi-Fi deployment, as is
intuitively sensible.

2.5.1 Homogeneous PPP

We have calculated protection distance $d_{opt}(\theta)$, using antenna pattern provided in (2.29).
Table 2.7 provides simulation results for different cases of $I_{max}$. Comparing theoretical
mean from Campbell (2.15) with sample averages verifies Campbell theorem for the case
of directional antenna as well. In addition, Table 2.7 shows outage probability, mini-
mum/maximum protection distance $d_{opt}$ (corresponding to minimum/maximum $G(\theta)$) for
both cases of Monte-Carlo simulation as well as Gaussian assumption method.

Figures 2.14 and 2.15 presents $d_{opt}(\theta)$ as a function of azimuth for two cases of $I_{max} =
-50, -70$ dBm. In each figure, Monte-Carlo results are compared with analytical results
from Gaussian assumption. We can see that for $I_{max} = -60, -70$dBm, both methods have

![Figure 2.13: Radar antenna gain vs. Azimuth](image)
very similar outcomes and Gaussian assumption provides an accurate estimate of $d_{opt}(\theta)$. However, for $I_{\text{max}} = -50$, Gaussian assumption leads to a more conservative protection region with larger distances and lower outage probability.

<table>
<thead>
<tr>
<th>$I_{\text{max}}$</th>
<th>-50 dBm</th>
<th>-60 dBm</th>
<th>-70 dBm</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Iterative Monte-Carlo</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d_{opt}$ km (mainlobe)</td>
<td>509.14</td>
<td>2055</td>
<td>5173.1</td>
</tr>
<tr>
<td>$d_{opt}$ km (sidelobe)</td>
<td>53.16</td>
<td>214.6</td>
<td>540.1</td>
</tr>
<tr>
<td>$(P_{\text{out}})$</td>
<td>(0.100)</td>
<td>(0.103)</td>
<td>(0.104)</td>
</tr>
<tr>
<td><strong>Gaussian assumption</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d_{opt}$ km (mainlobe)</td>
<td>845.75</td>
<td>2023.9</td>
<td>5242.8</td>
</tr>
<tr>
<td>$d_{opt}$ km (sidelobe)</td>
<td>88.32</td>
<td>211.31</td>
<td>547.41</td>
</tr>
<tr>
<td>$(P_{\text{out}})$</td>
<td>(0.057)</td>
<td>(0.125)</td>
<td>(0.098)</td>
</tr>
<tr>
<td><strong>Theoretical mean (dBm)</strong></td>
<td>-55.76</td>
<td>-63.33</td>
<td>-71.60</td>
</tr>
<tr>
<td><strong>Equation (2.15)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Sample mean (dBm)</strong></td>
<td>-56.38</td>
<td>-63.26</td>
<td>-71.63</td>
</tr>
</tbody>
</table>

The conservative result from Gaussian assumption can also be understood from a different point of view, notably p.d.f. of $I_{\text{aggr}}$. Figure 2.16 and 2.17 shows histogram of $I_{\text{aggr}}$ (calculated from simulation results) and compares that with Gaussian and Inverse-Gaussian p.d.f. (with same mean and variance). It is observed that Gaussian p.d.f mimics $I_{\text{aggr}}$ histogram with very high accuracy, for the case of $I_{\text{max}} = -70$ dBm. On the other hand, Gaussian p.d.f is a very poor approximation for the histogram in case of $I_{\text{max}} = -50$ dBm. Inverse-Gaussian (2.28), however, presents a very accurate model for p.d.f. of $I_{\text{aggr}}$ in both cases.

For smaller protection distances (where Gaussian is a poor model), the heavier tails in
For $I_{\text{max}} = -50 \text{ dBm}$, the Gaussian assumption is not a good approximation for distribution of $I_{\text{aggr}}$.

The distribution of $I_{\text{aggr}}$ arise from the contributions to the aggregate interference power from Wi-Fi users that are at/close to the boundary of the protection region, which are possibly orders of magnitudes larger than the far-off sources. As the protection region increases, the effect of the interference from Wi-Fi sources in the vicinity of the protection region (relative to the rest) reduces and the distribution becomes less heavy-tailed or more symmetric.

### 2.5.2 Inhomogeneous PPP

Following the approach in Section 2.4.4, the protection region for an inhomogeneous distribution of Wi-Fi nodes, and with a directional antenna pattern at radar, is calculated and plotted in Figure 2.18. The result is compared with the protection region of a homogeneous PPP with intensity $\frac{\lambda_0}{2}$. Intensity $\frac{\lambda_0}{2}$ is chosen so that the average number of Wi-Fi APs in both processes (homogeneous and inhomogeneous) are equal. The computed protection
Figure 2.15: For $I_{\text{max}} = -70$ dBm, the Gaussian assumption is a good approximation for distribution of $I_{\text{aggr}}$.

Figure 2.16: Gaussian fit is not a good approximation

Figure 2.17: Gaussian fit is a reasonable approximation

region area for both cases is shown in Table 2.8.
Figure 2.18: Protection region for the inhomogeneous PPP Wi-Fi deployment, compared with the homogeneous PPP Wi-Fi deployment ($\lambda_0 = 2 \times 10^{-6}$, $I_{\text{max}} = -60$ dBm)

Table 2.8: Fair comparison with a homogeneous PPP having $\frac{\lambda_0}{2}$ intensity

<table>
<thead>
<tr>
<th>Poisson Point Process</th>
<th>Intensity</th>
<th>Area of protection region (sq. km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inhomogeneous</td>
<td>$\lambda(x)$</td>
<td>4.1664x10^5</td>
</tr>
<tr>
<td>Homogeneous</td>
<td>$\frac{\lambda_0}{2}$</td>
<td>4.1105x10^5</td>
</tr>
</tbody>
</table>

2.6 Statistics of optimal protection region

Due to the simulation model used and the stochastic nature of the Wi-Fi deployments, the statistics of the protection region need to be characterized.
2.6.1 Numerical Brute force

For simulating the brute-force method, we utilize parameters in Section 2.4, with $I_{\text{max}} = -50$ dBm. The algorithm is shown below:

i Initially, pick $d(\theta) = 100$ km. For a given value of $d(\theta)$, a histogram of $I_{\text{aggr}}$ is generated for 1000 snapshots.

ii Using the $I_{\text{max}}$ value, the outage probability $P_{out} = P[I_{\text{aggr}} > I_{\text{max}}]$ is calculated and compared against $P_{out,\text{max}}$.
   - If $P_{out} \leq P_{out,\text{max}}$, then stop; the current value of $d(\theta)$ is the optimum $d_{\text{opt}}(\theta)$.
   - If $P_{out} > P_{out,\text{max}}$, then repeat experiment after incrementing $d(\theta)$ by $\Delta = 0.5$ km.

iii The above steps are repeated to plot a histogram as shown in Figure 2.19.

The above method is repeated and the distribution of the protection region is found for the non-omni gain pattern described in Section 1.

2.6.1.1 Distribution of protection region

The protection region is calculated 2000 times for $I_{\text{max}} = -50$ dBm and $\lambda = 10^{-6}$. This is shown in Figure 2.19. It was found that $d_{\text{opt}}$ has a Gaussian fit, and therefore can be characterized as a Gaussian random variable.

2.6.2 Iterative Monte Carlo simulations

In this section, we adopt an adaptive on-line algorithm for updating the values of $d(\theta)$ based on the Adam algorithm proposed in [27]. Adaptive moment estimation (Adam) computes an adaptive learning rates for each parameter by using an exponentially decaying average of past squared gradients ($v_t$) and the past gradients ($m_t$). The algorithm steps are outlined below:

i The gradient is chosen as $g_t = P_{out} - P_{out,\text{max}}$. 
Figure 2.19: Numerical Brute Force: P.D.F. of $d_{opt}$ with mean 637.64 km and standard deviation 74.41 km

Figure 2.20: C.D.F. of $d_{opt}$

ii Initialize: Stepsize $\alpha = 50$, $\beta_1 = 0.9$, $\beta_2 = 0.999$ and $\epsilon = 10^{-8}$.

iii The first moment vector ($m$), second moment vector ($v$) and timestep vector ($t$) are initialized as

$m \leftarrow 0, v \leftarrow 0, t \leftarrow 0$ and $d(\theta) = 10$ km.

iv while $d(\theta)$ has not converged, do

$t \leftarrow t + 1$

run Monte Carlo experiment 10000 times to obtain $P_{out-t}$

$g_t \leftarrow P_{out-t} - P_{out,\text{max}}$

$m_t \leftarrow \beta_1 m_{t-1} + (1 - \beta_1)g_t$

$v_t \leftarrow \beta_2 v_{t-1} + (1 - \beta_2)g_t^2$

$\alpha_t \leftarrow \alpha(1 - \beta_2)^{t}/(1 - \beta_1^t)$

$m_t \leftarrow \beta_1 m_{t-1} + (1 - \beta_1)g_t$

$v_t \leftarrow \beta_2 v_{t-1} + (1 - \beta_2)g_t^2$

$d(\theta)_t \leftarrow d(\theta)_{t-1} - \alpha_t m_t / (v_t)^{t/2} + \epsilon$

end while
v stop: \( d(\theta) \) is optimal protection distance

In order to check convergence at each iteration, a histogram of \( I_{\text{aggr}} \) is generated, using 10000 snapshot results for a given value of \( d(\theta) \). Using \( I_{\text{max}} \) value, the outage probability \( (P_{\text{out}} = P[I_{\text{aggr}} > I_{\text{max}}]) \) is calculated and compared against \( P_{\text{out, max}} \). If \(|P_{\text{out}} - P_{\text{out, max}}| \leq \delta = 0.001\), then algorithm stops; keeping current value of \( d(\theta) \) as \( d_{\text{opt}}(\theta) \).

2.6.2.1 Distribution of protection region

The protection region is calculated 5000 times for \( I_{\text{max}} = -50 \text{ dBm} \) and \( \lambda = 10^{-6} \). This is shown as a histogram in Figure 2.21.

![Histogram of Protection region](image)

Figure 2.21: Iterative Monte Carlo: P.D.F of \( d_{\text{opt}} \) with mean 645.65 km and standard deviation 27.25 km

2.7 WLAN Network Activity and its Implications

2.7.1 Part 1 - All co-channel WLAN Networks are non-overlapping

The number of Wi-Fi nodes associated with an access point affects the WLAN network activity. In order to model network activity, some assumptions are made:
(a) The number of nodes associated with an AP is fixed, and each node always has a packet for transmission (saturation condition).

(b) Regardless of the number of retransmissions that a packet may have already suffered, the collision probability of a packet transmitted by each node is assumed to be independent and constant.

(c) Packet errors occur only due to transmission collisions, i.e. the channel conditions are ideal. Also, there are no hidden nodes considered in the analysis.

To use these results, the access points must be distributed such that no two co-channel cells overlap with high probability.

A stationary Poisson point process $\Phi$ of intensity $\lambda_{AP}$ is characterized by

$$P(\Phi(A) = n) = \frac{(\lambda_{AP}|A|)^n}{n!} e^{-\lambda_{AP}|A|}$$  \hspace{1cm} (2.30)

Let

$$P(\Phi(\pi r_{CS}^2) > 1) < 0.1$$  \hspace{1cm} (2.31)

where $r_{CS}$ is the carrier sense range of an access point.

Carrier sensing range is the range within which the other stations can sense transmitted power. From the IEEE standard for 802.11g, a receiver sensitivity of -87 dBm is chosen as the threshold for carrier sense range. For a free-space path loss exponent 4, the carrier sense range is 266 m.

This implies

$$P(\Phi(\pi r_{CS}^2) = 0) + P(\Phi(\pi r_{CS}^2) = 1) > 0.9$$  \hspace{1cm} (2.32)

$$e^{-\lambda_{AP}\pi r_{CS}^2} + \lambda_{AP}\pi r_{CS}^2 e^{-\lambda_{AP}\pi r_{CS}^2} > 0.9$$  \hspace{1cm} (2.33)

On solving we get

$$\lambda_{AP} < \frac{-W_{-1}(\frac{-9}{10}) - 1}{\pi r_{CS}^2}$$  \hspace{1cm} (2.34)
where $W_k(z)$ is the analytic continuation of the product log function (Lambert $W$ function).

From [22], the probability $\tau$ that a station transmits in a randomly chosen time slot is found. For simplicity, the constant backoff window problem is considered. That is, the number of backoff stages $m = 0$. In this case,

$$\tau = \frac{2}{W + 1} \quad (2.35)$$

where $W$ is the contention window.

In every Wi-Fi network, let $P_{tr}$ be the probability that there is at least one transmission in a randomly chosen time slot. If $n$ stations contend on the channel, and each transmits with probability $\tau$,

$$P_{tr}(n) = 1 - (1 - \tau)^n \quad (2.36)$$

From the perspective of coexistence between a radar and Wi-Fi networks, the number of

Figure 2.22: $P_{tr}$ vs $n$
transmissions in a Wi-Fi network directly impact the total interference at the radar receiver. By increasing the number of clients associated with a Wi-Fi AP, the probability that there is at least one transmission in a randomly chosen time slot is given in Figure 2.22.

2.7.1.1 Characterization of aggregate interference at Radar

The aggregate interference from secondary users to radar (located at \( y \)) is described as a generalized shot noise process in space as shown in (2.37). It is assumed that every AP is independently active with probability \( p \). In this thesis, this probability is characterized according to (2.36).

\[
I_{aggr}(y) = \sum_{x \in \Phi} \frac{P_x G(\theta_x)}{FDR(\Delta f)} l(||y - x||)
\]

where \( P_x \) is the SU transmit power at AP location \( x \) and \( ||y - x|| \) describes the distance between radar and secondary access point. The transmit power for all Wi-Fi networks is assumed to be fixed, \( P_x = P_{SU} \). \( l(.) \) is the impulse response function that models signal attenuation (inverse of path loss). For a PPP of density \( \lambda \), the mean and variance of the sum \( \sum_{x \in \Phi} f(x) \) is calculated from Campbell’s theorem as

\[
E[\sum_{x \in \Phi} f(x)] = \lambda \int_{\mathbb{R}^d} f(x) dx \tag{2.38}
\]

\[
var[\sum_{x \in \Phi} f(x)] = \lambda \int_{\mathbb{R}^d} f^2(x) dx \tag{2.39}
\]

The average interference is calculated using (8) by integrating over the two dimensional plane. It is assumed that the radar receiver is at the origin, i.e. \( y = 0 \).

\[
\mu_{I_{aggr}} = E[I_{aggr}] = \frac{p \lambda P_{SU}}{FDR(\Delta f)} \int_{\mathbb{R}^2} G(\theta_x) l(||x||) dx \tag{2.40}
\]

Here, each network is independently active with probability \( p \). In this thesis, this is calculated according to (7).

A minimum separation distance between radar and SU as a function of \( \theta \), \( d(\theta) \). Using polar coordinates and considering Longley Rice path loss model \( l(r) = K_0 r^{-\alpha} \),

\[
\mu_{I_{aggr}} = \frac{p \lambda P_{SU} K_0}{FDR(\Delta f)} \int_{\theta} \int_{d(\theta)}^\infty G(\theta) r^{1-\alpha} dr d\theta = \frac{p \lambda P_{SU} K_0}{FDR(\Delta f)(\alpha - 2)} \int_{\theta} G(\theta) d^{2-\alpha}(\theta) d\theta \tag{2.41}
\]
Similarly, the variance is calculated as
\[
\sigma_{\text{aggr}}^2 = \frac{p\lambda P^2_{\text{SU}} K_0^2}{\text{FDR}^2(\Delta f)} \int \int G^2(\theta) r^{1-\alpha} dr d\theta = \frac{p\lambda P^2_{\text{SU}} K_0^2}{\text{FDR}^2(\Delta f)(2\alpha - 2)} \int G^2(\theta) d^{2-\alpha}(\theta) d\theta \tag{2.42}
\]

The Wi-Fi APs are represented by a PPP $\Phi$ with density $\lambda_{\text{AP}}$ and is characterized below.

\[
P(\Phi(A) = n) = \frac{(P_{\text{tr}}(M)\lambda_{\text{AP}}|A|)^n}{n!} e^{-P_{\text{tr}}(M)\lambda_{\text{AP}}|A|} \tag{2.43}
\]

The number of stations associated with an access point is a Poisson random variable $M$ with intensity $\lambda_{\text{node}}$. (2.7) is re-written for $P_{\text{tr}}(M)$ as

\[
P_{\text{tr}}(M) = 1 - (1 - \tau)^M \tag{2.44}
\]

The interference caused by one Wi-Fi network at the radar receiver is given by

\[
I(y|M = m) = \frac{P_{\text{tr}}(m)P_x G(\theta_x)}{\text{FDR}(\Delta f)} I(||y - x||) \tag{2.45}
\]

where $m \in M$ and $x \in \Phi$.

Therefore, (2.37) is re-written as

\[
I_{\text{aggr}}(y) = \sum_{m \in M} P(M = m)I(y|M = m) \tag{2.46}
\]

where

\[
P(M = m) = \frac{(\lambda_{\text{node}})^m}{m!} e^{-\lambda_{\text{node}}} \tag{2.47}
\]

By conditioning on $M$ in (2.43) we get

\[
P(\Phi(A) = n|M = m) = \frac{(1 - (1 - \tau)^m)\lambda_{\text{AP}}|A|)^n}{n!} e^{-(1-(1-\tau)^m)\lambda_{\text{AP}}|A|} \tag{2.48}
\]

To express (2.43) using the above equation,

\[
P(\Phi(A_1) = n) = \sum_{m=0}^{\infty} P(\Phi(A_1) = n|M = m)P(M(A_2) = m) \tag{2.49}
\]

where $A_1$ is the area considered around the radar (city or town) and $A_2$ is the circular area around the access point with radius $r_{CS}$ (i.e. $A_2 = \pi r_{CS}^2$).

\[
P(\Phi(A_1) = n) = \sum_{m=0}^{\infty} \frac{(1 - (1 - \tau)^m)\lambda_{\text{AP}}|A_1|)^n}{n!} e^{-(1-(1-\tau)^m)\lambda_{\text{AP}}|A_1|} \frac{\lambda_{\text{node}}|A_2|^m}{m!} e^{-\lambda_{\text{node}}|A_2|} \tag{2.50}
\]
2.7.2 Part 2: Overlapping co-channel WLAN Networks

In this subsection, we consider downlink traffic, i.e., packets sent from the APs to the associated nodes. We also assume that it is equally likely for the AP to transmit to all the nodes. In these scenarios, the location of the neighbouring APs determine the network activity. Consider Figure 2.23, the neighbouring APs are within the carrier-sense range. $P_{tr}$, i.e. the probability that there is at least one transmission in a randomly chosen time slot is given by (2.51) where $n_1$ and $n_2$ are the number of nodes (stations) associated with AP$_1$ and AP$_2$ respectively. Similar to the previous section, nodes contend on the channel, and each receives a packet with probability $\tau$,

$$P_{tr}(n) = 1 - (1 - \tau)^{n_1+n_2} \tag{2.51}$$

To find the average number of active links in a dense Wi-Fi AP deployments, the following algorithm is used:

i) Draw circles with radius equal to the carrier sense range by applying a suitable circle packing algorithm [23] in the considered 2-D space.
ii If APs lie inside a circle, then it is assumed that they are all within carrier sense range of each other.

iii If APs lie outside all the circles, they are associated to the nearest circle in their vicinity.

iv The number of circles with APs inside them correspond to the number of active links at a given time instant.

In the previous sections, we assumed that all the APs are simultaneously active at a given instant of time. By using the algorithm provided in this section we can find the number of Wi-Fi APs that have an active link. Figures 2.24, 2.25 and 2.26 show the above algorithm being used to find the number of active links for a given $\lambda_{AP}$. The $\lambda_{AP}$ values are relevant with real-world datasets as explained in the next section.

\textit{Wi-Fi APs within carrier sense range} $\lambda_{AP} = 10^2$

![Diagram of Wi-Fi APs within carrier sense range](image)

Figure 2.24: On average, only 5% of the APs are active
2.8 Real-world example

A sophisticated software program was designed to compute protection regions in the US. The determination of protection regions is impacted by a multiplicity of factors on both the secondary (notably, the transmit power, deployment geometry, transmit antenna patterns) and primary (receiver sensitivity and operational margins, receive antenna patterns) side as well as the channel characteristics (distance and frequency dependant pathloss, and any statistical effects such as small-scale fading and large-scale shadowing).

2.8.1 Path loss model and Propagation losses

In this analysis, the channel characteristics were modeled using the Longley-Rice path loss model [24]. The topological terrain was taken from SRTM-3, an international research effort that resulted in the most high-resolution digital topological database of Earth. Using the Longley Rice model would therefore provide a very accurate computation of the path loss. The propagation losses considered are building attenuation losses mentioned in [12]. It is assumed that 90 percent of the APs and associated clients will operate indoors. The transmit
power for outdoor and indoor APs is chosen as 30 dBm and 23 dBm respectively. For the indoor APs, a building attenuation loss in addition to the propagation loss was used in the protection region computation. The following random distribution was used to represent the building attenuation loss in the analysis: 20 percent were assigned a building attenuation loss of 20 dB; 60 percent were assigned a building attenuation loss of 15 dB; and 20 percent were assigned a building attenuation loss of 10 dB.

2.8.2 Location of WLANs and Network activity

Various open-source databases [25] provide locations (in terms of latitudes and longitudes) of WLANs across the world. These databases are populated by known research efforts and mobile applications that send AP information for users. Using the datasets, the locations of the active APs in the US was used for the protection region computation. Around 9 million locations were considered for the US as shown in Figure 2.27 using the Mercator projection [26]. Since we are interested in the number of Wi-Fi APs that are co-channel to the Radar, the channel used by the Wi-Fi APs is crucial. The dataset has an aggregation of APs in all channels, and the percentage of APs in the given channels are shown in Table 2.9. By considering WLANs in the 2.4 GHz bands (Channels 1, 6 and 11), we assume that 20% of the APs are co-channel to the Radar.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Percentage of Wi-Fi APs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21.79</td>
</tr>
<tr>
<td>6</td>
<td>21.75</td>
</tr>
<tr>
<td>11</td>
<td>19.66</td>
</tr>
<tr>
<td>Other</td>
<td>26.69</td>
</tr>
<tr>
<td>Unknown</td>
<td>10.11</td>
</tr>
</tbody>
</table>
Using the location of the Wi-Fi APs, $\lambda_{AP}$ values were estimated for various areas along the West coast of the US. This is shown in Table 2.10. The main takeaway from this analysis is that if WLANs are deployed to coexist with Radars, the density of these deployments can be similar to the density of networks currently being operated in the 2.4 GHz bands.

2.8.3 Determination of Protection region

The determination of the protection region is deterministic for a given main beam direction of the Radar for a given deployment. Using the given dataset and the algorithms described in this chapter, a realistic computation of the protection region can be obtained by assuming that the location and main-beam direction of the Radar is known. Then, the mobile operators can synchronize with WLANs across cities to operate based on the protection regions specified. In reality, the location and mainbeam direction of the Radar is not known. In the next chapter, we estimate these unknown parameters.
Table 2.10: $\lambda_{AP}$ values along the West coast

<table>
<thead>
<tr>
<th>Area</th>
<th>Type of area</th>
<th>$\lambda_{AP}$ (per sq. km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>San Francisco (SF)</td>
<td>Dense Urban</td>
<td>$1.45 \times 10^2$</td>
</tr>
<tr>
<td>Seattle (SEA)</td>
<td>Dense Urban</td>
<td>$8.12 \times 10^1$</td>
</tr>
<tr>
<td>Los Angeles (LA)</td>
<td>Dense Urban</td>
<td>$8.35 \times 10^1$</td>
</tr>
<tr>
<td>Portland</td>
<td>Dense Urban</td>
<td>$1.12 \times 10^1$</td>
</tr>
<tr>
<td>SF Bay area</td>
<td>Urban</td>
<td>$5.01 \times 10^0$</td>
</tr>
<tr>
<td>SEA Metropolitan</td>
<td>Urban</td>
<td>$6.05 \times 10^0$</td>
</tr>
<tr>
<td>LA Metropolitan</td>
<td>Urban</td>
<td>$4.08 \times 10^0$</td>
</tr>
<tr>
<td>Coastline between SF and LA</td>
<td>Suburban</td>
<td>$4 \times 10^{-2}$</td>
</tr>
<tr>
<td>Coastline between SF and Portland</td>
<td>Suburban</td>
<td>$2 \times 10^{-3}$</td>
</tr>
<tr>
<td>Washington (excluding Seattle Metropolitan area)</td>
<td>Suburban</td>
<td>$7.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>Oregon and Idaho (excluding Portland and coastline)</td>
<td>Rural</td>
<td>$6.9 \times 10^{-4}$</td>
</tr>
<tr>
<td>Nevada (excluding Las Vegas)</td>
<td>Rural</td>
<td>$6.3 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

Chapter 3

ESTIMATION OF RADAR PARAMETERS USING WI-FI USERS

3.1 Introduction

Radars are mainly used for military, weather and air-traffic control. Due to the sensitive nature of Radar operation, the instantaneous location and main beam direction are not revealed. In Chapter 2, all the studies rely on the assumption that these parameters are known. Here, a method to predict these parameters and characterize the implications on the protection region are discussed. The assumptions made here are that there is no synchronization between the Radar and WLANs.

3.2 Prediction of Main Beam Direction

3.2.1 System Set-up and Dataset generation

The Radar is placed at a known location. Around it, the Wi-Fi users are scattered according to a homogeneous Poisson point process as shown in Figure 3.1.
Figure 3.1: Homogeneous PPP with intensity $\lambda = 10^{-2}$

The antenna gain and Wi-Fi specifications are taken from (2.29) and [9]. Figure 3.3 shows antenna gain versus azimuth with a main lobe value of 33.5 dBi.

At the Wi-Fi receiver, the radar signal is treated as interference. The effective interference-to-noise ratio at the receiver is calculated according to (3.1) whose values are taken from [28].

$$\text{INR}_{\text{eff}} = 10 \log_{10} \left( \frac{PW f_R P_T G_w G(\theta) l(r)}{N_0 BW} \right)$$  \hspace{1cm} (3.1)

where $l(d) = K_0 r^{-\alpha}$ is the path loss model between radar and Wi-Fi receiver [9], $d$ is the distance between radar and Wi-Fi receiver, $G_w$ is the antenna gain of the Wi-Fi access point and $BW$ is the Wi-Fi channel bandwidth. $P_T$ is the power of the transmitted radar signal and $f_R$ is the pulse repetition frequency.

For a given main beam direction, the Wi-Fi users detect the Radar signal during normal operation, with a probability of detection given in [28]. This is shown in Figure 3.2. By considering the probability of detection during normal operation of Wi-Fi, we assume the worst-case scenario in terms of the number of users that detect the signal. If Wi-Fi users were idle and sensing the channel for Radar transmissions, the probability of detection will be higher.
The algorithm steps are outlined below:

i A random Radar location and main beam direction is chosen.

ii Using the antenna gain in (2.29), we compute the INR$_{eff}$ at every Wi-Fi receiver using (3.1).

iii To choose the subset of all Wi-Fi APs that have detected the Radar signal, the following decision is made. An AP is chosen with a probability of detection given by the results in [28], i.e., Figure 2. Once this operation is complete, we have the set of all Wi-Fi receivers that have detected the Radar signal.

iv The true location of the Radar, the true main beam direction of the Radar and the location of all the Wi-Fi APs form one dataset. One realization of a dataset is shown in Figure 3.4.

v The above operation is repeated 10000 times to obtain a dataset with 10000 realizations.
At a given time instant, the set of all Wi-Fi users that detect the Radar signal report their locations to a data center. One realization of this is shown in Figure 3.4.

The data center implements the technique described in this paper, using the location of these users.

Figure 3.3: Radar Antenna Gain pattern with $\theta_M = -\frac{\pi}{6}$
Figure 3.4: The Radar location, main beam direction and Wi-Fi user locations comprise the dataset

3.2.2 Simulation set-up

For a radar receiver located at the origin, Wi-Fi nodes are scattered according to a PPP with density $\lambda = 10^{-6}$. The total area of deployment is a circle with outer radius of 5,000 km.

Secondary user power (including its antenna gain) is set to $P_{su} = 1.0$ watt, in line with commercial Wi-Fi devices. We consider a path-loss component of $\alpha = 4$ and outage probability of $P_{out,max} = 0.1$.

3.2.3 Huber M-estimator

The objective function of the Huber M-estimator [29] is given below and shown in Figure 3.5.

$$\rho(e) = \begin{cases} 
\frac{1}{2}e^2 & |e| \leq k \\
ke - \frac{1}{2}k^2 & |e| > k 
\end{cases} \quad (3.2)$$

To choose the value of $k$, the following algorithm was used.

i Initialize $k = 0.1, \delta = 0.1$.

ii For a given realization of the dataset, the line of best fit is the line that minimizes the objective function $\rho$ in (3.2).

iii while $\rho$ has not been minimized, do

\[ k \leftarrow k + \delta \]
\[ t \leftarrow t + 1 \]

Compute minimum $(\rho)_t$

If $(\rho)_t < (\rho)_{t-1}$: $\rho \leftarrow (\rho)_t$

end while
iv stop: $\rho$ is minimized, and corresponding value of $k$ is used for every dataset.

Figure 3.5: Objective function for $k = 1$.

3.2.4 Error in Main beam direction

The error in main beam direction was computed by using the Huber M-estimator as the predicted direction of the main beam. The error is given by the angle between the true main beam direction and the predicted main beam direction. For 10000 observations, a histogram of the error was computed and modelled as a Gaussian random variable with mean -0.001 and $\sigma = 2.865$ degrees. It is shown in Figure 3.6.

Figure 3.6: Histogram of error
3.2.4.1 Gaussian assumption for distribution of $I_{aggr}$

In [9], the distribution of $I_{aggr}$ was approximated as a normal distribution $N(\mu_I, \sigma_I^2)$, as it allows further inroads towards an analytical determination of the optimum protection region as summarized below. With the Gaussian assumption, the outage probability can be determined as

$$P_{\text{outage}} = \Pr\{I_{aggr} > I_{\max}\} = Q\left(\frac{I_{\max} - \mu_I}{\sigma_I}\right)$$

(3.3)

where $Q(.)$ function is the tail probability of the standard normal distribution. It is desired to set an upper bound for probability of outage, $P_{out,\max}$, i.e.,

$$P_{\text{outage}} \leq P_{out,\max} \rightarrow Q\left(\frac{I_{\max} - \mu_I}{\sigma_I}\right) \leq P_{out,\max}$$

$$I_{\max} \geq \mu_I + \sigma_I Q^{-1}(P_{out,\max})$$

(3.4)

Here, the coexistence criteria is defined by (3.4). This inequality is trivially achieved by letting $d(\theta) \rightarrow \infty$. In order to avoid this, we minimize the total protection area subject to net interference limit as formulated in following optimization problem:

$$d_{opt} = \arg \min_{d(\theta)} \frac{1}{0} d^2(\theta) d\theta$$

(3.5)

subject to:

$$\mu_I + \sigma_I Q^{-1}(P_{out,\max}) \leq I_{\max}$$

(3.6)

For a general antenna pattern $G(\theta)$, it is proven in the appendix that optimum protection distance $d_{opt}(\theta)$ is proportional to $G^{1/\alpha}(\theta)$ with a constant $\gamma$ that is determined by numerically solving following equation:

$$d_{opt}(\theta) = \gamma G^{\frac{1}{\alpha}}(\theta)$$

where

$$\mathcal{A} \gamma^{2-\alpha} + \mathcal{B} \gamma^{1-\alpha} - I_{\max} = 0$$

(3.7)

(3.8)

in which $\mathcal{A}$ and $\mathcal{B}$ are determined by:

$$\mathcal{A} = C_{\mu_I} \int_{0}^{2\pi} G^{\frac{1}{\alpha}}(\theta) d\theta$$

$$\mathcal{B} = Q^{-1}(P_{out,\max}) \sqrt{C_{\sigma_I^2} \int_{0}^{2\pi} G^{\frac{2}{\alpha}}(\theta) d\theta}$$

(3.9)
3.2.5 Distribution of $d_{\text{opt}}(\theta)$

Let the error in the main beam direction be a Gaussian random variable $\epsilon \sim \mathcal{N}(0, \sigma_m^2)$ given in Figure 3.6. In the presence of $\epsilon$, (3.7) is re-written as

$$d_{\text{opt}}(\theta + \epsilon) = \gamma G^{\frac{1}{\alpha}}(\theta + \epsilon)$$  \hfill (3.10)

Using Taylor series expansion of $G^{\frac{1}{\alpha}}(\theta + \epsilon)$ we get

$$d_{\text{opt}}(\theta + \epsilon) = \gamma \left[ G(\theta) + \epsilon G^{(1)}(\theta) + \frac{\epsilon^2}{2} G^{(2)}(\theta) + \frac{\epsilon^3}{6} G^{(3)}(\theta) + O(h^4) \right]^{\frac{1}{\alpha}}$$  \hfill (3.11)

$$d_{\text{opt}}(\theta + \epsilon) \simeq \gamma \left[ G(\theta) + \epsilon G^{(1)}(\theta) \right]^{\frac{1}{\alpha}}$$  \hfill (3.12)

by approximating to the first-degree. Further, by using Binomial approximation we get

$$d_{\text{opt}}(\theta + \epsilon) = \gamma G^{\frac{1}{\alpha}}(\theta) \left[ 1 + \epsilon \frac{G^{(1)}(\theta)}{G(\theta)} \right]^{\frac{1}{\alpha}}$$  \hfill (3.13)

$$d_{\text{opt}}(\theta + \epsilon) \simeq \gamma G^{\frac{1}{\alpha}}(\theta) \left[ 1 + \frac{\epsilon}{\alpha} \frac{G^{(1)}(\theta)}{G(\theta)} \right]$$  \hfill (3.14)

for $\alpha > 1$ and $G^{(1)}(\theta) < G(\theta)$.

This implies that $d_{\text{opt}}(\theta + \epsilon)$ has a Gaussian distribution, i.e.

$$d_{\text{opt}}(\theta + \epsilon) \sim \mathcal{N} \left( d_{\text{opt}}(\theta), \frac{\sigma_m}{\alpha} \frac{G^{(1)}(\theta)}{G(\theta)} \right)^2$$

3.3 Prediction of Location

The simulation set-up is described in the previous section. Here, using the predicted main beam direction of the Radar, a prediction for the location is made.

3.3.1 Dataset

The dataset used to evaluate the error in location of the Radar is same as the previous section (Figure 3.4).
3.3.2 Two regression lines at random angles

If we draw two regression lines using the m-estimator at random angles, the point of intersection of these lines is chosen as the location of the Radar.

The predicted location of the radar has a 2-D Gaussian distribution with

\[
\mu = \begin{bmatrix} -0.16 \\ -0.11 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 95.70 & 0.28 \\ 0.28 & 97.91 \end{bmatrix}
\]

where the \( \begin{bmatrix} 0 \\ 0 \end{bmatrix} \) is the true location of the radar. A histogram of the distance to the points from the mean of the 2-D Gaussian distribution is plotted in Figure 3.8.

3.3.3 N regression lines at random angles

By choosing angles randomly, multiple lines drawn may be parallel to each other (Figures 3.9 and 3.10). This will increase the location error. To quantify this, we use the root-mean squared error (RMSE). Using the RMSE as the performance metric, we find the optimal
angle to choose between any two lines. It is found that 90 degrees is the optimal angle when we choose to construct two lines. This is shown in Figure 3.11.

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (y_i - \hat{y}_i)^2}$$

where $y_i$ and $\hat{y}_i$ are the location of the Radar and predicted location of the Radar.

3.3.4 $N$ regression lines at $\frac{\pi}{N}$ angles

By choosing the lines at $\frac{\pi}{N}$ angles, we improve the accuracy of our prediction of the Radar location. This is shown in Figure 3.15. Further, we compare the results obtained in this section with the previous section in Table 3.1, i.e. to compare the effect of choosing angles at $\frac{\pi}{N}$ radians, versus choosing the angles at random.

3.3.5 Protection region due to location error

The implication of the location error on the average protection region was shown using simulations in Figure 3.16. The following algorithm was used to determine the same.
Figure 3.11: RMSE of location estimator vs. Angle between the two lines.

Table 3.1: RMSE comparison

<table>
<thead>
<tr>
<th>Number of lines</th>
<th>RMSE (km) Lines at random angles</th>
<th>RMSE (km) Lines at $\frac{\pi}{N}$ radians</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>8.592</td>
<td>2.516</td>
</tr>
<tr>
<td>3</td>
<td>7.150</td>
<td>2.108</td>
</tr>
<tr>
<td>5</td>
<td>5.385</td>
<td>1.760</td>
</tr>
<tr>
<td>10</td>
<td>3.316</td>
<td>1.491</td>
</tr>
</tbody>
</table>
Figure 3.12: Three lines - RMSE vs Angles between successive lines. 60 degrees is the optimal angle

Figure 3.13: Four lines - RMSE vs Angles between successive lines. 45 degrees is the optimal angle

Figure 3.14: Probability of detection of Radar at a given distance from predicted location
Figure 3.15: Probability of detection of Radar at a given distance from predicted location

i A 2-D Gaussian distribution is used to determine the Radar location. It is assumed that
the main beam direction is known.

ii Given the Radar location, the $d_{opt}$ is found using the Iterative Monte Carlo simulations.

iii The above steps are repeated with increasing values of $\sigma_L$ where the covariance matrix
of the 2-D Gaussian distribution is $\Sigma = \begin{bmatrix} \sigma^2_L & 0 \\ 0 & \sigma^2_L \end{bmatrix}$.

Figure 3.16 shows the increase in the protection region due to the error in location of the
Radar. Figure 3.17 shows the increase in protection distance along the main-beam direction
with an increase in $\sigma_L$. 
Figure 3.16: Comparison of Protection region with location error

Figure 3.17: Effect of $\sigma_L$ on the protection region
Chapter 4

CONCLUSIONS AND FUTURE WORK

4.1 Conclusions

The primary objective of our analysis has been to explore the impact of various stochastic point process models on $I_{aggr}$. Since map data for actual real-world deployment of Wi-Fi APs is available, a useful sanity check would be to explore which (or if) the above models represent such deployments. This will potentially result in actual prescriptions for the protection region for a specific metro area, which will be of likely interest to mobile operators. By using an open-source dataset and expecting the deployed networks to be as dense as the networks in the 2.4 GHz bands, we can produce instantaneous computations of the protection region given the Radar location and Mainbeam direction. Further, as noted - $I_{aggr}$ is also impacted by the probability of a wireless network transmitting at a given instant of time, which is related to the MAC protocol employed. Integrating the probability of transmission into the above gives a more complete analysis. Furthermore, it is important to estimate various parameters regarding radar operation (the location of the radar and main beam direction) since this information is not readily known. Extending the analysis to when such parameters are unknown and must be estimated on-line - and its consequent effect on the protection region - is also discussed in this thesis. The analysis emphasizes that the determination of protection regions is significantly impacted by a multiplicity of factors on both the secondary (notably, the transmit power, deployment geometry, transmit antenna patterns) and primary (receiver sensitivity and operational margins, receive antenna patterns) side as well as the channel characteristics (distance and frequency dependant pathloss, and any statistical effects such as small-scale fading and large-scale shadowing).
4.2 Future Work

It is important to compute more accurate Wi-Fi AP densities. This can be done by considering population metrics and building layouts. This allows for a more precise characterization of WLAN network activity and the aggregate interference at the Radar receiver. Also, the path loss model considered in this thesis is obtained from [9]. A more accurate channel model for instantaneous protection region calculation can be obtained by using SRTM-3 (an international research effort that obtained digital elevation models on a global scale to generate the most complete high-resolution digital topographic database of the Earth) with the Longley-Rice pathloss model. To make the WLAN network activity characterization more precise, the exponential backoff scenario can be considered using system-level simulators like NS-3 instead of computing based on the simplistic constant backoff scenario. Since the mainbeam direction is being estimated, this allows for estimating the Radar rotation speed. Although this parameter is not required for protection region calculations, it is an important parameter to estimate for other practical purposes. Another important effort would be to compute the exclusion region along the coastline and compare with NTIA’s calculations.
BIBLIOGRAPHY


APPENDIX

Based on equations (2.24) and (3.6), optimal protection distance by limiting maximum outage probability is obtained as:

\[ d_{opt} = \arg \min_{d(\theta)} \int_{0}^{2\pi} d^2(\theta) d\theta \]

\[ \mu_I + \sigma_I Q^{-1}(P_{out,max}) \leq I_{max} \]

where \( \mu_I \) and \( \sigma_I \) are calculated in (2.18) and (2.19):

\[ \mu_I = C_{\mu_I} \int_{\theta} G'(\theta) d^{2-\alpha}(\theta) d\theta \]

\[ \sigma_I = \sqrt{C_{\sigma^2} \int_{\theta} G^2(\theta) d^{2-2\alpha}(\theta) d\theta} \]

Optimal \( d(\theta) \) is attained by converting the inequality constraint to equality. This follows because for any \( d(\theta) \) for which the strict inequality constraint holds, we can scale down \( d(\theta) \) accordingly to increase \( \mu_I, \sigma_I \) and achieve equality constraint (note that \( 2 - \alpha < 0 \) and \( 2 - 2\alpha < 0 \)). This will clearly result in a smaller objective function.

With equality constraint, we use Lagrange multiplier method with a dummy variable \( \epsilon \) to redefine objective function as

\[ d_{opt} = \arg \min_{d(\theta)} \int_{0}^{2\pi} d^2(\theta) d\theta + \epsilon \left( \mu_I + \sigma_I Q^{-1}(P_{out,max}) - I_{max} \right) \]

Taking partial derivatives of the new objective function with respect to \( d(\theta) \) results in:

\[ \frac{\partial f}{\partial d(\theta)} = 0, \quad d(\theta) + \epsilon \left[ \frac{\partial \mu_I}{\partial d(\theta)} + Q^{-1}(P_{out,max}) \frac{\partial \sigma_I}{\partial d(\theta)} \right] = 0 \]

Replacing \( \mu_I \) and \( \sigma_I \):

\[ d(\theta) + \epsilon \left[ (2 - \alpha)C_{\mu_I} G(\theta) d^{1-\alpha}(\theta) + \frac{Q^{-1}(P_{out,max})C_{\sigma^2}(2 - 2\alpha)G^2(\theta) d^{1-2\alpha}(\theta)}{2 \sqrt{C_{\sigma^2} \int_{\theta} G^2(\theta) d^{2-2\alpha}(\theta) d\theta}} \right] = 0 \]
Appendix A

WHERE TO FIND THE FILES

All the code and documentation can be found on:

- FuNLAB webpage.
  
  https://depts.washington.edu/funlab/research

- My webpage
  
  https://www.anishashok.com
VITA

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