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FINITE ELEMENT ANALYSIS OF PROPAGATING INTERFACE CRACKS IN COMPOSITES

University of Washington

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FINITE ELEMENT ANALYSIS OF PROPAGATING INTERFACE CRACKS IN COMPOSITES

by

Mohammad Ali Aminpour

A dissertation submitted in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

University of Washington

1986

Approved by

(Chairperson of Supervisory Committee)

Program Authorized to Offer Degree

Department of Aeronautics and Astronautics

Date 3/10/86
Doctoral Dissertation

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Abstract

FINITE ELEMENT ANALYSIS OF PROPAGATING INTERFACE CRACKS IN COMPOSITES

by Mohammad Ali Aminpour

Chairperson of the Supervisory Committee: Prof. Keith A. Holsapple
Department of Aeronautics and Astronautics

A complex variable formulation has been developed to describe the near-field state of stresses and displacements for a propagating crack along the interface of two dissimilar anisotropic materials. It is shown that the formulation is general and can be reduced to all the other subordinate cases without any difficulty. The crack can be propagating or stationary and each of the materials on the sides of the crack can be anisotropic, orthotropic or isotropic. The near-field stresses contain the regular square root singularity and the oscillatory behavior in case of dissimilar materials on the sides of the crack.

Due to the complexity of the problem it was not possible to use the conventional definitions of the stress intensity factors. Therefore a new definition for the stress intensity factors is proposed. It is proportional to the coefficient of the lowest order term of the near-field state of stresses and reduces to all the subordinate cases to within a multiplicative factor.

A detailed description of the development of the near-field state of stresses and displacements is presented. A finite element procedure has been developed to provide solution. The finite element
procedure utilizes a singular element which gives the direct solution of the time-dependent stress intensity factors. The procedure for the finite element formulation including a detailed description of the development of the singular element is presented. The element matrices are derived from a variational principle involving a modified functional for elastodynamic problems.

The resulting discretized dynamic equations of motion are solved by an implicit method of temporal integration using Nemar-β formulas. Local asymmetries in the matrices which arise due to crack propagation are dealt with by modifying the finite difference formulation and by the use of an iterative procedure for convergence of the solution. Crack propagation is accomplished by moving the crack-tip inside the singular element according to a prescribed crack-tip position history. A local redefinition of the finite element mesh is required when the crack-tip reaches an extreme position inside the singular element. When the local mesh redefinition takes place, an extra node is created. This is accomplished using a method of double noding technique.

The accuracy of the finite element formulation is evaluated by solving problems for which analytical and numerical solutions are available. The solutions are found to compare well with widely accepted solutions in the literature. The solutions to some problems involving propagating and stationary cracks at the interface of two dissimilar anisotropic materials, for which there are no known solutions, are presented. Finally, recommendations are made for further development of the finite element procedure.
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NOMENCLATURE

Subscript \( k \) corresponds to \( k^{th} \) material

Subscript \( \xi \) corresponds to \( \xi^{th} \) eigenvalue

I. Roman symbols

\( a \)  
Half crack length

\( \mathbf{A} \), \( \mathbf{A}_k \)  
Matrices of elastic constants

\( a_i \)  
Elements of matrix \( \mathbf{A} \)

\( a_{ij} \)  
Elastic constants for a state of plane stress

\( a_{ij} \) \( \mathbf{A} \)  
Elements of matrix \( \mathbf{A} \)

\( \mathbf{A}_i, A_{ij} \)  
Real parameters (Appendix A)

\( A_1, A_2, A_3 \)  
Complex parameters (Chapter 6)

\( b \)  
A parameter defined by material properties and crack-tip speed (Chapter 3 and Appendix A)

\( b \)  
Half plate width (Chapter 9)

\( b_{ij} \)  
Elements of matrix \( \mathbf{A} \) (Chapter 3)

\( \mathbf{B}, \mathbf{B}_k \)  
Matrices relating stresses to \( \phi \) (Chapter 3 and Appendix A)

\( \mathbf{E} \)  
Matrix relating \( \mathbf{a} \) to \( \mathbf{g} \) (Chapter 5)

\( c_{ijmn} \)  
Elements of compliance tensor

\( c(t) \)  
Crack-tip speed as a function of time
NOMENCLATURE
(Continued)

\( C_i \) Wave amplitudes (Chapter 8)

\( C_1-C_8 \) Complex parameters (Appendix A)

\( C_{ik}, (C_{ik})_\ell \) Complex coefficients of complex power functions \( \Omega_{ik} \)

\( C_T, C_L \) Transverse and longitudinal wave speeds of an isotropic material, respectively

\( cm \) Centimeter, unit of length

\( d_0 \) Determinant of matrix \( \sim \)

\( \sim \) Matrix of differential operators

\( d_{mk\ell} \) Parameters defining \( D_{mk\ell} \)

\( dv \) Differential volume element

\( ds \) Differential surface element

\( dt \) Differential time element

\( dz, dz_k, dz_{ik} \) Differential complex plane element

\( D_{ik}, (D_{ik})_\ell \) Complex coefficients of complex power functions \( \Omega_{ik} \)

\( D_{mk\ell} \) Functions defining near-field displacements

\( \sim_k \) Matrix

\( \sim_{ik} \) \( i \)th row of matrix \( \sim_k \)

\( (\text{DIS}_{ij})_k \) Elements of matrix \( \sim_k \)

\( dyne \) Dyne, unit of force

\( e \) Base of natural logarithm

\( e_i, e_{ij} \) Strain components

\( E_1, E_2, E_3 \) Moduli of elasticity
NOMENCLATURE
(Continued)

\( E_{ik} \) = \( C_{ik} + D_{ik} \), for real eigenvalues

\( \dot{E} \) Matrix of derivatives of \( \dot{U} \)

\( \dot{E}_{ik} \) Matrix

\( \dot{E}_{ik} \) \( i \)th row of matrix \( \dot{E}_{k} \)

\( (\dot{E}_{ij})_k \) Elements of matrix \( \dot{E}_{k} \)

\( f \) Parameter

\( f(t) \) Crack-tip position as a function of time

\( F_{ik} \) Complex parameters defining \( C_{ik} \) and \( D_{ik} \)

\( \dot{F} \) Body force integral

\( \dot{F}_{R} \) Regular element force vector

\( \dot{F}_{S} \) Singular element force vector

\( \tilde{F}_{R} \) Global form of \( \dot{F}_{R} \)

\( \tilde{F}_{S} \) Global form of \( \dot{F}_{S} \)

\( g \) Parameter (Chapter 8)

\( g \) Gram, unit of mass

G Lame's constant

\( G_{12}, G_{13}, G_{23} \) Elastic shear moduli

\( \tilde{G} \) Boundary integral

\( H_{ik} \) Complex parameters

\( \tilde{H} \) Volume integral
NOMENCLATURE

(Continued)

$H_1$  Boundary integral
$i$  $\sqrt{-1}$

in.  Inch, unit of length

$k$  Parameter

$k_1,k_2$  Stress intensity factors

$K,K_1,K_2$  Stress intensity factors

$K_S$  $\sqrt{\frac{\sigma}{\nu K}}$, static stress intensity factor for an infinite uniform medium

$K_{ik},K^*_{ik}$  Complex parameters

$\tilde{K}$  Global stiffness matrix

$\tilde{\bar{K}}_1$  Matrix

$\tilde{\bar{K}}_1$  Global form of $\tilde{K}_1$

$K_S$  Singular element stiffness matrix

$K_R$  Regular element stiffness matrix

$\tilde{\bar{K}}_S$  Global form of $\tilde{K}_S$

$\tilde{\bar{K}}_R$  Global form of $\tilde{K}_R$

$K_{eq},K_{eff},K_{eff}$  Stiffness matrices

$\tilde{K}_{sym}$  Symmetric part of $\tilde{K}$

$\tilde{K}_{asym}$  Asymmetric part of $\tilde{K}$

$Kg$  Kilogram, unit of mass

$L$  Half height of plate

x
NOMENCLATURE

(Continued)

\( L \)  
Matrix of interpolating functions

\( L_{ij} \)  
Elements of matrix \( L \)

\( m \)  
Parameter (Chapter 8)

\( m \)  
Meter, unit of length

\( M \)  
Global mass matrix

\( M_{1}, M_{2}, M_{3}, M_{4}, M_{5}, M_{6} \)  
Matrices of volume and boundary integrals

\( \tilde{M}^{*} \)  
Matrix

\( \tilde{M}_{1}^{*} \)  
Global form of \( \tilde{M}^{*} \)

\( M_{S} \)  
Singular element mass matrix

\( M_{R} \)  
Regular element mass matrix

\( \tilde{M}_{S} \)  
Global form of \( M_{S} \)

\( \tilde{M}_{R} \)  
Global form of \( M_{R} \)

\( N \)  
Newton, unit of force

\( \tilde{N} \)  
Matrix of bilinear functions

\( N_{i} \)  
Bilinear functions

\( n, n_{\xi} \)  
Complex exponent

\( n_{\alpha}, n_{\beta} \)  
Number of unknown coefficients \( \alpha \) and \( \beta \), respectively

\( \tilde{n} \)  
Matrix of unit normal vector components

\( n_{i}, n_{x}, n_{y} \)  
Components of unit normal vector

xi
NOMENCLATURE

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<td>( P )</td>
<td>Matrix relating strains to stress function ( \phi ) (Chapter 3)</td>
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<td>( P_{ij} )</td>
<td>Elements of ( P )</td>
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<td>( \psi_i )</td>
<td>Pounds per square inch, unit of pressure</td>
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<tr>
<td>( q )</td>
<td>A parameter defining ( \epsilon ) (Chapter 6)</td>
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<td>( g(t), g^*, g, g^- )</td>
<td>Vectors of nodal displacements</td>
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<tr>
<td>( q_i, q_j )</td>
<td>Elements of ( g ) and ( g^- ), respectively</td>
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<td>( Q )</td>
<td>Parameter</td>
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<td>( Q_0 )</td>
<td>Parameter</td>
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<td>( Q_{1k}, Q_{2k}, Q_{k} )</td>
<td>Matrices</td>
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<td>( Q_{eq}, Q_{eff}, Q^- )</td>
<td>Force matrices</td>
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<td>( r, \theta, r_k, \theta_k, r_{jk}, \theta_{jk} )</td>
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<td>( R )</td>
<td>Parameter</td>
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<td>Region occupied by cracked body</td>
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<td>( R_0 )</td>
<td>Subregion of ( R )</td>
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<td>$s_{nkL}$</td>
<td>Parameters defining $S_{nkL}$</td>
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<td>$S_n$</td>
<td>Boundary of subregion $R_n$</td>
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<tr>
<td>$S_{in}$</td>
<td>Interior interface between elements</td>
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<tr>
<td>$S_t$</td>
<td>Part of boundary where tractions are prescribed</td>
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<tr>
<td>$S_u$</td>
<td>Part of boundary where displacements are prescribed</td>
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<td>$\sim_k$</td>
<td>Matrix</td>
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<td>$\sim_{ik}$</td>
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<td>Components of $t^0$</td>
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<td>T</td>
<td>Superscript denoting transpose of a matrix</td>
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<td>$T_1, T_2$</td>
<td>Constant tractions in X and Y directions, respectively</td>
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NOMENCLATURE

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$I$  Boundary force integral (Chapter 5)
$\tilde{I}$  Transformation matrix for eliminating double nodes (Chapter 7)
$T_{mn}$  Elements of matrix $\tilde{I}$
$\hat{u}$  Matrix of displacement components
$u_{ik}, u_{ik}$  Displacements
$\hat{u}^0$  Prescribed boundary displacements
$u_{i}^0$  Elements of $\hat{u}^0$
$u_{mk\ell}$  Displacement eigenfunctions
$\hat{U}$  Matrix of displacement functions
$U_1, U_2$  Submatrices of $\hat{U}$
$\hat{U}_{ij}$  Elements of $U_1$
$v, v_1, v_2$  Wave propagation speeds (Chapter 8)
$\tilde{u}$  Boundary displacements
$v_i$  Components of matrix $\tilde{u}$
$\tilde{v}^0$  Prescribed boundary displacements
$\tilde{v}_{i}^0$  Components of $\tilde{v}^0$
$V_1, V_2$  Functions (Chapter 3)
$V$  Strain energy density function
$\gamma$  "Damping" matrix
$\gamma_I$  Matrix

xiv
NOMENCLATURE
(Continued)

$\tilde{V}_1$  Global form of $V_1$
$V_S$  Singular element "damping" matrix
$\tilde{V}_{S}$  Global form of $V_{S}$
$V_{sym}$  Symmetric part of $V$
$V_{asym}$  Asymmetric part of $V$
$X_i,x-y$  Global rectilinear coordinate system fixed to the body
$x_i,x-y$  Local rectilinear coordinate system moving with crack-tip
$X',Y'$  Rotated rectilinear coordinate system
$z_i,\dot{z}_i,k,z_{ik}$  Coordinates of complex plane

II. Greek symbols

$\alpha,\alpha_k$  Complex parameters
$\xi$  Column matrix of unknown coefficients
$\xi_1,\xi_2$  Submatrices of $\xi$
$\alpha_{ij}$  Elements of $\xi$
$\xi$  Column matrix of unknown coefficients
$\beta_1,\beta_2$  Submatrices of $\xi$
$\beta_{ij}$  Elements of $\xi$
$\beta_{ij}$  Elastic constants for a state of plane strain (Chapter 2)
$\beta_{ij}$  Unknown complex coefficients
NOMENCLATURE

(Continued)

\((b_1)_x, (b_2)_x\)
Real and imaginary parts of \(b_x\)

\(\gamma_{ij}\)
Shear strain

\(r, r_0\)
Functions (Chapter 3)

\(\Delta t\)
Time increment

\(\varepsilon\)
Bielastic parameter

\(\xi, \xi_k\)
Matrices of strains

\(\varepsilon_{ij}, (\varepsilon_{ij})_k\)
Components of strain tensor

\(\zeta, \nu\)
Local rectilinear coordinate system for transformation

\(\eta_k\)
A parameter defined by \(\nu_k\)

\(\theta, r, \theta_k, r_k, \theta_{kj}, r_{kj}\)
Polar coordinate variables

\(\theta\)
Angle from \(X_1\) axis (Chapter 8)

\(\theta\)
Angle from \(Z\) axis (Chapter 9)

\(\lambda\)
Lame's constant

\(\mu_k\)
Shear moduli of elasticity (Chapter 6)

\(\mu, \mu_i, \mu_k, \mu_{ik}\)
Complex parameters

\(\nu, \nu_k, \nu_{12}, \nu_{13}, \nu_{23}\)
Poisson's ratios

\(\Pi\)
Functional

\(\Pi_S\)
Functional for singular element

\(\Pi_R\)
Functional for regular element

\(\rho\)
Mass density
NOMENCLATURE

(Continued)

\( \sigma, \sigma_k \)
Matrices of stresses

\( \sigma_{ij}, (\sigma_{ij})_k \)
Components of stress tensor

\( \sigma_y, \sigma_{xx}, \sigma_{xx1}, \sigma_{xx2} \)
Uniformly applied stresses

\( \sigma_{nk} \)
Stress eigenfunctions

\( \tau_i \)
Components of matrix of stresses

\( T, T_1, T_2 \)
Complex analytic functions

\( \phi \)
Angle of rotation (Chapter 2)

\( \phi_0, \phi_1, \phi_k \)
Stress functions

\( \phi_0, \phi_2, \phi_k \)
Matrices of 2nd derivatives of \( \phi \)

\( \psi_1, \psi_2 \)
Functions (Chapter 3)

\( \omega \)
Real circular frequency

\( \Omega, \Omega_{ik} \)
Complex analytic functions

\( \Omega_\sigma, \Omega_{\sigma k}, \Omega_u, \Omega_{uk} \)
Matrices of derivatives of \( \Omega_i \) and \( \Omega_{ik} \)
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CHAPTER I
INTRODUCTION

In recent years there has been much interest in employing anisotropic materials in industry, because of their desirable properties. In contrast to isotropic materials, for which the material properties are independent of directions, anisotropic material properties are directional. For example, materials such as plywood have large differences in their elastic moduli in the principal directions. Composite materials, which are usually formed from several layers of different orientation of the same material bonded together, are being used extensively in the aerospace industry. The problem of debonding or delamination of the layers of composites under various types of loads is of major concern. The study of this problem requires a description of the stress and displacement fields in the vicinity of the crack-tip in the delamination process, at the interface of two dissimilar anisotropic materials.

To the author's knowledge, there has been no attempt to formulate the problem of a propagating interface crack between two dissimilar anisotropic materials. In this dissertation this problem is formulated, and the complete state of stresses, displacements and the stress intensity factors are found. The solution technique employs a hybrid-displacement finite element formulation. In addition, a new and more general definition for stress intensity factors is proposed. It can be reduced, to within a multiplicative factor, to all other
subordinate cases.

M.L. Williams [1]* was among the first to derive the form of the stress singularity for interface cracks between two dissimilar isotropic materials for static problems. Later, F. Erdogan [2] and G.C. Sih and J.R. Rice [3-4] successfully formulated the stress state near the crack-tip and derived a formula for the stress intensity factors for the problem considered by Williams [1]. The stress intensity factors are of special interest in fracture mechanics, since they are parameters governing the onset of rapid crack extension and they characterize the near-field stresses and displacements.

K.Y. Lin and J.W. Mar [5] derived the complete expressions for the eigenfunctions of a stationary interface crack between two dissimilar isotropic materials under static loads. Also, employing a finite element formulation, they successfully solved for the state of stresses and displacements and the stress intensity factors (using Sih's definition) for a variety of material combinations and loadings.

P.S. Theocaris [6] and others [21] have questioned the form of the eigenfunctions used by [1-5], in which the exponents "n" of the eigenfunctions are implicitly assumed to be real, since when they are solved for as eigen-values, some of the n's turn out to be complex numbers. Theocaris [6] proposes the use of a more complete form of eigenfunctions involving both \(n\) and \(\overline{n}\). (The bar indicates the

*Numbers in brackets indicate references.
complex conjugate). However, the present author has verified that when all the analysis is carried out and the expressions for stresses and displacements are derived, for this particular problem of a stationary interface crack between two dissimilar isotropic materials under static loads, both methods result in the same final expressions. However, there is no reason to believe that this will be the case for the more general problem of an interface crack (stationary or running) between two dissimilar materials (each can be anisotropic, orthotropic or isotropic) under static or dynamic loads, as considered in this dissertation.

It would be a cumbersome and time consuming task to prove or disprove that the two methods will have the same results for the problem considered here. However, after some analysis it is seen that both methods will involve the same number of unknowns in the final equations. In the method used by [1-5], four unknown complex coefficients of the eigenfunctions and their complex conjugates, i.e. a total of eight unknown complex coefficients, will be involved, while in the method proposed by Theocaris, of the eight unknown complex coefficients of the eigenfunctions, either the complex coefficient itself or its complex conjugate will be involved, resulting also in eight unknown complex coefficients. Therefore, the amount of analysis and calculations will be the same for both methods. Here the more complete form of the eigenfunctions proposed by Theocaris will be used, with no further verification of the equivalency of the two methods.
D.B. Bogy [7-8] and also D.N. Fenner [9] calculated the eigenvalues of the exponent n for a more general problem of a stationary crack terminating at the interface of two dissimilar isotropic materials at an angle under static loads, a particular case of which would be the problem considered by [1-6]. Fenner also calculated the near-field stress distribution for that problem.

Recently An-yu Kuo has solved the problem of stationary interface crack in infinite media composed of two dissimilar orthotropic [26] and anisotropic [27] materials under impact loading on the crack surfaces. Singular integral equations and Jacobi polynomials were used to obtain the solution. Furthermore An-yu Kuo and Su-Su Wang [10] have reported the problem of a stationary interface crack between two dissimilar anisotropic materials under dynamic loads by employing a hybrid-stress finite element formulation. This author however, has no knowledge of their results having been published to date.

J. Aboudi [11] has solved for the near-field stresses for a moving interface crack between two dissimilar isotropic materials. An implicit three-level (in time and space) numerical method was used to solve the equations of motion. An arbitrary definition for the stress intensity factors as the ratio of the near-field stresses to a reference stress and a reference dimension was given. This definition of the stress intensity factors is different from that of Sih's derivation. In Aboudi's definition the mode I (the opening mode) stress intensity factor was given in terms of only the normal stresses, while in Sih's derivation, both the mode I and the mode II
(the sliding mode) stress intensity factors are functions of both normal and shear stresses.

S.N. Atluri and T. Nishioka [14-15] and C.K. Gunther and K.A. Holsapple [16] have employed a hybrid-displacement finite element formulation for running cracks in a single isotropic material body. The finite element formulation in this dissertation is structured after the Gunther and Holsapple [16] formulation. Many modifications and extra subroutines were implemented in the computer program described in [16] to make the program more efficient, and to generalize to the problem of this research.

In the finite element mesh used here, the crack-tip is embedded in a "singular element", and all other elements away from the crack-tip and around the singular element are regular elements. Different approximating functions for the singular and regular elements are considered in Chapters 3 and 4.

In the literature it is customary to formulate the singular element using two displacement potentials for dynamic problems. However, we will use a totally different and new approach here and formulate the singular element using stress functions. The stress functions developed here are new and resemble the complex variable formulation presented by N.I. Muskhelishvili [12] and used by S.G. Lekhnitskii [13] for anisotropic materials to formulate static problems. Complex power-series eigenfunctions are assumed for Muskhelishvili's functions.

If the crack-tip is not stationary, it will run inside the
singular element until it reaches an extreme position, at which time a local remeshing takes place and the position of the singular element is moved forward in the direction of crack propagation. Thus the crack can continue to propagate inside the singular element.

In the process of remeshing as the crack-tip runs, new nodes have to be created in the finite element mesh. A double noding technique proposed by B.M. Liaw, A.S. Kobayashi and A.F. Emery [17] and Santosh K. Arya [18] is employed for the creation of new nodes. In this technique the mesh-points along the crack-line, i.e. the mesh-points on the interface have two nodes, so that when the crack-tip passes through a mesh-point that point becomes two separate nodes. However, before the crack-tip reaches such points, one must to eliminate the extra node at such points by using the equality of displacements of the two nodes so that the final equivalent stiffness matrix is not singular.

In the following chapters a detailed derivation of the near-field functions are presented, and the implementation of these functions into a hybrid-displacement finite element is described. The solution to some problems are compared to the known theoretical and numerical results, and some examples of stationary and running interface cracks between two dissimilar anisotropic materials under static and dynamic loads are presented. A detailed description of the finite element program and the double noding technique are given. Finally a summary and recommendations for further research is presented.
CHAPTER II
THE THEORY OF ELASTICITY FOR AN ANISOTROPIC MATERIAL

1. The basic assumptions:

For the linearized theory of elasticity for small displacements, the deformations of an elastic body are given by

\[ 2\varepsilon_{ij} = u_{i,j} + u_{j,i} \quad i, j = 1, 2, 3 \]  
(2.1)

where \( \varepsilon_{ij} \) are the elements of the strain tensor and \( u_i \) are the components of the displacement vector with

\[ u_{i,j} = \frac{\partial u_i}{\partial x_j} \quad i, j = 1, 2, 3 \]  
(2.2)

It is obvious from equations (2.1) that the strain tensor is symmetric, i.e.

\[ \varepsilon_{ij} = \varepsilon_{ji} \quad i, j = 1, 2, 3 \]  
(2.3)

The dynamic equilibrium equations are taken to be Navier's equations of motion

\[ \sigma_{ij,j} + \rho b_i = \rho \ddot{u}_i \quad i, j = 1, 2, 3 \]  
(2.4)

where \( \sigma_{ij} \) are the elements of the stress tensor, \( \rho \) is the mass density of the material, \( b_i \) are the elements of body force per unit mass vector, and

\[ \dot{u}_i = \frac{\partial u_i}{\partial t} \quad i = 1, 2, 3 \]  
(2.5)

\[ \ddot{u}_i = \frac{\partial^2 u_i}{\partial t^2} \]

represent the particle velocity and acceleration. The components of surface tractions \( t_i \) are given by
\[ t_{ij} = \sigma_{ij} n_j \quad i, j = 1, 2, 3 \] (2.6)

where \( n_j \) are the components of an outward unit vector normal to the surface. The stress tensor is taken to be symmetric, i.e.

\[ \sigma_{ij} = \sigma_{ji} \quad i, j = 1, 2, 3 \] (2.7)

For linear elastic materials the constitutive relations are given by

\[ \varepsilon_{ij} = c_{ijmn} \sigma_{mn} \quad i, j, m, n = 1, 2, 3 \] (2.8)

where \( c_{ijmn} \) represent 81 elastic constants. From the considerations of equations (2.3) and (2.7) \( c_{ijk1} \) will reduce to 36 independent constants. Define \( e_{ij} \) as

\[ e_{ij} = \begin{cases} 
\varepsilon_{ij} & \text{when } i = j \\
\gamma_{ij} = 2\varepsilon_{ij} & \text{when } i \neq j 
\end{cases} \quad i, j = 1, 2, 3 \] (2.9)

The existence of a "strain energy density function" \( \mathcal{W} \), is assumed so that

\[ \sigma_{ij} = \partial \mathcal{W} / \partial e_{ij} \quad i, j = 1, 2, 3 \] (2.10)

Then the number of independent elastic constants reduces to 21. Furthermore, let

\[ [\varepsilon_1 \ v_2 \ v_3 \ v_4 \ v_5 \ v_6] = [\varepsilon_{11} \ \varepsilon_{22} \ \varepsilon_{33} \ 2\varepsilon_{23} \ 2\varepsilon_{31} \ 2\varepsilon_{12}] \]

and

\[ [\tau_1 \ \tau_2 \ \tau_3 \ \tau_4 \ \tau_5 \ \tau_6] = [\sigma_{11} \ \sigma_{22} \ \sigma_{33} \ \sigma_{23} \ \sigma_{31} \ \sigma_{12}] \] (2.11)

so that

\[ e_i = a_{ij} \tau_j \quad i, j = 1, 2, \ldots 6 \] (2.12)

with

\[ a_{ij} = a_{ji} \quad i, j = 1, 2, \ldots 6 \] (2.13)
Equation (2.12), "Generalized Hooke's Law", represent the constitutive relation for an anisotropic material in 3-dimensions which involves 21 independent elastic constants. If it is assumed that there exist three orthogonal planes of symmetry in the material, then the material is called an "orthotropic material", and the number of independent constants is reduced to 9. Furthermore, for an "isotropic material", where the material is symmetric in all directions, the number of independent elastic constants is reduced to 2. For a detailed treatment of these derivations and reductions in the number of elastic constants refer to [13].

2. The generalized state of plane stress

Consider an elastic uniform anisotropic flat plate of constant thickness, in equilibrium under applied loads along its edges. The following conditions (see S.G. Lekhnitskii [19]) define the generalized state of plane stress.

a) At each point of the plate there is a plane of elastic symmetry parallel to the middle surface.

b) The applied forces along the edges, and the body forces, act on planes parallel to the middle surface, are distributed symmetrically about that surface and suffer only slight changes through the thickness.

c) The strains are small.

Under these conditions, the middle surface remains flat under the applied loads, and equation (2.12) reduces to
\[ e_{ij} = a_{ij} \tau_j \]  
\[ i,j = 1, 2, 6 \]  
(2.14)

where \( \tau_i \) and \( e_i \) are now the average stresses and strains over the plate thickness [19]. Also \( \tau_3 = \sigma_{33} \) will be neglected in comparison with \( \tau_1, \tau_2, \) and \( \tau_6. \)

3. The state of plane strain

Consider an elastic uniform anisotropic body of cylindrical shape of arbitrary cross-section, in equilibrium under applied loads along its surface. The following conditions (see S.G. Lekhnitskii [19]) define the state of plane strain.

a) The forces act in planes normal to the cylinder and do not vary along it (its length is supposed large by comparison with its cross-sectional dimensions).

b) At each point there is a plane of elastic symmetry perpendicular to the cylinder.

c) The strains are small.

For plane sections far from the ends it is assumed that

\[ u_1 = u_1(X,Y), u_2 = u_2(X,Y), \text{ and } u_3 = 0 \]  
(2.15)

where \( X-Y \) is a cross-sectional plane and the \( Z \) axis is considered to be parallel to the cylinder. Under these conditions, equations (2.12) reduce to

\[ e_{ij} = \beta_{ij} \tau_j \]  
\[ i,j = 1, 2, 6 \]  
(2.16)

where

\[ \beta_{ij} = a_{ij} - a_{13} a_{3j} / a_{33} \]  
\[ i,j = 1, 2, 6 \]  
(2.17)

4. Orthotropic and isotropic materials

For orthotropic materials, it is necessary that
\[
\begin{align*}
a_{16} = a_{26} &= 0 \quad \text{and} \quad \beta_{16} = \beta_{26} = 0 \\
(2.18)
\end{align*}
\]

For an isotropic material, it is necessary that

a) For plane stress
\[
\begin{align*}
a_{11} &= a_{22} \\
a_{16} &= a_{26} = 0 \\
a_{66} &= 2(a_{11} - a_{12}) \\
(2.19a)
\end{align*}
\]

b) For plane strain
\[
\begin{align*}
\beta_{11} &= \beta_{22} \\
\beta_{16} &= \beta_{26} = 0 \\
\beta_{66} &= 2(\beta_{11} - \beta_{12}) \\
(2.19b)
\end{align*}
\]

5. Transformation of elastic constants under a transformation of the coordinate system.

In some problems it is necessary to recalculate the elastic constants in a new \(X'-Y'\) coordinate system from the elastic constants in a different \(X-Y\) coordinate system. This can be done by considering that the value of the strain energy density function remains the same for any coordinate system. Let \(\phi\) be the angle through which the old coordinate system has been rotated to obtain the new coordinate system. Then, when the elastic strain energy is calculated for each coordinate system and equated term by term (see [13 and 19]) the following will result for a plane problem.

\[
\begin{align*}
a_{11}' &= a_{11}\cos^4\phi + (2a_{12} + a_{66})\sin^2\phi\cos^2\phi + a_{22}\sin^4\phi + (a_{16}\cos^2\phi + a_{26}\sin^2\phi)\sin2\phi \\
(2.20)
\end{align*}
\]

\[
\begin{align*}
a_{22}' &= a_{11}\sin^4\phi + (2a_{12} + a_{66})\sin^2\phi\cos^2\phi + a_{22}\cos^4\phi - (a_{16}\sin^2\phi + a_{26}\cos^2\phi)\sin2\phi \\
\end{align*}
\]
\[ a_{12}^* = (a_{11} + a_{22} - 2a_{12} - a_{66}) \sin^2 \phi \cos^2 \phi + a_{12}^2 + 1/2(a_{26} - a_{16}) \sin 2\phi \cos 2\phi \]

\[ a_{66}^* = 4(a_{11} + a_{22} - 2a_{12} - a_{66}) \sin^2 \phi \cos^2 \phi + a_{66} + 1/2(a_{26} - a_{16}) \sin 2\phi \cos 2\phi \]

\[ a_{16}^* = (a_{22} \sin^2 \phi - a_{11} \cos^2 \phi + 1/2(2a_{12} + a_{66}) \cos 2\phi) \sin 2\phi + a_{16}(\cos^2 \phi - 3\sin^2 \phi) \cos^2 \phi + a_{26}(3\cos^2 \phi - \sin^2 \phi) \sin^2 \phi \]

\[ a_{26}^* = (a_{22} \cos^2 \phi - a_{11} \sin^2 \phi - 1/2(2a_{12} + a_{66}) \cos 2\phi) \sin 2\phi + a_{16}(3\cos^2 \phi - \sin^2 \phi) \sin^2 \phi + a_{26}(\cos^2 \phi - 3\sin^2 \phi) \cos^2 \phi \]
CHAPTER III

A FORMULATION FOR THE STRESSES AND DISPLACEMENTS OF A PROPAGATING CRACK ALONG THE INTERFACE OF TWO DISSIMILAR ANISOTROPIC MATERIALS

In this chapter a detailed derivation of the expressions for the approximating eigenfunctions which govern the near-field solution for a propagating crack at the interface of two dissimilar anisotropic materials is presented. This formulation is new. All previous formulations for interface cracks have been concerned with stationary cracks or for propagating cracks in homogeneous isotropic materials only. Also, in the literature for propagating cracks, traditionally two displacement potentials are used to formulate the problem. Here we derive a single stress function and formulate the near-field solution for propagating cracks using this stress function. Although this formulation is for the more general problem of a propagating interface crack between two dissimilar anisotropic materials, the formulation will reduce to all the subordinate cases, since all the possible solutions are accounted for. Therefore this is a single formulation in which each of the materials on the side of the interface can be isotropic, orthotropic or anisotropic, the crack can be stationary or propagating and the loading can be static or dynamic. In the previous formulations in the literature the formulation for an anisotropic material could not be reduced to that of an isotropic material and the formulation for a propagating crack could not be reduced to that of a stationary crack.
Let \( X, Y \) be a reference coordinate system with the \( X \)-axis parallel to the crack-line. Also let \( x, y \) be a moving coordinate system with its origin at the crack-tip and with the \( x \)-axis coinciding with the crack-line. Assuming Fig. 3.1 represents the situation at time \( t = t_0 \), then

\[
\begin{align*}
    y &= Y - Y_0 \\
    x &= X - f(t)
\end{align*}
\]

where \( f(t) \) represents the position of the crack-tip with

\[
    f(t_0) = X_0
\]

and

\[
    \dot{f}(t) = c(t)
\]

where \( c(t) \) is the crack-tip velocity as a function of time. Using the chain rule, one has

\[
\begin{align*}
    \frac{\partial \tau}{\partial Y} &= \frac{\partial \tau_0}{\partial y} \\
    \frac{\partial \tau}{\partial X} &= \frac{\partial \tau_0}{\partial x} \\
    \frac{\partial \tau}{\partial t} &= \frac{\partial \tau_0}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial \tau_0}{\partial t} = -\frac{\partial \tau_0}{\partial x} \ddot{f}(t) + \frac{\partial \tau_0}{\partial t} \\
    \frac{\partial^2 \tau}{\partial t^2} &= \frac{\partial^2 \tau_0}{\partial x^2} \dot{f}^2(t) - \frac{\partial^2 \tau_0}{\partial x \partial t} \dot{f}(t) - \frac{\partial \tau_0}{\partial x} \ddot{f}(t) + \frac{\partial^2 \tau_0}{\partial x^2} \dddot{t}
\end{align*}
\]

where \( \tau \) represents functions such as displacements or stresses and \( \tau = \tau(X, Y, t) = \tau_0(x, y, t) \).

When an asymptotic expansion is made as \( x \to 0 \), only the first term in the expansion is taken as an approximation, so that the time derivatives in equations (3.3) simplify as (see Freund [20])

\[
\begin{align*}
    \frac{\partial \tau}{\partial t} &= -\frac{\partial \tau_0}{\partial x} \ddot{f}(t) = -c(t) \frac{\partial \tau_0}{\partial x} \\
    \frac{\partial^2 \tau}{\partial t^2} &= \frac{\partial^2 \tau_0}{\partial x^2} \dot{f}^2(t) = c^2(t) \frac{\partial^2 \tau_0}{\partial x^2}
\end{align*}
\]
Fig. 3.1 Coordinate System Definition for a Cracked Body at Some Fixed Instant of Time
L. B. Freund [20] using asymptotic expansions and taking only the first term of the expansion as an approximation, shows that it is possible to solve problems of curved cracks with variable crack-tip velocities. Furthermore he shows that the resulting equations are the same as those for straight cracks with respect to a local coordinate system at the crack-tip with the x-axis being tangent to the path of the crack, and with the constant crack-tip speed replaced by the instantaneous crack-tip speed.

Using the equilibrium equations (2.4) and neglecting the body forces, one has

$$\sigma_{ij,j} = \rho \ddot{u}_i$$  \hspace{1cm} (3.5)

with respect to the global coordinate system X-Y. Using equations (3.4) for the displacement field in equation (3.5) gives

$$\sigma_{ij,j} = \rho c^2(t) \frac{\partial^2 u_i}{\partial x^2}$$  \hspace{1cm} (3.6)

with respect to the local coordinate system x-y. Expanding equation (3.6) for plane problems and letting $R = \rho c^2(t)$ for convenience, yields

$$\sigma_{11,1} + \sigma_{12,2} = R \frac{\partial^2 u_1}{\partial x^2}$$  \hspace{1cm} (3.7a)

$$\sigma_{12,1} + \sigma_{22,2} = R \frac{\partial^2 u_2}{\partial x^2}$$  \hspace{1cm} (3.7b)

These equations will be solved by introducing a new stress function which governs the near field solution for a propagating crack in an anisotropic material. From that the formulation for a propagating crack at the interface of two dissimilar anisotropic
materials is derived.

The right-side of equations (3.7a) and (3.7b) can be written as

\[ \frac{\partial^2 u_1}{\partial x^2} = \frac{\partial}{\partial x} \left( \frac{\partial u_1}{\partial x} \right) = \frac{\partial}{\partial x} (e_{11}) \]

\[ = \frac{\partial}{\partial x} \left( a_1 \sigma_{11} + a_2 \sigma_{22} + a_3 \sigma_{12} \right) \]

\[ \frac{\partial^2 u_2}{\partial x^2} = \frac{\partial}{\partial x} \left( \frac{\partial u_2}{\partial x} \right) = \frac{\partial}{\partial x} (\gamma_{12} - \frac{\partial u_1}{\partial y}) \]

\[ = \frac{\partial}{\partial x} (\gamma_{12}) - \frac{\partial}{\partial y} \left( \frac{\partial u_1}{\partial x} \right) \]

\[ = \frac{\partial}{\partial x} (a_3 \sigma_{11} + a_5 \sigma_{22} + a_6 \sigma_{12}) \]

\[- \frac{\partial}{\partial y} (a_1 \sigma_{11} + a_2 \sigma_{22} + a_3 \sigma_{12}) \]

(3.8)

In the above, use has been made of equations (2.1), (2.14), and (2.16) with the following conventions for simplicity:

\[ \mathbf{a} = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_2 & a_4 & a_5 \\ a_3 & a_5 & a_6 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{16} \\ b_{12} & b_{22} & b_{26} \\ b_{16} & b_{26} & b_{66} \end{bmatrix} \]

(3.9)

where \( b_{ij} = a_{ij} \) defined by equation (2.14) for plane stress and \( b_{ij} = \beta_{ij} \) defined by equation (2.16) for plane strain. In what follows and the rest of this dissertation we will not differentiate between plane stress and plane strain, because only \( a_{ij} \) has to change to \( \beta_{ij} \) to alternate from plane stress to plane strain.

Substituting equation (3.8) into equations (3.7) results in

\[ \frac{\partial}{\partial x} [(1-a_1 R) \sigma_{11} - a_2 R \sigma_{22} - a_3 R \sigma_{12}] + \frac{\partial}{\partial y} (\sigma_{12}) = 0 \]

(3.9a)

\[ \frac{\partial}{\partial x} [-(a_3 R \sigma_{11} - a_5 R \sigma_{22} + (1-a_6 R) \sigma_{12})] \frac{\partial}{\partial y} [a_4 R \sigma_{11} + (1+a_5 R) \sigma_{22} + a_6 R \sigma_{12}] = 0 \]

(3.9b)
These equations have the general form of
\[ \frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} = 0 \]  
(3.10)

for appropriate \( V_1 \) and \( V_2 \), and can therefore be derived from potential function \( \psi(x,y) \) such that
\[ V_1 = \frac{\partial \psi}{\partial y} \quad \text{and} \quad V_2 = -\frac{\partial \psi}{\partial x} \]  
(3.11)

so that equation (3.10) is identically satisfied. Using this idea in equation (3.9), let
\[ (1-a_1R)\sigma_{11} - a_2R\sigma_{22} - a_3R\sigma_{12} = \frac{\partial \psi_1}{\partial y} \]  
(3.12a)

\[ \sigma_{12} = -\frac{\partial \psi_1}{\partial x} \]  
(3.12b)

\[ -a_3R\sigma_{11} - a_5R\sigma_{22} + (1-a_6R)\sigma_{12} = -\frac{\partial \psi_2}{\partial y} \]  
(3.12c)

and
\[ a_1R\sigma_{11} + (1+a_2R)\sigma_{22} + a_3R\sigma_{12} = \frac{\partial \psi_2}{\partial x} \]  
(3.12d)

where \( \psi_1 \) and \( \psi_2 \) are two arbitrary potential functions. Using equation (3.12b) in equations (3.12a) and (3.12d) and solving for stresses in terms of the potential \( \psi_1 \) and \( \psi_2 \) yields
\[ \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} -a_3R & 1+a_2R & a_2R \\ a_3R & -a_1R & 1-a_1R \\ -(1-(a_1-a_2)R) & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial \psi_1}{\partial x} \\ \frac{\partial \psi_1}{\partial y} \\ \frac{\partial \psi_2}{\partial x} \end{bmatrix} \]  
(3.13)
When equation (3.13) and (3.12b) are substituted into (3.12c), the result is

\[
\frac{\partial}{\partial x} [d_1 \psi_1 + d_3 \psi_2] + \frac{\partial}{\partial y} [d_2 \psi_1 + d_4 \psi_2] = 0
\]  

(3.14)

where

\[
d_1 = a_3(a_3 - a_5)R^2 - (1-a_6)(1-(a_1 - a_2)R)
\]

\[
d_2 = -a_3R - (a_2a_3 - a_1a_5)R^2
\]

\[
d_3 = -a_5R - (a_2a_3 - a_1a_5)R^2
\]

\[
d_4 = 1 - (a_1 - a_2)R
\]

Now, again let

\[
d_1 \psi_1 + d_3 \psi_2 = \frac{\partial \phi_0}{\partial y}
\]

(3.15)

and

\[
d_2 \psi_1 + d_4 \psi_2 = \frac{\partial \phi_0}{\partial x}
\]

Solving for potentials \( \psi_1 \) and \( \psi_2 \) in terms the potential \( \phi_0 \) gives

\[
\begin{bmatrix}
\psi_1 \\
\psi_2
\end{bmatrix} =
\begin{bmatrix}
de_3 & -d_4 \\
d_1 & d_2
\end{bmatrix}
\begin{bmatrix}
\frac{\partial \phi_0}{\partial x} \\
\frac{\partial \phi_0}{\partial y}
\end{bmatrix}
= \frac{1}{d_1d_4 - d_2d_3}
\]

(3.16)

Substituting equations (3.16) into equations (3.13) yields

\[
\mathcal{G} = \frac{B_{\phi}}{\partial \sigma}
\]

(3.17)

where

\[
\mathcal{G} = [\sigma_{11} \sigma_{22} \sigma_{12}]^T
\]

\[
\phi_0 = [\phi_{,11} \phi_{,22} \phi_{,12}]^T
\]
\[
\begin{pmatrix}
 a_2 R + (a_3 a_5 - a_2 a_6) R^2 & 1 + a_2 R & - (a_3 + a_5) R \\
 1 - (a_1 + a_6) R + (a_1 a_6 - a_3^2) R^2 & -a_1 R & 2 a_3 R \\
 a_5 R + (a_2 a_3 - a_1 a_5) R^2 & 0 & - (1 - a_1 a_2) R \\
\end{pmatrix}
\]

where \( \phi = -\frac{1}{d_1 d_4 - d_2 d_3} \phi_0 \) has been rescaled for convenience.

Let the constitutive equations (2.12) and (2.13) be written as

\[
\varepsilon = \sigma = P \phi \sigma
\]

where \( p = \bar{\varepsilon} \bar{\sigma} \)

\[
\bar{\varepsilon} = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{pmatrix}, \quad \bar{\sigma} = \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{pmatrix}, \quad \bar{\varepsilon} = \begin{pmatrix} a_1 & a_2 & a_3 \\ a_2 & a_4 & a_5 \\ a_3 & a_5 & a_6 \end{pmatrix}
\]

From equations (2.1) the usual compatibility equation in 2-D is written as

\[
2 \varepsilon_{12,12} = \gamma_{12,12} = \varepsilon_{11,22} + \varepsilon_{22,11}
\]

substituting equation (3.18) into (3.20) yields

\[
P_5 \phi_{,2222} + p_4 \phi_{,1222} + p_3 \phi_{,1122} + p_2 \phi_{,1112} + p_1 \phi_{,1111} = 0
\]

where

\[
p_5 = p_{12} = a_1 \\
p_4 = p_{13} - p_{32} = -2 a_3 \\
p_3 = p_{11} + p_{22} - p_{33} = 2 a_2 + a_6 - R [(a_1 a_6 - a_3^2) + (a_1 a_4 - a_2^2)] \\
p_2 = p_{23} - p_{31} = -2 (a_5 + R [(a_2 a_5 - a_3 a_4) + (a_2 a_3 - a_1 a_5)]) \\
p_1 = p_{21} = a_4 - R [(a_4 a_6 - a_2^2) + (a_1 a_4 - a_2^2)] + R^2 d_0
\]

and where, \( d_0 = |a| \), is the determinant of matrix \( a \) and \( p_{ij} \) are the elements of matrix \( p \) as defined in equation (3.19).

When \( c(t) \), the crack tip velocity, vanishes, so that \( R = p c^2(t) \) vanishes, equation (3.21) reduces to that of Lekhnitskii [19] for anisotropic materials. Furthermore when the material becomes
isotropic, equation (3.21) reduces to the well known equation
\[ \nabla^2 \phi = \phi_{,222} + 2\phi_{,2211} + \phi_{,1111} = 0 \]
in which \( \phi \) assumes the role of Airy stress function. Therefore the function \( \phi \) in equation (3.21) can be considered as a new stress function which governs the near field solution for a propagating crack in an anisotropic material.

It is assumed that the function \( \phi \) can be represented by an analytic complex function \( \tau \) as
\[ \phi = \text{Real} (\tau) \]  \hspace{1cm} (3.22)
where
\[ \tau = \tau(z) \]
with
\[ z = x + \mu y \]
and where \( \mu \) is a complex parameter to be determined. When equation (3.22) is substituted in equation (3.21), one obtains
\[ p_5 \mu^6 + p_4 \mu^3 + p_3 \mu^2 + p_2 \mu + p_1 = 0 \]  \hspace{1cm} (3.23)
The characteristic equation (3.23) has, in general, four roots. When the crack-tip speed \( c(t) \) vanishes, i.e. for static problems, Lekhnitskii [19] has proven that equation (3.23) has either pairs of complex roots or pairs of purely imaginary roots, with real roots being impossible. Furthermore, if the material becomes isotropic, equation (3.23) will have only two distinct roots \( \mu = \pm i \), with \( i = \sqrt{-1} \), [19].

However, as the crack-tip speed \( c(t) \) increases from zero, there becomes a situation in which, equation (3.23) will have two real roots and one pair of complex roots. When the crack-tip speed is further increased, there becomes a situation in which equation (3.23) will have four real roots. Substituting the isotropic material properties,
\( a_3 = a_5 = 0, \ a_4 = a_1 \) and \( 2(a_1 - a_2) = a_6 \) into equation (3.23) one obtains

\[
A_1 A_6 u^4 + (2A_1 A_6 - R(A_1 + A_6)) u^2 \\
+ (A_1 A_6 - R(A_1 + A_6) + R^2) = 0
\]

where \( A_i \) are the elements of matrix

\[
A = A^{-1} = \begin{bmatrix}
A_1 & A_2 & A_3 \\
A_2 & A_4 & A_5 \\
A_3 & A_5 & A_6
\end{bmatrix} = \begin{bmatrix}
A_{11} & A_{12} & A_{16} \\
A_{12} & A_{22} & A_{26} \\
A_{16} & A_{26} & A_{66}
\end{bmatrix}
\] (3.24)

For isotropic materials in plane strain \( A_1 = \lambda + 2G \) and \( A_6 = G \) where \( \lambda \) and \( G \) are the Lame's constants. The above equation gives

\[
u_1^2 = -1 + \left( \frac{R}{A_6} \right) \quad \nu_2^2 = -1 + \left( \frac{R}{A_1} \right)
\]

or

\[
u_1^2 = -1 + \left( \frac{c(t)}{C_T} \right)^2 \quad \nu_2^2 = -1 + \left( \frac{c(t)}{C_L} \right)^2
\]

where

\[
C_T = \sqrt{G/\rho} \quad \text{and} \quad C_L = \sqrt{(\lambda + 2G)/\rho}
\]

are the transverse (shear) and longitudinal wave speeds of the material respectively. Therefore, for isotropic materials the transitions from complex roots to real roots for the characteristic equation (3.23) take place when the crack-tip speeds are the transverse wave speed and the longitudinal wave speed respectively. However, for a freely propagating crack the crack-tip speed is only a fraction of the shear wave speed of the material. Therefore real roots of the characteristic equation (3.23) do not occur in the usual problems and are not of interest.

Therefore, here it is assumed that the crack-tip speed remains below the first transition, so that equation (3.23) has either two
distinct pairs of complex roots or only one distinct pair of complex roots of multiplicity two.

If the equation (3.23) has two distinct pairs of complex roots, \( \mu_1 \), \( \mu_2 \), \( \overline{\mu}_1 \), and \( \overline{\mu}_2 \), then equation (3.22) can be written as

\[
\phi = \text{Real}(T_1 + T_2)
\]

where we choose

\[
T_1 = \int \Omega_1 dz_1, \quad T_2 = \int \Omega_2 dz_2
\]

so that

\[
\phi = \text{Real} \left( \int \Omega_1 dz_1 + \int \Omega_2 dz_2 \right)
\]

where

\[
\Omega_i = \Omega_i(z_i) \quad \text{no sum on } i
\]

\[
z_i = x + \mu_i y \quad i = 1, 2
\]

If equation (3.23) has only one distinct pair of complex roots, \( \mu \) and \( \overline{\mu} \), then the equation (3.22) can be written as

\[
\phi = \text{Real}(T_1 + T_2)
\]

where we choose

\[
T_1 = \overline{\Omega}_1
\]

\[
T_2 = \int \Omega_2 dz
\]

so that

\[
\phi = \text{Real}(\overline{\Omega}_1 + \int \Omega_2 dz)
\]

where

\[
\Omega_i = \Omega_i(z) \quad i = 1, 2
\]

\[
z = x + \mu y
\]

and the bar over \( z \) indicates the complex conjugate. In equations (3.25) and (3.26) we have assumed the above definitions for \( T_i \) for convenience so that the displacement and stresses are expressed in terms of \( \Omega_i \) and \( \overline{\Omega}_i \) instead of \( \Omega_i^* \) and \( \Omega_i^{*\prime} \) respectively, see [12,13,19].

Equation (3.26) is in the same form as Muskhelishvili's [12] complex variable formulation for isotropic materials in static
problems, and equation (3.25) is in the same form as that of Lekhinskii's [13,19] for anisotropic materials in static problems.

We shall now turn attention to the problem of a propagating crack at the interface of two dissimilar anisotropic materials and derive the eigenvalues and eigenfunctions for this problem. Assume that in Fig. 3.1 the material in the positive y axis and the material in the negative y axis are different materials (see Fig. 3.2).

The equations thus far derived in this chapter are valid for either of the two materials, but from now on we shall use an index to differentiate between the two materials. We shall consider the two following cases.

Case (a). Consider the case in which equation (3.23) has two distinct pairs of complex roots \( u_{1k}, \bar{u}_{1k} \) and \( u_{2k}, \bar{u}_{2k} \). Then let

\[
\phi_k = \text{Real} \left( \int \Omega_{1k} dz_{1k} + \int \Omega_{2k} dz_{2k} \right) \tag{3.27}
\]

where

\[
z_{ik} = x + u_{ik} y \quad i, k = 1, 2
\]

and \( k \) identifies the material under consideration. Let \( \Omega_{ik} \) have a complex power representation as

\[
\Omega_{1k} = C_{1k} z_{1k}^n + D_{1k} \bar{z}_{1k}^n \tag{3.28}
\]

\[
\Omega_{2k} = C_{2k} z_{2k}^n + D_{2k} \bar{z}_{2k}^n
\]

Case (b). Equation (3.23) has only one distinct pair of complex roots \( u_k \) and \( \bar{u}_k \). Let

\[
\phi_k = \text{Real} \left( \bar{z}_k \Omega_{1k} + \int \Omega_{2k} dz_k \right) \tag{3.29}
\]
Fig. 3.2 A Propagating Crack Along the Interface of Two Dissimilar Anisotropic Materials
where \[ z_k = x + \nu_k y \quad k = 1,2 \]

Assume that \( \Omega_{ik} \) have power representation as

\[
\Omega_{1k} = c_{1k} z_k^n + d_{1k} \bar{z}_k^n \\
\Omega_{2k} = c_{2k} z_k^n + d_{2k} \bar{z}_k^n
\]

Later it will be shown that there are an infinite number of solutions for the exponent \( n \) and the functions \( \Omega_{ik} \) actually become complex power series. Since \( n \) is a complex parameter, we assume that \( \Omega_{ik} \) has the above form of representation which involves both \( n \) and \( \bar{n} \). As discussed in the introduction chapter, M.L. Williams and others [1-5] have solved the problem of a stationary interface crack between two dissimilar isotropic material under static loads, assuming that \( \Omega_{ik} = c_{ik} z_k^n \) and implicitly regarding \( n \) as a real variable and then recovering its complex values. However P.S. Theocaris [6] and others [21] have questioned this method and Theocaris [6] has suggested the use of a more complete form of function as in equations (3.30) and regarding \( n \) as a complex parameter. However, this author has verified that the two methods will result in identical values of \( n \) and identical expressions for displacement and stress fields for the problem of a stationary interface crack between two dissimilar isotropic materials under static loads, as considered by [1-6,21]. There is, however, no reason to believe that, for a more general problem such as the one considered here, these two methods will have identical results. It might seem that the first method, assuming implicity \( n \) to be real, will result in
less calculations, since it involves only four unknown complex coefficients, $C_{ik}$, as compared to eight unknown complex coefficients, $C_{ik}$ and $D_{ik}$ involved in the second method. But this is not the case. When the restrictive conditions of traction-free surfaces and continuity of stresses and displacements ahead of the crack tip are imposed, the first method will involve $C_{ik}$ and $\bar{C}_{ik}$ as eight unknowns with eight equations, while the second method will involve only $C_{ik}$ and $D_{ik}$ as eight unknowns with eight equations, as will be shown later (in addition to the solution for the unknown complex exponent $n$). Thus either method involves the same number of unknowns and the same amount of calculations, and there is no need for further verification to see whether the two methods will truly have identical results for the problem considered here. Thus we have accepted that the functions $\eta_{ik}$ be presented in the form given by equations (3.28) and (3.30).

To begin the problem, with two different materials there are nine unknowns to be solved for, namely $C_{ik}$, $D_{ik}$, $i,k=1,2$ and the exponent $n$. However, expressing the conditions of zero tractions on the crack surfaces and the equality of tractions and displacements directly ahead of the crack-tip across the interface will result in eight complex homogeneous equations. To simplify the problem, four of the unknowns $C_{2k}$ and $D_{2k}$ are solved for and substituted in the remaining four equations, so that we will have four equations in terms of 5 unknowns $C_{2k}$, $\bar{D}_{2k}$, $k=1,2$ and the exponent $n$.

The algebra is very involved and the detailed derivations are presented in Appendix A. For each of the cases (a) and (b) the stresses and
strains are derived according to equations (3.17) and (3.18) using the
definitions of equation (3.27) and (3.29) for the stress function $\phi$. The displacements are derived by integrating the strains. Then the
expressions for $\sigma_{22} - i\sigma_{12}$ and $u_1 + iu_2$ are formed. Following the
conditions of zero tractions on the crack surfaces, i.e. $\sigma_{22} - i\sigma_{12} = 0$ on
the crack surfaces, we will solve for $C_{2k}$ and $\overline{D}_{2k}$ in terms of $C_{1k}$ and
$\overline{D}_{1k}$. Then by substituting $C_{2k}$ and $\overline{D}_{2k}$, the expressions $\sigma_{22} - i\sigma_{12}$ and
$u_1 + iu_2$ for each material are expressed in terms of $C_{1k}$ and $\overline{D}_{1k}$. It is
shown that these expressions are of the same form for either case (a)
or case (b). Then by applying the conditions of equal tractions and
displacements directly ahead of the crack-tip across the interface, we
will have four homogeneous equations in terms of $C_{2k}$, $\overline{D}_{2k}$ and the
exponent $n$.

Non-trivial solutions for the complex coefficients $C_{ik}$ and $D_{ik}$ are
possible only when the determinant of the resulting matrix in equation
(A.29) vanishes, which gives

$$(1-x)^2 (1+2bx+x^2)=0$$

(3.31)

where $x = e^{2i\pi n}$ and $b$ is a complex combination of material properties
of the materials involved and the crack-tip speed.

From equations (3.31) the acceptable eigenvalues to give finite
displacements at the crack-tip are

$$n = 1, 2, 3, ...$$

$$n = \frac{1}{2} \pm i\varepsilon, \frac{3}{2} \pm i\varepsilon, \frac{5}{2} \pm i\varepsilon, ...$$
where
\[ \varepsilon = \frac{1}{2\pi} \log (b + \sqrt{b^2 - 1}) \] (3.32)

For static problems the parameter \( \varepsilon \) has been called the "bielastic constant" [3-4]. For crack propagation problems however, \( \varepsilon \) is a function of the crack-tip speed as well as the material properties, therefore, here we call it the "bielastic parameter".

As discussed in Appendix A the eigenvalues \( n = 1/2 + i\varepsilon \) and \( n = 1/2 - i\varepsilon \) lead to the same solutions, so that only \( n = 1/2 + i\varepsilon \) need to be considered. Thus the eigenvalues are taken to be

\[ n_\lambda = \frac{1}{2} + i\varepsilon , \frac{3}{2} + i\varepsilon , \frac{5}{2} + i\varepsilon \ldots , \lambda = 1, 3, 5 \ldots \]

\[ n_\lambda = 1, 2, 3 \ldots \lambda = 2, 4, 6 \ldots \] (3.33)

Then by using each eigenvalue \( n_\lambda \) we will solve for the corresponding eigenvectors. As is common in any eigenvector problem, the unknowns \( C_{ik} \) and \( D_{ik} \) are determined to within an unknown scaling complex parameter \( \beta_\lambda \). The near-field state of stresses and displacements are then expressed in terms of only the complex unknowns \( \beta_\lambda \). The unknowns \( \beta_\lambda \) will be determined in the finite element formulation using the external boundary conditions.

For the state of displacements the eigenfunctions corresponding to the eigenvalue \( n_2 = 1 \) include the appropriate terms for a rigid body rotation. Therefore to complete the equations for the state of displacements, one has to add the terms corresponding to rigid body translations in \( X \) and \( Y \) directions. These terms involve the eigenfunctions corresponding to \( n_0 = 0 \) which has not been accounted for.
We have deliberately separated these terms for the rigid body translations in order to be able to carry out the necessary integrations for the finite element formulation in Chapter 5. Therefore, the complete form of the displacement functions are given by

\[ \mathbf{u} = \mathbf{U} \mathbf{\xi} \quad (3.34) \]

where

\[ \mathbf{U} = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \]

\[ \mathbf{\xi} = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} \]

with

\[ U_1 = \begin{bmatrix} U_{11} & U_{12} & U_{13} & \ldots \end{bmatrix} \]

and

\[ \xi_1 = \begin{bmatrix} \xi_{11} & \xi_{12} & \xi_{13} & \ldots \end{bmatrix}^T \]

\[ U_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]

\[ \xi_2 = \begin{bmatrix} \xi_{21} & \xi_{22} \end{bmatrix}^T \quad (3.35) \]

where \( U_{mk} \) are the real and the imaginary parts of the complex eigenfunctions \( D_{mk\xi} \) given by equations (A.36) for the \( \xi \)th eigenvalue and the \( k \)th material. The elements of \( \mathbf{U}_2 \) correspond to a rigid body translational motion and \( \xi_{ij} \)'s (real and imaginary parts of \( \xi_{ij} \)'s renamed) are the real unknowns to be determined from the prescribed boundary conditions from the finite element formulation. Similarly the complete form of the state of stresses derived from the above displacements are given by

\[ \mathbf{\sigma} = \mathbf{S} \mathbf{\xi} \quad (3.36) \]
where
\[ S = [S_1 \ S_2] \]
with
\[ S_1 = \begin{bmatrix}
    (\sigma_{11})_1 & (\sigma_{11})_2 & (\sigma_{11})_3 & \cdots \\
    (\sigma_{22})_1 & (\sigma_{22})_2 & (\sigma_{22})_3 & \cdots \\
    (\sigma_{12})_1 & (\sigma_{12})_2 & (\sigma_{12})_3 & \cdots 
\end{bmatrix} \]
and
\[ S_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \]
where \((\sigma_{ij})_k\) are the real and the imaginary parts of the complex eigenfunctions \(S_{nk2}\) given by equations (A.36) for the \(k\)th eigenvalue and the \(k\)th material, and elements of \(S_2\) are the stresses corresponding to the rigid body translations \(U_2\) which are identically zero.

This concludes the derivations of the complete state of near-field stresses and displacements for a propagating crack at the interface of two dissimilar anisotropic materials.
CHAPTER IV

DERIVATION OF SINGULAR AND REGULAR ELEMENT FUNCTIONS

For the purpose of finite element formulation, the body containing the crack is divided into subregions. The singular elements are placed at the regions containing the crack tip and regular elements fill the remainder. It is necessary to formulate the internal stresses and displacements. Also, the boundary tractions and boundary displacements for the elements must be formulated independently from the internal stresses and displacements.

For the singular elements the internal displacements and stresses are as expressed in equations (A.36). In the functional equation (Chapter 5) it is also necessary to formulate the boundary displacements and tractions for the singular element, independently from the internal stresses and displacements. To derive the boundary displacement functions, we consider the singular element shown in Fig. 4.1. The element has 10 nodes and the crack-tip is not associated with any node. Although nodes 5 and 6 could be considered as one node, we have deliberately treated this point as two separate nodes. As discussed earlier in Chapter 1, when the tip of a propagating crack passes through this point, two separated nodes are needed. But before the crack-tip passes through this point, these two nodes are one and the same. One should note that, since the tip of a running crack always stays inside the singular element by a local re-meshing process, see Chapter 7, nodes 5 and 6 always coincide. Therefore, it is possible to formulate the singular element with 9 nodes instead
Fig. 4.1 Nodal Displacement Vectors for the Singular Element
of 10 nodes, i.e. taking nodes 5 and 6 to be one single node. However, for the purpose of computer programming it seemed easier to have two nodes at this mesh point.

Letting \( q_1 \) to \( q_{20} \) represent the displacements at the nodes of the singular element, it is assumed that the boundary displacements are given by

\[
\mathbf{v} = \mathbf{L} \mathbf{g}
\]

(4.1)

where

\[
\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \quad \mathbf{g} = [q_1 \ldots q_{20}]^T
\]

\[
\mathbf{L} = \begin{bmatrix} L_{11} & L_{12} & L_{13} & \cdots & L_{120} \\ L_{21} & L_{22} & L_{23} & \cdots & L_{220} \end{bmatrix}
\]

and the elements of matrix \( \mathbf{L} \) are chosen such that the boundary displacements \( v_i \) vary linearly along the boundary and are equal to the nodal displacements at the nodes. For example, for the segment between nodes 7 and 8, one has

\[
\begin{align*}
L_{11} &= 1-s \\
L_{12} &= s \\
L_{21} &= 1-s \\
L_{22} &= s
\end{align*}
\]

(4.2)

with all \( L_{ij} = 0 \), and \( s \) is a parameter on the boundary chosen so that \( s = 0 \) at node 7 and \( s = 1 \) at node 8. This gives

\[
\begin{align*}
v_1 &= (1-s)q_{13} + sq_{15} \\
v_2 &= (1-s)q_{14} + sq_{16}
\end{align*}
\]

(4.3)

which is the desired form of the boundary displacement functions described above.
For the formulation of the boundary tractions, it is assumed that
\[ \mathbf{t} = R \mathbf{a} \]  
(4.4)
where
\[ \mathbf{t} = [t_1^T, t_2^T]^T, \quad R = \begin{bmatrix} R_{11} & R_{12} & R_{13} & \cdots \\ R_{21} & R_{22} & R_{23} & \cdots \end{bmatrix} \]
\[ \mathbf{a} = [a_1, a_2, a_3, \ldots]^T \]
where \( t_i \) are tractions and \( R_{ij} \) are the functions discussed below, and \( a_i \) are unknown constants to be determined from the finite element formulation.

The functions \( R_{ij} \) can be chosen such that the boundary tractions are distributed linearly along the boundary and are equal to nodal values \( a_i \) at the nodes, in the same manner as the boundary displacement functions \( L_{ij} \) were chosen. In that case there will be 20 unknown \( a_i \)'s to be determined.

The functions \( R_{ij} \) could also be chosen to give constant boundary tractions along all edges, in which case one has
\[ \mathbf{t} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}[a_1, a_2]^T \]
with only two unknown \( a_i \)'s representing the constant tractions along all edges. Other forms of traction distributions are acceptable as well.

However, following the discussion in Chapter 5, it is preferable to choose the number of unknowns \( a_i \) to be equal to the number of unknowns \( \beta_i \), i.e., the unknowns for the internal displacement functions. One way to do this, is to derive the functions \( R_{ij} \) from the displacement functions \( U_{ij} \), as
\[
\vec{R} = n \vec{S} = n A(d \vec{U}) \tag{4.5}
\]

where

\[
\vec{n} = \begin{bmatrix} n_x & 0 & n_y \\ 0 & n_y & n_x \end{bmatrix} \tag{4.6}
\]

\[
\vec{d} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial y} \\ 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \tag{4.7}
\]

Here, \(n_x\) and \(n_y\) are the components of an outward unit vector normal to the sides of the singular element, \(S\), A and \(U\) are defined by equations (3.36), (3.24) and (3.34) respectively.

In doing so, one should note that the terms corresponding to rigid body displacements do not contribute to these tractions. Thus, if \(\vec{t}\) is represented as

\[
\vec{t} = \vec{R} \vec{a} \tag{4.8}
\]

where

\[
\vec{R} = \begin{bmatrix} R_1 & R_2 \\ R_2 & R_2 \end{bmatrix}
\]

and

\[
\vec{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}
\]

with

\[
R_1 = \begin{bmatrix} R_{11} & R_{12} & R_{13} & \cdots & R_{1n} \\ R_{21} & R_{22} & R_{23} & \cdots & R_{2n} \end{bmatrix}
\]

\[
R_2 = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}
\]

\[
a_1 = [a_{11} \ a_{12} \ a_{13} \ a_{14} \ \cdots \ a_{1n}]^T
\]

and

\[
a_2 = [a_{21} \ a_{22}]^T
\]

so that the elements of \(R_2\) corresponding to the rigid body translational motion all vanish. One should also note that since the
displacement eigenfunctions corresponding to \( n_2 = 1 \) contain rigid body rotation terms, the third and the fourth columns of \( R_1 \) are not independent, with the possibility that the elements of the fourth column are all zero. This happens when the roots of the characteristic equation (3.23) are pure imaginary, e.g. when isotropic materials are involved.

As discussed in Chapter 5 the elements of matrix \( \tilde{R}_1 \) should be chosen so that the matrix \( \tilde{P} \) (Chapter 5) is invertible. Therefore, we shall replace the elements of matrix \( \tilde{R}_2 \) with some functions that will make the matrix \( \tilde{P} \) non-singular. One way to do this is to choose the elements of matrix \( \tilde{R}_2 \) as if the body had a purely translational motion, i.e.

\[
\tilde{R}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{4.9}
\]

and choose the elements of the 4th column of matrix \( \tilde{R}_1 \) as if the body had a purely rotational motion, i.e.

\[
\begin{bmatrix} R_{14} \\
R_{24} \end{bmatrix} = \begin{bmatrix} T_1(X,Y) \\
T_2(X,Y) \end{bmatrix}
\]

so that the function \( T_1(X,Y) \) and \( T_2(X,Y) \) will result in a non-zero resultant moment, e.g., see Fig. 4.2, for a square singular element for which

\[
T_1 = \begin{cases} 
1 & \text{on side 1} \\
0 & \text{on sides 2 and 4} \\
-1 & \text{on side 3}
\end{cases}
\]

and

\[
T_2 = \begin{cases} 
1 & \text{on side 2} \\
0 & \text{on sides 1 and 3} \\
-1 & \text{on side 4}
\end{cases}
\]

When the elements of matrix \( \tilde{R} \) are chosen as described here, the matrix \( \tilde{P} \) (Chapter 5) will be square and invertible.
Fig. 4.2 Traction Distribution on a Square Singular Element for a Purely Rotational Motion
This concludes the derivations of all the necessary functions needed for the singular element.

The regular elements are assumed to be four-noded quadrilateral elements. These elements are subsequently transformed into four noded square "parent elements", for which

\[
X = N_iX_i \\
Y = N_iY_i
\]

where \(X-Y\) is a set of global coordinate system and \(X_i, Y_i\) represent the positions of the nodes of the element in \(X-Y\) coordinate system. \(N_i\) are the familiar bilinear shape functions as follows

\[
\begin{align*}
N_1 &= 1/4(1-\zeta)(1-\eta) \\
N_2 &= 1/4(1+\zeta)(1-\eta) \\
N_3 &= 1/4(1+\zeta)(1+\eta) \\
N_4 &= 1/4(1-\zeta)(1+\eta)
\end{align*}
\]

where \(\zeta-\eta\) is a set of coordinate systems with its origin at the center of the square parent element and with \(\zeta\) and \(\eta\) axis parallel to the sides of the square, see Fig. 4.3.

Furthermore, it is assumed that the regular elements are isoparametric, i.e., the field functions are assumed to be in terms of the bilinear functions \(N_i\), as follows

\[
\tilde{u} = Nq
\]

where \(\tilde{u} = [u_1 u_2]^T\) is the displacement matrix and \(q = [q_1 q_2 \ldots q_8]^T\) is a matrix containing the eight nodal displacements, and

\[
\tilde{N} = \begin{bmatrix}
N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 \\
0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4
\end{bmatrix}
\]

The boundary displacements \(\tilde{u}\) are assumed to be given by the values \(\tilde{u}\) at the element boundary. For two adjacent regular elements the values of
Fig. 4.3 (a) A Four Noded Quadrilateral, (b) A Square Parent Element
nodal displacements at a common boundary are the same, so that the
constraint integral in the functional equation (Chapter 5)
automatically vanishes. Therefore there is no need to formulate
boundary traction functions \( \tilde{t} \) for regular elements.

The stresses are assumed to be derived from the displacement
functions as

\[ \sigma = A\tilde{\varepsilon} = A(du) \quad (4.14) \]

where \( A \) is defined by equation (3.24), \( \tilde{\varepsilon} \) is strain matrix, and \( d \) is
defined by equation (4.7). Since the \( \zeta-n \) is a material coordinate
system, then the time derivatives of the displacements are given by

\[ \ddot{u} = N\dot{q} \quad \text{and} \quad \dddot{u} = N\ddot{q} \quad (4.15) \]

With the above assumed functions for the singular and regular elements,
it is now possible to employ the finite element method to formulate the
problem under consideration. In the next chapter a detailed analysis
of the finite element formulation is presented.
CHAPTER V

FINITE ELEMENT FORMULATION

Since the finite element technique has proven to be a powerful method for analyzing a wide range of complicated problems, such as elasticity problems, etc., its extension to crack problems seems natural. The finite element methods are generally classified, according to the unknowns to be found at the nodal points, into the force method, the displacement method, and the mixed method [22-24]. For the problem considered here, a type of displacement finite element method called the "hybrid-displacement" finite element method is employed. The body is assumed to occupy a planar region $\mathcal{R}$, with the boundary $\mathcal{S}$ which is subdivided into subregions $\mathcal{R}_n$ with the boundaries $\mathcal{S}_n$, $n=1,2,3...p$, where, $p$ is the number of elements or subregions in $\mathcal{R}$. The elements $\mathcal{R}_n$ are divided into "singular elements" and "regular elements." The elements containing the crack-tips are called the singular elements and the elements not containing the crack-tips are called the regular elements; see Fig. 5.1. The body is assumed to contain a crack which may grow with time. Also, each crack tip is assumed to be embedded in one of the subregions, say $\mathcal{R}_1$ and $\mathcal{R}_2$, with boundaries $\mathcal{S}_1$ and $\mathcal{S}_2$. Subregions $\mathcal{R}_n$, $n=3,4,...p$ may border on part of the crack surface. For convenience, it is assumed that the subregions $\mathcal{R}_1$ and $\mathcal{R}_2$ are squares and that the other subregions are quadrilaterals. The boundary $\mathcal{S}$ is assumed to consist of straight line segments.

Since different approximating functions for displacement fields are considered for the singular and regular elements, discontinuous
Fig. 5.1  A Cracked Body at Some Fixed Instant of Time
displacement fields across the boundaries of the singular and regular elements will have to be considered. To address this problem, the so-called "hybrid displacement" functional [14-16] is used as follows. Define a functional $\Pi$ as

$$\Pi = \int_{t_0}^{t_1} \sum_{n=1}^{p} \{ \int_{R_n} \left[ 1/2\sigma_{ij} \varepsilon_{ij} - \rho \dot{b}_i u_i - 1/2 \rho \dot{u}_i u_i \right] dv \\
+ \int_{S_{in}} t_i (v_i - u_i) ds - \int_{S_{tn}} t_i^0 v_i ds \} dt$$

(5.1)

Where $S_{in}$ are the boundaries of the subregions and $S_{tn}$ is the part of the boundary where traction boundary conditions $t_i^0$ are prescribed.

The displacements $v_i$ on the boundary of each element are assumed to be linear functions of the nodal displacements. The integral $\int_{S_{in}} t_i (v_i - u_i) ds$ in equation (5.1) is added to the functional to enforce the continuity of displacements between the singular elements and the regular elements, which use different approximating displacement functions. This constraint integral will, in the limit, force the interior displacements $u_i$ to equal the boundary displacements $v_i$ for each element. Since the displacements $v_i$ are the same for two adjacent elements, the constraint integral therefore will insure, in the limit, continuity between all elements. It is assumed that the boundary displacements $v_i$ will satisfy the displacement boundary conditions prescribed on the part of the boundary $S_u$, i.e.

$$v_i = v_i^0 \text{ on } S_u$$

(5.2)

It follows then that the variation $\delta v_i = 0$ on $S_u$. Using the constitutive equations (2.8) and the boundary condition (5.2) and also assuming that the variation $\delta u_i$ vanishes at times $t_0$ and $t_1$, one can
show that the vanishing of the variation $\delta \Pi$ for arbitrary $\delta u_i$ in $R,$ for arbitrary $\delta t_i$ on $S_n$ and for admissible $\delta v_i$ on $S_{in}$, such that $\delta v_i = 0$ on $S_u$, gives the following

$$\sigma_{ij,j} + \rho \delta b_i = \rho \delta u_i \text{ in } R$$

$$u_i = v_i \text{ on } S_{in}$$

$$\sigma_{ij} n_j = t_i \text{ on } S_{in}$$

$$t_i = t_i^0 \text{ on } S_{tn}$$

The first equation states that the displacement functions $u_i$ in $R$ generate stresses $\sigma_{ij}$, from equations (2.1) and (2.8) that satisfy the local equilibrium equation in $R$. The second equation states that the values of $u_i$ at the boundary $S_n$ coincide with the inter-element boundary displacement $v_i$, which is treated as an independent unknown, thus enforcing interelement continuity for displacements. The third equation states that the boundary tractions $t_i$, which are treated as independent unknowns in the problem, coincide with the tractions $\sigma_{ij} n_j$ generated at the boundary $S_n$ by the function $u_i$. The fourth equation states that the tractions generated at the boundary $S_{tn}$ coincides with the tractions specified on this part of the boundary.

Thus, of all the admissible functions $u_i$ in $R$ and $v_i$ and $t_i$ on $S_n$, those functions that render the functional $\Pi$ of equation (5.1) stationary will satisfy all the necessary requirements stated above. Substituting the assumed functions $u_i$, $v_i$, $t_i$ for the singular element derived in Chapter 3 and 4 into equation (5.1) one obtains

$$\Pi_S = \int_{t_0}^{t_1} (1/2 \bar{\varepsilon}^T H \bar{\varepsilon} - \bar{\varepsilon}^T F - 1/2 \bar{\varepsilon}^T M \bar{\varepsilon} - 1/2 \bar{\varepsilon}^T M \bar{\varepsilon} \dot{\theta}^T) dt$$

$$- \bar{\varepsilon}^T M \dot{\theta}^T + \bar{\varepsilon}^T G \dot{\theta} - \bar{\varepsilon}^T P \dot{\theta} - \bar{\varepsilon}^T T \dot{\theta}) dt$$

(5.3)
where

\[ H = \int_{R_n} E^{TAE} dv \]

\[ F = \int_{R_n} U^{T} p dv \]

\[ M_{1} = \int_{R_n} U^{T} p \dot{U} dv \]

\[ M_{2} = \int_{R_n} U^{T} p U dv \]

\[ M_{3} = \int_{R_n} U^{T} p U dv \]

\[ G = \int_{S_{in}} R^{T} L ds \]

\[ P = \int_{S_{in}} R^{T} U ds \]

\[ I = \int_{S_{tn}} L^{0} T^{0} ds \]

and where \( A \) is the stress-strain constitutive relation matrix defined by equation (3.24), and \( E \) is defined by

\[ E = dU \]

with \( d \) as defined by equation (4.7). The dots indicate the time derivatives defined by equations (3.4a) as

\[ \ddot{U} = -c(t) \frac{\partial U}{\partial x} \]

(5.3b)

Since the coefficients \( a \) and \( b \) of the singular element are assumed to be independent of the nodal displacements \( \bar{u} \), then the vanishing of the variation of equation (5.3) with respect to the independent coefficients \( a \) and \( b \), for each singular element, yields
\[ \int_{t_0}^{t_1} \left[ \delta \omega^T \delta \omega - \delta \omega \delta F - \delta \omega \delta T_M \omega - \delta \omega \delta T_{M_2} \omega - \delta \omega \delta T_{M_3} \omega - \delta \omega \delta T_M \omega \right] \, dt = 0 \quad (5.4) \]

and
\[ \int_{t_0}^{t_1} \delta \omega^T [G \omega - P \omega] \, dt = 0 \quad (5.5) \]

Assuming that the variations of $\omega$ vanish at times $t_0$ and $t_1$, it is possible to integrate equation (5.4) to yield

\[ H \delta \omega - \delta F - M_1 \delta \omega - M_2 \delta \omega - M_3 \delta \omega + M_T \delta \omega + M_T \delta \omega - P^T \delta \omega = 0 \quad (5.6) \]

Also, equation (5.5) gives
\[ G \delta \omega - P \delta \omega = 0 \quad (5.7) \]

Equation (5.6) is a set of $n_\beta$ equations and equation (5.7) is a set of $n_\alpha$ equations, where, $n_\alpha$ and $n_\beta$ are the number of unknown coefficients $\omega$ and $\beta$ respectively. Therefore it is possible to solve for the coefficients $\omega$ and $\beta$ in terms of the nodal displacements $\omega$, using equations (5.6) and (5.7). From equation (5.7) it is also observed that one should select $n_\beta > n_\alpha$.

It is noted that the simultaneous solution of equations (5.6) and (5.7) is not an easy task. The matrices have to be partitioned, as done in [41] for static problems. In addition, temporal integration numerical techniques have to be employed to solve for the coefficients $\omega$ and $\beta$ in terms of the nodal displacements $\omega$. However, it is noted that if one selects $n_\alpha = n_\beta$, then the matrix $P$ becomes square and assuming $\omega$ is invertible, then, from equation (5.7) one obtains
\[ \bar{\varepsilon} = \bar{p}^{-1}\tilde{\varepsilon} \]  

(5.8)

Equation (5.8) can be used in equation (5.7) to solve for the coefficients \( \tilde{\varepsilon} \), but this is not necessary since, when equation (5.8) is substituted in the functional equation (5.3), the coefficients \( \tilde{\varepsilon} \) are automatically eliminated, and thus the solution for \( \tilde{\varepsilon} \) is not needed. Therefore the advantage of selecting \( n_\alpha = n_\beta \) is obvious. The invertibility of the matrix \( \bar{p} \) must be assured by proper selection of the assumed eigenfunctions for \( \tilde{\varepsilon} \) and \( \varphi_\lambda \) as discussed in Chapter 3 and 4.

Introducing

\[ \bar{B} = \bar{p}^{-1}\tilde{\varepsilon} \]  

(5.9)

equation (5.8) becomes

\[ \bar{B} = \bar{p}\tilde{\varepsilon} \]  

(5.10)

Introducing equation (5.10) into the function equation (5.3) yields

\[ \Pi_s = \int_0^t [1/2 \bar{g}^T K_1 \bar{g} - 1/2 \bar{q}^T M_1 \bar{g} - \bar{g}^T V_1 \bar{g} - \bar{g}^T F_s] dt \]  

(5.11)

where

\[ K_1 = B^T H B - B^T M_1 B - B^T M_2 \bar{\varepsilon} - 2B^T M_3 \bar{\varepsilon} \]

\[ M_1 = B^T M_2 \bar{\varepsilon} \]

\[ V_1 = B^T M_2 \bar{\varepsilon} + B^T M_3 \bar{\varepsilon} \]

\[ F_s = B^T F + T \]  

(5.12)

For a propagating crack the matrix \( M_1^* \) is symmetric, while the matrices \( \bar{K}_1 \) and \( \bar{V}_1 \) are not. For a stationary crack the matrix \( \bar{V}_1 \) vanishes and the matrix \( \bar{K}_1 \) becomes symmetric. Equation (5.11) is the functional equation valid for the singular elements.
For regular elements, substituting equation of Chapter 4 into the functional equation (5.1) yields

$$\Pi_R = \int_{t_0}^{t_1} \left[ 1/2 \dot{q}^T K_g \dot{q} - 1/2 \dot{q}^T M_g \ddot{q} - \dot{q}^T F_R \right] dt \quad (5.13)$$

where

$$K_R = \int_{R_n} (dN)^T A (dN) dv$$

$$M_R = \int_{R_n} N^T \rho N dv \quad (5.14)$$

$$F_R = \int_{S_{tn}} N^T t_0 ds + \int_{R_n} N^T \rho dv$$

Denoting the global nodal displacements by \( g^* \), and combining all the element matrices into corresponding global matrices yields

$$\Pi = \int_{t_0}^{t_1} \sum_{n=1}^{2} \left( 1/2 \dot{g}^* T K_q \dot{g}^* - 1/2 \dot{g}^* T M_q \ddot{g}^* - \dot{g}^* T V_q \dot{g}^* - \dot{g}^* T F_q \right) + \sum_{n=3}^{p} \left( 1/2 \dot{g}^* T K_q \dot{g}^* - 1/2 \dot{g}^* T M_q \ddot{g}^* - \dot{g}^* T V_q \dot{g}^* \right) dt \quad (5.15)$$

where the bar over the matrices denote the global matrices corresponding to the element matrices.

The variation of \( \Pi \) with respect to \( g^* \), yields

$$\delta \Pi = \int_{t_0}^{t_1} \sum_{n=1}^{2} \left( 1/2 \dot{g}^* T \left( K_q + K_{\gamma} \right) \dot{g}^* - \delta g^* \dot{M} \dot{g}^* - \delta g^* V \dot{g}^* \right. \left. - \delta g^* F_q \right) + \sum_{n=3}^{p} \left( \delta g^* T K_q \dot{g}^* - \delta g^* T M_q \ddot{g}^* - \delta g^* T V_q \dot{g}^* \right) dt$$

Assuming that the variation of \( \delta g^* \) vanishes at times \( t_0 \) and \( t_1 \), integration in time yields
\[ \delta \Pi = \int_{t_0}^{t_1} \left\{ \delta g^T \left[ \sum_{n=1}^{2} (K_S g^* + V_S g^* + \bar{W}_S g^* - F_S) \right] + \delta g^T \left[ \sum_{n=3}^{p} (K_R g^* + \bar{W}_R g^* - F_R) \right] \right\} dt \]

where

\[ K_S = 1/2(K_1 + K_1^T) + \ddot{V}_1 \]

\[ V_S = \dot{M}_1 + V_1 - \dot{V}_1 \]

\[ M_S = \bar{M}_1^* \]

The variation \( \delta \Pi \) should vanish for arbitrary \( \delta g^* \), yielding

\[ \bar{M}_S \dot{g}^* + V_S \dot{g}^* + K_S g^* = Q \]

where

\[ M = \sum_{n=1}^{2} M_S + \sum_{n=3}^{p} W_R \]

\[ V = \sum_{n=1}^{2} V_S \]

\[ K = \sum_{n=1}^{2} K_S + \sum_{n=3}^{p} K_R \]

\[ Q = \sum_{n=1}^{2} F_S + \sum_{n=3}^{p} F_R \]

The global matrices \( M, V, K \) and \( Q \) are obtained by summing all the element matrices, a process conventionally referred to as the merging of the element matrices.

Equation (5.17) represents the governing equation of motion for the elastodynamic problem considered here. It is noted that although
there is no damping system present in the problem, the matrix \( \mathbf{Y} \) called the "pseudo-damping" matrix is present. From equations (5.3a), (5.12) and (5.18) it is also noted that the stiffness matrix \( \mathbf{K} \) is not symmetric. Furthermore from equation (5.18) it is deduced that only the singular elements contribute to the pseudo-damping matrix \( \mathbf{Y} \) and only the singular elements contribute to the non-symmetricity of the stiffness matrix \( \mathbf{K} \). These complexities arise due to the fact that the eigen functions for the singular element were derived with respect to a moving local coordinate system \( x-y \) at the crack-tip and therefore, matrices \( \mathbf{M}^*, \mathbf{K} \) and \( \mathbf{V} \) are functions of time. An examination of equations (5.3a), (5.3b) and (5.12) reveals that when the instantaneous crack-tip velocity \( c(t) \) vanishes, so that the local crack-tip coordinate system \( x-y \) becomes stationary, then the pseudo-damping matrix \( \mathbf{V} \) vanishes and the stiffness matrix \( \mathbf{K} \) becomes symmetric.

From equation (5.16) the corresponding element matrix for the singular elements can be written as

\[
\begin{align*}
\mathbf{K}_s &= \frac{1}{2}(\mathbf{K}_1 + \mathbf{K}_1^T) + \mathbf{Y} \\
\mathbf{V}_s &= \mathbf{M}^* + \mathbf{V}_1 - \mathbf{V}_1^T \\
\mathbf{M}_s &= \mathbf{M}^*
\end{align*}
\]

Calculating the time derivatives of \( \mathbf{V}_1 \) and \( \mathbf{M}^* \) and using the fact that

\[
\begin{align*}
\dot{\mathbf{M}}_2 &= \mathbf{M}_3 + \mathbf{M}_3^T \\
\dot{\mathbf{M}}_3 &= \mathbf{M}_1 + \mathbf{M}_4
\end{align*}
\]
with

\[ M_4 = \int_{R_n} \tilde{u}_p \tilde{u}_d \, dv \]  \hspace{2cm} (5.20)

equation (5.19) yields

\[ K_S = B^T H B + \tilde{B}^T (M_2 \tilde{\bar{B}} + 2M_3 \tilde{\bar{B}} + \tilde{\bar{B}}^T) \]

\[ V_S = 2\tilde{B}^T (M_2 \tilde{\bar{B}} + M_3 \tilde{\bar{B}}) \]

\[ M_S = \tilde{B}^T M_2 \tilde{B} \]

\[ F_S = \tilde{B}^T F + \tilde{T} \]  \hspace{2cm} (5.21)

The regular element matrices can be evaluated from equation (5.14) by evaluating the integrals using numerical techniques such as the Gaussian integration method without any difficulty. However, in evaluating the singular element matrices of equation (5.20) it is noted that when the various functions for the singular element derived in Chapter 3 and 4 are substituted into equations (5.3a) and (5.20), matrices \( \tilde{H} \), \( \tilde{M}_3 \), and \( \tilde{M}_4 \) contain various orders of singularities. Therefore, some modifications have to be made before one can perform the integration task. From equations (5.3a) and (5.20) writing the matrices \( \tilde{H} \) and \( \tilde{M}_4 \) in decomposed form one has

\[ \tilde{H} = \int_{R_n} \tilde{E}^T A \tilde{E} \, dv = \begin{bmatrix} \int_{R_n} (\tilde{d}\tilde{U}_1)^T A (\tilde{d}\tilde{U}_1) \, dv & 0 \\ 0 & 0 \end{bmatrix} \]

\[ \tilde{M}_4 = \int_{R_n} \tilde{u}_p \tilde{U}_d \, dv = \begin{bmatrix} \int_{R_n} \tilde{u}_p \tilde{U}_1 \, dv & \int_{R_n} \tilde{u}_p \tilde{U}_2 \, dv \\ 0 & 0 \end{bmatrix} \]
Integrating by parts, one has

$$\int_{R_n} (dU_1)^T \omega_1 (dU_1) dv = \int_{S_n} (nU_1)^T \omega_1 (dU_1) ds - \int_{R_n} U_1^T (d\omega_1^T (dU_1)) dv$$

Therefore

$$\omega + \omega_4^T = \omega_1 + \omega_5^T$$  \hspace{1cm} (5.22)

with

$$\omega_1 = \begin{bmatrix} \int_{S_n} U_1^T n^T (dU_1) ds & 0 \\ 0 & 0 \end{bmatrix}$$ \hspace{1cm} (5.23)

$$\omega_5 = \begin{bmatrix} 0 & \int_{R_n} U_1^T \rho \omega_2 dv \\ 0 & 0 \end{bmatrix}$$

in the above derivations use was made of the fact that $\tilde{U}_2 = \tilde{U}_2 = 0$, see equation (3.35), and also

$$\rho \omega_1 = \omega_1^T \omega_1$$

since $\omega_1$ was chosen to satisfy the dynamic linear momentum equations of (3.5).

The matrix $\omega_5$ still contains the singularity, but since $\omega_2$ is a constant coefficient matrix, one is able to write

$$\int_{R_n} U_1^T \rho \omega_2 dv = \int_{R_n} (-c(t) \tilde{U}_1^T \tilde{U}_2) \omega_2 dv = -c(t) \int_{S_n} U_1^T \rho \omega_2 ds$$

Therefore

$$\omega_5 = \begin{bmatrix} 0 & -c(t) \int_{S_n} U_1^T \rho \omega_2 ds \\ 0 & 0 \end{bmatrix}$$ \hspace{1cm} (5.24)

The integral $\omega_3$ can be treated in a similar manner. Writing $\omega_3$ in the decomposed form one has
\[
M_3 = \begin{bmatrix}
\int_{R_1^\infty} \tilde{U}_1^T \rho \tilde{U}_1 \, dv & \int_{R_1^\infty} \tilde{U}_2^T \rho \tilde{U}_1 \, dv \\
0 & 0
\end{bmatrix}
\]

The only integral containing a singularity is \( \int_{R_1^\infty} \tilde{U}_1^T \rho \tilde{U}_1 \, dv \). Again since \( \tilde{U}_2 \) is a constant coefficient matrix, \( M_3 \) is replaced by

\[
M_5 = \begin{bmatrix}
\int_{R_1^\infty} \tilde{U}_1^T \rho \tilde{U}_1 \, dv & -c(t)\int_{S_1}\tilde{U}_1^T \rho \tilde{U}_2 n_1 ds \\
0 & 0
\end{bmatrix}
\]

Also since the body forces were ignored, the singular element matrices of equation (5.21) change to

\[
K_S = B^T \tilde{H} B + B^T \tilde{M} \tilde{B} + 2B^T \tilde{M} \tilde{B} + B^T \tilde{M} \tilde{B} + B^T \tilde{M} \tilde{B}
\]

\[
Y_S = 2B^T (M_2 \tilde{B} + M_6 \tilde{B})
\]

\[
M_S = B^T \tilde{M} \tilde{B}
\]

\[
F_S = \tilde{T}
\]

With the above modifications, it is now possible to perform all the necessary integrations and, by combining all the element matrices, the total mass matrix \( \tilde{M} \), pseudo-damping matrix \( \tilde{Y} \) and the stiffness matrix \( \tilde{K} \) of equation (5.17) can be evaluated. The solution procedure of the governing dynamic equation of motion (5.17) is discussed in Chapter 7.
CHAPTER VI
STRESS INTENSITY FACTORS

Stress intensity factors are parameters capable of characterizing the near-field displacements and stresses and enables one to make judgements concerning the behavior of crack under different load conditions, since they govern the onset of rapid crack extension. The stress intensity factors in plane problems are usually defined by removing the stress singularities as

\[
K_1 = \lim_{r \to 0} \sqrt{2 \pi r} \sigma_{22}(r,0) \tag{6.1}
\]

\[
K_2 = \lim_{r \to 0} \sqrt{2 \pi r} \sigma_{12}(r,0)
\]

where \(K_1\) and \(K_2\) are the opening mode and the sliding mode stress intensity factors respectively, and \(r\) is a distance directly ahead of the crack-tip with respect to the local coordinate system at the crack-tip.

The above definition is valid for problems with only one material, in which case normal loads will result in only the opening mode, and shear loads will result in only the sliding mode. In the bimaterial case, however, either normal or shear loads will result in both the opening mode and the sliding mode. Therefore the parameters \(K_1\) and \(K_2\) can no longer be regarded as the crack-tip stress intensity factors for symmetrical and skew-symmetrical stress distributions. The parameters \(K_1\) and \(K_2\) may in general be considered as the strength of the stress singularities at the crack-tip [4].

For static interface crack problems with two dissimilar isotropic
materials, the stress intensity factors are defined [3] by removing the stress singularities as,
\[ k_1 - ik_2 = 2 \sqrt{\pi} \ e^{-\pi \epsilon} \lim_{z_1 \to 0} z_1^{1/2 - i\epsilon} \Omega_{11}(z_1) \] (6.2)
with \( \Omega_{11}(z) \) and \( \epsilon \) as defined by equations (A.17) and (A.31) respectively. For this case they are given by
\[ \Omega_{11}(z_1) = \sum_{k=1}^{m} \beta_k z_1^{n_k} \]
\[ \epsilon = -\frac{1}{2\pi} \log(q) \] (6.3)
with \( q = \frac{n_1 + u_1/\mu_2}{(u_1/\mu_2)n_2+1} \)
where
\[ n_k = \begin{cases} 3-4v_k & \text{for plane strain} \\ 3-v_k & \text{k=1,2} \\ 1+v_k & \text{for plane stress} \end{cases} \]
and where \( \mu_k \) and \( v_k \) are the shear moduli and the Poisson's ratios of the two isotropic materials in the problem, \( \beta_k \) are the unknown complex coefficients discussed in Appendix A and \( n_k \) are given by equation (A.32).

Substituting equations (6.3) into equation (6.2), yields
\[ k_1 - ik_2 = 2 \sqrt{\pi} \ e^{-\pi \epsilon n_1 \beta_1} \] (6.4)

For this problem the first term of the stresses directly ahead of the crack-tip is given by
\[ \sigma_{22} = \text{Real} \ [(1+q) n_1 \beta_1 r^{-\frac{1}{2} + i\epsilon}] \] (6.5)
\[ \sigma_{12} = \text{Real} \ [(i(1+q) n_1 \beta_1 r^{-\frac{1}{2} + i\epsilon}] = -\text{Im}[(1+q)n_1 \beta_1 r^{-\frac{1}{2} + i\epsilon}] \]

From equation (6.3), one obtains
\[ q = e^{-2\pi \varepsilon} \quad \text{and} \quad (1+q) = 2\cosh(\pi \varepsilon)e^{-\pi \varepsilon} \]

so that equation (6.2) can be defined in an alternative way as

\[ k_1 - ik_2 = \frac{\sqrt{2}}{\cosh(\pi \varepsilon)} \lim_{r \to 0} \left[ r^{\frac{1}{2}}e^{-i\varepsilon} (\sigma_{22} - i\sigma_{12}) \right] \quad (6.6) \]

Turning attention to the more general problem of dynamic interface cracks with two dissimilar anisotropic materials considered in this dissertation, it is noted that the first term of \( \Omega_{11}(z) \) is given by equations (3.28), (A.17), and (A.33) as

\[ \Omega_{11}(z) = z \frac{n_1}{\beta_1} + \frac{F_{211}}{z} \frac{n_{11}}{\beta_1} \quad (6.7) \]

where in general \( F_{211} \neq 0 \).

Therefore, it is seen that the stress singularities cannot be removed in the same manner as in equation (6.2), since for the general case both \( n_1 \) and \( \overline{n}_1 \) are involved, while in the special case of equation (6.2), \( F_{211} \) turned out to be zero and only \( n_1 \) was involved.

Therefore, a new definition is proposed here for the stress intensity factors as follows. Let the first term of the stresses directly ahead of the crack-tip be given by equation (A.36) as

\[ \sigma_{22} = \text{Re} \left( A_2 r^{n_1-1} \beta_1 \right) \quad (6.8) \]

\[ \sigma_{12} = \text{Re} \left( A_3 r^{n_1-1} \beta_1 \right) \]

where \( A_2 \) and \( A_3 \) are known complex constants from equations (A.36a) and (A.36b) as follows. For case (a) let \( z_1 = z_2 = r, \varepsilon = 1 \) and \( k = 1 \) in equation (A.36a) to get
\[ A_i = n_i \left[ s_{i11} F_{111} + s_{i12} F_{112} + s_{i21} F_{211} + s_{i22} F_{221} \right] \quad i=2,3 \]

and for case (b) let \( z_k = r \), \( z=1 \) and \( k=1 \) in equation (A.36b) to get

\[
A_i = n_i \left[ (s_{i11} F_{111} + s_{i21} F_{112}) + (s_{i12} F_{121} + s_{i22} F_{221}) \right] + s_{i11} (n_i - 1) F_{111} + s_{i21} (n_i - 1) F_{211} \]

\[ i=2,3 \]

Now, we define

\[
K_1 = \sqrt{2\pi} \text{ \text{Real} } (A_2 \beta_1) \quad \quad (6.9)
\]

\[
K_2 = \sqrt{2\pi} \text{ \text{Real} } (A_3 \beta_1)
\]

It is readily seen that this definition reduces to the definition of equation (6.1) for the problems with one material, for which \( \varepsilon = 0 \).

For the static interface crack problem with two dissimilar isotropic materials, from equations (6.5) and (6.8) one has

\[
A_2 = -i A_3 = (1+q)n_1 = 2\cosh(\pi \varepsilon )e^{-\pi \varepsilon n_1} \quad (6.10)
\]

Substituting equation (6.10) into (6.9) yields

\[
K_1 - iK_2 = \sqrt{2\pi} \quad 2\cosh(\pi \varepsilon )e^{-\pi \varepsilon n_1 \beta_1} \quad (6.11)
\]

Comparing equation (6.11) and (6.4) one obtains

\[
k_1 - ik_2 = \frac{K_1 - iK_2}{\sqrt{\pi \cosh(\pi \varepsilon )}}
\]

so that definition (6.9) reduces to that of (6.2) to within a multiplicative factor of \( \sqrt{\pi \cosh(\pi \varepsilon )} \). Thus we adopt the new definition of equation (6.9) for the stress intensity factors, which reduces to the previous definitions, to within a multiplicative factor.
CHAPTER VII
SOLUTION PROCEDURE

The complete solution to the elastodynamic problem considered here is obtained by solving the discretized governing equation of motion (5.17).

There are two classes of methods for the direct integration of the equation of motion: explicit methods, in which accelerations are found from the equations of motion and then integrated to obtain displacements; and the implicit methods, in which the equations of motion are combined with the time integration operator so that displacements are found directly. Broadly speaking, implicit methods permit larger time steps, whereas explicit methods are restricted to small time steps by numerical stability requirements.

An implicit method of temporal integration which is developed from finite difference formulas and known as the Newmark-β formulas [22] is used here. The Newmark-β formulas can be written as

\[ \ddot{q}(t+\Delta t) = \ddot{q}(t) + \frac{\Delta t}{2} [\ddot{q}(t) + \ddot{q}(t+\Delta t)] \]  

\[ q(t+\Delta t) = q(t) + \Delta t \dot{q}(t) + \Delta t^2 [\frac{1}{2} - \beta] \ddot{q}(t) + \beta \ddot{q}(t+\Delta t)] \]  

In these formulas a value of β=1/4 corresponds to the assumption of a constant acceleration equal to the average of the accelerations at the ends of the interval between \(t\) and \(t+\Delta t\). A value of \(\beta=1/6\) corresponds to a piecewise linear acceleration within each time interval, and a value of \(\beta=1/8\) corresponds to a step function with a uniform value equal to the initial value for the first half of the
time interval and a uniform value equal to the final value for the second half of the time interval. A value of $\beta=0$ corresponds to double pulses of acceleration at the beginning and end of the time interval with each double pulse consisting of parts equal to 1/2 of the acceleration times the time interval, one occurring just before the end of the preceding interval and the other just after the beginning of the next interval. It can also be shown that the Newmark-$\beta$ formulas of (7.1) with $\beta=0$ are equivalent to the central difference formulas

$$\ddot{q}(t+ \frac{\Delta t}{2}) = \ddot{q}(t- \frac{\Delta t}{2}) + \Delta t \dddot{q}(t)$$

(7.2a)

$$q(t+\Delta t) = q(t) + \Delta t \dot{q}(t + \frac{\Delta t}{2})$$

provided that we assume

$$\dot{q}(t+ \frac{\Delta t}{2}) = \dot{q}(t) + \frac{\Delta t}{2} \ddot{q}(t)$$

(7.2b)

When used in an implicit method, the Newmark-$\beta$ formula of equation (7.1) yields unconditionally stable algorithms in linear problems for $\beta>1/4$. The central difference formulas of equations (7.2), i.e. equations (7.1) with $\beta=0$, are only conditionally stable and give an explicit procedure.

To formulate an implicit method of temporal integration, it is assumed that $\beta>0$. Another aspect of the temporal integration methods which should be considered is their artificial damping, which may be considered as the tendency of the difference formulas to damp certain components of the response. The artificial damping may or may not be
desirable depending on the desired solution [23]. The Newmark-β formulas have no artificial damping.

Substituting equations (7.1) into equation (5.17) and dropping the "*" for convenience, yields

\[
K_{eq}(t+\Delta t) = \omega_{eq}
\]  
(7.3)

where

\[
K_{eq} = M + \frac{\Delta t}{2} V + \beta \Delta t^2 K
\]

\[
\omega_{eq} = M \left[ g(t) + \Delta t \dot{q}(t) + \left( \frac{1}{2} - \beta \right) \Delta t^2 \ddot{q}(t) \right] + V \left[ \frac{\Delta t}{2} g(t) + \left( \frac{1}{2} - \beta \right) \Delta t^2 \dot{q}(t) + \left( \frac{1}{4} - \beta \right) \Delta t^3 \ddot{q}(t) \right] + \beta \Delta t^2 \ddot{Q}(t+\Delta t)
\]

The solution to equation (5.17) is obtained by solving equation (7.3) at each time interval \( \Delta t \). However, it is seen that \( K_{eq} \) is not symmetric, since the matrices \( V \) and \( K \) in equation (5.17) are not symmetric as discussed earlier. In order to be able to use a procedure for the solution of a set of algebraic equations with symmetric coefficient matrix, equation (7.3) is modified in the following manner. Matrices \( V \) and \( K \) are divided into symmetric and asymmetric parts as

\[
V = \frac{1}{2} (V + V^T) + \frac{1}{2} (V - V^T)
\]

i.e.

\[
V = V_{sym} + V_{asym}
\]

with similar formulas for \( K \). Substituting these expressions into equation (7.3) yields
\[ K_{\text{eff}} \dot{g}(t+\Delta t) = Q_{\text{eff}} \tag{7.4} \]

where

\[ K_{\text{eff}} = M + \frac{\Delta t}{2} V_{\text{sym}} + \beta \Delta t^2 K_{\text{sym}} \]

and

\[ Q_{\text{eff}} = V_{\text{sym}} \left[ \Delta t \dot{g}(t) + \left( \frac{1}{2} - \beta \right) \Delta t^2 \ddot{g}(t) \right] 
+ V_{\text{sym}} \left[ \frac{\Delta t}{2} \dot{q}(t) + \left( \frac{1}{2} - \beta \right) \Delta t^2 \dddot{q}(t) + \left( \frac{1}{4} - \beta \right) \Delta t^3 \ddot{q}(t) \right] 
+ \beta \Delta t^2 [Q(t + \Delta t) - V_{\text{sym}} \ddot{g}(t + \Delta t) - K_{\text{sym}} \ddot{q}(t + \Delta t)] \]

It is seen that the evaluation of \( Q_{\text{eff}} \) at \( t + \Delta t \) requires a knowledge of \( g(t + \Delta t) \) and \( \dot{g}(t + \Delta t) \), which we are about to solve. Therefore an iterative procedure is employed as follows. In the the expression for \( Q_{\text{eff}} \), change \( \dot{g}(t + \Delta t) \) and \( g(t + \Delta t) \) to \( \dot{g}(t) \) and \( g(t) \) respectively, solve for \( g(t + \Delta t) \) from equation (7.4) and for \( \dot{g}(t + \Delta t) \) from equation (7.1), then use these values of \( g(t + \Delta t) \) and \( \dot{g}(t + \Delta t) \) in the expression for \( Q_{\text{eff}} \) to find a new set of \( g(t + \Delta t) \) and \( \dot{g}(t + \Delta t) \). Repeat this process until some convergence criteria has been met. Therefore equation (7.4) can be rewritten as

\[ K_{\text{eff}} g^{(p)}(t + \Delta t) = Q^{(p-1)}_{\text{eff}} \tag{7.5} \]

for any \( p \)th iteration.

However, since the asymmetry of \( K \) relates to the small part of matrix \( K \) which corresponds to the singular element, and the matrix \( V \) corresponds only to the singular element, therefore, a fast convergence for equation (7.5) is to be expected. In fact experimentation by this author indicated that only one iteration was adequate for convergence and subsequent iterations hardly improved the solution.
It remains now to devise a method of crack-propagation in the finite element mesh for dynamic problems with crack extensions. A method for crack propagation can be described as follows. The singular element is placed in the finite element mesh such that the initial crack-tip position is situated inside the singular element. Then the crack-tip is advanced at each time step according to the prescribed crack-tip position history. When the crack-tip reaches an extreme forward position, i.e. the 3/4 point position, inside the singular element, a local remeshing takes place and the position of the singular element is moved forward, i.e. in the direction of crack propagation, by half the size of the singular element. Thus the crack-tip is located at the 1/4 point position of the singular element and the crack-tip can continue to extend inside the singular element (Fig. 7.1). Therefore, two elements that were regular at the previous time step turn into a part of the new singular element and half of the region of the previous singular element is replaced with two new regular elements.

As shown in Fig. 7.1, this procedure requires a certain degree of regularity in the finite element mesh. In fact, the finite element program requires the singular element to be a square composed of four square regular elements and for the propagating cracks it is required that the elements adjacent to the interface be squares for the distance required for remeshing as long as the crack is extending. Therefore as long as remeshing takes place, the singular element is a square composed of four square regular elements. These restrictions
Fig. 7.1 Finite Element Mesh Redefinition Procedure
are not the results of the procedure developed here, but are imposed for programming ease and economic reasons.

As it is seen from Fig. 7.1, in the process of remeshing new nodes have to be created in the finite element mesh. A method of double noding proposed by B.M. Liaw, A.S. Kobayashi and A.F. Emery [17] and also Santosh K. Arya [18] was employed to create new nodes. In this technique the mesh-points along the crack-tip, i.e. the mesh points on the interface, have two nodes, so that when remeshing takes place the necessary extra node is available. Although the center of the singular element is not associated with any node, nevertheless, there have to be two imaginary nodes at that point. These nodes are, of course eliminated in the calculation process in order to make \( k_{\text{eff}} \) non-singular. Also one of the nodes on all the mesh points ahead of the crack-tip which are associated with two nodes are eliminated by using the equality of the displacements of the two nodes at such points, to ensure that the final \( k_{\text{eff}} \) is not singular and thus invertible.

This procedure is implemented as follows. Assume that \( k_{\text{eff}} \) and \( q_{\text{eff}} \) in equation (7.5) are formed. Then for the mesh points ahead of the crack-tip which are associated with two nodes, the horizontal and vertical degrees of freedom of these two nodes have to be equal. Now, assuming that the \( i^{\text{th}} \) and \( j^{\text{th}} \) degrees of freedom are constrained to be equal, i.e.

\[
q_i = q_j \tag{7.6}
\]

define the relative displacement vector \( g^r \) as

\[
g = g - g^r \tag{7.7}
\]
and
\[ Q_{\text{eff}} = T^T Q_{\text{eff}} \]  \hspace{1cm} (7.8)

where
\[ T_{mn} = \begin{cases} 
1 & \text{when } m=n \\
1 & \text{when } m=i \text{ and } n=j \\
0 & \text{for all other } m \text{ and } n 
\end{cases} \]

and
\[ q_k^- = \begin{cases} 
q_i - q_j = 0 & \text{when } k=i \\
q_k & \text{when } k \neq i 
\end{cases} \]

Substituting equations (7.7) and (7.8) into equation (7.5) yields
\[ K_{\text{eff}} \ddot{q} = Q_{\text{eff}} \]  \hspace{1cm} (7.9)
\[ = T^T Q_{\text{eff}} \]
\[ = T^T K_{\text{eff}} \dot{\tilde{q}} \]
\[ = T^T K_{\text{eff}} - T \dot{\tilde{q}} \]

The above equation requires that
\[ K_{\text{eff}}^- = T^T K_{\text{eff}}^- T \]  \hspace{1cm} (7.10)

Therefore equation (7.9) is solved instead of equation (7.5) and then the displacements \( \tilde{q} \) are found from equation (7.7).

A computer program was developed that carries out the procedure described in this dissertation and solves for the displacements \( \tilde{q} \). The velocities and accelerations \( \dot{\tilde{q}} \) and \( \ddot{\tilde{q}} \) are found from the difference equations (7.1). The stresses at the center of regular elements are found according to equation (4.14), and the stresses at four points of the singular element corresponding to the center points of the regular elements which form the singular element
are found according to equation (3.34). The finite element mesh definition, initial conditions, boundary conditions, external load history, crack-tip velocity history, crack-tip position history and the material properties are provided as input. The program obtains the eigenvalues and eigenfunctions for the singular element and evaluates all the regular and singular element matrices and stores them into global matrices $\tilde{K}$, $\tilde{V}$ and $\tilde{M}$ of equations (5.17). Then, $K_{\text{eff}}$ and $Q_{\text{eff}}$ are formed from which $Z_{\text{eff}}$ and $Q_{\text{eff}}$ are obtained. Then using the iterative procedure described earlier, the solution for $\phi$, and hence the complete solution, including the stress intensity factors as defined by equation (6.9) are obtained. A more detailed description of the computer program and its complete listing are presented in the Appendices.

In the next chapter solutions to some simple problems are presented. These results are compared with known theoretical and numerical results. Finally the solutions to some stationary and propagating interface cracks between two dissimilar anisotropic materials under static and dynamic loads, for which there are no known solutions, are presented.
CHAPTER VIII

WAVE PROPAGATION IN ANISOTROPIC MEDIUM

When a structure is subjected to dynamic loads such as impacts, generated waves travel through it. It is necessary to understand the response characteristics of the material body and the way the generated waves propagate through the material.

It is well known that for isotropic materials there are two characteristic wave speeds [24], $C_T$ and $C_L$, where $C_T = \sqrt{G/\rho}$ is the transverse (also called shear or distortional) wave speed, and $C_L = \sqrt{(\lambda+2G)/\rho}$ is the longitudinal or dilatational wave speed. In the above $\lambda$ and $G$ are Lame's constants and $\rho$ is the mass density of the material. For isotropic materials, the waves propagate through the medium with the same speeds in all directions. This is obviously due to the fact that isotropic material properties are independent of direction.

For anisotropic materials, which exhibit different properties in different directions, the wave propagation is directional, i.e., the wave speeds are a function of direction as well as material properties.

The displacement form for a plane harmonic wave can be written as [25]

$$u_n = C_n \exp[i\omega(n_m \cdot \mathbf{v} - t)] \quad n, m = 1, 2, 3 \quad (8.1)$$

where $\omega$ is the real circular frequency, $n_m$ are the elements of a unit vector representing the direction of propagation, $C_n$ are the wave
amplitudes, and \( v \) is the velocity of wave propagation.

For a plane problem substituting equation (8.1) into the strain-displacement equations (2.1), the constitutive equations (2.12) and the equilibrium equations (2.4) and ignoring the body forces, one obtains

\[
\begin{bmatrix}
A_{11}n_1^2 + 2A_{16}n_1n_2 + A_{66}n_2^2 - Q & A_{16}n_1^2 + (A_{12} + A_{66})n_1n_2 + A_{26}n_2^2 \\
A_{16}n_1^2 + (A_{12} + A_{66})n_1n_2 + A_{26}n_2^2 & A_{66}n_1^2 + 2A_{26}n_1n_2 + A_{22}n_2^2 - Q
\end{bmatrix}
\begin{bmatrix}
C_1 \\
C_2
\end{bmatrix} = 0
\]

(8.2)

where \( Q = \rho v^2 \), \( A_{ij} \) are the elements of the matrix \( \mathbf{A} \) in the constitutive equation \( \mathbf{e} = \mathbf{A} \mathbf{s} \) and \( C_1 \) and \( C_2 \) are the wave amplitudes for \( u_1 \) and \( u_2 \) in equation (8.1).

For non-trivial solutions of \( C_1 \) and \( C_2 \), the determinant of the matrix in equation (8.2) should vanish, which gives

\[
Q^2 - Q[A_{11}n_1^2 + 2(A_{16} + A_{26})n_1n_2 + A_{22}n_2^2 + A_{66}]
+ [(A_{11}A_{66} - A_{16}^2)n_1^2 + 2(A_{11}A_{26} - A_{16}A_{12})n_1n_2 + (A_{11}A_{22} + 2A_{16}A_{26} - A_{12} - 2A_{16}^2 - A_{66})n_2^2 + 2(A_{11}A_{26} - A_{16}A_{12})n_1n_2 + (A_{22}A_{66} - A_{26}^2)n_2^4] = 0
\]

(8.3)

For any given direction \( n_1 \) and \( n_2 \) equation (8.3) gives two roots for \( Q \) which in turn gives two values for the wave speed \( v \). These two values correspond to the shear wave and longitudinal wave speeds in the specified direction for the material. For example, taking the \( X_1 \) direction one obtains

\[
v_1^2 = \frac{A_{11} + A_{66} + \sqrt{(A_{11} - A_{66})^2 + 4A_{26}^2}}{2\rho}
\]

\[
v_2^2 = \frac{A_{11} + A_{66} - \sqrt{(A_{11} - A_{66})^2 + 4A_{26}^2}}{2\rho}
\]
and in the \( X_2 \) direction one obtains

\[
v_1^2 = \frac{A_{22} + A_{66} + \sqrt{(A_{22} - A_{66})^2 + 4A_{26}^2}}{2p}
\]

\[
v_2^2 = \frac{A_{22} + A_{66} - \sqrt{(A_{22} - A_{66})^2 + 4A_{26}^2}}{2p}
\]

where \( v_1 \) and \( v_2 \) are the longitudinal and shear wave speeds in the indicated directions, respectively.

To find the directions in which the wave speeds are minimum or maximum, equation (8.2) should be differentiated with respect to the angle \( \theta \) from \( X_1 \) direction and \( \frac{dQ}{d\theta} \) be set to zero. Recalling that

\[
\begin{align*}
n_1 &= \cos \theta \\
n_2 &= \sin \theta
\end{align*}
\]

one obtains

\[
\begin{align*}
n_1^2 &= -n_2 \\
n_2^2 &= n_1
\end{align*}
\]

The differentiation of equation (8.2) yields

\[
Q[A_{11}n_1n_2-(A_{16}+A_{26})(n_1^2-n_2^2)] \\
+[-2(A_{11}A_{66}-A_{12}^2)n_1^3n_2+(A_{11}A_{26}-A_{16}A_{12})(n_1^4-3n_1^2n_2^2)] \\
+(A_{11}A_{22}+2A_{16}A_{26}-A_{12}^2-2A_{12}A_{66})(n_1^3n_2-n_1n_2^3) \\
+(A_{16}A_{22}-A_{26}A_{12})(3n_1^2n_2^2-n_2^4)+2(A_{22}A_{66}-A_{26}^2)n_2^3n_1]=0
\]  

(8.4)

The simultaneous solution of equations (8.3) and (8.4) gives the minimum or maximum values of \( Q \) and the corresponding directions. Obviously the solution has to be obtained by numerical means. To see whether a value of the wave speed found corresponds to a minimum or a
maximum, one could twice differentiate equation (8.3) and then set 
\( Q' = 0 \) and check for the sign of \( Q'' \), or more simply the values of \( Q \) can 
be evaluated from equation (8.3) for slightly different directions to 
determine whether the wave speed corresponds to a minimum or a maximum 
in that direction.

It is noted that if the material is orthotropic, i.e. \( A_{16} = A_{26} = 0 \), 
equations (8.3) and (8.4) are considerably simplified. Equation (8.3) 
gives

\[
Q^2 - Q[A_{11} - A_{22}]n_1^2 + (A_{22} + A_{66}) \]
\[
+ [(A_{11}A_{66}n_1^4) + (A_{11}A_{22} - A_{12}^2 - 2A_{12}A_{66})(n_1^2 - n_1^4) + (A_{22}A_{66})(1 - n_1^2)^2] = 0
\]

and equation (8.4) gives

\[
Q[A_{11}] + [-2(A_{11}A_{66} - A_{12}^2)n_1^2 + (A_{11}A_{22} - A_{12}^2 - 2A_{12}A_{66})(2n_1^2 - 1) \\
+ 2(A_{22}A_{66} - A_{26}^2)(1 - n_1^2)] = 0
\]

and

\[ n_1n_2 = 0 \]  \hspace{1cm} (8.7)

Here, use was made of the fact that \( n_1^2 + n_2^2 = 1 \).

Equation (8.7) states that for orthotropic materials the wave 
propagation speeds in \( X_1 \) and \( X_2 \) directions correspond to minima or 
maxima. The results from equation (8.5) are

\[
v_1 = \sqrt{\frac{A_{11}}{\rho}}
\]
\[
v_2 = \sqrt{\frac{A_{66}}{\rho}}
\]

in the \( X_1 \) direction and
\[ v_1 = \sqrt{\frac{A_{22}}{\rho}} \]
\[ v_2 = \sqrt{\frac{A_{66}}{\rho}} \]

in the \( X_2 \) direction.

The \( v_1 \)'s correspond to longitudinal wave speeds and \( v_2 \)'s correspond to transverse wave speeds in the indicated directions.

For convenience define

\[ f = A_{11} - A_{22} \]
\[ g = A_{22} + A_{66} \]
\[ k = -A_{11} A_{22} + A_{12}^2 + (2A_{12} + A_{11} + A_{22}) A_{66} \]
\[ m = A_{11} A_{22} - A_{12}^2 - 2(A_{12} + A_{22}) A_{66} \]
\[ s = A_{22} A_{66} \]

Then, substituting the above into equations (8.5) and (8.6) gives

\[ Q^2 - Q[fn_1^2 + g] + (kn_1^4 + mn_1^2 + s) = 0 \]  \hspace{1cm} (8.8a)

and

\[ Qf = 2kn_1^2 + m \]  \hspace{1cm} (8.8b)

Substituting equation (8.8b) into equation (8.8a) yields

\[ n_1^6[k(4k-f^2)] + 2n_1^2[k(2m-gf)] + [m^2-mf+gf] = 0 \]  \hspace{1cm} (8.9)

For a given set of material properties the solution to equation (8.9) is easily obtained. This solution gives the directions in which the wave speeds are minimum or maximum. Substituting these directions into equation (8.8b) the magnitude of the wave speeds in the
respective directions are obtained.

In the case of isotropic materials, one has

\[ A_{22} = A_{11} \]
\[ A_{12} = A_{11} - 2A_{66} \]  \hspace{1cm} (8.10)

Substituting the above into equation (8.5) yields

\[ Q^2 - Q(A_{11} + A_{66}) + A_{11}A_{66} = 0 \]  \hspace{1cm} (8.11)

and equation (8.8b) is identically satisfied as expected.

The solution to equation (8.11) gives

\[ v_1 = \sqrt{\frac{A_{11}}{\rho}} \]  \hspace{1cm} (8.12)
\[ v_2 = \sqrt{\frac{A_{66}}{\rho}} \]

As it is seen equation (8.11) does not contain the directional parameters \( n_1 \) and \( n_2 \), which indicates the fact that wave propagation in isotropic materials is independent of direction. Equations (8.12) represent the solution to equation (8.11), in which \( v_1 \) is the longitudinal wave speed and \( v_2 \) is the transverse wave speeds. These equations are the same as the ones discussed at the beginning of this chapter with \( A_{11} = \lambda + 2G \) and \( A_{66} = G \).
CHAPTER IX

RESULTS

The correctness of the singular element stiffness, psuedo-damping, and mass matrices were evaluated by calculating the corresponding forces due to a unit rigid body displacement, velocity, and acceleration of the singular element respectively. This was done by addition of all the elements of the matrices corresponding to horizontal and vertical forces to produce the above unit rigid body motion in the horizontal and vertical directions. From Newton's law of motion the sum of the forces applied to the nodes of the element should be equal to zero in each direction with a zero net moment about the center of mass for rigid body displacements and velocities. Furthermore the sum of the forces applied to the nodes of the element in each direction should be equal to the mass of the element with a zero net moment about the center of mass for a unit rigid body acceleration in the respective direction.

All the above conditions were satisfied with at least six digits of accuracy. The above test was conducted for several combinations of different materials and crack-tip speeds.

9.1 Static problems

As a first test, the problem of a stationary center crack in a homogeneous isotropic plate is solved. Fig. 9.1 shows the finite element mesh for one half of the plate.

Six Gaussian integration points and the first fifteen terms of
the singular element eigenfunctions were used. Computed results of stress intensity factor $K_I$ for various crack length to width ratios $a/b$ are plotted in Fig. 9.2. These results are compared with a solution by Isida [28] using a boundary collocation method. It is seen that the results of the present finite element solution are in good agreement with Isida's results with a maximum difference of less than 2% for small values of $a/b$. The mesh shown in Fig. 9.1 is for $a/b = .1$. For other values of $a/b$ the singular element should be moved to proper positions to incorporate the crack-tip. It was also noted here that the results for the same crack-length but different location inside the singular element differed with a maximum of 1.5% for extreme positions. This is also shown in Fig. 9.2. A state of plane strain was assumed and the material properties $G=2.94 \times 10^7 \text{N/m}^2$, $\nu = .292$ were used.

As a second test, the problem of a center crack in a homogeneous orthotropic plate is solved. It is assumed that the elastic axes for $E_1$ and $E_2$ coincide with $X$ and $Y$ axes, respectively. Fig. 9.3 shows the finite element mesh for one half of the plate. Five Gaussian integration points and the first thirteen terms of the eigenfunctions were employed. The results for the stress intensity factor $K_I$ for various crack-length to width ratios $a/b$ are plotted in Fig. 9.4. These results are compared with a solution by Bowie [29] obtained through a mapping-collocation method and also with a solution by K.Y. Lin and Pin Tong [30] using a finite element method with a 17 noded singular element. It is seen that the present results are in good
Fig. 9.1 Finite Element Mesh for Center Crack in Isotropic Medium
Fig. 9.2 Stress Intensity Factor for Center Crack Tension Plate in Isotropic Medium
Fig. 9.3 Finite Element Mesh for Center Crack in Orthotropic Medium
Fig. 9.4 Stress Intensity Factor for Center Crack in Orthotropic Medium
agreement with both solutions with a maximum difference of less than 2%. A state of plane stress was assumed and the following material properties $E_1 = 1$, $E_2 = 10$, $v_{21} = .21$, $G_{12} = .8757$ were used.

As another example of a static problem, the problem of a centrally cracked plate along the interface of two dissimilar isotropic materials is solved. A state of plane stress is assumed. Fig. 9.5 shows the finite element mesh for one half of the plate. Six Gaussian integration points and eighteen terms of the singular element eigenfunctions were used. Computed results of stress intensity factors $K_1$ and $K_2$ are listed in Table 9.1. These results are compared with classical exact solutions by Sih [4] and also with the K.Y. Lin and J.W. Mar [5] finite element method with a 17 noded singular element. As discussed in Chapter 6, the results shown in Table 9.1 are scaled by a factor of $\sqrt{\pi \cosh(\pi \varepsilon)}$ for comparison with the results of Sih [4] and Lin [5].

<table>
<thead>
<tr>
<th>$\nu_1$</th>
<th>$\nu_2$</th>
<th>$\sigma_{xx2}$</th>
<th>$k_1$</th>
<th>$k_2$</th>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Lin</td>
<td>exact</td>
</tr>
<tr>
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<td>.97</td>
<td>.96</td>
</tr>
<tr>
<td>1000</td>
<td>.3</td>
<td>.30</td>
<td>.96</td>
<td>.95</td>
</tr>
</tbody>
</table>

Table 9.1 Stress intensity factors $k_1$ and $k_2$ for a crack along the interface of two dissimilar isotropic materials. $E_1 = 1$psi, $v_1 = .3$, $\sigma_{yy} = 1$psi, $\sigma_{xx1} = 1$psi, 20"x20" plate, crack-length = 2", plane-stress.
Fig. 9.5 Finite Element Mesh for Center Crack at the Interface of Two Dissimilar Isotropic Materials

\[
\frac{L}{b} = 1
\]

\[
\frac{a}{b} = 0.1
\]

\[
L = 10\text{in.}
\]
From Table 9.1 it is seen that the results for $k_1$ are in good agreement, but the error in the results for $k_2$ are somewhat greater. This is due to the fact that the singular element used by [5] is a 17 noded element, while the singular element used presently is in fact a 9 noded element. A 17 noded singular element was not used here because it would have required a large core storage and been uneconomical for this general problem. However, one should note that the values of $k_2$ are small compared to the values of $k_1$. Therefore, this inaccuracy is not important.

9.2 Stationary cracks under impact

As a first test of the accuracy of the procedure for dynamic problems, the problem of a rectangular plate with a centrally located crack in a homogeneous isotropic plate is solved. The material properties are taken as: $G = 2.94 \times 10^{11}$ dyne/cm$^2$; $\nu = 0.286$; and $\rho = 2.45$ g/cm$^3$. A state of plane strain is assumed. Uniformly distributed uniaxial tensile stresses, with a Heaviside step-function time dependence were assumed to act at the edges of the plate parallel to the crack-axis. The crack is assumed to be stationary under the action of the applied load. Fig. 9.6 shows the finite element mesh for one half of the plate. For this problem a value of $\beta = 1/4$ and a value of time step $\Delta t = 1.0 \times 10^{-6}$ sec. were used. This time step allows the longitudinal waves to travel 63.3% of the smallest dimension in the mesh. Five Gaussian integration points and the first eighteen terms of the singular element eigenfunctions were used. The computed stress intensity factor $K_1$ is plotted in Fig. 9.7.
Fig. 9.6 Finite Element Mesh for Impact Loaded Homogeneous Isotropic Plate With Stationary Center Crack and for Impact Loaded Plate With Stationary Center Crack Along the Interface of Two Dissimilar Orthotropic Materials

L = 5cm
b = 13cm
a = 3cm
Fig. 9.7 Stress Intensity Factor for Impact Loaded Homogeneous Isotropic Plate With Stationary Center Crack
The problem of a semi-infinite crack subjected to a sudden impact at the crack surface was solved by B.R. Baker [31], and the problem of a finite size crack in an infinite medium subjected to a sudden impact loading at the crack surface was solved by G.C. Sih and G.T. Embley [32]. Transform methods, such as the Wiener-Hopf and Cagniard methods were used to obtain the solutions.

The solution given by Baker is valid from the time that the longitudinal wave created by the impact reaches the crack-tip to the time that the longitudinal wave created at one crack-tip reaches the other. The solution by Sih is valid up to the time that the longitudinal wave travels the length of the plate and reflects from the loaded edges of the plate and reaches the crack axis. The results of Baker and Sih are also plotted in Fig. 9.7 and it is seen that the present results are in good agreement with both solutions for valid times. These times are shown by vertical arrows in Fig. 9.7. Also shown in fig. 9.7 are the numerical results by T. Nishioka and S.N. Atluri [15], and S. Aoki [33]. It is seen that the present results are closer to those of Nishioka [15], and higher than the results by Aoki [33]. However, the solution by Aoki, et. al., appears to be lower than the theoretical results by Baker [31] and Sih [32]. It is of interest to note that the numerical results plotted in Fig. 9.7 show a non-zero stress intensity factor even before the time that the longitudinal waves arrive from the loaded boundary to the crack axis, as computed from the material wave speeds. This is attributed partly to the fact that in the finite element formulation the crack-tip is
not associated with a node, and when the first longitudinal wave reaches the nodes of the singular element, the formulation will give rise to some value for the stress intensity factor. In addition, Atluri [15] suggests that the use of the consistent mass matrix may contribute to this problem.

Next, the problem of a rectangular plate with a centrally located crack at the interface of two dissimilar orthotropic materials under impact loading is solved. The composite media is assumed to be made of two dissimilar unidirectional fiber reinforced graphite-epoxy composite materials. The fiber directions of the upper and lower media are and parallel to Z and X axes, i.e., \( \theta = 0^\circ \), and \( \theta = 90^\circ \), respectively, see Fig. 9.8. The following material properties of graphite fiber-epoxy composites [27] are used:

\[
\rho = 7.44 \text{ g/cm}^3
\]

\[
E_1 = 1.378 \times 10^{12} \text{ dyne/cm}^2
\]

\[
E_2 = E_3 = 0.14469 \times 10^{12} \text{ dyne/cm}^2
\]

\[
G_{12} = G_{13} = G_{23} = 0.058565 \times 10^{12} \text{ dyne/cm}^2
\]

\[
\nu_{12} = \nu_{13} = \nu_{23} = 0.21
\]

After respective rotations the material properties for the composite media becomes:

\[
a_{11}^{(1)} = 6.9113 \times 10^{-12} \text{ cm}^2/\text{dyne}
\]

\[
a_{12}^{(1)} = -1.4510 \times 10^{-12} \text{ cm}^2/\text{dyne}
\]

\[
a_{22}^{(1)} = 6.9113 \times 10^{-12} \text{ cm}^2/\text{dyne}
\]

\[
a_{66}^{(1)} = 17.075 \times 10^{-12} \text{ cm}^2/\text{dyne}
\]

\[
a_{16}^{(1)} = a_{26}^{(1)} = 0
\]

\[
\rho^{(1)} = 7.44 \text{ g/cm}^3
\]
for the upper medium and

\[
\begin{align*}
    a^{(2)}_{11} &= 0.72569 \times 10^{-12} \text{ cm}^2/\text{dyne} \\
    a^{(2)}_{12} &= -0.15240 \times 10^{-12} \text{ cm}^2/\text{dyne} \\
    a^{(2)}_{22} &= 6.9113 \times 10^{-12} \text{ cm}^2/\text{dyne} \\
    a^{(2)}_{66} &= 17.075 \times 10^{-12} \text{ cm}^2/\text{dyne} \\
    a^{(2)}_{16} &= a^{(2)}_{26} = 0 \\
    \rho^{(2)} &= 7.44 \text{ g/cm}^3
\end{align*}
\]

for the lower medium.

The finite element mesh used for this problem is the same as the one shown in Fig. 9.6, except that the half crack length \( a \) is taken to be 1cm and the singular element is relocated accordingly. Five Gaussian integration points and the first eighteen terms of the singular element approximating eigenfunctions were used. Uniformly distributed uniaxial tensile stresses, with a Heaviside step-function time dependence were assumed to act at the edges of the plate parallel to the crack axis. The crack is assumed to be stationary under the action of the applied loads.

For this problem values of \( \beta = 1/4 \) and \( \Delta t = 1.5 \times 10^{-6} \) sec. were used. This time step allows the fastest wave to propagate a distance of 64.7% of the smallest dimension in the mesh. The results of \( K = \sqrt{K_1^2 + K_2^2} \) are plotted in Fig. 9.9. It is seen that similar behavior to the previous problem are exhibited. For the given material properties the first wave to reach the crack tip is the longitudinal wave in the second material at an angle of \( \theta = 46^\circ \), from \( X_1 \) axis, from
Fig. 9.8 Unidirectional Graphite-Epoxy Composites
Fig. 9.9 Results of Stress Intensity Factors for a Center Crack at the Interface of Two Dissimilar Orthotropic Media Under Impact
the edges parallel to the crack axis (see Chapter 8). The wave reaches the crack-tip approximately 23 µsec after the application of the load. Therefore, the stress intensity factors should be zero for times \( t < 23 \) µsec, but for the reasons explained earlier, non-zero values arise even before the waves reach the crack-tip.

9.3 Propagating crack problems

As a test of the procedure for dynamic crack propagation, the problem of the sudden appearance of a crack in a homogeneous, isotropic plate in a uniform tension field is solved. The finite element mesh used for this problem is shown in Fig. 9.10. Five Gaussian integration points and the first eighteen terms of the singular element eigenfunctions were used. Broberg [34] has investigated the problem of a suddenly appearing crack, which propagates at constant velocity in a uniform tension field. Although Broberg’s solution is for an infinite medium, the results apply to a finite size plate for the times until the waves created at the crack-tip by the sudden appearance of the crack reflect from the boundaries of the plate and reach the crack-tip. Broberg’s solution reduces to \( K_1 / K_S = 0.505 \) for steel, for which \( G = 2.94 \times 10^{11} \) dyne/cm², \( v = 0.292 \), \( \rho = 2.45 \) g/cm³ for a crack speed of \( c(t) = 0.6c_T \). Here \( K_S = \sigma \sqrt{\pi a} \) is the static stress intensity factor for an infinite uniform medium with a half crack length of \( a \) and applied stresses of \( \sigma \).

The finite element solution starts with a zero crack length with the static solution as the initial condition. Therefore the solution
for the stress intensity factor starts with a value of $K_1/K_S = 1.0$ and approaches the Broberg's solution of $K_1/K_S = .505$ as the steady state solution. The results are plotted in Fig. 9.11 and it is seen that the finite element solution approaches Broberg's steady state solution in about thirty time steps. Also shown in Fig. 9.11 are the results of Aoki [33] and Gunther [16]. For this problem, values of $\beta = 1/4$ and $\Delta t = .481125 \times 10^{-6}$ sec. were used. This time step allows the crack to propagate a distance of .05cm or 1/20 of the singular element dimension. Also the time step corresponds to the longitudinal wave travelling a distance of 30.8% of the smallest dimension in mesh.

Next the problem of a centrally located crack at the interface of two dissimilar orthotropic media is solved. It is assumed that the crack is at rest under the action of uniformly applied loads parallel to the crack axis, then suddenly starts to propagate at a constant velocity. The composite media is the same as the one described for the problem of a stationary crack at the interface of two dissimilar orthotropic materials under impact.

The finite element mesh used for this problem is the same as the one shown in Fig. 9.10. Five Gaussian integration points and the first eighteen terms of the singular element eigenfunctions are used.

It is assumed that the initial crack-length is 2cm and the crack suddenly starts to propagate at a constant velocity of $.5(C_P)_{min}$, where $(C_P)_{min}$ is the minimum shear wave speed of the media, i.e.
Fig. 9.10 Finite Element Mesh for Propagating Center Crack in a Homogeneous Isotropic Plate and for Propagating Center Crack Along the Interface of Two Dissimilar Orthotropic Materials
Fig. 9.11 Stress Intensity Factor for a Propagating Center Crack in a Homogeneous Isotropic Medium
\( (C_T)_{\text{min}} = \sqrt{\frac{A_{66}}{\rho}} = .08872 \times 10^6 \text{ cm/sec.} \) This is the speed of the shear wave propagation in \( X_1 \) and \( X_2 \) directions.

The time step is chosen so that the crack will propagate a distance equal to \( 1/30 \) of the length of the singular element, i.e. \( \Delta t = 1.50282 \times 10^{-6} \text{ sec.} \) This time step allows the fastest longitudinal wave, i.e. \( (C_L)_{\text{max}} = \sqrt{\frac{A_{11}}{\rho}} = .43137 \times 10^6 \text{ cm/sec.} \), to travel a distance of 64.8% of the smallest dimension in the mesh. Again a value of \( \beta = 1/4 \) was used. The computed results for \( K = \sqrt{K_1^2 + K_2^2} \) are plotted in Fig. 9.12. It is seen, as in the previous problem, that when the crack starts to propagate the stress intensity factor starts to decrease from the static value and finally assumes a constant value as the steady state solution. For this problem the steady state solution, Fig. 9.12, is \( K_1 = .635K \), where \( K_S = \sigma \sqrt{\pi a} \) is the static stress intensity factor for an infinite uniform medium with a half crack length of \( a \) and applied stresses of \( \sigma \).

It is seen that the rate of convergence to the steady state solution is much faster than in the previous problem. This is at least partly due to the fact that in the previous problem the initial crack length was taken to be zero, which causes the numerical algorithm to produce rather poor results at the start of the problem. In any case it is seen that the results do converge to a steady state solution.

### 9.4 Crack arrest problems

For safe design of structures, it is important to provide mechanisms for crack arrests. Therefore, it is of interest to know
Fig. 9.12 Results of Stress Intensity Factors for a Propagating Center Crack at the Interface of Two Dissimilar Orthotropic Media
the values of stress intensity factors after a propagating crack is suddenly stopped.

The propagating cracks in the previous section were suddenly stopped at a half crack-length of 4cm. The computed results of the stress intensity factor $K_1$ for the first problem are plotted in Fig. 9.13. It is seen that the stress intensity factor $K_1$ rises sharply after the crack has been suddenly stopped. The stress intensity factor then reaches a peak value of 1.46 times the static value and then starts to decrease. For long times however, the stress intensity factor should assume the static value, see [35-36].

The peak value of the stress intensity factor occurs after the longitudinal waves created at the crack-tip by the sudden stopping of the crack reflect from the loaded boundary and reach the crack axis. Also plotted in Fig. 9.13 are the results of C.K. Gunther [16].

The computed results for the second problem of the previous section which involves the suddenly stopping of an interface crack between two dissimilar orthotropic materials are plotted in Fig. 9.14. It is seen that the same behavior as in the previous problem are exhibited. The peak value for the stress intensity factor in this problem is 1.48 times the static value.
Fig. 9.13 Stress Intensity Factor for the Propagating Center Crack in a Homogeneous Isotropic Medium After Arrest
Fig. 9.14 Results of Stress Intensity Factors for the Propagating Center Crack at the Interface of Two Dissimilar Orthotropic Media After Arrest
CHAPTER X

SUMMARY AND CONCLUSION

A complete procedure for the elastodynamic analysis of interface cracks between two dissimilar anisotropic media has been formulated. It was shown that for all cases the stresses near the crack-tip exhibit the familiar oscillatory singularity of the type $r^{-1/2+i\varepsilon}$, where $-1/2$ represents the conventional square root singularity and the imaginary part represents the oscillatory behavior. When the two materials embedding the crack-tip become identical then $\varepsilon = 0$ and there is no oscillatory behavior for the stresses and displacements near the crack-tip.

The above derivations were successfully implemented into a hybrid-displacement finite element formulation. The resulting discretized equations of motion were solved using Newmark-$\beta$ formulas. A Fortran computer code was then developed which is capable of solving a wide range of problems. Cracks can be either stationary or propagating at a prescribed rate and the materials can be isotropic, orthotropic or fully anisotropic. Due to the complexity of the problem analyzed here it was not possible to define the stress intensity factors by removing the stress singularity near the crack-tip by conventional definitions. Therefore a new definition for stress intensity factors is proposed which reduces to all the previous definitions to within a multiplicative factor.

In the finite element mesh, the crack-tip is embedded in a relatively large singular element, and all other elements away from
the crack-tip are regular elements. If the crack-tip is not stationary, it will propagate inside the singular element until it reaches an extreme position, at which time a local remeshing takes place, the position of the singular element is moved forward, so that the crack-tip is relocated inside the singular element and can continue to propagate inside the singular element.

In the process of re-meshing, as the crack-tip propagates, new nodes have to be created in the finite element mesh. This requirement was satisfied by use of a method which was referred to as the double noding technique.

The results obtained on the basis of the developed procedure are very satisfying and further development is encouraged. In the next chapter some recommendations concerning further developments are presented.
CHAPTER XI
RECOMMENDATIONS FOR FURTHER DEVELOPMENT

The procedure described in this dissertation represents the development of a very complex numerical algorithm. Recommendations are made concerning the accuracy of the algorithm, the economics of computer code and increasing the capability of the procedure.

As it was mentioned the results for the same crack-length but different position of the crack-tip inside the singular element were slightly different. In order to modify this discrepancy it is recommended to study the effects of

(1) The number of Gaussian points for the integration procedure.
(2) The number of terms of the approximating eigenfunctions for the singular element.
(3) The use of singular elements with higher number of nodes such as the one used by Lin [5 and 30]. Also see [37].
(4) For dynamic problems the effect of the size of the time step and the use of difference formulas with artificial damping should be studied, see [22, 23, 38].

To reduce the cost of executing the computer code it is recommended to study the following

(1) Every effort has been made to keep the core storage requirement to a minimum. However, since the program is designed to handle dynamic problems involving propagating cracks, the core requirement is considerable. It might be worthwhile to
modify this program and make three separate programs, one for static problems, one for stationary cracks with dynamic loads and one for propagating cracks.

(2) The effect of the number of Gaussian points for the integration procedure and to see how many are needed for sufficient accuracy.

(3) The effect of the number of terms of the singular element eigenfunctions and to see how many terms are required for sufficient accuracy.

(4) The feasibility of a lumped mass matrix instead of a consistent mass matrix for both the regular elements and singular elements should be studied. This would allow the use of an explicit time integration procedure instead of an implicit one, which causes the elimination of the requirement of solving a large number of simultaneous equations. Also, if a lumped mass matrix for the singular element proves to be inaccurate, it is recommended to study the feasibility of a mixture of implicit and explicit time integration procedure. The subregion around the crack-tip could be treated implicitly while the region away from the crack-tip could be treated explicitly, thus resulting in considerable reduction in the number of simultaneous equations.

(5) Atluri, et al. [14] have used a different crack propagation method. Their procedure uses a fixed crack-tip position inside the singular element and crack propagation is
accomplished by moving the singular element which requires deforming the adjacent regular elements. In this procedure, for a given crack-tip speed, the singular element matrices have to be calculated only once. This saving in computing time is partially offset by the fact that the deforming regular element matrices have to be computed at each time step. However for a constant crack-tip speed the savings are expected to be considerable and it is recommended that this method of crack propagation be studied for the problem considered here.

As it was mentioned in the previous chapter the crack-tip velocity and position histories are prescribed a priori in the present analysis. However, in practical problems this information is not available. Thus it is recommended that the equation of motion for the crack-tip for interface cracks be implemented in the present code. This would allow the crack-tip velocity and position to be determined from the equation of motion according to a prescribed fracture criterion, see [39-40].
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APPENDIX A

THE DERIVATION OF THE EIGENVALUES AND EIGENFUNCTIONS FOR THE STRESSES AND DISPLACEMENTS OF A PROPAGATING CRACK AT THE INTERFACE OF TWO DISSIMILAR ANISOTROPIC MATERIALS

As discussed in Chapter 3, the characteristic equation (3.23) has either two distinct pairs of complex roots or one pair of complex root of multiplicity two. Both cases are considered here. For each case we will show that using the conditions of zero tractions on the crack surfaces and the equality of the stresses and displacements ahead of the crack-tip, we shall obtain similar equations. Then we will solve for the eigenvalues and the corresponding eigenvectors and eigenfunctions. The two cases of interest are now considered.

Case (a). Consider the case in which equation (3.23) has two distinct pairs of complex roots \( \mu_{1k}, \bar{\mu}_{1k} \) and \( \mu_{2k}, \bar{\mu}_{2k} \). Then let

\[
\phi_k = \text{Real} \left( \int \Omega_{1k} \, dz_{1k} + \int \Omega_{2k} \, dz_{2k} \right) \tag{A.1}
\]

where

\[
z_{ik} = x + \mu_{ik} y \quad i, k = 1, 2
\]

and \( k \) identifies the material under consideration. Let \( \Omega_{ik} \) have a complex power representation as

\[
\Omega_{1k} = C_{1k} \, z_{1k}^n + D_{1k} \, \bar{z}_{1k}^n \tag{A.2}
\]

\[
\Omega_{2k} = C_{2k} \, z_{2k}^n + D_{2k} \, \bar{z}_{2k}^n
\]

Later it will be shown that there are an infinite number of solutions for the exponent \( n \) and the functions \( \Omega_{ik} \) actually become complex power series.
Introducing equation (A.1) into equation (3.17) gives

\[ \mathcal{G}_k = B_k \mathcal{G}_{\Phi k} = B_k \text{Real} (\mathcal{Q}_{1k} \mathcal{G}_{\Phi k}) = \text{Real} (\Sigma G_k \mathcal{G}_{\Phi k}) \] (A.3)

where

\[ \mathcal{Q}_{1k} = \begin{bmatrix} 1 & 1 \\ \nu_{1k} & \nu_{2k} \\ \mu_{1k} & \mu_{2k} \end{bmatrix}, \quad \mathcal{G}_{\Phi k} = \begin{bmatrix} \phi_{1k,11} \\ \phi_{1k,22} \\ \phi_{1k,12} \end{bmatrix}, \quad \mathcal{Q}_k = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} \] (A.3a)

and

\[ \Sigma G_k = B_k \mathcal{Q}_{\Phi k} \]

with \(B_k\) being the matrix \(B\) given by equation (3.17a) for material \(k\).

Introducing equation (A.3) into the constitutive equation (3.18) yields

\[ \varepsilon_k = \mathcal{A}_k \mathcal{Q}_k = \mathcal{A}_k \text{Real} (\Sigma G_k \mathcal{G}_{\Phi k}) = \text{Real} (\mathcal{E}PS_k \mathcal{G}_{\Phi k}) \] (A.4)

where \(\mathcal{E}PS_k = \mathcal{A}_k \Sigma G_k\)

and

\[ \varepsilon_k = \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \end{bmatrix}_k, \quad \mathcal{A}_k = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_2 & a_4 & a_5 \\ a_3 & a_5 & a_6 \end{bmatrix}_k \] (A.4a)

Using the displacement - strain equations (2.1) one has

\[ u_{1k} = \int (\varepsilon_{11})_k dx, \quad u_{2k} = \int (\varepsilon_{22})_k dy \]

Using equation (A.4) in the above gives

\[ u_{ik} = \text{Real} (\text{DIS}_{ik} \mathcal{G}_{\Phi k}) \quad i = 1,2 \] (A.5)

where

\[ \mathcal{G}_{\Phi k} = \begin{bmatrix} \Omega_{1k} \\ \Omega_{2k} \end{bmatrix} \] (A.5a)
\[ \text{DIS}_{1k} = \text{EPS}_{1k} \]
\[ \text{DIS}_{2k} = \text{EPS}_{2k} \text{O}_{2k} \]

where \( \text{EPS}_{i_k} \) is the \( i \)th row of matrix \( \text{EPS}_{k} \) defined in equation (A.4)

and
\[ \text{O}_{2k} = \begin{bmatrix} 1/\mu_{1k} & 0 \\ 0 & 1/\mu_{2k} \end{bmatrix} \]

Equations (A.5) can be written as
\[ u_k = \text{Real}(\text{DIS}_{k}\text{O}_{uk}) \quad (A.6) \]

with
\[ u_k = \begin{bmatrix} u_{1k} \\ u_{2k} \end{bmatrix} \quad \text{and} \quad \text{DIS}_k = \begin{bmatrix} \text{DIS}_{1k} \\ \text{DIS}_{2k} \end{bmatrix} \quad (A.6a) \]

Now let us form the expressions for \( \sigma_{22} - i\sigma_{12} \) and \( u_1 + iu_2 \) in order to express the conditions of zero tractions on the crack surfaces and the conditions of equality of stresses and displacements ahead of the crack-tip across the interface.

\[ \sigma_{22} - i\sigma_{12} = \text{Real}(\text{SIG}_{2k}\text{O}_{2k}) - i\text{Real}(\text{SIG}_{3k}\text{O}_{2k}) \]

\[ = \frac{\text{SIG}_{2k} - i\text{SIG}_{3k}}{2} \text{O}_{2k} + \frac{\overline{\text{SIG}}_{2k} - i\overline{\text{SIG}}_{3k}}{2} \text{O}_{2k} \]

or
\[ \sigma_{22} - i\sigma_{12} = K_{1k}\sigma_{1k} + K_{2k}\overline{\sigma}_{1k} + K_{3k}\sigma_{2k} + K_{4k}\overline{\sigma}_{2k} \quad (A.7) \]

where \( \text{SIG}_{i_k} \) is the \( i \)th rows of matrix \( \text{SIG}_{k} \) and

\[ K_{1k} = \frac{(\text{SIG}_{21})_k - i(\text{SIG}_{31})_k}{2} \quad (A.7a) \]

\[ K_{2k} = \frac{(\text{SIG}_{22})_k - i(\text{SIG}_{32})_k}{2} \]

\[ K_{3k} = \frac{(\text{SIG}_{23})_k - i(\text{SIG}_{33})_k}{2} \]
\[ K_{4k} = \frac{(\text{SIG}_{22})_k - i(\text{SIG}_{32})_k}{2} \]

where \((\text{SIG}_{ij})_k\) are the elements of \(\text{SIG}_k\).

\[ u_{1k} + iu_{2k} = \text{Real}\left[\text{DIS}_{1k}\bar{\eta}_{uk}\right] + i\text{Real}\left[\text{DIS}_{2k}\bar{\eta}_{uk}\right] \]

\[ = \frac{\text{DIS}_{1k} + i\text{DIS}_{2k}}{2}\bar{\eta}_{uk} + \frac{\text{DIS}_{1k} + i\text{DIS}_{2k}}{2}\bar{\eta}_{uk} \]

or

\[ u_{1k} + iu_{2k} = K_{5k}\bar{\eta}_{1k} + K_{6k}\bar{\eta}_{1k} + K_{7k}\bar{\eta}_{2k} + K_{8k}\bar{\eta}_{2k} \]  \( (A.8) \)

where

\[ K_{5k} = \frac{(\text{DIS}_{11})_k + i(\text{DIS}_{21})_k}{2} \]

\[ K_{6k} = \frac{(\text{DIS}_{11})_k + i(\text{DIS}_{21})_k}{2} \]

\[ K_{7k} = \frac{(\text{DIS}_{12})_k + i(\text{DIS}_{22})_k}{2} \]

\[ K_{8k} = \frac{(\text{DIS}_{12})_k + i(\text{DIS}_{22})_k}{2} \]

with \((\text{DIS}_{ij})_k\) being the elements of \(\text{DIS}_k\).

The condition of zero tractions on the crack surfaces can be expressed as

\[ \sigma_{22} - i\sigma_{12} = 0 \]  \( (A.9) \)

on the negative x-axis.

Let us express \(z_{ik}\) in terms of polar coordinate variables \(r, \theta\) as
\[ z_{jk} = r_{jk}e^{i\theta_{jk}} \quad (A.10) \]

However, on the crack surface
\[ z_{1k} = z_{2k} = re^{2\pi i} \quad (A.10a) \]

with \( \alpha \) defined as \( \alpha_k = \begin{cases} 2i\pi & \text{for } k = 1 \\ -2i\pi & \text{for } k = 2 \end{cases} \quad (A.10b) \)

Then, introducing the above into equation (A.3a) and using equation (A.2) gives
\[ \Omega_{ok} = \begin{bmatrix} \alpha_k & \bar{\alpha}_k \\ C_{1k}n e^{\frac{2\alpha_k}{2}} & D_{1k} \bar{n} e^{-\frac{\alpha_k}{2}} \\ \alpha_k & \bar{\alpha}_k \\ C_{2k}n e^{\frac{2\alpha_k}{2}} & D_{2k} \bar{n} e^{-\frac{\alpha_k}{2}} \end{bmatrix} \begin{bmatrix} n^{-1} \\ \bar{n}^{-1} \end{bmatrix} \quad (A.11) \]

substituting equations (A.11) into equations (A.8) and, using equation (A.7) yields
\[ [K_{1k}C_{1k}e^{\alpha_k} + K_{2k}D_{1k} + K_{3k}C_{2k}e^{\alpha_k} + K_{4k}D_{2k}]r^{-1}ne^{-\frac{\alpha_k}{2}} + [K_{1k}C_{1k} + K_{2k}D_{1k} + K_{3k}C_{2k} + K_{4k}D_{2k}]\bar{n}^{-1}ne^{-\frac{\bar{\alpha}_k}{2}} = 0 \]

This equation has to hold for all values of \( r \). Since \( r^n \) and \( \bar{r}^\frac{n}{2} \) are independent, each of the expressions in the brackets must vanish, which gives
\[ K_{3k}C_{2k}e^{\alpha_k} + K_{4k}D_{2k} + K_{1k}C_{1k}e^{\alpha_k} + K_{2k}D_{1k} = 0 \]
and
\[ K_{4k}C_{2k}e^{\alpha_k} + K_{3k}D_{2k} + K_{2k}C_{1k}e^{\alpha_k} + K_{1k}D_{1k} = 0 \]
Forming the complex conjugate of the second equation and solving for $C_{2k}$ and $\overline{D}_{2k}$ in terms of $C_{1k}$ and $\overline{D}_{1k}$ gives

\[
\begin{bmatrix}
C_{2k} \\
\overline{D}_{2k}
\end{bmatrix} = \begin{bmatrix}
\overline{KK}_{1k} & \overline{KK}_{2k} e^{-\alpha_k} \\
\overline{KK}_{2k} e^{\alpha_k} & \overline{KK}_{1k}
\end{bmatrix} \begin{bmatrix}
C_{1k} \\
\overline{D}_{1k}
\end{bmatrix}
\]  
(A.12)

where

\[
\overline{KK}_{1k} = (K_{1k} \overline{K}_{3k} - K_{4k} \overline{K}_{2k})/R_0
\]

\[
\overline{KK}_{2k} = (K_{2k} \overline{K}_{3k} - K_{4k} \overline{K}_{1k})/R_0
\]

with

\[
R_0 = K_{4k} \overline{K}_{4k} - K_{3k} \overline{K}_{3k}
\]

The conditions of equality of tractions and displacements across the interface ahead of the crack-tip requires

\[
(\sigma_{22} - i\sigma_{12})_1 = (\sigma_{22} - i\sigma_{12})_2
\]  
(A.13)

and

\[
(u_1 + iu_2)_1 = (u_1 + iu_2)_2
\]

on the positive x-axis, where, equation (A.10) becomes

\[
z_{1k} = z_{2k} = r
\]  
(A.14)

Introducing equation (A.14) into equations (A.3a) and (A.5a), and using equations (A.12) and substituting the results into equations (A.13) gives

\[
(\sigma_{22} - i\sigma_{12})_k = \{(H_{1k} + H_{2k} e^{\alpha_k})C_{1k} + (H_{3k} + H_{4k} e^{-\alpha_k})\overline{D}_{1k}\}(-n)\overline{r}^{n-1}
\]

\[
+ \{(H_{3k} + H_{4k} e^{-\alpha_k})\overline{C}_{1k} + (H_{1k} + H_{2k} e^{\alpha_k})\overline{D}_{1k}\}(-\overline{n})\overline{r}^{n-1}
\]  
(A.15a)
\[ (u_1 + iu_2)_k = \{(H_{5k} + H_{6k}e^{-\alpha_1 k})C_{1k} + (H_{7k} + H_{8k}e^{-\alpha_1 k})D_{1k}\}(-n)^{\gamma-1} \\
+ \{(H_{7k} + H_{6k}e^{-\alpha_1 k})C_{1k} + (H_{5k} + H_{6k}e^{-\alpha_1 k})D_{1k}\}(-\bar{n})^{\gamma-1} \]  

(A.15b)

where

\[ H_{1k} = K_{1k} + K_{3k}K_{1k} = -K_{4k} \bar{K}_{2k} \]

\[ H_{2k} = K_{4k} \bar{K}_{2k} \]

\[ H_{3k} = K_{2k} + K_{4k} \bar{K}_{1k} = -K_{3k} \bar{K}_{2k} \]

\[ H_{4k} = K_{3k} \bar{K}_{2k} \]

\[ H_{5k} = K_{5k} + K_{7k} \bar{K}_{1k} \]

\[ H_{6k} = K_{8k} \bar{K}_{2k} \]

\[ H_{7k} = K_{6k} + K_{8k} \bar{K}_{1k} \]

\[ H_{8k} = K_{7k} \bar{K}_{2k} \]

It should be noted that the parameters \( H_{1k} \) are functions of material properties and crack-tip speed and are known quantities.

We will come back to this point in our analysis later, but for now, let us leave this case here, and let us do the same treatment for the case (b) where equation (3.23) has only one distinct pair of complex roots. It will be shown that case (b) will also result in similar set of equations as in (A.15) for case (a).

Case (b). Equation (3.23) has only one distinct pair of complex roots \( \mu_k \) and \( \bar{\mu}_k \). Let

\[ \phi_k = \text{Real}(\bar{Z}_{k} \Omega_{1k} + \int \Omega_{2k} dz_k) \]  

(A.16)
where \( z_k = x_k + u_k y_k \quad \text{k} = 1, 2 \)

Assume that \( \Omega_{1k} \) have power series representation as

\[
\Omega_{1k} = C_{1k} z_k^n + D_{1k} z_k^n
\]

\( \Omega_{2k} = C_{2k} z_k^n + D_{2k} z_k^n \)

Introducing equations (A.17) into equations (3.17) gives

\[
\mathcal{G}_k = B_k \phi_{0k} = B_k \text{Real}(\Omega_{1k} \Omega_{0k}) = \text{Real}(\text{SIG}_k \Omega_{0k}) \quad (A.18)
\]

Where

\[
\mathcal{Q}_{1k} = \begin{bmatrix}
1 & 2 & 1 \\
\mu_k & -2i \mu_k & \mu_k^2 \\
\mu_k & \mu_k^{-1} & \mu_k
\end{bmatrix}
\]

\[
\mathcal{Q}_{0k} = \begin{bmatrix}
\Omega_{2k}^n \\
\Omega_{1k}^n \\
\Omega_{1k}^n
\end{bmatrix}
\]

\[
\text{SIG}_k = B_k \mathcal{Q}_{1k}
\]

and \( \mathcal{G}_k, \phi_{0k} \) and \( B_k \) are as defined in equation (A.3a).

Introducing equation (A.18) into the constitutive equation (3.18) yields

\[
\mathcal{E}_k = \bar{a}_k \mathcal{G}_k = \bar{a}_k \text{Real}(\text{SIG}_k \Omega_{0k}) = \text{Real}(\text{EPS}_k \Omega_{0k}) \quad (A.19)
\]

where

\[
\text{EPS}_k = \bar{a}_k \text{SIG}_k
\]

and \( \mathcal{E}_k \) and \( \mathcal{G}_k \) are as defined in (A.4a).

Substituting equation (A.19) into the displacement-strain relations of (2.1) and (A.5) gives

\[
u_{1k} = \int (\varepsilon_{11})_k dx = \int \text{Real}[\text{EPS}_{1k} \phi_{0k}] dx = \text{Real}[\text{DIS}_{1k} \Omega_{2k} \Omega_{0k}]
\]

or

\[
u_{1k} = \text{Real}[\text{DIS}_{1k} \Omega_{0k}] \quad (A.20a)
\]
and

\[ u_{2k} = f(\varepsilon_{2k}) \int dy = \text{Real}[\mathcal{EPS}_{2k} \Omega_{ok}] = \text{Real}[\mathcal{EPS}_{2k} \Omega_{3k} \Omega_{uk}] \]

or

\[ u_{2k} = \text{Real}[\mathcal{DIS}_{2k} \Omega_{uk}] \quad (A.20b) \]

where

\[ \Omega_{uk} = \begin{bmatrix} \Omega_{2k} \\ \Omega_{1k} \\ \Omega_{3k} \end{bmatrix}, \quad \Omega_{2k} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \]

\[ \Omega_{3k} = \begin{bmatrix} 1/\mu_k & 0 & 0 \\ 0 & 1/\mu_k & 0 \\ 0 & \mu_k & 1/\mu_k \end{bmatrix} \quad (A.20c) \]

\[ \mathcal{DIS}_{1k} = \mathcal{EPS}_{1k} \Omega_{1k} \]
\[ \mathcal{DIS}_{2k} = \mathcal{EPS}_{2k} \Omega_{2k} \]

with \( \mathcal{EPS}_{ik} \) being the \( i \)th row of matrix \( \mathcal{EPS}_k \) defined in (A.4).

Equations (A.20a) and (A.20b) can be written as

\[ u_{2k} \sim_k = \text{Real}(\mathcal{DIS}_{uk}) \quad (A.21) \]

with \( u_{2k} \sim_k \) and \( \mathcal{DIS}_{uk} \) as defined in equation (A.6a).

Now let us form the expressions for \( \sigma_{22} - i\sigma_{12} \) and \( u_1 + iu_2 \)

\[ \sigma_{22} - i\sigma_{12} = \text{Real}[\mathcal{SIG}_{2k} \Omega_{ok}] - i\text{Real}[\mathcal{SIG}_{3k} \Omega_{ok}] \]

\[ = K_{1k} \Omega_{2k} + K_{2k} \Omega_{2k} + K_{3k} \Omega_{1k} + K_{4k} \Omega_{1k} + K_{5k} \Omega_{1k} + K_{6k} \Omega_{1k} + K_{7k} \Omega_{1k} \quad (A.22) \]

where \( K_{1k}, K_{2k}, K_{3k}, K_{4k} \) are defined in the same manner as in equations (A.7a), but with \( (\mathcal{SIG}_{ij})_k \) being the elements of matrix \( \mathcal{SIG}_k \) as defined in equation (A.18a).

Note that the first column and the third column of the matrix \( \Omega_{1k} \) in equation (A.18a) are identical, which causes the matrix \( \mathcal{SIG}_k \)
defined in equation (A.18a); \( \text{EPS}_k \) defined in equation (A.19a) and \( \text{DIS}_{1k} \) and \( \text{DIS}_{2k} \) defined in equation (A.20c), with definitions of \( \bar{Q}_{2k} \) and \( \bar{Q}_{3k} \) in equation (A.20c), have the same property of identical first and third columns. Therefore, in equation (A.22), the coefficients of \( \bar{\Omega}_{2k}^c \) and \( \overline{\Omega}_{2k}^c \) are identical to the coefficients of \( \bar{z}_{k\Omega_{1k}}^c \) and \( z_{k\Omega_{1k}}^c \) respectively.

Now
\[
u_{1k} + i u_{2k} = \text{Real}[\text{DIS}_{1k} Q_{uk}] + i \text{Real}[\text{DIS}_{2k} Q_{uk}] \tag{A.23}
\]

\[= K_{5k} \bar{\Omega}_{2k}^c + K_{6k} \Omega_{2k}^c + K_{7k} \Omega_{1k}^c + K_{8k} \bar{\Omega}_{1k}^c + K_{9k} \bar{\Omega}_{1k}^c + K_{10k} \bar{\Omega}_{1k}^c
\]

with \( K_{5k}, K_{6k}, K_{7k} \) and \( K_{8k} \) as defined in equations (A.8a); but, with the elements of \( (\text{DIS}_{ij})_k \) being the elements of matrix \( \text{DIS}_k \) as defined in equation (A.20c). Again, in equation (A.23) in coefficients of \( \bar{\Omega}_{2k}^c \) and \( \overline{\Omega}_{2k}^c \) are identical to those of \( \bar{z}_{k\Omega_{1k}}^c \) and \( z_{k\Omega_{1k}}^c \) respectively, because of the above reasoning.

The condition of traction free surfaces is expressed by equation (A.9). Let us express \( z_k \) in terms of the polar coordinates in the same manner as in equations (A.10) as

\[z_k = r_k e^{i \theta_k} \tag{A.24}\]

However, on the crack surfaces

\[z_k = r_k e^{2 \pi \alpha_k} \tag{A.25}\]

with \( \alpha_k \) defined as in equation (A.10b).

Introducing equation (A.24a) into equations (A.18a), and using equation (A.17), gives
\[
\Omega_{0k} = \begin{bmatrix}
\Omega_{2k}^o \\
\Omega_{1k}^o \\
\bar{z}_k^o \Omega_{1k}^o
\end{bmatrix} = - \begin{bmatrix}
\frac{\alpha_k}{C_{2k} n_e} & \frac{\bar{\alpha}_k}{D_{2k} n_e} & \frac{\alpha_k}{C_{1k} n_e} \\
\frac{\alpha_k}{C_{1k} n_e} & \frac{\bar{\alpha}_k}{D_{1k} n_e} & \frac{\alpha_k}{C_{1k} n(n-1)e} \\
\frac{\alpha_k}{C_{1k} n(n-1)e} & \frac{\bar{\alpha}_k}{D_{1k} n(n-1)e} & \frac{\alpha_k}{C_{1k} n(n-1)e}
\end{bmatrix} \begin{bmatrix}
r^{n-1} \\
r^{-1}
\end{bmatrix}
(A.26)
\]

Substituting equation (A.26) into equation (A.9) and carrying out the same process and the same reasoning as for case (a) one obtains

\[
\begin{bmatrix}
C_{2k} \\
D_{2k}
\end{bmatrix} = \begin{bmatrix}
\frac{KK_{1k} - (n-1)}{KK_{1k} - (n-1)} & \frac{KK_{2k}}{KK_{1k} - (n-1)} \\
\frac{KK_{2k}}{KK_{1k} - (n-1)} & \frac{KK_{2k}}{KK_{1k} - (n-1)}
\end{bmatrix} \begin{bmatrix}
C_{1k} \\
D_{1k}
\end{bmatrix}
(A.27)
\]

where

\[
KK_{1k} = (K_{3k} K_{1k} - K_{2k} K_{4k})/R_0 \\
KK_{2k} = (K_{4k} K_{1k} - K_{2k} K_{3k})/R_0
\]

with

\[
R_0 = K_{2k} K_{2k} - K_{1k} K_{1k}
\]

where \( K_{1k}, K_{2k}, K_{3k}, K_{4k} \) are as defined in equation (A.22) for case (b).

The conditions of equality of tractions and displacements ahead of the crack-tip are as expressed by equations (A.13). Also expressing \( \bar{z}_k \) along the positive x-axis as \( \bar{z}_k = r \) and using equations (A.27) yields

\[
(\sigma_{22} - i\sigma_{12})_k = \{(H_{1k} + H_{2k} e^{\alpha_k}) C_{1k} + (H_{3k} + H_{4k} e^{-\alpha_k}) D_{1k}\} (-n) r^{n-1}
\]

\[
+ \{(H_{3k} + H_{4k} e^{\alpha_k}) C_{1k} + (H_{1k} + H_{2k} e^{-\alpha_k}) D_{1k}\} (-\bar{n}) \bar{r}^{n-1}
\]

(A.28a)
and

\[ (u_1 + iu_2)_k = (H_{5k} + H_{6k}e^{\alpha_k})C_{1k} + (H_{7k} + H_{8k}e^{-\alpha_k})D_{1k} \{ -n \} \bar{n}^{-1} \]  
\[ + (H_{7k}^{\pi} + H_{8k}^{\pi}e^{-\alpha_k})\bar{C}_{1k} + (H_{5k}^{\pi} + H_{6k}^{\pi}e^{\alpha_k})\bar{D}_{1k} \{ -\bar{n} \} \bar{n}^{-1} \]  

where

\[ H_{1k} = K_{1k}KK_{1k} + K_{3k} = - K_{2k}\bar{KK}_{2k} \]
\[ H_{2k} = K_{2k}\bar{KK}_{2k} \]
\[ H_{3k} = K_{2k}\bar{KK}_{2k} + K_{4k} = - K_{1k}KK_{2k} \]
\[ H_{4k} = K_{1k}KK_{2k} \]
\[ H_{5k} = K_{5k}[KK_{1k} + 1] + K_{7k} \]
\[ H_{6k} = K_{6k}\bar{KK}_{2k} \]
\[ H_{7k} = K_{6k}[\bar{KK}_{1k} + 1] + K_{8k} \]
\[ H_{8k} = K_{5k}\bar{KK}_{2k} \]

It is observed that equations (A.15) of case (a) and (A.28) of case (b) are in the same form. Therefore, no matter whether the characteristic equation of (3.23) has one, or two distinct pairs of complex roots for either of the two materials involved, the restrictions expressed by equations (A.9) and (A.13) result in similar sets of equations.

In the literature for the formulation of anisotropic materials and also the formulation of propagating cracks only case (a) has been considered. Therefore the formulations could not have been reduced to isotropic materials and stationary cracks, since for problems
involving isotropic materials and stationary cracks case (b) has to be considered. Since the nature of the solution for case (b) is different than for case (a) this reduction has not been possible. However, we have shown here that for both cases the formulations reduce to the same form and therefore it is possible to have one single formulation, as presented here, for all the possible cases, i.e. each one of the materials on the sides of the interface can be isotropic, orthorhombic, or anisotropic and the crack can be stationary or propagating.

Now using the earlier argument of independency of $r^n$ and $r^m$, the following sets of equations will result from conditions of zero tractions on the crack surfaces and the continuity of stresses and displacements across the interface ahead of the crack-tip expressed by equations (A.9) and (A.13)

$$
\begin{bmatrix}
H_{11}(1-\bar{e}^\alpha) & H_{31}(1-\bar{e}^\alpha) & -H_{12}(1-\bar{e}^\alpha) & -H_{32}(1-\bar{e}^\alpha) \\
\bar{H}_{31}(1-\bar{e}^\alpha) & H_{11}(1-\bar{e}^\alpha) & -\bar{H}_{32}(1-\bar{e}^\alpha) & -\bar{H}_{12}(1-\bar{e}^\alpha) \\
H_{51}+H_{61}^\alpha & H_{71}+H_{81}^\alpha & -(H_{52}+H_{62}^\alpha) & -(H_{72}+H_{82}^\alpha) \\
\bar{H}_{71}+\bar{H}_{91}^\alpha & \bar{H}_{51}+\bar{H}_{61}^\alpha & -(\bar{H}_{72}+\bar{H}_{82}^\alpha) & -(\bar{H}_{52}+\bar{H}_{62}^\alpha)
\end{bmatrix}
\begin{bmatrix}
C_{11} \\
D_{11} \\
C_{12} \\
D_{12}
\end{bmatrix} = 0 \quad (A.29)
$$

where $\alpha=2\text{in}_\pi$

Non-trivial solutions for the complex coefficients $C_{ik}$ and $D_{ik}$ are possible only when the determinant of the matrix in equation (A.29) vanishes, which gives

$$(1-x)^2 \ (1+2bx+x^2)=0 \quad (A.30)$$
where

\[ x = e^\alpha \]

\[ b = \frac{R_0}{2Q_0} \]

where

\[ R_0 = -A_{11}A_{22} - A_{12}A_{21} + 2\text{Real}(C_5C_7 - C_1C_2) \]

\[ Q_0 = -A_{11}B_2 - A_{12}B_1 + (-C_1C_3 + C_5C_8 + \overline{C_5C_6} - \overline{C_1C_4}) \]

where

\[ A_{1k} = H_{1k}H_{1k} - H_{3k}H_{3k} \quad \text{no sum } k=1,2 \]

\[ A_{2k} = H_{5k}H_{5k} + H_{6k}H_{6k} - H_{7k}H_{7k} - H_{8k}H_{8k} \]

\[ B_1 = H_{51}H_{61} - H_{81}H_{71} \]

\[ B_2 = H_{62}H_{52} - H_{71}H_{82} \]

\[ C_1 = H_{11}H_{11} - H_{12}H_{12} \]

\[ C_2 = H_{72}H_{51} + H_{82}H_{61} - H_{71}H_{52} - H_{81}H_{62} \]

\[ C_3 = H_{72}H_{61} - H_{81}H_{52} \]

\[ C_4 = H_{82}H_{51} - H_{71}H_{62} \]

\[ C_5 = H_{32}H_{31} - H_{11}H_{12} \]

\[ C_6 = H_{52}H_{51} - H_{71}H_{72} \]

\[ C_7 = H_{62}H_{51} + H_{52}H_{61} - H_{71}H_{72} - H_{81}H_{82} \]

\[ C_8 = H_{62}H_{61} - H_{81}H_{82} \]

It is noted that the parameters \( A_{ik}, \) \( i,k=1,2 \) are real so that the parameter \( R_0 \) will be real. It can be shown, after some algebra,
that the parameters $B_1$ and $B_2$ and also the expression
$(-C_1 C_3 + C_3 C_5 + C_5 C_7 - C_1 C_4)$ are all real so that the parameter $Q_0$ is also
real. Furthermore it can be shown that $b = \frac{R_0}{2Q_0} \geq 1$, with the equality
holding when the two materials became identical.

From equations (A.30) the acceptable eigenvalues to give finite
displacements at the crack-tip are

$$n = 1, 2, 3, \ldots$$

\[ n = \frac{1}{2} \pm i\varepsilon, \frac{3}{2} \pm i\varepsilon, \frac{5}{2} \pm i\varepsilon \ldots \]

where

$$\varepsilon = \frac{1}{2\pi} \log (b + \sqrt{b^2 - 1}) \quad (A.31)$$

For static problems the parameter $\varepsilon$ has been called the "bielastic
constant" [3-4]. For crack propagation problems however, $\varepsilon$ is a
function of the crack-tip speed as well as the material properties,
therefore, here we call it the "bielastic parameter".

It will be shown later that $n = 1/2 + i\varepsilon$ and $n = 1/2 - i\varepsilon$ lead to
the same solutions, so that only $n = 1/2 + i\varepsilon$ needs to be considered.
Thus the eigenvalues are taken to be

$$\eta_\lambda = \frac{1}{2} + i\varepsilon, \frac{3}{2} + i\varepsilon, \frac{5}{2} + i\varepsilon \ldots, \lambda = 1, 3, 5 \ldots$$

$$\eta_\lambda = 1, 2, 3 \ldots \quad \lambda = 2, 4, 6 \ldots \quad (A.32)$$

Now, for each eigenvalue $\eta_\lambda$, we can use equations (A.29) along
with equation (A.27) or equation (A.12), whichever appropriate, to
solve for all the unknown coefficients in terms of only one set of as
yet unknown coefficients, say $\rho_\lambda$'s, as follows. If the eigenvalue $\eta_\lambda$
is complex eliminate one row, e.g. the third row, of equations (A.29)
and from the remaining three equations solve for $D_{11}$, $C_{12}$, $D_{12}$ in terms of $C_{11}$ and then use equations (A.27) or (A.12), whichever is appropriate, to solve for the other unknown coefficients, so that one has

\[
\begin{pmatrix}
C_{11} \\
D_{11} \\
C_{21} \\
D_{21} \\
C_{12} \\
D_{12} \\
C_{22} \\
D_{22}
\end{pmatrix} = \begin{pmatrix}
F_{111} \\
F_{211} \\
F_{311} \\
F_{411} \\
F_{121} \\
F_{221} \\
F_{321} \\
F_{421}
\end{pmatrix} \beta_l \quad \text{or} \quad \begin{pmatrix}
C_{11} & D_{11} \\
C_{21} & D_{21} \\
C_{12} & D_{12} \\
C_{22} & D_{22}
\end{pmatrix} \ell = \begin{pmatrix}
F_{111} & F_{211} \\
F_{311} & F_{411} \\
F_{121} & F_{221} \\
F_{321} & F_{421}
\end{pmatrix} \begin{pmatrix}
\beta_l \\
0
\end{pmatrix}
\]

(A.33)

with $F_{111} = 1$ and where we have renamed the unknown coefficients $C_{111}$ as $\beta_l$. All the parameters $F_{ikk}$'s are the result of calculations and are known quantities.

However, if the eigenvalue $\eta_l$ is real, all the unknown coefficients can not be determined independently and only the following partial sums can be determined

\[
\begin{align*}
E_{11} &= C_{11} + D_{11} \\
E_{21} &= C_{21} + D_{21} \\
E_{12} &= C_{12} + D_{12} \\
E_{22} &= C_{22} + D_{22}
\end{align*}
\]

(A.34)
To solve for $E_{ik}$'s eliminate the first two rows in equation (A.29) and solve for $C_{12}$ and $D_{12}$ in terms of $C_{11}$ and $D_{11}$ from the last two equations and then solve for $E_{12}$ in terms of $E_{11}$ and $E_{11}$. Then use equations (A.27) or (A.12) to find $E_{21}$ and $E_{22}$ in terms of $E_{11}$ and $E_{11}$. Therefore,

$$
\begin{bmatrix}
E_{11} \\
E_{21} \\
E_{12} \\
E_{22}
\end{bmatrix} =
\begin{bmatrix}
F_{11i} & F_{21i} \\
F_{31i} & F_{41i} \\
F_{12i} & F_{22i} \\
F_{32i} & F_{42i}
\end{bmatrix}
\begin{bmatrix}
\beta_1 \\
\beta_2
\end{bmatrix}
$$

(A.35)

with $F_{11i} = 1$, $F_{22i} = 0$ and the unknown coefficient $E_{11}$ is renamed as $\beta_1$. Again, all the parameters $F_{ik}$'s are the result of calculations and are known quantities.

It is proper now to show that $n = 1/2 + i\varepsilon$ and $n = 1/2 - i\varepsilon$ actually lead to the same solution, so that one needs to consider only one of them, namely, $n = 1/2 + i\varepsilon$.

For $n = 1/2 + i\varepsilon$ the result is as in equations (A.33). A careful study of equation (A.29) and (A.27) and (A.12) reveals that $n_m = 1/2 - i\varepsilon$ will have the result

$$
\begin{bmatrix}
C_{11} & D_{11} \\
C_{21} & D_{21} \\
C_{12} & D_{12} \\
C_{22} & D_{22}
\end{bmatrix}_m =
\begin{bmatrix}
F_{21m} & F_{11m} \\
F_{41m} & F_{31m} \\
F_{22m} & F_{12m} \\
F_{42m} & F_{32m}
\end{bmatrix}_m
\begin{bmatrix}
\beta_m \\
0
\end{bmatrix}
$$

Substituting a linear combination of the above and equations (A.33)
into equations (A.2) and substituting only the equations (A.33) into equations (A.2) one obtains similar expressions with only the unknown complex coefficient $\beta_\ell$'s looking different. Therefore the two eigenvalues of $n=1/2 + i\varepsilon$ will lead to the same solution and only one of the eigenvalues needs to be considered.

Consider now the limiting case when one of the materials become rigid. It should be noted that for a rigid material all of the displacements and strains are identically zero by definition. Therefore, in the finite element formulation a rigid material does not contribute to the functional equation (Chapter 5, equation 5.1).

Assuming that material $k$ is the rigid material, in which case $H_{5k}, H_{6k}, H_{7k}$ and $H_{8k}$ in equation (A.29) vanish. Then, it is seen that for the eigenvalues $\lambda = 1, 2, 3, \ldots$ The complex coefficients corresponding to the non-rigid material vanish, while the complex coefficients corresponding to the rigid material remain to be determined. In other words, for these eigenvalues the non-rigid material does not contribute to the functional equation, and since the rigid material is discussed does not contribute to the functional equation either, these terms should not be accounted for.

However, for the eigenvalues $\lambda = \ell/2 + i\varepsilon$, $\ell = 1, 3, 5, \ldots$, the complex coefficients $C_{i\ell k}$'s and $D_{i\ell k}$'s do not vanish for either of the two materials. Although the rigid material does not contribute to the functional equation, these terms have to be accounted for since the terms corresponding to the non-rigid material do contribute to the functional equation.
Now let us derive the expressions for the state of stresses and displacements. Substituting equations (A.33) and (A.35) into equations (A.3) and (A.6) or (A.18) and (A.21) one obtains

$$\sigma_{pk\ell} = \text{Real}(S_{pk\ell} \epsilon_{k\ell}) = \begin{bmatrix} \text{Real}(S_{pk\ell}) & -\text{Im}(S_{pk\ell}) \end{bmatrix} \begin{bmatrix} \epsilon_{1\ell} \\ \epsilon_{2\ell} \end{bmatrix}$$ (A.36)

$$u_{mk\ell} = \text{Real}(D_{mk\ell} \epsilon_{k\ell}) = \begin{bmatrix} \text{Real}(D_{mk\ell}) & -\text{Im}(D_{mk\ell}) \end{bmatrix} \begin{bmatrix} \epsilon_{1\ell} \\ \epsilon_{2\ell} \end{bmatrix}$$

where \((\epsilon_{1\ell})\) and \((\epsilon_{2\ell})\) are two real unknown coefficients to be determined from the prescribed boundary conditions, and

$$S_{pk\ell} = n_{k}[(s_{p1k}F_{1k\ell}z_{k\ell}^{n_{k}-1} + \overline{s}_{p1k}F_{1k\ell}z_{k\ell}^{n_{k}-1} + s_{p2k}F_{2k\ell}z_{k\ell}^{n_{k}-1} + \overline{s}_{p2k}F_{2k\ell}z_{k\ell}^{n_{k}-1})]$$ (A.36a)

$$D_{mk\ell} = d_{m1k}F_{1k\ell}z_{k\ell}^{n_{k}} + \overline{d}_{m1k}F_{1k\ell}z_{k\ell}^{n_{k}} + d_{m2k}F_{2k\ell}z_{k\ell}^{n_{k}} + \overline{d}_{m2k}F_{2k\ell}z_{k\ell}^{n_{k}}$$

for case (a) and

$$S_{pk\ell} = n_{k}[(s_{p1k}F_{1k\ell}z_{k\ell}^{n_{k}-1} + \overline{s}_{p1k}F_{1k\ell}z_{k\ell}^{n_{k}-1} + (s_{p2k}F_{2k\ell}z_{k\ell}^{n_{k}-1} + \overline{s}_{p2k}F_{2k\ell}z_{k\ell}^{n_{k}-1})]$$ (A.36b)

$$D_{mk\ell} = [d_{m1k}F_{1k\ell}z_{k\ell}^{n_{k}} + \overline{d}_{m1k}F_{1k\ell}z_{k\ell}^{n_{k}} + (d_{m2k}F_{2k\ell}z_{k\ell}^{n_{k}-1} + \overline{d}_{m2k}F_{2k\ell}z_{k\ell}^{n_{k}-1})]$$

for case (b), with \(p=1,2,3\) for \(\sigma_{pk\ell}\) representing

$$\begin{bmatrix} \sigma_{1k\ell} \\ \sigma_{2k\ell} \\ \sigma_{3k\ell} \end{bmatrix} = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix}_{k\ell}$$

for the \(k\)th material and the \(\ell\)th eigenvalue \(n_{k}\) and with \(m=1,2\) for
$u_{mk\ell}$ representing $[u_{1k\ell}]^T=[u_{1\ell}^T]$ with $k$ and $\ell$ as above. Also $s_{pjk}$ and $d_{mjk}$ with $j=1,2$ represent the elements of matrices $\tilde{\Sigma}_k$ and $\tilde{D}_k$ in equations (A.3), (A.6), (A.13) and (A.21).

In the finite element formulation of the problem the first and second time derivatives of the stresses and displacements are needed, but their derivations from equations (A.36) and (3.4) are straightforward and will not be produced here.

This concludes the derivation of the eigenvalues and the corresponding eigenfunctions for the near-field stresses and displacements of a propagating crack at the interface of two dissimilar anisotropic materials.
APPENDIX B

COMPUTER PROGRAM DESCRIPTION

The computer program, Finite Element Analysis of Propagating Interface Cracks in Composites (FEAPICC), is written to be compatible with both FORTRAN IV and FORTRAN V compilers. The program is operable on the CDC (Cyber) 180-855. Many subroutines were newly developed, some others were taken from [16] with the necessary modifications implemented. The program is lengthy and complicated. Part of the complication is due to the efforts to make the program as efficient as possible in terms of both memory management and execution time. In the following paragraphs a brief description of the program subroutines is presented.

FEAPICC

This is the main program. It reads, checks and writes the problem title and a number of control parameters. It calculates for \( \Sigma = \Sigma^{-1} \) of equation (3.24) for each material. It calls several subroutines to initiate and to proceed with solution to the problem.

GAUSSPT

This subroutine is called by the main program FEAPICC and sets the values of the necessary parameters for Gaussian quadrature formulas to perform the necessary integrations. Up to 10 points for the Gaussian quadrature can be selected.

DATAIN

This subroutine is called by the main program FEAPICC and reads the bulk of the input data and checks for errors. In case of a
restart problem, this subroutine calls subroutine TAPIN to read the restart values from unit TAPE16.

**SOLVE**

This subroutine is called by the main program FEAPICC. The entire solution process is directed from this subroutine. It contains a loop on timesteps which is executed until a final specified time is reached. During the execution of each loop it calls other subroutines which assemble the global matrices and solve the system of equations.

**MATPROD**

This subroutine is called by subroutine SOLVE and performs matrix multiplication \( \mathbf{Ab} = \mathbf{C} \) for a given banded symmetric matrix \( \mathbf{A} \), which is stored in upper triangular form and a given column matrix \( \mathbf{b} \).

**FORMK**

This subroutine is called by subroutine SOLVE and stores the element mass and stiffness matrices in upper triangular form into the global matrices. Storage is performed in blocks with dimensions of one bandwidths by one bandwidths. The core requirement for program execution is thus kept to a minimum. The blocks are written onto disk files after the completion of each block storage operation. Subroutines STIFEL and MASSEL are called to obtain the regular element stiffness and mass matrices, respectively. The singular element stiffness, damping and mass matrices are provided by subroutine SINGEL.

**ESOLVE**

This subroutine is called by subroutine SOLVE at each time step.
It accounts for the displacement boundary conditions and solves the system of equations

$$Kg = Q$$

for $g$ (the nodal displacements). Matrix $K$ is banded symmetric and stored in upper triangular form. The solution process is by Gaussian elimination which is carried out block by block compatible with storage procedure in FORMK. This subroutine calculates the nodal velocities and accelerations using the difference formulas (7.1). It calculates the displacements, velocities and accelerations of the internal nodes of the singular element using equations (3.34) and (5.10) and their derivatives. The stress intensity factors are also calculated in this subroutine.

**MASSEL**

It is called by subroutine FORMK and calculates the regular element mass matrices.

**FUNCTR**

It is called by subroutine MASSEL and calculates values of shape functions for regular isoparametric elements.

**STIFEL**

It is called by subroutine FORMK and calculates regular element stiffness matrices.

**DIFFB**

It is called by subroutines STIFEL and DATOUT and calculates values of derivatives of shape functions for regular isoparametric elements.
POSIT

It is called by subroutines DATAIN and SOLVE. It determines the crack-tip position from a given crack-tip position history. It also redefines the finite element mesh when necessary.

VELOC

It is called by subroutines DATAIN and SOLVE. It determines the crack-tip velocity from a given crack-tip velocity history. It also determines the time step for the next cycle in conjunction with subroutine LOAD.

LOAD

It is called by subroutines DATAIN and SOLVE. It stores the external element loads into the global external force vector from the given load history. It also determines the time step for the next cycle in conjunction with subroutine VELOC.

DATOUT

This subroutine is called by subroutine SOLVE. It calculates the stresses at the center of specified elements and writes the nodal displacements, velocities, accelerations, and center-element stresses on the output at specified times.

TAPOUT

It is called by subroutine SOLVE. It writes nodal displacements, velocities, accelerations and all the other necessary matrices and parameters to unit TAPE16, for a restart.

TAPIN

It is called by subroutine DATAIN. It reads nodal displacements,
velocities, accelerations, and all the other necessary matrices and parameters from unit TAPE16 for a restart.

**SINGEL**

This subroutine is called by subroutine SOLVE. This subroutine is another executive routine calling a number of subroutines which perform the necessary integrations to assemble the singular element mass, damping and stiffness matrices. It also calculates the symmetric and the asymmetric parts of the singular element matrices. This subroutine also calls the subroutine "LINV2F" of the IMSL library of the University of Washington to invert the generally non-symmetric matrix \( \mathbf{P} \) of equation (5.3).

**LINEI**

It is called by subroutine SINGEL. It performs all the necessary integrations along the boundary of the singular element.

**STIFK**

It is called by subroutine SINGEL. It mainly assembles the singular element stiffness matrix. It also calls the subroutine "LINV2F" of the IMSL Library described above (see description for subroutine SINGEL).

**MAREAI**

It is called by subroutines SINGEL. It performs the necessary integrations over the surface of the singular element.

**MASSM**

It is called by subroutine SINGEL. It assembles singular element mass and damping matrices.
INPOL

It is called by subroutine LINEI. It calculates values of interpolating coefficients of boundary displacement functions for the singular element.

TRANS

It is called by subroutines ESOLVE, DATOUT, LINEI and MAREAI. It performs coordinate transformation from $\xi$, $\eta$ into physical coordinates $X,Y$ and calculates values of differentials for the singular element.

NORMAL

It is called by subroutine LINEI. It calculates the components of a unit vector normal to the sides of the singular element.

PAIR

It is called by subroutine ESOLVE. It eliminates the redundant degrees of freedom for the double nodes in the finite element mesh.

FUNCTS

Is called by subroutines ESOLVE, DATOUT, LINEI and MAREAI. It calculates values of singular element approximating functions and their derivatives.

PRECRCK

Is called by subroutine SOLVE. This subroutine is another executive routine calling a number of subroutines to calculate the complex roots of the characteristic equation for each material and to calculate the complex exponent eigen-values for the assumed eigen-functions of the singular element.
ROOTS

Is called by subroutine PRECRCK. It calculates the complex roots of the characteristic equation for a given set of material properties and a crack-tip speed. If the roots cannot be found analytically, this subroutine calls the subroutine "PROOT" of the BMATH library of the Boeing Company to find the distinct complex roots by numerical methods.

MULT1

Is called by subroutine PRECRCK. It calculates the values of parameters $K_{ik}(i=1,8)$ of equations (A.7a) and (A.7b) for material $k$ whose characteristic equation has two pairs of distinct complex roots.

MULT2

Same as MULT1, except that the characteristic equation for the material has one pair of complex roots of multiplicity 2.

EIGEN

Is called by subroutine PRECRCK. It calls a number of other subroutines and calculates the complex exponent eigen-values for the assumed eigen-functions of the singular element.

INV22

Is called by subroutine EIGEN. It calculates the inverse of a 2x2 complex coefficient matrix to solve for the complex parameters $E_{ik}$ of equation (A.35) for real values of the exponent $n$.

INV33

Is called by subroutine EIGEN. It calculates the inverse of a
3x3 complex coefficient matrix to solve for the complex parameters $C_{ik}$ and $D_{ik}$ of equation (A.33) for complex values of the exponent $n$.

**MAT**

Is called by subroutine EIGEN. It calculates the elements of the 4x4 complex coefficient matrix of equation (A.29).

**COEFF**

Is called by subroutine FUNCTS. It calls other subroutines to calculate the complex parameters $S_{nk\varepsilon}$ and $D_{mk\varepsilon}$ of equations (A.36a) and (A.36b).

**CONS1**

Is called by subroutine COEFF. It calculates the values of the complex parameters $F_{ik\varepsilon}$ of equations (A.36a).

**CONS2**

Same as CONS1 for equation (A.36b).

**MULT**

Is called by subroutine COEFF. It calculates the complex parameters $S_{nk\varepsilon}$ and $D_{mk\varepsilon}$ of equations (A.36a) and (A.36b) for the assumed singular element eigen-functions. It also calculates similar complex parameters for the derivatives of the assumed singular element eigen-functions to be used by subroutine FUNCTS.

**EQUATE**

Is called by subroutine PRECRCK. It equates the elements of certain matrices in order to prevent unnecessary calculations in case the singular element is composed of only one material.
APPENDIX C

INPUT INSTRUCTIONS

CARD SET 1  Title Card (8A10)

80 column problem identification - Any BCD information. For a restart
this card should be the same as the original, otherwise the program
will not run.

Note: For a restart CARD SETS 3 through 8 must be eliminated.

CARD SET 2  Control Parameters (8I5, 2E10.0)

NBT       Number of terms for eigen-functions.
          If NBT < 0, the code sets NBT = 15.
          If 0 < NBT < 12, the code sets NBT = 12.
          Must have, NBT < 18.

NINT      Number of Gaussian points for evaluation of integrals of
          singular element.
          If NINT < 0, the code sets NINT = 6.
          If 0 < NINT < 4, the code sets NINT = 4.
          Must have, NINT < 10.

NITER     Number of iterations per cycle for solution convergence.
          If NITER < 0, the code sets NITER = 0.
          If NITER > 2, the code sets NITER = 2.
          Default, NITER = 1.

          This input is ignored for stationary cracks, where no
          iterations are needed.

ICOND     = 0, initial nodal displacements, velocities and
accelerations are set to zero by the code.

= 1, static solution is desired (as initial condition or not).

=2, restart run, read initial nodal displacements, velocities and accelerations and also mesh definition cards from unit TAP16.

NUMPC  Number of pressure boundary condition cards. Must have, $1 < \text{NUMPC} < 100$.

NUMLP  Number of pressure time-history points. Must have, $2 < \text{NUMLP} < 20$.

NUMCV  Number of velocity time-history points. If NUMCV = 0, the code assumes a stationary crack. Otherwise, must have, $2 < \text{NUMCV} < 20$.

NUMPS  Number of position time-history points. Must have, $2 < \text{NUMPS} < 20$.

NOTE: For parameters NUMLP, NUMCV (if not zero), NUMPS the minimum value must be 2 for interpolation purposes.

$\beta$  Newmark-$\beta$, if $\beta < 0$, the code sets $\beta = .25$.

DT3  Time Step.

If DT3 < 0, the code calculates a time step from material properties.

NOTE: If ICOND=2, go to card set 9. Do not input card sets 3 through 8.

CARD SET 3  Control Parameters (1015, E10.0)

NUMMAT  Number of materials in the mesh. Must have, NUMMAT < 6.
NELTYP  Number of different elements in the mesh (two elements with same geometric shapes but two different materials are considered as two different elements, not one).

NUMNP  Number of nodal points (including the double nodes and the two nodes at the center of the singular element).

Must have, NUMLP < 300.

Remember, as long as remeshing takes place, the singular element must be a square composed of four square regular elements and the center mesh point on the right side of the singular element must be double noded.

NUMEL  Number of elements (including the four square elements which form the singular element, i.e. the singular element is counted as four element and not one).

Must have, NUMEL < 250.

IALL  > 0  all elements are squares.

< 0  non-square elements are present.

ISTAT  > 0  only the static solution is desired.

< 0  dynamic solution is also desired.

This input is ignored for ICOND = 0.

NBAND  Bandwidth for the singular element. Must have, NBAND < 96. If NBAND < 0, NBAND is calculated by the code.

NBRED  Bandwidth for regular elements. Must have, NBRED < 96.

Remember, for the elements adjacent to the interface (i.e. the double noded line) the node on one side of the interface is considered as the node for an element on the
other side of the interface.

NBRED must be < NBAND.

If NBRED < 0, NBRED is calculated by the code.

NOTE: NBAND and NBRED are calculated internally and checked against the input values, so it is good practice to input these values to make sure that you and the code understand each other.

IPLANE

> 0, plane strain problem.

< 0, plane stress problem.

ISYMT

> 1, the problem is symmetric about a vertical axis coinciding with the left side of the mesh. In this case the conditions of symmetry is set by the code and the input values of the array ICODE (see nodal input cards) for nodes on the left side of the mesh are ignored. This parameter is intended to help reduce the number of input cards required in case of a symmetry.

< 0, read the values of ICODE for all nodes from input cards and do not impose any conditions of symmetry.

TT

Starting time (i.e. actual starting problem time).

CARD SET 4

Material Property Card(s) (8E10.0)

\( p \)

Material mass density.

\( a_{11}, a_{12}, a_{16}, a_{22}, a_{26}, a_{66} \)

Material elastic constants in \( \varepsilon = \sigma \) for a given \( X', Y' \) coordinate system.

ANG

The angle through which the \( X', Y' \) coordinate system has to rotate to become parallel to the global X-Y coordinate
system of the problem.

NOTE: Card set 4 should be repeated NUMMAT times.

CARD SET 5 Material Property Card(s) (4E10.0)

NOTE: This card set must be eliminated for plane stress problems, i.e. for IPLANE < 0.

$\alpha_{13}, \alpha_{23}$ Material elastic constants in $\varepsilon = \alpha_0$. These are needed for transformation to a plane strain problem.

NOTE: If card set 5 is present, it must be repeated NUMMAT times.

CARD SET 6 Nodal Point Cards (I5, F5.0, 2E10.0, I5)

N Node number.

CODE(N) = 0, the node is not restrained in either X or Y directions.

= 1, the node is restrained in X direction but is free in Y direction.

= 2, the node is restrained in Y direction but is free in X direction.

= 3, the node is restrained in both X and Y directions.

R(N) X-coordinate value.

Z(N) Y-coordinate value.

ND Interval for generation of nodal points, if zero, the interval is set to one by the code.

NOTE: Cards must be in increasing nodal number sequence. When nodal points are skipped in the data file, the program
fills the missing data by linear interpolation. First and last nodal cards must be supplied. Repeat these cards until finished.

**CARD SET 7**  
**Element Cards (815)**

- **M**  
  Element number.

- **IX(J,M)**  
  J=1,4, nodal point numbers i,j,k,l of element M, starting counterclockwise from bottom left corner.

- **IX(5,M)**  
  Element type number.

- **IX(6,M)**  
  Material type (i.e. element material number).

- **ND**  
  Interval for generation of element cards, if zero, the interval is set to one by the code.

**NOTE:** Cards must be in increasing element number sequence. When elements are skipped in the data file, the program fills in the missing data by linear interpolation. First and last element cards must be supplied. Repeat these cards until finished.

**CARD Set 8**  
**Singular Element Card (315)**

- **NCR1, NCR2**  
  Element numbers for top and bottom regular elements forming the left half of the singular element.

- **NELX**  
  Increment of element numbers in X-direction, so that one has

  \[ NCR3 = NCR1 + NELX \]

  \[ NCR4 = NCR2 + NELX \]

  where NCR3 and NCR4 are the top and bottom regular elements forming the right half of the singular element.
CARD SET 9 Pressure Card(s) (2I5, 3E10.0)

INI(K) Nodal point 1
JNJ(K) Nodal point 2
PI(K) Pressure multiplier $p_1$ for node 1.
PJ(K) Pressure multiplier $p_2$ for node 2.
T(K) Arrival time of pressure at the center of the element surface.

NOTE: Card set 9 must be repeated NUMPC times.

CARD SET 10 Pressure Time-History (2E10.0)

P(1,M) Time $t$.
P(2,M) Pressure value $p(t)$.

NOTE: Repeat NUMLP times.

CARD SET 11 Velocity Time-History (2E10.0)

CVH(1,M) Time $t$.
CVH(2,M) Velocity $c(t)$.

NOTE: Repeat NUMCV times.

CARD SET 12 Position Time-History (2E10.0)

POST(1,M) Time $t$.
POST(2,M) Position $f(t)$.

NOTE: Repeat NUMPS times.
CARD Set 13  Output Parameters (5E10.0/16I5/16I5)

T01  First time of printed output.

T02  Last time of printed output.

T03  Time interval of printed output.

S100  Time interval for restart tape output.

TMACH  Machine STOP time (i.e. computer (cpu) stop time).

NDSOUT(I)  I=1,16, nodal numbers for which output (displacements, velocities and accelerations) is desired.

If NDSOUT(I)=0, for all I, then all nodes are printed.

NSTOUT(I)  I=1,16 element numbers for which output (stresses) is desired.

If NSTOUT(I)=0, for all I, then all elements are printed.
APPENDIX D

COMPUTER PROGRAM LISTING

PROGRAM F B A C C ( INPUT, OUTPUT, TAPE11, TAPE12, TAPE14, TAPE15,
  1 TAPE21, TAPE24, TAPE5=INPUT, TAPE8=OUTPUT, TAPE16 )
    COMMON /BK1/NUMMAT, NUMNP, NUMEL, NUMPC, NUMLP, IHED(0), NTAPI, NBG, ICOND
    1, ISTAT, THACG, IHED(0), ISYMT
    COMMON /VRU/VRV, VNCG, CVU(2,20)
    COMMON /POS/NUMPS, POST(2,20)
    COMMON /BS/NSOUT(16), NSTOUT(16), TID1, TID2, TID3, SIOD, NUMDS, NUMST
    COMMON /RNS/ R(300), Z(300), CODE(300), IX(8,300)
    COMMON /BE12/BET1, BET2, BET3, BET4, BET5, BET6, WIND, WND2
    COMMON / Implementation of Fourier series for diffusion
    COMMON /SHRT/ VSHU, VDLS, LASTB, M, NBKED, RSTYP, ISK, RCODE
    COMMON /RED/RED(0), ERED
    COMMON /SUMAN/AI(3,5,6), ALL(3,5,6), ADEL(6), ASIZE(6), CL
    COMMON /INTG/PT(10), W(10), PTZ(10), WGZ(10), PZ
    COMMON /DIN/MAC, NW, N1, N2, N3, KINT, KINT2, KALL, KITER, SMCOD
    COMMON /RIG/RIG1(0), RIG2
    COMMON /TOLER/TOLER1, TOLER2
    COMMON /IFL/IFL(0), IFLX1(0)

C C MAX NO. OF MATERIALS ALLOWED IS 6
C C TO INCREASE THE NUMBER OF MATERIALS INCREASE THE DIMENSION OF THE ARRAYS
C C IN COMMON BLOCKS 'SUMAN,RED,RED' TO APPROPRIATE NUMBER OF DESIRED MATERIALS
C C
C C MAX NO. OF ELEMENT TYPES ALLOWED IS 64
C C TO INCREASE THE NUMBER OF ELEMENT TYPES INCREASE THE DIMENSION OF THE ARRAYS
C C IN COMMON BLOCK 'RIG' TO APPROPRIATE NUMBER OF DESIRED ELEMENT TYPES
C C
C C MAX BANDWIDTH IS 96
C C TO INCREASE THE BANDWIDTH INCREASE THE DIMENSION OF THE ARRAY
C C IN COMMON BLOCK 'STORE' AND ALSO THE DIMENSIONS OF THE ARRAY 'MS'
C C IN SUBROUTINE 'SOLVE' TO APPROPRIATE DESIRED BANDWIDTH, ALSO CHANGE
C C THE VARIABLE 'WIND' BELOW TO THE DESIRED BANDWIDTH
C C
NBD1=96
NAA=56
NQ=20
NE=2
TERROR=1
NTAPI=5
NINT2=2
ISK=5
TOLER1=1.0E-5
TOLER2=1.0E-1
PE1=ACOS(-1.0)
SINCOD=6.
RCODE=6.

C C READ AND WRITE TITLE
C C
READ (NTAPI,116) IHED
WRITE (8,166) IHED

C C READ AND WRITE CONTROL PARAMETERS
C C
READ (NTAPI,126) NBG, KINT, KITER, ICOND, NUMPC, NUMLP, NUMCV, NUMPS
1, BET1, PT2
IP(NBG.GT.18) GO TO 481
IF (NINT.GT.10) GO TO 482
IF (ICONL.LT.0 OR ICOND.GT.2) GO TO 483
IF (NUMPC.GT.1000) GO TO 484
IF (NUMLF.GT.29) GO TO 485
IF (NUMOV.GT.29) GO TO 486
IF (NUMPS.GT.29) GO TO 487
IF (ICONR.EQ.2) READ (RTAPE,136) NUMMAT,NELTYP,NUMNP,NUMB,IA LL
1,ISTAT,IBAND,IMERD,IFLANE,ISYM,T,T
IF (ICONL.EQ.2) GO TO 2
IF (NUMMAT.GT.50) GO TO 488
IF (NELTYP.GT.50) GO TO 489
IF (NUMNP.GT.2999) GO TO 490
IF (NUMB.GT.256) GO TO 491
IF (NUMBRD.GT.800) GO TO 492
2
IF (NINT.LE.0) NINT=0
IF (NBT.LT.0) NBT=15
IF (NITER.EQ.0) NITER=1
IF (NITER.LT.0) NITER=0
IF (BETA.LE.0.) BETA=.25
IF (ISTAT.LT.0) ISTAT=0
IF (ISTAT.GT.0) ISTAT=1
C THE FOLLOWING THREE STATEMENTS ARE ADDED FOR SAFETY, ACCURACY AND SPEED
C THESE STATEMENTS CAN BE REMOVED OR CHANGED FOR EXPERIMENTAL PURPOSES
C NINT.BE.NE.0 MeANS THE MATRIX 'P', WHICH IS NAMED AS 'B2' IN
C SUBROUTING SINGL, SINGULAR AND TOUS NOT INVERTIBLE
C HOWEVER FOR NINT.LT.4 THE ACCURACY IS LOST
C ALSO FOR NBT.LT.12 THE ACCURACY IS LOST
C NITER.GT.2 DOES NOT IMPROVE THE SOLUTION AND ONLY WASTES COMPUTER TIME
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
IF (NINT.LT.4) NINT=4
IF (NBT.LT.12) NBT=12
IF (NITER.LT.2) NITER=2
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
IF (ICONL.EQ.2) CALL TAPFIN(TT)
NBq=2[NUMNP
IF (ICONL.EQ.2) GO TO 22
IRIG=0
DO 28 I=1,NUMMAT
READ (RTAPE,136) EHD(I),A11,A12,A16,A22,A26,A66,ANG
WRITE (6,310) I
WRITE (6,311) EHD(I),ANG
WRITE (6,312) A11,A12,A16,A22,A26,A66,ANG
IRIG1(I)=8
IF (ABS(A11)+ABS(A22)+ABS(A66)+ABS(A12)+ABS(A16)+ABS(A26) .EQ. 8.)
19!
IRIG1(I)=1
IRIG=IRIG-IRIG1(I)
IF (IRIG1(I).EQ.1) GO TO 19
IF (IFLANE.LE.0) GO TO 18
READ (RTAPE,136) A12,A22,A33,A36,ANG
WRITE (6,313) A12,A22,A33,A36,ANG
B11=A11-A13-A15/A33
B12=A12-A13+A22/A33
B16=A16-A13-A26/A33
B22=A22-A23+A26/A33
B33=A26-A23+A33/A36
B66=A36-A35+A36/A33
A11=B11
A12=B12
A16=B16
A22=B22
A26=B26
A33=B33
A36=B66
19 IF (ANG.EQ.8.) GO TO 12
ANG = ANG + ACOS(-1.) / 180.
C = COS(ANG)
C2 = C * C
C4 = C2 * C2
S = SIN(ANG)
S2 = S * S
S4 = S2 * S2
SC = S + C
1
A16 + C2 = (C2 - 3. + S2) + A26 + S2 + (3. + C2 - S2)
1
A16 + S2 = (3. + C2 - S2) + A26 + C2 + (C2 - S2)
B11 = B11
A12 = B12
A16 = B16
A22 = B22
A26 = B26
A86 = B86

12
A1(1,1,E) = A11
A1(2,1,E) = A12
A1(3,1,E) = A16
A1(1,2,E) = A12
A1(2,2,E) = A22
A1(3,2,E) = A26
A1(1,3,E) = A16
A1(2,3,E) = A26
A1(3,3,E) = A66

ASIZE(K) = (A11 + A22 + A86) / 3.
A11(1,1,E) = A22 + A86 - A36 - A26
A11(2,1,E) = A16 - A26 + A36
A11(3,1,E) = A13 + A26 - A16 - A22
ADET(K) = A11 + A11(1,1,E) + A12 + A11(2,1,E) + A16 + A11(3,1,E)
IF (ADET(K) .EQ. 6.) WRITE (9, 346) K
IF (ADET(K) .EQ. 0.) STOP 6
A11(1,1,E) = A11(1,1,E)/ADET(K)
A11(2,1,E) = A11(2,1,E)/ADET(K)
A11(3,1,E) = A11(3,1,E)/ADET(K)
A11(2,2,E) = (A11 + A66 - A16 + A16)/ADET(K)
A11(3,2,E) = (A16 + A12 - A11 - A36)/ADET(K)
A11(3,3,E) = (A11 + A22 + A22 + A12)/ADET(K)
A11(1,2,E) = A11(2,1,E)
A11(1,3,E) = A11(3,1,E)
A11(2,3,E) = A11(3,2,E)
IF (ISTAT .EQ. 1) GO TO 26
A13 = A11(1,1,E)
A13 = A11(1,1,E)
A13 = A11(2,2,E)
A13 = A11(3,3,E)
AR8 = A11(3,3,E)
BR1 = BR1
IF (BR1 .LT. BR2) BR1 = BR2
BR1 = SQRT(BR1) END(K)
IF (CL.LT.ER1) CL = ER1
GO TO 26

18
A1(1,1,E) = 6.
A1(2,1,E) = 6.
A1(3,1,E) = 6.
A1(1,2,E) = 6.
A1(2,2,E) = 6.
A1(3,2,1)=S.
A1(1,3,1)=S.
A1(2,3,1)=S.
A1(3,1,1)=S.  
A1(3,2,2)=S.  
A1(2,3,2)=S.  
ASIZE(3)=S.  
ADEX1=3.  
SA=S.  
A1I(1,1,1)=SA  
A1I(2,1,1)=SA  
A1I(2,2,1)=SA  
A1I(3,1,1)=SA  
A1I(3,2,1)=SA  
A1I(1,2,1)=SA  
A1I(2,3,1)=SA  
A1I(3,1,2)=SA
A1I(3,2,2)=SA
A1I(2,3,2)=SA
28 CONTINUE
IF (IRIG.EQ.NUMMAT) GO TO 500
INPLN(1)=4SHPLA
INPLN(2)=4EN ST
INPLN(3)=4ERBSS
INPLN(4)=4H PRO
INPLN(5)=4EBLEM
IF (IFPLANE.GT.0) INPLN(9)=4ERAIN
22 WRITE(6,F93) INPLN
IF (ICOND.EQ.2) GO TO 24
WRITE(6,F325) ISISNT
24 WRITE(6,F168) NINT, NBT, NITER
IF (ICOND.EQ.2) GO TO 25
INPLN(1)=4H NO
IF (TALL.LT.0) TALL=S
IF (TALL.GT.0) TALL=1
IF (TALL.GT.0) INPLN(1)=4H YES
IF (ICOND.NE.1) GO TO 25
INPLN(2)=4H NO
IF (ISTRG.GT.0) INPLN(2)=4H YES
WRITE (6,F208) NUMMAT, NELTP, NUMNP, NUMEL, INPLN(1), NUMPC, NUMLP, NUMCV
1.NUMPS, ICOND, INPLN(2)
GO TO 27
25 ISTAT=0
WRITE (6,F208) NUMMAT, NELTP, NUMNP, NUMEL, INPLN(1), NUMPC, NUMLP, NUMCV
1.NUMPS, ICOND
C
27 CONTINUE
C
SELECT GAUSSIAN POINTS
CALL GAUSSPT(NINT2)
DO 30 I=1,NINT2
PT2(I)=PT(I)
WG2(I)=WG(I)
30 CONTINUE
CALL GAUSSPT(NINT)
C
READ AND WRITE DATA
C
CALL DATAIN(ITER,TT,MBD1)
C
CHECK FOR DATA ERROR
C
IF (ITERE. EQ. 0) GO TO 100
IF (ICOND.EQ.1) GO TO 65
IF (TT.GE.TID2) GO TO 98
IF (ICOND.EQ.0) GO TO 65
DO 62 I=1,30
IF (IHEO(I) .NE. IHEO1(I)) GO TO 63
C
CONTINUE
GO TO 65
C
WRITE(6,880)
GO TO 68
C
SET CONSTANTS
C
WRITE(6,256)
C
CALL SOLVE(TT)
C
STOP 1
CONTINUE
STOP 2
C
WRITE(6,481) NBT
GO TO 561
C
WRITE(6,482) NINT
GO TO 561
C
WRITE(6,483) ICOND
GO TO 561
C
WRITE(6,484) NUMPC
GO TO 561
C
WRITE(6,485) NUMLP
GO TO 561
C
WRITE(6,486) NUMCV
GO TO 561
C
WRITE(6,487) NUMPS
GO TO 561
C
WRITE(6,488) NUMMAT
GO TO 561
C
WRITE(6,489) NETYP
GO TO 561
C
WRITE(6,490) NUMNP
GO TO 561
C
WRITE(6,491) NMBEL
GO TO 561
C
WRITE(6,492) NSAND
GO TO 561
C
WRITE(6,328)
C
STOP 3
C
FORMAT(20A4)
C
FORMAT((E15.2,E15.2))
C
FORMAT((E15.2,E15.2))
C
FORMAT(1HE,20X,3F7.4,2F11.2F)
C
FORMAT(1HE,20X,3F7.4,2F11.2F)
C
FORMAT(1HE,20X,3F7.4,2F11.2F)
C
FORMAT(1HE,20X,3F7.4,2F11.2F)
C
FORMAT("/46X,27HNUMBER OF MATERIALS-------,I4/1HE,30X,27HNUMBE"
18 OF ELEMENT TYPES------,I4/1HE,30X,27HNUMBE"
18 OF ELEMENT TYPES------,I4/1HE,30X,27HNUMBE"
18 OF ELEMENT TYPES------,I4/1HE,30X,27HNUMBE"
18 OF ELEMENT TYPES------,I4/1HE,30X,27HNUMBE"
18 OF ELEMENT TYPES------,I4/1HE,30X,27HNUMBE"
18 OF ELEMENT TYPES------,I4/1HE,30X,27HNUMBE"
18 OF ELEMENT TYPES------,I4/1HE,30X,27HNUMBE"
18 OF ELEMENT TYPES------,I4/1HE,30X,27HNUMBE"
18 OF ELEMENT TYPES------,I4/1HE,30X,27HNUMBE"
18 OF ELEMENT TYPES------,I4/1HE,30X,27HNUMBE"
18 OF ELEMENT TYPES------,I4/1HE,30X,27HNUMBE"
18 OF ELEMENT TYPES------,I4/1HE,30X,27HNUMBE"
18 OF ELEMENT TYPES------,I4/1HE,30X,27HNUMBE"
18 OF ELEMENT TYPES------,I4/1HE,30X,27HNUMBE"
18 OF ELEMENT TYPES------,I4/1HE,30X,27HNUMBE"
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18 OF ELEMENT TYPES------,I4/1HE,30X,27HNUMBE"
18 OF ELEMENT TYPES------,I4/1HE,30X,27HNUMBE"
18 OF ELEMENT TYPES------,I4/1HE,30X,27HNUMBE"
18 OF ELEMENT TYPES------,I4/1HE,30X,27HNUMBE"
18 OF ELEMEN...
2R OF NODAL POINTS----.I4//153,30X,27.I4/NUMBER OF ELEMENTS--------.I4
34/153,30X,27.I4//ALL ELEMENTS ARE SQUARES----.I4,4
/153,30X,27.I4//NUMBER OF PRESSURE CARDS----.I4//153,30X,27.I4//NUMBER OF L
50AD POINTS------.I4//153,30X,27.I4//NUMBER OF VELOCITY CARDS----.I4//153,
353X,27.I4/NUMBER OF POSITION CARDS----.I4//153,30X,27.I4//INITIAL CONDI
70N CODE----.I4//

200 FORMAT (  
1.153,30X,27.I4//NUMBER OF PRESSURE CARDS----.I4//153,30X,27.I4//NUMBER OF L
20AD POINTS------.I4//153,30X,27.I4//NUMBER OF VELOCITY CARDS----.I4//153,
353X,27.I4//NUMBER OF POSITION CARDS----.I4//153,30X,27.I4//INITIAL CONDI
70N CODE----.I4//

301 FORMAT (/56X,6A4)
302 FORMAT (/153,30X,37HY-AXIS SYMMETRIC CODE-----------------.I4//)
318 FORMAT (/56X,11HMATERIAL # .I2//)
311 FORMAT (/46X,25H DENSITY---------------------1PE12.4//)
1 /46X,25H OBTUSITIVE ANGLE------------------1PE12.4//)
312 FORMAT (/41X,35HSTRESS-STRAIN CONSTITUTIVE RELATION,
1/8(46X,21PE12.4//))
313 FORMAT(29X,4BA13=.1PE12.4,7H A23=1PE12.4,7H A33=1PE12.4,
1 7H A23=.1PE12.4//)
336 FORMAT (/46X,42H ALL MATERIALS CAN NOT BE RIGID *****
440 FORMAT(1EI.///,46X,15H ERROR*****
1,46X,46H THE STRAIN-STRESS CONSTITUTIVE LAW FOR MATIERIAL #.I3,///
440X,6XTHE CASE OF A ZERO DETERMINANT. CHANGE THESE MATERIAL CONSTANTS.)
306 FORMAT(1EI.///,46X,15H ERROR*****
1,46X,36H THIS IS THE RESTART OF A WRONG PROBLEM,///
2,46X,36XTHE HEADER CARD FOR THIS PROBLEM WAS: ///,26X,26A4)

C
401 FORMAT(1EI.///,26X,17THE VALUE OF,15,63H FOR 'NBT ' IS NO
10R THAN MAX. ALLOWABLE OF 'NBT = 10'*****
402 FORMAT(1EI.///,26X,17THE VALUE OF,15,63H FOR 'NINT ' IS NO
10R THAN MAX. ALLOWABLE OF 'NINT = 10'*****
403 FORMAT(1EI.///,26X,17THE VALUE OF,15,63H FOR 'ICOND ' IS NO
15D THE ALLOWABLE RANGE OF '0,LE.ICOND.LE.2 BYTE
404 FORMAT(1EI.///,26X,17THE VALUE OF,15,63H FOR 'NUMPC ' IS NO
10R THAN MAX. ALLOWABLE OF 'NUMPC =10'*****
405 FORMAT(1EI.///,26X,17THE VALUE OF,15,63H FOR 'NULPL ' IS NO
10R THAN MAX. ALLOWABLE OF 'NULPL = 10'*****
406 FORMAT(1EI.///,26X,17THE VALUE OF,15,63H FOR 'NUNCV ' IS NO
10R THAN MAX. ALLOWABLE OF 'NUNCV = 20'*****
407 FORMAT(1EI.///,26X,17THE VALUE OF,15,63H FOR 'NUMPS ' IS NO
10R THAN MAX. ALLOWABLE OF 'NUMPS = 20'*****
408 FORMAT(1EI.///,26X,17THE VALUE OF,15,63H FOR 'NUMAT ' IS NO
10R THAN MAX. ALLOWABLE OF 'NUMAT = 20'*****
409 FORMAT(1EI.///,26X,17THE VALUE OF,15,63H FOR 'NELTYP ' IS NO
10R THAN MAX. ALLOWABLE OF 'NELTYP = 50'*****
410 FORMAT(1EI.///,26X,17THE VALUE OF,15,63H FOR 'NUNPP ' IS NO
10R THAN MAX. ALLOWABLE OF 'NUNPP = 50'*****
411 FORMAT(1EI.///,26X,17THE VALUE OF,15,63H FOR 'NUME ' IS NO
10R THAN MAX. ALLOWABLE OF 'NUME = 20'*****
412 FORMAT(1EI.///,26X,17THE VALUE OF,15,63H FOR 'NBAND ' IS NO
10R THAN MAX. ALLOWABLE OF 'NBAND = 90'*****
END
SUBROUTINE GAUSSPT(NINT)
COMMON/INTGZ/PT(16),WG(16),PT2(2),WG2(2),PHI

GO TO (1,2,3,4,5,6,7,8,9,10) NINT

1 PT(1)=9.
   WG(1)=3.
   GO TO 11

2 PT(2)=.5773502892
   PT(1)=PT(2)
   WG(5)=1.
   WG(1)=WG(2)
   GO TO 11

3 PT(3)=.7745966592
   PT(2)=0.
   PT(1)=PT(3)
   WG(5)=.55555555555
   WG(2)=.88888888889
   WG(1)=WG(3)
   GO TO 11

4 PT(4)=.86118583118
   PT(3)=.35997958488
   PT(2)=PT(3)
   PT(1)=PT(4)
   WG(5)=.3478548451
   WG(3)=.6621461549
   WG(2)=WG(3)
   WG(1)=WG(4)
   GO TO 11

5 PT(5)=.9891798459
   PT(4)=.6568698181
   PT(3)=0.
   PT(2)=PT(4)
   PT(1)=PT(5)
   WG(5)=.23602888889
   WG(4)=.4788290789
   WG(2)=.6888888889
   WG(1)=WG(5)
   GO TO 11

6 PT(6)=.9324695142
   PT(5)=.6512938655
   PT(4)=.2368191861
   PT(3)=PT(4)
   PT(2)=PT(5)
   PT(1)=PT(6)
   WG(5)=.1713244924
   WG(4)=.3599795839
   WG(3)=.4788290789
   WG(2)=WG(4)
   WG(1)=WG(6)
   GO TO 11

7 PT(7)=.991079123
   PT(6)=.7415111555
   PT(5)=.4058451514
   PT(4)=0.
   PT(3)=PT(6)
   PT(2)=PT(6)
PT(1) = PT(7)
WG(7) = 1294849862
WG(6) = 2797853915
WG(5) = 3818388666
WG(4) = 4176591927
WG(3) = WG(6)
WG(2) = WG(6)
WG(1) = WG(7)
GO TO 11

8 PT(8) = .0022986585
PT(7) = .7066844774
PT(6) = .5325834999
PT(5) = .183446425
PT(4) = PT(5)
PT(3) = PT(6)
PT(2) = PT(7)
PT(1) = PT(8)
WG(6) = .10123285383
WG(5) = .22238210945
WG(4) = .3137896459
WG(3) = .5626887834
WG(2) = WG(6)
WG(1) = WG(7)
GO TO 11

9 PT(9) = .0061662395
PT(8) = .8366311673
PT(7) = .6138714327
PT(6) = .3242584234
PT(5) = 0
PT(4) = PT(6)
PT(3) = PT(7)
PT(2) = PT(8)
PT(1) = PT(9)
WG(6) = .9013749984
WG(5) = .1868461887
WG(4) = .250106964
WG(3) = .3123476770
WG(2) = .3293335569
WG(1) = WG(6)
WG(0) = WG(7)
WG(2) = WG(8)
WG(1) = WG(9)
GO TO 11

10 PT(10) = .0739965285
PT(9) = .8659638667
PT(8) = .0794595833
PT(7) = .4338553941
PT(6) = .1488748290
PT(5) = PT(6)
PT(4) = PT(7)
PT(3) = PT(8)
PT(2) = PT(9)
PT(1) = PT(10)
WG(10) = .0066713443
WG(9) = .1494518492
WG(8) = .2158988235
WG(7) = .292267193
WG(6) = .2965242247
WG(5) = WG(6)
WG(4) = WG(7)
WG(2) = WG(9)
WG(3) = WG(0)
WG(1) = WG(10)

11 RETURN
END
SUBROUTINE DATAIN(IERROR,TT,NBD1)

COMMON/BX1/NUMAT,NUMP,HUML,NUMPC,NUMP,L, IHED(8), NTAPE, NEQ, ICOND
1, ISTAT, TMACH, IHED(8), ISYST
COMMON/BX3/NDSOUT(16), NOUT(16), TIO1, TIO2, TIOD, SIOD, NUMBS, NUMST
COMMON/BX11/DELT,DT1,DT2,DTA, BETA,BET1,BET2,BET4,BET5,KBAND,NBD2
COMMON/DIM,NA,MAA,MBB,M,N,MN2,IA1L, NITER,SINCOD
COMMON/PAIR1/PAIR,LPAIR(3,40)
COMMON/BX16/ M(865), Z(865), CODE(865), IX(8,255)
COMMON/SPLIT/NSIL, NSIL, LASTB, NP, NRED, NELTF, ISK, RCODE
COMMON/OLDISP/U(665), V(665), A(665)
COMMON/PRESS/INT(100), JNJ(100), PJ(100), PI(100), T(100), F(2,28), PF
COMMON/REHD/REH(6), RODUM
COMMON/SUM/N1(2,3,6), N1(3,3,6), ASIZE(6), CL
COMMON/TP/NC1,NCE2,NCE3,NCE4, NELX, CTFX, CTFY, SIF1, SIF2
COMMON/VEL/CV, VNOV, CV(2,20)
COMMON/FOS/NUMPS, POST(2,20)
COMMON/IG/IXIG(6), IXIG2
COMMON/TOOL/TOLER1, TOLER2
COMMON/WSL/IP(20)
COMMON/MAIN/CORD(10,2)

C IF (ICOND.EQ.2) GO TO 282
C
C READ OF NODAL POINT DATA
C
RMIN=1.E100
I=1
ID=1
29 CONTINUE
READ (NTAPE,640) N, CODE(N), R(N), Z(N), ND
IF (R(N) .LT. RMIN) RMIN=R(N)
C
IF (ND) 60, 70, 60
60 ID=ND
70 IF (I) 80, 140, 80
80 NL=N-I
IF (NL) 140, 130, 90
90 NL=NL/ID
IF (I+NL.ID-1) 520, 100, 520
100 IF (CODE(I).NE.CODF(N)) GO TO 522
110 IF (NL) 120, 130, 110
120 ANL=NL
DR=(R(N)-R(I))/ANL
DZ=(Z(N)-Z(I))/ANL
DL=NL/2.ID
D0 120 J=I,NL, ID
11 I=J+ID
R(J)=R(J)+DR
Z(J)=Z(J)+DZ
130 CODE(I)=CODE(J)
140 IF (NUMAP.NE.550, 150, 140
150 N=1
160 CONTINUE
C
C READ OF ELEMENT DATA
C
NUMATI=6
NELTP1=9
ID=1
I=8
190 READ (NTAPE, 665) M, (IX(J,M), J=1,6), ND
IF (NELTP1.LT.IX(6,M)) NELTP1=IX(6,M)
IF (NUMATI.LT.IX(6,M)) NUMATI=IX(6,M)
IF (ND) 195, 198, 195
195 ID=ND
198 I=I+1
210 IF (M-I) 546, 239, 216
216 IF (IX(5,M)-NE.IX(5,M1)) GO TO 546
IF (IX(5,M)-NE.IX(6,M1)) GO TO 546
IF ((IX(1,M)-ID1)/(M-M1)) .NE. ID GO TO 546
IF ((IX(2,M)-ID2)/(M-M1)) .NE. ID GO TO 546
IF ((IX(3,M)-ID3)/(M-M1)) .NE. ID GO TO 546
IF ((IX(4,M)-ID4)/(M-M1)) .NE. ID GO TO 546
218 IX(1,1)=IX(1,1)+ID
IX(2,1)=IX(2,1)+ID
IX(3,1)=IX(3,1)+ID
IX(4,1)=IX(4,1)+ID
IX(5,1)=IX(5,1)+ID
IX(6,1)=IX(6,1)+ID
I=I+1
220 M1=M
226 ID=IX(1,M1)
ID2=IX(2,M1)
ID3=IX(3,M1)
ID4=IX(4,M1)
IF (NUMEL-M) 556, 226, 215
230 CONTINUE
235 IF (NELTYP.LE.0) NELTYP=NELTP1
IF (NELTYP.NE.NELTP1) GO TO 572
IF (NUMMAT.NE.NUMMAT1) GO TO 571
IF (NELTYP.LT.NUMMAT) GO TO 575
C READ INITIAL SPECIAL ELEMENT NUMBERS
READ (NTAPE, 656) NCR1, NCR2, NELX
NCR8=NCR1+NELX
NCR4=NCR2+NELX
C CHECK FOR THE SINGULAR ELEMENT TO BE INSIDE THE MESH
IF (NELX.EQ.0) GO TO 586
IF (NCR1.LE.0 .OR. NCR1.GT.NUMEL) GO TO 586
IF (NCR2.LE.0 .OR. NCR2.GT.NUMEL) GO TO 586
IF (NCR4.LE.0 .OR. NCR4.GT.NUMEL) GO TO 586
C CHECK TO SEE IF MATERIALS IN SINGULAR ELEMENT IS CORRECT
IF (IX(6,NCR1).NE.IX(6,NCR2)) GO TO 679
IF (IX(6,NCR2).NE.IX(6,NCR4)) GO TO 679
C CTPY=IZ(4,NCR2))
C IF (2)=IX(4,NCR2)+2
IF (4)=IX(1,NCR2)+2
IF (6)=IX(3,NCR2)+2
IF (8)=IX(5,NCR2)+2
IF (10)=IX(3,NCR4)+2
IF (12)=IX(2,NCR3)+2
IF (14)=IX(5,NCR3)+2
IF (16)=IX(4,NCR8)+2
IF (18)=IX(4,NCR1)+2
IF (20)=IX(1,NCR1)+2
IF (1)=IP(2)-1
IF (3)=IP(4)-1
IF (5)=IP(6)-1
IF (7)=IP(8)-1
IF (9)=IP(10)-1
IF (11)=IP(12)-1
IF (13)=IP(14)-1
IF (15)=IP(16)-1
IF (17)=IP(18)-1
IF (19) = IF(20) - 1
MAX = IF(2)
MIN = IF(3)
DO 2323 I = 4, NQ, 2
IF (IF(I).GT.MAX) MAX = IF(I)
IF (IF(I).LT.MIN) MIN = IF(I)
2323 CONTINUE
NBAND1 = MAX - MIN + 2
IF (NBAND.LE.0) NBAND = NBAND1
IF (NBAND.ME.NBAND) GO TO 573
IF (NBAND.GT.NBD1) GO TO 583
NBD2 = 2 * NBAND
NBRED1 = IABS(IF(4) - 2 * IX(2, NCR1)) + 2
NBRED2 = IABS(IF(20) - IF(6)) + 2
IF (NBRED1.LT.NBRED2) NBRED1 = NBRED2
IF (NBRED1.ME.NBRED) GO TO 574
IF (NBRED.GT.NBAND) STOP 13
WHITE(6, 547) NBAND, NBRED
NBSL = (MAX - 1) / NBAND + 1
MVDEL = (NBSL - 1) * NBAND - 2 - MIN
LASTB = (NBQ + NDEL - 1) / NBAND + 1
NF = LASTB * NBAND
DO 2325 I = 2, NQ, 2
KH = IF(I)/2
II = I/2
CORD(II, 1) = K(K)
CORD(II, 2) = K(K)
2325 CONTINUE
IF (IRIG1(IX(6, NCR1)).EQ.1 .AND. IRIG1(IX(6, NCR2)).EQ.1) GO TO 578
IRIG2 = 0
IF (IRIG1(IX(6, NCR1)).EQ.1 .OR. IRIG1(IX(6, NCR2)).EQ.1) IRIG2 = 1
232 CONTINUE
IF (IRIG2.EQ.1) NBT = (NBT + 1)/2
NB = NBT + 2
IF (ISTAT.EQ.1) DT1 = DT2
IF (ISTAT.EQ.1) GO TO 2329
CLENGT = ABS(K(IF(16)/2) - K(IF(2)/2))/2.
DT2 = (CLENGT - 2)/CL
2329 CONTINUE
C
C READ OF PRESSURE R.C. DATA
IF (NUMPC.LT.1) GO TO 582
DO 2408 K = 1, NUMPC
READ (NTAPE, 868) INI(K), JNJ(K), PI(K), PJ(K), T(K)
KFF = 8
DO 2407 J = 1, NUMBL
IF (J.EQ.NCR1) GO TO 2403
IF (J.EQ.NCR2) GO TO 2402
IF (INI(K).EQ.IX(1, J) .AND. JNJ(K).EQ.IX(2, J)) .OR. 
1 (INI(K).EQ.IX(2, J) .AND. JNJ(K).EQ.IX(1, J))) KFF = 1
IF (J.EQ.NCR2) GO TO 2404
2402 IF (INI(K).EQ.IX(2, J) .AND. JNJ(K).EQ.IX(3, J)) .OR.
1 (INI(K).EQ.IX(3, J) .AND. JNJ(K).EQ.IX(2, J))) KFF = 1
IF (J.EQ.NCR4) GO TO 2407
2403 IF (INI(K).EQ.IX(3, J) .AND. JNJ(K).EQ.IX(4, J)) .OR.
1 (INI(K).EQ.IX(4, J) .AND. JNJ(K).EQ.IX(3, J))) KFF = 1
IF (J.EQ.NCR3) GO TO 2407
2404 IF (INI(K).EQ.IX(4, J) .AND. JNJ(K).EQ.IX(1, J)) .OR.
1 (INI(K).EQ.IX(1, J) .AND. JNJ(K).EQ.IX(4, J))) KFF = 1
2407 CONTINUE
IF (KFF.EQ.8) GO TO 564
2408 CONTINUE
C READ OF PRESSURE LOAD HISTORY
IF (NUMLP.LT.2) GO TO 592
M=0
NUMLP1=NUMLP
DO 286 N=1,NUMLP1
M=M+1
READ (NTAPE,670) (P(X,K),K=1,2)
IF (M.EQ.1) GO TO 286
IF (P(1,M).LT.P(1,M-1)) GO TO 596
IF (P(1,M).NE.P(1,M-1).OR. P(2,M).NE.P(2,M-1)) GO TO 285
M=M-1
NUMLP=NUMLP-1
286 CONTINUE
C READ OF CRACK TIP VELOCITY HISTORY
IF (NUMCV.EQ.0) GO TO 2861
IF (NUMCV.LT.2) GO TO 597
M=0
NUMCV1=NUMCV
DO 2855 N=1,NUMCV1
M=M+1
READ (NTAPE,670) (CVEH(C,K),K=1,2)
IF (CVEH(2,M).LT.0) GO TO 596
IF (M.EQ.1) GO TO 2865
IF (CVEH(1,M).LT.CVEH(1,M-1)) GO TO 598
IF (CVEH(1,M).NE.CVEH(1,M-1).OR. CVEH(2,M).NE.CVEH(2,M-1)) GO TO 2855
M=M-1
NUMCV=NUMCV-1
2855 CONTINUE
IF (NUMCV.LT.2) GO TO 597
IF (ISTAT.EQ.1) GO TO 289
VMAX=CVEH(2,1)
DO 286 X=2,NUMCV
IF (CVEH(2,I).GT. VMAX) VMAX=CVEH(2,I)
286 CONTINUE
IF (VMAX.EQ.0.) GO TO 2861
DT4=(CLENG/10.)/VMAX
DT0=DT4
DT7=DT4
IF (DT6.GT.DT2) DT5=DT2
IF (DT7.LT.DT2) DT7=DT2
GO TO 2862
2861 IF (ISTAT.EQ.1) GO TO 289
DT6=DT2
DT7=DT2
2882 IF (DT5.LE.0.) DT1=DT5
IF (DT5.LE.0.) GO TO 289
IF (DT5.LT.TOLER2+DT6) WRITE(6,988) DT8
IF (DT5.GT.2.-DT7) WRITE(6,989) DT8
DT1=DT3
289 CONTINUE
WRITE(6,988) BETA,DT1
C READ OF CRACK TIP POSITION HISTORY
IF (NUMPS.LT.2) GO TO 504
M=0
NUMPS1=NUMPS
DO 246 N=1,NUMPS1
M=M+1
READ (NTAPE,670) (POST(X,K),K=1,2)
IF (M.EQ.1) GO TO 246
IF (POST(1,M).LT.POST(1,M-1)) GO TO 596
IF (POST(2,M).LT.POST(2,M-1)) GO TO 596
IF (POST(1,M).NE.POST(1,M-1).OR.POST(2,M).NE.POST(2,M-1)) GO TO 246
M=M-1
246 CONTINUE
NUNPS=NUNPS-1  
2440 CONTINUE  
   IF (NUNPS.LT.2) GO TO 594  
   IF (ICOND.EQ.2) GO TO 269  
   XL=R((IX(2,NCR1))-R(IX(1,NCR1)))  
   IF (XL.EQ.0.) GO TO 569  
C  
C  CORRECT THE VALUE OF "CODE" FOR NODES ON THE AXIS OF SYMMETRY FOR  
C  SYMMETRIC PROBLEMS  
   IF (ISYMT.LE.0) GO TO 246  
   DO 245 I=1,NUNPS  
   RE=(R(I)-RMIN)/XL  
245 CONTINUE  
246 CONTINUE  
C  
C  FIND CRACK-TIP POSITION  
   CALL POSIT(IY,ILF,1)  
C  CHECK FOR THE CRACK-TIP TO BE INSIDE THE SINGULAR ELEMENT  
   IF(R(IX(4,NCR2)).GT.CTX) GO TO 587  
   IF(R(IX(8,NCR4)).LT.CTX) GO TO 587  
C  CHECK TO SEE IF ALL THE ELEMENTS ARE SQUARE AND EQUAL DIMENSION  
   DO 2481 I=1,NUNPL  
   M1=IX(1,1)  
   M2=IX(2,1)  
   M3=IX(3,1)  
   M4=IX(4,1)  
   R1=R(M1)  
   R2=R(M2)  
   R3=R(M3)  
   R4=R(M4)  
   ZZ1=2*(M1)  
   ZZ2=2*(M2)  
   ZZ3=2*(M3)  
   ZZ4=2*(M4)  
   IF(ABS(ABS((R2-R1)/XL)-1.).GT.TOLER1) GO TO 2462  
   IF(ABS((R3-R2)/XL).GT.TOLER1) GO TO 2462  
   IF(ABS(ABS((R3-R4)/XL)-1.).GT.TOLER1) GO TO 2462  
   IF(ABS((R4-R1)/XL).GT.TOLER1) GO TO 2462  
   IF(ABS((ZZ2-ZZ2)/XL).GT.TOLER1) GO TO 2462  
   IF(ABS(ABS((ZZ2-ZZ2)/XL)-1.).GT.TOLER1) GO TO 2462  
   IF(ABS((ZZ2-ZZ2)/XL).GT.TOLER1) GO TO 2462  
   IF(ABS(ABS((ZZ2-ZZ2)/XL)-1.).GT.TOLER1) GO TO 2462  
2481 CONTINUE  
IALL=1  
GO TO 2483  
2482 CONTINUE  
IALL=0  
2483 CONTINUE  
   IF(IALL.NE.IALL) GO TO 591  
   IF(IALL.EQ.1) GO TO 2485  
C  CHECK FOR THE ELEMENTS OF THE SINGULAR ELEMENT TO BE SQUARES  
   DO 2485 N=1,4  
   IF(N.EQ.1) I=NCR1  
   IF(N.EQ.2) I=NCR2  
   IF(N.EQ.3) I=NCR3  
   IF(N.EQ.4) I=NCR4  
   M1=IX(1,1)  
   M2=IX(2,1)  
   M3=IX(3,1)  
   M4=IX(4,1)  
   R1=R(M1)  
   R2=R(M2)
R3=R(M3)
R4=R(M4)
Z12=Z(M1)
Z22=Z(M2)
Z23=Z(M3)
Z24=Z(M4)
IF(ABS(ABS((R3-R1)/XL))-.1.) CT. TOLE1) GO TO 589
IF(ABS((R3-R2)/XL) .CT. TOLE1) GO TO 589
IF(ABS((R3-R4)/XL) .CT. TOLE1) GO TO 589
IF(ABS((R4-R1)/XL) .CT. TOLE1) GO TO 589
IF(ABS((Z22-Z21)/XL) .CT. TOLE1) GO TO 589
IF(ABS((Z23-Z22)/XL) .CT. TOLE1) GO TO 589
IF(ABS((Z23-Z24)/XL) .CT. TOLE1) GO TO 589
IF(ABS((Z24-Z21)/XL) .CT. TOLE1) GO TO 589
2485 CONTINUE
C CHECK FOR THE SINGULAR ELEMENT TO BE A SQUARE(I.E. CHECK TO SEE IF THE
C ELEMENTS OF THE SINGULAR ELEMENT ARE NUMBERED CORRECTLY)
M1=IX(1,NCR2)
M2=IX(2,NCR4)
M3=IX(3,NCR8)
M4=IX(4,NCR1)
R1=R(M1)
R2=R(M2)
R3=R(M3)
R4=R(M4)
Z12=Z(M1)
Z22=Z(M2)
Z23=Z(M3)
Z24=Z(M4)
XL2=3.*XL
IF(ABS(ABS((R2-R1)/XL))-.1.) CT. TOLE1) GO TO 589
IF(ABS((R3-R2)/XL) .CT. TOLE1) GO TO 589
IF(ABS((R3-R4)/XL) .CT. TOLE1) GO TO 589
IF(ABS((R4-R2)/XL) .CT. TOLE1) GO TO 589
IF(ABS((Z22-Z21)/XL) .CT. TOLE1) GO TO 589
IF(ABS((Z23-Z22)/XL) .CT. TOLE1) GO TO 589
IF(ABS((Z23-Z24)/XL) .CT. TOLE1) GO TO 589
IF(ABS((Z24-Z21)/XL) .CT. TOLE1) GO TO 589
C C CORRECT THE VALUES OF "CODE" FOR NODES OF THE RIGID MATERIAL
C DO 270 I=1,NUMEL
IIX=IX(I)
IF(IXIG(IX).EQ.0) GO TO 270
DO 285 J=1,4
CODE(I,J)=3.
285 CONTINUE
C C WRITE OF NODAL POINT DATA
MPRINT=0
J=0
DO 288 N=1,NUMNP
J=J+1
IF(MPRINT.NE.0) GO TO 284
IF(NUMNP.LT.J+56.AND.J.GT.1) GO TO 288
IF(NUMNP.GT.J+49.AND.J.GT.1) J=J+56
WRITE(6,738)
WRITE(6,738)
MPRINT=MPRINT-1
NN=N+56
IF(NUMNP.LT.NN) NN=NUMNP
IF(J.GT.NUMNP) GO TO 288
288 WRITE(6,748)(I,CODE(I),R(I),Z(I),I=J,NN,56)

288 CONTINUE
C
C WRITE OF ELEMENT DATA
  MPRINT=0
  J=0
  DO 295 N=1,NUMEL
   J=J+1
   IF (MPRINT.NE.0) GO TO 292
   IF (NUMEL.LT.J+50.AND.J.GT.1) GO TO 292
   IF (NUMEL.GT.J+49.AND.J.GT.1) J=J+50
   WRITE (6,769) MPRINT=N
  292 MPRINT=MPRINT-1
  NN=J-50
  IF (NUMEL.LT.NN) NN=NUMEL
  IF (J.GT.NUMEL) GO TO 292
  WRITE (6,768) (I,(II(NJ(I)),I=1,6),I=J,NN,50)
  298 CONTINUE
C
299 CONTINUE
C WRITE SPECIAL ELEMENT NUMBERS
  WRITE(6,765) NCR1,NCR2,NCR3,NCR4
C
C WRITE OF PRESSURE B.C. DATA AND PRESSURE LOAD HISTORY
  IF (NUMPC.EQ.0) GO TO 325
  WRITE (6,778) NN=NUMPC
  IF (NUMLP.GT.NN) NN=NUMLP
  K=1
  300 WRITE (6,880)
  IF (K.GT.NUMPC) GO TO 315
  WRITE (6,780) INI(K),INJ(K),PI(K),PJ(K),T(K)
  315 IF (K.GT.NUMLP) GO TO 325
  WRITE (6,790) F(1,K),F(2,K)
  325 K=K+1
  IF (K.LE.NN) GO TO 300
C
C WRITE OF CRACK TIP VELOCITY HISTORY
  325 IF (NUMCV.EQ.0) GO TO 328
  WRITE (6,775)
  WRITE (6,776) ((CVH(K,W),I=1,2),I=1,NUMCV)
C
C WRITE OF CRACK TIP POSITION HISTORY
  328 WRITE(6,777)
  DO 329 M=1,NUMPS
  WRITE (6,778) (POST(K,W),I=1,2)
  329 CONTINUE
  330 CONTINUE
  IF (ICOND.EQ.2) GO TO 444
C
C CORRECT THE VALUE OF "CODE" FOR THE INTERNAL NODES OF
C THE SINGULAR ELEMENT
  CODE(IX(2,NCR1))=3.
  CODE(IX(8,NCR2))=3.
C
C FIND THE DOUBLE NODES
  MPAIR=0
  NUMNP=NUMNP-1
  DO 255 I=1,NUMNP
   R1=E(I)
   Z1=Z(I)
   II=II+1
  255 DO J=II,NUMNP
   R2=E(J)
   Z2=Z(J)
   SR=ABS(R1-R2)/XI

SZ = ABS((Z1 - Z2) / XL)  
IF (SR.GT.TOLER1 .OR. SZ.GT.TOLER1) GO TO 254  
IF (ABS((Z1-CITY)/XL).GT.TOLER1) GO TO 577  
IF (R1.LT. R(I)(IX(3,NCR4))) GO TO 254  
MPAIR = MPAIR + 1  
NPAIR = MPAIR + MPAIR  
LPAIR(1,NPAIR-1) = I + I - 1  
LPAIR(3,NPAIR-1) = J + J - 1  
LPAIR(5,NPAIR-1) = I + I  
LPAIR(2,NPAIR) = J + J  
LPAIR(3,NPAIR-1) = 1  
LPAIR(5,NPAIR) = 1  
IF (CODE(I).EQ.8) CODE(J) = 3  
IF (CODE(J).EQ.8) CODE(I) = 3  
IF (CODE(I).NE.CODE(J)) GO TO 512  
CODE(J) = 8.  
254 CONTINUE  
255 CONTINUE  
IF (MPAIR.LT.1) GO TO 581  
C  
C WRITE OF DOUBLE NODES  
WRITE(6,786) NPAIR/2  
WRITE(6,787) (LPAIR(1,J)/2,J=2,NPAIR,2)  
WRITE(6,788) (LPAIR(2,J)/2,J=2,NPAIR,2)  
C CHECK TO SEE IF THE NODAL POINTS OF THE ELEMENTS ARE NUMBERED CORRECTLY  
NUMEL1 = NUMEL - 1  
DO 256 I = 1, NUMEL1  
II = I + 1  
DO 257 J = 1, 4  
LJS = IX(I,J)  
E1 = E(IJS)  
Z1 = Z(IJS)  
DO 257 K = II, NUMEL  
DO 256 L = 1, 4  
KL1 = IX(L,K)  
KL2 = IX(L+1,K)  
IF (L.EQ.4) KL2 = IX(1,K)  
E2 = E(KL1)  
Z2 = Z(KL1)  
E3 = E(KL2)  
Z3 = Z(KL2)  
E12 = (E1 - E2) / XL  
Z12 = (Z1 - Z2) / XL  
E13 = (E1 - E3) / XL  
Z13 = (Z1 - Z3) / XL  
ZK = ABS(Z12 - Z12 + E13)  
IF (ZK.GT.TOLER1) GO TO 256  
IF (ABS(E12).LT.TOLER1) GO TO 256  
IF (ABS(E13).LT.TOLER1) GO TO 256  
IF (R1.LT.RS .AND. R1.GT.RS) GO TO 578  
IF (R1.LT.RS .AND. R1.GT.RS) GO TO 578  
256 CONTINUE  
257 CONTINUE  
258 CONTINUE  
259 CONTINUE  
C  
C FIND CRACK-TIP POSITION AND RE-MESH IF NECESSARY  
CALL POSIT(TT,ILP,0)  
IF (ILP.EQ.1) GO TO 589  
C  
C IF (ICOND.EQ.1) GO TO 442  
C  
C INITIAL CONDITIONS FOR ICOND=0  
332 DO 255 I = 1, NSEQ  
U(I) = 0.  

V(I)=0.
A(I)=0.
335 CONTINUE
SIF1=0.
SIF2=0.
442 CONTINUE
CALL LOAD(TT,TTV,2,0)
CALL VELOC(TT,TTV,1,0)
IF(CV.NE.0.) GO TO 582
IF(ICOND.EQ.1) GO TO 444
IF(FF.NE.0.) GO TO 584
444 CONTINUE
C
C READ AND WRITE OF PRINTED OUTPUT PARAMETERS
C
READ(WTAPF,600)TIO1,TIO2,TIOD,SIOD,TMACH,NDSOUT,NSTOUT
IF(SIOD.EQ.0.) SIOD=TIOD
IF(TMACH.EQ.0.) TMACH=90.0.
NUMDS=0
NUMST=0
DO 500 I=1,10
IF(NDSOUT(I).NE.0) NUMDS=NUMDS+1
IF(NSTOUT(I).NE.0) NUMST=NUMST+1
500 CONTINUE
IF(NUMDS.EQ.0) NUMDS=NUMNP
IF(NUMST.EQ.0) NUMST=NUMEL
WRITE (6,825) TT
WRITE (6,830) TIO1,TIO2,TTIOD,SIOD,TMACH
WRITE (6,850) NUMDS,NUMST
WRITE (6,700) IF(ICOND.NE.2) GO TO 581
CALL PRCRCK(CV,NUMMAT,TT,IL)
IF(IL.EQ.1) STOP 14
581 IF(ICOND.NE.1) CALL DATOUT(6,TT)

C
510 RETURN
C
C INPUT DATA ERROR EXITS
C
512 WRITE (6,900) I,J
GO TO 505
520 WRITE (6,870) I,ND,N
GO TO 505
522 WRITE (6,872) I,N
GO TO 505
530 WRITE (6,880) N,NUMNP
GO TO 505
540 WRITE (6,890) I,M
GO TO 505
545 WRITE (6,895) M,MI
GO TO 505
550 WRITE (6,900) M,NUMEL
GO TO 505
564 WRITE (6,912) INI(K),KNI(K)
GO TO 505
571 WRITE (6,922)
GO TO 505
572 WRITE (6,924)
GO TO 505
573 WRITE (6,928)
GO TO 505
574 WRITE (6,928)
GO TO 505
575 WRITE (6,927)
GO TO 695
578 WRITE (6,933) I,K
GO TO 695
577 WRITE (6,932) I,J
GO TO 695
578 WRITE (6,929)  
GO TO 695
579 WRITE (6,935)  
GO TO 695
581 WRITE (6,937)  
GO TO 695
582 WRITE (6,940)  
GO TO 695
583 WRITE (6,945) NBRAND,NBD1
GO TO 596
584 WRITE (6,950)  
GO TO 695
586 WRITE (6,960)  
GO TO 695
587 WRITE (6,965)  
GO TO 695
588 WRITE (6,976)  
GO TO 695
589 WRITE (6,975)  
GO TO 695
591 IF(ILL1.EQ.0) WRITE (6,970) I 
IF(ILL1.EQ.1) WRITE (6,979) 
GO TO 595
592 WRITE (6,981)  
GO TO 695
593 WRITE (6,982)  
GO TO 695
594 WRITE (6,983)  
GO TO 695
595 WRITE (6,984)  
GO TO 695
597 WRITE (6,985)  
GO TO 695
598 WRITE (6,988)  
599 IERROR=0  
GO TO 510
C
600 FORMAT (1H1)
640 FORMAT (I6,F5.0,2E16.8,I5)
650 FORMAT (8I5)
660 FORMAT (2I6,3E16.0)
670 FORMAT (2E16.0)
671 FORMAT (2E16.0,25I2)
680 FORMAT (1I0,6E16.0,6I0)
690 FORMAT (6E16.0,16I6/16I6)
700 FORMAT (1H1)
720 FORMAT (1H1//=44X,23HXNODAL POINT COORDINATES//=6X,2HEM,4X,4ECODE,8X 
1,68R-ORD,11X,68Z-ORD,14X,2HEM,4X,4ECODE,8X,5ER-ORD,11X,5EZ-ORD/)
740 FORMAT (1X,2,4X,F8.1,2E16.4,4X)
750 FORMAT (1H1//=46X,10HELEMENT DEFINITIONS//=6X,7HELEMENT,5X,1HI,6X,1 
1DJ,5X,1DJ,6X,1DJ,5X,1DJ,6X,1DJ,6X,1DJ,5X,1DJ,6X,1DJ,6X,1DJ,5X,1DJ) 
765 FORMAT (/25X,17ELEMELEMENTS NUMBERED,I6,I5,I5,6H AND,I5,24H ARE S 
1PICAL ELEMENTS)
766 FORMAT (1H1//=20X,9THERE ARE,I6,12H DOUBLE NODES)
767 FORMAT (/1X,10SHNODES OF FIRST ROW,20I6)
770 FORMAT (/1X,10SHNODES OF SECOND ROW,20I6)
770 FORMAT (1H1//=11X,28HEPRESSURE BOUNDARY CONDITIONS,28X,28HEPRESSURE H 
1ISTORY DESCRIPTION//=5X,1HI,6X,1HJ,7X,4HPI/P,8X,4HPI/J,P,6X,18HSTART
2 TIME, 23X, 48TIME, 0X, 125PRESSURE P/
775 FORMAT (1BI/*48X, 20BREACK TIP VELOCITY HISTORY, /*37X,
14TIME, 12X, 18BREACK TIP VELOCITY/)
776 FORMAT (1BI/*12X, 1PE15.7, 8X, 1PE15.7)
777 FORMAT (1BI/*48X, 20BREACK TIP POSITION HISTORY, /*20X,
14TIME, 12X, 20BREACK-TIP POSITION-2, 12X, 20BREACK-TIP POSITION-2, /*)
778 FORMAT (1BI/*12X, 1PE15.7, 8X, 1PE15.7, 12X, 1PE15.7)
780 FORMAT (18+, 6X, 12F12.3, 8X, 12F12.4)
789 FORMAT (18+, +6X, 1PE15.7)
825 FORMAT (1BI/*48X, 124CURRENT TIME = 1PE15.7, /*)
830 FORMAT (1BI/*48X, 20BREPRINTED OUTPUT PARAMETERS)
840 FORMAT (/*48X, 22BSTOP OUT PUT AT-------, 1PE15.7, /*48X, 22BSTOP OUT PUT AT-------, 1PE15.7, /*)
1UT AT----------, 1PE15.7, /*48X, 22BSTOP OUT PUT AT----------, 1PE15.7, /*
2, 48X, 22BSTOP OUTPUT AT-----, 1PE15.7, /*
850 FORMAT (/*48X, 8ENDONAL POINTS TO BE PRINTED-------, 16X, /*48X, 8ENDONAL POINTS TO BE PRINTED------, 16X,
18ELEHEMT STRESSES TO BE PRINTED------, 16X)
870 FORMAT (23H INCREMENTING FROM N.P., 14, 28H BY 13, 22H, 8S WILL NOT REAC
1N N.P., 14)
872 FORMAT (23H CODE FOR NODAL POINTS, 15, 6H AND 15,
127 ARE NOT THE SAME, CANNOT INTERPOLATE)
880 FORMAT (6H N.P., 14, 28H IS GREATER THAN NUMP=, 14)
890 FORMAT (25H ELEMENT DEFINITION CARDS, 214, 13H OUT OF ORDER)
895 FORMAT (84H ELEMENT TYPE AND/OR MATERIAL TYPE ON DEFINITION CARDS,
AND I4, 73H ARE NOT THE SAME, CANNOT INTERPOLATE)
900 FORMAT (19H ELEMENT DEFINITION, 14, 29H IS GREATER THAN NUMEL=, 14)
910 FORMAT (25H *PRESSURE CARD FOR NODS, I4, 6H AND I4, 11H IS WRONG)
920 FORMAT (23H INCREMENTING FROM N.P., 14, 28H BY 13, 22H WILL NOT REAC
1N N.P., 14)
922 FORMAT (1BI/*48X, 44H INPUT FOR NUMBER OF MATERIALS IS WRONG***)
924 FORMAT (1BI/*48X, 44H INPUT FOR NUMBER OF ELEMENT TYPES IS WRONG***)
926 FORMAT (1BI/*48X, 64H INPUT FOR BANDWIDTH OF SINGULAR ELEMENTS IS
1 WRONG***)
927 FORMAT (1BI/*48X, 64H INPUT FOR "NUMMAT AND/OR MELTYP AND/OR IX(
15, 1)" ON ELEMENT CARDS IS WRONG***)
928 FORMAT (1BI/*48X, 64H INPUT FOR BANDWIDTH OF REGULAR ELEMENTS IS
1 WRONG***)
929 FORMAT (/*48X, 69H BOTH MATERIALS OF THE SINGULAR ELEMENT CAN
1 NOT BE RIGID***)
930 FORMAT (6H N.P., 14, 28H IS GREATER THAN NUMP=, 14)
931 FORMAT (1BI/*48X, 22H NODAL CARDS WRONG. NODS, I5, I5, 6H NOT
1 ON THE CRACK-LINE HAVE THE IDENTICALCOORDINATES***)
933 FORMAT (1BI/*48X, 22H ELEMENT CARDS ARE NOT THE SAME, CANNOT INTERPOLATE)
935 FORMAT (1BI/*38X, 93H THE SINGULAR ELEMENT IS WRONG****, 11,
16X, 64H THE MATERIAL TYPES IN SINGULAR ELEMENT ARE NOT CORRECT)
937 FORMAT (1BI/*48X, 59H THE SINGULAR ELEMENT IS WRONG******, 1,
16X, 75H THE LEAST, NODES 5 AND 6 OF THE SINGULAR ELEMENT HAVE T
20 BE DOUBLE NODED)
940 FORMAT (1BI/*48X, 74H FOR ICOND=0 OR ICOND=1 INITIAL CRACK-TI
1P VELOCITY HAS TO BE ZERO****)
945 FORMAT (1BI/*48X, 62H THE BAND WIDTH OF THE PROBLEM FOR THE S
1N SINGULAR ELEMENT IS 15, 12X, 28X, 46H WHICH IS MORE THAN THE ALLOWABLE BA
2ND WIDTH OF 15, 18H IN THIS PROGRAM, 16X, 122H YOU NEED TO INCREASE
3 THE SIZE OF MATRIX "SE" IN COMMON BLOCK "STORE" AND ALSO THE SIZ
48H OF MATRIX "SM" IN SUBROUTINE FORM)
947 FORMAT (48X, 137H BAND WIDTH FOR SINGULAR ELEMENT------, I4, +18X, 28X, 37H BAND WIDTH
2 FOR REGULAR ELEMENTS------, I4)
950 FORMAT (1BI/*48X, 49H FOR ICOND=0 INITIAL LOAD HAS TO BE ZERO
1****)
955 FORMAT (1BI/*48X, 39H THE SINGULAR ELEMENT IS WRONG******, 1,
11X, 64H, SENC1= 12, 5X, SENC2= 13, 5X, SENC3= 15,
2, 6X, SENC4= 13, 6X, SENC5= 13)
959 FORMAT (1BI/*38X, 59H THE SINGULAR ELEMENT IS WRONG******, 1,
1,36X,58H------THE CRACK-TIP IS NOT INSIDE THE SINGULAR ELEMENT------

2)

979 FORMAT (IH1,,/36X,91H------THE CRACK-TIP LOCATION HAS PAST THE CENTER
1ST ELEMENT ON THE CRACK-LINE,,/36X,
2nd ELEMENT IN THE MESH AND RE-MESHING IS NOT POSSIBLE------)

975 FORMAT (IH1,,/36X,92H------THE SINGULAR ELEMENT IS WRONG------,
1,36X,44H------THE SINGULAR ELEMENT IS NOT SQUARE------)

978 FORMAT (IH1,,/36X,44H------INPUT FOR "IALL" IS "1" BUT ELEMENT NO.
1,16,2H IS NOT SQUARE------,
2,16X,41HE THIS IS THE CASE SET "IALL=0" ON INPUT)

979 FORMAT (IH1,,/36X,66H------INPUT FOR "IALL" IS "0" BUT ALL THE ELEMENTS ARE SQUARES------,
2,16X,41HE THIS IS THE CASE SET "IALL=1" ON INPUT)

989 FORMAT (/36X,69H="1PB15.7,7X,11BTIME STEP =1PB15.7")

981 FORMAT (IH1,,/36X,64H------PRESSURE CARDS MUST BE PRESENT WITH "N
1UMP.C.GE.1" AND "NUMLP.GE.2" FOR INTERPOLATION------)

982 FORMAT (IH1,,/36X,69H------PRESSURE CARDS P(1,K),K=1,NUMLP MUST BE IN INCREASING ORDER------)

983 FORMAT (IH1,,/36X,76H------POSITION CARDS MUST BE PRESENT WITH "N
1UMP.S.GE.2" FOR INTERPOLATION------)

984 FORMAT (IH1,,/36X,119H------POSITION CARDS POST(1,K),K=1,NUMPS MUS
1T BE IN INCREASING ORDER AND CRACK-TIP POSITION MUST INCREASE WITH
2 TIME------)

985 FORMAT (IH1,,/36X,88H------IF VELOCITY CARDS ARE PRESENT, THEN WE
1MUST HAVE "NUNCV.GE.2" FOR INTERPOLATION------)

986 FORMAT (IH1,,/36X,116H------VELOCITY CARDS CVH(1,K),K=1,NUNCV MUST
1 BE IN INCREASING ORDER AND CRACK-TIP VELOCITY MUST BE NON-NEGATIV
2------)

988 FORMAT (IH1,,/36X,10H------A TIME STEP OF 1PB15.5,72H MAYBE TOO
1 SMALL AND WILL PROBABLY CREATE NUMERICAL DIFFICULTY------)

990 FORMAT (IH1,,/36X,10H------A TIME STEP OF 1PB15.6,86H MAYBE TOO
1 LARGE AND WILL PROBABLY CREATE UNRELIABLE RESULTS------)

998 FORMAT (IH1,,/36X,10H------NODULES,1D,18,71H ARE DOUBLE NODES BUT T
1HEY HAVE DIFFERENT VALUES FOR THEIR 'CUBB'------)

C

END
SUBROUTINE SOLVE(TT)

COMMON/BK1/NUMMAT, NUMNP, NUMEL, NUMPC, NUMLP, IHED(9), NTAE, NEQ, ICOND
1, ISTAT, TMAX, IHED1(8), ISYMT
COMMON/BR3/NSUT(16), NSUTC(16), TIO1, TIO2, TIOD, SID=, NUMDS, NUMST
COMMON/BRED/E(300), S(300), CDDS(300), IX(6, 300)
COMMON/OILLSP1(U(600), V(600), A(600)
COMMON/RHED/RH0(8), RODUM
COMMON/SUMAH/A1(3, 3, 8), AII(3, 3, 8), ADST(6), ASIZE(6), CL
COMMON/BE11/DBL1, DT1, DT8, BETA, BETA1, BETA2, BETA3, BETA4, BETA5, NBAND, NBD2
COMMON/TIP/NCR1, NCR2, NCR3, NCR4, NMLX, CTRY, CTRY1, SIF1, SIF2
COMMON/VKL/CV, NUNCV, CVE(2, 20)
COMMON/POS/NUMPS, POST(2, 20)
COMMON/PRESS/INI(100), JN1(100), FY(100), PJ(100), T(100), P(2, 20), FP
COMMON/ELM/ESTYM(8, 8, 50), ELMASS(8, 8, 50), VSS(20, 20)
COMMON/AMAT/SMS(20, 20), VIS(20, 20), AIAS(20, 20)
COMMON/BOUND/NXIF(200), NBC(200), NBB
COMMON/SEIF/NSBL, NDEL, LASTB, NF, NBRED, NELYP, ISK, ROBE
COMMON/DIM/NA, NAA, NB, NW, NINT, NINT2, IALL, NITER, SINCOD
COMMON/STORE/SE(192, 00)
COMMON/KSL/IP(20)
COMMON/TOLE/TOLER1, TOLER2
REAL MS(192, 00)
EQUIVALENCE(MS(1, 1), SK(1, 1))

C
COMMON/SSS/S1S(36, 20), S1SD(36, 20), S1SDD(36, 20)
COMMON/MASS/AMRR(36, 20), AMR1(36, 20), AMR2(2, 2), AMR3(36, 2)
1, AMRED(36, 2), AMRAD(36, 2)
COMMON/VK/ceptar(104)
COMMON/LOB/52 (36, 20), S4(36, 20), S5(36, 20), S2DD(36, 20)
DIMENSION E(600), EXLOAD(600), EV(600), EMA(600)
DIMENSION US(600), VS(600), AS(600)
DIMENSION SX1(2160), EXG(1484), EXG(184)
EQUIVALENCE(R(1), EXG(1))
EQUIVALENCE(EXLOAD(1), EXG(1))
EQUIVALENCE(EXM(1), EXG(1))
EQUIVALENCE(VS(1), EXG(1))
EQUIVALENCE(AS(1), EXG(1))
EQUIVALENCE(E(1), EXG(1))
EQUIVALENCE(S1S1S(1, 1), EXG(1))
EQUIVALENCE(S1S/SEKRA(1), EXG(1))
EQUIVALENCE(S1S(2, 1), EXG(1))

C
TIOT=TIO1
IF(TIOT.LT.TT) TIOT=TT
IF(TIOT.<.NE.1) TIOT=TIO1+TIOD
SITD=TT
IF((ICOND.<.NE.1) SITD=SITD+SID0
NB=NBAND
I=0
DO 10 N=1, NUMNP
IPHI-IPIX(CODE(N))
  IF(IPHI.EQ.0) GO TO 10
  I=I+1
  NB(N)=N
  IF(IPHI.EQ.1) NFXI(N)=16
  IF(IPHI.EQ.2) NFXI(N)=8
  IF(IPHI.EQ.3) NFXI(N)=11
10 CONTINUE
NB=1
RODUM=1.
LJR=0
IF((ICOND.<.NE.1 .OR. (ICOND.<.NE.2 .AND. SINCOD.EQ.0.) ) JR=1
C OBTAIN STATIC SOLUTION
C
IF(ICOND.NE.1) GO TO 58
WRITE(6,2000)
CV=0.
IF(ISTAT.GT.0) RODM=0.
CALL PCHECK(CV,NUMMAT,TT,IL)
IF(IL.EQ.1) RETURN
CALL SINGEL
CALL FORMX(S)
RODM=1.
CALL LOAD(TT,TTF,3,LL)
DO 38 I=1,NI
38 CONTINUE
E(I)=ELOAD(I)
CONTINUE
CALL PAIR(S,S)
CALL ESOLVE(S,S)
DO 48 N=1,NEQ
U(N)=US(N)
V(N)=S.
A(N)=S.
CONTINUE
CALL SECOND(TMACH1)
IF(ISTAT.LT.0) GO TO 42
CALL DATOUT(S,TT)
RETURN
42 IF(TT.LT.T102 .AND. TMACH1.LT.PMACH) GO TO 45
CALL DATOUT(S,TT)
CALL TAPOUT(PT)
RETURN
45 IF(TT.LT.TIOT) GO TO 48
CALL DATOUT(S,TT)
WRITE(6,488)
TIOT=TIOT+TIOD
48 CALL TAPOUT(PT)
SIOT=SIOT+SIOD
CONTINUE
C LOOP ON TIME STEP
C
LJUMP=0
LJUMP=S
CONTINUE
C
ITER=S
LOOP=LOOP+1
TEMP=TT
CPIX=CPIX
CV1=CV
C LOOK FOR JUMP CONDITION AND CALCULATE PROPER TIME,LOAD,AND VELOCITY
C
CALL VELOC(TT,TTF,S,LL)
CALL LOAD(TT,TTF,S,LL)
C LOAD AND VELOCITY JUMP OR VELOCITY JUMP ONLY
C IF(LL.EQ.1) GO TO 61
C LOAD JUMP ONLY
C IF(LL.EQ.1 .AND. LL.EQ.1) GO TO 64
C NO JUMP
C TT=TTF
C IF(TTF.GT.TTF) TT=TTF
C DELT=TT-TEMP
C DELEP=DELT/DT1
LOOP = LOOP + 1
IF (DLR > R.TOL) GO TO 80
LOOP = LOOP + 1
JUMP = 8
CALL VELOC(TT, TV, 1, LV)
IF (LOGIC.EQ.1 .AND. ICOND.EQ.1) OR. CV.NE.CV1) GO TO 691
GO TO 78
691 WRITE(6, 496)
CALL PRECCK(CV, NUMMAT, TT, IL)
IF (IL.EQ.1) RETURN
GO TO 78
61 CALL VELOC(TT, TV, 1, LV)
WRITE(6, 496)
CALL PRECCK(CV, NUMMAT, TT, IL)
IF (IL.EQ.1) RETURN
CALL SINGEL
CALL FORM(2)
CALL FORM(8)
CALL LOAD(TT, TV, 1, LL)
GO TO 812
64 IF (LOGIC.EQ.1) GO TO 644
CALL LOAD(TT, TV, 1, LL)
GO TO 812
644 IF (ICOND.EQ.1) GO TO 6444
WRITE(6, 496)
CALL PRECCK(CV, NUMMAT, TT, IL)
IF (IL.EQ.1) RETURN
IF (KJR.EQ.1) GO TO 8444
CALL SINGEL
6444 CALL FORM(2)
CALL FORM(8)
CALL LOAD(TT, TV, 1, LL)
GO TO 812
C END OF LOOP FOR JUMP CONDITION AND FINDING PROPER TIME
C UPDATE CRACK TIP LOCATION
78 CALL POSIT(TT, ILP, 0)
IF (ILP.EQ.1) RETURN
BET1 = DELT + DELT + BETA
BET2 = DELT + BETA
BET3 = (.5 - BETA) / BETA
BET4 = (.5 - BETA) + DELT + DELT
BET5 = (.25 - BETA) * DELT = 3
C IF (CTPX.NE.CTPX1) GO TO 71
IF (CV.NE.CV1) GO TO 71
IF (LOGIC.EQ.1 .AND. KJR.EQ.1) GO TO 72
IF (LOGIC.EQ.1) GO TO 71
GO TO 78
71 CALL SINGEL
72 CALL FORM(2)
CALL FORM(8)
73 CALL FORM(1)
C UPDATE SURFACE TRACTIONS
C 75 CALL LOAD(TT, TV, 1, LL)
C MODIFY EXTERNAL FORCE VECTOR FOR ASYMMETRIC STIFFNESS AND
C DO 82 I = 1, NP
EM(I) = 0.
82 CONTINUE
IF (LOGIC.EQ.1 .AND. ICOND.EQ.6) GO TO 120
DO 118 I=1,NQ
II=IF(I)+NDEL
DO 186 J=1,NQ
JJ=IF(J)
EM(II)=EM(II)-(V(S(J),I)+V(J,J)+A(4S(I,J)+U(J,J))
186 CONTINUE
EM(II)=EM(II)+BET1
118 CONTINUE
128 CONTINUE

C

MM=0
REWIND 14

140 CONTINUE
MM=MM+1
NB1=(MM-1)+NBAND+1
NB2=MM+NBAND
DO 150 I=NB1,NB2
EMAT(I)=0.
150 CONTINUE
IF(MM.EQ.1) NB1=NB1+NDEL
IF(MM.EQ.LASTB) NB2=NB2+NDEL
K=NBAND+1
READ(14) (MS(I,J),I=1,NBD2,J=1,NBG)
CALL MATPROD(NB1,NB2,MM,1)
IF(MM.EQ.LASTB) GO TO 162
DO 160 J=1,NBG
DO 160 I=1,NBAND
MS(I,J)=MS(I+NBAND,J)
160 CONTINUE
GO TO 140
162 CONTINUE

C

END LOOP ON BLOCKS

C

IF(CV.EQ.,0.)GO TO 162
DO 180 I=1,NQ
II=IF(I)+NDEL
DO 180 J=1,NQ
JJ=IF(J)
EMAT(J)=EMAT(J)+V(S(I,J)+U(J,J)+DELTA+.6*V(J,J)+BET4+A(J,J)+BET5)
180 CONTINUE
182 CONTINUE
DO 190 K=1,NBQ
II=K-NDEL
EMAT(II)=EMAT(II)+BET1+ELoad(II)
190 CONTINUE
192 CONTINUE
DO 196 K=1,NF
E(K)=EMAT(K)+EM(N)
195 CONTINUE

C

SOLVE SYSTEM OF EQUATIONS

C

IF(ITER.EQ.,0) CALL PAIR(0,0)
IF(ITER.EQ.,1) CALL ESOLVE(0,0)
IF(CV.EQ.,0)GO TO 200
IF(ITER.EQ.,1) KITER) GO TO 200
GO TO 220
200 DO 210 I=1,NBQ
U(I)=US(I)
V(I)=VS(I)
A(I)=AS(I)
210 CONTINUE
GO TO 320
220  ITE=ITE+1
  WRITE(9,2001) ITE,TT,CV,CTPX,CTPY,PP
  WRITE(6,408) SIP1,SIP2
C
C      CALCULATE ERROR VECTOR
C
DO 280 I=1,NF
   EM(I)=0.
280    CONTINUE
DO 300 I=1,NQ
   II=IF(I)+NDE
   DO 300 J=1,NQ
      JJ=IF(J)
      EM(J)=EM(J)-(V1S(I,J)+VS(JJ)+AK4S(I,J)+US(JJ))
300    CONTINUE
EM(I)=EM(I)+HET1
305    CONTINUE
GO TO 192
C
C      THIS PORTION SOLVES FOR A JUMP CONDITION
C
312    CONTINUE
   DELT=TT-TTEMP
   REWIND 15
   IJUMP1=1
   IJUMP=1
   MM=#
   DO 3121 I=1,NBQ
      US(I)=U(I)
3121   CONTINUE
   MM=MM+1
   NB1=(MM-1)+NBAND+1
   NB2=MM+NBAND
   DO 314 I=NB1,NB2
514    EMAT(I)=0.
      IF(MM.EQ.1) NB1=NB1+NDE
      IF(MM.EQ.LASTB) NB2=NB2+NDE
      K=NBAND+1
      READ (15) ((SK(I,J),I=1,K),J=1,NBG)
      CALL MATPROD(NB1,NB2,MM,2)
      IF(MM.EQ.LASTB) GO TO 316
      DO 315 J=1,NBG
      DO 315 I=1,NBAND
         SK(I,J)=SK(I,NBAND,J)
      315    CONTINUE
      GO TO 318
316    CONTINUE
C
IF(CV.EQ.0.)GO TO 318
IF(LOOP.EQ.1 .AND. ICOND.EQ.0)GO TO 318
DO 317 I=1,NQ
   II=IF(I)+NDE
   DO 317 J=1,NQ
      JJ=IF(J)
      EMAT(J)=EMAT(J)+(V1S(I,J)+VSS(I,J)+US(JJ)+AK4S(I,J)+U(JJ))
317    CONTINUE
318    CONTINUE
   DO 319 I=1,NF
      E(I)=ELOAD(I)-EMAT(I)
319    CONTINUE
   CALL PAIR(1,.0)
   CALL ESOLVE(1,1)
   DO 1210 I=1,NBQ
A(I)=US(I)
1210 CONTINUE
    GO TO 321
C
C END OF SOLUTION FOR A JUMP CONDITION
C
C
C ASSIGN DISPLACEMENTS, VELOCITIES AND ACCELERATIONS
C
320 JUMP1=0
321 CONTINUE
    CALL SECOND(TMACH1)
C
C PRINT SOLUTION
C
    IF(TT.LT.TI02 .AND. TMACH1.LT.TMACH) GO TO 340
    CALL DATOUT(1,TT)
    CALL TAPOUT(TT)
    RETURN
340 IF(TT.LT.TI0T) GO TO 350
    CALL DATOUT(1,TT)
    TI0T=TI0T+TI0D
    WRITE(6,400)
350 IF(TT.LT.SI0T) GO TO 355
    CALL TAPOUT(TT)
    SI0T=SI0T+SI0D
355 CONTINUE
    GO TO 35
C
C END LOOP ON TIME STEP
C
400 FORMAT(5X,1P16E12.4)
401 FORMAT(1H1)
406 FORMAT(/,46X,6H K1=,1PE12.4,6H K2=,1PE12.4)
2000 FORMAT(/,46X,37ESTATIC SOLUTION FOR INITIAL CONDITION)
2001 FORMAT(1H1, //,16X,47THE STRESS INTENSITY FACTORS BEFORE ITERATION
1 #,1S, //,16X,12HCURRENT TIME,16X,16HCRACK-TIP VELOCITY,16X,
220HCRACK-TIP POSITION-R,16X,28HCRACK-TIP POSITION-2,16X,4ELOAD,/,
316X,1PE13.4,12X,1PE13.4,16X,1PE13.4,16X,1PE13.4,9X,1PE13.4)
C
C END
SUBROUTINE MATPROD(N1, N2, MM, LL)

COMMON/SHIFT/NSNL, NDEL, LASTB, NF, NBRED, NBLET, ISK, ROCODE
COMMON/BE1/NUMMAT, NUMNP, NUMPC, NUMLF, THERD(6), NTAFE, NRQ, ICOND
COMMON/EG/DEL1, DELT, DT1, DT2, ETA, ETA1, ETA2, ETA3, ETA4, ETA5, ETA6, NBD2
COMMON/STORES/8(122,80), V(080), A(080)

COMMON/SSS/S15(38,20), S1SD(38,38), S1SDD(38,38)
COMMON/MASS/AMDC(38,38), AMEDR(38,38), AM22(3,2), AM2R(38,2)
COMMON/WX/WKAREA(1464)
COMMON/LOH/SZ(38,38), S1(38,38), S2D(38,38), S2DD(38,38)
DIMENSION E(8000), EL(8000), EM(8000), EMAT(8000)
DIMENSION UK(800), VS(8000), AS(8000)
DIMENSION EX1(2168), EX2(1464), EX5(5184)
EQUIVALENCE(E(1), EX1(1))
EQUIVALENCE(E(1), EX1(1))
EQUIVALENCE(EMAT(1), EX5(1))
EQUIVALENCE(US(1), EXS(1))
EQUIVALENCE(VS(1), EXS(1))
EQUIVALENCE(AS(1), EXS(1))
EQUIVALENCE(EX1(1), EXS(1))
EQUIVALENCE(WKNRA1(1), EXS(1))
EQUIVALENCE(SZ(1,1), EXS(1))

NBA=NBRED
IF (NF.EQ.NSPL) NBA=NBAND
NMAX=2+NBA-1
DO 100 I=N1,N2
IF (IGF.LT.6) IGF=0
L=I-NBRED
IF (L.GT.NBA+1) L=NBA+1
DO 90 J=1,NMAX
IGF=IGF+1
K=IGF+NBD2
IF (L.GT.1) GO TO 66
L=L-1
GO TO 70
66 CONTINUE
K=I
L=L-1
70 CONTINUE
IF (L.GT.NBA) GO TO 100
IF (IGF.GT.NSPL) GO TO 100
EX=X-(MM-2)*NBAND
IF (LL.EQ.2) GO TO 66
EMAT(I)=EMAT(I)+B(K,L)*(U(IGF)+DELT*V(IGF)+BET4*A(IGF))
GO TO 90
66 CONTINUE
EMAT(I)=EMAT(I)+B(K,L)*U(IGF)
90 CONTINUE
100 CONTINUE
RETURN
END
SUBROUTINE FORM(L)
C
COMMON/EX1/NUMMAT, NUMNP, NUMEL, NUMPC, NUMLP, IEHD(8), NTAPB, NEQ, ICOND
1, ISTAT, IMAGE, IEHD1(8), ISYM
COMMON/DIM/NA, NAA, NEB, NB, NQ, NR, NINT, NINT2, LALL, NITER, SINCOD
COMMON/BE11/DELT, DT1, DT3, BETA, BETA1, BETA2, BETA3, BETA4, BET5, NBAND, NBD2
COMMON/TIP/NCR1, NCR2, NCR3, NCR4, NREL, CTPX, CTPF, SIF1, SIF2
COMMON/EFL/EFLP(F, 8, 56), ELMASS(F, 8, 56), VBS(28, 28)
COMMON/AMAT/SMS(28, 28), VIS(28, 28), AXS(28, 28)
COMMON/ION/SK(28, 28)
COMMON/SHIFT/NSHL, NDEL, LASTB, NF, NBERD, NELY, ITK, RCODE
COMMON/BE16/IF(100), Z(100), CODE(100), IX(8, 256)
COMMON/VESL/IF(28)
COMMON/STORE/SSK(102, 56)
COMMON/VEL/CV, NUMCV, CVR(2, 28)
COMMON/RECD/BO(8), RODUM
DIMENSION IX(100)
C
C L=1 CALCULATES K EFFECTIVE MATRIX AND WRITES THE RESULT ON TAPE11
C L=2 CALCULATES MASS MATRIX AND WRITES THE RESULT ON TAPE14
C L=3 CALCULATES K MATRIX AND WRITES THE RESULT ON TAPE16
C L=4 CALCULATES K MATRIX, WHICH IS THE SAME AS K EFFECTIVE FOR STATIC
C CASE, BUT WRITES THE RESULTS ON TAPE11, WHICH IS FOR K EFFECTIVE.
C
REWIND 11
REWIND 14
REWIND 15
NBO=NBAND
DO 16 J=1, NBO
DO 16 I=1, NBD2
16 SK(I, J)=0.
MM=0
ISING=0
NBAL2=0
20 CONTINUE
NBAL1=NBAL2
C
C LOOP ON BLOCKS
C
MM=M+1
NBAL2=(((M+1)*NBAND-NDEL)/2
IX=0
DO 28 I=1, NUMEL
IN=IX(1, I)
DO 28 J=2, 4
IF (IX(I, J).EQ. IN) IN=IX(J, I)
25 CONTINUE
IF (IN.GT.NBAL1 .AND. IN.LE.NBAL2) GO TO 27
GO TO 30
27 IX=IX+1
28 CONTINUE
DO 288 IP=1, IX
NE=IX(IP)
IF (NE.EQ. NCR1) GO TO 33
IF (NE.EQ. NCR2) GO TO 33
IF (NE.EQ. NCR3) GO TO 33
IF (NE.EQ. NCR4) GO TO 33
GO TO 128
33 IF (ISING .EQ. 0) GO TO 50
GO TO 128
C
STORE SPECIAL ELEMENT MATRICES
58 CONTINUE
   ISINC=1
   DO 110 I=1,NQ
      NRWB=IP(I)-(MM-1)*NBAND+NDEL
   DO 110 J=1,NQ
      NCOL=IP(J)-(MM-1)*NBAND-NROWB+1+NDEL
      IF(NCOL) 106,106,78
58 CONTINUE
   IF(L.EQ.2) GO TO 88
   IF(L.GE.3) GO TO 96
   IF(CY_NE.8) GO TO 75
   SK(NROWB,NCOL)=SK(NROWB,NCOL)+(SKS(I,J)*BET1+BET1*VSS(I,J)+DELT*.5
   +SM5(I,J))
   GO TO 166
88 CONTINUE
   SK(NROWB,NCOL)=SK(NROWB,NCOL)+SM5(I,J)
   GO TO 166
96 CONTINUE
   SK(NROWB,NCOL)=SK(NROWB,NCOL)+SKS(I,J)
106 CONTINUE
   GO TO 226
128 CONTINUE
   IF(N.EQ.NC2) GO TO 226
   IF(N.EQ.NC3) GO TO 226
   IF(N.EQ.NCB) GO TO 226
   IF(N.EQ.NCM) GO TO 226
   IF(ISK_EQ.1) GO TO 146
   IF(ICOND_EQ.2 .AND. ECODE_EQ.0) GO TO 146
   IF(ICOND_EQ.2 .AND. IALL_EQ.1) GO TO 146
   CALL STIF!EL
   IF(ICOND_EQ.2) GO TO 146
   IF(ISTAT.LE.6) CALL MASEL
146 CONTINUE
   ISK=1
158 CONTINUE
   II=II(5,NE)

C
C DO 216 JJ=1,4
   NN=J(J,NE)
   NROWB=(NN-1)+2-(MM-1)*NBAND+NDEL
   DO 216 J=1,2
      NROWB=NROWB-1
   I=(II-1)+J
   DO 206 NII=1,4
      NN=II(K,NE)
      NCOL=(NN-1)+2*NDEL
   DO 196 I=1,2
      NROWB=NROWB-1
   NCOL=NCOL+K-1-NROWB-(MM-1)*NBAND
   IF(NCOL) 106,106,186
186 CONTINUE
   IF(L.EQ.2) GO TO 170
   IF(L.GE.3) GO TO 186
   SK(NROWB,NCOL)=SK(NROWB,NCOL)+BSTIFM(I,M,II)+BET1+ELMASS(I,M,II)
   GO TO 196
170 CONTINUE
   SK(NROWB,NCOL)=SK(NROWB,NCOL)+ELMASS(I,M,II)
   GO TO 196
186 CONTINUE
   SK(NROWB,NCOL)=SK(NROWB,NCOL)+BSTIFM(I,M,II)
190 CONTINUE
200 CONTINUE
210 CONTINUE
220 CONTINUE
230 CONTINUE
240 CONTINUE
240 CONTINUE
IF(L.EQ.2) GO TO 248
IF(L.EQ.3) GO TO 250
WRITE(11) ((SK(I,J),I=1,NBAND),J=1,NBG)
GO TO 260
248 WRITE(14) ((SK(I,J),I=1,NBAND),J=1,NBG)
GO TO 260
250 WRITE(15) ((SK(I,J),I=1,NBAND),J=1,NBG)
260 CONTINUE
IF(LASTB.EQ.1) RETURN
C
C SHIFT UP ONE BLOCK
C
DO 330 I=1,NBAND
K=I-1,NBAND
DO 330 J=1,NBG
SK(I,J)=SK(K,J)
330 SK(I,J)=S.
IF(MM.EQ.LASTB-1) GO TO 380
C
C RETURN FOR NEXT BLOCK
C
GO TO 28
380 IF(L.EQ.2) GO TO 378
IF(L.EQ.3) GO TO 376
WRITE(11) ((SK(I,J),I=1,NBAND),J=1,NBG)
GO TO 380
378 WRITE(14) ((SK(I,J),I=1,NBAND),J=1,NBG)
GO TO 380
376 WRITE(15) ((SK(I,J),I=1,NBAND),J=1,NBG)
380 CONTINUE
RETURN
C
END
SUBROUTINE ESOLVE(L, LP)

C

COMMON/PAIR1/PAIR, LPAIR(3, 48)
COMMON/EK1/NNUMAT, NUMP, NUMEL, NUMPC, NUMLP, IHEX(8), NTAPE, NEQ, ICOND
1, ISTAT, IMACH, IHEX0(8), ISYM
COMMON/B11/DV2, BT1, BT2, BET1, BET2, BET3, BET4, BET5, NEQ, NDAB, ND2
COMMON/LMP1/PJ1, PS1, PJ2, PS2, PJ4, JY1, JY2
COMMON/SUMA/A1(3, 48), AII(3, 48), ALR(3, 48), ASIZE(8), CL
COMMON/BOUNDF/NNB(256), NNB(256), NGB
COMMON/SHEFT/NSDB, NDDEL, LASTB, NP, NRQ, NRQ, NINT, LINT, LIALL, NITER, SINCOD
COMMON/OLDDISP/U(688), V(688), AA(688)
COMMON/AB/PIS(36, 36), PI3(36, 36), PI3DD(36, 36)
1, AT(2, 36), ATD(2, 36), ATDD(2, 36)
COMMON/MISL/IP(26)
COMMON/DISP/UP(2, 36), UDD(2, 36), UDD(2, 36), UDE(2, 2)

C

COMMON/SSS/S18(36, 36), S1SD(36, 36), S1SD(36, 36)
COMMON/MASS/AMOR(36, 36), AMOR(36, 36), AM22(2, 2), AM22(2, 2)
1, AMPD(36, 36), AMPD2(36, 36)
COMMON/KE/WLAMERA(1484)
COMMON/IOR(36, 36), S4(36, 36), S2D(36, 36), S2DD(36, 36)
DIMENSION B(688), ELLOAD(688), EM(688), EMAT(688)
DIMENSION US(688), VS(688), AS(688)
DIMENSION EK(2186), EKS(1484), EKS(6194)
EQUIVALENCE(E(1), EXI (1))
EQUIVALENCE(ELOAD(1), EXI (1))
EQUIVALENCE(ELMA(1), EXI (1))
EQUIVALENCE(US(1), EXI (1))
EQUIVALENCE(VS(1), EXI (1))
EQUIVALENCE(AS(1), EXI (1))
EQUIVALENCE(EM(1), EXI (1))
EQUIVALENCE(S1S(1, 1), EXI (1))
EQUIVALENCE(WLAMERA(1), EKS(1))
EQUIVALENCE(S2(1, 1), EKS(1))
COMMON/STRESS/EX(3, 36), EXD(3, 36), RXDD(3, 36)
DIMENSION B(102)
DIMENSION B1(324)
EQUIVALENCE(B(1), B1(1))
EQUIVALENCE(B(1), EXI (1))
DIMENSION BETAADD(36)
DIMENSION BETAAD(2), BETAADD(2)
EQUIVALENCE(BETAADD(1), B1(288))
EQUIVALENCE(BETAAD(1), B1(288))
COMMON/BETA/BETAD(36), BETAADD(36), BETAADD(2)

C

COMPLEX LAM, SI, SI, SIII, DIS, DISX, DISXX
COMPLEX PACT1, PACT2

C

NGC=NEQ
NGQDEL=NGC=NEQ
DO 18 J=1, NGC
DO 18 I=1, NEQ
18 A(I, J)=8.
DO 20 I=1, NBRAND
20 B(I)=0.
   IF (L.EQ.0) REWIND 21
   IF (L.EQ.1) REWIND 24
   REWIND 12
   NBLOC=0
   CONTINUE
   NBLOC=NBLOC+1
   NBR=NBR+1
   IF (NBLOC.EQ.NSBL) NBR=NBRAND
   IF (NBLOC.EQ.LASTB-1) 40, 140, 40
   K=NBAND+1
   IF (L.EQ.0) READ (21) ((A(I,J), I=1, NBRD), J=1, NBG)
   IF (L.EQ.1) READ (24) ((A(I,J), I=1, NBRD), J=1, NBG)
   II=(NBLOC-2)+NBRAND
   DO 50 J=1, NBRD
   B(I)=B(I)+II
   50 CONTINUE
C
C   INSERT BC
C
   NB1=NBRAND*(NBLOC-1)+1
   NB2=NBRAND+NBLOC
   DO 100 I=1, NBB
   IF (NFIX(I).EQ.0) GO TO 100
   K1=NFIX(I)/10
   K2=NFIX(I)-K1
   K=2+NBRD(I)-K1+NDBL
   IF (K.LT.NB1.O.R.K.GT.NB2) GO TO 100
   100 NES=NBR-(NBLOC-2)+NBRAND
   A(NES,1)=1.
   B(NES)=0.
   DO 70 L1=2, NES
   70 A(NES,L1)=0.
   DO 80 L1=2, NBRAND
   L1=NES-L1+1
   80 A(L1,L1)=0.
   IF (K2) 100, 100, 90
   90 K=K+1
   K2=K
   GO TO 60
   100 CONTINUE
   IF (NDBL.EQ.0) GO TO 120
   IF (NBLOC.EQ.1) GO TO 140
   DO 120 I=1, NDBL
   120 A(I)=NBRAND+1.
   GO TO 200
   130 IF (NBLOC.EQ.1) GO TO 200
   140 CONTINUE
   NBR=NBRAND
   IF (NBLOC.EQ.NSBL+1) NBR=NBRAND
   DO 160 I=1, NBRAND
   PIVOT=A(I,1)
   IF (PIVOT.EQ.0) 160, 160, 160
   DO 160 J=2, NBR
   C=A(I,J)/PIVOT
   K=J-1-1
   IF (K.LT.NBRD) GO TO 170
   DO 160 K=I-JJB, NBA
   A(K,III+J)=A(K,III+J)+C*A(I,III)
   A(I,J)=0
   160 B(K)=B(K)-C*B(I)
   170 B(I)=B(I)/PIVOT
   180 CONTINUE
   190 CONTINUE
IF(NBLOC.EQ.LASTB+1) GO TO 280
WRITE(12) (((I,J),I=1,NBAND),J=1,NBG)
WRITE(12) (B(I),I=1,NBAND)
260 CONTINUE
DO 260 I=1,NBAND
K=I+NBAND
DO 210 J=1,NBG
A(I,J)=A(K,J)
210 CONTINUE
B(I)=B(K)
220 B(K)=B(I).
GO TO 260
230 CONTINUE
K=NBEK-(NF-NBQDEL)
280 CONTINUE
NBA=NBERED
IP(NBLOC.EQ.NSBL+1) NBA=NBAND
DO 300 III=1,K
JJJ=K-III+1
DO 300 J=2,NBA
B(JJJ)=B(JJJ)-A(JJJ,J)*B(JJJ,J-1)
JJ=NBAND+(NBLOC-2)
III=1
IF(JJ.EQ.0) III=NBQDEL+1
DO 300 I=III,K
J=JJ+I-NDEL
310 US(J)=B(I)
NBLOC=NBLOC-1
IF(NBLOC.EQ.1) GO TO 380
DO 320 I=1,NBAND
320 B(I+NBAND)=B(I)
BACKSPACE 12
READ(12) ((A(I,J),I=1,NBAND),J=1,NBG)
READ(12) (B(I),I=1,NBAND)
BACKSPACE 12
K=NBEK
GO TO 280
380 CONTINUE
C EQUATE DISPLACEMENTS FOR DOUBLE NODES
DO 340 I=1,NPAIR
IF(LPFAIR(I,1).LE.0) GO TO 340
US(LPFAIR(I,1))=US(LPFAIR(2,1))
340 CONTINUE
IF(LP.EQ.1) GO TO 410
IF(LP.EQ.0) GO TO 420
DO 460 N=1,NEQ
AN=A1(N)
AS(N)=(US(N)-U(N))/BET1-V(N)/BET2-AN*SBT3
VS(N)=V(N)+(AN+AS(N))*DELT/2.
460 CONTINUE
GO TO 4640
420 CONTINUE
DO 480 I=1,NEQ
AS(I)=0.
VS(I)=0.
480 CONTINUE
C SOLVE FOR CENTER POINT DISPLACEMENT, VELOCITY, AND ACCELERATION
C OF THE SINGULAR ELEMENT
C CALC DISPLACEMENT COEFFICIENTS FOR THE SINGULAR ELEMENT BETA=B+Q, ETC.
DO 350 I=1,NB
BETA1(K)=S.
DO 350 J=1,NQ
BETA1(K)=BETA1(K)+P1S(K,J)*US(IF(J))
350 CONTINUE
DO 355 K=1, NR
BETA2(K)=0.
DO 355 J=1,NQ
BETA2(K)=BETA2(K)+AT(K,J)*US(IF(J))
355 CONTINUE
IF(CV.NQ.EQ.0.) GO TO 4100
DO 4000 I=1,NB
BETA1D(I)=0.
DO 4000 J=1,NQ
BETA1D(I)=BETA1D(I)+P1SD(I,J)*US(IF(J))+P1S(I,J)*VS(IF(J))
4000 CONTINUE
DO 4070 I=1,NR
BETA2D(I)=0.
DO 4070 J=1,NQ
BETA2D(I)=BETA2D(I)+ATD(I,J)*US(IF(J))+AT(I,J)*VS(IF(J))
4070 CONTINUE
DO 4090 I=1,NB
BETA1DDD(I)=0.
DO 4090 J=1,NQ
BETA1DDD(I)=BETA1DDD(I)+P1SDD(I,J)*US(IF(J))+2.*P1SD(I,J)*VS(IF(J))+1.*P1S(I,J)*AS(IF(J))
4090 CONTINUE
DO 4090 I=1,NB
BETA2DDD(I)=0.
DO 4090 J=1,NQ
BETA2DDD(I)=BETA2DDD(I)+ATDD(I,J)*US(IF(J))+2.*ATD(I,J)*VS(IF(J))+1.*AT(I,J)*AS(IF(J))
4090 CONTINUE
GO TO 4200
4100 CONTINUE
DO 4100 I=1,NB
BETA1D(I)=0.
DO 4100 J=1,NQ
BETA1D(I)=BETA1D(I)+P1S(I,J)*VS(IF(J))
4100 CONTINUE
DO 4170 I=1, NR
BETA2D(I)=0.
DO 4170 J=1,NQ
BETA2D(I)=BETA2D(I)+AT(I,J)*VS(IF(J))
4170 CONTINUE
DO 4190 I=1, NR
BETA1DDD(I)=0.
DO 4190 J=1,NQ
BETA1DDD(I)=BETA1DDD(I)+P1S(I,J)*AS(IF(J))
4190 CONTINUE
DO 4190 I=1, NR
BETA2DDD(I)=0.
DO 4190 J=1,NQ
BETA2DDD(I)=BETA2DDD(I)+AT(I,J)*AS(IF(J))
4190 CONTINUE
4200 CONTINUE
C CALC DISPLACEMENTS OF THE INTERNAL NODES OF THE SINGULAR ELEMENT

NN=LX(N,NCB2)+2
X1=2.*(NN/(NN+1.0)-CORD(1,1))/((CORD(0,1)-CORD(1,1))-1.0)
IF(CFX.GT.E(NN/2.0)) GO TO 870
CX=(2.*CFX-CORD(1,1)-CORD(0,1))/((CORD(0,1)-CORD(1,1))
IF(X1-CX .LT. 1.E-10) X1=1.E-10+CX
Y1=6.
CALL TRANS(X1,Y1,N,2,1)
CALL FUNCTS(X1,Y1,0)
IF(CV,EQ.,0.) GO TO 362
DO 360 K=1,2
US(NN-2+K)=0.
VS(NN-2+K)=0.
AS(NN-2+K)=0.
DO 360 J=1,NB
US(NN-2+K)=US(NN-2+K)+UU(K,J)*BETA1(J)
VS(NN-2+K)=VS(NN-2+K)+UUD(K,J)+BETA1(J)+UU(K,J)+BETA1D(J)
AS(NN-2+K)=AS(NN-2+K)+UDDD(K,J)+BETA1(J)+2.*UUD(K,J)+BETA1D(J)
1+U(U,K,J)+BETA1DD(J)
DO 360 K=1,2
DO 363 J=1,NB
US(NN-2+K)=US(NN-2+K)+UU(K,J)*BETA1(J)
VS(NN-2+K)=VS(NN-2+K)+UUD(K,J)+BETA1D(J)
AS(NN-2+K)=AS(NN-2+K)+UUD(K,J)+BETA1DD(J)
DO 363 K=1,2
DO 365 J=1,NB
US(NN-2+K)=US(NN-2+K)+UUD(K,J)+BETA1D(J)
VS(NN-2+K)=VS(NN-2+K)+UUD(K,J)+BETA1DD(J)
AS(NN-2+K)=AS(NN-2+K)+UUD(K,J)+BETA1DD(J)
DO 370 I=1,2
IF(X,EQ.2) GO TO 375
NN=XX(2,NGC1)+2
Y1=1.1-18
CALL TRANS(X1,Y1,N,2,1)
CALL FUNCTS(X1,Y1,0)
DO 370 K=1,2
US(NN-2+K)=0.
VS(NN-2+K)=0.
AS(NN-2+K)=0.
DO 380 J=1,NB
US(NN-2+K)=US(NN-2+K)+UU(K,J)*BETA1(J)
VS(NN-2+K)=VS(NN-2+K)+UUD(K,J)+BETA1(J)+UU(K,J)+BETA1D(J)
AS(NN-2+K)=AS(NN-2+K)+UDDD(K,J)+BETA1(J)+2.*UUD(K,J)+BETA1D(J)
1+U(U,K,J)+BETA1DD(J)
DO 380 K=1,2
DO 385 J=1,NB
US(NN-2+K)=US(NN-2+K)+UU(K,J)*BETA1(J)
VS(NN-2+K)=VS(NN-2+K)+UUD(K,J)+BETA1D(J)
AS(NN-2+K)=AS(NN-2+K)+UUD(K,J)+BETA1DD(J)
9181
VS(N-2+K)=VS(N-2+K)+UU(K,J)*BETA1D(J)
A5(N-2+K)=A5(N-2+K)+UU(K,J)*BETA1DD(J)

383 CONTINUE

384 DO 385 K=1,2
   DO 385 J=1,NR
   US(N-2+K)=US(N-2+K)+UBR(K,J)*BETA2(J)
   VS(N-2+K)=VS(N-2+K)+UBR(K,J)*BETA2D(J)
   A5(N-2+K)=A5(N-2+K)+UBR(K,J)*BETA2DD(J)
   CONTINUE

396 CONTINUE

C CALC STRESS INTENSITY FACTORS (SQUARE ELEMENT ONLY)
C C= (2.*CTFX-CURD(1,1)-CURD(5,1))/(CURD(6,1)-CURD(1,1))
C XI=CI
Y1=0.
CALL TRANS(X1,Y1,N,2,1)
CALL COEFF(1,1)
FACTX2=LAM(1)+ALOG(PJ1)
FACTX2=EXP(FACTX2)
FACTX1=(SI(5)+SI(9)+SI(7)+SI(6))*FACTX2
FACTX2=(SI(9)+SI(13)+SI(11)+SI(12))*FACTX2
SIF1=(REAL(FACTX1)+BETA1(1)-AIMAG(FACTX1)+BETA1(2))+SQRT(2.*PFL)
SIF2=(REAL(FACTX2)+BETA1(1)-AIMAG(FACTX2)+BETA1(2))+SQRT(2.*PFL)

C RETURN
C
C SOLVE FOR CENTER POINT ACCELERATION OF SINGULAR ELEMENT FOR JUMP
C CALC DISPLACEMENT COEFFICIENTS FOR THE SINGULAR ELEMENT BETA=B+Q, ETC.

410 CONTINUE
   DO 420 I=1,NB
      BETA1(I)=0.
   DO 420 J=1,NQ
      BETA1(I)=BETA1(I)+P1S(I,J)*U(IP(J))
   CONTINUE

420 CONTINUE
   DO 430 I=1,NR
      BETA2(I)=0.
   DO 430 J=1,NQ
      BETA2(I)=BETA2(I)+AT(I,J)*U(IP(J))
   CONTINUE

430 CONTINUE
   IF(CV.EQ.0.) GO TO 4900
   DO 440 I=1,NB
      BETA1D(I)=0.
   DO 440 J=1,NQ
      BETA1D(I)=BETA1D(I)+P1SD(I,J)*U(IP(J))+P1S(I,J)*V(IP(J))
   CONTINUE

440 CONTINUE
   DO 460 I=1,NR
      BETA2D(I)=0.
   DO 460 J=1,NQ
      BETA2D(I)=BETA2D(I)+ATD(I,J)*U(IP(J))+AT(I,J)*V(IP(J))
   CONTINUE

460 CONTINUE
   DO 480 I=1,NB
      BETA1DD(I)=0.
   DO 480 J=1,NQ
      BETA1DD(I)=BETA1DD(I)+P1SDD(I,J)*U(IP(J))+2.*P1SD(I,J)*V(IP(J))
   CONTINUE

480 CONTINUE
   DO 470 I=1,NR
      BETA2DD(I)=0.
   DO 470 J=1,NQ
      BETA2DD(I)=BETA2DD(I)+ATDD(I,J)*U(IP(J))+2.*ATD(I,J)*V(IP(J))
   CONTINUE

470 CONTINUE
   GO TO 4900

4900 CONTINUE
   DO 4900 I=1,NB
BETA1D(I)=S.
DO 4848 J=1,NQ
BETA1D(I)=BETA1D(I)+P1S(I,J)*V(IF(J))

4848 CONTINUE
DO 4850 I=1,MR
BETA2D(I)=S.
DO 4850 J=1,NQ
BETA2D(I)=BETA2D(I)+AT(I,J)*V(IF(J))

4850 CONTINUE
DO 4850 I=1,NR
BETA1DD(I)=S.
DO 4850 J=1,NQ
BETA1DD(I)=BETA1DD(I)+P1S(I,J)*US(IF(J))

4850 CONTINUE
DO 4850 I=1,MR
BETA2DD(I)=S.
DO 4850 J=1,NQ
BETA2DD(I)=BETA2DD(I)+AT(I,J)*US(IF(J))

4850 CONTINUE

C CALC ACCELERATION OF THE INTERNAL NODES OF THE SINGULAR ELEMENT
I=IX(3,NCR2)+2-2
XI=2.*((I/6+1)-CORD(1,1))/(CORD(5,1)-CORD(1,1))-1.
IF (CTFX.GT.X(11)/9)) GO TO 471
CX=(1.*CTFX-CORD(1,1)-CORD(5,1))/(CORD(5,1)-CORD(1,1))
IF (XI-CX .LT. 1.E-10) XI=1.E-10+CX
YI=0.
CALL TRANS(XI,YI,N,2,1)
CALL FUNCTS(XI,YI,6)
IF (TV.EQ. 0) GO TO 6862
DO 6860 I=1,2
JJ=I+K
US(JJ)=S.
DO 6862 J=1,MR
US(JJ)=US(JJ)+UUD(X,J)+BETA1D(J)+2.*UUD(X,J)+BETA1DD(J)
1=UU(X,J)+BETA1DD(J)

6860 CONTINUE
GO TO 6864

6862 DO 6862 I=1,2
JJ=I+K
US(JJ)=S.
DO 6862 J=1,MR
US(JJ)=US(JJ)+UU(X,J)+BETA1DD(J)

6862 CONTINUE
DO 6864 K=1,2
JJ=I+K
DO 6864 K=1,MR
US(JJ)=US(JJ)+UUR(X,J)+BETA2DD(J)

6864 CONTINUE
US(IX(2,NCR1)+2-1)=US(I-1)
US(IX(2,NCR1)+2)=US(I+2)
GO TO 518

471 DO 589 M=1,2
IF(N.EQ.2) GO TO 475
I=2*IX(3,NCR2)-2
YI=1.E-10
CALL TRANS(XI,YI,N,2,1)
CALL FUNCTS(XI,YI,6)
GO TO 478

475 I=2*IX(3,NCR1)-2
YI=1.E-10
CALL TRANS(XI,YI,N,2,1)
CALL FUNCTS(XI,YI,6)
478 IF (CV.EQ. 0) GO TO 482
DO 485 J=1,2
JJ=I+J
US(JJ)=S.
DO 469 K=1,NB
US(JJ)=US(JJ)+UDD(J,K)+BETA1(K)+2.*UUD(J,K)+BETA1D(K)
1+UU(J,K)+BETA1DD(K)
485 CONTINUE
GO TO 486
486 DO 483 J=1,2
JJ=I+J
US(JJ)=S.
DO 469 K=1,NB
US(JJ)=US(JJ)+UU(J,K)+BETA1DD(K)
483 CONTINUE
486 DO 465 J=1,2
JJ=I+J
DO 468 K=1,NB
US(JJ)=US(JJ)+UUR(J,K)+BETA2DD(K)
465 CONTINUE
500 CONTINUE
510 CONTINUE

C CALC STRESS INTENSITY FACTORS (SQUARE ELEMENT ONLY)
CX=(2.*CTFX-CORD(1,1)-CORD(5,1))/(CORD(5,1)-CORD(1,1))
XI=0.
YI=0.
CALL TRANS(X1,Y1,N,2,1)
CALL OBEFF(1,1)
FACTI2=LAG(1)+ALOG(PJ1)
FACTI2=EXP(FACTI2)
FACTI1=(SI(3)+SI(8)+SI(7)+SI(9))+FACTI2
FACTI1=(SI(3)+SI(18)+SI(11)+SI(12))+FACTI2
SIF1=(REAL(FACTI1)+BETA1(1)-AIMAG(FACTI1)+BETA1(2))*SQR(2.*PI)
SIF2=(REAL(FACTI2)+BETA1(1)-AIMAG(FACTI2)+BETA1(2))*SQR(2.*PI)
RETURN

C END
SUBROUTINE MASEL

COMMON/ELM/ESTIPM(8,8,50), ELMASS(5,8,50), Y(26,26)
COMMON/INTGE/PT(16), WG(16), PT2(2), WC2(2), PRI
COMMON/DIF/DXX2, DXX3, DXXY, DXYY, DXY2, DXY2, DXYY, DXY, DXY, DXY, DEM
COMMON/SHIFT/NSK, NDEL, LASTB, NF, NBRD, NELTP, ISK, RCDB
COMMON/BK10/(5(800)), 2(800), CODE(800), IX(8,250)
COMMON/REH2/REH(8), REDUM
COMMON/DIM/HA, HAL, NB, NQ, NR, NINT, NINT2, IALL, NITER, SINCOD
COMMON/WE2/WE2(720)
COMMON/BK1/NUMMAT, NUMNP, NUMEL, NUMPC, NUMLP, IHED(8), NTAPF, NB4, ICOND
1, ISTAT, TMACH, IHED1(8), ISYMT
DIMENSION U(2,8)
EQUIVALENCE(U(1,1)), WKAR(1))

DO 16 I=1, NELTP
DO 16 J=1, 8
DO 16 K=1, 8
16 ELMASS(I,J,K)=0.
DO 26 I=1, NELTP
DO 26 K=1, NUMEL
IF (IX(I,J).EQ.K) GO TO 25
25 CONTINUE
GO TO 26
26 N=1
KK=IX(8,N)
DO 46 M=1, NINT2
X=PT2(L)
DO 46 M=1, NINT2
Y=PT2(M)
WT=WG2(L)*WG2(M)
CALL TRANS(X,Y,N,2,2)
CALL FUNCTR(X,Y)

WTDEM=W+DEM+RHO(KK)
DO 36 J=1, 8
DO 36 I=1, 8
36 ELMASS(I,J,K)=ELMASS(I,J,K)+U(KP,I)*U(KP,J)+WTDEM
46 CONTINUE
56 CONTINUE
99 CONTINUE
RETURN

END
SUBROUTINE FUNCTM(X,Y)

COMMON/W12/WEAR(728)
DIMENSION U(2,8)
EQUIVALENCE(U(1,1),WEAR(1))

DO 10 J=1,8
   DO 10 I=1,2
      10 U(I,J)=0.
      U(1,1)=.25*(1.-X)*(1.-Y)
      U(1,3)=.25*(1.+X)*(1.-Y)
      U(1,5)=.25*(1.+X)*(1.+Y)
      U(1,7)=.25*(1.-X)*(1.+Y)
      U(2,2)=U(1,1)
      U(2,4)=U(1,3)
      U(2,6)=U(1,5)
      U(2,8)=U(1,7)
RETURN

END
SUBROUTINE STIFEL

COMMON/BLM/ESTIFM(8,8,56),ELMASS(8,8,56),VSS(20,20)
COMMON/DIM/NA,NAA,NBT,KB,RQ,RK,NINT,NINT2,IALL, NITER,SINCOD
COMMON/SHIFT/NSBL,NDEL,LASTB,RF, NHREO,MLETP,ISEK,RCODE
COMMON/EM,N(300),E(300),CODE(300),IX(8,256)
COMMON/SUMN(3,3,8),RI(3,3,8),ADIS(6),ASIZE(8),CL
COMMON/INTG/PT(19),WG(19),PT2(2),WG2(2),PEI
COMMON/DIF/DY2;DXY3,DXY4,DY2;DXY2;DXY3,DXY4,DY2,DXY,DEM
COMMON/BK1/NUMMAT,NUMNP,NUMEL,NUMPC,NUMLP, IHELD(9),NTAPE,NEQ,ICOND
1,ISTAT,TMAG2, IHELD1(9),ISYM

C

DIMENSION A(2,2),C(3,8)
DIMENSION B(2,4),AJJ(3,8)
DIMENSION B1(68)
COMMON/WK2,WLAB(720)
EQUIVALENCE(B(1,1),B1(1))
EQUIVALENCE(AJJ(1,1),B1(9))
EQUIVALENCE(B1(1),B1(3))
EQUIVALENCE(A(1,1),B1(87))
EQUIVALENCE(B1(1),WLAB(1))

C

NIN=NINT
IF (IALL.GT.0) NIN=NINT2
DO 18 K=1,NLTYP
DO 18 J=1,8
DO 18 I=1,8
18 ESTIFM(I,J,K)=0.
DO 88 L=1,NLTYP
DO 88 M=1,NUMEL
IF (IX(5,L).EQ.K) GO TO 25
20 CONTINUE
GO TO 88
25 N=IX(8,N)
DO 70 L=1,NIN
X=PT(L)
IF (IALL.GT.0) X=PT2(L)
DO 68 M=1,NIN
Y=PT(M)
IF (IALL.GT.0) Y=PT2(M)
WT=WG(L)*WG(M)
IF (IALL.GT.0) WT=WG2(L)*WG2(M)
C
CALL TRANS(X,Y,N,2,2)
C
CALL DIPTR(X,Y)
C
DO 48 J=1,8
DO 48 I=1,3
AJJ(I,J)=0.
DO 48 KF=1,8
48 AJJ(I,J)=AJJ(I,J)+EI(I,KF,XX)+C(KF,J)
WTDEM=WT+DEM
DO 58 J=1,8
DO 58 I=1,2
58 WTDEM=WTDEM+
CONTINUE
70 CONTINUE
88 CONTINUE
RETURN
C
END
SUBROUTINE DIFFB(X,Y)

COMMON/IPAR/PJ1,PJ2,PJ3,PJ4,DXY1,DXY2
DIMENSION A(2,2),C(8,8)
DIMENSION B(2,4),AJJ(8,8)
DIMENSION B1(88)

COMMON/WK2/WKAR(728)
EQUIVALENCE(B(1,1),B1(1))
EQUIVALENCE(AJJ(1,1),B1(9))
EQUIVALENCE(C(1,1),B1(88))
EQUIVALENCE(A(1,1),B1(67))
EQUIVALENCE(B1(1),WKAR(1))

A(1,1)=PJ1
A(1,2)=PJ2
A(2,1)=PJ3
A(2,2)=PJ4
C(1,1)=-.25*(1.-Y)
C(1,2)=-.25*(1.-Y)
C(1,3)=-.25*(1.+Y)
C(1,4)=-.25*(1.+Y)
C(2,1)=-.25*(1.-X)
C(2,2)=-.25*(1.-X)
C(2,3)=-.25*(1.+X)
C(2,4)=-.25*(1.+X)
DO 16 J=1,4
DO 16 I=1,2
B(I,J)=S
DO 16 K=1,2
B(I,J)=B(I,J)+A(I,K)*C(K,J)
CONTINUE
DO 28 J=1,8
DO 28 I=1,3
28
C(1,1)=S
C(1,2)=B(1,1)
C(1,3)=B(1,2)
C(1,4)=B(1,3)
C(1,7)=B(1,4)
C(2,1)=B(2,1)
C(2,2)=B(2,2)
C(2,6)=B(2,3)
C(2,8)=B(2,4)
C(3,1)=C(2,2)
C(3,2)=C(1,1)
C(3,3)=C(2,4)
C(3,4)=C(1,3)
C(3,5)=C(2,6)
C(3,6)=C(1,5)
C(3,7)=C(2,8)
C(3,8)=C(1,7)
RETURN

END
SUBROUTINE POSIT(TT, ILP, LS)

COMMON/BL1/DELT, DT1, DT2, BETA, BET1, BET2, BET3, BET4, BET5, NBRAND, NB2
COMMON/TIP/NCR1, NCR2, NCR3, NCR4, NELX, CTPX, CTPY, STIFF, STIPS
COMMON/VEL/CV, NUMCV, CVH(2, 28)
COMMON/POS/NUMP, POST(2, 28)
COMMON/EK1, NUMMAT, NUMN, NUML, NUMLP, IMADE, NTAPE, NB, ICUND
1, ISTAT, TIMEC, IHED(9), I5YMT
COMMON/BOUND/NEPX(200), NBC(200), NNB
COMMON/DIM/RAD, NAA, NBT, NE, NI, NINT2, IALL, NITER, SINGOD
COMMON/SHIFT/NSBL, NSBL, LAST, NV, NBRED, NELTY, ISK, BCODE
COMMON/EX1/R(200), E(200), CODE(200), IX(8, 265)
COMMON/ULDISP/H(800), Y(800), A(800)
COMMON/MISL/IF(28)
COMMON/PAIR1//PAIR2, LPAIR(3, 40)
COMMON/MAIN/CURD(15, 2)
COMMON/EX1/T(IX(1, NCR1))
COMMON/TOLE/TOLE1, TOLE2

C FIND NEW CRACK TIP POSITION

C

ILP=0
IF(TT .GE. POST(1, 1) .AND. TT .LE. POST(1, NUMPS)) GO TO 5
WRITE(8, 548)
STUP 21
5 DO 10 I = 2, NUMPS
IF(TT .LE. POST(I, 1)) GO TO 10
10 CONTINUE
12 DELT1=TT-POST(1, I-1)
D1=POST(1, I)-POST(1, I-1)
DELP=POST(2, I)-POST(2, I-1)
IF(D1 .NE. 0.) GO TO 13
IF(DELP .NE. 0.) GO TO 300
IF(I .EQ. NUMPS) RETURN
I=I+1
GO TO 12
13 CTPX=POST(2, I-1)+DELP+DELT1/D1
IF(LS .EQ. 1) RETURN
M1=IX(4, NCR1)
MP=IX(4, NCR2)
DELTA=(E(WP)-E(M1))*0.75
DIST=R(M1)+DELTA
IF(CTPX.GT.DIST) GO TO 15
GO TO 300
15 CONTINUE
C ADVANCE CRACK TIP ELEMENTS
C NCR1=NCR1+NELX
NCR2=NCR2+NELX
NCR3=NCR3+NELX
NCR4=NCR4+NELX
WRITE(6, 545) NCR1, NCR2, NCR3, NCR4
C CHECK FOR THE SINGULAR ELEMENT TO BE INSIDE THE MESH
IF(NCR1 .LE. 0 .OR. NCR1 .GT. NUMEL) GO TO 28
IF(NCR2 .LE. 0 .OR. NCR2 .GT. NUMEL) GO TO 28
IF(NCR3 .LE. 0 .OR. NCR3 .GT. NUMEL) GO TO 28
IF(NCR4 .LE. 0 .OR. NCR4 .GT. NUMEL) GO TO 28
C CHECK TO SEE IF MATERIALS IN SINGULAR ELEMENT IS CORRECT
IF(IX(6, NCR1).NE. IX(6, NCR2)) GO TO 679
IF(IX(6, NCR1).NE. IX(6, NCR3)) GO TO 679
C CHECK FOR THE ELEMENTS OF THE SINGULAR ELEMENT TO BE SQUARES
X1=E(IX(2, NCR1))-E(IX(1, NCR1))
IF(IALL .EQ. 1) GO TO 2456
DO 2465 N=1,2
   IF(M.EQ.1) I=NCR2
   IF(M.EQ.2) I=NCR4
   M1=IX(1,1)
   M2=IX(2,1)
   M3=IX(3,1)
   M4=IX(4,1)
   RR1=R(M1)
   RR2=R(M2)
   RR3=R(M3)
   RR4=R(M4)
   ZZ1=Z(M1)
   ZZ2=Z(M2)
   ZZ3=Z(M3)
   ZZ4=Z(M4)
   IF(ABS(ABS((RR2-RR1)/XL)-1.) .GT. TOLER1) GO TO 21
   IF(ABS(ABS((RR2-RR4)/XL) .GT. TOLER1) GO TO 21
   IF(ABS(ABS((RR1-RR4)/XL) .GT. TOLER1) GO TO 21
   IF(ABS(ABS((ZZ3-ZZ1)/XL) .GT. TOLER1) GO TO 21
   IF(ABS(ABS((ZZ2-ZZ1)/XL) .GT. TOLER1) GO TO 21
   IF(ABS(ABS((ZZ2-ZZ4)/XL) .GT. TOLER1) GO TO 21
   IF(ABS(ABS((ZZ3-ZZ4)/XL) .GT. TOLER1) GO TO 21
   CONTINUE
   CONTINUE
   C CHECK FOR THE SINGULAR ELEMENT TO BE A SQUARE (I.E., CHECK TO SEE IF THE C ELEMENTS OF THE SINGULAR ELEMENT ARE NUMBERED CORRECTLY)
   M1=IX(1,NCR2)
   M2=IX(2,NCR4)
   M3=IX(3,NCR8)
   M4=IX(4,NCR1)
   RR1=R(M1)
   RR2=R(M2)
   RR3=R(M3)
   RR4=R(M4)
   ZZ1=Z(M1)
   ZZ2=Z(M2)
   ZZ3=Z(M3)
   ZZ4=Z(M4)
   XL2=2.*XL
   IF(ABS(ABS((RR2-RR1)/XL2)-1.) .GT. TOLER1) GO TO 21
   IF(ABS(ABS((RR2-RR4)/XL2) .GT. TOLER1) GO TO 21
   IF(ABS(ABS((RR1-RR4)/XL2) .GT. TOLER1) GO TO 21
   IF(ABS(ABS((ZZ2-ZZ1)/XL2) .GT. TOLER1) GO TO 21
   IF(ABS(ABS((ZZ3-ZZ1)/XL2) .GT. TOLER1) GO TO 21
   IF(ABS(ABS((ZZ3-ZZ4)/XL2) .GT. TOLER1) GO TO 21
   IF(ABS(ABS((ZZ4-ZZ2)/XL2) .GT. TOLER1) GO TO 21
   GO TO 24
20 WRITE(6,970)
   GO TO 23
21 WRITE(6,975)
   GO TO 23
579 WRITE(6,985)
23 ILP=1
   RETURN
24 CONTINUE
   IF(2)=IX(4,NCR2)+2
   IF(4)=IX(1,NCR2)+2
   IF(6)=IX(2,NCR3)+2
   IF(8)=IX(2,NCR4)+2
   IF(10)=IX(3,NCR4)+2
   IF(12)=IX(2,NCR8)+2
   IF(14)=IX(8,NCR8)+2
IF (18) = IX(4, NCR1) + 2
IF (20) = IX(1, NCR1) + 2
IF (1) = IF (2) - 1
IF (9) = IF (4) - 1
IF (6) = IF (8) - 1
IF (7) = IF (8) - 1
IF (9) = IF (18) - 1
IF (11) = IF (12) - 1
IF (13) = IF (14) - 1
IF (15) = IF (16) - 1
IF (17) = IF (18) - 1
IF (19) = IF (20) - 1
MAX = IF (2)
MIN = IF (2)
DO 25 I = 4, NQ, 2
IF (IF (I) .GT. MAX) MAX = IF (I)
IF (IF (I) .LT. MIN) MIN = IF (I)
25 CONTINUE
NSBL = (MAX - 1) / WBL + 1
NDBL = (NSBL - 1) * WBL + 2 - MIN
LASTB = (NSQ + NDBL - 1) / WBL + 1
NF = LASTB - WBL
DO 30 I = 2, NQ, 2
K = IF (I) / 2
II = I / 2
CORD (II, 1) = K (X)
CORD (II, 2) = 2 (X)
30 CONTINUE
MC1 = IX(2, NCR1)
MC2 = IX(5, NCR2)
NC1 = IX(1, NCR1)
NC2 = IX(4, NCR2)
CODE (MC1) = S.
CODE (MC2) = S.
CODE (NC1) = S.
CODE (NC2) = S.
IF (IIIG1 (IX (6, NCR1)) .EQ. 1) CODE (NC1) = 3.
CODE (NC2) = S.
IF (IIIG1 (IX (6, NCR2)) .EQ. 1) CODE (NC2) = 3.
DO 180 I = 1, NPAIR
N = (LPAIR (1, 1) + 1) / 2
IF (N .EQ. NC2) LPAIR (3, 1) = 0
180 CONTINUE
I = 0
DO 120 N = 1, NUMP
IPHI = IFIX (CODE (N))
IF (IPHI .EQ. 0) GO TO 120
I = I + 1
NBC (I) = N
IF (IPHI .EQ. 1) NFIX (I) = 15
IF (IPHI .EQ. 2) NFIX (I) = 16
IF (IPHI .EQ. 3) NFIX (I) = 17
120 CONTINUE
NBB = I
200 RETURN
320 WRITE (6, 320)
STOP 4
320 FORMAT (1XL, '***** 16HWRONG DATA, 16H *****
1 34HPOSITION CARDS CAN NOT HAVE JUMPS.')
330 FORMAT (1XL, '**** 28X, 2EH SINGULAR ELEMENT RELOCATION AND RE-RESH
28 THE NEW SPECIAL ELEMENTS. ///////////)
340 FORMAT (1XL, '28X, 71H **** CRACK-TIP POSITION IS NOT DEFINED AT TH
15s TIME ON INPUT CARDS.*****
350 FORMAT (1XL, '28X, 96H ***** THE SINGULAR ELEMENT IS WRONG *****
1, 16X, 16H THE MATERIAL TYPES IN SINGULAR ELEMENT ARE NO LONGER CORRE
SUBROUTINE VELOC(TT,TTV,L,LV)
C THIS SUBROUTINE COMPUTES THE CRACK TIP VELOCITY
C FROM A GIVEN VELOCITY HISTORY
C
COMMON/VEL,CV,NUMCV,CVE(2,28)
COMMON/BE11/DELT,DT1,DT2,BETA,BETA1,BET2,BET4,BET6,BAND,NBD2
C L=0 FIND APPROPRIATE TIME ONLY AND DETERMINE IF THERE IS A JUMP
C L=1 GIVEN TIME AND JUMP CONDITION FIND VELOCITY MAGNITUDE
C LV=0 NO JUMP
C LV=1 JUMP

IF(L.EQ.0) GO TO 2
IF(L.EQ.1) GO TO 66
2 IF(NUMCV.NE.0) GO TO 4
TTV=TT+DT1
LV=0
GO TO 100
4 IF(TT.LT.CVH(1,1)) GO TO 8
IF(TT.GT.CVH(1,NUMCV)) GO TO 8
GO TO 16
6 TTV=TT+DT1
IF(TTV.GT.CVH(1,1)) TTV=CVH(1,1)
LV=5
GO TO 100
8 TTV=TT+DT1
LV=6
GO TO 100
16 DO 88 1=1,NUMCV
J=1
IF(I.EQ.NUMCV) GO TO 28
IF(TT.GT.CVH(1,I)) GO TO 58
IF(TT.EQ.CVH(1,I) .AND. TT.EQ.CVH(1,I+1)) GO TO 48
IF(TT.EQ.CVH(1,I)) GO TO 58
28 J=J-1
88 TTV=TT+DT1
IF(TTV.GT.CVH(1,J+1)) TTV=CVH(1,J+1)
LV=5
GO TO 100
48 IF(CVH.EQ.CVH(2,I)) GO TO 42
TTV=TT+DT1
IF(TTV.GT.CVH(1,I+2)) TTV=CVH(1,I+2)
LV=9
GO TO 100
42 TTV=TT
LV=1
GO TO 100
58 CONTINUE
66 IF(NUMCV.NE.0) GO TO 67
CV=0.
GO TO 100
67 IF(TT.GE.CVH(1,1) .AND. TT.LE.CVH(1,NUMCV)) GO TO 68
CV=0.
GO TO 100
68 DO 78 M=2,NUMCV
IF(TT.GE.CVH(1,M)) 88,88,78
78 CONTINUE
88 DT=TT-CVH(1,M-1)
D1=CVH(1,M)-CVH(1,M-1)
D2=CVH(2,M)-CVH(2,M-1)
CV=CVH(2,M-1)+DT*D2/D1
GO TO 100
90 IF(CVH(1,M) .EQ. CVE(1,M-1)) GO TO 92
IF(M.EQ.NUMCV) GO TO 91
IF(CVE(1,N) .EQ. CVEH(1,N+1)) GO TO 93
91 CV=CVEH(2,N)
GO TO 100
92 IF(LV.EQ.1) CV=CVEH(2,N)
IF(LV.EQ.0) CV=CVEH(2,N-1)
GO TO 100
93 IF(LV.EQ.1) CV=CVEH(2,N+1)
IF(LV.EQ.0) CV=CVEH(2,N)
100 RETURN
END
SUBROUTINE LOAD (TT, TDP, L, LL)
C
COMMON/BE1/NUMMAT, NUMNP, NUMEL, NUMPC, NUMLP, IHED (8), NTAPE, NEQ, ICOND
C
COMMON/BE11/DINC, DT1, DT2, BET4, BET6, NDD2, NDD3, NDD4, NDD5, NDD6
C
COMMON/SHIFT/NDEL, NDEL, LHSTB, NF, NRRED, HELTE, ISX, ICODE
C
COMMON/PRESS/INQ (100), T1 (100), T2 (100), T1 (100), P (2, 20), PP
C
COMMON/EX16/IX (1000), Z (600), CODE (600), IX (6, 250)
C
COMMON/SSS/S15 (86, 28), S1SD (86, 28), S1SSD (86, 28)
COMMON/MASS/AMR (86, 86), AMRDR (86, 86), AM22 (2, 2), AMR2 (86, 2)
C
1, AMR2 (86, 2), AMR22 (86, 2)
C
COMMON/VV/AREA (1484)
C
COMMON/LLS /L (36, 36), S4 (36, 36), S2D (36, 36), S2DD (36, 36)
C
DIMENSION B (666), ELOAD (666), EM (366), EMA (366)
C
DIMENSION US (666), VS (666), AS (666)
C
DIMENSION EXI (2168), EXH (1484), EXG (5184)
C
EQUIVALENCE (E (1), EXI (1))
C
EQUIVALENCE (ELOAD (1), EXI (1001))
C
EQUIVALENCE (EMAT (1), EXI (1))
C
EQUIVALENCE (US (1), EXG (1))
C
EQUIVALENCE (VS (1), EXG (1001))
C
EQUIVALENCE (AS (1), EXG (2001))
C
EQUIVALENCE (EM (1), EXG (3001))
C
EQUIVALENCE (S1S (1, 1), EXI (1))
C
EQUIVALENCE (VAREA (1), EXG (1))
C
EQUIVALENCE (S2 (1, 1), EXG (1))
C
C
L=0 FIND APPROPRIATE TIME ONLY AND DETERMINE IF THERE IS A JUMP
C
L=1 GIVEN TIME AND JUMP CONDITION FIND LOAD MAGNITUDE AND LOAD MATRIX
C
L=2 GIVEN TIME AND JUMP CONDITION FIND LOAD MAGNITUDE ONLY
C
L=3 GIVEN TIME AND LOAD MAGNITUDE FIND LOAD MATRIX ONLY
C
LL=0 NO JUMP
C
LL=1 JUMP
C
IF (L.EQ.0) GO TO 4
IF (L.EQ.1) GO TO 66
IF (L.EQ.2) GO TO 66
IF (L.EQ.8) GO TO 66
C
4 IF (TT.LT.P (1, 1)) GO TO 6
IF (TT.GT.P (1, 1)) GO TO 8
GO TO 18
C
66 TTP=TT-DT1
IF (TTP.GT.P (1, 1)) TTP=P (1, 1)
LL=6
GO TO 166
C
166 TTP=TT-DT1
LL=6
GO TO 166
C
DO 56 I=1, NUMLP
C
56 IF (I.EQ.NUMLP) GO TO 56
IF (TT.GT.P (I, 1)) GO TO 56
IF (TT.EQ.P (I, 1)) GO TO 46
IF (TT.EQ.P (I, 1)) GO TO 56
C
DO 56 I=1, NUMLP
C
56 TTP=TT-DT1
IF (TTP.GT.P (I, 1)) TTP=P (I, 1)
LL=6
GO TO 166
C
46 IF (PP.EQ.P (2, I)) GO TO 42
TTP=TT-DT1
IF (TTP.GT.P (1, 1)) TTP=P (1, 1)
LL=6
GO TO 166
42 TTP=TT
   LL=1
   GO TO 100
50 CONTINUE
60 DO 65 I=1,NF
65 ELOAD(I)=S.
   IF(L.EQ.2) GO TO 95
C
C APPLY PRESSURE LOAD
C
65 IF (NUMPC.NE.0) GO TO 67
   PP=S.
   GO TO 100
67 IF (TT.GE.P(1,1) .AND. TT.LE.P(1,NUMLP)) GO TO 68
   PP=S.
   GO TO 100
68 DO 70 M=2,NUMLP
   IF (TT=P(1,M)) 80,90,70
70 CONTINUE
80 DT=TT-P(1,M-1)
   DI=P(1,M)-P(1,M-1)
   D2=P(2,M)-P(2,M-1)
   PF=P(2,M-1)*DT+DZ/D1
   IF (L.EQ.2) GO TO 100
   GO TO 95
90 IF (P(1,M) .EQ. P(1,M-1)) GO TO 92
   IF (M.EQ.NUMLP) GO TO 91
   IF (P(1,M) .EQ. P(1,M+1)) GO TO 93
91 PF=P(2,M)
   GO TO 94
92 IF (LL.EQ.1)PF=P(2,M)
   IF (LL.EQ.2)PF=P(2,M-1)
   GO TO 94
93 IF (LL.EQ.1)PF=P(2,M+1)
   IF (LL.EQ.2)PF=P(2,M)
94 IF (L.EQ.2) GO TO 100
95 IF (NUMPC.EQ.0) GO TO 100
DO 98 N=1,NUMPC
   IF (TT.LT.T(N)) GO TO 98
   IF (T(N).GT.P(1,1) .OR. T(N).LT.P(1,NUMLP)) GO TO 98
   I=1
   J=1
   DR=E(R(J)-R(I))/6.0
   DZ=(Z(J)-Z(I))/6.0
   Q1=FI(N)
   Q2=FI(N)
   RX=Q1+Q2
   IX=Q1+Q2
   I=I+1
   END
98 CONTINUE
100 RETURN
SUBROUTINE DATOUT (L, TT)

C

COMMON /BK1, NUMMAT, NUMNP, NUMEL, NUMPC, NUMLP, IXED (8), ITAPE, NREQ, ICOND
1, ISTAT, TBC, IBER (8), IYMT
COMMON /BK2, NDOUT (16), NSTOUT (16), TIX, TIX2, TIXD, SIDD, NUMDS, NUMST
COMMON /DIMHA, NAA, NBT, NBI, NQ, NDE, NITM, N2T2, IALL, NITER, SINCOD
COMMON /INTC/ PT (16), WG (16), PTZ (2), WZ (2), PEI
COMMON /DEP/DX1, DX2, DXY, DX2, FXY2, DXY, DUX, DUE, DUX, DSU
COMMON /IFP/ FJ1, PJ2, PJ3, FJ4, PJ4, DJ1, DJX
COMMON /TIP/NCR1, NCR2, NCR3, NCR4, NELX, CTPX, CTPY, SFP1, SFP2
COMMON /B1/E0 (300), Z (300), CODE (300), IX (6, 256)
COMMON /PRESS/ INJ (1000), INJ (1000), PI (1000), PJ (1000), T (1000), P (2, 20), PP
COMMON /VEL/CV, NUMOV, CVE (2, 256)
COMMON /STAN/E (3, 3, 6), BI (3, 3, 6), AD (3, 6), ASIZ (6), CL
COMMON /SEPIF/ NSB, NSBL, MAST, NF, NRED, NJEP, ISP, IPIC
COMMON /RRE/ RHO (8), RODUM
COMMON /RIG/ RIG (6), RIG (6)
COMMON /ؤول/ U (300), V (300), A (300)
DIMENSION C (3, 6), A (2, 2), B (3, 3), AJJ (3, 3)
DIMENSION U (3), BI (3, 3), S (3)
DIMENSION B1 (74)
COMMON /WE2/ NWE (728)
EQUIVALENCE (B1 (1), BI (1))
EQUIVALENCE (AJJ (1), BI (9))
EQUIVALENCE (C (1), BI (88))
EQUIVALENCE (A (1), BI (57))
EQUIVALENCE (U (1), BI (81))
EQUIVALENCE (B1 (1), BI (69))
EQUIVALENCE (S (1), BI (72))
EQUIVALENCE (B1 (1), WE2 (1))
COMMON /STRESS/ RX (3, 36), RXD (3, 36), RXDD (3, 36)
COMMON /BET/ BETA1 (36), BET1 (36), BET1D (36), BET1D (36)

C

WRITE (6, 2001) TT, CV, CTPX, CTPY, PF
WRITE (6, 400) SFP1, SFP2
WRITE (6, 100) N
DO 10 I = 1, N
WRITE (6, 128) N, U(I), U(J), V(J), V(J), A(I), A(J), N
CONTINUE
WRITE (6, 188)
IF (L.EQ.0 .AND. ICOND.EQ.0) GO TO 20
DO 12 I = 1, NUMAT
IF (B1 (I) .EQ. 0) GO TO 15
CONTINUE
GO TO 28
10 CONTINUE
WRITE (6, 510)
20 CONTINUE
IF (L.EQ.0 .AND. ICOND.EQ.0) GO TO 25
GO TO 28
CONTINUE
S (1) = W
S (2) = S
S (3) = W
DO 29 M = 1, NUMAT
N = W
IF (NUMST.EQ.0) GO TO 27
N = NSTOUT (M)
CONTINUE
WRITE (6, 148) N, (S (I), I = 1, 3), N
CONTINUE
GO TO 165

C 30 DO 165 M=1, NUMST
N=M
IF (NUMST.EQ.NUMSEL) GO TO 32
N=NSTOUT(I)
32 CONTINUE
IF (N.EQ.NC81) GO TO 161
IF (N.EQ.NC82) GO TO 162
IF (N.EQ.NC83) GO TO 163
IF (N.EQ.NC84) GO TO 164

X=S.
Y=S.
CALL TRANS(X,Y,N,2,2)
CALL DIFFB(X,Y)
N1=IX(1,N)+2
N2=IX(2,N)+2
N3=IX(3,N)+2
N4=IX(4,N)+2
U1(1)=U(N1-1)
U1(2)=U(N1)
U1(3)=U(N3-1)
U1(4)=U(N2)
U1(5)=U(N3)
U1(7)=U(N4-1)
U1(8)=U(N4)
DO 60 I=1,3
60 E1(I)=S.
DO 61 J=1,8
E1(I)=E1(I)+G(I,J)+U1(J)
61 CONTINUE
K=IX(6,N)
DO 65 I=1,3
S(I)=S.
DO 65 J=1,3
S(I)=S(I)+E1(I,J,K)+E1(I)
65 CONTINUE
GO TO 165

C 101 X=+5
Y=+5
GO TO 165
102 X=-5
Y=-5
GO TO 165
103 X=+5
Y=-5
GO TO 165
104 X=-5
Y=-5
105 CONTINUE
CALL TRANS(X,Y,N,2,1)
CALL FUNCTS(X,Y,8)
DO 110 J=1,8
S(J)=S.
DO 110 I=1,NA
S(J)=S(J)+RX(J,I)*BETA1(I)
110 CONTINUE
155 WRITE(*,148) N,(S(I),I=1,8),N
160 CONTINUE
165 CONTINUE
RETURN

C 168 FORMAT (1H1)
100 FORMAT (               //,6X,11SH NODAL POINT R-DISPLACEMENT Z-DISPLA
1CMENT R-VELOCITY Z-VELOCITY R-ACCELERATION Z-ACCELERA
2ON NODAL POINT)
120 FORMAT (18X,4,1PE13.4,I0)
130 FORMAT (               //,25X,9SH ELEMENT NO. XX-STRESS YY-STRES
15 XY-STRESS ELEMENT NO.)
140 FORMAT (25X,4,1PE13.4,I0)
408 FORMAT (/45X,8H K1=,1PE12.4,8H K2=,1PE12.4)
518 FORMAT (/25X,36H THE STRESSES IN,
1 SOME RIGID MATERIALS, EXCEPT FOR THE SINGULAR ELEMENT, ARE SET,
2 /,25X,37H ARBITRARILY EQUAL TO ZERO AND DO NOT,
302HERESENT THE TRUE VALUES OF THE STRESSES IN THIS MATERIAL-----)
2001 FORMAT (1X1, //,10X,12HCURRENT TIME,10X,16HCURRACK-TIP VELOCITY,10X,
12HCRACK-TIP POSITION-X,10X,12HCRACK-TIP POSITION-Z,10X,4HLOAD,/,216X,4,1PE13.4,12X,1PE13.4,10X,1PE13.4,10X,1PE13.4,9X,1PE13.4)
C
END
SUBROUTINE TAPOUT(II)
COMMON/VRL/CV,NUMCV,CVBS(2,20),
COMMON/PRESS/INFJXN100,FIN100,PIJ100,PIJ2100,T(100),P(2,20),PF
COMMON/EK18/ E(200),E(200),CODE(200),IE(0,250)
COMMON/FAIR1/NFAIR,IFAIR(3,40)
COMMON/IPLN/NIPLN(5),IPLN(2)
COMMON/RENO/RENO(5),RODON
COMMON/SUMBA/AI(3,3,3),AI(3,3,3),ADET(3,3),ASIZE(3),CL
COMMON/SHFT/NSHL,NDEL,LASTB,FB,NBRED,NELTP,IKB,UCODE
COMMON/BK1/DHT1,DHT2,BETA1,BETA2,BETA3,BETA4,BETP,NBAND,NBND2
COMMON/TIP/NCOR,NCOR2,NCOR4,NCOR5,NCOR6,NCOR7,CTPY,CTPF,SIP1,SIP2
COMMON/DIM/NA,NAA,NBT,NNB,NQ,NN,NINT,NINT2,ILL,IKT,SMC
COMMON/BI/KNUMMAT,NUMNP,NUMEL,NUMPC,NUMLP,INRED(8),NFAPE,NNQ,IOND
1,ISTAT,TMAGC,INRED(8),ISYM
COMMON/OLDDISP/GU(400),V(400),A(400)
COMMON/NSHL/IP(20)
COMMON/MAIN/COORD(10,2)
COMMON/RIG/IRIG1(5),IRIG2

C
COMMON/AB/P15(26,26),P15D(26,26),P15DD(26,26)
1,AT(2,26),ATD(2,26),ATDD(2,26)
COMMON/MMAT/AM1(26,26),A1(26,26),AK4(26,26)
COMMON/IXMAT/AXIS(26,26)
COMMON/ELM/RSTFIM(8,8,68),RELMASS(8,8,68),YSS(20,20)
COMMON/BET/BEBAI(30),CBETAID(30),CBETAID2(2)

C
REWIND 1
WRITE(16) NUMMAT,NELTP,NUMNP,NNQ,NUMEL,IALL,NBAND,NBRED,INRED1
WRITE(16) IPI1,IPNL1
WRITE(16) ((AI(J,J,K),J=1,3),K=1,3),I=1,NUMMAT
WRITE(16) ((AI1(I,J,K),I=1,3),J=1,3),K=1,NUMMAT
WRITE(16) (KHO(K),ADET(K),ASIZE(K),K=1,NUMMAT),CL
WRITE(16) (IIRIG1(K),K=1,NUMMAT),IRIG2
WRITE(16) NRED2,NSHL,NDEL,LASTB,NN
WRITE(16) (IP(I),I=1,NQ)
WRITE(16) (NNQ(I),I=1,NNQ)
WRITE(16) ((CDER(I,J),I=1,NQ1),J=1,2)
WRITE(16) T1,CV,CTPX,CTPF,PF,SIP1,SIP2,NBT,NINT,NA
WRITE(16) NCOR1,NCOR2,NCOR4,NCOR5,NCOR6,NCOR7
WRITE(16) (U(I),I=1,NNQ)
WRITE(16) (V(I),I=1,NNQ)
WRITE(16) (A(I),I=1,NNQ)
WRITE(16) ((AK4(I,J),I=1,NNQ),J=1,NNQ)
WRITE(16) ((AK1(I,J),I=1,NNQ),J=1,NNQ)
WRITE(16) ((V1(I,J),I=1,NNQ),J=1,NNQ)
WRITE(16) ((YSS(I,J),I=1,NNQ),J=1,NNQ)
WRITE(16) ((AM1(I,J),I=1,NNQ),J=1,NNQ)
WRITE(16) ((RSTFIM(I,J,K),I=1,8),K=1,8),I=1,NELTP
WRITE(16) ((RELMASS(I,J,K),I=1,8),K=1,8),I=1,NELTP
WRITE(16) (BEBAI(I),I=1,NA)
WRITE(16) (PI1(I,J),I=1,NA),J=1,NNQ)
WRITE(16) (AT(I,J),I=1,NNQ),J=1,NNQ)
WRITE(16) (PI1D(I,J),I=1,NA),J=1,NNQ)
WRITE(16) (P15D(I,J),I=1,NA),J=1,NNQ)
WRITE(16) (P15DD(I,J),I=1,NA),J=1,NNQ)
WRITE(16) (ATD(I,J),I=1,NNQ),J=1,NNQ)
WRITE(16) (ATDD(I,J),I=1,NNQ),J=1,NNQ)
WRITE(16) (IIX(I,J),I=1,8),J=1,NUMEL)
WRITE(16) ((NFAIR(I,J),I=1,3),J=1,NFAIR)
RETURN
END
SUBROUTINE TAPIX(T)
COMMON/VEL/CY,NUMCV,CYH(2,28)
COMMON/PRESS/I(100),J(100),F(100),T(100),P(2,28),PF
COMMON/BK16/ Z(300),X(300),CODE(300),IX(6,350)
COMMON/PAIR/PAIR,IPAIR(3,40)
COMMON/IPLM/IPLM(5),IPLM1(2)
COMMON/EHBO/EBD(3),NATOM
COMMON/SUMM/AI(3,3,3),AI(3,3,3),ADET(6),ASIC(6),CL
COMMON/SIGT/NSIG,NSIGL,LASTB,NF,MBRED,STETYP,ISIG,rcode
COMMON/BX1/B1,B2,B3,B1A,B1T,B2T1,B2T2,B3T4,MBAND,MB2
COMMON/TYP/NCR1,NCRC2,NCRC2,NCRC4,NCRL,CTPX,CTPY,SIP1,SIP2
COMMON/DIM/NA,NA,MBT,MBT,NQ,NS,NINT,NT1,NAAT,MBRED,MNCOD
COMMON/BK1/RUNMAT,NUMNP,NUMEL,NUMPC,NUMLP,THED(16),NTAPE,NEQ,TOCOND
1,ISTAT,TMACI,THEHD(16),ISYM
COMMON/OLDISP/U(N),V(N),A(N)
COMMON/MISL/IP(2)
COMMON/MAIN/CORD(16,2)
COMMON/IRIG/IRIG1(6),IRIG2

C
COMMON/AB/P15(16,28),P1SD(36,28),P1SD(28,28)
1, AT(2,28),ATD(2,28),ATDD(2,28)
COMMON/MMAT/AM1(28,28),V1(28,28),AK4(28,28)
COMMON/EMAT/AXIS(28,28)
COMMON/BLM/BSTIFM(8,8,38),BELMASS(8,8,38),VSS(28,28)
COMMON/BETA/BETA1(28),BETA1P(28),BETA1D(28)

C
REWRITE 16
READ (16) NUMMAT,STETYP,NUMNP,NEQ,NUMEL,IA,MBAND,MBRED,THEHD
READ (16) IPLM,IPLM1
READ (16) (*91(T(I,J),I=1,3),J=1,3),K=1,NUMMAT
READ (16) (*91(AI(I,J,K),I=1,3),J=1,3),K=1,NUMMAT
READ (16) (*91(AI(I,J,K),I=1,3),J=1,3),K=1,NUMMAT
READ (16) (*91(BEO(X),ADEX(X),ASIZE(X),K=1,NUMMAT),CL
DO 5 K=1,NUMMAT
WRITE(6,816) K
 WRITE(6,816) K
CONTINUE
READ (16) (IRIG1(K),K=1,NUMMAT),IRIG2
READ (16) NBD2,NSD,NSD,LASTF,NF
READ (16) (IF(I),I=1,NQ)
NQ1=NQ/2
READ (16) ((CGBD(I,J),I=1,NQ1),J=1,2)
READ (16) TT,CV,CTP,F,CTPY,PF,SIP1,SIP,STET1,STET2,NA1
READ (16) NC1,NC2,NCB,NCR4,NSLE
READ (16) (U(I),I=1,NEQ)
READ (16) (V(I),I=1,NEQ)
READ (16) (A(I),I=1,NEQ)
READ (16) ((AK4(I,J),I=1,NQ),J=1,NQ)
READ (16) ((AXIS(I,J),I=1,NQ1),J=1,NQ)
READ (16) (V1(I,J),I=1,NQ),J=1,NQ)
READ (16) ((VSS(I,J),I=1,NQ),J=1,NQ)
READ (16) ((AM1(I,J),I=1,NQ),J=1,NQ)
READ (16) ((BSTIFM(1,J,K),I=1,8),J=1,NQ)
READ (16) ((BELMASS(1,J,K),I=1,8),J=1,NQ)
READ (16) (BETA1(I),I=1,NA1)
READ (16) ((PIS(I,J),I=1,NA1),J=1,NQ)
READ (16) ((ST(I,J),I=1,NE),J=1,NQ)
IF(CV.EQ.0.) GO TO 16
READ (16) ((PISD(I,J),I=1,NA1),J=1,NQ)
READ (16) ((P1SDD(I,J),I=1,NA1),J=1,NQ)
READ (16) ((ATD(I,J),I=1,NE),J=1,NQ)
READ (16) ((ATDD(I,J),I=1,NE),J=1,NQ)
READ (16) (CDDB(I,J),I=1,NUMNP)
READ (16) ((T(I,J),I=1,6),J=1,NUE)
READ (16) NFAE,((IPAIR(I,J),I=1,3),J=1,NFAE)
IF(IRIG2.EQ.0.) GO TO 12
NBT1=NBT1+2
IF(NBT.EQ.NBT1 .OR. (NBT+1).EQ.NBT1) GO TO 13
SINCOD=1.
GO TO 14
12 IF(NBT1.NE.NBT) SINCOD=1.
13 IF(MINTE.NE.MINT) SINCOD=1.
14 IF(MINTE.NE.MINT) RCODE=1.
RETURN
816 FORMAT(/,66X,11H MATERIAL # ,I2)
811 FORMAT(/,46X,26H DENSITY-------------------,1PE12.4)
END
SUBROUTINE SINGEL

COMMON/ER16/, Z(300), IX(300), CODE(300), IX(6,255)
COMMON/EREG/IX(6), RMOD
COMMON/TIP/NCR1(NCR2,NCR4), MELX, CTFX, CTFY, SIE1, SIF2
COMMON/AMAT/ANM1(28,28), VI(28,28), AK4(28,28)
COMMON/EMAT/AK15(28,28)
COMMON/ELMF,STIFM(8,8,58), RLMAS(8,8,58), VSS(28,28)
COMMON/VEL/CV, NUMCV, CVX(2,28)
COMMON/DIM/NA, NAA, NBB, NBQ, NQ, NEK, NINT, NINT2, IALL, NITER, SINCOD
COMMON/LON/S2(28,28), S4(28,28), S2D(28,28), S2DD(28,28)
COMMON/NI/NIAREA(1484)
COMMON/MISL/IP(28)

CALL LINEI

NORMALIZE S2 FOR NUMERICAL ACCURACY IN THE INVERSION
SSS=S2(1,1)
IF(SSS.EQ.0.) GO TO 91
DO 12 J=1, NB
DO 12 I=1, NB
S2(I,J)=S2(I,J)/SSS
12 CONTINUE

IDICT=2
IDICT1=3
CALL LINVF(S2,NA,NAA,S4,IDICT1,NIAREA,IER)
IF(IER.EQ.129 .OR. IER.EQ.131) GO TO 96
DO 14 J=1, NB
DO 14 I=1, NB
S2(I,J)=S2(I,J)+SSS
S4(I,J)=S4(I,J)/SSS
14 CONTINUE

CALL STIF

IF(RMOD.EQ.5.) GO TO 45

CALL MAREA

CALL MASSW

IF(CV.EQ.0.) GO TO 45
DO 28 J=1, NQ
DO 28 I=1, NQ
AK4(I,J)=AK4(I,J)+AK15(I,J)
28 CONTINUE

DO 26 J=1, NQ
DO 26 I=1, NQ
AK15(I,J)=.5*(AK4(I,J)+AK4(J,I))
VSS(I,J)=.5*(V1(I,J)+V1(J,I))
25 CONTINUE

DO 28 J=1, NQ
DO 28 I=1, NQ
AK4(I,J)=AK4(I,J)-AK15(I,J)
V1(I,J)=V1(I,J)-VSS(I,J)
26 CONTINUE

GO TO 88
45 CONTINUE

C FOR CV=5, "AK15" IS AUTOMATICALLY SYMMETRIC EXCEPT MAYBE FOR NUMERICAL
C TRUNCATIONS; THIS DO THE FOLLOWING TO GET A PERFECTLY SYMMETRIC "AK15"
C AND THEN SET "AK4(I,J)=0." DOWN BELOW
DO 45 J=1,NQ
DO 45 I=1,NQ
AX4(I,J)=.5*(AX1S(I,J)-AX1S(J,I))
45 CONTINUE
DO 50 J=1,NQ
DO 50 I=1,NQ
AX1S(I,J)=AX1S(I,J)-AX4(I,J)
50 CONTINUE
TF(ROD,UM.R,Q.) GO TO 68
DO 65 J=1,NQ
DO 65 I=1,NQ
VSS(I,J)=0.
V1(I,J)=0.
AX4(I,J)=0.
65 CONTINUE
RETURN
90 WRITE(6,301) IER,IDIGT,IDIGT1
STOP 5
91 WRITE(6,302)
STOP 6
301 FORMAT(/,26X,29H*****INVERS FAILED ON S2*****/,26X,9HERROR NO.,4,
1/2X,36HPROPOSED DECIMAL DIGITS OF ACCURACY WAS,I4,2X,51H BUT THE
2-IMSL LIBRARY ROUTINE "LINV2F" FOUND ONLY,I4,2X,36H DECIMAL DIGITS
3OF ACCURACY.
4/2X,47HERROR NO. 34=DECIMAL DIGIT ACCURACY TEST FAILED,
5/2X,48HERROR NO. 129=MATRIX IS ALGORITHMICALLY SINGULAR,
6/2X,46HERROR NO. 181=MATRIX IS TOO ILL-CONDITIONED FOR ITERATIVE
7PROGRESSION TO BE EFFECTIVE,
8/18X,26H*****TRY CHANGING MATERIAL PROPERTIES OR CRACK-TIP SPEED****
9/38X,26H*****INVERS FAILED ON S2*****/,13H S2(1,1)=0.)
C
END
SUBROUTINE LINR1

COMMON/REHO/REHO(9), RDMUW
COMMON/LO/H(36,36), S4(36,36), S2D(36,36), S2DD(36,36)
COMMON/SS/SS(36,36), S1SD(36,36), S1SDD(36,36)
COMMON/INTGR/PT(16), NG(16), PT(2), NG2(2), PI
COMMON/SUM/AN(3,3), AI(3,3), ADI(3,3), ASI(3,3), CL
COMMON/DIF/DX, DX1, DX2, DXY, DYY, DXY1, DYY1, DX1Y, DXY, DYY, DXY, DYY, DXY1, DYY1, DX1Y1, DXY1Y, DYY1Y
COMMON/IPAR/P, P11, P12, P3, P4, DXY1, DXY2
COMMON/TIP/NCR1, NCR2, NCR3, NCR4, NCR5, NCR6, NCR7, CTIP, CTIP, S1P1, S1P2
COMMON/VEL/CV, NUNCV, CV(2,80)
COMMON/DIM/NA, NAA, NBT, NB, NQ, NNN, NINT2, IALL, NTER, SINCOD
COMMON/PHYS/ANX, ANX, A(2)
COMMON/RIH(I2(36,36), I3(36,36), CODE(36,36), IX(3,280)
COMMON/DISP/UD(2,36), UDE(2,36), UDR(2,36), UDR(2,36)
COMMON/STRESS/IX(3,36), IEX(3,36), IEDD(3,36)
COMMON/ELF/G(2,28), PGT(2,28), PST(2,28), PSTD(2,28), PSTDD(2,28)
1, P4(36,36), P4D(36,36), P4RD(36,36)
COMMON/MASS/AMRE(36,36), AMIRE(36,36), AME(2,2), AM2(36,36)
A, AMERD2(36,36), AMERDD2(36,36)
COMMON/WK/WKAREA(1484)
COMMON/INTPOL/ALS(2,36)
COMMON/KR/KR(6), KR(6)
DIMENSION R(2,36), RD(2,36), RDD(2,36)
DIMENSION R2(2,2)
EQUIVALENCE (R(1,1), WKAREA(1))
EQUIVALENCE (RD(1,1), WKAREA(1))
EQUIVALENCE (RDD(1,1), WKAREA(2))
EQUIVALENCE (R2(1,1), WKAREA(3))
COMMON/S2K/R2(36)
DIMENSION K2G(2)
DIMENSION EPS(3,36)
EQUIVALENCE (K2G(1), WKAREA(486))
EQUIVALENCE (EPS(1,1), WKAREA(581))

C NOTE THAT MATR EXH1 AND MATRIX P11=INTGR(11) ARE THE TRANSPOSE
C OF EACH OTHER EXCEPT FOR THE FORTH ROW OF MATRIX P11 WHICH CORRESPONDS
C TO RIGID BODY ROTATION
C IN THIS SUBROUTINE THE MATRIX S2 IS CALCULATED AS THE INTEGRAL OF
C R1U1 WHERE THE ELEMENTS OF SURFACE TRACTION MATRIX R ARE ASSUMED
C TO BE DERIVED FROM THE INTERNAL STRESSES OF THE SINGULAR ELEMENT
C BUT SINCE THE TERMS CORRESPONDING TO RIGID BODY ROTATIONS DO NOT
C CONTRIBUTE TO THESE TRACTIONS, SOME ARBITRARY TRACTIONS CORRESPONDING
C TO A RIGID BODY ROTATION ARE CHosen IN ORDER TO MAKE THE MATRIX
C S2=P11 NON-SINGULAR, THESE TRACTIONS ARE PLACED IN THE FORTH COLUMN
C OF MATRIX R. HOWEVER THIS WILL CHANGE THE TRUE VALUES OF THE ELEMENTS
C IN THE FORTH ROW OF S2 THEREFORE WE HAVE CALCULATED THE MATRIX S2K
C WHICH CONTAINS THE TRUE VALUES OF THE FORTH ROW OF S2. THESE TRUE
C VALUES OF S2 ARE REPLACED WITH THE PSKDD VALUES AT THE END OF
C THE SUBROUTINE STALE, WHERE THE TRUE VALUES OF S2-H1 ARE NEEDED TO
C CALCULATE BBD

IF(CV EQ .0.) GO TO 400
DO 8 J=1,NQ
DO 8 I=1,NA
S1S(J,J)=0.
S1SD(J,J)=0.
S1SDD(J,J)=0.
8 CONTINUE
DO 4 J=1,NR
DO 4 I=1,NA
P4(J,J)=0.
P4D(J,J)=0.
4 CONTINUE
DO 5 J=1,NB
DO 5 I=1,NA
S2(I,J)=0.
S2D(I,J)=0.
S2DD(I,J)=0.
5 CONTINUE
DO 6 I=1,NA
S2EIG(I)=0.
6 CONTINUE
DO 7 J=1,NQ
DO 7 I=1,NE
G2(I,J)=0.
7 CONTINUE
DO 8 I=1,NE
DO 8 J=1,NQ
AMRD2(I,J)=0.
AMRDD2(I,J)=0.
P3T(I,J)=0.
P3TD(I,J)=0.
P3TDD(I,J)=0.
8 CONTINUE
DO 9 J=1,NQ
DO 9 I=1,NE
P2T(I,J)=0.
9 CONTINUE
CA=0.

C
C LOOP ON SIDES
C
DO 300 LSIDE=1,4
CALL NORMAL(LSIDE)
IF(LSIDE.EQ.1) GO TO 10
IF(LSIDE.EQ.2) GO TO 15
IF(LSIDE.EQ.3) GO TO 20
IF(LSIDE.EQ.4) GO TO 25
10 CONTINUE
Y=-1.
E2(1,1)=0.
E2(2,1)=0.
E2(1,2)=0.
E2(2,2)=1.
REIG(1)=1.
REIG(2)=0.
GO TO 30
15 CONTINUE
X=1.
E2(1,1)=1.
E2(2,1)=0.
E2(1,2)=0.
E2(2,2)=0.
REIG(1)=0.
REIG(2)=1.
GO TO 30
20 CONTINUE
Y=1.
E2(1,1)=0.
E2(2,1)=0.
E2(1,2)=0.
E2(2,2)=1.
REIG(1)=1.
REIG(2)=0.
GO TO 30
25 CONTINUE
X=1.
E2(1,1)=1.
E2(2,1)=0.
E2(1,2)=.5.
E2(2,2)=.5.
ERIG(1)=.5.
ERIG(2)=-1.
30 CONTINUE
DO 200 L=1,2
DO 100 I=1,NINT
IF(LSIDE.EQ.1.AND.LH.EQ.1) GO TO 3001
IF(LSIDE.EQ.1.AND.LH.EQ.2) GO TO 3002
IF(LSIDE.EQ.2.AND.LH.EQ.1) GO TO 3003
IF(LSIDE.EQ.2.AND.LH.EQ.2) GO TO 3004
IF(LSIDE.EQ.3.AND.LH.EQ.1) GO TO 3005
IF(LSIDE.EQ.3.AND.LH.EQ.2) GO TO 3006
IF(LSIDE.EQ.4.AND.LH.EQ.1) GO TO 3007
IF(LSIDE.EQ.4.AND.LH.EQ.2) GO TO 3008
3001 X=.5*(PT(L)-1.)
GO TO 3009
3002 X=.5*(PT(L)+1.)
GO TO 3009
3003 Y=.5*(PT(L)-1.)
GO TO 3009
3004 Y=.5*(PT(L)+1.)
GO TO 3009
3005 X=-.5*(PT(L)-1.)
GO TO 3009
3006 X=-.5*(PT(L)+1.)
GO TO 3009
3007 Y=-.5*(PT(L)-1.)
GO TO 3009
3008 Y=-.5*(PT(L)+1.)
3009 ES=IX(6,NCE1)
IF(Y.LT.0.) KS=IX(8,NCE2)
RD=RHO(KS)
WT=WC(I)
CALL TRANS(X,Y,N,2,1)
C SCALE TO E2 AND ERIG TO COMPARE WITH THE VALUES OF OTHER ELEMENTS
C OF MATRIX R (THIS IS ONLY FOR NUMERICAL ACCURACY, ESPECIALLY FOR
C (THIS IS ONLY FOR NUMERICAL ACCURACY, ESPECIALLY FOR INVERSION PROCESS)
R2(1,1)=E2(1,1)+PJ1
R2(1,2)=E2(1,2)+PJ1
R2(2,1)=E2(2,1)+PJ1
R2(2,2)=E2(2,2)+PJ1
ERIG(1)=ERIG(1)+PJ1
ERIG(2)=ERIG(2)+PJ1
SCALE=E2Y1
IF(LSIDE.EQ.2.OR.LSIDE.EQ.4) SCALE=D2Y2
DY=AXY-SCALE
WS=PT+SCALE-GA
Wy=-WT*DY+GA-CV
Wy=Y1+RD
CALL FUNCTS(X,Y,2)
DO 35 M=1,NA
ED(1,M)=ANX*EDD(1,M)+ANY*EDD(3,M)
EDD(1,M)=ANX*EDDD(1,M)+ANY*EDDD(3,M)
ED(2,M)=ANX*EDD(2,M)+ANY*EDD(3,M)
EDD(2,M)=ANX*EDDD(2,M)+ANY*EDDD(2,M)
R(1,M)=ANX*RX(1,M)+ANY*RX(3,M)
R(2,M)=ANX*RX(2,M)+ANY*RX(2,M)
35 CONTINUE
C REPLACE THE ELEMENTS OF THE FOURTH COLUMN OF R WITH THE PSUDO-VALUES
C CORRESPONDING TO RIGID BODY ROTATION AND SAVE THE TRUE VALUES OF THE
C FOURTH COLUMN OF R IN MATRIX ERIG IN ORDER TO CALCULATE THE TRUE
C VALUES OF THE FOURTH ROW OF MATRIX S2 IN MATRIX S2RIG
IF(ERIG2.EQ.1) GO TO 36
IF(NBT.LT.2) GO TO 36
IF(NBT.GT.2) GO TO 36
BB1=R(1,4)
BB2=R(2,4)
R(1,4)=BBIG(1)
R(2,4)=BBIG(2)
BBIG(1)=BB1
BBIG(2)=BB2

CONTINUE
DO 40 J=1,NB
DO 40 I=1,NA
DO 40 K=1,2
S2(I,J)=S2(I,J)+R(K,I)*U(K,J)*WS
S2D(I,J)=S2D(I,J)+R(K,I)*U(K,J)*UD(K,J)*WS
S2DD(I,J)=S2DD(I,J)+R(K,I)*U(K,J)*UD(K,J)*
R(K,I)+UDD(K,J)*WS

CONTINUE
IF(IRIG2.EQ.1) GO TO 46
IF(NBT.LT.2) GO TO 46
DO 46 I=1,NA
DO 46 J=1,2
S2RIG(I)=S2RIG(I)+BBIG(J)*U(J,I)*WS

CONTINUE
DO 46 J=1,NR
DO 46 I=1,NA
DO 46 K=1,2
F4(I,J)=F4(I,J)+UR(K,J)*WS
F4D(I,J)=F4D(I,J)+UR(K,J)*UD(K,J)*WS
F4DD(I,J)=F4DD(I,J)+UR(K,J)*UD(K,J)*WS

CONTINUE
CALL INPOL(X,Y,LSIDE,LI)
DO 60 J=1,NQ
DO 60 I=1,NA
DO 60 K=1,2
S1S(I,J)=S1S(I,J)+R(K,I)*ALS(K,J)*WS
S1SD(I,J)=S1SD(I,J)+RD(K,I)*ALS(K,J)*WS
S1SDD(I,J)=S1SDD(I,J)+R(K,I)*ALS(K,J)*WS

CONTINUE
DO 60 J=1,NQ
DO 60 I=1,NA
DO 60 K=1,2
S2(I,J)=S2(I,J)+R2(K,I)*ALS(K,J)*WS
DO 60 J=1,NB
DO 60 I=1,NA
DO 60 K=1,2
AMRD2(I,J)=AMRD2(I,J)+UR(K,I)*WS
AMRD2D(I,J)=AMRD2D(I,J)+UR(K,I)*UD(K,J)*WS
AMRD2DD(I,J)=AMRD2DD(I,J)+UR(K,I)*UD(K,J)*WS

CONTINUE
PSTD(I,J)=PSTD(I,J)+R2(K,I)*UD(K,J)*WS
PSTD2D(I,J)=PSTD2D(I,J)+R2(K,I)*UD(K,J)*WS

CONTINUE
EPS(I,J)=EPS(I,J)+AL(I,K,KS)*RX(K,J)
CONTINUE
CONTINUE
CONTINUE
RETURN

CONTINUE
DO 460 J=1,NQ
DO 493 I=1,NA
   S15(I,J)=0.
493 CONTINUE
   DO 494 J=1,NB
   DO 494 I=1,NA
   P4(I,J)=0.
494 CONTINUE
   DO 495 J=1,NB
   DO 495 I=1,NA
   S2(I,J)=0.
495 CONTINUE
   DO 496 I=1,NA
   S2RIG(I)=5.
496 CONTINUE
   DO 497 J=1,NQ
   DO 497 I=1,NE
   CA(I,J)=5.
   DO 498 J=1,NQ
   DO 498 I=1,NE
   PST(I,J)=0.
498 CONTINUE
   DO 499 J=1,NE
   DO 499 I=1,Q
   PST(I,J)=5.
499 CONTINUE
   GA=5.
C
C   LOOP ON SIDES
C
   DO 700 LSIDE=1,4
      CALL NORMAL(LSIDE)
      IF (LSIDE.EQ.1) GO TO 410
      IF (LSIDE.EQ.2) GO TO 415
      IF (LSIDE.EQ.3) GO TO 420
      IF (LSIDE.EQ.4) GO TO 425
410 CONTINUE
   X=-1.
   Z2(1,1)=5.
   Z2(2,1)=5.
   Z2(1,2)=5.
   Z2(2,2)=1.
   BRIG(1)=1.
   BRIG(2)=0.
   GO TO 430
415 CONTINUE
   X=1.
   Z2(1,1)=1.
   Z2(2,1)=5.
   Z2(1,2)=5.
   Z2(2,2)=5.
   BRIG(1)=5.
   BRIG(2)=1.
   GO TO 430
420 CONTINUE
   X=-1.
   Z2(1,1)=5.
   Z2(2,1)=5.
   Z2(1,2)=5.
   Z2(2,2)=1.
   BRIG(1)=1.
   BRIG(2)=0.
   GO TO 430
425 CONTINUE
   X=-1.
   Z2(1,1)=1.
DO 445 J=1, NB
    DO 445 I=1, NA
    DO 445 K=1, 2
445 S2(I,J)=S2(I,J)+R(K,I)*U(K,J)*WS
    IF(K.EQ.2 .AND. K.GT.1) GO TO 445
    IF(NBT.LT.2 .AND. K.EQ.1) GO TO 445
    DO 445 I=1, NA
    DO 445 J=1, 2
    S2RIG(I)=S2RIG(I)+RRIG(J)*U(J,I)*WS
445 CONTINUE
    DO 450 J=1, NR
        DO 450 I=1, NA
        DO 450 K=1, 2
        P4(I,J)=P4(I,J)+R(K,I)*UR(K,J)*WS
450 CONTINUE
    CALL IMPOL(X,Y,LSIDE,LIB)
    DO 460 J=1, NQ
        DO 460 I=1, NA
            DO 460 K=1, 2
                S1S(I,J)=S1S(I,J)+R(K,I)*ALS(K,J)*WS
460 CONTINUE
    DO 485 J=1, NR
        DO 485 I=1, NA
            DO 485 K=1, 2
                G2(I,J)=G2(I,J)+R2(K,I)*ALS(K,J)*WS
485 CONTINUE
    DO 490 J=1, NR
        DO 490 I=1, NA
            DO 490 K=1, 2
                P3T(I,J)=P3T(I,J)+R2(K,I)*UT(K,J)*WS
490 CONTINUE
    DO 500 J=1, NR
        DO 500 I=1, NA
            DO 500 K=1, 2
                P3T(I,J)=P3T(I,J)+R2(K,I)*UR(K,J)*WS
500 CONTINUE
    700 CONTINUE
RETURN
END
SUBROUTINE STIPK

COMMON/MASS/AMRK(86,38), AMDR(86,38), AM22(2,2), AM22(86,2)
1, AMRD2(86,2), AMRDM2(86,2)
COMMON/EXAT/AKS(28,28)
COMMON/LEN/S2(36,36), S4(86,36), S2D(86,36), S2DD(86,36)
COMMON/SS1/S1S(86,28), S1SD(86,28), S1SDD(86,28)
COMMON/AB/B(86,28), B(86,28), B(86,28),
1 A(2,28), AD(2,28), ADD(2,28)
COMMON/DIM/NA, NPB, NLP, NQ, NR, N1R, NINF, NINT2, NALL, NITER, SINCOD
COMMON/ELIN/C(2,28), P(2,28), P(2,28), P(2,28), P(2,28), P(2,28), P(2,28)
1, P(2,28), P(2,28), P(2,28)
COMMON/VEL/CV, NUNCV, CVE(2,28)
COMMON/WR/WAREA(1464)
COMMON/WK2/WEAR(728)
DIMENSION PAA(86,28), BB1(86,28), BDD1(86,28), DDD1(86,28)
DIMENSION DD1(86,28), DDD2(86,28)
DIMENSION PGI(86,28)
DIMENSION AJP(86,28)
DIMENSION D(2,28), DD(2,28), DDD(2,28)
DIMENSION CDA(2,28), ADD1(2,28)
DIMENSION CDI(86,2), CDD1(86,2), CDD2(86,2)
DIMENSION PIP(86,2)
DIMENSION C1(2,28)
DIMENSION P(2,2), CD(2,2), CDD(2,2), CI(2,2)
EQUIVALENCE(PAA(1,1), BB1(1,1), BDD1(1,1), DDD1(1,1))
EQUIVALENCE(DD1(1,1), DDD2(1,1), AMDR(1,1))
EQUIVALENCE(PGI(1,1), AJP(1,1), WEAR(1))
DIMENSION BF(728)
EQUIVALENCE(BF(1,1), BF(1))
EQUIVALENCE(DI(1,1), DDD1(1,1), BF(1))
EQUIVALENCE(CDA(1,1), ADD1(1,1), BF(41))
EQUIVALENCE(CD1(1,1), BF(91))
EQUIVALENCE(CDD1(1,1), BF(124))
EQUIVALENCE(CDD2(1,1), BF(241))
EQUIVALENCE(PIP(1,1), BF(321))
EQUIVALENCE(C1(1,1), BF(481))
EQUIVALENCE(C(1,1), BF(481))
EQUIVALENCE(DD(1,1), BF(581))
EQUIVALENCE(C1(1,1), BF(611))
COMMON/SEIR/SEIRG(86)
COMMON/RI2/IR1G1(8), IR1G2

C

A

DO 5 J=1,NA
DO 5 I=1, NR
C(I,J)=S.
DO 5 K=1, NB
5 C(I,J)=C(I,J)+P(K,I)+S4(I,J)
DO 10 J=1, NR
DO 10 I=1, NR
C(I,J)=S.
DO 10 K=1, NA
10 C(I,J)=C(I,J)+C(I,K)+P(K,J)
DO 15 J=1, NR
DO 15 I=1, NR
C(I,J)=C(I,J)-P2(T,I,J)
15 CONTINUE

C

NORMALIZE C FOR NUMERICAL ACCURACY IN THE INVERSION
CCC=0(1,1)
IF(CCC.EQ.8.) GO TO 321
DO 16 J=1, NR

DO 16 I=1, NR
   C(I,J)=C(I,J)/CCC
  16 CONTINUE

   DI0=3
   IDICT=3
   CALL LINV2F(C,NR,NR,CI,IDICT,WER,IBR,IER)
   IF (IER.EQ.129) GOTO 226
   DO 17 J=1, NR
   DO 17 I=1, NR
      C(I,J)=C(I,J)*CCC
      CI(I,J)=CI(I,J)/CCC
   17 CONTINUE

C
   DO 28 J=1, NQ
   DO 28 I=1, NR
      D(I,J)=S
   DO 28 K=1, NA

   DO 25 J=1, NQ
   DO 25 I=1, NR

25 D(I,J)=D(I,J)-G2(I,J)
   DO 28 J=1, NQ
   DO 28 I=1, NR
      A(I,J)=S
   DO 28 K=1, NR

35 A(I,J)=A(I,J)+CI(I,K)*D(I,J)

   DO 35 J=1, NQ
   DO 35 I=1, NR
   P4A(I,J)=S
   DO 35 K=1, NA

35 P4A(I,J)=P4A(I,J)+P4(I,K)*A(K,J)
   DO 40 J=1, NQ
   DO 40 I=1, NR
   B(I,J)=S
   DO 40 K=1, NA

40 B(I,J)=B(I,J)+S4(I,K)*(S1S(I,J)-P4A(K,J))

   DO 45 J=1, NR
   DO 45 I=1, NA
   P1P4(I,J)=S
   DO 45 K=1, NA

   DO 49 J=1, NR
   DO 49 I=1, NA
      CD1(I,J)=S
   DO 49 K=1, NA

49 CD1(I,J)=CD1(I,J)+S2D(I,K)*P1P4(K,J)
   DO 48 J=1, NR
   DO 48 I=1, NA
      CD(I,J)=S
   DO 48 K=1, NA

58 CD(I,J)=CD(I,J)+C1(I,K)*CD1(K,J)
   DO 46 J=1, NR
   DO 46 I=1, NA
   CD(I,J)=S
   DO 46 K=1, NA

58 CD(I,J)=CD(I,J)+C1(I,K)*CD1(K,J)
   DO 46 J=1, NR
   DO 46 I=1, NA
   CD(I,J)=S
   DO 46 K=1, NA

58 CD(I,J)=CD(I,J)+P3TD(I,K)*P1P4(K,J)
   DO 46 J=1, NQ
DO 65 I=1,NA
   P1G1(I,J)=.F.
   DO 66 K=1,NA
65 P1G1(I,J)=P1G1(I,J)+S4(I,K)+S1S(K,J)
   DO 70 J=1,NQ
   DO 70 I=1,NA
   DD1(I,J)=.F.
   DO 69 K=1,NA
69 DD1(I,J)=DD1(I,J)+P2D(I,K)+P1G1(I,J)
70 DD1(I,J)=DD1(I,J)+S1SD(I,J)
   DO 75 J=1,NQ
   DO 75 I=1,NE
   DD(I,J)=.F.
   DO 76 K=1,NA
75 DD(I,J)=DD(I,J)+C1(I,K)+DD1(K,J)
   DO 85 J=1,NQ
   DO 85 I=1,NE
   CD(I,J)=.F.
   DO 86 K=1,NE
85 CD(I,J)=CD(I,J)+CD(I,K)+A(K,J)
   DO 95 J=1,NQ
   DO 95 I=1,NE
   AD(I,J)=.F.
   DO 96 K=1,NE
95 AD(I,J)=AD(I,J)+C1(I,K)+(DD(K,J)-CD(K,J))
C
   BD
   DO 96 J=1,NQ
   DO 96 I=1,NA
   BD1(I,J)=.F.
   DO 95 K=1,NE
95 BD1(I,J)=BD1(I,J)+P4D(I,K)+A(K,J)
   DO 100 J=1,NQ
   DO 100 I=1,NA
   DO 100 K=1,NE
100 BD1(I,J)=BD1(I,J)+P4(I,K)+AD(K,J)
   DO 105 J=1,NQ
   DO 105 I=1,NA
   DO 105 K=1,NE
105 BD1(I,J)=BD1(I,J)+S2D(I,K)+B(K,J)
   DO 110 J=1,NQ
   DO 110 I=1,NE
   BD(I,J)=.F.
   DO 115 K=1,NA
110 BD(I,J)=BD(I,J)+S4(I,K)+(S1SD(K,J)-BD1(K,J))
C
   ADD
   DO 115 J=1,NE
   DO 115 I=1,NA
   CDD1(I,J)=.F.
   DO 116 K=1,NA
116 CDD1(I,J)=CDD1(I,J)+S4(I,K)+C1(K,J)
   DO 120 J=1,NE
   DO 120 I=1,NA
   CDD2(I,J)=.F.
   DO 125 K=1,NA
120 CDD2(I,J)=CDD2(I,J)+2.*S2D(I,K)+CDD1(K,J)
   DO 125 J=1,NE
   DO 125 I=1,NA
DO 124 K=1,NA
124 CDD2(I,J)=CDD2(I,J)+S2DD(I,K)=P1P4(K,J)
125 CDD2(I,J)=CDD2(I,J)+P4DD(I,J)
   DO 130 J=1,NR
   DO 130 I=1,SR
   CDD(I,J)=S.
   DO 130 K=1,SB
130 CDD(I,J)=CDD(I,J)+CI(I,K)*CDD2(K,J)
   DO 130 J=1,NR
   DO 130 I=1,SR
   CDD(I,J)=S.
   DO 130 K=1,SB
135 CDD(I,J)=CDD(I,J)+2.*PSTD(I,K)*CDD1(K,J)
   DO 140 J=1,NR
   DO 140 I=1,SR
   CDD(I,J)=S.
   DO 140 K=1,SB
140 CDD(I,J)=CDD(I,J)+PSSTD(I,K)+P1P4(K,J)
   DO 150 J=1,SR
   DO 150 I=1,SR
   DDD1(I,J)=S.
   DO 150 K=1,SB
145 DDD1(I,J)=BDD1(I,J)+S4(I,K)=DDD(K,J)
   DO 160 J=1,SR
   DO 160 I=1,SR
   DDD2(I,J)=S.
   DO 160 K=1,SB
150 DDD2(I,J)=BDD2(I,J)+2.*S2D(I,K)+DDD1(K,J)
   DO 165 J=1,SR
   DO 165 I=1,SR
   DDD2(I,J)=S.
   DO 165 K=1,SB
155 DDD2(I,J)=BDD2(I,J)+2.*S2D(I,K)+DDD1(K,J)
   DO 165 J=1,SR
   DO 165 I=1,SR
   DDD2(I,J)=S.
   DO 165 K=1,SB
160 DDD2(I,J)=BDD2(I,J)+S1DD(K,K)+S2DD(K,J)
   DO 165 J=1,SR
   DO 165 I=1,SR
   DDD(K,J)=S.
   DO 165 K=1,SB
165 DDD2(I,J)+S1DD(K,K)+S2DD(K,J)
   DO 165 J=1,SR
   DO 165 I=1,SR
   DDD(K,J)=S.
   DO 165 K=1,SB
170 DDD(I,J)=DDD1(I,J)+PSTD(I,K)+P1G1(K,J)
   DO 175 J=1,SR
   DO 175 I=1,SR
   ADD1(I,J)=S.
   DO 175 K=1,SB
175 ADD1(I,J)=ADD1(I,J)+CDD(I,K)+P2DD(I,K)
   DO 180 J=1,SR
   DO 180 I=1,SR
   ADD1(I,J)=S.
   DO 180 K=1,SB
180 ADD1(I,J)=ADD1(I,J)+P2DD(I,K)
   DO 185 J=1,SR
   DO 185 I=1,SR
   ADD1(I,J)=S.
   DO 185 K=1,SB
185 ADD1(I,J)=ADD1(I,J)+CDD(I,K)+ADD1(I,J)
C
   DO 190 J=1,SR
   DO 190 I=1,SR
   ADD1(I,J)=S.
   DO 190 K=1,SB
190 ADD1(I,J)=ADD1(I,J)+S2DD(I,K)=B(K,J)
DO 105 J=1,NQ
DO 105 I=1,NA
DO 105 K=1,NA

105  BDD1(I,J)=BDD1(I,J)+2.*S2D(I,K)*BD(I,K)
DO 205 J=1,NQ
DO 205 I=1,NA
DO 205 K=1,NA

205  BDD1(I,J)=BDD1(I,J)+P4(I,K)*ADD(K,J)
DO 285 J=1,NQ
DO 285 I=1,NA
DO 285 K=1,NA

285  BDD1(I,J)=BDD1(I,J)+2.*P4D(I,K)*AD(K,J)
DO 210 J=1,NQ
DO 210 I=1,NA
DO 210 K=1,NA

210  BDD1(I,J)=BDD1(I,J)+P4ADD(I,K)*A(K,J)
DO 215 J=1,NQ
DO 215 I=1,NA
DO 215 K=1,NA

BDD(I,J)=0.
DO 215 K=1,NA

215  BDD(I,J)=BDD(I,J)+S4(I,K)*((S1SDD(I,K)-BDD1(I,K)))
C
C  B=E=B
C
C REPLACE THE FOURTH ROW OF MATRIX S2 WITH ITS TRUE VALES, REMEMBER THE
C FOURTH ROW OF S2 RIGHT NOW IS FOR BOUNDARY TRactions FOR RIGID BODY
C ROTATION TERMS IN ORDER TO MAKE MATRIX "F" NONSINGULAR
C
C SUBROUTINE LINES

218  CONTINUE
IF (TRIG2.EQ.1) GO TO 2191
IF (INTL.LT.2) GO TO 2101
DO 210 I=1,NA
S2(4,I)=S2RIG(4)

219  CONTINUE

2191 CONTINUE
DO 220 J=1,NQ
DO 220 I=1,NB
AJF(I,J)=8.
DO 225 I=1,NA

220  AJF(I,J)=AJF(I,J)+S2(I,I)*B(I,J)
DO 220 J=1,NQ
DO 220 I=1,NA
AKS(I,J)=8.
DO 220 I=1,NB

225  AKS(I,J)=AKS(I,J)+B(I,I)*AJF(I,J)
RETURN

328  WRITE(6,482) TEB, IDIGT, IDIGT1
STOP 7

321  WRITE(8,482)
STOP 10

482 FORMAT(/,26X,2SH-----INVERS FAILED ON C-----,/26X,9HERROR NO.,14,
1/2X,30HPROPOSED DECIMAL DIGITS OF ACCURACY WAS,I4,2X,51H BUT THE UW
2-INSL LIBRARY ROUTINE "LINV2F" FOUND ONLY,I4,2X,26H DECIMAL DIGITS
3OF ACCURACY,
4/2X,4HERROR NO. 34=DECIMAL DIGIT ACCURACY TEST FAILED,
5/2X,4HERROR NO. 129=MATRIX IS ALGORITHMICALLY SINGULAR,
6/2X,4HERROR NO. 131=MATRIX IS TOO ILL-CONDITIONED FOR ITERATIVE IM
7PROVEMENTS TO BE EFFECTIVE,
8//,1DX,6SH-----TRY CHANGING MATERIAL PROPERTIES OR CRACK-TIP SPEED= 9-----)

483 FORMAT(/,26X,2SH-----INVERS FAILED ON C-----,12H C(I,J)=8.)
C
END
SUBROUTINE MAREAI

COMMON/ERHO/RHO(6), RODUM
COMMON/TIP/NCR1, NCR2, NCR3, X2, Y2, Z2, IX(6,35)
COMMON/MASS/AMRR(36,36), AMRR(36,36), AMRR(2,2), AMRR2(36,2)
COMMON/DISP/U(2,36), UD(2,36), UDD(2,36), UE(2,2)
COMMON/STRESS/IX(8,8), EIX(8,8), EIXD(8,8)
COMMON/NUM/CV, NUMCV, CVH(2,2)
COMMON/STRESS/IX(8,36), EIX(8,36), EIXD(8,36)
COMMON/SUMAN/AX1(3,3,6), AXI(3,3,6), AXB(6), AXSB(6), CL

C IF (RODUM.EQ.'S') RETURN
DO 5 J=1, NB
DO 5 I=1, NB
5 AMR2(I,J)=0.
DO 15 J=1, NB
DO 15 I=1, NB
15 AMR2(I,J)=0.
DO 20 J=1, NB
DO 20 I=1, NB
AMRDR(I,J)=0.
AMRR(I,J)=0.

30 CONTINUE
G1=.5
G2=.5
G=G1+G2
DO 80 LE=1,2
DO 80 LLE=1,2
80 LE=LLE,MINT
X=.5*(PT(L)-1.)
IF (LE.EQ.2) X=.5*(PT(L)+1.)
DO 70 M=1, MINT
Y=.5*(PT(M)-1.)
IF (LE.EQ.2) Y=.5*(PT(M)+1.)
EG=ERHO(IX(6,NCR1))
IF (Y.LT.0.) RD=ERHO(IX(6,NCR2))
WT=UG(L)/UG(M)
CALL TRANS(X,Y,N,2,1)
WT=WT+DEM+RD
CALL FUNCTS(X,Y,4)
DO 55 J=1,2
DO 55 I=1, NB
55 J=I+1,2
AMR2(I,J)=AMR2(I,J)+U(I,J)+UE(I,J)+WA

50 CONTINUE
IF (CV.EQ.'S') GO TO 51
GO TO 65

51 CONTINUE
DO 62 J=1, NB
DO 62 I=1, NB
DO 62 K=1,2
AMRR(I,J)=AMRR(I,J)+U(I,J)+U(I,J)+WA
52 CONTINUE
GO TO 62

55 CONTINUE
DO 68 J=1, NB
DO 68 I=1, NB
DO 68 K=1,2
AMRR(I,J)=AMRR(I,J)+U(I,J)+U(I,J)+WA
AMRDR(I,J)=AMRDR(I,J)+UD(I,J)+U(I,J)+WA
68 CONTINUE
63 CONTINUE
DO 65 J=1, NR
   DO 65 I=1, NR
      DO 65 K=1,2
      65 AM22(I,J)=AM22(I,J)+UR(I,I)*UR(K,J)*WA
      CONTINUE
   CONTINUE
CONTINUE
CONTINUE
CONTINUE
RETURN
C
END
SUBROUTINE MASSW

COMMON/MASS/AMR(36,36), AMREDR(36,36), AM22(2,2), AMR2(36,2)

1. AMRED2(36,2), AMRED2(36,2)
COMMON/VEL/CV, NUMCV, CVR(2,29)
COMMON/KERD/KRD(6), KRDUM
COMMON/BK16/2(366), Z(366), CODE(366), IX(6,256)
COMMON/SUMAT/A1(3,3,6), AI(3,3,6), ABST(6), ASIZE(6), CL
COMMON/LUS/S2(36,36), S4(36,36), S2D(36,36), S2DD(36,36)
COMMON/SUS/S1E(36,29), S1SD(36,29), S1SDD(36,29)
COMMON/ANAT/AM1(28,28), V1(28,28), AK4(28,28)
COMMON/DIM/NA, NAA, NBT, NB, NQ, NK, NINT, NINT2, IALL, NITER, SIMCOD
COMMON/TIP/NCR1, NCR2, NCR3, NCR4, RELX, CTPX, CTPY, SIP1, SIP2
COMMON/AD/B(36,36), BD(36,29), SDD(36,29)

1. A(2,28) AD(2,29), ADD(2,28)

DIMENSION AM11(36,28)

DIMENSION AMD(36,28)


DIMENSION V16(28,2)

DIMENSION AE4(28,36), AE42(28,2), AE43(28,36), AE44(28,2)

DIMENSION AE45(28,36), AE46(28,2), AE47(28,2)

DIMENSION AMB(28,28)

DIMENSION AM1R(28,28)

DIMENSION AMC(2,28)

EQUIVALENCE(AK41(1,1), V11(1,1), AM11(1,1), S1S(1,1))

EQUIVALENCE(AK43(1,1), V13(1,1), AMD(1,1), S1SD(1,1))

EQUIVALENCE(AK46(1,1), V16(1,1), S1SDD(1,1))

EQUIVALENCE(AK42(1,1), V12(1,1))

EQUIVALENCE(AK44(1,1), V14(1,1), S2DD(1,1))

EQUIVALENCE(AK45(1,1), V15(1,1), S2DD(1,1))

EQUIVALENCE(AM1(1,1), S4(1,1))

EQUIVALENCE(AM1R(1,1), S4(1,1))

EQUIVALENCE(AK47(1,1), S4(1,1))

M1=B+MRR+B

IF (BODIN.EQ. 0.) RETURN

DO 16 J=1,NQ

DO 16 I=1,NB

AM11(I,J)=B.

DO 16 K=1,NB

16 AM11(I,J)=AM11(I,J)+AMRED(I,K)*B(K,J)

DO 20 J=1,NQ

DO 20 I=1,NQ

AM1(I,J)=B.

DO 20 K=1,NB

20 AM1(I,J)=AM1(I,J)+B(K,I)*AM11(K,J)

RIGID BODY TERMS FOR M1

M1R=A=M22+A =B=MRR=A +A=M2R=B

DO 38 J=1,NQ

DO 38 I=1,NR

AMC(I,J)=B.

DO 38 K=1,NR

38 AMC(I,J)=AMC(I,J)+AM22(I,K)*A(K,J)

DO 46 J=1,NQ

DU 46 I=1,NQ

AM1R(I,J)=B.

DU 46 K=1,NR

46 AM1R(I,J)=AM1R(I,J)+A(K,I)*AMC(K,J)

DO 59 J=1,NQ
DO 60 I=1,NB
   AMD(I,J)=0.
DO 60 K=1,NR
50 AMD(I,J)=AMD(I,J)+AMB2(I,X)*A(K,J)
DO 60 J=1,NQ
DO 60 I=1,NQ
   AMD(I,J)=0.
DO 60 K=1,NR
60 AMR(I,J)=AMR(I,J)+B(K,I)*AMD(K,J)
DO 70 J=1,NQ
DO 70 I=1,NQ
70 AMR(I,J)=AMR(I,J)+AMR(I,J)+AMB(I,J)
DO 80 J=1,NQ
DO 80 I=1,NQ
80 AM1(I,J)=AM1(I,J)+AMR(I,J)
IF(CV_EQ.) RETURN

C  Damping and Convection Terms
  C  
   X4  
   DO 110 J=1,NB
   DO 110 I=1,NQ
      AX41(I,J)=0.
      AX46(I,J)=0.
   DO 110 K=1,NR
      AX41(I,J)=AX41(I,J)+BDD(K,I)*AMR(K,J)
      AX45(I,J)=AX45(I,J)+BD(K,I)*AMRD2(K,J)
   110 CONTINUE
   DO 120 J=1,NB
   DO 120 I=1,NQ
      AX42(I,J)=0.
   DO 120 K=1,NR
      AX42(I,J)=AX42(I,J)+ADD(K,I)*AM22(K,J)
   120 CONTINUE
   DO 130 J=1,NB
   DO 130 I=1,NQ
      AX48(I,J)=0.
   DO 130 K=1,NR
      AX48(I,J)=AX48(I,J)+ADD(K,I)*AMR2(J,K)
   130 CONTINUE
   DO 140 J=1,NB
   DO 140 I=1,NQ
      AX44(I,J)=0.
      AX46(I,J)=0.
   DO 140 K=1,NR
      AX44(I,J)=AX44(I,J)+BDD(K,I)*AMR2(K,J)
      AX46(I,J)=AX46(I,J)+BD(K,I)*AMRD2(K,J)
      AX47(I,J)=AX47(I,J)+B(K,I)*AMRD2(K,J)
   140 CONTINUE
   DO 150 J=1,NQ
   DO 150 I=1,NQ
      AX4(I,J)=0.
   DO 150 K=1,NR
      AX4(I,J)=AX4(I,J)+(2.*AX45(J,K)+AX44(J,K)+AX43(J,K))* B(K,I)
   150 CONTINUE

C  V1  
   DO 160 J=1,NQ
   DO 160 I=1,NQ
      AX4(I,J)=AX4(I,J)+(AX42(J,K)+AX44(J,K)+2.*AX46(J,K)+AX47(J,K))
      1 =A(K,I)
   160 CONTINUE
DO 170 J=1,NB
DO 170 I=1,NQ
V11(I,J)=0.
V15(I,J)=0.
DO 170 K=1,NB
V15(I,J)=V15(I,J)+B(K,I)*AMRD(K,J)
170 V11(I,J)=V11(I,J)+BD(K,I)*AMRE(K,J)
DO 180 J=1,NB
DO 180 I=1,NQ
V12(I,J)=0.
DO 180 K=1,NB
V12(I,J)=V12(I,J)+AD(K,I)*AM22(K,J)
180 V13(I,J)=V13(I,J)+AD(K,I)*AMR2(J,K)
DO 190 J=1,NB
DO 190 I=1,NQ
V13(I,J)=0.
V16(I,J)=0.
DO 190 K=1,NB
V16(I,J)=V16(I,J)+B(K,I)*AMRD2(K,J)
200 V14(I,J)=V14(I,J)+BD(K,I)*AMRE2(K,J)
DO 210 I=1,NQ
DO 210 J=1,NQ
V1(I,J)=0.
V16(I,J)=V16(I,J)+2.*V11(J,K)+V13(J,K)+V15(J,K)-B(K,I)
210 CONTINUE
DO 220 I=1,NQ
DO 220 J=1,NQ
DO 220 K=1,NB
V1(I,J)=V1(I,J)+2.*(V12(J,K)+V14(J,K)+V16(J,K)) *A(K,I)
220 CONTINUE
RETURN
C
END
SUBROUTINE INPOL(X,Y,LSIDE,LE)

COMMON/DIM/MA, NAA, NBT, NE, NQ, NR, NINT, NINT2, IALL, NITER, SINCOD
COMMON/INTPOL/ALS(2,20)

C

DO 18 J=1,NQ
   DO 18 I=1,2

18 ALS(I,J)=0.
   IF( (LSIDE.EQ.1) ) GO TO 28
   IF( (LSIDE.EQ.2) ) GO TO 48
   IF( (LSIDE.EQ.3) ) GO TO 68
   IF( (LSIDE.EQ.4) ) GO TO 88

28 CONTINUE
   IF( (LS.EQ.2) ) GO TO 38
   ALS(1,8)=-X
   ALS(1,9)=1.+X
   ALS(2,4)=ALS(1,3)
   ALS(2,6)=ALS(1,5)
   GO TO 188

38 ALS(1,5)=1.-X
   ALS(1,7)=X
   ALS(2,6)=ALS(1,5)
   ALS(2,8)=ALS(1,7)
   GO TO 188

48 CONTINUE
   IF( (LS.EQ.2) ) GO TO 58
   ALS(1,7)=-Y
   ALS(1,9)=-X
   ALS(2,6)=ALS(1,7)
   ALS(2,10)=ALS(1,9)
   GO TO 188

58 ALS(1,11)=1.-Y
   ALS(1,13)=Y
   ALS(2,12)=ALS(1,11)
   ALS(2,14)=ALS(1,13)
   GO TO 188

68 CONTINUE
   IF( (LS.EQ.2) ) GO TO 78
   ALS(1,13)=X
   ALS(1,15)=1.-X
   ALS(2,14)=ALS(1,13)
   ALS(2,16)=ALS(1,15)
   GO TO 188

78 ALS(1,15)=1.+X
   ALS(1,17)=-X
   ALS(2,16)=ALS(1,15)
   ALS(2,18)=ALS(1,17)
   GO TO 188

88 CONTINUE
   IF( (LS.EQ.2) ) GO TO 98
   ALS(1,17)=-Y
   ALS(1,19)=-Y
   ALS(2,18)=ALS(1,17)
   ALS(2,20)=ALS(1,19)
   GO TO 188

98 ALS(I,1)=1.-Y
   ALS(I,3)=-Y
   ALS(2,2)=ALS(I,1)
   ALS(2,4)=ALS(I,3)

188 RETURN
C
END
SUBROUTINE TRANS(X,Y,N,JC,JB)

COMMON/EX16,E(800),Z(800),CODE(800),IX(8,256)
COMMON/DIF/DXX2,DXY1,DXY2,DYY2,DYYY2,DXX,DXY,DYY,DYY2,DEM
COMMON/IPAR/PJ1,PJ2,PJ3,PJ4,DY1,DXY2
COMMON/PHYS/ANX,ANY,A(2)
COMMON/MAIN/CORD(10,2)
COMMON/DM/HA,NAA,NBT,NB,NQ,NL,NIT2,NIT3,ITALL,NITRE,SINCOD
DIMENSION D(2),G(2),P(2)
REAL KORD(10,2)

IF(JR.EQ.2) GO TO 28
L1=2
L2=4
L3=7
L4=9
NQ1=NQ/2
DO 10 I=1,2
DO 10 I=1,NQ1
10 KORD(I,J)=CORD(I,J)
GO TO 48

28 CONTINUE
L1=1
L2=2
L3=3
L4=4
DO 28 I=1,4
K=IX(I,N)
KORD(I,1)=K
28 CONTINUE

48 CONTINUE
DO 58 K=1,2
A(K)=(1.-X)*(1.-Y)*.25*KORD(L1,K)
A(K)=A(K)*(1.+X)*(1.-Y)*.25*KORD(L2,K)
A(K)=A(K)*(1.+X)*(1.+Y)*.25*KORD(L3,K)
A(K)=A(K)*(1.-X)*(1.+Y)*.25*KORD(L4,K)
58 CONTINUE
IF(JC.EQ.1) GO TO 200
T1 =1.-Y
T3 =1.+Y
DO 101 K=1,2
G(K)=0.
F(K)= (KORD(L1,K)-KORD(L2,K)+KORD(L3,K)-KORD(L4,K))/4.
101 D(K)=-(T1*KORD(L1,K)+T1*KORD(L2,K)+T3*KORD(L3,K)-T3*KORD(L4,K))/4.
DXX2=G(1)
DXY2=G(2)
DXX=F(P)
DXX2=F(2)
DXY=D(1)
DXX=D(2)
T1 =1.-X
T3 =1.+X
DO 202 K=1,2
G(K)=0.
202 D(K)=-(T1*KORD(L1,K)-T3*KORD(L2,K)+T3*KORD(L3,K)+T1*KORD(L4,K))/4.
DXX2=G(1)
DYY2=G(2)
DXY=D(1)
DYY=D(2)
DEM = DXX*DXY-DXX2*DYY
P1 =DYY2/DEM
P3 =DXY2/DEM
P3 =-DXY2/DEM
P3 = DXY/DEM
DYX1 = SQRT(DXX*DXX-DYX*DYY)
\[ \text{DXY2} = \sqrt{(\text{DXY}^2 + \text{DXY}^2 + \text{DYY}^2)} \]

CONTINUE
RETURN

END
SUBROUTINE NORMAL(L)
COMMON/MAIN/CORD(16,2)
COMMON/PHYS/ANX,ANY,A(2)
C
IF(L-1) 25,18,26
16 CONTINUE
N1=2
N2=4
GO TO 88
25 IF(L-2) 45,39,49
39 CONTINUE
N1=4
N2=7
GO TO 88
49 IF(L-3) 60,50,69
59 CONTINUE
N1=7
N2=9
GO TO 88
69 CONTINUE
N1=9
N2=2
88 CONTINUE
X1=CORD(N1,1)
X2=CORD(N2,1)
Y1=CORD(N1,2)
Y2=CORD(N2,2)
B=X2-X1
c=Y2-Y1
AR=SQRT(B**2+C**2)
ANX=(Y2-Y1)/AR
ANY=(X1-X2)/AR
RETURN
C
END
SUBROUTINE PAIR (L, NTER)
COMMON/STORE/SK(192, 6)
COMMON/B11/DELT, DT1, DT2, BET1, BET2, BET3, BET4, BET5, NBAND, NBD2
COMMON/BK16/E(300), Z(300), CODE(300), IX(0, 256)
COMMON/SHIFT/NSBL, NDEL, LASTB, IF, NBRED, NESTEP, ISE, BCODE
COMMON/PAIR1/NPAIR, LPAIR(2, 40)
COMMON/STRESS/EY(4, 38), EXD(4, 38), XDDD(3, 38)
DIMENSION B(102)
EQUIVALENCE (B(1), EX(1, 1))
C
COMMON/SSS/S15(36, 36), S1SD(36, 36), S1SDD(36, 36)
COMMON/MASS/AM00E(36, 36), AM00D(36, 36), AM22(2, 2), AM22(36, 2)
1, AM002(36, 2), AM00D2(36, 2)
COMMON/WK/WKAREA(1484)
COMMON/LS/L2(36, 36), L4(36, 36), S2D(36, 36), S2DD(36, 36)
DIMENSION B(300), BLOAD(300), EM(300), EMAT(300)
DIMENSION US(300), US(300), AS(300)
DIMENSION EX1(2104), EX3(1484), EX3(5184)
EQUIVALENCE (E(1), EX1(1))
EQUIVALENCE (LOAD(1), EX1(1))
EQUIVALENCE (EMAT(1), EX1(1))
EQUIVALENCE (US(1), EX1(1))
EQUIVALENCE (VS(1), EX1(1))
EQUIVALENCE (AS(1), EX1(1))
EQUIVALENCE (EM(1), EX1(1))
EQUIVALENCE (S1S(1, 1), EX1(1))
EQUIVALENCE (WKAREA(1), EX1(1))
EQUIVALENCE (S2(1, 1), EX1(1))
C
NBG=NBAND
IF (NTERR.EQ.1) GO TO 500
DO 1001 J=1, NBG
DO 1001 I=1, NBAND
SK(I, J) = 0.
1001 CONTINUE
DO 1002 I=1, NBAND
B(I) = 0.
1002 CONTINUE
IF (L.EQ.0) REWIND 11
IF (L.EQ.1) REWIND 14
IF (L.EQ.2) REWIND 21
IF (L.EQ.3) REWIND 24
NB1=1
NB2=NBAND
NB3=NBAND+NBAND
NBLOC=1
2 CONTINUE
NBLOC=NBLOC+1
II=(NBLOC-2)+NBAND
IF (NBLOC-LASTB-1) 5, 110, 6
5 NB=NBRED
IF (NBLOC.EQ.NSBL) NB=NBAND
K=2-NBAND
IF (L.EQ.0) READ(11) ((SK(I, J), I=K, NBD2), J=1, NBG)
IF (L.EQ.1) READ(14) ((SK(I, J), I=K, NBD2), J=1, NBG)
DO 6 I=K, NBD2
B(I) = E(II+1)
6 CONTINUE
DO 100 N=1, NPAIR
IF (LPAIR(2, N).LE.0) GO TO 100
N1=LPAIR(1, N)+NDL1-II
N2-LPAIR(2, N)+NDL1-II
IF (N1.LT.NB1 .OR. N1.GT.NB3) GO TO 100
IF (N2.LT.NB1 .OR. N2.GT.NB3) GO TO 100
IF (N1 LE NB2 .AND. N2 LE NB2) GO TO 100
100 CONTINUE
ND=N2-N1
IF (ND.LT.0) GO TO 46
ND=2+ND
N2P=N2+1
DO 20 K=1,NBA
K2=N2P-K
KD=ND2-K
IF (KD.LE.0) GO TO 10
SK(I2,K)=SK(I2,K)+SK(N1,KD)
GO TO 20
10 KD=K-ND
SK(I2,K)=SK(I2,K)+SK(I2,KD)
20 CONTINUE
NBS=NBA-ND
DO 80 K=1,NBS
KD=K-ND
SK(N2,K)=SK(N2,K)+SK(N1,KD)
30 CONTINUE
GO TO 60
40 ND=2-ND
N2P=N2+1
N2M=N2-1
DO 60 K=1,NBA
KD=ND2-K
IF (KD.LE.0) GO TO 50
K2=N2M+K
SK(N2,K)=SK(N2,K)+SK(K2,KD)
GO TO 60
50 KD=K-ND
SK(N2,K)=SK(N2,K)+SK(N1,KD)
60 CONTINUE
NBS=NBA+ND
DO 70 K=1,NBS
K2=N2P-K
KD=K-ND
SK(I2,K)=SK(I2,K)+SK(I2,KD)
70 CONTINUE
80 B(N2)=B(N2)+B(N1)
100 CONTINUE
IF (NBLOC.EQ.1) GO TO 200
110 CONTINUE
IF (L.EQ.0) WRITE(21) ((SK(I,J),I=1,NBAND),J=1,NBG)
IF (L.EQ.1) WRITE(24) ((SK(I,J),I=1,NBAND),J=1,NBG)
DO 120 I=1,NBAND
E(I+I1)=B(I)
120 CONTINUE
IF (NBLOC.EQ.LASTB+1) RETURN
200 DO 220 I=1,NBAND
K=I+NBAND
DO 210 J=1,NBAND
SK(I,J)=SK(I,J)
210 CONTINUE
B(I)=0(K)
220 CONTINUE
GO TO 2
C
500 CONTINUE
DO 4992 I=1,NBAND
B(I)=0.
4992 CONTINUE
NB1=1
NB2=NBAND
NBS=NBAND+NBAND
NBLOC=0
500 CONTINUE
NBLOC=NBLOC+1
II=(NBLOC-2)*NBAND
IF(NBLOC.LT.LASTB-1) 505,619,65
505 K=1+NBAND
DO 508 I=1,NBD2
   B(I)=B(II+I)
508 CONTINUE
DO 619 N=1,NPAIR
   IF(LPAIR(1,N).LE.0) GO TO 619
   N1=LPAIR(1,N)+NDEL-II
   N2=LPAIR(2,N)+NDEL-II
   IF(N1.LT.NB1 .OR. N1.GT.NBS) GO TO 619
   IF(N2.LT.NB1 .OR. N2.GT.NBS) GO TO 619
   IF(N1.LE.NB2 .AND. N2.LE.NB2) GO TO 619
   B(N2)=B(N2)+B(N1)
619 CONTINUE
IF(NBLOC.EQ.1) GO TO 705
619 CONTINUE
DO 629 I=1,NBAND
   B(I+II)=B(I)
629 CONTINUE
IF(NBLOC.EQ.LASTB-1) RETURN
705 DO 725 I=1,NBAND
   B(I)=B(I+NBAND)
725 CONTINUE
GO TO 502
C END
SUBROUTINE FUNCT5(X1, Y1, L)
COMMON/TIP/NCR1, NCR2, NCR3, NCR4, NELX, CTPX, CTPY, SIF1, SIF2
COMMON/MAIN/CORD(1,2)
COMMON/VEL/CV, HMCV, CVH(2,25)
COMMON/XM/XA, NAA, NP, MB, NS, MX, MR, MINT, MINT2, IALL, MITER, SINCOD
COMMON/PHYS/ANX, ANY, A(2)
COMMON/INTGB/PT(10), WQ(10), PT2(2), WC2(2), PHE
COMMON/IPAR/PJ1, PJ2, PJ8, PJ4, DXY1, DXY2
COMMON/IKG/IKIGA(6), IKIGB
COMMON/DISP/UR(2,28), UQ(2,28), UG(2,28), UR(2,2)
COMMON/STRESS/S(3,66), SD(3,66), SDD(3,66)
COMPLEX UU(2,28), UUD(2,28)
COMPLEX SS(3,28), SDD(3,28), SDD(3,28)
COMMON/W2/WEAR(720)
EQUIVALENCE (WVAR(1), UU(1,1))
EQUIVALENCE (WVAR(81), UU(1,1))
EQUIVALENCE (WVAR(181), UDD(1,1))
EQUIVALENCE (WVAR(281), SS(1,1))
EQUIVALENCE (WVAR(321), SDD(1,1))
EQUIVALENCE (WVAR(481), SDD(1,1))

COMMON /CT/ICASE(2)
COMMON /C18/LAM(28)
COMMON /C7/P(6)
COMMON /C13/SI(12), SIX(12), SIXX(12)
COMMON /C14/MU(2,2)
COMMON /C14/MU(2,2)
COMPLEX MU, LAM, SI, SIX, SIXX, DIS, DISX, DISXX, F
COMPLEX LM/XA, NAA, NELX, ZBL1, ZBL2, ZBL3, ZBL4, ZBL5, ZBL6, ZBL7
1
2
3
4

COMMON /C/ICASE(2)
COMMON /C/ICASE(2)
COMPLEX CX, CV1, CV2
COMMON /C/ICASE(2)
COMMON /C/ICASE(2)
COMMON /C/ICASE(2)
COMMON /C/ICASE(2)
COMMON /C/ICASE(2)

C THIS SUBROUTINE CONTAINS THE MODE I AND II FUNCTIONS FOR SQUARE ELEMENT ONLY

C stationary and movig cracks

UR(1,1)=1.
UR(2,1)=6.
UR(1,2)=6.
UR(2,2)=1.

CX=(2*CPRX-CORD(1,1)-CORD(6,1))/(CORD(6,1)-CORD(1,1))
Y=CY
Y=CV1
CV2=CY+CV
P1=P1
P2=P2+P1
P1=P2+P1
P2=P2+P1
P2=P2+P1
P2=P2+P1
II=1
IF(7.LT.0.) II=2
IF(ICASE(II).EQ.2) GO TO 1000
IF(L.EQ.8) GO TO 1000
IF(CV.EQ.0. AND. L.EQ.4) GO TO 200
IF(CV.EQ.0.) GO TO 500
IF(L.EQ.2) GO TO 1
IF(L.EQ.4) GO TO 100
IF(L.EQ.8) GO TO 300

DO 10 I=1,2
DO 10 J=1,NBT
SS(I,J)=(0.,0.)
SSD(I,J)=(0.,0.)
10 CONTINUE
SSDD(I,J) = (0,0)

CONTINUE
DO 20 I=1,2
DO 20 J=1,NBT
UU(I,J) = (0,0)
UUD(I,J) = (0,0)

CONTINUE
DO 70 M=1,2
N=M-1
Z=X+MUU(M,II)*Y
ZB=CONJG(Z)
LM=LAM(N)
LJ=LM-1
ZLJ=LJ+CLOG(2)
ZLJ=CEXP(ZLJ)
ZBLJ=CONJG(LJ)*CLOG(2)
ZBLJ=CONJG(CEXP(ZBLJ))
ZL2=ZLJ-2
ZBL2=ZBLJ-2
ZL1=ZL2-2
ZBL1=ZBL2-2
ZL0=ZL1-2
ZBL0=ZBL1-2
DO 30 I=1,3
J=(I-1)*4
J=J+M1+2
J1=J+1
J2=J1+1
SS(J,I,N)=SS(I,N)+(SI(J1)+ZL1+ZL2+ZBL1)*P1
SSD(J,I,N)=SSD(I,N)-(SIX(J1)+ZL2+SIX(J2)+ZBL2)*P2CV1
SSDD(J,I,N)=SSDD(I,N)-(SIXX(J1)+ZL3+SIXX(J2)+ZBL3)*P2CV2

CONTINUE
DO 40 I=1,2
J=(I-1)*4
J=J+M1+2
J1=J+1
J2=J1+1
UU(J,I,N) = UU(J,I,N) + DIS(J1)*ZL0 + DIS(J2)*ZBL0
UUD(J,I,N) = UUD(J,I,N) + (DISX(J1) + ZL1 + DISX(J2) + ZBL1) * P1CV1
UDDD(J,I,N) = UDDD(J,I,N) + (DISXX(J1) + ZL2 + DISXX(J2) + ZBL2) * P2CV2

CONTINUE
DO 60 I=1,2
DO 60 J=1,NBT
JJ=J+2
JJ1=JJ-1
S(J,J1) = REAL(SS(I,J))
S(J,J2) = AIMAG(SS(I,J))
SD(J,J1) = REAL(SSD(I,J))
SD(J,J2) = AIMAG(SSD(I,J))
SDD(J,J1) = REAL(SSDD(I,J))
SDD(J,J2) = AIMAG(SSDD(I,J))

CONTINUE
DO 80 I=1,2
DO 80 J=1,NBT
JJ=J+2
JJ1=JJ-1
U(J,J1) = REAL(UU(I,J))
U(J,J2) = AIMAG(UU(I,J))
UD(J,J1) = REAL(UUD(I,J))
UD(J,J2) = AIMAG(UUD(I,J))
UD(J,J1) = REAL(UDDD(I,J))
UD(J,J2) = AIMAG(UDDD(I,J))
UDDL(I,J) = REAL(UDDL(I,J))
UDDL(I,J) = -AIMAG(UDDL(I,J))
CONTINUE
RETURN
100 CONTINUE
DO 120 J = 1, NBT
UU(I,J) = (0., 0.)
UDDL(I,J) = (0., 0.)
CONTINUE
DO 170 K = 1, NBT
CALL CDEFF(N,II)
DO 160 N = 1, 2
MI = N - 1
Z = X * MUU(N,II) + Y
ZB = CONJG(Z)
LM = LAM(N)
LJ = LM - 1
ZL1 = LJ + CLOG(Z)
ZL1 = CEXP(ZL1)
ZBL1 = CONJG(LJ) + CLOG(Z)
ZBL1 = CONJG(CEXP(ZBL1))
ZLS = ZL1 * Z
ZBL1 = ZBL1 * ZB
DO 140 I = 1, 2
J = (I - 1) + 4
J = J + M1 + 2
J1 = J + 1
J2 = J1 + 1
UU(I,N) = UU(I,N) + DIS(J1) * ZLS * DIS(J2) * ZBL1
UDDL(I,N) = UDDL(I,N) - (DISX(J1) * ZL1 + DISX(J2) * ZBL1) * P1CV1
CONTINUE
140 CONTINUE
170 CONTINUE
DO 180 I = 1, 2
DO 180 J = 1, NBT
JJ = J + 1
JJ1 = JJ + 1
U(I,JJ1) = REAL(UU(I,J))
U(I,JJ1) = -AIMAG(UU(I,J))
UD(I,JJ1) = REAL(UDDL(I,J))
UD(I,JJ1) = -AIMAG(UDDL(I,J))
CONTINUE
RETURN
CONTINUE
DO 220 J = 1, NBT
UU(I,J) = (0., 0.)
CONTINUE
DO 270 N = 1, NBT
CALL CDEFF(N,II)
DO 260 N = 1, 2
MI = N - 1
Z = X * MUU(N,II) + Y
ZB = CONJG(Z)
LM = LAM(N)
LJ = LM
ZLS = LJ + CLOG(Z)
ZLS = CEXP(ZLS)
ZBL1 = CONJG(LJ) + CLOG(Z)
ZBL1 = CONJG(CEXP(ZBL1))
DO 240 I = 1, 2
J = (I - 1) + 4
J = J + M1 + 2
J1 = J + 1
J2=J1+1
UU(I,N)=UU(I,N)+DIS(J1)+ZL0+DIS(J2)+ZBL0
240 CONTINUE
260 CONTINUE
270 CONTINUE
DO 260 I=1,2
DO 260 J=1,NBT
JJ=J+2
JJ1=JJ-1
U(I,JJ1)=REAL(UU(I,J))
U(I,JJ)=-AIMAG(UU(I,J))
260 CONTINUE
RETURN
300 CONTINUE
DO 320 I=1,2
DO 320 J=1,NBT
UU(I,J)=(0.,0.)
UUD(I,J)=(0.,0.)
320 CONTINUE
DO 370 N=1,NBT
CALL COEFF(N,N1)
DO 350 K=1,2
W1=W-1
Z=+MUW(N,N1)*Y
ZB=CONJG(Z)
LM=LAM(N)
LJ=LJ-2.
ZL2=LJ+4L0
ZL3=CONJG(LJ)*CLOG(Z)
ZL6=CONJG(CONJG(ZL6))
ZL1=ZL2+Z
ZL2=ZL2+Z
ZL3=ZL3+Z
ZL6=ZL6+Z
DO 340 I=1,2
J=(I-1)*4
J=J-W1+2
JJ=J+1
UU(I,N)=UU(I,N)+DIS(J1)+ZL0+DIS(J2)+ZBL0
UUD(I,N)=UUD(I,N)+(DISX(J1)+ZL1+DISX(J2)+ZBL1)+P1CV1
UDD(I,N)=UDD(I,N)+(DISXX(J1)+ZL2+DISXX(J2)+ZBL2)+P2CV2
340 CONTINUE
370 CONTINUE
DO 380 I=1,2
DO 380 J=1,NBT
JJ=J+2
JJ1=JJ-1
U(I,JJ1)=REAL(UU(I,J))
U(I,JJ)=-AIMAG(UU(I,J))
380 CONTINUE
RETURN
500 CONTINUE
DO 510 I=1,3
DO 510 J=1,NBT
SS(I,J)=(0.,0.)
510 CONTINUE
DO 520 I=1,2
520
DO 520 J=1,NBT
UU(I,J)=(0.,0.)
CONTINUE

520 DO 570 N=1,NBT
CALL COEFF(N,II)
DO 550 M=1,2
M1=M-1
Z=X*MUU(N,II)*Y
ZB=CONJG(Z)
LM=LAM(N)
LJ=LJ-1
ZL1=LJ+CLOG(Z)
ZL1=CEXP(ZL1)
ZBL1=CONJG(LJ)*CLOG(Z)
ZBL1=CONJG(CEXP(ZBL1))
ZL0=ZL1+Z
ZBL0=ZBL1+ZB
DO 530 I=1,3
J=(I-1)*4
J=J+M1+2
J1=J-1
J2=J1+1
SS(I,N)=SS(I,N)+((SI(J1)+ZL1*SI(J2)+ZBL1)*P1
CONTINUE
DO 540 I=1,2
J=(I-1)*4
J=J+M1+2
J1=J-1
J2=J1+1
UU(I,N)=UU(I,N)+DIS(J1)+ZL0*DIS(J2)+ZBL0
CONTINUE
540 CONTINUE
560 DO 570 N=1,NBT
DO 550 M=1,2
M1=M-1
Z=X*MUU(N,II)*Y
ZB=CONJG(Z)
LM=LAM(N)
LJ=LJ-1
ZL1=LJ+CLOG(Z)
ZL1=CEXP(ZL1)
ZBL1=CONJG(LJ)*CLOG(Z)
ZBL1=CONJG(CEXP(ZBL1))
ZL0=ZL1+Z
ZBL0=ZBL1+ZB
DO 530 I=1,3
J=(I-1)*4
J=J+M1+2
J1=J-1
J2=J1+1
SS(I,N)=SS(I,N)+((SI(J1)+ZL1*SI(J2)+ZBL1)*P1
CONTINUE
DO 540 I=1,2
J=(I-1)*4
J=J+M1+2
J1=J-1
J2=J1+1
UU(I,N)=UU(I,N)+DIS(J1)+ZL0*DIS(J2)+ZBL0
CONTINUE
540 CONTINUE
560 CONTINUE
570 DO 575 I=1,3
DO 570 J=1,NBT
JJ=J-1
S(I,JJ)=REAL(SS(I,J))
S(I,JJ)=AIMAG(SS(I,J))
CONTINUE
DO 580 I=1,2
DO 580 J=1,NBT
JJ=J+2
JJ1=JJ-1
U(I,JJ1)=REAL(UU(I,J))
U(I,JJ)=AIMAG(UU(I,J))
CONTINUE
RETURN
580 CONTINUE
CONTINUE
DO 610 I=1,3
DO 610 J=1,NBT
SS(I,J)=(0.,0.)
CONTINUE
610 CONTINUE
DO 670 N=1,NBT
CALL COEFF(N,II)
DO 660 M=1,2
M1=M-1
Z=X*MUU(N,II)*Y
ZB=CONJG(Z)
LM=LAM(N)
LJ=LJ-1
ZL1=LJ+CLOG(Z)
ZL1=CEXP(ZL1)
ZBL1=CONJG(LJ)*CLOG(Z)
ZBL1=CONJG(CEXP(ZBL1))
ZL=ZL+Z
ZBL2=ZBL1*ZBL
DO 635 I=1,3
J=(I-1)*4
J=J+MJ+2
JJ=J+1
SS(I,J)=SS(I,J)+ZL+SI(J2)*ZBL1*P1
635 CONTINUE
636 CONTINUE
637 CONTINUE
DO 675 I=1,3
DO 675 J=1,NBT
JJ=J+1
SS(I,J)=BBAL(SS(I,J))
S(I,J)=AIMAG(SS(I,J))
675 CONTINUE
RETURN
1600 CONTINUE
IF(L.BQ.8) GO TO 1600
IF(CV.BQ.8 AND L.GE.4) GO TO 1200
IF(CV.BQ.8) GO TO 1600
IF(L.BQ.2) GO TO 1001
IF(L.BQ.4) GO TO 1100
IF(L.BQ.6) GO TO 1300
1001 Z=X-MUL(I,II)+Y
ZB=CONJG(Z)
DO 1870 N=1,NBT
CALL CDEFF(N,II)
LM=LM+4
LJ=LM+4
ZL=ZL+CLG(T)
ZL=CEXP(ZL)
ZBL4=ZB+ZL4
ZBL1=CONJG(LJ)+CLG(T)
ZBL4=CONJG(CEXP(ZBL4))
ZBL1=ZBL1+ZBL4
ZL=ZL+Z
ZBL2=ZBL2+ZBL2
ZBL3=ZBL3+Z
ZBL3=ZBL3+ZBL3
ZBL2=ZBL2+ZBL2
ZBL1=ZBL1+ZBL1
ZL=ZL+Z
ZBL1=ZBL1+ZBL1
ZBL3=ZBL3+Z
ZBL3=ZBL3+ZBL3
ZBL2=ZBL2+ZBL2
ZBL1=ZBL1+ZBL1
ZL=ZL+Z
ZBL1=ZBL1+ZBL1
DO 1838 I=1,3
J=(I-1)*4
J=J+1
J2=J+1
J3=J+1
J4=J+1
SS(I,J)=ZL+SI(J2)*ZBL1
1+SI(J3)*ZBL2+SI(J4)*ZBL2)*P1
SSD(I,J)=(SIX(J1)+ZL+SI(J2)*ZBL2+ZBL2+ZBL3*SIX(J2)*ZBL2)
1+ZBL2+ZBL3+ZBL2+ZBL3)*P2CV1
SSDD(I,J)=SIXX(J1)+ZL+SIXX(J2)*ZBL2
1 CONTINUE
DO 1045 I=1,3
  J=(I-1)*4
  J1=J+1
  J2=J1+1
  J3=J2+1
  J4=J3+1
UU(I,N)=DIS(J1)*ZL*DIS(J2)*ZBL*
1 CONTINUE
DO 1045 I=1,3
  J=(I-1)*4
  J1=J+1
  J2=J1+1
  J3=J2+1
  J4=J3+1
UU(I,N)=DIS(J1)*ZL*DIS(J2)*ZBL*
1 CONTINUE
DO 1075 J=1,NBT
  JJ=J+2
  JJ1=JJ-1
  S(I,JJ1)=REAL(SS(I,J))
  S(I,JJ)=AIMAG(SS(I,J))
  SD(I,JJ1)=REAL(SSD(I,J))
  SD(I,JJ)=AIMAG(SSD(I,J))
1 CONTINUE
DO 1065 I=1,2
  DO 1065 J=1,NBT
    JJ=J+2
    JJ1=JJ-1
    U(I,JJ1)=REAL(UU(I,J))
    U(I,JJ)=AIMAG(UU(I,J))
    UD(I,JJ1)=REAL(UUD(I,J))
    UD(I,JJ)=AIMAG(UUD(I,J))
1 CONTINUE
RETURN
1 CONTINUE
2=X-MUU(1,II)*Y
ZE=CONJG(Z)
DO 1170 N=1,NBT
CALL GUEFF(N,II)
LM=LM(N)
LI=LM-2
ZL2=LJ*CLOG(Z)
ZL2=CEXP(ZL2)
ZBL2=ZB+ZL2
ZBL2=CONJG(ZL2)
ZBL2=CONJG(CEXP(ZBL2))
ZBL2=ZB+ZBL2
ZL1=LZ2+Z
ZBL1-ZBL2*LB
ZBL1-ZBL2*LB
ZBL1-ZBL2*LB
ZBL1-ZBL2*LB
ZBL1-ZBL2*LB
ZBL1-ZBL2*LB
ZBL1-ZBL2*LB
ZBL1-ZBL2*LB
DO 1145 I=1,2
  J=(I-1)*4
  J1=J+1
  J2=J1+1
  J3=J2+1
  J4=J3+1
UU(I,N)=DIS(J1)*ZL*DIS(J2)*ZBL*
1
\*DIS(J3)+ZBL1+DIS(J4)+ZBL1
UUD(I,N)=DIS(J1)+ZLI+DIS(J2)+ZBL1
1
\*DIS(J3)+ZBL2+DIS(J4)+ZBL2+P1CV1

1140 CONTINUE
1170 CONTINUE
DO 1180 I=1,2
DO 1180 J=1,NBT
JJ=J-1
U(I,JJ)=REAL(U(U(I,J))
U(I,JJ)=AIMAG(U(U(I,J))
UD(I,JJ)=REAL(UD(I,J))
UD(I,JJ)=AIMAG(UUD(I,J))

1180 CONTINUE
RETURN

1200 CONTINUE
Z=X-MUU(1,II)*Y
ZB=CONJG(Z)
DO 1270 N=1,NBT
CALL COEFF(N,II)
LM=LAN(N)
LJ=LM-1
ZL=LI+LJ+LZL
ZL=CONJG(ZL)
ZB1=ZB+ZL
ZB1=CONJG(LJ)+LJ+LZL
ZB1=CONJG(ZB1)
ZB1=Z+ZBL1
ZL=ZL1=Z
ZBL=ZB1=ZB
DO 1240 I=1,2
J=(I-1)*4
J1=J+1
J2=J+1
J3=J+1
J4=J+1
U(I,J)=DIS(J1)+ZL+DIS(J2)+ZBL1
1
\*DIS(J3)+ZB1+DIS(J4)+ZBL1

1
\*DIS(J3)+ZBL1+DIS(J4)+ZBL1
1340 CONTINUE
1270 CONTINUE
DO 1280 I=1,2
DO 1280 J=1,NBT
JJ=J+2
JJ1=JJ-1
U(I,JJ1)=REAL(UU(I,J))
U(I,JJ)=AIMAG(UU(I,J))

1280 CONTINUE
RETURN

1300 CONTINUE
Z=X-MUU(1,II)*Y
ZB=CONJG(Z)
DO 1270 N=1,NBT
CALL COEFF(N,II)
LM=LAN(N)
LJ=LM-1.
ZL=LI+LJ+LZL
ZL=CONJG(ZL)
ZB2=ZB+ZL
ZB2=CONJG(LI)+LJ+LZL
ZB2=CONJG(ZB2)
ZB2=Z+ZBL3
ZL=ZL2=Z
ZBL=ZB2=ZB
ZBL2=ZB2+ZB
ZB2=ZBL2+ZB
ZBL2=Z+ZBL2
ZL1=ZL2+2
ZBL1=ZB+ZL1
ZBL1=ZBL2+2B
ZBL1=Z+ZBL1
ZL=ZL1+Z
ZBL=ZBL1+Z
DO 1340 I=1,2
J=(I-1)*4
J1=J+1
J2=J1+1
J3=J2+1
J4=J3+1
UU(I,N)=DIS(J1)+ZL*DIS(J2)+ZBL
1 +DIS(J3)+ZBZL+DIS(J4)+ZZBL1
UD(I,N)=-(DISX(J1)+ZL+DISX(J2)+ZBL1
1 +DISX(J3)+ZBZL+DISX(J4)+ZZBL2)+P1CV1
UDDD(I,N)=(DISXX(J1)+ZL2+DISXX(J2)+ZBL2
1 +DISXX(J3)+ZBZL3+DISXX(J4)+ZZBL3)*P2CV2
CONTINUE
1340
CONTINUE
DO 1380 J=1,2
DO 1380 J=1,NBT
JJ=J+2
J1=JJ
UI(J,J1)=REAL(UU(I,J))
1 UJ=AJMAG(UU(I,,))
1 UD(J,J1)=REAL(UDD(I,J))
UDD(I,J1)=AJMAG(UDDD(I,J))
CONTINUE
1380
RETURN
1500
CONTINUE
Z=X*MUU(I,II)*Y
ZB=CONJG(Z)
DO 1570 N=1,NBT
CALL COEFF(N,II)
LM=LAM(N)
LJ=LJ-2
ZL=ZL1+CLOG(2)
ZL2=CEXP(ZL2)
ZBL2=ZB+ZL2
ZBL2=CONJG(LJ)*CLOG(2)
ZBL2=CONJG(CEXP(ZBL2))
ZBL2=Z+ZBL2
ZL1=ZL2+2
ZBL1=ZB+ZL1
ZBL1=ZBL2+ZB
ZBL1=Z+ZBL1
ZL=ZL1+Z
ZBL=ZBL1+Z
DO 1580 I=1,3
J=(I-1)*4
J1=J+1
J2=J1+1
J3=J2+1
J4=J3+1
SS(I,N)=(SI(J1)+ZL1+SI(J2)+ZBL1
1 +SI(J3)+ZBL2+SI(J4)+ZZBL2)*P1
CONTINUE
1580
CONTINUE
DO 1640 I=1,2
J=(I-1)*4
J1=J+1
J2=J1+1
J3=J2+1
J4=J3+1
U(I,N)=DIS(J1)*ZL5+DIS(J2)*ZBL0
1=DIS(J3)*ZBL1+DIS(J4)*ZBLL1

1540 CONTINUE
1570 CONTINUE
DO 1575 I=1,3
DO 1575 J=1,NBT

1575 CONTINUE
DO 1580 J=1,NBT
JJI=J+1
S(I,JJI)=REAL(SS(I,J))
S(I,JJI)=-AIMAG(SS(I,J))

1580 CONTINUE
RETURN
1600 DO 1605 N=1,NBT
CALL COEFF(N,II)
1605 CONTINUE
Z=A+WXU(1,II)*Y
ZB=CONJG(Z)

DO 1630 J=1,3
J+1=J+1

1630 CONTINUE
DO 1675 I=1,3
DO 1675 J=1,NBT
JJ=J+1
S(I,JJI)=REAL(SS(I,J))
S(I,JJI)=-AIMAG(SS(I,J))

1675 CONTINUE
RETURN

C END
SUBROUTINE PRECRCK(C,NUMMAT,TT,IL)

C
COMMON/RI28,R(280),R(280),CODE(280),IX(6,250)
COMMON/TIP/NCR1,NCR2,NCR3,NCR4,RELX,CTPX,CTPY,SIP1,SIP2
COMMON /C7/ICASE(2)
COMMON /C1/IY,P2IY,PEI2
COMPLEX MU(2),IY,P2IY
IY=(0.,1.)
PEI2=2.*ACOS(-1.)
P2IY=PEI2*IY
WRITE(6,120) TT,C
NMAT=2
IF(IX(6,NCR1).EQ.IX(6,NCR2)) NMAT=1
DO 10 II=1,NMAT
10 IF(II.EQ.1) II=IX(6,NCR1)
IF(II.EQ.2) II=IX(6,NCR2)
CALL ROOTS(C,II,IIS,MU)
IF(ICASE(II).EQ.1) CALL MULTI(MU,II,IIS)
IF(ICASE(II).EQ.2) CALL MULT2(MU,II,IIS)
CONTINUE
IF(NMAT.EQ.2) GO TO 20
CALL EQUATE
20 CALL EIGEN(IL)
RETURN
130 FORMAT(///,38X,6BTIME =,1PE13.4,6X,28HCRACK-TIP VELOCITY =,
1 1PE13.4,//)
END
SUBROUTINE ROOTS(C,II,IIS,MU)
COMPLEX MU(2),MUI,IV,P2IY
DIMENSION P(6),X(4),A1(4),E(6),BB(5),CC(6),ALPHA(2),BETA(2)
COMMON/SUMA/A(3,3,3),AJY(3,1,6),ADET(6),ASIZE(6),CL
COMMON/HRO/HRO(6),BODUM
COMMON/EIG/EIG1(6),EIG2
COMMON/TOLR/TOLER1,TOLER2
COMMON /C1/IV,P2IY,PRI2
COMMON /C8/AA(3,3)
COMMON /C7/ICASE(2)
COMMON /C14/MUU(2,2)
IF(EIG1(IIS).EQ.0) GO TO 2
AA(1,1)=-1.
AA(2,1)=0.
AA(1,2)=0.
AA(2,2)=-1.
AA(1,3)=0.
AA(2,3)=0.
AA(3,3)=0.
C FOR A RIGID MATERIAL THE CHARACTERISTIC EQUATION BECOMES AN IDENTITY
C AND THEREFORE THE ROOTS CAN BE ARBITRARILY CHOSEN TO HAVE ANY VALUES
C HERE WE ARBITRARILY CHOOSE A MULTIPLE ROOT OF MU(1)=(0,0) JUST TO
C HAVE SOME VALUE FOR THE ROOTS OF THE CHARACTERISTIC EQUATION SO THAT
C THE CALCULATIONS CAN GO ON. ANY OTHER VALUES FOR MU(1) AND MU(2)
C COULD HAVE BEEN CHOSEN AS WELL. THE SOLUTION WILL NOT DEPEND ON THESE
C ROOTS AND THE SOLUTION WILL BE THE SAME REGARDLESS OF THE ROOTS
C CHOSEN HERE.
ICASE(II)=2
MU(1)=(0,1) GO TO 32
2 B1=2*B0(IIS)+C*C
  A1=A(1,1,II)
  A2=A(1,2,II)
  A3=A(2,1,II)
  A4=A(2,2,II)
  A5=A(3,1,II)
  A6=A(3,2,II)
  B1=A4*A5-A5*A6
  B2=A3*A5-A4*A6
  B3=A2*A5-A3*A6
  B4=A1*A5-A2*A6
  B5=A2*A6-A3*A6
  B6=A1*A6-A4*A6
  D4=1.-(A1-A2)**R1
  IF (ABS(B4).LT.TOLER1) GO TO 34
  D1=AS=[A(3,2,II)]**R1-(1.-A5*A1)*D4
  D2=-(B6-R1+A5)**R1
  D3=-(A5+B5)**R1
  DD=-(D1*D4-D2*D5)
  IF (ABS(DD).LT.TOLER1) GO TO 34
  AA(1,1)=(A2-B2)**R1
  AA(2,1)=(1.-A1-A5+B4)**R1
  AA(3,1)=(A5+B5)**R1
  AA(1,2)=(1.-A2)**R1
  AA(2,2)=A1**R1
  AA(3,2)=0.
  AA(1,3)=(A2-A5)**R1
  AA(2,3)=2.-A5**R1
  AA(3,3)=D4
  IF (AA(1,3).EQ.0) GO TO 34
  P(1)=(A4-R1*[(B1+B6)**R1*(R1+1)*ADET(IIS)])/A1
  P(2)=-2.*(A5-R1*(B3+B6))/A1
  P(3)=(A5+2.*A2-R1*(B4+B6))/A1
P(4) = -2. * AS/A1
P(6) = 1.
IF (P(1) .LE. 0.) GO TO 34
C SOLVE FOR THE ROOTS ANALITICALLY IF P(2) = P(4) = 0.
IF (ABS(P(2)) .LT. TOLER1) AND. ABS(P(4)) .LT. TOLER1) GO TO 210
GO TO 255
210 IF (ABS(P(3)) .GT. TOLER1) GO TO 220
MU(1) = (P(1) / 4.) ** .25 * (1 + IY)
MU(2) = CONJG(MU(1))
ILOCAL(II) = 1
GO TO 23
220 D = P(3) * P(3) - 4. * P(1)
IF (D .LT. 0.) GO TO 230
GO TO 246
230 D = -P(3) + 2. * SQRT(P(1))
IF (D .LT. 0.) GO TO 34
MU(1) = (SQRT(D) * IY + SQRT(P(3) + 2. * SQRT(P(1)))) / 2.
IF (REAL(MU(1)) .GT. TOLER1) GO TO 235
MU(1) = IY * SQRT(P(3) / 2.)
ILOCAL(II) = 3
GO TO 32
235 MU(2) = CONJG(MU(1))
ILOCAL(II) = 1
GO TO 23
246 IF (P(3) .LT. 6.) GO TO 34
D = SQRT(D)
D1 = (P(3) + D) / 2.
D2 = (P(3) - D) / 2.
D1 = SQRT(D1)
D2 = SQRT(D2)
IF ((D1 + D2) .LT. TOLER1 + D1) GO TO 247
MU(1) = IY * D1
MU(2) = IY * D2
ILOCAL(II) = 2
GO TO 23
247 MU(1) = IY * (D1 + D2) / 2.
ILOCAL(II) = 2
GO TO 32
C SOLVE FOR THE ROOTS ANALITICALLY IF P(3) = (P(4) ** 2) / 4. + 2. * SQRT(P(1))
C AND P(2) = P(4) + SQRT(P(1)) WHICH GIVES ONLY ONE PAIR OF COMPLEX
C ROOTS OF MULTIPLICITY 2
C NOTE THAT THE BOXING LIBRARY ROUTINE "PROOT" (BELOW) IS NOT GOOD
C IN FINDING MULTIPLE ROOTS
250 IF (ABS(P(3)) * P(4) = (P(4) / 4.) - 2. * SQRT(P(1))) .LT. TOLER1) AND.
1 ABS(P(2) - P(4) + SQRT(P(1))) .LT. TOLER1) GO TO 250
GO TO 270
250 D = SQRT(P(1)) + P(4) / 16.
IF (D .LT. 0.) GO TO 34
D = SQRT(D)
MU(1) = P(4) / 4. * IY + D
IF (ABS(REAL(MU(1))) .LT. TOLER1) MU(1) = IY + D
ILOCAL(II) = 2
GO TO 32
C SOLVE FOR THE ROOTS NUMERICALLY
270 CALL PROOT(4, P, R, R, I, R, CC, CONV)
IF (CONV .LT. 0.) GO TO 38
N = 0
DO 5 N = 1, 4
IF (ABS(R(N)) .LT. TOLER1) R(N) = 0.
BI = AI(N)
IF (BI .LT. 0.) GO TO 5
EN = NN + 1
BETA(NN) = BI
ALPHA(NN) = R(N)
5 CONTINUE
IF (KN .NE. 2) GO TO 30
R1=ABS(ALPHA(2)-ALPHA(1))
RR1=TOLE1*ABS(ALPHA(2))
R2=ABS(BETA(2)-BETA(1))
RR2=TOLE2*ABS(BETA(2))
ICASE(II)=1
IF (R1.LT. RR1 .AND. R2.LT. RR2) ICASE(II)=2
IF (ICASE(II) .EQ. 1) GO TO 10
IF (ICASE(II) .EQ. 2) GO TO 30
10 DO 12 I=1,2
MU(I)=CMPLX(ALPHA(I),BETA(I))
12 CONTINUE
20 DO 25 N=1,2
MU(N,II)=MU(N)
25 CONTINUE
WRITE(6,48) IIS,MU
GO TO 40
30 MU(1)=CMPLX((ALPHA(1)+ALPHA(2))/2.,(BETA(1)+BETA(2))/2.)
31 MU(1,II)=MU(1)
IF (IIG1(IIS).EQ.1) WRITE(6,60) IIS,MU(1)
WRITE(6,60) IIS,MU(1)
GO TO 40
34 WRITE(6,92) IIS
GO TO 39
35 WRITE(6,94) IIS
GO TO 39
36 WRITE(6,98) IIS
STOP
11 RETURN
40 FORMAT(///,15X,41H THE CHARACTERISTIC EQUATION FOR MATERIAL#.,I1,
1 1X,15EXHAS ROOTS OF://,36X,1H(),F16.8,1H(),F16.8,1H(),6X,5HHAND 
21H(),F16.8,1H(),F16.8,1H))
45 FORMAT(///,15X,41H THE CHARACTERISTIC EQUATION FOR MATERIAL#.,I1,IX,
1 25EHAS MULTIPLE ROOT OF:EX,1H(),F16.8,1H(),F16.8,1H))
46 FORMAT(///,15X,41H THE CHARACTERISTIC EQUATION FOR MATERIAL#.,I1,IX,
1 1,15X,6HAS ARBITRARILY BEEN CHOSEN TO HAVE A MULTIPLE ROOT OF:
25X,1H(),F16.8,1H(),F16.8,1H))
50 FORMAT(///,15X,13HP FOR MATERIAL#.,I1,/. 
1 6X,8H GIVEN COMBINATION OF THE MATERIAL PROPERTIES A 
2ND THE CRACK-TIP SPEED WILL NOT/.6X,72H RESULT IN COMPLEX CONJUGAT 
3E ROOTS FOR THE CHARACTERISTIC EQUATION.//,6X,69H TRY LOWERIN 
4G THE CRACK-TIP SPEED OR CHANGING THE MATERIAL PROPERTIES.)
54 FORMAT(///,15X,13HP FOR MATERIAL#.,I1,/. 
1 5X,69H THE BORING LIBRARY ROUTINE 'PROOT' WAS NOT ABLE TO 
2 FIND THE ROOTS OF THE CHARACTERISTIC EQUATION//,6X,69H TRY LOWERIN 
3G THE CRACK-TIP SPEED OR CHANGING THE MATERIAL PROPERTIES.)
59 FORMAT(///,15X,13HP FOR MATERIAL#.,I1,/. 
1 6X,114H THE BORING LIBRARY ROUTINE 'PROOT' DID NOT FIND P 
2AIR (5) OF COMPLEX CONJUGATE ROOTS FOR THE CHARACTERISTIC EQUATON. 
3//,6X,69H TRY LOWERIN 
4G THE CRACK-TIP SPEED OR CHANGING THE MATERIAL PROPERTIES.)
END
SUBROUTINE MULT1(MU,II,II2)
COMMON /C1/TY,P2TY,PI2
COMMON /SUM/PA(3,3,8),AI(3,3,8),A0(8,8),ADFT(8),ASIZEB(8),CL
COMMON/RHOD/R0D(6),R0UM
COMMON/IRIG/IRIG1(8),IRIG2
COMMON /C5/AA(3,3)
COMMON /S4/SIG(3,2,2),DISP(2,2,2)
COMMON /C6/K2(2,2),K2B(2,2),K(8,2),KB(8,2)
COMPLEX MU(2),K1(8),K1B(4),F(3,2)
COMPLEX X2,K2B,X,KB,MIS1,MIS2,IT,P2ITY,SIG,DISP
P(1,1)=(1.,0.)
P(2,1)=MU(1)*P(1,1)
P(1,2)=MU(1)*P(1,2)
P(2,2)=MU(2)*P(2,2)
P(3,2)=MU(2)*P(3,2)
DO 20 J=1,2
DO 20 I=1,3
SIG(I,J,II)=0.
DO 20 L=1,3
SIG(I,J,II)=SIG(I,J,II)+AA(I,L)*P(L,J)
20 CONTINUE
DO 30 J=1,2
DO 30 I=1,3
DISP(I,J,II)=(0.,0.)
DO 30 L=1,3
DISP(I,J,II)=DISP(I,J,II)+A(I,L,II)*SIG(L,J,II)
30 CONTINUE
DO 40 I=1,2
II2=II1
II1=II1+1
MIS1=SIG(2,I,II)
MIS2=SIG(3,I,II)*IY
K1(II1)=(MIS1-MIS2)/2.
MIS2=(MIS1+MIS2)/2.
K1(II2)=CONJG(MIS2)
DISP(2,I,II)=DISP(2,I,II)/MU(I)
MIS1=DISP(1,I,II)
MIS2=DISP(3,I,II)-IY
K1(I1+4)=(MIS1+MIS2)/2.
MIS2=(MIS1-MIS2)/2.
K1(II2+4)=CONJG(MIS2)
40 CONTINUE
DO 50 I=1,4
K1B(I)=CONJG(K1(I))
50 CONTINUE
E=REAL(K1(4)+K1B(4)-K1(3)-K1B(3))
K2(1,II)=K1(1,II)+K1B(3)+K1(4)-K1B(2))/R
K2(2,II)=K1(2,II)+K1B(3)-K1(4)+K1B(1))/R
DO 60 I=1,2
K2B(1,II)=CONJG(K2(1,II))
60 CONTINUE
K(1,II)=K1(4)+K2B(2,II)
K(2,II)=K1(1,II)
K(3,II)=K1(8)+K2(2,II)
K(4,II)=K(3,II)
K(5,II)=K(8,II)+K2(3,II)
K(7,II)=K(8,II)+K2(1,II)
K(8,II)=K(7,II)+K2(2,II)
DO 80 I=1,8
KB(I,II)=CONJG(K(I,II))
80 CONTINUE
RETURN
END
SUBROUTINE MUL2(MU,II,IIIS)
COMMON /CI/IY,P2TY,PEIS
COMMON /SUMAN/A(3,3,6),AI(3,3,6),ADET(8),ASIZE(8),CL
COMMON/RH0/RH0(6),R0DUM
COMMON/ISIG/ISIG(6),ISIG2
COMMON /CB/AA(3,3)
COMMON /CS/SIG(3,2,2),DISP(2,2,2)
COMMON /CS/K2(2,2),K2B(2,2),K(8,2),KB(8,2)
COMPLEX MU(2),K1(8),K1B(4),P(8,2),EPS(2,2)
COMPLEX K2,K2B,K,KB,MISS1,MISS2,MV,IVY,P2TY,SIG,DISP
MV=MU(I)
P(1,1)=(1.,0.)
P(2,1)=MV/IV
P(3,1)=MV
P(1,2)=(2.,0.)
P(2,2)=2*IV/IV
P(3,2)=MV/IV
DO 28 J=1,2
DO 28 I=1,3
SIG(I,J,II)=0(.,0.)
DO 28 L=1,3
SIG(I,J,II)=SIG(I,J,II)+AA(I,L)*P(L,J)
28 CONTINUE
DO 38 J=1,2
DO 38 I=1,2
EPS(I,J)=(0.,0.)
DO 38 L=1,3
EPS(I,J)=EPS(I,J)+A(I,L,II)*SIG(L,J,II)
38 CONTINUE
DISP(1,1,II)=EPS(1,1)
DISP(1,2,II)=EPS(1,2)-EPS(1,1)
DISP(2,1,II)=EPS(2,1)/MV
DISP(2,2,II)=(EPS(2,2)+EPS(2,1)+IV/MV)/MV
DO 48 I=1,2
I2=2*I
I1=I2-1
MISS1=SIG(2,1,II)
MISS2=SIG(8,1,II)*IV
K1(I1)=(MISS1-MISS2)/2.
MISS=(MISS+MISS2)/2.
K1(I2)=CONJG(MISS)
MISS1=DISP(1,1,II)
MISS2=DISP(8,1,II)*IV
K1(I2+4)=(MISS+MISS2)/2.
MISS=(MISS-MISS2)/2.
K1(I2+4)=CONJG(MISS2)
48 CONTINUE
DO 58 I=1,4
K1B(I)=CONJG(K1(I))
58 CONTINUE
R=REAL(K1(2)*K1B(2)-K1(1)*K1B(1))
K2(1,II)=(K1(8)*K1B(1)-K1(2)*K1B(4))/R
K2(2,II)=(K1(4)*K1B(1)-K1(2)*K1B(3))/R
DO 68 I=1,3
K2B(I,II)=CONJG(K2(I,II))
68 CONTINUE
K(1,II)=-K(2)*K2B(2,II)
K(3,II)=K(1,II)
K(4,II)=-K(3,II)
K(6,II)=K(6)*K2(1,II)+1.*K1(7)
K(8,II)=K(8)*K2B(2,II)
K(7,II)=K(8)*K2B(1,II)+1.*K1(8)
K(8,II)=K(8)*K2(2,II)
DO 68 I=1,8
\[ KB(I, II) = \text{CONJG}(K(I, II)) \]

CONTINUE
RETURN
END
SUBROUTINE EIGEN(IL)
COMPLEX LAM
COMPLEX B1,B2,B1,C1,C2,C3,C4,C5,C6,C7,C8,CC
COMPLEX E2,E2B,E1,B1,IB1,IP1,FP1,F,FP
COMMON /SUMA/A(8,8,8),AI(8,8,8),ADDET(8),ASIZE(8),CL
COMMON/XL/NA,RAA,A1,KB,KB,NK,NI,NI2,T21,IL,IT,IT2,IC,IC1,
COMMON/ERG/ERG1(8),ERG2
COMMON/TOLB/TOLER1,TOLER2
COMMON /C1/IT,F2/FP2,KZ2
COMMON /C6/EZ(2,2),KZB(2,2),K(8,2),KB(8,2)
COMMON /C8/LAM(26)
COMMON /C12/F(8)
COMMON /C16/F(4,2)
COMMON /C8/E(4,4)
COMMON /C8/Z(2)
COMPLEX Z, ZH
A11-E=REAL(K(1,1)-KB(1,1)-K(8,1)+KB(8,1))
A21-E=REAL(K(6,1)-KB(6,1)+K(8,1)-KB(8,1))
K(7,1)-KB(7,1)+K(8,1)-KB(8,1))
A12-E=REAL(K(1,2)-KB(1,2)-K(8,2)+KB(8,2))
A22-E=REAL(K(5,2)-KB(5,2)+K(8,2)-KB(8,2))
K(7,2)-KB(7,2)-K(8,2)-KB(8,2))
B1=E-K(6,1)+KB(6,1)-K(8,1)-KB(8,1))
B3=E-K(6,2)+KB(6,2)-K(8,1)-KB(8,1))
C1=E-K(1,1)-KB(1,1)-K(8,2)+KB(8,2))
C2=E-K(6,1)-KB(6,1)-K(8,2)+KB(8,2))
C3=E-K(7,1)-KB(7,1)-K(8,2)+KB(8,2))
C4=E-K(7,2)-KB(7,2)-K(8,2)+KB(8,2))
C5=E- KB(2,1)+K(8,1)-KB(8,1))
C6=E-KB(2,2)-K(8,1)-KB(8,1))
C7=E- KB(6,1)+K(8,1)-KB(8,1))
C8=E-KB(8,1)+K(8,1)-KB(8,1))
CC=C1+C6+C5+C8+CONJG(C5+C6-C1-C4)
IF (REAL (B1).EQ.0 .AND. AIMAG (B1).NE.0.) STOP 12
IF (REAL (B2).EQ.0 .AND. AIMAG (B2).NE.0.) STOP 12
IF (REAL (CC).EQ.0 .AND. AIMAG (CC).NE.0.) STOP 12
IF (REAL (B1).EQ.0.) GO TO 2
IF (ABS (AIMAG (B1)).EQ.1).GT.TOLER1) STOP 12
IF (REAL (B2).EQ.0.) GO TO 4
IF (ABS (AIMAG (B2)).EQ.1).GT.TOLER1) STOP 12
IF (REAL (CC).EQ.0.) GO TO 6
IF (ABS (AIMAG (CC)).EQ.1).GT.TOLER1) STOP 12
CONTINUE
R=A11-A22-A12=2-A21=2=REAL(C5-C7-C1-C2)
Q=REAL(-A11=2-B12=2=1-C1-C2)
R=E/(2.+Q)
D=R=1.
IF (D.LT.-TOLER1) STOP 12
IF (D.GE.-TOLER1 .AND. D.LT.0.) D=0.
SIZE=R-SQRT(D)
EPS=ALOG (SIZE)/PEI2
IF (EPS.GE.-TOLER1 .AND. EPS.LT.0.) EPS=0.
LAM(1)=COMPLEX (0,EPS)
LAM(2)=(-1,0)
DO 30 J=1,2
IF (J.EQ.2) GO TO 30
IF (EPS.EQ.0.) GO TO 30
CALL MAT (LAM(1),1,8)
CALL MAT (LAM(1),2,8)
IDET=0
CALL INV2 (IDET)
IF (IDET.EQ.0) GO TO 20
EPS=EPS
LAM(1)=CONJG (LAM(1))
CALL MAT (LAM(1),1,8)
CALL MAT(LAM(1), 2, 3)
IDENT=0
CALL INVS(IDENT)
IF(IDENT.EQ.0) GO TO 20
WRITE(6, 140) EPS
WRITE(6, 150)
IL=1
RETURN
20 IL=0
WRITE(6, 140) EPS
FF(1,1)=F(1)
FF(2,1)=F(2)
FF(3,1)=F(5)
FF(4,1)=F(6)
GO TO 50
30 CONTINUE
IF(IRIG2.EQ.1) GO TO 100
CALL MAT(LAM(1), 1, 2)
CALL MAT(LAM(1), 2, 2)
IDENT=0
CALL INVS(IDENT)
IF(IDENT.EQ.0) GO TO 32
WRITE(6, 150)
IL=1
RETURN
32 IL=0
FF(1,1)=F(1)
FF(2,1)=F(2)
FF(3,1)=F(5)
FF(4,1)=F(6)
50 CONTINUE
KK=0
IF(NBT.LT.2) RETURN
DO 70 I=3, NBT, 2
KK=KK+1
LAM(I)=LAM(1)+KK
LAM(I+1)=LAM(2)+KK
70 CONTINUE
RETURN
100 CONTINUE
FF(1,2)=FF(1,1)
FF(2,2)=FF(2,1)
FF(3,2)=FF(3,1)
FF(4,2)=FF(4,1)
DO 110 I=2, NBT
LAM(I)=LAM(I-1)
110 CONTINUE
RETURN
140 FORMAT//(///, 36X, 36X'THE ELASTIC BIMATERIAL COEFFICIENT IS:', F16.8, 1 ' //////)
150 FORMAT//(///, 36X, 36X'THE COMBINATION OF MATERIAL(S) PROPERTIES AND T
1HE GIVEN CRACK-TIP VELOCITY PRESENTS AN EXTREME SITUATION.,///, 16X, 216X'THE DETERMINANT OF EIGEN-FUNCTION COEFFICIENT MATRIX VANISH
3FOR THE EIGEN-VALUES, THEREFORE MAKING IT,, 16X, 80X'IMPOSSIBLE TO D
4TERMINE THE RELATIVE VALUES OF THE COEFFICIENTS OF THE EIGEN-FUNCT
6ION.,///, 16X, 80X'TRY LOWERING THE CRACK-TIP SPEED OR CHANGING THE MAT
8ERIAL PROPERTIES.')
C
END
SUBROUTINE INV22(IDET)
COMPLEX H(I,2), H(2,I), P2Y
COMMON/X, Y, P2Y, P2Y
COMMON/EX/EX(400),E(800),CDE(800),I(9,256)
COMMON/TIP/NCR1, NCR2, NCR3, NCR4, NHEL, CTX, CTPY, SIF1, SIPE
COMMON/SUM/X(3,3), AI(3,3), ADET(3), ASIZE(3), CL
COMMON/TULX/TULY, TULZ
COMMON/CX/CX(4,4)
COMMON/C12/F(8)
R=REAL(H(3,8)*H(4,4)-H(4,3)*H(3,4))
SA=ASIZE(I(9,256))
SAS=SA+SA+TOLER1
IFABS(R) .GT. SA) GO TO 5

IDET=1
RETURN

5
IF(I)=H(4,4)/R
IF(2)=H(2,4)/E
F(1)=(1,0,0)
F(2)=(0,1,0)
DO 10 I=1,2
J=I+4
F(J)=(0,0,0)
DO 10 J=I+4,10
F(J)=F(J)+H(I,2)+H(I,2,1)
10 CONTINUE
F(8)=CONJG(F(8))
RETURN
END
SUBROUTINE INV2(IDET)
COMMON/XH(2,2),D2(2,2),CMODE(2,2),IX(8,256)
COMMON/TIP,NCR1,NCR2,NCR3,NCR4, NLXL,CTPX,CTPY,SIF1,SIP2
COMPLEX DET, HH(12,3),HI(1,3),H,F,IT,P2Y
COMMON /SUMMA/A(2,3,6),AI(2,3,6),ADET(6),ASIZE(6),CL
COMMON/TOLR/TOLER1,TOLER2
COMMON /CI/IT,P2Z,PEI2
COMMON /CS/Hi(1,1)
COMMON /C12/F(10)
EQUIVALENCE (H(1,1),HI(1,1))
H(1) = H(1,1)
H(2) = H(2,1)
H(3) = H(4,1)
H(1,1) = H(1,2)
H(1,2) = H(2,1)
H(1,3) = H(4,3)
H(2,1) = H(1,3)
H(2,2) = H(2,3)
H(2,3) = H(1,4)
H(3,2) = H(4,2)
H(3,3) = H(4,4)
HI(1,1) = HI(2,2)*HH(3,3)-HI(3,2)*HI(2,3)
HI(1,2) = HI(2,1)*HI(3,2)-HI(3,1)*HI(2,3)
HI(1,3) = HI(2,1)*HI(3,3)-HI(3,1)*HI(2,2)
DTER=<HI(1,1)*HH(1,1)+HI(1,2)*HH(2,1)+HI(1,3)*HH(3,1)/2.
IF (CABS(DTER).GT. SA*TOLER1) GO TO 15
IDET=3
RETURN
15 DO 20 I=1,3
HI(1,1)=HI(1,1)/DTER
20 CONTINUE
HI(2,1) = (HI(2,1)*HI(3,2)-HI(3,1)*HI(2,3))/DTER
HI(2,2) = (HI(2,1)+HI(3,3)-HI(3,1)+HI(2,3))/DTER
HI(2,3) = (HI(2,1)+HI(3,3)-HI(3,1)+HI(2,2))/DTER
HI(3,1) = (HI(2,1)+HI(3,3)-HI(3,1)+HI(2,2))/DTER
HI(3,2) = (HI(3,1)+HI(1,2)-HI(1,1)+HI(3,2))/DTER
HI(3,3) = (HI(3,1)+HI(1,2)-HI(1,1)+HI(2,2))/DTER
F(1) = (1..8)
DO 30 I=1,3
IF (I .EQ. 1) J=2
F(J) = (1...8)
30 IF (J .EQ. 1) HI(I,K)+HI(K)
CONTINUE
RETURN
END
SUBROUTINE MAT(X, II, IFLAG)
COMPLEX X, Z, K, E2, EB, K, KB, IY, P2iY
COMMON /C1/IY, P2iY,P5I2
COMMON /C5/E2(2,2),EB(2,2),K(8,2),KB(8,2)
COMMON /C8/1(4,4)
IM=1
IF (IFLAG .EQ. 2) IM=3
Z(1)=COSH(P2iY*K)
Z(2)=1/Z(1)
DO 98 J=IM,4
I2=2*I
JJ=2
III=3
JP=I2-8
IF (I .LE. 2) III=1
IF (I .EQ. 1 .OR. I .EQ. 3) GO TO 10
GO TO 28
10 JJ=1
JP=I2+1
28 K=JJ
IF (I .EQ. 2) K=K-KJ
JK=(I-I-1)+2
JJ=JJ+JK
IF (IFLAG .EQ. 3 .AND. III .EQ. 3) GO TO 85
H(III,JJJ)=I(I-1,II)+K(I2,II)+2(K)
85 H(III+1,JJJ)=KB(JF,II)+EB(JF-1,II)+2(K)
90 CONTINUE
RETURN
END
SUBROUTINE CORFF(I,II)
COMPLEX LAM,X,F,IFY,P2IY,FF
COMMON /C1/IY,P2IY,PIH2
COMMON /C7/ICASE(2)
COMMON /C18/LAM(28)
COMMON /C12/F(6)
COMMON /C16/FF(4,2)
IF(I-I/2+2 .EQ.8) GO TO 36
F(1)=FF(1,1)
F(2)=FF(2,1)
F(5)=FF(3,1)
F(6)=FF(4,1)
GO TO 46
36 F(1)=FF(1,2)
F(2)=FF(2,2)
F(5)=FF(3,2)
F(6)=FF(4,2)
46 CONTINUE
X=LAM(I)
IF(ICASE(II) .EQ.1) CALL CONS1(X,II)
IF(ICASE(II) .EQ.2) CALL CONS2(X,II)
CALL MUL(T,X,II)
RETURN
END
SUBROUTINE CONS1(IX,II)
COMPLEX X, Z, F, K2, KB, I, KB, IY, P2IY
COMMON /C1/IY, P2IY, P12
COMMON /C5/K2(2,2), KB(2,2), I(2,2), KB(2,2)
COMMON /C6/I2(2)
COMMON /C7/F(6)
Z(1) = CEPS(P2IY * X)
Z(2) = 1 / Z(1)
IF (II .EQ. 2) GO TO 26
F(3) = K2(1,1) * F(1) + K2(2,1) * Z(2) * F(2)
F(4) = K2B(2,1) * Z(1) + F(1) + K2B(1,1) * F(2)
GO TO 166
26 F(7) = K2(1,2) * F(5) + K2(2,2) * Z(4) * F(6)
F(8) = K2B(2,2) * Z(2) + F(6) + K2B(1,2) * F(8)
166 RETURN
END
SUBROUTINE CONS2(I,II)
COMPLEX X,X1,X2,I2,BB,E6,FP,IV,II
COMMON /C1/IV,IP2I,PE1P
COMMON /G5/E6,E2,EB(2,2),EB(2,1),EB(1,1),EB(1,2)
COMMON /G2/E6,E2
COMMON /C12/P(0)
Z(1)=CEEP(P2IY+X)
Z(2)=1/Z(1)
X1=X-1.
IF(I1 .EQ. 2) GO TO 28
F(0)=(X1(1,1)-X1)*F(1)+X1(2,1)*Z(2)*F(2)
F(1)=EB(2,1)*Z(1)*F(1)+(EB(1,1)-X1)*F(2)
GO TO 100
28
F(7)=(X1(1,2)-X1)*F(6)+X1(2,2)*Z(1)*F(6)
F(8)=EB(2,2)*Z(2)*F(6)+(EB(1,2)-X1)*F(8)
100 RETURN
END
SUBROUTINE MULT(X,II)
COMPLEX X,XX,XXI,XXII,XXIII,WM,WM1,WM2,WM3,MIS(2),MISB(2)
COMPLEX SIG,DISP,SI,SIX,SIXX,DIS,DISX,DISXX,P,PY,PYY,MMU
COMMON/DIM/HA,MAA,NBT,MB,NG,NS,NINT,NINT2,IALL,NINT2,SINCUD
COMMON /CI/TY,PM2Y,PRE2
COMMON /C4/SIG(3,2,2),DISP(2,2,2)
COMMON /CT/ICASE(2)
COMMON /CM/F(4)
COMMON /C18/SI(12),SIX(12),SIXX(12)
,,DIS(8),DISX(8),DISXX(8)
COMMON /C14/MU(2,2)
J=(II-1)+4
J1=J+1
J2=J1+1
J3=J2+1
J4=J3+1
XX=XX(II-1)
XXI=XXI(II-1)
XXII=XXII(II-1)
IF(ICASE(II))EQ.2 GO TO 56
DO 10 I=1,3
DO 6 N=1,2
MIS(N)=SIG(I,N,II)
MISB(N)=CONJG(MIS(N))
CONTINUE
K=(I-1)+4
K1=K+1
K2=K1+1
K3=K2+1
K4=K3+1
WM=MIS(1)+P(J1)
SI(K1)=WM+X
SIX(K1)=WM+XX
SIXX(K1)=WM+XXX
WM=MISB(1)+P(J2)
SI(K2)=WM+X
SIX(K2)=WM+XX
SIXX(K2)=WM+XXX
WM=MISB(2)+P(J3)
SI(K3)=WM+X
SIX(K3)=WM+XX
SIXX(K3)=WM+XXX
WM=MISB(3)+P(J4)
SI(K4)=WM+X
SIX(K4)=WM+XX
SIXX(K4)=WM+XXX
CONTINUE
DO 20 I=1,2
DO 15 N=1,2
MIS(N)=DIS(I,N,II)
MISB(N)=CONJG(MIS(N))
CONTINUE
K=(I-1)+4
K1=K+1
K2=K1+1
K3=K2+1
K4=K3+1
WM=MIS(1)+P(J1)
DIS(K1)=WM
DISX(K1)=WM+X
DISXX(K1)=WM+XX
WM=MISB(1)+P(J2)
DIS(K2)=WM
DISX(K2)=WM+X
DISXX(K2)=WM+XX
MM = MIS (2) * F (J3)
DIS (K3) = MM
DISX (K3) = MM * X
DISXX (K3) = MM * XX1
MM = MISB (2) * F (J4)
DIS (K4) = MM
DISX (K4) = MM * X
DISXX (K4) = MM * XX1
CONTINUE
RETURN

50 CONTINUE
DO 56 I = 1, 3
DO 56 N = 1, 2
MIS (N) = SIG (I, N, II)
MISB (N) = CONJG (MIS (N))
CONTINUE
K = (I - 1) * 4
K1 = K + 1
K2 = K + 1
K3 = K + 1
K4 = K3 + 1
MM = MIS (1) * F (J3) * MIS (2) * F (J1)
MM1 = MIS (1) * F (J3)
MM2 = MM1
SI (K1) = MM * X
SIX (K1) = (MM + MM1) * XX1
SIXX (K1) = (MM + MM1 + MM1) * XX1X2
MM = MISB (1) * F (J4)
MM1 = MISB (1) * F (J4)
MM2 = MM1
SI (K2) = MM * X
SIX (K2) = (MM + MM1) * XX1
SIXX (K2) = (MM + MM1 + MM1) * XX1X2
MM = MM2
SI (K3) = MM * XX1
SIX (K3) = MM * XX1X2
SIXX (K3) = MM * XX1X2X3
MM = MM2
SI (K4) = MM * XX1
SIX (K4) = MM * XX1X2
SIXX (K4) = MM * XX1X2X3
CONTINUE
DO 60 I = 1, 2
DO 60 N = 1, 2
MIS (N) = DISP (I, N, II)
MISB (N) = CONJG (MIS (N))
CONTINUE
K = (I - 1) * 4
K1 = K + 1
K2 = K + 1
K3 = K + 1
K4 = K3 + 1
MM = MIS (1) * F (J3) * MIS (2) * F (J1)
MM1 = MIS (1) * F (J3)
MM2 = MM1
DIS (K1) = MM
DISX (K1) = (MM + MM1) * X
DISXX (K1) = (MM + MM1 + MM1) * XX1
MM = MISB (1) * F (J4) * MISB (2) * F (J2)
MM1 = MISB (1) * F (J4)
MM2 = MM1
DIS (K2) = MM
DISX (K2) = (MM + MM1) * X
DISXX (K2) = (MM + MM1 + MM1) * XX1
\[ \text{MM} = \text{MM2} \]
\[ \text{DIS} (x2) = \text{MM} + X \]
\[ \text{DIS}^2 (x2) = \text{MM} + XX1 \]
\[ \text{DIS}^3 (x2) = \text{MM} + XX1X2 \]
\[ \text{MM} = \text{MM3} \]
\[ \text{DIS} (x4) = \text{MM} + X \]
\[ \text{DIS}^2 (x4) = \text{MM} + XX1 \]
\[ \text{DIS}^3 (x4) = \text{MM} + XX1X2 \]

70 CONTINUE
RETURN
END
SUBROUTINE EQUATE
COMMON /C4/SIG(8,2,2),DISP(2,2,2)
COMMON /C5/I2(2,2),K2B(2,2),I(8,2),KB(8,2)
COMMON /C7/ICASE(2)
COMMON /C14/MUU(2,2)
COMPLEX SIG,DISP,MUU,I2,I2B,I,KB
ICASE(2)=ICASE(1)
DO 10 I=1,2
  K2(I,2)=K2(I,1)
  K2B(I,2)=K2B(I,1)
10 CONTINUE
DO 20 J=1,2
  SIG(I,J,2)=SIG(I,J,1)
20 CONTINUE
N=1
IF (ICASE(1).EQ.1) N=2
DO 25 I=1,N
  MUU(I,2)=MUU(I,1)
25 CONTINUE
DO 30 J=1,2
  DISP(I,J,2)=DISP(I,J,1)
30 CONTINUE
DO 40 I=1,8
  I(I,2)=I(I,1)
  KB(I,2)=KB(I,1)
40 CONTINUE
RETURN
END
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