Modeling of Test Section Conditions in a High Enthalpy Flow Facility

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Abstract

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The Shock Wave Reactor laboratory, or SWR, at the University of Washington is a high enthalpy flow facility capable of a wide range of test conditions. In this study, an analytical model has been prepared using MATLAB in an effort to predict the envelope of test conditions of which the SWR is capable for a given range of input conditions. The SWR modeling assumed one-dimensional, steady, ideal gas flow, and considered the effects of heat transfer and friction on stagnation pressure and stagnation temperature, as well as other values such as Mach number. The SWR can receive steam from a pebble bed heater at temperatures up to 1400 K and $\text{H}_2(\text{g})$ and $\text{O}_2(\text{g})$ at room temperature in a combustor section. The resulting fluid flows through a converging-diverging nozzle, which accelerates the flow to supersonic Mach numbers, typically at about Mach 2.5. Downstream of the throat, feedstock such as $\text{N}_2(\text{g})$ can be injected into the flow. Downstream of feedstock injection, the flow proceeds through a mixer section into a test section with a diameter of 4”. It is the flow properties in the test section that were attempted to be predicted by the model in this study. The model was run using known or assumed input conditions for existing SWR experimental data, and the modeled results for the test section data were compared against the experimental data. Although the model showed good ability to predict the trends of test section conditions relative to input conditions, the model mostly did not show good ability to predict the envelope of test conditions that the SWR is capable of producing.
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GLOSSARY

A: Area, m²

A*: Sonic throat area, m²

a: Speed of sound, m/s

b: Time constant, 1/s

C₁: Dimensionless constant in Nusselt number relationship

C₂: Dimensionless constant in Nusselt number relationship

cₚ: Specific heat at constant pressure, J/(kg*K)

cᵥ: Specific heat at constant volume, J/(kg*K)

D: Diameter, m

f: Friction factor

h: Specific enthalpy, J/kg

hₑ: Convective heat transfer coefficient, W/(m²*K) (unless otherwise stated)

k: Conductive heat transfer coefficient, W/(m*K) (unless otherwise stated)

L: Length, m

M: Mach number

Mᵣ: Molecular weight, kg/kmol (unless otherwise stated)

m: Mass, kg
\( \dot{m} \): Mass flow rate, kg/s (unless otherwise stated)

\( Nu \): Nusselt number

\( OF \): Ratio of oxidizer to fuel mass flow rates

\( OF_{st} \): Stoichiometric ratio of oxidizer to fuel mass flow rates

\( p \): Static pressure, Pa (unless otherwise stated)

\( Pr \): Prandtl number

\( \dot{Q} \): Heat transfer rate, W

\( \dot{Q}_f \): Heat flux rate, W/m\(^2\)

\( \dot{q} \): Specific heat transfer rate, W/kg

\( Q \): Heat transfer, J

\( q \): Specific heat transfer, J/kg

\( q_f \): Specific heat flux, J/(kg*m\(^2\))

\( R \): Gas constant for a particular gas, J/(kg*K)

\( R_u \): Universal gas law constant, J/(kmol*K)

\( Re \): Reynolds number

\( T \): Static temperature, K (unless otherwise stated)

\( T_{ad} \): Adiabatic flame temperature, K (unless otherwise stated)

\( T_s \): Wall temperature, K (unless otherwise stated)

\( u \): Velocity, m/s

\( V \): Volume, m\(^3\)
Greek Letters:

$\alpha$: Ratio of feedstock mass flow rate to freestream mass flow rate

$\phi$: Equivalence ratio

$\gamma$: Ratio of specific heats

$\mu$: Dynamic viscosity, kg/(m$^*$/s) (unless otherwise stated)

$\rho$: Static density, kg/(m$^3$)

Subscripts:

$\infty$: Freestream values

0: Stagnation conditions

7: Mixed fluid conditions

a: Conditions of fluid prior to feedstock mixing

am: Conditions at a temperature that is an arithmetic mean of wall temperature and the freestream fluid temperature of a particular station

b: Feedstock conditions

t: Throat conditions

TS: Test section conditions
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DEDICATION

to my parents, Barb and Jerry Drabiak, without whose unending love and support none of this would have been possible.
Chapter 1

INTRODUCTION

The Shock Wave Reactor laboratory, or SWR, at the University of Washington is a high enthalpy flow facility, and is capable of a wide range of test conditions. Examples of projects that have been conducted in the SWR include extraction of hydrogen from hydrocarbons; a rotating detonation engine, or RDE; and experiments involving downstream injection of additional feedstock mass flow. However, never before has the complete scope of these test conditions been modeled. In order to better inform those who might seek to employ the services of the SWR to run their experiments, an analytical modeling package written using the MATLAB software has been prepared.

In particular, the primary goal in preparing the model was to be able to predict the flow properties, including the transport properties, inside of a 4" diameter test section in the SWR. The freestream flow is typically some combination of steam and H\(_2\)(g) and O\(_2\)(g), and the feedstock is typically N\(_2\)(g). Feedstock, such as N\(_2\), is injected with the goal of lowering the stagnation temperature, raising the stagnation pressure, and raising the Mach number. The steam in the freestream comes from a pebble bed heater, which can raise the temperatures of the steam up to about 1400 K. Additional H\(_2\)(g) and O\(_2\)(g) can be injected at room temperature. When injected, the H\(_2\)(g) and O\(_2\)(g) combust, producing more steam and raising the adiabatic flame temperature in the combustor to as high as 3300 K. For the purposes of this study, the H\(_2\)(g) and O\(_2\)(g) are assumed to react completely, with pure steam as the products of the stoichiometric reaction.

A butterfly valve downstream of the test section can be used to vary the position of a shockwave in the SWR. The shockwave can be placed in the diverging section of the nozzle, a 3" diameter feedstock injector section, or the 4" diameter mixer section, all of which would
produce subsonic flow in the test section. The shock can also be placed downstream of the test section, which would allow for supersonic flow in the test section. Depending on flow conditions, supersonic Mach numbers can be as high as Mach 2.5 for the 3" diameter section and Mach 3 for the 4" diameter section.

One of the biggest issues with the SWR facility is acquiring measurements. The walls of the SWR are solid steel and are completely opaque, prohibiting visual observations. Due to the high temperatures of the flow, only Type C thermocouples, made out of mixtures of tungsten and rhenium, can be used to record the temperatures in or near the test section, which is typically around 2300 K. However, the Type C thermocouples burn out in oxidizing flow; thus, frequent spot-welding to repair the thermocouples is necessary. The high temperatures also make acquiring pressure measurements in the test section difficult; pitot probes were used to measure the pressure but, like the thermocouples, frequently melted and needed to be repaired. PCBs and pressure transducers were used for pressure measurements upstream of the test section.

Another issue with the SWR facility is the fact that the vacuum pump can only reach pressures as low as about 3 kPa. This is due to the saturation temperature of the water vapor inside being about room temperature at about 3 kPa, meaning that some of the water vapor inside begins to liquefy at that point.
Chapter 2

THEORETICAL BACKGROUND

2.1 Isentropic, Adiabatic Calculations

Despite the name of the facility, the structure of shock waves is not considered in the modeling used. Furthermore, one-dimensional, axisymmetric, isentropic flow is assumed to occur and is used as the basis for later, more rigorous modeling, which is also part of the current study. As such, the conventional isentropic equations, shown below, are used:

\[ p = p_0 \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{\gamma}{\gamma - 1}} \]  
\[ T = T_0 \left(1 + \frac{\gamma - 1}{2} M^2 \right) \]  
\[ \frac{A}{A^*} = \sqrt{\frac{1}{M^2} \left[ \frac{2}{\gamma + 1} \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{\gamma}{\gamma - 1}} \right]} \]

Equations (2.1) and (2.2) can be found in Ref. [1], and Eq. (2.3) can be found in Ref. [2]. Note that in Eq. (2.3), the relation for Mach number is a function of the specific heat ratio \( \gamma \) and area ratio, exclusively. Furthermore, Eq. (2.3) is only valid under adiabatic, isentropic, one-dimensional, steady-state flow.

The model used in the current study is designed such that the user is expected to define the freestream conditions, including the properties of the incoming gas. The user also has
the opportunity to modify the geometry and thus the area ratio. The stagnation values for
temperature and pressure are calculated from the user-defined freestream conditions using
Eqs. (2.1) and (2.2). An iterative process is then used to calculate the isentropic Mach
number using Eq. (2.3), in which the area ratio is taken from the user-defined geometry.
Eqs. (2.1) and (2.2) are used again to calculate the static values for temperature and pressure
at all points downstream of the inlet, under the assumption that the stagnation values for
temperature and pressure remain constant and using the Mach numbers given by Eq. (2.3).

Unless stated otherwise, the following Reynolds number is used throughout the analysis,
most notably to determine the friction factor and Nusselt number:

\[ Re = \frac{\rho u D}{\mu} \]  \hspace{1cm} (2.4)

where \( D \) is the diameter of the duct at a given point, \( \mu \) is the viscosity of the flow, \( \rho \) is the
local freestream density, and \( u \) is the local freestream velocity. The viscosity \( \mu \) is calculated
from CEA and is assumed to be constant for a given gas composition. Chemical reactions
are assumed to be frozen. The density \( \rho \) is calculated according to the ideal gas law:

\[ \rho = \frac{p}{RT} \]  \hspace{1cm} (2.5)

Moreover, the velocity \( u \) is calculated according to the definition of Mach number:

\[ u = aM \]  \hspace{1cm} (2.6)

It is worth noting that the values on the right hand side of both Eqs. (2.5) and (2.6) are
the values obtained from the adiabatic, isentropic calculations. This is worth noting because
the Reynolds number is derived from adiabatic, isentropic values is the same Reynolds num-
ber that is used for calculating the friction factor and Nusselt number. Applying the friction factor and Nusselt number to the analysis would change velocity $u$ and density $\rho$, among other variables, and thus the Reynolds number. Thus, this approach is not perfectly accurate. However, this approach does give a close enough approximation for current modeling purposes.

2.2 Heat Flux between Fluid and Walls

For the calculation of the specific heat flux to the walls, more rigorous calculations were necessary. The Chemical Equilibrium with Applications program, or CEA, was identified early on as a potential aid for the modeling efforts. A version of CEA compatible with MATLAB, known as CEAM, is available, and this is the version of CEA used in the model.

In order to compute transport properties, which are then used to calculate the convective heat transfer coefficient, the "hp" case of CEAM, in which the specific enthalpy and the pressure are held constant, is used to calculate the properties of the freestream fluid mixture at the entrance of the combustor, including calculating the results of combustion, if $H_2(g)$ and $O_2(g)$ have been added. Values that were calculated using CEAM included adiabatic flame temperature, $T_{ad}$; speed of sound, $a$; specific heat at constant pressure, $c_p$; conductive heat transfer coefficient, $k$; Prandtl number, $Pr$; the ratio of specific heats, $\gamma$; the molecular weight, $M$; and the dynamic viscosity, $\mu$.

It was understood prior to this project that the SWR experienced a streamwise stagnation temperature loss of approximately 100 K per meter. However, a more rigorous modeling of the stagnation temperature losses was desired. To calculate this, the heat transfer needed to be calculated.

Two different sets of empirically-derived relations were used to determine the value of the convective heat transfer coefficient, depending on the section’s geometry.

For all regions of the flow except the contraction section of the throat, the Petukhov equation for friction factor and the Gnielinski equation for Nusselt number were used, both of which can be found in Ref. [3].
The following value of Petukhov friction factor is valid for Reynolds numbers of $3000 < Re < 5 \times 10^6$:

$$f = (0.79 \log(Re) - 1.64)^{-2}$$

(2.7)

The following Gnielinski equation is valid for Reynolds numbers of $10^4 < Re < 5 \times 10^6$ and Prandtl numbers of $0.5 < Pr < 2000$:

$$Nu = \frac{(f/8)(Re - 1000)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}$$

(2.8)

In the contraction section of the throat, the geometry was known to be similar to a quarter circle, and was thus approximated as a perfect quarter circle for the purposes of the current study. It was decided that the Bartz equation, shown in Eq. (2.9), was better suited for a case with curved wall geometry than the Gnielinski and Petukhov relations.

The following expression for the Bartz equation can be found in Ref [8]:

$$h_c = 0.026 \left( \frac{c_p \mu^{0.2}}{Pr^{0.6}} \right) (\rho u)^{0.8} \left( \frac{\rho_{am}}{\rho} \right) \left( \frac{\mu_{am}}{\mu_0} \right)^{0.2}$$

(2.9)

In the expression for the Bartz equation given in Eq. (2.9), $D$ is the inner diameter of the duct, a subscript "am" denotes the properties calculated at a temperature that is an arithmetic mean of the local freestream temperature and the wall temperature. The subscript zero indicates stagnation conditions.

It is worth noting that use of the Bartz equation in the subsonic region of the throat produced values of Mach number and stagnation temperature that agreed well with the Petukhov and Gnielinski version of the analysis.

For all sections other than the contraction area of the nozzle, the definition of Nusselt
number, Nu, was used to calculate the value of the convective heat transfer coefficient, $h_c$:

$$Nu = \frac{h_c L}{k}$$ (2.10)

which becomes

$$h_c = \frac{(Nu)k}{L}$$ (2.11)

Using a user-defined temperature for the wall, the model then calculates the rate of heat flux into the walls due to convection using the following equation, derived from Ref. [3]:

$$\dot{Q}_A = h_c (T_s - T_\infty)$$ (2.12)

It is useful to define a rate of heat flux term, $Q_f$, as follows:

$$\dot{Q}_f = \frac{\dot{Q}}{A}$$ (2.13)

Note that $\dot{Q}_f$ has units of W/m$^2$. By dividing Eq. (2.12) by mass flow rate, one arrives at an equation for specific heat flux into the walls due to convection:

$$q_f = \frac{\dot{Q}_f}{\dot{m}}$$ (2.14)

Note that $q_f$ has units of J/(kg*m$^2$). By multiplying the specific heat flux given in Eq. (2.14) by area, one arrives at the specific heat transfer $q$:
Thus, by Eq. (2.15), it is possible to determine the specific heat transfer from the fluid to the walls.

2.2.1 Steady, 1-D, Compressible Flow Equations

Knowing the specific heat transfer, and assuming that \( c_p \) is constant, the stagnation temperature can then be calculated using the following relationship:

\[
dT_0 = \frac{\delta q}{c_p} \tag{2.16}
\]

In discrete form, Eq. (2.16) becomes

\[
T_{0,i} = T_{0,i-1} + \frac{q_{i-1}}{c_p} \tag{2.17}
\]

In Eq. (2.17), the subscript \( i \) denotes a particular point in an iterative process that begins at the inlet and proceeds downstream.

With the change in stagnation temperature acquired, it is now possible to acquire a change in Mach number using the following relationship, which can be found in Ref. [5]:

\[
\frac{dM^2}{M^2} = \frac{(1 + \frac{\gamma - 1}{2} M^2)}{1 - M^2} \left( -2 \frac{dA}{A} + (1 + \gamma M^2) \frac{dT_0}{T_0} + \gamma M^2 \frac{fdx}{D} \right) \tag{2.18}
\]

Assuming \( dx \) is constant, Eq. (2.18) can be discretized as
\[ M_i = \sqrt{M_{i-1}^2 (1 + \frac{2 \gamma - 1}{2} M_{i-1}^2) \left(-2 \frac{dA_{i-1}}{A_{i-1}} + (1 + \gamma M_{i-1}^2) \frac{dT_{0,i-1}}{T_{0,i-1}} + \gamma M_{i-1}^2 \frac{f_{i-1} dx}{D_{i-1}} \right)} \] (2.19)

With the Mach number known, the stagnation pressure can be calculated from the following relationship, also found in Ref. [5]:

\[ \frac{dP_0}{P_0} = -\frac{\gamma M_{i-1}^2}{2} \left( \frac{dT_{0,i-1}}{T_{0,i-1}} + \frac{f_{i-1} dx}{D_{i-1}} \right) \] (2.20)

In discretized form, Eq. (2.20) becomes

\[ P_{0,i} = P_{0,i-1} \left( 1 - \frac{\gamma M_{i-1}^2}{2} \left( \frac{dT_{0,i-1}}{T_{0,i-1}} + \frac{f_{i-1} dx}{D_{i-1}} \right) \right) \] (2.21)

### 2.3 Mixing of Freestream with Injectant

The mixing of the injectant with the freestream was assumed to be instantaneous and complete. Additionally, the effects of momentum loss due to the sidewall injection of the fluid into the freestream were ignored. The injector nozzles were angled relative to the freestream, four at angles of 60° and four at angles of 30°. Only the components of momentum that were parallel to the freestream were considered.

The notation used for the mixing process is shown in Fig. (2.1). The properties with a subscript \( b \) represent the injectant, the properties with a subscript \( a \) represent the freestream fluid, and the properties with a subscript \( 7 \) represent the mixed fluid.
It was desired to determine the flow properties of the mixed fluid, including, but not limited to, Mach number, pressure, temperature, and transport properties such as viscosity. The laws of conservation of mass, momentum, and energy were used as a starting point from which relations for the properties of the mixed fluid could be determined based on the known properties of the freestream and the injectant fluids.

\[
\dot{m}_a + \dot{m}_b = \dot{m}_7 \quad (2.22)
\]

Eq. (2.22) gives the mass conservation. Furthermore, it is helpful to define a mass flow rate fraction \( \alpha \) such that

\[
\alpha = \frac{\dot{m}_b}{\dot{m}_a} \quad (2.23)
\]

Eq. (2.23) allows us to re-write \( \dot{m}_b \) in terms of \( \dot{m}_a \) by saying that \( \dot{m}_b = \alpha \dot{m}_a \). By substituting this into the mass conservation, we can also re-write \( \dot{m}_7 \) in terms of \( \dot{m}_a \), as follows:

\[
\dot{m}_7 = (1 + \alpha)\dot{m}_a \quad (2.24)
\]
Keeping in mind the mass flow rate relationships given in Eqs. (2.24), the conservation of energy given in the form of a balance of enthalpy and kinetic energy is as follows:

\[ \dot{m}_7(h_7 + \frac{u_7^2}{2}) = \dot{m}_b(h_b + \frac{u_b^2}{2}) + \dot{m}_a(h_a + \frac{u_a^2}{2}) \]  

(2.25)

Using the definition of enthalpy,

\[ h = c_p T \]  

(2.26)

one can re-write Eq. (2.25) as

\[ \dot{m}_7(c_{p,7}T_7 + \frac{u_7^2}{2}) = \dot{m}_b(c_{p,b}T_b + \frac{u_b^2}{2}) + \dot{m}_a(c_{p,a}T_a + \frac{u_a^2}{2}) \]  

(2.27)

By rearranging Eq. (2.27), one can express \( T_7 \) as follows:

\[ \frac{T_7}{T_a} = -\frac{u_7^2}{2c_{p,7}T_a} + \frac{\alpha}{c_{p,7}T_a(\alpha + 1)}(c_{p,b}T_b + \frac{u_b^2}{2}) + \frac{1}{c_{p,7}T_a(\alpha + 1)}(c_{p,a}T_a + \frac{u_a^2}{2}) \]  

(2.28)

Eq. (2.28) is important, because it allows one to define the static temperature of the mixed fluid exclusively in terms of properties of the injectant and the freestream.

It was also desired to find the Mach number of the mixed fluid. By assuming that all fluids involved are calorically perfect gases, the following definition for mass flow rate, found in Ref. [6], can be used:

\[ \dot{m} = pA\sqrt{\frac{\gamma}{R\sqrt{T_0}}}M(1 + \frac{\gamma - 1}{2}M^2)^{1/2} \]  

(2.29)
Additionally, if wall friction is neglected, the conservation of momentum can be written as the following equation, found in Ref. [7]:

\[ p_b A_b (1 + \gamma_b M_b^2) + p_a A_a (1 + \gamma_a M_a^2) = p_\tau A_\tau (1 + \gamma_\tau M_\tau^2) \] (2.30)

By redefining Eq. (2.29) in terms of \( pA \), as follows,

\[ pA = \dot{m} \sqrt{\frac{R}{\gamma}} \sqrt{T_0} \frac{1}{M} (1 + \frac{\gamma - 1}{2} M^2)^{-1/2} \] (2.31)

and then combining Eqs. (2.30) and (2.31), and assuming that flow is uniform at all stations, one can then derive an equation for the mixed Mach number:

\[
\begin{align*}
\dot{m}_7 & \sqrt{\frac{R_7}{\gamma_7}} \sqrt{T_{0,7}} \frac{1}{M_7} (1 + \frac{\gamma_7 - 1}{2} M_7^2)^{-1/2} (1 + \gamma_7 M_7^2) \\
= \dot{m}_b & \sqrt{\frac{R_b}{\gamma_b}} \sqrt{T_{0,b}} \frac{1}{M_b} (1 + \frac{\gamma_b - 1}{2} M_b^2)^{-1/2} (1 + \gamma_b M_b^2) \\
+ \dot{m}_a & \sqrt{\frac{R_a}{\gamma_a}} \sqrt{T_{0,a}} \frac{1}{M_a} (1 + \frac{\gamma_a - 1}{2} M_a^2)^{-1/2} (1 + \gamma_a M_a^2)
\end{align*}
\] (2.32)

At this stage, it becomes important to consider the angles of the injector ports. Four of the injector ports are angled at 60° relative to the freestream flow and the other four are angled at 30°. The transverse components of the velocity are ignored. The area of the injectant can be expressed as an average of the mass flow rates from the 60° ports and the 30° ports that are projected onto a plane that is perpendicular to the freestream flow direction, as follows:
\[ \dot{m}_b = \frac{1}{2} \dot{m}_{b,full} \cos(30^\circ) + \frac{1}{2} \dot{m}_{b,full} \cos(60^\circ) \]  

where \( m_{b,full} \) is the full mass flow rate of the injectant through the original area. Equation (2.33) simplifies to

\[ \dot{m}_b = \frac{1}{2} \dot{m}_{b,full} (\cos(30^\circ) + \cos(60^\circ)) \]  

In order to simplify the mathematical expressions, it is helpful to define a term \( f = f(M) \), found in Ref. [7] such that

\[ f(M) = M^2(1 + \frac{\gamma - 1}{2} M^2)(1 + \gamma M^2)^{-2} \] (2.35)

Eq. (2.35) allows Eq. (2.32) to be re-written as follows:

\[ \dot{m}_\gamma \sqrt{\frac{R_7}{\gamma_7}} \sqrt{T_{0,7}} \frac{1}{\sqrt{f(M_7)}} \]

\[ = \frac{1}{2} \dot{m}_{b,full} (\cos(30^\circ) + \cos(60^\circ)) \sqrt{\frac{R_b}{\gamma_b}} \sqrt{T_{0,b}} \frac{1}{\sqrt{f(M_b)}} + \dot{m}_a \sqrt{\frac{R_a}{\gamma_a}} \sqrt{T_{0,a}} \frac{1}{\sqrt{f(M_a)}} \] (2.36)

Eq. (2.36) can be rearranged to produce an expression in terms of \( f(M_\gamma) \), as follows:

\[ f(M_\gamma) = \frac{(\alpha + 1)^2 R_7 \frac{T_{0,7}}{T_{0,a}}}{[\alpha \frac{1}{2}(\cos(30^\circ) + \cos(60^\circ)) \sqrt{\frac{R_b}{\gamma_b}} \sqrt{\frac{T_{0,b}}{T_{0,a}}} \frac{1}{\sqrt{f(M_b)}} + \sqrt{\frac{R_a}{\gamma_a}} \frac{1}{\sqrt{f(M_a)}}]^2} \] (2.37)

Furthermore, by re-writing \( T_{0,7} \) in terms of the properties of the freestream and the
injectant, as expressed in Eq. (2.28), Eq. (2.37) becomes

\[ f(M_f) = \frac{(\alpha + 1)^2 R_b}{\gamma_f} \left( -\frac{u_f^2}{2\gamma_f T_u} + \frac{\alpha}{\gamma_f T_u(\alpha + 1)}(c_{p,b} T_b + \frac{u_f^2}{2}) + \frac{1}{c_{p,f} T_a}(c_{p,a} T_a + \frac{u_f^2}{2}) \right) + \left[ \alpha \frac{1}{2} (\cos(30^\circ) + \cos(60^\circ)) \right] \sqrt{\frac{R_a}{\gamma_f} \frac{1}{f(M_b)}} + \sqrt{\frac{R_b}{\gamma_f} \frac{1}{f(M_a)}} \right]^2 \] (2.38)

Moreover, since the value of \( u_f \) is expected to be unknown, one must re-write the equation even further by using the definition of Mach number,

\[ u = a M \] (2.39)

where \( a \) is the speed of sound.

It is at this stage that the angles of the injector ports are again considered. Four of the injector ports are angled at 60\(^\circ\) relative to the freestream flow and the other four are angled at 30\(^\circ\). The transverse components of the velocity are ignored. The velocity of the injectant can be expressed as an average of the velocities from the 60\(^\circ\) ports and the 30\(^\circ\) ports, as follows:

\[ u_b = \frac{1}{2} a_{b,30^\circ} M_{b,30^\circ} \cos(30^\circ) + \frac{1}{2} a_{b,60^\circ} M_{b,60^\circ} \cos(60^\circ) \] (2.40)

Noting that speed of sound \( a \) and Mach number \( M \) are the same for all injector ports, regardless of angle, Eq. (2.40) becomes

\[ u_b = \frac{1}{2} a_b M_b (\cos(30^\circ) + \cos(60^\circ)) \] (2.41)

Thus, Eq. (2.38) becomes
\[ f(M_7) = \frac{(\alpha + 1)^2 R_7}{[\alpha \frac{1}{2}(\cos(30^\circ) + \cos(60^\circ))\sqrt{\frac{R_7}{\gamma_7}} + \sqrt{\frac{T_{0,b}}{\gamma_7}} \sqrt{\frac{1}{J(M_b)}} + \sqrt{\frac{R_a}{\gamma_a}} \frac{1}{f(M_a)}]^2} \times \frac{-(a_7 M_7)^2}{2c_{p,7} T_a} \]

\[ + \frac{\alpha}{c_{p,7} T_a (\alpha + 1)} (c_{p,b} T_b + \frac{\frac{1}{2} a_b M_b (\cos(30^\circ) + \cos(60^\circ))^2}{2}) \]

\[ + \frac{1}{c_{p,7} T_a} (c_{p,a} T_a + \frac{(a_a M_a)^2}{2}) \]  

(2.42)

where

\[ f(M_7) = M_7^2 (1 + \frac{\gamma_7 - 1}{2} M_7^2) (1 + \gamma_7 M_7^2)^{-2} \]  

(2.43)

Eq. (2.43) is a quadratic expression for \( M_7 \), and can be rearranged into the following, given in Ref. [7]:

\[ M_7^2 = \frac{2 f(M_7)}{1 - 2 \gamma_7 f(M_7) \pm \sqrt{1 - 2 (\gamma_7 + 1) f(M_7)}} \]  

(2.44)

where the + sign is used in the case of subsonic flow and the - sign is used in the case of supersonic flow, as described in Ref. [6]. Unfortunately, there is a slight problem the formula contained in Eqs. (2.42) through (2.39), and that is that, as a result of accounting for the change in kinetic energy, the Mach number for the mixed fluid appears in the definition of both variants of \( f(M_7) \). This means that the equation would be very difficult or even impossible to solve analytically. Therefore, the fzero function in MATLAB was used to find \( M_7 \) using the value of \( M_a \) as the initial guess, since it is assumed that \( M_7 \) will be close to \( M_a \).

Furthermore, by combining Eqs. (2.31) and (2.28), it can be easily shown that
\[ p_{0,7} = (1 + \alpha) \frac{M_a}{M_7} \sqrt{\frac{T_{0,7} R \gamma_a}{T_{0,a} R \gamma_7}} \left( 1 + \frac{\gamma-1}{2} M_7^2 \right)^{\frac{\gamma+1}{\gamma(\gamma-1)}} \] (2.45)

Eq. (2.45) was used for calculating the stagnation pressure of the mixed fluid immediately prior to mixing. However, unlike Eq. (2.42), Eq. (2.45) was derived without considering the angles of the injector ports.

### 2.4 Normal Shocks

In cases in which a normal shock was considered, the following relations, found in Ref. [1], were used:

\[ M_2^2 = \frac{1 + \frac{\gamma-1}{2} M_1^2}{\gamma M_1^2 - \frac{\gamma-1}{2}} \] (2.46)

Eq. (2.46) was used to find the Mach number after the shock, \( M_2 \), based on the Mach number just before the shock, \( M_1 \), assuming a constant \( \gamma \). Static temperature was calculated using Eq. (2.47), shown below:

\[ \frac{T_2}{T_1} = [1 + \frac{2\gamma}{\gamma + 1} (M_1^2 - 1)]^2 + \frac{(\gamma - 1) M_1^2}{(\gamma + 1) M_1^2} \] (2.47)

Static pressure change across the shock was calculated using Eq. (2.48)

\[ \frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1} (M_1^2 - 1) \] (2.48)
2.5 A Sample Case: Heat Transfer into a Test Article

2.5.1 Description of Problem

As stated above, the SWR lab is capable of a wide range of test functions and experiments. One such experiment, which has been conducted, was to calculate the heat transfer into a test article placed in the test section. The test articles in question were stainless steel cylinders approximately 0.5" in diameter and 4" in length. The test objects were exposed to two different types of freestream flows. In the first case, the freestream flow consisted of a mixture of steam from the pebble bed heater and steam resulting from the combustion of H$_2$ and O$_2$ in the combustor section. The second case was the same, except that no pebble bed steam was used. During the course of these experiments, the combustor section was changed from a tubular deflagration combustion chamber to an annular combustion chamber that could operate in both deflagration mode, to produce subsonic exhaust flow, or in rotating detonation mode, to produce supersonic exhaust. No injection of N$_2$ feedstock was used for these experiments. For these tests the range of flow conditions used included velocities from 200 to 600 m/s, corresponding to Mach numbers of about 0.2 to about 0.4, and temperatures from 800 °C to 2400 °C, with static pressures ranging from 0.6 bar to 1.1 bar. Cylindrical test articles supplied by a sponsor were exposed axially to the flow in the test section.

2.5.2 Methods Used

Empirically derived Nusselt number correlations for axially-oriented cylinders exposed to turbulent flow were used in combination with the experimental temperature and pressure data and calculated flow properties to find the convective heat transfer coefficient, and thence the heat transfer, of the test samples. The correlation for Nusselt number was found in Ref. [9], and is shown below:

$$Nu = C_1 Re^{C_2}$$  \hspace{1cm} (2.49)
where \( C_1 \) and \( C_2 \) are empirically-derived constants that differ for different parts of the geometry. For the upstream face of the cylinder, the values are \( C_1 = 0.662 \) and \( C_2 = 0.534 \). For the side of the cylinder, the values are \( C_1 = 0.14 \) and \( C_2 = 0.686 \). For the downstream face of the cylinder, the values are \( C_1 = 0.14 \) and \( C_2 = 0.623 \). Wiberg and Lior give three different sets of values for \( C_1 \) and \( C_2 \), but it is the second of their three cases, which they call case (B), that matches the conditions of the current study most closely. Furthermore, for the current study, only the side of the cylinder was considered.

In order to find the Reynolds number, \( Re \), values for density and velocity were necessary. The definition of Reynolds number is

\[
Re = \frac{\rho u L}{\mu}
\]  

(2.50)

where \( L \) is some characteristic length. In this case, the characteristic length used was the diameter of the cylindrical test article. The density \( \rho \), velocity \( u \), and viscosity \( \mu \) are all the values of the freestream flow.

The velocity of the freestream was calculated according to the following equation:

\[
u = \frac{\dot{m}_\infty RT_\infty}{P_\infty A_{TS}}
\]

(2.51)

where the freestream values are all either empirically derived or assumed, and the test section cross sectional area is calculated from the measured radius.

Once the Nusselt number was derived, Eq. (2.11) was used to calculate the convective heat transfer coefficient. The convective heat transfer coefficient was then used to calculate the final temperature of the cylinder using a lumped system analysis, found in Ref. [4], as shown below:
\[
\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}
\]

where

\[
b = \frac{h_c A_s}{\rho V c_p}
\]

Eq. (2.53) gives the value of a time constant, \( b \), for which the dimensions are 1/s. Furthermore, \( \rho \), \( V \), and \( c_p \) refer to the properties of the test article, \( T_i \) refers to the initial temperature of the test article, and \( A_s \) refers to the surface area being considered. In this case, \( A_s \) was equal to the surface area of the side of the cylinder. Eq. (2.52) can be rearranged to give the final temperature \( T(t) \) after time \( t \), as follows:

\[
T(t) = T_\infty + (T_i - T_\infty)e^{-bt}
\]

Using the value for final temperature after time \( t \) found using Eq. (2.54), the total heat transfer \( Q \) into the test article after time \( t \) was then found using the following equation:

\[
Q = mc_p[T(t) - T_i]
\]

where the values for \( m \) and \( c_p \) correspond to the values for the test article. The mass was derived by multiplying the density \( \rho \) by the volume \( V \).
Chapter 3

EXPERIMENTAL APPARATUS

3.1 Description of the SWR Capabilities

Figure 3.1: SWR Setup
Steam is the main working fluid of the SWR facility, and the steam is heated up to a maximum of approximately 1100 °C, or 1370 K, with an electrically-heated pebble bed heater, or can be produced as a result of combustion of H₂ and O₂ in the combustor or as part of a rotating detonation engine, or RDE. Furthermore, it is possible to insert additional H₂ and O₂ at room temperature (298 K). Without water cooling, the combustor can run for as long as 4 seconds, and mass flow rates as high as 0.4 kg/s are possible, though that maximum mass flow rate varies with the nature of the experiment. The mass flow in the combustor can consist of either pure steam or some mixture of steam and H₂(g) or O₂(g), depending on the equivalence ratio φ of the experiment. The equivalence ratio is given as
\[
\phi = \frac{O_{st}}{O_F}
\]  

(3.1)

where

\[
O_F = \frac{\dot{m}_{\text{oxidizer}}}{\dot{m}_{\text{fuel}}}
\]  

(3.2)

and \(O_{st}\) is the stoichiometric \(O_F\) ratio, which is the \(O_F\) ratio for a perfectly balanced chemical reaction. For example, for the combustion of \(\text{H}_2(\text{g})\) with \(\text{O}_2(\text{g})\), \(O_{st} = 8\).

Due to the limitations imposed by the pressure rating of the pebble bed heater, the maximum pressure at which one can operate the combustion chamber is approximately 4.5 bar. A butterfly valve with a diameter of 0.15 m is used to position the normal shock wave at almost any desired position in the flow channels; for the purposes of the current study, when subsonic flow was desired in the test section, a normal shock was assumed to take place either midway through the feedstock injector section or midway through the mixer section. At the end of the test apparatus is a dump tank with a volume of approximately 5 m\(^3\), which contains approximately 500 kg of aluminum plates and maintains the pressure gradient necessary for steady-state operation by condensing the steam flow.

Figure (3.1) shows a detailed schematic of the SWR facility. In Fig. (3.1), the flow moves from left to right. At the far left end of Fig. (3.1) is the pebble bed heater. Immediately to the right of the pebble bed heater are the \(\text{H}_2\) and \(\text{O}_2\) injector ports. The combustor lies to the right of the \(\text{H}_2\) and \(\text{O}_2\) injector ports. At the exit of the combustor is a nozzle throat. To the right of the throat is a feedstock injection section, where the \(\text{N}_2\) is injected, to the right of which lies the mixer section. Feedstock \(\text{N}_2\) is injected with the goal of lowering the stagnation temperature, raising the stagnation pressure, and raising the Mach number. Immediately downstream of the mixer section lies the test sample holder.
These sections are of interest because the test section is the area being modeled in the current study, and the flow must pass through the mixer, throat, feedstock injector, and mixing section in order to reach the test section. Although it is not noted in Fig. (3.1), the inner radius of the feedstock injector is approximately 0.0381 m.

The flow enters the combustor 1 m prior to entering a converging-diverging nozzle. At the narrowest point of the nozzle is a throat with a diameter of 0.0397 m. The intended design capability was to accelerate steam to supersonic velocities with Mach numbers as high as 2.8. After exiting the nozzle, the flow enters a feedstock injection section section that is 0.0381 m in radius. Note that not all cases considered in this study incorporated feedstock injection. After leaving the feedstock injection chamber, the steam or other mixed fluid enters a mixer section with a radius of 0.0508 m. At the end of the mixer section is the test section in which test samples can be placed on a sting mount. Type C thermocouples placed just upstream of the test section, which in the results covered in this study were capable of recording temperatures as high as approximately 2600 °C. To measure pressure, pressure transducers were stood off from the test section using short tubes, which heated continuously in order to prevent effects from condensation on the measurements. Furthermore, PID-controlled external heat tapes were used to maintain a constant temperature on the walls of the sections including and downstream of the combustor of at least 150 °C.

Fig. (3.2) shows a schematic of the steam generator used for the experiments covered in the current study. The steam accumulator has a volume of approximately 2 m³ and stores saturated steam at pressures up to 590 kPa, corresponding to a saturation temperature of 85 °C. Heat-taped 3" pipe routes steam through a flow regulator which isenthalpically drops pressure to set the point of the experiment, and thereby deliver dry steam to the pebble bed.
3.2 Geometry

For the purposes of the current study, a simplified variant of the SWR geometry was considered. Fig. (3.3) shows the notation used for each of the stations in a simplified rendition of the SWR geometry. Station 1 represents the incoming freestream, as indicated by $U_\infty$. Station 2 marks the beginning of the contraction section, leading to a throat at station 3. Choked flow with Mach number equal to 1 is assumed to occur at the throat. Station 4 marks the end of the diffuser section, in which supersonic flow is assumed to occur. Additionally, the mixing of the freestream with the injectant is assumed to occur at station 4. At station 5, the radius of the duct widens from 1.5" to 2". Though aphysical, normal shock waves are assumed to either midway between stations 4 and 5 or midway between stations 5 and 6. Station 6 represents the beginning of the test section of the SWR.
4.1 Assumptions Made

4.1.1 Nitrogen Feedstock Injection

Various assumptions were made regarding the nature of the mixing of N$_2$ feedstock with the freestream fluid. Firstly, the mixing was assumed to occur instantaneously and completely. Secondly, it was assumed that the components the feedstock momentum injected perpendicular to the freestream flow canceled each other out; thus, only the components of momentum injected parallel to the freestream flow were considered. However, following on the first assumption, there were assumed to be no plume interactions. This approximation is shown in Fig (2.1). Worthy of note is that the area of the injector port was not neglected; documentation from the SWR lab indicates that the injector ports had radii of 0.0970", and that there were eight of them, thus the area of the feedstock in Fig (2.1) was assumed to be the sum of the areas of the eight injector ports. Thirdly, four of the injectors were assumed to be angled at an angle of 60° relative to the freestream flow and the other four were assumed to be angled at an angle of 30° relative to the freestream flow. Finally, the feedstock was assumed to be injected at 298 K and at Mach 1.

4.1.2 Injection of Hydrogen and Oxygen into Combustor

Many of the same assumptions made for the N$_2$ feedstock injection were also used for the insertion of H$_2$ and O$_2$ into the combustor. First of all, the combustion of the H$_2$ and O$_2$ was assumed to be instantaneous and complete. Secondly, it was assumed that the H$_2$ and O$_2$ were inserted at a temperature of 298 K. Thirdly, it was assumed that the temperature
output from the use of the "hp" case in CEAM, in which the pressure and specific enthalpy were held constant, was the adiabatic flame temperature. Fourthly, it was assumed that although the initial pressure and the adiabatic flame temperature were stagnation values, they were close enough to the static values due to low Mach numbers that they could be used as substitutes for the static values at station 1, as indicated in Fig. (3.3).

4.1.3 Transient Heat Transfer into a Test Article

For the heat transfer calculations, various assumptions were made regarding the properties of the freestream flow and the stainless steel test article. The freestream pressure was assumed to be 80 kPa. The freestream mass flow rates varied depending on the run but ranged from approximately 90 g/s to 120 g/s. Likewise, the freestream temperatures varied depending on the run but ranged from approximately 1700 K to 2200 K. The freestream mixture's molecular mass was assumed to be that of pure steam: 18 g/mol. The viscosity of the freestream was assumed to be the viscosity of water vapor at 2000 °C, which is 0.00007808 kg/(m*s). The thermal conductivity of the steam was assumed to be the corresponding value at 2000 °C: 0.00029183 kW/(m*K).

The test article was treated as a lumped system. The initial temperature of the test article was assumed to be 45 °C. This is slightly higher than room temperature and accounts for the consideration that the test article will not have fully cooled down to room temperature between the end of one run and the start of the next. The specific heat at constant pressure of the test article was assumed to be \( c_p = 0.500 \text{ kJ/(kg*K)} \). The density of stainless steel was assumed to be approximately 8000 kg/(m³). Finally, only the sides of the cylinders were considered during the heat transfer calculations of this study.

4.2 Techniques Used

MATLAB was used for the majority of the calculations as well as for plotting the results. Additionally, the MATLAB version of CEA, a chemical combustion software package, was used for calculating flow properties. The model used in this study calls the "tp" and "hp"
cases of the CEAM code. In the former, temperature and pressure are among the inputs and are held constant; in the latter, pressure and specific enthalpy are the inputs and are held constant.

In order to compute transport properties, which are then used to calculate the convective heat transfer coefficient, the "hp" case was used to calculate the properties of the freestream fluid mixture at the entrance of the combustor, including calculating the results of combustion, if H$_2$(g) and O$_2$(g) have been added. Values that were calculated using CEAM included adiabatic flame temperature, $T_{ad}$; speed of sound, $a$; specific heat at constant pressure, $c_p$; conductive heat transfer coefficient, $k$; Prandtl number, $Pr$; the ratio of specific heats, $\gamma$; the molecular weight, $\mathfrak{M}$; and the dynamic viscosity, $\mu$. The "hp" case was also used to calculate the same properties of the mixed fluid in cases with N$_2$ feedstock injection.

The "tp" case was used to calculate the arithmetic mean properties and the stagnation viscosity for the Bartz equation. The outputs procured in this case were viscosities $\mu_{am}$ and $\mu_0$ and density, $\rho_{am}$. 

Chapter 5

RESULTS

5.1 Study A: Case with H2, O2, Steam, and N2 Injection

Study A considers injection of additional N\textsubscript{2} feedstock in the feedstock injector section and is the only study considered that has this attribute. Input conditions that varied in Study A included pebble bed steam temperature; Equivalence ratio \( \phi \); combustor freestream temperature \( p_\infty \); mass flow rates of pebble bed steam, H\textsubscript{2}, O\textsubscript{2}, and N\textsubscript{2}; and the ratio of mass flow rates \( \alpha \). In study A, a normal shock was assumed to occur midway through the feedstock injector section.

Figure 5.1: Mach number inside the SWR plotted along the streamwise, axial coordinate from the entrance of the combustor to the test section, for the same case with and without considering N\textsubscript{2} mixing
Figure 5.2: Temperature inside the SWR plotted along the streamwise, axial coordinate from the entrance of the combustor to the test section, for the same case with and without considering N\textsubscript{2} mixing.
Figure 5.3: Pressure inside the SWR plotted along the streamwise, axial coordinate from the entrance of the combustor to the test section, for the same case with and without considering N$_2$ mixing.
Figure 5.4: Temperature of the test section plotted against mass flow rate ratio $\alpha$ for cases with N$_2$ mixing
Figure 5.5: Pressure of the test section plotted against mass flow rate ratio $\alpha$ for cases with N$_2$ mixing

Figure 5.6: Nusselt number of the test section plotted against mass flow rate ratio $\alpha$ for cases with N$_2$ mixing
Figure 5.7: Velocity of the test section plotted against mass flow rate ratio $\alpha$ for cases with $N_2$ mixing

Figure 5.8: Mach number of the test section plotted against mass flow rate ratio $\alpha$ for cases with $N_2$ mixing
Figure 5.9: Viscosity of the test section plotted against mass flow rate ratio $\alpha$ for cases with $N_2$ mixing.
Figure 5.10: Thermal conductivity of the test section plotted against mass flow rate ratio $\alpha$ for cases with $N_2$ mixing.
5.2 Study B: First Case with H2, O2, and Steam

In Study B, no mixing with N2 was considered. Input conditions that were held constant include the mass flow rate of the pebble bed steam, the freestream pressure in the combustor, and the freestream temperature in the combustor. Input conditions that were varied include the mass flow rate of the H2 and O2 injected into the combustor and the equivalence ratio φ. In study B, a normal shock was assumed to occur midway through the mixer section.
5.3 Study C: Second Case with H2, O2, and Steam

In Study C, no mixing with N\textsubscript{2} was considered. Input conditions that were held constant include the mass flow rate of the pebble bed steam, the mass flow rate of the H\textsubscript{2} and O\textsubscript{2} injected into the combustor, the freestream pressure in the combustor. Input conditions that
were varied include the equivalence ratio $\phi$ and the freestream temperature in the combustor. In study C, a normal shock was assumed to occur midway through the mixer section.

Figure 5.14: Temperature vs. $T_\infty$

Figure 5.15: Temperature vs. $\phi$
5.4 Study D: Case with H2 and O2

In Study D, no mixing with N\textsubscript{2} was considered. Additionally, no pebble bed steam was used; only molecular H\textsubscript{2} and O\textsubscript{2} were inserted into the combustor. Input conditions that were varied include the mass flow rate of the H\textsubscript{2} and O\textsubscript{2} injected into the combustor, the freestream pressure in the combustor, and the equivalence ratio \( \phi \). In study B, a normal shock was assumed to occur midway through the mixer section.

![Test Section Pressure vs. \( \dot{m}_\infty \)](image)

(a) Full view

![Test Section Pressure vs. \( \dot{m}_\infty \)](image)

(b) Zoomed in

![Test Section Pressure vs. \( \dot{m}_\infty \)](image)

(c) Zoomed in further

Figure 5.16: Pressure vs. \( \dot{m}_\infty \)
Figure 5.17: Pressure in the test section plotted against the freestream pressure $P_\infty$

Figure 5.18: Pressure in the test section plotted against the equivalence ratio $\phi$
Figure 5.19: Temperature in the test section plotted against mass flow rate of the combustion reactants
5.5 Transient Heat Flow into a Test Article

Fig. (5.21) shows a representative run from these experiments. In the left chart, one can see that the steam is at approximately 740 °C during the preheat phase, as indicated by the
red line denoting the readings from the test section thermocouple. Upon insertion of the gaseous H$_2$ and O$_2$ into the combustor, the gases ignite, and the temperature and pressure throughout the system rise. The release of energy from the combustion process completely stops the influx of steam from the pebble-bed heater, despite the fact that the combined mass flow rate for the gaseous H$_2$ and O$_2$ is less than that of the pebble-bed steam. In this experiment, the temperature rises sharply to approximately 1950 °C and continues rising gradually to about 2100 °C until the burner is shut off. The downstream flow consists of the combustion products, whose composition depends on the stoichiometry of the gaseous H$_2$ and O$_2$ flow.

The graph on the right side shows the data from the pressure sensors in the gas manifold and the thermocouple in the test section. In order to feed the igniter system that generated a pilot flame, the O$_2$ flow was turned on 0.5 seconds before the H$_2$. This allowed the H$_2$ to be ignited smoothly as the H$_2$ and O$_2$ were injected at the mass flow rates desired.
The average heat transfer into the cylinder for the cases with four 0.7 s pulses was 1.95 kJ. The average heat transfer into the cylinder for the cases with four 1 s pulses was 2.54 kJ. This is a difference of 30.4 percent from the value for the 0.7 s pulses.
Figure 5.23: Total heat transfer into a test article plotted against average freestream temperature, in °C.

In Fig. (5.23), the "mean freestream temperature" is the average of all the freestream temperatures given in the "corrected temperature" column in Tables (E.1) and (E.2) across all pulses for a particular run.
Chapter 6

DISCUSSION

6.1 Study A: Case with H2, O2, Steam, and N2 Injection

It is difficult to draw rigorous conclusions from Study A, simply because so few data points were able to be collected for the case of interest, which was the case with N2 mixing. Although the data from only five experimental runs were used, there were an additional three runs with N2 mixing that were available but could not be used. This is because, unlike the five runs that were used, the three unusable runs featured values of $\alpha$ that were greater than one. For reasons that are not entirely clear, the model used in the current study is incapable of producing results for cases with such values of $\alpha$; the approach described in section 2 produces complex values of Mach number when $\alpha > 1$. This is an aphysical result and indicates that something is likely wrong with the N2 mixing portion of the model. Therefore, it is recommended that one proceed with caution when assessing the results of Study A.

Among the conclusions that can be drawn from Study A are general observations of the model’s ability to predict trends for temperature. For example, in Fig. (5.4), one can see that both the model and the experimental results appear to show an association between temperature and $\alpha$, and that both appear to show that temperature decreases as $\alpha$ increases. This is consistent with the theory. One of the objectives of injecting the N2 feedstock was to decrease the stagnation temperature, which would in turn lead to a decrease in static temperature. It makes sense that for greater proportions of N2 relative to the freestream flow, the temperature would continue to decrease.

Figures (5.1) through (5.3) display the values of Mach number, temperature, and pressure throughout the length of the SWR from the entrance of the combustor to the test section. The data in these figures are taken from the model prepared for this study. All three plots
are plotted with the input conditions of the same case, and all three display the results with and without the consideration of N₂ mixing. For temperature and pressure, both the static and stagnation values are plotted. Recall that the main objectives with injecting N₂ feedstock were to lower the stagnation temperature, raise the stagnation pressure, and raise the Mach number. One can observe that the Mach number does indeed increase at the point of N₂ injection in Fig. (5.1), that the stagnation temperature does indeed drop at the point of N₂ injection in Fig. (5.2), and that the stagnation pressure does indeed rise at the point of N₂ injection in Fig. (5.3),

As one can see in Fig. (5.5), the model appears to do a good job of matching the trends in the experimental data for Pressure. In Fig. (5.5), pressure in the test section is plotted against the ratio of mass flow rates α. In both the modeled and experimental results, the pressure appears to decrease as mass flow rate ratio α increases. However, this apparent relationship is tenuous at best. Due to the very limited number of data points, it is recommended to proceed with caution.

As one can see in Fig. (5.6), the model appears to do a good job of matching the trends in the experimental data for Nusselt number. In Fig. (5.6), Nusselt number is plotted against the ratio of mass flow rates α. In both the modeled and experimental results, the Nusselt number appears to increase sharply at about α = 0.4 and to decrease sharply at about α = 0.6; however, due to the very limited number of data points, it is recommended to proceed with caution.

As one can see in Fig. (5.7), the model appears to do a good job of matching the trends in the experimental data for velocity. In Fig. (5.7), velocity is plotted against the ratio of mass flow rates α. In both the modeled and experimental results, the velocity appears to decrease sharply at about α = 0.4 and to increase sharply at about α = 0.6; however, due to the very limited number of data points, it is recommended to proceed with caution.

As one can see in Fig. (5.8), the model does not appear to do a good job of matching the trends in the experimental data for Mach number. In Fig. (5.8), Mach number is plotted against the ratio of mass flow rates α. In the experimental results, shown in a full view
with the modeled results in Fig. (5.8a), the Mach number appears to decrease until about \( \alpha = 0.42 \) and then increase thereafter. On the other hand, in the modeled results, shown in a zoomed in view in Fig. (5.8b), the Mach number appears to steadily increase. However, due to the very limited number of data points, it is recommended to proceed with caution.

As one can see in Fig. (5.9), the model does not appear to do a good job of matching the trend in the experimental data for viscosity. In Fig. (5.9), viscosity is plotted against the ratio of mass flow rates \( \alpha \). In the experimental results, the viscosity appears to decrease sharply at about \( \alpha = 0.4 \) and to increase sharply at about \( \alpha = 0.6 \), whereas the model shows the opposite trend. However, due to the very limited number of data points, it is recommended to proceed with caution. That the model’s trends are inverted from the experimental trends, however, suggests that there might be something wrong with the model that is relatively easy to fix, such as a missing negative sign.

As one can see in Fig. (5.10), the model appears to do a good job of matching the trends in the experimental data for thermal conductivity. In Fig. (5.10), thermal conductivity is plotted against the ratio of mass flow rates \( \alpha \). In both the modeled and experimental results, the thermal conductivity appears to decrease sharply at about \( \alpha = 0.4 \) and to increase sharply at about \( \alpha = 0.6 \); however, due to the very limited number of data points, it is recommended to proceed with caution.

As one can see in Fig. (5.11), the model appears to do a good job of matching the trends in the experimental data for molecular mass. In Fig. (5.11), molecular mass is plotted against the ratio of mass flow rates \( \alpha \). In both the modeled and experimental results, the molecular mass appears to increase sharply at about \( \alpha = 0.4 \) and to decrease sharply at about \( \alpha = 0.6 \); however, due to the very limited number of data points, it is recommended to proceed with caution.

### 6.2 Study B: First Case with H2, O2, and Steam

In Study B, many more data points were available than in Study A. Recall that in Study B, no mixing with \( \text{N}_2 \) was considered. Input conditions that were held constant include
the mass flow rate of the pebble bed steam, the freestream pressure in the combustor, and the freestream temperature in the combustor. Input conditions that were varied include the mass flow rate of the H$_2$ and O$_2$ injected into the combustor and the equivalence ratio $\phi$.

Like in Study A, Study B’s results allow an assessment of the model’s ability to predict trends. In Fig. (5.12), temperature for both the model’s results and the experimental results is plotted against the freestream mass flow rate, which is the sum of the mass flow rates for both the pebble bed steam and the H$_2$ and O$_2$. In this case, the trend in the model shows good agreement with the trend in experimental results. Both the model and the experiments appear to show that the temperature increases as mass flow rate increases. This makes sense in the context of the current experiment, since the mass flow rate of the pebble bed steam was held constant, but the mass flow rate of the combustion reactants was varied. If more combustion reactants are inserted, the adiabatic flame temperature should increase. If the adiabatic flame temperature increases, then the static temperature in the test should also increase, according to the theory.

The other case considered in Study B was one in which temperature was plotted against the equivalence ratio $\phi$, as shown in Fig. (5.13). Like the results shown in Fig. (5.12), the model shows good agreement with the experimental results in terms of the trends of the data. According to both the model and the experiments, as the equivalence ratio $\phi$ increases for values of $\phi > 1$, the test section temperature decreases. This makes sense according to the theory, since increases in equivalence ratios for $\phi > 1$ corresponds to further departures from stoichiometric mass fractions of the combustion reactants, and thus lower adiabatic flame temperatures. Lower adiabatic flame temperatures would lead, in turn, to lower test section temperatures.

Overall, however, the model does not show very good agreement with the experimental values in terms of the temperature as opposed to the trends. In terms of the values, the model predicts values for temperature that are about 600 °C higher than the experimental values. This suggest one of three possibilities. The first possibility is that something is wrong with the model. The second possibility is that, at least under conditions to those similar in
Study B, the SWR undergoes a stagnation temperature loss that is not predicted very well by the theory used. This could include 2-D or even 3-D flow effects, as well as turbulent flow effects, all of which would likely produce greater frictional losses in stagnation temperature.

It is also worth noting that in Study B, the model invariably predicted test section pressures of 81 kPa. This is worthy of note mainly because the value is exactly the same, at least to two significant figures, for all runs. Admittedly, as one can see in Table (B.1), the pressure measurements in the experimental data only vary from 103 kPa to 114 kPa. It is possible that pressure in the test section has little or no dependence on $\phi$ or mass flow rate of the $\text{H}_2$ and $\text{O}_2$, which were the two input variables altered in Study B. Furthermore, it is suspected from previous experimental studies that the pressure sensors in the test section might be recording pressures that are excessively high by about 20 to 30 kPa, and the modeled results from this study seem to corroborate that.

### 6.3 Study C: Second Case with H2, O2, and Steam

Recall that in Study C, no mixing with $\text{N}_2$ was considered. Input conditions that were held constant include the mass flow rate of the pebble bed steam, the mass flow rate of the $\text{H}_2$ and $\text{O}_2$ injected into the combustor, the freestream pressure in the combustor. Input conditions that were varied include the equivalence ratio $\phi$ and the freestream temperature in the combustor. Like in Study B, a much greater amount of data points was available than in Study A.

Shown in Fig. (5.14) is a plot of the temperature in the test section plotted against the freestream temperature of the pebble bed steam. Neither the experimental values nor the modeled value show any easily discernible dependence. This makes sense according to the theory, since the temperature in the test section is understood to be dependent on the adiabatic flame temperature in the combustor, and the adiabatic flame temperature in the combustor is only dependent on the initial temperatures of the combustion reactants, in which the pebble bed steam is not included.

Shown in Fig. (5.15) is a plot of temperature in the test section plotted against the
equivalence ratio $\phi$. There appears to be good agreement between model, and the experimental results in terms of the trends predicted, and just like in Study B, the model and the experiments both appear to indicate that temperature in the test section decreases as equivalence ratio $\phi$ increases for $\phi > 1$.

In Study C, just as in Study B, the model invariably predicted test section pressures of 81 kPa. This is worthy of note mainly because the value is exactly the same, at least to two significant figures, for all runs. Admittedly, as one can see in Table (C.1), the pressure measurements in the experimental data only vary from 104 kPa to 115 kPa, which is comparable to the test envelope described by the experimental results in Study B. It is possible that pressure in the test section has little or no dependence on $\phi$ or freestream temperature of the pebble bed steam, which were the two input variables altered in Study C. Furthermore, as stated in the previous subsection, it is suspected from previous experimental studies that the pressure sensors in the test section might be recording pressures that are excessively high by about 20 to 30 kPa, and the modeled results from this study seem to corroborate that.

### 6.4 Study D: Case with H2 and O2

Recall that in Study D, no mixing with N$_2$ was considered. Additionally, no pebble bed steam was used; only molecular H$_2$ and O$_2$ were inserted into the combustor. Input conditions that were varied include the mass flow rate of the H$_2$ and O$_2$ injected into the combustor, the freestream pressure in the combustor, and the equivalence ratio $\phi$.

Shown in Fig. (5.16) are three plots of pressure in the test section plotted against the mass flow rate of the combustion reactants. In Figs. (5.16a) and (5.16b), the latter of which is zoomed in on an area of interest in the former, one can see that pressure in the test section appears to increase as mass flow rate of the combustion reactants increases. As one can see in Fig. (5.16c), which shows a zoomed in region of Fig. (5.16b), the modeled values appear to agree with the experimental values in this regard. This makes sense in terms of the theory, if the ideal gas law is assumed to be valid, since temperature and pressure are directly proportional according to the ideal gas law, and the temperature in the test
section increases as a function of the adiabatic flame temperature, which in turn increases as a function of the mass of combustion reactants present in the combustor.

Shown in Fig. (5.17) are plots of the pressure in the test section plotted against the pressure in the combustor. In all three plots, one can see that both the model and the experiments show tight correlations between the two pressures, and that the pressure in the test section appears to increase linearly with the combustor pressure.

Shown in Fig. (5.18) is a plot of the test section pressure plotted against the equivalence ratio $\phi$. Both the experimental results and the modeled results appear to show that the pressure decreases as $\phi$ increases for $\phi > 1$, although the relationships found are admittedly tenuous.

Shown in Fig. (5.19) is a plot of the temperature in the test section plotted against the mass flow rate of the combustion reactants. Note that very few experimental data points exist below about 2250 °C. This is an artifact of an engineering approximation that was made during the data collection process. Due to the high temperatures in the test section during this study, the thermocouples used to measure the temperature frequently melted, typically after about three runs. For runs in which thermocouple data was unavailable, temperatures from runs with similar input conditions were used as approximations. Nevertheless, above about 2250 °C, the experimental data appears to suggest that temperature in the test section increases as freestream mass flow rate increases. The model also appears to predict the same trend throughout. This is consistent with the theory, because if there is a higher mass of reactants present in the combustor, one should expect the adiabatic flame temperature to be higher.

Shown in Fig. (5.20) is a plot of temperature plotted against the equivalence ratio $\phi$. Above about 2250 °C, the experimental results appear to suggest that temperature decreases as equivalence ratio. The model also predicts the same trend throughout.

No apparent association was found to exist between temperature in the test section and the temperature of the freestream pebble bed steam.
6.5 Test Envelope

Overall, while the model used in the current study mostly showed good agreement predicting the trends of the experimental data, the model did not show very good agreement in predicting the test section values for a particular data point. In other words, the model did not do a very good job of predicting the test envelope for a given set of input conditions.

6.5.1 Study A: Case with H2, O2, Steam, and N2 Injection

In Study A, the model’s predicted temperatures ranged from 131 kPa to 158. The experimental pressures ranged from 111 kPa to 125 kPa. The model’s predicted range came close to the experimental range but was higher and did not overlap.

The model’s predicted temperatures ranged from 1030 °C to 1400 °C. The experimental temperatures ranged from 1730 °C to 1940 °C. The model’s predicted range was lower than the predicted range for the experimental values and the two did not overlap.

The model’s predicted Mach numbers ranged from 0.183 to 0.186. The experimental Mach numbers ranged from 0.394 to 0.451. The model’s predicted range was lower than the range for the experimental values and the two did not overlap.

The model’s predicted velocity ranged from 250 m/s to 273 m/s. The experimental velocities ranged from 444 m/s to 535 m/s. The model’s predicted range was lower than the range of the experimental values and the two did not overlap.

The model’s predicted range for molecular weight ranged from 13.4 to 16.5. The experimental values for molecular weight ranged from 14.2 g/mol to 17.6 g/mol. The model’s predicted range was only slightly lower than the experimental range, and the two ranges overlapped rather well.

The model’s predicted values for viscosity ranged from 0.927 millipoise to 0.978 millipoise. The experimental values for viscosity ranged from 0.502 millipoise to 0.621 millipoise. The model’s predicted range was higher than the experimental range and the two did not overlap.

The model’s predicted values for thermal conductivity ranged from 4.05 mW/cm/K to
4.92. The experimental values for thermal conductivity ranged from 1.54 mW/cm/K to 2.88 mW/cm/K. The model’s predicted range was higher than the experimental range and the two did not overlap.

The model’s predicted values for Nusselt number ranged from 86.1 to 97.4. The experimental values for Nusselt number ranged from 165 to 227. The model’s predicted range was lower than the experimental range and the two did not overlap.

The model’s predicted value for conductive heat transfer coefficient ranged from 388 W/m²/K to 420 W/m²/K. The experimental values for conductive heat transfer coefficient ranged from 344 W/m²/K to 477 W/m²/K.

Among the input conditions that were considered for Study A, the values of α used ranged from 0.26 to 0.61. The total mass flow rates used ranged from 353 g/s to 461 g/s. The freestream pressures ranged from 320 kPa to 400 kPa. The freestream pebble bed steam temperatures ranged from 670 °C to 750 °C. The mass flow rates of pebble bed steam ranged from 11.4 g/s to 106 g/s. The mass flow rates of H₂ ranged from 28.5 g/s to 46.4 g/s. The mass flow rates of O₂ ranged from 171 g/s to 255 g/s.

6.5.2 Study B: First Case with H₂, O₂, and Steam

In Study B, the model’s predicted temperatures ranged from 2560 °C to 2580 °C. The experimental values of temperature ranged from 1960 °C to 2150 °C. The model’s predicted range was higher than the experimental range and the two did not overlap.

The model’s predicted pressures were all exactly 81 kPa to two significant figures. The experimental values for pressure ranged from 103 kPa to 114 kPa. The model’s predicted range was lower than the experimental range and the two did not overlap.

Among the input conditions that were varied in Study B, the equivalence ratio φ ranged from 1.32 to 1.52. The values for mass flow rate of the combustion reactants varied from 0.197 kg/s to 0.232 kg/s.
6.5.3 Study C: Second Case with H2, O2, and Steam

In Study C, the model’s predicted temperatures ranged from 2550 °C to 2580 °C. The experimental values of temperature ranged from 1840 °C to 2160 °C. The model’s predicted range was higher than the experimental range and the two did not overlap.

The model’s predicted pressures were all exactly 81 kPa to two significant figures. The experimental values for pressure ranged from 104 kPa to 115 kPa. The model’s predicted range was lower than the experimental range and the two did not overlap.

Among the input conditions that were varied in Study C, the equivalence ratio varied from 1.25 to 2. The freestream pebble bed steam temperature varied from 540 °C to 751 °C.

6.5.4 Study D: Case with H2 and O2

In Study D, the model’s predicted temperatures ranged from 1540 °C to 2490 °C. The experimental values of temperature ranged from 1900 °C to 2440 °C. The model’s predicted range enclosed the experimental range.

The model’s predicted pressures in the test section ranged from 15 kPa to 37 kPa, but the experimentally measured pressures ranged from 49.7 kPa to 172 kPa. The model’s predicted values for pressure were much lower than the experimental values for pressure and the two did not overlap.

Among the input conditions that were varied in Study D, the equivalence ratio varied from 0.77 to 1.55. The combustor pressures ranged from 60.5 kPa to 176 kPa. The freestream mass flow rates of the combustion reactants varied from 43.4 g/s to 284 g/s. Additionally, the time of each pulse was either 0.7 s or 1 s.

6.6 Transient Heat Transfer into a Test Article

The data from the heat transfer model is tabulated in Tables (E.1) and (E.2). As can be seen in Figs. (5.22) and (5.23), the total heat transfer absorbed by the test article varied from about 0.5 kJ to about 3 kJ. The mean freestream temperatures varied from about 2000
°C to about 2450 °C. The number of pulses was 1, 2, 4, or 5. The pulses, in this case, were pulsed firings of a rotating detonation engine upstream of the combustor section.

In Fig. (5.22), one can see a dependence of total heat transfer on the number of pulses. For one-pulse runs, the heat transfer absorbed by the cylinder was low, at about 0.7 kJ. For two-pulse firings, the heat transfer was higher, at about 1 kJ. In the case of four pulse firings, however something interesting appears in the data. Apart from one outlier, most of the runs with 1 s pulses resulted in higher heat transfer than the cases with 0.7 s pulses. In fact, the average value of the 1 s pulses in the four pulse case was 2.54 kJ, which was about 30 percent higher than the average of 1.95 kJ for the 0.7 s pulse runs. In the current study, the only number of pulses for which data was recorded for both 0.7 s pulses and 1 s pulses was the four pulse case.

The observation that an increase in the number of pulses produces an increase in the total heat transfer agrees with the theory. Specifically, this observation suggests that the heat transfer into the object is dependent on the amount of time that the test article is exposed to the high enthalpy flow. In the theory, Eqs. (2.54) and (2.55) indicate that final temperature and heat transfer have time dependencies. The observation that the runs with longer pulses produce higher heat transfer than the runs with shorter pulses also agrees with the same part of the theory.

Interestingly, there does not appear to be any easily discernible dependence of the total heat transfer on the mean freestream temperature. The theory suggests that such a dependence should exist; however, this dependence is likely obscured by the wide range of other variables that were being adjusted in the experiments considered. Such a dependence is perhaps further masked by the fact that the dependence indicated by the theory is on the freestream temperature of a specific pulse, not on the mean of all the freestream temperatures for all pulses. One recommendation for future research would be to apply elementary statistical learning algorithms, such as a k-nearest neighbors search or linear discriminant analysis, to the current data set to find the dominant modes in the data. Another recommendation for future research would be to run more tests with only one pulse and to calculate
the heat transfer for the additional one-pulse cases.
Chapter 7

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

7.1 Transient Heat Transfer into a Test Article

In Fig. (5.22), one can see a dependence of total heat transfer on the number of pulses. However, in the current study, the only number of pulses for which data was recorded for both 0.7 s pulses and 1 s pulses was the four pulse case. Therefore, one recommendation for future research would be to investigate the difference in average heat transfer absorbed by the test article for multiple pulse lengths at various numbers of pulses.

There does not appear to be any easily discernible dependence of the total heat transfer on the mean freestream temperature, although the theory suggests that such a dependence should exist; however, this dependence is likely obscured by the wide range of other variables that were being adjusted in the experiments considered. One recommendation for future research would be to apply elementary statistical learning algorithms, such as a k-nearest neighbors search or linear discriminant analysis, to the current data set to find the dominant modes in the data. Another recommendation for future research would be to run more tests with only one pulse and to calculate the heat transfer for the additional one-pulse cases.

A final recommendation for future research would be to adapt the heat transfer calculations to more exotic geometries. The current study only considered the sides of a cylinder exposed axially to the flow. However, examples of other geometries that might be of interest include a cylinder exposed axially to the flow with a hemispherical cap on the upstream end, a hexagonal prism exposed axially to the flow, and stacked coaxial cylinders and frusta of varying radii, all exposed axially to the flow.

In order to produce heat transfer models for such exotic geometries, it is recommended
that a literature search be conducted to find existing, empirically-derived Nusselt number correlations for such geometries. If a Nusselt number correlation with satisfactory accuracy cannot be found in the existing literature, it is recommended that the researcher conduct new experimental studies to find new, empirically-derived Nusselt number correlations for whatever geometries are of interest.

In such experiments, heat sensors of some kind should be mounted inside the test article, in order to measure the final temperature of the object at the end of the pulse. However, the temperature measured inside the object will not be the same as the corresponding temperature on the outside walls. Therefore, knowledge of the material properties of the test article would be necessary in order to calculate the temperature on the outside of the test article using basic conductive heat transfer relations. If a high-enthalpy test facility with windows is accessible, infrared cameras are highly recommended as a reliable method of measuring temperature surface temperature of the object via remote sensing.

Finally, some knowledge of the material properties of the test article both before and after the experiment, including mass loss due to ablation, would be necessary to determine how much heat transfer the object absorbed.

7.2 Study D: Case with H2 and O2

In Study D, it is interesting that the test section pressure appears to show such a strong linear dependence on the combustor pressure. This is interesting because according to the theory, the drop in stagnation pressure is a function of numerous variables, including the local values of stagnation temperature, Mach number, and friction factor, among others. Nevertheless, the relationship between test section pressure and initial pressure in the combustor appears to be linear. This seems to suggest that all of the other variables upon which the drop in stagnation pressure depends are themselves also linear functions of combustor pressure. One recommendation for future study would be to investigate the dependences of Mach number, stagnation temperature drop, friction factor, and the other variables on which stagnation pressure depends and see whether or not such dependences are linear. This could likely be
done adequately through computational modeling.

In Study D, it was found that test section temperature appeared to increase with freestream mass flow rate, but appeared to decrease with equivalence ratio. One recommendation for future study would be to conduct more controlled experiments similar to Study D but with either equivalence ratio or freestream mass flow rate held constant.

7.3 Considerations across Multiple Cases

Interestingly, the model appears to show a much tighter correlation between temperature in the test section and equivalence ratio $\phi$ in Study C, shown in Fig. (5.15b), than in Study B, shown in Fig. (5.13b). This is interesting because only two input variables were altered in Study B, two were altered in Study C, and one of the two was the same for both. The one that was the same for both was equivalence ratio $\phi$. For Study B, the other input variable was mass flow rate of the combustion reactants $H_2$ and $O_2$. For Study C, the second input variable that was altered was the freestream pebble bed steam temperature. This seems to suggest that the test section temperature has a greater dependence on mass flow rate of the combustion reactants than on the pebble bed steam temperature. This is consistent with the existing theory, from which it is understood that temperature in the test section is dependent on the adiabatic flame temperature in the combustor. The adiabatic flame temperature in the combustor is a function of the amount of combustion reactants present, but not on the temperature of any non-reacting chemical species that might be present.

7.4 Final Conclusions

It is recommended that extreme caution be used when considering the results and conclusions from Study A. Unfortunately, only eight data points were available for cases with $N_2$ mixing, and of those eight, the model only worked for five of them. Therefore, the number one recommendation for future research would be to conduct more experimental studies with $N_2$ feedstock injection. This would allow more rigorous examination of the model used in the current study as well as any future models.
Overall, the model does not do a very good job of predicting the values, or even the test envelope, of the experimental data, despite doing a good job of predicting the trends. This could be due to imperfections in the model, but it could also be due to the existence of additional flow effects that were neglected in the construction of the model. For example, the model assumed 1-D flow. Although 1-D approximations for turbulent flow were used to determine the Nusselt number and thereby the heat transfer, it is entirely possible that the presence of turbulence could be inducing 2-D or even 3-D effects on the flow that lead to disagreement with the model. Another flow consideration that was neglected in the current study was the effect of boundary layers, including both thermal and velocity boundary layers. The presence of such boundary layers, especially thermal boundary layers, could be partially responsible for some of the disagreement between the experimental values and the modeled values. These include, notably, the temperatures in the test section, which in many of the studies were higher for the modeled predictions than for the experimental values. Considering boundary layer effects could potentially lead to lower predicted temperatures. Therefore, another recommendation for future study would be to reconstruct the model incorporating 2-D boundary layers of both the thermal and velocity varieties in the pursuit of better agreement between the modeled and experimental values. It may also be necessary to consider non-ideal gas effects such as chemically reacting flows. Considering the effects of plume interactions due to transverse injection of N$_2$ on the momentum conservation may also be necessary.
BIBLIOGRAPHY


Appendix A

DATA FROM STUDY A: CASE WITH H2, O2, STEAM, AND N2 INJECTION

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Table A.1: Tabulated data from a second study with H₂ and O₂ inserted into the combustor, with steam from the pebble bed heater and injection of N₂ feedstock, part 1
Table A.2: Tabulated data from a second study with H\textsubscript{2} and O\textsubscript{2} inserted into the combustor, with steam from the pebble bed heater and injection of N\textsubscript{2} feedstock, part 2

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<th>Cond mW/cm\cdot K</th>
<th>Nu</th>
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Table A.3: Tabulated data from a second study with H\textsubscript{2} and O\textsubscript{2} inserted into the combustor, with steam from the pebble bed heater and injection of N\textsubscript{2} feedstock, part 3

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<th>N\textsubscript{2} g/s</th>
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Appendix B

DATA FROM STUDY B: FIRST CASE WITH H2, O2, AND STEAM

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Table B.1: Tabulated data from a study with H2 and O2 inserted into the combustor, with steam from the pebble bed heater but no injection of N2 feedstock
Appendix C

DATA FROM STUDY C: SECOND CASE WITH H₂, O₂, AND STEAM

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Table C.1: Tabulated data from a second study with H₂ and O₂ inserted into the combustor, with steam from the pebble bed heater but no injection of N₂ feedstock.
Appendix D
Table D.1: Tabulated data from a study with H$_2$ and O$_2$ inserted into the combustor, with no steam from the pebble bed heater and no injection of N$_2$ feedstock, part 1
Table D.2: Tabulated data from a study with H\textsubscript{2} and O\textsubscript{2} inserted into the combustor, with no steam from the pebble bed heater and no injection of N\textsubscript{2} feedstock, part 2
Appendix E

DATA FROM STUDY OF TRANSIENT HEAT FLOW INTO A
Table E.1: Tabulated data from transient heat flow into a stainless steel cylinder exposed axially to high enthalpy flow, part 1. There is $\text{H}_2$ and $\text{O}_2$ inserted into the combustor, with no steam from the pebble bed heater and no injection of $\text{N}_2$ feedstock.
Table E.2: Tabulated data from transient heat flow into a stainless steel cylinder exposed axially to high enthalpy flow, part 2. There is $\text{H}_2$ and $\text{O}_2$ inserted into the combustor, with no steam from the pebble bed heater and no injection of $\text{N}_2$ feedstock.

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Appendix F

MATLAB CODES

Transient_Flow_Problem_v8c.m:

```matlab
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%---University of Washington---%
%---CRDE Lab-------------------%
%---Dan Drabiak---------------%
%---03 April 2016-------------%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Don't mess with this version! It works!

% This code calculates heat transfer into a cylinder
% of steel in axial flow using transient flow analysis.

clear

% Choose your run
RunNumber = 166;
```
% Select a value from 166 through 170 or 172 through 192.
Compressible = 1;
% Set to 1 for compressible venturi meter mass flow rates,
% otherwise manifold

% Set up pulse square waves
if Compressible == 1
    TFP_MdotT_Comp;
    mDot_gs = mDot_v_gs;
else
    TFP_MdotT_Man;
    mDot_gs = mDot_m_gs;
end

% Convert mass flow to kg/s
mDot = mDot_gs*.001; % Mass flow of freestream, kg/s

% Record the number of pulses
[~,PulseNo] = size(mDot);

% Given values
P = 80000; % Pressure of freestream, Pa
TiC = 45; % Initial temperature of cylinder (e.g. air temperature), C
Ti = TiC + 273.15; % Initial temperature of cylinder, K
Ti2 = Ti; % Preserve value for later
M = 18; % Molecular mass of freestream mixture, kg/(kmol)
Mu = .00007808; % Viscosity of water, kg/(m*s), at T = 2000 C
% See Cengel, et al Table A-23
Rbar = 8314; % Universal Gas Constant, J/(kmol K), see Cengel, et al pg. 127 to verify units
R = Rbar/M; % Gas constant for working mixture, J/(kg K)
Cp = 500; % Specific heat of steel (Cengel, et al Table A-3), J/(kg K)
r1E = .25; % Radius of cylinder, inches
r1 = r1E*.0254; % Convert from inches to meters
r2E = 2; % Radius of test section, inches
r2 = r2E*.0254; % Convert from inches to meters
LE = 2; % Length of cylinder, inches
L = LE*.0254; % Convert from inches to meters
k = .29183; % Thermal conductivity, W/(m K) of steam at T = 2000 C
% See Cengel, et al Table A-23
t = PulseW; % Time of experiment, s

% Geometric bookkeeping
A1 = (r1^2)*pi; % Area of circular face of cylinder, m^2
A2 = 2*r1*pi*L; % Area of side of cylinder, m^2
A3 = (r2^2)*pi; % Cross-sectional area of cylindrical test section, m^2
V = A1*L; % Volume of cylinder, m^3

% Temperature bookkeeping
T0C = T; % Assume for now; ultimately, T0 is freestream stagnation temperature
T0 = T0C + 273.15; % Convert from Celsius to Kelvin
TC = T; % Redefine variable T while retaining old value (see next line)
T = TC + 273.15; % Convert from Celsius to Kelvin

% Radiative losses from thermocouple
T = T + 200;

% Nusselt number relations
u = mDot.*R.*T./(P*A3); % Freestream velocity according to continuity, m/s
Rho = P./(R.*T); % Density according to ideal gas law, kg/(m^3)
Rho_st = 8000; % Density of steel, approximate, kg/(m^3)
m = Rho_st*V; % Mass, kg
Re = Rho.*u.*2*r1/Mu; % Reynolds Number with respect to cylinder diameter
Nu2 = .140*(Re.^.686); % See Wiberg and Lior, Table 1, Configuration (B), section b-c
% Note: Config. (B) best approximation given our turbulent freestream

% Side walls of cylinder
h2 = Nu2.*k./L;  % Heat transfer coefficient
b2 = h2.*A2./(m*Cp);  % Exponential coefficient, 1/s
Ti = Ti2;  % Reset initial temperature
Tt2 = zeros(1,PulseNo);
Q2 = zeros(1,PulseNo);
Tt2(1) = T(1) + (Ti - T(1))*exp(-b2(1)*t);  % Final temperature, K
Q2(1) = m.*Cp.*(Tt2(1) - Ti);  % Total heat transfer into cylinder front surface, kJ
for K = 2:PulseNo
    Ti = Tt2(K-1);
    Tt2(K) = T(K) + (Ti - T(K))*exp(-b2(K)*t);  % Final temperature, K
    Q2(K) = m.*Cp.*(Tt2(K) - Ti);  % Total heat transfer into cylinder front surface, kJ
end
Tt2total = Tt2(PulseNo);

% Total heat transfer into cylinder
Q = sum(Q2,2);  % See above, J

% Note: Heat transfer output is "integrating as you go", i.e. total heat
% transfer into the surface up to a given point in time.

% Display results
% subplot(3,1,1)
% plot(t,Tt1,'-','Color',[0 .77 .57])
% grid on
% xlabel('Time, [s]'),ylabel('Temperature, [K]'),title('Surface Temperature on Front Surface of Cylinder as a Function of Time')
% subplot(3,1,2)
% plot(t,Tt2,'-','Color',[.22 .27 .51])
% grid on
% xlabel('Time, [s]'),ylabel('Temperature, [K]'),title('Surface Temperature on Side Wall of Cylinder as a Function of Time')
% subplot(3,1,3)
% plot(t,Q,'-','Color',[.5 .3 .8])
% grid on
xlabel('Time, [s]'),ylabel('Heat Transfer, [J]'),title('Heat Transfer into Cylinder as a Function of Time')
nprintf('−−−−− Run No. %.f −−−−−
',RunNumber) % Display run number

if Compressible == 1
    fprintf('
Using compressible Venturi meter mass flow')
else
    fprintf('
Using manifold mass flow')
end

fprintf('
Velocity u = %.f m/s',u) % Display velocity
fprintf('
Temperature along sides of cylinder T = %.f K',Tt2total) % Display temperature along sides T
fprintf('
Heat transfer Q = %.3f kJ

',Q*.001) % Display heat transfer Q

TFP_MdotT_Comp.m:

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%—University of Washington—%
%—CRDE Lab—-%
%—Dan Drabiak—-%
%—22 April 2016—-%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% This code computes the compressible venturi meter mass flow rates and the
% temperatures for the file Transient_Flow_Problem_v2b.m

if RunNumber == 192
    mDot_v_gs(1) = 93.32;
    mDot_v_gs(2) = 103.41;
    mDot_v_gs(3) = 104.13;
    mDot_v_gs(4) = 105.5;
elseif RunNumber == 191
    mDot_v_gs(1) = 100.31;
    mDot_v_gs(2) = 107.3;
    mDot_v_gs(3) = 117.6;
mDot_v_gs(4) = 110.9;

elseif RunNumber == 190
  mDot_v_gs(1) = 127.5;
  mDot_v_gs(2) = 120.51;
  mDot_v_gs(3) = 106.69;
  mDot_v_gs(4) = 109.18;

elseif RunNumber == 189
  mDot_v_gs(1) = 90.85;
  mDot_v_gs(2) = 105.54;
  mDot_v_gs(3) = 106.94;
  mDot_v_gs(4) = 110.44;

elseif RunNumber == 188
  mDot_v_gs(1) = 105.64;
  mDot_v_gs(2) = 106.08;
  mDot_v_gs(3) = 112.98;
  mDot_v_gs(4) = 115.75;

elseif RunNumber == 187
  mDot_v_gs(1) = 111.96;
  mDot_v_gs(2) = 115.2;
  mDot_v_gs(3) = 116.14;
  mDot_v_gs(4) = 118.5;

elseif RunNumber == 186
  mDot_v_gs(1) = 108.97;
  mDot_v_gs(2) = 115.09;
  mDot_v_gs(3) = 119.05;
  mDot_v_gs(4) = 118.44;

elseif RunNumber == 185
  mDot_v_gs(1) = 105.83;
  mDot_v_gs(2) = 107.88;
  mDot_v_gs(3) = 112.5;
  mDot_v_gs(4) = 113.04;

elseif RunNumber == 184
  mDot_v_gs = 108.75;
```plaintext
53  elseif RunNumber == 183
54    mDot_v_gs = 101.3;
55  elseif RunNumber == 182
56    mDot_v_gs = 121.08;
57  elseif RunNumber == 181
58    mDot_v_gs(1) = 121.08;
59    mDot_v_gs(2) = 111.44;
60    mDot_v_gs(3) = 119.74;
61  elseif RunNumber == 180
62    mDot_v_gs(1) = 118.33;
63    mDot_v_gs(2) = 121.08;
64    mDot_v_gs(3) = 111.44;
65    mDot_v_gs(4) = 119.74;
66  elseif RunNumber == 179
67    mDot_v_gs(1) = 115.93;
68    mDot_v_gs(2) = 120.25;
69  elseif RunNumber == 178
70    mDot_v_gs(1) = 114.6;
71    mDot_v_gs(2) = 119.72;
72  elseif RunNumber == 177
73    mDot_v_gs(1) = 119.08;
74    mDot_v_gs(2) = 120.2;
75  elseif RunNumber == 176
76    mDot_v_gs(1) = 112.08;
77    mDot_v_gs(2) = 110.83;
78    mDot_v_gs(3) = 112.5;
79    mDot_v_gs(4) = 112.51;
80    mDot_v_gs(5) = 112.59;
81  elseif RunNumber == 175
82    mDot_v_gs(1) = 95.28;
83    mDot_v_gs(2) = 90.7;
84    mDot_v_gs(3) = 94.74;
85    mDot_v_gs(4) = 94.54;
```
mDot_v_gs(5) = 94.16;

elseif RunNumber == 174
  mDot_v_gs(1) = 97.37;
  mDot_v_gs(2) = 98.84;
  mDot_v_gs(3) = 94.94;
  mDot_v_gs(4) = 97.44;
  mDot_v_gs(5) = 95.97;

elseif RunNumber == 173
  mDot_v_gs(1) = 100.29;
  mDot_v_gs(2) = 99.93;
  mDot_v_gs(3) = 104.37;
  mDot_v_gs(4) = 103.6;
  mDot_v_gs(5) = 105.95;

elseif RunNumber == 172
  mDot_v_gs(1) = 107;
  mDot_v_gs(2) = 106.66;
  mDot_v_gs(3) = 104.59;
  mDot_v_gs(4) = 105.87;
  mDot_v_gs(5) = 101.88;

elseif RunNumber == 170
  mDot_v_gs(1) = 105.37;
  mDot_v_gs(2) = 103.94;
  mDot_v_gs(3) = 107.31;
  mDot_v_gs(4) = 107.13;

elseif RunNumber == 169
  mDot_v_gs(1) = 102.17;
  mDot_v_gs(2) = 107.85;
  mDot_v_gs(3) = 102.09;
  mDot_v_gs(4) = 105.27;

elseif RunNumber == 168
  mDot_v_gs(1) = 106.93;
  mDot_v_gs(2) = 109.18;
  mDot_v_gs(3) = 108.39;
mDot_v_gs(4) = 109.93;

elseif RunNumber == 167
mDot_v_gs(1) = 111.54;
mDot_v_gs(2) = 109.06;
mDot_v_gs(3) = 111.03;
mDot_v_gs(4) = 108.11;

elseif RunNumber == 166
mDot_v_gs(1) = 111.11;
mDot_v_gs(2) = 111.83;
mDot_v_gs(3) = 111.97;
mDot_v_gs(4) = 109.24;
end

% Now do it all over again for temperature, you lucky duck.

if RunNumber == 192
T(1) = 1785;
T(2) = 1997;
T(3) = 2091;
T(4) = 2091;

elseif RunNumber == 191
T(1) = 1785;
T(2) = 1997;
T(3) = 2091;
T(4) = 2091;

elseif RunNumber == 190
T(1) = 2109;
T(2) = 1724;
T(3) = 1700;
T(4) = 1796;

elseif RunNumber == 189
T(1) = 2104;
T(2) = 2148;
T(3) = 2148;
T(4) = 2148;

elseif RunNumber == 188
    T(1) = 1846;
    T(2) = 2013;
    T(3) = 2000;
    T(4) = 2068;

elseif RunNumber == 187
    T(1) = 2225;
    T(2) = 2242;
    T(3) = 2242;
    T(4) = 2242;

elseif RunNumber == 186
    T(1) = 2094;
    T(2) = 2088;
    T(3) = 2143;
    T(4) = 2143;

elseif RunNumber == 185
    T(1) = 2121;
    T(2) = 2121;
    T(3) = 2121;
    T(4) = 2121;

elseif RunNumber == 184
    T = 2079;

elseif RunNumber == 183
    T = 1997;

elseif RunNumber == 182
    T = 2180;

elseif RunNumber == 181
    T(1) = 2180;
    T(2) = 2266;
    T(3) = 2011;

elseif RunNumber == 180
    T(1) = 2139;
T(2) = 2199;
T(3) = 2199;
T(4) = 2199;
elseif RunNumber == 179
  T(1) = 2134;
  T(2) = 2164;
elseif RunNumber == 178
  T(1) = 2167;
  T(2) = 2156;
elseif RunNumber == 177
  T(1) = 2089;
  T(2) = 2121;
elseif RunNumber == 176
  T(1) = 2048;
  T(2) = 2048;
  T(3) = 2048;
  T(4) = 2048;
  T(5) = 2048;
elseif RunNumber == 175
  T(1) = 2060;
  T(2) = 2062;
  T(3) = 2077;
  T(4) = 2088;
  T(5) = 2095;
elseif RunNumber == 174
  T(1) = 2079;
  T(2) = 2113;
  T(3) = 2145;
  T(4) = 2141;
  T(5) = 2141;
elseif RunNumber == 173
  T(1) = 2079;
  T(2) = 2113;
T(3) = 2145;
T(4) = 2141;
T(5) = 2141;
elseif RunNumber == 172
T(1) = 2079;
T(2) = 2113;
T(3) = 2145;
T(4) = 2141;
T(5) = 2141;
elseif RunNumber == 170
T(1) = 2092;
T(2) = 2092;
T(3) = 2092;
T(4) = 2092;
elseif RunNumber == 169
T(1) = 2208;
T(2) = 2208;
T(3) = 2208;
T(4) = 2208;
elseif RunNumber == 168
T(1) = 2068;
T(2) = 2102;
T(3) = 2102;
T(4) = 2102;
elseif RunNumber == 167
T(1) = 2068;
T(2) = 2102;
T(3) = 2102;
T(4) = 2102;
else
T(1) = 2068;
T(2) = 2102;
T(3) = 2102;
if RunNumber < 182
    PulseW = .7;
else
    PulseW = 1;
end

% This code computes the manifold mass flow rates and the temperatures for
% the file Transient_Flow_Problem_v2b.m

if RunNumber == 192
    mDot_m_gs(1) = 126.3;
    mDot_m_gs(2) = 110.8;
    mDot_m_gs(3) = 106.16;
    mDot_m_gs(4) = 102.32;
elseif RunNumber == 191
    mDot_m_gs(1) = 108.51;
    mDot_m_gs(2) = 109.98;
    mDot_m_gs(3) = 106.05;
    mDot_m_gs(4) = 96.48;
elseif RunNumber == 190
mDot_m_gs(1) = 134.67;
mDot_m_gs(2) = 126.09;
mDot_m_gs(3) = 123.24;
mDot_m_gs(4) = 120.23;
elseif RunNumber == 189
mDot_m_gs(1) = 144.47;
mDot_m_gs(2) = 139.19;
mDot_m_gs(3) = 135.56;
mDot_m_gs(4) = 127.81;
elseif RunNumber == 188
mDot_m_gs(1) = 134.02;
mDot_m_gs(2) = 142.69;
mDot_m_gs(3) = 135.06;
mDot_m_gs(4) = 130.37;
elseif RunNumber == 187
mDot_m_gs(1) = 127.34;
mDot_m_gs(2) = 128.53;
mDot_m_gs(3) = 129.43;
mDot_m_gs(4) = 124.58;
elseif RunNumber == 186
mDot_m_gs(1) = 137.83;
mDot_m_gs(2) = 130.5;
mDot_m_gs(3) = 128.24;
mDot_m_gs(4) = 126.66;
elseif RunNumber == 185
mDot_m_gs(1) = 142.41;
mDot_m_gs(2) = 141.46;
mDot_m_gs(3) = 139.79;
mDot_m_gs(4) = 138.63;
elseif RunNumber == 184
mDot_m_gs = 137.54;
elseif RunNumber == 183
mDot_m_gs = 144.12;
elseif RunNumber == 182
mDot_m_gs(1) = 147.21;
elseif RunNumber == 181
mDot_m_gs(1) = 147.21;
mDot_m_gs(2) = 138.15;
mDot_m_gs(3) = 136.79;
elseif RunNumber == 180
mDot_m_gs(1) = 139.6;
mDot_m_gs(2) = 147.21;
mDot_m_gs(3) = 138.15;
mDot_m_gs(4) = 136.79;
elseif RunNumber == 179
mDot_m_gs(1) = 140.47;
mDot_m_gs(2) = 142.87;
elseif RunNumber == 178
mDot_m_gs(1) = 140.66;
mDot_m_gs(2) = 144.57;
elseif RunNumber == 177
mDot_m_gs(1) = 141.34;
mDot_m_gs(2) = 145.78;
elseif RunNumber == 176
mDot_m_gs(1) = 133.46;
mDot_m_gs(2) = 134.26;
mDot_m_gs(3) = 134.79;
mDot_m_gs(4) = 134.86;
mDot_m_gs(5) = 134.61;
elseif RunNumber == 175
mDot_m_gs(1) = 118.55;
mDot_m_gs(2) = 119.29;
mDot_m_gs(3) = 118.43;
mDot_m_gs(4) = 117.1;
mDot_m_gs(5) = 116.7;
elseif RunNumber == 174
mDot_m_gs(1) = 121.35;
mDot_m_gs(2) = 123.54;
mDot_m_gs(3) = 123;
mDot_m_gs(4) = 122.98;
mDot_m_gs(5) = 121.79;

elseif RunNumber == 173
mDot_m_gs(1) = 123.63;
mDot_m_gs(2) = 124.41;
mDot_m_gs(3) = 124.83;
mDot_m_gs(4) = 124.02;
mDot_m_gs(5) = 123.18;

elseif RunNumber == 172
mDot_m_gs(1) = 124.49;
mDot_m_gs(2) = 128.55;
mDot_m_gs(3) = 126.59;
mDot_m_gs(4) = 124.99;
mDot_m_gs(5) = 124.76;

elseif RunNumber == 170
mDot_m_gs(1) = 130.86;
mDot_m_gs(2) = 128.39;
mDot_m_gs(3) = 128.28;
mDot_m_gs(4) = 127.03;

elseif RunNumber == 169
mDot_m_gs(1) = 135.24;
mDot_m_gs(2) = 131.78;
mDot_m_gs(3) = 131.47;
mDot_m_gs(4) = 130.86;

elseif RunNumber == 168
mDot_m_gs(1) = 136.8;
mDot_m_gs(2) = 133.35;
mDot_m_gs(3) = 132.62;
mDot_m_gs(4) = 132.18;

elseif RunNumber == 167
mDot_m_gs(1) = 139.5;
mDot_m_gs(2) = 135.6;
mDot_m_gs(3) = 134.34;
mDot_m_gs(4) = 134.61;
elseif RunNumber == 166
  mDot_m_gs(1) = 129.29;
  mDot_m_gs(2) = 134.54;
  mDot_m_gs(3) = 134.71;
  mDot_m_gs(4) = 133.43;
end

% Now do it all over again for temperature, you lucky duck.
if RunNumber == 192
  T(1) = 1785;
  T(2) = 1997;
  T(3) = 2091;
  T(4) = 2091;
elseif RunNumber == 191
  T(1) = 1785;
  T(2) = 1997;
  T(3) = 2091;
  T(4) = 2091;
elseif RunNumber == 190
  T(1) = 2109;
  T(2) = 1724;
  T(3) = 1700;
  T(4) = 1796;
elseif RunNumber == 189
  T(1) = 2104;
  T(2) = 2148;
  T(3) = 2148;
  T(4) = 2148;
elseif RunNumber == 188
T(1) = 1846;
T(2) = 2013;
T(3) = 2000;
T(4) = 2068;
elseif RunNumber == 187
  T(1) = 2225;
  T(2) = 2242;
  T(3) = 2242;
  T(4) = 2242;
elseif RunNumber == 186
  T(1) = 2094;
  T(2) = 2088;
  T(3) = 2143;
  T(4) = 2143;
elseif RunNumber == 185
  T(1) = 2121;
  T(2) = 2121;
  T(3) = 2121;
  T(4) = 2121;
elseif RunNumber == 184
  T(1) = 2079;
else
  T(1) = 2180;
  T(2) = 2266;
  T(3) = 2011;
else
  T(1) = 2139;
  T(2) = 2199;
  T(3) = 2199;
T(4) = 2199;

elseif RunNumber == 179
    T(1) = 2134;
    T(2) = 2164;

elseif RunNumber == 178
    T(1) = 2167;
    T(2) = 2156;

elseif RunNumber == 177
    T(1) = 2089;
    T(2) = 2121;

elseif RunNumber == 176
    T(1) = 2048;
    T(2) = 2048;
    T(3) = 2048;
    T(4) = 2048;
    T(5) = 2048;

elseif RunNumber == 175
    T(1) = 2060;
    T(2) = 2062;
    T(3) = 2077;
    T(4) = 2088;
    T(5) = 2095;

elseif RunNumber == 174
    T(1) = 2079;
    T(2) = 2113;
    T(3) = 2145;
    T(4) = 2141;
    T(5) = 2141;

elseif RunNumber == 173
    T(1) = 2079;
    T(2) = 2113;
    T(3) = 2145;
    T(4) = 2141;
T(5) = 2141;

elseif RunNumber == 172
T(1) = 2079;
T(2) = 2113;
T(3) = 2145;
T(4) = 2141;
T(5) = 2141;

elseif RunNumber == 171
fprintf('Lost run')

elseif RunNumber == 170
T(1) = 2092;
T(2) = 2092;
T(3) = 2092;
T(4) = 2092;

elseif RunNumber == 169
T(1) = 2208;
T(2) = 2208;
T(3) = 2208;
T(4) = 2208;

elseif RunNumber == 168
T(1) = 2068;
T(2) = 2102;
T(3) = 2102;
T(4) = 2102;

elseif RunNumber == 167
T(1) = 2068;
T(2) = 2102;
T(3) = 2102;
T(4) = 2102;

else
T(1) = 2068;
T(2) = 2102;
T(3) = 2102;
T(4) = 2102;
end

if RunNumber < 182
    PulseW = .7;
else
    PulseW = 1;
end

SWR_Modeling_Function.m:

% Don't mess with this version of the code. It works!

% This is the base version of the code from which all of the variants used for
% the final calculations were derived. The nuts and bolts are the same
% throughout; it's just some of the input conditions that vary, and thus
% require slight modifications.

% Note: This code assumes isentropic flow; thus, oblique shocks and
% expansion fans are ignored.

% References:
close all; clear

ShockStation = '3';

% Determine station of the normal shock that approximates the oblique
% shock train.
% Possible values are '4' for the 4" duct and '3' for the 3" duct.

I2M = .0254; % Inches to meters conversion factor; 1 inch = .0254 meters
M2I = 1./I2M; % Meters to inches conversion factor; 1 inch = .0254 meters

% Outer radius of the point at the beginning of each station, in inches
r(1) = 2;
r(2) = 2;
r(3) = (.0397/2).*M2I;
r(4) = 1.5;  
r(5) = 1.5;  
r(6) = 2;  
r(7) = 2;  

r(4) = 1.5;  
r(5) = 1.5;  
r(6) = 2;  
r(7) = 2;  

% Axial coordinates of each point, in inches  
x(1) = 0;  
x(2) = 1/I2M;  
x(3) = x(2) + 2.5;  
x(4) = x(3) + 1.5*sqrt(3);  
x(5) = x(4) + .22/I2M;  
x(6) = x(5);  
x(7) = x(6) + 1/I2M;  

xthroat = x(3);  

NS = length(r); % Number of corners  

% Axial length of each station, in inches  
L = zeros(NS,1);  
L(1) = x(1);  
for j = 2:NS  
    L(j) = x(j) - x(j-1);  
end  

% Convert everything to SI units  
r = r.*I2M;  
x = x.*I2M;  
L = L.*I2M;  

NP = 2000; % Multiplier per each increment  

% Outer radius as a function of axial length, m
Rc = cell(NS-1,1);
for j = 1:NS-1
    Rc(j) = linspace(r(j),r(j+1),round(L(j+1)*NP));
end

% Cumulative length vector
Lc = cell(NS,1);
Lc{1} = 0;
for j = 2:NS
    Lc{j} = length(Rc{j-1}) + Lc{j-1};
end

% Outer radius vector
% Rv(1:length(Rc{1})) = Rc{1};
for j = 1:NS-1
    Rv(Lc{j} + 1:Lc{j+1}) = Rc{j};
end

for j = 1:NS-1
    X(Lc{j}+1:Lc{j+1}) = linspace(x(j),x(j+1),Lc{j+1}−Lc{j}); % Axial position vector, m
end

%---Define circular subsonic region of throat---%

rstar_m = r(3); % Throat radius, m
rcurv = r(2)−r(3); % Radius of curvature, m
kcircle = rcurv + rstar_m; % y-co-ordinate of the center of the circle, m
y_ne = r(2); % y-co-ordinate of the entrance of the nozzle, m
jcircle = L(2) + rcurv;
% jcircle = L(2) + sqrt((rcurv^2)−((y_ne − rcurv − rstar_m)^2));

%---Re-define geometry to include circular region---%
% Axial coordinates of each point, in inches
x(1) = 0;
\( x(2) = \frac{1}{I^2M} \);
\( x(3) = j\text{circle}\cdot M^2 I \);
\( x(4) = x(2) + 4.925; \) % From drawings
\( x(5) = x(4) + \frac{.22}{I^2M}; \)
\( x(6) = x(5); \)
\( x(7) = x(6) + \frac{1}{I^2M}; \)
\( x_{\text{throat}} = x(3); \)

% Axial length of each station, in inches
L = zeros(NS,1);
\( L(1) = x(1); \)
\textit{for} j = 2:NS
\( L(j) = x(j) - x(j-1); \)
\textit{end}

% Convert everything to SI units
x = x.*I2M;
L = L.*I2M;

NP = 2000; % Multiplier per each increment

% Outer radius as a function of axial length, m
Rc = cell(NS-1,1);
\textit{for} j = 1:NS-1
\( \text{Rc}(j) = \text{linspace}(r(j),r(j+1),\text{round}(L(j+1)\cdot NP)); \)
\textit{end}

% Cumulative length vector
Lc = cell(NS,1);
\( Lc(1) = 0; \)
for j = 2:NS
    Lc{j} = length(Rc{j-1}) + Lc{j-1};
end

% Outer radius vector
% Rv(1:length(Rc{1})) = Rc{1};
for j = 1:NS-1
    Rv(Lc{j} + 1:Lc{j+1}) = Rc{j};
end

for j = 1:NS-1
    X(Lc{j}+1:Lc{j+1}) = linspace(x(j),x(j+1),Lc{j+1}-Lc{j}); % Axial position vector, m
end

Rv = real(Rv);

ycurv = kcircle - sqrt((rcurv^2)-((X(Lc{2}:Lc{3}) - jcircle).^2));
y2 = Rv;
y2(Lc{2}:Lc{3}) = ycurv;
Rv = y2;
Rv(Lc{3}) = rstar_m;

Rv = real(Rv);
NT = length(Rv);
% Corrected total number of points to account for rounding when calculating % Rx vector

% Define index of the nozzle entrance in the X vector, among other vectors.
for j = 1:NT
    if X(j) == L(2)
        Idx2 = j;
    end
end
% Define index of the exit of the diffuser in the X vector, among other vectors.
Idx4 = Lc{4};

% Define index of the exit of the diffuser in the X vector, among other vectors.
Idx5 = Lc{5};

dx = x(end)/NT; % Length increment from one point to the next, in
% [~,Idx3] = (min(abs(Rv)));
Idx3 = Lc(3);

% Index of the throat in the X vector, among other vectors
Rv = Rv';

%---Input Conditions---%
mdotH2O = .06367; % Mass flow rate of steam, kg/s
 phi = 1.49; % Equivalence Ratio, []
mdotO2_tot = .17346;

% Mass flow rate of O2, kg/s
mdotH2_tot = .03787;

% Mass flow rate of H2, kg/s
mdotinj = mdotO2_tot + mdotH2_tot; % Mass flow rate of H2 and O2, kg/s
mdotinf = mdotH2O + mdotinj;

mdotinf_g = 1000*mdotinf; % Freestream mass flow rate, g/s
T0inf_C = 730; % Freestream temperature, degrees C
P0inf_kPa = 370; % Freestream pressure, kPa

gamma = 1.22; % Ratio of specific heats, []

rstar = .0397/2; % Radius of the throat, m

MWH2O = 18; % Molecular weight, kg/kmol

Ts_C = 150; % Temperature of the wall, degrees C

Z = mdotinj/mdotinf; % Proportion by mass of (2H2+O2) in the H2O, H2, and O2 mixture.
% Note: Z ranges from 0 to 1.
dotH2_extra_g = 0; % Extra amount of H2 beyond stoichiometric amount, g/s
mdot4 = .106; % Mass flow of injectant, kg/s

%---End Input Conditions---%

% Universal Constants
Rbar = 8314; % Universal gas law constant, J/(kmol K)
Patm = 101325; % Standard atmospheric pressure, Pa

%---End Universal Constants---%

mdotinf = mdotinf_g./1000; % Freestream mass flow rate, kg/s
Ts = Ts_C + 273.15; % Wall temperature, K
T0inf = T0inf_C + 273.15; % Freestream temperature, K
P0inf = P0inf_kPa*1000; % Freestream pressure, Pa
P0inf_bar = P0inf_kPa/100; % Freestream pressure, bar
rstar = rstar_in*I2M;
% Convert radius of the throat to meters
Astar = pi.*(rstar^2); % Area of the throat, m^2
RH2O = Rbar/MWH2O; % Gas constant for freestream mixture, J/(kg K)

A = pi.*(Rv.^2); % Cross-sectional area, m^2
A2 = 2.*pi.*Rv.*dx; % Circumferential area, m^2
rho0inf = P0inf./(RH2O.*T0inf); % Freestream density, kg/(m^3)
Uinf = mdotinf./(rho0inf.*A(1)); % Freestream velocity, m/s

% Intermediate constants to make the code less jumbled
B = 2/(gamma+1);
C = (gamma-1)/2;
G = (gamma+1)/(2.*(gamma-1));

Cp = (gamma*RH2O)/(gamma-1); % Specific heat at constant pressure, same units as R
Cv = RH2O/(gamma-1); % Specific heat at constant volume, same units as R
ainf = sqrt(gamma.*RH2O.*T0inf); % Speed of sound at the inlet, m/s
Minf = Uinf./ainf; % Freestream Mach number, []

Marea = .001:.001:5;
Marea = Marea';
Amach = (Astar./Marea).*((B.*(1+C.*Marea.^2)).^G);
% Produce a range of areas for all possible Mach numbers over an interval
% of .001 from 0 to 5.

IdxM1 = find(Marea == 1);
M(1:Idx3) = interp1(Amach(1:IdxM1),Marea(1:IdxM1),A(1:Idx3));
M(Idx3+1:Idx5) = interp1(Amach(IdxM1:end),Marea(IdxM1:end),A(Idx3+1:Idx5));

% However, it does account for a normal shock at the end of the feedstock
% injector:
Mps = ((1+((gamma−1)/2).*M(Idx5).^2)./(gamma.*M(Idx5).^2)−... 
  ((gamma−1)/2))).^.5;
% Post−shock Mach number, []
M(Idx5+1:NT) = Mps*ones(NT−Idx5,1);
% From Anderson, pg. 597

M = M';

T = T0inf./(1+C.*(M.^2)); % Stagnation temperature, K
P = P0inf./((1+C.*(M.^2)).^(gamma/(gamma−1)))); % Stagnation pressure, Pa
rho = rho0inf.*((1+C.*(M.^2)).^(1/(gamma−1))); % Stagnation density, kg/(m^3)
% Remember, this code assumes no oblique shockwaves!
Pps = P(Idx5).*((1+((gamma−1)/2).*M(Idx5).^2) − 1));
P(Idx5+1:end) = Pps.*ones(NT−Idx5,1);
rhops = rho(Idx5).*((gamma + 1).*M(Idx5).^2)/...
  (2 + (gamma − 1).*M(Idx5).^2));
%---H2 and O2 insertion in combustor---%

OFst = 8; % Stoichiometric O/F ratio for H2 and O2
OF = mdotO2_tot/mdotH2_tot;
% Oxidizer to fuel ratio for H2 and O2 at entrance

if OF > OFst
    mdotH2_extra = 0;
    mdotO2_extra = ((OF-OFst)/OF)*mdotO2_tot;
elseif OF < OFst
    mdotH2_extra = ((OFst-OF)/OFst)*mdotH2_tot;
    mdotO2_extra = 0;
elseif OF == OFst
    mdotH2_extra = 0;
    mdotO2_extra = 0;
end

mdotH2 = mdotH2_tot - mdotH2_extra;
mdotO2 = mdotO2_tot - mdotO2_extra;
mdotSteam = mdotinf - mdotH2_extra - mdotO2_extra;

gammaH2 = 1.405; % From Table A-20 on pg. 925 in 7.
MWH2 = 2; % Molecular mass of H2, g/mol
MH2 = 1; % Mach number of injectant, []
% Assume injectant enters at sonic velocity

gammaO2 = 1.395; % From Table A-20 on pg. 925 in 7.
MWO2 = 16; % Molecular mass of O2, g/mol
MO2 = 1; % Mach number of incoming freestream, []
% Assume adiabatic isentropic Mach number for now.
MWH2O = 18; % Molecular mass of O2, g/mol
MH2O = M(1); % Mach number of incoming freestream, []
% Assume adiabatic isentropic Mach number for now.

ndotH2 = mdotH2/MWH2; % Molar flow rate of H2, kmol/s
ndotO2 = mdotO2/MWO2; % Molar flow rate of O2, kmol/s
ndotH2O = mdotH2O/MWH2O; % Molar flow rate of steam, kmol/s

ndotH2_tot = mdotH2_tot/MWH2; % Molar flow rate of H2, kmol/s
ndotO2_tot = mdotO2_tot/MWO2; % Molar flow rate of O2, kmol/s
ndot_tot = ndotH2_tot + ndotO2_tot + ndotH2O; % Total molar flow rate, kmol/s

ndotH2_extra = mdotH2_extra/MWH2; % Molar flow rate of H2, kmol/s
ndotO2_extra = mdotO2_extra/MWO2; % Molar flow rate of O2, kmol/s

yH2 = ndotH2/ndot_tot;
yO2 = ndotO2/ndot_tot;
yH2O = ndotH2O/ndot_tot;
yH2_tot = ndotH2_tot/ndot_tot;
yO2_tot = ndotO2_tot/ndot_tot;
yH2_extra = ndotH2_extra/ndot_tot;
yO2_extra = ndotO2_extra/ndot_tot;

PH2 = yH2*P0inf; % Partial pressure of H2, Pa
PO2 = yO2*P0inf; % Partial pressure of O2, Pa
PH2O = P0inf*(1 - yH2_tot - yO2_tot); % Partial pressure of steam, Pa
PSteam = P0inf*(1 - yH2_extra - yO2_extra);

PH2_extra = yH2_extra*P0inf; % Partial pressure of H2, Pa
PO2_extra = yO2_extra*P0inf; % Partial pressure of O2, Pa
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% Convert pressures from Pascals to bar

\begin{align*}
PH2\text{\_bar} &= \frac{PH2}{100000}; \\
PO2\text{\_bar} &= \frac{PO2}{100000}; \\
PH2O\text{\_bar} &= \frac{PH2O}{100000}; \\
PSteam\text{\_bar} &= \frac{PSteam}{100000}; \\
\end{align*}

% Remember, these are intermediate variables to make the code less
% jumbled.

\begin{align*}
\text{RH2} &= \frac{Rbar}{MWH2}; \quad \% \text{Gas law constant for injectant} \\
\text{RO2} &= \frac{Rbar}{MWO2}; \quad \% \text{Gas law constant for injectant} \\
\end{align*}

\begin{align*}
CH2 &= \frac{(\gamma H2-1)}{2}; \\
BH2 &= \frac{2}{(\gamma H2+1)}; \\
\text{GH2} &= \frac{(\gamma H2+1)}{(2\times(\gamma H2-1))}; \\
\end{align*}

\begin{align*}
\text{Tatm} &= 298.15; \quad \% \text{Atmospheric temperature, K} \\
\text{TH2} &= \text{Tatm}; \\
\text{TO2} &= \text{Tatm}; \\
\text{CEAout2} &= \text{CEA('problem','hp','frozen','p,bar',P0inf\_bar,...}
\quad \text{'reactants','name','H2O(g)','H',2,'O',1,...}
\quad \text{'wt\%','mdotH2O','t(K)',T0inf,'name','H2(g)','H',2,...}
\quad \text{'wt\%','mdotH2\_tot','t(K)',TH2,'name','O2(g)','O',2,...}
\quad \text{'wt\%','mdotO2\_tot','t(K)',TO2,...}
\quad \text{'output','transport','short','end');}
\end{align*}
% Run CEAM
Cp2_kJ = CEAout2.output.cp_tran.froz; % Specific heat at constant pressure, [kJ/(kg*K)]
Cp2 = Cp2_kJ.*1000; % Convert to J/(kg*K)
mu2 = CEAout2.output.viscosity./10000; % Dynamic Viscosity, kg/(m*s)
% Note that default output from CEAM is in millipoise.
% 10 P = 1 kg/(m*s), 1 P = 1000 mP
cond2 = CEAout2.output.conduct.froz.*.1; % Conductive heat transfer coefficient, W/(m*K)
% Note that the default output from CEAM is in mW/(cm*K)
Pr2 = CEAout2.output.prandtl.froz; % Specific heat at constant pressure, kJ/(kg*K)
gamma2 = CEAout2.output.gamma; % Ratio of specific heats from CEAM, []
rho02 = CEAout2.output.density; % Density, kg/(m^3)
MW2 = CEAout2.output.mw; % Molecular weight, check units
% Educated guess based on order of magnitude (10^1): kg/kmol
a2 = CEAout2.output.sonvel; % Sonic velocity, m/s
Tad = CEAout2.output.temperature;
% Adiabatic flame temperature of mixture, K
h2_kJ = CEAout2.output.enthalpy; % Specific enthalpy, kJ/kg
h2 = h2_kJ.*1000; % Convert specific enthalpy to J/kg

R2 = Rbar/MW2;

% Intermediate constants to make the code less jumbled
B2 = 2/(gamma2+1);
C2 = (gamma2-1)/2;
G2 = (gamma2+1)/(2.*(gamma2-1));

% Define indices of possible shock locations
IdxShk3 = round((Idx4 + Idx5)/2);
% Assumes shock occurs halfway through the feedstock injector section
IdxShk4 = round((Idx5 + NT)/2);
% Assumes shock occurs halfway through the mixer section
Amach2 = (Astar./Marea).*((B2.*(1+C2.*Marea.^2)).^G2);
if strcmp(ShockStation,'4') == 1
    M2(1:Idx3)= interp1(Amach2(1:IdxM1),Marea(1:IdxM1),A(1:Idx3));
    M2(Idx3+1:IdxShk4) = interp1(Amach2(IdxM1:end),Marea(IdxM1:end),A(Idx3+1:IdxShk4));

    % Account for a normal shock midway through the mixer section:
    Mps2_b = ((1+C2.*M2(IdxShk4).^2)./((gamma2.*M2(IdxShk4).^2)-C2)).^.5;
    P2 = P0inf./((1+C2.*(M2.^2)).^(gamma2/(gamma2-1))); % Static pressure, Pa
    Pratio2 = (1+((2.*gamma2)/(gamma2+1))*((M2(IdxShk4).^2)-1));
    Pps2 = Pratio2*P2(IdxShk4);
    P2(IdxShk4+1:end) = Pps2.*ones(NT-IdxShk4,1);
    rho2 = P2./(R2.*T2); % Static density, kg/(m^3)

elseif strcmp(ShockStation,'3') == 1
    M3(1:Idx3)= interp1(Amach2(1:IdxM1),Marea(1:IdxM1),A(1:Idx3));
    M3(Idx3+1:IdxShk3) = interp1(Amach2(IdxM1:end),Marea(IdxM1:end),A(Idx3+1:IdxShk3));

    % Account for a normal shock midway through the feedstock injector:
    Mps3 = ((1+C2.*M3(IdxShk3).^2)./((gamma2.*M3(IdxShk3).^2)-C2)).^.5;
    P3 = P0inf./((1+C2.*(M3.^2)).^(gamma2/(gamma2-1))); % Static pressure, Pa
    Pratio3 = (1+((2.*gamma2)/(gamma2+1))*((M3(IdxShk3).^2)-1));
    Pps3 = Pratio3*P3(IdxShk3);
    P3(IdxShk3+1:end) = Pps3.*ones(NT-IdxShk3,1);
    rho3 = P3./(R3.*T3); % Static density, kg/(m^3)
M3 = M3';

P3 = P0inf./((1+C2.*(M3.^2)).^((gamma2/(gamma2-1)))); % Static pressure, Pa

Pratio3 = (1+((2.*gamma2)./(gamma2+1))*(M3(IdxShk3).^2) - 1));
Pps3 = Pratio3*P3(IdxShk3);
P3(IdxShk3+1:end) = Pps3.*ones(NT-IdxShk3,1);

% Calculate the Mach number post area expansion (pae = post area expansion).
Mpaetest = M3(Idx5+1);
Mpaet = 0;
while abs(Mpaet - Mpaetest) > .00002
    Mpaet = M3(Idx5).*((A(Idx5)./(A(Idx5+1))).*(((1 + C2.*M3(Idx5)).^(-G2))./(1 + C2.*Mpaet^2).^(-G2)));
    if Mpaet > Mpaet
        Mpaet = Mpaetest -.00001;
    elseif Mpaet < Mpaet
        Mpaet = Mpaetest + .00001;
    end
end
M3(Idx5+1:NT) = Mpaet*ones(NT-Idx5,1);

P03 = P3(Idx5).*((1 + C2*M3(Idx5)^2)^((gamma2/(gamma2 - 1))));
% Stagnation pressure after the shock, Pa
Ppaeb = P03/((1 + C2*Mpaet^2)^((gamma2/(gamma2 - 1))));
% Post area expansion static pressure, Pa
P3(Idx5+1:NT) = Ppaeb*ones(NT-Idx5,1);
% Static pressure, Pa

T3 = Tad./(1+C2.*(M3.^2)); % Static temperature, K
rho3 = P3./(R2.*T3); % Static density, kg/(m^3)
else
    fprintf('\n\nNot a valid shock station.\nShockStation must equal either ''3'' or ''4''.
\n')
end

%---End H2 and O2 insertion in combustor---%

if strcmp(ShockStation, '4') == 1
    Tf = T2;
Pf = P2;
rhof = rho2;
Mf = M2;
elseif strcmp(ShockStation, '3') == 1
    Tf = T3;
Pf = P3;
rhof = rho3;
Mf = M3;
end

U = a2.*Mf;
P0f = P0inf*ones(NT,1);
P0f_bar = P0f/100000;
Pf_bar = Pf/100000;

Re = (rhof.*U.*2.*Rv)./mu2;
% Reynolds number
f = (.79*log(Re)-1.64).^(-2); % First Petukhov equation, (2, pg. 805).
% Friction factor, valid for 3000 < Re < 5*10^6 in smooth tubes.
Nu = (f/8).*(Re-1000).*Pr2./(1+12.7.*(f/8).^5.*(Pr2.^(2/3)-1));
% Nusselt number, Gnielinski equation (2, pg. 806).
% Valid for .5 <= Pr <= 2000 and 3000 < Re < 5*10^6
h = Nu.*cond2./(2.*Rv); % Convective heat transfer coefficient, W/(m^2)*K
Qfdotconv = h.*(Tf-Ts);
% Rate of heat flux to fluid, W/(m^2), (2, pg. 777)
qf = Qfdotconv./mdotinf; % Heat flux per unit mass, J/(kg*m^2)
q = qf.*A2; % Specific heat transfer, J/kg
T0f = Tad*ones(NT,1); % Use frictionless T0 as the value for T0f at X = Idx2

% Note that the lowercase letter "f" at the end of a variable name
% indicates use in the frictional calculations, unless stated
% otherwise. For instance, Qf indicates heat flux, and qf indicates
% specific heat flux, as two examples of exceptions.

%---Preallocation---%
dT0f = zeros(NT,1);
dA = zeros(NT,1);

%---End Preallocation---%

%---Station 1 to 2---%
% For loop to assign stagnation temperatures accounting for friction
for m = 2:Idx2
    T0f(m) = T0f(m-1) + (1./Cp2).*q(m-1);
    dT0f(m) = T0f(m) - T0f(m-1); % Differential of stagnation temperature, K
    dA(m) = A(m) - A(m-1); % Differential of cross-sectional area, m^2
end

D = 2.*Rv;
% Diameter of the duct, m

for j = 2:Idx2
    Mf(j) = SWR_Steady1D_Mach(gamma2,Mf(j-1),dA(j),A(j),dT0f(j),T0f(j),f(j),dx,D(j));
    P0f(j) = SWR_Steady1D_Pressure(gamma2,Mf(j),P0f(j-1),dT0f(j),T0f(j),f(j),dx,D(j));
end
muam = zeros(NT,1);
rhoam = zeros(NT,1);
Tam = zeros(NT,1);

for j = Idx2:Idx3
    Tam(j) = (Tf(j)+Ts)/2;
    if mdotH2_extra > 0 && mdotO2_extra == 0
        TempOut2 = CEA('problem','tp','equilibrium','p,bar',Pf_bar(j),... 
                     't,K',Tam(j),'reactants','name','H2O(g)','H',2,'O',1,... 
                     'wt%',mdotSteam,'reactants','name','H2(g)','H',2,... 
                     'wt%',mdotH2_extra,'output','transport','short','end');
    elseif mdotO2_extra > 0 && mdotH2_extra == 0
        TempOut2 = CEA('problem','tp','equilibrium','p,bar',Pf_bar(j),... 
                     't,K',Tam(j),'reactants','name','H2O(g)','H',2,'O',1,... 
                     'wt%',mdotSteam,'reactants','name','O2(g)','O',2,... 
                     'wt%',mdotO2_extra,'output','transport','short','end');
    else
        fprintf('

Invalid values for mdotH2_extra or mdotO2_extra.

')
    end
    muam(j) = TempOut2.output.viscosity/10000;
% Dynamic viscosity at the wall temperature, kg/(m*s)
rhoam(j) = TempOut2.output.density;
% Density at the wall temperature, kg/(m^3)
% Value of the property at the arithmetic mean temperature of the
% local freestream static temperature and the wall temperature
TempOut3 = CEA('problem','tp','equilibrium','p,bar',P0inf_bar,...
    't,K',Tad,'reactants','name','H2O(g)','H',2,'O',1,...
    'wt%',100.,'output','transport','short','end');

mu0 = TempOut3.output.viscosity/10000;
% Dynamic viscosity at stagnation conditions, kg/(m*s)

hB = h;

for m = Idx2+1:Idx3

    hB(m) = (.026./((2.*Rv(m-1)).^.2)).*((Cp2.*...
        (mu2.^.2))./(Pr2.^.6)).*((rhof(m-1).*...
        U(m-1)).^.8).*(rhoam(m-1)./rhof(m-1)).*((muam(m-1)./...
        mu0).^.2);
    % Bartz equation; computes convective heat transfer coefficient of the
    % gas, W/((m^2)*K)
    % From (6, pg. 314.)

    Qfdotconv(m-1) = hB(m).*(Ts-Tf(m-1));
    % Rate of heat flux to fluid, W/(m^2), (2, pg. 777)

    qf(m-1) = Qfdotconv(m-1)./mdotinf; % Heat flux per unit mass, J/(kg*m^2)
    q(m-1) = qf(m-1).*A2(m-1); % Specific heat transfer, J/kg
    % Note that the capital letter "B" at the end of a variable name
    % indicates use in the Bartz equation calculations, unless stated
    % otherwise.

    T0f(m) = T0f(m-1) + (1./Cp2).*q(m-1);
    % Stagnation temperature accounting for friction, K

    dT0f(m) = T0f(m) - T0f(m-1); % Differential of stagnation temperature, K
    dA(m) = A(m) - A(m-1); % Differential of cross-sectional area, m^2

end
for j = Idx2+1:Idx3
    Mf(j) = SWR_Steady1D_Mach(gamma2,Mf(j-1),dA(j),A(j),dT0f(j),T0f(j),f(j),dx,D(j));
    P0f(j) = SWR_Steady1D_Pressure(gamma2,Mf(j),P0f(j-1),dT0f(j),T0f(j),f(j),dx,D(j));
end

%——End Bartz equation——%

%——End Station 2 to 3——%

%——Station 3 to Shock Station——%

% For loop to assign stagnation temperatures accounting for friction
if strcmp(ShockStation,'4') == 1
    IdxShk = IdxShk4;
elseif strcmp(ShockStation,'3') == 1
    IdxShk = IdxShk3;
end

for m = Idx3+1:NT
    T0f(m) = T0f(m-1) + (1./Cp2).*q(m-1); % Stagnation temperature accounting for friction, K
    dT0f(m) = T0f(m) - T0f(m-1); % Differential of stagnation temperature, K
    dA(m) = A(m) - A(m-1); % Differential of cross-sectional area, m^2
end

Mf(Idx3) = 1;

% Assume Mach number at the throat is exactly 1.
P0f(Idx3:Idx3+2) = P0f(Idx3);

% Assume stagnation pressure one and two increments downstream of the
% throat is equal to the value at the throat.
Mf(Idx3+2) = M(Idx3+2);

% Assume Mach number two increments downstream of the throat is equal to
% the adiabatic, isentropic Mach number at that point.

for j = Idx3+3:IdxShk
    Mf(j) = SWR_Steady1D_Mach(gamma2,Mf(j-1),dA(j),A(j),dT0f(j),T0f(j),f(j),dx,D(j));
    P0f(j) = SWR_Steady1D_Pressure(gamma2,Mf(j),P0f(j-1),dT0f(j),T0f(j),f(j),dx,D(j));
end

%---Shock Station to Test Section---%

%---Shock relations---%

% Account for a normal shock midway through the feedstock injector:
Mfps = ((1+C2.*Mf(IdxShk).^2)./((gamma2.*Mf(IdxShk).^2)-C2)).^.5;
% Post-shock Mach number, []
Mf(IdxShk+1) = Mfps;
% From Anderson, pg. 597

Pf(IdxShk) = P0f(IdxShk)./((1+C2.*(Mf(IdxShk).^2)).^(gamma2/(gamma2-1)));
% Static pressure immediately prior to the normal shock, Pa
Pfratio = (1+((2.*gamma2)./(gamma2+1))*((Mf(IdxShk).^2)-1));
Pfps = Pfratio*Pf(IdxShk);
% Static pressure immediately after the normal shock, Pa
Pf(IdxShk+1) = Pfps;

P0f(IdxShk+1) = Pf(IdxShk+1).*((1+C2.*(Mf(IdxShk+1).^2)).^(gamma2/(gamma2-1)));
% Static pressure immediately prior to the normal shock, Pa

if strcmp(ShockStation,'3')==1
for j = IdxShk+2:Idx5
    Mf(j) = SWR_Steady1D_Mach(gamma2,Mf(j-1),dA(j),A(j),dT0f(j),T0f(j),f(j),dx,D(j));
    P0f(j) = SWR_Steady1D_Pressure(gamma2,Mf(j),P0f(j-1),dT0f(j),T0f(j),f(j),dx,D(j));
end

% Calculate the Mach number post area expansion (pae = post area
% expansion).
Mfpaetest = Mf(Idx5+1);
Mfpae = 0;
while abs(Mfpae - Mfpaetest) > .00002
    Mfpae = Mf(Idx5).*((A(Idx5)./A(Idx5+1)).*(((1 + C2.*Mf(Idx5)).^(-G2))./(1 + C2.*Mfpaetest.^2).^(-G2)));
    if Mfpae > Mfpaetest
        Mfpaetest = Mfpaetest - .00001;
    elseif Mfpae < Mfpaetest
        Mfpaetest = Mfpaetest + .00001;
    end
end

Mf(Idx5+1) = Mfpae;
P0f(Idx5+1) = P0f(Idx5);

for j = Idx5+2:NT
    Mf(j) = SWR_Steady1D_Mach(gamma2,Mf(j-1),dA(j),A(j),dT0f(j),T0f(j),f(j),dx,D(j));
    P0f(j) = SWR_Steady1D_Pressure(gamma2,Mf(j),P0f(j-1),dT0f(j),T0f(j),f(j),dx,D(j));
end

elseif strcmp(ShockStation,'4') == 1
    for j = IdxShk+2:NT
        Mf(j) = SWR_Steady1D_Mach(gamma2,Mf(j-1),dA(j),A(j),dT0f(j),T0f(j),f(j),dx,D(j));
        P0f(j) = SWR_Steady1D_Pressure(gamma2,Mf(j),P0f(j-1),dT0f(j),T0f(j),f(j),dx,D(j));
    end
end

%---End Shock Station to Test Section---%
%---Static values with friction ---%

\[ T_f = T_0_f / (1 + C_2 \cdot M_f^2); \]
% Static temperature, K

\[ P_f = P_0_f / \left( (1 + C_2 \cdot M_f^2)^{\gamma_2 / (\gamma_2 - 1)} \right); \]
% Static pressure, Pa

\[ \rho_f = P_f / (R_2 \cdot T_f); \]
% Static density assuming ideal gas, kg/(m^3)

%---End static values with friction---%

if mdot4 > 0;

%---N2 mixing---%

d4_in = .194; % Diameter of injection ports, in
r4_in = d4_in/2; % Radius of injector port, in
r4 = r4_in.*I2M; % Radius of injector port, m
N = 8; % Number of injector ports

\[ \theta_{a, \text{deg}} = 60; \] % Angle of four of the injector ports, degrees
\[ \theta_{b, \text{deg}} = 30; \] % Angle of the other four injector ports, degrees

\[ \theta_{a} = \theta_{a, \text{deg}} \cdot \pi / 180; \] % Angle of four of the injector ports, radians
\[ \theta_{b} = \theta_{b, \text{deg}} \cdot \pi / 180; \] % Angle of the other four injector ports, radians

\[ \text{CosCor} = 0.5 \cdot (\cos(\theta_{a}) + \cos(\theta_{b})); \]
% Cosine correction term to account for angled injector ports.

\[ A_{4a} = N \cdot \pi \cdot (r_4^2); \]
A4 = N.*pi.*(r4.^2).*CosCor;
A5 = A(Idx4);

alpha = mdot4./mdotinf; % Ratio of mass flow rates, mdot3/mdotinf, []
% Vary from .1 to 3

gamma4 = 1.4;
Cp4 = 1070;
% Specific heat at constant pressure of N2 at 300K and 1 atm, J/(kg*K)
MW4 = 28; % Molecular mass of injectant, g/mol
M4 = 1; % Mach number of injectant, []
% Assume injectant enters at sonic velocity
T4 = 298.15; % Temperature of injectant, K
R4 = Rbar./MW4; % Gas law constant for injectant
a4 = sqrt(gamma4*R4*T4); % Speed of sound of N2, m/s

gamma5 = gamma2;
M5 = Mf(Idx4); % Mach number of incoming freestream, []
% Assume adiabatic isentropic Mach number for now.

C4 = (gamma4-1)/2;
B4 = 2/(gamma4+1);
G4 = (gamma4+1)/(2*(gamma4-1));

% Remember, these are intermediate variables to make the code less % jumbled.
C5 = (gamma5-1)/2;
B5 = 2/(gamma5+1);
G5 = (gamma5+1)/(2*(gamma5-1));

Astar4 = pi.*(r4.^2); % Area of just one of the injectors, m^2
T04 = T4*(1+C4*M4^2);
% Stagnation temperature of injectant, K
P4 = .5.*((mdot4./A4a).*sqrt(R4./gamma4).*sqrt(T04).*(1/M4).*... 
  ((1+C4.*M4.^2).^(-.5))).*(cos(thetaa) + cos(thetab));
P04 = (mdot4./A4a).*sqrt(R4.*T04./gamma4).*((1./B4).^(G4));
P4 = P04./((1+C4.*M4.^2).^(gamma4./(gamma4-1)));

% Pressure of injectant, Pa
P4_bar = P4/100000;

% Stagnation temperature of the incoming freestream, K
T05 = T0f(Idx4);

% Stagnation pressure of the incoming freestream, Pa
P05 = P0f(Idx4);

% Static pressure of the incoming freestream, Pa
P5 = Pf(Idx4);

% Convert P5 from Pascals to bar
P5_bar = P5/100000;

P7_old = P4 + P5;
P7_bar = P4_bar + P5_bar;

Cp7a = (mdotinf*Cp2 + mdot4*Cp4)/(mdotinf + mdot4);
R7a = (mdotinf*R2 + mdot4*R4)/(mdotinf + mdot4);
gamma7a = Cp7a/(Cp7a-R7a);

CEAout7 = CEA('problem','hp','frozen','p,bar',P7_bar,...
  'reactants','name','H2O(g)', 'H', '2', 'O', '1', ...
  'wt%', mdotSteam, 't(K)', T5, 'name', 'H2(g)', 'H', '2', ...
  'wt%', mdotH2_extra, 't(K)', T5, 'name', 'O2(g)', 'O', '2', ...
  'wt%', mdotO2_extra, 't(K)', T5, 'name', 'N2(g)', 'N', '2', ...
  'wt%', mdot4, 't(K)', T4, 'output', 'transport', 'short', 'end');
Cp7_kJ = CEAout7.output.cp_tran.froz; % Specific heat at constant pressure, [kJ/(kg*K)]
Cp7 = Cp7_kJ.*1000; % Convert to J/(kg*K)
mu7 = CEAout7.output.viscosity./10000; % Dynamic Viscosity, kg/(m*s)
% Note that default output from CEAM is in millipoise.
% 10 P = 1 kg/(m*s), 1 P = 1000 mP
cond7 = CEAout7.output.conduct.froz*10; % Conductive heat transfer coefficient, W/(m*K)
% Note that the default output from CEAM is in mW/(cm*K)
Pr7 = CEAout7.output.prandtl.froz; % Specific heat at constant pressure, kJ/(kg*K)
gamma7 = CEAout7.output.gamma; % Ratio of specific heats from CEAM, []
MW7 = CEAout7.output.mw; % Molecular weight, check units
% Educated guess based on order of magnitude (10^1): kg/kmol
a7 = CEAout7.output.sonvel; % Sonic velocity, m/s
T7ad = CEAout7.output.temperature;
% Adiabatic flame temperature of mixture, K

R7 = Rbar/MW7;

C7 = (gamma7−1)/2;
B7 = 2/(gamma7+1);
G7 = (gamma7+1)/(2*(gamma7−1));

a5 = a2;
Cp5 = Cp2;
R5 = R2;

u4a = M4*a4*cos(thetaa);
u4b = M4*a4*cos(thetab);

u4 = (u4a + u4b)/2;
u5 = M5*a5;

ke4 = .5*(u4^2);
ke5 = .5*(u5^2);
fM4 = (M4.^2).*(1+C4.*M4.^2).*(1+gamma4.*M4.^2).^(-2));
fM5 = (M5.^2).*(1+C5.*M5.^2).*(1+gamma5.*M5.^2).^(-2));

% M7test = .0001:.0001:5;

a7M72fun = @(M7,a7) .5.*(a7.*M7).^2;
alpha4 = alpha./(alpha+1);
alpha5 = 1./(alpha+1);
CpTke4 = alpha4.*(Cp4.*T4+ke4);
CpTke5 = alpha5.*(Cp5.*T5+ke5);

T7fun = @(M7,CpTke4,CpTke5,Cp7) (-a7M72fun(M7,a7) + CpTke4 + CpTke5)./Cp7;

T07fun = @(M7,C7) T7fun(M7,CpTke4,CpTke5,Cp7).*((1+C7.*M7.^2);

% Stagnation temperature of mixed fluid, K

fM7 = @(M7,alpha,R7,gamma7,T05,CosCor,R4,gamma4,fM4,R5,gamma5,fM5) ((1+alpha).^2).*(R7./gamma7).*(T7fun(M7,C7))./(1-2.*gamma7.*fM7(M7,alpha,R7,gamma7,T05,CosCor,R4,gamma4,fM4,R5,gamma5,fM5)).^2);

M7fun = @(M7) M7 - sqrt(2.*fM7(M7,alpha,R7,gamma7,T05,CosCor,R4,gamma4,fM4,R5,gamma5,fM5)).^2);

% Mach number of mixed fluid, []

% M7Real = real(M7);

options = optimset('TolX',.0002);
M7 = fzero(M7fun,M5,options);

T7 = T7fun(M7,CpTke4,CpTke5,Cp7);
T07 = T07fun(M7,C7);

BlkA = (1+alpha).*((M5./M7);
BlkB = sqrt(T07.*R7.*gamma5./(T05.*R5.*gamma7));
BlkC = (1 + C7.*M7.^2).^G7;
BlkD = (1 + C5.*M5.^2).^G5;
P07 = P05.*BlkA.*BlkB.*BlkC./BlkD;
P7 = P07/((1+C7*M7^2)^(gamma7/(gamma7-1)));

%---End N2 mixing---%

%---Shocks with N2---%
Amach7 = (Astar./Marea).*((B7.*(1+C7.*Marea.^2)).^G7);
if strcmp(ShockStation,'4') == 1
M2pm = M2; % Assign a post mixing Mach number, []
M2pm(Idx4+1:Idx5) = M7;
Mpaetest = M2pm(Idx5);
Mpae = 0;
while abs(Mpae - Mpaetest) > .00002
    D = (M2pm(Idx5)./Mpaetest).*A(Idx5)./A(Idx5+1));
    E = 1;
    DE = D.*E;
    F = 1+C7.*(M2pm(Idx5).^2);
    DEG = DE.^(-1./G7);
    DEGF = DEG.*F;
    CDEGF = (1./C7).*(DEGF-1);
    Mpae = CDEGF.^5;
    if Mpaetest > Mpae
        Mpaetest = Mpaetest - .00001;
    elseif Mpaetest < Mpae
        Mpaetest = Mpaetest + .00001;
    end
end

M2pm(Idx5+1:IdxShk4) = Mpa;
% Mach number with mixing inside the 4" section but upstream of the shock.

% Account for a normal shock midway through the mixer section:
Mps2 = ((1+C7.*M2pm(IdxShk4).^2)./((gamma7.*M2pm(IdxShk4).^2)−C7)).^0.5;
% Post-shock Mach number, []
M2pm(IdxShk4+1:NT) = Mps2*ones(NT−IdxShk4,1);
% From Anderson, pg. 597

P2pm = P2;
P2pm(Idx4+1:IdxShk4) = P07./((1+C7.*M2pm(Idx4+1:IdxShk4).^2).^(gamma7./(gamma7−1)));

Pratio2pm = (1+((2.*gamma7)/(gamma7+1))*((M2pm(IdxShk4).^2)−1));
Pps2pm = Pratio2pm*P2pm(IdxShk4);
P2pm(IdxShk4+1:end) = Pps2pm.*ones(NT−IdxShk4,1);

T2pm = T2;
T2pm(Idx4+1:NT) = T07./(1+C7.*(M2pm(Idx4+1:NT).^2)); % Static temperature, K

elseif strcmp(ShockStation,'3') == 1
M3pm = M3; % Assign a post mixing Mach number, []
M3pm(Idx4+1:IdxShk3) = M7;

% Account for a normal shock midway through the feedstock injector:
Mps3pm = ((1+C7.*M3pm(IdxShk3).^2)./((gamma7.*M3pm(IdxShk3).^2)−C7)).^0.5;
% Post-shock Mach number, []
M3pm(IdxShk3+1:NT) = Mps3pm*ones(NT−IdxShk3,1);
% From Anderson, pg. 597

P3pm = P3;
P3pm(Idx4+1:IdxShk3) = P07./((1+C7.*(M3pm(Idx4+1:IdxShk3).^2)).^...
(gamma7/(gamma7-1))); % Static pressure, Pa

Pratio3pm = (1+((2.*gamma7)./(gamma7+1))*((M3pm(IdxShk3).^2) - 1));
Pps3pm = Pratio3pm*P3pm(IdxShk3);
P3pm(IdxShk3+1:NT) = Pps3pm.*ones(NT-IdxShk3,1);

% Calculate the Mach number post area expansion (pae = post area expansion).
Mpaetest = M3(Idx5+1);
Mpaes = 0;

while abs(Mpaes - Mpaetest) > .00002
    Mpaes = M3pm(Idx5).*((A(Idx5)./A(Idx5+1)).*( ((1 + C7.*M3pm(Idx5)).^(-G7))./((1 + C7.*Mpaetest.^(2)).^(-G7)) ));
    if Mpaetest > Mpaes
        Mpaetest = Mpaetest - .00001;
    elseif Mpaetest < Mpaes
        Mpaetest = Mpaetest + .00001;
    end
end

M3pm(Idx5+1:NT) = Mpaes*ones(NT-Idx5,1);

P03ps = P3pm(Idx5)*((1 + C7*M3pm(Idx5).^2)^(gamma7/(gamma7-1))); % Stagnation pressure after the shock, Pa
Ppae = P03ps/((1 + C7*Mpaes.^2)^(gamma7/(gamma7-1))); % Post area expansion static pressure, Pa
P3pm(Idx5+1:NT) = Ppae*ones(NT-Idx5,1);

% Static pressure, Pa
T3pm = T3;
T3pm(Idx4+1:NT) = T07./(1+C7.*(M3pm(Idx4+1:NT).^2)); % Static temperature, K
else
    fprintf('

Not a valid shock station.
ShockStation must equal either 3 or 4.

')
end
end

%---End shocks with N2---%

%---Frictional considerations for mixed flow---%

%---Shocks with N2 and friction---%

Amach7 = (Astar./Marea).*((B7.*(1+C7.*Marea.^2)).^G7);
if strcmp(ShockStation,'4') == 1
M2pm_b = Mf; % Assign a post mixing Mach number, []

M2pm_b(Idx4+1:Idx5) = M7;
Mpaetest = Mf(Idx5+1);
Mpaed = 0;
while abs(Mpaed - Mpaetest) > .00002
    D = (M2pm_b(Idx5)./Mpaetest).*(A(Idx5)./A(Idx5+1));
    E = 1;
    DE = D.*E;
    F = 1+C7.*(M2pm_b(Idx5).^2);
    DEG = DE.^(-1./G7);
    DEGF = DEG.*F;
    CDEGF = (1./C7).*((DEGF-1);

    Mpaed = CDEGF.^ .5;
if Mpaetest > Mpaed
    Mpaetest = Mpaetest - .00001;
elseif Mpaetest < Mpaed
    Mpaetest = Mpaetest + .00001;
end
end
M2pm_b(Idx5+1:IdxShk4) = Mpa;

% Mach number with mixing inside the 4" section but upstream of the shock.

% Account for a normal shock midway through the mixer section:
Mps2_b = ((1+C7.*M2pm_b(IdxShk4).^2)./(gamma7.*M2pm_b(IdxShk4).^2 - C7)).^.5;

% Post-shock Mach number, [ ]
M2pm_b(IdxShk4+1:NT) = Mps2_b*ones(NT-IdxShk4,1);

% From Anderson, pg. 597

P2pm_b = Pf;
P2pm_b(Idx4+1:IdxShk4) = P07./((1+C7.*(M2pm_b(Idx4+1:IdxShk4).^2)).^(gamma7./(gamma7-1)));

Pratio2pm_b = (1+((2.*gamma7)/(gamma7+1))*((M2pm_b(IdxShk4).^2) - 1));
Pps2pm_b = Pratio2pm_b*P2pm_b(IdxShk4);
P2pm_b(IdxShk4+1:end) = Pps2pm_b.*ones(NT-IdxShk4,1);

T2pm_b = Tf;
T2pm_b(Idx4+1:NT) = T07./(1+C7.*(M2pm_b(Idx4+1:NT).^2)); % Static temperature, K

elseif strcmp(ShockStation,'3') == 1
M3pm_b = Mf; % Assign a post mixing Mach number, [ ]
M3pm_b(Idx4+1:IdxShk3) = M7;

% Account for a normal shock midway through the feedstock injector:
Mps3pm_b = ((1+C7.*M3pm_b(IdxShk3).^2)./(gamma7.*M3pm_b(IdxShk3).^2 - C7)).^.5;

% Post-shock Mach number, [ ]
M3pm_b(IdxShk3+1:NT) = Mps3pm_b*ones(NT-IdxShk3,1);

% From Anderson, pg. 597

P3pm_b = Pf;
P3pm_b(Idx4+1:IdxShk3) = P07./((1+C7.*(M3pm_b(Idx4+1:IdxShk3).^2)).^(gamma7/(gamma7-1)));

% Static pressure, Pa
```matlab
Pratio3pm_b = 1 + ((2.*gamma7)./(gamma7+1))*((M3pm_b(IdxShk3).^2) - 1));
Pps3pm_b = Pratio3pm_b*P3pm_b(IdxShk3);
P3pm_b(IdxShk3+1:NT) = Pps3pm_b.*ones(NT-IdxShk3,1);

% Calculate the Mach number post area expansion (pae = post area expansion).
Mpaetest = Mf(Idx5+1);
Mpae = 0;
while abs(Mpae - Mpaetest) > .00002
    Mpae = M3pm(Idx5).*((A(Idx5)/A(Idx5+1)).*((1 + C7.*M3pm(Idx5).^2)./(1 + C7.*Mpaetest.^2)));
    if Mpaetest > Mpae
        Mpaetest = Mpaetest - .00001;
    elseif Mpaetest < Mpae
        Mpaetest = Mpaetest + .00001;
    end
end
M3pm_b(Idx5+1:NT) = Mpae.*ones(NT-Idx5,1);
P03ps_b = P3pm_b(Idx5)*((1 + C7*M3pm_b(Idx5).^2)^(gamma7/(gamma7 - 1)));
% Stagnation pressure after the shock, Pa
Ppae_b = P03ps_b/((1 + C7*Mpae^2)^(gamma7/(gamma7 - 1)));
% Post area expansion static pressure, Pa
P3pm_b(Idx5+1:NT) = Ppae_b.*ones(NT-Idx5,1);
% Static pressure, Pa
T3pm_b = Tf;
T3pm_b(Idx4+1:NT) = T07./(1+C7.*(M3pm_b(Idx4+1:NT).^2)); % Static temperature, K
end
```

```
elseif mdot4 < 0;
    fprintf('

Not a valid value for mdot4.

')
```
```
end

%---End shocks with N2 and friction---%

if strcmp(ShockStation, '4') == 1
    Tfpm = T2pm_b;
    Pfpm = P2pm_b;
    Mfpm = M2pm_b;
elseif strcmp(ShockStation, '3') == 1
    Tfpm = T3pm_b;
    Pfpm = P3pm_b;
    Mfpm = M3pm_b;
end

mdot7 = mdotinf + mdot4;

rhofpm = rhof;
Upm = U;
P0fpm = P0f;
P0fpm_bar = P0f_bar;
Pfpm_bar = Pf_bar;
Repm = Re;
fpm = f;
Nupm = Nu;
hpm = h;
qfpm = qf;

Upm(Idx4+1:NT) = a7.*Mfpm(Idx4+1:NT);
P0fpm(Idx4+1:NT) = Pfpm(Idx4+1:NT).*((1+C7.*Mfpm(Idx4+1:NT).^2).^(gamma7./(gamma7-1)));
P0fpm_bar(Idx4+1:NT) = P0fpm(Idx4+1:NT)/100000;
Pfpm_bar(Idx4+1:NT) = Pfpm(Idx4+1:NT)/100000;
rhofpm(Idx4+1:NT) = Pfpm(Idx4+1:NT)./(R7.*Tfpm(Idx4+1:NT));
```
Repm(Idx4+1:NT) = (rhofpm(Idx4+1:NT).*Upm(Idx4+1:NT).*2.*Rv(Idx4+1:NT))./mu7;

% Reynolds number

fpm(Idx4+1:NT) = (.79*log(Repm(Idx4+1:NT))-1.64).^(-2); % First Petukhov equation, (2, pg. 805).

% Friction factor, valid for 3000 < Re < 5*10^6 in smooth tubes.

Nupm(Idx4+1:NT) = (fpm(Idx4+1:NT)/8).*(Repm(Idx4+1:NT)-1000).*Pr7./(1+12.7.*(fpm(Idx4+1:NT)/8).^.5).*((Pr7.^(2/3))-1));

% Nusselt number, Gnielinski equation (2, pg. 806).

% Valid for .5 <= Pr <= 2000 and 3000 < Re < 5*10^6

hpm(Idx4+1:NT) = Nupm(Idx4+1:NT).*cond7./(2.*Rv(Idx4+1:NT)); % Convective heat transfer coefficient, W/((m^2)*K)

Qfdotconvpm = hpm.*(Ts-Tfpm);

% Rate of heat flux to fluid, W/(m^2), (2, pg. 777)

qfpm(Idx4+1:NT) = Qfdotconvpm(Idx4+1:NT)./mdot7; % Heat flux per unit mass, J/(kg*m^2)

qpm = qfpm.*A2; % Specific heat transfer, J/kg

T0fpm = T0f;

dT0fpm = dT0f;

T0fpm(Idx4+1) = T07;

dT0fpm(Idx4+1) = T0fpm(Idx4+1) - T0fpm(Idx4); % Differential of stagnation temperature

Mfpm(Idx4+1) = M7;

P0fpm(Idx4+1) = P07;

for m = Idx4+2:NT

T0fpm(m) = T0fpm(m-1) + (1./Cp7).*qpm(m-1);

% Stagnation temperature accounting for friction, K

dT0fpm(m) = T0fpm(m) - T0fpm(m-1); % Differential of stagnation temperature, K

dA(m) = A(m) - A(m-1); % Differential of cross-sectional area, m^2
end

for j = Idx4+2:IdxShk

Mfpm(j) = SWR_Steady1D_Mach(gamma7,Mfpm(j-1),dA(j),A(j),dT0fpm(j),T0fpm(j),fpm(j),dx,D(j));

P0fpm(j) = SWR_Steady1D_Pressure(gamma7,Mfpm(j),P0fpm(j-1),dT0fpm(j),T0fpm(j),fpm(j),dx,D(j));
end

%---Shock Station to Test Section---%
%---Shock relations---%

% Account for a normal shock midway through the feedstock injector:
Mfpspm = ((1+C7.*Mfpm(IdxShk).^2)./((gamma7.*Mfpm(IdxShk).^2) - C7)).^.5;
% Post-shock Mach number, [ ]
Mfpm(IdxShk+1) = Mfpspm;
% From Anderson, pg. 597

Pfpm(IdxShk) = P0fpm(IdxShk)./((1+C7.*(Mfpm(IdxShk).^2))^(gamma7/(gamma7-1)));
% Static pressure immediately prior to the normal shock, Pa

Pfpmratio = (1+(2.*gamma7)/(gamma7+1))*((Mfpm(IdxShk).^2) - 1));
Pfpspm = Pfpmratio*Pfpm(IdxShk);
% Static pressure immediately after the normal shock, Pa
Pfpm(IdxShk+1) = Pfpspm;

P0fpm(IdxShk+1) = Pfpm(IdxShk+1).*((1+C7.*(Mfpm(IdxShk+1).^2))^(gamma7/(gamma7-1)));
% Static pressure immediately prior to the normal shock, Pa

%---End shock relations---%

if strcmp(ShockStation,'3') == 1
  for j = IdxShk+2:Idx5
    Mfpm(j) = SWR_Steady1D_Mach(gamma7,Mfpm(j-1),dA(j),A(j),dT0fpm(j),T0fpm(j),fpm(j),dx,D(j));
    P0fpm(j) = SWR_Steady1D_Pressure(gamma7,Mfpm(j),P0fpm(j-1),dT0fpm(j),T0fpm(j),fpm(j),dx,D(j));
  end
  % Calculate the Mach number post area expansion (pae = post area
  % expansion).
  Mfpaepmtst = Mfpm(Idx5+1);
  Mfpaepm = 0;
  while abs(Mfpaepm - Mfpaepmtst) > .00002
Mfpaem = Mfpm(Idx5).*((A(Idx5)./A(Idx5+1)).*(1 + C7.*Mfpm(Idx5)).^(-G7))./((1 + C7.*Mfpaemtest.^2).^(-G7));

if Mfpaemtest > Mfpaem
    Mfpaemtest = Mfpaemtest - .00001;
elseif Mfpaemtest < Mfpaem
    Mfpaemtest = Mfpaemtest + .00001;
end

Mfpm(Idx5+1) = Mfpaem;
P0fpm(Idx5+1) = P0fpm(Idx5);

for j = Idx5+2:NT
    Mfpm(j) = SWR_Steady1D_Mach(gamma7,Mfpm(j-1),dA(j),A(j),dT0fpm(j),T0fpm(j),fpm(j),dx,D(j));
P0fpm(j) = SWR_Steady1D_Pressure(gamma7,Mfpm(j),P0fpm(j-1),dT0fpm(j),T0fpm(j),fpm(j),dx,D(j));
end

elseif strcmp(ShockStation,'4') == 1
    for j = IdxShk+2:NT
        Mfpm(j) = SWR_Steady1D_Mach(gamma7,Mfpm(j-1),dA(j),A(j),dT0fpm(j),T0fpm(j),fpm(j),dx,D(j));
P0fpm(j) = SWR_Steady1D_Pressure(gamma7,Mfpm(j),P0fpm(j-1),dT0fpm(j),T0fpm(j),fpm(j),dx,D(j));
    end
end

%---End shock station to test section---%

%---Static values with friction ---%

Tfpm = Tf;
Tfpm(Idx4+1:NT) = T0fpm(Idx4+1:NT)./(1+C7.*Mfpm(Idx4+1:NT).^2);

% Static temperature, K

Pfpm = Pf;
Pfpm(Idx4+1:NT) = P0fpm(Idx4+1:NT)./((1+C7.*Mfpm(Idx4+1:NT).^2).^((gamma7./(gamma7-1)));

% Static pressure, Pa
rhofpm = rhof;
rhofpm(Idx4+1:NT) = Pfpm(Idx4+1:NT)./(R7.*Tfpm(Idx4+1:NT));

% Static density assuming ideal gas, kg/(m^3)

%---End static values with friction---%

%---End frictional considerations for mixed flow---%

%---Data output---%

if mdot4 > 0
    fprintf('

Test section M = %.4f',Mfpm(end))
    fprintf('
Test section T = %.f K',Tfpm(end))
    fprintf('
Test section P = %.f kPa',Pfpm(end)/1000)
    fprintf('
Test section U = %.f m/s',Upm(end))
    fprintf('
Test section mdotSteam = %.f g/s',mdotSteam*1000)
    fprintf('
Test section mdotH2_extra = %.2f g/s',mdotH2_extra*1000)
    fprintf('
Test section mdotN2 = %.f g/s',mdot4*1000)
    fprintf('
Test section MW = %.2f g/mol',MW7)
    fprintf('
Test section mu = %.4f millipoise',CEAout7.output.viscosity)
    fprintf('
Test section k = %.3f mW/(cm*K)',CEAout7.output.conduct.froz)
    fprintf('
Test section Nu = %.1f ',Nupm(end))
    fprintf('
Test section h_c = %.1f W/(m^2*K)

',hpm(end))
else mdot4 == 0
    fprintf('

Test section M = %.4f',Mf(end))
    fprintf('
Test section T = %.f K',Tf(end))
    fprintf('
Test section P = %.f kPa',Pf(end)/1000)
    fprintf('
Test section U = %.f m/s',U(end))
    fprintf('
Test section mdotSteam = %.f g/s',mdotSteam*1000)
    fprintf('
Test section mdotH2_extra = %.2f g/s',mdotH2_extra*1000)
    fprintf('
Test section mdotN2 = %.f g/s',mdot4*1000)
    fprintf('
Test section MW = %.2f g/mol',MW2)
fprintf('\nTest section mu = %.4f millipoise',CEAout2.output.viscosity)
fprintf('\nTest section k = %.3f mW/(cm*K)',CEAout2.output.conduct.froz)
fprintf('\nTest section Nu = %.1f ',Nu(end))
fprintf('\nTest section h_c = %.1f W/(m^2*K)\n\n',h(end))
end

%---Plots!---%

%---Plot as a function of axial position---%
figure
plot(X,Rv,'-','Color',[.9 .4 .09],'LineWidth',2)
title('{\fontname{Times New Roman}Mach Number}')
title('{\fontname{Times New Roman}Geometry}')
xlabel('{\fontname{Times New Roman}Axial Position, [m]}')
ylabel('{\fontname{Times New Roman}Radius, [m]}')
ax = gca;
ax.FontName = 'Times New Roman';
figure
% plot(X,M,'-','Color',[.45 .5 1],'LineWidth',2)
% hold on
if strcmp(ShockStation,'4') == 1
plot(X,M2,'-','Color',[.2 .4 .8],'LineWidth',2)
elseif strcmp(ShockStation,'3') == 1
plot(X,M3,'-','Color',[.2 .4 .8],'LineWidth',2)
end
if mdot4 > 0;
hold on
if strcmp(ShockStation,'4') == 1
plot(X,M2pm,'-','Color',[.45 .5 1],'LineWidth',2)
elseif strcmp(ShockStation,'3') == 1
plot(X,M3pm,'-','Color',[.45 .5 1],'LineWidth',2)
end
hold on
plot(X,Mf,'-','Color',[.7 .2 .65], 'LineWidth',2)
if mdot4 > 0;
hold on
plot(X,Mfpm,'-','Color',[.58 .75 .68],'LineWidth',2)
end
title('{\fontname{Times New Roman}Mach Number}')
xlabel('{\fontname{Times New Roman}Axial Position, [m]}')
ylabel('{\fontname{Times New Roman}Mach number, []}')
legend('No Mixing','Mixing','With Friction','With Friction and Mixing')
l = legend;
l.FontName = 'Times New Roman';
ax = gca;
ax.FontName = 'Times New Roman';
figure
 plot(X,T,'-','Color',[.2 .9 .7], 'LineWidth',2)
 hold on
 if strcmp(ShockStation,'4') == 1
  plot(X,T2,'-', 'Color',[.2 .4 .8], 'LineWidth',2)
 elseif strcmp(ShockStation,'3') == 1
  plot(X,T3,'-','Color',[.2 .4 .8], 'LineWidth',2)
 end
 if mdot4 > 0;
  hold on
  if strcmp(ShockStation,'4') == 1
   plot(X,T2pm,'-','Color',[.45 .5 1], 'LineWidth',2)
  elseif strcmp(ShockStation,'3') == 1
   plot(X,T3pm,'-','Color',[.45 .5 1], 'LineWidth',2)
  end
 end
 hold on
 plot(X,T0f,'-','Color',[.7 .2 .65], 'LineWidth',2)
 hold on
plot(X,Tf,':','Color',[.7 .2 .65],'LineWidth',2)
if mdot4 > 0;
hold on
plot(X,T0fpm,'-','Color',[.58 .75 .68],'LineWidth',2)
hold on
plot(X,Tfpm,':','Color',[.58 .75 .68],'LineWidth',2)
end
title('{\fontname{Times New Roman}Temperature}')
xlabel('{\fontname{Times New Roman}Axial Position, [m]}')
ylabel('{\fontname{Times New Roman}Temperature, [K]}')
legend('No Mixing','Mixing','T_0 with Friction',...
'T with Friction','T_0 with Friction and Mixing',...
'T with Friction and Mixing')
l = legend;
l.FontName = 'Times New Roman';
ax = gca;
ax.FontName = 'Times New Roman';
figure
if strcmp(ShockStation,'4') == 1
plot(X,P2,'-','Color',[.2 .4 .8],'LineWidth',2)
elseif strcmp(ShockStation,'3') == 1
plot(X,P3,'-','Color',[.2 .4 .8],'LineWidth',2)
end
if mdot4 > 0;
hold on
if strcmp(ShockStation,'4') == 1
plot(X,P2pm,'-','Color',[.45 .5 1],'LineWidth',2)
elseif strcmp(ShockStation,'3') == 1
plot(X,P3pm,'-','Color',[.45 .5 1],'LineWidth',2)
end
end
hold on
plot(X,P0f,'-','Color',[.7 .2 .65],'LineWidth',2)
hold on
plot(X,Pf,':','Color',[.7 .2 .65],'LineWidth',2)
if mdot4 > 0;
hold on
plot(X,P0fpm,'-','Color',[.58 .75 .68],'LineWidth',2)
hold on
plot(X,Pfpm,':','Color',[.58 .75 .68],'LineWidth',2)
end
title('{\fontname{Times New Roman}Pressure}')
xlabel('{\fontname{Times New Roman}Axial Position, [m]}')
ylabel('{\fontname{Times New Roman}Pressure, [Pa]}')
legend('No Mixing','Mixing','P_0 with Friction','P with Friction',...
    'P_0 with Friction and Mixing','P with Friction and Mixing')
l = legend;
l.FontName = 'Times New Roman';
ax = gca;
ax.FontName = 'Times New Roman';

SWR_Steady1D_Mach.m:

function [M2] = SWR_Steady1D_Mach(gamma,M1,dA,A,dT0,T0,f,dx,D)
```matlab
BlkA = ((1 + ((gamma-1)/2).*M1.^2)./(1-M1.^2));
BlkB = -2.*dA./A;
BlkC = (1 + gamma.*M1.^2).*dT0./T0;
BlkD = gamma.*(M1.^2).*f.*dx./D;
BlkE = BlkA.*(BlkB + BlkC + BlkD);
M2 = sqrt((M1.^2).*(1+BlkE));
end

SWR_Steady1D_Pressure.m:

```
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20   end